For Thought

1. True **2.** True

3. True, since one complete revolution is $360°$.

4. False, the number of degrees in the intercepted arc of a circle is the degree measure.

5. False, the degree measure is negative if the rotation is clockwise.

6. True, the terminal side of $540°$ lies in the negative x-axis.

7. True

8. False, since $-365°$ lies in quadrant IV while $5°$ lies in quadrant I.

9. True, since $25°60' + 6' = 26°6'$.

10. False, since $25 + 20/60 + 40/3600 = 25.3444....$

1.1 Exercises

1. angle

2. central

3. standard position

4. acute

5. obtuse

6. right

7. coterminal

8. quadrantal

9. minute

10. second

11. Substitute $k = 1, 2, -1, -2$ into $60° + k \cdot 360°$. Coterminal angles are $420°, 780°, -300°, -660°$. There are other coterminal angles.

12. Substitute $k = 1, 2, -1, -2$ into $45° + k \cdot 360°$. Coterminal angles are $405°, 765°, -315°, -675°$. There are other coterminal angles.

13. Substitute $k = 1, 2, -1, -2$ into $30° + k \cdot 360°$. Coterminal angles are $390°, 750°, -330°, -690°$. There are other coterminal angles.

14. Substitute $k = 1, 2, -1, -2$ into $90° + k \cdot 360°$. Coterminal angles are $450°, 810°, -270°, -630°$. There are other coterminal angles.

15. Substitute $k = 1, 2, -1, -2$ into $225° + k \cdot 360°$. Coterminal angles are $585°, 945°, -135°, -495°$. There are other coterminal angles.

16. Substitute $k = 1, 2, -1, -2$ into $300° + k \cdot 360°$. Coterminal angles are $660°, 1020°, -60°, -420°$. There are other coterminal angles.

17. Substitute $k = 1, 2, -1, -2$ into $-45° + k \cdot 360°$. Coterminal angles are $315°, 675°, -405°, -765°$. There are other coterminal angles.

18. Substitute $k = 1, 2, -1, -2$ into $-30° + k \cdot 360°$. Coterminal angles are $330°, 690°, -390°, -750°$. There are other coterminal angles.

19. Substitute $k = 1, 2, -1, -2$ into $-90° + k \cdot 360°$. Coterminal angles are $270°, 630°, -450°, -810°$. There are other coterminal angles.

20. Substitute $k = 1, 2, -1, -2$ into $-135° + k \cdot 360°$. Coterminal angles are $225°, 585°, -495°, -855°$. There are other coterminal angles.

21. Substitute $k = 1, 2, -1, -2$ into $-210° + k \cdot 360°$. Coterminal angles are $150°, 510°, -570°, -930°$. There are other coterminal angles.

22. Substitute $k = 1, 2, -1, -2$ into $-315° + k \cdot 360°$. Coterminal angles are $45°, 405°, -675°, -1035°$. There are other coterminal angles.

23. Yes, since $40° - (-320°) = 360°$

24. Yes, since $380^\circ - 20^\circ = 360^\circ$

25. No, since $4^\circ - (-364^\circ) = 368^\circ \neq k \cdot 360^\circ$ for any integer k.

26. No, since $8^\circ - (-368^\circ) = 376^\circ \neq k \cdot 360^\circ$ for any integer k.

27. Yes, since $1235^\circ - 155^\circ = 3 \cdot 360^\circ$.

28. Yes, since $272^\circ - 1712^\circ = -4 \cdot 360^\circ$.

29. Yes, since $22^\circ - (-1058)^\circ = 3 \cdot 360^\circ$.

30. Yes, since $-128^\circ - 592^\circ = -2 \cdot 360^\circ$.

31. No, since $312.4^\circ - (-227.6^\circ) = 540^\circ \neq k \cdot 360^\circ$ for any integer k.

32. No, since $-87.3^\circ - 812.7^\circ = -900^\circ \neq k \cdot 360^\circ$ for any integer k.

33. Quadrant I **34.** Quadrant II

35. -125° lies in Quadrant III since $-125^\circ + 360^\circ = 235^\circ$ and $180^\circ < 235^\circ < 270^\circ$

36. Quadrant II

37. -740 lies in Quadrant IV since $-740^\circ + 2 \cdot 360^\circ = -20^\circ$ and -20° lies in Quadrant IV.

38. -1230° lies in Quadrant III since $-1230^\circ + 4 \cdot 360^\circ = 210^\circ$ and 210° lies in Quadrant III.

39. 933° lies in Quadrant III since $933^\circ - 2 \cdot 360^\circ = 213^\circ$ and 213° lies in Quadrant III.

40. 1568° lies in Quadrant II since $1568^\circ - 4 \cdot 360^\circ = 128^\circ$ and 128° lies in Quadrant II.

41. -310°, since $50^\circ - 360^\circ = -310^\circ$

42. -320°, since $40^\circ - 360^\circ = -320^\circ$

43. -220°, since $140^\circ - 360^\circ = -220^\circ$

44. -170°, since $190^\circ - 360^\circ = -170^\circ$

45. -90°, since $270^\circ - 360^\circ = -90^\circ$

46. -150°, since $210^\circ - 360^\circ = -150^\circ$

47. 40°, since $400^\circ - 360^\circ = 40^\circ$

48. 180°, since $540^\circ - 360^\circ = 180^\circ$

49. 20°, since $-340^\circ + 360^\circ = 20^\circ$

50. 180°, since $-180^\circ + 360^\circ = 180^\circ$

51. 340°, since $-1100^\circ + 4 \cdot 360^\circ = 340^\circ$

52. 240°, since $-840^\circ + 3 \cdot 360^\circ = 240^\circ$

53. 180.54°, since $900.54^\circ - 2 \cdot 360^\circ = 180.54^\circ$

54. 155.6°, since $1235.6^\circ - 3 \cdot 360^\circ = 155.6^\circ$

55. c **56.** f **57.** e **58.** a

59. h **60.** b **61.** g **62.** d

63. $13^\circ + \dfrac{12}{60}^\circ = 13.2^\circ$

64. $45^\circ + \dfrac{6}{60}^\circ = 45.1^\circ$

65. $-8^\circ - \dfrac{30}{60}^\circ - \dfrac{18}{3600}^\circ = -8.505^\circ$

66. $-5^\circ - \dfrac{45}{60}^\circ - \dfrac{30}{3600}^\circ \approx -5.7583^\circ$

67. $28^\circ + \dfrac{5}{60}^\circ + \dfrac{9}{3600}^\circ \approx 28.0858^\circ$

68. $44^\circ + \dfrac{19}{60}^\circ + \dfrac{32}{3600}^\circ \approx 44.3256^\circ$

69. $155^\circ + \dfrac{34}{60}^\circ + \dfrac{52}{3600}^\circ \approx 155.5811^\circ$

70. $200^\circ + \dfrac{44}{60}^\circ + \dfrac{51}{3600}^\circ = 200.7475^\circ$

71. $75.5^\circ = 75^\circ 30'$ since $0.5(60) = 30$

72. $39.25^\circ = 39^\circ 15'$ since $0.25(60) = 15$

73. $39.4^\circ = 39^\circ 24'$ since $0.4(60) = 24$

74. $17.8^\circ = 17^\circ 48'$ since $0.8(60) = 48$

75. $-17.33^\circ = -17^\circ 19' 48''$ since $0.33(60) = 19.8$ and $0.8(60) = 48$

76. $-9.12^\circ = -9^\circ 7' 12''$ since $0.12(60) = 7.2$ and $0.2(60) = 12$

77. $18.123^\circ \approx 18^\circ 7' 23''$ since $0.123(60) = 7.38$ and $0.38(60) \approx 23$

78. $122.786^\circ = 122^\circ 47' 10''$ since $0.786(60) = 47.16$ and $0.16(60) \approx 10$

79. $24^\circ 15' + 33^\circ 51' = 57^\circ 66' = 58^\circ 6'$

80. $99^\circ 35' + 66^\circ 48' = 165^\circ 83' = 166^\circ 23'$

81. $55^\circ 11' - 23^\circ 37' = 54^\circ 71' - 23^\circ 37' = 31^\circ 34'$

82. $76^\circ 6' - 18^\circ 54' = 75^\circ 66' - 18^\circ 54' = 57^\circ 12'$

83. $16^\circ 23' 41'' + 44^\circ 43' 39'' = 60^\circ 66' 80'' = 60^\circ 67' 20'' = 61^\circ 7' 20''$

84. $7^\circ 55' 42'' + 8^\circ 22' 28'' = 15^\circ 77' 70'' = 15^\circ 78' 10'' = 16^\circ 18' 0''$

85. $90^\circ - 7^\circ 44' 35'' = 89^\circ 59' 60'' - 7^\circ 44' 35'' = 82^\circ 15' 25''$

86. $179^\circ 59' 60'' - 49^\circ 39' 45'' = 130^\circ 20' 15''$

87. $66^\circ 43' 6'' - 5^\circ 51' 53'' = 65^\circ 102' 66'' - 5^\circ 51' 53'' = 60^\circ 51' 13''$

88. $33^\circ 98' 72'' - 9^\circ 49' 18'' = 24^\circ 49' 54''$

89. $2(32^\circ 36' 37'') = 64^\circ 72' 74'' = 64^\circ 73' 14'' = 65^\circ 13' 14''$

90. $267^\circ 123' 168'' = 267^\circ 125' 48'' = 269^\circ 5' 48''$

91. $3(15^\circ 53' 42'') = 45^\circ 159' 126'' = 45^\circ 161' 6'' = 47^\circ 41' 6''$

92. $36^\circ 144' 160'' = 36^\circ 146' 40'' = 38^\circ 26' 40''$

93. $(43^\circ 13' 8'')/2 = (42^\circ 73' 8'')/2 = (42^\circ 72' 68'')/2 = 21^\circ 36' 34''$

94. $(33^\circ 100' 15'')/3 = (33^\circ 99' 75'')/3 = 11^\circ 33' 25''$

95. $(13^\circ 10' 9'')/3 = (12^\circ 70' 9'')/3 = (12^\circ 69' 69'')/3 = 4^\circ 23' 23''$

96. $(4^\circ 78' 40'')/4 = (4^\circ 76' 160'')/4 = 1^\circ 19' 40''$

97. $\alpha = 180^\circ - 88^\circ 40' - 37^\circ 52' = 180^\circ - 126^\circ 32' = 179^\circ 60' - 126^\circ 32' = 53^\circ 28'$

98. $\alpha = \dfrac{179^\circ 59' 60'' - 30^\circ 24' 12''}{2} = \dfrac{149^\circ 35' 48''}{2} = \dfrac{148^\circ 95' 48''}{2} = \dfrac{148^\circ 94' 108''}{2} = 74^\circ 47' 54''$

99. $\alpha = 180^\circ - 90^\circ - 48^\circ 9' 6'' = 89^\circ 59' 60'' - 48^\circ 9' 6'' = 41^\circ 50' 54''$

100. $\alpha = 180^\circ - 90^\circ - 61^\circ 1' 48'' = 89^\circ 59' 60'' - 61^\circ 1' 48'' = 28^\circ 58' 12''$

101. $\alpha = 180^\circ - 140^\circ 19' 16'' = 179^\circ 59' 60'' - 140^\circ 19' 16'' = 39^\circ 40' 44''$

102. $\alpha = 360^\circ - 72^\circ 21' 35'' = 359^\circ 59' 60'' - 72^\circ 21' 35'' = 287^\circ 38' 25''$

103. $\alpha = 90^\circ - 75^\circ 5' 6'' = 89^\circ 59' 60'' - 75^\circ 5' 6'' = 14^\circ 54' 54''$

104. $\alpha = 270^\circ - 243^\circ 36' 29'' = 269^\circ 59' 60'' - 243^\circ 36' 29'' = 26^\circ 23' 31''$

105. Since $0.17647(60) = 10.5882$ and $0.5882(60) = 35.292$, we find $21.17647^\circ \approx 21^\circ 10' 35.3''$.

106. Since $0.243(60) = 14.58$ and $0.58(60) = 34.8$, we find $37.243^\circ \approx 37^\circ 14' 34.8''$.

107. Since $73^\circ 37' \approx 73.6167^\circ$, $49^\circ 41' \approx 49.6833^\circ$, and $56^\circ 42' = 56.7000^\circ$, the sum of the numbers in decimal format is 180°. Also,

$$73^\circ 37' + 49^\circ 41' + 56^\circ 42' = 178^\circ 120' = 180^\circ.$$

108. Since $27^\circ 23' \approx 27.3833^\circ$ and $125^\circ 14' \approx 125.2333^\circ$, the sum of the three angles is

$$2(27.3833^\circ) + 125.2333^\circ = 179.9999^\circ.$$

On the other hand,

$$2(27^\circ 23') + 125^\circ 14' = 180^\circ.$$

109. We find $108^\circ 24' 16'' \approx 108.4044^\circ$, $68^\circ 40' 40'' \approx 68.6778^\circ$, $84^\circ 42' 51'' \approx 84.7142^\circ$, and $98^\circ 12' 13'' \approx 98.2036^\circ$. The sum of these four numbers in decimal format is 360°. Also, $108^\circ 24' 16'' + 68^\circ 40' 40'' + 84^\circ 42' 51'' + 98^\circ 12' 13'' = 360^\circ$.

110. We find $64°41'5'' \approx 64.6847°$, $140°28'7'' \approx 140.4686°$, $62°40'35'' \approx 62.6764°$, and $92°10'13'' \approx 92.1703°$. The sum of these four numbers in decimal format is $360°$. Also, $64°41'5'' + 140°28'7'' + 62°40'35'' +$ $92°10'13'' = 360°$.

111. At 3:20, the hour hand is at an angle

$$\frac{20}{60} \times 30° = 10°$$

below the 3 in a standard clock.

Since the minute hand is at 4 and there are $30°$ between 3 and 4, the angle between the hour and minute hands is

$$30° - 10° = 20°.$$

112. At 7:10:18, the second hand will be at the angle

$$\frac{18}{60} \times 360° = 108°$$

from the 12 o'clock.

The minute hand will be at the angle

$$\frac{10 + 18/60}{60} \times 360° = 61.8°$$

Then the angle between the minute and second hands is

$$108° - 61.8° = 46.2°$$

113. $f(g(2)) = f(4) = 15$ and $g(f(2)) = g(3) = 7$

114. Reverse the oder of operations. Then

$$f^{-1}(x) = \frac{x+9}{5}.$$

115. $\sqrt{(4-1)^2 + (5-1)^2} = \sqrt{9+16} = 5$

116. $(x-4)^2 + (y+8)^2 = 3$

117. $\sqrt{5^2 + 5^2} = 5\sqrt{2}$ feet

118. Since $x + 3 \geq 0$, the domain is $[-3, \infty)$.

The range is $[5, \infty)$.

119. Let r be the radius of the circle. Let p be the distance from the lower left most corner of the 1-by-1 square to the point of tangency of the left most, lower most circle at the base of the 1-by-1 square. By the Pythagorean Theorem,

$$(p+r)^2 + (r+1-p)^2 = 1$$

or equivalently

$$p^2 + r^2 + r - p = 0. \tag{1}$$

Since the area of the four triangles plus the area of the small square in the middle is 1, we obtain

$$4r^2 + 4\left[\frac{1}{2}(p+r)(r+1-p)\right] = 1$$

or

$$6r^2 - 2p^2 + 2p + 2r = 1. \tag{2}$$

Multiply (1) by two and add the result to (2). We obtain

$$8r^2 + 4r = 1.$$

The solution is $r = (\sqrt{3} - 1)/4$.

120. If a number is divisible by 11 and 13, it is of the form $11^a 13^b n$ where a, b, n are integers greater than or equal to 1.

Start with $a = b = 1$ and solve $2000 < 11(13)(n) < 2300$ to get $13.98 < n < 16.08$. Then $n = 14$, $n = 15$, or $n = 16$. Since the number is odd, $n = 15$ and the number is $11 \cdot 13 \cdot 15$ or 2145.

If either a or b is 2 or larger there is no solution to $2000 < 11^a 13^b n < 2300$. So there is only one answer and it is 2145.

1.1 Pop Quiz

1. Quadrant III

2. $1267° - 1080° = 187°$

3. $720° - 394° = 326°$

4. No, since $-240° - 60° = -300°$ is not a multiple of $360°$.

5. $70° + (30/60)° + (36/3600)° = 70.51°$

6. Since $0.82(60) = 49.2$ and $0.2(60) = 12$, we find $32.82° = 32°49'12''$.

7. The sum is $51°83'87'' = 51°84'27'' = 52°24'27''$.

For Thought

1. False, in a negative angle the rotation is clockwise.

2. False, the radius is 1.

3. True, since the circumeference is $2\pi r$ where r is the radius.

4. True, since $s = \alpha r$ **5.** True

6. False, one must multiply by $\frac{\pi}{180}$.

7. True **8.** False, rather $45° = \frac{\pi}{4}$ rad.

9. True

10. True, rather the length of arc is $s = \alpha \cdot r = \frac{\pi}{4} \cdot 4 = 1$.

1.2 Exercises

1. unit

2. radian

3. $s = \alpha r$

4. $A = \alpha r^2/2$

5. $30° = \frac{\pi}{6}$, $45° = \frac{\pi}{4}$, $60° = \frac{\pi}{3}$, $90° = \frac{\pi}{2}$,
$120° = \frac{2\pi}{3}$, $135° = \frac{3\pi}{4}$, $150° = \frac{5\pi}{6}$, $180° = \pi$,
$210° = \frac{7\pi}{6}$, $225° = \frac{5\pi}{4}$, $240° = \frac{4\pi}{3}$,
$270° = \frac{3\pi}{2}$, $300° = \frac{5\pi}{3}$, $315° = \frac{7\pi}{4}$,
$330° = \frac{11\pi}{6}$, $360° = 2\pi$

6. $-30° = -\frac{\pi}{6}$, $-45° = -\frac{\pi}{4}$, $-60° = -\frac{\pi}{3}$,
$-90° = -\frac{\pi}{2}$, $-120° = -\frac{2\pi}{3}$, $-135° = -\frac{3\pi}{4}$,
$-150° = -\frac{5\pi}{6}$, $-180° = -\pi$, $-210° = -\frac{7\pi}{6}$,
$-225° = -\frac{5\pi}{4}$, $-240° = -\frac{4\pi}{3}$, $-270° = -\frac{3\pi}{2}$,
$-300° = -\frac{5\pi}{3}$, $-315° = -\frac{7\pi}{4}$,
$-330° = -\frac{11\pi}{6}$, $-360° = -2\pi$

7. $45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$

8. $\frac{\pi}{6}$

9. $90 \cdot \frac{\pi}{180} = \frac{\pi}{2}$

10. $\frac{\pi}{3}$

11. $120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$

12. π

13. $150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$

14. $\frac{5\pi}{4}$

15. $18 \cdot \frac{\pi}{180} = \frac{\pi}{10}$

16. $48 \cdot \frac{\pi}{180} = \frac{4\pi}{15}$

17. $\frac{\pi}{3} \cdot \frac{180}{\pi} = 60°$

18. $30°$

19. $\frac{5\pi}{12} \cdot \frac{180}{\pi} = 75°$

20. $\frac{17\pi}{12} \cdot \frac{180}{\pi} = 255°$

21. $\frac{3\pi}{4} \cdot \frac{180}{\pi} = 135°$

22. $\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225°$

23. $-6\pi \cdot \frac{180}{\pi} = -1080°$

24. $-9\pi \cdot \frac{180}{\pi} = -1620°$

25. $2.39 \cdot \frac{180}{\pi} \approx 136.937°$

26. $0.452 \cdot \dfrac{180}{\pi} \approx 25.898^\circ$

27. $-0.128 \cdot \dfrac{180}{\pi} \approx -7.334^\circ$

28. $-1.924 \cdot \dfrac{180}{\pi} \approx -110.237^\circ$

29. $37.4\left(\dfrac{\pi}{180}\right) \approx 0.653$

30. $125.3\left(\dfrac{\pi}{180}\right) \approx 2.187$

31. $\left(-13 - \dfrac{47}{60}\right) \cdot \dfrac{\pi}{180} \approx -0.241$

32. $\left(-99 - \dfrac{15}{60}\right) \cdot \dfrac{\pi}{180} \approx -1.732$

33. $\left(53 + \dfrac{37}{60} + \dfrac{6}{3600}\right) \cdot \dfrac{\pi}{180} \approx 0.936$

34. $\left(187 + \dfrac{49}{60} + \dfrac{36}{3600}\right) \cdot \dfrac{\pi}{180} \approx 3.278$

35. Substitute $k = 1, 2, -1, -2$ into $\dfrac{\pi}{3} + k \cdot 2\pi$,
coterminal angles are $\dfrac{7\pi}{3}, \dfrac{13\pi}{3}, -\dfrac{5\pi}{3}, -\dfrac{11\pi}{3}$.
There are other coterminal angles.

36. Substitute $k = 1, 2, -1, -2$ into $\dfrac{\pi}{4} + k \cdot 2\pi$,
coterminal angles are $\dfrac{9\pi}{4}, \dfrac{17\pi}{4}, -\dfrac{7\pi}{4}, -\dfrac{15\pi}{4}$.
There are other coterminal angles.

37. Substitute $k = 1, 2, -1, -2$ into $\dfrac{\pi}{2} + k \cdot 2\pi$,
coterminal angles are $\dfrac{5\pi}{2}, \dfrac{9\pi}{2}, -\dfrac{3\pi}{2}, -\dfrac{7\pi}{2}$.
There are other coterminal angles.

38. Substitute $k = 1, 2, -1, -2$ into $\pi + k \cdot 2\pi$,
coterminal angles are 3π, 5π, $-\pi$, -3π.
There are other coterminal angles.

39. Substitute $k = 1, 2, -1, -2$ into $\dfrac{2\pi}{3} + k \cdot 2\pi$,
coterminal angles are $\dfrac{8\pi}{3}, \dfrac{14\pi}{3}, -\dfrac{4\pi}{3}, -\dfrac{10\pi}{3}$.
There are other coterminal angles.

40. Substitute $k = 1, 2, -1, -2$ into $\dfrac{5\pi}{6} + k \cdot 2\pi$,
coterminal angles are $\dfrac{17\pi}{6}, \dfrac{29\pi}{6}, -\dfrac{7\pi}{6}, -\dfrac{19\pi}{6}$.
There are other coterminal angles.

41. Substitute $k = 1, 2, -1, -2$ into $1.2 + k \cdot 2\pi$,
coterminal angles are about 7.5, 13.8, -5.1, -11.4. There are other coterminal angles.

42. Substitute $k = 1, 2, -1, -2$ into $2 + k \cdot 2\pi$,
coterminal angles are about 8.3, 14.6, -4.3, -10.6. There are other coterminal angles.

43. Quadrant I **44.** Quadrant III

45. Quadrant III

46. $-\dfrac{39\pi}{20}$ lies in Quadrant I since
$-\dfrac{39\pi}{20} + 2\pi = \dfrac{\pi}{20}$

47. $-\dfrac{13\pi}{8}$ lies in Quadrant I since
$-\dfrac{13\pi}{8} + 2\pi = \dfrac{3\pi}{8}$

48. $-\dfrac{11\pi}{8}$ lies in Quadrant II since
$-\dfrac{11\pi}{8} + 2\pi = \dfrac{5\pi}{8}$

49. $\dfrac{17\pi}{3}$ lies in Quadrant IV since
$\dfrac{17\pi}{3} - 4\pi = \dfrac{5\pi}{3}$

50. $\dfrac{19\pi}{4}$ lies in Quadrant II since
$\dfrac{19\pi}{4} - 4\pi = \dfrac{3\pi}{4}$

51. 3 lies in Quadrant II since $\dfrac{\pi}{2} < 3 < \pi$

52. 23.1 lies in Quadrant III since $23.1 - 6\pi \approx 4.3$ and 4.3 lies in Qudrant III

53. -7.3 lies in Quadrant IV since $-7.3 + 4\pi \approx 5.3$ and 5.3 lies in Qudrant IV

54. -2 lies in Quadrant III since $-2 + 2\pi \approx 4.3$ and 4.3 lies in Qudrant III

55. g **56.** e **57.** b **58.** a

59. h **60.** c **61.** d **62.** f

63. π, since $3\pi - 2\pi = \pi$ and π is the smallest positive coterminal angle

64. 2π, since $6\pi - 4\pi = 2\pi$ and π is the smallest positive coterminal angle

65. $\frac{3\pi}{2}$, since $-\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$

66. $\frac{\pi}{2}$, since $-\frac{3\pi}{2} + 2\pi = \frac{\pi}{2}$

67. $\frac{9\pi}{2} - 4\pi = \frac{\pi}{2}$

68. $\frac{19\pi}{2} - 8\pi = \frac{3\pi}{2}$

69. $-\frac{5\pi}{3} + 2\pi = \frac{\pi}{3}$

70. $-\frac{7\pi}{6} + 2\pi = \frac{5\pi}{6}$

71. $-\frac{13\pi}{3} + 6\pi = \frac{5\pi}{3}$

72. $-\frac{19\pi}{4} + 6\pi = \frac{5\pi}{4}$

73. $8.32 - 2\pi \approx 2.04$

74. $-23.55 + 8\pi \approx 1.58$

75. $\frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$

76. $\frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

77. $\frac{3\pi}{6} + \frac{2\pi}{6} = \frac{5\pi}{6}$

78. $\frac{3\pi}{6} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

79. $\frac{2\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4}$

80. $\frac{4\pi}{12} + \frac{3\pi}{12} = \frac{7\pi}{12}$

81. $\frac{\pi}{3} - \frac{3\pi}{3} = -\frac{2\pi}{3}$

82. $\frac{7\pi}{6} - \frac{12\pi}{6} = -\frac{5\pi}{6}$

83. $s = 12 \cdot \frac{\pi}{4} = 3\pi \approx 9.4$ ft

84. $s = 4 \cdot 1 = 4$ cm

85. $s = 4000 \cdot \frac{3\pi}{180} \approx 209.4$ miles

86. $s = 2 \cdot \frac{\pi}{3} \approx 2.1$ m

87. $s = (26.1)(1.3) \approx 33.9$ m

88. $s = 30 \cdot \frac{\pi}{8} \approx 11.8$ yd

89. radius is $r = \frac{s}{\alpha} = \frac{1}{1} = 1$ mile.

90. radius is $r = \frac{s}{\alpha} = \frac{99}{0.004} = 24,750$ km

91. radius is $r = \frac{s}{\alpha} = \frac{10}{\pi} \approx 3.2$ km

92. radius is $r = \frac{s}{\alpha} = \frac{8}{2\pi} \approx 1.3$ m

93. radius is $r = \frac{s}{\alpha} = \frac{500}{\pi/6} \approx 954.9$ ft

94. radius is $r = \frac{s}{\alpha} = \frac{7}{\pi/3} \approx 6.7$ in.

95. $A = \frac{\alpha r^2}{2} = \frac{(\pi/6)6^2}{2} = 3\pi$

96. $A = \frac{\alpha r^2}{2} = \frac{(\pi/4)4^2}{2} = 2\pi$

97. $A = \frac{\alpha r^2}{2} = \frac{(\pi/3)12^2}{2} = 24\pi$

98. $A = \frac{\alpha r^2}{2} = \frac{(\pi/12)8^2}{2} = \frac{8\pi}{3}$

99. Distance from Peshtigo to the North Pole is

$$s = r\alpha = 3950\left(45 \cdot \frac{\pi}{180}\right) \approx 3102 \text{ miles}$$

100. Assume the helper is an arc in a circle whose radius is the distance r between the helper and the surveyor. The angle in radians subtended is

$$\frac{37}{60} \cdot \frac{\pi}{180} \approx 0.01076.$$

Then $r = \frac{s}{\alpha} \approx \left(6 + \frac{2}{12}\right) \div 0.01076 \approx 573$ ft.

101. The central angle is $\alpha = \dfrac{2000}{3950} \approx$

0.506329 radians $\approx 0.506329 \cdot \dfrac{180}{\pi} \approx 29.0^\circ$

102. Fifty yards from the goal, the angle is

$$\alpha = \frac{18.5}{50(3)} \approx 0.1233 \text{ radians} \approx 7.06^\circ.$$

The deviation from the actual trajectory is

$$\frac{7.06^\circ}{2} = 3.53^\circ.$$

While 20 yd from the goal, the angle is $\alpha = \dfrac{18.5}{20(3)} \approx 0.3083$ radians $\approx 17.66^\circ$.

The deviation from 20 yd is

$$\frac{17.66^\circ}{2} = 8.83^\circ.$$

103. Since $7^\circ \approx 0.12217305$, the radius of the earth according to Eratosthenes is

$$r = \frac{s}{\alpha} \approx \frac{800}{0.12217305} \approx 6548.089 \text{ km}.$$

Thus, using Eratosthenes' radius, the circumference is

$$2\pi r \approx 41,143 \text{ km}.$$

Using $r = 6378$ km, circumference is 40,074 km.

104. Since $7^\circ \approx 0.12217305$, the radius of the earth according to Eratosthenes is

$$r = \frac{s}{\alpha} \approx \frac{500}{0.12217305} \approx 4092.56 \text{ stadia}$$

Thus, using Eratosthenes' above radius, the circumference is

$$2\pi r \approx 25,714 \text{ stadia}$$

105. The length of the arc intercepted by the central angle $\alpha = 20^\circ$ in a circle of radius 6 in. is

$$s = 6\left(20 \cdot \frac{\pi}{180}\right) = \frac{2\pi}{3}.$$

Since the radius of the cog is 2 in., the cog rotates through an angle

$$\alpha = \frac{s}{r} = \frac{2\pi/3}{2} = \frac{\pi}{3} \text{ radians} = 60^\circ.$$

106. The angular velocity of the cyclist's cadence is 80π radians per minute. The linear velocity on a circle with radius 6 inches is

$$s_6 = 6(80\pi) = 480\pi \frac{\text{in.}}{\text{min}}$$

On a circle with radius 2 inches, if the linear velocity is s_6, then the angular velocity is

$$\alpha_2 = \frac{s_6}{2} = 240\pi \frac{\text{radians}}{\text{min}}$$

Note, 700 mm ≈ 27.5591 in.

On a circle with radius $r_0 = \frac{27.5591}{2}$ inches, if the angular velocity is α_2, then the linear velocity is

$$\begin{aligned} s_0 &= r_0\alpha_2 \\ &= \frac{27.5591}{2}(240\pi)\frac{\text{inches}}{\text{min}} \\ &= 120\pi(27.5591)\frac{\text{inches}}{\text{min}} \end{aligned}$$

But 1 in/min is equivalent to 0.0009469 miles per hour. Thus, her speed is

$$120\pi(27.5591) \times 0.0009469 \approx 9.8 \text{ mph}.$$

107. If s is the arc length, the radius is $r = \frac{6-s}{2}$. Let θ be the central angle such that

$$s = r\theta = \frac{(6-s)\theta}{2}.$$

Solving for s, we find

$$s = \frac{6\theta}{\theta + 2}.$$

The area of the sector is

$$A = \frac{\theta r^2}{2} = \frac{\theta}{2}\left(\frac{s}{\theta}\right)^2 = \frac{s^2}{2\theta}.$$

Substituting, we obtain

$$\begin{aligned} A &= \frac{1}{2\theta}\left(\frac{6\theta}{\theta+2}\right)^2 \\ &= \frac{18\theta}{(\theta+2)^2}. \end{aligned}$$

108. Using a calculator, the area A in Exercise 107 is maximized when $\theta = 2$.

109. Note, the fraction of the area of the circle of radius r intercepted by the central angle $\frac{\pi}{6}$ is $\frac{1}{12} \cdot \pi r^2$. So, the area watered in one hour is

$$\frac{1}{12} \cdot \pi 150^2 \approx 5890 \text{ ft}^2.$$

110. The area scanned by a $75°$ angle is

$$\frac{75}{360} \cdot \pi 30^2 \approx 589 \text{ mi}^2.$$

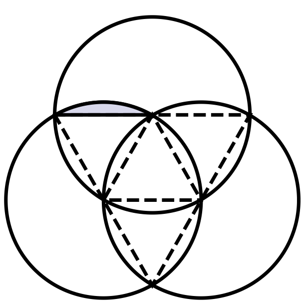
111. Note, the radius of the pizza is 8 in. Then the area of each of the six slices is

$$\frac{1}{6} \cdot \pi 8^2 \approx 33.5 \text{ in.}^2$$

112. The area of a slice with central angle $\frac{\pi}{7}$ and radius 10 in. is $\frac{\pi/7}{2\pi} \cdot \pi 10^2 \approx 22.4 \text{ in.}^2$

113. A region S bounded by a chord and a circle of radius 10 meters is shaded below. The central angle is $60°$. The area A_s of S may be obtained by subtracting the area of an equilateral triangle from the area of a sector. That is,

$$A_s = 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right).$$



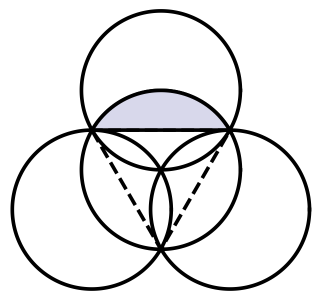
The common region bounded by two or three circles consists of 4 equilateral triangles and six regions like S.

The area of the region inside the higher circle but outside the common region is $100(\pi/2 - 2A_s)$. There are two other such regions for the two circles on the left and right.

Then the total area A watered by the three circular sprinklers is

$$\begin{aligned} A &= \left[100\sqrt{3} + 6A_s + 3\left(\frac{100\pi}{2} - 2A_s\right)\right] \\ &= 100\left[\frac{3\pi}{2} + \sqrt{3}\right] \text{ m}^2 \\ &\approx 644.4 \text{ m}^2. \end{aligned}$$

114. A shaded region below is bounded by a chord and a circular arc of a circle of radius 10 meters. Let A_{sr} be the area of the region.



a) If we subtract the area of an isosceles triangle from the area of a sector, we find

$$\begin{aligned} A_{sr} &= \frac{2\pi/3(10)^2}{2} - \frac{1}{2}(10)^2\frac{\sqrt{3}}{2} \\ &= 10^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right). \end{aligned}$$

Then the area of the region inside the circle at the top, and that lies above the shaded region is

$$A_t = 10^2\pi - 2A_{sr}.$$

Thus, the total area A watered by the four sprinklers is the area of the middle circle plus the area from the other three circles. That is,

$$\begin{aligned} A &= 10^2\pi + 3A_t \\ &= 10^2\pi + 3\left(10^2\pi - 2\left[10^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)\right]\right) \\ &= 10^2\left(\pi + 3\left(\pi - 2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\right)\right) \end{aligned}$$

Simplifying, we obtain

$$\begin{aligned} A &= 10^2\left(2\pi + \frac{3\sqrt{3}}{2}\right) \\ &\approx 888.1 \text{ m}^2. \end{aligned}$$

b) Using Cartesian coordinates, position the center of the middle circle at $(0, 0)$. The centers of the other circles are at $(0, 10)$, $(5\sqrt{3}, -5)$, and $(-5\sqrt{3}, -5)$.

Draw the horizontal lines $y = -5$ and $y = 10$ that pass though the centers of the two lower circles and the upper circle, respectively, The centers of these three circles are sprinklers. We ignore the middle circle. The two sprinklers on $y = -5$ are $10\sqrt{3} \approx 17.32$ meters apart.

Hence, the sprinklers should be placed in rows that are 15 meters apart. In each row, the sprinklers are 17.32 meters apart. Moreover, each sprinkler lies 15 meters above the midpoint of two sprinklers in an adjacent row.

115. a) Given angle α (in degrees) as in the problem, the radius r of the cone must satisfy $2\pi r = 8\pi - 4\alpha\dfrac{\pi}{180}$; note, 8π in. is the circumference of a circle with radius 4 in. Then $r = 4 - \dfrac{\alpha}{90}$. Note, $h = \sqrt{16 - r^2}$ by the Pythagorean theorem. Since the volume $V(\alpha)$ of the cone is $\dfrac{\pi}{3}r^2h$, we find

$$V(\alpha) = \frac{\pi}{3}\left(4 - \frac{\alpha}{90}\right)^2\sqrt{16 - \left(4 - \frac{\alpha}{90}\right)^2}$$

This reduces to

$$V(\alpha) = \frac{\pi(360 - \alpha)^2\sqrt{720\alpha - \alpha^2}}{2,187,000}$$

If $\alpha = 30°$, then $V(30°) \approx 22.5$ inches^3.

b) As shown in part a), the volume of the cone obtained by an overlapping angle α is

$$V(\alpha) = \frac{\pi(360 - \alpha)^2\sqrt{720\alpha - \alpha^2}}{2,187,000}$$

116. a) The volume $V(\alpha)$, see Exercise 115 b), of the cone is maximized when $\alpha \approx 66.06°$.

b) The maximum volume is approximately $V(66.06°) \approx 25.8$ cubic inches.

119. angle, rays

120. Since $0.23(60) = 13.8$ and $0.8(60) = 48$, we find

$$48.23° = 48°13'48''$$

121. $x = \pi$

122. $\left(\dfrac{\pi/2 + \pi}{2}, \dfrac{0 + 0}{2}\right) = \left(\dfrac{3\pi}{4}, 0\right)$

123. right

124. Since $-3w - 7 = 5$, we find $-3w = 12$ or $w = -4$.

125. a) The contestants that leave the table are # 1, # 3, # 5, # 7, # 9, # 11, # 13, # 4, # 8, # 12, # 2, #6 (in this order). Thus, contestant # 10 is the unlucky contestant.

b) Let $n = 8$. The contestants that leave the table are # 1, # 3, # 5, # 7, # 2, # 6, # 4 (in this order). Thus, contestant # 8 is the unlucky contestant.

Let $n = 16$. The contestants that leave the table are # 1, # 3, # 5, # 7, # 9, # 11, # 13, # 15, # 2, # 6, # 10, # 14, # 4, # 12, # 8 (in this order). Thus, contestant # 16 is the unlucky contestant.

Let $n = 41$. The contestants that leave the table are # 1, # 3, # 5, # 7, # 9, # 11, # 13, # 15, # 17, # 19, # 21, # 23, # 25, # 27, # 29, # 31, # 33, # 35, # 37, # 39, # 4, # 8 # 12, # 16, # 20, # 24, # 28, # 32, # 36, # 40, # 6, # 14, # 22, # 30, # 38, # 10, # 26, # 2, # 34 (in this order). Thus, contestant # 18 is the unlucky contestant.

c) Let $m \geq 1$ and let n satisfy

$$2^m < n \leq 2^{m+1} \tag{3}$$

where

$$n = 2^m + k. \quad (4)$$

We claim the unlucky number is $2k$. One can check the claim is true for all n when $m = 1$. Suppose the claim is true for $m - 1$ and all such n.

Consider the case when k is an even integer satisfying (3) and (4). After selecting the survivors in round 1, the remaining contestants are

$$2, 4, 6, ..., 2^m + k.$$

Renumber, the above contestants by the rule

$$f(x) = x/2$$

so that the remaining contestants are renumbered as

$$1, 2, 3, ..., 2^{m-1} + k/2.$$

Note,

$$2^{m-1} < 2^{m-1} + k/2 \leq 2^m$$

Since the claim is true for $m - 1$, the unlucky contestant in the renumbering is

$$2\left(\frac{k}{2}\right) = k.$$

But by the renumbering, we find

$$f(x) = \frac{x}{2} = k$$

or the unlucky contestant is $2k$. In particular, if $n = 2^m$ then the unlucky contestant is

$$2k = 2 \cdot 2^{m-1} = 2^m.$$

Finally, let k be an odd integer satisfying (3) and (4). After selecting the survivors in round 1, the remaining contestants are

$$2, 4, 6, ..., 2^m + (k - 1).$$

Note, the next survivor is 4 since the last survivor chosen is $2^m + k$.

Renumber, the above contestants by the rule

$$g(x) = \frac{x}{2} - 1$$

so the remaining contestant are renumbered as follows:

$$0, 1, 2, 3, ..., [2^{m-1} + (k - 1)/2 - 1].$$

Since the claim is true for $m - 1$, the unlucky contestant using the previous renumbering is

$$2\left(\frac{k-1}{2}\right) \quad \text{or} \quad k - 1$$

Using the original numbering, the unlucky contestant is

$$g(x) = \frac{x}{2} - 1 = k - 1$$

or

$$x = 2k = 2(n - 2^m).$$

126. Let (x, y, xy) be an ordered triple such that $x^2y = y$ and $xy^2 = x$. Notice, $x = 0$ if and only $y = 0$. If $y \neq 0$, then $x = \pm 1$ and $y = \pm 1$. The ordered triples are $(0, 0, 0)$, $(1, 1, 1)$, $(1, -1, -1)$, $(-1, 1, -1)$, and $(-1, -1, 1)$.

There are five ordered triples.

1.2 Pop Quiz

1. $270 \cdot \dfrac{\pi}{180°} = \dfrac{3\pi}{2}$

2. $\left(\dfrac{7\pi}{4} \cdot \dfrac{180}{\pi}\right)^{\circ} = 315°$

3. Yes, since the difference

$$-\frac{3\pi}{4} - \frac{5\pi}{4} = -2\pi$$

is a multiple of 2π.

4. Since $-\dfrac{3\pi}{4}$ lies in Quadrant 3, the smallest positive angle that is coterminal is $\dfrac{5\pi}{4}$.

5. $s = r\alpha = 30 \cdot \dfrac{\pi}{3} = 10\pi$ ft

6. $\alpha = \frac{s}{r} = \frac{1}{8}$ radians $= \left(\frac{1}{8} \cdot \frac{180}{\pi}\right)^\circ \approx 7.2^\circ$

7. $A = \frac{\alpha r^2}{2} = \frac{(\pi/2)8^2}{2} = 16\pi$ square inches

For Thought

1. False, $\frac{240 \text{ rev}}{\text{min}} = \frac{(240)(6) \text{ rev}}{\text{hr}}$.

2. True

3. False, since $\frac{4 \text{ rev}}{\text{sec}} \cdot \frac{2\pi \text{ ft}}{1 \text{ rev}} = \frac{8\pi \text{ ft}}{\text{sec}}$.

4. False, since $\frac{5\pi \text{ rad}}{1 \text{ hr}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = \frac{300\pi \text{ rad} \cdot \text{min}}{\text{hr}^2}$.

5. False, it is angular velocity.

6. False, it is linear velocity.

7. False, since 40 inches/second is linear velocity.

8. True, since 1 rev/sec is equivalent to $\omega = 2\pi$ radians/sec, we get that the linear velocity is $v = r\omega = 1 \cdot 2\pi = 2\pi$ ft/sec.

9. True

10. False, Miami has a faster linear velocity than Boston. Note, Miami's distance from the axis of the earth is farther than that of Boston's.

1.3 Exercises

1. angular velocity

2. linear velocity

3. linear velocity

4. linear, angular

5. $\frac{300 \text{ rad}}{60 \text{ min}} = 5$ rad/min

6. $\frac{2(60) \text{ rad}}{3 \text{ min}} = 40$ rad/min

7. $\frac{4(2\pi) \text{ rad}}{\text{sec}} = 8\pi$ rad/sec

8. $\frac{99.6 \text{ rev}}{2(2\pi) \text{ sec}} \approx 7.9$ rev/sec

9. $\frac{55(6\pi) \text{ ft}}{\text{min}} \approx 1036.7$ ft/min

10. $\frac{33.3(5.6) \text{ ft}}{\text{sec}} \approx 186.5$ ft/sec

11. $\frac{10(60) \text{ rev}}{2\pi \text{ hr}} \approx 95.5$ rev/hr

12. $\frac{4400(0.87) \text{ yd}}{45(3) \text{ sec}} \approx 28.4$ yd/sec

13. $\frac{30 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} = 60\pi \approx 188.5$ rad/min

14. $\frac{60 \text{ rev}}{\text{sec}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} = 120\pi \approx 377.0$ rad/sec

15. $\frac{120 \text{ rev}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = 2$ rev/min

16. $\frac{150 \text{ rev}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} = 540,000$ rev/hr

17. $\frac{180 \text{ rev}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \approx$
4,071,504.1 rad/hr

18. $\frac{3000 \text{ rad}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \approx$
0.1 rev/sec

19. $\frac{30 \text{ mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \approx$
44 ft/sec

20. $\frac{150 \text{ ft}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \approx$
102.3 mi/hr

21. $\frac{500 \text{ rev}}{\text{sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 1000\pi \approx 3141.6$ rad/sec

22. $\frac{300 \text{ rev}}{\text{sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 600\pi \approx 1885.0$ rad/sec

23. $\frac{433.2 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \approx$
45.4 rad/sec

24. $\frac{11,000 \text{ rev}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \approx$
19.2 rad/sec

25. $\frac{50,000 \text{ rev}}{\text{day}} \cdot \frac{1 \text{ day}}{3600(24) \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \approx$
3.6 rad/sec

26. $\dfrac{999,000 \text{ rev}}{\text{mo}} \cdot \dfrac{1 \text{ mo}}{3600(24)(30) \text{ sec}} \cdot \dfrac{2\pi \text{ rad}}{1 \text{ rev}} \approx$
2.4 rad/sec assuming 30 days in a month

27. Convert rev/min to rad/hr:
$$\frac{3450 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}.$$
Since arc length is $s = r\alpha$, linear velocity is
$$v = \left((3 \text{ in.}) \cdot \frac{1 \text{ mi}}{5280(12) \text{ in.}}\right) \cdot \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
≈ 61.6 mph.

28. Convert rev/min to rad/hr:
$$\frac{3450 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}.$$
Since arc length is $s = r\alpha$, linear velocity is
$$v = \left((4 \text{ in.}) \cdot \frac{1 \text{ mi}}{5280(12) \text{ in.}}\right) \cdot \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
≈ 82.1 mph.

29. Convert rev/min to rad/hr:
$$\frac{3450 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}.$$
Since arc length is $s = r\alpha$, linear velocity is
$$v = \left((5 \text{ in.}) \cdot \frac{1 \text{ mi}}{5280(12) \text{ in.}}\right) \cdot \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
≈ 102.6 mph.

30. Note,
$$\frac{3450 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
and arc length is $s = r\alpha$.
The linear velocity is
$$v = \left((6 \text{ in.}) \cdot \frac{1 \text{ mi}}{5280(12) \text{ in.}}\right) \cdot \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
≈ 123.2 mph

31. Note,
$$\frac{3450 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
and arc length is $s = r\alpha$.
The linear velocity is
$$v = \left((7 \text{ in.}) \cdot \frac{1 \text{ mi}}{5280(12) \text{ in.}}\right) \cdot \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
≈ 143.7 mph

32. Note,
$$\frac{3450 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
and arc length is $s = r\alpha$.
The linear velocity is
$$v = \left((8 \text{ in.}) \cdot \frac{1 \text{ mi}}{5280(12) \text{ in.}}\right) \cdot \frac{120\pi(3450) \text{ rad}}{1 \text{ hr}}$$
≈ 164.2 mph

33. The angular velocity is
$w = 45\,(2\pi) = 90\pi \approx 282.7$ rad/min.
Sinc arc length $s = r\alpha$, linear velocity is
$v = 6.25(90\pi) \approx 918.9$ in./min.

34. The angular velocity is
$w = \dfrac{(33 + 1/3)\,(2\pi)}{60}$ rad/sec $=$
$\dfrac{10\pi}{9}$ rad/sec ≈ 3.5 rad/sec.
Sinc arc length $s = r\alpha$, linear velocity is
$v = 6$ in. $\left(\dfrac{10\pi}{9}\text{ in./sec}\right) \cdot 60$ sec/min $=$
1256.6 in./min.

35. The radius of the circle is $r = 424/(2\pi)$.
Applying the formula $s = r\alpha$, the linear velocity is
$$v = \left(\frac{424 \text{ m}}{2\pi}\right) \cdot \frac{2\pi}{30 \text{ min.}} \cdot \frac{1 \text{ min.}}{60 \text{ sec}} \cdot \frac{3.28 \text{ ft}}{1 \text{ m}} \approx$$
0.8 ft/sec.

36. The linear velocity is
$$v = (125 \text{ ft}) \cdot \frac{2 \text{ rev}}{\text{hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \approx$$
0.4 ft/sec.

37. The radius of the blade is
$$10 \text{ in.} = 10 \cdot \frac{1}{12 \cdot 5280} \approx 0.0001578 \text{ miles.}$$
Since the angle rotated in one hour is
$$\alpha = 2800 \cdot 2\pi \cdot 60 = 336,000\pi,$$
the linear velocity is
$v = r\alpha \approx (0.0001578) \cdot 336,000\pi \approx 166.6$ mph.

38. The radius of the bit is $r = 0.5$ in. $=$ $0.5 \cdot \dfrac{1}{12 \cdot 5280}$ mi. ≈ 0.0000079 mi.
Since the angle made in one hour is $\alpha = 45,000 \cdot 2\pi \cdot 60 = 5,400,000\pi$, we get
$v = r\alpha \approx (0.0000079) \cdot (5,400,000\pi)$
≈ 133.9 mph.

39. In 1 hr, the saw rotates through an angle $\alpha = 3450(60) \cdot 2\pi$. Note $s = r\alpha$ is a formula for the arc length. Then the difference in the linear velocity is
$\dfrac{6}{12}\alpha - \dfrac{5}{12}\alpha$ or $\dfrac{3450(60) \cdot 2\pi}{(3600)12} \approx 30.1$ ft/sec.

40. The radius of the tire in miles is
$r = \dfrac{13}{12(5280)} \approx 0.00020517677$ mi.
So $w = \dfrac{\alpha}{t} = \dfrac{\alpha}{1} = \alpha = \dfrac{s}{r} \approx$
$\dfrac{55}{0.00020517677} \approx 268,061.5$ radians/hr.

41. The angular velocity of any point on the surface of the earth is $w = \dfrac{\pi}{12}$ rad/hr.
A point 1 mile from the North Pole is approximately 1 mile from the axis of the earth. The linear velocity of that point is $v = w \cdot r = \dfrac{\pi}{12} \cdot 1 \approx 0.3$ mph.

42. The angular velocity is $w = \dfrac{\pi}{12}$ rad/hr
or about 0.26 rad/hr. Let r be the distance between Peshtigo and the point on the x-axis closest to Peshtigo. Since Peshtigo is on the 45th parallel, that point on the x-axis is r miles from the center of the earth.
By the Pythagorean Theorem,
$r^2 + r^2 = 3950^2$ or $r \approx 2,793.0718$ miles.
The linear velocity is
$v = w \cdot r \approx \dfrac{\pi}{12} \cdot 2,793.0718 \approx 731.2$ mph.

43. Since $\alpha = 15° \approx 0.261799$ radians, the linear velocity is $v = \dfrac{r\alpha}{t} = \dfrac{r\alpha}{1} = r\alpha \approx$
$6.5(3950) \cdot 0.261799 \approx 6721.7$ mph.

44. **a)** The linear velocity is $v = r\alpha =$ (10 meters)$(3(2\pi)$ rad/min$) =$ 60π meters/minute.

b) Angular velocity is $w = 3(2\pi) =$ 6π rad/min.

c) The arc length between two adjacent seats is $s = r\alpha = 10 \cdot \dfrac{2\pi}{8} = \dfrac{5\pi}{2}$ meters.

45. The linear velocity is given by $v = r\omega$.
If $v = 2$ ft/sec, then the angular velocity is $\omega = \frac{2}{r}$ radians/sec.
As the radius increases, the angular velocity decreases.

46. Angular velocity is $\omega = \dfrac{1 \text{ rev}}{2 \text{ sec}} = \dfrac{2\pi \text{ rad}}{2 \text{ sec}} =$ π rad/sec.

47. Since the velocity at point A is 10 ft/sec, the linear velocity at B and C are both 10 ft/sec. The angular velocity at C is

$$\omega = \frac{v}{r} = \frac{10 \text{ ft/sec}}{5/12 \text{ ft}} = 24 \text{ rad/sec}$$

and the angular velocity at B is

$$\omega = \frac{v}{r} = \frac{10 \text{ ft/sec}}{3/12 \text{ ft}} = 40 \text{ rad/sec.}$$

48. The linear velocity at A, B, and C are all equal to $v = \omega r = \dfrac{1000(2\pi)}{60}\dfrac{3}{12}$ ft/sec
$= \dfrac{25\pi}{3}$ ft/sec ≈ 26.2 ft/sec.
Note, $5\pi/6$ ft is the circumference of a circle with radius 5 in. Then
$\dfrac{25\pi \text{ ft}}{3 \text{ sec}} \cdot \dfrac{60 \text{ sec}}{1 \text{ min}} \cdot \dfrac{1 \text{ rev}}{5\pi/6 \text{ ft}} = 600$ rev/min.

49. Since the chain ring which has 52 teeth turns at the rate of 1 rev/sec, the cog with 26 teeth will turn at the rate of 2 rev/sec. Thus, the linear velocity of the bicycle with 13.5-in.-radius wheels is
$\dfrac{2\text{rev}}{\text{sec}} \cdot \dfrac{2\pi(13.5)\text{in.}}{\text{rev}} \cdot \dfrac{1\text{mile}}{63,360\text{in.}} \cdot \dfrac{3600\text{sec}}{1\text{hr}} \approx 9.6$ mph.

50. If a chain ring with 44 teeth turns at the rate of 1 rev/sec, then a cog with x teeth will turn at the rate of $\dfrac{44}{x}$ rev/sec. Thus, the linear velocity of a bicycle with 13-in.-radius wheels is

$$\nu(x) = \frac{44\text{rev}}{x\text{sec}} \cdot \frac{2\pi(13)\text{in.}}{\text{rev}} \cdot \frac{1\text{mile}}{63,360\text{in.}} \cdot \frac{3600\text{sec}}{1\text{hr}}.$$

The maximum linear velocity among $x = 13, 15, 17, 20, 23$ is

$$v(13) \approx 15.7 \text{ mph}.$$

53. **a)** $12°65'96'' = 12°66'36'' = 13°6'36''$
b) $27°77'68'' - 9°19'29'' = 18°58'39''$

54. $5\pi/4$

55. $210°$

56. αr

57. $A = \pi r^2$

58. $r = \sqrt{A/\pi}$, solve for r in Exercise 57.

59. **a)** Let t be a fraction of an hour, i.e., $0 \le t \le 1$. If the angle between the hour and minute hands is 120°, then

$$\begin{aligned} 360t - 30t &= 120 \\ 330t &= 120 \\ t &= \frac{12}{33}\text{hr} \\ t &\approx 21 \text{ min}, 49.1 \text{ sec.} \end{aligned}$$

Thus, the hour and minute hands will be 120° apart when the time is 12:21:49.1. Moreover, this is the only time between 12 noon and 1 pm that the angle is 120°.

b) We measure angles clockwise from the 12 0'clock position.

At 12:21:49.1, the hour hand is pointing to the 10.9°-angle, the minute hand is pointing to the 130.9°-angle, and the second hand is pointing to the 294.5°-angle. Thus, the three hands of the clock cannot divide the face of the clock into thirds.

c) No, as discussed in part b).

60. Let x, y, z be the dimensions of the rectangular box. The surface area satisfies

$$2(xy + yz + xz) = 225.$$

From the identity

$$x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + xz)$$

and the given $x + y + z = 25$, we obtain

$$x^2 + y^2 + z^2 = 625 - 225 = 400.$$

The distance from A to B is

$$\sqrt{x^2 + y^2 + z^2} = 20 \text{ inches}.$$

1.3 Pop Quiz

1. 300 rad/sec · 60 sec/min = 18,000 rad/min

2. $200 \ \dfrac{\text{miles}}{\text{hour}} \cdot \left(5280 \ \dfrac{\text{feet}}{\text{mile}}\right)\left(\dfrac{1}{3600} \ \dfrac{\text{hour}}{\text{sec}}\right) \approx$ 293.3 ft/sec

3. $240(2\pi) \ \dfrac{\text{radians}}{\text{minute}} = \dfrac{240(2\pi)}{60} \ \dfrac{\text{radians}}{\text{second}} = 8\pi \ \dfrac{\text{radians}}{\text{second}}$

4. The angular velocity is

$$\omega = 200(2\pi)(60) \ \frac{\text{radians}}{\text{hour}}$$

The linear velocity is

$$v = \frac{60}{5280} \text{ feet} \cdot w = 856.8 \text{ mph}$$

5. $2\pi \ \dfrac{\text{radians}}{\text{day}} = \dfrac{2\pi}{24} \ \dfrac{\text{radians}}{\text{hour}} = \dfrac{\pi}{12} \ \dfrac{\text{radians}}{\text{hour}}$

1.3 Linking Concepts

a) Since the circumference is 20π and the ferris wheel makes three revolutions in one minute, the linear velocity is 60π $(= 3 \cdot 20\pi)$ m/min.

b) Since the angle made in one revolution is 2π, the angular velocity is 6π $(= 3 \cdot 2\pi)$ rad/min.

c) The length of the arc between adjacent seats (represented as points) is the circumference divided by 8, i.e.,

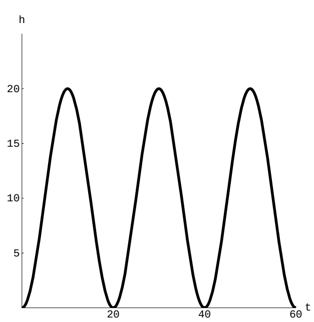
$\frac{20\pi}{8}$ meters or 2.5π meters.

d) Note, the ferris wheel makes one revolution in 20 seconds. For the following times, given in seconds, one finds the following heights in meters

t	0	2.5	5	7.5	10	12.5
$h(t)$	0	2.9	10	17.1	20	17.1

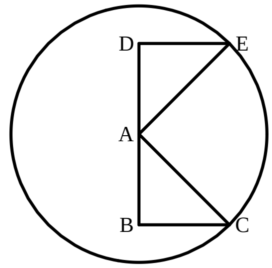
t	15	17.5	20
$h(t)$	10	2.9	0

e) By connecting the points $(t, h(t))$ provided in part d) and by repeating this pattern, one obtains a sketch of a graph of $h(t)$ versus t for $0 \le t \le 60$.



f) There are six solutions to $h(t) = 18$ in the interval $[0, 60]$ since the ferris wheel makes one revolution every 20 seconds and in each revolution one attains the height of 18 meters twice (one on the way up and the other on the way down).

g) In the picture below, the ferris wheel is represented by the circle with radius 10 meters. Triangles $\triangle ABC$ and $\triangle ADE$ are isosceles triangles. By the Pythagorean Theorem, one finds $AD = DE = AB = BC = 5\sqrt{2}$.



After 2.5 seconds, one's position on the ferris wheel will be at C; and after 7.5 seconds the location will be at E. Thus, $h(2.5) = 10 - 5\sqrt{2}$ and $h(7.5) = 10 + 5\sqrt{2}$.

For Thought

1. True, since $(0, 1)$ is on the positive y-axis and $\sin 90° = \frac{y}{r} = \frac{1}{1} = 1$.

2. True, since $(1, 0)$ is on the positive x-axis and $\sin 0° = \frac{x}{r} = \frac{0}{1} = 0$.

3. True, since $(1, 1)$ is on the terminal side of 45° and $r = \sqrt{2}$, and so

$$\cos 45° = \frac{x}{r} = \frac{1}{\sqrt{2}}.$$

4. True, since $(1, 1)$ is on the terminal side of 45° we get

$$\tan 45° = \frac{y}{x} = \frac{1}{1} = 1.$$

5. True, since $(\sqrt{3}, 1)$ is on the terminal side of 30° and $r = 2$ we find

$$\sin 30° = \frac{y}{r} = \frac{1}{2}.$$

6. True, since $(0, 1)$ is on the positive y-axis we get $\cos(\pi/2) = \frac{x}{r} = \frac{0}{1} = 1$.

7. True, since the terminal sides of 390° and 30° are the same.

8. False, since $\sin(-\pi/3) = -\frac{\sqrt{3}}{2}$ and $\sin(\pi/3) = \frac{\sqrt{3}}{2}$.

9. True, since $\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{1/5} = 5$.

10. True, since $\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{1}{2/3} = 1.5$.

1.4 Exercises

1. $\sin\alpha$, $\cos\alpha$

2. trigonometric ratios

3. reciprocals

4. quadrant I

5. Note $r = \sqrt{1^2+2^2} = \sqrt{5}$. Then

$\sin\alpha = \dfrac{y}{r} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$,

$\cos\alpha = \dfrac{x}{r} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$,

$\tan\alpha = \dfrac{y}{x} = \dfrac{2}{1} = 2$, $\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{\sqrt{5}}{2}$,

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{\sqrt{5}}{1} = \sqrt{5}$, and

$\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{2}$.

6. Note $r = \sqrt{(-1)^2+(-2)^2} = \sqrt{5}$. Then

$\sin\alpha = \dfrac{y}{r} = \dfrac{-2}{\sqrt{5}} = -\dfrac{2\sqrt{5}}{5}$,

$\cos\alpha = \dfrac{x}{r} = \dfrac{-1}{\sqrt{5}} = -\dfrac{\sqrt{5}}{5}$,

$\tan\alpha = \dfrac{y}{x} = \dfrac{-2}{-1} = 2$, $\csc\alpha = \dfrac{1}{\sin\alpha} = -\dfrac{\sqrt{5}}{2}$,

$\sec\alpha = \dfrac{1}{\cos\alpha} = -\dfrac{\sqrt{5}}{1} = -\sqrt{5}$, and

$\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{2}$.

7. Note $r = \sqrt{0^2+1^2} = 1$. Then

$\sin\alpha = \dfrac{y}{r} = \dfrac{1}{1} = 1$, $\cos\alpha = \dfrac{x}{r} = \dfrac{0}{1} = 0$,

$\tan\alpha = \dfrac{y}{x} = \dfrac{1}{0} =$ undefined,

$\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{1} = 1$,

$\sec\alpha = \dfrac{1}{\cos\alpha} = -\dfrac{1}{0} =$ undefined, and

$\cot\alpha = \dfrac{x}{y} = \dfrac{0}{1} = 0$.

8. Note $r = \sqrt{1^2+0^2} = 1$. Then

$\sin\alpha = \dfrac{y}{r} = \dfrac{0}{1} = 0$, $\cos\alpha = \dfrac{x}{r} = \dfrac{1}{1} = 1$,

$\tan\alpha = \dfrac{y}{x} = \dfrac{0}{1} = 0$,

$\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{0} =$ undefined,

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{1}{1} = 1$, and

$\cot\alpha = \dfrac{x}{y} = \dfrac{1}{0} =$ undefined.

9. Note $r = \sqrt{1^2+1^2} = \sqrt{2}$. Then

$\sin\alpha = \dfrac{y}{r} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$,

$\cos\alpha = \dfrac{x}{r} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$,

$\tan\alpha = \dfrac{y}{x} = \dfrac{1}{1} = 1$,

$\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2}$,

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2}$, and

$\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{1} = 1$.

10. Note $r = \sqrt{1^2+(-1)^2} = \sqrt{2}$. Then

$\sin\alpha = \dfrac{y}{r} = \dfrac{-1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$,

$\cos\alpha = \dfrac{x}{r} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$,

$\tan\alpha = \dfrac{y}{x} = \dfrac{-1}{1} = -1$,

$\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{-1/\sqrt{2}} = -\sqrt{2}$,

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2}$, and

$\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{-1} = -1$.

11. Note $r = \sqrt{(-2)^2+2^2} = 2\sqrt{2}$. Then

$\sin\alpha = \dfrac{y}{r} = \dfrac{2}{2\sqrt{2}} = \dfrac{\sqrt{2}}{2}$,

$\cos\alpha = \dfrac{x}{r} = \dfrac{-2}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$,

$$\tan\alpha = \frac{y}{x} = \frac{2}{-2} = -1,$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{1}{2/(2\sqrt{2})} = \sqrt{2},$$

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{-2/(2\sqrt{2})} = -\sqrt{2}, \text{ and}$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{-1} = -1.$$

12. Note $r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$. Then

$$\sin\alpha = \frac{y}{r} = \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2},$$

$$\cos\alpha = \frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2},$$

$$\tan\alpha = \frac{y}{x} = \frac{-2}{2} = -1,$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{1}{-2/(2\sqrt{2})} = -\sqrt{2},$$

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{2/(2\sqrt{2})} = \sqrt{2}, \text{ and}$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{-1} = -1.$$

13. Note $r = \sqrt{(-4)^2 + (-6)^2} = 2\sqrt{13}$. Then

$$\sin\alpha = \frac{y}{r} = \frac{-6}{2\sqrt{13}} = -\frac{3\sqrt{13}}{13},$$

$$\cos\alpha = \frac{x}{r} = \frac{-4}{2\sqrt{13}} = \frac{-2\sqrt{13}}{13},$$

$$\tan\alpha = \frac{y}{x} = \frac{-6}{-4} = \frac{3}{2},$$

$$\csc\alpha = \frac{1}{\sin\alpha} = -\frac{13}{3\sqrt{13}} = -\frac{\sqrt{13}}{3},$$

$$\sec\alpha = \frac{1}{\cos\alpha} = -\frac{13}{2\sqrt{13}} = -\frac{\sqrt{13}}{2}, \text{ and}$$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{2}{3}.$$

14. Note $r = \sqrt{(-5)^2 + 10^2} = 5\sqrt{5}$. Then

$$\sin\alpha = \frac{y}{r} = \frac{10}{5\sqrt{5}} = \frac{2\sqrt{5}}{5},$$

$$\cos\alpha = \frac{x}{r} = \frac{-5}{5\sqrt{5}} = \frac{-\sqrt{5}}{5},$$

$$\tan\alpha = \frac{y}{x} = \frac{10}{-5} = -2,$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{5\sqrt{5}}{10} = \frac{\sqrt{5}}{2},$$

$$\sec\alpha = \frac{1}{\cos\alpha} = -\frac{5\sqrt{5}}{5} = -\sqrt{5}, \text{ and}$$

$$\cot\alpha = \frac{1}{\tan\alpha} = -\frac{1}{2}.$$

15. 0 **16.** -1 **17.** -1 **18.** 0

19. 0 **20.** Undefined

21. Undefined **22.** Undefined

23. -1 **24.** 1 **25.** -1 **26.** 1

27. $\frac{\sqrt{2}}{2}$ **28.** $\frac{\sqrt{2}}{2}$ **29.** $\frac{\sqrt{2}}{2}$ **30.** $-\frac{\sqrt{2}}{2}$

31. -1 **32.** -1 **33.** $\sqrt{2}$ **34.** $-\sqrt{2}$

35. $\frac{1}{2}$ **36.** $\frac{\sqrt{3}}{2}$ **37.** $\frac{1}{2}$ **38.** $-\frac{1}{2}$

39. $-\frac{\sqrt{3}}{3}$ **40.** $-\sqrt{3}$ **41.** -2 **42.** $-\frac{2\sqrt{3}}{3}$

43. 2 **44.** $-\frac{2\sqrt{3}}{3}$ **45.** $\sqrt{3}$ **46.** $-\frac{\sqrt{3}}{3}$

47. $\dfrac{\cos(\pi/3)}{\sin(\pi/3)} = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{\sqrt{3}}{3}$

48. $\dfrac{\sin(-5\pi/6)}{\cos(-5\pi/6)} = \dfrac{-1/2}{-\sqrt{3}/2} = \dfrac{\sqrt{3}}{3}$

49. $\dfrac{\sin(7\pi/4)}{\cos(7\pi/4)} = \dfrac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

50. $\dfrac{\sin(-3\pi/4)}{\cos(-3\pi/4)} = \dfrac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$

51. $\sin\left(\dfrac{\pi}{3} + \dfrac{\pi}{6}\right) = \sin\left(\dfrac{\pi}{2}\right) = 1$

52. $\cos\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

53. $\dfrac{1-\cos(5\pi/6)}{\sin(5\pi/6)} = \dfrac{1-(-\sqrt{3}/2)}{1/2} \cdot \dfrac{2}{2} = 2 + \sqrt{3}$

54. $\dfrac{\sin(5\pi/6)}{1+\cos(5\pi/6)} = \dfrac{1/2}{1+(-\sqrt{3}/2)} \cdot \dfrac{2}{2} =$

$$\frac{1}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

55. $\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} = \sqrt{2}$

56. $\sin(\pi/6) + \cos(\pi/3) = \dfrac{1}{2} + \dfrac{1}{2} = 1$

57. $\cos(45°)\cos(60°) - \sin(45°)\sin(60°) =$
$\dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{2} - \sqrt{6}}{4}$

58. $\sin(30°)\cos(135°) + \cos(30°)\sin(135°) =$
$\dfrac{1}{2} \cdot \dfrac{-\sqrt{2}}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6} - \sqrt{2}}{4}$

59. $\dfrac{1 - \cos(\pi/3)}{2} = \dfrac{1 - 1/2}{2} = \dfrac{1}{4}.$

60. $\dfrac{1 + \cos(\pi/4)}{2} = \dfrac{1 + \sqrt{2}/2}{2} = \dfrac{2 + \sqrt{2}}{4}.$

61. $2\cos(210°) = 2 \cdot \dfrac{-\sqrt{3}}{2} = -\sqrt{3}.$

62. $\cos(-270°) - \sin(-270°) = 0 - 1 = -1$

63. 0.6820 **64.** 0.8192

65. -0.6366 **66.** 0.9617

67. 0.0105 **68.** -114.5887

69. $\dfrac{1}{\sin(23°48')} \approx 2.4780$

70. $\dfrac{1}{\sin(49°13')} \approx 1.3207$

71. $\dfrac{1}{\cos(-48°3'12'')} \approx 1.4960$

72. $\dfrac{1}{\cos(-9°4'7'')} \approx 1.0127$

73. $\dfrac{1}{\tan(\pi/9)} \approx 2.7475$

74. $\dfrac{1}{\tan(\pi/10)} \approx 3.0777$

75. 0.8578 **76.** 1.1626

77. 0.2679 **78.** -0.2286

79. 0.9894 **80.** -0.2794

81. 2.9992 **82.** 0.9726

83. $\sin(2 \cdot \pi/4) = \sin(\pi/2) = 1$

84. $\sin(\pi/4) = \dfrac{\sqrt{2}}{2}$

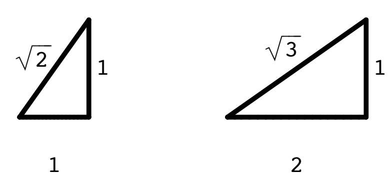
85. $\cos(2 \cdot \pi/6) = \cos(\pi/3) = \dfrac{1}{2}$

86. $\cos(2\pi/3) = -\dfrac{1}{2}$

87. $\sin((3\pi/2)/2) = \sin(3\pi/4) = \dfrac{\sqrt{2}}{2}$

88. $\sin(\pi/3) = \dfrac{\sqrt{3}}{2}$

89.



90. **a)** $(-1, \sqrt{3})$, $r = 2$
b) $(-1, 1)$, $r = \sqrt{2}$
c) $(-\sqrt{3}, -1)$, $r = 2$
d) $(\sqrt{3}, -1)$, $r = 2$
e) $(-1, -1)$, $r = \sqrt{2}$
f) $(-1, -1)$, $r = \sqrt{2}$

91. $\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{3/4} = \dfrac{4}{3}$

92. $\dfrac{1}{1/70} = 70$

93. $\cos\alpha = \dfrac{1}{\sec\alpha} = \dfrac{1}{10/3} = \dfrac{3}{10}$

94. Undefined, since $\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{0}$

95. **a)** II, since $y > 0$ and $x < 0$ in Quadrant II
b) IV, since $y < 0$ and $x > 0$ in Quadrant IV
c) III, since $y/x > 0$ and $x < 0$ in Quadrant III
d) II, since $y/x < 0$ and $y > 0$ in Quadrant II

96. **a)** II, since $2\alpha = 132°$
b) III, since $2\alpha = 4\pi/3$

c) IV, since $2\alpha = 300°$

d) I, since $2\alpha = -\dfrac{5\pi}{3}$

97. If $R = 3$, $r = 14$, and $\theta = 60°$, then

$$\begin{aligned} T &= \frac{14\cos 60° - 3}{\sin 60°} \\ &= \frac{14(1/2) - 3}{\sqrt{3}/2} \\ &\approx 4.6 \text{ inches} \end{aligned}$$

98. $F = 10\sin 39° \cos 39° \approx 4.9$

99. $180° - 2\,(9°38'52'') = 180° - (19°17'44'') =$ $179°59'60'' - 19°17'44'' = 160°42'16''$

100. degrees, 180

101. $\dfrac{13\pi}{12} \cdot \dfrac{180°}{\pi} = 195°$

102. The area is $A = \theta\dfrac{r^2}{2} = \dfrac{\pi}{8} \cdot \dfrac{8^2}{2} \approx 12.6 \text{ m}^2$

103. The radius is 15 inches. The linear velocity is

$$v = \omega r = 2000(2\pi) \cdot 15 \text{ inches/minute}$$

Equivalently,

$$v = 2000(2\pi) \cdot 15\frac{60}{12(5280)} \approx 178.5 \text{ mph}$$

104. $\omega = 200(2\pi)$ radians/sec = $200(2\pi)(60) \approx 75{,}398.2$ radians/min

105. Let P stand for Porsche, N for Nissan, and C for Chrysler. The fifteen preferences could be the following:

Six preferences: P, N, C (1st, 2nd, 3rd respectively)

Five preferences: C, N, P

Three preferences: N, C, P

One preference: N, P, C

106. Since $-1 \le \cos^3 x \le 1$, we find

$$\sqrt{-12+37} \le \sqrt{12\cos^3 x + 37} \le \sqrt{12+37}$$

Note, $\sqrt{-12+37} = 5$ and $\sqrt{12+37} = 7$. Then the range of $1 + \sqrt{12\cos^3 x + 37}$ is $[6, 8]$.

1.4 Pop Quiz

1. Since $r = \sqrt{4^2 + 3^2} = 5$, we find

$$\sin\alpha = \frac{y}{r} = \frac{3}{5},\ \cos\alpha = \frac{x}{r} = \frac{4}{5},$$
$$\tan\alpha = \frac{y}{x} = \frac{3}{4}.$$

2. 0

3. $-\dfrac{\sqrt{3}}{2}$

4. $-\dfrac{\sqrt{2}}{2}$

5. 0

6. $\sqrt{3}$

7. $-\dfrac{\sqrt{3}}{3}$

8. $\sqrt{2}$

9. $\dfrac{1}{\cos 60°} = 2$

1.4 Linking Concepts

a) One obtains the sequence of numbers,

$$\pi/6 \approx 0.5235987756,$$
$$(\pi/6) - \frac{(\pi/6)^3}{3!} \approx 0.4996741794,$$
$$(\pi/6) - \frac{(\pi/6)^3}{3!} + \frac{(\pi/6)^5}{5!} \approx 0.5000021326,$$
$$(\pi/6) - \frac{(\pi/6)^3}{3!} + \frac{(\pi/6)^5}{5!} - \frac{(\pi/6)^7}{7!} \approx 0.4999999919,$$

and

$$\frac{\pi}{6} - \frac{(\pi/6)^3}{3!} + \frac{(\pi/6)^5}{5!} - \frac{(\pi/6)^7}{7!} + \frac{(\pi/6)^9}{9!} \approx 0.5$$

The last expression (the one with five nonzero terms) is the first expression to give 0.5 .

b) One obtains the sequence of numbers,

$$(13\pi/6) - \frac{(13\pi/6)^3}{3!} \approx -45.756,$$
$$(13\pi/6) - \frac{(13\pi/6)^3}{3!} + \frac{(13\pi/6)^5}{5!} \approx 76.011,$$

and the first expression which gives 0.5 is
$(13\pi/6) - \dfrac{(13\pi/6)^3}{3!} + ... - \dfrac{(13\pi/6)^{33}}{65!} \approx 0.5.$
Thus, the expression with 17 nonzero terms is the first expression which gives 0.5

c) One finds
$1 - \dfrac{(\pi/4)^2}{2} \approx 0.6915748625,$
$1 - \dfrac{(\pi/4)^2}{2} + \dfrac{(\pi/4)^4}{4!} \approx 0.7074292067,$
$1 - \dfrac{(\pi/4)^2}{2} + \dfrac{(\pi/4)^4}{4!} - \dfrac{(\pi/4)^6}{6!} \approx 0.7071032148,$
and the first expression to agree with a calculator's value of $\cos\left(\dfrac{\pi}{4}\right)$ is
$1 - \dfrac{(\pi/4)^2}{2} + ... + \dfrac{(\pi/4)^{12}}{12!} \approx 0.7071067812.$
Thus, the expression with 7 nonzero terms is the first expression which agrees with $\cos(\pi/4)$.

d) One finds
$1 - \dfrac{(9\pi/4)^2}{2} \approx -23.98243614,$
$1 - \dfrac{(9\pi/4)^2}{2} + \dfrac{(9\pi/4)^4}{4!} \approx 80.03791644,$
$1 - \dfrac{(9\pi/4)^2}{2} + \dfrac{(9\pi/4)^4}{4!} - \dfrac{(9\pi/4)^6}{6!} \approx -93.20753794,$
and the first expression to agree with a calculator's value of $\cos\left(\dfrac{9\pi}{4}\right)$ is
$1 - \dfrac{(9\pi/4)^2}{2} + ... - \dfrac{(9\pi/4)^{34}}{34!} \approx 0.7071067812$
Thus, the expression with 18 nonzero terms is the first expression to agree with $\cos(9\pi/4)$.

e) As x gets larger, more terms are needed to find the value of $\sin x$ or $\cos x$.

f) Since $\sin\left(\dfrac{601\pi}{3}\right) = \sin\left(\dfrac{\pi}{3}\right)$, one can find $\sin\left(\dfrac{\pi}{3}\right)$ instead.

For Thought

1. True

2. False, rather $\alpha = 30°$ since $\cos 30° = \dfrac{\sqrt{3}}{2}$.

3. False, since $\sin^{-1}(\sqrt{2}/2) = 45°$.

4. False, since $\cos^{-1}(1/2) = 60°$.

5. False, since $\tan^{-1}(1) = 45°$.

6. False, since $c = \sqrt{2^2 + 4^2} = \sqrt{20}$.

7. True, since $c = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

8. False, since $c = \sqrt{4^2 + 8^2} = \sqrt{80}$.

9. True, since $\alpha + \beta = 90°$.

10. False, otherwise $1 = \sin 90° = \dfrac{\text{hyp}}{\text{adj}}$ and we find hyp = adj, which is impossible. The hypotenuse is longer than each of the legs of a right triangle.

1.5 Exercises

1. right

2. adjacent, hypotenuse

3. inverse sine

4. opposite side, hypotenuse

5. adjacent side, hypotenuse

6. opposite side, adjacent side

7. 45° **8.** 30° **9.** 60° **10.** 90°

11. 60° **12.** 45° **13.** 0° **14.** 90°

15. 83.6° **16.** 23.6°

17. 67.6° **18.** 80.0°

19. 26.1° **20.** 22.6°

21. 29.1° **22.** 53.1°

23. Note, the hypotenuse is hyp $= \sqrt{13}$.

Then $\sin\alpha = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{2}{\sqrt{13}} = \dfrac{2\sqrt{13}}{13}$,

$\cos\alpha = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{3}{\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$,

$\tan\alpha = \dfrac{\text{opp}}{\text{adj}} = \dfrac{2}{3}$,

$\csc\alpha = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{13}}{2}$,

$\sec\alpha = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{13}}{3}$, and

$\cot\alpha = \dfrac{\text{adj}}{\text{opp}} = \dfrac{3}{2}$.

24. Note, the adjacent side is adj $= \sqrt{24} = 2\sqrt{6}$.

Then $\sin\alpha = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1}{5}$

$\cos\alpha = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{2\sqrt{6}}{5}$,

$\tan\alpha = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$,

$\csc\alpha = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{5}{1} = 5$,

$\sec\alpha = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{5}{2\sqrt{6}} = \dfrac{5\sqrt{6}}{12}$, and

$\cot\alpha = \dfrac{\text{adj}}{\text{opp}} = \dfrac{2\sqrt{6}}{1} = 2\sqrt{6}$.

25. Note, the hypotenuse is $4\sqrt{5}$.
Then $\sin(\alpha) = \sqrt{5}/5, \cos(\alpha) = 2\sqrt{5}/5$,
$\tan(\alpha) = 1/2$, $\sin(\beta) = 2\sqrt{5}/5$,
$\cos(\beta) = \sqrt{5}/5$, and $\tan(\beta) = 2$.

26. $\sin(\alpha) = 7\sqrt{58}/58$, $\cos(\alpha) = 3\sqrt{58}/58$,
$\tan(\alpha) = 7/3$, $\sin(\beta) = 3\sqrt{58}/58$,
$\cos(\beta) = 7\sqrt{58}/58$, $\tan(\beta) = 3/7$

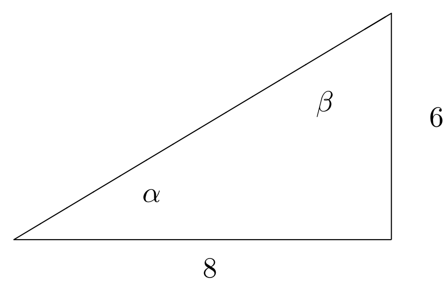
27. Note, the hypotenuse is $2\sqrt{34}$.
$\sin(\alpha) = 3\sqrt{34}/34$, $\cos(\alpha) = 5\sqrt{34}/34$,
$\tan(\alpha) = 3/5$, $\sin(\beta) = 5\sqrt{34}/34$,
$\cos(\beta) = 3\sqrt{34}/34$, and $\tan(\beta) = 5/3$.

28. $\sin(\alpha) = 2\sqrt{13}/13$, $\cos(\alpha) = 3\sqrt{13}/13$,
$\tan(\alpha) = 2/3$, $\sin(\beta) = 3\sqrt{13}/13$,
$\cos(\beta) = 2\sqrt{13}/13$, $\tan(\beta) = 3/2$

29. Note, the side adjacent to β has length 12.
Then $\sin(\alpha) = 4/5$, $\cos(\alpha) = 3/5$,
$\tan(\alpha) = 4/3$, $\sin(\beta) = 3/5$,
$\cos(\beta) = 4/5$, and $\tan(\beta) = 3/4$.

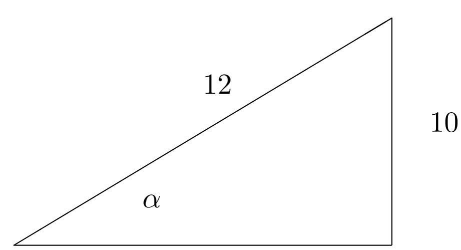
30. Note, the side adjacent to β has length 20.
Then $\sin(\alpha) = 20\sqrt{481}/481$,
$\cos(\alpha) = 9\sqrt{481}/9$, $\tan(\alpha) = 20/9$,
$\sin(\beta) = 9\sqrt{481}/481$,
$\cos(\beta) = 20\sqrt{481}/481$, and $\tan(\beta) = 9/20$.

31. Form the right triangle with $a = 6$, $b = 10$.



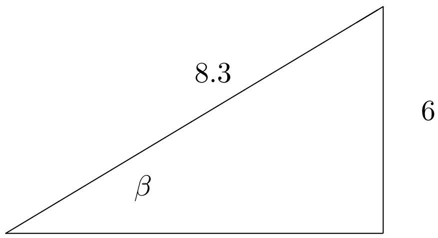
Note: $c = \sqrt{6^2 + 8^2} = 10$, $\tan(\alpha) = 6/8$,
so $\alpha = \tan^{-1}(6/8) \approx 36.9^\circ$ and $\beta \approx 53.1^\circ$.

32. Form the right triangle with $a = 10$, $c = 12$.



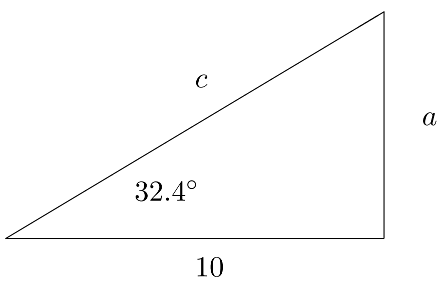
Note: $b = \sqrt{12^2 - 10^2} = \sqrt{44}$, $\sin(\alpha) = 10/12$,
so $\alpha = \sin^{-1}(5/6) \approx 56.4^\circ$ and $\beta \approx 33.6^\circ$.

33. Form the right triangle with $b = 6$, $c = 8.3$.



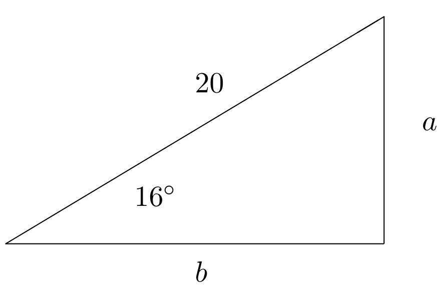
Note: $a = \sqrt{8.3^2 - 6^2} \approx 5.7$, $\sin(\beta) = 6/8.3$, so $\beta = \sin^{-1}(6/8.3) \approx 46.3°$ and $\alpha \approx 43.7°$.

34. Form the right triangle with $\alpha = 32.4°$, $b = 10$.



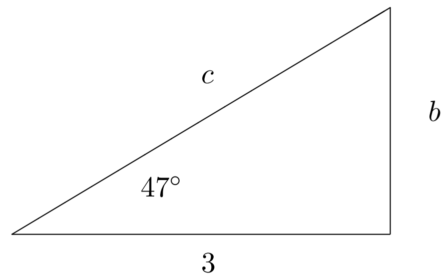
Since $\tan(32.4°) = a/10$ and $\cos(32.4) = 10/c$, $a = 10\tan(32.4°) \approx 6.3$ and $c = 10/\cos(32.4°) \approx 11.8$. Also $\beta = 57.6°$.

35. Form the right triangle with $\alpha = 16°$, $c = 20$.



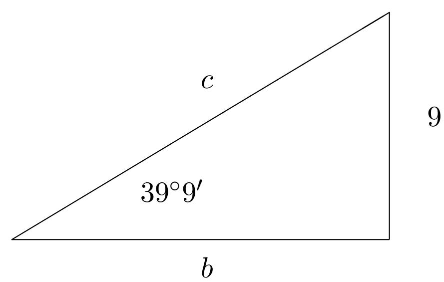
Since $\sin(16°) = a/20$ and $\cos(16°) = b/20$, $a = 20\sin(16°) \approx 5.5$ and $b = 20\cos(16°) \approx 19.2$. Also $\beta = 74°$.

36. Form the right triangle with $\beta = 47°$, $a = 3$.



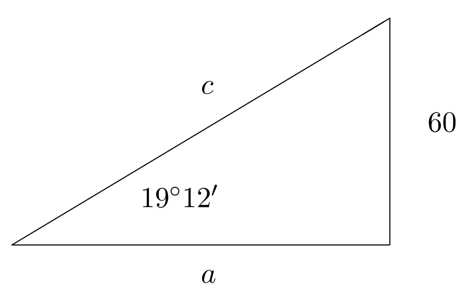
Since $\tan(47°) = b/3$ and $\cos(47°) = 3/c$ then $b = 3 \cdot \tan(47°) \approx 3.2$ and $c = 3/\cos(47°) \approx 4.4$. Also $\alpha = 43°$.

37. Form the right triangle with $\alpha = 39°9'$, $a = 9$.



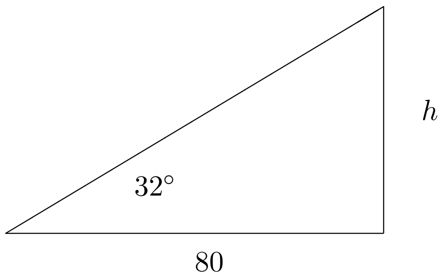
Since $\sin(39°9') = 9/c$ and $\tan(39°9') = 9/b$, then $c = 9/\sin(39°9') \approx 14.3$ and $b = 9/\tan(39°9') \approx 11.1$. Also $\beta = 50°51'$.

38. Form the right triangle with $\beta = 19°12'$, $b = 60$.



Since $\sin(19°12') = 60/c$ and $\tan(19°12') = 60/a$, then $c = 60/\sin(19°12') \approx 182.4$ and $a = 60/\tan(19°12') \approx 172.3$. Also $\alpha = 70°48'$.

39. Let h be the height of the buliding.



Since $\tan(32°) = h/80$, we obtain

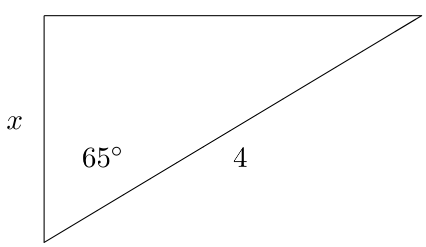
$$h = 80 \cdot \tan(32°) \approx 50 \text{ ft.}$$

40. If h is the height of the tree, then

$$\tan 75° = \frac{h}{80}.$$

Then $h = 80 \tan 75° \approx 299$ ft.

41. Let x be the distance between Muriel and the road at the time she encountered the swamp.



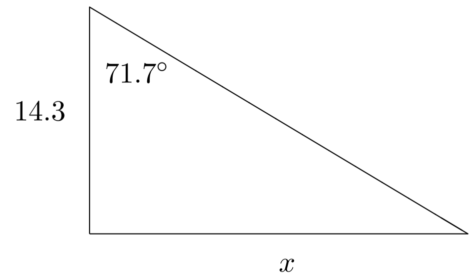
Since $\cos(65°) = x/4$, we obtain $x = 4 \cdot \cos(65°) \approx 1.7$ miles.

42. Let h be the height of the antenna. Then

$$\sin 62° = \frac{h}{100}.$$

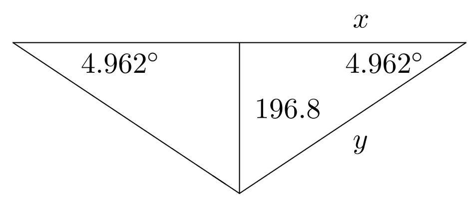
Thus, $h = 100 \sin 62° \approx 88$ ft.

43. Let x be the distance between the car and a point on the highway directly below the overpass.



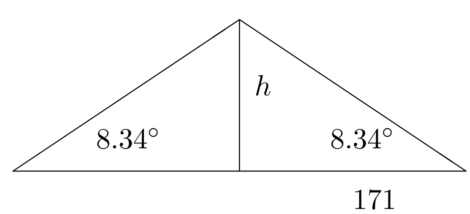
Since $\tan(71.7°) = x/14.3$, we obtain $x = 14.3 \cdot \tan(71.7°) \approx 43.2$ meters.

44. Let x and y be as in the picture below.



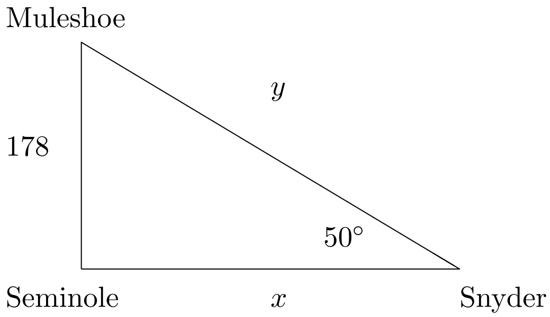
Since $\tan(4.962°) = 196.8/x$, the distance on the surface between the entrances is $2x = 2 \cdot \dfrac{196.8}{\tan(4.962°)} \approx 4533$ ft.
Similarly, since $\sin(4.962°) = 196.8/y$, we get that the length of the tunnel is $2y = 2 \cdot \dfrac{196.8}{\sin(4.962°)} \approx 4551$ ft.

45. Let h be the height as in the picture below.



Since $\tan(8.34°) = h/171$, we obtain $h = 171 \cdot \tan(8.34°) \approx 25.1$ ft.

46.



Note that

$$x = \frac{178}{\tan 50^\circ} \approx 149.36,$$

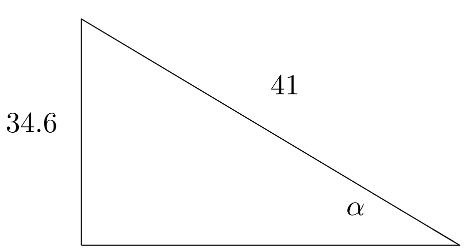
$$y = \frac{178}{\sin 50^\circ} \approx 232.36$$

and

$$178 + 149.36 - 232.36 = 95.$$

Harry drove 95 more miles than Harriet. The towns are in Texas.

47. Let α be the angle the guy wire makes with the ground.

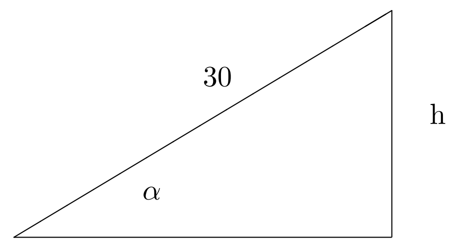


From the Pythagorean Theorem, the distance of the point to the base of the antenna is

$$\sqrt{41^2 - 34.6^2} \approx 22 \text{ meters.}$$

Also, $\alpha = \sin^{-1}(34.6/41) \approx 57.6^\circ$.

48. Let α be an angle of elevation of the ladder.



Since $\sin(\alpha) = h/30$, the minimum and maximum heights are $30\sin(55^\circ) \approx 24.6$ ft and $30\sin(70^\circ) \approx 28.2$ ft, respectively.

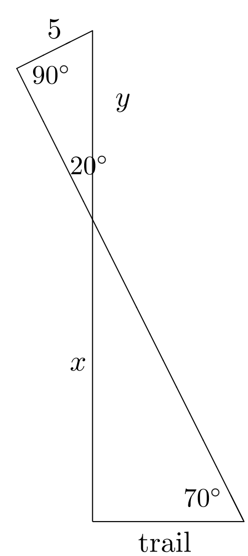
49. Consider a right triangle with a height of 13.3 in., and the base is the length of the trail. Let 64° be the angle opposite the height. If x is the length of the trail, then

$$\tan 64^\circ = \frac{13.3}{x}.$$

Thus, $x \approx 6.5$ in.

50. The radius of the axle is 350 mm or 35 cm.

In the figure below, $x + y = 35$ cm.



We find $y = \frac{5}{\sin 20^\circ}$ and $x = 35 - y \approx 20.38$ cm. Then the trail is given by

$$\text{trail} = \frac{x}{\tan 70^\circ} \approx 7.418 \text{ cm} \approx 2.9 \text{ in.}$$

51. Choose the ball in the left corner of the triangle. From the center A of the ball draw the perpendicular line segment to the point B below and on the outside perimeter of the rack.

Since the diameter of the ball is 2.25 in., and the sides of the triangular rack is 0.25 in., we obtain $AB = 2.25/2 + 0.25 = 1.375$ in.

Let x be the length from the left vertex of the rack to B. By right triangle trigonometry,

$$\begin{aligned} \tan 30^\circ &= \frac{1.375}{x} \\ x &= \frac{1.375}{\tan 30^\circ} \approx 2.38. \end{aligned}$$

Then the length of the horizontal outside perimeter of the rack is $2.25 \times 4 + 2x$. The total length of the outside perimeter is

$$3\,(2.25(4) + 2x) \approx 41.3 \text{ in.}$$

52. Choose the ball in the left corner of the triangle. From the center A of the ball draw the perpendicular line segment to the point B below and on the outside perimeter of the rack. Also, draw a line segment from A to point C that is perpendicular to the left side, outside perimeter of the rack. The length of the circular arc joining B to C along the ball is

$$\widehat{BC} = \frac{2\pi}{3} \times 1.375 \approx 2.88$$

Then the total length of the outside perimeter is

$$2.25(12) + 3\widehat{BC} \approx 35.6 \text{ in.}$$

53. Draw a line segment from the center A of the pentagon to the midpoint C of one of the sides of the regular pentagon. This line segment should be perpendicular to the side. Consider a right triangle with vertices at A, C, and an adjacent vertex B of the hexagon near C. The angle at A of the right triangle is 36°, and the hypotenuse r is the distance from the center A to vertex B. Using right triangle trigonometry, we obtain

$$r = \frac{1}{\sin 36^\circ} \approx 1.701.$$

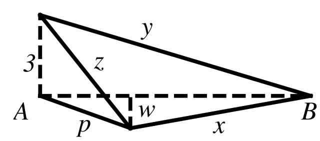
Furthermore, the side adjacent to 36° is $h - r$. Applying right triangle trigonometry, we have

$$\cos 36^\circ = \frac{h-r}{r}$$

Solving for h, we find

$$h = r(1 + \cos 36^\circ) \approx 3.08 \text{ m}$$

54. Let x, y, z be the dimensions of the triangle. Let A and B be the endpoints of the projection of side y to the horizontal, as shown below. Let w be the perpendicular distance to the horizontal radius $r = AB$.



Using the 36 in. width, we obtain $x + w = 18$. The angle at B is 18°, and

$$\sin 18^\circ = \frac{w}{x}.$$

Thus,

$$x = \frac{18}{1 + \sin 18^\circ} \approx 13.75 \text{ in.}$$

Notice, $w \approx 4.24922$.

The angle between the tips of the stars is 72°. Also, 36 in. is the length of a chord extended by a central angle of 144°. Then the radius r of the star satisfies

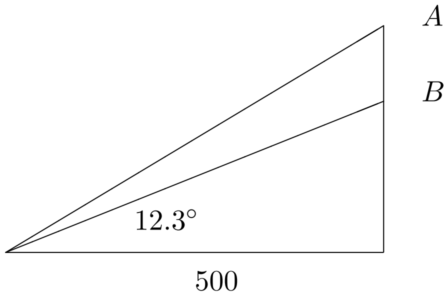
$$\begin{aligned} \cos 144^\circ &= 1 - \frac{36^2}{2r^2} \\ r^2 &= \frac{36^2}{2(1 - \cos 144^\circ)}. \end{aligned}$$

By the Pythagorean theorem, $r^2 + 3^2 = y^2$. Then $y \approx 19.16$ in.

Again, applying the Pythagorean theorem, we have $(r - x\cos 18^\circ)^2 + w^2 = p^2$ and $p^2 + 3^2 = z^2$. Thus, $z \approx 7.83$ in.

Hence, the sides of the triangle are 13.75 in., 19.16 in., and 7.83 in.

55. Note, 1.75 sec. = 1.75/3600 hour.



The distance in miles between A and B is

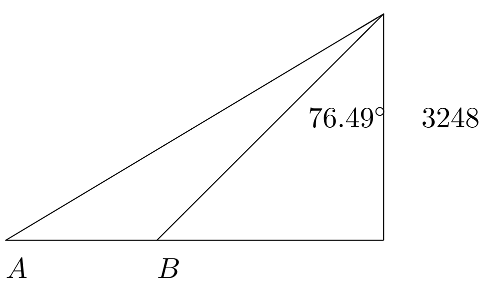
$$\frac{500\,(\tan(15.4^\circ) - \tan(12.3^\circ))}{5280} \approx 0.0054366$$

The speed is

$$\frac{0.0054366}{(1.75/3600)} \approx 11.2 \text{ mph}$$

and the car is not speeding.

56. Note, 5 min. = 1/12 hour.

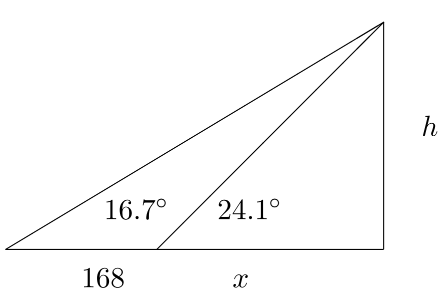


The distance in miles between A and B is

$$\frac{3248\,(\tan(78.66^\circ) - \tan(76.49^\circ))}{5280} \approx 0.50706$$

The speed is $\dfrac{0.50706}{1/12} \approx 6.1$ mph.

57. Let h be the height.



Note, $\tan 24.1^\circ = \dfrac{h}{x}$ and $\tan 16.7^\circ = \dfrac{h}{168+x}$.

Solve for h in the second equation and substitute $x = \dfrac{h}{\tan 24.1^\circ}$.

$$h = \tan(16.7^\circ) \cdot \left(168 + \frac{h}{\tan 24.1^\circ}\right)$$

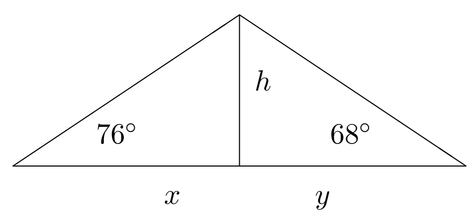
$$h - \frac{h\tan(16.7^\circ)}{\tan 24.1^\circ} = \tan(16.7^\circ) \cdot 168$$

$$h = \frac{168 \cdot \tan(16.7^\circ)}{1 - \tan(16.7^\circ)/\tan(24.1^\circ)}$$

$$h \approx 153.1 \text{meters}$$

The height is 153.1 meters.

58. Let h be the height of the balloon.

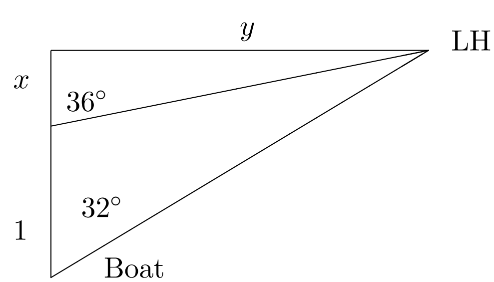


Note that $1.2 = x + y = \dfrac{h}{\tan(76^\circ)} + \dfrac{h}{\tan(68^\circ)}$.

If we factor h, then we obtain

$$h = \frac{1.2}{1/\tan(76^\circ) + 1/\tan(68^\circ)} \approx 1.8 \text{ mile.}$$

59. Let y be the closest distance the boat can come to the lighthouse LH.



Since $\tan(36^\circ) = y/x$ and $\tan(32^\circ) = y/(1+x)$, we obtain

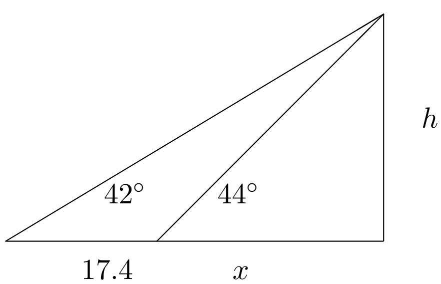
$$\tan(32^\circ) = \frac{y}{1 + y/\tan(36^\circ)}$$

$$\tan(32^\circ) + \frac{\tan(32^\circ)y}{\tan(36^\circ)} = y$$

$$\begin{aligned}\tan(32^\circ) &= y\left(1 - \frac{\tan(32^\circ)}{\tan(36^\circ)}\right)\\ y &= \frac{\tan(32^\circ)}{1 - \tan(32^\circ)/\tan(36^\circ)}\\ y &\approx 4.5 \text{ km.}\end{aligned}$$

The closest the boat will come to the lighthouse is 4.5 km.

60. Let h be the height of the Woolworth skyscraper.



Since $\tan(44^\circ) = h/x$ and

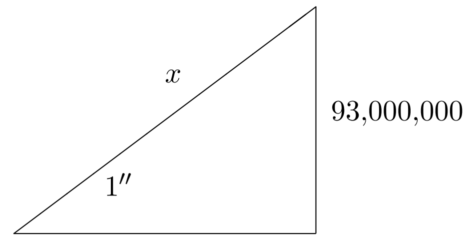
$\tan(42^\circ) = \dfrac{h}{17.4 + x}$, we find

$$\tan(42^\circ) = \frac{h}{17.4 + h/\tan(44^\circ)}$$

$$17.4 \cdot \tan(42^\circ) + \frac{\tan(42^\circ)h}{\tan(44^\circ)} = h$$

$$\begin{aligned}17.4 \cdot \tan(42^\circ) &= h\left(1 - \tan(42^\circ)/\tan(44^\circ)\right)\\ h &= \frac{17.4 \cdot \tan(42^\circ)}{1 - \tan(42^\circ)/\tan(44^\circ)}\\ h &\approx 231.7 \text{ meters.}\end{aligned}$$

61. Let x be the number of miles in one parsec.



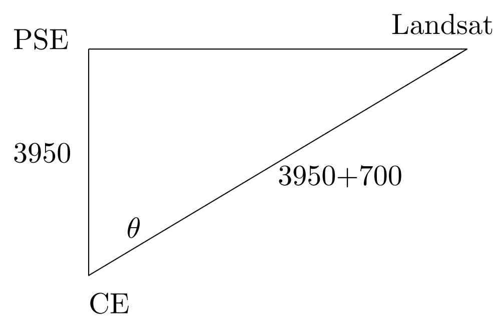
Since $\sin(1'') = \dfrac{93,000,000}{x}$, we obtain

$$x = \frac{93,000,000}{\sin(1/3600^\circ)} \approx 1.9 \times 10^{13} \text{ miles.}$$

Light travels one parsec in 3.26 years since

$$\frac{x}{193,000(63,240)} \approx 3.26 \text{ years.}$$

62. In the triangle below CE stands for the center of the earth and PSE is a point on the surface of the earth on the horizon of the cameras of Landsat.



Since $\cos(\theta) = \dfrac{3950}{3950 + 700}$, we have

$\theta = \cos^{-1}\left(\dfrac{3950}{3950 + 700}\right) \approx 0.5558$ radians.

But 2θ is the central angle, with vertex at CE, intercepted by the path on the surface of the earth as can be seen by Landsat. The width of this path is the arclength subtended by 2θ, i.e., $s = r \cdot 2\theta = 3950 \cdot 2 \cdot 0.5558 \approx 4391$ miles.

63. Let h be the height of the building, and let x be the distance between C and the building. Using right triangle trigonometry, we obtain

$$\frac{1}{\sqrt{3}} = \tan 30^\circ = \frac{h}{40+x}$$

and

$$1 = \tan 45^\circ = \frac{h}{20+x}.$$

Solving simultaneously, we find $x = 10(\sqrt{3}-1)$ and $h = 10(\sqrt{3}+1)$. However,

$$\tan C = \frac{h}{x} = \frac{10(\sqrt{3}+1)}{10(\sqrt{3}-1)} = 2+\sqrt{3}$$

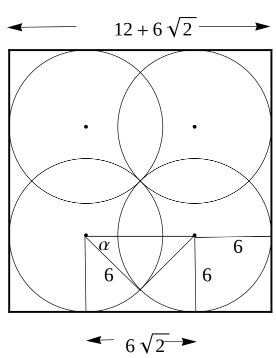
Since $\tan 75^\circ = 2+\sqrt{3}$ by the addition formula for tangent, we obtain

$$C = 75^\circ$$

64. In two minutes, the height increases by $250(\tan 58^\circ - \tan 24^\circ)$. Then the rate of ascent is

$$\frac{250(\tan 58^\circ - \tan 24^\circ)}{2(60)} \approx 2.4 \ \frac{\text{m}}{\text{sec}}$$

65. a) The distance from a center to the nearest vertex of the square is $6\sqrt{2}$ by the Pythagorean theorem. Then the diagonal of the square is $12+12\sqrt{2}$. From which, the side of the square is $12+6\sqrt{2}$ by the Pythagorean theorem as shown below.



The area A_c of the region in one corner of the box that is *not* watered is obtained by subtracting one-fourth of the area of a circle of radius 6 from the area of a 6-by-6 square:

$$A_c = 36 - 9\pi.$$

Note, the distance between two horizontal centers is $6\sqrt{2}$ as shown above. Then the angle α between the line joining the centers and the line to the intersection of the circles is $\alpha = \pi/4$.

Thus, the area A_b of the region between two adjacent circles that is not watered is the area of a 6-by-$6\sqrt{2}$ square minus the area, 18, of the isosceles triangle with base angle $\alpha = \pi/4$, and minus the combined area 9π of two sectors with central angle $\pi/4$:

$$A_b = 36\sqrt{2} - 18 - 9\pi$$

Hence, the total area not watered is

$$\begin{aligned} 4(A_c + A_b) &= 4\left(36 - 9\pi + 36\sqrt{2} - 18 - 9\pi\right) \\ &= 72\left(1 + 2\sqrt{2} - \pi\right) \ \text{m}^2 \end{aligned}$$

b) Since the side of the square is $12+6\sqrt{2}$, the area that is watered by at least one sprinkler is

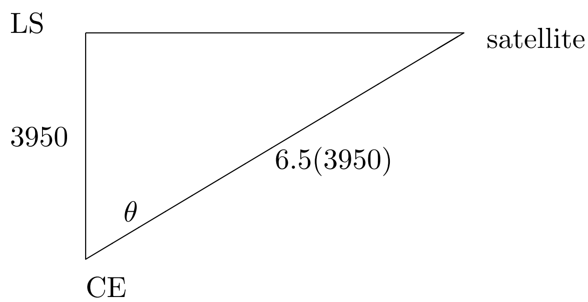
$$100\% - \frac{4(A_c + A_b)}{(12+6\sqrt{2})^2} \cdot 100\% \approx 88.2\%$$

66. The triangle that encloses the three circles is an equilateral triangle. From this, it follows that the central angle subtended by the metal band along the circumference of a circle is 120°. Then the length of the metal band along the circumference of a circle of radius 1 ft is

$$s = r\alpha = 1 \cdot \frac{2\pi}{3} = \frac{2\pi}{3}$$

Thus, the length of the band around the three circles is 2π. Note, the length of the remaining metal band (i.e., part of triangle) is 12 ft. Thus, the length of the metal band around the circles is $2\pi + 12$ ft.

67. In the triangle below CE stands for the center of the earth, and LS is a point on the surface of the earth lying in the line of sight of the satellite.



Since $\cos(\theta) = \dfrac{3950}{6.5(3950)} = 1/6.5$, we get $\theta = \cos^{-1}(1/6.5) \approx 1.41634$ radians. But 2θ is the widest angle formed by a sender and receiver of a signal with vertex CE. The maximum distance is the arclength subtended by 2θ, i.e., $s = r \cdot 2\theta = 3950 \cdot 2 \cdot 1.41634 \approx 11,189$ miles.

68.

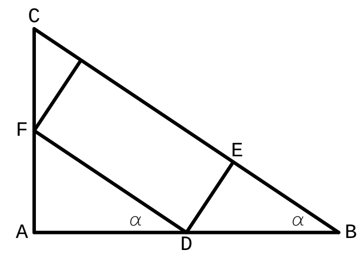
a) $\cos^{-1}\left(\dfrac{4000(5280)}{4000(5280)+2}\right) \approx 0.0249°$

b) $\cos^{-1}\left(\dfrac{4000(5280)}{4000(5280)+6}\right) \approx 0.0432°$

c) Let t be the number of seconds between the moment Diane saw the green flash and the moment when Ed saw the green flash. Assume the earth rotates every 24 hours. Then t is given by

$$\begin{aligned} \frac{360}{24(60)(60)} &= \frac{0.0432 - 0.0249}{t} \\ t &= (0.0432 - 0.0249) \cdot \frac{24(60)(60)}{360} \\ t &\approx 4.4 \text{ seconds.} \end{aligned}$$

69. First, consider the figure below.



Suppose $AC = s$, $AB = \dfrac{3s}{2}$, $BD = h$, $DE = w$, and $DF = L$. Note, $\tan\alpha = \dfrac{AC}{AB} = \dfrac{2}{3}$. Since $\sin\alpha = \dfrac{2}{\sqrt{13}}$ and $\sin\alpha = \dfrac{DE}{BD}$, we obtain $w = \dfrac{2h}{\sqrt{13}}$. Since $\cos\alpha = \dfrac{AD}{DF}$, we get

$$\cos\alpha = \frac{\frac{3s}{2} - h}{L}.$$

Solving for L. we find $L = \dfrac{\frac{3s}{2} - h}{\cos\alpha}$ and since $\cos\alpha = \dfrac{3}{\sqrt{13}}$, we get

$$L = \frac{\sqrt{13}}{3}\left(\frac{3s}{2} - \frac{w\sqrt{13}}{2}\right).$$

So, the area of the parking lot is

$$wL = \frac{w\sqrt{13}}{3}\left(\frac{3s}{2} - \frac{w\sqrt{13}}{2}\right).$$

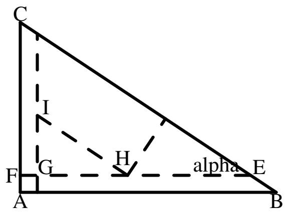
Since this area represents a quadratic function of w, one can find the vertex of its graph and conclude that the maximum area of the rectangle is obtained if one chooses $w = \dfrac{3s}{2\sqrt{13}}$.

Correspondingly, we obtain $L = \dfrac{s\sqrt{13}}{4}$.

Finally, given $s = 100$ feet, the dimensions of the house with maximum area are

$$w = \frac{3(100)}{2\sqrt{13}} \approx 41.60 \text{ ft and } L = \frac{100\sqrt{13}}{4} \approx 90.14 \text{ ft.}$$

70. Consider the right triangle below.



The triangle with vertices at G, H, and I is the part of the lot where the parking lot must be built so that the parking lot is 10 feet from each side of the property and 40 feet from the street.

Note, $AC = 100$, $AB = 150$, and $AF = FG = 10$. Since the ratios of corresponding sides of similar triangles are equal, we have

$$\begin{aligned} \frac{100}{150} &= \frac{90}{EF} \\ EF &= 90\left(\frac{3}{2}\right) \\ EF &= 135. \end{aligned}$$

Since H is 40 feet from BC, we get

$$\begin{aligned} \sin\alpha &= \frac{40}{EH} \\ \frac{2}{\sqrt{13}} &= \frac{40}{EH} \\ EH &= 20\sqrt{13}. \end{aligned}$$

Then

$$\begin{aligned} GH &= EF - EH - 10 \\ &= 125 - 20\sqrt{13} \end{aligned}$$

and by using the ratios of similar triangles one finds

$$\begin{aligned} \frac{100}{150} &= \frac{GI}{GH} \\ GI &= \frac{2}{3}(125 - 20\sqrt{13}). \end{aligned}$$

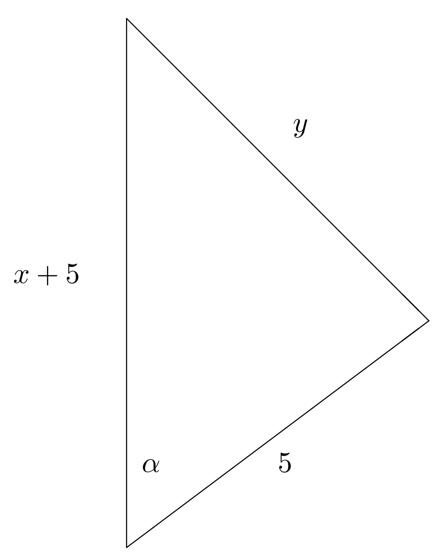
From the discussion in Exercise 61, the dimensions of the parking lot with maximum area that can be built inside the triangle with vertices G, H, and I are (where $s = GI$ in this case)

$$\begin{aligned} w &= \frac{3s}{2\sqrt{13}} \\ &= \frac{3(GI)}{2\sqrt{13}} \\ &= \frac{3\left(\frac{2}{3}(125 - 20\sqrt{13})\right)}{2\sqrt{13}} \\ &= \frac{125 - 20\sqrt{13}}{\sqrt{13}} \\ w &\approx 14.7 \text{ feet} \end{aligned}$$

and

$$\begin{aligned} L &= \frac{s\sqrt{13}}{4} \\ &= \frac{\left(\frac{2}{3}(125 - 20\sqrt{13})\right)\sqrt{13}}{4} \\ &= \frac{\sqrt{13}}{6}(125 - 20\sqrt{13}) \\ L &\approx 31.8 \text{ feet.} \end{aligned}$$

71. Consider the right triangle formed by the hook, the center of the circle, and a point on the circle where the chain is tangent to the circle.



Then $\tan\alpha = \dfrac{y}{5}$ or $y = 5\tan\alpha$. Since the chain is 40 ft long and the angle $2\pi - 2\alpha$ intercepts an arc around the pipe where the chain wraps around the circle, we obtain

$$2y + 5(2\pi - 2\alpha) = 40.$$

By substitution, we get

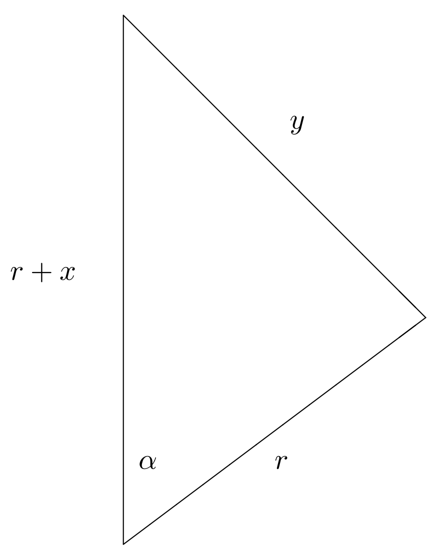
$$10\tan\alpha + 10\pi - 10\alpha = 40.$$

With a graphing calculator, we obtain $\alpha \approx 1.09835$ radians. From the figure above, we get $\cos\alpha = \dfrac{5}{5+x}$. Solving for x, we obtain

$$x = \frac{5 - 5\cos\alpha}{\cos\alpha} \approx 5.987 \text{ ft}.$$

72. Consider the right triangle where

$$r = 6,400,000 \text{ meters}$$



Then $\cos\alpha = \dfrac{r}{r+x}$ or

$$\alpha = \cos^{-1}\left(\frac{r}{r+x}\right)$$

By the Pythagorean Theorem, we find $r^2 + y^2 = (r+x)^2$ from which we derive

$$y = \sqrt{x^2 + 2rx}.$$

Since the length of the rope is 1 meter longer than the earth's diameter, we obtain

$$2y + (2\pi - 2\alpha)r = 2\pi r + 1$$

or by substitution

$$2\sqrt{x^2+2rx} + 2r\left(\pi - \cos^{-1}\left(\frac{r}{r+x}\right)\right) = 2\pi r + 1$$

Using a calculator, we obtain $r = 121.6$ meters

73. Assume the circle is given by

$$x^2 + (y-r)^2 = r^2$$

where r is the radius. Suppose the points where the blocks touch the circle are at the points $(-x_2, 1)$ and $(x_1, 2)$ where $x_1, x_2 > 0$. Substitute the points into the equation of the circle. Then

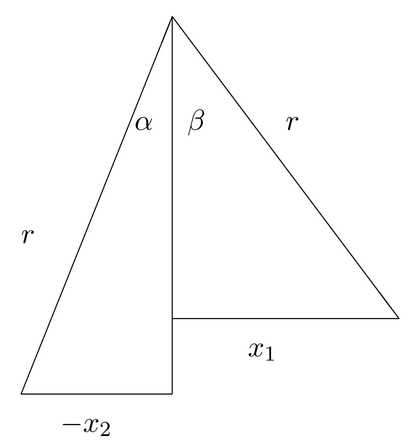
$$x_1^2 + (2-r)^2 = r^2$$

and

$$x_2^2 + (1-r)^2 = r^2.$$

From which we obtain $x_1^2 + 4 - 4r = 0$ and $x_2^2 + 1 - 2r = 0$. Thus, $x_1 = \sqrt{4r-4}$ and $x_2 = -\sqrt{2r-1}$.

Consider the two triangles below.



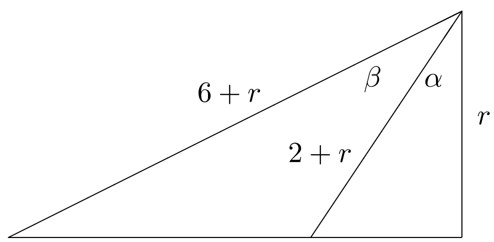
Note, $\alpha = \sin^{-1}\left(\dfrac{-x_2}{r}\right)$ and $\beta = \sin^{-1}\left(\dfrac{x_1}{r}\right)$.

Since 6 ft is the arclength between the blocks, we have $r(\alpha + \beta) = 6$. By substitution, we obtain

$$r\left(\sin^{-1}\left(\frac{\sqrt{2r-1}}{r}\right) + \sin^{-1}\left(\frac{\sqrt{4r-4}}{r}\right)\right) = 6$$

With the aid of a graphing calculator, we find $r \approx 2.768$ ft.

74. In the figure, r is the radius of the circle.



Note,

$$\alpha = \cos^{-1}\left(\frac{r}{2+r}\right) \text{ and } \beta = \cos^{-1}\left(\frac{r}{6+r}\right)$$

which we assume are in degree measure. With the aid of a graphing calculator, we find that the solutions to

$$\cos^{-1}\left(\frac{r}{6+r}\right) - \cos^{-1}\left(\frac{r}{2+r}\right) - 18 = 0$$

are $r \approx 3.626$ ft and $r \approx 9.126$ ft.

When we use $19°$, with a graphing calculator we see that

$$\cos^{-1}\left(\frac{r}{6+r}\right) - \cos^{-1}\left(\frac{r}{2+r}\right) - 19 = 0$$

has no solution.

75. Let r be the radius of the earth. Consider the angles shown in Exercises 68 a) and b).

Namely, $\alpha = \cos^{-1}\left(\dfrac{r(5280)}{r(5280)+2}\right)$ and

$\beta = \cos^{-1}\left(\dfrac{r(5280)}{4000(5280)+6}\right)$. Since it takes 4 seconds for the green flash to travel from Diane's eyes to Ed's eyes, we obtain

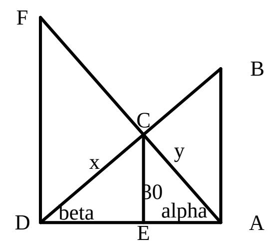
$$\begin{aligned} \frac{360}{24(60)(60)} &= \frac{\beta-\alpha}{4} \\ \beta-\alpha &= \frac{1}{60}. \end{aligned}$$

With a calculator, we get that the solution to

$$\cos^{-1}\left(\frac{r(5280)}{4000(5280)+6}\right) - \cos^{-1}\left(\frac{r(5280)}{r(5280)+2}\right) = \frac{1}{60}$$

is $r \approx 4798$ miles.

76. In the figure below,



let $AF = 120$, $AC = y$, $BD = 90$, and $CD = x$. Also, let α be the angle formed by AE and AC, and let β be the angle formed by CD and DE.

Choose the point Q in DF so that DF is perpendicular to CQ. In $\triangle CQF$, we have

$$\sin(\pi/2-\alpha) = \frac{QC}{120-y}.$$

In $\triangle CQD$, we get

$$\sin(\pi/2-\beta) = \frac{QC}{x}.$$

Therefore,

$$\begin{aligned} \sin(\pi/2-\alpha)(120-y) &= x\sin(\pi/2-\beta) \\ \frac{x}{\sin(\pi/2-\alpha)} &= \frac{120-y}{\sin(\pi/2-\beta)} \\ \frac{x}{\cos\alpha} &= \frac{120-y}{\cos\beta}. \end{aligned}$$

Similarly, $\dfrac{90-x}{\cos\alpha} = \dfrac{y}{\cos\beta}$. Thus,

$$\frac{x}{120-y} = \frac{\cos\alpha}{\cos\beta} = \frac{90-x}{y}.$$

Consequently, $4x + 3y = 360$ or equivalently

$$y = \frac{360-4x}{3}.$$

From the right triangles $\triangle$ ACE and $\triangle$ CDE, one finds $\sin\alpha = \dfrac{30}{y}$ and $\sin\beta = \dfrac{30}{x}$.

Consequently,

$$\cos\alpha = \frac{\sqrt{y^2-30^2}}{y} \quad \text{and} \quad \cos\beta = \frac{\sqrt{x^2-30^2}}{x}.$$

Moreover, using triangles $\triangle$ ABD and $\triangle$ ADF, one obtains

$$\cos\beta = \frac{AD}{90}$$

and

$$\cos\alpha = \frac{AD}{120}.$$

Combining all of these, one derives

$$\begin{aligned}
90\cos\beta &= 120\cos\alpha \\
\frac{3}{4} &= \frac{\cos\alpha}{\cos\beta} \\
\frac{3}{4} &= \frac{\frac{\sqrt{y^2-30^2}}{y}}{\frac{\sqrt{x^2-30^2}}{x}} \\
\frac{3}{4} &= \frac{\sqrt{y^2-30^2}}{\sqrt{x^2-30^2}} \cdot \frac{x}{y} \\
\frac{3}{4} &= \frac{\sqrt{\left(\frac{360-4x}{3}\right)^2 - 30^2}}{\sqrt{x^2-30^2}} \cdot \frac{x}{\frac{360-4x}{3}} \\
\frac{3}{4} &= \frac{3x}{360-4x} \cdot \frac{\sqrt{\left(\frac{360-4x}{3}\right)^2 - 30^2}}{\sqrt{x^2-30^2}} \\
\frac{1}{16} &= \frac{x^2}{(360-4x)^2} \cdot \frac{\left(\frac{360-4x}{3}\right)^2 - 30^2}{x^2-30^2}
\end{aligned}$$

$$(360-4x)^2(x^2-30^2) = \\ = 16x^2\left[\left(\frac{360-4x}{3}\right)^2 - 30^2\right]$$

Using a graphing calculator, for $0 < x < 90$, one finds $x \approx 60.4$. Working backwards, one derives

$$\sin\beta = \frac{30}{60.4} \quad \text{or} \quad \beta \approx 29.8^\circ$$

and

$$\cos(29.8) = \frac{AD}{90}.$$

Hence, the width of the property is $AD = 78.1$ feet.

79. Let $r = \sqrt{4^2+3^2} = 5$

a) $\sin\alpha = \frac{y}{r} = \frac{3}{5}$

b) $\cos\alpha = \frac{x}{r} = \frac{4}{5}$

c) $\tan\alpha = \frac{y}{x} = \frac{3}{4}$

80. **a)** $\frac{\sqrt{2}}{2}$ **b)** $\frac{\sqrt{2}}{2}$ **c)** -1

81. **a)** $\frac{\sqrt{3}}{2}$ **b)** $-\frac{\sqrt{3}}{2}$ **c)** $-\sqrt{3}$

82. -210°

83. Quadrant III since $\frac{17\pi}{12} = 255^\circ$

84. Linear velocity

$$v = \frac{2\pi(3950)}{24} \cdot \frac{5280}{3600} \approx 1517 \text{ ft/sec}$$

85. We assume that the center of the bridge moves straight upward to the center of the arc where the bridge expands. The arc has length $100 + \frac{1}{12}$ feet. To find the central angle α, we use $s = r\alpha$ and $\sin(\alpha/2) = 50/r$. Then

$$\alpha = 2\sin^{-1}(50/r).$$

Solving the equation

$$100 + \frac{1}{12} = 2r\sin^{-1}\left(\frac{50}{r}\right)$$

with a graphing calculator, we find

$$r \approx 707.90241 \text{ ft}.$$

The distance from the chord to the center of the circle can be found by using the Pythagorean theorem, and it is

$$d = \sqrt{r^2 - 50^2} \approx 706.1344221 \text{ ft}.$$

Then the distance from the center of the arc to the cord is

$$r - d \approx 1.767989 \text{ ft} \approx 21.216 \text{ inches}.$$

86. Draw a triangle with vertices at the center of the circle, and two other vertices at two adjacent vertices of the hexagon. All the angles in this triangle are 60°'s, and the triangle is an equilateral triangle.

Notice, the height from the center to the opposite side of the equilateral triangle is $\frac{1}{2}$. Consequently, the length of each side of the equilateral triangle is $\frac{1}{\sqrt{3}}$.

Recall, $\frac{1}{2}ab\sin C$ is the area of a triangle with sides a and b, and included angle C. The area A of the hexagon is six times the area of the equilateral triangle. Then

$$\begin{aligned} A &= 6\left(\frac{1}{2}ab\sin C\right) \\ &= 3ab\sin C \\ &= 3\left(\frac{1}{\sqrt{3}}\right)^2 \sin 60^\circ \\ A &= \frac{\sqrt{3}}{2}. \end{aligned}$$

1.5 Pop Quiz

1. 45°

2. 60°

3. 120°

4. The hypotenuse is

$$\sqrt{3^2+6^2} = \sqrt{45} = 3\sqrt{5}.$$

If α is the angle opposite the side with length 3, then

$$\sin\alpha = \frac{3}{3\sqrt{5}} = \frac{\sqrt{5}}{5},$$

$$\cos\alpha = \frac{6}{3\sqrt{5}} = \frac{2\sqrt{5}}{5}, \quad \text{and}$$

$$\tan\alpha = \frac{3}{6} = \frac{1}{2}.$$

5. If h is the height of the building, then

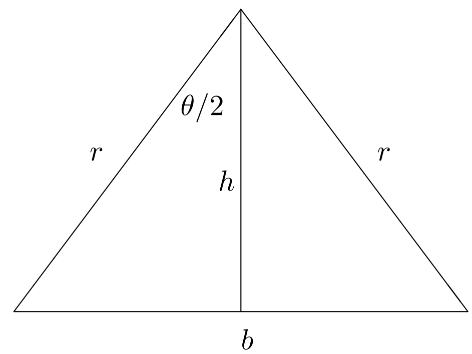
$$\tan\alpha = \frac{h}{1000}$$

or

$$h = 1000\tan 36^\circ \approx 727 \text{ ft}.$$

1.5 Linking Concepts

a) Consider the isosceles triangle below.



A pentagon inscribed in a circle of radius r consists of 5 triangles each one like the one shown above with radius r, $\theta = 72^\circ$ or $\theta/2 = 36^\circ$, and where b is the base. Since $h = r\cos(36^\circ)$ and $\sin(36^\circ) = \dfrac{b/2}{r}$, we get $b = 2r\sin(36^\circ)$ and the area of the triangle is

$$\begin{aligned} \frac{1}{2}bh &= \frac{1}{2}(2r\sin(36^\circ))(r\cos(36^\circ)) \\ &= r^2\sin(36^\circ)\cos(36^\circ). \end{aligned}$$

Hence, the area of the pentagon is

$$5r^2\sin(36^\circ)\cos(36^\circ).$$

b) An n-gon consists of n triangles like the one shown in part a) where $\theta/2 = \dfrac{360^\circ/n}{2}$.

For this n-gon, $h = r\cos\left(\dfrac{360^\circ}{2n}\right)$,

$\sin\left(\dfrac{360^\circ}{2n}\right) = \dfrac{b/2}{r}$, or $b = 2r\sin\left(\dfrac{360^\circ}{2n}\right)$, and the area of the triangle is given by

$$\begin{aligned} \frac{1}{2}bh &= \frac{1}{2}\left(2r\sin\left(\frac{360^\circ}{2n}\right)\right)\left(r\cos\left(\frac{360^\circ}{2n}\right)\right) \\ &= r^2\sin\left(\frac{360^\circ}{2n}\right)\cos\left(\frac{360^\circ}{2n}\right) \\ &= r^2\sin\left(\frac{180^\circ}{n}\right)\cos\left(\frac{180^\circ}{n}\right). \end{aligned}$$

Thus, the area of an n-gon inscribed in a circle of radius r is $nr^2 \sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$.

c) The area of an n-gon varies directly with the square of the radius r (of the circle in which the n-gon was inscribed).

d) For an n-gon, the constant of proportionality

is $n\sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$. For a decagon ($n = 10$), kilogon ($n = 10^3$), and megagon ($n = 10^6$), the constants of proportion are 2.9389, 3.14157, 3.141592654, respectively.

e) As n increases, the shape of the n-gon approaches the shape of a circle. Thus, when n is a large number, the area of a circle of radius r is approximately

$nr^2 \sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$ or $3.141592654r^2$.

f) If $n = 10^6$, then $\pi \approx n\sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$

$= 10^6 \sin\left(\frac{180°}{10^6}\right)\cos\left(\frac{180°}{10^6}\right) \approx 3.141592654.$

g) As derived in part b), the base of the triangle is $b = 2r\sin\left(\frac{180°}{n}\right)$. Thus, the perimeter P of an n-gon is nb or equivalently

$$P = 2nr\sin\left(\frac{180°}{n}\right).$$

h) When n is a large number, the shape of an n-gon approximates the shape of a circle. Consequently, the circumference C of a circle of radius r is approximately

$$C \approx 2nr\sin\left(\frac{180°}{n}\right).$$

If $n = 10^6$, then $n\sin\left(\frac{180°}{n}\right) \approx 3.141592654$.

Thus, $C = 2r(3.141592654)$.

For Thought

1. True, since $\sin^2\alpha + \cos^2\alpha = 1$ for any real number α.

2. True

3. False, since α is in Quadrant IV.

4. True

5. False, rather $\sin\alpha = -\frac{1}{2}$.

6. True **7.** True

8. False, since the reference angle is $\frac{\pi}{3}$.

9. True, since $\cos 120° = -\frac{1}{2} = -\cos 60°$.

10. True, since $\sin(7\pi/6) = -\frac{1}{2} = -\sin(\pi/6)$.

1.6 Exercises

1. fundamental

2. reference angle

3. $\sin\alpha = \pm\sqrt{1-\cos^2\alpha} = \pm\sqrt{1-1^2} = 0$

4. $\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-0^2} = -1$

5. Use the Fundamental Identity.

$$\begin{aligned} \left(\frac{5}{13}\right)^2 + \cos^2(\alpha) &= 1 \\ \frac{25}{169} + \cos^2(\alpha) &= 1 \\ \cos^2(\alpha) &= \frac{144}{169} \\ \cos(\alpha) &= \pm\frac{12}{13} \end{aligned}$$

Since α is in quadrant II, $\cos(\alpha) = -12/13$.

6. Use the Fundamental Identity.

$$\begin{aligned} \left(-\frac{4}{5}\right)^2 + \sin^2(\alpha) &= 1 \\ \frac{16}{25} + \sin^2(\alpha) &= 1 \\ \sin^2(\alpha) &= \frac{9}{25} \\ \sin(\alpha) &= \pm\frac{3}{5} \end{aligned}$$

Since α is in quadrant III, $\sin(\alpha) = -3/5$.

7. Use the Fundamental Identity.

$$\begin{aligned}\left(\frac{3}{5}\right)^2+\sin^2(\alpha) &= 1\\ \frac{9}{25}+\sin^2(\alpha) &= 1\\ \sin^2(\alpha) &= \frac{16}{25}\\ \sin(\alpha) &= \pm\frac{4}{5}\end{aligned}$$

Since α is in quadrant IV, $\sin(\alpha)=-4/5$.

8. Use the Fundamental Identity.

$$\begin{aligned}\left(-\frac{12}{13}\right)^2+\cos^2(\alpha) &= 1\\ \frac{144}{169}+\cos^2(\alpha) &= 1\\ \cos^2(\alpha) &= \frac{25}{169}\\ \cos(\alpha) &= \pm\frac{5}{13}\end{aligned}$$

Since α is in quadrant IV, $\cos(\alpha)=5/13$.

9. Use the Fundamental Identity.

$$\begin{aligned}\left(\frac{1}{3}\right)^2+\cos^2(\alpha) &= 1\\ \frac{1}{9}+\cos^2(\alpha) &= 1\\ \cos^2(\alpha) &= \frac{8}{9}\\ \cos(\alpha) &= \pm\frac{2\sqrt{2}}{3}\end{aligned}$$

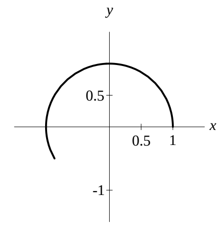
Since $\cos(\alpha)>0$, $\cos(\alpha)=\dfrac{2\sqrt{2}}{3}$.

10. Use the Fundamental Identity.

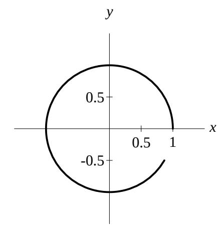
$$\begin{aligned}\left(\frac{2}{5}\right)^2+\sin^2(\alpha) &= 1\\ \frac{4}{25}+\sin^2(\alpha) &= 1\\ \sin^2(\alpha) &= \frac{21}{25}\\ \sin(\alpha) &= \pm\frac{\sqrt{21}}{5}\end{aligned}$$

Since $\sin(\alpha)<0$, $\sin(\alpha)=-\dfrac{\sqrt{21}}{5}$.

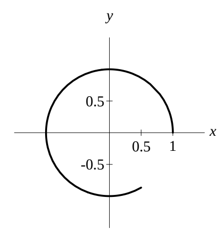
11. $30°, \pi/6$



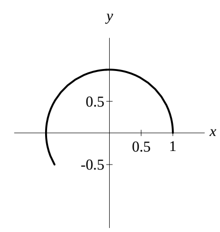
12. $30°, \pi/6$



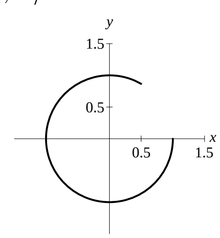
13. $60°, \pi/3$



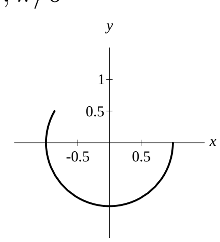
14. $30°, \pi/6$



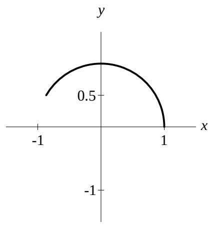
15. $60°, \pi/3$



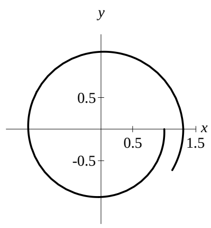
16. $30°, \pi/6$



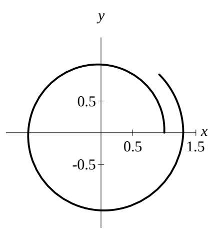
17. $30°, \pi/6$



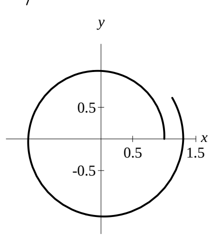
18. $30°, \pi/6$



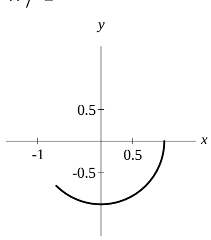
19. $45°, \pi/4$



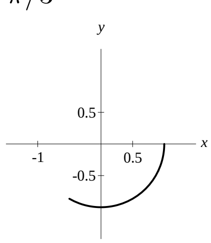
20. $30°, \pi/6$



21. $45°, \pi/4$



22. $60°, \pi/3$



23. $\sin(135°) = \sin(45°) = \dfrac{\sqrt{2}}{2}$

24. $\sin(420°) = \sin(60°) = \dfrac{\sqrt{3}}{2}$

25. $\cos\left(\dfrac{5\pi}{3}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

26. $\cos\left(\dfrac{11\pi}{6}\right) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

27. $\sin\left(\dfrac{7\pi}{4}\right) = -\sin\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

28. $\sin\left(-\dfrac{13\pi}{4}\right) = \sin\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

29. $\cos\left(-\dfrac{17\pi}{6}\right) = -\cos\left(\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

30. $\cos\left(-\dfrac{5\pi}{3}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

31. $\sin(-45°) = -\sin(45°) = -\dfrac{\sqrt{2}}{2}$

32. $\cos(-120°) = -\cos(60°) = -\dfrac{1}{2}$

33. $\cos(-240°) = -\cos(60°) = -\dfrac{1}{2}$

34. $\sin(-225°) = \sin(45°) = \dfrac{\sqrt{2}}{2}$

35. The reference angle of $3\pi/4$ is $\pi/4$.

Then $\sin(3\pi/4) = \sin(\pi/4) = \dfrac{\sqrt{2}}{2}$,

$\cos(3\pi/4) = -\cos(\pi/4) = -\dfrac{\sqrt{2}}{2}$,

$\tan(3\pi/4) = -\tan(\pi/4) = -1$,

$\csc(3\pi/4) = \csc(\pi/4) = \sqrt{2}$,

$\sec(3\pi/4) = -\sec(\pi/4) = -\sqrt{2}$, and

$\cot(3\pi/4) = -\cot(\pi/4) = -1$.

36. The reference angle of $2\pi/3$ is $\pi/3$.

Then $\sin(2\pi/3) = \sin(\pi/3) = \dfrac{\sqrt{3}}{2}$,

$\cos(2\pi/3) = -\cos(\pi/3) = -\dfrac{1}{2}$,

$\tan(2\pi/3) = -\tan(\pi/3) = -\sqrt{3}$,

$\csc(2\pi/3) = \csc(\pi/3) = \dfrac{2\sqrt{3}}{3}$,
$\sec(2\pi/3) = -\sec(\pi/3) = -2$, and
$\cot(2\pi/3) = -\cot(\pi/3) = -\dfrac{\sqrt{3}}{3}$.

37. The reference angle of $4\pi/3$ is $\pi/3$.
Then $\sin(4\pi/3) = -\sin(\pi/3) = -\dfrac{\sqrt{3}}{2}$,
$\cos(4\pi/3) = -\cos(\pi/3) = -\dfrac{1}{2}$,
$\tan(4\pi/3) = \tan(\pi/3) = \sqrt{3}$,
$\csc(4\pi/3) = -\csc(\pi/3) = -\dfrac{2\sqrt{3}}{3}$,
$\sec(4\pi/3) = -\sec(\pi/3) = -2$, and
$\cot(4\pi/3) = \cot(\pi/3) = \dfrac{\sqrt{3}}{3}$.

38. The reference angle of $7\pi/6$ is $\pi/6$.
Then $\sin(7\pi/6) = -\sin(\pi/6) = -\dfrac{1}{2}$,
$\cos(7\pi/6) = -\cos(\pi/6) = -\dfrac{\sqrt{3}}{2}$,
$\tan(7\pi/6) = \tan(\pi/6) = \dfrac{\sqrt{3}}{3}$,
$\csc(7\pi/6) = -\csc(\pi/6) = -2$,
$\sec(7\pi/6) = -\sec(\pi/6) = -\dfrac{2\sqrt{3}}{3}$, and
$\cot(7\pi/6) = \cot(\pi/6) = \sqrt{3}$.

39. The reference angle of 300° is 60°.
Then $\sin(300^\circ) = -\sin(60^\circ) = -\dfrac{\sqrt{3}}{2}$,
$\cos(300^\circ) = \cos(60^\circ) = \dfrac{1}{2}$,
$\tan(300^\circ) = -\tan(60^\circ) = -\sqrt{3}$,
$\csc(300^\circ) = -\csc(60^\circ) = -\dfrac{2\sqrt{3}}{3}$,
$\sec(300^\circ) = \sec(60^\circ) = 2$, and
$\cot(300^\circ) = -\cot(60^\circ) = -\dfrac{\sqrt{3}}{3}$.

40. The reference angle of 315° is 45°.
Then $\sin(315^\circ) = -\sin(45^\circ) = -\dfrac{\sqrt{2}}{2}$,
$\cos(315^\circ) = \cos(45^\circ) = \dfrac{\sqrt{2}}{2}$,
$\tan(315^\circ) = -\tan(45^\circ) = -1$,
$\csc(315^\circ) = -\csc(45^\circ) = -\sqrt{2}$,
$\sec(315^\circ) = \sec(45^\circ) = \sqrt{2}$, and
$\cot(315^\circ) = -\cot(45^\circ) = -1$.

41. The reference angle of -135° is 45°.
Then $\sin(-135^\circ) = -\sin(45^\circ) = -\dfrac{\sqrt{2}}{2}$,
$\cos(-135^\circ) = -\cos(45^\circ) = -\dfrac{\sqrt{2}}{2}$,
$\tan(-135^\circ) = \tan(45^\circ) = 1$,
$\csc(-135^\circ) = -\csc(45^\circ) = -\sqrt{2}$,
$\sec(-135^\circ) = -\sec(45^\circ) = -\sqrt{2}$, and
$\cot(-135^\circ) = \cot(45^\circ) = 1$.

42. The reference angle of 135° is 45°.
Then $\sin(135^\circ) = \sin(45^\circ) = \dfrac{\sqrt{2}}{2}$,
$\cos(135^\circ) = -\cos(45^\circ) = -\dfrac{\sqrt{2}}{2}$,
$\tan(135^\circ) = -\tan(45^\circ) = -1$,
$\csc(135^\circ) = \csc(45^\circ) = \sqrt{2}$,
$\sec(135^\circ) = -\sec(45^\circ) = -\sqrt{2}$, and
$\cot(135^\circ) = -\cot(45^\circ) = -1$.

43. False, since $\sin 210^\circ = -\sin 30^\circ$.

44. True

45. True, since $\cos 330^\circ = \dfrac{\sqrt{3}}{2} = \cos 30^\circ$.

46. True

47. False, since $\sin 179^\circ = \sin 1^\circ$.

48. False

49. True, for the reference angle is $\pi/7$, $6\pi/7$ is in Quadrant II, and cosine is negative in Quadrant II.

50. True, for the reference angle is $\pi/12$, $13\pi/12$ is in Quadrant III, and cosine is negative in Quadrant III.

51. False, since $\sin(23\pi/24) = \sin(\pi/24)$.

52. False, since $\sin(25\pi/24) = -\sin(\pi/24)$.

53. True, for the reference angle is $\pi/7$, $13\pi/7$ is in Quadrant IV, and cosine is positive in Quadrant IV.

54. True, for the reference angle is $\pi/5$, $9\pi/5$ is in Quadrant IV, and cosine is positive in Quadrant IV.

55. If $h = 18$, then

$$\begin{aligned} T &= 18\sin\left(\frac{\pi}{12}(6)\right) + 102 \\ &= 18\sin\left(\frac{\pi}{2}\right) + 102 = 18 + 102 \\ &= 120°\text{F}. \end{aligned}$$

If $h = 6$, then

$$\begin{aligned} T &= 18\sin\left(\frac{\pi}{12}(-6)\right) + 102 \\ &= 18\sin\left(-\frac{\pi}{2}\right) + 102 = -18 + 102 \\ &= 84°\text{F}. \end{aligned}$$

56. If $h = 14$, then

$$\begin{aligned} T &= 13\cos\left(\frac{\pi}{12}(48)\right) - 7 \\ &= 13\cos(4\pi) - 7 = 13 - 7 \\ &= 6°\text{F}. \end{aligned}$$

If $h = 2$, then

$$\begin{aligned} T &= 13\cos\left(\frac{\pi}{12}(36)\right) - 7 \\ &= 13\cos(3\pi) - 7 = -13 - 7 \\ &= -20°\text{F}. \end{aligned}$$

57. Note, $x(t) = 4\sin(t) + 3\cos(t)$.

a) Initial position is

$$x(0) = 3\cos 0 = 3.$$

b) If $t = 5\pi/4$, the position is

$$\begin{aligned} x(5\pi/4) &= 4\sin(5\pi/4) + 3\cos(5\pi/4) \\ &= -2\sqrt{2} - \frac{3\sqrt{2}}{2} \\ &= -\frac{7\sqrt{2}}{2}. \end{aligned}$$

58. Note, $x(t) = -\dfrac{\sqrt{3}}{3}\sin\left(\dfrac{\pi}{3}t\right) - \cos\left(\dfrac{\pi}{3}t\right)$.

a) Initial position is

$$x(0) = -\cos(0) = -1.$$

b) If $t = 2$, the position is

$$\begin{aligned} x(2) &= -\frac{\sqrt{3}}{3}\sin\left(\frac{2\pi}{3}\right) - \cos\left(\frac{2\pi}{3}\right) \\ &= -\frac{\sqrt{3}}{3}\frac{\sqrt{3}}{2} + \frac{1}{2} \\ &= 0. \end{aligned}$$

59. The angle between the tips of two adjacent teeth is $\dfrac{2\pi}{22} = \dfrac{\pi}{11}$. The actual distance is

$$c = 6\sqrt{2 - 2\cos(\pi/11)} \approx 1.708 \text{ in.}$$

The length of the arc is

$$s = 6 \cdot \frac{\pi}{11} \approx 1.714 \text{ in.}$$

60. The central angle determined by an edge of a stop sign is $\dfrac{2\pi}{8}$. If r is the radius of the circular drum, then

$$10 = r\sqrt{2 - 2\cos(2\pi/8)} \approx r(0.765367).$$

Thus, $r \approx 13.07$ in.

61. Solving for v_o, one finds

$$\begin{aligned} 367 &= \frac{v_o^2}{32}\sin 86° \\ \sqrt{\frac{32(367)}{\sin 86°}} \text{ ft/sec} &= v_o \\ \sqrt{\frac{32(367)}{\sin 86°}\frac{3600}{5280}} \text{ mph} &= v_o \\ 74 \text{ mph} &\approx v_o. \end{aligned}$$

62. Note, $\cos(\alpha) = \pm\sqrt{1 - \sin^2\alpha}$. Then

$$\cos\alpha = \sqrt{1 - \sin^2\alpha}$$

if the terminal side of α lies in the 1st or 4th quadrant, while

$$\cos\alpha = -\sqrt{1-\sin^2\alpha}$$

if the terminal side of α lies in the 2nd or 3rd quadrant.

63. Since $r = \sqrt{3^2+4^2} = 5$, we find

a) $\sec\alpha = \dfrac{r}{x} = \dfrac{5}{3}$

b) $\csc\alpha = \dfrac{r}{y} = \dfrac{5}{4}$

c) $\cot\alpha = \dfrac{x}{y} = \dfrac{3}{4}$

64. **a)** $-\sqrt{2}$ **b)** $-\sqrt{2}$ **c)** 1

65. **a)** 2 **b)** -2 **c)** $\dfrac{\sqrt{3}}{3}$

66. opposite, hypotenuse

67. $-30°$

68. Let h be the height of the building. Since $\tan 30° = h/2000$, we obtain

$$h = 2000\tan 30° \approx 1155 \text{ ft.}$$

69. We begin by writing the length of a diagonal of a rectangular box. If the dimensions of a box are a-by-b-by-c, then the length of a diagonal is $\sqrt{a^2+b^2+c^2}$.

Suppose the ball is placed at the center of the field and 60 feet from the goal line. Then the distance between the ball and the right upright of the goal is

$$A = \sqrt{90^2+10^2+9.25^2} \approx 91.025.$$

Consider the triangle formed by the ball, and the left and right uprights of the goal. Opposite the angle θ_1 is the 10-ft horizontal bar. To simplify the calculation, we use the cosine law in Chapter 5. Then

$$18.5^2 = 2A^2 - 2A^2\cos\theta$$

and $\theta_1 \approx 11.66497°$.

Now, place the ball on the right hash mark which is 9.25 ft from the centerline. The ball is also 60 feet from the goal line. Then the distance between the ball and the right upright of the goal is

$$B = \sqrt{90^2+10^2} \approx 90.554.$$

And, the distance between the ball and the left upright of the goal is

$$C = \sqrt{90^2+10^2+18.5^2} \approx 92.424.$$

Similarly, consider the triangle formed by the ball, and the left and right uprights of the goal. Opposite the angle θ is the 10-ft horizontal bar. Using the cosine law, we find

$$18.5^2 = B^2 + C^2 - 2BC\cos\theta_2$$

and $\theta_2 \approx 11.54654°$.

Thus, the difference between the values of θ is

$$\theta_1 - \theta_2 \approx 11.66497° - 11.54654° \approx 0.118°$$

70. Place 16 red cubes on opposite faces but not in the center of a face. Place the 17th red cube anywhere except on the center of a face. The percentage of the surface area that is red is

$$\frac{42}{54} \times 100\% = 77\frac{7}{9}\%.$$

1.6 Pop Quiz

1. $\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\dfrac{9}{25}} = -\sqrt{\dfrac{16}{25}} = -\dfrac{4}{5}$

2. $60°$ **3.** $\dfrac{\pi}{6}$

4. $\sin 210° = -\sin 30° = -\dfrac{1}{2}$

5. $\sin(-5\pi/4) = \sin(\pi/4) = \dfrac{\sqrt{2}}{2}$

6. $\cos 135° = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

7. $\cos(11\pi/6) = \cos(\pi/6) = \dfrac{\sqrt{3}}{2}$

Chapter 1 Review Exercises

1. $388° - 360° = 28°$

2. $-840° + 3 \cdot 360° = 240°$

3. $-153°14'27'' + 359°59'60'' = 206°45'33''$

4. $455°39'24'' - 360° = 95°39'24''$

5. $180°$ **6.** $-35\pi/6 + 6\pi = \pi/6 = 30°$

7. $13\pi/5 - 2\pi = 3\pi/5 = 3 \cdot 36° = 108°$

8. $29\pi/12 - 2\pi = 5\pi/12 = 5 \cdot 15° = 75°$

9. $5\pi/3 = 5 \cdot 60° = 300°$ **10.** $-135°$

11. $270°$ **12.** $150°$ **13.** $11\pi/6$

14. $9\pi/4$ **15.** $-5\pi/3$ **16.** $-7\pi/6$

17., 18.

θ deg	0	30	45	60	90	120	135	150	180
θ rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

19. $-\sqrt{2}/2$ **20.** $-1/2$ **21.** $\sqrt{3}$

22. $2\sqrt{3}/3$ **23.** $-2\sqrt{3}/3$ **24.** -1

25. 0 **26.** 0 **27.** 0 **28.** 2

29. -1 **30.** $-\sqrt{3}/3$ **31.** $\cot(60°) = \sqrt{3}/3$

32. $\sin(30°) = 1/2$ **33.** $-\sqrt{2}/2$ **34.** 1

35. -2 **36.** $-\sqrt{2}$ **37.** $-\sqrt{3}/3$ **38.** $-1/2$

39. 0.6947 **40.** 0.4226 **41.** -0.0923

42. -1.0000 **43.** 0.1869 **44.** -1.0538

45. $\dfrac{1}{\cos(105°4')} \approx -3.8470$ **46.** 1.7458

47. $\dfrac{1}{\sin(\pi/9)} \approx 2.9238$ **48.** 1.7793

49. $\dfrac{1}{\tan(33°44')} \approx 1.4975$ **50.** 0.2180

51. $45°$ **52.** $45°$ **53.** $0°$ **54.** $90°$

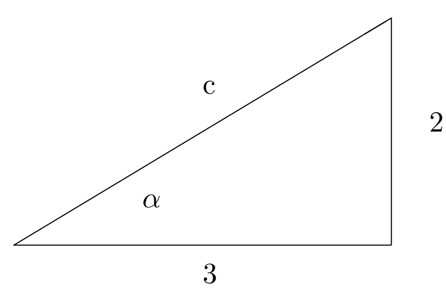
55. $30°$ **56.** $60°$ **57.** $30°$ **58.** $60°$

59. Note, the the length of the hypotenuse is 13.
Then $\sin(\alpha) = \text{opp}/\text{hyp} = 5/13$,
$\cos(\alpha) = \text{adj}/\text{hyp} = 12/13$,
$\tan(\alpha) = \text{opp}/\text{adj} = 5/12$,
$\csc(\alpha) = \text{hyp}/\text{opp} = 13/5$,
$\sec(\alpha) = \text{adj}/\text{hyp} = 13/12$, and
$\cot(\alpha) = \text{adj}/\text{opp} = 12/5$.

60. Note, the the length of a diagonal is $2\sqrt{13}$.

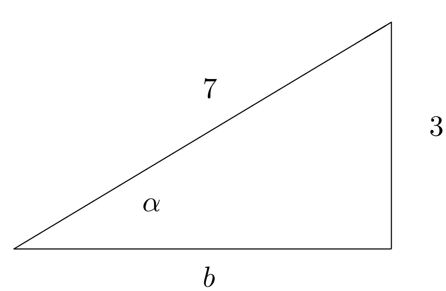
$$\sin(\alpha) = \frac{6}{2\sqrt{13}} = \frac{3\sqrt{13}}{13},$$
$$\cos(\alpha) = \frac{4}{2\sqrt{13}} = \frac{2\sqrt{13}}{13},$$
$$\tan(\alpha) = \frac{6}{4} = \frac{3}{2},$$
$$\csc(\alpha) = \frac{2\sqrt{13}}{6} = \frac{\sqrt{13}}{3},$$
$$\sec(\alpha) = \frac{2\sqrt{13}}{4} = \frac{\sqrt{13}}{2}, \text{ and}$$
$$\cot(\alpha) = \frac{4}{6} = \frac{2}{3}.$$

61. Form the right triangle with $a = 2$, $b = 3$.



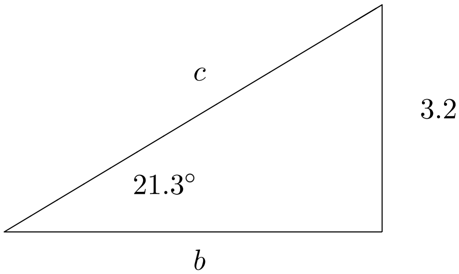
Note that $c = \sqrt{2^2 + 3^2} = \sqrt{13}$, $\tan(\alpha) = 2/3$, so $\alpha = \tan^{-1}(2/3) \approx 33.7°$ and $\beta \approx 56.3°$.

62. Form the right triangle with $a = 3$, $c = 7$.



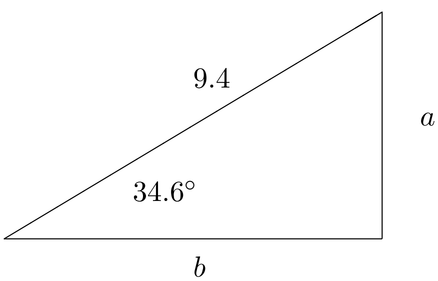
Note that $b = \sqrt{7^2 - 3^2} = 2\sqrt{10}$, $\sin(\alpha) = 3/7$, so $\alpha = \sin^{-1}(3/7) \approx 25.4°$ and $\beta \approx 64.6°$.

63. Form the right triangle with $a = 3.2$ and $\alpha = 21.3°$.



Since $\sin 21.3° = \dfrac{3.2}{c}$ and
$\tan 21.3° = \dfrac{3.2}{b}$, $c = \dfrac{3.2}{\sin 21.3°} \approx 8.8$
and $b = \dfrac{3.2}{\tan 21.3°} \approx 8.2$
Also, $\beta = 90° - 21.3° = 68.7°$

64. Form the right triangle with $c = 9.4$ and $\alpha = 34.6°$.



Since $\sin 34.6° = \dfrac{a}{9.4}$ and $\cos 34.6° = \dfrac{b}{9.4}$,
we get $a = 9.4 \cdot \sin 34.6° \approx 5.3$ and
$b = 9.4 \cdot \cos 34.6° \approx 7.7$
Also, $\beta = 90° - 34.6° = 55.4°$

65. $\sin(\alpha) = -\sqrt{1 - \left(\dfrac{1}{5}\right)^2} = -\sqrt{\dfrac{24}{25}} = \dfrac{-2\sqrt{6}}{5}$

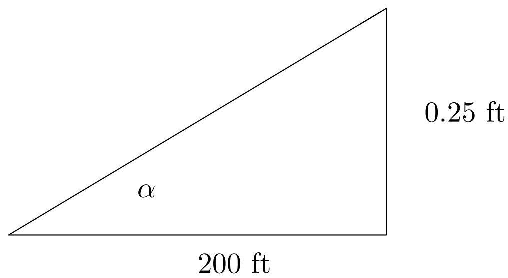
66. $\cos(\alpha) = -\sqrt{1 - \left(\dfrac{1}{3}\right)^2} = -\sqrt{\dfrac{8}{9}} = -\dfrac{2\sqrt{2}}{3}$

67. In one hour, the nozzle revolves through an angle of $\dfrac{2\pi}{8}$. The linear velocity is
$v = r \cdot \alpha = 120 \cdot \dfrac{2\pi}{8} \approx 94.2$ ft/hr.

68. Note 1 mile $= 5280 \cdot 12$ inches. Then
$\omega = \dfrac{s}{r} = \dfrac{16 \cdot 5280 \cdot 12}{13} \approx 77,981.5$ rad/hr.

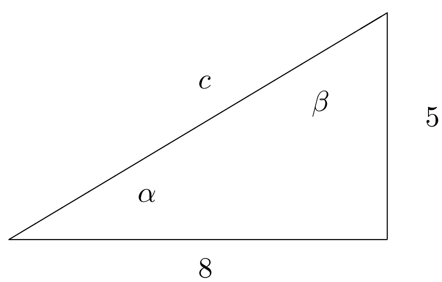
69. The height of the man is
$s = r \cdot \alpha = 1000(0.4) \cdot \dfrac{\pi}{180} \approx 6.9813$ ft.

70. Form the right triangle below.



Since $\tan(\alpha) = 0.25/200$, $\alpha = \tan^{-1}(0.25/200)$ $\approx 0.0716°$. She will not hit the target if she deviates by $0.1°$ from the center of the circle.

71. Form the right triangle below.

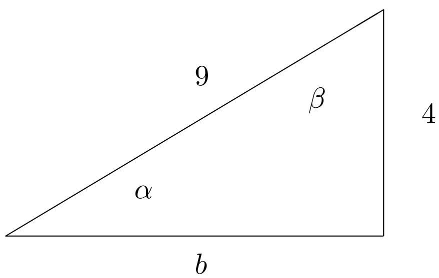


By the Pythagorean Theorem, we get

$$c = \sqrt{8^2 + 5^2} = \sqrt{89} \text{ ft}.$$

Note, $\alpha = \tan^{-1}\left(\dfrac{5}{8}\right) \approx 32.0°$ and
$\beta = 90° - \alpha \approx 58.0°$.

72. Form the right triangle below.

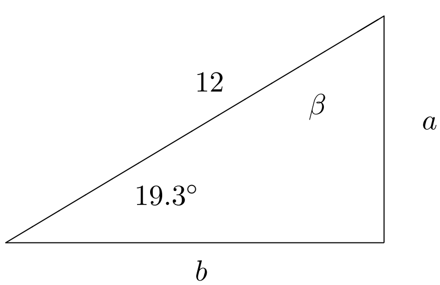


By the Pythagorean Theorem, we get

$$b = \sqrt{9^2 - 4^2} = \sqrt{65} \text{ in.}$$

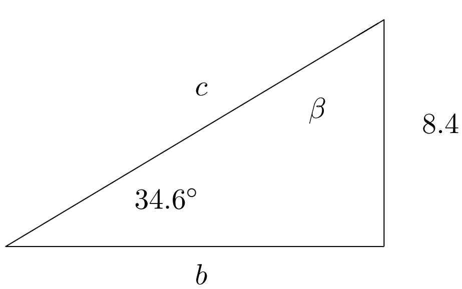
Note, $\alpha = \sin^{-1}\left(\dfrac{4}{9}\right) \approx 26.4^\circ$ and $\beta = 90^\circ - \alpha \approx 63.6^\circ$.

73. Form the right triangle below.



Note, $\beta = 90^\circ - 19.3^\circ = 70.7^\circ$.
Also, $a = 12\sin(19.3^\circ) \approx 4.0$ ft and $b = 12\cos(19.3^\circ) \approx 11.3$ ft.

74. Form the right triangle below.

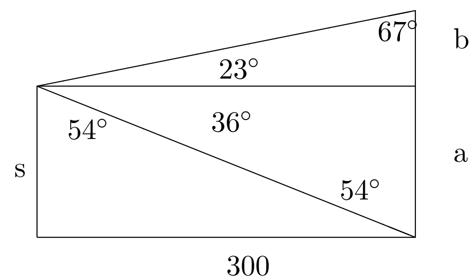


Note, $\beta = 90^\circ - 34.6^\circ = 55.4^\circ$.

Also, $c = \dfrac{8.4}{\sin 34.6^\circ} \approx 14.8$ m and

$b = \dfrac{8.4}{\tan 34.6^\circ} \approx 12.2$ m.

75. Let s be the height of the shorter building and let $a + b$ the height of the taller building.



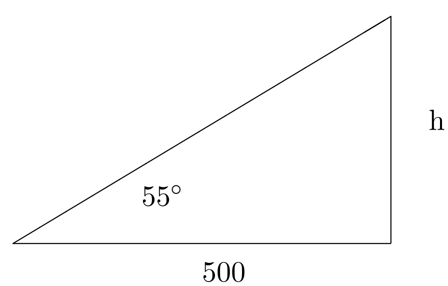
Since $\tan 54^\circ = \dfrac{300}{s}$, get

$$s = \frac{300}{\tan 54^\circ} \approx 218 \text{ ft.}$$

Similarly, since $\tan 36^\circ = \dfrac{a}{300}$ and $\tan 23^\circ = \dfrac{b}{300}$, the height of the taller building is

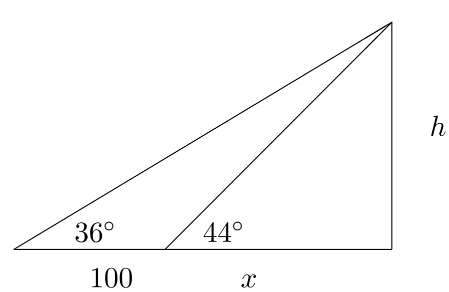
$$a + b = 300\tan 36^\circ + 300\tan 23^\circ \approx 345 \text{ ft.}$$

76. Let h be the height of the cloud cover.



Since $\tan 55^\circ = \dfrac{h}{500}$, $h = 500\tan 55^\circ \approx 714$ ft.

77. Let h be the height of the tower.



Since

$$h = (100 + x)\tan 36^\circ$$

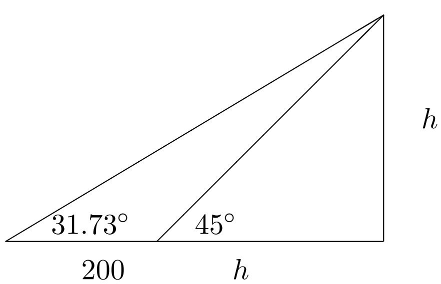
and

$$x = \frac{h}{\tan 44^\circ}$$

we find

$$\begin{aligned} h &= \left(100 + \frac{h}{\tan 44^\circ}\right)\tan 36^\circ \\ h &= 100\tan 36^\circ + h\frac{\tan 36^\circ}{\tan 44^\circ} \\ h &= \frac{100\tan 36^\circ}{1 - \frac{\tan 36^\circ}{\tan 44^\circ}} \\ h &\approx 293 \text{ ft} \end{aligned}$$

78. Let h be the height of the Eiffel tower.



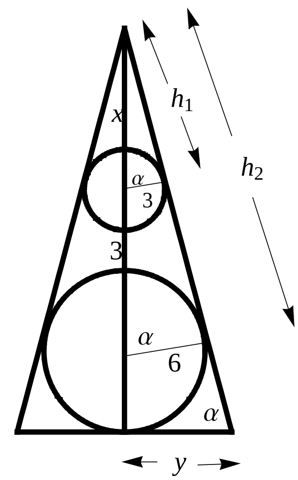
Since

$$\tan 31.73^\circ = \frac{h}{200 + h}$$

we obtain

$$\begin{aligned} (200 + h)\tan 31.73^\circ &= h \\ h\tan 31.73^\circ - h &= -200\tan 31.73^\circ \\ h &= \frac{-200\tan 31.73^\circ}{\tan 31.73^\circ - 1} \\ h &\approx 324 \text{ meters} \end{aligned}$$

79. Draw an isosceles triangle containing the two circles as shown below. The indicated radii are perpendicular to the sides of the triangle.



Using similar triangles, we obtain

$$\frac{x+3}{3} = \frac{x+15}{6}$$

from which we solve $x = 9$. Applying the Pythagorean theorem, we find

$$h_1 = 3\sqrt{15}, \quad h_2 = 6\sqrt{15}$$

Note, the height of the triangle is 30 ft. By similar triangles,

$$\frac{3\sqrt{15}}{3} = \frac{30}{y}$$

or $y = 2\sqrt{15}$. Thus, the base angle α of the isosceles triangle is $\alpha = \arctan\sqrt{15}$.

Then the arc lengths of the belt that wrap around the two pulleys are (using $s = r\alpha$)

$$s_1 = 6 \cdot (2\pi - 2\alpha)$$

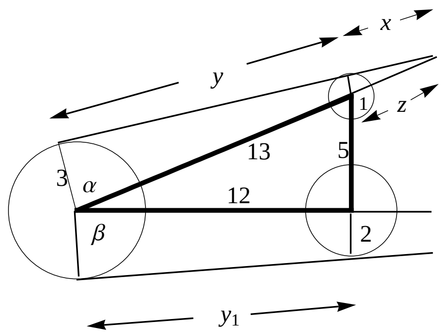
and

$$s_2 = 3 \cdot 2\alpha = 6\alpha.$$

Hence, the length of the belt is

$$\begin{aligned} s_1 + s_2 + 2(h_2 - h_1) &= \\ 12\pi - 6\arctan(\sqrt{15}) + 6\sqrt{15} &\approx \\ &53.0 \text{ in.} \end{aligned}$$

80. In the figure below, the radii are perpendicular to the belts.



By the Pythagorean theorem and similar triangles, we obtain

a) $1 + x^2 = z^2$

b) $3^2 + (x + y)^2 = (13 + z)^2$

c) $\frac{x}{1} = \frac{x + y}{3}$

Solving, we find $x = \sqrt{165}/2$, $y = \sqrt{165}$, and $z = 13/2$. Then $\alpha = \arctan(\sqrt{165}/2)$.

Repeating the above process, we find $y_1 = \sqrt{143}$ and $\beta = \arctan\sqrt{143}$.

Then the length of the belt that wraps around the circle of radius 3 inches is

$$s_3 = 3\,(2\pi - \alpha - \beta - \arctan(5/12))$$

We repeat the calculations.

Then the length y_2 of the belt between the points of tangency for the circles with radii 1 and 2 is $y_2 = 2\sqrt{6}$. Likewise, the length of the belt that wraps around the circle of radius 2 is

$$s_2 = 2\left(\beta + \frac{\pi}{2} - \arctan(2\sqrt{6})\right).$$

Also, the length of the belt that wraps around the circle of radius 1 is

$$s_1 = \alpha + \arctan(2\sqrt{6}) - \arctan\frac{12}{5}.$$

Finally, the total length of the belt is

$$s_3 + s_2 + s_1 + y + y_1 + y_2 \approx 43.6 \text{ in.}$$

81. Consider a right triangle whose height is h, the hypotenuse is 2 ft, and the angle between h and the hypotenuse is 18°. Then

$$h = 2\cos 18^\circ \approx 1.9 \text{ ft} = 22.8 \text{ in.}$$

82. Consider a right triangle with hypotenuse a, with an angle of 18°, and the side opposite 18° is $b/2$. Since $2a + b = 2$, we find

$$\begin{aligned} \sin 18^\circ &= \frac{b/2}{a} \\ \sin 18^\circ &= \frac{1-a}{a} \\ a &= \frac{1}{1+\sin 18^\circ} \text{ ft} \\ a &\approx 9.2 \text{ in.} \end{aligned}$$

Then $b = 2 - 2a \approx 5.7$ in.

83.

a) Note, a right triangle is formed when two diametrically oppposite points on a circle and a third point on the circle are chosen as vertices of a triangle.
Then the angle spanned by the sector is

$$\theta = \cos^{-1}\frac{x}{2}.$$

Since the area of the sector is $A = \frac{\theta r^2}{2}$, we find

$$A = \frac{x^2}{2}\cos^{-1}\left(\frac{x}{2}\right).$$

b) Note, the area of a circle with radius 1 is π. If the area of the sector in part a) is one-half the area of a circle with radius one, then

$$\frac{x^2}{2}\cos^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{2}.$$

Thus, the radius of the blade is either $x = \sqrt{2}$ ft or $x = \sqrt{3}$ ft.

c) Using a graphing calculator, we find that the area

$$A = \frac{x^2}{2}\cos^{-1}\left(\frac{x}{2}\right)$$

is maximized when $x \approx 1.5882$ ft.

84. The line $y = -x + 5$ intersects $y = x - 5$ at $(5, 0)$. The line $y = x+5$ is parallel to $y = x-5$. Then any line that intersect $y = x - 5$ in the first quadrant should be between $y = -x + 5$ and $y = x + 5$. The possible slopes of the lines that intersect $y = x - 5$ in the 1st quadrant is the interval $(-1, 1)$.

Chapter 1 Test

1. Since $60°$ is the reference angle, we get

$$\cos 420° = \cos(60°) = 1/2.$$

2. Since $30°$ is the reference angle, we get

$$\sin(-390°) = -\sin(30°) = -\frac{1}{2}.$$

3. $\frac{\sqrt{2}}{2}$ **4.** $\frac{1}{2}$ **5.** $\frac{\sqrt{3}}{3}$ **6.** $\sqrt{3}$

7. Undefined, since $\frac{1}{\cos(\pi/2)} = \frac{1}{0}$

8. $\frac{1}{\sin(-\pi/2)} = \frac{1}{-1} = -1$

9. Undefined, since $(-1, 0)$ lies on the terminal side of angle -3π and

$$\cot(-3\pi) = \frac{x}{y} = \frac{-1}{0}.$$

10. Since $(-1, -1)$ lies on the terminal side of angle $225°$, we get

$$\cot(225°) = \frac{x}{y} = \frac{-1}{-1} = 1.$$

11. $\frac{\pi}{4}$ or $45°$ **12.** $\frac{\pi}{6}$ or $30°$

13. Since $46°24'6'' \approx 0.8098619$, the arclength is $s = r\alpha = 35.62(0.8098619) \approx 28.85$ meters.

14. $2.34 \cdot \frac{180°}{\pi} \approx 134.07°$

15. Coterminal since $2200° - 40° = 2160° = 6 \cdot 360°$.

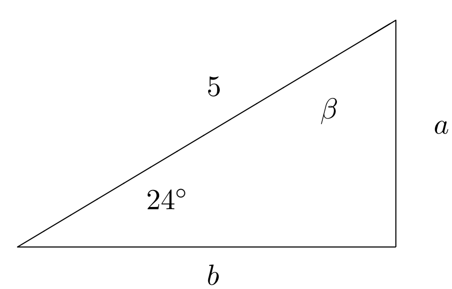
16. $\cos(\alpha) = -\sqrt{1 - \left(\frac{1}{4}\right)^2} = -\frac{\sqrt{15}}{4}$

17. $\omega = 103 \cdot 2\pi \approx 647.2$ radians/minute

18. In one minute, the wheel turns through an arclength of $13(103 \cdot 2\pi)$ inches. Multiplying this by $\frac{60}{12 \cdot 5280}$ results in the speed in mph which is 7.97 mph.

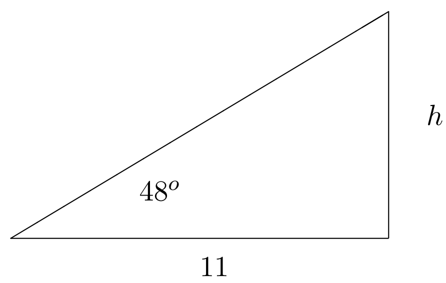
19. Since $r = \sqrt{x^2 + y^2} = \sqrt{5^2 + (-2)^2} = \sqrt{29}$, we find $\sin\alpha = \frac{y}{r} = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$,
$\cos\alpha = \frac{x}{r} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$,
$\tan\alpha = \frac{y}{x} = \frac{-2}{5}$,
$\csc\alpha = \frac{r}{y} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$,
$\sec\alpha = \frac{r}{x} = \frac{\sqrt{29}}{5}$, and
$\cot\alpha = \frac{x}{y} = \frac{5}{-2} = -\frac{5}{2}$.

20. Consider the right triangle below.



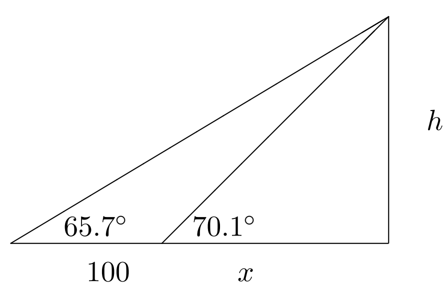
Note, $\beta = 90° - 24° = 66°$.
Then $a = 5\sin(24°) \approx 2.0$ ft and $b = 5\cos(24°) \approx 4.6$ ft.

21. Let h be the height of the head.



Since
$$\tan(48^o) = \frac{h}{11}$$
we find
$$h = 11 \tan 48^o \approx 12.2 \quad \text{m.}$$

22. Let h be the height of the building.



Since $\tan 70.1^o = \dfrac{h}{x}$ and $\tan 65.7^\circ = \dfrac{h}{100+x}$,
we obtain

$$\tan(65.7^\circ) = \frac{h}{100 + h/\tan(70.1^\circ)}$$

$$100 \cdot \tan(65.7^\circ) + h \cdot \frac{\tan(65.7^o)}{\tan(70.1^\circ)} = h$$

$$100 \cdot \tan(65.7^\circ) = h\left(1 - \frac{\tan(65.7^\circ)}{\tan(70.1^\circ)}\right)$$

$$h \quad = \quad \frac{100 \cdot \tan(65.7^\circ)}{1 - \tan(65.7^\circ)/\tan(70.1^\circ)}$$

$$h \quad \approx \quad 1117 \text{ ft.}$$