|  |  |  |
| --- | --- | --- |
| 1. The potential energy  of a pendulum of length 1 and mass 2, relative to its rest position is . Compute the average rate of change of the potential energy over the angle interval .

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 2. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the equation . Calculate the average rate of change of  for

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 3. The flight time of a shell shot at an angle  and initial velocity  is . Compute the average rate of change of the flight time for  in the interval .

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| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 4. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the equation . Calculate the average rate of change of  for

|  |  |
| --- | --- |
| *ANSWER:* |  |

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| 5. The volume of a cone of radius  and height  is .What is the average rate of change of  if the radius increases from 1 to 3 and the height remains unchanged?

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| --- | --- |
| *ANSWER:* |  |

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| 6. The electrical field due to an infinite rod at a point at distance  from the rod is perpendicular to the rod and has a magnitude of  ( is a constant and  is the longitudinal charge density).Find the average rate of change of the field along the interval .

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 7. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the semi-ellipse. Calculate the average rate of change of  for

|  |  |
| --- | --- |
| *ANSWER:* | -0.206 per unit length |

 |

|  |  |  |
| --- | --- | --- |
| 8. The electrical field caused by an electrical charge  at a point at distance  is  ( is a constant).Find the average rate of change of the field along the interval .

|  |  |
| --- | --- |
| *ANSWER:* |  |

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| --- | --- | --- |
| 9. The volume of a sphere of radius  is . What is the average rate of change of the volume when the radius increases from  to ?

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 10. Let  denote the slope of the line segment connecting the origin to the point  on the graph of the equation . Calculate the average rate of change of  for

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 11. The position of a particle is given by. Compute the average velocity over the time interval . Estimate the instantaneous velocity at .

|  |  |
| --- | --- |
| *ANSWER:* | Average velocity over : 20Instantaneous velocity at : 16 |

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| --- | --- | --- |
| 12. A balloon is blown up and takes the shape of a sphere. What is the average rate of change of the surface area of the balloon as the radius increases from 3 to 4 cm?

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| --- | --- |
| *ANSWER:* |  |

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| --- | --- | --- |
| 13. Determine  and  for the function shown in the figure.

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| --- | --- |
| *ANSWER:* |   |

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| --- | --- | --- |
| 14. The greatest integer function is defined by , where  is the unique integer such that .The graph of  is shown in the figure.A) For which values of  does  exist?B) For which values of  does  exist?C) For which values of  does  exist?

|  |  |
| --- | --- |
| *ANSWER:* | A) All *c*B) All *c*C) Every real number *c* that is not an integer |

 |

|  |  |  |
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| 15. The graph of a function  is shown in the figure.Determine the following limits or state that the limit does not exist (if the limit is infinite, write  or  ):

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| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 16. Determine the one-sided limits at  of the function  shown in the figure and state whether the limit exists at these points.

|  |  |
| --- | --- |
| *ANSWER:* | ; limit exists; limit does not exist; limit does not exist  |

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|  |  |  |
| --- | --- | --- |
| 17. Consider the function  for . (Here, [*x*] denotes the greatest integer function.)A) Write  in piecewise form.    What is  for positive integers ?B) Determine  and .C) For which values of  does  exist?

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C) The limit exists for all positive real numbers that are not integers. |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 18. Determine the one-sided limits at  of the function shown in the figure and state whether the limit exists at these points.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *ANSWER:* |

|  |  |  |  |
| --- | --- | --- | --- |
| *c* | Left-sided | Right-sided | Limit |
| 1 | 2.5 | 2.5 | Exists (2.5) |
| 2 | 3 | 2 | Does not exist |
| 3 | 1 | 3.5 | Does not exist |
| 4 | 2.5 | 3 | Does not exist |

​ |

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|  |  |  |
| --- | --- | --- |
| 19. Consider the function  for . (Here, [*x*] denotes the greatest integer function.)A) Write  in piecewise form. What is  for positive integers ?B) Find  and .C) For which values of  does the limit  fail to exist?

|  |  |
| --- | --- |
| *ANSWER:* | A)  for , B) C) The limit fails to exist for all positive integers. |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 20. Determine the one-sided limits at  of the function shown in the figure and state whether the limit exists at these points.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *ANSWER:* |

|  |  |  |  |
| --- | --- | --- | --- |
| *c* | Left-sided | Right-sided | Limit |
| 1 | 3 | 4 | Does not exist |
| 2 | 5 | 5 | Exists (5) |
| 3 | 3.5 | 5 | Does not exist |
| 4 | 3 | 1.5 | Does not exist |

  |

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|  |  |  |
| --- | --- | --- |
| 21. Let  be the following function defined for :Write  as a piecewise-defined function where the intervals are in terms of *x* instead of , sketch its graph, and determine the points where the limit of  does not exist. Find the one-sided limits at these points.

|  |  |
| --- | --- |
| *ANSWER:* |  |

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| --- | --- | --- |
| 22. Find a real number  such that  exists and compute the limit.​​

|  |  |
| --- | --- |
| *ANSWER:* |  |

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| --- | --- | --- |
| 23. Let . Determine whether each of the following statements is always true, never true, or sometimes true.A) B) C) D)

|  |  |
| --- | --- |
| *ANSWER:* | A) AlwaysB) SometimesC) NeverD) Sometimes (note the case when  ) |

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| --- | --- | --- |
| 24. Compute the following one-sided limits:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) 0B) 0C) 0 |

 |

|  |  |  |
| --- | --- | --- |
| 25. Evaluate the limits using the Limit Laws:A) B) C) D)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C) D)  |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 26. Which of the following functions are examples of the existence of the limit , although the limits of  and  as  do not exist?

|  |  |  |
| --- | --- | --- |
|   | a.  |  |
|   | b.  |  |
|   | c.  |  |
|   | d.  |  |
|   | e.  |  |

|  |  |
| --- | --- |
| *ANSWER:* | c |

 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 27. Let ,  be functions and let . Consider the following statement: If  and  exist, then  also exists. To prove this statement, we should use which of the following?

|  |  |  |
| --- | --- | --- |
|   | a.  | The statement is not true. |
|   | b.  | The Product Rule applied to  and . |
|   | c.  | The Quotient Rule applied to  and . |
|   | d.  | The Sum Rule applied to  and . |

|  |  |
| --- | --- |
| *ANSWER:* | d |

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| --- | --- | --- |
| 28. Evaluate the limits using the Limit Laws:A) B) C) D)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C) D)  |

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| --- | --- | --- |
| 29. Determine whether the following statement is correct: If , then  exists. If yes, prove it; otherwise, give a counter example

|  |  |
| --- | --- |
| *ANSWER:* | False;  |

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| --- | --- | --- |
| 30. Evaluate the limits using the Limit Laws:A) B)   C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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|  |  |  |
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| 31. A) Can the Product Rule be used to compute the limit ? (Here, [*x*] denotes the greatest integer function.) Explain.B) Show that  exists and find it. *Hint:* Compute the one-sided limits.

|  |  |
| --- | --- |
| *ANSWER:* | A) No. The limit  does not exist.B) 0 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 32. Let , , and . To prove that if  and  exist then also  exists, we should use which of the following?

|  |  |  |
| --- | --- | --- |
|   | a.  | The Product Rule applied to  and . |
|   | b.  | The Quotient Rule applied to  and . |
|   | c.  | The Sum Rule applied to  and . |
|   | d.  | The statement is not true. |
|   | e.  | Both A and C |

|  |  |
| --- | --- |
| *ANSWER:* | c |

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| --- | --- | --- |
| 33. Evaluate the limits using the Limit Laws:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 34. Consider this statement: If  and , then  does not converge to a finite limit as .To prove this statement, we assume that  exists and is finite. Then, by the Quotient Rule,  and by the Product Rule, .Which of the following statements completes the proof?

|  |  |  |
| --- | --- | --- |
|   | a.  | From , it follows that 1 = 0, which is a contradiction. |
|   | b.  | From , we can conclude that , which contradicts our assumption. |
|   | c.  | From , we can conclude that , which contradicts our assumption. |
|   | d.  | From , we can conclude that , which contradicts our assumption. |

|  |  |
| --- | --- |
| *ANSWER:* | a |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 35. Which of the following functions are examples of the existence of the limit , although the limits  and  do not exist?

|  |  |  |
| --- | --- | --- |
|   | a.  |  |
|   | b.  |  |
|   | c.  |  (Here, [*x*] denotes the greatest integer function.) |
|   | d.  |  |
|   | e.  |  |

|  |  |
| --- | --- |
| *ANSWER:* | b |

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| 36. Assume  and  are nonzero real numbers. If  and , calculate the following limits, if possible. If not, state that it is not possible.A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) 0B) Not possibleC)  |

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| --- | --- | --- |
| 37. Determine the points at which the following functions are not continuous and state the type of discontinuity: removable, jump, infinite, or none of these.A) *f*(*x*) =B) *g*(*x*) = C) *h*(*x*) =  (Here, [*x*] denotes the greatest integer function.)D) *j*(*x*) = E) *k*(*x*) =

|  |  |
| --- | --- |
| *ANSWER:* | A) ; infiniteB) ; removableC) Integers; jumpD) ; none of theseE) *x* = –3; removable |

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| --- | --- | --- |
| 38. At each point of discontinuity, state whether the function is left or right continuous:

|  |  |
| --- | --- |
| *ANSWER:* | A) ; left continuousB) ; left continuous    ; right continuous |

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| --- | --- | --- |
| 39. Determine real numbers *a, b,* and c that make the function continuous:

|  |  |
| --- | --- |
| *ANSWER:* |  |

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| --- | --- | --- |
| 40. Find the points of discontinuity for each of these functions and state the type of discontinuity: removable, jump, infinite, or none of these.A) *f*(*x*) = B) *g*(*x*) =  (Here, [*x*] denotes the greatest integer function.)C) *h*(*x*) =

|  |  |
| --- | --- |
| *ANSWER:* | A) *x* = –4; jumpB) *x* = positive integer; jumpC)  removable;  infinite |

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| 41. Determine whether the function is left or right continuous at each of its points of discontinuity:A) B)  (Here, [*x*] denotes the greatest integer function.)

|  |  |
| --- | --- |
| *ANSWER:* | A)  right continuousB) Right continuous at the positive integers |

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| --- | --- | --- |
| 42. Determine real numbers  and  that make the following function continuous:

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 43. Determine the points where the function is not continuous and state the type of the discontinuity: removable, jump, infinite, or none of these.A) *f*(*x*) = B) *g*(*x*) =  (Here, [*x*] denotes the greatest integer function.)C) *h*(*x*) = D) *j*(*x*) =

|  |  |
| --- | --- |
| *ANSWER:* | A) , jumpB)  ; jumpC) ; removableD) *x* = 3, *x* = –3; infinite |

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| --- | --- | --- |
| 44. At each point of discontinuity, state whether the function is left or right continuous.A) B)

|  |  |
| --- | --- |
| *ANSWER:* | A) ; right continuous    ; right continuousB) ; left continuous    ; right continuous |

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|  |  |  |
| --- | --- | --- |
| 45. Determine real numbers  and  that make the following function continuous:(Here, [*x*] denotes the greatest integer function.)

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 46. Determine the points where the function is not continuous and state the type of discontinuity: removable, jump, infinite, or none of these:A) *f*(*x*) = B) *g*(*x*) = C) *h*(*x*) =  (Here, [*x*] denotes the greatest integer function.)D) *j*(*x*) =

|  |  |
| --- | --- |
| *ANSWER:* | A) ; removableB) ; infinite    ; none of theseC) Nonzero integers; jumpD) ; infinite |

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|  |  |  |
| --- | --- | --- |
| 47. At each point of discontinuity state whether the function is left continuous, right continuous, or neitherA) B)

|  |  |
| --- | --- |
| *ANSWER:* | A) ; left continuous    ; none of theseB) ; right continuous    ; left continuous |

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| --- | --- | --- |
| 48. Determine real numbers  and  that make the function continuous:

|  |  |
| --- | --- |
| *ANSWER:* |  |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 49. Consider the functionThe function  is continuous for which of the following functions *g*?

|  |  |  |
| --- | --- | --- |
|   | a.  |  if   |
|   | b.  |  if   |
|   | c.  |  if   if  |
|   | d.  |  if   if  |
|   | e.  | A and C both correct |

|  |  |
| --- | --- |
| *ANSWER:* | c |

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| --- | --- | --- |
| 50. Let  be a discontinuous function. Is it possible to find a continuous function  such that  is continuous? Explain.

|  |  |
| --- | --- |
| *ANSWER:* | No. If *F*(*x*) = *f*(*x*) + *g*(*x*) is continuous, then *f*(*x*) = *F*(*x*) – *g*(*x*) is continuous by the continuity laws. |

 |

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| --- | --- | --- |
| 51. Sketch the graph of a function  that satisfies all of the following conditions:

|  |  |
| --- | --- |
| *ANSWER:* |  |

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| --- | --- | --- |
| 52. Evaluate each limit or state that it does not exist:A) B) ​C)

|  |  |
| --- | --- |
| *ANSWER:* | A) 33B) C) Does not exist |

 |

|  |  |  |
| --- | --- | --- |
| 53. Evaluate each limit or state that it does not exist:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

 |

|  |  |  |
| --- | --- | --- |
| 54. Evaluate the limits in terms of the constants involved:A) B)

|  |  |
| --- | --- |
| *ANSWER:* | A) B)  |

 |

|  |  |  |
| --- | --- | --- |
| 55. Evaluate each limit or state that it does not exist:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) 28B) C) Does not exist |

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| --- | --- | --- |
| 56. Evaluate the limit:

|  |  |
| --- | --- |
| *ANSWER:* |  |

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|  |  |  |
| --- | --- | --- |
| 57. Evaluate each limit or state that it does not exist:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) 7B) C) Does not exist |

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| --- | --- | --- |
| 58. Determine a real number  for which the limit exists and then compute the limit:

|  |  |
| --- | --- |
| *ANSWER:* | , the limit is 0 |

 |

|  |  |  |
| --- | --- | --- |
| 59. Evaluate each limit or state that it does not exist:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

 |

|  |  |  |
| --- | --- | --- |
| 60. Determine a real number  for which the limit exists and then compute the limit:

|  |  |
| --- | --- |
| *ANSWER:* | ; limit is  |

 |

|  |  |  |
| --- | --- | --- |
| 61. Let *f*(*x*) = 2*x* + 3. Compute  .

|  |  |
| --- | --- |
| *ANSWER:* | 2 |

 |

|  |  |  |
| --- | --- | --- |
| 62. Compute .

|  |  |
| --- | --- |
| *ANSWER:* |  |

 |

|  |  |  |
| --- | --- | --- |
| 63. Compute .

|  |  |
| --- | --- |
| *ANSWER:* |  |

 |

|  |  |  |
| --- | --- | --- |
| 64. Evaluate the limits:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) 1B) 0C) 0 |

 |

|  |  |  |
| --- | --- | --- |
| 65. Show that  for all . (Here, [*x*] denotes the greatest integer function.)Then use the above inequality and the Squeeze Theorem to evaluate .

|  |  |
| --- | --- |
| *ANSWER:* | 0 |

 |

|  |  |  |
| --- | --- | --- |
| 66. Evaluate the limits in terms of the constants involved:A) B)

|  |  |
| --- | --- |
| *ANSWER:* | A) B)  |

 |

|  |  |  |
| --- | --- | --- |
| 67. Evaluate the limits using the Squeeze Theorem, trigonometric identities, and trigonometric limits, as necessary:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) 0B) 1C) 1 |

 |

|  |  |  |
| --- | --- | --- |
| 68. Show that  for all . (Here, [*x*] denotes the greatest integer function.) Then use this inequality with the Squeeze Theorem to evaluate .

|  |  |
| --- | --- |
| *ANSWER:* | 0 |

 |

|  |  |  |
| --- | --- | --- |
| 69. Determine a real number  such that the following limit exists, and then evaluate the limit for this value:

|  |  |
| --- | --- |
| *ANSWER:* | ; limit is  |

 |

|  |  |  |
| --- | --- | --- |
| 70. Evaluate each limit or state that it does not exist:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

 |

|  |  |  |
| --- | --- | --- |
| 71. Evaluate the limits:A) B)

|  |  |
| --- | --- |
| *ANSWER:* | A) B)  |

 |

|  |  |  |
| --- | --- | --- |
| 72. Evaluate the limits:A) *Hint:* Factor the denominator.B) *Hint:* Factor the two expressions.C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

 |

|  |  |  |
| --- | --- | --- |
| 73. Use the Squeeze Theorem to evaluate the limit

|  |  |
| --- | --- |
| *ANSWER:* | 0 |

 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 74. If  on the interval [0,4], then  must exist.

|  |  |  |
| --- | --- | --- |
|   | a.  | True |
|   | b.  | False |

|  |  |
| --- | --- |
| *ANSWER:* | b |

 |

|  |  |  |
| --- | --- | --- |
| 75. Calculate the limits:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

 |

|  |  |  |
| --- | --- | --- |
| 76. Calculate the limits:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

 |

|  |  |  |
| --- | --- | --- |
| 77. Calculate the following limits:A)   B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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|  |  |  |
| --- | --- | --- |
| 78. Compute the following limits:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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| --- | --- | --- |
| 79. Compute the following limits:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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| --- | --- | --- |
| 80. Compute the following limits:A) *Hint:* Multiply and divide by the conjugate expression.B) *Hint:* For .C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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|  |  |  |
| --- | --- | --- |
| 81. Compute the following limits:A) B) C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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| 82. Compute the following limits:A) B) (C)

|  |  |
| --- | --- |
| *ANSWER:* | A) B) C)  |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 83. The Intermediate Value Theorem guarantees that the equation  has a solution in which of the following intervals?

|  |  |  |
| --- | --- | --- |
|   | a.  |  |
|   | b.  |  |
|   | c.  |  |
|   | d.  |  |
|   | e.  |  |

|  |  |
| --- | --- |
| *ANSWER:* | a |

 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 84. The polynomial  must have a root in which of the following intervals?

|  |  |  |
| --- | --- | --- |
|   | a.  |  |
|   | b.  |  |
|   | c.  |  |
|   | d.  |  |
|   | e.  |  |

|  |  |
| --- | --- |
| *ANSWER:* | b |

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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 85. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If  assumes all the values between  and  in the interval , then  is continuous on .

|  |  |  |
| --- | --- | --- |
|   | a.  |  on  |
|   | b.  |  on  |
|   | c.  |  on  on [0,2]​ |
|   | d.  |  on  (Here, [*x*] denotes the greatest integer function.) |
|   | e.  |  on  |

|  |  |
| --- | --- |
| *ANSWER:* | e |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 86. Which of the following functions has a zero in the interval ?

|  |  |  |
| --- | --- | --- |
|   | a.  |  |
|   | b.  |  |
|   | c.  |  |
|   | d.  |  |
|   | e.  | Both A and C |

|  |  |
| --- | --- |
| *ANSWER:* | e |

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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 87. The Intermediate Value Theorem guarantees that the equation  has a solution in which of the following intervals?

|  |  |  |
| --- | --- | --- |
|   | a.  |  |
|   | b.  |  |
|   | c.  |  |
|   | d.  |  |
|   | e.  | Both A and C |

|  |  |
| --- | --- |
| *ANSWER:* | e |

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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 88. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem, which states: If  assumes all the values between  and  in the interval , then  is continuous on .

|  |  |  |
| --- | --- | --- |
|   | a.  |  for , ,  on [1,3] |
|   | b.  |  on  |
|   | c.  |  on  on  |
|   | d.  |  on  (Here, [*x*] denotes the greatest integer function.) |
|   | e.  | Both A and C |

|  |  |
| --- | --- |
| *ANSWER:* | a |

 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 89. Which of the following functions is a counterexample for the converse of the Intermediate Value Theorem: If  assumes all the values between  and  in the interval , then  is continuous on .

|  |  |  |
| --- | --- | --- |
|   | a.  |  if  on [1,3] |
|   | b.  |  on  (Here, [*x*] denotes the greatest integer function.) |
|   | c.  |  on  |
|   | d.  |  for  and  |
|   | e.  | Both A and D |

|  |  |
| --- | --- |
| *ANSWER:* | a |

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| --- | --- | --- |
| 90. Assume  is continuous on , , and . Determine whether each of the following statements is always true, never true, or sometimes true.A) : no solution with B) : no solution with C) : no solution with D) : exactly one solution with E) : a solution with

|  |  |
| --- | --- |
| *ANSWER:* | A) Sometimes trueB) Sometimes trueC) Never trueD) Sometimes trueE) Always true |

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| 91. Draw the graph of a function  on  such that the graph does not satisfy the conclusion of the Intermediate Value Theorem.

|  |  |
| --- | --- |
| *ANSWER:* | Answers may vary. A sample answer is: |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 92. Which of the following properties can be used to prove that  is continuous for all ?

|  |  |  |
| --- | --- | --- |
|   | a.  |  for all  |
|   | b.  |  for all  and  |
|   | c.  |  for all  and  |
|   | d.  | The limit  exists |
|   | e.  |  for all  and  |

|  |  |
| --- | --- |
| *ANSWER:* | b |

 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 93. Which of the following statements imply that  is not continuous at ?

|  |  |  |
| --- | --- | --- |
|   | a.  |  has opposite signs on the two sides of  |
|   | b.  |  implies that  |
|   | c.  | For any ,  implies that  |
|   | d.  | If , then  . |
|   | e.  | A and C are correct. |

|  |  |
| --- | --- |
| *ANSWER:* | c |

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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 94. To show that  is not the limit of  as , we should show that:

|  |  |  |
| --- | --- | --- |
|   | a.  | For any , there exists  such that if  then . |
|   | b.  | For any , there exists  such that if  then . |
|   | c.  | There exists , such that for any  the inequalities  and  have a solution . |
|   | d.  | There exist  and  such that if , then . |
|   | e.  | A and C are both correct. |

|  |  |
| --- | --- |
| *ANSWER:* | c |

 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 95. Suppose there exists a value of  so that for any value of , we can find a value of  satisfying  and . We may conclude that:

|  |  |  |
| --- | --- | --- |
|   | a.  |  is the limit of  as . |
|   | b.  |  is not the limit of  as . |
|   | c.  | The limit of  as  does not exist. |
|   | d.  | The limit of  as  exists but is not equal to L. |
|   | e.  | None of the above. |

|  |  |
| --- | --- |
| *ANSWER:* | b |

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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 96. To show that  is not the limit of  as , we should show that:

|  |  |  |
| --- | --- | --- |
|   | a.  | There exists  such that for any  there exists a solution to the inequalities  and . |
|   | b.  | There exists  such that for any  there exists a solution to the inequalities  and . |
|   | c.  | There exists  such that for any , if , then . |
|   | d.  | For any  and , if , then . |
|   | e.  | A and D are both correct. |

|  |  |
| --- | --- |
| *ANSWER:* | a |

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