Discrete Mathematics with Applications, 5th Edition
by Susanna S. Epp

Answers for Test Bank Questions: Chapters 1-4

Please use caution when using these answers. Small differences in wording, notation, or choice of examples or counterexamples may be acceptable.

Chapter 1

1. a. a remainder of 1 when it is divided by 4 and a remainder of 3 when it is divided by 7

   b. an integer n; n is divided by 7 the remainder is 3

2. a. a positive real number; smaller than r

   b. positive real number r; there is a positive real number s

   Fill in the blanks to rewrite the following statement with variables:

3. There is an integer whose reciprocal is also an integer.

4. a. have three sides

   b. has three sides

   c. has three sides

   d. is a triangle; has three sides

   e. T has three sides

5. a. have additive inverses

   b. an additive inverse

   c. y is an additive inverse for x

6. a. less than or equal to every positive integer

   b. positive integer m; less than or equal to every positive integer

   c. less than or equal to n

7. (a) The set of all integers n such that n is a factor of 9.

   Or: The set of all elements n in $\mathbf{Z}$ such that n is a factor of 9.

   Or: The set of all elements n in the set of all integers such that n is a factor of 9.

   (b) $\{1, 3, 9\}$

8. (a) No

   (b) Yes

   (c) Yes

   (d) No

9. a. $\{(a, u), (a, v), (b, u), (b, v), (c, u), (c, v)\}$

   b. $\{(u, a), (v, a), (u, b), (v, b), (u, c), (v, c)\}$

10. a. Yes; No; No; Yes

    b. $\{(3, 15), (3, 18), (5, 15)\}$

    c. domain is $\{3, 5, 7\}$; co-domain is $\{15, 16, 17, 18\}$.

    d. Draw an arrow diagram for R.

    e. No: R fails both conditions for being a function from A to B. (1) Elements 5 and 7 in A are not related to any elements in B, and (2) there is an element in A, namely 3, that is related to two different elements in B, namely 15 and 18.

11. a. No; Yes; No; Yes

    b. Draw the graph of R in the Cartesian plane.

    c. No: R fails both conditions for being a function from **R** to **R**. (1) There are many elements in **R** that are not related to any element in **R**. For instance, none of 0, 1/2, and -1 is related to any element of **R**. (2) there are elements in **R** that are related to two different elements in **R**. For instance 2 is related to both 1 and -1.

12. a. $G(2) = c$

    b. Draw an arrow diagram for G.

13. $F \neq G$. Note that for every real number x,

$$G(x) = (x-2)^2 - 7 = x^2 - 4x + 4 - 7 = x^2 - 4x - 3,$$

whereas

$$F(x) = (x+1)(x-3) = x^2 - 2x - 3.$$

Thus, for instance,

$$F(1) = (1+1)(1-3) = -4 \quad \text{whereas} \quad G(1) = (1-2)^2 - 7 = -6.$$

Chapter 2

1. e

2. e

3. a. The variable S is not undeclared or the data are not out of order.

   b. The variable S is not undeclared and the data are not out of order.

   c. Al was with Bob on the first, and Al is not innocent.

   d. $-5 > x$ or $x \geq 2$

4. The statement forms are not logically equivalent.

   Truth table:

p	q	$\sim p$	$p \vee q$	$\sim p \wedge q$	$p \vee q \to p$	$p \vee (\sim p \wedge q)$
T	T	F	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	T	F	T
F	F	T	F	F	T	F

   Explanation: The truth table shows that $p \vee q \to p$ and $p \vee (\sim p \wedge q)$ have different truth values in rows 3 and 4, i.e, when p is false. Therefore $p \vee q \to p$ and $p \vee (\sim p \wedge q)$ are not logically equivalent.

5. *Sample answers:*

   Two statement forms are logically equivalent if, and only if, they always have the same truth values.

   Or: Two statement forms are logically equivalent if, and only if, no matter what statements are substituted in a consistent way for their statement variables the resulting statements have the same truth value.

6. *Solution 1:* The given statements are not logically equivalent. Let p be "Sam bought it at Crown Books," and q be "Sam didn't pay full price." Then the two statements have the following form:

$$p \to q \quad \text{and} \quad p \vee \sim q.$$

The truth tables for these statement forms are

p	q	$\sim q$	$p \to q$	$p \vee \sim q$
T	T	F	T	T
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

Rows 2 and 3 of the table show that $p \to q$ and $p \vee \sim q$ do not always have the same truth values, and so $p \to q \not\equiv p \vee \sim q$.

Solution 2: The given statements are not logically equivalent. Let p be "Sam bought it at Crown Books," and q be "Sam paid full price." Then the two statements have the following form:

$$p \to \sim q \quad \text{and} \quad p \vee q.$$

The truth tables for these statement forms are

p	q	$\sim q$	$p \to \sim q$	$p \vee q$
T	T	F	F	T
T	F	T	T	T
F	T	F	T	T
F	F	T	T	F

Rows 1 and 4 of the table show that $p \to \sim q$ and $p \vee q$ do not always have the same truth values, and so $p \to \sim q \not\equiv p \vee q$.

7. The given statements are not logically equivalent. Let p be "Sam is out of Schlitz," and q be "Sam is out of beer." Then the two statements have the following form:

$$p \to q \quad \text{and} \quad \sim q \vee \sim p.$$

The truth tables for these statement forms are

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \vee \sim q$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	T	T

The table shows that $p \to q$ and $\sim p \vee \sim q$ sometimes have opposite truth values (shown in rows 1 and 2), and so $p \to q \not\equiv \sim p \vee \sim q$.

8. *Converse*: If Jose is Jan's cousin, then Ann is Jan's mother

   Inverse: If Ann is not Jan's mother, then Jose is not Jan's cousin.

   Contrapositive: If Jose is not Jan's cousin, then Ann is not Jan's mother.

9. *Converse*: If Liu is Sue's cousin, then Ed is Sue's father.

   Inverse: If Ed is not Sue's father, then Liu is not Sue's cousin

   Contrapositive: If Liu is not Sue's cousin, then Ed is not Sue's father.

10. *Converse*: If Jim is Tom's grandfather, then then Al is Tom's cousin.

    Inverse: If Al is not Tom's cousin, then Jim is not Tom's grandfather

    Contrapositive: If Jim is not Tom's grandfather, then then Al is not Tom's cousin.

11. If someone does not get an answer of 10 for problem 16, then the person will not have solved problem 16 correctly.

    Or: If someone solves problem 16 correctly, then the person got an answer of 10.

12. *Sample answers:*

    For a form of argument to be valid means that no matter what statements are substituted for its statement variables, if the resulting premises are all true, then the conclusion is also true.

    Or: For a form of argument to be valid means that no matter what statements are substituted for its statement variables, it is impossible for all the premises to be true at the same time that the conclusion is false.

    Or: For a form of argument to be valid means that no matter what statements are substituted for its statement variables, it is impossible for conclusion to be false if all the premises are true.

13. The given form of argument is invalid.

				premises		*conclusion*
p	q	$\sim p$	$\sim q$	$p \to \sim q$	$q \to \sim p$	$p \vee q$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	F

Row 4 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion.

14. The given form of argument is invalid.

					premises			*conclusion*
p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge \sim q \to r$	$p \vee q$	$q \to p$	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	T	F	T
F	T	F	F	F	T	T	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	T	F	T	F

Row 2 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion.

15. Let p be "Hugo is a physics major," q be "Hugo is a math major," and r be "Hugo needs to take calculus." Then the given argument has the following form:

$$\begin{array}{rl} & p \vee q \to r \\ & r \vee q \\ \text{Therefore} & p \vee q. \end{array}$$

Truth table:

				premises		*conclusion*
p	q	r	$p \vee q$	$p \vee q \to r$	$r \vee q$	$p \vee q$
T	T	T	F	T	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	F	T	T	F
F	F	F	F	T	F	F

Row 7 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion. Therefore, the given argument is invalid.

16. Let p be "12 divides 709,438," q be "3 divides 709,438," and r be "The sum of the digits of 709,438 is divisible by 9." Then the given argument has the following form:

$$\begin{array}{rl} & p \to q \\ & r \to q \\ & \sim r \\ \text{Therefore} & \sim p. \end{array}$$

Truth table:

					premises			*conclusion*
p	q	r	$\sim q$	$p \wedge \sim q$	$p \to q$	$r \to q$	$\sim r$	$\sim p$
T	T	T	F	F	T	T	F	F
T	T	F	F	F	T	T	T	F
T	F	T	T	T	F	F	F	F
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	T	F	T
F	T	F	F	F	T	T	T	T
F	F	T	T	F	T	F	F	T
F	F	F	T	F	T	T	T	T

Row 2 of the truth table shows that it is possible for an argument of this form to have true premises and a false conclusion. Therefore, the given argument is invalid.

17. The argument has the form

$$\begin{array}{rl} & p \to q \\ & \sim q \\ \text{Therefore} & \sim p, \end{array}$$

which is valid by modus tollens (and the fact that the negation of "17 is not a divisor of 54,587" is "17 is a divisor of 54,587").

18. The argument has the form

$$\begin{array}{rl} & p \to q \\ & q \\ \text{Therefore} & p, \end{array}$$

which is invalid; it exhibits the converse error.

19. A and B are knights, and C is a knave.

Reasoning: A cannot be a knave because if A were a knave his statement would be true, which is impossible for a knave. Hence A is a knight, and at least one of the three is a knave. That implies that at most two of the three are knaves, which means that B's statement is true. Hence B is a knight. Since at least one of the three is a knave and both A and B are knights, it follows that C is a knave.

20. a. $S = 1$

b. $\sim (P \wedge Q) \wedge (Q \wedge R)$

21. $110101_2 = 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2^0 = 32 + 16 + 4 + 1 = 53_{10}$

22. $75_{10} = 64 + 8 + 2 + 1 = 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 1001011_2$.

23. The following circuit corresponds to the given Boolean expression: