Chapter 2
Past, Present and Future World Energy Use

Problem 2.1 Consider an island with a current population density of 50 people/km^2 (about equal to the present world average). If the annual growth rate is 2%, determine the year in which the population density be equal to 20,000 people/km^2 (approximately the population density of Macau, the world's most densely populated nation).

Solution As the problem deals with population density, the actual area of the island is not relevant. Solving equation (2.8) for *a* as a function of *R* gives

$$a = \ln[1+R/100]$$

Substituting in the annual growth rate of 2% gives

$$a = \ln[1+0.02] = 1.98\times10^{-2}\ \mathrm{y}^{-1}$$

From equation (2.4) the quantity as a function of time (relative to the present value) is

$$N(t)/N_0 = \exp(at)$$

Solving for *t* gives

$$t = (1/a)\times\ln[N(t)/N_0]$$

In this problem *N* is taken to be the population in 1 km^2 of land area so

$$N(t)/N_0 = 20{,}000/50 = 400.$$

Using the above value for *a* and solving for *t* gives

$$t = (\ln 400)/(1.98\times10^{-2}\ \mathrm{y}^{-1}) = 303\ \mathrm{y}$$

Relative to the year 2013, this population density will be reached in the year 2316.

Problem 2.2 A quantity has a doubling time of 110 years. Estimate the annual percent increase in the quantity.

Solution From equation (2.9) the annual rate of increase *R* is given as

$$R = \frac{100 \ln 2}{t_D}$$

where t_D is the doubling time. If t_D is 110 years then

$$R = \frac{(100)\times(0.693)}{110\ \text{y}} = 0.63\%\ \text{per year}$$

This is much less than 10% so the approximation given in equation (2.9) is valid.

Problem 2.3 The population of a particular country has a doubling time of 45 years. When will the population be three times its present value?

Solution From equation (2.7) the constant a can be determined from the doubling time as

$$t_D = \frac{1n2}{a}$$

so

$$a = \frac{1n2}{t_D}$$

For t_D=45 years then

$$a = \frac{0.693}{45} = 0.0154\text{y}^{-1}$$

From equation (2.4) the quantity of any time is given in terms of the initial value as

$$N(t) = N_0 \exp(at)$$

so solving for t we get

$$t = \frac{1}{a}\ln\left(\frac{N(t)}{N_0}\right)$$

for $N(t) = 3N_0$ then we get

$$t = \left(\frac{1}{0.0154\text{y}^{-1}}\right)\ln(3) = 71.3\text{years}$$

Problem 2.4 Assume that the historical growth rate of the human population was constant at 1.6% per year. For a population of 7 billion in 2012, determine the time in the past when the human population was 2.

Solution As the annual percentage growth rate is small then we can use the approximation of equation (2.4) to get the doubling time from R so

$$t_D = \left(\frac{100 \ln 2}{R} \right) = \frac{(100) \times (0.693)}{1.6} = 43.31 \text{years}$$

from equation (2.7) the constant a can be found to be

$$a = \left(\frac{\ln 2}{t_D} \right) = \frac{0.693}{43.31\text{y}} = 0.016\text{y}^{-1}$$

from equation (2.4) we start with an initial population of $N_0 = 2$ at $t = 0$ then $N(t) = 6.7 \times 10^9$ then from

$$N(t) = N_0 \exp(at)$$

so

$$t = \frac{1}{a} \ln \frac{N(t)}{N_0} = \frac{1}{0.016} \ln \frac{7 \times 10^9}{2} = 1374\text{y}$$

in the past or at year 2012-1374 = 638 (obviously growth rate was not constant)

Problem 2.5 What is the current average human population density (i.e., people per square kilometer) on earth?

Solution The radius of the Earth is 6378 km (assumed spherical). The total area (including oceans) is $A = 4\pi r^2 = (4) \times (3.14) \times (6378\text{km})^2 = 5.1 \times 10^8 \text{ km}^2$. The total current population is 6.7×10^9, so the population density is

$$\frac{6.7 \times 10^9}{5.1 \times 10^8 \text{ km}^2} = 13.1 \text{people/km}^2$$

If only land area is included, the land area on Earth is from various values given on the web range from 1.483×10^8 km^2 to 1.533×10^8 km^2. Using 1.5×10^8 km^2 we find

$$\frac{6.7 \times 10^9}{1.5 \times 10^8 \text{km}^2} = 44.7 \text{people/km}^2$$

Problem 2.6 The total world population in 2012 was about 7 billion, and Figure 2.11 shows that at that time the actual world population growth rate was about 1% per year. The figure also shows an anticipated roughly linear decrease in growth rate that extrapolates to zero growth in about the year 2080. Assuming an average growth rate of 0.5% between 2012 and 2080, what would the world population be in 2080? How does this compare with estimates discussed in the text for limits to human population?

Solution If R = 0.5% per year then the doubling time is found from equation (2.9) to be

$$t_D = \frac{10\ln 2}{R} = \frac{(100)\times(0.693)}{0.5} = 138.6\text{y}$$

using equation (2.7) to get the constant a

$$a = \frac{\ln 2}{t_D} = \frac{0.693}{138.6\text{y}} = 0.005\text{y}^{-1}$$

then equation (2.4) gives

$$N(t) = N_0 \exp(at)$$

so from $N_0 = 7\times10^9$ people and $t = 2080 - 2012 = 68\text{years}$ we find

$$N(t) = (7\times10^9)\exp((0.005\text{y}^{-1})\times(68\text{y})) = 9.8\times10^9\,\text{people}$$

This is consistent with comments in the text which suggest that the limit to human population can not be much more than 10 billion.

Problem 2.7 The population of a state is 25,600 in the year 1800 and 218,900 in the year 1900. Calculate the expected population in the year 2000 if (a) the growth is linear and (b) the growth is exponential.

Solution If population growth is linear then for 100 years between 1800 and 1900 it grows by $(218.9 - 25.6)\times10^3 = 193.3\times10^3$, so the population would grow by another 193.3×10^3 during the 100 years from 1900 to 2000 for a total of

$$(218.9 + 193.3)\times10^3 = 412.2\times10^3\,\text{people}$$

If the population growth is exponential then from equation (2.4) for $N_0 = 6.7\times10^9$ in 1800 then for t=100 years, $N(t)$ is 218.9×10^3. From this a can be found to be

$$a = \frac{1}{t}\ln\frac{N(t)}{N_0} = \left(\frac{1}{100\text{y}}\right)\times\ln\left(\frac{218.9\times10^3}{25.6\times10^3}\right) = 0.0215\text{y}^{-1}$$

Then using $N_0 = 218.9\times10^3$ in year 1900 the population at 100y (i.e. in year 2000) is

$$N(t) = \left(218.9\times10^3\right)\times\exp\left(\left(0.0215\text{y}^{-1}\right)\times\left(100\text{y}\right)\right) = 1.87\times10^6$$

about 4.5 times the value for linear growth.

Problem 2.8 The population of a country as a function of time is shown in the following table. Is the growth exponential?

year	population (millions)
1700	0.501
1720	0.677
1740	0.891
1760	1.202
1780	1.622
1800	2.163
1820	2.884
1840	3.890
1860	5.176
1880	6.761
1900	8.702
1920	10.23
1940	11.74
1960	13.18
1980	14.45
2000	15.49

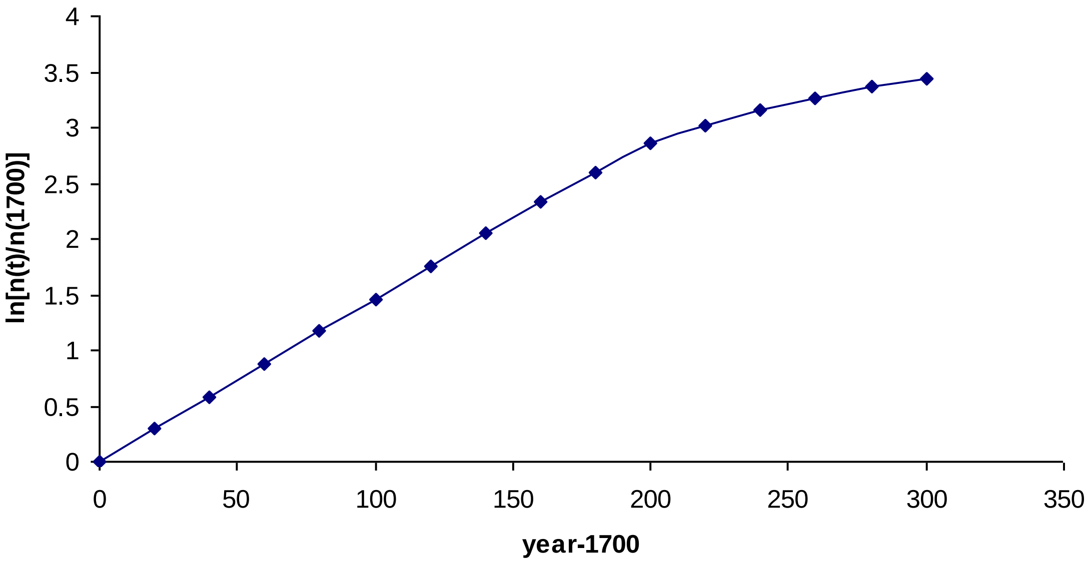
Solution For exponential growth

$$N(t) = N_0\exp\left(a(t-t_0)\right) \text{ so } \ln\left(\frac{N(t)}{N_0}\right) = a(t-t_0)$$

and the ln of the related population should be linear in time. Calculating $N(t)/N_0$ from the values above gives the tabulated values. They are plotted as a function of t - t_D as shown

year	population (millions)	year - 1700	ln[*N*(*t*)/*N*(1700)]
1700	0.501	0	0
1720	0.677	20	0.301065
1740	0.891	40	0.575738
1760	1.202	60	0.875136
1780	1.622	80	1.174809
1800	2.163	100	1.462645
1820	2.884	120	1.750327
1840	3.89	140	2.049558
1860	5.176	160	2.335182
1880	6.761	180	2.60232
1900	8.702	200	2.854702
1920	10.23	220	3.016474
1940	11.74	240	3.154151
1960	13.18	260	3.26985
1980	14.45	280	3.361844
2000	15.49	300	3.431344

The graph shows that the ln is linear and hence the population is exponential until ~ 1900 when the increase is less than exponential.



Problem 2.9 Consider a solar photovoltaic system with a total rated output of 10 MW_e and a capacity factor of 29%. If the total installation cost is $35,000,000, calculate the decrease in the cost of electricity per kilowatt-hour if the payback period is 25 years instead of 15 years. Assume a constant interest rate of 5.8%.

Solution From Example 2.3 the contribution to the cost of electricity per kWh due to the capital cost is

$$\frac{1}{Rf\left(8760\text{h/y}\right)} \times \frac{i(1+i)^T}{\left((1+i)^T - 1\right)}$$

Using I = 35,000,000, i = 0.058, $R = 10^4$ kW, f =0.29, then for a payback period of 15 years the cost per kWh is

$$\frac{3.5\times 10^7}{10^4 \times 0.29 \times 8760} \times \frac{0.058\times(1.058)^{15}}{\left((1.058)^{15} - 1\right)} = 1.378 \times 0.102 = \$0.140\text{/kWh}$$

For a payback period of 25 years the cost is

$$1.378 \times \frac{0.058\times(1.058)^{25}}{\left((1.058)^{25} - 1\right)} = 1.378 \times 0.0767 = \$0.106\,/\,\text{kWh}$$

or a decrease of (0.140-0.106)=\$0.034 per kWh.

Problem 2.10 If a quantity has a doubling time of 30 days and its value is 1000 units at noontime today, what will be its value at noontime tomorrow?

Solution From equation (2.7)

$$t_D = (\ln 2)/a$$

so

$$a = (\ln 2)/t_D = (0.693)/(30\text{ d}) = 0.0231\text{ d}^{-1}$$

The quantity as a function of time is

$$N(t) = N_0 \cdot \exp(at)$$

so for t = 1 day

$$N(1\text{ d}) = (1000)\cdot\exp(0.0231\text{ d}^{-1} \times 1\text{d}) = 1023$$

Problem 2.11 If a facility has a capital recovery factor of 0.12 and the annual interest rate is 5%, what is the payback period?

Solution From equation (2.12)

$$CRF = \frac{i(1+i)^T}{[(1+i)^T - 1]}$$

For i = 0.05 and CRF =0.12 then solve T for

$$CRF = \frac{0.05 \times (1.05)^T}{[(1.05)^T - 1]}$$

Easiest to do this numerically

T (guess)	*CRF (calculated)*
1	1.05
10	0.129
20	0.080
12	0.113
11	0.120

Numerical solution shows T = 11 years.

Problem 2.12 (a) If the total amount of a resource available is 10^{10} kg, calculate its lifetime if the rate of use is constant at 10^8 kg/y.
(b) Repeat part (a) if the initial rate of use is 10^8 kg/y and this increases at an annual rate of 5%.
(c) Repeat part (b) for an initial rate of 10^8 kg/y and an increase of 10% per year.

Solution (a) For a constant rate

time = (10^{10} kg)/(10^8 kg/y) = 100 years

(b) From Energy Extra 2.1

$$t_0 = \frac{1}{r}\left[\ln\left(\frac{rR}{C} + 1\right)\right]$$

where

C is the initial rate 10^8 kg/y
R is the total resource 10^{10} kg
r is the growth rate 0.05

So

$$t_0 = \frac{1}{0.05}\left[\ln\left(\frac{0.05 \times 10^{10}}{10^8} + 1\right)\right] = 35.8\text{y}$$

(c) For 10% growth, $r = 0.10$ so

$$t_0 = \frac{1}{0.10}\left[\ln\left(\frac{0.10 \times 10^{10}}{10^8} + 1\right)\right] = 24.0\text{y}$$

Problem 2.13 (a) Assume that human population growth is described as exponential growth and the current population is 7.5 billion. If the current daily increase in population is 250,000, calculate the doubling time.
(b) For the conditions in part (a), what is the current annual growth rate?

Solution (a) Doubling time is

$$t_D = (\ln 2)/a$$

and from equation (2.3)

$$\frac{dN}{dt} = aN$$

then

$$a = \frac{1}{N}\frac{dN}{dt} = \frac{1}{7.5 \times 10^9\,\text{y}} \times 250{,}000 = 3.33 \times 10^{-5}\,\text{d}^{-1}$$

Then

$$t_D = (\ln 2)/a = (0.693)/(3.33\times10^{-5}\ \text{d}^{-1}) = 20{,}815\ \text{d} = 57\ \text{years}$$

(b) For t_D = 57 y then

$$\text{R} = 100\times(\ln 2)/t_D = (69.3)/57 = 1.22\%$$

Problem 2.14 If world energy use increases linearly, use Figure 2.13 to estimate the annual percentage increase for OECD and non-OECD countries from 1990 to 2035 relative to the use in 1990.

Solution Drawing a straight line through the data in the graph for OECD and non-OECD countries gives

1990	OECD	217 QBtu
	total	355 QBtu
	non-OECD	138 QBtu
2035	OECD	293 QBtu
	total	774 QBtu
	no-OECD	481 QBtu

So for OECD

$$\frac{\Delta E \times 100}{E(1990)} \times \frac{1}{45\text{years}} = \frac{293\text{QBtu} - 217\text{QBtu}}{217\text{QBtu}} \cdot \frac{1}{45\text{years}} = 0.78\%\text{per year}$$

For non-OECD

$$\frac{\Delta E \times 100}{E(1990)} \times \frac{1}{45\text{years}} = \frac{481\text{QBtu} - 138\text{QBtu}}{138\text{QBtu}} \cdot \frac{1}{45\text{years}} = 5.52\%\text{per year}$$

Problem 2.15 Assume that the combustion of coal is approximated by the burning of carbon. If a coal-fired generating station operates at 40% efficiency, calculate the amount of CO_2 emitted per MWh of electricity produced.

Solution By the reaction

$$C + O_2 \rightarrow CO_2$$

then 1 mole of carbon (12 g) produces 1 mole (44 g) of CO_2. Coal produces 32.8 MJ/kg and converting to MWh using 1 MWh = 3.6 GJ then 1 MWh of electricity at generation of 40% efficiency requires

$$\frac{1}{0.4} \cdot \frac{3.6 \times 10^9\ \text{J}}{3.28 \times 10^7\ \text{J/kg}} = 274\text{kg of coal}$$

Since 1 mole (12 g) of carbon produces 1 mole (44 g) of CO2 then 1 kg od coal produces (1 kg)×(44 g/mol)/(12 g/mol) = 3.67 kg of CO_2, then 274 kg coal will produce

$$(274\text{ kg})\times(3.67\text{ kg/kg}) = 1007\text{ kg } CO_2.$$

Problem 2.16 Repeat problem 2.15 for the combustion of natural gas (methane). How does the use of the HHV affect your answer?

Solution The combustion of methane is given by the equation

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O + 55.5\text{ MJ/kg}$$

The molecular masses are

CH_4 = 16 g/mol
CO_2 = 44 g/mol

So 1kg of methane produces (1 kg)×(44 g/mol)/(16 g/mol) = 2.75 kg of CO_2. Since 1 MWh = 3.6 GJ then at 40% efficiency the production of 1 MWh requires

$$\frac{1}{0.4}\cdot\frac{3.6\times10^{9}\text{ J}}{5.55\times10^{7}\text{ J/kg}} = 162\text{kg of methane}$$

The combustion of 162 kg methane will produce

$$(162\text{ kg})\times(2.75\text{ kg/kg}) = 446\text{ kg of } CO_2$$

Using the molecular mass of water 18 g/mol, we see that the combustion of 1 kg of methanol produces (1 kg)×(2×18 g/mol)/(16 g/mol) = 2.25 kg water. Since the latent heat of combustion of water is (from the Web) 2.26 MJ/kg then the LHV of methanol will be

$$(55.5\text{ MJ/kg}) - (2.25\text{ kg/kg})\times(2.26\text{ MJ/kg}) = 50.42\text{ MJ/kg}$$

The mass of CO_2 produced per MWh will therefore be

$$\frac{1}{0.4}\cdot\frac{3.6\times10^{9}\text{ J}}{5.04210^{7}\text{ J/kg}}\times(2.75\text{ kg/kg}) = 491\text{ kg of } CO_2$$

Problem 2.17 In the example of energy use on page 31, leading to equation (2.1), consider a family (two people) who use gasoline at the rate indicated but who heat their home electrically in accordance with the efficiencies given in the text above equation (2.1). How would this affect the per person power consumption?

Solution Assuming an efficiency of 40% for electricity generation the primary energy use for two people will be

gasoline 9.8×10^{10} J
heating $(1/0.4)\times(1.04\times10^{11}$ J$) = 2.6\times10^{11}$ J
electricity (non-heating) $(1/0.4)\times(4.32\times10^{10}$ J$) = 1.1\times10^{11}$ J

Therefore the per capita primary power consumption will be

$$0.5\times\frac{(0.98+2.6+1.1)\times10^{11}\ \text{J}}{3.15\times10^{7}\ \text{s/y}}=7.4\ \text{kW}$$

compared to 5.0 kW for oil heating.

Problem 2.18 A person in the United States consumes 2000 Calories of food per day. What fraction of the average per capita energy use, as indicated in Table 2.1, does this represent?

Solution From the Table the per capita power consumption is 11,730 W. This represents

$$(11730\ \text{J/s})\times(24\ \text{h/d})\times(3600\ \text{s/h}) = 1.01\times10^{9}\ \text{J/d}$$

Food consumption represents

$$(2000\ \text{Cal/d})\times(4180\ \text{Cal/J}) = 8.37\times10^{6}\ \text{J/d}$$

or

$$100\times(8.37\times10^{6}\ \text{J/d})/(1.01\times10^{9}\ \text{J/d}) = 0.83\ \%\ \text{of energy use}$$

Problem 2.19 From the data in Table 2.1 and Figure 2.3, estimate the total energy production per year from hydroelectricity in the United States.

Solution Table 2.1 gives the per capita primary power consumption as 11,730 W. This gives the total per capita energy use per year as

$$(11{,}730\ \text{J/s})\times(3.15\times10^{7}\ \text{s/y}) = 3.69\times10^{11}\ \text{J/y}$$

The current population of the U.S. (from the Web) is 3.189×10^{8}, giving the total primary energy use per year

$$(3.189\times10^{8})\times(3.69\times10^{11}\ \text{J/y}) = 1.18\times10^{20}\ \text{J/y}$$

From Figure 2.3 hydroelectricity represents 2.45% of primary energy use in the U.S. Therefore, the total hydroelectricity production will be

$$(1.18\times10^{20}\ \text{J/y})\times(0.0245) = 2.89\times10^{18}\ \text{J/y}.$$

Problem 2.20 For an annual growth rate of 1.6%, use the data in Table 2.3 to estimate the year in which the human population will reach 200 per km^2 of land area.

Solution The total land area of the world is $1.5\times10^{14}\ m^2 = 1.5\times10^8\ km^2$ so a population density of 200 per km^2 would represent a total population of

$$(1.5\times10^8\ \text{km}^2)\times(200\ \text{km}^{-2}) = 3\times10^{10}$$

We need to determine when the population will reach this number if the population in 2016 is 7.43×10^9 and the growth rate is 1.6%. From Example 2.2

$$a = \ln(1 + R/100) = \ln(1 + 0.016) = 0.01587\ \text{y}^{-1}$$

The population at time t years from the present will be

$$N(t) = N_0\cdot\exp(at)$$

So

$$t = (1/a)\cdot\ln[N(t)/N_0]$$

Using values from above gives

$t = (1/0.01587\ \text{y}^{-1})\cdot\ln[3\times10^{10}/7.43\times10^9] = 88$ years

or in year 2016 + 88 = 2104