Chapter 3

Test Bank

3.0 Chapter 0

1. Convert the binary number 10110 to base ten.

   Answer. 22

2. Write the base ten number 37 in binary.

   Answer. 100101

3. Write the base ten number 19 in binary.

   Answer. 10011

4. Convert the base 8 number 75 to base ten.

   Answer. 61

5. Write the base ten number 75 in base 8.

   Answer. 113

6. Convert the base 16 number $a7$ to base ten.

   Answer. 167

7. Write the base ten number 436 in base 16.

   Answer. $1b4$

8. How is 8^n expressed in binary?

   Answer. A 1 followed by $3n$ 0's.

3.1 Chapter 1

Section 1.1

1. Is the sentence "There are no true sentences." a statement? Explain.

   Answer. Yes. It is false.

2. Make a truth table for $p \to q \vee r$.

   Answer.

p	q	r	$q \vee r$	$p \to q \vee r$
F	F	F	F	T
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

3. Is the statement form $p \to \neg p$ a contradiction? Explain.

   Answer. No. It is true when p is false.

4. Determine if $\neg(p \to q)$ and $\neg p \to \neg q$ are logically equivalent? Justify your answer.

   Answer. They are not logically equivalent. They differ when p is true and q is true.

5. Write and simplify the contrapositive of $p \to \neg q \wedge r$.

   Answer. $q \vee \neg r \to \neg p$

6. Given the statement

   If Tara is not studying, then Tara is sleeping.

   Write its

   (a) converse.
   (b) contrapositive.
   (c) inverse.
   (d) negation.

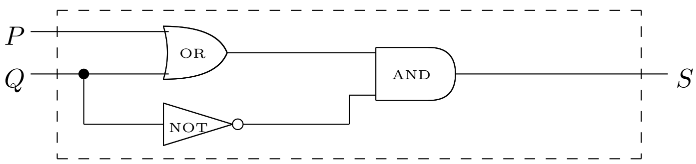
Answer.
(a) If Tara is sleeping, then Tara is not studying.
(b) If Tara is not sleeping, then Tara is studying.
(c) If Tara is studying, then Tara is not sleeping.
(d) Tara is not studying, and Tara is not sleeping.

7. Verify that $\neg p \wedge (\neg q \vee p) \equiv \neg(p \vee q)$ not by making a truth table but by using known basic logical equivalences.

Answer.

$\neg p \wedge (\neg q \vee p)$	$\equiv$	$(\neg p \wedge \neg q) \vee (\neg p \wedge p)$	Distributivity
	$\equiv$	$(\neg p \wedge \neg q) \vee \underline{f}$	Contradiction Rule
	$\equiv$	$\neg p \wedge \neg q$	Contradiction Rule
	$\equiv$	$\neg(p \vee q)$	De Morgan's Law

8. Trace the pictured circuit

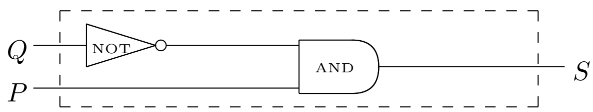


(a) to determine an expression for the output in terms of the input,
(b) and make an input-output table.
(c) Explain how the same input-output table can be accomplished by a circuit using fewer basic gates.

Answer.
(a) $(P \vee Q) \wedge \neg Q = S$.
(b)

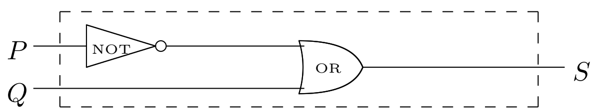
P	Q	S
0	0	0
0	1	0
1	0	1
1	1	0

(c) $S \equiv P \wedge \neg Q$.



9. Draw a circuit that realizes the expression $\neg P \vee Q = S$.

Answer.



Section 1.2

1. Express in set notation the set of integers smaller than 5.

   Answer. $\{n : n \in \mathbb{Z} \text{ and } n < 5\}$.

2. Express in interval notation the set of real numbers greater than or equal to -3.

   Answer. $[-3, \infty)$.

   For Exercises 3 through 8, determine if each of the the following relations is True or False.

3. $\{1, 3, 5, 3, 1, 7, 1\} \subseteq \{1, 3, 5, 7\}$.

   Answer. True.

4. $\{7\} \in \mathbb{N}$.

   Answer. False.

5. $3 \subset \{1, 2, 3, 4\}$.

   Answer. False.

6. $\emptyset = 0$.

   Answer. False.

7. $[-1, 1]$ is infinite.

   Answer. True.

8. $|\{2, 3, 7, 8, 5, 3\}| = 6$.

   Answer. False.

9. Write the expression for the "set" given in Russell's Paradox.

   Answer. $\{S : S \text{ is a set and } S \notin S\}$.

Section 1.3

For Exercises 1 through 3, write the given statement as efficiently as possible using quantifiers and standard notation. Determine if the statement is True or False.

1. Every real number is smaller than twice itself.

   Answer. $\forall\, x \in \mathbb{R}, x < 2x$.

2. There is an integer whose square is odd.

Answer. $\exists\, n \in \mathbb{Z}$ such that n^2 is odd.

3. There is an integer n such that the n^{th} power of every real number is negative.

   Answer. $\exists\, n \in \mathbb{Z}$ such that $\forall\, x \in \mathbb{R}, x^n < 0$.

For Exercises 4 through 6, write the negation of the given statement. Determine which of the statement or its negation is True.

4. For every integer n, if n is positive then $2n - 1$ is positive.

   Answer. $\exists\, n \in \mathbb{Z}$ such that $n > 0$ and $2n - 1 \leq 0$.
   The original statement is True.

5. There is a real number whose cube is negative.

   Answer. $\forall\, x \in \mathbb{R}, x^3 \geq 0$.
   The original statement is True.

6. The product of any two real numbers is positive.

   Answer. $\exists\, x, y \in \mathbb{R}$ such that $xy \leq 0$.
   The negation is True.

7. Negate the statement

$$\exists\, n \in \mathbb{Z} \text{ such that } \forall\, x \in \mathbb{R}, x^n < 0.$$

   Answer. $\forall\, n \in \mathbb{Z}, \exists\, x \in \mathbb{R}$ such that $x^n \geq 0$.

8. Negate the statement

   All good things come to an end.

   Answer. There is a good thing that does not end.

For Exercises 9 and 10, let f and g be real functions. Use quantifiers to precisely express the definition of the given notion.

9. f is periodic.

   Answer. $\exists\, p \in \mathbb{R}^+$ such that $\forall\, x \in \mathbb{R}, f(x + p) = f(x)$.

10. The composite function $g \circ f$.

   Answer. The function $g \circ f$ is defined by

$$\forall\, x \in \mathbb{R}, (g \circ f)(x) = g(f(x)).$$

Section 1.4

For Exercises 1 and 2, find A^c, $A \cap B$, $A \cup B$, $A \setminus B$, and $A \triangle B$ for the given sets.

1. $A = \{2, 3, 7\}$, $B = \{1, 2, 7, 9\}$, and $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

   Answer. $A^c = \{1, 4, 5, 6, 8, 9, 10\}$, $A \cap B = \{2, 7\}$, $A \cup B = \{1, 2, 3, 7, 9\}$, $A \setminus B = \{3\}$, and $A \triangle B = \{1, 3, 9\}$.

2. $A = (0, 3]$, $B = (2, 4)$, and $\mathcal{U} = \mathbb{R}$.

   Answer. $A^c = (-\infty, 0] \cup (3, \infty)$, $A \cap B = (2, 3]$, $A \cup B = (0, 4)$, $A \setminus B = (0, 2]$, and $A \triangle B = (0, 2] \cup (3, 4)$.

3. Are $(0, 3)$ and $(2, 4)$ disjoint? Justify your answer.

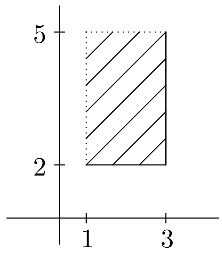
   Answer. No. $2.5 \in (0, 3) \cap (2, 4) \neq \emptyset$.

4. Find $\{0, 1\} \times \{2, 4, 6\}$.

   Answer. $\{(0, 2), (0, 4), (0, 6), (1, 2), (1, 4), (1, 6)\}$.

5. Sketch $(1, 3] \times [2, 5)$.

   Answer.



6. Find $\mathcal{P}(\{0, 1, 2\})$.

   Answer. $\{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$.

7. Decide if the proposed identity $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$ is True or False.

   Answer. True.

8. Use definitions and basic set identities to verify the identity

$$A^c \cap (B^c \cup A) = (A \cup B)^c.$$

   Answer.

$A^c \cap (B^c \cup A)$	$=$	$(A^c \cap B^c) \cup (A^c \cap A)$	Distributivity
	$=$	$(A^c \cap B^c) \cup \emptyset$	An $\emptyset$ Rule
	$=$	$A^c \cap B^c$	An $\emptyset$ Rule
	$=$	$(A \cup B)^c$	De Morgan's Law

Section 1.5

1. Determine if the given argument form is valid. Justify your answer.

$$p \to q$$
$$r \to p$$
$$q \vee r$$
$$\therefore \quad q$$

Answer.

p	q	r	$p \to q$	$r \to p$	$q \vee r$	q
F	F	F	T	T	F	
F	F	T	T	F	T	
F	T	F	T	T	T	T
F	T	T	T	F	T	
T	F	F	F	T	F	
T	F	T	F	T	T	
T	T	F	T	T	T	T
T	T	T	T	T	T	T

Rows 3, 7, and 8 demonstrate the validity of the argument form.

2. Show that the given argument form is valid without using a truth table.

$$q \to p$$
$$\neg q \to p$$
$$\therefore \quad p$$

Answer.

	Statement Form	Justification
1.	$q \to p$	Given
2.	$\neg q \to p$	Given
3.	$q \vee \neg q$	a tautology
4.	$\therefore \quad p$	(1),(2),(3), Two Separate Cases

3. Determine if the given argument is valid or invalid. Justify your answer.

$$\text{If } e > 0, \text{ then } \frac{1}{e} > 0.$$
$$\frac{1}{e} > 0.$$
$$\therefore \quad e > 0.$$

Answer. The argument's form

$$p \to q$$
$$q$$
$$\therefore \quad p$$

is not valid, as can be seen when p is false (and q is arbitrary). So the argument is not valid.

4. Verify that the given argument form is valid.

$$\begin{aligned}&\forall\, x \in \mathcal{U}, p(x) \wedge q(x)\\ &a \in \mathcal{U}\\ &\therefore\;\; p(a)\end{aligned}$$

Answer.

	Statement Form	Justification
1.	$\forall\, x \in \mathcal{U}, p(x) \wedge q(x)$	Given
2.	$a \in \mathcal{U}$	Given
3.	$p(a) \wedge q(a)$	(1),(2), Principle of Specification
4.	$\therefore\;\; p(a)$	(3), In Particular

5. Verify that the given argument form is valid.

$$\begin{aligned}&\forall\, x \in \mathcal{U}, p(x)\\ &\therefore\;\; \forall\, x \in \mathcal{U}, p(x) \vee q(x)\end{aligned}$$

Answer.

	Statement Form	Justification
1.	$\forall\, x \in \mathcal{U}, p(x)$	Given
2.	Let $a \in \mathcal{U}$ be arbitrary	Assumption
3.	$p(a)$	(1),(2), Principle of Specification
4.	$p(a) \vee q(a)$	(3), Obtaining Or
5.	$\therefore\;\; \forall\, x \in \mathcal{U}, p(x) \vee q(x)$	(2), (4), Principle of Generalization

6. Show that the given argument form is invalid.

$$\begin{aligned}&\forall\, x \in \mathcal{U}, p(x) \to q(x)\\ &\forall\, x \in \mathcal{U}, q(x)\\ &\therefore\;\; \forall\, x \in \mathcal{U}, p(x)\end{aligned}$$

Answer. Let $\mathcal{U} = \mathbb{R}^+$, $p(x) =$ "$x > 1$", and $q(x) =$ "$x > 0$". The resulting argument

$$\begin{aligned}&\forall\, x \in \mathbb{R}^+, \text{ if } x > 1 \text{ then } x > 0\\ &\forall\, x \in \mathbb{R}^+, x > 0\\ &\therefore\;\; \forall\, x \in \mathbb{R}^+, x > 1\end{aligned}$$

has all of its premises true but its conclusion false.