|  |  |  |
| --- | --- | --- |
| 1. The potential energy  of a pendulum of length 1 and mass 2, relative to its rest position is . Compute the average rate of change of the potential energy over the angle interval .   |  |  | | --- | --- | | *ANSWER:* |  | |

|  |  |  |
| --- | --- | --- |
| 2. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the equation . Calculate the average rate of change of  for     |  |  | | --- | --- | | *ANSWER:* |  | |

|  |  |  |
| --- | --- | --- |
| 3. The flight time of a shell shot at an angle  and initial velocity  is . Compute the average rate of change of the flight time for  in the interval .   |  |  | | --- | --- | | *ANSWER:* |  | |

|  |  |  |
| --- | --- | --- |
| 4. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the equation . Calculate the average rate of change of  for   |  |  | | --- | --- | | *ANSWER:* |  | |

|  |  |  |
| --- | --- | --- |
| 5. The volume of a cone of radius  and height  is . What is the average rate of change of  if the radius increases from 1 to 3 and the height remains unchanged?   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 6. The electrical field due to an infinite rod at a point at distance  from the rod is perpendicular to the rod and has a magnitude of  ( is a constant and  is the longitudinal charge density). Find the average rate of change of the field along the interval .   |  |  | | --- | --- | | *ANSWER:* |  | |

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| 7. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the semi-ellipse. Calculate the average rate of change of  for     |  |  | | --- | --- | | *ANSWER:* | -0.206 per unit length | |

|  |  |  |
| --- | --- | --- |
| 8. The electrical field caused by an electrical charge  at a point at distance  is  ( is a constant). Find the average rate of change of the field along the interval .   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 9. The volume of a sphere of radius  is . What is the average rate of change of the volume when the radius increases from  to ?   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 10. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the equation . Calculate the average rate of change of  for   |  |  | | --- | --- | | *ANSWER:* |  | |

|  |  |  |
| --- | --- | --- |
| 11. The position of a particle is given by. Compute the average velocity over the time interval . Estimate the instantaneous velocity at .   |  |  | | --- | --- | | *ANSWER:* | Average velocity over : 20 Instantaneous velocity at : 16 | |

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| --- | --- | --- |
| 12. A balloon is blown up and takes the shape of a sphere. What is the average rate of change of the surface area of the balloon as the radius increases from 3 to 4 cm?   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 13. Determine  and  for the function shown in the figure.   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 14. The greatest integer function is defined by , where  is the unique integer such that . The graph of  is shown in the figure. A) For which values of  does  exist? B) For which values of  does  exist? C) For which values of  does  exist?   |  |  | | --- | --- | | *ANSWER:* | A) All *c*  B) All *c*  C) Every real number *c* that is not an integer | |

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| 15. The graph of a function  is shown in the figure. Determine the following limits or state that the limit does not exist (if the limit is infinite, write  or  ):   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 16. Determine the one-sided limits at  of the function  shown in the figure and state whether the limit exists at these points.   |  |  | | --- | --- | | *ANSWER:* | ; limit exists ; limit does not exist ; limit does not exist | |

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| 17. Consider the function  for . (Here, [*x*] denotes the greatest integer function.) A) Write  in piecewise form.     What is  for positive integers ? B) Determine  and . C) For which values of  does  exist?   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) The limit exists for all positive real numbers that are not integers. | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 18. Determine the one-sided limits at  of the function shown in the figure and state whether the limit exists at these points.   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *ANSWER:* | |  |  |  |  | | --- | --- | --- | --- | | *c* | Left-sided | Right-sided | Limit | | 1 | 2.5 | 2.5 | Exists (2.5) | | 2 | 3 | 2 | Does not exist | | 3 | 1 | 3.5 | Does not exist | | 4 | 2.5 | 3 | Does not exist |   ​ | |

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| --- | --- | --- |
| 19. Consider the function  for . (Here, [*x*] denotes the greatest integer function.) A) Write  in piecewise form. What is  for positive integers ? B) Find  and . C) For which values of  does the limit  fail to exist?   |  |  | | --- | --- | | *ANSWER:* | A)  for ,  B)  C) The limit fails to exist for all positive integers. | |

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| 20. Determine the one-sided limits at  of the function shown in the figure and state whether the limit exists at these points.   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *ANSWER:* | |  |  |  |  | | --- | --- | --- | --- | | *c* | Left-sided | Right-sided | Limit | | 1 | 3 | 4 | Does not exist | | 2 | 5 | 5 | Exists (5) | | 3 | 3.5 | 5 | Does not exist | | 4 | 3 | 1.5 | Does not exist | | |

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| 21. Let  be the following function defined for :  Write  as a piecewise-defined function where the intervals are in terms of *x* instead of , sketch its graph, and determine the points where the limit of  does not exist. Find the one-sided limits at these points.   |  |  | | --- | --- | | *ANSWER:* |  | |

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| 22. Find a real number  such that  exists and compute the limit.  ​  ​   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 23. Let . Determine whether each of the following statements is always true, never true, or sometimes true. A)  B)  C)  D)   |  |  | | --- | --- | | *ANSWER:* | A) Always B) Sometimes C) Never D) Sometimes (note the case when  ) | |

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| --- | --- | --- |
| 24. Compute the following one-sided limits: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A) 0 B) 0 C) 0 | |

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| --- | --- | --- |
| 25. Evaluate the limits using the Limit Laws: A)  B)  C)  D)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C)  D) | |

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| 26. Which of the following functions are examples of the existence of the limit , although the limits of  and  as  do not exist?   |  |  |  | | --- | --- | --- | |  | a. |  | |  | b. |  | |  | c. |  | |  | d. |  | |  | e. |  |  |  |  | | --- | --- | | *ANSWER:* | c | |

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| 27. Let ,  be functions and let . Consider the following statement: If  and  exist, then  also exists. To prove this statement, we should use which of the following?   |  |  |  | | --- | --- | --- | |  | a. | The statement is not true. | |  | b. | The Product Rule applied to  and . | |  | c. | The Quotient Rule applied to  and . | |  | d. | The Sum Rule applied to  and . |  |  |  | | --- | --- | | *ANSWER:* | d | |

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| 28. Evaluate the limits using the Limit Laws: A)  B)  C)  D)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C)  D) | |

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| 29. Determine whether the following statement is correct: If , then  exists. If yes, prove it; otherwise, give a counter example   |  |  | | --- | --- | | *ANSWER:* | False; | |

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| 30. Evaluate the limits using the Limit Laws: A)  B)    C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| 31. A) Can the Product Rule be used to compute the limit ? (Here, [*x*] denotes the greatest integer function.) Explain. B) Show that  exists and find it. *Hint:* Compute the one-sided limits.   |  |  | | --- | --- | | *ANSWER:* | A) No. The limit  does not exist. B) 0 | |

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| 32. Let , , and . To prove that if  and  exist then also  exists, we should use which of the following?   |  |  |  | | --- | --- | --- | |  | a. | The Product Rule applied to  and . | |  | b. | The Quotient Rule applied to  and . | |  | c. | The Sum Rule applied to  and . | |  | d. | The statement is not true. | |  | e. | Both A and C |  |  |  | | --- | --- | | *ANSWER:* | c | |

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| 33. Evaluate the limits using the Limit Laws: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| 34. Consider this statement: If  and , then  does not converge to a finite limit as . To prove this statement, we assume that  exists and is finite. Then, by the Quotient Rule,  and by the Product Rule, . Which of the following statements completes the proof?   |  |  |  | | --- | --- | --- | |  | a. | From , it follows that 1 = 0, which is a contradiction. | |  | b. | From , we can conclude that , which contradicts our assumption. | |  | c. | From , we can conclude that , which contradicts our assumption. | |  | d. | From , we can conclude that , which contradicts our assumption. |  |  |  | | --- | --- | | *ANSWER:* | a | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 35. Which of the following functions are examples of the existence of the limit , although the limits  and  do not exist?   |  |  |  | | --- | --- | --- | |  | a. |  | |  | b. |  | |  | c. | (Here, [*x*] denotes the greatest integer function.) | |  | d. |  | |  | e. |  |  |  |  | | --- | --- | | *ANSWER:* | b | |

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| --- | --- | --- |
| 36. Assume  and  are nonzero real numbers. If  and , calculate the following limits, if possible. If not, state that it is not possible. A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A) 0 B) Not possible C) | |

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| 37. Determine the points at which the following functions are not continuous and state the type of discontinuity: removable, jump, infinite, or none of these. A) *f*(*x*) = B) *g*(*x*) =  C) *h*(*x*) =  (Here, [*x*] denotes the greatest integer function.) D) *j*(*x*) =  E) *k*(*x*) =   |  |  | | --- | --- | | *ANSWER:* | A) ; infinite B) ; removable C) Integers; jump D) ; none of these E) *x* = –3; removable | |

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| 38. At each point of discontinuity, state whether the function is left or right continuous:   |  |  | | --- | --- | | *ANSWER:* | A) ; left continuous B) ; left continuous     ; right continuous | |

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| 39. Determine real numbers *a, b,* and c that make the function continuous:     |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 40. Find the points of discontinuity for each of these functions and state the type of discontinuity: removable, jump, infinite, or none of these.  A) *f*(*x*) =  B) *g*(*x*) =  (Here, [*x*] denotes the greatest integer function.) C) *h*(*x*) =   |  |  | | --- | --- | | *ANSWER:* | A) *x* = –4; jump B) *x* = positive integer; jump C)  removable;  infinite | |

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| 41. Determine whether the function is left or right continuous at each of its points of discontinuity: A)  B)  (Here, [*x*] denotes the greatest integer function.)   |  |  | | --- | --- | | *ANSWER:* | A)  right continuous B) Right continuous at the positive integers | |

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| 42. Determine real numbers  and  that make the following function continuous:   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 43. Determine the points where the function is not continuous and state the type of the discontinuity: removable, jump, infinite, or none of these. A) *f*(*x*) =  B) *g*(*x*) =  (Here, [*x*] denotes the greatest integer function.) C) *h*(*x*) =  D) *j*(*x*) =   |  |  | | --- | --- | | *ANSWER:* | A) , jump B)  ; jump C) ; removable D) *x* = 3, *x* = –3; infinite | |

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| 44. At each point of discontinuity, state whether the function is left or right continuous. A)  B)   |  |  | | --- | --- | | *ANSWER:* | A) ; right continuous     ; right continuous B) ; left continuous     ; right continuous | |

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| --- | --- | --- |
| 45. Determine real numbers  and  that make the following function continuous:  (Here, [*x*] denotes the greatest integer function.)   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 46. Determine the points where the function is not continuous and state the type of discontinuity: removable, jump, infinite, or none of these: A) *f*(*x*) =  B) *g*(*x*) =  C) *h*(*x*) =  (Here, [*x*] denotes the greatest integer function.) D) *j*(*x*) =   |  |  | | --- | --- | | *ANSWER:* | A) ; removable B) ; infinite     ; none of these C) Nonzero integers; jump D) ; infinite | |

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| 47. At each point of discontinuity state whether the function is left continuous, right continuous, or neither A)  B)   |  |  | | --- | --- | | *ANSWER:* | A) ; left continuous     ; none of these B) ; right continuous     ; left continuous | |

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| 48. Determine real numbers  and  that make the function continuous:   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 49. Consider the function  The function  is continuous for which of the following functions *g*?   |  |  |  | | --- | --- | --- | |  | a. | if | |  | b. | if | |  | c. | if   if | |  | d. | if   if | |  | e. | A and C both correct |  |  |  | | --- | --- | | *ANSWER:* | c | |

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| 50. Let  be a discontinuous function. Is it possible to find a continuous function  such that  is continuous? Explain.   |  |  | | --- | --- | | *ANSWER:* | No. If *F*(*x*) = *f*(*x*) + *g*(*x*) is continuous, then *f*(*x*) = *F*(*x*) – *g*(*x*) is continuous by the continuity laws. | |

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| 51. Sketch the graph of a function  that satisfies all of the following conditions:   |  |  | | --- | --- | | *ANSWER:* |  | |

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| 52. Evaluate each limit or state that it does not exist:  A)  B)  ​  C)   |  |  | | --- | --- | | *ANSWER:* | A) 33  B)  C) Does not exist | |

|  |  |  |
| --- | --- | --- |
| 53. Evaluate each limit or state that it does not exist: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| --- | --- | --- |
| 54. Evaluate the limits in terms of the constants involved: A)  B)   |  |  | | --- | --- | | *ANSWER:* | A)  B) | |

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| 55. Evaluate each limit or state that it does not exist: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A) 28 B)  C) Does not exist | |

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| 56. Evaluate the limit:   |  |  | | --- | --- | | *ANSWER:* |  | |

|  |  |  |
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| 57. Evaluate each limit or state that it does not exist: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A) 7 B)  C) Does not exist | |

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| --- | --- | --- |
| 58. Determine a real number  for which the limit exists and then compute the limit:   |  |  | | --- | --- | | *ANSWER:* | , the limit is 0 | |

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| --- | --- | --- |
| 59. Evaluate each limit or state that it does not exist: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| --- | --- | --- |
| 60. Determine a real number  for which the limit exists and then compute the limit:   |  |  | | --- | --- | | *ANSWER:* | ; limit is | |

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| --- | --- | --- |
| 61. Let *f*(*x*) = 2*x* + 3. Compute  .   |  |  | | --- | --- | | *ANSWER:* | 2 | |

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| --- | --- | --- |
| 62. Compute .   |  |  | | --- | --- | | *ANSWER:* |  | |

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| --- | --- | --- |
| 63. Compute .   |  |  | | --- | --- | | *ANSWER:* |  | |

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| 64. Evaluate the limits: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A) 1 B) 0 C) 0 | |

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| --- | --- | --- |
| 65. Show that  for all . (Here, [*x*] denotes the greatest integer function.) Then use the above inequality and the Squeeze Theorem to evaluate .   |  |  | | --- | --- | | *ANSWER:* | 0 | |

|  |  |  |
| --- | --- | --- |
| 66. Evaluate the limits in terms of the constants involved: A)  B)   |  |  | | --- | --- | | *ANSWER:* | A)  B) | |

|  |  |  |
| --- | --- | --- |
| 67. Evaluate the limits using the Squeeze Theorem, trigonometric identities, and trigonometric limits, as necessary: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A) 0 B) 1 C) 1 | |

|  |  |  |
| --- | --- | --- |
| 68. Show that  for all . (Here, [*x*] denotes the greatest integer function.) Then use this inequality with the Squeeze Theorem to evaluate .   |  |  | | --- | --- | | *ANSWER:* | 0 | |

|  |  |  |
| --- | --- | --- |
| 69. Determine a real number  such that the following limit exists, and then evaluate the limit for this value:   |  |  | | --- | --- | | *ANSWER:* | ; limit is | |

|  |  |  |
| --- | --- | --- |
| 70. Evaluate each limit or state that it does not exist: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| 71. Evaluate the limits: A)  B)   |  |  | | --- | --- | | *ANSWER:* | A)  B) | |

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| --- | --- | --- |
| 72. Evaluate the limits: A)  *Hint:* Factor the denominator. B)  *Hint:* Factor the two expressions. C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| 73. Use the Squeeze Theorem to evaluate the limit   |  |  | | --- | --- | | *ANSWER:* | 0 | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 74. If  on the interval [0,4], then  must exist.   |  |  |  | | --- | --- | --- | |  | a. | True | |  | b. | False |  |  |  | | --- | --- | | *ANSWER:* | b | |

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| --- | --- | --- |
| 75. Calculate the limits: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| --- | --- | --- |
| 76. Calculate the limits: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

|  |  |  |
| --- | --- | --- |
| 77. Calculate the following limits: A)    B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

|  |  |  |
| --- | --- | --- |
| 78. Compute the following limits: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

|  |  |  |
| --- | --- | --- |
| 79. Compute the following limits: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| --- | --- | --- |
| 80. Compute the following limits: A)  *Hint:* Multiply and divide by the conjugate expression. B)  *Hint:* For . C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

|  |  |  |
| --- | --- | --- |
| 81. Compute the following limits: A)  B)  C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| --- | --- | --- |
| 82. Compute the following limits: A)  B)  (C)   |  |  | | --- | --- | | *ANSWER:* | A)  B)  C) | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 83. The Intermediate Value Theorem guarantees that the equation  has a solution in which of the following intervals?   |  |  |  | | --- | --- | --- | |  | a. |  | |  | b. |  | |  | c. |  | |  | d. |  | |  | e. |  |  |  |  | | --- | --- | | *ANSWER:* | a | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| 84. The polynomial  must have a root in which of the following intervals?   |  |  |  | | --- | --- | --- | |  | a. |  | |  | b. |  | |  | c. |  | |  | d. |  | |  | e. |  |  |  |  | | --- | --- | | *ANSWER:* | b | |

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| 85. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If  assumes all the values between  and  in the interval , then  is continuous on .   |  |  |  | | --- | --- | --- | |  | a. | on | |  | b. | on | |  | c. | on  on [0,2]  ​ | |  | d. | on  (Here, [*x*] denotes the greatest integer function.) | |  | e. | on |  |  |  | | --- | --- | | *ANSWER:* | e | |

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| 86. Which of the following functions has a zero in the interval ?   |  |  |  | | --- | --- | --- | |  | a. |  | |  | b. |  | |  | c. |  | |  | d. |  | |  | e. | Both A and C |  |  |  | | --- | --- | | *ANSWER:* | e | |

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| 87. The Intermediate Value Theorem guarantees that the equation  has a solution in which of the following intervals?   |  |  |  | | --- | --- | --- | |  | a. |  | |  | b. |  | |  | c. |  | |  | d. |  | |  | e. | Both A and C |  |  |  | | --- | --- | | *ANSWER:* | e | |

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| 88. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If  assumes all the values between  and  in the interval , then  is continuous on .   |  |  |  | | --- | --- | --- | |  | a. | for , ,  on [1,3] | |  | b. | on | |  | c. | on  on | |  | d. | on  (Here, [*x*] denotes the greatest integer function.) | |  | e. | Both A and C |  |  |  | | --- | --- | | *ANSWER:* | a | |

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| 89. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem: If  assumes all the values between  and  in the interval , then  is continuous on .   |  |  |  | | --- | --- | --- | |  | a. | if  on [1,3] | |  | b. | on  (Here, [*x*] denotes the greatest integer function.) | |  | c. | on | |  | d. | for  and | |  | e. | Both A and D |  |  |  | | --- | --- | | *ANSWER:* | a | |

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| --- | --- | --- |
| 90. Assume  is continuous on , , and . Determine whether each of the following statements is always true, never true, or sometimes true. A) : no solution with  B) : no solution with  C) : no solution with  D) : exactly one solution with  E) : a solution with   |  |  | | --- | --- | | *ANSWER:* | A) Sometimes true B) Sometimes true C) Never true D) Sometimes true E) Always true | |

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| 91. Draw the graph of a function  on  such that the graph does not satisfy the conclusion of the Intermediate Value Theorem.   |  |  | | --- | --- | | *ANSWER:* | Answers may vary. A sample answer is: | |

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| 92. Which of the following properties can be used to prove that  is continuous for all ?   |  |  |  | | --- | --- | --- | |  | a. | for all | |  | b. | for all  and | |  | c. | for all  and | |  | d. | The limit  exists | |  | e. | for all  and |  |  |  | | --- | --- | | *ANSWER:* | b | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 93. Which of the following statements imply that  is not continuous at ?   |  |  |  | | --- | --- | --- | |  | a. | has opposite signs on the two sides of | |  | b. | implies that | |  | c. | For any ,  implies that | |  | d. | If , then  . | |  | e. | A and C are correct. |  |  |  | | --- | --- | | *ANSWER:* | c | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 94. To show that  is not the limit of  as , we should show that:   |  |  |  | | --- | --- | --- | |  | a. | For any , there exists  such that if  then . | |  | b. | For any , there exists  such that if  then . | |  | c. | There exists , such that for any  the inequalities  and  have a solution . | |  | d. | There exist  and  such that if , then . | |  | e. | A and C are both correct. |  |  |  | | --- | --- | | *ANSWER:* | c | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 95. Suppose there exists a value of  so that for any value of , we can find a value of  satisfying  and . We may conclude that:   |  |  |  | | --- | --- | --- | |  | a. | is the limit of  as . | |  | b. | is not the limit of  as . | |  | c. | The limit of  as  does not exist. | |  | d. | The limit of  as  exists but is not equal to L. | |  | e. | None of the above. |  |  |  | | --- | --- | | *ANSWER:* | b | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 96. To show that  is not the limit of  as , we should show that:   |  |  |  | | --- | --- | --- | |  | a. | There exists  such that for any  there exists a solution to the inequalities  and . | |  | b. | There exists  such that for any  there exists a solution to the inequalities  and . | |  | c. | There exists  such that for any , if , then . | |  | d. | For any  and , if , then . | |  | e. | A and D are both correct. |  |  |  | | --- | --- | | *ANSWER:* | a | |