Instructors' Manual to accompany

THEORY OF MACHINES AND MECHANISMS

Fifth Edition

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PART 1

KINEMATICS AND MECHANISMS

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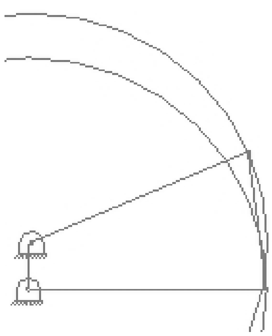
Chapter 1
The World of Mechanisms

1.1 Sketch at least six different examples of the use of a planar four-bar linkage in practice. These can be found in the workshop, in domestic appliances, on vehicles, on agricultural machines, and so on.

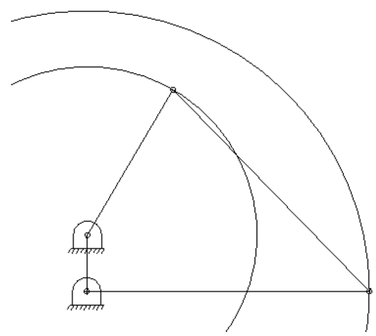
Since the variety is unbounded no standard solutions are provided here.

1.2 The link lengths of a planar four-bar linkage are 1 in, 3 in, 5 in, and 5 in. Assemble the links in all possible combinations and sketch the four inversions of each. Do these linkages satisfy Grashof's law? Describe each inversion by name, for example, a crank-rocker linkage or a drag-link linkage.

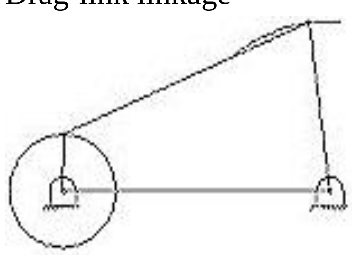
$s = 1$ in, $l = 5$ in, $p = 3$ in, $q = 5$ in; these linkages all satisfy Grashof's law since $1 \text{ in} + 5 \text{ in} < 3 \text{ in} + 5 \text{ in}$. *Ans.*



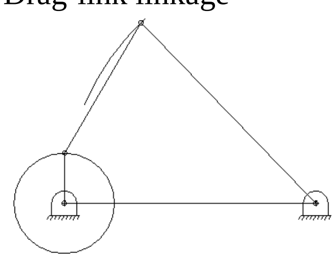
Drag-link linkage



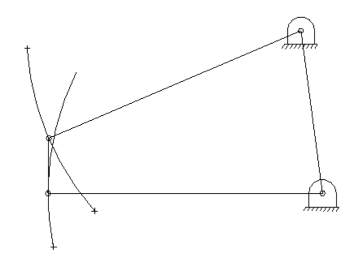
Drag-link linkage *Ans.*



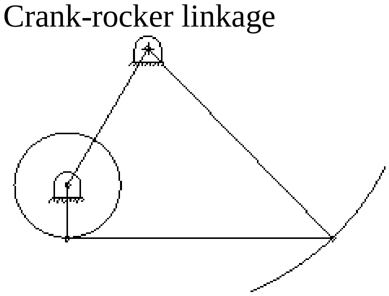
Crank-rocker linkage



Crank-rocker linkage *Ans.*

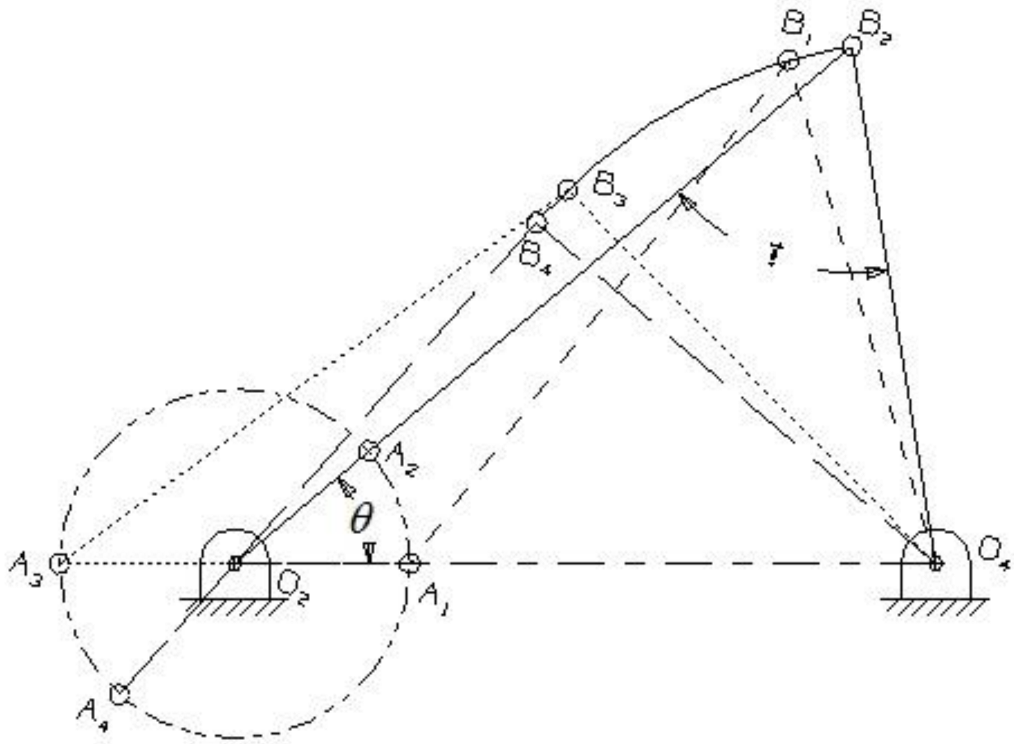


Double-rocker linkage.



Crank-rocker linkage *Ans.*

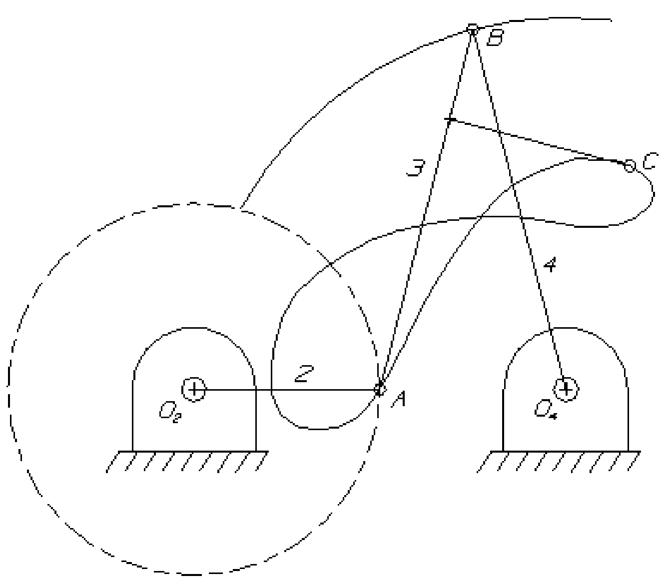
1.3 A crank-rocker linkage has a 100-mm frame, a 25-mm crank, a 90-mm coupler, and a 75-mm rocker. Draw the linkage and find the maximum and minimum values of the transmission angle. Locate both toggle postures and record the corresponding crank angles and transmission angles.



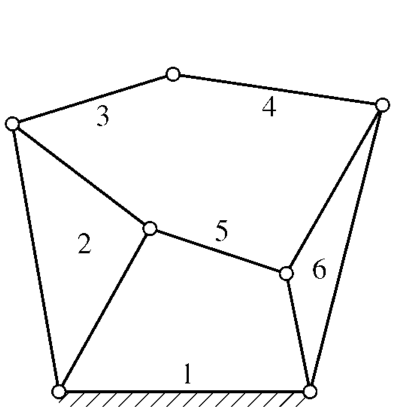
Extremum transmission angles: $\gamma_{min} = \gamma_1 = 53.1°$; $\gamma_{max} = \gamma_3 = 98.1°$ *Ans.*

Toggle postures: $\theta_2 = 40.1°; \gamma_2 = 59.1°$; $\theta_4 = 228.6°; \gamma_4 = 90.9°$ *Ans.*

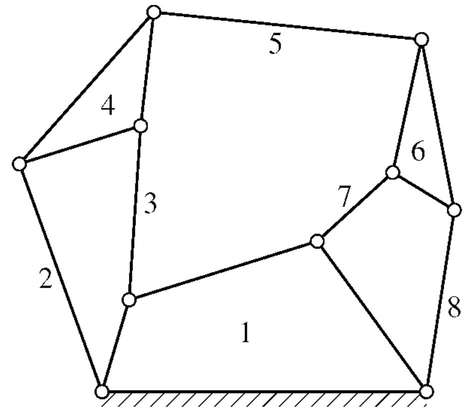
1.4 Plot the complete path of coupler point *C*.



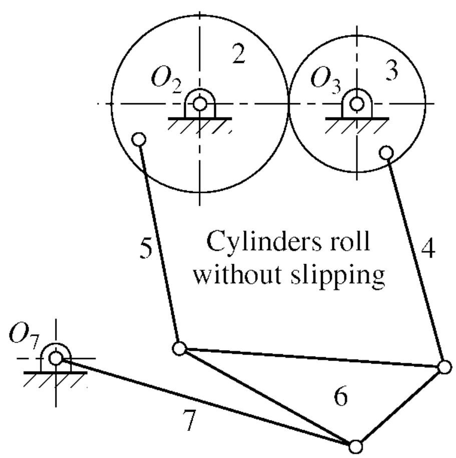
1.5 Find the mobility of each mechanism.



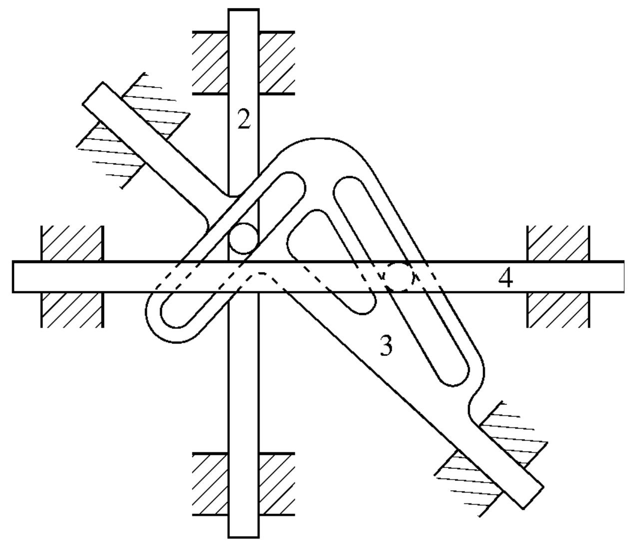
(*a*)



(*b*)



(*c*)



(*d*)

(*a*) $n = 6,\ j_1 = 7,\ j_2 = 0;$ $m = 3(6-1) - 2(7) - 1(0) = 1$ <u>*Ans.*</u>

(*b*) $n = 8,\ j_1 = 10,\ j_2 = 0;$ $m = 3(8-1) - 2(10) - 1(0) = 1$ <u>*Ans.*</u>

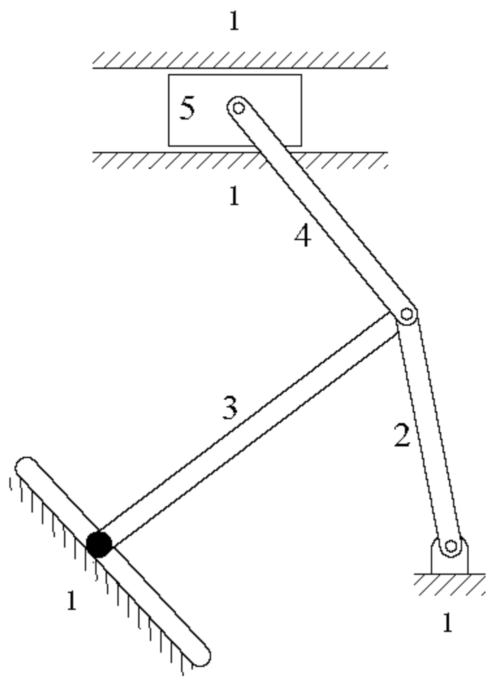
(*c*) $n = 7,\ j_1 = 9,\ j_2 = 0;$ $m = 3(7-1) - 2(9) - 1(0) = 0$ <u>*Ans.*</u>

Note that the Kutzbach criterion fails in the case of part (*c*); the true mobility is m=1. The exception is due to a redundant constraint. The assumption that the rolling contact joint does not allow links 2 and 3 to separate duplicates the constraint of the fixed link length O_2O_3.

(*d*) $n = 4,\ j_1 = 3,\ j_2 = 2;$ $m = 3(4-1) - 2(3) - 1(2) = 1$ <u>*Ans.*</u>

Note in part (*d*) that each pair of coaxial sliding ground joints is counted as only a single prismatic pair.

1.6 Use the Kutzbach criterion to determine the mobility of the mechanism.

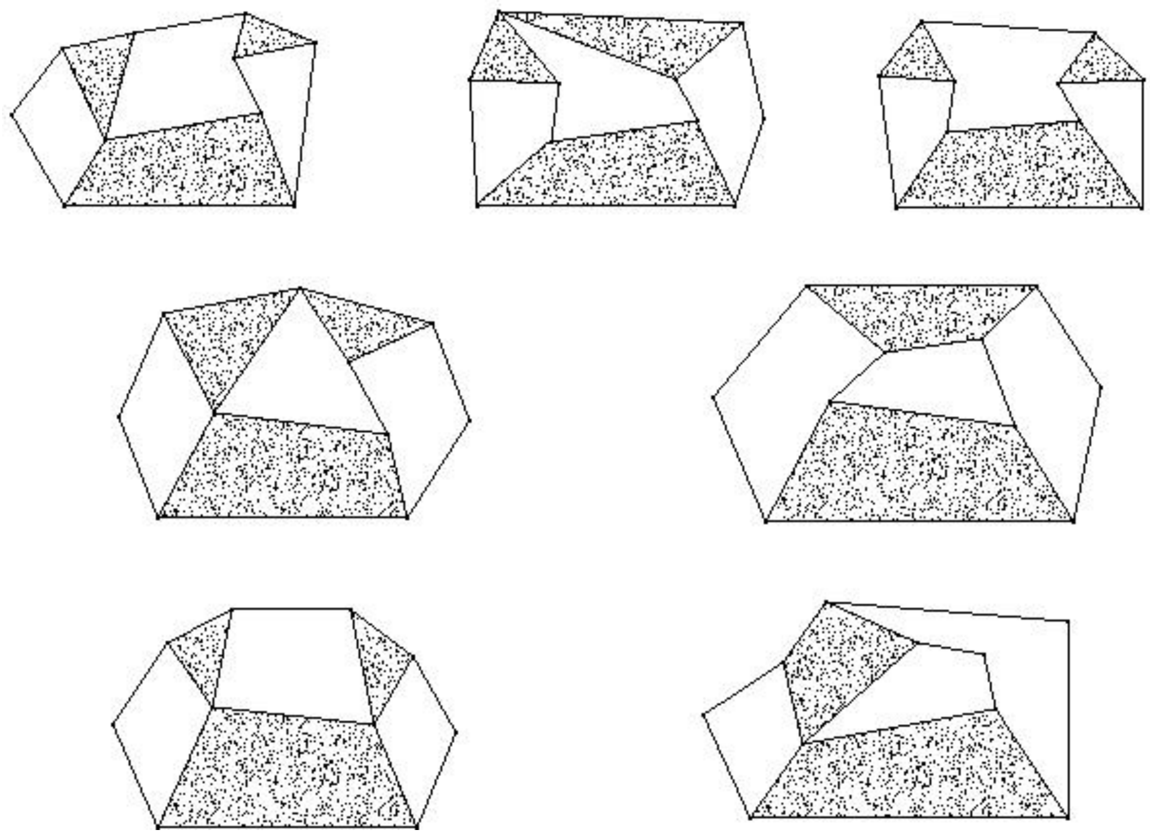


$n = 5,\ j_1 = 5,\ j_2 = 1;$ $m = 3(5-1) - 2(5) - 1(1) = 1$ ***Ans.***

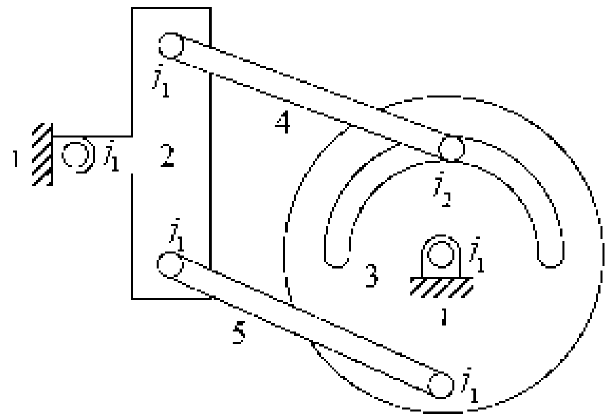
Note that the double pin is counted as two single pin j_1 joints.

1.7 Sketch a planar linkage with only revolute joints and a mobility of m=1 that contains a moving quaternary link. How many distinct variations of this linkage can you find?

To have at least one quaternary link, a planar linkage must have at least eight links. The Kutzbach criterion then indicates that ten single-freedom joints are required for mobility of m = 1. According to H. Alt, 1955. "Die Analyse und Synthese der achtgleidrigen Gelenkgetriebe", *VDI-Berichte*, **5**, pp. 81-93, there are a total of sixteen distinct eight-link planar linkages having ten revolute joints, seven of which contain a quaternary link. These seven are illustrated here: *Ans.*

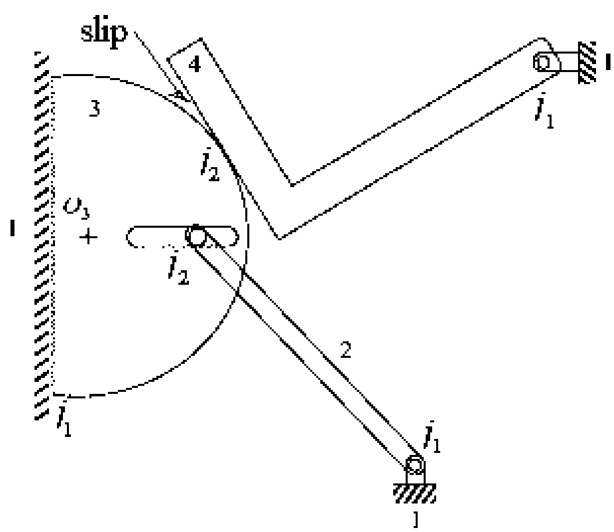


1.8 Use the Kutzbach criterion to detemine the mobility of the mechanism. Clearly number each link and label the lower pairs (j_1 joints) and higher pairs (j_2 joints).



$n = 5,\ j_1 = 5,\ j_2 = 1;$ $m = 3(5-1) - 2(5) - 1(1) = 1$ *Ans.*

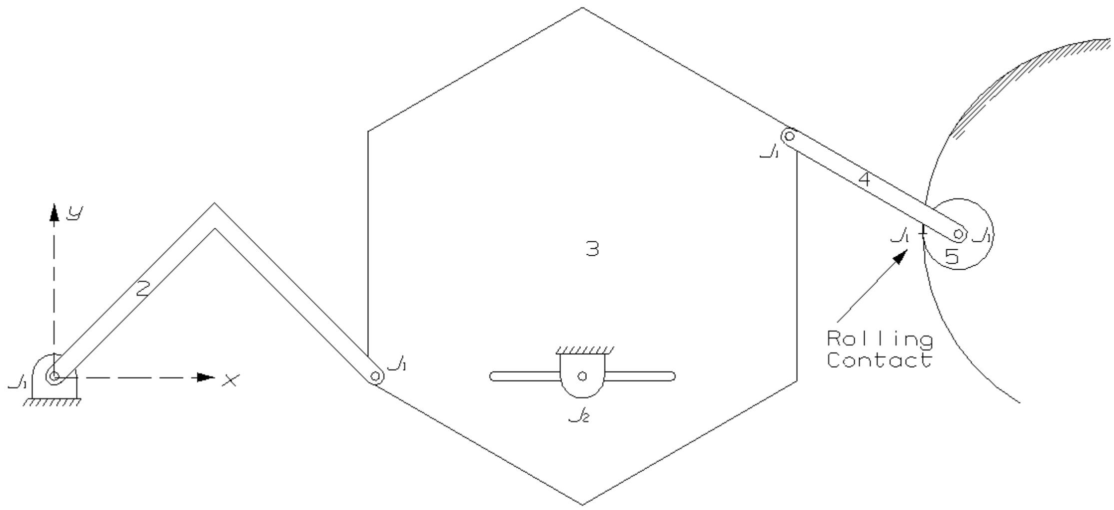
1.9 Determine the number of links, the number of lower pairs, and the number of higher pairs. Use the Kutzbach criterion to determine the mobility of the mechanism. Is the answer correct? Briefly explain.



$n = 4,\ j_1 = 3,\ j_2 = 2;$ $m = 3(4-1) - 2(3) - 1(2) = 1$ *Ans.*

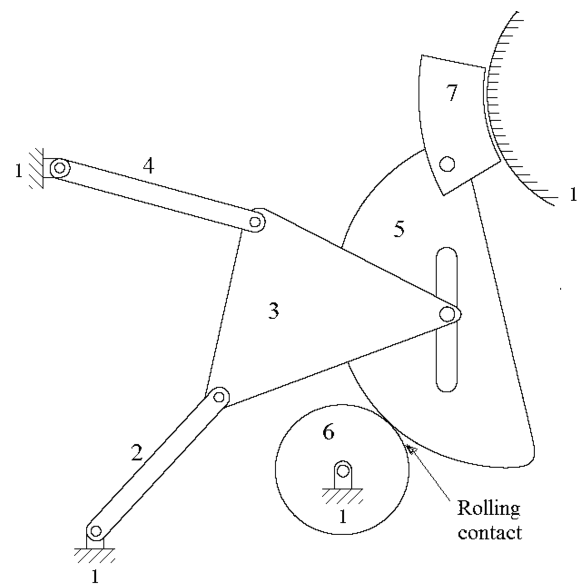
If it is not evident visually that link 3 can be incremented upward without jamming, then consider incrementing link 3 downward. Since it is clear visually that this determines the position of all other links, this verifies that mobility of one is correct.

1.10 Use the Kutzbach criterion to detemine the mobility of the mechanism. Clearly number each link and label the lower pairs and higher pairs.



$n = 5,\ j_1 = 5,\ j_2 = 1;$ $m = 3(5-1) - 2(5) - 1(1) = 1$ *Ans.*

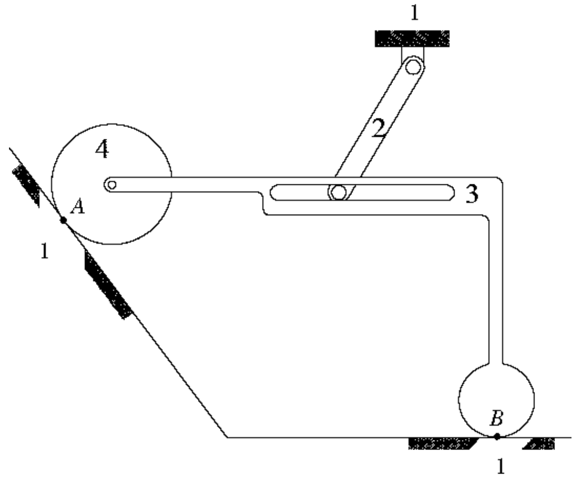
1.11 Determine the number of links, the number of lower pairs, and the number of higher pairs. Treat rolling contact to mean rolling with no slipping. Using the Kutzbach criterion determine the mobility. Is the answer correct? Briefly explain.



$n = 7,\ j_1 = 8,\ j_2 = 1;$ $m = 3(7-1) - 2(8) - 1(1) = 1$ *Ans.*

This result appears to be correct. If all parts remain assembled (connected), then within the limits of travel of the joints illustrated, it appears that when any one joint is locked the total system becomes a structure.

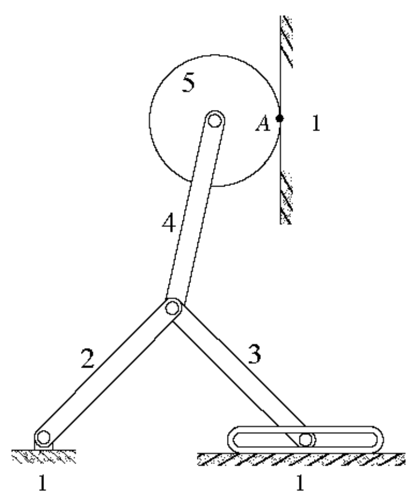
1.12 Does the Kutzbach criterion provide the correct result for this mechanism? Briefly explain why or why not.



$n = 4,\ j_1 = 2,\ j_2 = 3;$ $m = 3(4-1) - 2(2) - 1(3) = 2$ *Ans.*

The joints at *A* and *B* are both assumed to allow slipping and are j_2 joints. This results in $m = 2$ which appears to be correct. If any part except wheel 4 is moved, all other parts are required to follow. However, after all other parts are in a certain posture, wheel 4 is still able to rotate while slipping against the frame at *A*.

1.13 The mobility of the mechanism is $m = 1$. Use the Kutzbach criterion to determine the number of lower pairs and the number of higher pairs. Is the wheel rolling without slipping, or rolling and slipping, at point *A* on the wall?

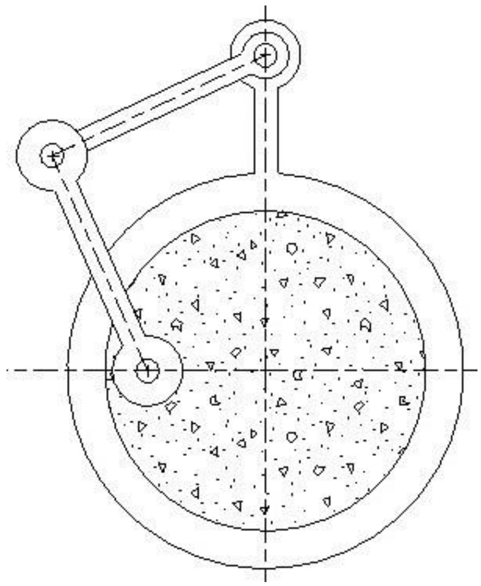


Suppose that we identify the number of independent freedoms at *A* by the symbol *k*. Then if we account for all links and all other joints as follows, the Kutzbach criterion gives

$n = 5;\ j_1 = 4;\ j_2 = 1;\ j_k = 1;$ $m = 3(5-1) - 2(4) - 1(1) - (3-k)(1) = k;$

Therefore, to have mobility of $m = 1$, we must have $k = 1$ independent freedom at *A*. The wheel must be rolling without slipping. Then, $n = 5;\ j_1 = 5;\ j_2 = 1;$ and $m = 1;$ *Ans.*

1.14 Devise a practical working model of the drag-link linkage



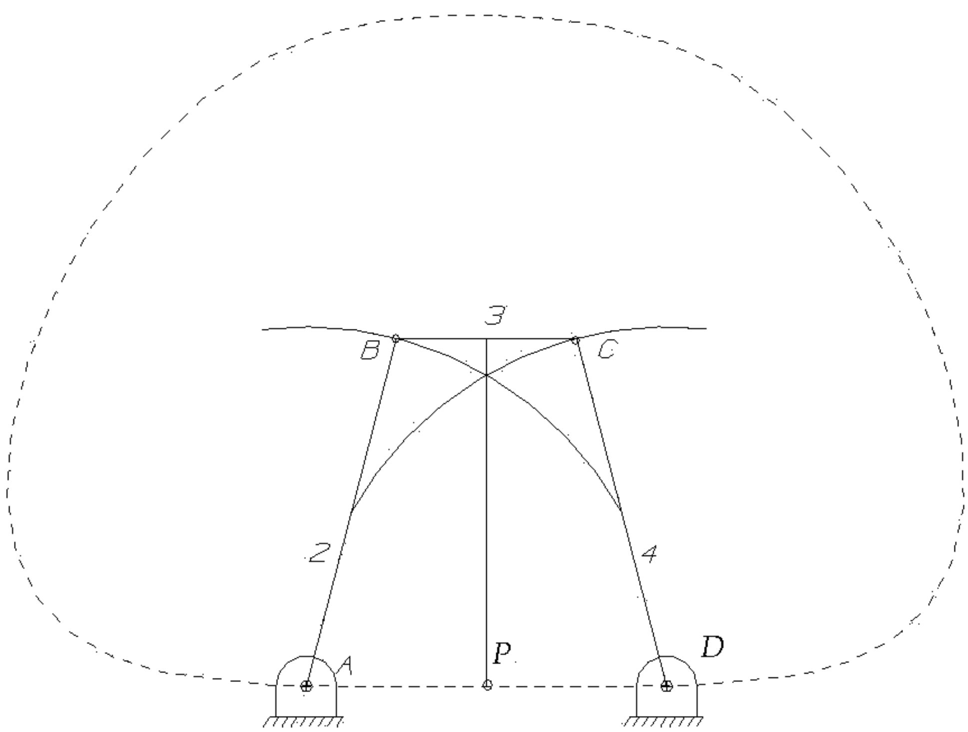
Ans.

1.15 Find the advance-to-return ratio of the linkage of Prob. 1.3.

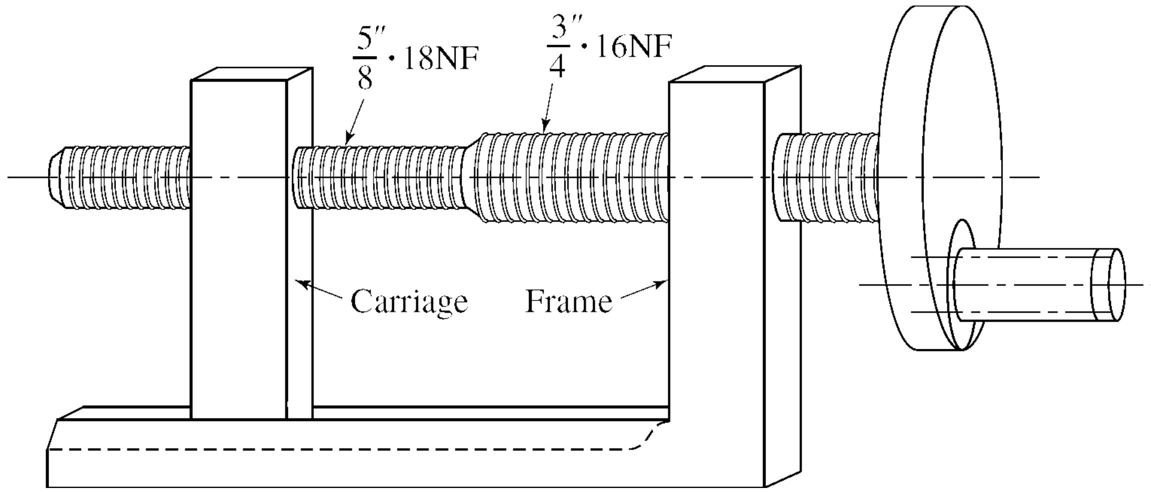
From the values of θ_2 and θ_4 we find $\alpha = 188.5°$ and $\beta = 171.5°$.

Then, from Eq. (1.5), $Q = \alpha/\beta = 1.099$. *Ans.*

1.16 Plot the complete coupler curve of Roberts' linkage illustrated in Fig. 1.24*b*. Use *AB* = *CD* = *AD* = 2.5 in and *BC* = 1.25 in.

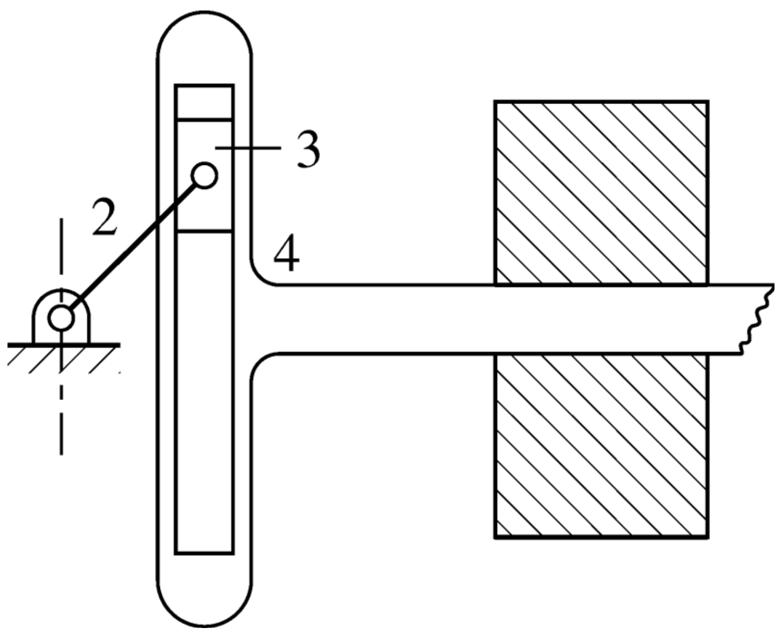


1.17 If the handle of the differential screw in Fig. 1.11 is turned 15 revolutions clockwise, how far and in what direction does the carriage move?



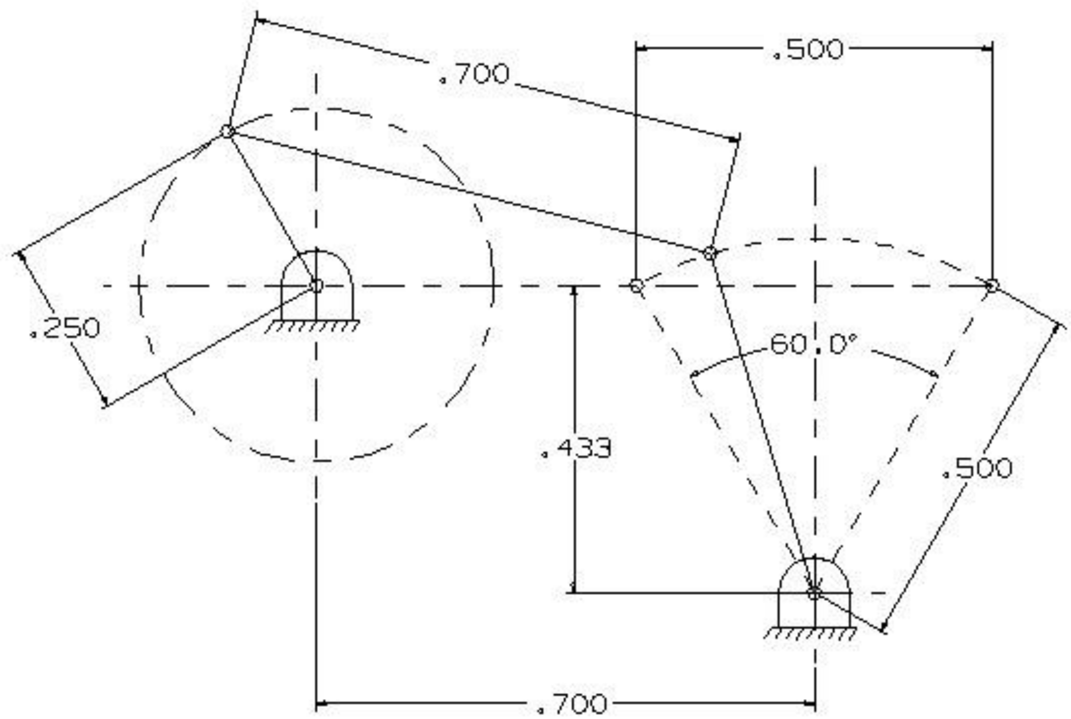
Screw and carriage move by (15 rev)/(16 rev/in) = 0.937 50 in to the left.
Carriage moves (15 rev)/(18 rev/in) = 0.833 33 in to the right with respect to the screw.
Net motion of carriage = 15/16 in – 15/18 in = 15/144 = 0.104 17 in to the left. *Ans.*

1.18 Show how the linkage of Fig. 1.15*b* can be used to generate a sine wave.



With the length and angle of crank 2 designated as R and θ_2, respectively, the horizontal motion of link 4 is $x_4 = R\cos\theta_2 = R\sin(\theta_2 + 90°)$.

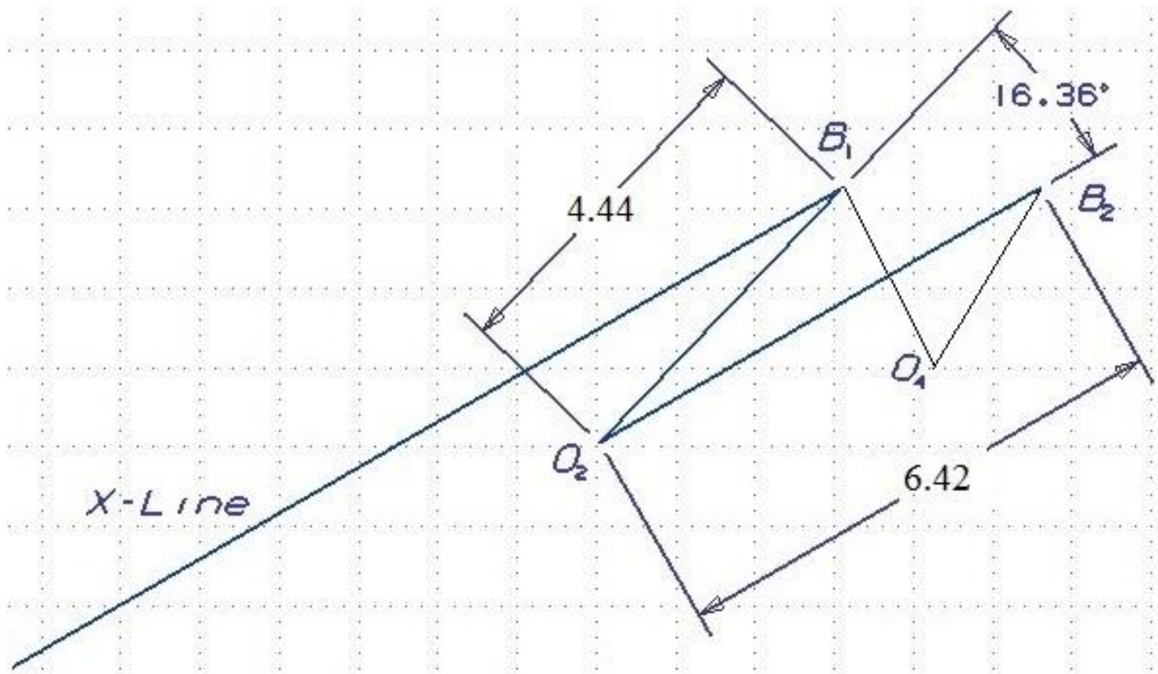
1.19 Devise a crank-rocker four-bar linkage, as in Fig. 1.14*c*, having a rocker angle of 60°. The rocker length is to be 0.50 m.



Distances shown are in meters. *Ans.*

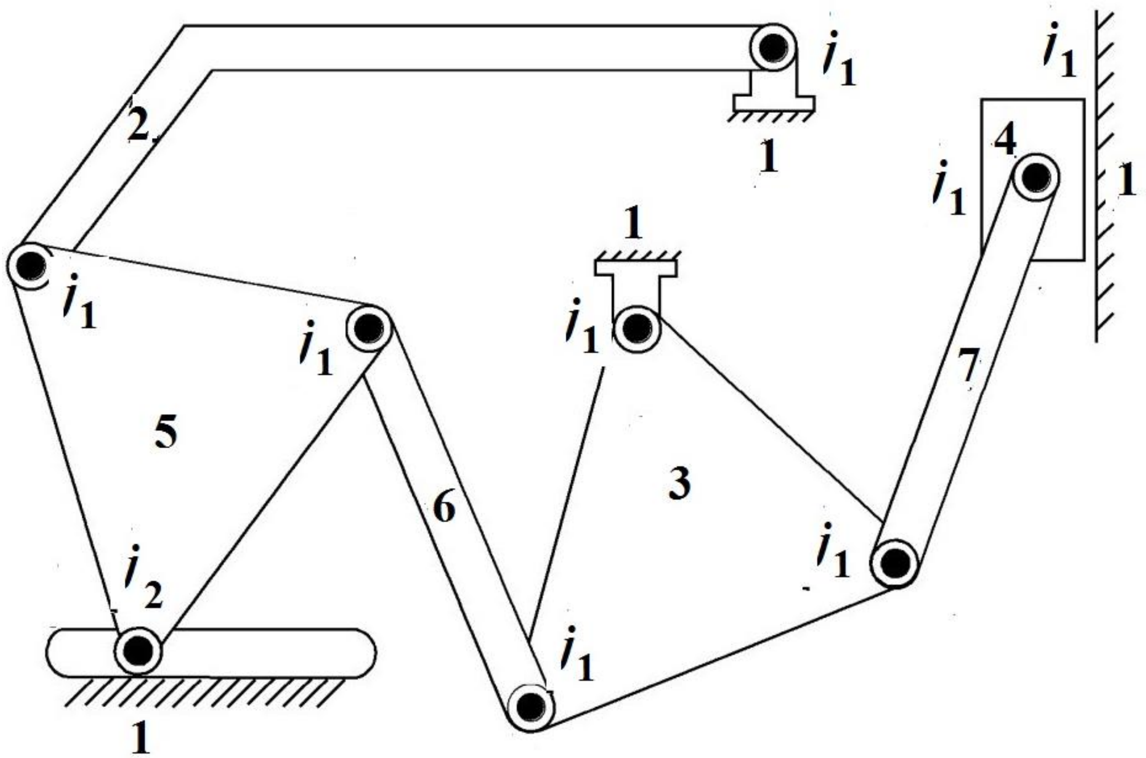
1.20 A crank-rocker four-bar linkage is required to have an advance-to-return ratio Q = 1.2. The rocker is to have a length of 2.5 in and oscillate through a total angle of 60°. Determine a suitable set of link lengths for the remaining three links of the four-bar linkage.

Following the procedure of Example 1.4, the required advance-to-return ratio gives $Q = (180° + \phi)/(180° - \phi) = 1.2$ and, therefore, we must have $\phi = 16.36°$. Then, with the X-line chosen at 30°, the drawing shown below gives measured distances of $R_{O_4O_2} = r_1 = 4.34$ in, $R_{B_2O_2} = r_3 + r_2 = 6.42$ in, and $R_{B_1O_2} = r_3 - r_2 = 4.44$ in. From these we get one possible solution, which has link lengths of $R_{O_4O_2} = r_1 = 4.34$ in, $R_{AO_2} = r_2 = 0.99$ in, $R_{BA} = r_3 = 5.43$ in, and $R_{BO_4} = r_4 = 2.50$ in. *Ans.*



Distances shown are in inches

1.21 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 7$, $j_1 = 8$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

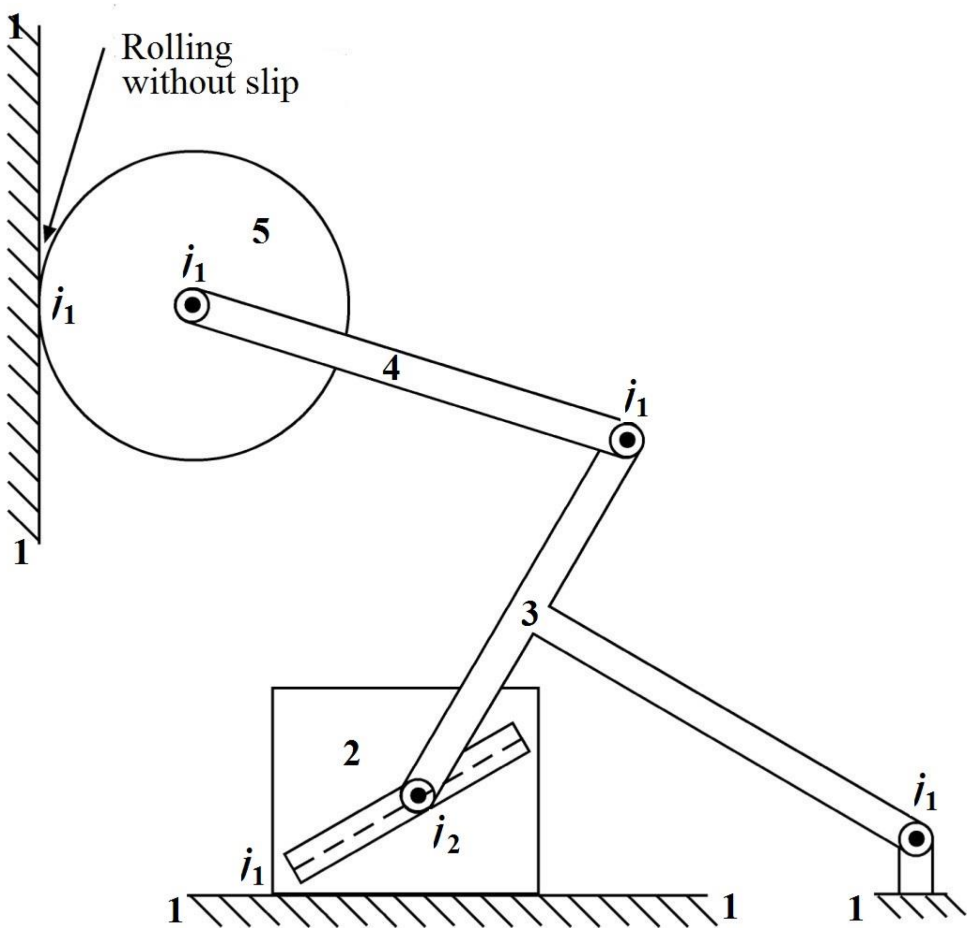
$$m = 3(7-1) - 2(8) - 1(1) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 3 would be suitable as inputs since these are pinned to the ground link. Translation of the slider (link 4) would also be suitable as an input. Other choices for the input are not particularly practical. *Ans.*

1.22 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 5 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$$n = 5,\ j_1 = 5,\text{ and } j_2 = 1.$$

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

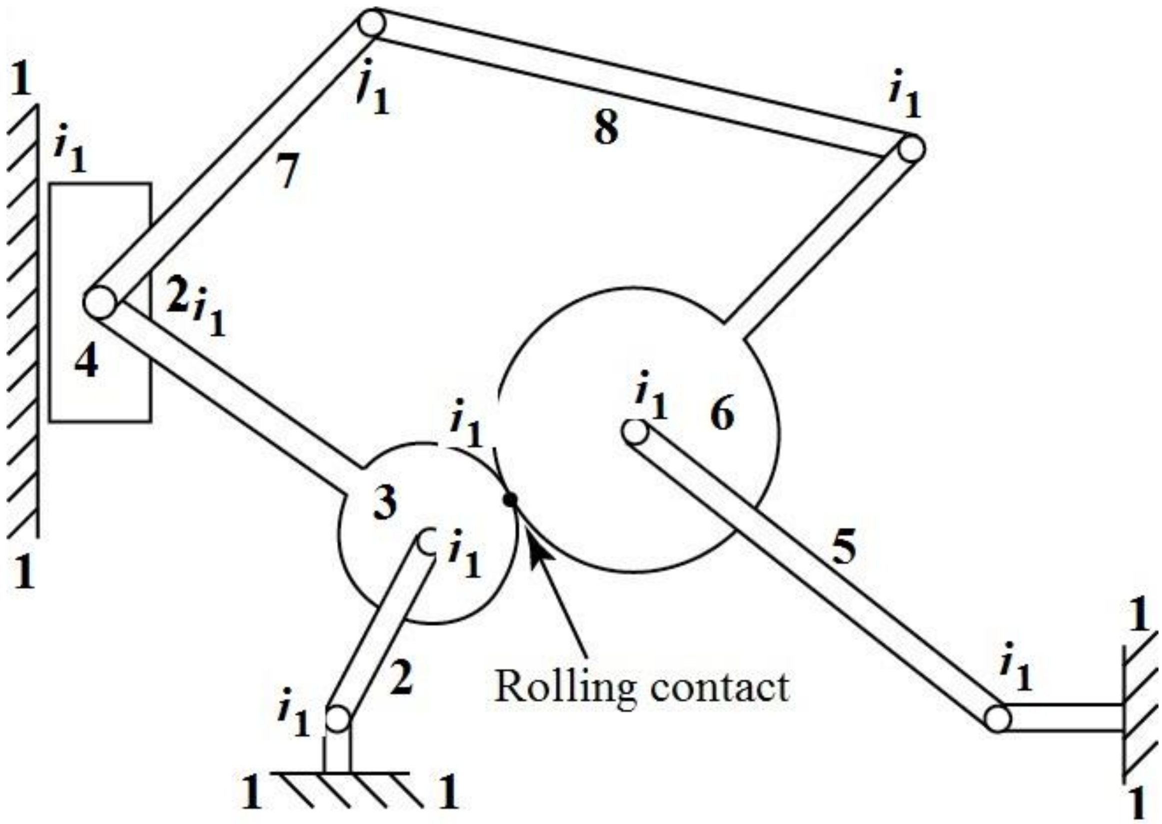
$$m = 3(5-1) - 2(5) - 1(1) = 1$$ *Ans.*

This is the correct answer for this mechanism, that is, for a single input value there is a unique posture.

Rotation of link 3 would be a suitable input since it is pinned to the ground link. Translation of the slider (link 2) would also be suitable as an input. Other choices for the input are not particularly practical. *Ans.*

1.23 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 5 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 8$, $j_1 = 10$, and $j_2 = 0$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

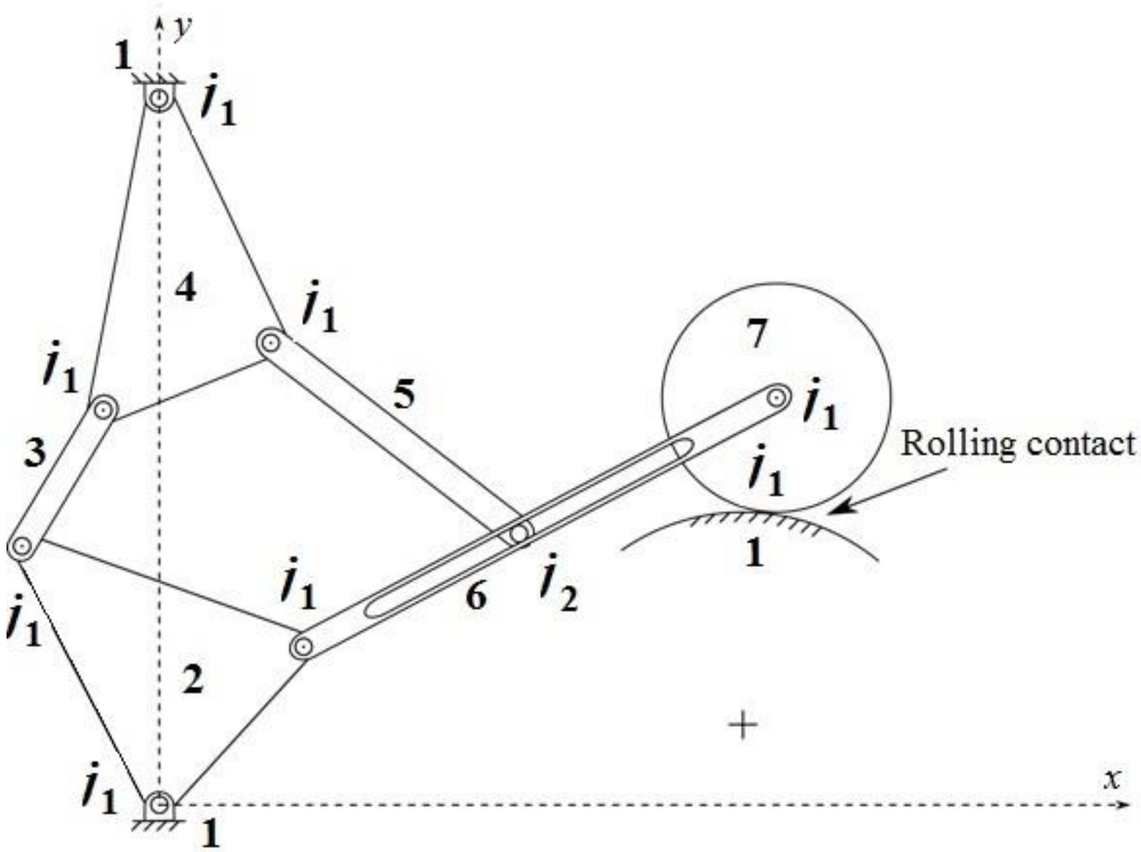
$$m = 3(8-1) - 2(10) - 1(0) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 5 would be suitable inputs since they are pinned to the ground link. Translation of the slider (link 4) would also be a suitable input. Other choices for the input are not particularly practical. *Ans.*

1.24 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 7$, $j_1 = 8$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

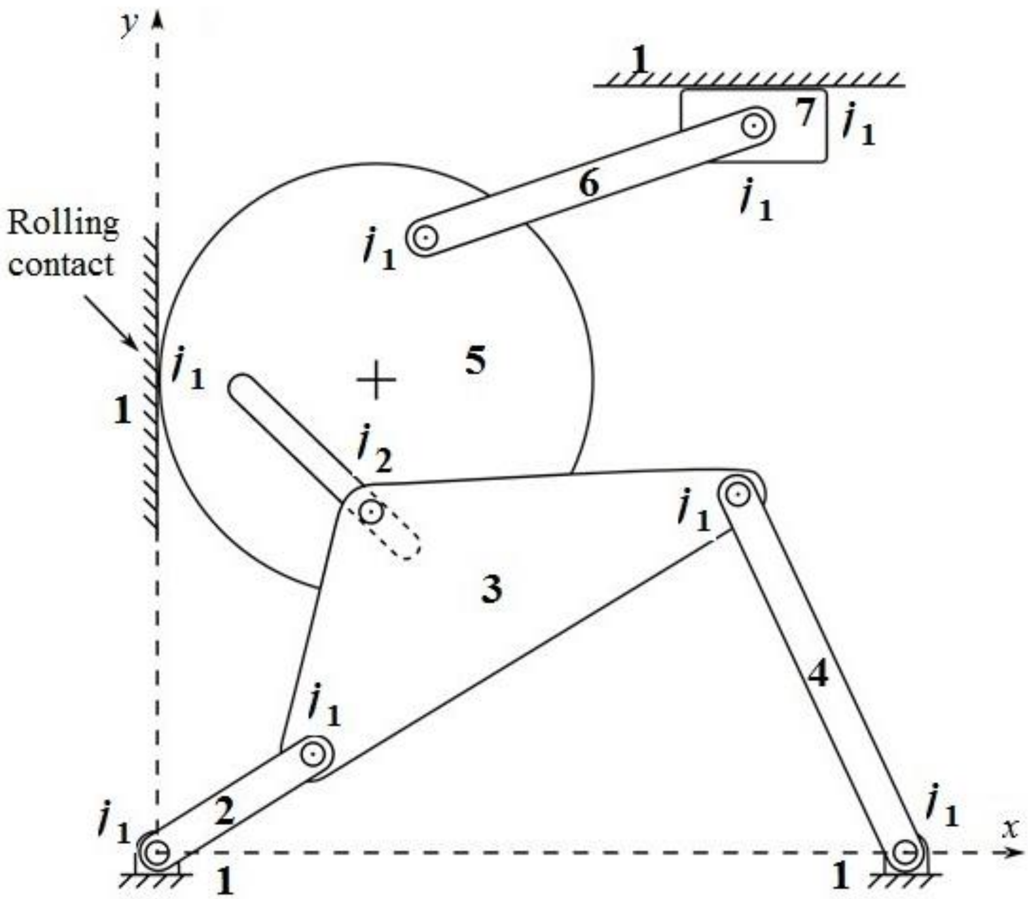
$$m = 3(7-1) - 2(8) - 1(1) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 4 would be suitable inputs since they are pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.25 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are illustrated in the figure below. *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$$n = 7,\ j_1 = 8,\ \text{and}\ j_2 = 1.$$

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

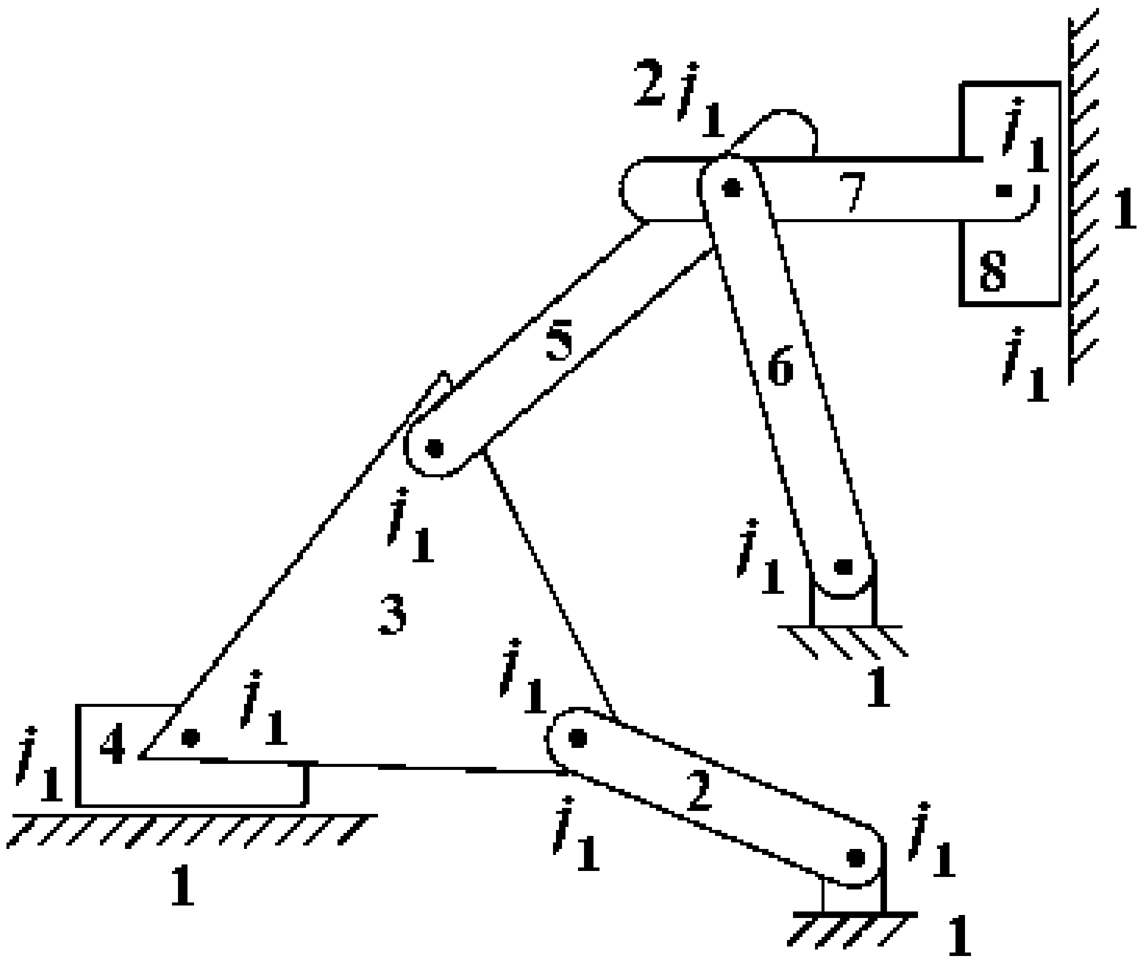
$$m = 3(7-1) - 2(8) - 1(1) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 4 would be suitable inputs since they are pinned to the ground link. Translation of the slider (link 7) would also be suitable as an input. Other choices for the input are not particularly practical. *Ans.*

1.26 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 8 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 8$, $j_1 = 10$, and $j_2 = 0$.

Note that the double-pin joint between links 5, 6, and 7 is counted as 2 j_1 joints.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

$$m = 3(8-1) - 2(10) - 1(0) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

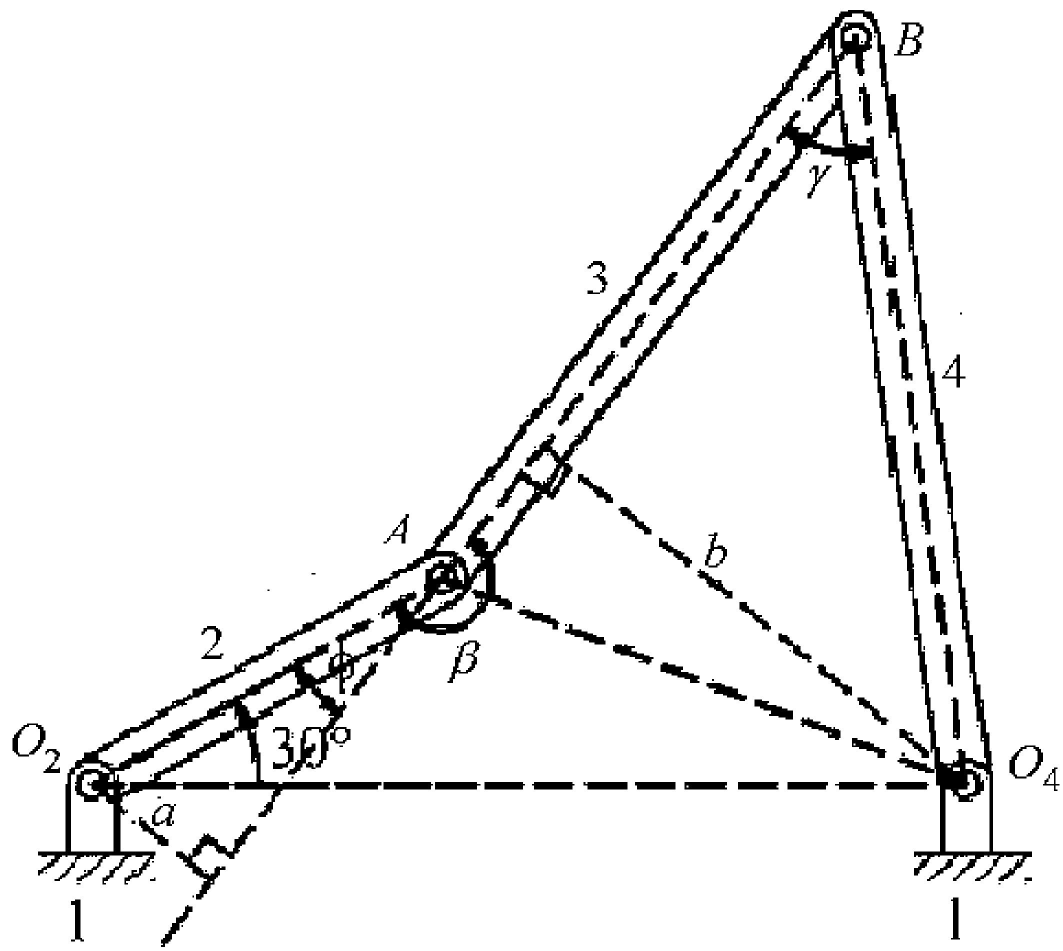
Rotation of either link 2 or link 6 would be suitable inputs since they are pinned to the ground link. Translation of either of the sliders (link 4 or link 8) would also be suitable as inputs. Other choices for the input are not particularly practical. *Ans.*

1.27 Determine the mechanical advantage of the four-bar linkage in the posture illustrated.

The mechanical advantage, Eq. (3.41), can be written as

$$MA = \frac{b}{a} = \frac{R_{BO_4}\sin\gamma}{R_{AO_2}\sin\phi} = -\frac{R_{BO_4}\sin\gamma}{R_{AO_2}\sin\beta} \tag{1}$$

where the angles β, γ, and ϕ are as shown in Fig. P1.27.



$O_2O_4 = 120$ mm, $O_2A = 60$ mm, $AB = 100$ mm, and $O_4B = 130$ mm.

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To determine the angles β, ϕ, and γ, the law of cosines for the triangle O_2AO_4 can be written as

$$\overline{AO_4}^2 = R_{O_4O_2}^2 + R_{AO_2}^2 - 2R_{O_4O_2}R_{AO_2}\cos(\measuredangle O_4O_2A)$$

$$= (120\text{ mm})^2 + (60\text{ mm})^2 - 2(120\text{ mm})(60\text{ mm})\cos 30° = 5529.234\ \mu\text{m}^2$$

Therefore,

$$\overline{AO_4} = 74.359\text{ mm}$$

The law of sines for the triangle O_2AO_4 can be written as

$$\frac{\overline{AO_4}}{\sin(\measuredangle AO_2O_4)} = \frac{R_{O_2O_4}}{\sin(\measuredangle O_2AO_4)}$$

Rearranging this equation gives

$$\sin(\measuredangle O_2AO_4) = \frac{R_{O_2O_4}}{AO_4}\sin(\measuredangle AO_2O_4) = \frac{120 \text{ mm}}{74.359 \text{ mm}}\sin 30° = 0.8069$$

Therefore, the angle is either

$\measuredangle O_2AO_4 = 53.79°$ or $\measuredangle O_2AO_4 = 126.21°$

Note that $\measuredangle O_2AO_4 = 53.79°$ can be eliminated since it is not a physically possible result for the open configuration of the four-bar linkage. Therefore, the correct result for the angle is

$$\measuredangle O_2AO_4 = 126.21° \tag{2}$$

The law of cosines for the triangle ABO_4 can be written as

$$R_{BO_4}^2 = R_{BA}^2 + \overline{AO_4}^2 - 2R_{BA}\overline{AO_4}\cos(\measuredangle O_4AB)$$

Rearanging this equation gives

$$\cos(\measuredangle O_4AB) = \frac{(100 \text{ mm})^2 + (74.359 \text{ mm})^2 - (130 \text{ mm})^2}{2(100 \text{ mm})(74.359 \text{ mm})} = -0.092\,17$$

Therefore, the angle is either

$\measuredangle O_4AB = 95.29°$ or $\measuredangle O_4AB = -84.71°$

Note that the value $\measuredangle O_4AB = -84.71°$ can be eliminated since it is not physically possible for the open configuration of the four-bar linkage. Therefore, the correct result is

$$\measuredangle O_4AB = 95.29° \tag{3}$$

The angle

$$\beta = \measuredangle O_2AO_4 + \measuredangle O_4AB \tag{4}$$

Substituting Eqs. (2) and (3) into Eq. (4) gives

$$\beta = 126.21° + 95.29° = 221.50° \tag{5}$$

The law of cosines for the triangle ABO_4 can be written as

$$\overline{AO_4}^2 = R_{BA}^2 + R_{BO_4}^2 - 2R_{BA}R_{BO_4}\cos\gamma$$

Rearanging this equation gives

$$\cos\gamma = \frac{(100 \text{ mm})^2 + (130 \text{ mm})^2 - (74.359 \text{ mm})^2}{2(100 \text{ mm})(130 \text{ mm})} = 0.821\,95$$

Therefore, the transmission angle is

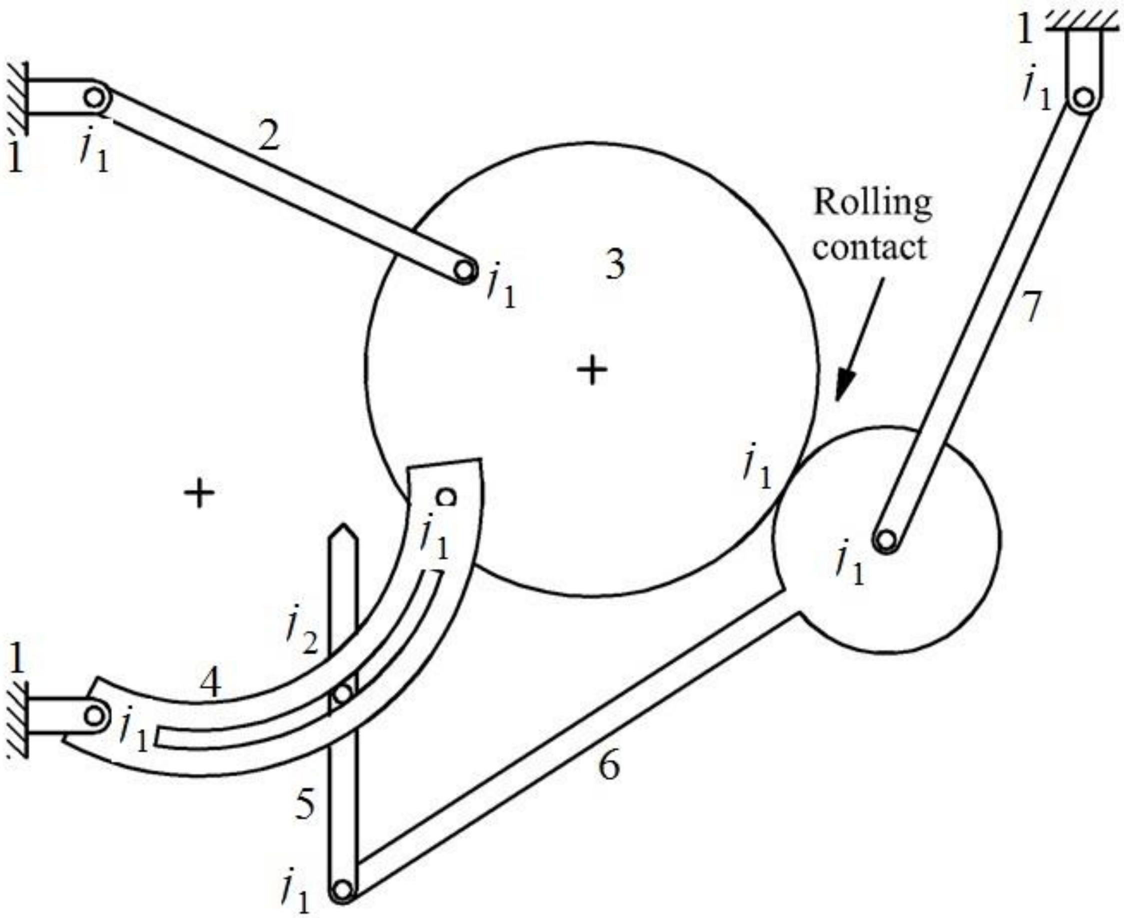
$$\gamma = 34.72° \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (1), the mechanical advantage is

$$MA = -\frac{(130 \text{ mm})\sin 34.72^o}{(60 \text{ mm})\sin 221.50^o} = -\frac{74.044 \text{ mm}}{-39.757 \text{ mm}} = 1.86$$ *Ans.*

1.28 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are indicated in the figure below *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 7$, $j_1 = 8$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

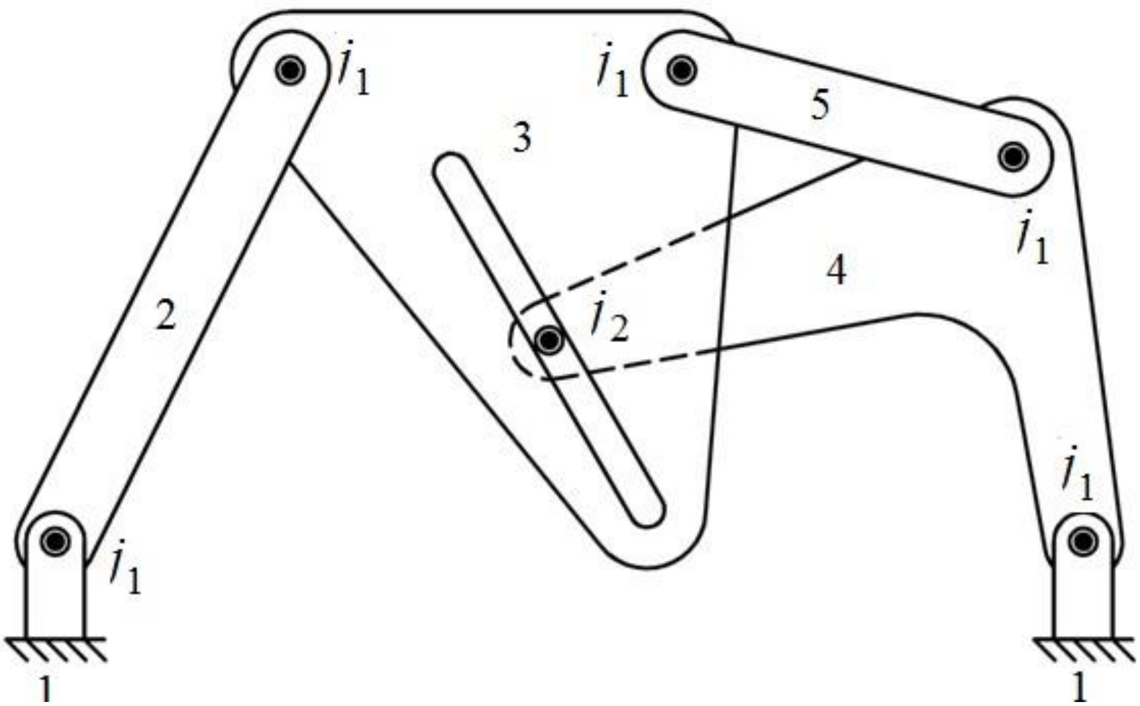
$$m = 3(7-1) - 2(8) - 1(1) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2, link 4, or link 7 would be suitable inputs since they are each pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.29 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are illustrated in the figure below. *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 5$, $j_1 = 5$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

$$m = 3(5-1) - 2(5) - 1\,(1) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 4 would be suitable inputs since they are each pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.30 The rocker of a crank-rocker four-bar linkage is required to have a length of 6 in and swing through a total angle of 30°. Also, the advance-to-return ratio of the linkage is required to be 1.75. Determine a suitable set of link lengths for the remaining three links.

The advance-to-return-ratio must be

$$Q = \alpha/\beta = 1.75 \tag{1}$$

where

$$\alpha = 180^{\circ} + \phi \qquad \text{and} \qquad \beta = 180^{\circ} - \phi \tag{2}$$

Substituting Eqs. (2) into Eq. (1) gives

$$180^{\circ} + \phi = 1.75(180^{\circ} - \phi)$$

Therefore,

$$\phi = 49.09^{\circ}$$

$$\beta = 130.91^{\circ} \qquad \text{and} \qquad \alpha = 229.09^{\circ}$$

Note that the dimensions of the synthesized four-bar linkage to satisfy the given design constraints are not unique. However, the graphic procedure follows the steps shown in Example 1.4. (For this problem, refer to the figure below):

(1) Draw the rocker $r_4 = 6\text{ in}$ of the crank-rocker four-bar linkage to a suitable scale, for example, full scale. Draw the rocker in the two extreme positions, that is, show the swing angle of the rocker of 30 degrees. Label the ground pivot O_4 and label the pin B in the two positions B_1 and B_2.

(2) Through point B_1 draw an arbitrary line (labeled the X-line). Through B_2 draw a line parallel to the X-line.

(3) Lay out the angle $\phi = 49.09^{\circ}$ counterclockwise from the X-line through point B_1. The intersection of this line with the line parallel to the X-line through point B_2 is the input crank pivot O_2.

(4)*i* The length O_2O_4 of the ground link can be measured from the drawing. That is

$$r_1 = O_2O_4 = 8.94\text{ in} \qquad \underline{\textit{Ans.}}$$

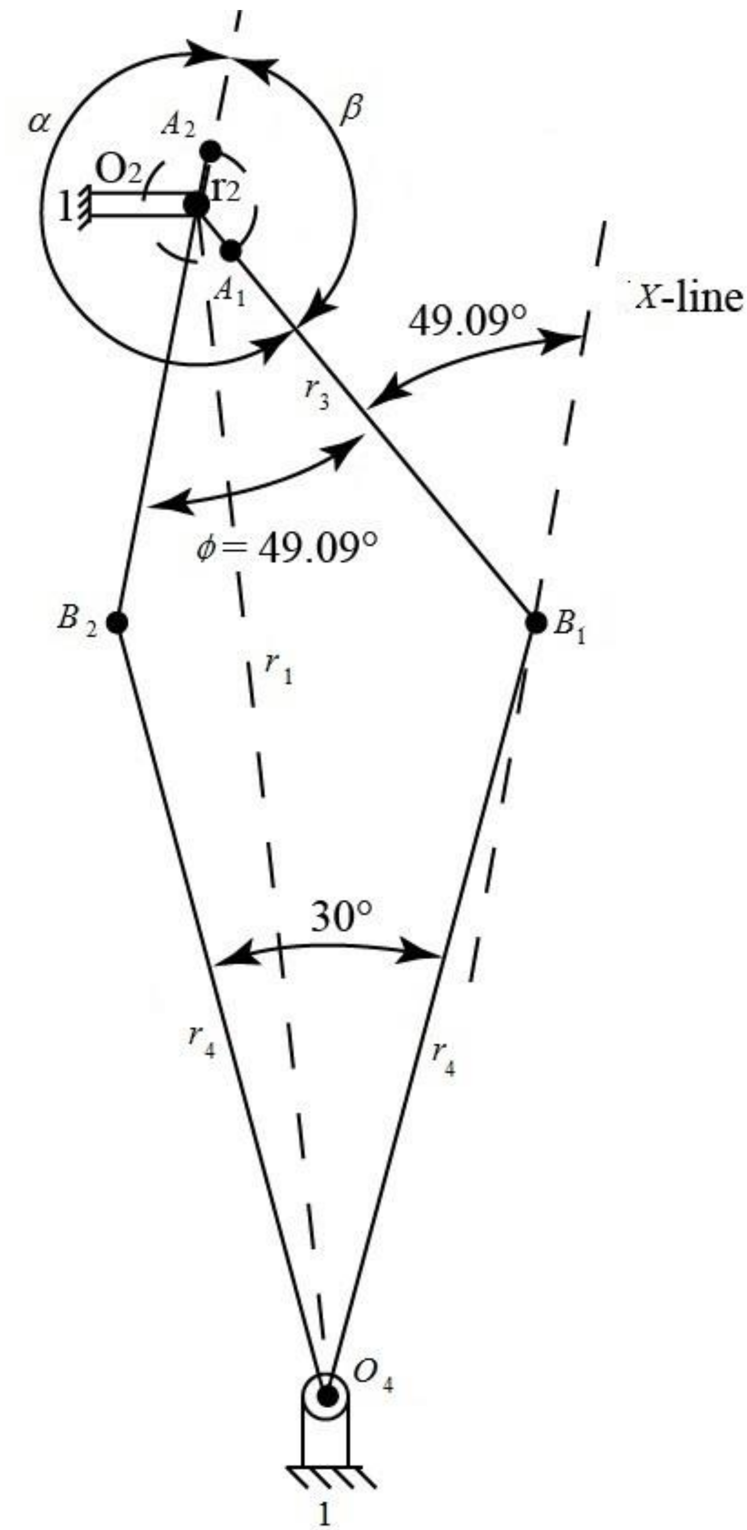
The other measurements are

$$O_2B_1 = r_3 + r_2 = 4.02\text{ in} \qquad \text{and} \qquad O_2B_2 = r_3 - r_2 = 3.18\text{ in}$$

Therefore, the length of the input link and the length of the coupler link, respectively, must be

$$r_2 = O_2A = 0.42\text{ in} \qquad \text{and} \qquad r_3 = AB = 3.60\text{ in} \qquad \underline{\textit{Ans.}}$$

This solution of the synthesized four-bar linkage is shown in the figure below.



The synthesized four-bar linkage.

1.31 Determine a suitable set of link lengths for a slider-crank linkage such that the stroke will be 500 mm and the advance-to-return ratio will be 1.8.

The advance-to-return ratio must be

$$Q = \alpha/\beta = 1.8 \tag{1}$$

where

$$\alpha = 180^{\circ} + \phi \qquad \text{and} \qquad \beta = 180^{\circ} - \phi \tag{2}$$

Substituting Eqs. (2) into Eq. (1) gives

$$180^{\circ} + \phi = 1.8(180^{\circ} - \phi)$$

Therefore,

$$\phi = 51.43^{\circ}$$

$$\beta = 128.57^{\circ} \qquad \text{and} \qquad \alpha = 231.43^{\circ}$$

Note that the dimensions of the synthesized slider-crank linkage to satisfy the given design constraints are not unique. However, the graphic procedure follows the steps shown in Example 1.5. (For this problem, refer to the figure below):

(1) Draw the stroke of the slider-crank linkage to a suitable scale, for example, 1 in ~ 100 mm. The length of the stroke of this mechanism is specified as $r_4 = 500\text{ mm}$. Label the pin *B* in its two extreme positions as B_1 and B_2.

(2) Through point B_2 draw an arbitrary line (labeled the *X*-line). Through point B_1 draw a line parallel to the *X*-line.

(3) Lay out the angle $\phi = 51.43^{\circ}$ clockwise from the *X*-line. The intersection of this line with the line parallel to the *X*-line through point B_1 is the ground pivot O_2.

(4) From the scale drawing below, the ground link; i.e., the offset (the vertical distance), is measured as

$$r_1 = 102.5\text{ mm} \qquad \underline{\textit{Ans.}}$$

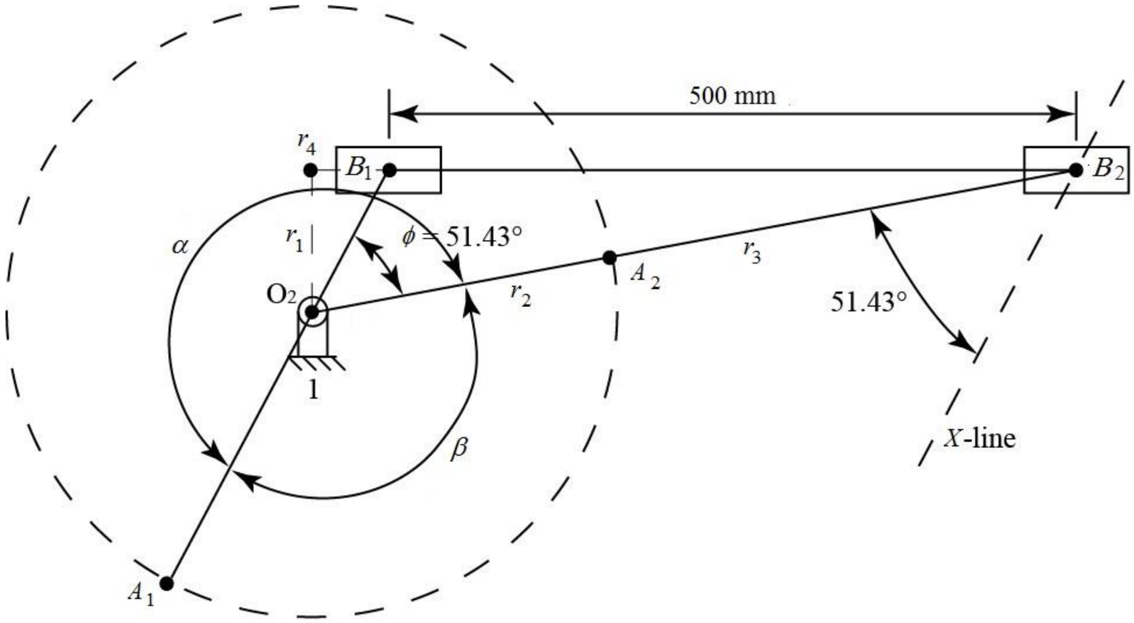
The other measurements are

$$O_2B_2 = r_3 + r_2 = 562.5\text{ mm} \qquad \text{and} \qquad O_2B_1 = r_3 - r_2 = 118.5\text{ mm}$$

Therefore, the length of the input link and the length of the coupler link, respectively, are

$$r_2 = O_2A = 222\text{ mm} \qquad \text{and} \qquad r_3 = AB = 340.5\text{ mm} \qquad \underline{\textit{Ans.}}$$

The solution of the synthesized slider-crank linkage is shown in the figure below.



The synthesized slider-crank linkage.

1.32 Determine the transmission angle and the mechanical advantage of the four-bar linkage in the posture illustrated. What type of four-bar linkage is this?

Grashof's law for a planar four-bar linkage, Sec. 1.9, Eq. (1.6), states that in order for the four-bar linkage to be a Grashof chain, the dimensions must satisfy

$$s + l \leq p + q \tag{1}$$

The lengths of the four links of the four-bar linkage are

$s = 20$ mm, $l = 90$ mm, $p = 60$ mm, and $q = 70$ mm..

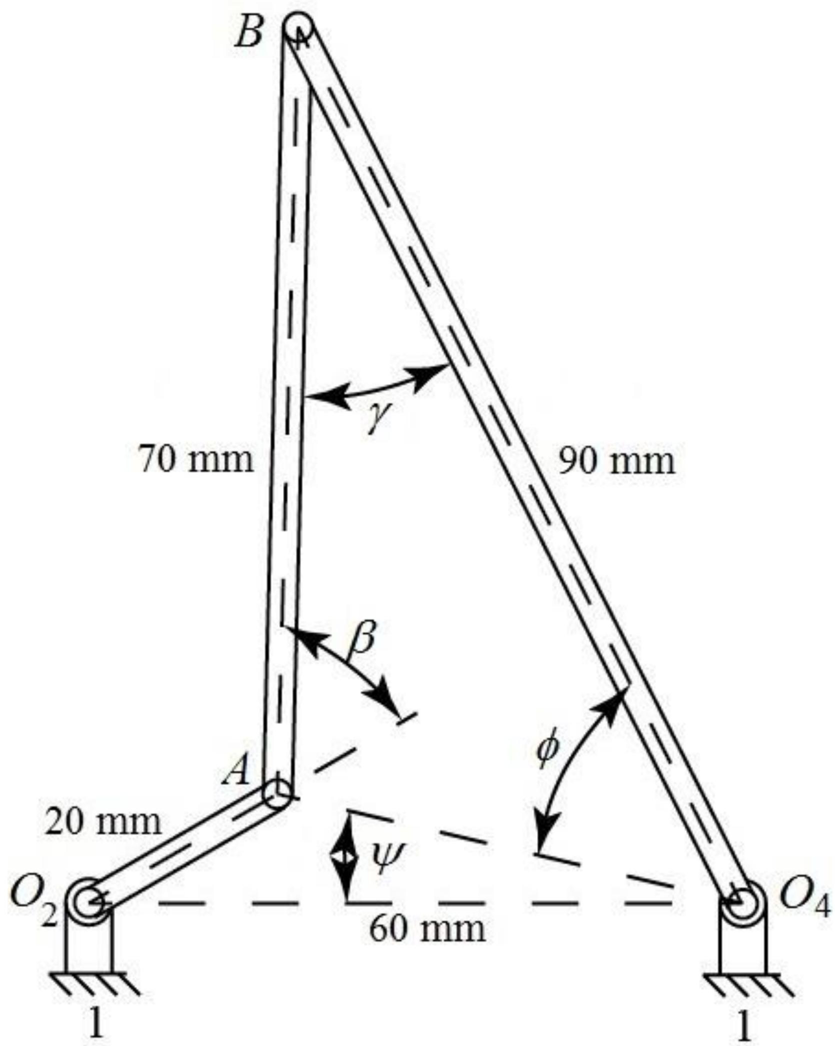
Substituting these dimensions into Eq. (1) gives

$$20\text{ mm} + 90\text{ mm} \leq 60\text{ mm} + 70\text{ mm}$$

or

$$110\text{ mm} \leq 130\text{ mm}$$

Since this inequality is satisfied, the four-bar linkage is a Grashof chain. Also, since the shortest link s is adjacent to the ground link, the shortest link is a crank; see Sec. 1.9. Therefore, this is a crank-rocker four-bar linkage. *Ans.*



$r_2 = 20$ mm, $r_3 = 70$ mm, $r_4 = 90$ mm, and $r_1 = 60$ mm.

In the posture illustrated, the distance between points A and O_4 can be found from the triangle AO_2O_4. The law of cosines can be written as

$$\overline{AO_4}^2 = r_2^2 + r_1^2 - 2r_2r_1\cos\theta_2$$

Substituting the given dimensions gives

$$\overline{AO_4}^2 = (20\text{ mm})^2 + (60\text{ mm})^2 - 2(20\text{ mm})(60\text{ mm})\cos 30^\circ = 1\,921.54\,(\text{mm})^2$$

Therefore, $\overline{AO_4} = 43.84\text{ mm}$.

To determine the transmission angle γ consider the triangle ABO_4. The law of cosines can be written as

$$\overline{AO_4}^2 = r_3^2 + r_4^2 - 2r_3r_4\cos\gamma$$

Substituting the given dimensions gives

$$1921.54\,(\text{mm})^2 = (70\text{ mm})^2 + (90\text{ mm})^2 - 2(70\text{ mm})(90\text{ mm})\cos\gamma$$

This equation gives $\cos\gamma = 0.879\,24$

Therefore, the transmission angle is $\gamma = 28.45^\circ$ *Ans.* (2)

The angle ψ can be found from the triangle O_2O_4A. From the law of sines,

$$\psi = \sin^{-1}\left[\frac{(20\text{ mm})\sin 30^\circ}{43.84\text{ mm}}\right] = 13.19^\circ$$

To determine the angle ϕ, consider the triangle BO_4A. From the law of sines, this angle is

$$\phi = \sin^{-1}\left[\frac{(70\text{ mm})\sin 28.45^\circ}{43.84\text{ mm}}\right] = 49.52^\circ$$

The angle between link 3 (link AB) and the ground link O_2O_4 can be written as

$$\theta_3 = 180^o - \gamma - \phi - \psi = 180^o - 28.45^o - 49.52^o - 13.19^o = 88.84^o$$

The angle β can be written as

$$\beta = \theta_3 - 30^\circ = 88.84^\circ - 30^\circ = 58.84^\circ \quad (3)$$

The mechanical advantage of a four-bar linkage, Sec.1.10, can be written as

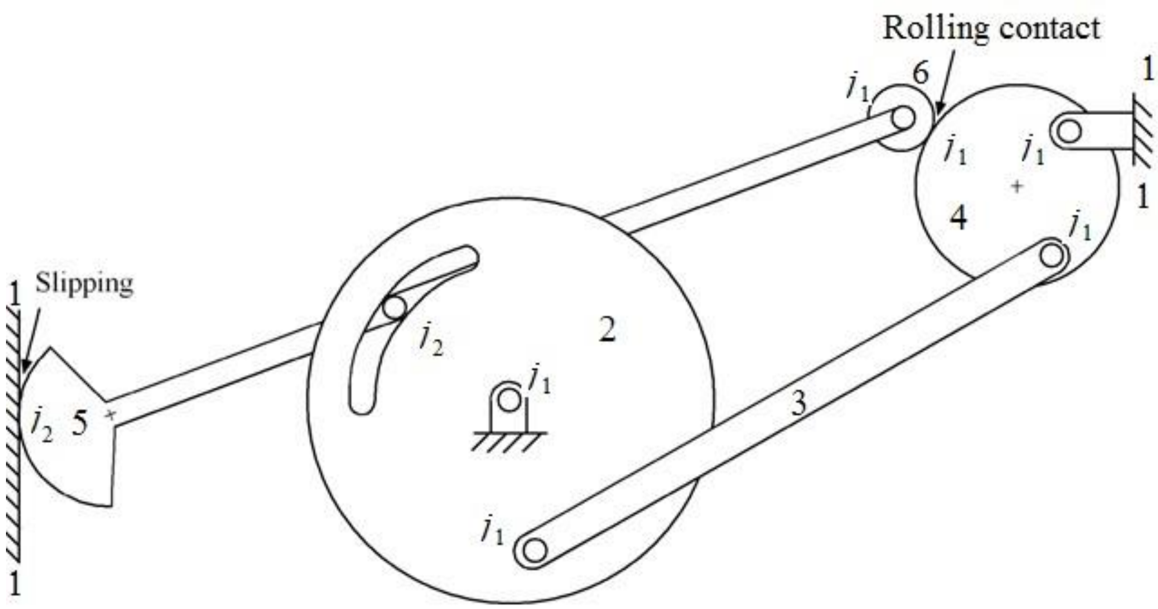
$$MA = \frac{r_4 \sin\gamma}{r_2 \sin\beta}$$

Substituting Eqs. (2) and (3), and the given link lengths, the mechanical advantage of the four-bar linkage (in the given posture) is

$$MA = \frac{(90\text{ mm})\sin 28.45^o}{(20\text{ mm})\sin 58.84^o} = 2.51$$ *Ans.*

1.33 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 6 links and the joint types are illustrated in the following figure *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 6$, $j_1 = 6$, and $j_2 = 2$.

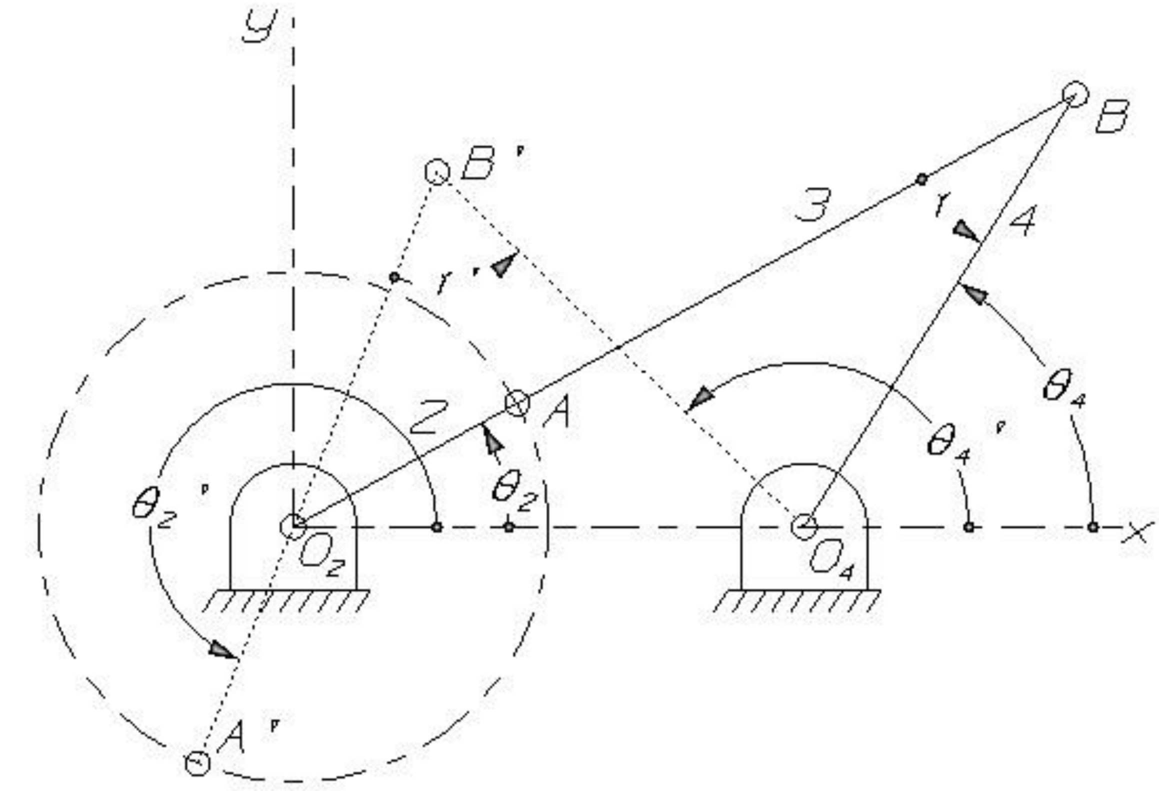
Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

$$m = 3(6-1) - 2(6) - 1(2) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

The rotation of either link 2 or link 4 would be suitable inputs since they are pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.34 A crank-rocker four-bar linkage is illustrated in one of its two toggle postures. Find θ_2 and θ_4 corresponding to each toggle posture. What is the total rocking angle of link 4? What are the transmission angles at the extremes?



$r_2 = 8$ in, $r_3 = 20$ in, and $r_4 = r_1 = 16$ in.

(*a*) From isosceles triangle O_4O_2B we can calculate

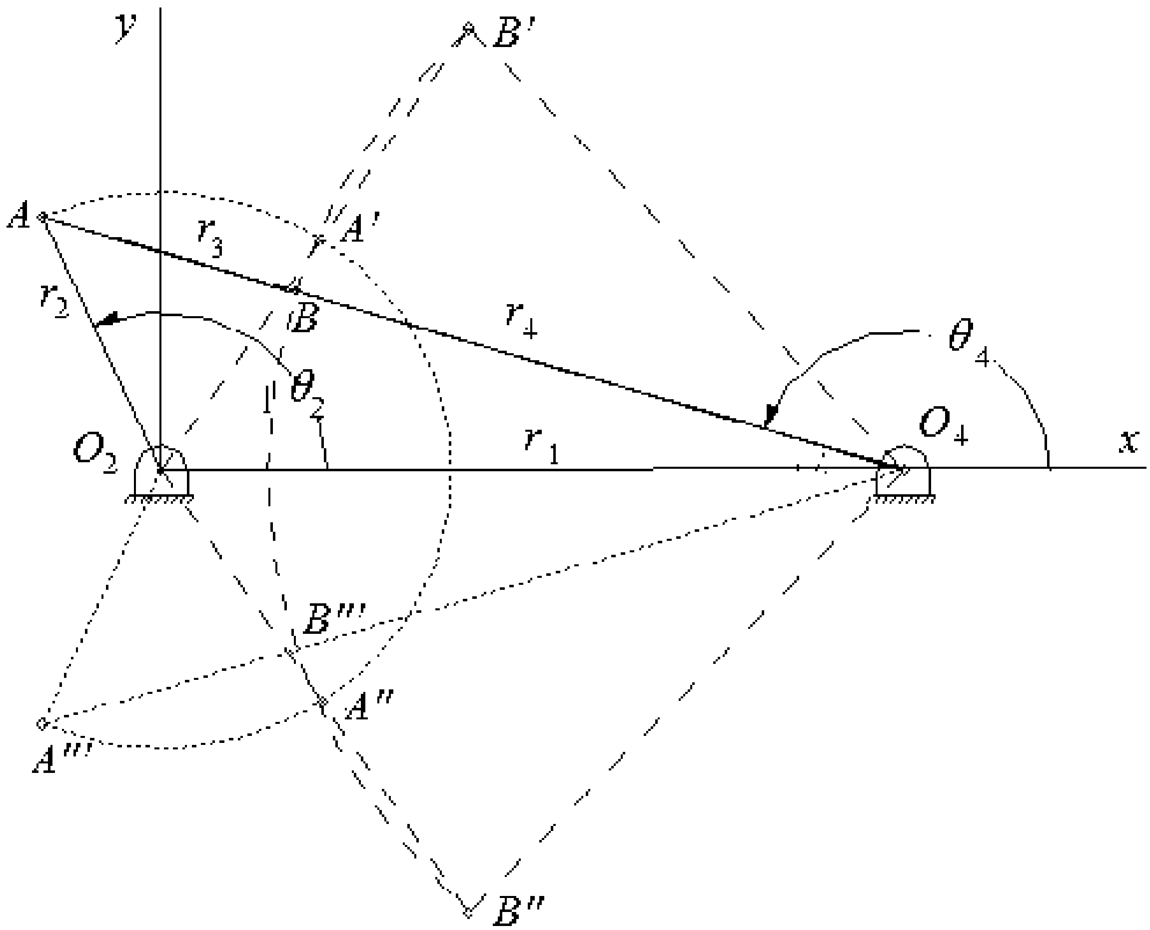
$$\theta_2 = \cos^{-1}\frac{(r_2 + r_3)/2}{r_1} = \cos^{-1}\frac{8\text{ in} + 20\text{ in}}{2(16\text{ in})} = 28.955°\,,\ \theta_4 = 57.910°, \qquad \underline{Ans.}$$

$$\theta_2' = \cos^{-1}\frac{r_3 - r_2}{2r_1} = \cos^{-1}\frac{20\text{ in} - 8\text{ in}}{2(16\text{ in})} = 247.976°\,,\ \theta_4' = 135.951°\ . \qquad \underline{Ans.}$$

(*b*) Then $\Delta\theta_4 = \theta_4' - \theta_4 = 78.041°$ <u>Ans.</u>

(*c*) Finally, from isosceles triangle O_2BO_4, $\gamma = 28.955°$ and $\gamma' = 67.976°$. <u>Ans.</u>

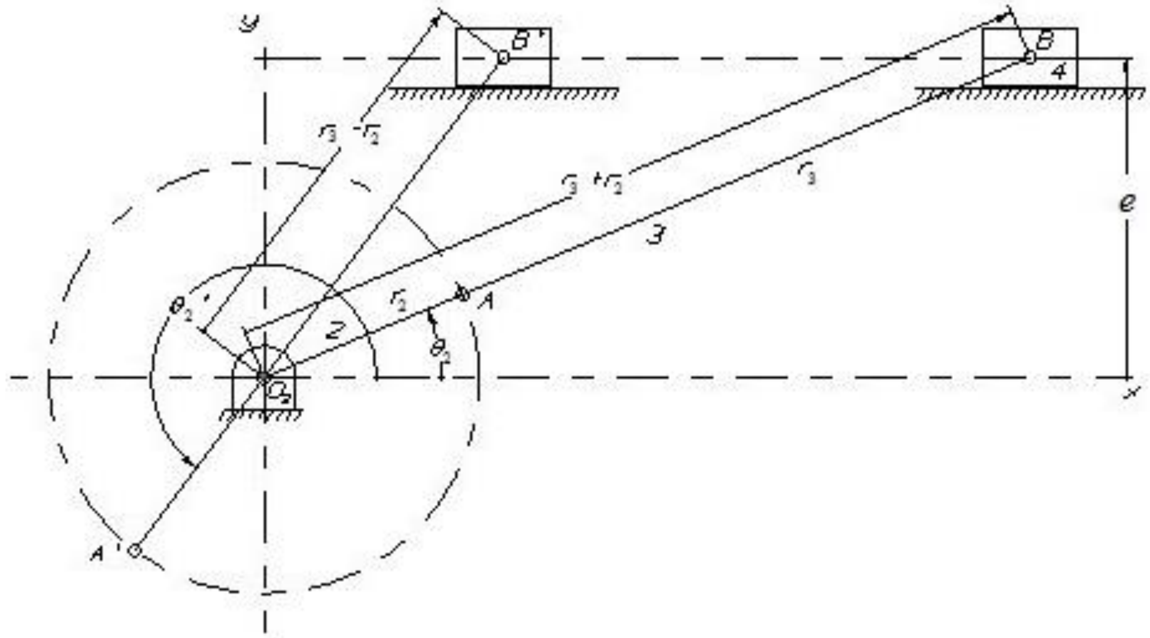
1.35 Find θ_2 and θ_4 corresponding to a dead-center posture. Is there a toggle posture?



$r_2 = 110$ mm, $r_3 = 100$ mm, $r_4 = 240$ mm, and $r_1 = 280$ mm.

For the given dimensions, there are two dead-center postures, and they correspond to the two extreme travel postures of crank O_2A. From ΔO_4AO_2 using the law of cosines, we can find $\theta_2 = 114.05°$, $\theta_4 = 162.82°$ and, symmetrically, $\theta_2''' = -114.05°$, $\theta_4''' = -162.82°$. There are also two toggle postures; these occur at $\theta_2' = 56.50°$, $\theta_4' = 133.14°$ and, symmetrically, at $\theta_2'' = -56.50°$, $\theta_4'' = -133.14°$. <u>Ans.</u>

1.36 Determine the advance-to-return ratio for the slider-crank linkage with the offset *e*. Also, determine in which direction the crank should rotate to provide quick return.



An offset slider-crank linkage in the two dead-center postures.

From the figure we can see that $e = (r_3 + r_2)\sin\theta_2 = (r_3 - r_2)\sin(\theta_2' - 180°)$ or

$$\theta_2 = \sin^{-1}\left(\frac{e}{r_3 + r_2}\right), \qquad \theta_2' = 180° + \sin^{-1}\left(\frac{e}{r_3 - r_2}\right)$$

$$\Delta\theta_{drive} = \theta_2' - \theta_2 = 180° + \sin^{-1}\left(\frac{e}{r_3 - r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 + r_2}\right)$$

$$\Delta\theta_{return} = \theta_2 + 360° - \theta_2' = 180° + \sin^{-1}\left(\frac{e}{r_3 + r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 - r_2}\right)$$

The advance-to-return ratio is

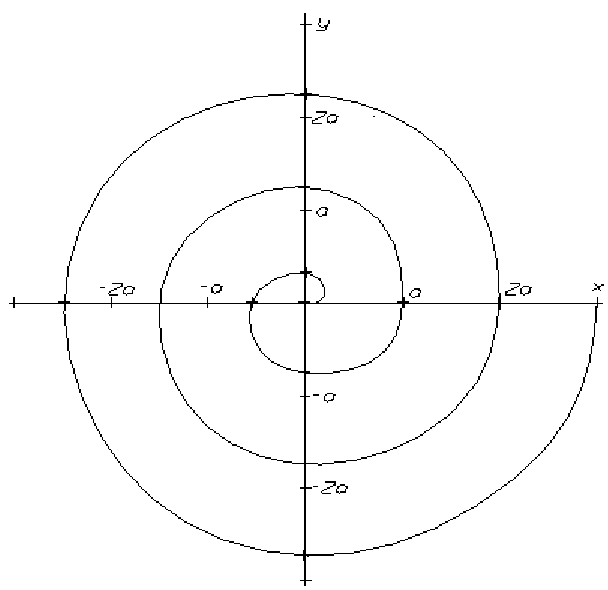
$$Q = \frac{180° + \sin^{-1}\left(\frac{e}{r_3 - r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 + r_2}\right)}{180° + \sin^{-1}\left(\frac{e}{r_3 + r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 - r_2}\right)} \qquad \underline{\textit{Ans.}}$$

Assuming driving is when *B* is sliding to the right, the crank should rotate clockwise. *Ans.*

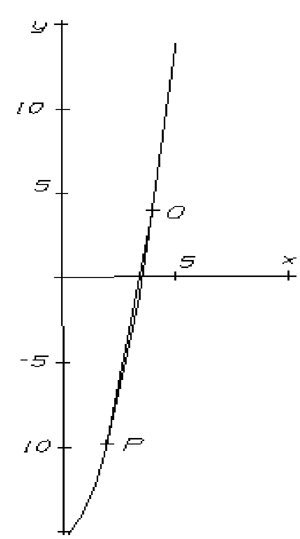
Chapter 2
Position and Displacement

2.1 Describe and sketch the locus of a point A which moves according to the equations $R_A^x = at\cos(2\pi t)$, $R_A^y = at\sin(2\pi t)$, and $R_A^z = 0$.

The locus is the spiral shown. *Ans.*



2.2 Find the position difference from point P to point Q on the curve $y = x^2 + x - 16$, where $R_P^x = 2$ and $R_Q^x = 4$.

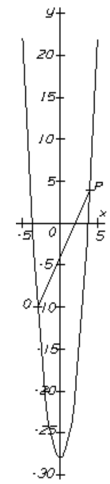


$$R_P^y = (2)^2 + 2 - 16 = -10;\quad \mathbf{R}_P = 2\hat{\mathbf{i}} - 10\hat{\mathbf{j}}$$

$$R_Q^y = (4)^2 + 4 - 16 = 4;\quad \mathbf{R}_Q = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$$

$$\mathbf{R}_{QP} = \mathbf{R}_Q - \mathbf{R}_P = 2\hat{\mathbf{i}} + 14\hat{\mathbf{j}} = 14.142\angle 81.87^\circ$$ *Ans.*

2.3 The path of a moving point is defined by the equation $y = 2x^2 - 28$. Find the position difference from point P to point Q if $R_P^x = 4$ and $R_Q^x = -3$.

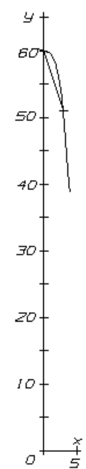


$$R_P^y = 2(4)^2 - 28 = 4\,; \qquad \mathbf{R}_P = 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$$

$$R_Q^y = 2(-3)^2 - 28 = -10\,; \; \mathbf{R}_Q = -3\hat{\mathbf{i}} - 10\hat{\mathbf{j}}$$

$$\mathbf{R}_{QP} = \mathbf{R}_Q - \mathbf{R}_P = -7\hat{\mathbf{i}} - 14\hat{\mathbf{j}} = 15.652\angle 243.43°$$ *Ans.*

2.4 The path of a moving point P is defined by the equation $y = 60 - x^3/3$. What is the displacement of the point if its motion begins at $R_P^x = 0$ and ends at $R_P^x = 3$?



$$R_P^y(0) = 60 - (0)^3/3 = 60\,; \mathbf{R}_P(0) = 60\hat{\mathbf{j}}$$

$$R_P^y(3) = 60 - (3)^3/3 = 51\,; \; \mathbf{R}_P(3) = 3\hat{\mathbf{i}} + 51\hat{\mathbf{j}}$$

$$\Delta\mathbf{R}_P = \mathbf{R}_P(3) - \mathbf{R}_P(0) = 3\hat{\mathbf{i}} - 9\hat{\mathbf{j}} = 9.487\angle -71.57°$$ *Ans.*

2.5 If point A moves on the locus of Problem 2.1, find its displacement from $t = 2$ to $t = 2.5$.

$$\mathbf{R}_A(2.0) = 2.0a\cos 4\pi\hat{\mathbf{i}} + 2.0a\sin 4\pi\hat{\mathbf{j}} = 2.0a\hat{\mathbf{i}}$$

$$\mathbf{R}_A(2.5) = 2.5a\cos 5\pi\hat{\mathbf{i}} + 2.5a\sin 5\pi\hat{\mathbf{j}} = -2.5a\hat{\mathbf{i}}$$

$$\mathbf{\Delta R}_A = \mathbf{R}_A(2.5) - \mathbf{R}_A(2.0) = -4.5a\hat{\mathbf{i}}$$ *Ans.*

2.6 The position of a point is given by the equation $\mathbf{R} = 100e^{j2\pi t}$. What is the path of the point? Determine the displacement of the point from $t = 0.10$ to $t = 0.40$.

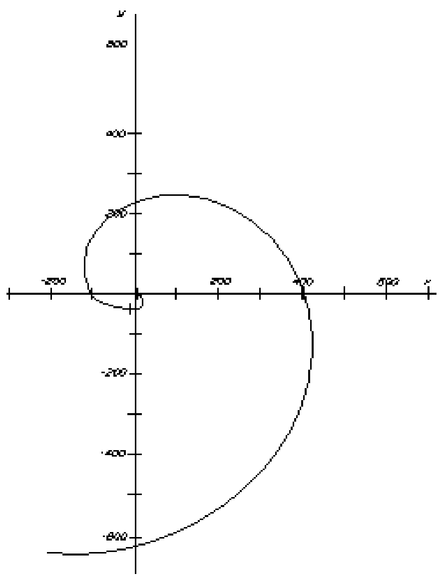
The point moves on a circle of radius 100 units with center at the origin. *Ans.*

$$\mathbf{R}(0.10) = 100e^{j0.628} = 80.902\hat{\mathbf{i}} + 58.779\hat{\mathbf{j}}$$

$$\mathbf{R}(0.40) = 100e^{j2.513} = -80.902\hat{\mathbf{i}} + 58.779\hat{\mathbf{j}}$$

$$\mathbf{\Delta R} = \mathbf{R}(0.40) - \mathbf{R}(0.10) = -161.804\hat{\mathbf{i}} = 161.804\angle 180°$$ *Ans.*

2.7 The equation $\mathbf{R} = \left(t^2 + 4\right)e^{-j\pi t/10}$ defines the position of a point. In which direction is the position vector rotating? Where is the point located when t = 0? What is the next value t can have if the orientation of the position vector is to be the same as it is when t = 0? What is the displacement from the first position of the point to the second?



Since the polar angle for the position vector is $\theta = -\pi t/10$, then $d\theta/dt$ is negative and therefore the position vector is rotating clockwise. *Ans.*

$$\mathbf{R}(0) = \left(0^2 + 4\right)e^{-j0} = 4\angle 0°$$

The position vector will next have the same orientation when $\pi t/10 = 2\pi$, that is, when t=20. *Ans.*

$$\mathbf{R}(20) = \left(20^2 + 4\right)e^{-j2\pi} = 404\angle 0°$$

$$\Delta\mathbf{R} = \mathbf{R}(20) - \mathbf{R}(0) = 400\angle 0°$$ *Ans.*

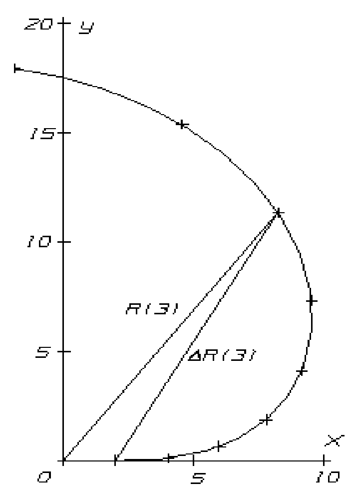
2.8 The location of a point is defined by the equation $\mathbf{R} = (4t + 2)e^{j\pi t^2/30}$, where t is time in seconds. Motion of the point is initiated when t = 0. What is the displacement during the first 3 s? Find the change in angular orientation of the position vector during the same time interval.

$$\mathbf{R}(0) = (0 + 2)e^{j0} = 2\angle 0° = 2\hat{\mathbf{i}}$$

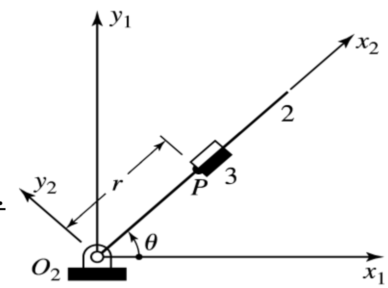
$$\mathbf{R}(3) = (12 + 2)e^{j\pi 9/30} = 14\angle 54° = 8.229\hat{\mathbf{i}} + 11.326\hat{\mathbf{j}}$$

$$\Delta\mathbf{R} = \mathbf{R}(3) - \mathbf{R}(0) = 6.229\hat{\mathbf{i}} + 11.326\hat{\mathbf{j}} = 12.926\angle 61.19°$$ *Ans.*

$$\Delta\theta = 54° - 0° = 54° \text{ ccw}$$ *Ans.*

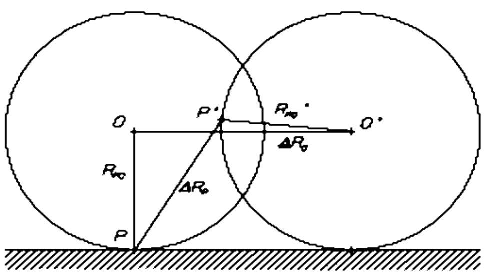


2.9 Link 2 rotates according to the equation $\theta = \pi t/4$. Block 3 slides outward on link 2 according to the equation $r = t^2 + 2$. What is the absolute displacement $\Delta\mathbf{R}_{P_3}$ from t = 1 to t = 2? What is the apparent displacement $\Delta\mathbf{R}_{P_{3/2}}$?



$$\mathbf{R}_{P_3} = re^{j\theta} = \left(t^2+2\right)e^{j\pi t/4}$$
$$\mathbf{R}_{P_3}(1) = 3\angle 45° = 2.121\hat{\mathbf{i}} + 2.121\hat{\mathbf{j}}$$
$$\mathbf{R}_{P_3}(2) = 6\angle 90° = 6\hat{\mathbf{j}}$$
$$\Delta\mathbf{R}_{P_3} = \mathbf{R}_{P_3}(2) - \mathbf{R}_{P_3}(1) = -2.121\hat{\mathbf{i}} + 3.879\hat{\mathbf{j}} = 4.421\angle 118.67° \quad \underline{\textit{Ans.}}$$
$$\mathbf{R}_{P_3/2} = re^{j0} = \left(t^2+2\right)\hat{\mathbf{i}}_2$$
$$\mathbf{R}_{P_3/2}(1) = 3\hat{\mathbf{i}}_2$$
$$\mathbf{R}_{P_3/2}(2) = 6\hat{\mathbf{i}}_2$$
$$\Delta\mathbf{R}_{P_3/2} = \mathbf{R}_{P_3/2}(2) - \mathbf{R}_{P_3/2}(1) = 3\hat{\mathbf{i}}_2 \quad \underline{\textit{Ans.}}$$

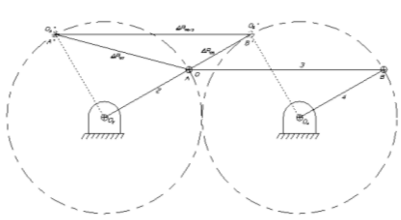
2.10 A wheel with center at O rolls without slipping on the ground at point P. If point O is displaced 10 in to the right, determine the displacement of point P during this interval.



Since the wheel rolls without slipping,

$$\Delta R_O = -\Delta\theta R_{PO}.$$
$$\Delta\theta = -\Delta R_O / R_{PO}$$
$$= -10 \text{ in}/6 \text{ in} = -1.667 \text{ rad} = -95.51°$$
$$\theta' = \theta + \Delta\theta = 270° - 95.51° = 174.49°$$
$$\mathbf{R}'_{PO} = 6 \text{ in}\angle 174.49° = -5.972\hat{\mathbf{i}} + 0.576\hat{\mathbf{j}} \text{ in}$$
$$\Delta\mathbf{R}_P = \Delta\mathbf{R}_O + \left(\mathbf{R}'_{PO} - \mathbf{R}_{PO}\right)$$
$$= 10\hat{\mathbf{i}} - 5.972\hat{\mathbf{i}} + 0.576\hat{\mathbf{j}} + 6\hat{\mathbf{j}} \text{ in}$$
$$\Delta\mathbf{R}_P = 4.028\hat{\mathbf{i}} + 6.576\hat{\mathbf{j}} = 7.712 \text{ in}\angle 58.51° \quad \underline{\textit{Ans.}}$$

2.11 A point Q moves from A to B along link 3 while link 2 rotates from $\theta_2 = 30°$ to $\theta'_2 = 120°$ Find the absolute displacement of Q.



$R_{AO_2} = R_{BO_4} = 3$ in and $R_{BA} = R_{O_4O_2} = 6$ in

$$\mathbf{R}_{Q_3} = 3 \text{ in}\angle 30° = 2.598\hat{\mathbf{i}} + 1.500\hat{\mathbf{j}} \text{ in}$$
$$\mathbf{R}'_{Q_3} = 3 \text{ in}\angle 120° = -1.500\hat{\mathbf{i}} + 2.598\hat{\mathbf{j}} \text{ in}$$
$$\Delta\mathbf{R}_{Q_3} = \mathbf{R}'_{Q_3} - \mathbf{R}_{Q_3} = -4.098\hat{\mathbf{i}} + 1.098\hat{\mathbf{j}} \text{ in}$$
$$\Delta\mathbf{R}_{Q_5/3} = \mathbf{R}_{BA} = 6.000\hat{\mathbf{i}} \text{ in}$$
$$\Delta\mathbf{R}_{Q_5} = \Delta\mathbf{R}_{Q_3} + \Delta\mathbf{R}_{Q_5/3}$$
$$\Delta\mathbf{R}_{Q_5} = 1.902\hat{\mathbf{i}} + 1.098\hat{\mathbf{j}} \text{ in} = 2.196 \text{ in}\angle 30° \quad \underline{\textit{Ans.}}$$

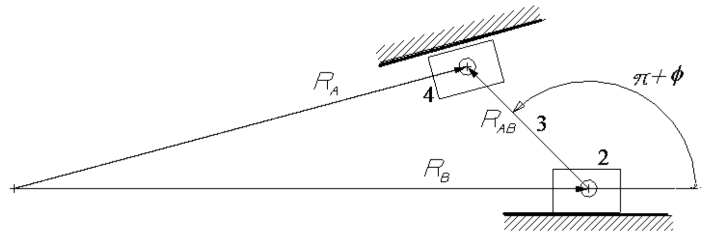
2.12 The double-slider linkage is driven by moving sliding block 2. Write the loop-closure equation. Solve analytically for the position of sliding block 4. Check the result graphically for the posture where $\phi = -45°$..

The loop-closure equation is

$$\overset{?\surd}{\mathbf{R}_A} = \overset{I\surd}{\mathbf{R}_B} + \overset{\surd ?}{\mathbf{R}_{AB}} \qquad \underline{\textit{Ans.}}$$

$$R_A e^{j\pi/12} = R_B + R_{AB} e^{j(\pi+\phi)}$$

$$= R_B - R_{AB} e^{j\phi}$$

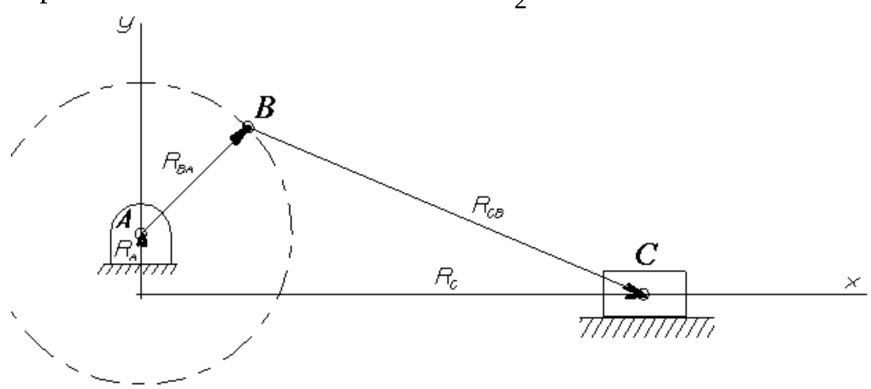


Taking the imaginary components of this, we get $\qquad R_{AB} = 200$ mm and $\psi = 15°$

$$R_A \sin 15° = -R_{AB} \sin\phi$$

$$R_A = -R_{AB}\frac{\sin\phi}{\sin 15°} = -200 \text{ mm}\frac{\sin -45°}{\sin 15°} = 546.4 \text{ mm} \qquad \underline{\textit{Ans.}}$$

2.13 The offset slider-crank linkage is driven by crank 2. Write the loop-closure equation. Solve for the position of slider 4 as a function of θ_2.



$$R_{AO} = 1 \text{ in},\ R_{BA} = 2.5 \text{ in}, \text{ and } R_{CB} = 7 \text{ in}.$$

$$\overset{?\surd}{\mathbf{R}_C} = \overset{\surd\surd}{\mathbf{R}_A} + \overset{\surd I}{\mathbf{R}_{BA}} + \overset{\surd ?}{\mathbf{R}_{CB}}$$

$$R_C = R_A e^{j\pi/2} + R_{BA} e^{j\theta_2} + R_{CB} e^{j\theta_3}$$

Taking real and imaginary parts,

$$R_C = R_{BA}\cos\theta_2 + R_{CB}\cos\theta_3 \quad \text{and} \quad 0 = R_A + R_{BA}\sin\theta_2 + R_{CB}\sin\theta_3$$

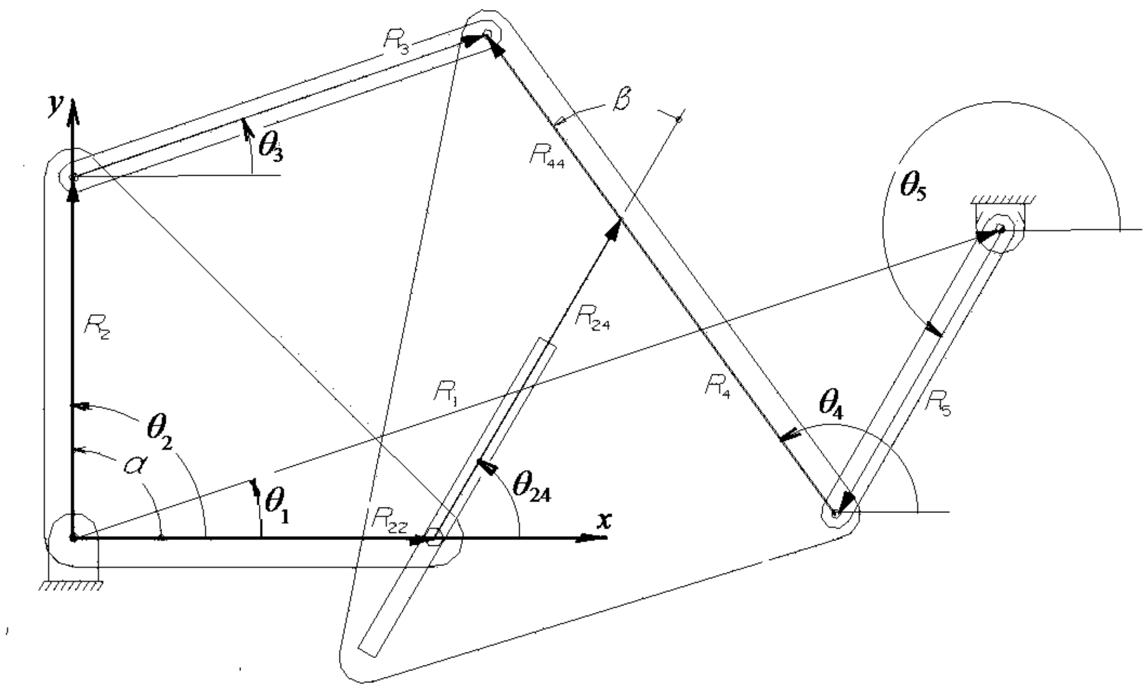
and, solving simultaneously, we get

$$\theta_3 = \sin^{-1}\left(\frac{-R_A - R_{BA}\sin\theta_2}{R_{CB}}\right) \text{with } -90° < \theta_3 < 90°$$

$$R_C = R_{BA}\cos\theta_2 + \sqrt{R_{CB}^2 - \left(R_A + R_{BA}\sin\theta_2\right)^2}$$

$$= 2.5\cos\theta_2 + \sqrt{48 - 5\sin\theta_2 - 6.25\sin^2\theta_2} \text{ in} \qquad \underline{\textit{Ans.}}$$

2.14 Define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).



One suitable set of two vector loop equations is

$$\overset{\surd I}{\mathbf{R}_2}+\overset{\surd ?}{\mathbf{R}_3}-\overset{\surd ?}{\mathbf{R}_4}-\overset{\surd ?}{\mathbf{R}_5}-\overset{\surd\surd}{\mathbf{R}_1}=\mathbf{0} \quad \text{and} \quad \overset{\surd I}{\mathbf{R}_2}+\overset{\surd ?}{\mathbf{R}_3}-\overset{\surd C1}{\mathbf{R}_{44}}-\overset{?C2}{\mathbf{R}_{24}}-\overset{\surd C3}{\mathbf{R}_{22}}=\mathbf{0}$$ *Ans.*

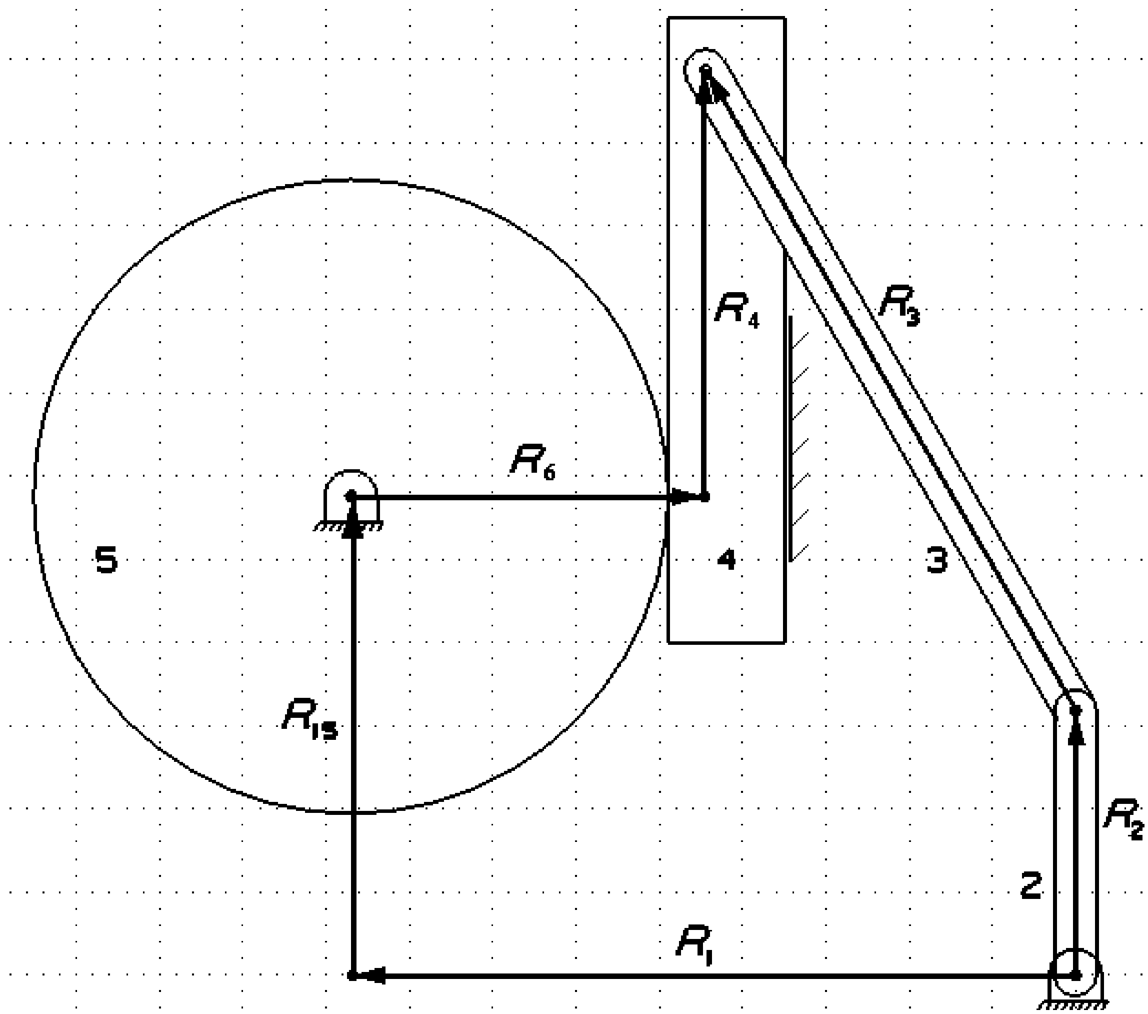
The angle θ_2 is a suitable input. Three constraint equations are required. *Ans.*

$$\theta_{44}=\theta_4 \quad (C1) \qquad \theta_{24}=\theta_4-\beta \quad (C2) \qquad \theta_{22}=\theta_2-\alpha \quad (C3)$$ *Ans.*

The known quantities are R_1, θ_1, R_2, R_3, R_4, R_5, R_{22}, R_{44}, α and β. *Ans.*

The unknown quantities are θ_3, θ_4, θ_5, θ_{22}, θ_{24}, θ_{44}, and R_{24}. *Ans.*

2.15 Define a set of vectors that is suitable for a complete kinematic analysis of the rack-pinion mechanism. Label and show the sense and orientation of each vector. Assuming rolling with no slip between rack 4 and pinion 5, write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).



One suitable set of vectors is as shown in the figure. The vector loop equation is

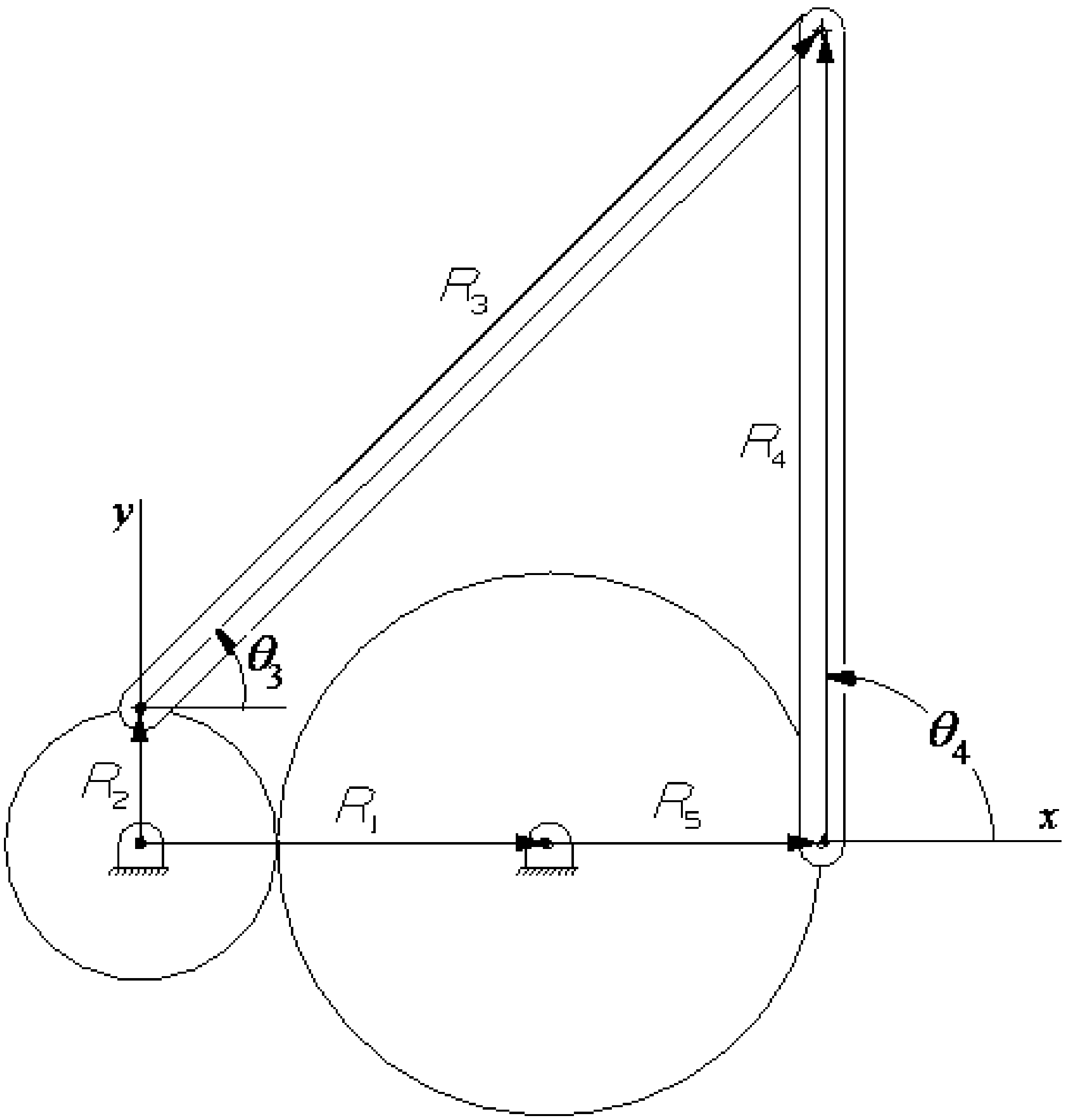
$$\overset{\surd I}{\mathbf{R}_2} + \overset{\surd ?}{\mathbf{R}_3} - \overset{?\surd}{\mathbf{R}_4} - \overset{\surd\surd}{\mathbf{R}_6} - \overset{\surd\surd}{\mathbf{R}_{15}} - \overset{\surd\surd}{\mathbf{R}_1} = \mathbf{0} \quad \text{with} \quad \rho_5 \Delta\theta_5 = -\Delta R_4 \text{ (C1)}$$ <u>*Ans.*</u>

The angle θ_2 is a suitable input. <u>*Ans.*</u>

The known quantities are R_1, θ_1=180°, R_2, R_3, θ_4=90°, ρ_5, R_6, θ_6=0, R_{15}, θ_{15}=90°. <u>*Ans.*</u>

The unknown variables are, θ_3, R_4, and $\Delta\theta_5$. <u>*Ans.*</u>

2.16 Define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Assuming rolling with no slipping between gears 2 and 5, write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).



One suitable set of vectors is as shown in the figure. The vector loop equation is

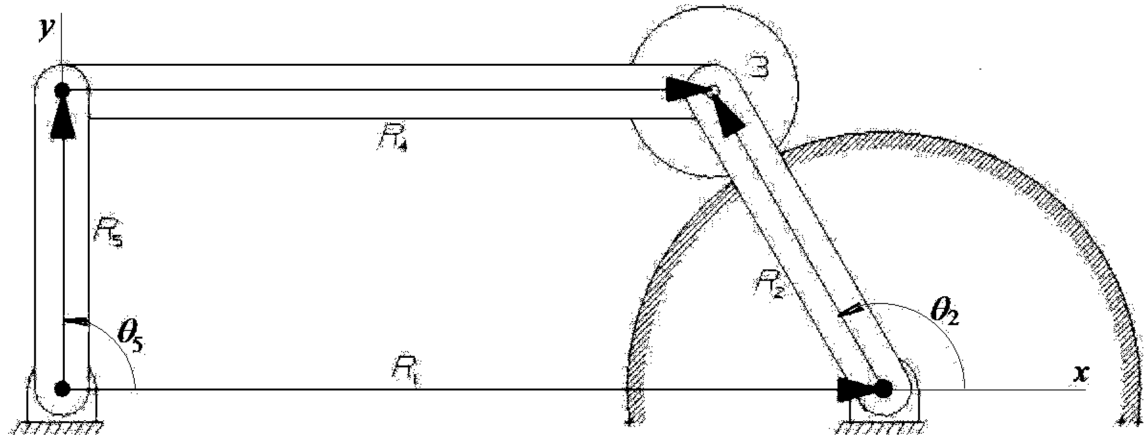
$$\overset{\surd I}{\mathbf{R}_2} + \overset{\surd ?}{\mathbf{R}_3} - \overset{\surd ?}{\mathbf{R}_4} - \overset{\surd C}{\mathbf{R}_5} - \overset{\surd\surd}{\mathbf{R}_1} = \mathbf{0} \quad \text{with} \quad \rho_2\Delta\theta_2 + \rho_5\Delta\theta_5 = 0 \text{ (C1)}$$ <u>*Ans.*</u>

The angle θ_2 is a suitable input. <u>*Ans.*</u>

The known quantities are R_1, θ_1=0, R_2, ρ_2, R_3, R_4, ρ_5, and R_5. <u>*Ans.*</u>

The unknown variables are θ_3, θ_4, and θ_5. <u>*Ans.*</u>

2.17 Gear 3, which is pinned to link 4 at point *B*, is rolling without slipping on semi-circular ground link 1. The radius of gear 3 is ρ_3 and the radius of the ground link is ρ_1. Define a set of vectors that are suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).



One suitable set of vectors is shown in the figure. The vector loop equation is

$$\overset{\surd I}{\mathbf{R}_2} - \overset{\surd ?}{\mathbf{R}_4} - \overset{\surd ?}{\mathbf{R}_5} + \overset{\surd \surd}{\mathbf{R}_1} = \mathbf{0} \quad \text{with} \quad R_2\Delta\theta_2 = \rho_3\Delta\theta_3 \text{ (C1)}$$ *Ans.*

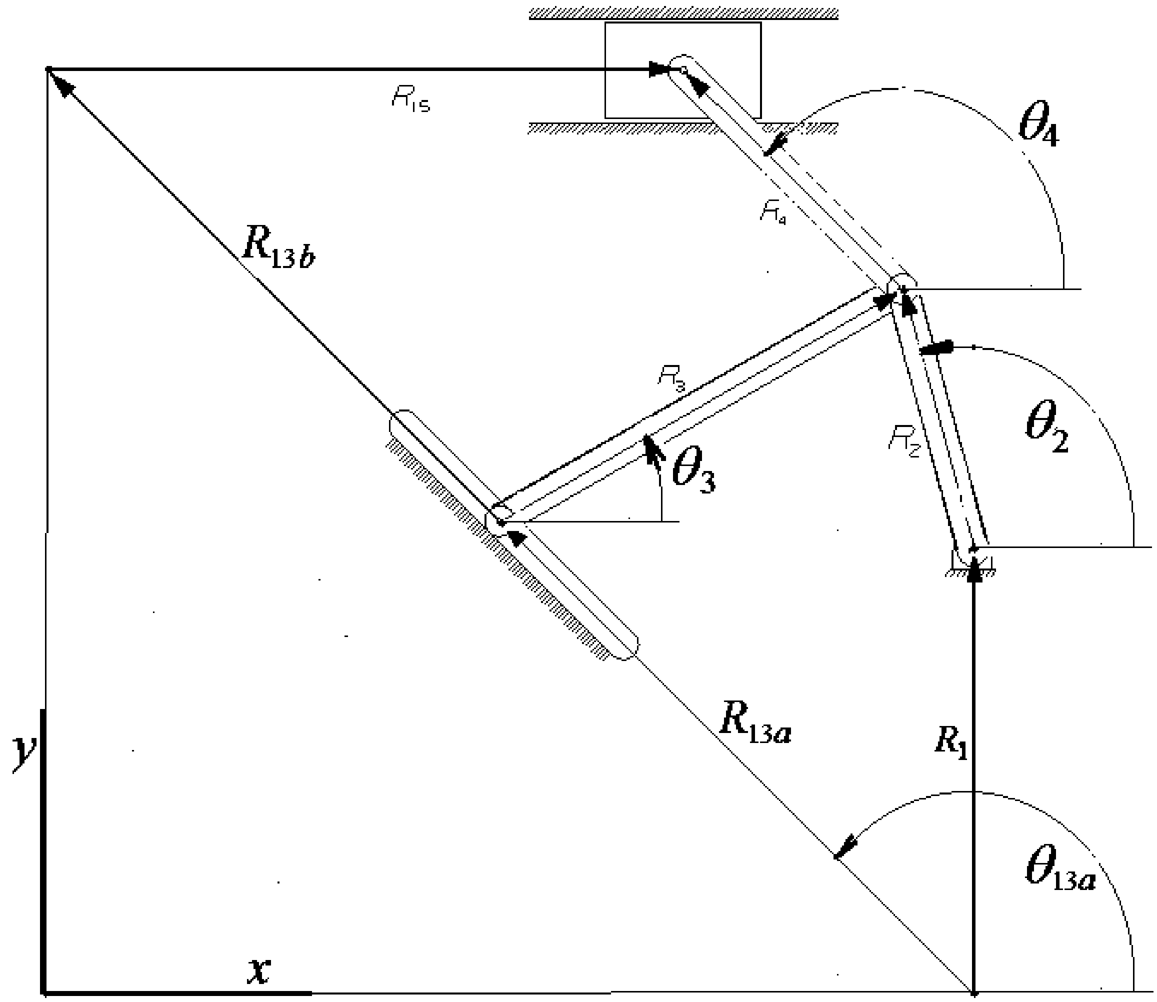
The angle θ_2 is a suitable input. *Ans.*

The known quantities are R_1, θ_1=0, R_2, ρ_3, R_4, and R_5. *Ans.*

The unknown variables are θ_3, θ_4, and θ_5. *Ans.*

2.18 For the mechanism in Figure P1.6, define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).

One set of vectors suitable for a kinematic analysis of the mechanism is shown below.



The corresponding vector loop equations are

$$\overset{\surd\surd}{\mathbf{R}_1}+\overset{\surd I}{\mathbf{R}_2}-\overset{\surd ?}{\mathbf{R}_3}-\overset{?\surd}{\mathbf{R}_{13a}}=\mathbf{0} \quad \text{and} \quad \overset{\surd ?}{\mathbf{R}_3}+\overset{\surd ?}{\mathbf{R}_4}-\overset{?\surd}{\mathbf{R}_{15}}-\overset{C1C2}{\mathbf{R}_{13b}}=\mathbf{0} \qquad \underline{Ans.}$$

with the constraint equation(s) $R_{13a}+R_{13b}=$ constant. (C1) and $\theta_{13b}=\theta_{13a}$ (C2). *Ans.*

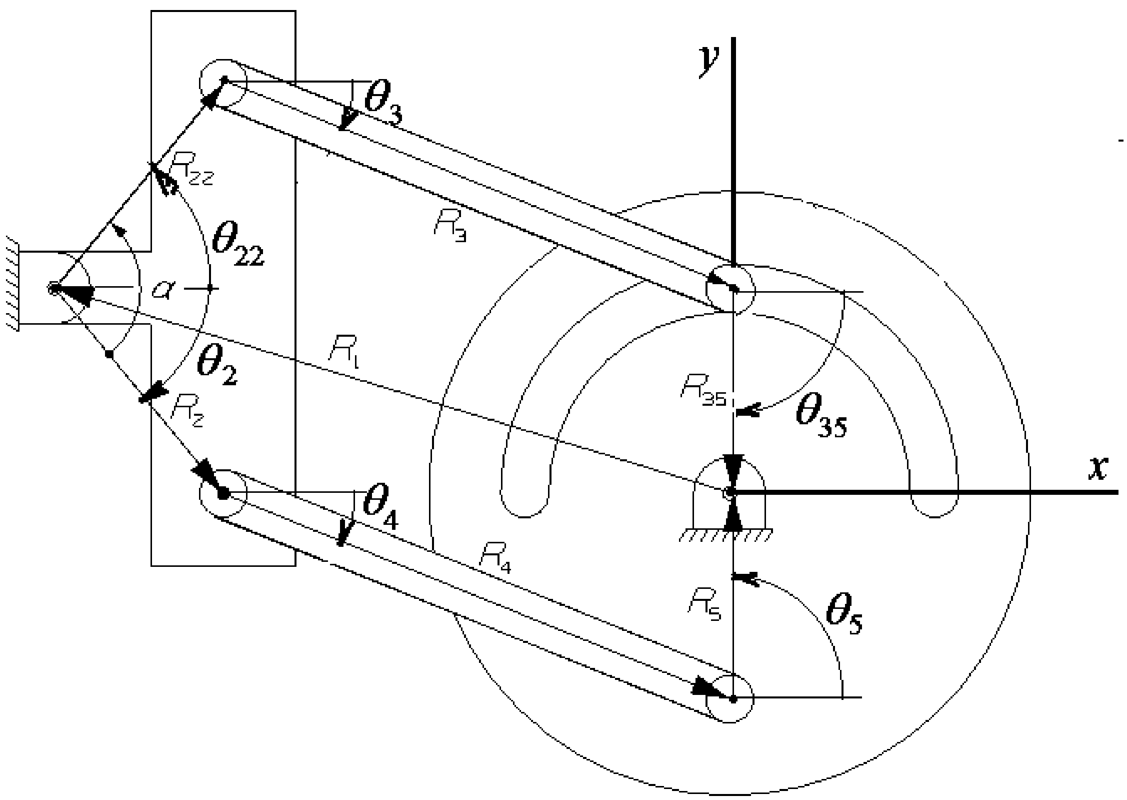
Angle θ_2 is a suitable input. *Ans.*

Known quantities are R_1, θ_1=90°, R_2, R_3, θ_{13a}, R_4, and θ_{15}=0. *Ans.*

Unknown variables are θ_3, R_{13a}, R_{13b}, θ_{13b}, θ_4, and R_{15}. *Ans.*

2.19 For the mechanism in Figure P1.8, define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).

One set of vectors suitable for a kinematic analysis of the mechanism is shown here.



The corresponding set of vector loop equations is

$$\overset{\surd\surd}{\mathbf{R}_1}+\overset{\surd ?}{\mathbf{R}_2}+\overset{\surd ?}{\mathbf{R}_4}+\overset{\surd I}{\mathbf{R}_5}=\mathbf{0} \quad \text{and} \quad \overset{\surd\surd}{\mathbf{R}_1}+\overset{\surd C1}{\mathbf{R}_{22}}+\overset{\surd ?}{\mathbf{R}_3}+\overset{\surd ?}{\mathbf{R}_{35}}=\mathbf{0}$$ *Ans.*

with the constraint equation $\theta_{22}=\theta_2+\alpha$ (C1). *Ans.*

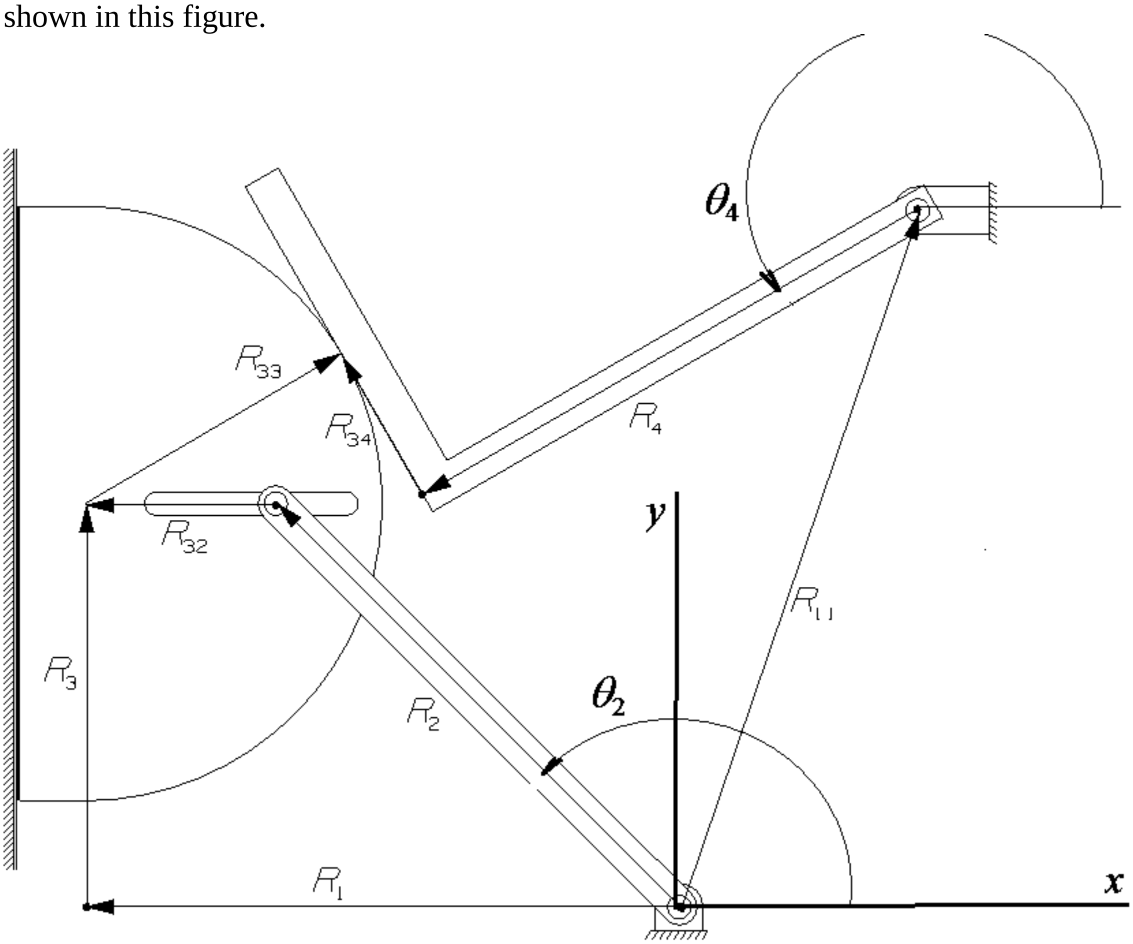
Angle θ_5 is a suitable input. *Ans.*

Known quantities are R_1, θ_1, R_2, R_3, R_4, R_5, R_{22}, and R_{35}. *Ans.*

Unknown variables are θ_2, θ_3, θ_4, θ_{22}, and θ_{35}. *Ans.*

2.20 For the mechanism in Figure P1.9, define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).

One set of vectors suitable for a complete kinematic analysis of this mechanism is as shown in this figure.



The corresponding set of vector loop equations is

$$\overset{\surd\surd}{\mathbf{R}_1}+\overset{?\surd}{\mathbf{R}_3}-\overset{?\surd}{\mathbf{R}_{32}}-\overset{\surd I}{\mathbf{R}_2}=\mathbf{0} \quad \text{and} \quad \overset{\surd\surd}{\mathbf{R}_{11}}+\overset{\surd ?}{\mathbf{R}_4}+\overset{?C1}{\mathbf{R}_{34}}-\overset{\surd C2}{\mathbf{R}_{33}}-\overset{?\surd}{\mathbf{R}_{32}}-\overset{\surd I}{\mathbf{R}_2}=\mathbf{0} \qquad \underline{\textit{Ans.}}$$

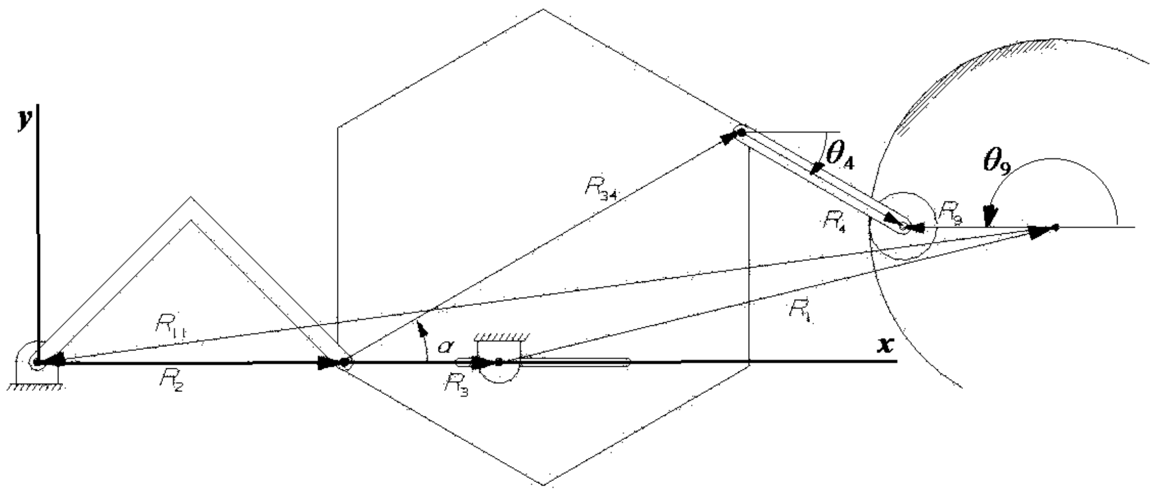
with the two constraint equations

$$\theta_{34}=\theta_4-90^\circ \quad (C1) \qquad \text{and} \qquad \theta_{33}=\theta_4-180^\circ \quad (C2). \qquad \underline{\textit{Ans.}}$$

The angle θ_2 is a suitable input. *Ans.*
Known quantities are R_1, θ_1=180°, R_2, θ_3=90°, R_4, R_{11}, θ_{11}, θ_{32}=180°, and R_{33}. *Ans.*
Unknown variables are R_3, θ_4, R_{32}, θ_{33}, R_{34}, and θ_{34}. *Ans.*

2.21 For the mechanism in Figure P1.10, define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).

One set of vectors suitable for a kinematic analysis of the mechanism is shown here.



The corresponding set of vector loop equations is

$$\overset{\surd\surd}{\mathbf{R}_1}+\overset{\surd\surd}{\mathbf{R}_{11}}+\overset{\surd I}{\mathbf{R}_2}+\overset{??}{\mathbf{R}_3}=\mathbf{0} \quad \text{and} \quad \overset{\surd\surd}{\mathbf{R}_{11}}+\overset{\surd I}{\mathbf{R}_2}+\overset{\surd C1}{\mathbf{R}_{34}}+\overset{\surd ?}{\mathbf{R}_4}-\overset{\surd ?}{\mathbf{R}_9}=\mathbf{0}$$ *Ans.*

with the constraint equation $\theta_{34}=\theta_3+\alpha$. ($C1$) *Ans.*

Angle θ_2 is a suitable input. *Ans.*

The known quantities are R_1, θ_1, R_2, R_4, R_9, R_{11}, θ_{11}, and R_{34}. *Ans.*

The unknown variables are R_3, θ_3, θ_4, θ_9, and θ_{34}.

However, these equations do not analyze the angular displacement of the small wheel, body 5. In order to do this, we might consider the apparent angular displacement as seen by an observer fixed on vector 9 and viewing the point of contact between bodies 5 and 1. The non-slip condition would provide the constraint

$$\rho_1\Delta\theta_{1/9}=\rho_5\Delta\theta_{5/9} \quad \text{(C2)}$$
$$\rho_1\left(\Delta\not{\theta}_1-\Delta\theta_9\right)=\rho_5\left(\Delta\theta_5-\Delta\theta_9\right)$$
$$\rho_5\Delta\theta_5+\left(\rho_1-\rho_5\right)\Delta\theta_9=0$$
$$\rho_5\Delta\theta_5+R_9\Delta\theta_9=0$$ *Ans.*

where ρ_5 is the radius of wheel 5 and $\Delta\theta_5$ is the angular displacement of body 5.

2.22 Write a calculator program to find the sum of any number of two-dimensional vectors expressed in mixed rectangular or polar forms. The result should be obtainable in either form with the magnitude and angle of the polar form having only positive values.

Because the variety of makes and models of calculators is vast and no standards are known for programming them, no solution is shown here.

2.23 Write a computer program to plot the coupler curve of any crank-rocker or double-crank form of the four-bar linkage. The program should accept four link lengths and either rectangular or polar coordinates of the coupler point with respect to the coupler.

Again the variety of programming languages makes it impossible to provide a standard solution. However, one version, written in ANSI/ISO FORTRAN 77, is supplied here as an example. There are also no universally accepted standards for programming graphics. Therefore the Tektronix PLOT10 subroutine library, for display on Tektronix 4010 series displays, is chosen as an old but somewhat recognized alternative. The symbols in the program correspond to the notation shown in Fig. 2.15 of the text. The required input data are:

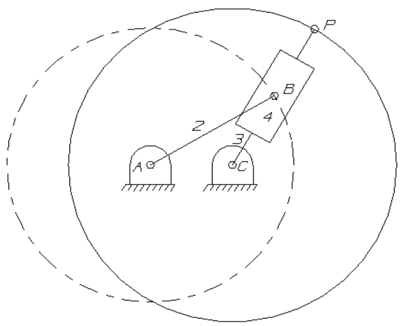
$$\text{R1, R2, R3, R4, } \begin{cases} \text{X5,Y5,} & \text{-1} \\ \text{R5,ALPHA,} & \text{1} \end{cases}$$

The program can be verified using the data of Example 2.6 and checking the results against those of Table 2.3.

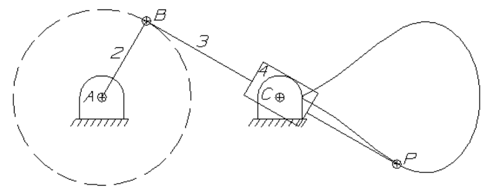
```
      PROGRAM CCURVE
C
C     A FORTRAN 77 PROGRAM TO PLOT THE COUPLER CURVE OF ANY CRANK-ROCKER
C       OR DOUBLE-CRANK FOUR-BAR LINKAGE, GIVEN ITS DIMESNIONS.
C     ORIGINALLY WRITTEN USING SUBROUTINES FROM TEKTRONIX PLOT10 FOR
C       DISPLAY ON 4010 SERIES DISPLAYS.
C     REF:J.J.UICKER,JR, G.R.PENNOCK, & J.E.SHIGLEY, 'THEORY OF MACHINES
C       AND MECHANISMS,'FIFTH EDITION, OXFORD UNIVERSITY PRESS, 2015.
C     EXAMPLE 2.7
C
C     WRITTEN BY: JOHN J. UICKER, JR.
C     ON:         01 JANUARY 1980
C
C     READ IN THE DIMENSIONS OF THE LINKAGE.
      READ(5,1000)R1,R2,R3,R4,X5,Y5,IFORM
 1000 FORMAT(6F10.0,I2)
C
C     FIND R5 AND ALPHA.
      IF(IFORM.LE.0)THEN
        R5=SQRT(X5*X5+Y5*Y5)
        ALPHA=ATAN2(Y5,X5)
      ELSE
        R5=X5
        ALPHA=Y5/57.29578
        X5=R5*COS(ALPHA)
        Y5=R5*SIN(ALPHA)
      END IF
C
C     INITIALIZE FOR PLOTTING AT 120 CHARACTERS PER SECOND.
      CALL INITT(1200)
```

```
C
C     SET THE WINDOW FOR THE PLOTTING AREA.
      CALL DWINDO(-R2,R1+R2+R4,-R4,R4+R4+Y5)
C
C     CYCLE THROUGH ONE CRANK ROTATION IN FIVE DEGREE INCREMENTS.
      TH2=0.0
      DTH2=5.0/57.29578
      IPEN=-1
      DO 2 I=1,73
        CTH2=COS(TH2)
        STH2=SIN(TH2)
C
C     CALCULATE THE TRANSMISSION ANGLE.
        CGAM=(R3*R3+R4*R4-R1*R1-R2*R2+2.0*R1*R2*CTH2)/(2.0*R3*R4)
        IF(ABS(CGAM).GT.0.99)THEN
          CALL MOVABS(100,100)
          CALL ANMODE
          WRITE(7,1001)
 1001     FORMAT(//' *** THE TRANSMISSION ANGLE IS TOO SMALL. ***')
          GO TO 1
        END IF
        SGAM=SQRT(1.0-CGAM*CGAM)
        GAM=ATAN2(SGAM,CGAM)
C
C     CALCULATE THETA 3.
        STH3=-R2*STH2+R4*SIN(GAM)
        CTH3=R3+R1-R2*CTH2-R4*COS(GAM)
        TH3=2.0*ATAN2(STH3,CTH3)
C
C     CALCULATE THE COUPLER POINT POSITION.
        TH6=TH3+ALPHA
        XP=R2*CTH2+R5*COS(TH6)
        YP=R2*STH2+R5*SIN(TH6)
C
C     PLOT THIS SEGMENT OF THE COUPLER CURVE.
        IF(IPEN.LT.0)THEN
          IPEN=1
          CALL MOVEA(XP,YP)
        ELSE
          IPEN=-1
          CALL DRAWA(XP,YP)
        END IF
        TH2=TH2+DTH2
    2 CONTINUE
C
C     DRAW THE LINKAGE.
      CALL MOVEA(0.0,0.0)
      CALL DRAWA(R2,0.0)
      XC=R2+R3*COS(TH3)
      YC=R3*SIN(TH3)
      CALL DRAWA(XC,YC)
      CALL DRAWA(XP,YP)
      CALL DRAWA(R2,0.0)
      CALL MOVEA(XC,YC)
      CALL DRAWA(R1,0.0)
    1 CALL FINITT(0,0)
      CALL EXIT
      STOP
      END
```

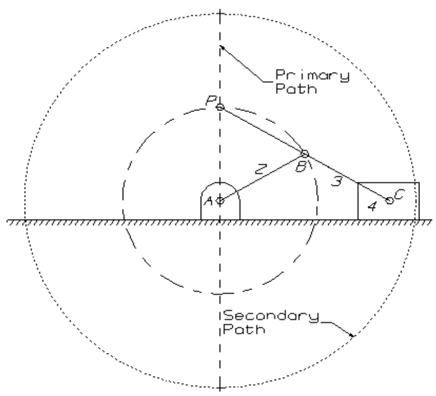
2.24 Plot the path of point *P* for: (*a*) inverted slider-crank linkage; (*b*) second inversion of the slider-crank linkage; (*c*) Scott-Russell straight-line linkage; and (*d*) drag-link linkage.



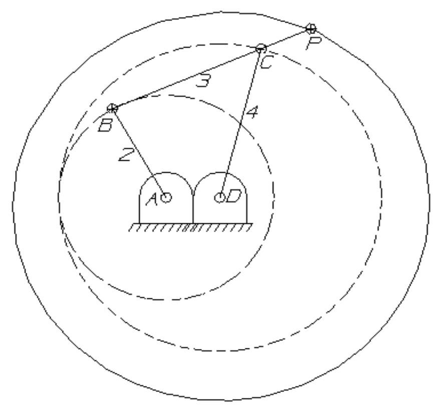
(a)



(*b*)



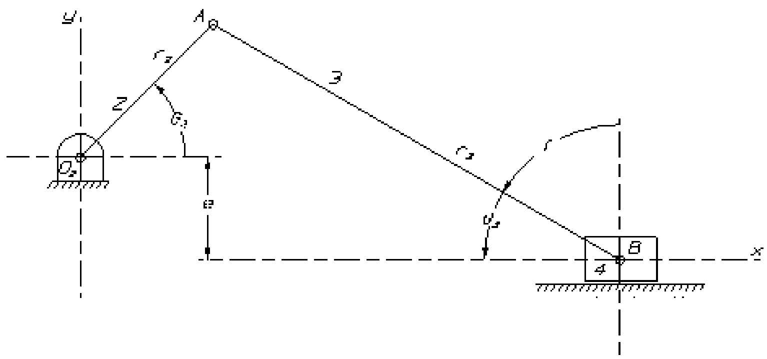
(*c*)



(*d*)

(*a*) $R_{CA} = 2$ in, $R_{BA} = 3.5$ in, and $R_{PC} = 4$ in; (*b*) $R_{CA} = 40$ mm, $R_{BA} = 20$ mm, and $R_{PB} = 65$ mm; (*c*) $R_{BA} = R_{CB} = R_{PB} = 25$ mm; (*d*) $R_{DA} = 1$ in, $R_{BA} = 2$ in, $R_{CB} = R_{CD} = 3$ in, and $R_{PB} = 4$ in.

2.25 Using the offset slider-crank linkage in Figure P2.13, find the crank angles corresponding to the extreme values of the transmission angle.



As shown, $\gamma = 90° - \theta_3$.
Also from the figure
$e + r_2 \sin\theta_2 = r_3 \cos\gamma$.
Differentiating with respect to θ_2;

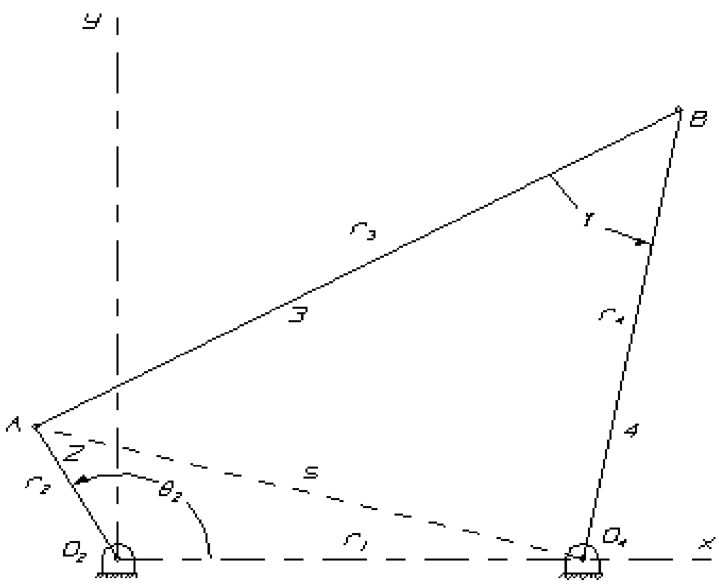
$$r_2 \cos\theta_2 = -r_3 \sin\gamma \frac{d\gamma}{d\theta_2};$$

$$\frac{d\gamma}{d\theta_2} = -\frac{r_2 \cos\theta_2}{r_3 \sin\gamma}.$$

Now, setting $d\gamma / d\theta_2 = 0$, we get $\cos\theta_2 = 0$.

Therefore, we conclude that $\theta_2 = \pm(2k+1)\pi/2 = \pm 90°, \pm 270°, \ldots$ <u>*Ans.*</u>

2.26 Section 1.10 states that the transmission angle reaches an extreme value for the four-bar linkage when the crank lies on the line between the fixed pivots. Referring to Figure 2.19, this means that γ reaches a maximum or minimum when crank 2 is collinear with the line O_2O_4. Show, analytically, that this statement is true.



From ΔO_4O_2A:
$s^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta_2$.
Also, from ΔABO_4:
$s^2 = r_3^2 + r_4^2 - 2r_3r_4\cos\gamma$.
Equating these we differentiate with respect to θ_2 to obtain

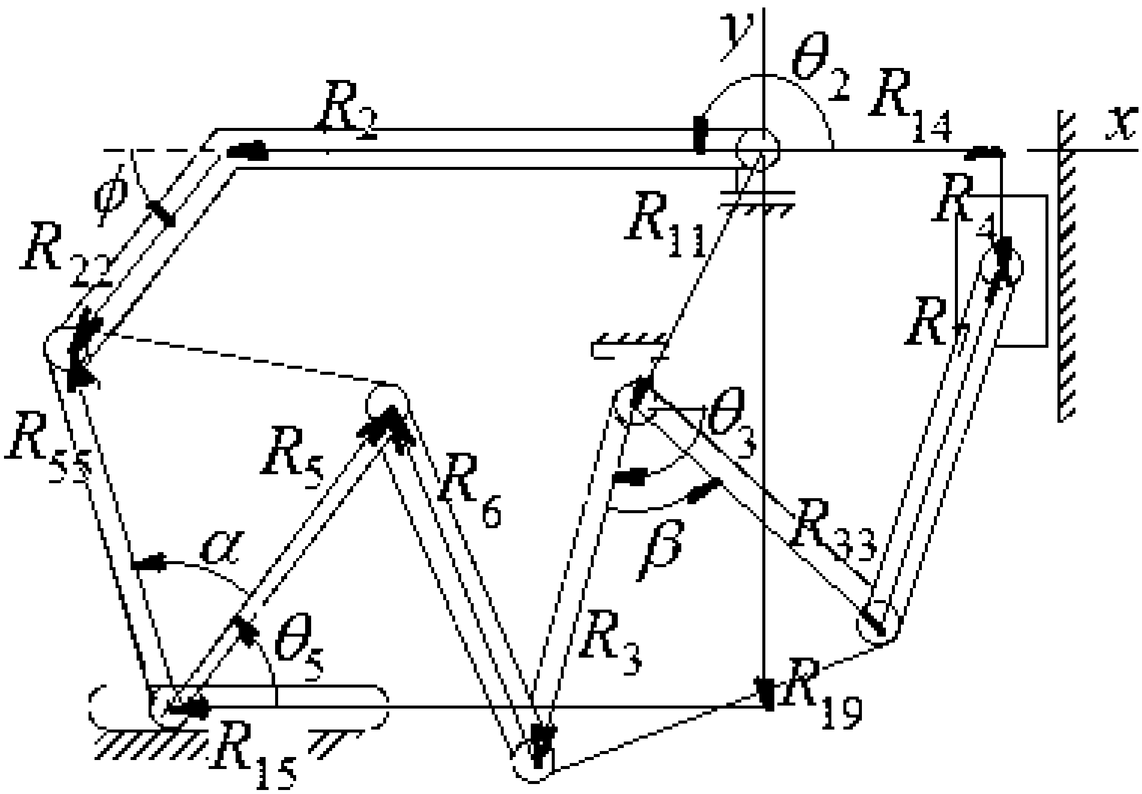
$$2r_1r_2\sin\theta_2 = 2r_3r_4\sin\gamma\frac{d\gamma}{d\theta_2} \quad \text{or}$$

$$\frac{d\gamma}{d\theta_2} = \frac{r_1r_2\sin\theta_2}{r_3r_4\sin\gamma}.$$

Now, for $\dfrac{d\gamma}{d\theta_2} = 0$, we have $\sin\theta_2 = 0$. Thus, $\theta_2 = 0, \ \pm 180°, \ \pm 360°, \ldots$ <u>*Q.E.D.*</u>

2.27 Define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).

One set of vectors suitable for a kinematic analysis of the mechanism is shown here.



The three vector loop equations are

$$\overset{\surd\surd}{\mathbf{R}_{11}} + \overset{\surd ?}{\mathbf{R}_{3}} + \overset{\surd ?}{\mathbf{R}_{6}} - \overset{\surd ?}{\mathbf{R}_{5}} - \overset{?\surd}{\mathbf{R}_{15}} - \overset{\surd\surd}{\mathbf{R}_{19}} = \mathbf{0}$$

$$\overset{\surd \mathrm{I}}{\mathbf{R}_{2}} + \overset{\surd \mathrm{C1}}{\mathbf{R}_{22}} - \overset{\surd \mathrm{C3}}{\mathbf{R}_{55}} - \overset{?\surd}{\mathbf{R}_{15}} - \overset{\surd\surd}{\mathbf{R}_{19}} = \mathbf{0}$$

$$\overset{\surd\surd}{\mathbf{R}_{11}} + \overset{\surd \mathrm{C2}}{\mathbf{R}_{33}} + \overset{\surd ?}{\mathbf{R}_{7}} - \overset{?\surd}{\mathbf{R}_{4}} - \overset{\surd\surd}{\mathbf{R}_{14}} = \mathbf{0}$$ *Ans.*

with three constraint equations

$\theta_{22} = \theta_2 + \phi$ $(C1)$ $\theta_{33} = \theta_3 + \beta$ $(C2)$ and $\theta_{55} = \theta_5 + \alpha$ $(C3)$. *Ans.*

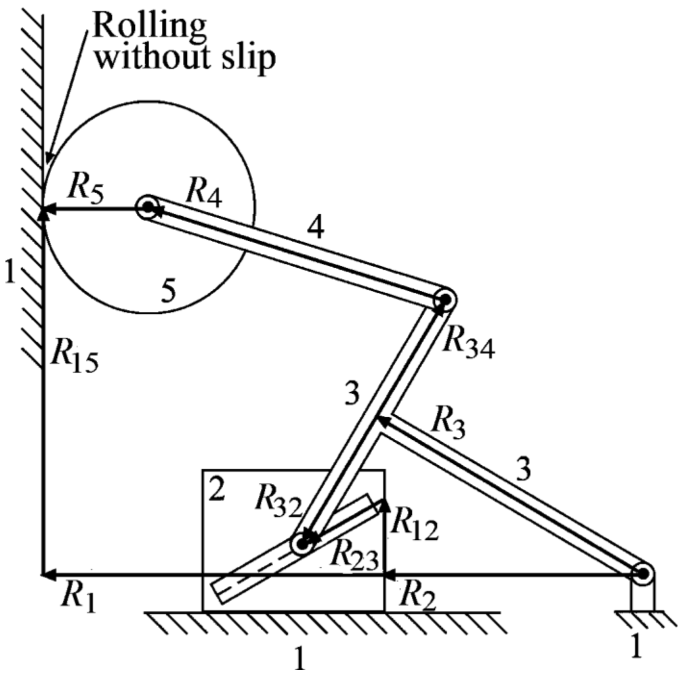
The angle θ_2 is a suitable input. *Ans.*

Known quantities are: R_2, R_3, θ_4=–90°, R_5, R_6, R_7, R_{11}, θ_{11}, R_{14}, θ_{14}=0, θ_{15}=180°, R_{19}, θ_{19}=–90°, R_{22}, R_{33}, and R_{55} *Ans.*

Unknown variables are θ_3, R_4, θ_5, θ_6, θ_7, R_{15}, θ_{22}, θ_{33}, and θ_{55}. *Ans.*

2.28 Define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraints then write the constraint equation(s).

One set of vectors suitable for a kinematic analysis of the mechanism is shown here.



The two vector loop equations are

$$\overset{I\surd}{\mathbf{R}_2} + \overset{\surd\surd}{\mathbf{R}_{12}} + \overset{?\surd}{\mathbf{R}_{23}} - \overset{\surd C1}{\mathbf{R}_{32}} - \overset{\surd ?}{\mathbf{R}_3} = \mathbf{0}$$

$$\overset{\surd ?}{\mathbf{R}_3} + \overset{\surd C2}{\mathbf{R}_{34}} + \overset{\surd ?}{\mathbf{R}_4} + \overset{\surd\surd}{\mathbf{R}_5} - \overset{?\surd}{\mathbf{R}_{15}} - \overset{\surd\surd}{\mathbf{R}_1} = \mathbf{0}$$ *Ans.*

with two constraint equations

$$\theta_{32} = \theta_3 + 90° \quad (C1) \qquad \text{and} \qquad \theta_{34} = \theta_3 - 90° \quad (C2).$$ *Ans.*

The magnitude R_2 is a suitable input. *Ans.*

Known quantities are: R_1, θ_1=180°, θ_2=180°, R_3, R_4, R_5, θ_5=180°, R_{12}, θ_{12}=90°, θ_{15}=90°, θ_{23}, R_{32}, and R_{34}. *Ans.*

Unknown variables are θ_3, θ_4, R_{15}, R_{23}, θ_{32}, and θ_{34}. *Ans.*

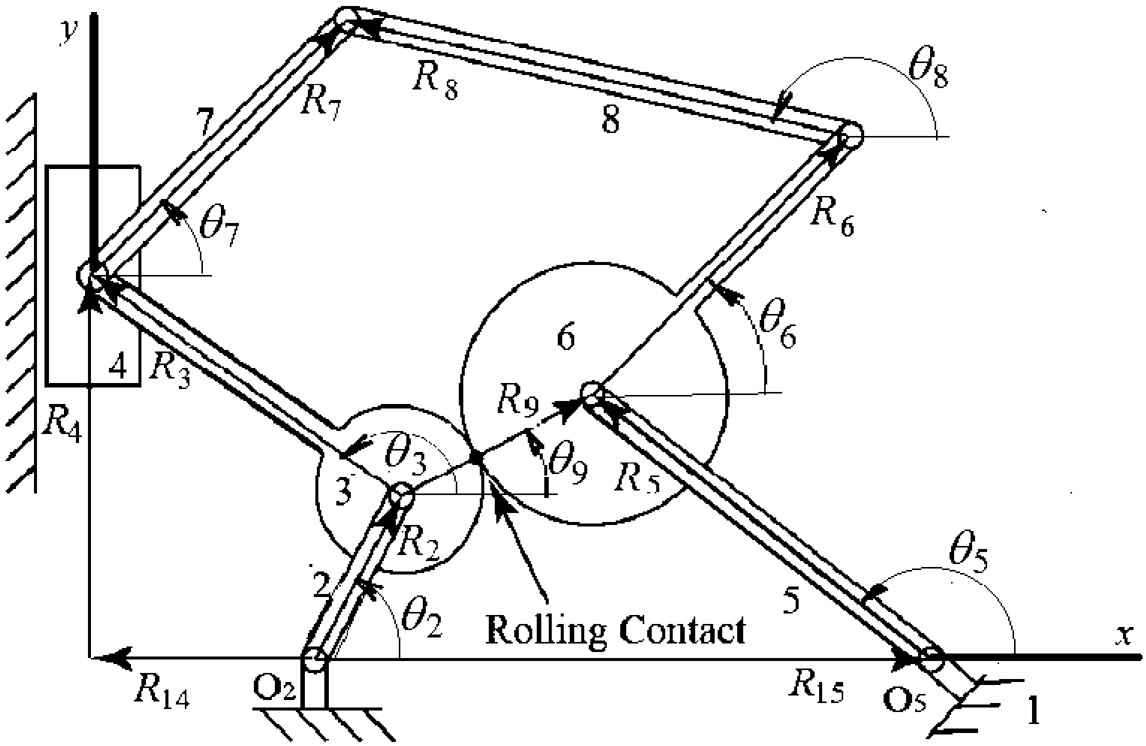
Note that the angular displacement of the wheel 5 is related to the distance R_{15} by rolling contact. The rolling contact equation between link 5 and the ground link 1 can be written as

$$\pm\,\Delta R_{15} = \rho_5\left(\Delta\theta_5 - \Delta\theta_{15}\right) = \rho_5\,\Delta\theta_5$$

The correct sign in this equation is positive because, for a positive (counterclockwise) rotation of wheel 5, the length of the vector R_{15} is increasing whereas, for a negative (clockwise) rotation of wheel 5, the length of the vector R_{15} is decreasing.

2.29 Define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraint(s) then write the constraint equation(s).

One set of vectors suitable for a kinematic analysis of the mechanism is shown here.



The three vector loop equations are

$$\overset{\surd I}{\mathbf{R}_2}+\overset{\surd ?}{\mathbf{R}_3}-\overset{?\surd}{\mathbf{R}_4}-\overset{\surd\surd}{\mathbf{R}_{14}}=\mathbf{0}$$

$$\overset{\surd I}{\mathbf{R}_2}+\overset{\surd ?}{\mathbf{R}_9}-\overset{\surd ?}{\mathbf{R}_5}-\overset{\surd\surd}{\mathbf{R}_{15}}=\mathbf{0}$$

$$\overset{\surd ?}{\mathbf{R}_3}+\overset{\surd ?}{\mathbf{R}_7}-\overset{\surd ?}{\mathbf{R}_8}-\overset{\surd C}{\mathbf{R}_6}-\overset{\surd ?}{\mathbf{R}_9}=\mathbf{0}$$ *Ans.*

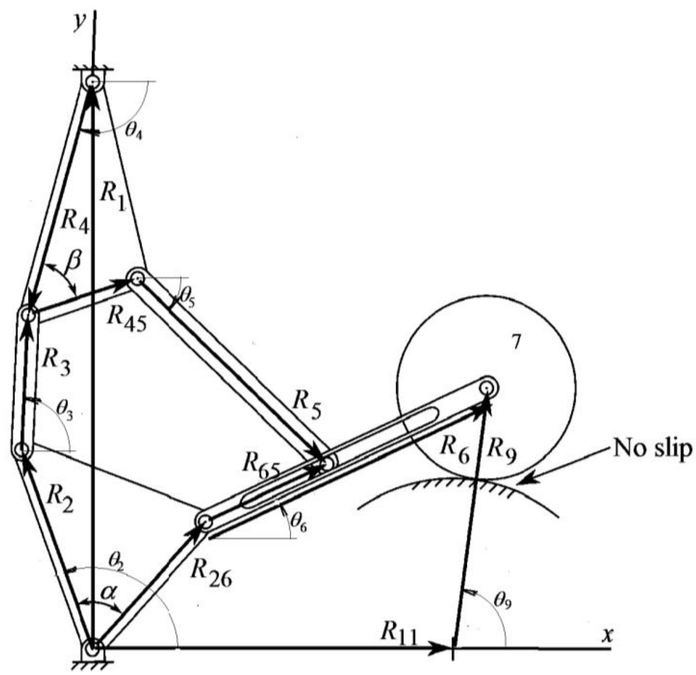
with one constraint equation

$$-\frac{\rho_6}{\rho_3}=\frac{\Delta\theta_3-\Delta\theta_9}{\Delta\theta_6-\Delta\theta_9}$$ *Ans.*

where the minus sign is used because gears 3 and 6 have external rolling contact.
The angle θ_2 is a suitable input. *Ans.*
Known quantities are:
R_2, R_3, θ_4=90°, R_5, R_6, R_7,R_8, $R_9=\rho_6+\rho_3$, R_{14}, θ_{14}=180°, R_{15}, and θ_{15}=0. *Ans.*
Unknown variables are θ_3, R_4, θ_5, θ_6, θ_7, θ_8, and θ_9. *Ans.*

2.30 Define a set of vectors that is suitable for a complete kinematic analysis of the mechanism. Label and show the sense and orientation of each vector.. Write the vector loop equation(s) for the mechanism. Identify suitable input(s), known quantities, unknown variables, and any constraints. If you identify constraint(s) then write the constraint equation(s).

One set of vectors suitable for a kinematic analysis of the mechanism is shown here.



The three vector loop equations are

$$\overset{\surd I}{\mathbf{R}_2} + \overset{\surd ?}{\mathbf{R}_3} - \overset{\surd ?}{\mathbf{R}_4} - \overset{\surd\surd}{\mathbf{R}_1} = \mathbf{0}$$

$$\overset{\surd C1}{\mathbf{R}_{26}} + \overset{\surd ?}{\mathbf{R}_6} - \overset{\surd ?}{\mathbf{R}_9} - \overset{\surd\surd}{\mathbf{R}_{11}} = \mathbf{0}$$

$$\overset{\surd I}{\mathbf{R}_2} + \overset{\surd ?}{\mathbf{R}_3} + \overset{\surd C2}{\mathbf{R}_{45}} + \overset{\surd ?}{\mathbf{R}_5} - \overset{?C3}{\mathbf{R}_{65}} - \overset{\surd C1}{\mathbf{R}_{26}} = \mathbf{0}$$ *Ans.*

There are four constraint equations

$$\theta_{26} = \theta_2 - \alpha \text{ (C1)}$$

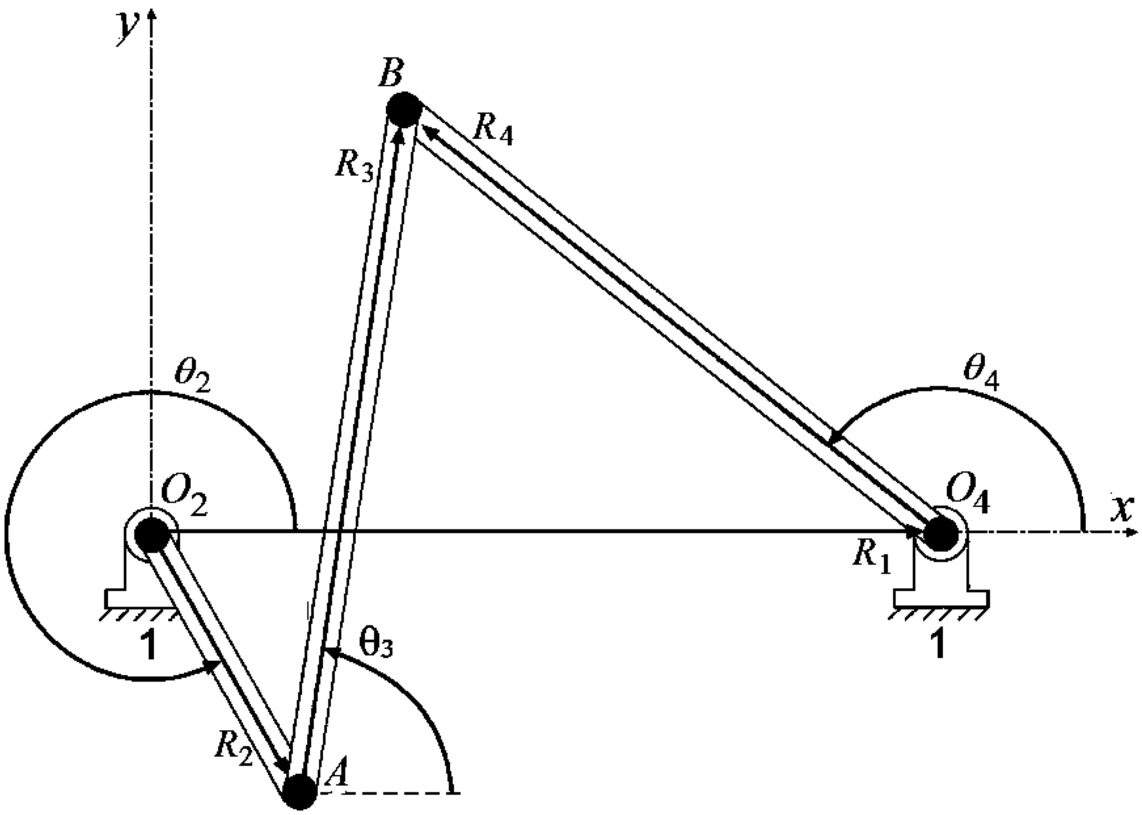
$$\theta_{45} = \theta_4 - \beta + 180° \text{ (C2)}$$

$$\theta_{65} = \theta_6 \quad \text{(C3)}$$

$$-\frac{\rho_1}{\rho_7} = \frac{\Delta\theta_7 - \Delta\theta_9}{\Delta\theta_1 - \Delta\theta_9} \text{ (C4)}$$ *Ans.*

where the minus sign is used because gears 1 and 7 have external rolling contact.
The angle θ_2 is a suitable input. *Ans.*
Known quantities are: R_1, θ_1=90°, R_2, R_3, R_4, R_5, R_6, R_9, R_{11}, θ_{11}=0, R_{26}, and R_{45}. *Ans.*
Unknown variables are θ_3, θ_4, θ_5, θ_6, θ_7, θ_9, θ_{26}, θ_{45}, R_{65}, and θ_{65}. *Ans.*

2.31 For the input angle $\theta_2 = 300^o$, measured counterclockwise from the x-axis, determine the two postures of link 4.



$r_2 = 60$ mm, $r_3 = 140$ mm, $r_4 = 140$ mm, and $r_1 = 160$ mm.

The vector loop equation can be written

$$\mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4 - \mathbf{r}_1 = \mathbf{0}$$

The horizontal and vertical components are

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

Squaring, adding, and rearranging these gives us Freudenstein's equation. That is

$$A\cos\theta_4 + B\sin\theta_4 = C \qquad (1)$$

where

$$A = 2r_1r_4 - 2r_2r_4\cos\theta_2$$

$$B = -2r_2r_4\sin\theta_2$$

$$C = r_3^2 - r_4^2 - r_1^2 - r_2^2 + 2r_1r_2\cos\theta_2$$

Substituting the known data into these equations gives

$$A = 2(160\text{ mm})(140\text{ mm}) - 2(60\text{ mm})(140\text{ mm})\cos 300° = 36\ 400\ (\text{mm})^2$$

$$B = -2(60\text{ mm})(140\text{ mm})\sin 300° = 14\ 549\ (\text{mm})^2$$

$$C = (140\text{ mm})^2 - (140\text{ mm})^2 - (160\text{ mm})^2 - (60\text{ mm})^2 + 2(160\text{ mm})(60\text{ mm})\cos 300°$$

$$= -19\ 600\ (\text{mm})^2$$

which reduces Eq. (1) to the form

$$36\ 400\cos\theta_4 + 14\ 549\sin\theta_4 + 19\ 600 = 0 \tag{2}$$

To solve this transcendental equation, we define

$$Z = \tan(\theta_4/2) \tag{3}$$

which gives

$$\sin\theta_4 = \frac{2Z}{1+Z^2} \qquad \text{and} \qquad \cos\theta_4 = \frac{1-Z^2}{1+Z^2}$$

Substituting these into Eq. (2), and rearranging, gives

$$16\ 800Z^2 - 29\ 098Z - 56\ 000 = 0$$

which has the solutions

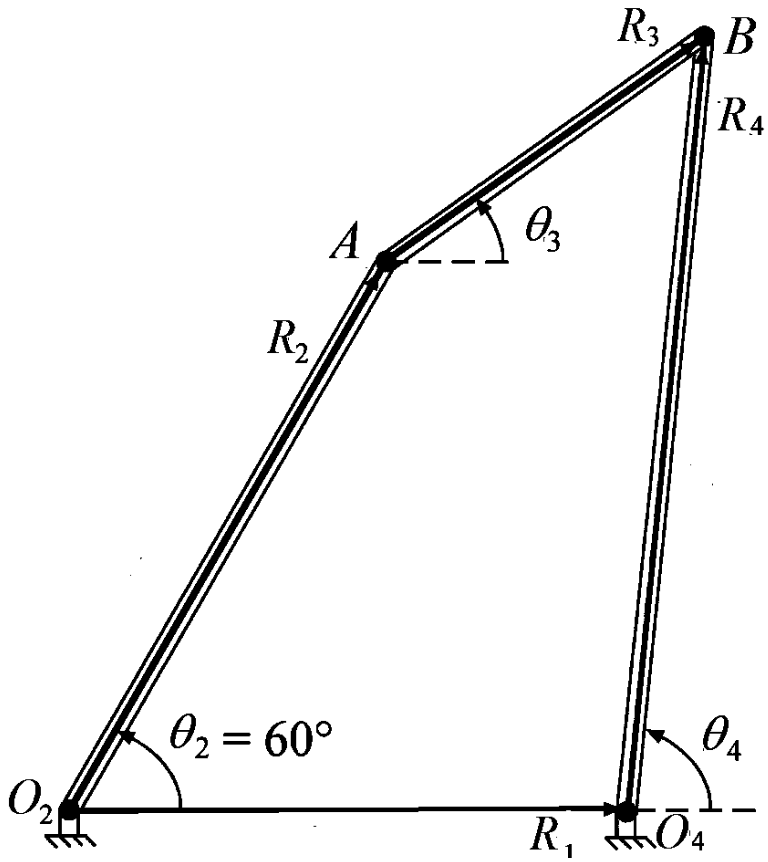
$$Z = \frac{14\ 549 \pm \sqrt{(14\ 549)^2 - (16\ 800)(-56\ 000)}}{16\ 800}$$

$$= 2.886\ 73 \quad \text{or} \quad -1.154\ 71$$

Substituting these two roots back into Eq. (3) gives the two solutions

$$\theta_4 = 141.79° \qquad \text{and} \qquad \theta_4 = -98.21°$$ *Ans.*

2.32 For the input angle $\theta_2 = 60^o$, measured counterclockwise from the *x*-axis, determine the two postures of link 4.



$r_2 = 80$ mm, $r_3 = 50$ mm, $r_4 = 100$ mm, and $r_1 = 70$ mm.

The vector loop equation can be written

$$\mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4 - \mathbf{r}_1 = \mathbf{0}$$

The horizontal and vertical components are

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

Squaring, adding, and rearranging these gives us Freudenstein's equation. That is

$$A\cos\theta_4 + B\sin\theta_4 = C \qquad (1)$$

where

$$A = 2r_1 r_4 - 2r_2 r_4 \cos\theta_2$$

$$B = -2r_2 r_4 \sin\theta_2$$

$$C = r_3^2 - r_4^2 - r_1^2 - r_2^2 + 2r_1 r_2 \cos\theta_2$$

Substituting the known data into these equations gives

$$A = 2(70\text{ mm})(100\text{ mm}) - 2(80\text{ mm})(100\text{ mm})\cos 60° = 6\ 000\ (\text{mm})^2$$

$$B = -2(80\text{ mm})(100\text{ mm})\sin 60° = -13\ 856.4\ (\text{mm})^2$$

$$C = (50\text{ mm})^2 - (100\text{ mm})^2 - (70\text{ mm})^2 - (80\text{ mm})^2 + 2(70\text{ mm})(80\text{ mm})\cos 60°$$
$$= -13\ 200\ (\text{mm})^2$$

which reduces Eq. (1) to the form

$$6\ 000\cos\theta_4 - 13\ 856\sin\theta_4 + 13\ 200 = 0 \qquad (2)$$

To solve this transcendental equation, we define

$$Z = \tan(\theta_4/2) \qquad (3)$$

which gives

$$\sin\theta_4 = \frac{2Z}{1+Z^2} \quad \text{and} \quad \cos\theta_4 = \frac{1-Z^2}{1+Z^2}$$

Substituting these into Eq. (2), and rearranging, gives

$$7\ 200Z^2 - 27\ 712Z + 19\ 200 = 0$$

which has the solutions

$$Z = \frac{13\ 856 \pm \sqrt{(-13\ 856)^2 - (7\ 200)(19\ 200)}}{7\ 200}$$
$$= 2.942\ 69 \quad \text{or} \quad 0.906\ 20$$

Substituting these two roots back into Eq. (3) gives the two solutions

$$\theta_4 = 142.46° \quad \text{and} \quad \theta_4 = 84.37°.$$

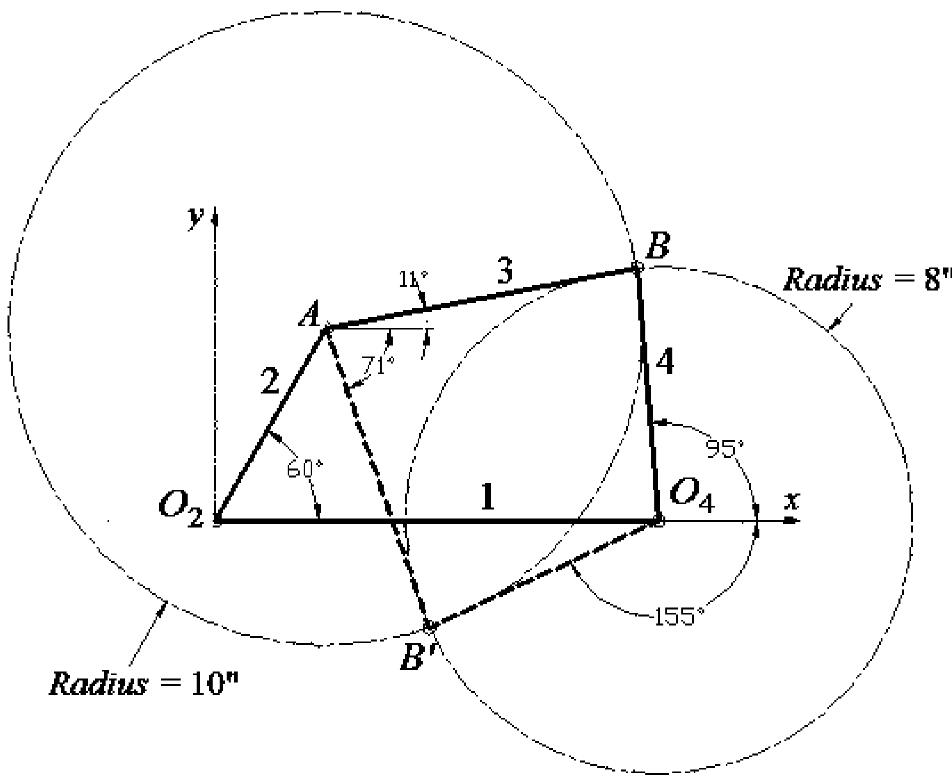
The angular position of link 4, for the open configuration shown is $\theta_4 = 84.37°$. *Ans.*

2.33 Consider a four-bar linkage for which ground link 1 is 14 in, input link 2 is 7 in, coupler link 3 is 10 in, and output link 4 is 8 in. The fixed *x* and *y* axes are specified as horizontal and vertical, respectively. The origin of this reference frame is coincident with the ground pivot of link 2, and the ground link is aligned with the *x* axis. For the input angle $\theta_2 = 60^\circ$ (counterclockwise from the *x* axis): (*a*) Using a suitable scale, draw the linkage in the open and crossed postures and measure the values of the variables θ_3 and θ_4 for each posture. (*b*) Use trigonometry (that is, the laws of sines and cosines) to determine θ_3 and θ_4 for the open posture. (*c*) Use Freudenstein's equation to determine θ_3 and θ_4 for both postures. (*d*) Use the Newton-Raphson iteration procedure to determine θ_3 and θ_4 for the open posture. Using the measurements in (*a*) as initial estimates for θ_3 and θ_4, iterate until the two variables converge to within 0.01°.

(*a*) Graphic Method. The link dimensions and angles are specified as:

Ground Link:	R_1 = 14.0 in	θ_1 = 0
Input Link:	R_2 = 7.0 in	$\theta_2 = 60^\circ$
Coupler Link:	R_3 = 10.0 in	θ_3 = ?
Output Link:	R_4 = 8.0 in	θ_4 = ?

For the specified input angle $\theta_2 = 60^\circ$, the two possible configurations of the four-bar linkage are as shown in the following figure. The loop O_2ABO_4 is the open posture of the four-bar linkage and the loop $O_2AB'O_4$ is the closed (crossed) posture.



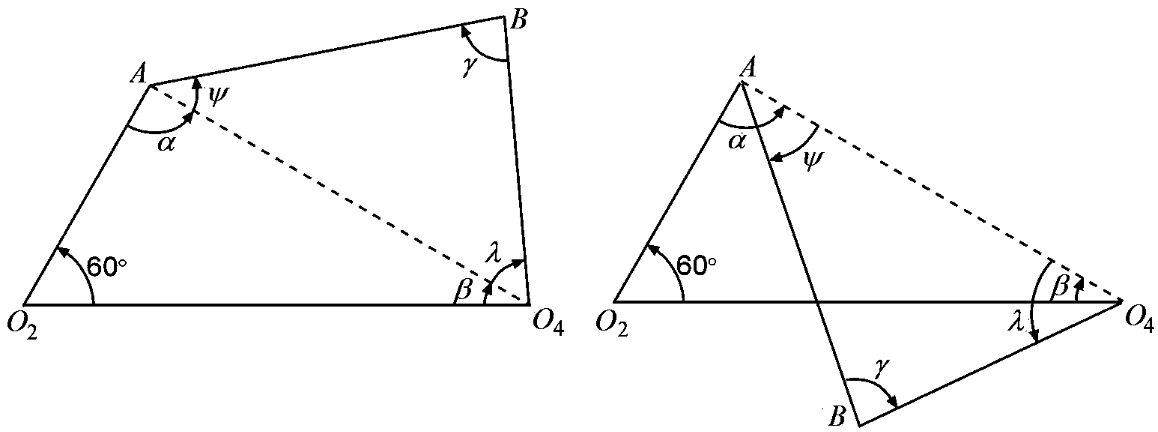
Graphic solution.

From measurements of the drawing, the answers for the coupler angle and the output angle are:

The Open Posture: $\theta_3 = 11^\circ$ and $\theta_4 = 95^\circ$

The Crossed Posture: $\theta_3 = -71^\circ$ or $+289^\circ$ and $\theta_4 = -155^\circ$ or $+205^\circ$

(*b*) Trigonometry. The notation for the open and crossed configurations of the four-bar linkage are shown in the following figure.



From the triangle AO_2O_4, the law of cosines gives

$$\overline{AO_4}^2 = R_1^2 + R_2^2 - 2R_1R_2\cos\theta_2$$

$$\overline{AO_4} = \sqrt{(14\text{ in})^2 + (7\text{ in})^2 - 2(14\text{ in})(7\text{ in})\cos 60^\circ} = 12.124\text{ in}$$

From the triangle O_2O_4A, the law of cosines gives

$$\beta = \cos^{-1}\frac{R_1^2 + \overline{AO_4}^2 - R_2^2}{2R_1\overline{AO_4}} = \cos^{-1}\frac{(14\text{ in})^2 + (12.124\text{ in})^2 - (7\text{ in})^2}{2(14\text{ in})(12.124\text{ in})} = -30^\circ$$

From the triangle O_4AB, the law of cosines gives

$$\psi = \cos^{-1}\frac{R_3^2 + \overline{AO_4}^2 - R_4^2}{2R_3\overline{AO_4}} = \cos^{-1}\frac{(10\text{ in})^2 + (12.124\text{ in})^2 - (8\text{ in})^2}{2(10\text{ in})(12.124\text{ in})} = \pm 41^\circ$$

$$\lambda = \cos^{-1}\frac{R_4^2 + \overline{AO_4}^2 - R_3^2}{2R_4\overline{AO_4}} = \cos^{-1}\frac{(8\text{ in})^2 + (12.124\text{ in})^2 - (10\text{ in})}{2(8\text{ in})(12.124\text{ in})} = \mp 55.10^\circ$$

From these, the angles for the open posture are

$$\theta_3 = \psi - \beta = 41^\circ - 30^\circ = 11^\circ \qquad \underline{\textit{Ans.}}$$

$$\theta_4 = \pi - \beta - \lambda = 180^\circ - 30^\circ - 55.10^\circ = 94.90^\circ \qquad \underline{\textit{Ans.}}$$

For the crossed posture, the angles are

$$\theta_3 = -\beta - \psi = -30^\circ - 41^\circ = -71^\circ = 289^\circ \qquad \underline{\textit{Ans.}}$$

$$\theta_4 = \pi - \beta + \lambda = 180° - 30° + 55.10° = 205.10° = -154.90°$$ <u>*Ans.*</u>

(*c*) The vector loop equation can be written

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = \mathbf{0}$$

The horizontal and vertical components are

$$R_2 \cos\theta_2 + R_3 \cos\theta_3 - R_4 \cos\theta_4 - R_1 = 0$$
$$R_2 \sin\theta_2 + R_3 \sin\theta_3 - R_4 \sin\theta_4 = 0 \qquad (1)$$

Squaring, adding, and rearranging these gives us Freudenstein's equation. That is,

$$A\cos\theta_4 + B\sin\theta_4 = C \qquad (2)$$

where

$$A = 2R_1R_4 - 2R_2R_4\cos\theta_2$$
$$B = -2R_2R_4\sin\theta_2$$
$$C = R_3^2 - R_4^2 - R_1^2 - R_2^2 + 2R_1R_2\cos\theta_2$$

Substituting the known data into these equations gives

$$A = 2(14\text{ in})(8\text{ in}) - 2(7\text{ in})(8\text{ in})\cos 60° = 168\text{ in}^2$$

$$B = -2(7\text{ in})(8\text{ in})\sin 60° = -97\text{ in}^2$$

$$C = (10\text{ in})^2 - (8\text{ in})^2 - (14\text{ in})^2 - (7\text{ in})^2 + 2(14\text{ in})(7\text{ in})\cos 60° = -111\text{ in}^2$$

which reduces Eq. (2) to the form

$$168\cos\theta_4 - 97\sin\theta_4 + 111 = 0 \qquad (3)$$

To solve this transcendental equation, we define

$$Z = \tan(\theta_4/2) \qquad (4)$$

which gives

$$\sin\theta_4 = \frac{2Z}{1+Z^2} \quad \text{and} \quad \cos\theta_4 = \frac{1-Z^2}{1+Z^2}$$

Substituting these into Eq. (3), and rearranging, gives

$$57Z^2 + 194 - 279 = 0$$

which has the solutions

$$Z = \frac{-97 \pm \sqrt{(-97)^2 - (57)(-279)}}{57}$$
$$= 1.089\ 43 \quad \text{or} \quad -4.492\ 94$$

Substituting these two roots back into Eq. (4) gives the two solutions

$$\theta_4 = 94.90° \quad \text{and} \quad \theta_4 = -154.90° = 205.10°.$$

The angular position of link 4, for the open posture shown is $\theta_4 = 94.90°$. <u>*Ans.*</u>

From this, Eqs. (1) gives $\theta_3 = 11.00°$. <u>*Ans.*</u>

The crossed posture gives $\theta_4 = -154.90°$ and $\theta_3 = -71.00° = 289.00°$. <u>*Ans.*</u>

(*d*) For the Newton-Raphson technique, the vector loop equation can be written

$$\mathbf{f} = \mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = \mathbf{0}$$

From this, the horizontal and vertical components are

$$f_x = R_2\cos\theta_2 + R_3\cos\theta_3 - R_4\cos\theta_4 - R_1 = 0$$
$$f_y = R_2\sin\theta_2 + R_3\sin\theta_3 - R_4\sin\theta_4 = 0$$

Expanding these to first order in Taylor series we find

$$R_2\cos\theta_2 + R_3\cos\theta_3 - R_3\sin\theta_3\Delta\theta_3 - R_4\cos\theta_4 + R_4\sin\theta_4\Delta\theta_4 - R_1 = 0$$
$$R_2\sin\theta_2 + R_3\sin\theta_3 + R_3\cos\theta_3\Delta\theta_3 - R_4\sin\theta_4 - R_4\cos\theta_4\Delta\theta_4 = 0$$

and, writing this in matrix format gives

$$\begin{bmatrix} R_3\sin\theta_3 & -R_4\sin\theta_4 \\ -R_3\cos\theta_3 & R_4\cos\theta_4 \end{bmatrix}\begin{bmatrix} \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix} = \begin{bmatrix} R_2\cos\theta_2 + R_3\cos\theta_3 - R_4\cos\theta_4 - R_1 \\ R_2\sin\theta_2 + R_3\sin\theta_3 - R_4\sin\theta_4 \end{bmatrix}$$

Substituting the given data this becomes

$$\begin{bmatrix} 10.0\sin\theta_3 & -8.0\sin\theta_4 \\ -10.0\cos\theta_3 & 8.0\cos\theta_4 \end{bmatrix}\begin{bmatrix} \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix} = \begin{bmatrix} -10.50000 + 10.0\cos\theta_3 - 8.0\cos\theta_4 \\ 6.06218 + 10.0\sin\theta_3 - 8.0\sin\theta_4 \end{bmatrix} \qquad (5)$$

Using the graphic solution of part (*a*) as an estimate, $\theta_3 = 11°$ and $\theta_4 = 95°$, the first iteration equations are

$$\begin{bmatrix} 1.90809 & -7.96956 \\ -9.81627 & -0.69725 \end{bmatrix}\begin{bmatrix} \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix} = \begin{bmatrix} 0.01352 \\ 0.00071 \end{bmatrix}$$

which give corrections of $\Delta\theta_3 = 0.000\ 0471\ \text{rad} = 0.002\ 70°$, $\Delta\theta_4 = -0.001\ 68\ \text{rad} = -0.096\ 26°$.

Therefore, after one iteration, we have $\theta_3 = 11.002\ 70°$ and $\theta_4 = 94.903\ 74°$.

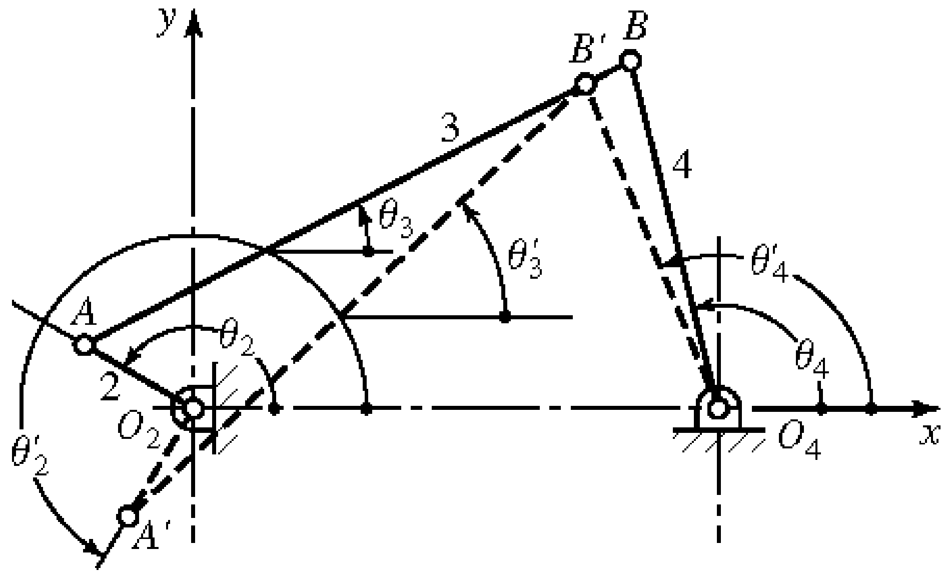
Substituting again into Eqs. (5) gives

$$\begin{bmatrix} 1.90855 & -7.97072 \\ -9.81618 & -0.68382 \end{bmatrix}\begin{bmatrix} \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix} = \begin{bmatrix} -0.00000099 \\ 0.000011312 \end{bmatrix}$$

This gives corrections of $\Delta\theta_3 = -1.142(10)^{-7}\ \text{rad} = -0.000\ 065\ 4°$, $\Delta\theta_4 = -1.49(10)^{-7}\ \text{rad} = -0.000\ 008\ 54°$..

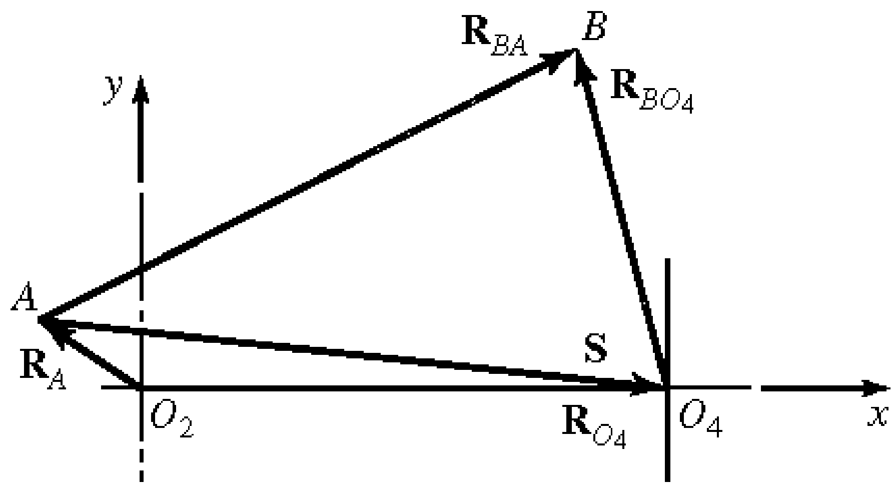
Therefore, after two iteration , we have $\theta_3 = 11.002\ 64°$ and $\theta_4 = 94.903\ 73°$. <u>*Ans.*</u>

2.34 A crank-rocker four-bar linkage is illustrated in two different postures for which $\theta_2 = 150°$ and $\theta_2' = 240°$. Determine θ_3 and θ_4 for the open posture and θ_3' and θ_4' for the crossed posture..



$R_{O_4O_2} = 600$ mm, $R_{AO_2} = 140$ mm, $R_{BA} = 690$ mm, and $R_{BO_4} = 400$ mm.

For the first input angle $\theta_2 = 150°$, to the following figure



and observe that

$$\mathbf{R}_A = 0.140 \text{ m}\angle 150°, \quad \text{and} \quad \mathbf{R}_{O_4} = 0.600 \text{ m}\angle 0°.$$

Therefore,

$$\mathbf{S} = \mathbf{R}_{O_4} - \mathbf{R}_A = 0.600 \text{ m}\angle 0° - 0.140 \text{ m}\angle 150° = 0.725 \text{ m}\angle -5.54°.$$

Referring again to the figure, we note that the vectors in the triangle ABO_4 are related by the equation

$$\overset{\surd\surd}{\mathbf{S}} = \overset{\surd ?}{\mathbf{R}_{BA}} - \overset{\surd ?}{\mathbf{R}_{BO_4}} \qquad (a)$$

There are two unknown orientations in this equation, and so we identify this as case 4. Using Eq. (2.49) and substituting **S** for **C**, R_{BA} for A, $-R_{BO4}$ for B, θ_S for θ_C, and θ_4 for θ_B

gives

$$\theta_4 = \theta_S \pm \cos^{-1}\frac{S^2 + R_{BO_4}^2 - R_{BA}^2}{2SR_{BO_4}}$$

$$= -5.54° \pm \cos^{-1}\frac{(0.725\text{ m})^2 + (-0.400\text{ m})^2 - (0.690\text{ m})^2}{2(0.725\text{ m})(-0.400\text{ m})}$$

$$= -5.54° \pm 111.18° = \underline{105.64°} \quad \text{or} \quad -116.72°$$ *Ans.*

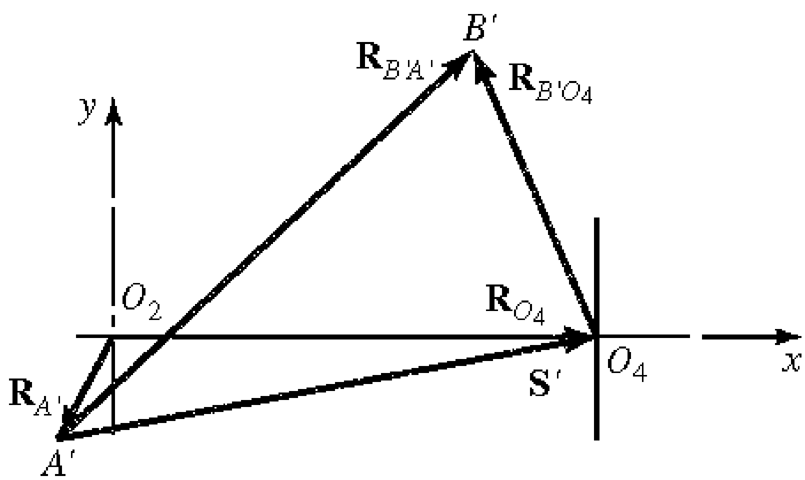
We note that we could have substituted R_{BO4}=+0.400 m for B and we would have obtained θ_4 + 180° for the final result.

Next, using Eq. (2.50) and substituting θ_3 for θ_A gives

$$\theta_3 = \theta_S \mp \cos^{-1}\frac{S^2 + R_{BA}^2 - R_{BO_4}^2}{2SR_{BA}}$$

$$= -5.54° \mp \cos^{-1}\frac{(0.725\text{ m})^2 + (0.690\text{ m})^2 - (-0.400\text{ m})^2}{2(0.725\text{ m})(0.690\text{ m})}$$

$$= -5.54° \mp 32.72° = -38.26° \quad \text{or} \quad \underline{27.18°}$$ *Ans.*



We follow the same procedure for the second input angle $\theta_2' = 240°$. Using the figure above yields

$$\mathbf{S}' = \mathbf{R}_{O_4} - \mathbf{R}_A' = 0.600\text{ m}\angle 0° - 0.140\text{ m}\angle 240° = 0.681\text{ m}\angle 10.26°.$$

$$\theta_4' = 10.26° \pm \cos^{-1}\frac{(0.681\text{ m})^2 + (-0.400\text{ m})^2 - (0.690\text{ m})^2}{2(0.681\text{ m})(-0.400\text{ m})}$$

$$= 10.26° \pm 105.73° = \underline{115.99°} \quad \text{or} \quad -95.47°$$ *Ans.*

$$\theta_3' = 10.26° \mp \cos^{-1}\frac{(0.681\text{ m})^2 + (0.690\text{ m})^2 - (-0.400\text{ m})^2}{2(0.681\text{ m})(0.690\text{ m})}$$

$$= 10.26° \mp 33.92° = -23.66° \quad \text{or} \quad \underline{44.18°}$$ *Ans.*

We recognize the positive angles as the solutions of interest in all cases.

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Chapter 3
Velocity

3.1 The position vector of a point is given by the equation $\mathbf{R} = 100e^{j\pi t}$, where R is in inches. Find the velocity of the point at $t = 0.40$ s.

$$\mathbf{R}(t) = 100e^{j\pi t} \text{ in}$$
$$\dot{\mathbf{R}}(t) = j\pi 100e^{j\pi t} \text{ in/s}$$
$$\dot{\mathbf{R}}(0.40\text{s}) = j\pi 100e^{j\pi 0.40} \text{ in/s}$$
$$= j\pi 100(\cos 0.40\pi + j\sin 0.40\pi) \text{ in/s}$$
$$= -100\pi \sin 72° \text{ in/s} + j100\pi \cos 72° \text{ in/s}$$
$$\dot{\mathbf{R}}(0.40\text{s}) = -298.783 + j97.080 \text{ in/s} = 314.159 \text{ in/s}\angle 162°$$ *Ans.*

3.2 The path of a point is defined by the equation $\mathbf{R} = (t^2 + 4)e^{-j\pi t/10}$, where R is in meters. Find the velocity of the point at $t = 20$ s.

$$\mathbf{R}(t) = (t^2 + 4)e^{-j\pi t/10}$$
$$\dot{\mathbf{R}}(t) = 2te^{-j\pi t/10} - (j\pi/10)(t^2 + 4)e^{-j\pi t/10}$$
$$\dot{\mathbf{R}}(20\text{ s}) = 40e^{-j\pi 20/10} - (j\pi/10)(20^2 + 4)e^{-j\pi 20/10}$$
$$= 40e^{-j2\pi} - j\pi 40.4e^{-j2\pi}$$
$$\dot{\mathbf{R}}(20\text{ s}) = 40.000 - j126.920 \text{ m/s} = 133.074 \text{ m/s}\angle -72.51°$$ *Ans.*