CHAPTER 2

P2.1. A W24×192 of A242 steel is to be used as a beam in a building structure. What are the values of the yield stress, F_y, ultimate tensile stress, F_u, and modulus of elasticity, E to be used in the design of this beam.

Solution

Enter LRFDM Table 2-4, and note that a W24×192 shape belongs to Group 3. In LRFDM Table 2-1, for W-shapes in A242 steel, the only alloy available for Group 3 shapes is of Grade 46 (see note k). Also, from this table, observe that A242 steel of Grade 46 has a yield stress F_y of 46 ksi and an ultimate tensile stress F_u of 67 ksi. Modulus of elasticity for all steels, E = 29, 000 ksi.

P2.2. A PL3×8 of A514 steel is to be used as a tension member in a truss. What are the values of the yield stress, F_y, and ultimate tensile stress, F_u, to be used in the design of this member?

Solution

Enter LRFDM Table 2-2, and note that a 3 in. thick A514 plate is available in Grade 90 only. Also, from this table, observe that A514 steel of Grade 90 has a yield stress F_y of 90 ksi and a minimum specified ultimate tensile stress F_u of 100 ksi.

P2.3. A chandelier weighing 2 kips hangs from the dome of a theater. The rod from which it hangs is 20 ft long and has a diameter of ½ in. Calculate the stress and strain in the rod and its elongation. Neglect the weight of the rod it self in the calculations.

Solution

Tensile force, T = 2 kips; Length, L = 20 ft = 240 in.

Diameter, d = ½ in.; Modulus of elasticity, E = 29,000 ksi

Cross-sectional area, $A = \pi d^2/4 = 0.196$ in.^{2} (LRFDM Table 1-21)

Stress, $f = T/A = 2.00 \div 0.196 = 10.2$ ksi (Ans.)

Strain, $\epsilon = f/E = 10.2 \div 29{,}000 = 0.000352$ in./in. (Ans.)

Elongation, e $= \epsilon L =$ 0.000352 (240) = 0.0845 in. (Ans.)

P2.4. The rod in Problem P2.3 is of A36 steel having a stress-strain diagram (see Figs. 2.6.1 and 2.6.2) with $\epsilon_{st} =$ 0.012 and $\epsilon_u =$ 0.18. Determine the load that causes the rod to yield; and the load that causes the rod to fracture. Also, determine the elongation of the bar corresponding to strains of ϵ_y, ϵ_{st}, and ϵ_u. Comment on your results.

Solution

For A36 steel: F_y = 36 ksi, and F_u = 58 ksi (from LRFDM Table 2-1)

Load that causes the rod to yield in tension, $T_y = A\,F_y =$ 0.196 (36.0) = 7.06 kips (Ans.)

Load that causes the rod to fracture in tension, $T_u = A\,F_u =$ 0.196 (58.0) = 11.4 kips (Ans.)

Elastic strain, $\epsilon_y = F_y \div E$ = 36.0 ÷ 29,000 = 0.00124 in./in.

Elongation of the rod corresponding to ϵ_y (maximum elastic elongation),

$$\Delta_y = \epsilon_y L = 0.00124(240) = 0.298 \text{ in.} \qquad \text{(Ans.)}$$

Strain hardening strain, $\epsilon_{st} =$ 0.012 in./in.

Elongation of the rod at onset of strain hardening, $\Delta_{st} = \epsilon_{st} L$ = 0.012 (240) = 2.88 in. (Ans.)

Fracture strain, ϵ_u = 0.18 in./in.

Elongation of the rod at fracture, $\Delta_f = \epsilon_u L =$ 0.18 (240) = 43.2 in. (Ans.)

The maximum elastic elongation of the member (0.3 in.) is (tolerably) small. The large elongations (43 in.) that precede the fracture give a visual warning of the impending failure of the member.

P2.5. Locate the principal axes for the beam sections given in Fig. P2.5. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and section moduli about the *x*- and *y*-axes.

See Figure P2.5 of text book

Solution

W24×68: A = 20.1 in.^{2}; d = 23.7 in.; I_x = 1830 in.^{4}; I_y = 70.4 in.^{4}

PL ½ ×12: A = 6.00 in.^{2}; $I_x = (1/12)\,b\,t^3 = (1/12)(12.0)(½)^3 =$ 0.125 in.^{4}

$I_y = (1/12)\,t\,b^3 = (1/12)(½)(12.0)^3 =$ 72.0 in.^{4}

Built-up section

$A = A_1 + A_2 = 20.1 + 6.00 = 26.1$ in.^{2} (Ans.)

Weight of the built-up section = 26.1(3.40) = 88.7 plf (Ans.)

The centers of gravity G_1, G_2 and G all lie on the vertical axis of symmetry.

Assume reference axis x'-x' at top fiber of the built-up section.

$$\bar{y} = \frac{(20.1)(11.85) + (6.00)(23.7 + 0.25)}{26.1} = 14.63 \text{ in.}$$

$d_1 = 14.63 - 11.85 = 2.78$ in.; $d_2 = 23.95 - 14.63$ in. $= 9.32$ in.

$I_x = [\ 1830 + 20.1\ (2.78)^2\ ] + [\ 0 + 6.00(9.32)^2\ ] = 2510$ in.^{4} (Ans.)

$c_t = 14.63$ in.; $c_b = 23.7 + 0.5 - 14.63 = 9.57$ in.

$S_{xt} = 2510 \div 14.63 = 172$ in.^{3}; $S_{xb} = 2510 \div 9.57 = 262$ in.^{3} (Ans.)

As G_1, G_2 and G are all located on the symmetry line,

$I_y = [70.4 + 0] + [72.0 + 0] = 142$ in.^{4} (Ans.)

$c = 6.00$ in.

$S_y = 142 \div 6.00 = 23.7$ in.^{3} (Ans.)

P2.6. Locate the principal axes for the beam sections given in Figs. P2.6. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and section moduli about the *x*- and *y*-axes.

See Figure P2.6 of text book.

Solution

The centers of gravity G_1, G_2, G_3 and G all lie on the vertical axis of symmetry. To locate G, take an arbitrary axis x'-x' coinciding with the bottom fibers of the bottom glange plate.

$A = A_1 + A_2 + A_3 = 2.00(8.00) + 1.00(16.0) + 2.00(12.0) = 56.0$ in.^{2} (Ans.)

Weight of the built-up section = 56.0 (3.40) = 190 plf (Ans.)

$$\bar{y} = \frac{16.0(19.0) + 16.0(10.0) + 24.0(1.00)}{56.0} = 8.71 \text{ in.}$$

$d_1 = 19.0 - 8.71 = 10.3$ in.; $d_2 = 10.0 - 8.71 = 1.29$ in.; $d_3 = 8.71 - 1.00 = -7.71$ in.

$I_x = [\ (1/12)\ (8.00)\ (2.00)^3 + 16.0(10.3)^2\ ] + [\ (1/12)\ (1.00)\ (16.0)^3 + 16.0\ (1.29)^2\ ]$

$+ [\ (1/12)\ (12.0)\ (2.00)^3 + 24.0\ (7.71)^2\ ] \ = \ 3510 \text{ in.}^4$ (Ans.)

$c_t = 20.0 - 8.71 = 11.3$ in.; $c_b = 8.71$ in.

$S_{xt} = 3510 \div 11.3 = 311 \text{ in.}^3$; $S_{xb} = 3510 \div 8.71 = 403 \text{ in.}^3$ (Ans.)

$I_y = (1/12)\ (2.00)\ (8.00)^3 + 0.0 + (1/12)\ (2.00)\ (12.0)^3 = 373 \text{ in.}^4$ (Ans.)

$S_y = 373 \div 6.00 = 62.2 \text{ in.}^3$ (Ans.)

P2.7. Locate the principal axes for the beam sections given in Fig. P2.7. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and section moduli about the *x*- and *y*-axes.

See Figure P2.7 of text book.

Solution

As the built-up section has two axes of symmetry, the center of gravity *G* of the built-up section coincides with the point of intersection of these two axes.

$A = 2(36.0)\ (4.00) + 2(2.00)\ (36.0) = 432 \text{ in.}^2$ (Ans.)

Weight = $432\ (3.40) \div 1000 = 1.47$ klf (Ans.)

$I_x = 2[\ (1/12)\ (36.0)\ (4.00)^3 + 144(20.0)^2\ ] + 2[\ (1/12)\ (2.00)\ (36.0)^3 + 0\ ] = 131{,}000 \text{ in.}^4$ (Ans.)

$I_y = 2[\ (1/12)\ (4.00)\ (36.0)^3 + 0\ ] + 2[\ (1/12)\ (36.0)\ (2.00)^3 + 72.0\ (15.0 + 1.0)^2\ ]$

$= 68{,}000 \text{ in.}^4$ (Ans.)

$S_x = 131{,}000 \div 22.0 = 5960 \text{ in.}^3$; $S_y = 68{,}000 \div 18.0 = 3780 \text{ in.}^3$ (Ans.)

P2.8. Locate the principal axes for the column section given in Fig P2.8. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the *x*- and *y*-axes.

See Figure P2.8 of text book.

Solution

As the built-up section has two axes of symmetry, the center of gravity *G* of the built-up section coincides with the point of intersection of these two axes.

$A = 2[\ 28.0\ (7.125)] + 5.0(16.5) = 482 \text{ in.}^2$ (Ans.)

Weight = $482\ (3.40) \div 1000 = 1.64$ klf (Ans.)

$I_x = 2[\,(1/12)(28.0)(7.125)^3 + 28.0(7.125)(8.25 + 0.5\times 7.125)^2\,] + (1/12)(5.00)(16.5)^3$

$= 59{,}200$ in.^{4} (Ans.)

$I_y = 2[(1/12)(7.125)(28.0)^3] + (1/12)16.5(5.00)^3 = 26{,}200$ in.^{4} (Ans.)

$$r_x = \sqrt{\frac{59{,}200}{482}} = 11.1 \text{ in.}; \quad r_y = \sqrt{\frac{26{,}200}{482}} = 7.37 \text{ in.}$$ (Ans.)

P2.9. Locate the principal axes for the column section given in Fig P2.9. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the x- and y-axes.

See Figure P2.9 of text book.

Solution

As the built-up section has two axes of symmetry, the center of gravity G of the built-up section coincides with the point of intersection of these two axes.

Built-up section

$A = (24.0)^2 - (15.0)^2 = 351$ in.^{2} (Ans.)

Weight = $351(3.40) \div 1000 = 1.19$ klf (Ans.)

$I_x = (1/12)(24.0)(24.0)^3 - (1/12)(15.0)(15.0)^3 = 23{,}400$ in.$^4 = I_y$ (Ans.)

$$r_x = \sqrt{\frac{23{,}400}{351}} = 8.17 \text{ in.} = r_y$$ (Ans.)

P2.10. Locate the principal axes for the column section given in Fig P2.10. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the x- and y-axes.

See Figure P2.10 of text book.

Solution

W14×730: $A = 215$ in.^{2}; $b_f = 17.9$ in.; $I_x = 14{,}300$ in.^{4}; $I_y = 4720$ in.^{4}

PL 3×24: $A = 3.00(24.0) = 72.0$ in.^{2}; $I_x = (1/12)(3.00)(24.0)^3 = 3456$ in.^{4}

$I_y = (1/12)(24.0)(3.00)^3 = 54.0$ in.^{4}

Built-up section

As the built-up section has two axes of symmetry, the center of gravity *G* of the built-up section coincides with the point of intersection of these two axes.

$A = 215 + 2(72.0) = 359$ in.^{2} (Ans.)

Weight = $359(3.40) \div 1000 = 1.22$ klf (Ans.)

$I_x = 14{,}300 + 2(3456) = 21{,}200$ in.^{4} (Ans.)

$I_y = 4720 + 2[54.0 + 72.0(0.5\times17.9 + 1.5)^2] = 20{,}600$ in.^{4} (Ans)

$$r_x = \sqrt{\frac{21{,}200}{359}} = 7.68 \text{ in.}; \quad r_y = \sqrt{\frac{20{,}600}{359}} = 7.58 \text{ in.} \quad \text{(Ans.)}$$

P2.11. Locate the principal axes for the column section given in Fig P2.11. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the *x*- and *y*-axes.

See Figure P2.11 of text book.

Solution

W14×145: $A = 42.7$ in.^{2}; $b_f = 15.5$ in.; $d = 14.8$ in.

$I_x = 1710$ in.^{4}; $I_y = 677$ in.^{4}

Built-up section

As the section has two axes of symmetry, the center of gravity *G* of the built-up section coincides with the point of intersection of these two axes.

$A = 4(42.7) = 171$ in.^{2} (Ans.)

Weight = $171(3.40) \div 1000 = 0.581$ klf (Ans.)

$I_x = 2[\,1710 + 42.7(0.5\times15.5 + 0.5\times14.8)^2 + 2[\,677 + 0\,] = 24{,}400$ in.$^4 = I_y$ (Ans.)

$$r_x = r_y = \sqrt{\frac{24{,}400}{171}} = 12.0 \text{ in.} \quad \text{(Ans.)}$$