Instructor's Manual

Statistics for Economics, Accounting and Business Studies

Seventh edition

Michael Barrow

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Statistics for Economics, Accounting and Business Studies

Answers and Commentary on Problems

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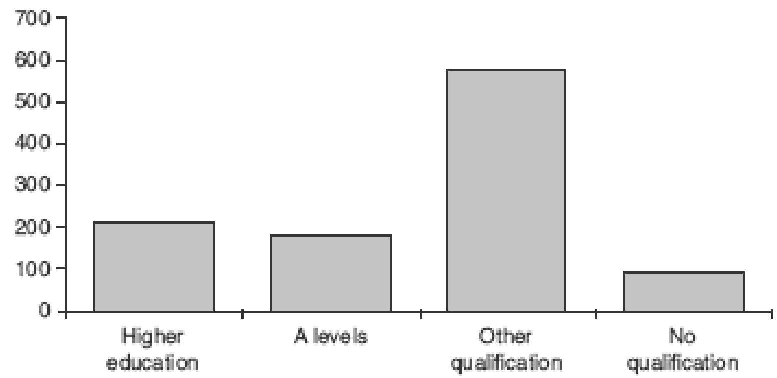
(Comments and corrections can be e-mailed to me at M.M.Barrow@sussex.ac.uk.)

Contents

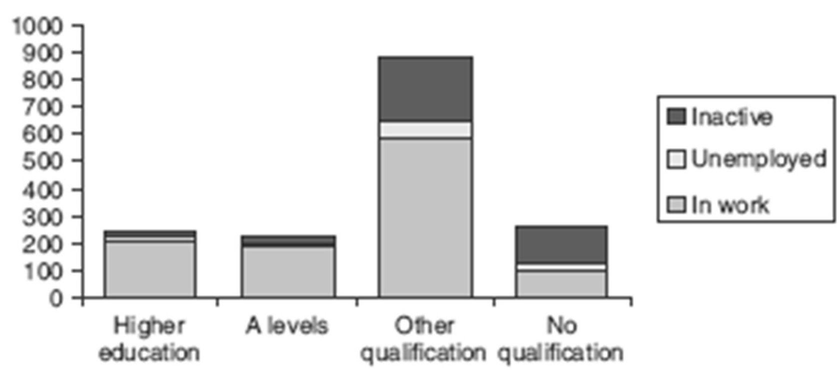
Descriptive statistics

1.1 (a)



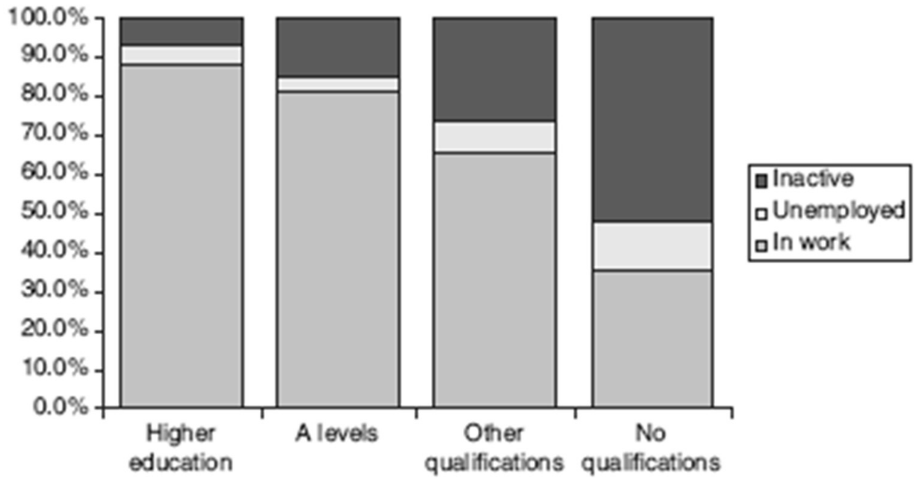
Comparison is complicated because the data for women only are from a survey, and hence the absolute numbers are much smaller. Also, we compare women here with the total for men and women in the text. We have to infer what the difference between men and women is. The major differences apparent are that there are relatively more women in the 'Other qualification' category and relatively fewer in the 'Higher education' category.

(b)



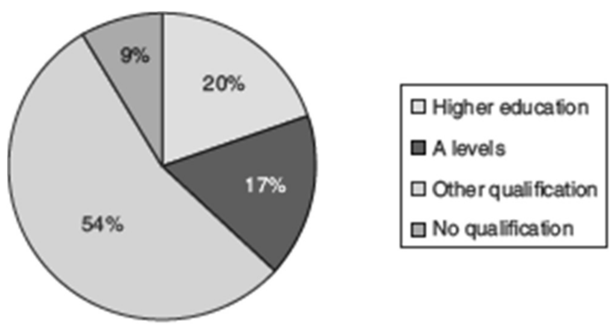
There is not a great difference apparent between this graph and the one in the text. Closer inspection of the figures suggests a higher proportion of women are 'inactive' and a lower proportion 'In work', but this detail is difficult to discern from the graph alone.

(c)



This chart brings out a little more clearly that there is a slightly higher degree of inactivity amongst women, particularly in the lower education categories.

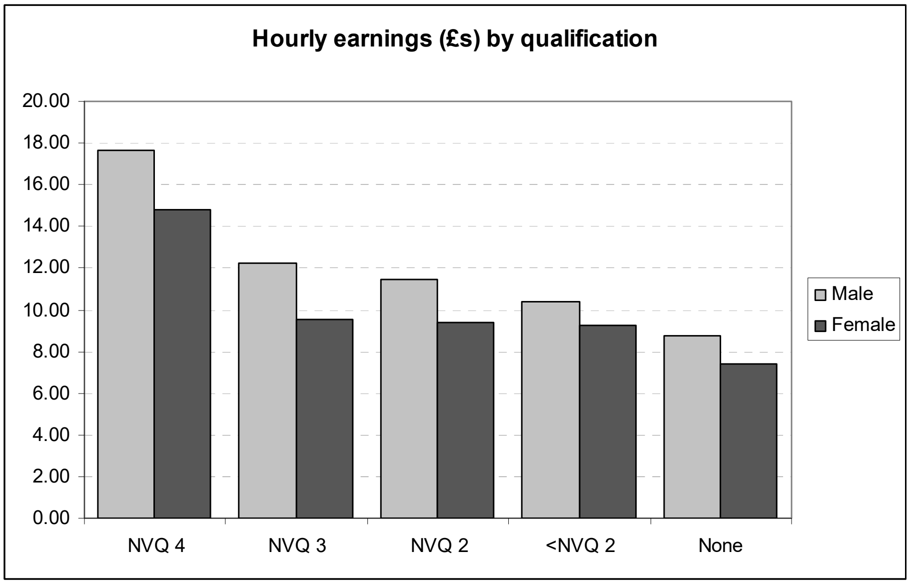
(d)



This again brings out the higher proportion with 'Other qualification' and the fewer with 'Higher education'.

1.2 (a) These data show the *average* (the mean) in each category, not the *count*. The data given are monetary amounts, not frequencies.

(b)



The bar chart shows that mean earnings decline by education category. This is true for both males and females, with the female values being lower than that for males. There appears to be a slightly non-linear relationship between earnings and education, but this is illusory: The education categories are drawn on an arbitrary scale.

(c) A stacked bar chart is not appropriate because you cannot add means together as you can with frequencies. To get the overall mean (i.e. for men and women combined), you would need to take a weighted average of the male and female values, with the weights given by the total numbers of men and women (which are not given here).

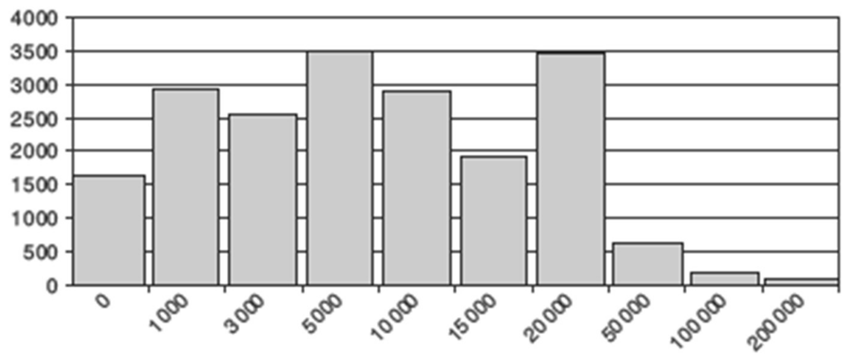
1.3 (a) Higher education, 88%.

(b) In work, 20%.

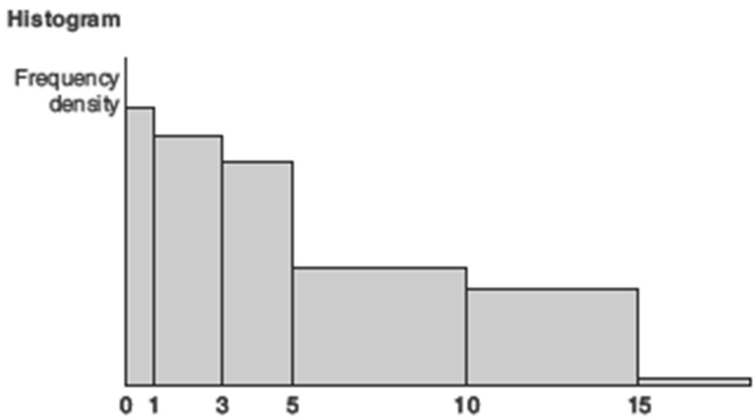
1.4 (a) The premiums are £5.45 for men (44% of the earnings of someone with NVQ 3) and £5.26 for women (55%). The premium is *proportionally* bigger for women.

(b) The median would almost certainly show a premium for a degree as well. As the median is less affected by outliers, the difference between the top two education categories might be smaller though this is not certain unless we actually look at the median values.

1.5 Bar chart:

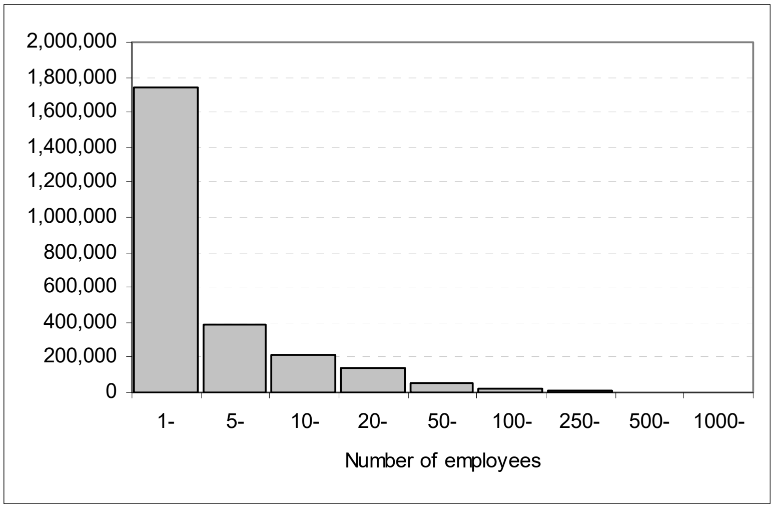


Histogram:

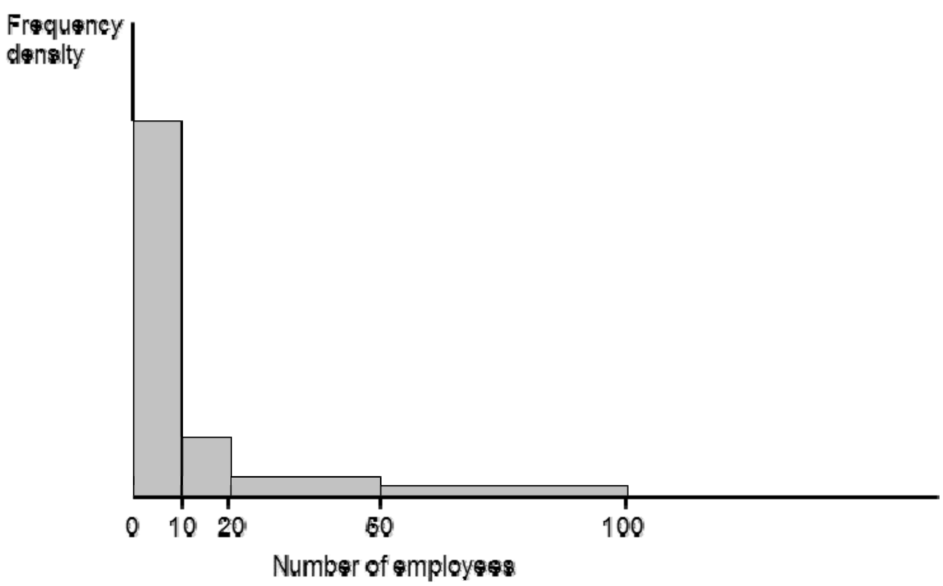


The difference between bar chart and histogram is similar to that for the 2005 distribution. The overall shape of the histogram is similar (heavily skewed to right). Comparison is difficult because of different wealth levels (because of inflation), and grouping into classes can affect the precise shape of the graph.

1.6 Bar chart:



Histogram:



Both show a highly skewed distribution once more. The histogram shows that there is a smooth but rapid decline in firm numbers as the number of employees rises.

1.7 (a) Mean 16.399 (£000); median 8.92; mode 0–1 (£000) group has the greatest frequency density. They differ because of skewness in the distribution.

(b) Q1 = 3.295, Q3 = 18.339, IQR = 15.044; variance = 652.88; standard deviation = 25.552; coefficient of variation = 1.56.

(c) $95{,}469.32/25.55^3 = 5.72 > 0$ as expected. Ask your class whether this number is revealing to them – is there any intuition?

(d) Comparison in text.

(e) This would increase the mean substantially (to 31.12), but the median and mode would be unaffected.

1.8 (a) Mean = $\Sigma fx/\Sigma f$ = 31,343,887.5/2,574,230 = 12.18. The median firm number is 1,287,115 in the first-class interval, and 0 + 1,287,115/1,740,685 × 4 = 3.96. The mode is also somewhere in the first-class interval, which has the highest frequency density. The differences are obviously down to the skewness of the distribution once again.

(b) Inter quartile range: Q1 = 2.48 (firm number 643,557.5). Q3 = 7.44 (firm number 1,930,673.5), so IQR is 4.96. Variance: Σfx^2 = 9,263,101,469, so σ^2 = 9,263,101,469/2,574,230 − 12.18^2 = 3,450.1. Hence, σ = 58.7 and coefficient of variation = 58.7/12.81 = 4.8. The extreme skewness of the data is reflected in the measures of dispersion. The IQR gives a very different picture from the standard deviation, which is affected by the high values.

(c) Coefficient of skewness: $\Sigma f(x - \mu)^3$ = 11,983,062,77,046, and dividing by N and by σ^3 (= 202,654.08) gives 22.97 > 0, and hence it is right skewed. This is a larger value than for wealth: The distribution is more skewed.

1.9 (33 × 134 + 40 × 139 + 25 × 137)/(33 + 40 + 25) = 136.8 pence/litre.

1.10 Since $w_i = p_i/\Sigma p_i$ (p_i is the number of pupils of type i), the weighted average may be expressed as $\Sigma px/\Sigma p$, which is total expenditure divided by pupils. Note its similarity to the formula for mean when using frequency data. The total expenditure is actually 23 million, and dividing by 18,000 gives 1,277.8.

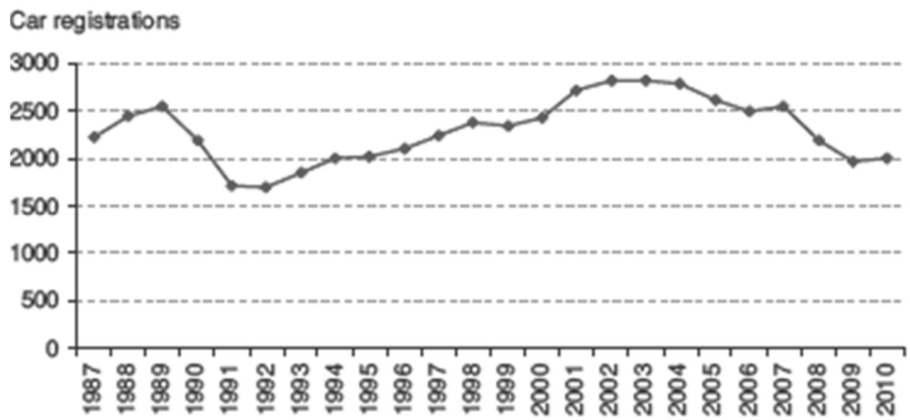
1.11 (a) z = 1.5 and −1.5 respectively.

(b) Using Chebyshev's theorem with k = 1.5, we have that at least $(1 - 1/1.5^2)$ = 0.56 (56%) lies within 1.5 standard deviations of the mean, so at most 0.44 (44 students) lies outside the range.

(c) As Chebyshev's theorem applies to *both* tails, we cannot answer this part. You cannot halve 0.44 as the distribution may be skewed.

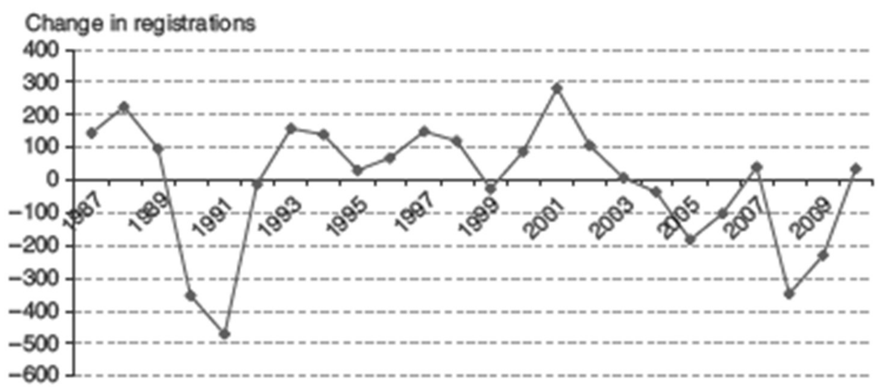
1.12 $(1 - 1/k^2)$ = 0.8 lie within k standard deviations. Hence, k = 2.23. Hence, 2.23s = 2,000 and so s = 896.86 (as a minimum).

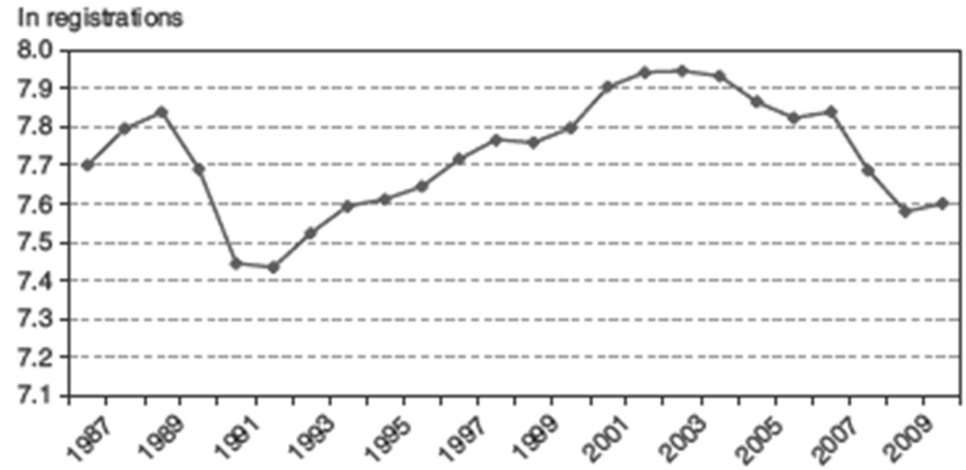
1.13 (a)

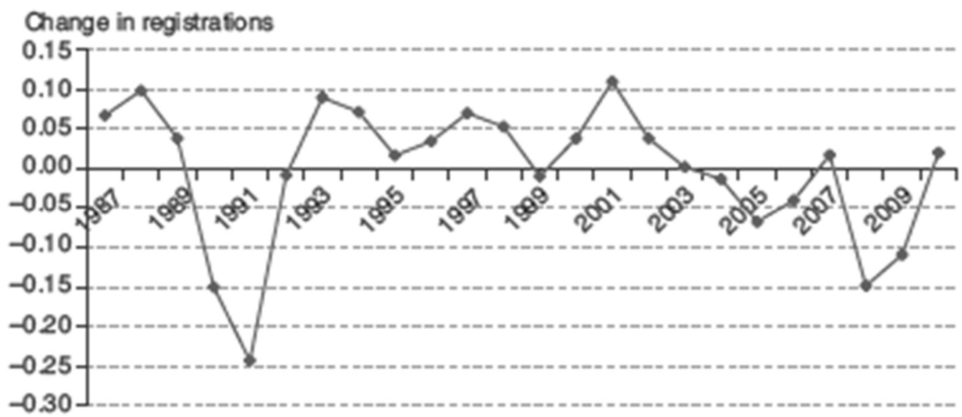


The series shows an initial peak in 1989, then a slow recovery from 1991 to 2003. After this the market turns down again. The series is quite volatile with long upward and downward swings.

(b)

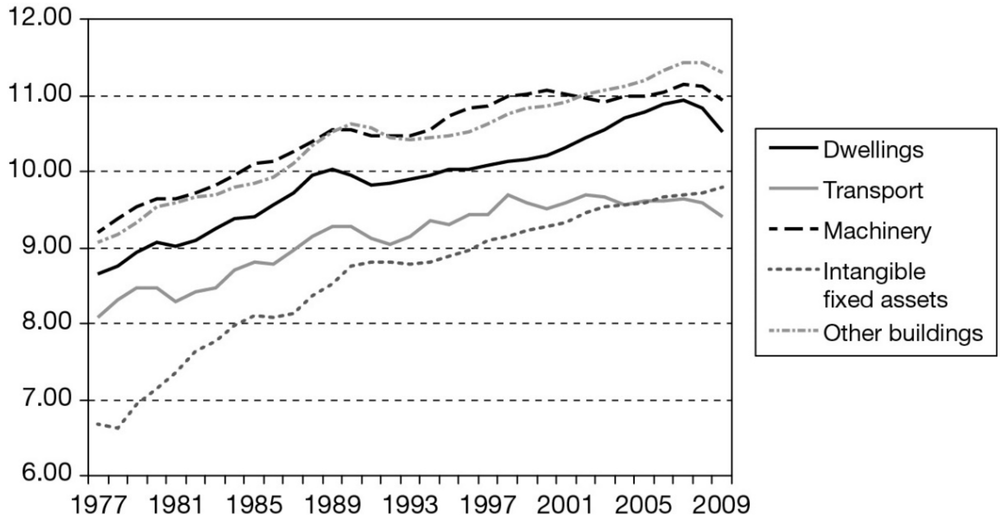






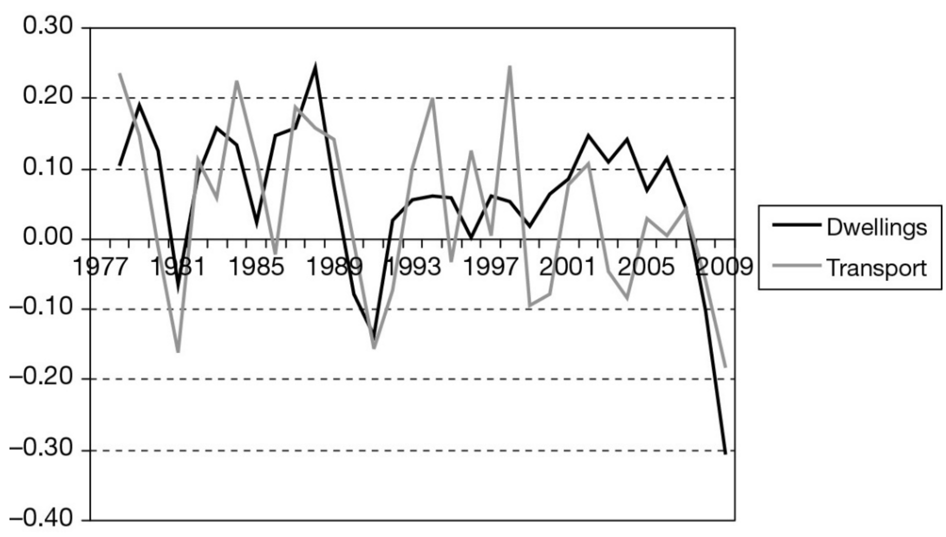
This form of the graph shows that recessions can be quite severe for the vehicle market, while upswings show smaller per annum increases. The log graphs are very similar to the levels graphs. There is not always an advantage in drawing these.

1.14 Figure 1.20 in the text shows line charts of the series and the main features were described. A chart of the log values reveals:



This reveals similar features, but highlights the more rapid growth rate of intangible fixed assets, as well as their lack of volatility (presumably because it is an estimated rather than tangible series of values). There is a suggestion of faster rates of growth up to around 1990 followed by a slower rate of increase, but this impressionistic conclusion should not be relied upon.

A graph of the change in the logs (only dwellings and transport are shown, for clarity) lends some support to the idea of a more rapid rate of growth before 1990. It is also apparent that both series (especially transport) are quite volatile. It raises the question of whether this is the most efficient form of investment for the long term.



1.15 (a) $(1{,}994.6/2{,}212.6)^{1/23} = -0.0045$ or −0.45% p.a. (note that this is not particularly representative – most years had positive growth, but there were sharp falls at the beginning and end of the period).

(b) 0.084 (around the geometric mean).

(c) The standard deviations of the two growth rates are similar (it was 0.0766 for investment), so this suggests a similar level of volatility. Note that, since the means are very different (6.37% p.a. for investment) and for registrations it is actually negative, there is a big difference in volatility if one relies upon the coefficient of variation. The latter is misleading in this case however – it reflects differences in the means rather than in volatility.

1.16 (a) 6.02% p.a. (use (37,044/5,699)1/32 − 1), similar to the growth of investment as a whole.

(b) 0.106.

(c) The coefficient of variation is 0.106/0.0602 = 1.76, slightly greater than the figure for investment as a whole (1.15). The latter is likely to be smoother since there are offsetting movements amongst the various investment categories. The general point is that the more disaggregated the data is, the more likely it is to fluctuate. When aggregating, random effects often cancel out (to some extent).

1.17 (a) Non-linear, upward trend. It is likely to be positively autocorrelated. Variation around the trend is likely to grow over time (heteroscedasticity).

(b) Similar to (a), except that the trend would be shallower after deflation. Probably there will be less heteroscedasticity because price variability has been removed, which may also increase the autocorrelation of the series.

(c) Unlikely to show a trend in the very long run, but there might be one over, say, five years, if inflation is increasing. Likely to be homoscedastic, with some degree of autocorrelation.

1.18 (a) The price level grows exponentially over time, at least in recent times. Historically, there have also been long periods of stable and even falling prices.

(b) The inflation rate should not have a trend, but should fluctuate around a figure of about 3% p.a. (for the UK, again, for recent history).

(c) Exchange rates are difficult to predict, both in the short and long term. The £/$ exchange rate has shown a rising trend over the past 50 years or so (depreciation of the £), interspersing periods of stability (fixed exchange rates), sudden changes (devaluation) and random fluctuations (floating exchange rate).

1.19 (a) Using $S_t = S_0\,(1 + r)^t$, hence $S_0 = S_t\,/(1 + r)^t$. Setting $S_t = 1{,}000$, $r = 0.07$ gives $S_0 = 712.99$. Price after two years: 816.30. If r rose to 10%, the bond would fall to 1,000/1.13 = 751.31.

(b) The income stream should be discounted to the present using $\frac{200}{1+r}+\frac{200}{(1+r)^2}+\frac{200}{(1+r)^3}+\frac{200}{(1+r)^4}+\frac{200}{(1+r)^5}=820.04$, so the bond should sell for £820.04. It is worth more than the previous bond because the return is obtained earlier.

1.20 This is the same, in principle, as a compound interest calculation. Using equation (1.26) and setting S_t = 3,000, S_0 = 30,000 and t = 10, we get $d = \sqrt[10]{\frac{3000}{30000}} - 1 = -0.2057$, i.e. 20.57% depreciation per annum. The value of the machine after one year is therefore 30,000 × (1 − 0.2057) = £23,830. After two years, its value is £23,830 × (1 − 0.2057) = £18,928, and after five years its value is 30,000 × (1 − 0.2057)5 = £9,485. You should check that its value is £3,000 after 10 years.

1.21 (a) 17.9% p.a. for BMW; 14% p.a. for Mercedes.

(b) Depreciated values are as shown in this table:

BMW 525i	22,275	18,284	15,008	12,319	10,112	8,300
Mercedes 200E	21,900	18,833	16,196	13,928	11,977	10,300

These are close to actual values. Depreciation is initially slower than the average, then speeds up, for both cars.

1.22 The discounted income stream is $400 \times \left\{ \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots \right\} = 400 \times \frac{1}{r} = 8{,}000$. An alternative way to see this is to note that if you had £8,000 and invested it at 5%, you would get 8,000 × 5% = £400 p.a., that is C × r = Y, where C is the capital sum, r is the interest rate and Y is the annual income. Rearranging this yields C = Y/r. This is a useful concept also in economics and finance. It relates income to wealth and so allows such things as the pricing of equities (discounted future profits). The share value of Tottenham Hotspurs recently (April 1995) fell by £5million when they lost an FA Cup semi-final. This reflects the loss of expected future profits (expected because they might have lost the final). Electricity shares also fell when the regulator announced a review of prices.

1.23 $$E(x+k) = \frac{\sum (x+k)}{n} = \frac{\sum x + nk}{n} = \frac{\sum x}{n} + k = E(x) + k.$$

1.24 $$V(kx) = \frac{\sum (kx - k\mu)^2}{n} = \frac{\sum k^2 (x-\mu)^2}{n} = \frac{k^2 \sum (x-\mu)^2}{n} = k^2 V(x).$$

It is useful if students can gain some confidence in doing these kinds of manipulation – they're not *that* difficult.

1.25 The mistake is comparing non-comparable averages. A first-time buyer would have an above-average mortgage and purchase a below-average-priced house, hence the amount of buyer's equity would be small. The original argument came from the *Morning Star* newspaper many years ago. With reasoning like this, no wonder communism failed.

1.26 If it is harder to get the popular jobs, then it is possible for Arts students to be more successful at getting both types of job than Science students, yet have a higher unemployment rate. Consider this example:

	Popular jobs			Unpopular jobs			Total		
	applications	successes	% success	applications	successes	% success	applications	successes	% success
Science	200	174	87	400	378	94.5	600	552	92
Arts	400	350	87.5	200	190	95	600	540	90

Arts students are more successful at both types of job but are less successful overall – a nice counter-intuitive conclusion. The inspiration for this question came from a careers service report at my university with precisely this flaw. Note that science students *may* be better than Arts students; the point is that the figures don't *prove* it.

Answers to exercises on Σ notation

1A.1 20, 90, 400, 5, 17, 11.

1A.2 40, 360, 1600, 25, 37, 22.

1A.3 88, 372, 16, 85.

1A.4 352, 2976, 208, 349.

1A.5 113, 14, 110.

1A.6 56, 8, 48.

1A.7 $$\frac{\sum f(x-k)}{\sum f} = \frac{\sum fx - k\sum f}{\sum f} = \frac{\sum fx}{\sum f} - k.$$

1A.8 $$\frac{\sum f(x-\mu)^2}{\sum f} = \frac{\sum f\left(x^2 - 2\mu x + \mu^2\right)}{\sum f} = \frac{\sum fx^2 - 2\mu \sum fx + \mu^2 \sum f}{\sum f}$$

$$= \frac{\sum fx^2}{\sum f} - 2\mu^2 + \mu^2 = \frac{\sum fx^2}{\sum f} - \mu^2.$$

Answers to exercises on logarithms

1C.1 −0.8239, 0.17609, 1.17609, 2.17609, 3.17609, 1.92284, 0.96142, impossible.

1C.2 −0.09691, 0.90309, 1.90309, 0.60206, 1.20412, impossible.

1C.3 −1.89712, 0.40547, 2.70705, 5.41610, impossible.

1C.4 −0.20397, 1, 1.09861, 3.49651, impossible.

1C.5 0.15, 12.58925, 125.8925, 1258.925, 10^{12}.

1C.6 0.8, 199.5262, 1995.2623, 1995262.3.

1C.7 15, 40.77422, 2.71828, 22026.4658.

1C.8 33, 1,202,604.284, 3,269,017.372, 0.36788.

1C.9 3.16228, 1.38692, 1.41421, 0.0005787, 0.008.

1C.10 3.1071, 1.6035, 1.6818, 1, 1, 0.6934.

ANSWERS TO CHAPTER 2

Probability

2.1 (a) 4/52 or 1/13. (There are 4 aces in a pack of 52 cards.)

(b) 12/52 or 3/13. Again, this is calculated simply by counting the possibilities, 12 court cards (3 in each suit).

(c) 1/2.

(d) $4/52 \times 3/52 \times 2/52 = 3/17{,}576$ (0.017%).

(e) (4/52)3 = 0.000455.

2.2 (a) Proportions unemployed in the various categories may be interpreted as probabilities according to the frequentist approach. Thus, given someone aged 16–19 who is out of work, the probability that they have been unemployed for ≤ eight weeks is 27.2%. Of course, this pattern of unemployment might change over time, altering the probabilities in the future.

(b) (i) True, (ii) false (19% is the probability of an unemployed person aged 16–19 being out of work for over one year, not the other way round), (iii) false (56.2% is the proportion of all currently unemployed (whenever they became unemployed) who are out of work for over one year), (iv) false (8.9% is the proportion of the unemployed, not the economically active, out of work for less than eight weeks), (v) true.

(c) 1,022.7 have been out of work over one year. (This is 19% × 273.4 + 36.8% × 442.5 + … and so on.) Of these, 51.9 (19% of 273.4, all in thousands) are aged 16–19, so the probability is 5.08% (= 51.9/1,022.7).

2.3 (a) 0.25 (three to one against means one win for every three losses, so one win in four races), 0.4, 5/9.

(b) 'Probabilities' are 0.33, 0.4, and 0.5, which sum to 1.23. Therefore, these cannot be real probabilities. The difference leads to a (expected) gain to the bookmaker.

(c) Suppose the true probabilities of winning are proportional to the odds, that is. 0.33/1.23, 0.4/1.23, 0.5/1.23, or 0.268, 0.325, 0.407. If £1 was bet on each horse, then the bookie would expect to pay out $0.268 \times 3 + 0.325 \times 1.5 + 0.407 \times 0.8 = 1.6171$, plus one of the £1 stakes, that is £2.62 in total. He would thus gain 38 pence on every £3 bet, or about 12.7%.

2.4 (a) 8/21 (0.381), 0.667, 0.231.

(b) $0.500 + 0.286 + 0.154 + 0.100 + 0.059 = 1.099$.

(c) Basing odds on amounts bet allows the bookie to make a guaranteed profit (no uncertainty). Diverging from this towards the true probabilities (presumably better known by the bookie than the punter) increases the expected gain, but involves increased risk of a loss to the bookmaker.

2.5 A number of factors might help: statistical ones such as the ratio of exports to debt interest, the ratio of GDP to external debt, the public sector deficit and so on, and political factors such as the policy stance of the government. More insight could be gained by looking at the current interest rate on the debt. A high interest rate suggests investors believe there is a greater chance of default, other things equal.

2.6 Factors such as profitability, expected future incomes, size of debt relative to earnings and so on. The credit default swap (CDS) market provides insurance against a company not repaying its bonds, and the premium charged would reflect the market's view of that risk.

2.7 (a) is the more probable, since it encompasses her being active or not active in the feminist movement. Many people get this wrong, which shows how one's preconceptions can mislead. People tend to read part (a) as 'Judy is a bank clerk, not active in the feminist movement'. A simple way of stating this mathematically is Pr(B) ≥ Pr(B and A) where B indicates a bank clerk and A indicates an activist. It has to be true since Pr(A) ≤ 1.

2.8 Not very good performance. If one was guessing at random (P = 0.5), the probability of getting two or less wrong is $6C0 \times 0.56 + 6C1 \times 0.56 + 6C2 \times 0.56 = 34\%$. Hence, the evidence is reasonably consistent with someone just guessing. If the clinic got only one wrong out of six, that would be more convincing. The chance of guessing correctly at random five out of six times is 11%.

2.9 The advertiser is a trickster and guesses at random. Every correct guess (P = 0.5) nets a fee, every wrong one costs nothing except reimbursing the fee. The trickster would thus keep half the money sent in. You should be wary of such advertisements!

2.10 Pr(winning) = (1/6)6 = 0.000021433 (about 2 in 100,000). Your expected winnings are therefore 0.000021433 × 40 000 − 1 = –£0.143. With 400 punters during the fair, the probability of the car not being won is therefore (1 − 0.000021433)400 = 0.991463; so, the probability of it being won is 0.008537. The expected loss is therefore 40,000 × 0.008537 = £340 approximately. This is less than the premium, but the premium also has to pay for someone independent to monitor the game, otherwise the organisers could cheat and claim the car had been won.

2.11 (a) E(winnings) = 0.520 × £1billion + (1 − 0.520) × −£100 = £853.67.

(b) Despite the positive expected value, most would not play because of their aversion to risk. Would you? The size of the prize is also not credible – would they actually pay up?

2.12 (a) Accident probabilities: four-engine plane: P = Pr(3 or 4 engines fail) = $4C3 \times 0.0013 \times 0.999 + 0.0014 = 0.000000004001$; two-engine plane: P = Pr(2 engines fail) = 0.0012 = 0.000001. Hence, a four-engine plane is 'safer' by a factor of 250, on this basis.

(b) However, the probabilities of not crashing are 0.999999996 and 0.999999 respectively. Thus, the former is 0.00009% safer, not a lot.

(c) The crucial assumption is independence. Since the engines share many features (e.g. fuel supply), this is probably not accurate. Dependence would tend to reduce the differences between the types of aircraft. Within the limit, if engines all worked or failed together, both types would effectively be single-engine planes.

Flying regulations dictate that aircraft with fewer engines are required to follow a flight path that remains closer to airports. Modern aircraft are much more reliable so these regulations have been relaxed over the years.

2.13 (a), (b) and (d) are independent, though legend says that rain on St Swithin's Day means rain for the next 40 days, so (d) is arguable according to legend.

2.14 (a) is likely to be independent. The others are not. IBM and Dell operate in the similar markets so their profits are likely to be correlated. In football, it is possible for teams to have a winning (or losing) 'streak' (the better the team is though, the less it should suffer from this phenomenon). No claims bonuses in car insurance reflect the dependence – if you have an accident, you reveal the information that you're more likely to have another.

2.15 (a) There are 15 ways where a 4–2 score could be arrived at, of which this is one. Hence, the probability is 1/15.

(b) Six of the routes through the tree diagram involve a 2–2 score at some stage, so the probability is 6/15.

2.16 (a) 97.03% (= 0.993).

(b) 99.9996% Pr(first three get it right) + Pr(any two get it right) + Pr(any one plus the fourth computer get it right) = 0.993 + 0.992 × 0.01 × 3*C*2 + 0.99 × 0.012 × 3*C*1 × 0.99).

(c) 0.014.

(d) 1 − 0.999996 − 0.014 = 0.00000397.

2.17 Pr(guessing all six) = 6/50 × 5/49 × ... × 1/45 = 1/15,890,700.

Pr(six from 10 guesses) = 10/50 × 9/49 × ... × 5/45 = 151,200/11,441,304,000. This is exactly 210 times the first answer, so there is no discount for bulk gambling.

2.18 (a) 6/49 × 5/48 × 4/47 ... × 1/44 = 1/13,983,816 = 0.000000072. Note this is 1/49*C*6, the number of ways of choosing 6 numbers from 49.

(b) 6/49 × ... × 2/45 × 43/44 × 6*C*5 × 1/43 = 1/2,330,636 = 0.000000429.

(c) Third prize: 6/49 × ... × 2/45 × 43/44 × 6*C*5 = 1/54,200 = 0.000018450; fourth prize: 6/49 × ... × 3/46 × 43/45 × 42/44 × 6*C*4 = 1/1,032 = 0.00096862; fifth prize: 6/49 × ... × 4/47 × 43/46 × 42/45 × 41/44 × 6*C*3 = 1/57 = 0.017650404.

(d) Summing the probabilities gives 0.018637974 or about a 1 in 54 chance of winning. Note that the events are mutually exclusive; you cannot win both the first and the second prize with the same ticket, for example, so the addition rule is valid.

(e) The second prize is six times more probable than the jackpot, yet the prize is only one-twentieth. Relatively, more is put into the jackpot since this is what tends to attract customers. This is true for all other prizes except the fifth. This pot too was 'over-weighted', presumably to get a lot of (small) winners and increase the attractiveness of the lottery.

(f) There would be a danger of guaranteeing the jackpot – you might get 20 winners and hence a huge payout. With the fifth prize, this is much less likely so the prize is guaranteed.

(g) The expected and actual numbers of winners were:

	Expected	Actual	Ratio
First	3.5	7	2.0
Second	21	39	1.9
Third	904	2,139	2.4
Fourth	47,462	76,731	1.6
Fifth	864,870	1,071,084	1.2
Total	913,261	11,50,000	

There were about 25% more winners than expected, probably because of the preponderance of low winning numbers in the draw (3, 5, 14, 22, 30, 44 and bonus – 10) and since many people used birth dates to select their numbers. This may account for the large number of third-prize winners (with five correct numbers) – five of the six numbers were 31 or below. The largest deviations from expectations (as one would expect) are for the first, second and third prizes. The number of fifth prize winners is very close to expected, given the large number of tickets sold.

2.19

	Prior	Likelihood	Prior × likelihood	Posterior
Fair coin	0.5	0.25	0.125	0.2
Two heads	0.5	1.00	0.500	0.8
			0.625	

2.20 Pr(A|+ test) = $\frac{0.99\times 0.01}{0.99\times 0.01+0.01\times 0.99}=0.5$. Half of all positive tests will be false ones. This is likely to spread undue alarm.

2.21 (a) Write the initial probability of guilt as Pr(G) = ½. The probability the witness says the defendant is guilty, given they are guilty, is Pr(W|G) = p. Using Bayes' theorem, the probability of guilt, given the witness's statement, Pr(G|W), is

$$\frac{\Pr(\mathrm{W}\,|\,\mathrm{G})\times\Pr(\mathrm{G})}{\Pr(\mathrm{W}\,|\,\mathrm{G})\times\Pr(\mathrm{G})+\Pr(\mathrm{W}\,|\,\mathrm{not-G})\times\Pr(\mathrm{not-G})}=\frac{p\times 0.5}{p\times 0.5+(1-p)\times 0.5}=p.$$

(b) Again using Bayes' theorem, and writing Pr(2W|G) for the probability that both witnesses claim the defendant is guilty and so on, we obtain Pr(G|2W) as

$$\frac{\Pr(\mathrm{2W}\,|\,\mathrm{G})\times\Pr(\mathrm{G})}{\Pr(\mathrm{2W}\,|\,\mathrm{G})\times\Pr(\mathrm{G})+\Pr(\mathrm{2W}\,|\,\mathrm{not-G})\times\Pr(\mathrm{not-G})}=\frac{p^2\times 0.5}{p^2\times 0.5+(1-p)^2\times 0.5}=$$

$$\frac{p^2}{p^2+(1-p)^2}.$$

(c) If $p < 0.5$, then the value in part (b) is less than the value in (a). The agreement of the second witness *reduces* the probability that the defendant is guilty. Intuitively, this seems unlikely. The fallacy is that they can lie in many different ways, so Bayes' theorem is not applicable here.

2.22 The probability of the assailant having red hair is, via Bayes' theorem, $(0.8 \times 0.1)/(0.8 \times 0.1 + 0.2 \times 0.9) = 0.31$. You probably guessed a much higher value, as would most jurors. This could be a contentious issue especially if applied to racial minorities, rather than the neutral issue of hair colour. The use of statistical evidence in court is sometimes a contentious issue as many people wrongly interpret the data.

2.23 (a) Expected values are 142, 148.75 and 146, respectively. Hence, B is chosen.

(b) The minima are 100, 130 and 110; so, B has the greatest minimum. The maxima are 180, 170 and 200; so, C is chosen.

(c) The regret table is

	low	middle	high	max
A	30	5	20	30
B	0	0	30	30
C	20	15	0	20

So, C has the minimax regret figure.

(d) The EV assuming perfect information is 157.75, against an EV of 148.75 for project B; so, the value of information is 9.

2.24 (a) EVs are 316, 365 and 260; hence, medium is preferred.

(b) The minima are 300, 270, 50, so small is preferred.

The maxima are 330, 420, 600, so large is chosen.

(c) The maximum regrets are 270, 180 and 250, and the min of these is associated with medium.

(d) The EV with perfect information is 410 against an EV of 365 for the medium factory; so, the value is 45.

2.25 The probability of no common birthday is $365/365 \times 364/365 \times 363/365 \times ... \times 341/365 = 0.43$. (This is obtained by noting that the probability of the second person having a birthday on a different day from the first is 364/365, the probability that the third has a birthday different from the first two is 363/365, etc.) Hence, the probability of at least one birthday in common is 0.57, or greater than one-half. Most people underestimate this probability by a large amount. (This result could form the basis of a useful source of income at parties.)

2.26 We start by calculating the odds for the one-against-one gunfights:

A v. B: If A goes first, it's all over. Pr(A wins) = 1, Pr(B) = 0.

If B goes first, then Pr(B) = 0.75, Pr(A) = 0.25 (fairly obvious).

A v. C: If A goes first, Pr(A) = 1, Pr(C) = 0.

If C goes first, Pr(C) = 0.5, Pr(A) = 0.5.

B v. C: More tricky. If B goes first: This is the sum of an infinite series (since there is a small but positive probability that neither ever kills the other).

Pr(B) = 0.75 + {0.25 × 0.5 × 0.75} + {0.25 × 0.5 × 0.25 × 0.5 × 0.75} + ... (i.e. the probability of B winning with the first, second, third, ... shot). This comes out as Pr(B) = 6/7, Pr(C) = 1/7.

C goes first: Pr(C) = 4/7, Pr(B) = 3/7 by a similar calculation.

Now in the first stage, if A starts, he shoots B (this is his optimal strategy, since he has a better chance against C than he would against B). There is then a 'Shoot-off' between A and C, with C shooting first. Hence, Pr(A wins the tournament|A starts) = 0.5, Pr(C|A) = 0.5. B has no chance.

C starts: He shoots in the air! This is his optimal strategy, surprisingly. This lets A in and we're back to the situation above. Hence, Pr(C|C) = 0.5, Pr(A|C) = 0.5. If he shot at A and hit him, then he'd have only a 1/7 chance against B. In a one-to-one shoot out with A, with C going first, he has a 50:50 chance. Better to deliberately miss and let A pick off B.

B starts: His best strategy is to shoot at A. If he hits, (P = 3/4) he has a 3/7 chance in the shootout against C. If he misses, he's had it. So Pr(B|B) = 9/28. In the case of a shootout between A and C (which occurs with probability 0.25), Pr(A|B) = 1/8 and Pr(C|B) = 1/8.

To find the ultimate probabilities, Pr(A wins) = Pr(A|A starts) × Pr(A starts) + Pr(A|B starts) × Pr(B starts) + Pr(A|C starts) × Pr(C starts) = 1/2 × 1/3 + 1/8 × 1/3 + 1/2 × 1/3 = 0.375.

Similarly, Pr(B) = 0.107 and Pr(C) = 0.518.

Note that C (the worst shot) has the best chance of winning.

Well done if you got this one right.

2.27 (a) Choose low confidence. The expected score is 0.6 × 1 + 0.4 × 0 = 0.6. For medium confidence the score would be 0.6 × 2 + 0.4 × −2 = −0.4, and for high confidence 0.6 × 3 + 0.4 × −6 = −0.6.

(b) Let p be the probability desired. We require E(score|medium) = $p \times 2 + (1 - p) \times -2 > p$ = E(score|low) – the payoff to medium confidence should be greater than low confidence. Hence, $4p - 2 > p \Rightarrow p > 2/3$. Similarly, we also require $p \times 2 + (1 - p) \times -2 > p \times 3 + (1 - p) \times -6$ (the payoff to medium has to exceed the payoff to high). This implies $p < 4/5$. Hence $2/3 < p < 4/5$.

(c) Given $p = 0.85$, the expected scores are $p = 0.85$ (low), $2p - 2(1 - p) = 1.4$ (medium) and $3p - 6(1 - p) = 1.65$. Hence, the expected loss would be 1.65 − 1.4 = 0.25 if opting for medium confidence and 1.65 − 0.85 = 0.8 if opting for low confidence.

2.28 (a) 5 out of 20.

(b) 8 + 3 (out of the remaining 12 questions) = 11.

(c) −1/3 (hence, the expected payoff to guessing is 0.25 × 1 + 0.25 × (−1/3) + 0.25 × (−1/3) + 0.25 × (−1/3) = 0).