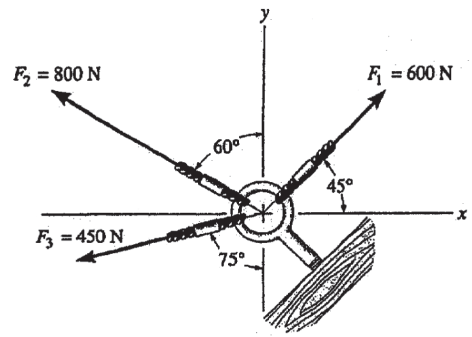
© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-1. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.

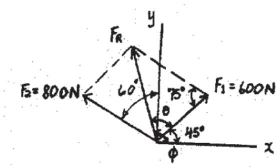


$$F_R = \sqrt{(600)^2 + (800)^2 - 2(600)(800)\cos 75°} = 866.91 = 867\text{ N} \qquad \textbf{Ans.}$$

$$\frac{866.91}{\sin 75°} = \frac{800}{\sin \theta}$$

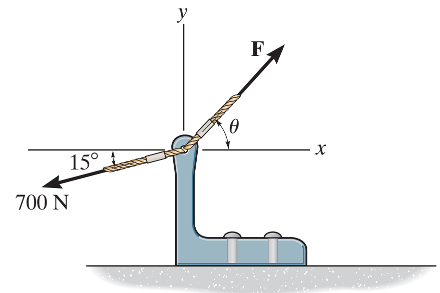
$$\theta = 63.05°$$

$$\phi = 63.05° + 45° = 108° \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–2.
If $\theta = 60^\circ$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45^\circ}$$

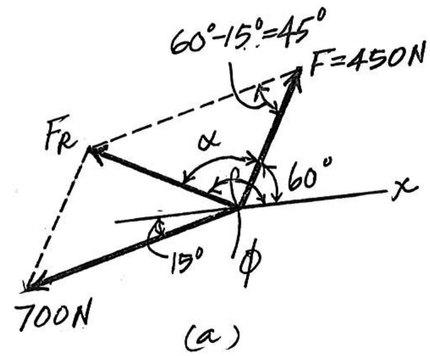
$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

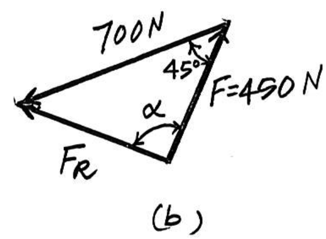
This yields

$$\frac{\sin \alpha}{700} = \frac{\sin 45^\circ}{497.01} \qquad \alpha = 95.19^\circ$$

Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60^\circ = 95.19^\circ + 60^\circ = 155^\circ \qquad \textbf{Ans.}$$

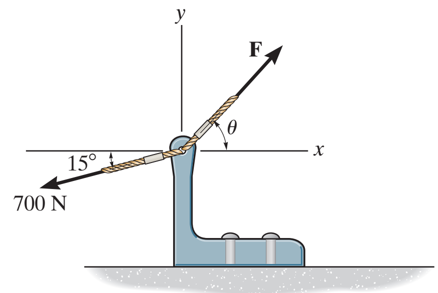




© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–3.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

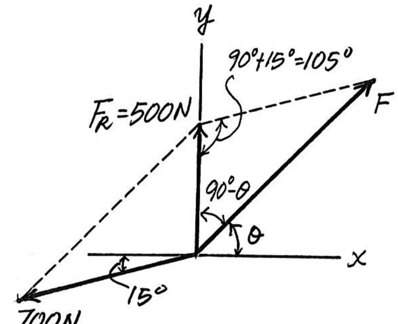
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105°}$$

$$= 959.78\text{ N} = 960\text{ N} \qquad \textbf{Ans.}$$

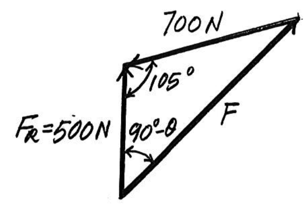
Applying the law of sines to Fig. *b*, and using this result, yields

$$\frac{\sin(90° + \theta)}{700} = \frac{\sin 105°}{959.78}$$

$$\theta = 45.2° \qquad \textbf{Ans.}$$



(a)

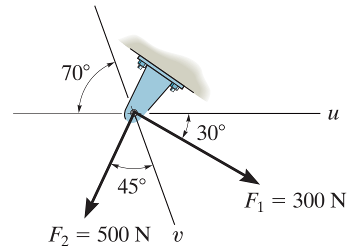


(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

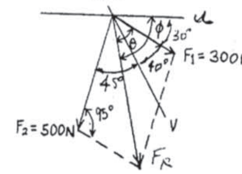
2–4.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

$$F_R = \sqrt{(300)^2 + (500)^2 - 2(300)(500)\cos 95^\circ} = 605.1 = 605\text{ N} \qquad \textbf{Ans.}$$



$$\frac{605.1}{\sin 95^\circ} = \frac{500}{\sin \theta}$$

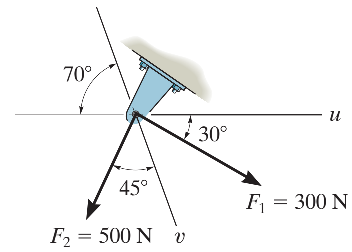
$$\theta = 55.40^\circ$$

$$\phi = 55.40^\circ + 30^\circ = 85.4^\circ \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–5.

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.



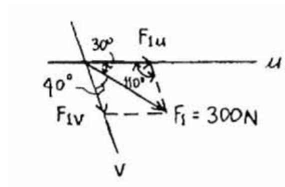
SOLUTION

$$\frac{F_{1u}}{\sin 40°} = \frac{300}{\sin 110°}$$

$$F_{1u} = 205 \text{ N}$$ **Ans.**

$$\frac{F_{1v}}{\sin 30°} = \frac{300}{\sin 110°}$$

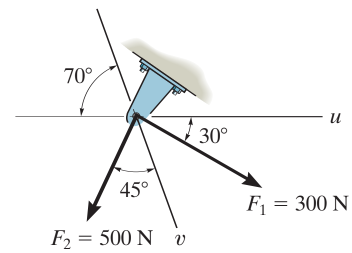
$$F_{1v} = 160 \text{ N}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–6.

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



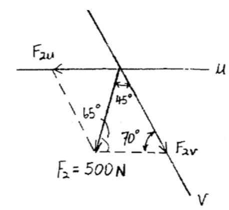
SOLUTION

$$\frac{F_{2u}}{\sin 45°} = \frac{500}{\sin 70°}$$

$F_{2u} = 376$ N **Ans.**

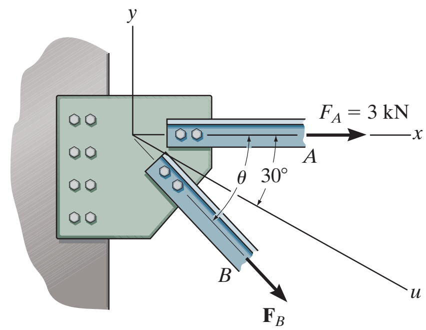
$$\frac{F_{2v}}{\sin 65°} = \frac{500}{\sin 70°}$$

$F_{2v} = 482$ N **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–7. If $F_B = 2$ kN and the resultant force acts along the positive u axis, determine the magnitude of the resultant force and the angle θ.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of sines to Fig. *b*, yields

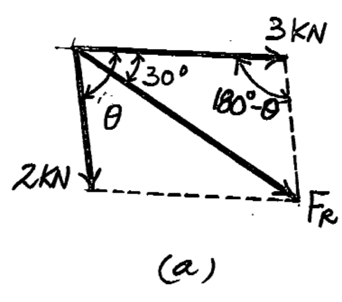
$$\frac{\sin\phi}{3} = \frac{\sin 30°}{2} \qquad \phi = 48.59°$$

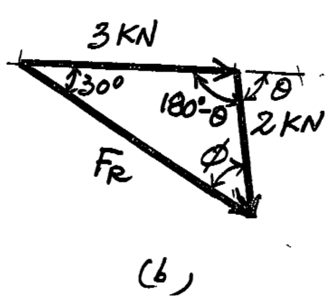
Thus,

$$\theta = 30° + \phi = 30° + 48.59° = 78.59° = 78.6° \qquad \textbf{Ans.}$$

With the result $\theta = 78.59°$, applying the law of sines to Fig. *b* again, yields

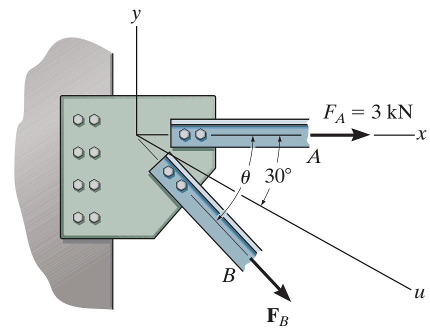
$$\frac{F_R}{\sin(180° - 78.59°)} = \frac{2}{\sin 30°} \qquad F_R = 3.92 \text{ kN} \qquad \textbf{Ans.}$$





© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–8. If the resultant force is required to act along the positive u axis and have a magnitude of 5 kN, determine the required magnitude of $\mathbf{F}_B$ and its direction θ.



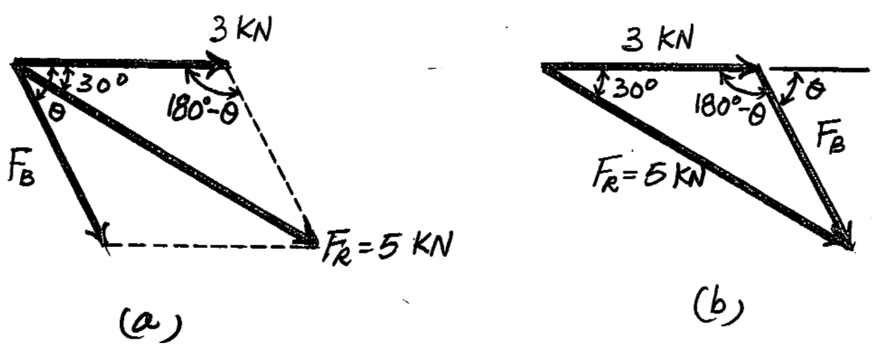
The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{3^2 + 5^2 - 2(3)(5)\cos 30°}$$
$$= 2.832 \text{ kN} = 2.83 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and realizing that $\sin(180° - \theta) = \sin\theta$, the application of the sine law to Fig. *b*, yields

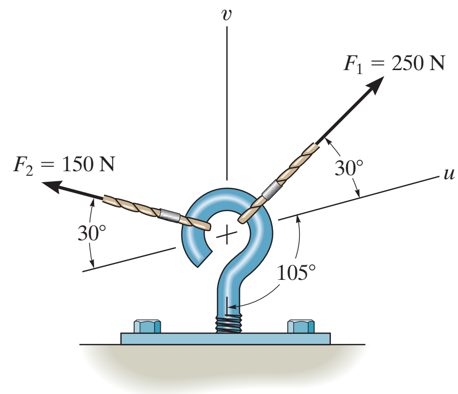
$$\frac{\sin\theta}{5} = \frac{\sin 30°}{2.832} \qquad \theta = 62.0° \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–9.

Resolve $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of these components.

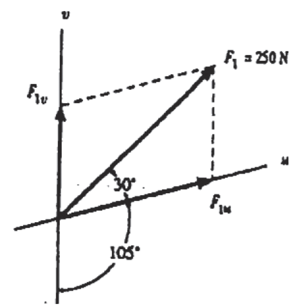


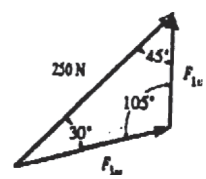
SOLUTION

Sine law:

$$\frac{F_{1v}}{\sin 30^\circ} = \frac{250}{\sin 105^\circ} \qquad F_{1v} = 129 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{1u}}{\sin 45^\circ} = \frac{250}{\sin 105^\circ} \qquad F_{1u} = 183 \text{ N} \qquad \textbf{Ans.}$$

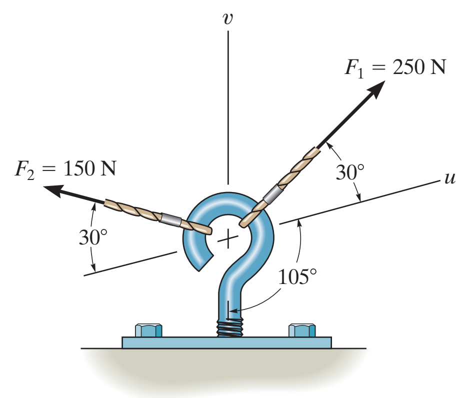




© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–10.

Resolve $\mathbf{F}_2$ into components along the u and v axes and determine the magnitudes of these components.

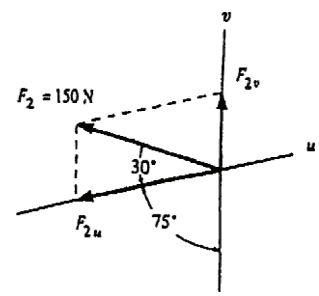


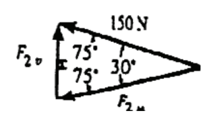
SOLUTION

Sine law:

$$\frac{F_{2v}}{\sin 30°} = \frac{150}{\sin 75°} \qquad F_{2v} = 77.6 \text{ N} \qquad \textbf{Ans.}$$

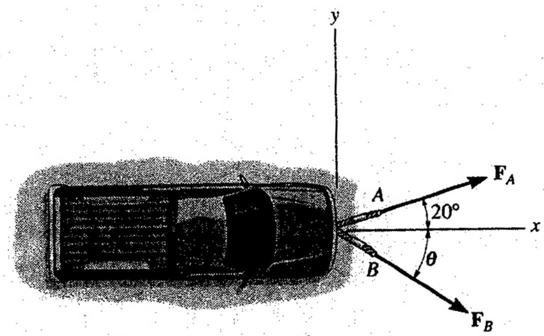
$$\frac{F_{2u}}{\sin 75°} = \frac{150}{\sin 75°} \qquad F_{2u} = 150 \text{ N} \qquad \textbf{Ans.}$$



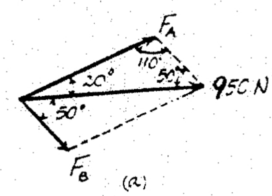


© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-11. The truck is to be towed using two ropes. Determine the magnitude of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope in order to develop a resultant force of 950 N directed along the positive x axis. Set $\theta = 50°$.



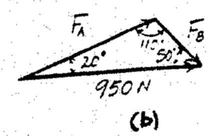
Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).



Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 50°} = \frac{950}{\sin 110°}$$

$$F_A = 774 \text{ N}$$ **Ans.**

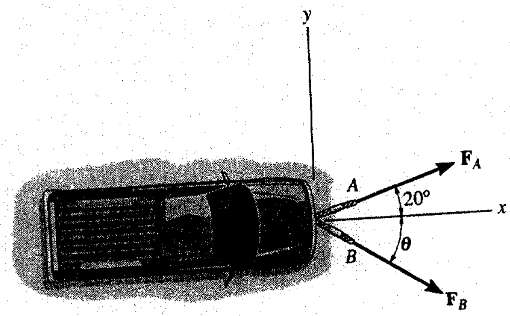


$$\frac{F_B}{\sin 20°} = \frac{950}{\sin 110°}$$

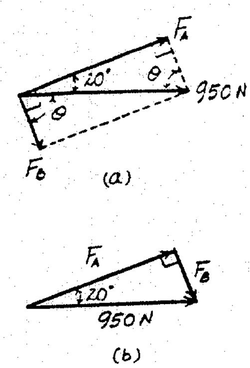
$$F_B = 346 \text{ N}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-12. The truck is to be towed using two ropes. If the resultant force is to be 950 N, directed along the positive x axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each rope and the angle of θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 20° from the x axis as shown.



Parallelogram Law : In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).



Trigonometry : Fig. (b).

$$F_B = 950\sin 20° = 325 \text{ N}$$ **Ans.**

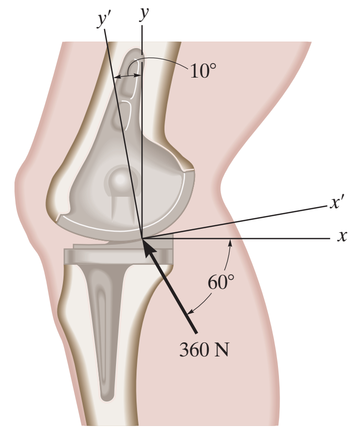
$$F_A = 950\cos 20° = 893 \text{ N}$$ **Ans.**

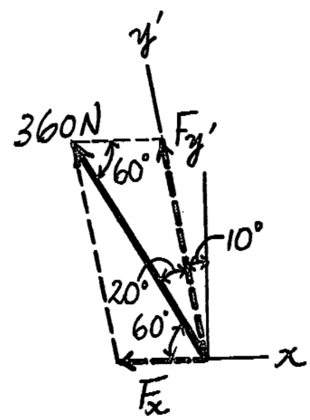
The angle θ is

$$\theta = 90° - 20° = 70.0°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–13. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x and y' axes.

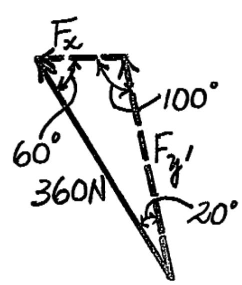




(a)

$$\frac{-F_x}{\sin 20^\circ} = \frac{360}{\sin 100^\circ}; \quad F_x = -125 \text{ N} \quad \textbf{Ans.}$$

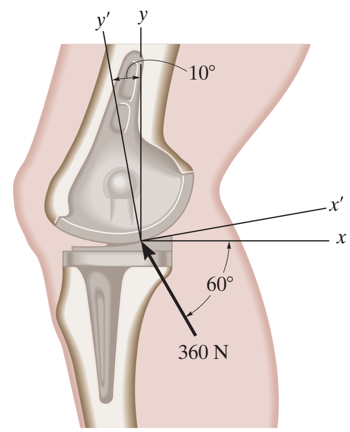
$$\frac{F_{y'}}{\sin 60^\circ} = \frac{360}{\sin 100^\circ}; \quad F_{y'} = 317 \text{ N} \quad \textbf{Ans.}$$

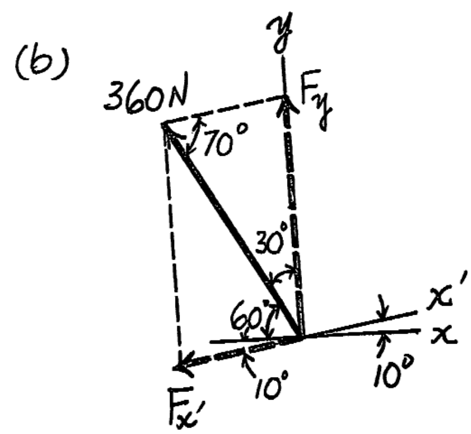


(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–14. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x' and y axes.

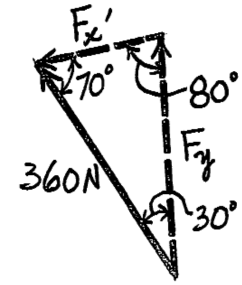




(a)

$$\frac{-F_{x'}}{\sin 30^\circ} = \frac{360}{\sin 80^\circ}; \quad F_{x'} = -183\text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_y}{\sin 70^\circ} = \frac{360}{\sin 80^\circ}; \quad F_y = 344\text{ N} \qquad \textbf{Ans.}$$

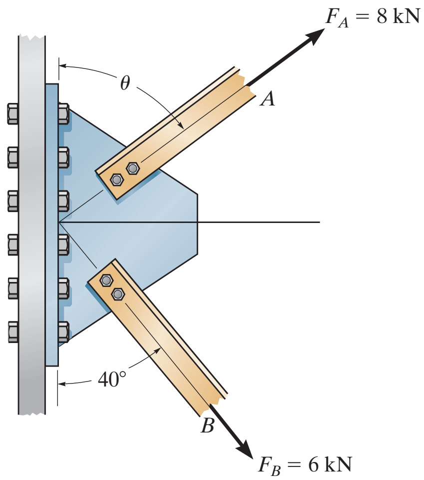


(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–15.

The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$ **Ans.**

The angle θ can be determined using law of sines (Fig. *b*).

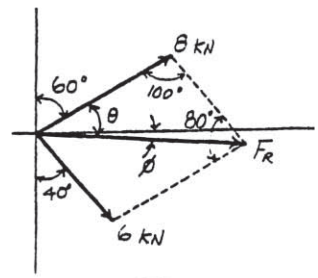
$$\frac{\sin\theta}{6} = \frac{\sin 100°}{10.80}$$

$$\sin\theta = 0.5470$$

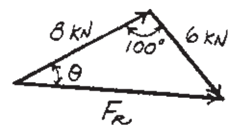
$$\theta = 33.16°$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16° - 30° = 3.16°$$ **Ans.**



(a)

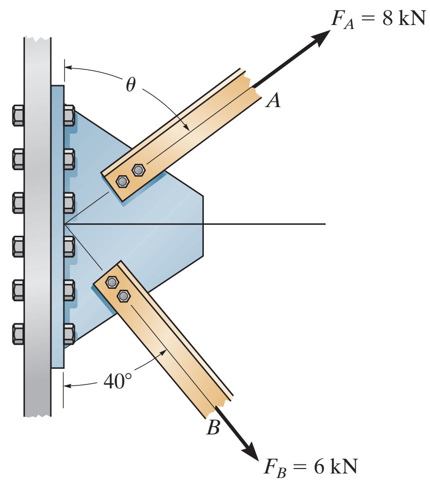


(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

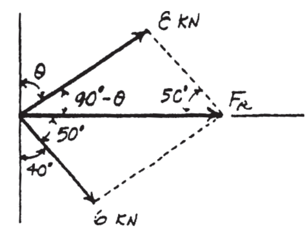
2–16.

Determine the angle of θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.



Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90° - \theta)}{6} = \frac{\sin 50°}{8}$$

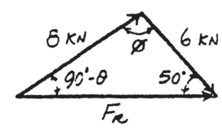
$$\sin(90° - \theta) = 0.5745$$

$$\theta = 54.93° = 54.9°$$ **Ans.**

From the triangle, $\phi = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$

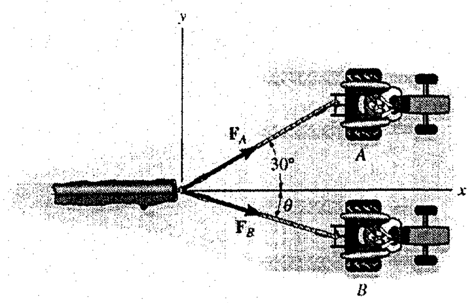
$$= 10.4 \text{ kN}$$ **Ans.**



(b)

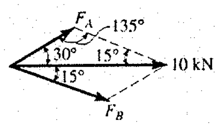
© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-17. The log is being towed by two tractors A and B. Determine the magnitude of the two towing forces $\mathbf{F}_A$ and $\mathbf{F}_B$ if it is required that the resultant force have a magnitude $F_R = 10$ kN and be directed along the x axis. Set $\theta = 15°$.

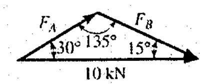


Parallelogram Law: The parallelogram law of addition is shown in Fig. (a).

Trigonometry: Using law of sines [Fig. (b)], we have



(a)



(b)

$$\frac{F_A}{\sin 15°} = \frac{10}{\sin 135°}$$

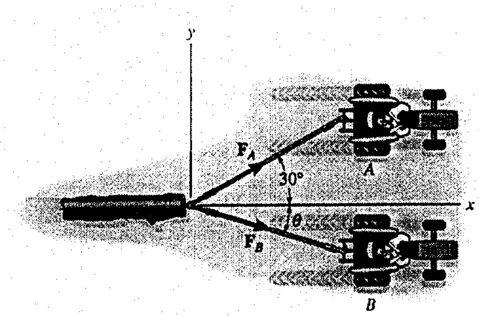
$F_A = 3.66$ kN **Ans.**

$$\frac{F_B}{\sin 30°} = \frac{10}{\sin 135°}$$

$F_B = 7.07$ kN **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-18. If the resultant $\mathbf{F}_R$ of the two forces acting on the log is to be directed along the positive x axis and have a magnitude of 10 kN, determine the angle θ of the cable, attached to B such that the force $\mathbf{F}_B$ in this cable is minimum. What is the magnitude of the force in each cable for this situation?



Parallelogram Law: In order to produce a *minimum* force $\mathbf{F}_B$, $\mathbf{F}_B$ has to act perpendicular to $\mathbf{F}_A$. The parallelogram law of addition is shown in Fig. (a).

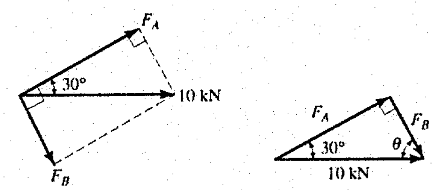
Trigonometry: Fig. (b).

$F_B = 10 \sin 30° = 5.00 \text{ kN}$ **Ans.**

$F_A = 10 \cos 30° = 8.66 \text{ kN}$ **Ans.**

The angle θ is

$\theta = 90° - 30° = 60.0°$ **Ans.**

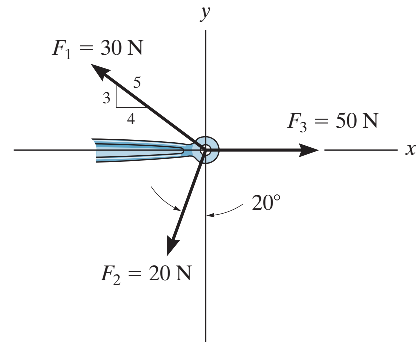


(a) (b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–19.

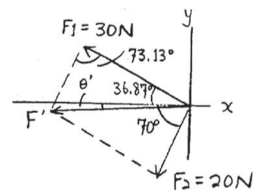
Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



SOLUTION

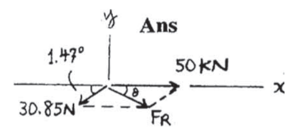
$$F' = \sqrt{(20)^2 + (30)^2 - 2(20)(30)\cos 73.13^\circ} = 30.85\text{ N}$$

$$\frac{30.85}{\sin 73.13^\circ} = \frac{30}{\sin(70^\circ - \theta')}; \qquad \theta' = 1.47^\circ$$



$$F_R = \sqrt{(30.85)^2 + (50)^2 - 2(30.85)(50)\cos 1.47^\circ} = 19.18 = 19.2\text{ N}$$ **Ans.**

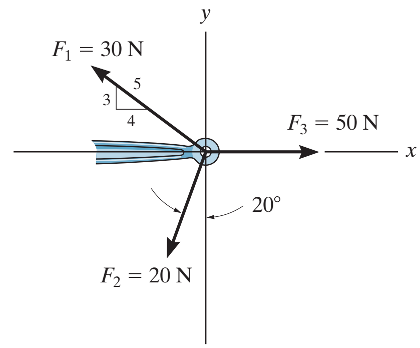
$$\frac{19.18}{\sin 1.47^\circ} = \frac{30.85}{\sin\theta}; \qquad \theta = 2.37^\circ \searrow$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–20.

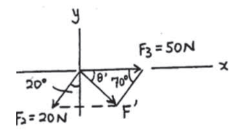
Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



SOLUTION

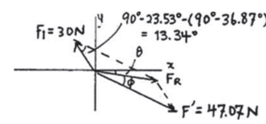
$$F' = \sqrt{(20)^2 + (50)^2 - 2(20)(50)\cos 70°} = 47.07 \text{ N}$$

$$\frac{20}{\sin \theta'} = \frac{47.07}{\sin 70°}; \qquad \theta' = 23.53°$$



$$F_R = \sqrt{(47.07)^2 + (30)^2 - 2(47.07)(30)\cos 13.34°} = 19.18 = 19.2 \text{ N}$$ **Ans.**

$$\frac{19.18}{\sin 13.34°} = \frac{30}{\sin \phi}; \qquad \phi = 21.15°$$

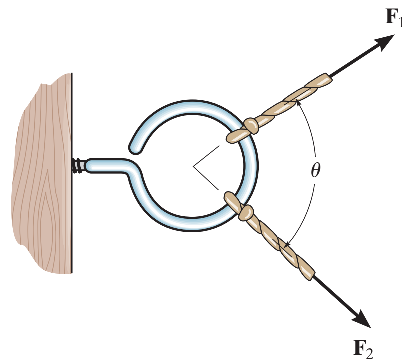


$$\theta = 23.53° - 21.15° = 2.37° \ ⦪$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–21.

Two forces act on the screw eye. If $F_1 = 400\text{ N}$ and $F_2 = 600\text{ N}$, determine the angle $\theta(0° \leq \theta \leq 180°)$ between them, so that the resultant force has a magnitude of $F_R = 800\text{ N}$.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

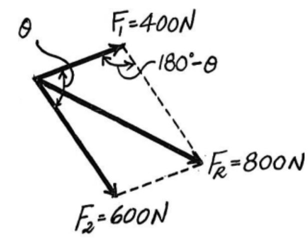
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - \theta°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - \theta)$$

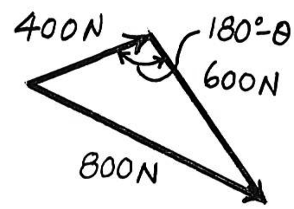
$$\cos(180° - \theta) = -0.25$$

$$180° - \theta = 104.48$$

$$\theta = 75.52° = 75.5° \qquad \textbf{Ans.}$$



(a)

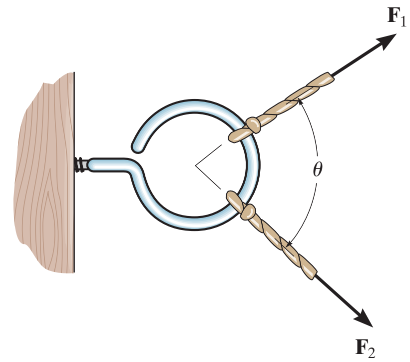


(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–22.

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



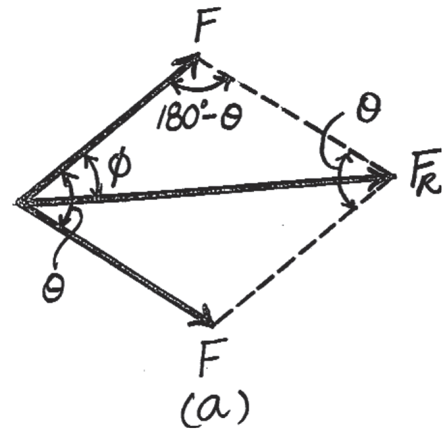
SOLUTION

$$\frac{F}{\sin\phi} = \frac{F}{\sin(\theta - \phi)}$$

$$\sin(\theta - \phi) = \sin\phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2}$$ **Ans.**



$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180^\circ - \theta)}$$

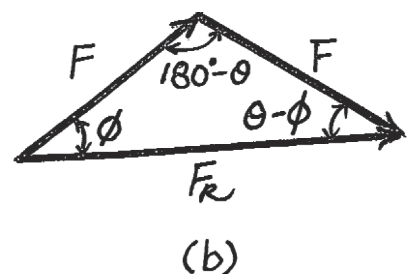
Since $\cos(180^\circ - \theta) = -\cos\theta$

$$F_R = F\left(\sqrt{2}\right)\sqrt{1 + \cos\theta}$$

Since $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos\theta}{2}}$

Then

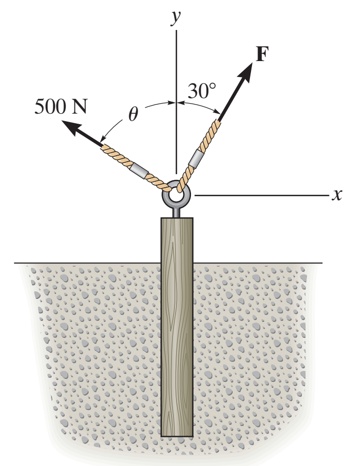
$$F_R = 2F\cos\left(\frac{\theta}{2}\right)$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–23.

Two forces act on the screw eye. If $F = 600$ N, determine the magnitude of the resultant force and the angle θ if the resultant force is directed vertically upward.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b* respectively. Applying law of sines to Fig. *b*,

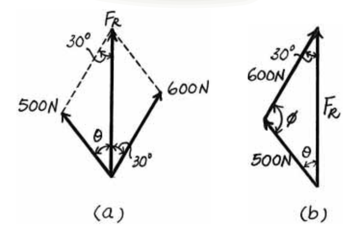
$$\frac{\sin\theta}{600} = \frac{\sin 30^\circ}{500}; \quad \sin\theta = 0.6 \quad \theta = 36.87^\circ = 36.9^\circ \qquad \textbf{Ans.}$$

Using the result of θ,

$$\phi = 180^\circ - 30^\circ - 36.87^\circ = 113.13^\circ$$

Again, applying law of sines using the result of ϕ,

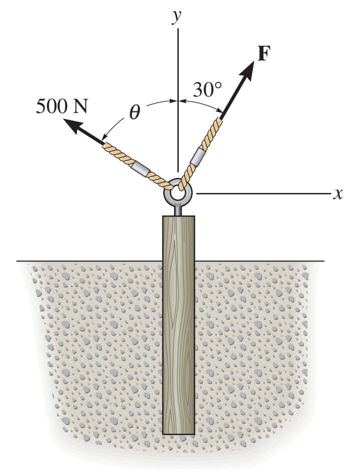
$$\frac{F_R}{\sin 113.13^\circ} = \frac{500}{\sin 30^\circ}; \quad F_R = 919.61\text{ N} = 920\text{ N} \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–24.

Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle θ $(0° \leq \theta \leq 90°)$ and the magnitude of force **F** so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin \phi}{750} = \frac{\sin 30°}{500}$$

$$\sin \phi = 0.750$$

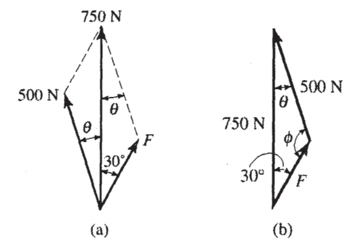
$$\phi = 131.41° \text{ (By observation, } \phi > 90°)$$

Thus,

$$\theta = 180° - 30° - 131.41° = 18.59° = 18.6°$$ **Ans.**

$$\frac{F}{\sin 18.59°} = \frac{500}{\sin 30°}$$

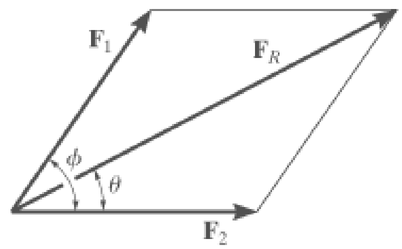
$$F = 319 \text{ N}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-25.

Determine the magnitude and direction of the resultant force $\mathbf{F_1}$. Express the result in terms of the magnitudes of the component $\mathbf{F_2}$ and resultant $\mathbf{F_R}$ and the angle θ.

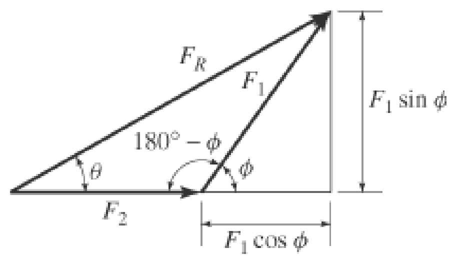


Solution:

$$F_1^{\,2} = F_R^{\,2} + F_2^{\,2} - 2F_R\,F_2\cos(\theta)$$

Since $\cos(180\,\text{deg} - \phi) = -\cos(\phi)$,

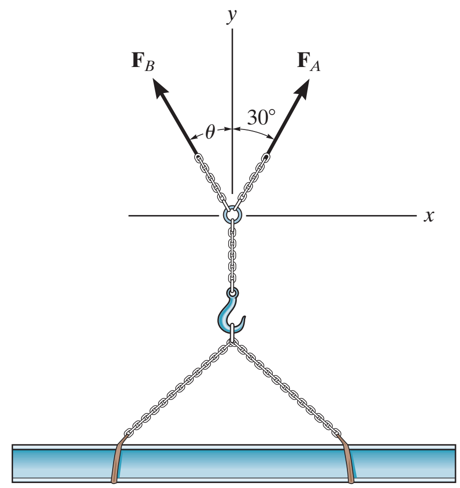
$$F_1 = \sqrt{F_R^{\,2} + F_2^{\,2} - 2\,F_R\,F_2\,\cos(\theta)}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–26.

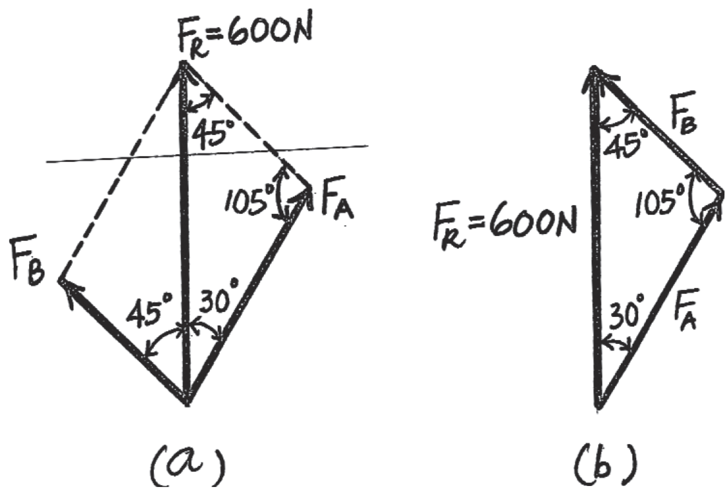
The beam is to be hoisted using two chains. Determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set $\theta = 45°$.



SOLUTION

$$\frac{F_A}{\sin 45°} = \frac{600}{\sin 105°}; \quad F_A = 439 \text{ N}$$ **Ans.**

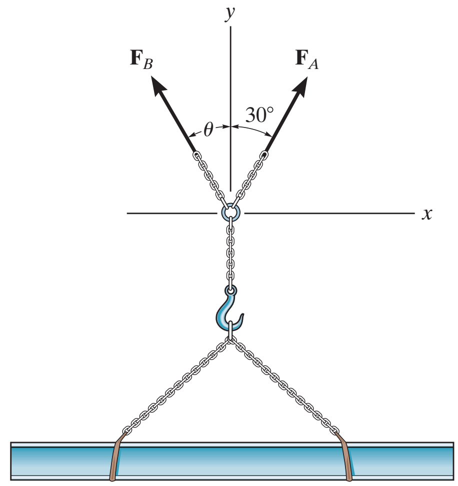
$$\frac{F_B}{\sin 30°} = \frac{600}{\sin 105°}; \quad F_B = 311 \text{ N}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–27.

The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive *y* axis, determine the magnitudes of forces $\mathbf{F}_A$ and $\mathbf{F}_B$ acting on each chain and the angle θ of $\mathbf{F}_B$ so that the magnitude of $\mathbf{F}_B$ is a *minimum*. $\mathbf{F}_A$ acts at 30° from the *y* axis, as shown.



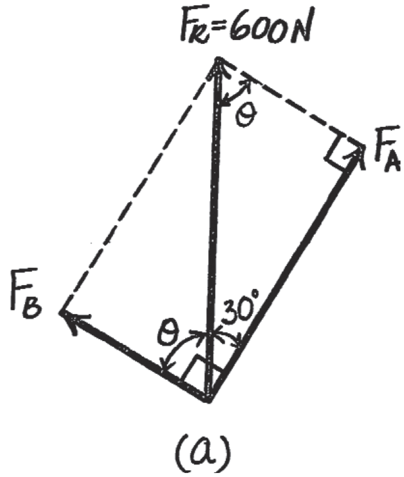
SOLUTION

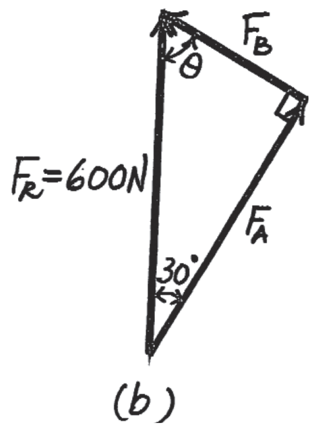
For minimum F_B, require

$\theta = 60°$ **Ans.**

$F_A = 600 \cos 30° = 520\text{ N}$ **Ans.**

$F_B = 600 \sin 30° = 300\text{ N}$ **Ans.**

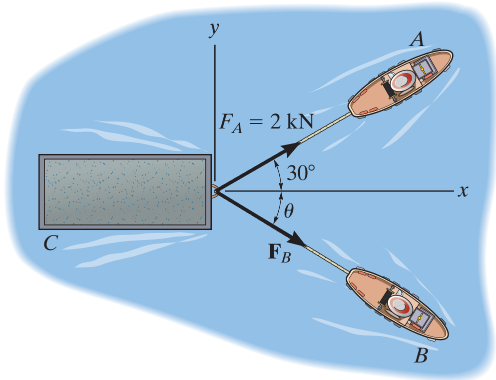




© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

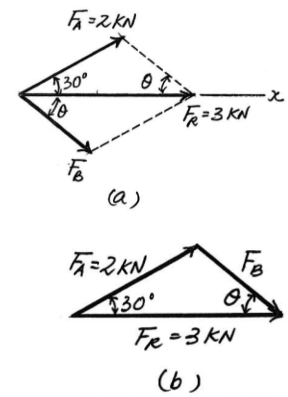
2–28.

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

$$= 1.615\text{kN} = 1.61 \text{ kN} \qquad \textbf{Ans.}$$

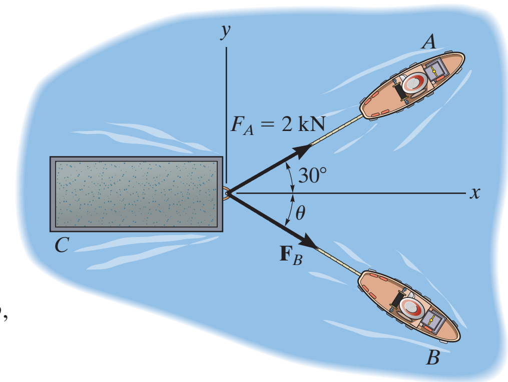
Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

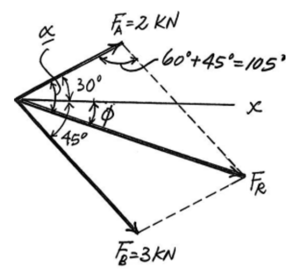
2–29.

If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.

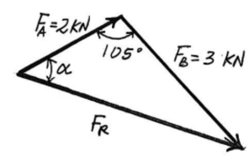


SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)



(b)

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

$$= 4.013 \text{ kN} = 4.01 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin \alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$

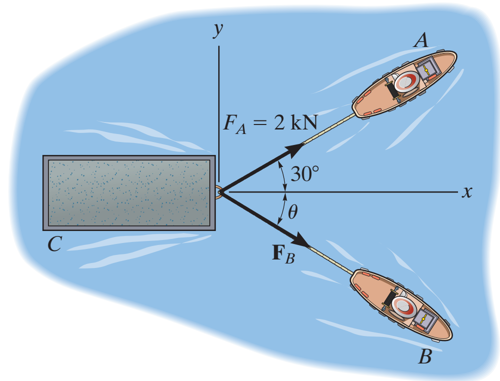
Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\phi = \alpha - 30° = 46.22° - 30° = 16.2° \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–30.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and F_B is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.



SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

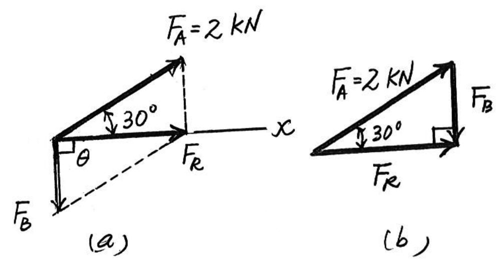
$$\theta = 90^\circ \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. a and b, respectively.

By applying simple trigonometry to Fig. b,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN} \qquad \textbf{Ans.}$$

$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN} \qquad \textbf{Ans.}$$

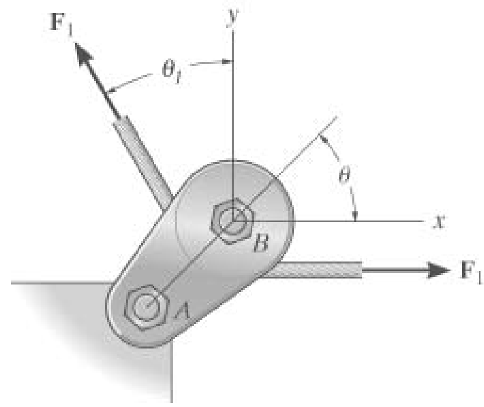


© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-31.

If the tension in the cable is $\mathbf{F_1}$, determine the magnitude and direction of the resultant force acting on the pulley. This angle defines the same angle θ of line *AB* on the tailboard block.

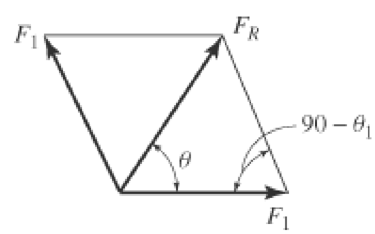
Given:



$F_1 = 400\ \text{N}$

$\theta_1 = 30\ \text{deg}$

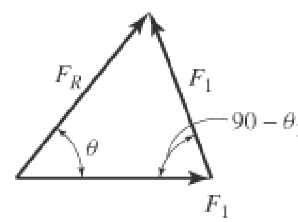
Solution:



$$F_R = \sqrt{F_1^2 + F_1^2 - 2F_1F_1\cos(90\ \text{deg} - \theta_1)}$$

$F_R = 400\ \text{N}$ **Ans.**

$$\frac{\sin(90\ \text{deg} - \theta)}{F_R} = \frac{\sin(\theta_1)}{F_1}$$



$$\theta = 90\ \text{deg} - \text{asin}\left(\frac{F_R}{F_1}\sin(\theta_1)\right)$$

$\theta = 60\ \text{deg}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-32.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.

Given:

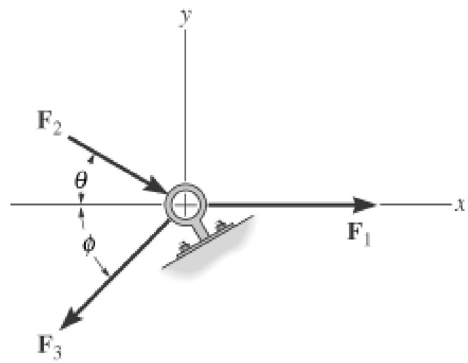
$F_1 = 70 \text{ N}$

$F_2 = 50 \text{ N}$

$F_3 = 65 \text{ N}$

$\theta = 30 \text{ deg}$

$\phi = 45 \text{ deg}$



Solution:

$\overset{+}{\rightarrow} \quad F_{Rx} = \Sigma F_x; \quad F_{RX} = F_1 + F_2\cos(\theta) - F_3\cos(\phi)$

$+\uparrow \quad F_{Ry} = \Sigma F_y; \quad F_{RY} = -F_2\sin(\theta) - F_3\sin(\phi)$

$$F_R = \sqrt{F_{RX}^2 + F_{RY}^2}$$

$$\theta = \operatorname{atan}\left(\frac{|F_{RY}|}{|F_{RX}|}\right)$$

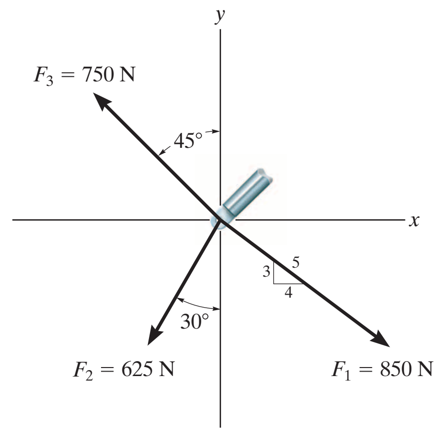
$F_R = 97.8 \text{ N}$ **Ans.**

$\theta = 46.5 \text{ deg}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–33.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive *x* axis.



SOLUTION

$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $\quad F_{R_x} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.8 \text{ N}$

$+\uparrow F_{R_y} = \Sigma F_y;$ $\quad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.9 \text{ N}$

$$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546 \text{ N} \qquad \textbf{Ans.}$$

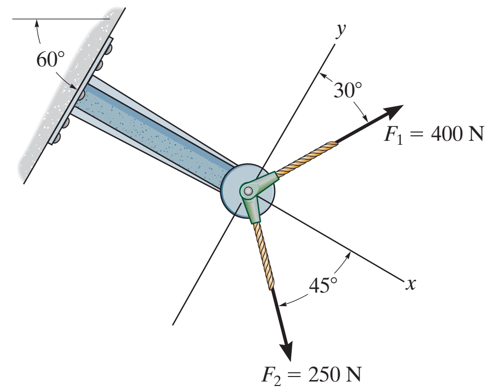
$$\phi = \tan^{-1}\left[\frac{-520.9}{-162.8}\right] = 72.64°$$

$$\theta = 180° + 72.64° = 253° \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–34.

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



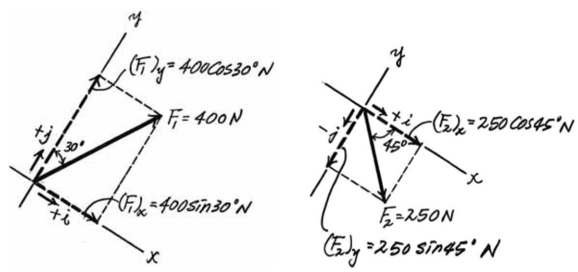
SOLUTION

$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\} \text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\} \text{ N}$$

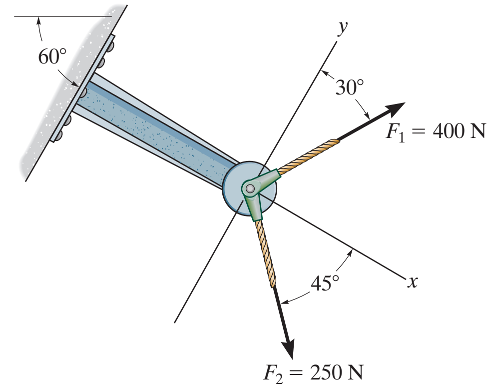
$$= \{177\mathbf{i} - 177\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–35.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$$(F_1)_x = 400 \sin 30^\circ = 200 \text{ N} \qquad (F_1)_y = 400 \cos 30^\circ = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45^\circ = 176.78 \text{ N} \qquad (F_2)_y = 250 \sin 45^\circ = 176.78 \text{ N}$$

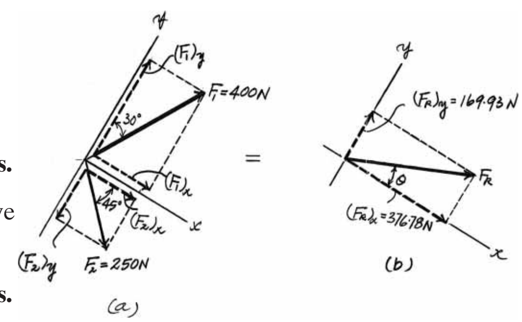
Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 + 176.78 = 376.78 \text{ N}$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N}$$ **Ans.**



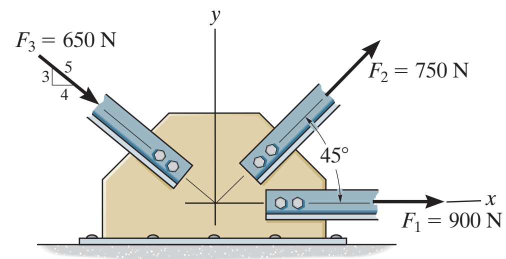
The direction angle θ of $\mathbf{F}_R$, Fig. b, measured counterclockwise from the positive axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{169.63}{376.78}\right) = 24.2^\circ$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–36.

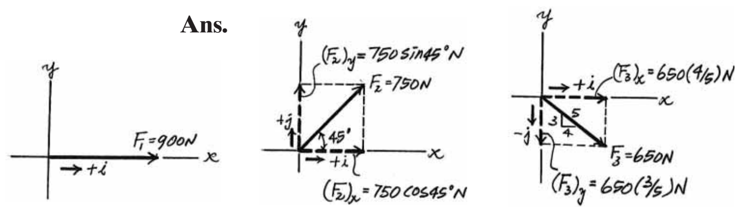
Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.



$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = \{750\cos 45^\circ(+\mathbf{i}) + 750\sin 45^\circ(+\mathbf{j})\}\text{ N}$$
$$= \{530\mathbf{i} + 530\mathbf{j}\}\text{ N}$$ **Ans.**

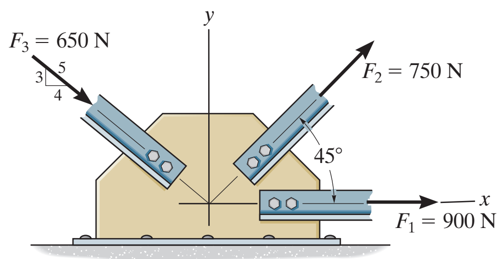
$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\text{ N}$$
$$= \{520\,\mathbf{i} - 390\mathbf{j})\}\text{ N}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–37.

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900 \text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750 \cos 45° = 530.33 \text{ N} \qquad (F_2)_y = 750 \sin 45° = 530.33 \text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520 \text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow$$

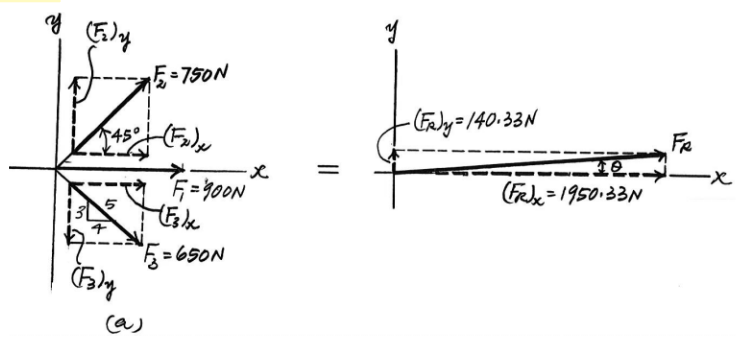
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \textbf{Ans.}$$

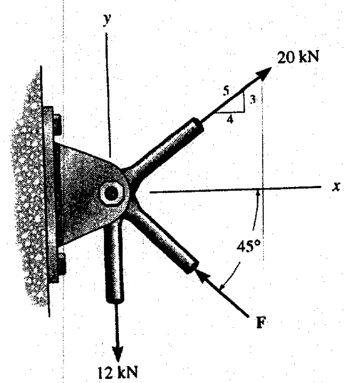
The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \quad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-38. Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 20\left(\frac{4}{5}\right) - F\cos 45^\circ = 16.0 - 0.7071F \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 20\left(\frac{3}{5}\right) - 12 + F\sin 45^\circ = 0.7071F \uparrow$$

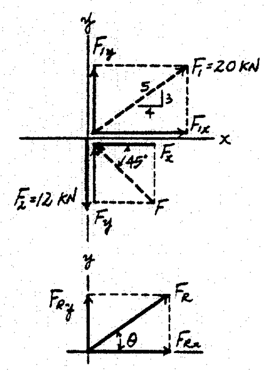
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{(16.0 - 0.7071F)^2 + (0.7071F)^2} = \sqrt{F^2 - 22.63F + 256} \quad [1]$$

$$F_R^2 = F^2 - 22.63F + 256$$

$$2F_R\frac{dF_R}{dF} = 2F - 22.63 \quad [2]$$

$$\left(F_R\frac{d^2F_R}{dF^2} + \frac{dF_R}{dF}\times\frac{dF_R}{dF}\right) = 1 \quad [3]$$



In order to obtain the *minimum* resultant force $\mathbf{F}_R$, $\frac{dF_R}{dF} = 0$. From Eq.[2]

$$2F_R\frac{dF_R}{dF} = 2F - 22.63 = 0$$

$$F = 11.31 \text{ kN} = 11.3 \text{ kN} \qquad \textbf{Ans.}$$

Substitute $F = 11.31$ kN into Eq.[1], we have

$$F_R = \sqrt{11.31^2 - 22.63(11.31) + 256} = \sqrt{128} \text{ kN}$$

Substitute $F_R = \sqrt{128}$ kN with $\frac{dF_R}{dF} = 0$ into Eq.[3], we have

$$\left(\sqrt{128}\frac{d^2F_R}{dF^2} + 0\right) = 1$$

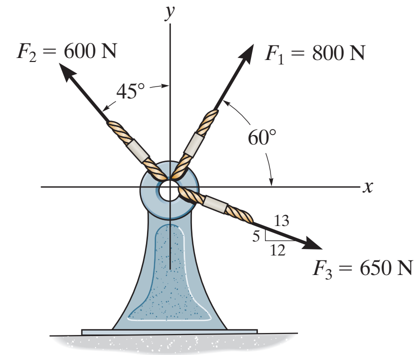
$$\frac{d^2F_R}{dF^2} = 0.0884 > 0$$

Hence, $F = 11.3$ kN is indeed producing a minimum resultant force.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–39.

Resolve each force acting on the support into its x and y components, and express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \{800 \cos 60°(+\mathbf{i}) + 800 \sin 60°(+\mathbf{j})\}\text{ N}$$

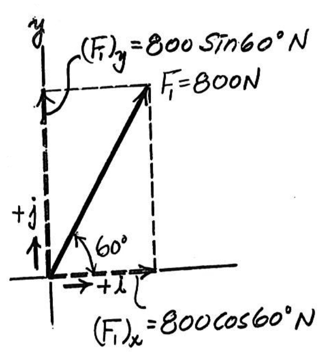
$$= \{400\mathbf{i} + 693\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

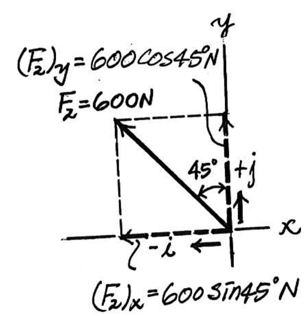
$$\mathbf{F}_2 = \{600 \sin 45°(-\mathbf{i}) + 600 \cos 45°(+\mathbf{j})\}\text{ N} \qquad \textbf{Ans.}$$

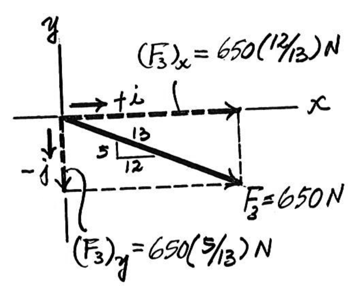
$$= \{-424\mathbf{i} + 424\mathbf{j}\}\text{ N}$$

$$\mathbf{F}_3 = \left\{650\left(\frac{12}{13}\right)(+\mathbf{i}) + 650\left(\frac{5}{13}\right)(-\mathbf{j})\right\}\text{ N} \qquad \textbf{Ans.}$$

$$= \{600\mathbf{i} - 250\mathbf{j}\}\text{ N}$$



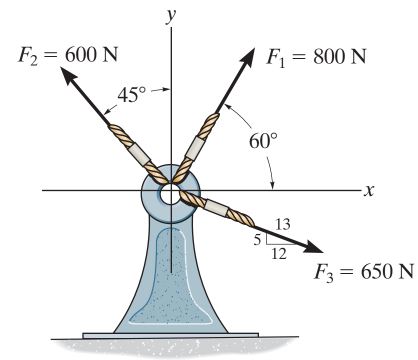




© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–40.

Determine the magnitude of the resultant force and its direction θ, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 800 \cos 60° = 400 \text{ N} \qquad (F_1)_y = 800 \sin 60° = 692.82 \text{ N}$$

$$(F_2)_x = 600 \sin 45° = 424.26 \text{ N} \qquad (F_2)_y = 600 \cos 45° = 424.26 \text{ N}$$

$$(F_3)_x = 650\left(\frac{12}{13}\right) = 600 \text{ N} \qquad (F_3)_y = 650\left(\frac{5}{13}\right) = 250 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 400 - 424.26 + 600 = 575.74 \text{ N} \rightarrow$$

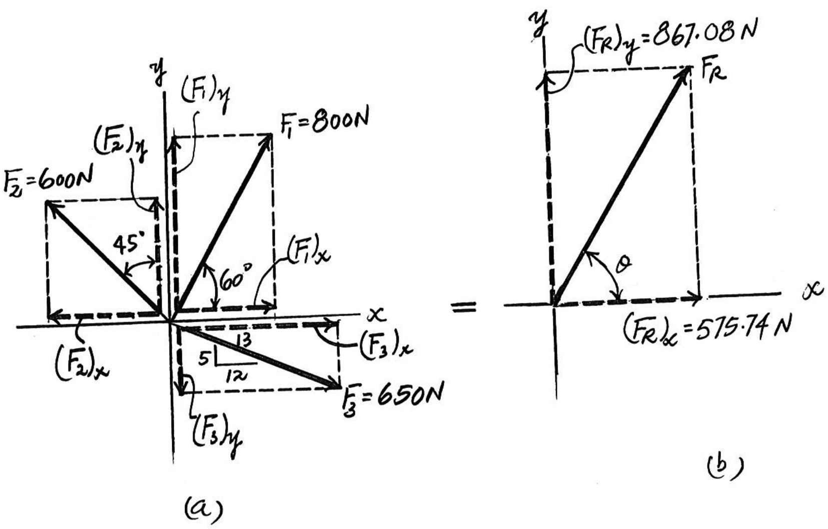
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = -692.82 + 424.26 - 250 = 867.08 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{575.74^2 + 867.08^2} = 1041 \text{ N} = 1.04 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. b, measured counterclockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{867.08}{575.74}\right) = 56.4° \quad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-41.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

Units Used:

$kN = 10^3\ N$

Given:

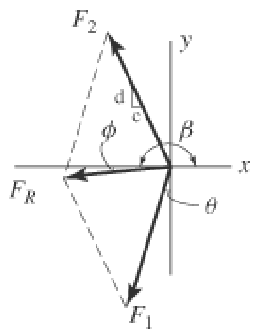
$F_1 = 30\ kN$

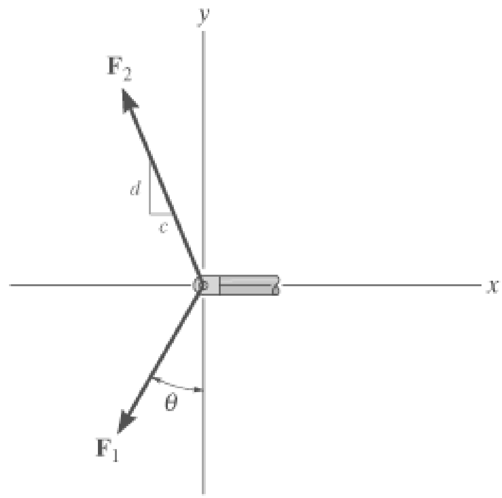
$F_2 = 26\ kN$

$\theta = 30\ deg$

$c = 5$

$d = 12$





Solution:

$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = -F_1 \sin(\theta) - \left(\frac{c}{\sqrt{c^2 + d^2}}\right) F_2 \quad F_{Rx} = -25\ kN$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -F_1 \cos(\theta) + \left(\frac{d}{\sqrt{c^2 + d^2}}\right) F_2 \quad F_{Ry} = -2\ kN$

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad F_R = 25.1\ kN$ **Ans.**

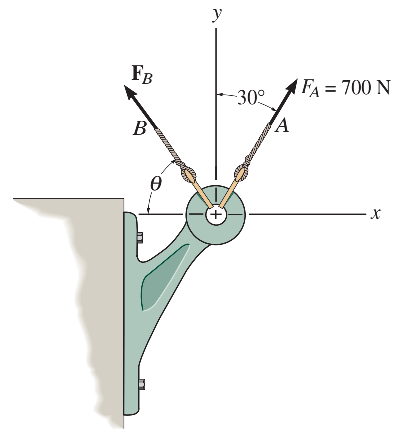
$\phi = \operatorname{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \quad \phi = 4.5\ deg$

$\beta = 180\ deg + \phi \quad \beta = 184.5\ deg$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–42.

Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad 0 = 700 \sin 30° - F_B \cos \theta$

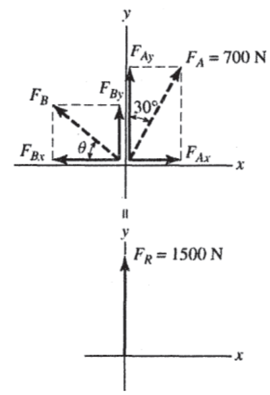
$$F_B \cos \theta = 350 \qquad \textbf{(1)}$$

$+\uparrow F_{R_y} = \Sigma F_y; \qquad 1500 = 700 \cos 30° + F_B \sin \theta$

$$F_B \sin \theta = 893.8 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields

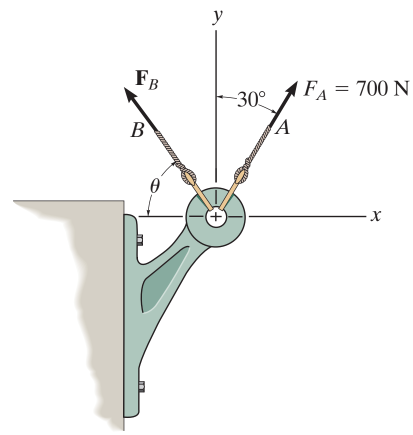
$$\theta = 68.6° \quad F_B = 960 \text{ N} \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–43.

Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.



SOLUTION

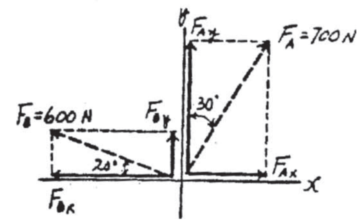
Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 700 \sin 30° - 600 \cos 20°$$

$$= -213.8 \text{ N} = 213.8 \text{ N} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 700 \cos 30° + 600 \sin 20°$$

$$= 811.4 \text{ N} \uparrow$$

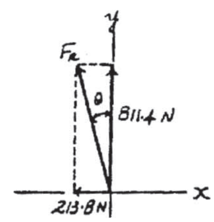


The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N} \qquad \textbf{Ans.}$$

The direction angle θ measured counterclockwise from the positive y axis is

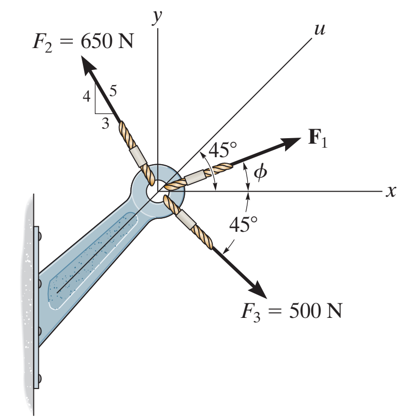
$$\theta = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–44.

The magnitude of the resultant force acting on the bracket is to be 400 N. Determine the magnitude of $\mathbf{F}_1$ if $\phi = 30°$.



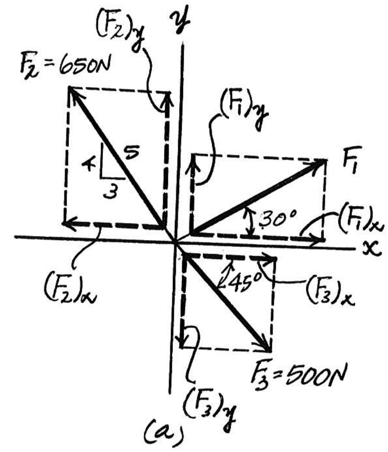
SOLUTION

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \cos 30° = 0.8660F_1 \qquad (F_1)_y = F_1 \sin 30° = 0.5F_1$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45° = 353.55 \text{ N}$$



Resultant Force: Summing the force components algebraically along the *x* and *y* axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 0.8660F_1 - 390 + 353.55 = 0.8660F_1 - 36.45$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 0.5F_1 + 520 - 353.55 = 0.5F_1 + 166.45$$

Since the magnitude of the resultant force is $\mathbf{F}_R = 400$ N, we can write

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$400 = \sqrt{(0.8660F_1 - 36.45)^2 + (0.5F_1 + 166.45)^2}$$

$$F_1^2 + 103.32F_1 - 130967.17 = 0$$ **Ans.**

Solving,

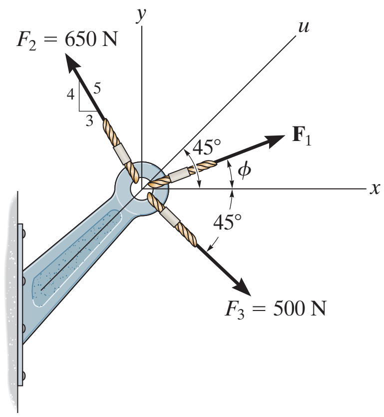
$$F_1 = 314 \text{ N} \qquad \text{or} \qquad F_1 = -417 \text{ N}$$ **Ans.**

The negative sign indicates that $\mathbf{F}_1 = 417$ N must act in the opposite sense to that shown in the figure.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–45.

If the resultant force acting on the bracket is to be directed along the positive u axis, and the magnitude of $\mathbf{F}_1$ is required to be *minimum*, determine the magnitudes of the resultant force and $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$

$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \qquad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$

$(F_3)_x = 500 \cos 45° = 353.55 \text{ N} \qquad (F_3)_y = 500 \sin 45° = 353.55 \text{ N}$

$(F_R)_x = F_R \cos 45° = 0.7071F_R \qquad (F_R)_y = F_R \sin 45° = 0.7071F_R$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad 0.7071F_R = F_1 \cos\phi - 390 + 353.55 \qquad$ **(1)**

$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad 0.7071F_R = F_1 \sin\phi + 520 - 353.55 \qquad$ **(2)**

Eliminating F_R from Eqs. (1) and (2), yields

$$F_1 = \frac{202.89}{\cos\phi - \sin\phi} \qquad \textbf{(3)}$$

The first derivative of Eq. (3) is

$$\frac{dF_1}{d\phi} = \frac{\sin\phi + \cos\phi}{(\cos\phi - \sin\phi)^2} \qquad \textbf{(4)}$$

The second derivative of Eq. (3) is

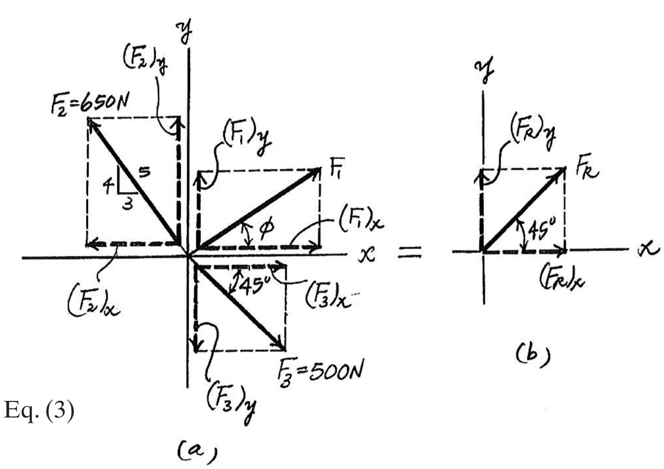
$$\frac{d^2F_1}{d\phi^2} = \frac{2(\sin\phi + \cos\phi)^2}{(\cos\phi - \sin\phi)^3} + \frac{1}{\cos\phi - \sin\phi} \qquad \textbf{(5)}$$

For $\mathbf{F}_1$ to be minimum, $\frac{dF_1}{d\phi} = 0$. Thus, from Eq. (4)

$$\sin\phi + \cos\phi = 0$$

$$\tan\phi = -1$$

$$\phi = -45°$$



Substituting $\phi = -45°$ into Eq. (5), yields

$$\frac{d^2F_1}{d\phi^2} = 0.7071 > 0$$

This shows that $\phi = -45°$ indeed produces minimum F_1. Thus, from Eq. (3)

$$F_1 = \frac{202.89}{\cos(-45°) - \sin(-45°)} = 143.47 \text{ N} = 143 \text{ N} \qquad \textbf{Ans.}$$

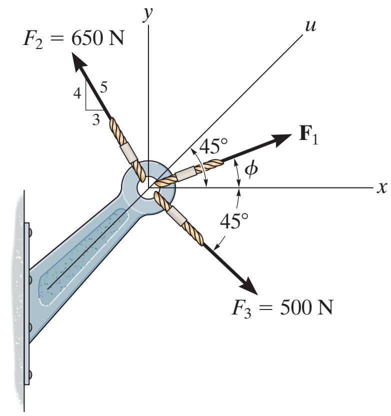
Substituting $\phi = -45°$ and $F_1 = 143.47$ N into either Eq. (1) or Eq. (2), yields

$$F_R = 91.9 \text{ N} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–46.

If the magnitude of the resultant force acting on the bracket is 600 N, directed along the positive u axis, determine the magnitude of **F** and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \cos\phi$ $\qquad$ $(F_1)_y = F_1 \sin\phi$

$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N}$ $\qquad$ $(F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$

$(F_3)_x = 500 \cos 45° = 353.55 \text{ N}$ $\qquad$ $(F_3)_y = 500 \cos 45° = 353.55 \text{ N}$

$(F_R)_x = 600 \cos 45° = 424.26 \text{ N}$ $\qquad$ $(F_R)_y = 600 \sin 45° = 424.26 \text{ N}$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x;$ $\qquad 424.26 = F_1 \cos\phi - 390 + 353.55$ **(1)**

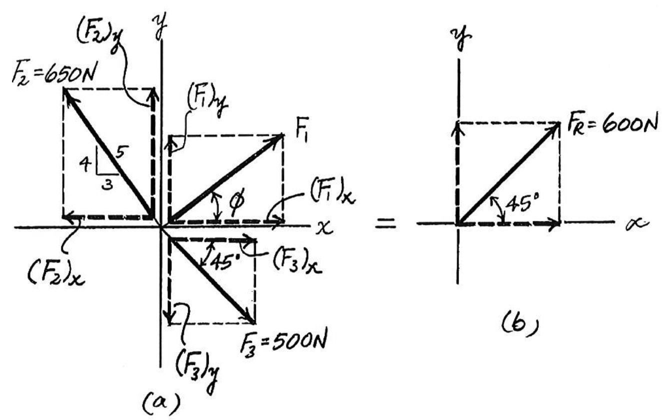
$$F_1 \cos\phi = 460.71$$

$+\uparrow \Sigma(F_R)_y = \Sigma F_y;$ $\qquad 424.26 = F_1 \sin\phi + 520 - 353.55$ **(2)**

$$F_1 \sin\phi = 257.82$$

Solving Eqs. (1) and (2), yields

$\phi = 29.2°$ $\qquad$ $F_1 = 528 \text{ N}$ **Ans.**

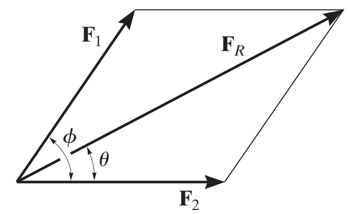


© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–47.

Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.

SOLUTION

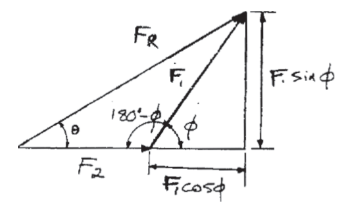


$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180° - \phi)$$

Since $\cos(180° - \phi) = -\cos\phi$,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$$ **Ans.**

From the figure,



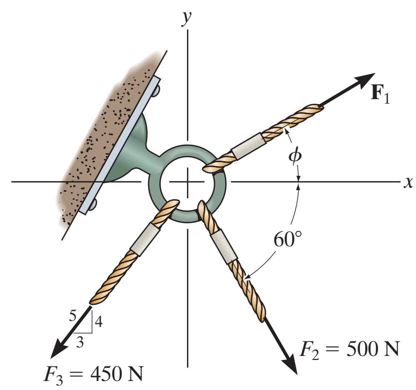
$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–48.

If $F_1 = 600$ N and $\phi = 30°$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of each force can be written as

$$(F_1)_x = 600 \cos 30° = 519.62 \text{ N} \quad (F_1)_y = 600 \sin 30° = 300 \text{ N}$$

$$(F_2)_x = 500 \cos 60° = 250 \text{ N} \quad (F_2)_y = 500 \sin 60° = 433.01 \text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270 \text{ N} \quad (F_3)_y = 450\left(\frac{4}{5}\right) = 360 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 519.62 + 250 - 270 = 499.62 \text{ N} \rightarrow$$

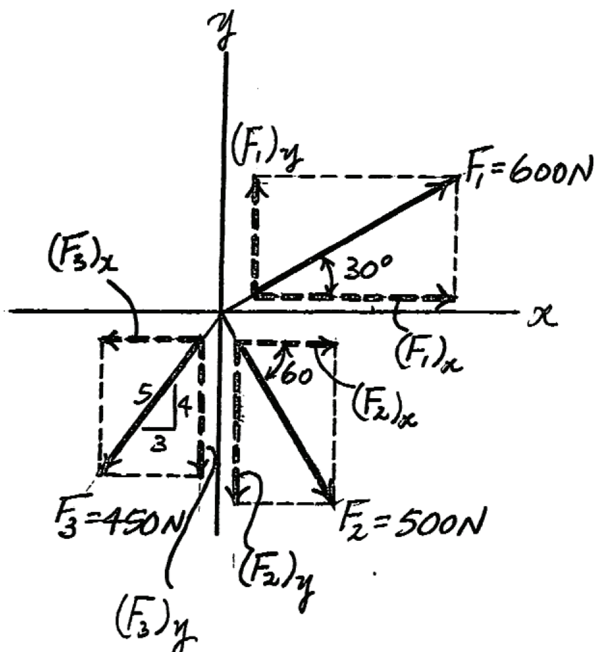
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 300 - 433.01 - 360 = -493.01 \text{ N} = 493.01 \text{ N} \downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

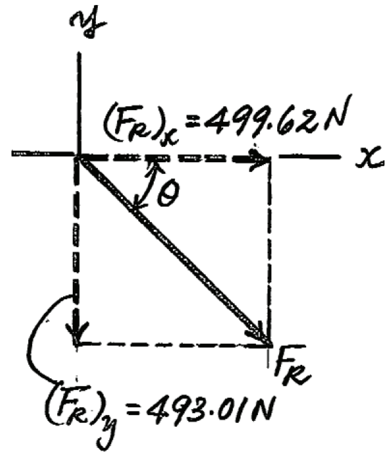
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{499.62^2 + 493.01^2} = 701.91 \text{ N} = 702 \text{ N}$$ **Ans.**

The direction angle θ of $\mathbf{F}_R$, Fig. *b*, measured clockwise from the x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{493.01}{499.62}\right) = 44.6°$$ **Ans.**



(a)

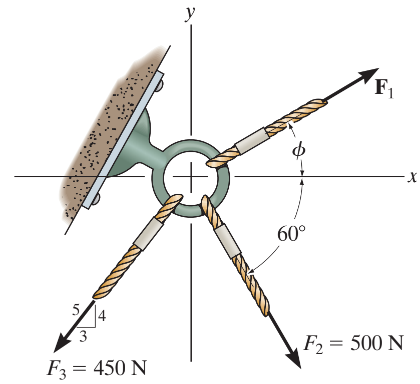


(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–49.

If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive x axis is $\theta = 30°$, determine the magnitude of $\mathbf{F}_1$ and the angle ϕ.



SOLUTION

Rectangular Components: By referring to Figs. a and b, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$$(F_1)_x = F_1 \cos\phi \qquad (F_1)_y = F_1 \sin\phi$$

$$(F_2)_x = 500\cos 60° = 250\text{ N} \qquad (F_2)_y = 500\sin 60° = 433.01\text{ N}$$

$$(F_3)_x = 450\left(\frac{3}{5}\right) = 270\text{ N} \qquad (F_3)_y = 450\left(\frac{4}{5}\right) = 360\text{ N}$$

$$(F_R)_x = 600\cos 30° = 519.62\text{ N} \qquad (F_R)_y = 600\sin 30° = 300\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad 519.62 = F_1\cos\phi + 250 - 270$$

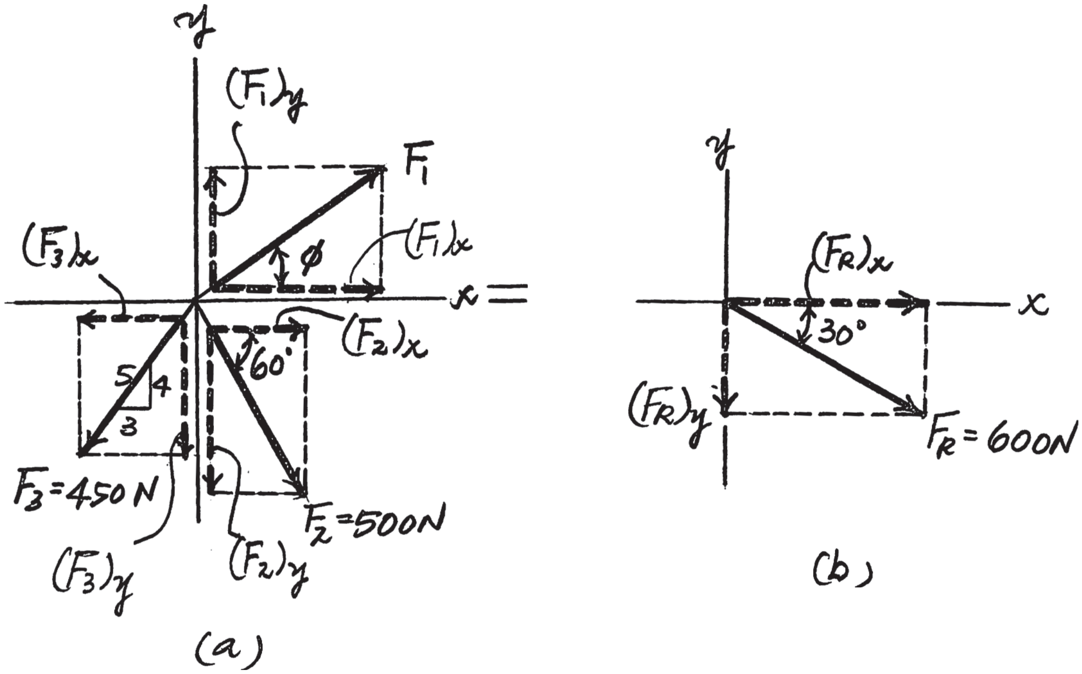
$$F_1\cos\phi = 539.62 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \quad -300 = F_1\sin\phi - 433.01 - 360$$

$$F_1\sin\phi = 493.01 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

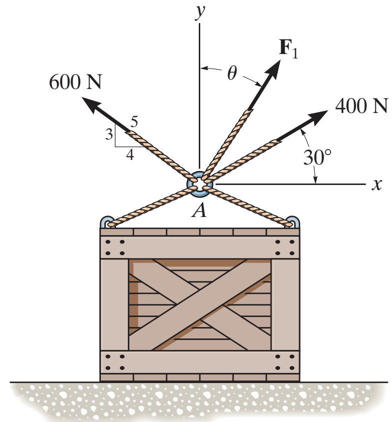
$$\phi = 42.4° \qquad F_1 = 731\text{ N} \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–50.

Determine the magnitude of $\mathbf{F}_1$ and its direction θ so that the resultant force is directed vertically upward and has a magnitude of 800 N.



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 0 = F_1 \sin\theta + 400\cos 30^\circ - 600\left(\frac{4}{5}\right)$$

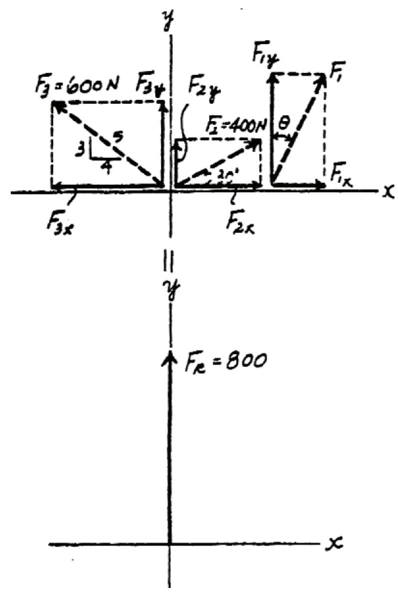
$$F_1 \sin\theta = 133.6 \qquad \textbf{(1)}$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 800 = F_1 \cos\theta + 400\sin 30^\circ + 600\left(\frac{3}{5}\right)$$

$$F_1 \cos\theta = 240 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

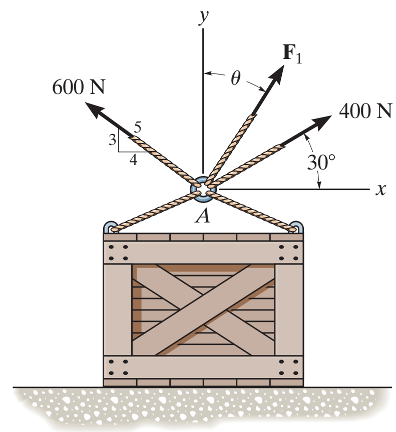
$$\theta = 29.1^\circ \qquad F_1 = 275 \text{ N} \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–51.

Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force of the three forces acting on the ring A. Take $F_1 = 500$ N and $\theta = 20°$.



SOLUTION

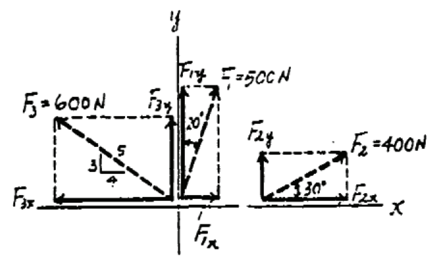
Scalar Notation: Summing the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 500 \sin 20° + 400 \cos 30° - 600\left(\frac{4}{5}\right)$$

$$= 37.42 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 500 \cos 20° + 400 \sin 30° + 600\left(\frac{3}{5}\right)$$

$$= 1029.8 \text{ N} \uparrow$$

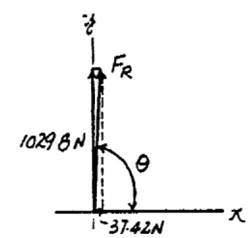


The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5 \text{ N} = 1.03 \text{ kN}$$ **Ans.**

The direction angle θ measured counterclockwise from positive x axis is

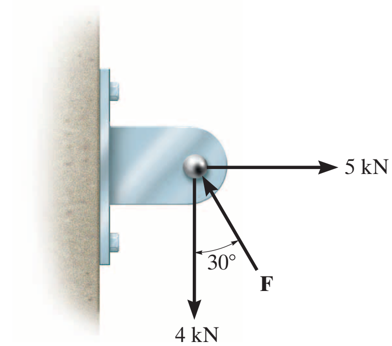
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{1029.8}{37.42}\right) = 87.9°$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–52.

Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?

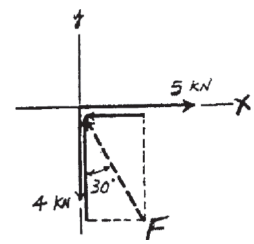


SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 5 - F\sin 30° = 5 - 0.50F \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = F\cos 30° - 4 = 0.8660F - 4 \uparrow$$

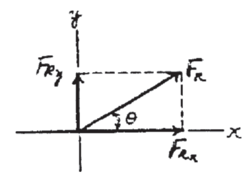


The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{(5 - 0.50F)^2 + (0.8660F - 4)^2}$$

$$= \sqrt{F^2 - 11.93F + 41} \qquad \textbf{(1)}$$



$$F_R^2 = F^2 - 11.93F + 41$$

$$2F_R\frac{dF_R}{dF} = 2F - 11.93 \qquad \textbf{(2)}$$

$$\left(F_R\frac{d^2F_R}{dF^2} + \frac{dF_R}{dF}\times\frac{dF_R}{dF}\right) = 1 \qquad \textbf{(3)}$$

In order to obtain the *minimum* resultant force $\mathbf{F}_R$, $\frac{dF_R}{dF} = 0$. From Eq. (2)

$$2F_R\frac{dF_R}{dF} = 2F - 11.93 = 0$$

$$F = 5.964 \text{ kN} = 5.96 \text{ kN} \qquad \textbf{Ans.}$$

Substituting $F = 5.964$ kN into Eq. (1), we have

$$F_R = \sqrt{5.964^2 - 11.93(5.964) + 41}$$

$$= 2.330 \text{ kN} = 2.33 \text{ kN} \qquad \textbf{Ans.}$$

Substituting $F_R = 2.330$ kN with $\frac{dF_R}{dF} = 0$ into Eq. (3), we have

$$\left[(2.330)\frac{d^2F_R}{dF^2} + 0\right] = 1$$

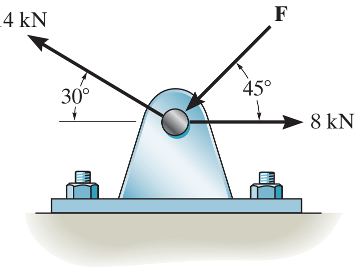
$$\frac{d^2F_R}{dF^2} = 0.429 > 0$$

Hence, $F = 5.96$ kN is indeed producing a minimum resultant force.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–53.

Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



SOLUTION

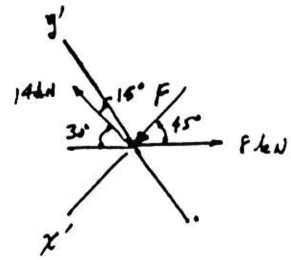
$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $\quad F_{Rz} = 8 - F\cos 45° - 14\cos 30°$

$= -4.1244 - F\cos 45°$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -F\sin 45° + 14\sin 30°$

$= 7 - F\sin 45°$

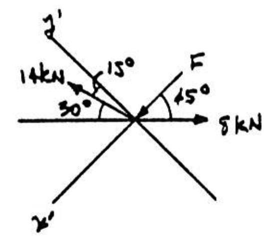
$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad \textbf{(1)}$$



$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$

$F = 2.03\ \text{kN}$ **Ans.**

From Eq. (1); $F_R = 7.87\ \text{kN}$ **Ans.**



Also, from the figure require

$(F_R)_{x'} = 0 = \Sigma F_{x'};$ $\quad F + 14\sin 15° - 8\cos 45° = 0$

$F = 2.03\ \text{kN}$ **Ans.**

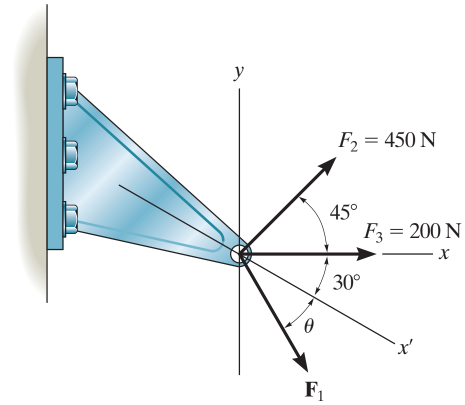
$(F_R)_{y'} = \Sigma F_{y'};$ $\quad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87\ \text{kN}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–54.

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 1 kN.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad 1000 \cos 30° = 200 + 450 \cos 45° + F_1 \cos(\theta + 30°)$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad -1000 \sin 30° = 450 \sin 45° - F_1 \sin(\theta + 30°)$

$F_1 \sin(\theta + 30°) = 818.198$

$F_1 \cos(\theta + 30°) = 347.827$

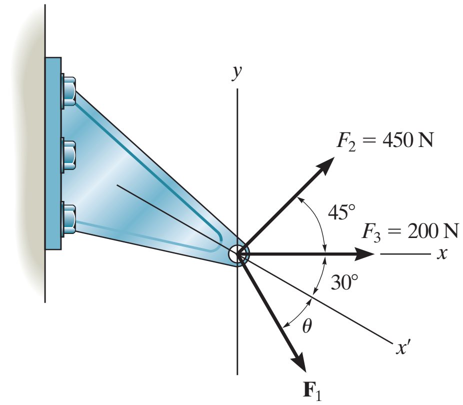
$\theta + 30° = 66.97°,\quad \theta = 37.0°$ **Ans.**

$F_1 = 889 \text{ N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–55.

If $F_1 = 300$ N and $\theta = 20°$, determine the magnitude and direction, measured counterclockwise from the x' axis, of the resultant force of the three forces acting on the bracket.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = 300 \cos 50° + 200 + 450 \cos 45° = 711.03 \text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -300 \sin 50° + 450 \sin 45° = 88.38 \text{ N}$

$$F_R = \sqrt{(711.03)^2 + (88.38)^2} = 717 \text{ N}$$ **Ans.**

$$\phi' \text{ (angle from } x \text{ axis)} = \tan^{-1}\left[\frac{88.38}{711.03}\right]$$

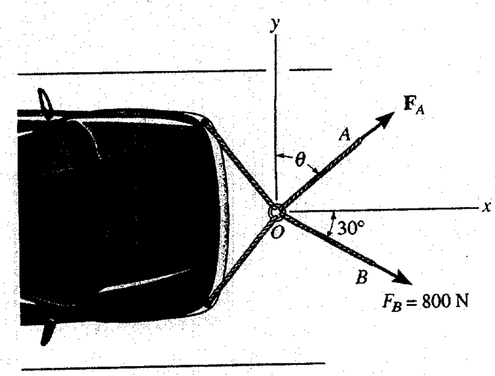
$$\phi' = 7.10°$$

$$\phi \text{ (angle from } x' \text{ axis)} = 30° + 7.10°$$

$$\phi = 37.1°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-56. Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = F_A \sin\theta + 800\cos 30^\circ = 1250$

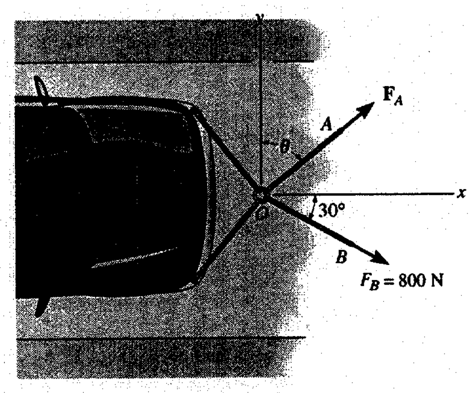
$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = F_A \cos\theta - 800\sin 30^\circ = 0$

$\theta = 54.3^\circ$ **Ans.**

$F_A = 686\text{ N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

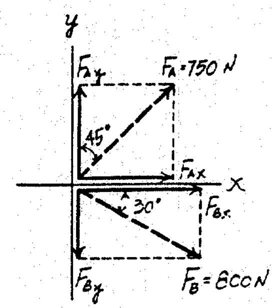
2-57. Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$.



Scalar Notation : Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 750\sin 45° + 800\cos 30° = 1223.15 \text{ N} \rightarrow$$

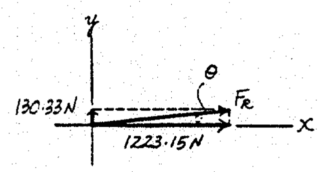
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 750\cos 45° - 800\sin 30° = 130.33 \text{ N} \uparrow$$



The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \qquad \textbf{Ans.}$$

The directional angle θ measured counterclockwise from positive x axis is

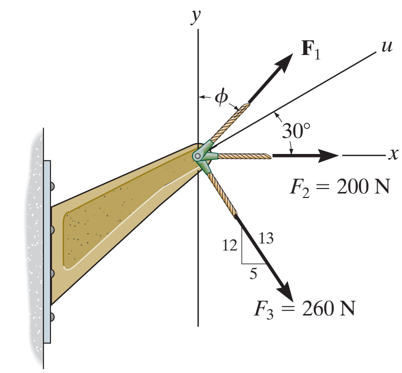


$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08° \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–58.

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \sin\phi$ $\qquad$ $(F_1)_y = F_1 \cos\phi$

$(F_2)_x = 200\ \text{N}$ $\qquad$ $(F_2)_y = 0$

$(F_3)_x = 260\left(\dfrac{5}{13}\right) = 100\ \text{N}$ $\qquad$ $(F_3)_y = 260\left(\dfrac{12}{13}\right) = 240\ \text{N}$

$(F_R)_x = 450\cos 30° = 389.71\ \text{N}$ $\qquad$ $(F_R)_y = 450\sin 30° = 225\ \text{N}$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x;\quad 389.71 = F_1\sin\phi + 200 + 100$$

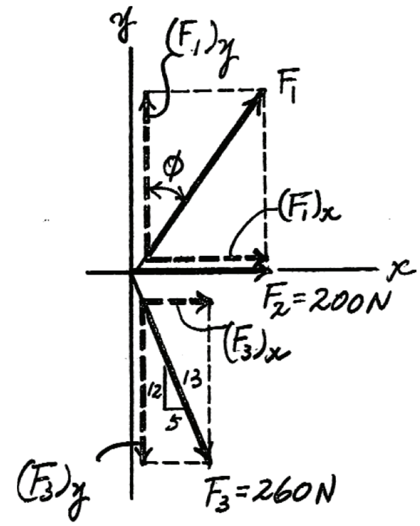
$$F_1\sin\phi = 89.71 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y;\quad 225 = F_1\cos\phi - 240$$

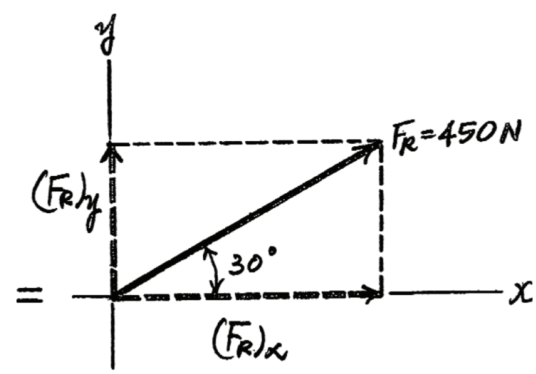
$$F_1\cos\phi = 465 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$\phi = 10.9° \qquad F_1 = 474\ \text{N} \qquad \textbf{Ans.}$$



(a)

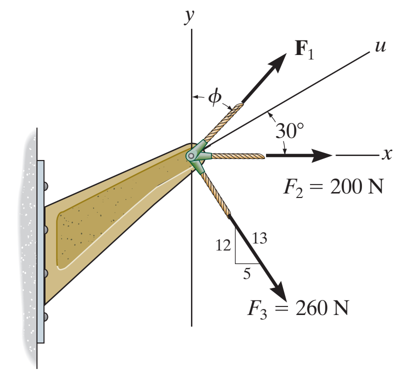


(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–59.

If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.



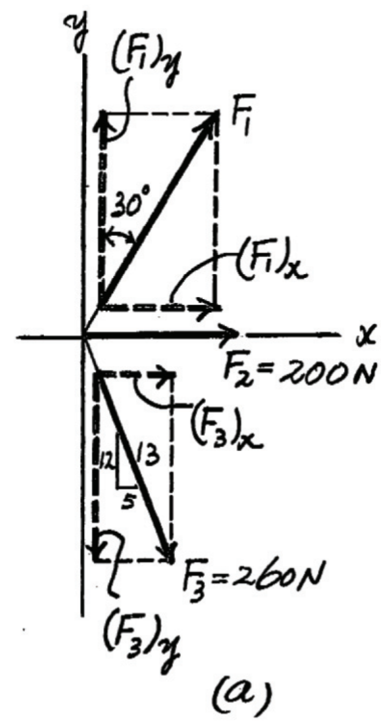
SOLUTION

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = F_1 \sin 30° = 0.5F_1 \qquad (F_1)_y = F_1 \cos 30° = 0.8660F_1$$

$$(F_2)_x = 200 \text{ N} \qquad (F_2)_y = 0$$

$$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ N} \qquad (F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ N}$$



Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 0.5F_1 + 200 + 100 = 0.5F_1 + 300$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 0.8660F_1 - 240$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2} \\ &= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2} \\ &= \sqrt{F_1^2 - 115.69F_1 + 147\,600} \end{aligned} \qquad \textbf{(1)}$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147\,600 \qquad \textbf{(2)}$$

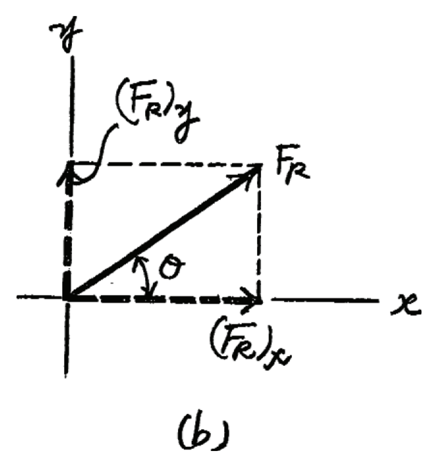
The first derivative of Eq. (2) is

$$2F_R \frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad \textbf{(3)}$$

For $\mathbf{F}_R$ to be minimum, $\dfrac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R \frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

$$F_1 = 57.846 \text{ N} = 57.8 \text{ N} \qquad \textbf{Ans.}$$



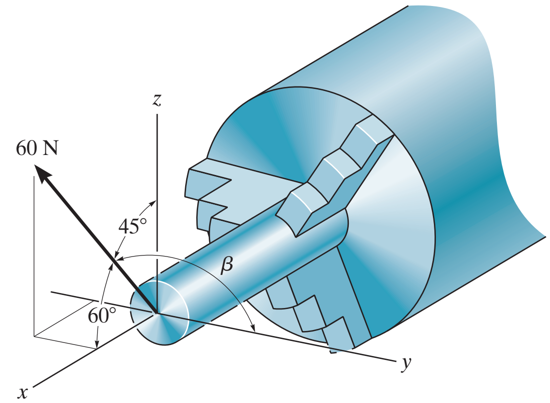
from Eq. (1),

$$F_R = \sqrt{(57.846)^2 - 115.69(57.846) + 147\,600} = 380 \text{ N} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–60.

The stock mounted on the lathe is subjected to a force of 60 N. Determine the coordinate direction angle β and express the force as a Cartesian vector.



SOLUTION

$$1 = \sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}$$

$$1 = \cos^2 60° + \cos^2\beta + \cos^2 45°$$

$$\cos\beta = \pm 0.5$$

$$\beta = 60°, 120°$$

Use

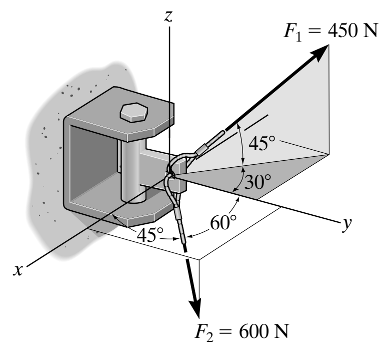
$$\beta = 120° \qquad \textbf{Ans.}$$

$$F = 60\text{ N}(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k})$$

$$= \{30\mathbf{i} - 30\mathbf{j} + 42.4\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

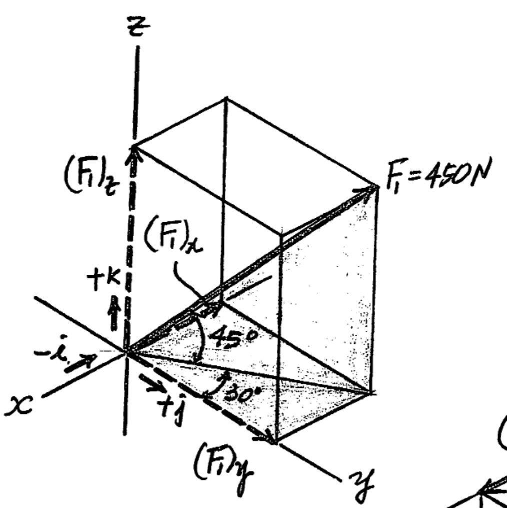
© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–61. Determine the coordinate angle γ for $\mathbf{F}_2$ and then express each force acting on the bracket as a Cartesian vector.

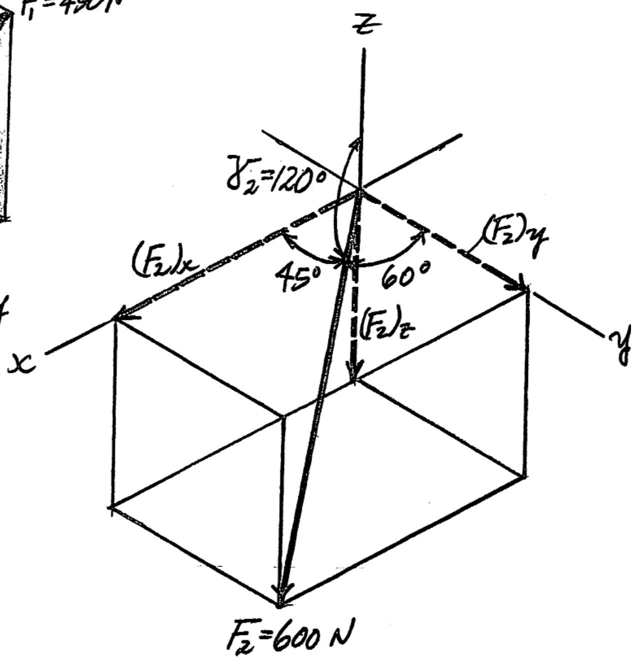


Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1-\cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\gamma_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$
$$= \{-159\mathbf{i} + 276\mathbf{j} + 318\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$



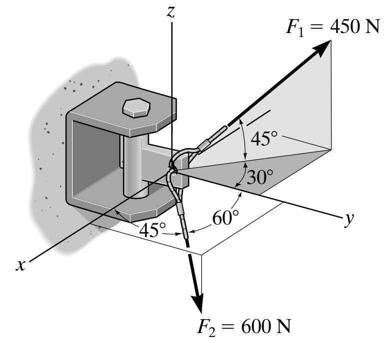
(a)



(b)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–62. Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



Rectangular Components: Since $\cos^2\alpha_2 + \cos^2\beta_2 + \cos^2\gamma_2 = 1$, then $\cos\gamma_{2z} = \pm\sqrt{1-\cos^2 45° - \cos^2 60°} = \pm 0.5$. However, it is required that $\alpha_2 > 90°$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120°$. By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$, can be expressed in Cartesian vector form, as

$$\mathbf{F}_1 = 450\cos 45°\sin 30°(-\mathbf{i}) + 450\cos 45°\cos 30°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$
$$= \{-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}\}\text{N} \qquad \textbf{Ans.}$$
$$\mathbf{F}_2 = 600\cos 45°\mathbf{i} + 600\cos 60°\mathbf{j} + 600\cos 120°\mathbf{k}$$
$$= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force: By adding $\mathbf{F}_1$ and $\mathbf{F}_2$ vectorally, we obtain $\mathbf{F}_R$.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}) + (424.26\mathbf{i} + 300\mathbf{j} - 300\mathbf{k})$$
$$= \{265.16\mathbf{i} + 575.57\mathbf{j} + 18.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{265.16^2 + 575.57^2 + 18.20^2} = 633.97\text{ N} = 634\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{265.16}{633.97}\right) = 65.3° \qquad \textbf{Ans.}$$

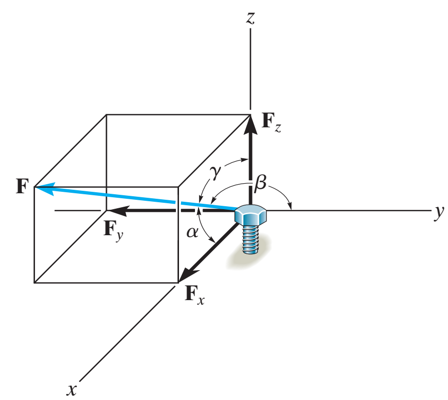
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{575.57}{633.97}\right) = 24.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{18.20}{633.97}\right) = 88.4° \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–63.

The bolt is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60°$ and $\gamma = 45°$, determine the magnitudes of its components.



SOLUTION

$$\cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$\beta = 120°$$

$F_x = |80 \cos 60°| = 40 \text{ N}$ **Ans.**

$F_y = |80 \cos 120°| = 40 \text{ N}$ **Ans.**

$F_z = |80 \cos 45°| = 56.6 \text{ N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-64.

Determine the magnitude and coordinate direction angles of the force **F** acting on the stake.

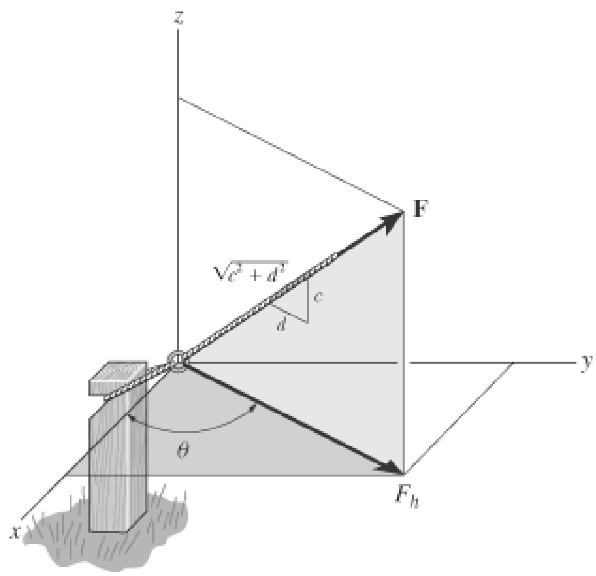
Given:

$F_h = 40\ \text{N}$

$\theta = 70\ \text{deg}$

$c = 3$

$d = 4$



Solution:

$$F = F_h\left(\frac{\sqrt{c^2 + d^2}}{d}\right)$$

$F = 50\ \text{N}$ **Ans.**

$$F_x = F_h\cos(\theta) \qquad F_y = F_h\sin(\theta) \qquad F_z = \left(\frac{c}{\sqrt{c^2 + d^2}}\right)F$$

$F_x = 13.7\ \text{N}$ $\quad F_y = 37.6\ \text{N}$ $\quad F_z = 30\ \text{N}$

$$\alpha = \text{acos}\left(\frac{F_x}{F}\right) \qquad \beta = \text{acos}\left(\frac{F_y}{F}\right) \qquad \gamma = \text{acos}\left(\frac{F_z}{F}\right)$$

$\alpha = 74.1\ \text{deg}$ $\quad \beta = 41.3\ \text{deg}$ $\quad \gamma = 53.1\ \text{deg}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-65.

Determine the magnitude and coordinate direction angles of the force **F** acting on the stake.

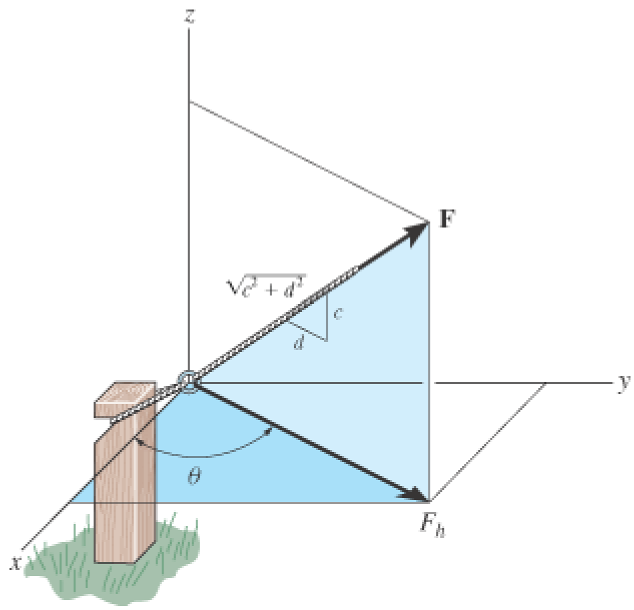
Given:

$F_h := 40N$

$\theta := 50deg$

$c := 3$

$d := 4$



Solution:

$$F := F_h \cdot \frac{\sqrt{c^2 + d^2}}{d}$$

$F = 50\,N$ **Ans.**

$F_x := F_h \cdot \cos(\theta)$ $F_y := F_h \cdot \sin(\theta)$ $F_z := \frac{c}{\sqrt{c^2 + d^2}} \cdot F$

$F_x = 25.7\,N$ $F_y = 30.6\,N$ $F_z = 30\,N$

$\alpha := acos\left(\frac{F_x}{F}\right)$ $\beta := acos\left(\frac{F_y}{F}\right)$ $\gamma := acos\left(\frac{F_z}{F}\right)$

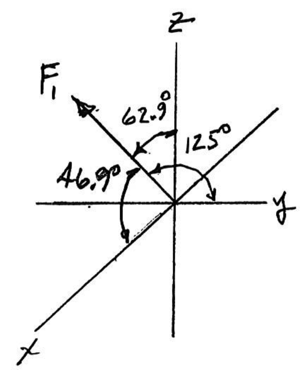
$\alpha = 59.1\,deg$ **Ans.** $\beta = 52.2\,deg$ **Ans.** $\gamma = 53.1\,deg$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–66.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_1 = \{60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}\}$ N. Sketch each force on an *x, y, z* reference.

SOLUTION



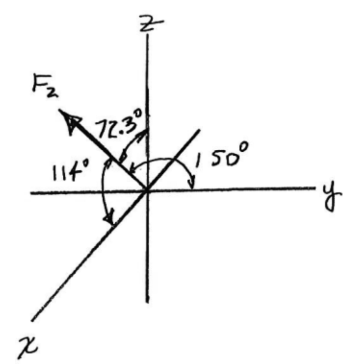
$\mathbf{F}_1 = 60\,\mathbf{i} - 50\,\mathbf{j} + 40\,\mathbf{k}$

$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.7496 = 87.7\text{ N}$ **Ans.**

$\alpha_1 = \cos^{-1}\left(\dfrac{60}{87.7496}\right) = 46.9°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\dfrac{-50}{87.7496}\right) = 125°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\dfrac{40}{87.7496}\right) = 62.9°$ **Ans.**



$\mathbf{F}_2 = -40\,\mathbf{i} - 85\,\mathbf{j} + 30\,\mathbf{k}$

$F_2 = \sqrt{(-40)^2 + (-85)^2 + (30)^2} = 98.615 = 98.6\text{ N}$ **Ans.**

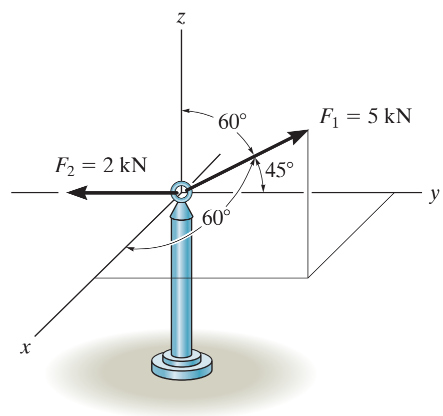
$\alpha_2 = \cos^{-1}\left(\dfrac{-40}{98.615}\right) = 114°$ **Ans.**

$\beta_2 = \cos^{-1}\left(\dfrac{-85}{98.615}\right) = 150°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\dfrac{30}{98.615}\right) = 72.3°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–67. Express each force in Cartesian vector form.



$(F_1)_x = 0$ $\qquad$ $(F_2)_x = 5\cos 60^\circ = 2.5$ kN

$(F_1)_y = 2$ kN $\qquad$ $(F_2)_y = 5\cos 45^\circ = 3.54$ kN

$(F_1)_z = 0$ $\qquad$ $(F_2)_z = 5\cos 60^\circ = 2.5$ kN

Thus, $\mathbf{F}_1 = 0(\mathbf{i}) + 2(-\mathbf{j}) + 0(\mathbf{k}) = \{-2\mathbf{j}\}$ N **Ans.**

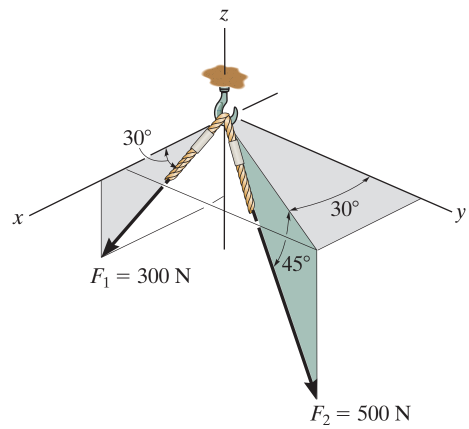
$\mathbf{F}_2 = 2.5(\mathbf{i}) + 3.54(\mathbf{j}) + 2.5(\mathbf{k})$

$= \{2.5\mathbf{i} + 3.54\mathbf{j} + 2.5\mathbf{k}\}$ N **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–68.

Express each force as a Cartesian vector.



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the *x*, *y*, and *z* components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$(F_1)_x = 300 \cos 30° = 259.8 \text{ N}$ $\quad (F_2)_x = 500 \cos 45° \sin 30° = 176.78 \text{ N}$

$(F_1)_y = 0$ $\quad (F_2)_y = 500 \cos 45° \cos 30° = 306.19 \text{ N}$

$(F_1)_t = 300 \sin 30° = 150 \text{ N}$ $\quad (F_2)_z = 500 \sin 45° = 353.55 \text{ N}$

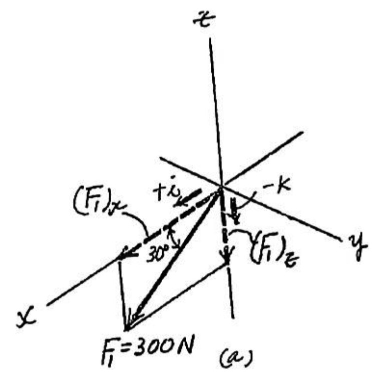
Thus, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written in Cartesian vector form as

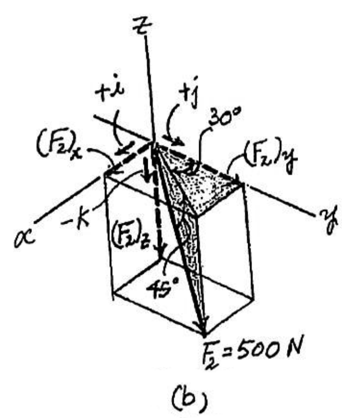
$$\mathbf{F}_1 = 259.81(+\mathbf{i}) + 0\mathbf{j} + 150(-\mathbf{k})$$

$$= \{260\mathbf{i} - 150\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 176.78(+\mathbf{i}) + 306.19(+\mathbf{j}) + 353.55(-\mathbf{k})$$

$$= 2\{177\mathbf{i} + 306\mathbf{j} - 354\mathbf{k}\} \text{ N} \qquad \textbf{Ans.}$$

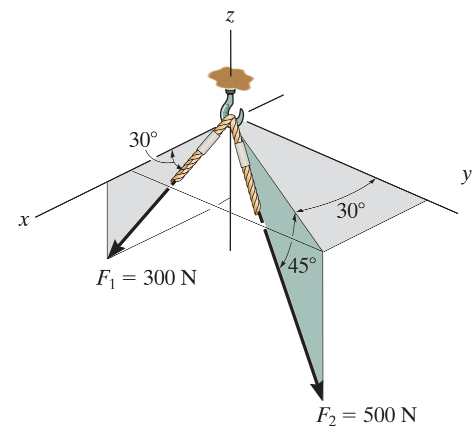




© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–69.

Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. *a* and *b*, respectively, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expessed in Cartesian vector form as

$$\mathbf{F}_1 = 300\cos 30°(+\mathbf{i}) + 0\mathbf{j} + 300\sin 30°(-\mathbf{k})$$
$$= \{259.81\mathbf{i} - 150\mathbf{k}\}\text{ N}$$

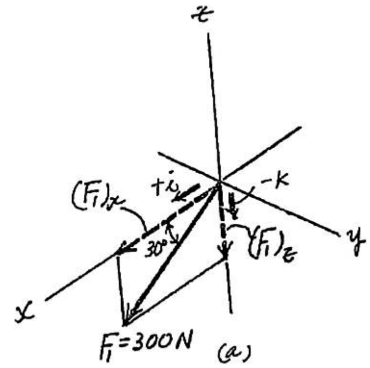
$$\mathbf{F}_2 = 500\cos 45°\sin 30°(+\mathbf{i}) + 500\cos 45°\cos 30°(+\mathbf{j}) + 500\sin 45°(-\mathbf{k})$$
$$= \{176.78\mathbf{i} - 306.19\mathbf{j} - 353.55\mathbf{k}\}\text{ N}$$

Resultant Force: The resultant force acting on the hook can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= (259.81\mathbf{i} - 150\mathbf{k}) + (176.78\mathbf{i} + 306.19\mathbf{j} - 353.55\mathbf{k})$$
$$= \{436.58\mathbf{i}) + 306.19\mathbf{j} - 503.55\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2(F_R)_z{}^2}$$
$$= \sqrt{(436.58)^2 + (306.19)^2 + (-503.55)^2} = 733.43\text{ N} = 733\text{ N} \qquad \textbf{Ans.}$$

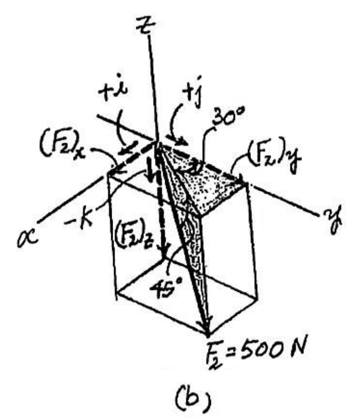


The coordinate direction angles of $\mathbf{F}_R$ are

$$\theta_x = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{436.58}{733.43}\right) = 53.5° \qquad \textbf{Ans.}$$

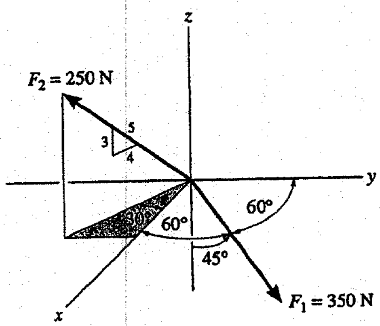
$$\theta_y = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{306.19}{733.43}\right) = 65.3° \qquad \textbf{Ans.}$$

$$\theta_z = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-503.55}{733.43}\right) = 133° \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-70. Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = 350\cos 60°\mathbf{i} + 350\cos 60°\mathbf{j} - 350\cos 45°\mathbf{k} + 250(\frac{4}{5})\cos 30°\mathbf{i} - 250(\frac{4}{5})\sin 30°\mathbf{j} + 250(\frac{3}{5})\mathbf{k}$$

$$\mathbf{F}_R = \{348.21\mathbf{i} + 75.0\mathbf{j} - 97.487\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(348.21)^2 + (75.0)^2 - (97.487)^2}$$

$$= 369.29 = 369\text{ N} \qquad \textbf{Ans.}$$

$$\alpha = \cos^{-1}(\frac{348.21}{369.29}) = 19.5° \qquad \textbf{Ans.}$$

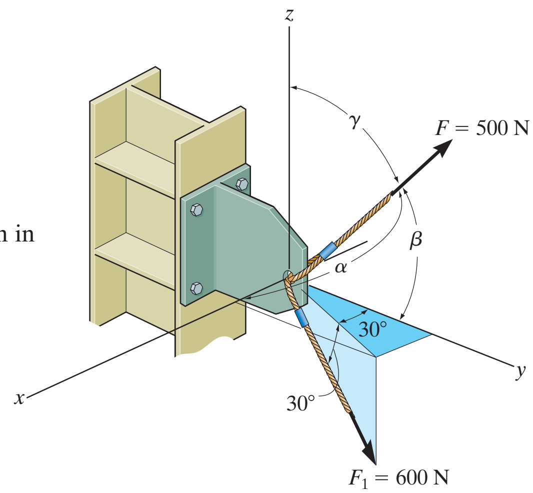
$$\beta = \cos^{-1}(\frac{75.0}{369.29}) = 78.3° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(\frac{-97.487}{369.29}) = 105° \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–71.

If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of **F** so that $\beta < 90°$.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}$ into their x, y, and z components, as shown in Figs. a and b, respectively, $\mathbf{F}_1$ and $\mathbf{F}$ can be expressed in Cartesian vector form as

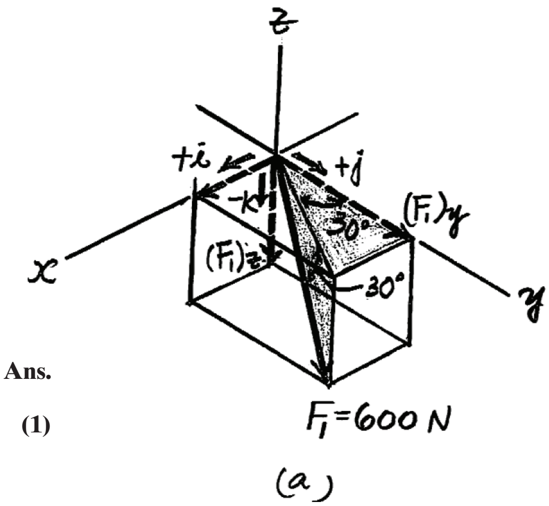
$$\mathbf{F}_1 = 600 \cos 30° \sin 30°(+\mathbf{i}) + 600 \cos 30° \cos 30°(+\mathbf{j}) + 600 \sin 30°(-\mathbf{k})$$

$$= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\} \text{ N}$$

$$\mathbf{F} = 500 \cos \alpha\mathbf{i} + 500 \cos \beta\mathbf{j} + 500 \cos \gamma\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed towards the positive y axis, then

$$\mathbf{F}_R = F_R\,\mathbf{j}$$



Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$F_R\,\mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500 \cos \alpha\mathbf{i} + 500 \cos \beta\mathbf{j} + 500 \cos \gamma\mathbf{k})$$

$$F_R\,\mathbf{j} = (259.81 + 500 \cos \alpha)\mathbf{i} + (450 + 500 \cos \beta)\mathbf{j} + (500 \cos \gamma - 300)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = 259.81 + 500 \cos \alpha$$

$$\alpha = 121.31° = 121° \qquad \textbf{Ans.}$$

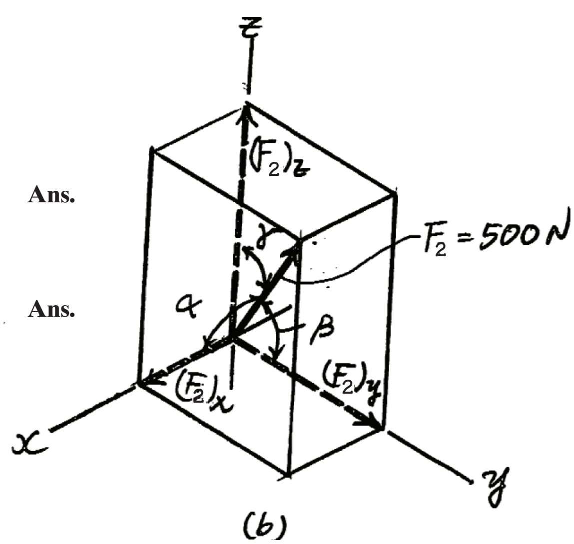
$$F_R = 450 + 500 \cos \beta \qquad \textbf{(1)}$$

$$0 = 500 \cos \gamma - 300$$

$$\gamma = 53.13° = 53.1° \qquad \textbf{Ans.}$$

However, since $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, $\alpha = 121.31°$, and $\gamma = 53.13°$,

$$\cos \beta = \pm\sqrt{1 - \cos^2 121.31° - \cos^2 53.13°} = \pm 0.6083$$



If we substitute $\cos \beta = 0.6083$ into Eq. (1),

$$F_R = 450 + 500(0.6083) = 754 \text{ N} \qquad \textbf{Ans.}$$

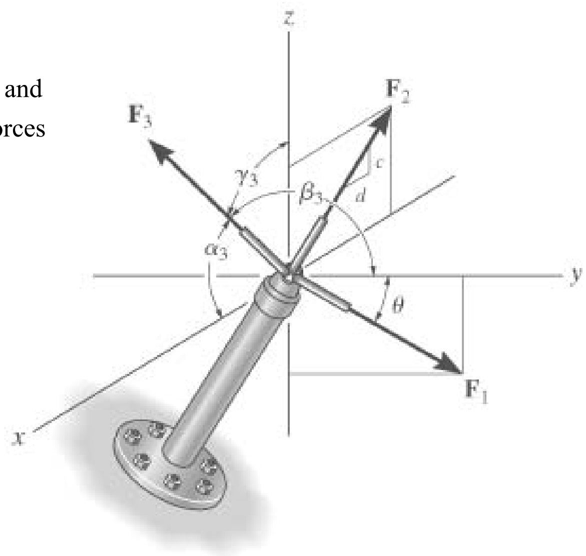
and

$$\beta = \cos^{-1}(0.6083) = 52.5° \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-72.

Specify the magnitude F_3 and directions α_3, β_3, and γ_3 of $\mathbf{F_3}$ so that the resultant force of the three forces is $\mathbf{F_R}$.



Units Used:

$\mathrm{kN} = 10^3\ \mathrm{N}$

Given:

$F_1 = 12\ \mathrm{kN}$ $\quad c = 5$

$F_2 = 10\ \mathrm{kN}$ $\quad d = 12$

$\theta = 30\ \mathrm{deg}$

$$\mathbf{F_R} = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} \mathrm{kN}$$

Solution:

Initial Guesses: $F_{3x} = 1\ \mathrm{kN}$ $\quad F_{3y} = 1\ \mathrm{kN}$ $\quad F_{3z} = 1\ \mathrm{kN}$

Given $$\mathbf{F_R} = \begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} + F_1 \begin{pmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{pmatrix} + \frac{F_2}{\sqrt{c^2 + d^2}} \begin{pmatrix} -d \\ 0 \\ c \end{pmatrix}$$

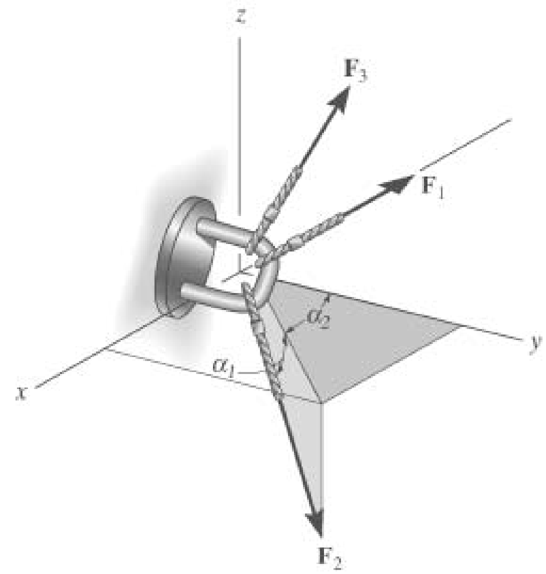
$$\begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} = \mathrm{Find}(F_{3x}, F_{3y}, F_{3z}) \qquad \mathbf{F_3} = \begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} \qquad \mathbf{F_3} = \begin{pmatrix} 9.2 \\ -1.4 \\ 2.2 \end{pmatrix} \mathrm{kN} \qquad |\mathbf{F_3}| = 9.6\ \mathrm{kN}$$ **Ans.**

$$\begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{pmatrix} = \mathrm{acos}\left(\frac{\mathbf{F_3}}{|\mathbf{F_3}|}\right) \qquad \begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 15.5 \\ 98.4 \\ 77.0 \end{pmatrix} \mathrm{deg}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-73.

Determine the magnitude and coordinate direction angles of $\mathbf{F_3}$ so that the resultant of the three forces acts along the positive *y* axis and has magnitude *F*.



Given:

$F = 600\ \text{N}$

$F_1 = 180\ \text{N}$

$F_2 = 300\ \text{N}$

$\alpha_1 = 30\ \text{deg}$

$\alpha_2 = 40\ \text{deg}$

Solution:

Initial guesses:

$\alpha = 40\ \text{deg} \qquad \gamma = 50\ \text{deg}$

$\beta = 50\ \text{deg} \qquad F_3 = 45\ \text{N}$

Given

$$F_{Rx} = \Sigma F_x; \qquad 0 = -F_1 + F_2\cos(\alpha_1)\sin(\alpha_2) + F_3\cos(\alpha)$$

$$F_{Ry} = \Sigma F_y; \qquad F = F_2\cos(\alpha_1)\cos(\alpha_2) + F_3\cos(\beta)$$

$$F_{Rz} = \Sigma F_z; \qquad 0 = -F_2\sin(\alpha_1) + F_3\cos(\gamma)$$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma) \qquad F_3 = 428\,\text{N} \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 88.3 \\ 20.6 \\ 69.5 \end{pmatrix}\text{deg}$$

Ans.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

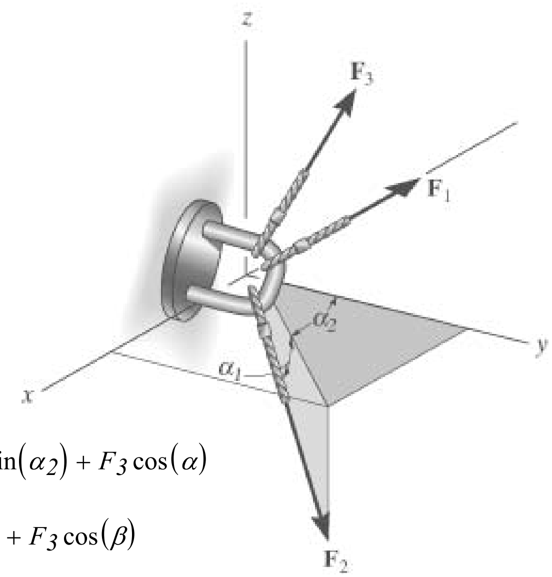
2-74.

Determine the magnitude and coordinate direction angles of $\mathbf{F_3}$ so that the resultant of the three forces is zero.

Given:

$F_1 = 180\ \text{N} \quad \alpha_1 = 30\ \text{deg}$

$F_2 = 300\ \text{N} \quad \alpha_2 = 40\ \text{deg}$



Solution:

Initial guesses:

$\alpha = 40\ \text{deg} \quad \gamma = 50\ \text{deg}$

$\beta = 50\ \text{deg} \quad F_3 = 45\ \text{N}$

Given

$F_{Rx} = \Sigma F_x;\qquad 0 = -F_1 + F_2\cos(\alpha_1)\sin(\alpha_2) + F_3\cos(\alpha)$

$F_{Ry} = \Sigma F_y;\qquad 0 = F_2\cos(\alpha_1)\cos(\alpha_2) + F_3\cos(\beta)$

$F_{Rz} = \Sigma F_z;\qquad 0 = -F_2\sin(\alpha_1) + F_3\cos(\gamma)$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

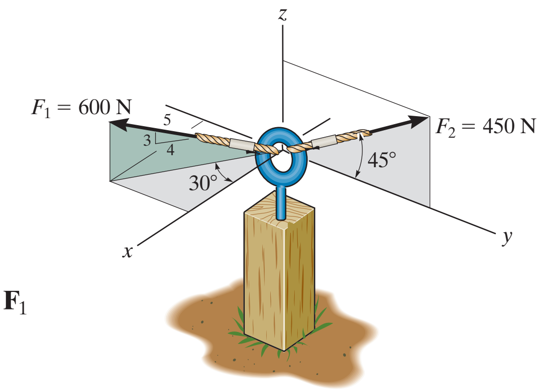
$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma) \qquad F_3 = 250\,\text{N} \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 87.0 \\ 142.9 \\ 53.1 \end{pmatrix} \text{deg}$$

Ans.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–75.

Determine the coordinate direction angles of force $\mathbf{F}_1$.



SOLUTION

Rectangular Components: By referring to Figs. a, the x, y, and z components of $\mathbf{F}_1$ can be written as

$$(F_1)_x = 600\left(\frac{4}{5}\right)\cos 30°\text{ N} \qquad (F_1)_y = 600\left(\frac{4}{5}\right)\sin 30°\text{ N} \qquad (F_1)_z = 600\left(\frac{3}{5}\right)\text{ N}$$

Thus, $\mathbf{F}_1$ expressed in Cartesian vector form can be written as

$$\mathbf{F}_1 = 600\left\{\frac{4}{5}\cos 30°(+\mathbf{i}) + \frac{4}{5}\sin 30°(-\mathbf{j}) + \frac{3}{5}(+\mathbf{k})\right\}\text{ N}$$
$$= 600[0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}]\text{ N}$$

Therefore, the unit vector for $\mathbf{F}_1$ is given by

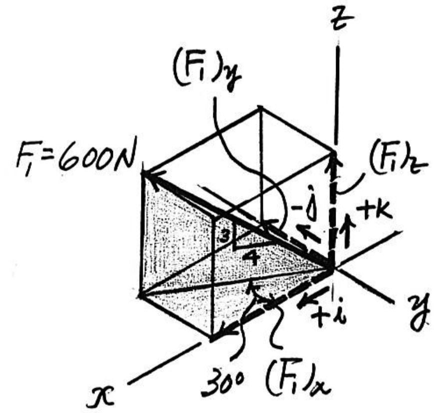
$$\mathbf{u}_{F_1} = \frac{\mathbf{F}_1}{F_1} = \frac{600(0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k})}{600} = 0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}$$

The coordinate direction angles of $\mathbf{F}_1$ are

$$\alpha = \cos^{-1}(u_{F_1})_x = \cos^{-1}(0.6928) = 46.1°$$ **Ans.**

$$\beta = \cos^{-1}(u_{F_1})_y = \cos^{-1}(-0.4) = 114°$$ **Ans.**

$$\gamma = \cos^{-1}(u_{F_1})_z = \cos^{-1}(0.6) = 53.1°$$ **Ans.**

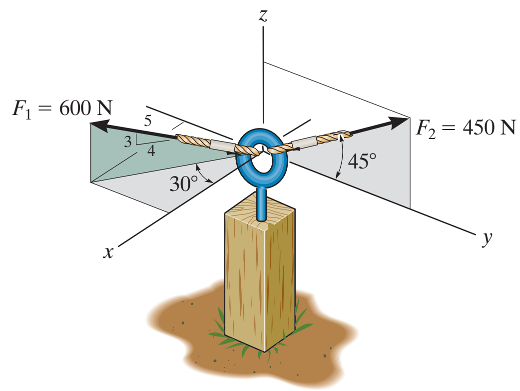


(a)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

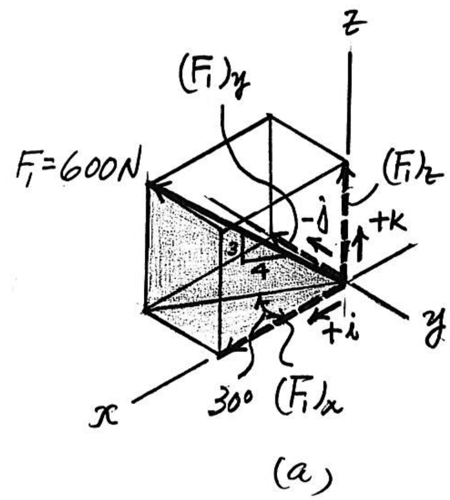
2–76.

Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, they are expressed in Cartesian vector form as



$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\sin 30°(-\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 450\cos 45°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$

$$= \{318.20\mathbf{j} + 318.20\mathbf{k}\}\text{ N}$$

Resultant Force: The resultant force acting on the eyebolt can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}) + (318.20\mathbf{j} + 318.20\mathbf{k})$$

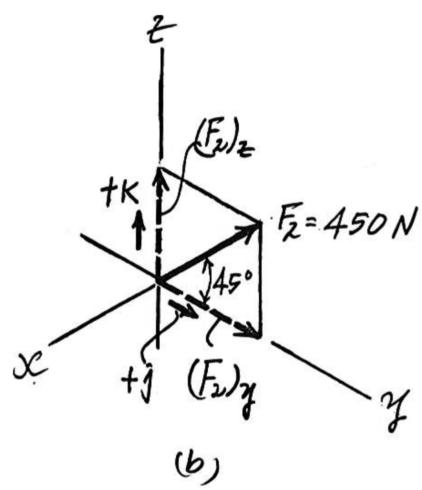
$$= \{415.69\mathbf{i} + 78.20\mathbf{j} + 678.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(415.69)^2 + (78.20)^2 + (678.20)^2} = 799.29\text{ N} = 799\text{ N}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are



$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{415.69}{799.29}\right) = 58.7°$$ **Ans.**

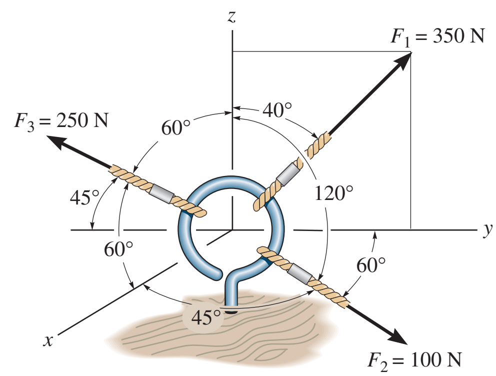
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{78.20}{799.29}\right) = 84.4°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{678.20}{799.29}\right) = 32.0°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–77.

The cables attached to the screw eye are subjected to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_1 = 350\{\sin 40°\mathbf{j} + \cos 40°\mathbf{k}\}\text{ N}$$
$$= \{224.98\mathbf{j} + 268.12\mathbf{k}\}\text{ N}$$
$$= \{225\mathbf{j} + 268\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 100\{\cos 45°\mathbf{i} + \cos 60°\mathbf{j} + \cos 120°\mathbf{k}\}\text{ N}$$
$$= \{70.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N}$$
$$= \{70.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_3 = 250\{\cos 60°\mathbf{i} + \cos 135°\mathbf{j} + \cos 60°\mathbf{k}\}\text{ N}$$
$$= \{125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}\}\text{ N}$$
$$= \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= \{(70.71 + 125.0)\mathbf{i} + (224.98 + 50.0 - 176.78)\mathbf{j} + (268.12 - 50.0 + 125.0)\mathbf{k}\}\text{ N}$$
$$= \{195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$
$$= \sqrt{195.71^2 + 98.20^2 + 343.12^2}$$
$$= 407.03\text{ N} = 407\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{195.71}{407.03} \qquad \alpha = 61.3° \qquad \textbf{Ans.}$$

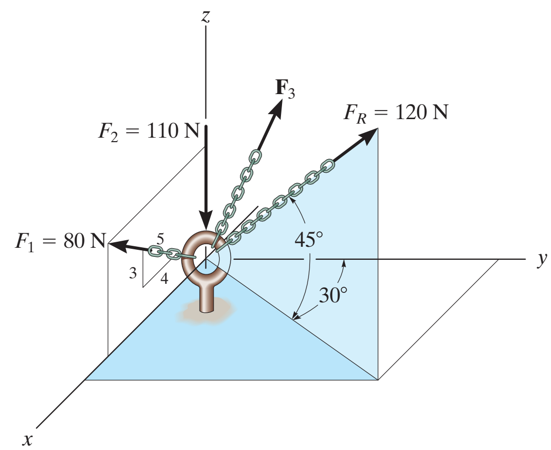
$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{98.20}{407.03} \qquad \beta = 76.0° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{343.12}{407.03} \qquad \gamma = 32.5° \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–78.

Three forces act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force $\mathbf{F}_3$.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_R = 120\{\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k}\}\text{ N}$$

$$= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\}\text{ N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\} = \left\{\left(64.0 + F_{3_x}\right)\mathbf{i} + F_{3_y}\mathbf{j} + \left(48.0 - 110 + F_{3_z}\right)\mathbf{k}\right\}$$

Equating **i**, **j** and **k** components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57\text{ N}$$

$$F_{3_y} = 73.48\text{ N}$$

$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85\text{ N}$$

The magnitude of force $\mathbf{F}_3$ is

$$F_3 = \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2}$$

$$= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2}$$

$$= 165.62\text{ N} = 166\text{ N}$$ **Ans.**

The coordinate direction angles for $\mathbf{F}_3$ are

$$\cos\alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5°$$ **Ans.**

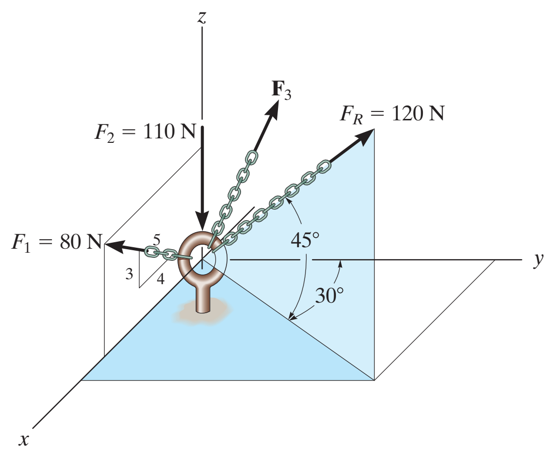
$$\cos\beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7°$$ **Ans.**

$$\cos\gamma = = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–79.

Determine the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_R$.



SOLUTION

Unit Vector of $\mathbf{F_1}$ ***and*** $\mathbf{F}_R$**:**

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{u}_R = \cos 45° \sin 30°\mathbf{i} + \cos 45° \cos 30°\mathbf{j} + \sin 45°\mathbf{k}$$

$$= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

Thus, the coordinate direction angles $\mathbf{F}_1$ and $\mathbf{F}_R$ are

$\cos \alpha_{F_1} = 0.8$ $\quad \alpha_{F_1} = 36.9°$ **Ans.**

$\cos \beta_{F_1} = 0$ $\quad \beta_{F_1} = 90.0°$ **Ans.**

$\cos \gamma_{F_1} = 0.6$ $\quad \gamma_{F_1} = 53.1°$ **Ans.**

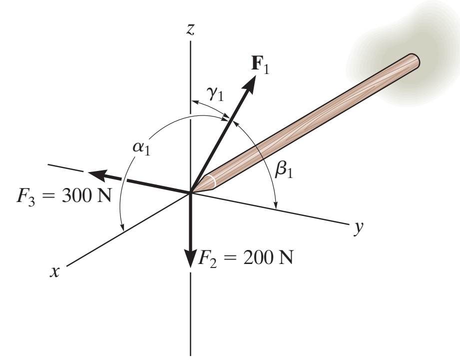
$\cos \alpha_R = 0.3536$ $\quad \alpha_R = 69.3°$ **Ans.**

$\cos \beta_R = 0.6124$ $\quad \beta_R = 52.2°$ **Ans.**

$\cos \gamma_R = 0.7071$ $\quad \gamma_R = 45.0°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–80. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\mathbf{F}_1$ so that the resultant force acting on the mast is $\mathbf{F}_R = \{350\mathbf{i}\}$ N.



$\mathbf{F}_1 = 500\cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500\cos\gamma_1\,\mathbf{k}$

$\mathbf{F}_R = \mathbf{F}_1 + (-300\mathbf{j}) + (-200\mathbf{k})$

$350\mathbf{i} = 500\cos\alpha_1\,\mathbf{i} + (500\cos\beta_1 - 300)\mathbf{j} + (500\cos\gamma_1 - 200)\mathbf{k}$

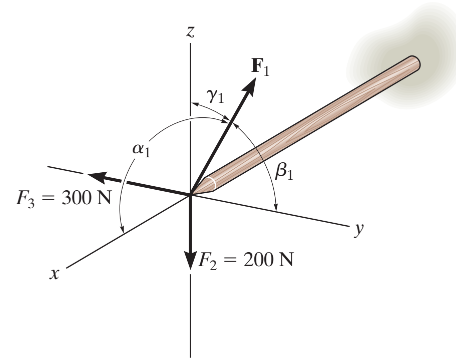
$350 = 500\cos\alpha_1;$ $\quad\alpha_1 = 45.6°$ **Ans.**

$0 = 500\cos\beta_1 - 300;$ $\quad\beta_1 = 53.1°$ **Ans.**

$0 = 500\cos\gamma_1 - 200;$ $\quad\gamma_1 = 66.4°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–81. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\mathbf{F}_1$ so that the resultant force acting on the mast is zero.



$\mathbf{F}_1 = \{500\cos\alpha_1\,\mathbf{i} + 500\cos\beta_1\,\mathbf{j} + 500\cos\gamma_1\,\mathbf{k}\}$ N

$\mathbf{F}_2 = \{-200\mathbf{k}\}$ N

$\mathbf{F}_3 = \{-300\mathbf{j}\}$ N

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$

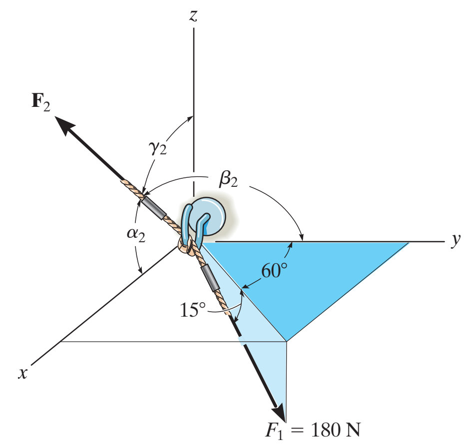
$500\cos\alpha_1 = 0;$ $\alpha_1 = 90°$ **Ans.**

$500\cos\beta_1 = 300;$ $\beta_1 = 53.1°$ **Ans.**

$500\cos\gamma_1 = 200;$ $\gamma_1 = 66.4°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–82. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces acts along the positive x axis and has a magnitude of 500 N.



$\mathbf{F}_1 = (180\cos 15°)\sin 60°\,\mathbf{i} + (180\cos 15°)\cos 60°\,\mathbf{j} - 180\sin 15°\,\mathbf{k}$

$= 150.57\,\mathbf{i} + 86.93\,\mathbf{j} - 46.59\,\mathbf{k}$

$\mathbf{F}_2 = F_2\cos\alpha_2\,\mathbf{i} + F_2\cos\beta_2\,\mathbf{j} + F_2\cos\gamma_2\,\mathbf{k}$

$\mathbf{F}_R = \{500\,\mathbf{i}\}$ N

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

i components :

$500 = 150.57 + F_2\cos\alpha_2$

$F_{2x} = F_2\cos\alpha_2 = 349.43$

j components :

$0 = 86.93 + F_2\cos\beta_2$

$F_{2y} = F_2\cos\beta_2 = -86.93$

k components :

$0 = -46.59 + F_2\cos\gamma_2$

$F_{2z} = F_2\cos\gamma_2 = 46.59$

Thus,

$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{(349.43)^2 + (-86.93)^2 + (46.59)^2}$

$F_2 = 363$ N **Ans.**

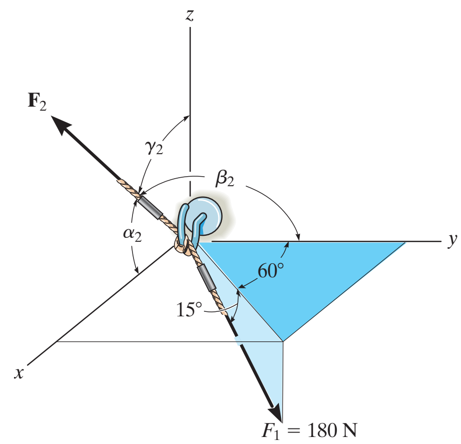
$\alpha_2 = 15.8°$ **Ans.**

$\beta_2 = 104°$ **Ans.**

$\gamma_2 = 82.6°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–83. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces is zero.



$$\mathbf{F}_1 = (180 \cos 15°) \sin 60° \, \mathbf{i} + (180 \cos 15°) \cos 60° \, \mathbf{j} - 180 \sin 15° \, \mathbf{k}$$

$$= 150.57 \, \mathbf{i} + 86.93 \, \mathbf{j} - 46.59 \, \mathbf{k}$$

$$\mathbf{F}_2 = F_2 \cos \alpha_2 \, \mathbf{i} + F_2 \cos \beta_2 \, \mathbf{j} + F_2 \cos \gamma_2 \, \mathbf{k}$$

$$\mathbf{F}_R = \mathbf{0}$$

i components :

$$0 = 150.57 + F_2 \cos \alpha_2$$

$$F_2 \cos \alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_2 \cos \beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_2 \cos \gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving,

$F_2 = 180 \text{ N}$ **Ans.**

$\alpha_2 = 147°$ **Ans.**

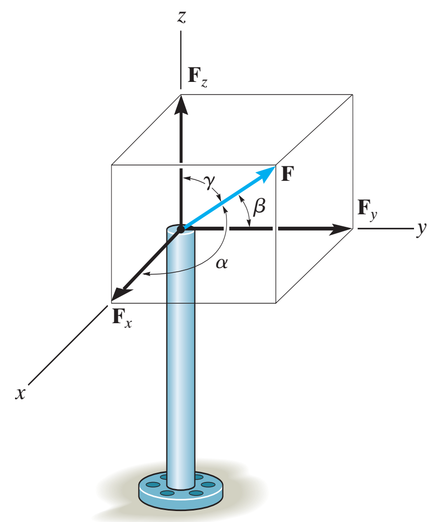
$\beta_2 = 119°$ **Ans.**

$\gamma_2 = 75.0°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–84.

The pole is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



SOLUTION

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 \alpha + \cos^2 30° + \cos^2 75° = 1$$

$$\alpha = 64.67°$$

$$F_x = 3 \cos 64.67° = 1.28 \text{ kN}$$ **Ans.**

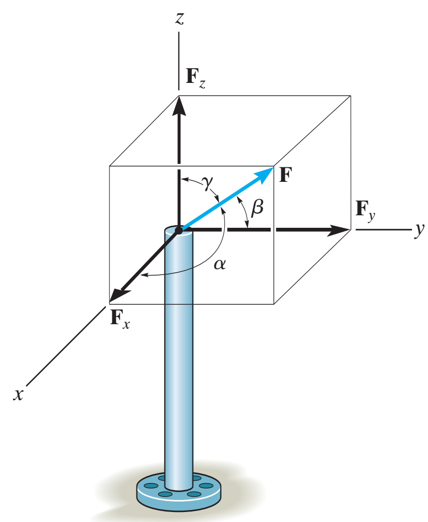
$$F_y = 3 \cos 30° = 2.60 \text{ kN}$$ **Ans.**

$$F_z = 3 \cos 75° = 0.776 \text{ kN}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–85.

The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



SOLUTION

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

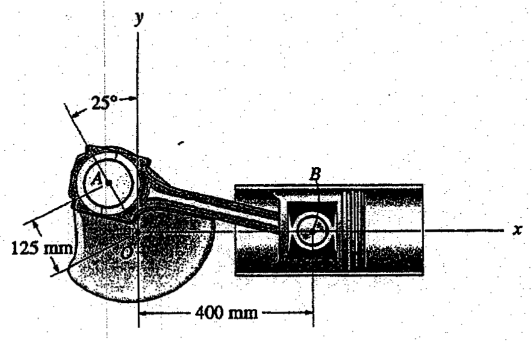
$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

$F = 2.02$ kN **Ans.**

$F_y = 2.02 \cos 75° = 0.523$ kN **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-86. Determine the length of the crankshaft AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



$$\mathbf{r}_{AB} = ((400 + 125\sin 25°)\mathbf{i} - 125\cos 25°\mathbf{j})$$

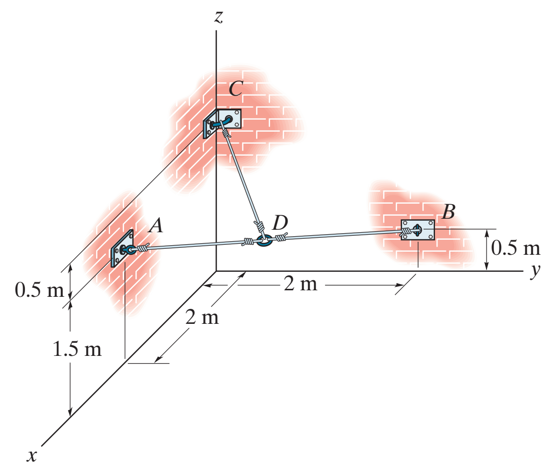
$$\mathbf{r}_{AB} = \{452.83\mathbf{i} - 113.3\mathbf{j}\}\text{ mm}$$

$$r_{AB} = \sqrt{(452.83)^2 + (-113.3)^2} = 467\text{ mm} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–87.

Determine the lengths of wires *AD*, *BD*, and *CD*. The ring at *D* is midway between *A* and *B*.



SOLUTION

$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right)\text{ m} = D(1, 1, 1)\text{ m}$$

$$\mathbf{r}_{AD} = (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k}$$

$$= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{r}_{BD} = (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k}$$

$$= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{r}_{CD} = (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k}$$

$$= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$$

$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50\text{ m}$ **Ans.**

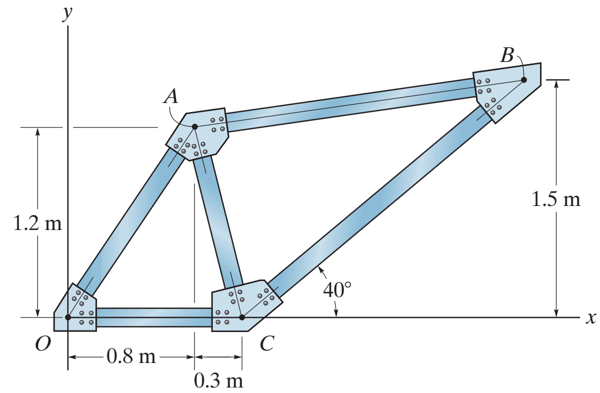
$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50\text{ m}$ **Ans.**

$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73\text{ m}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–88.

Determine the length of member AB of the truss by first establishing a Cartesian position vector from A to B and then determining its magnitude.



SOLUTION

$$\mathbf{r}_{AB} = (1.1) = \frac{1.5}{\tan 40^\circ} - 0.80)\mathbf{i} + (1.5 - 1.2)\mathbf{j}$$

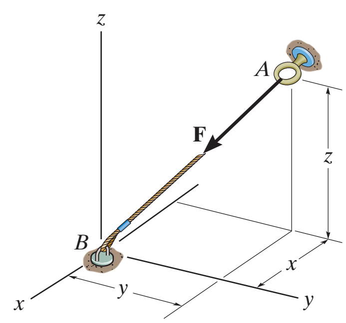
$$\mathbf{r}_{AB} = \{2.09\mathbf{i} + 0.3\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{AB} = \sqrt{(2.09)^2 + (0.3)^2} = 2.11\text{ m}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–89.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - (-x)]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$\frac{x}{9} = 0.5623$ $\qquad x = 5.06$ m **Ans.**

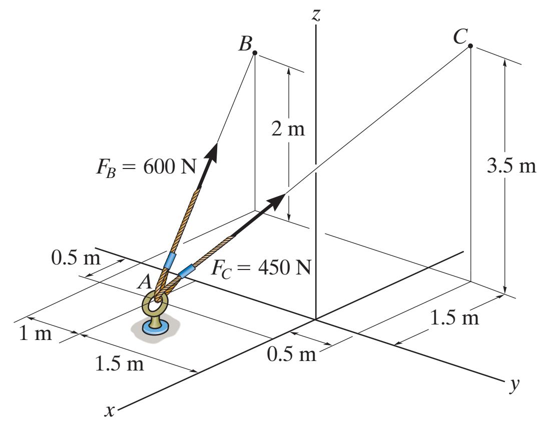
$\frac{-y}{9} = -0.4016$ $\qquad y = 3.61$ m **Ans.**

$\frac{-z}{9} = 0.7229$ $\qquad z = 6.51$ m **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–90.

Express $\mathbf{F}_B$ and $\mathbf{F}_C$ in Cartesian vector form.



SOLUTION

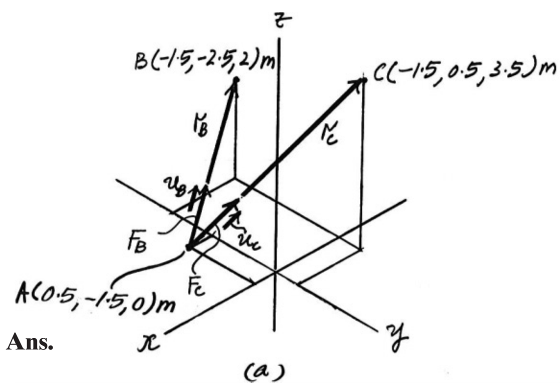
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$



Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

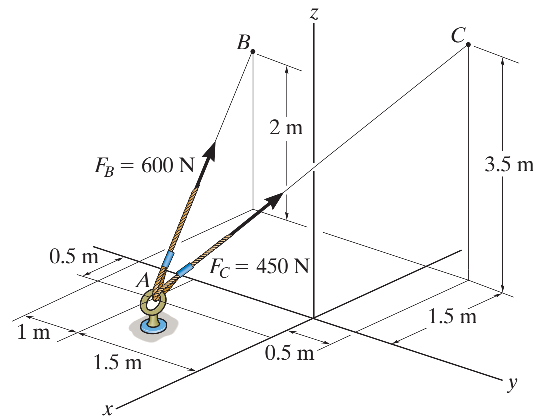
$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–91.

Determine the magnitude and coordinate direction angles of the resultant force acting at A.



SOLUTION

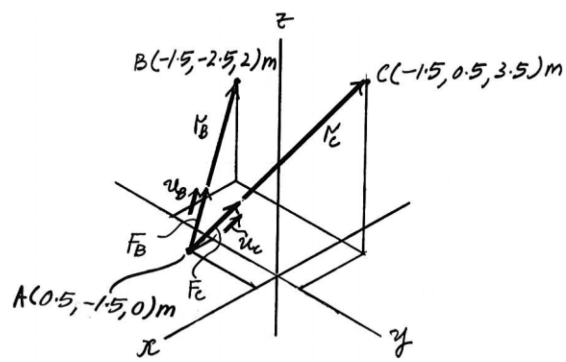
Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$



(a)

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}) + (-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k})$$

$$= \{-600\mathbf{i} + 750\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(-600)^2 + 0^2 + 750^2} = 960.47\text{ N} = 960\text{ N}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-600}{960.47}\right) = 129^\circ$$ **Ans.**

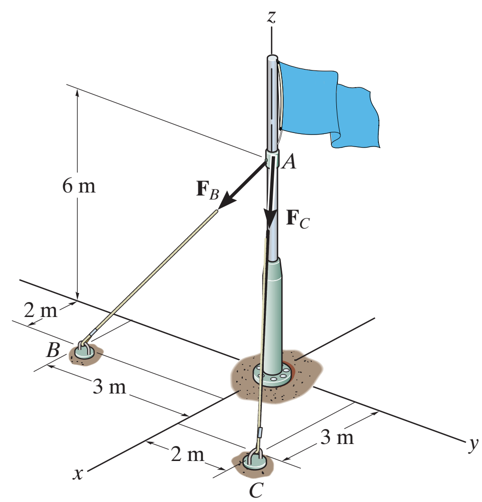
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{960.47}\right) = 90^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{960.47}\right) = 38.7^\circ$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–92.

If $F_B = 560\text{ N}$ and $F_C = 700\text{ N}$, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 560\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 700\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k})$$

$$= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

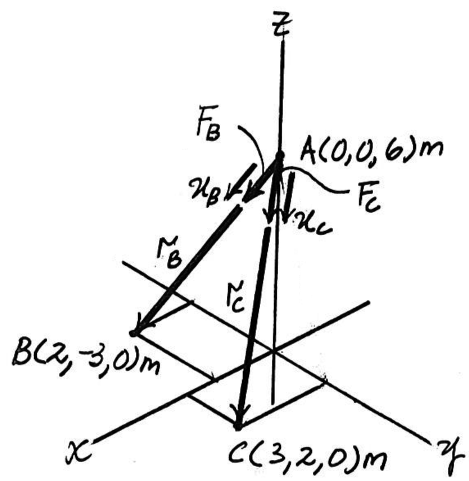
$$= \sqrt{(460)^2 + (-40)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9^\circ$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157^\circ$$ **Ans.**

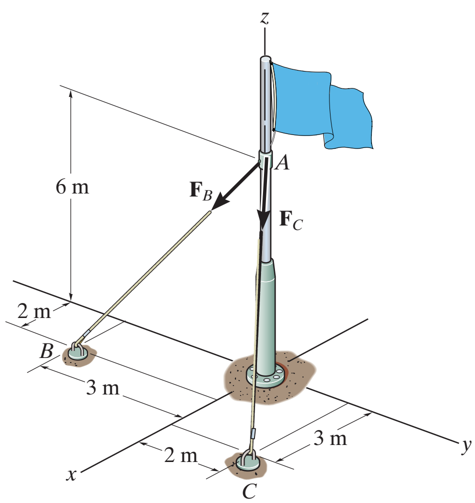


(a)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–93.

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$

$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

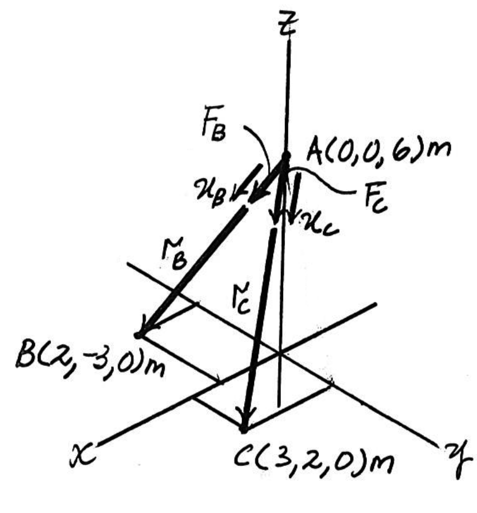
$$= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0°$$ **Ans.**

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157°$$ **Ans.**

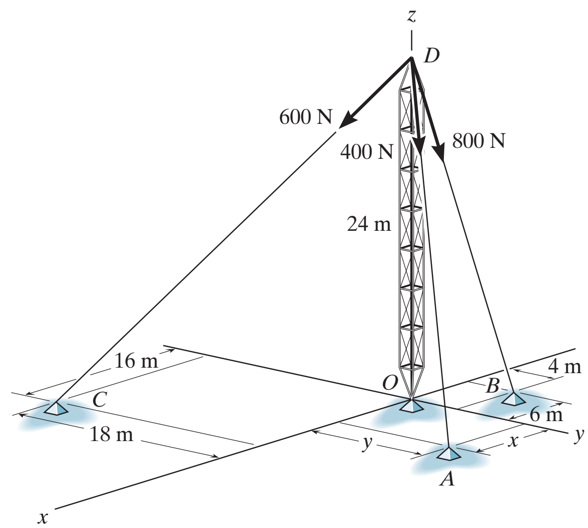


(a)

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–94.

The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 15$ m, $y = 20$ m.



SOLUTION

$$\mathbf{F}_{DA} = 400\left(\frac{15}{34.66}\mathbf{i} + \frac{20}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right)\text{N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right)\text{N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right)\text{N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{263.92\mathbf{i} + 40.86\mathbf{j} - 1466.71\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(263.92)^2 + (40.86)^2 + (-1466.71)^2}$$

$$= 1490.83\text{ N} = 1.49\text{ kN}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{263.92}{1490.83}\right) = 79.8^\circ$$ **Ans.**

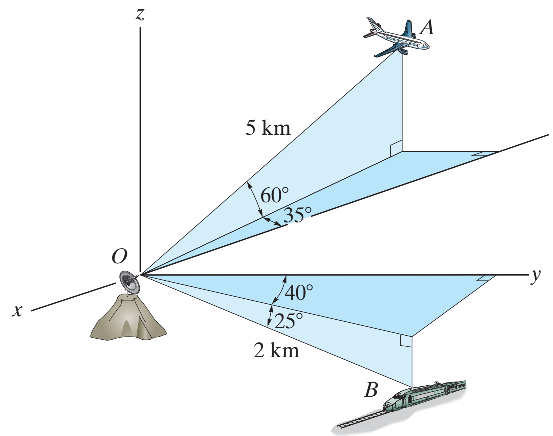
$$\beta = \cos^{-1}\left(\frac{40.86}{1490.83}\right) = 88.4^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1490.83}\right) = 169.7^\circ$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–95.

At a given instant, the position of a plane at A and a train at B are measured relative to a radar antenna at O. Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B, and then determine its magnitude.



SOLUTION

Position Vector: The coordinates of points A and B are

$$A(-5\cos 60^\circ \cos 35^\circ, -5\cos 60^\circ \sin 35^\circ, 5\sin 60^\circ)\text{ km}$$

$$= A(-2.048, -1.434, 4.330)\text{ km}$$

$$B(2\cos 25^\circ \sin 40^\circ, 2\cos 25^\circ \cos 40^\circ, -2\sin 25^\circ)\text{ km}$$

$$= B(1.165, 1.389, -0.845)\text{ km}$$

The position vector $\mathbf{r}_{AB}$ can be established from the coordinates of points A and B.

$$\mathbf{r}_{AB} = \{[1.165 - (-2.048)]\mathbf{i} + [1.389 - (-1.434)]\mathbf{j} + (-0.845 - 4.330)\mathbf{k}\}\text{ km}$$

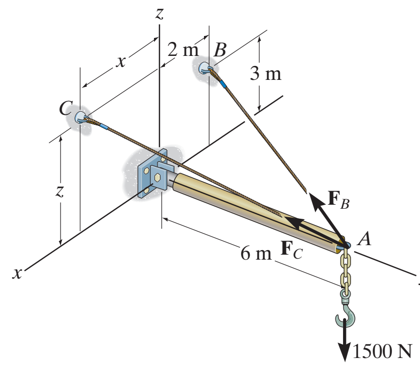
$$= \{3.213\mathbf{i} + 2.822\mathbf{j} - 5.175)\mathbf{k}\}\text{ km}$$

The distance between points A and B is

$$d = r_{AB} = \sqrt{3.213^2 + 2.822^2 + (-5.175)^2} = 6.71\text{ km}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–96. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the magnitudes of the resultant force and forces $\mathbf{F}_B$ and $\mathbf{F}_C$. Set $x = 3$ m and $z = 2$ m.



Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ must be determined first. From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i}+(0-6)\mathbf{j}+(2-0)\mathbf{k}}{\sqrt{(3-0)^2+(0-6)^2+(2-0)^2}} = \frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}$$

Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load $\mathbf{W}$ is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\ \text{N}$$

Resultant Force: The vector addition of $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{W}$ is equal to $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B\mathbf{i}-\frac{6}{7}F_B\mathbf{j}+\frac{3}{7}F_B\mathbf{k}\right)+\left(\frac{3}{7}F_C\mathbf{i}-\frac{6}{7}F_C\mathbf{j}+\frac{2}{7}F_C\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(-\frac{2}{7}F_B+\frac{3}{7}F_C\right)\mathbf{i}+\left(-\frac{6}{7}F_B-\frac{6}{7}F_C\right)\mathbf{j}+\left(\frac{3}{7}F_B+\frac{2}{7}F_C-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -\frac{2}{7}F_B+\frac{3}{7}F_C \qquad (1)$$

$$-F_R = -\frac{6}{7}F_B-\frac{6}{7}F_C \qquad (2)$$

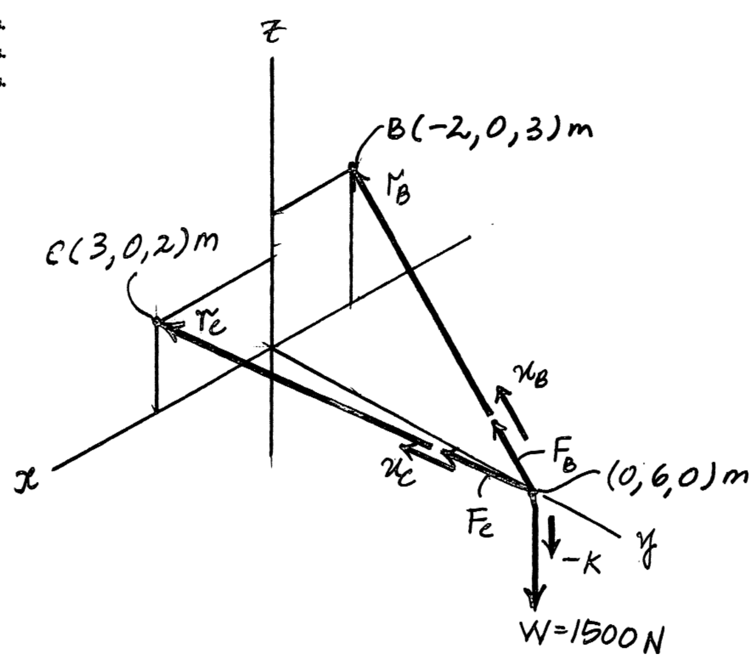
$$0 = \frac{3}{7}F_B+\frac{2}{7}F_C-1500 \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields

$F_C = 1615.38\ \text{N} = 1.62\ \text{kN}$ **Ans.**

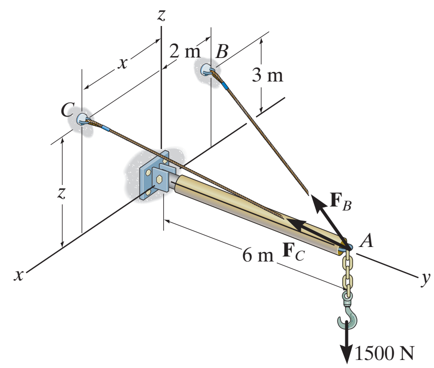
$F_B = 2423.08\ \text{N} = 2.42\ \text{kN}$ **Ans.**

$F_R = 3461.53\ \text{N} = 3.46\ \text{kN}$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–97. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O, determine the values of x and z for the coordinates of point C and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



Force Vectors: From Fig. *a*,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i}+(0-6)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(-2-0)^2+(0-6)^2+(3-0)^2}} = -\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}$$

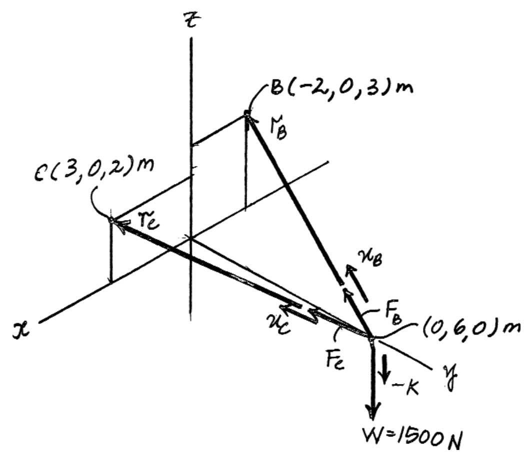
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i}+(0-6)\mathbf{j}+(z-0)\mathbf{k}}{\sqrt{(x-0)^2+(0-6)^2+(z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B\mathbf{u}_B = 1610\left(-\frac{2}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{3}{7}\mathbf{k}\right) = [-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k}]\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 2400\left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}$$



Since the resultant force $\mathbf{F}_R$ is directed along the negative y axis, and the load is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R\mathbf{j} \qquad \text{and} \qquad \mathbf{W} = [-1500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R\mathbf{j} = (-460\mathbf{i}-1380\mathbf{j}+690\mathbf{k})+\left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i}-\frac{14\,400}{\sqrt{x^2+z^2+36}}\mathbf{j}+\frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}\right)+(-1500\mathbf{k})$$

$$-F_R\mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}}-460\right)\mathbf{i}-\left(\frac{14\,400}{\sqrt{x^2+z^2+36}}+1380\right)\mathbf{j}+\left(690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}}-460 \qquad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \qquad (1)$$

$$-F_R = -\left(\frac{14400}{\sqrt{x^2+z^2+36}}+1380\right) \qquad F_R = \frac{14400}{\sqrt{x^2+z^2+36}}+1380 \qquad (2)$$

$$0 = 690+\frac{2400z}{\sqrt{x^2+z^2+36}}-1500 \qquad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \qquad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \qquad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

$$z = 2.197\text{ m} = 2.20\text{ m} \qquad \textbf{Ans.}$$

Substituting $z = 2.197$ m into Eq. (4), yields

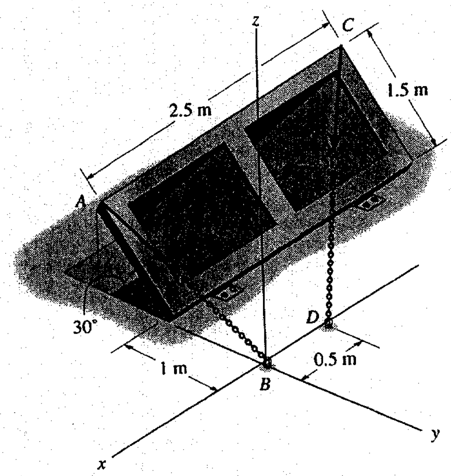
$$x = 1.248\text{ m} = 1.25\text{ m} \qquad \textbf{Ans.}$$

Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$$F_R = 3591.85\text{ N} = 3.59\text{ kN} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-98. The door is held opened by means of two chains. If the tension in AB and CD is $F_A = 300$ N and $F_C = 250$ N, respectively, express each of these forces in Cartesian vector form.



Unit Vector : First determine the position vector $\mathbf{r}_{AB}$ and $\mathbf{r}_{CD}$. The coordinates of points A and C are

$$A[0,\ -(1+1.5\cos 30°),\ 1.5\sin 30°]\ \text{m} = A(0,\ -2.299,\ 0.750)\ \text{m}$$
$$C[-2.50,\ -(1+1.5\cos 30°),\ 1.5\sin 30°]\ \text{m} = C(-2.50,\ -2.299,\ 0.750)\ \text{m}$$

Then

$$\mathbf{r}_{AB} = \{(0-0)\mathbf{i} + [0-(-2.299)]\mathbf{j} + (0-0.750)\mathbf{k}\}\ \text{m} = \{2.299\mathbf{j} - 0.750\mathbf{k}\}\ \text{m}$$
$$r_{AB} = \sqrt{2.299^2 + (-0.750)^2} = 2.418\ \text{m}$$
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.299\mathbf{j} - 0.750\mathbf{k}}{2.418} = 0.9507\mathbf{j} - 0.3101\mathbf{k}$$

$$\mathbf{r}_{CD} = \{[-0.5-(-2.5)]\mathbf{i} + [0-(-2.299)]\mathbf{j} + (0-0.750)\mathbf{k}\}\ \text{m} = \{2.00\mathbf{i} + 2.299\mathbf{j} - 0.750\mathbf{k}\}\ \text{m}$$
$$r_{CD} = \sqrt{2.00^2 + 2.299^2 + (-0.750)^2} = 3.138\ \text{m}$$
$$\mathbf{u}_{CD} = \frac{\mathbf{r}_{CD}}{r_{CD}} = \frac{2.00\mathbf{i} + 2.299\mathbf{j} - 0.750\mathbf{k}}{3.138} = 0.6373\mathbf{i} + 0.7326\mathbf{j} - 0.2390\mathbf{k}$$

Force Vector :

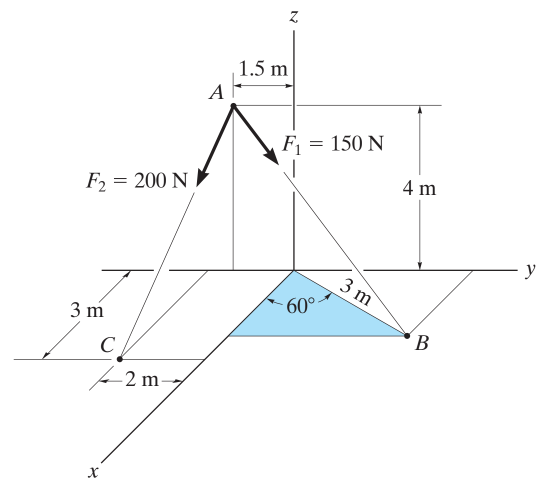
$$\mathbf{F}_A = F_A\mathbf{u}_{AB} = 300\{0.9507\mathbf{j} - 0.3101\mathbf{k}\}\ \text{N} = \{285.21\mathbf{j} - 93.04\mathbf{k}\}\ \text{N} = \{285\mathbf{j} - 93.0\mathbf{k}\}\ \text{N}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_{CD} = 250\{0.6373\mathbf{i} + 0.7326\mathbf{j} - 0.2390\mathbf{k}\}\ \text{N} = \{159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}\}\ \text{N} = \{159\mathbf{i} + 183\mathbf{j} - 59.7\mathbf{k}\}\ \text{N}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–99.

Determine the magnitude and coordinate direction angles of the resultant force acting at point A.



SOLUTION

$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200\left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494}\right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3\cos 60°\mathbf{i} + (1.5 + 3\sin 60°)\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} + 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150\left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198}\right) = (38.0079\mathbf{i} + 103.8396\mathbf{j} - 101.3545\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9389\mathbf{j} - 260.5607\mathbf{k})$$

$$F_R = \sqrt{(157.4124)^2 + (83.9389)^2 + (-260.5604)^2} = 315.7786 = 316\text{ N}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{157.4124}{315.7786}\right) = 60.100° = 60.1°$$ **Ans.**

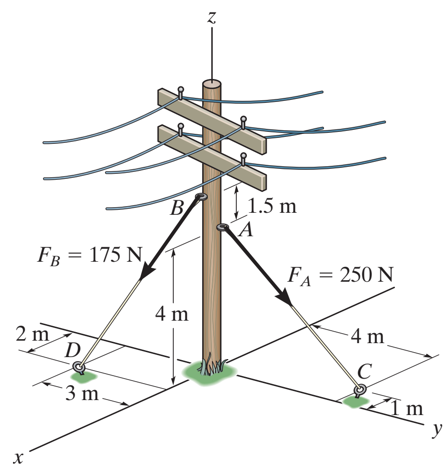
$$\beta = \cos^{-1}\left(\frac{83.9389}{315.7786}\right) = 74.585° = 74.6°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-260.5607}{315.7786}\right) = 145.60° = 146°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–100.

The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form. Neglect the diameter of the pole.



SOLUTION

Unit Vector:

$$\mathbf{r}_{AC} = \{(-1-0)\mathbf{i} + (4-0)\mathbf{j} + (0-4)\mathbf{k}\}\text{ m} = \{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(-1)^2 + 4^2 + (-4)^2} = 5.745\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}}{5.745} = -0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}$$

$$\mathbf{r}_{BD} = \{(2-0)\,\mathbf{i} + (-3-0)\mathbf{j} + (0-5.5)\mathbf{k}\}\text{ m} = \{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}\}\text{ m}$$

$$r_{BD} = \sqrt{2^2 + (-3)^2 + (-5.5)^2} = 6.576\text{ m}$$

$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i} - 0.4562\mathbf{j} - 0.8363\mathbf{k}$$

Force Vector:

$$\mathbf{F}_A = F_A\mathbf{u}_{AC} = 250\{-0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}\}\text{ N}$$

$$= \{-43.52\mathbf{i} + 174.08\mathbf{j} - 174.08\mathbf{k}\}\text{ N}$$

$$= \{-43.5\mathbf{i} + 174\mathbf{j} - 174\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_B = F_B\mathbf{u}_{BD} = 175\{0.3041\mathbf{i} + 0.4562\mathbf{j} - 0.8363\mathbf{k}\}\text{ N}$$

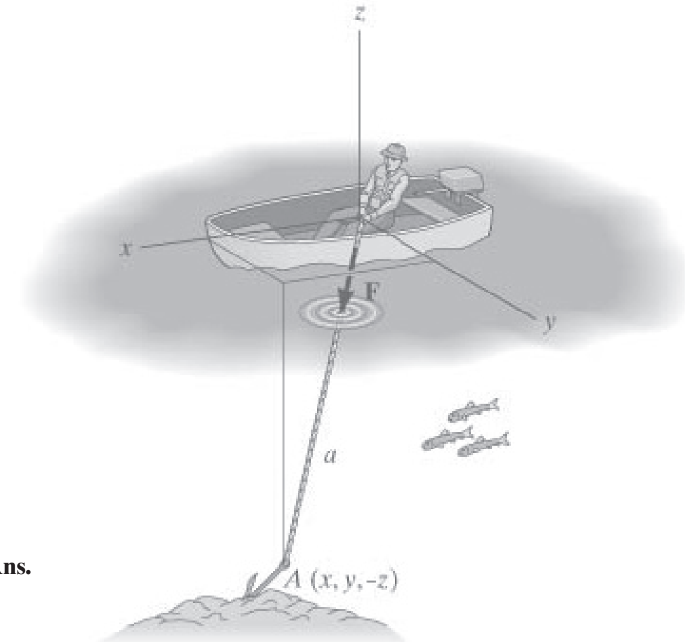
$$= \{53.22\mathbf{i} - 79.83\mathbf{j} - 146.36\mathbf{k}\}\text{ N}$$

$$= \{53.2\mathbf{i} - 79.8\mathbf{j} - 146\mathbf{k}\}\text{ N}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–101.

The force acting on the man, caused by his pulling on the anchor cord, is **F**. If the length of the cord is L, determine the coordinates $A(x, y, -z)$ of the anchor.



Given:

$$\mathbf{F} = \begin{pmatrix} 40 \\ 20 \\ -50 \end{pmatrix} \text{N}$$

$$L = 25 \text{ m}$$

Solution:

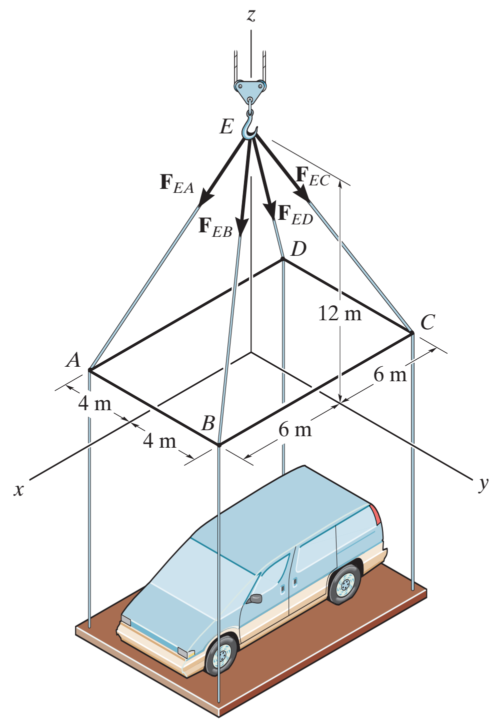
$$\mathbf{r} = L \frac{\mathbf{F}}{|\mathbf{F}|}$$

$$\mathbf{r} = \begin{pmatrix} 14.9 \\ 7.5 \\ -18.6 \end{pmatrix} \text{m}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–102.

Each of the four forces acting at E has a magnitude of 28 kN. Express each force as a Cartesian vector and determine the resultant force.



SOLUTION

$$\mathbf{F}_{EA} = 28\left(\frac{6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EA} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_{EB} = 28\left(\frac{6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EB} = \{12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_{EC} = 28\left(\frac{-6}{14}\mathbf{i} + \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{EC} = \{-12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_{ED} = 28\left(\frac{-6}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} - \frac{12}{14}\mathbf{k}\right)$$

$$\mathbf{F}_{ED} = \{-12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_{EA} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED}$$

$$= \{-96\mathbf{k}\}\text{ kN}$$ **Ans.**

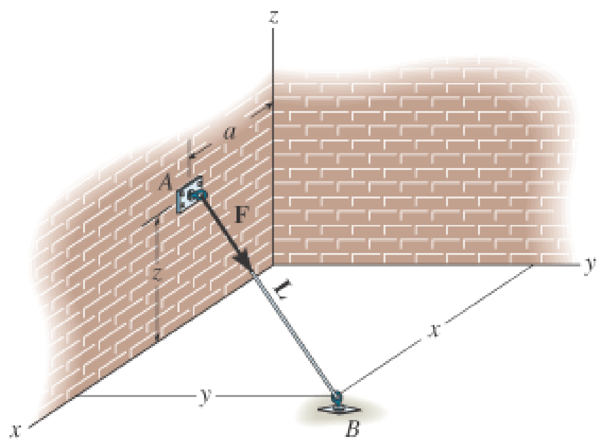
© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–103.

The cord exerts a force **F** on the hook. If the cord is length *L*, determine the location *x*,*y* of the point of attachment *B*, and the height *z* of the hook.

Units Used:

Given:



$$F := \begin{pmatrix} 12 \\ 9 \\ -8 \end{pmatrix} \text{kN}$$

$L := 4\text{m}$

$a := 1\text{m}$

Solution :

Initial guesses $x := 1\text{m}$ $y := 1\text{m}$ $z := 1\text{m}$

Given $\begin{pmatrix} x - a \\ y \\ -z \end{pmatrix} = L \cdot \frac{F}{|F|}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} := \text{Find}(x, y, z)$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3.82 \\ 2.12 \\ 1.88 \end{pmatrix} \text{m}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–104.

The cord exerts a force of magnitude F on the hook. If the cord length L, the distance z, and the x component of the force F_x are given, determine the location x, y of the point of attachment B of the cord to the ground.

Given:

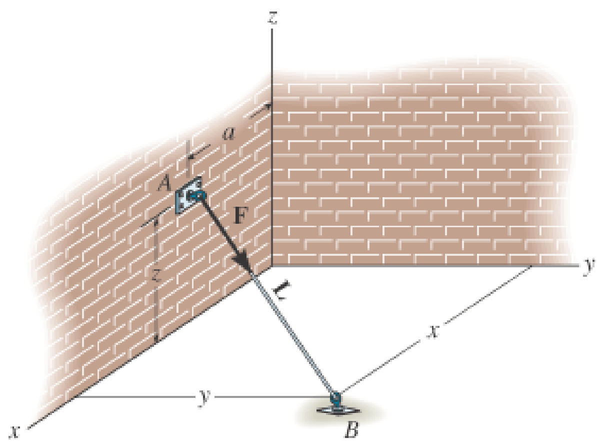
$F := 30\text{kN}$

$L := 4\text{m}$

$z := 2\text{m}$

$F_x := 25\text{kN}$

$a := 1\text{m}$



Solution :

Guesses

$x := 1\text{m}$

$y := 1\text{m}$

Given

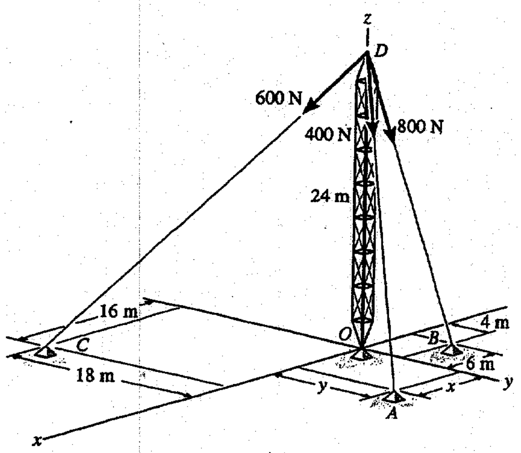
$$F_x = \frac{x - a}{L} \cdot F \qquad L^2 = (x-a)^2 + y^2 + z^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} := \text{Find}(x, y) \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4.33 \\ 0.94 \end{pmatrix} \text{m}$$

Ans.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2-105. The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



$$\mathbf{F}_{DA} = 400\left(\frac{20}{34.66}\mathbf{i} + \frac{15}{34.66}\mathbf{j} - \frac{24}{34.66}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_{DB} = 800\left(\frac{-6}{25.06}\mathbf{i} + \frac{4}{25.06}\mathbf{j} - \frac{24}{25.06}\mathbf{k}\right)\text{N}$$

$$\mathbf{F}_{DC} = 600\left(\frac{16}{34}\mathbf{i} - \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right)\text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC}$$

$$= \{321.66\mathbf{i} - 16.82\mathbf{j} - 1466.71\mathbf{k}\}\text{ N}$$

$$F_R = \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2}$$

$$= 1501.66\text{ N} = 1.50\text{ kN}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6°$$ **Ans.**

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–106.

The chandelier is supported by three chains which are concurrent at point *O*. If the force in each chain has magnitude **F**, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.

Given:

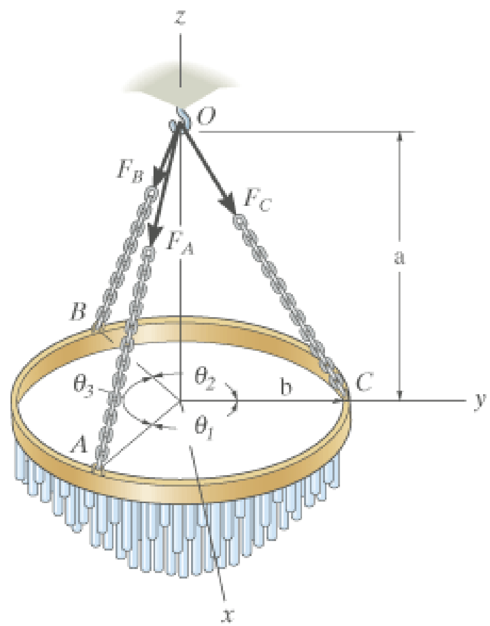
$\mathbf{F} := 300\mathbf{N}$

$\mathbf{a} := 1.8\mathbf{m}$

$\mathbf{b} := 1.2\mathbf{m}$

$\theta_1 := 120\mathbf{deg}$

$\theta_2 := 120\mathbf{deg}$



Solution:

$\theta_3 := 360\mathbf{deg} - \theta_1 - \theta_2$

$\mathbf{r_{OA}} := \begin{pmatrix} \mathbf{b}\cdot\sin(\theta_1) \\ \mathbf{b}\cdot\cos(\theta_1) \\ -\mathbf{a} \end{pmatrix}$ $\quad \mathbf{F_A} := \mathbf{F}\cdot\frac{\mathbf{r_{OA}}}{|\mathbf{r_{OA}}|}$ $\quad \mathbf{F_A} = \begin{pmatrix} 144.12 \\ -83.21 \\ -249.62 \end{pmatrix}\mathbf{N}$ **Ans.**

$\mathbf{r_{OB}} := \begin{pmatrix} \mathbf{b}\cdot\sin(\theta_1+\theta_2) \\ \mathbf{b}\cdot\cos(\theta_1+\theta_2) \\ -\mathbf{a} \end{pmatrix}$ $\quad \mathbf{F_B} := \mathbf{F}\cdot\frac{\mathbf{r_{OB}}}{|\mathbf{r_{OB}}|}$ $\quad \mathbf{F_B} = \begin{pmatrix} -144.12 \\ -83.21 \\ -249.62 \end{pmatrix}\mathbf{N}$ **Ans.**

$\mathbf{r_{OC}} := \begin{pmatrix} 0 \\ \mathbf{b} \\ -\mathbf{a} \end{pmatrix}$ $\quad \mathbf{F_C} := \mathbf{F}\cdot\frac{\mathbf{r_{OC}}}{|\mathbf{r_{OC}}|}$ $\quad \mathbf{F_C} = \begin{pmatrix} 0.00 \\ 166.41 \\ -249.62 \end{pmatrix}\mathbf{N}$ **Ans.**

$\mathbf{F_R} := \mathbf{F_A} + \mathbf{F_B} + \mathbf{F_C}$ $\quad |\mathbf{F_R}| = 748.85\,\mathbf{N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

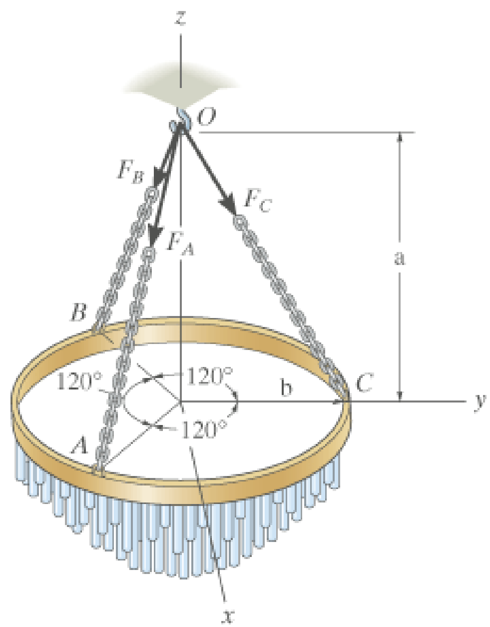
$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} := \mathbf{acos}\left(\frac{\mathbf{F_R}}{|\mathbf{F_R}|}\right) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 90.00 \\ 90.00 \\ 180.00 \end{pmatrix} \mathbf{deg}$$

Ans.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–107.

The chandelier is supported by three chains which are concurrent at point *O*. If the resultant force at *O* has magnitude F_R and is directed along the negative *z* axis, determine the force in each chain assuming $\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$.



Given:

$\mathbf{a} := 1.8\mathbf{m}$

$\mathbf{b} := 1.2\mathbf{m}$

$\mathbf{F_R} := 650\mathbf{N}$

Solution:

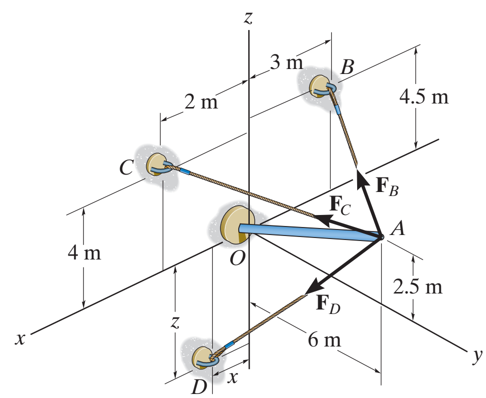
$$\mathbf{F} := \frac{\sqrt{\mathbf{a}^2 + \mathbf{b}^2}}{3\mathbf{a}}\mathbf{F_R}$$

$\mathbf{F} = 260.4\,\mathbf{N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

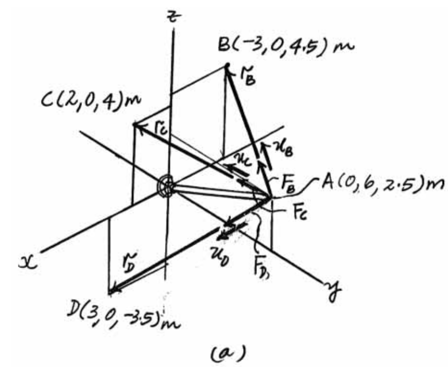
2–108.

Determine the magnitude and coordinate direction angles of the resultant force. Set $F_B = 630$ N, $F_C = 520$ N and $F_D = 750$ N, and $x = 3$ m and $z = 3.5$ m.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,



$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4.5-2.5)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4.5-2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2.5)\mathbf{k}}{\sqrt{(2-0)^2 + (0-6)^2 + (4-2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(3-0)\mathbf{i} + (0-6)\mathbf{j} + (-3.5-2.5)\mathbf{k}}{\sqrt{(0-3)^2 + (0-6)^2 + (-3.5-2.5)^2}} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 630\left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right) = \{-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 520\left(\frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}\right) = \{160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 750\left(\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \{250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}) + (160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}) + (250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k})$$
$$= [140\mathbf{i} - 1520\mathbf{j} - 200\mathbf{k}]\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$
$$= \sqrt{140^2 + (-1520)^2 + (-200)^2} = 1539.48\text{ N} = 1.54\text{ kN}$$ **Ans.**

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{140}{1539.48}\right) = 84.8°$$ **Ans.**

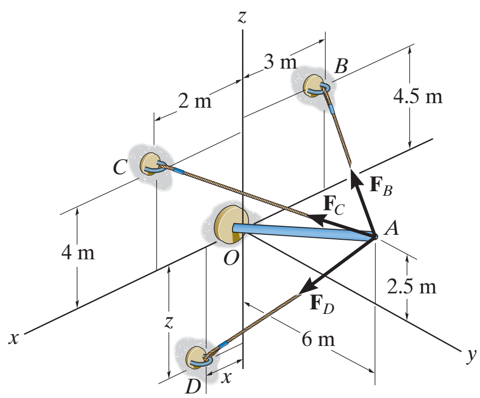
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-1520}{1539.48}\right) = 171°$$ **Ans.**

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-200}{1539.48}\right) = 97.5°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–109.

If the magnitude of the resultant force is 1300 N and acts along the axis of the strut, directed from point A towards O, determine the magnitudes of the three forces acting on the strut. Set $x = 0$ and $z = 5.5$ m.



SOLUTION

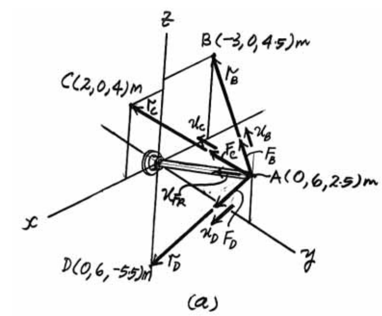
Force Vectors: The unit vectors $\mathbf{u}_B$, $\mathbf{u}_C$, $\mathbf{u}_D$, and $\mathbf{u}_{F_R}$ of $\mathbf{F}_B$, $\mathbf{F}_C$, $\mathbf{F}_D$, and $\mathbf{F}_R$ must be determined first. From Fig. a,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3-0)\mathbf{i} + (0-6)\mathbf{j} + (4.5-2.5)\mathbf{k}}{\sqrt{(-3-0)^2 + (0-6)^2 + (4.5-2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2-0)\mathbf{i} + (0-6)\mathbf{j} + (4-2.5)\mathbf{k}}{\sqrt{(2-0)^2 + (0-6)^2 + (4-2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (-5.5-2.5)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (-5.5-2.5)^2}} = -\frac{3}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$$

$$\mathbf{u}_{F_R} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{(0-0)\mathbf{i} + (0-6)\mathbf{j} + (0-2.5)\mathbf{k}}{\sqrt{(0-0)^2 + (0-6)^2 + (0-2.5)^2}} = -\frac{12}{13}\mathbf{j} + \frac{5}{13}\mathbf{k}$$



Thus, the force vectors $\mathbf{F}_B$, $\mathbf{F}_C$, $\mathbf{F}_D$, and $\mathbf{F}_R$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = -\frac{3}{7}F_B\mathbf{i} - \frac{6}{7}F_B\mathbf{j} + \frac{2}{7}F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = \frac{4}{13}F_C\mathbf{i} - \frac{12}{13}F_C\mathbf{j} + \frac{3}{13}F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = -\frac{3}{5}F_D\mathbf{j} - \frac{4}{5}F_D\mathbf{k}$$

$$\mathbf{F}_R = F_R\mathbf{u}_R = 1300\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right) = [-1200\mathbf{j} - 500\mathbf{k}]\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B\mathbf{i} - \frac{6}{7}F_B\mathbf{j} + \frac{2}{7}F_B\mathbf{k}\right) + \left(\frac{4}{13}F_C\mathbf{i} - \frac{12}{13}F_C\mathbf{j} + \frac{3}{13}F_C\mathbf{k}\right) + \left(-\frac{3}{5}F_D\mathbf{j} - \frac{4}{5}F_D\mathbf{k}\right)$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B + \frac{4}{13}F_C\right)\mathbf{i} + \left(-\frac{6}{7}F_B - \frac{12}{13}F_C - \frac{3}{5}F_D\mathbf{j}\right) + \left(\frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D\right)\mathbf{k}$$

Equating the **i**, **j**, and **k** components,

$$0 = -\frac{3}{7}F_B + \frac{4}{13}F_C \qquad (1)$$

$$-1200 = -\frac{6}{7}F_B - \frac{12}{13}F_C - \frac{3}{5}F_D\mathbf{j} \qquad (2)$$

$$-500 = \frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D \qquad (3)$$

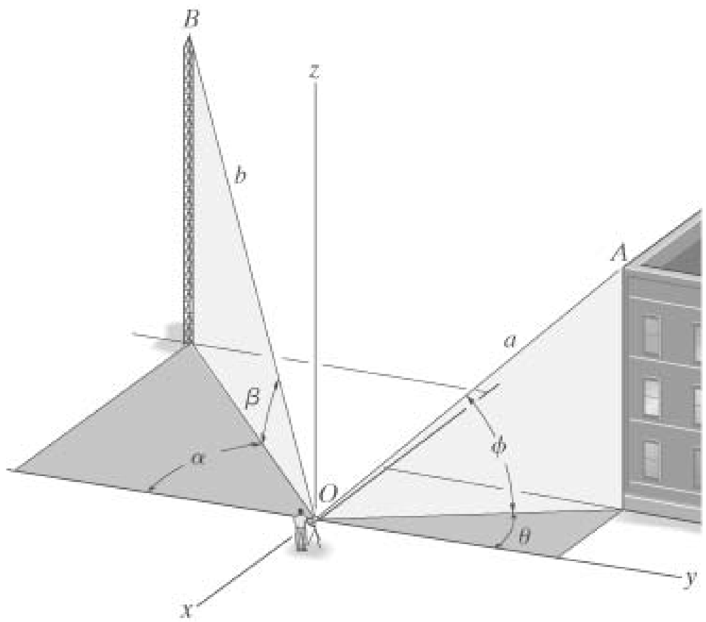
Solving Eqs. (1), (2), and (3), yields

$F_C = 442$ N $\quad F_B = 318$ N $\quad F_D = 866$ N **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–110.

The positions of point *A* on the building and point *B* on the antenna have been measured relative to the electronic distance meter (EDM) at *O*. Determine the distance between *A* and *B*. *Hint:* Formulate a position vector directed from *A* to *B*; then determine its magnitude.



Given:

$a = 460 \text{ m}$

$b = 653 \text{ m}$

$\alpha = 60 \text{ deg}$

$\beta = 55 \text{ deg}$

$\theta = 30 \text{ deg}$

$\phi = 40 \text{ deg}$

Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -a\cos(\phi)\sin(\theta) \\ a\cos(\phi)\cos(\theta) \\ a\sin(\phi) \end{pmatrix}$$

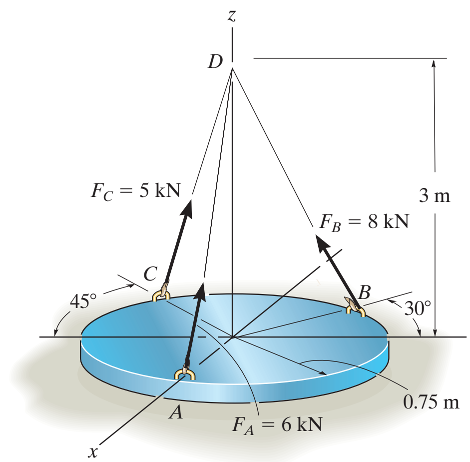
$$\mathbf{r_{OB}} = \begin{pmatrix} -b\cos(\beta)\sin(\alpha) \\ -b\cos(\beta)\cos(\alpha) \\ b\sin(\beta) \end{pmatrix}$$

$$\mathbf{r_{AB}} = \mathbf{r_{OB}} - \mathbf{r_{OA}} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -148.2 \\ -492.4 \\ 239.2 \end{pmatrix} \text{ m} \qquad |\mathbf{r_{AB}}| = 567.2 \text{ m}$$

Ans.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–111. The cylindrical plate is subjected to the three cable forces which are concurrent at point D. Express each force which the cables exert on the plate as a Cartesian vector, and determine the magnitude and coordinate direction angles of the resultant force.



$\mathbf{r}_A = (0-0.75)\mathbf{i}+(0-0)\mathbf{j}+(3-0)\mathbf{k} = \{-0.75\,\mathbf{i}+0\,\mathbf{j}+3\,\mathbf{k}\}$ m

$r_A = \sqrt{(-0.75)^2+0^2+3^2} = 3.0923$ m

$\mathbf{F}_A = F_A\left(\frac{\mathbf{r}_A}{r_A}\right) = 6\left(\frac{-0.75\,\mathbf{i}+3\,\mathbf{k}}{3.0923}\right)$

$= \{-1.4552\,\mathbf{i}+5.8209\,\mathbf{k}\}$ kN

$= \{-1.46\,\mathbf{i}+5.82\,\mathbf{k}\}$ kN **Ans.**

$\mathbf{r}_C = [0-(-0.75\sin 45°)]\mathbf{i}+[0-(-0.75\cos 45°)]\mathbf{j}+(3-0)\mathbf{k}$

$= \{0.5303\,\mathbf{i}+0.5303\,\mathbf{j}+3\,\mathbf{k}\}$ m

$r_C = \sqrt{(0.5303)^2+(0.5303)^2+3^2} = 3.0923$ m

$\mathbf{F}_C = F_C\left(\frac{\mathbf{r}_C}{r_C}\right) = 5\left(\frac{0.5303\,\mathbf{i}+0.5303\,\mathbf{j}+3\,\mathbf{k}}{3.0923}\right)$

$= \{0.8575\,\mathbf{i}+0.8575\,\mathbf{j}+4.8507\,\mathbf{k}\}$ kN

$= \{0.857\,\mathbf{i}+0.857\,\mathbf{j}+4.85\,\mathbf{k}\}$ kN **Ans.**

$\mathbf{r}_B = [0-(-0.75\sin 30°)]\mathbf{i}+(0-0.75\cos 30°)\mathbf{j}+(3-0)\mathbf{k}$

$= \{0.375\,\mathbf{i}-0.6495\,\mathbf{j}+3\,\mathbf{k}\}$ m

$r_B = \sqrt{(0.375)^2+(-0.6495)^2+3^2} = 3.0923$ m

$\mathbf{F}_B = F_B\left(\frac{\mathbf{r}_B}{r_B}\right) = 8\left(\frac{0.375\,\mathbf{i}-0.6495\,\mathbf{j}+3\,\mathbf{k}}{3.0923}\right)$

$= \{0.9701\mathbf{i}-1.6803\mathbf{j}+7.7611\mathbf{k}\}$ kN

$= \{0.970\mathbf{i}-1.68\mathbf{j}+7.76\mathbf{k}\}$ kN **Ans.**

$\mathbf{F}_R = \mathbf{F}_A+\mathbf{F}_B+\mathbf{F}_C$

$= \{-1.4552\,\mathbf{i}+5.8209\,\mathbf{k}\}+\{0.9701\,\mathbf{i}-1.6803\,\mathbf{j}+7.7611\,\mathbf{k}\}$

$+\{0.8575\,\mathbf{i}+0.8575\,\mathbf{j}+4.8507\,\mathbf{k}\}$

$= \{0.3724\,\mathbf{i}-0.8228\,\mathbf{j}+18.4327\,\mathbf{k}\}$ kN

$F_R = \sqrt{(0.3724)^2+(-0.8228)^2+(18.4327)^2}$

$= 18.4548$ kN $= 18.5$ kN **Ans.**

$\mathbf{u}_R = \frac{\mathbf{F}_R}{F_R} = \frac{0.3724\mathbf{i}-0.8228\mathbf{j}+18.4327\mathbf{k}}{18.4548}$

$= 0.02018\mathbf{i}-0.04459\mathbf{j}+0.9988\mathbf{k}$

$\cos\alpha = 0.02018$ $\quad \alpha = 88.8°$ **Ans.**

$\cos\beta = -0.04458$ $\quad \beta = 92.6°$ **Ans.**

$\cos\gamma = 0.9988$ $\quad \gamma = 2.81°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

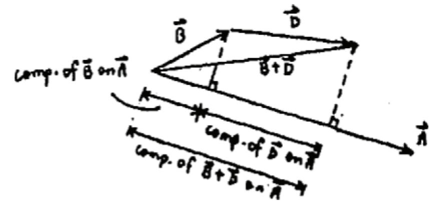
2–112.

Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.

SOLUTION

Since the component of $(\mathbf{B}+\mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \textbf{(QED)}$$



Also,

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

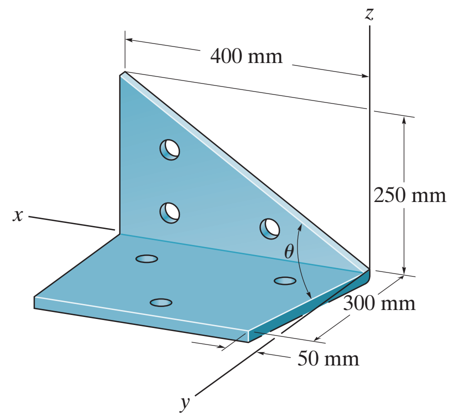
$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \textbf{(QED)}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–113.

Determine the angle θ between the edges of the sheet-metal bracket.



SOLUTION

$\mathbf{r}_1 = \{400\mathbf{i} + 250\mathbf{k}\}$ mm ; $\quad r_1 = 471.70$ mm

$\mathbf{r}_2 = \{50\mathbf{i} + 300\mathbf{j}\}$ mm ; $\quad r_2 = 304.14$ mm

$\mathbf{r}_1 \cdot \mathbf{r}_2 = (400)(50) + 0(300) + 250(0) = 20\,000$

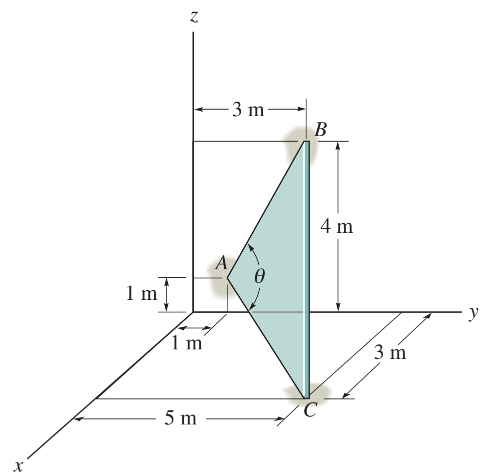
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}\right)$$

$$= \cos^{-1}\left(\frac{20\,000}{(471.70)(304.14)}\right) = 82.0^\circ$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–114.

Determine the angle θ between the sides of the triangular plate.



SOLUTION

$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}$ m

$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990$ m

$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}$ m

$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056$ m

$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$

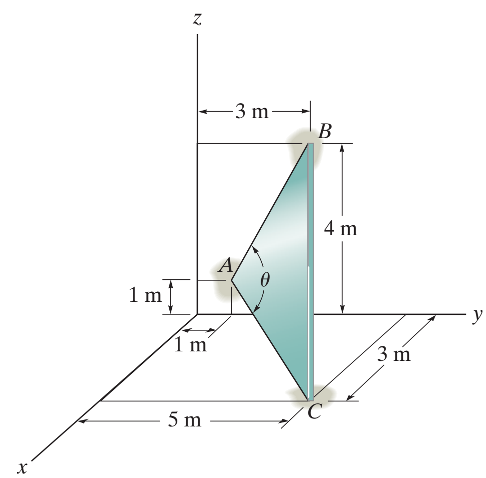
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC} r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$\theta = 74.219° = 74.2°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–115.

Determine the length of side BC of the triangular plate. Solve the problem by finding the magnitude of $\mathbf{r}_{BC}$; then check the result by first finding θ, r_{AB}, and r_{AC} and then using the cosine law.



SOLUTION

$$\mathbf{r}_{BC} = \{3\,\mathbf{i} + 2\,\mathbf{j} - 4\,\mathbf{k}\}\text{ m}$$

$$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39\text{ m} \qquad \textbf{Ans.}$$

Also,

$$\mathbf{r}_{AC} = \{3\,\mathbf{i} + 4\,\mathbf{j} - 1\,\mathbf{k}\}\text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990\text{ m}$$

$$\mathbf{r}_{AB} = \{2\,\mathbf{j} + 3\,\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056\text{ m}$$

$$\mathbf{r}_{AC}\cdot\mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC}\cdot\mathbf{r}_{AB}}{r_{AC}\,r_{AB}}\right) = \cos^{-1}\frac{5}{(5.0990)(3.6056)}$$

$$\theta = 74.219°$$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056)\cos 74.219°}$$

$$r_{BC} = 5.39\text{ m} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–116.

Determine the angle θ between the tails of the two vectors.

Given:

$r_1 = 9\ \text{m}$

$r_2 = 6\ \text{m}$

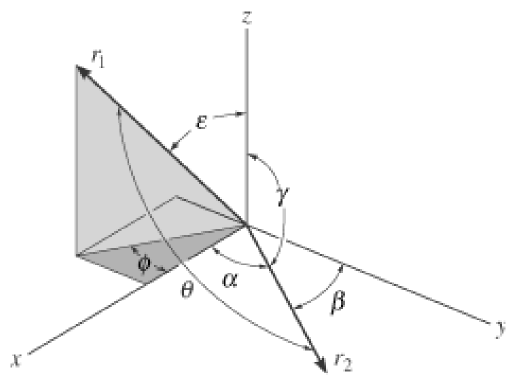
$\alpha = 60\ \text{deg}$

$\beta = 45\ \text{deg}$

$\gamma = 120\ \text{deg}$

$\phi = 30\ \text{deg}$

$\varepsilon = 40\ \text{deg}$



Solution:

Determine the two position vectors and use the dot product to find the angle

$$\mathbf{r_{1v}} = r_1 \begin{pmatrix} \sin(\varepsilon)\cos(\phi) \\ -\sin(\varepsilon)\sin(\phi) \\ \cos(\varepsilon) \end{pmatrix} \qquad \mathbf{r_{2v}} = r_2 \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix}$$

$$\theta = \text{acos}\left(\frac{\mathbf{r_{1v}} \cdot \mathbf{r_{2v}}}{|\mathbf{r_{1v}}|\ |\mathbf{r_{2v}}|}\right) \qquad \theta = 109.4\ \text{deg} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–117.

Determine the magnitude of the projected component of $\mathbf{r_1}$ along $\mathbf{r_2}$, and the projection of $\mathbf{r_2}$ along $\mathbf{r_1}$.

Given:

$r_1 = 9$ m

$r_2 = 6$ m

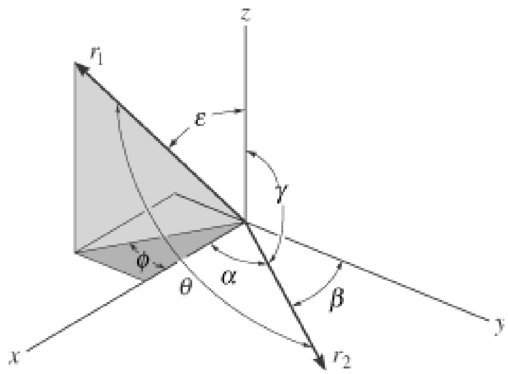
$\alpha = 60$ deg

$\beta = 45$ deg

$\gamma = 120$ deg

$\phi = 30$ deg

$\varepsilon = 40$ deg



Solution:

Write the vectors and unit vectors

$$\mathbf{r_{1v}} = r_1 \begin{pmatrix} \sin(\varepsilon)\cos(\phi) \\ -\sin(\varepsilon)\sin(\phi) \\ \cos(\varepsilon) \end{pmatrix} \qquad \mathbf{r_{1v}} = \begin{pmatrix} 5.01 \\ -2.89 \\ 6.89 \end{pmatrix} \text{ m}$$

$$\mathbf{r_{2v}} = r_2 \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix} \qquad \mathbf{r_{2v}} = \begin{pmatrix} 3 \\ 4.24 \\ -3 \end{pmatrix} \text{ m}$$

$$\mathbf{u_1} = \frac{\mathbf{r_{1v}}}{|\mathbf{r_{1v}}|} \qquad \mathbf{u_2} = \frac{\mathbf{r_{2v}}}{|\mathbf{r_{2v}}|} \qquad \mathbf{u_1} = \begin{pmatrix} 0.557 \\ -0.321 \\ 0.766 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} 0.5 \\ 0.707 \\ -0.5 \end{pmatrix}$$

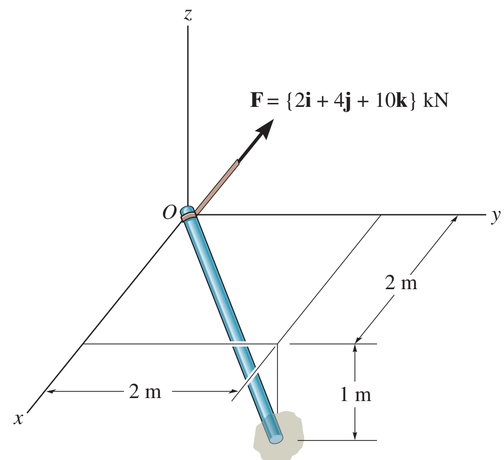
The magnitude of the projection of $\mathbf{r_1}$ along $\mathbf{r_2}$. $\quad |\mathbf{r_{1v}} \cdot \mathbf{u_2}| = 2.99$ m **Ans.**

The magnitude of the projection of $\mathbf{r_2}$ along $\mathbf{r_1}$. $\quad |\mathbf{r_{2v}} \cdot \mathbf{u_1}| = 1.99$ m **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–118.

Determine the projection of the force **F** along the pole.



SOLUTION

$$\text{Proj } F = \mathbf{F}\cdot\mathbf{u}_a = (2\,\mathbf{i} + 4\,\mathbf{j} + 10\,\mathbf{k})\cdot\left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right)$$

$$\text{Proj } F = 0.667\text{ kN}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–119.

Determine the angle θ between the two cords.

Given:

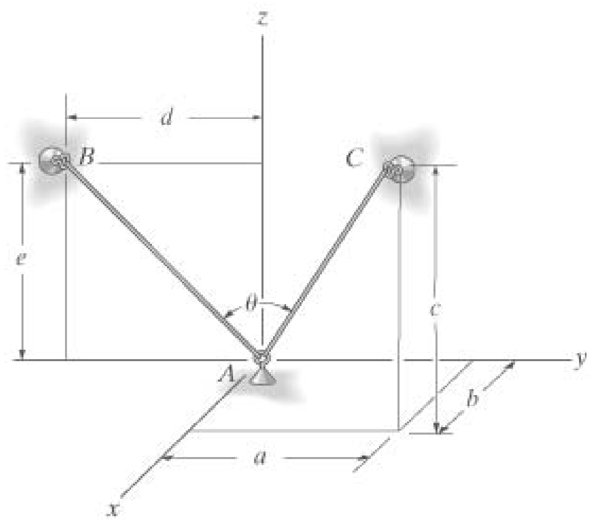
$a = 3$ m

$b = 2$ m

$c = 6$ m

$d = 3$ m

$e = 4$ m



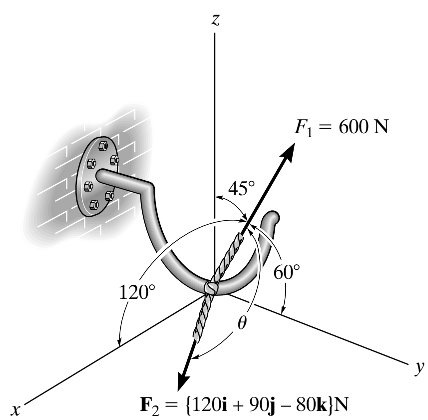
Solution:

$$\mathbf{r_{AC}} = \begin{pmatrix} b \\ a \\ c \end{pmatrix} \text{m} \qquad \mathbf{r_{AB}} = \begin{pmatrix} 0 \\ -d \\ e \end{pmatrix} \text{m} \qquad \theta = \operatorname{acos}\left(\frac{\mathbf{r_{AC}} \cdot \mathbf{r_{AB}}}{|\mathbf{r_{AC}}|\,|\mathbf{r_{AB}}|}\right) \qquad \theta = 64.6 \text{ deg}$$

Ans.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–120. Two forces act on the hook. Determine the angle θ between them. Also, what are the projections of $\mathbf{F}_1$ and $\mathbf{F}_2$ along the y axis?



$$\mathbf{F}_1 = 600\cos 120°\,\mathbf{i} + 600\cos 60°\,\mathbf{j} + 600\cos 45°\,\mathbf{k}$$

$$= -300\,\mathbf{i} + 300\,\mathbf{j} + 424.3\,\mathbf{k}; \quad F_1 = 600\text{ N}$$

$$\mathbf{F}_2 = 120\,\mathbf{i} + 90\,\mathbf{j} - 80\,\mathbf{k}; \quad F_2 = 170\text{ N}$$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = (-300)(120) + (300)(90) + (424.3)(-80) = -42\,944$$

$$\theta = \cos^{-1}\left(\frac{-42\,944}{(170)(600)}\right) = 115° \qquad \textbf{Ans.}$$

$$\mathbf{u} = \mathbf{j}$$

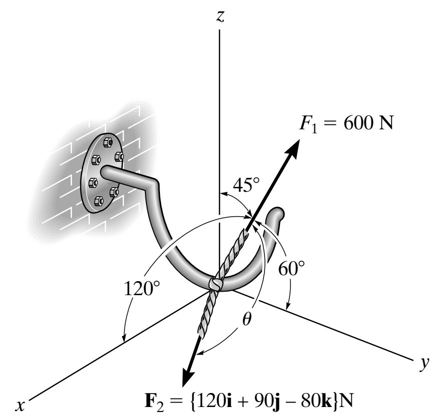
So,

$$F_{1y} = \mathbf{F}_1 \cdot \mathbf{j} = (300)(1) = 300\text{ N} \qquad \textbf{Ans.}$$

$$F_{2y} = \mathbf{F}_2 \cdot \mathbf{j} = (90)(1) = 90\text{ N} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–121. Two forces act on the hook. Determine the magnitude of the projection of $\mathbf{F}_2$ along $\mathbf{F}_1$.



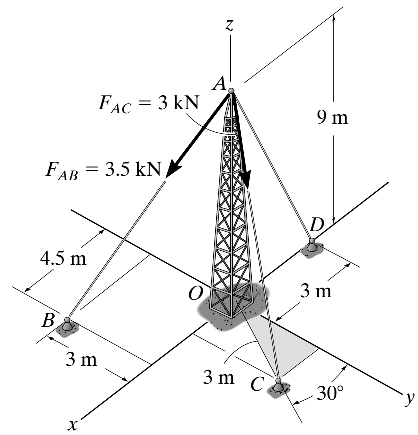
$\mathbf{u}_1 = \cos 120°\,\mathbf{i} + \cos 60°\,\mathbf{j} + \cos 45°\,\mathbf{k}$

Proj $F_2 = \mathbf{F}_2 \cdot \mathbf{u}_1 = (120)(\cos 120°) + (90)(\cos 60°) + (-80)(\cos 45°)$

$|\text{Proj } F_2| = 71.6$ N   **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–122. Determine the magnitude of the projected component of force $\mathbf{F}_{AB}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(4.5-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-9)\mathbf{k}}{\sqrt{(4.5-0)^2 + (-3-0)^2 + (0-9)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

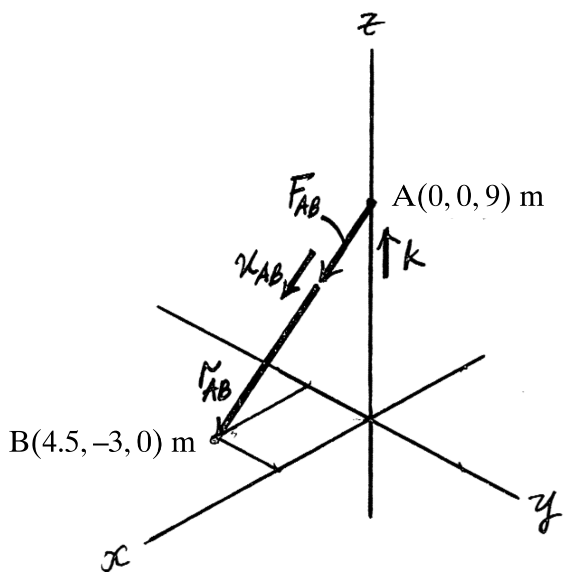
$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 3.5\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{1.5\mathbf{i} - 1\mathbf{j} - 3\mathbf{k}\}\text{ kN}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AB}$ along the z axis is

$$(F_{AB})_z = \mathbf{F}_{AB} \cdot \mathbf{k} = (1.5\mathbf{i} - 1\mathbf{j} - 3\mathbf{k}) \cdot \mathbf{k}$$
$$= -3\text{ kN}$$

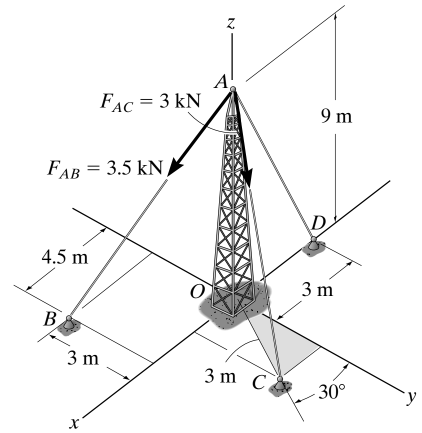
The negative sign indicates that $(F_{AB})_z$ is directed towards the negative z axis. Thus

$(F_{AB})_z = 3$ kN **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–123. Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(3\sin 30° - 0)\mathbf{i} + (3\cos 30° - 0)\mathbf{j} + (0-9)\mathbf{k}}{\sqrt{(3\sin 30° - 0)^2 + (3\cos 30° - 0)^2 + (0-9)^2}} = 0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

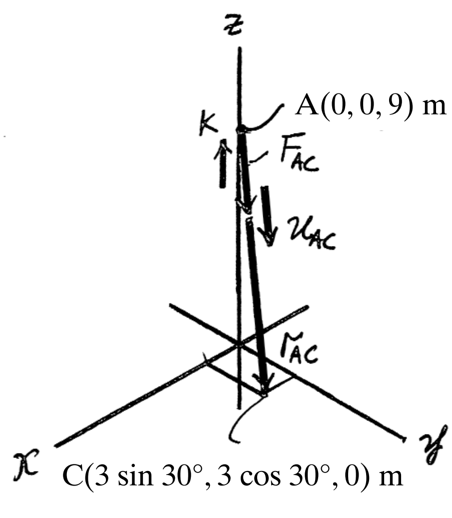
$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 3(0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}) = \{0.4743\mathbf{i} + 0.8217\mathbf{j} - 2.8461\mathbf{k}\}\text{ kN}$$

Vector Dot Product: The projected component of $\mathbf{F}_{AC}$ along the z axis is

$$(F_{AC})_z = \mathbf{F}_{AC} \cdot \mathbf{k} = (0.4743\mathbf{i} + 0.8217\mathbf{j} - 2.8461\mathbf{k}) \cdot \mathbf{k}$$
$$= -2.8461\text{ kN}$$

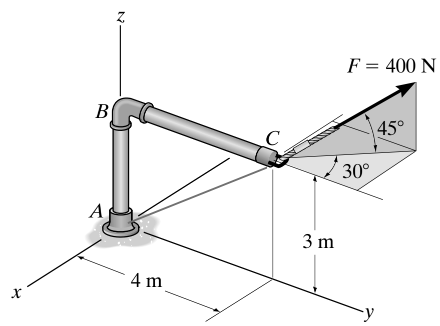
The negative sign indicates that $(F_{AC})_z$ is directed towards the negative z axis. Thus

$(F_{AC})_z = 2.846$ kN **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–124. Determine the projection of force $F = 400$ N acting along line AC of the pipe assembly. Express the result as a Cartesian vector.



Force and unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. (*a*)

$$\mathbf{F} = 400(-\cos 45° \sin 30° \mathbf{i} + \cos 45° \cos 30° \mathbf{j} + \sin 45° \mathbf{k})$$

$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line AC is

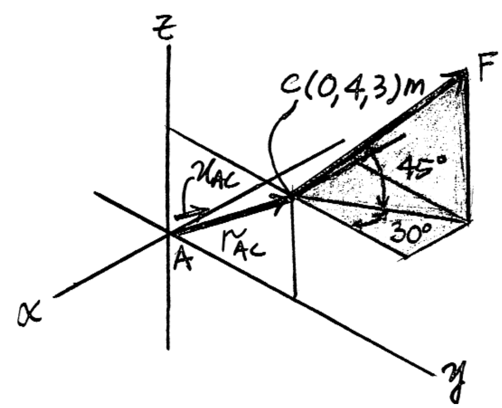
$$F_{AC} = \mathbf{F} \cdot \mathbf{u}_{AC} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right)$$

$$= (-141.42)(0) + 244.95\left(\frac{4}{5}\right) + 282.84\left(\frac{3}{5}\right)$$

$$= 365.66 \text{ N}$$ **Ans.**

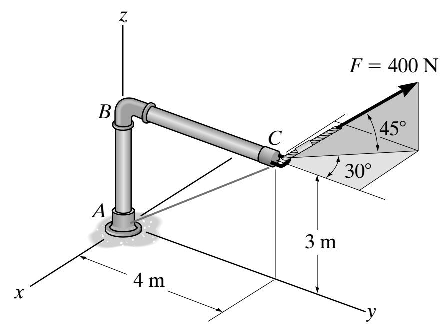
Thus, $\mathbf{F}_{AC}$ written in Cartesian vector form is

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 365.66\left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = \{293\mathbf{j} + 219\mathbf{k}\} \text{ N}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–125. Determine the magnitudes of the components of force $F = 400$ N acting parallel and perpendicular to segment BC of the pipe assembly.



Force Vector: The force vector **F** must be determined first. From Fig. *a*,

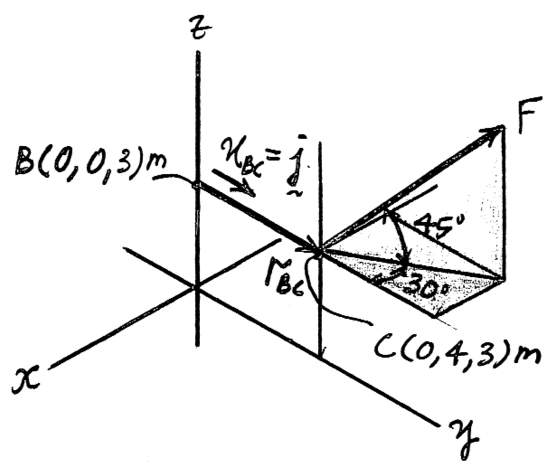
$$\mathbf{F} = 400(-\cos 45° \sin 30° \mathbf{i} + \cos 45° \cos 30° \mathbf{j} + \sin 45° \mathbf{k})$$
$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\} \text{ N}$$

Vector Dot Product: By inspecting Fig. (a) we notice that $\mathbf{u}_{BC} = \mathbf{j}$. Thus, the magnitude of the component of **F** parallel to segment BC of the pipe assembly is

$$(F_{BC})_{paral} = \mathbf{F} \cdot \mathbf{j} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \mathbf{j}$$
$$= -141.42(0) + 244.95(1) + 282.84(0)$$
$$= 244.95 \text{ N} = 245 \text{ N}$$ **Ans.**

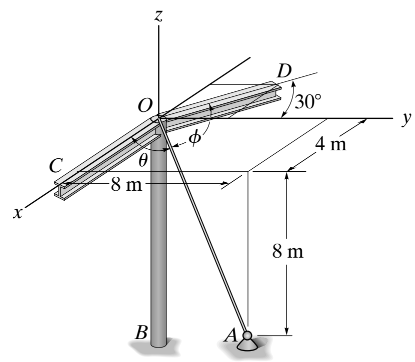
The magnitude of the component of **F** perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{per} = \sqrt{F^2 - (F_{BC})_{paral}^2} = \sqrt{400^2 - 244.95^2} = 316 \text{ N}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–126. Cable OA is used to support column OB. Determine the angle θ it makes with beam OC.



Unit Vector :

$$\mathbf{u}_{OC} = 1\mathbf{i}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\mathbf{i}+(8-0)\mathbf{j}+(-8-0)\mathbf{k}}{\sqrt{(4-0)^2+(8-0)^2+(-8-0)^2}} = \frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$$

The Angle Between Two Vectors θ:

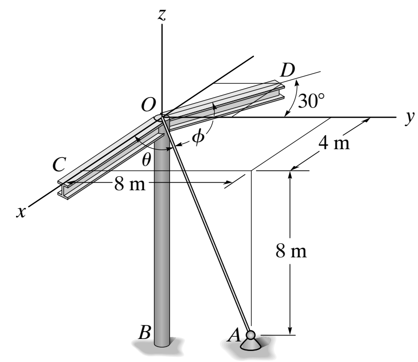
$$\mathbf{u}_{OC}\cdot\mathbf{u}_{OA} = (1\mathbf{i})\cdot\left(\frac{1}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right) = 1\left(\frac{1}{3}\right)+(0)\left(\frac{2}{3}\right)+0\left(-\frac{2}{3}\right) = \frac{1}{3}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{OC}\cdot\mathbf{u}_{OA}) = \cos^{-1}\frac{1}{3} = 70.5^\circ \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–127. Cable *OA* is used to support column *OB*. Determine the angle ϕ it makes with beam *OD*.



Unit Vector :

$$\mathbf{u}_{OD} = -\sin 30°\mathbf{i} + \cos 30°\mathbf{j} = -0.5\mathbf{i} + 0.8660\mathbf{j}$$

$$\mathbf{u}_{OA} = \frac{(4-0)\mathbf{i} + (8-0)\mathbf{j} + (-8-0)\mathbf{k}}{\sqrt{(4-0)^2 + (8-0)^2 + (-8-0)^2}} = \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

The Angles Between Two Vectors ϕ :

$$\mathbf{u}_{OD} \cdot \mathbf{u}_{OA} = (-0.5\mathbf{i} + 0.8660\mathbf{j}) \cdot \left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = (-0.5)\left(\frac{1}{3}\right) + (0.8660)\left(\frac{2}{3}\right) + 0\left(-\frac{2}{3}\right) = 0.4107$$

Then,

$$\phi = \cos^{-1}(\mathbf{u}_{OD} \cdot \mathbf{u}_{OA}) = \cos^{-1} 0.4107 = 65.8°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–128.

Determine the angles θ and ϕ between the axis OA of the pole and each cable, AB and AC.

Given:

$F_1 = 50\ \text{N}$

$F_2 = 35\ \text{N}$

$a = 1\ \text{m}$

$b = 3\ \text{m}$

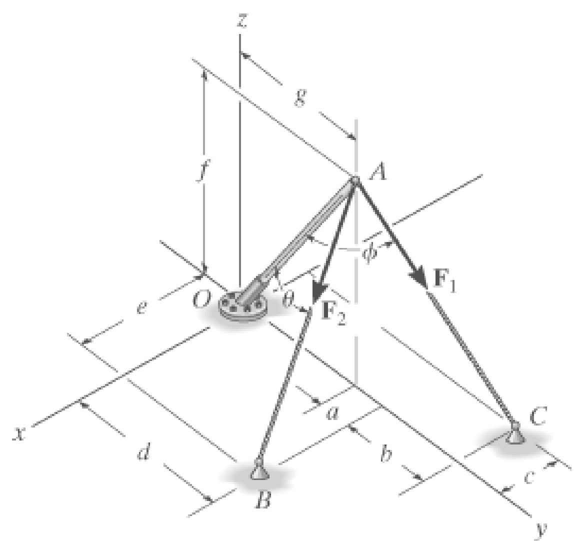
$c = 2\ \text{m}$

$d = 5\ \text{m}$

$e = 4\ \text{m}$

$f = 6\ \text{m}$

$g = 4\ \text{m}$



Solution:

$$\mathbf{r_{AO}} = \begin{pmatrix} 0 \\ -g \\ -f \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} e \\ a \\ -f \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -c \\ a+b \\ -f \end{pmatrix}$$

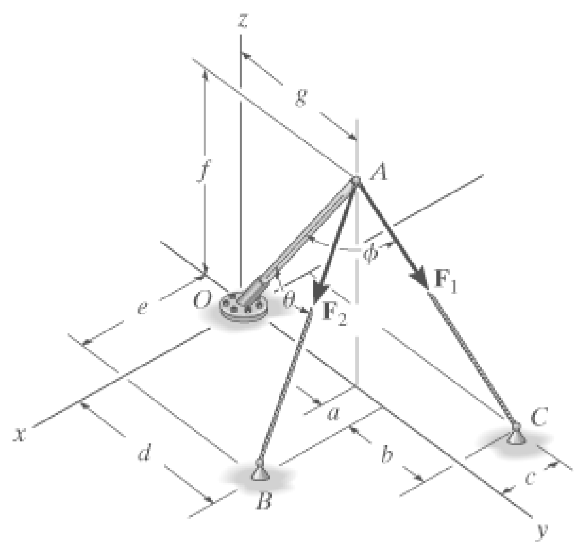
$$\theta = \operatorname{acos}\left(\frac{\mathbf{r_{AO}}\cdot\mathbf{r_{AB}}}{|\mathbf{r_{AO}}|\,|\mathbf{r_{AB}}|}\right) \qquad \theta = 52.4\ \text{deg} \qquad \textbf{Ans.}$$

$$\phi = \operatorname{acos}\left(\frac{\mathbf{r_{AO}}\cdot\mathbf{r_{AC}}}{|\mathbf{r_{AO}}|\,|\mathbf{r_{AC}}|}\right) \qquad \phi = 68.2\ \text{deg} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–129.

The two cables exert the forces shown on the pole. Determine the magnitude of the projected component of each force acting along the axis *OA* of the pole.



Given:

$F_1 = 50\ \text{N}$

$F_2 = 35\ \text{N}$

$a = 1\ \text{m}$

$b = 3\ \text{m}$

$c = 2\ \text{m}$

$d = 5\ \text{m}$

$e = 4\ \text{m}$

$f = 6\ \text{m}$

$g = 4\ \text{m}$

Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} e \\ a \\ -f \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -c \\ a+b \\ -f \end{pmatrix} \qquad \mathbf{r_{AO}} = \begin{pmatrix} 0 \\ -g \\ -f \end{pmatrix} \qquad \mathbf{u_{AO}} = \frac{\mathbf{r_{AO}}}{|\mathbf{r_{AO}}|}$$

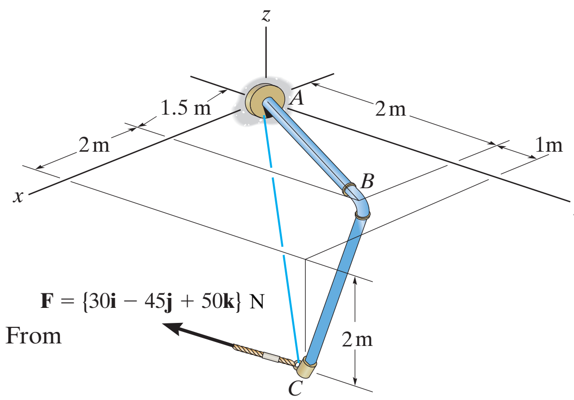
$$\mathbf{F_{1v}} = F_1 \frac{\mathbf{r_{AC}}}{|\mathbf{r_{AC}}|} \qquad F_{1AO} = \mathbf{F_{1v}} \cdot \mathbf{u_{AO}} \qquad F_{1AO} = 18.5\ \text{N}$$ **Ans.**

$$\mathbf{F_{2v}} = F_2 \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad F_{2AO} = \mathbf{F_{2v}} \cdot \mathbf{u_{AO}} \qquad F_{2AO} = 21.3\ \text{N}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–130.

Determine the angle θ between the pipe segments BA and BC.

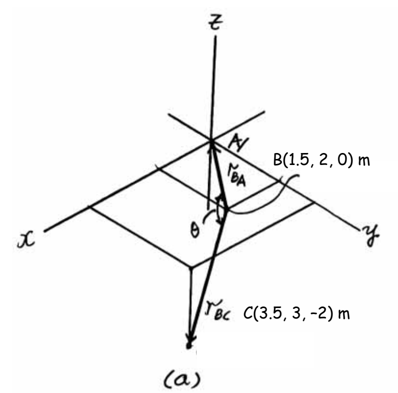


SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{BA} = (0 - 1.5)\mathbf{i} + (0 - 2)\mathbf{j} + (0 - 0)\mathbf{k} = \{-1.5\mathbf{i} - 2\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{BC} = (3.5 - 1.5)\mathbf{i} + (3 - 2)\mathbf{j} + (-2 - 0)\mathbf{k} = \{2\mathbf{i} + 1\mathbf{j} - 2\mathbf{k}\}\text{ m}$$



The magnitude of $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ are

$$\mathbf{r}_{BA} = \sqrt{(-1.5)^2 + (-2)^2} = 2.5\text{ m}$$

$$\mathbf{r}_{BC} = \sqrt{2^2 + 1^2 + (-2)^2} = 3\text{ m}$$

Vector Dot Product:

$$\mathrm{r}_{BA} \cdot \mathbf{r}_{BC} = (-1.5\mathbf{i} - 2\mathbf{j}) \cdot (2\mathbf{i} + 1\mathbf{j} - 2\mathbf{k})$$

$$= (-1.5)(2) + (-2)(1) + 0(-2)$$

$$= -5\text{ m}^2$$

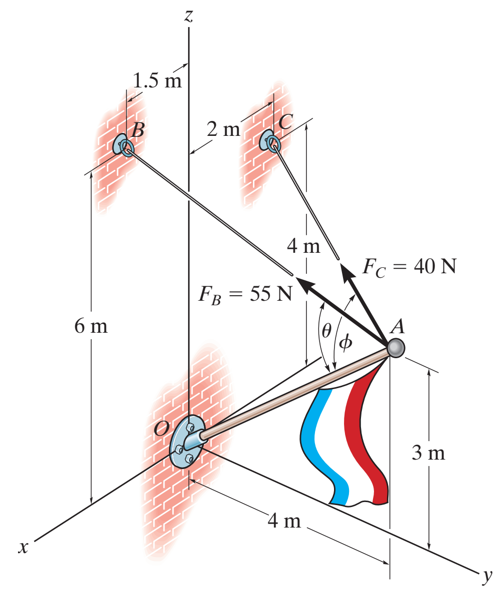
Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{\mathbf{r}_{BA}\,\mathbf{r}_{BC}}\right) = \cos^{-1}\left[\frac{-5}{2.5(3)}\right] = 132^\circ$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–131.

Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m ; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$ m

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4^\circ$$ **Ans.**

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

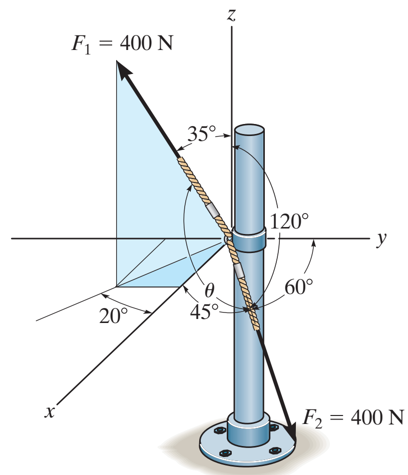
$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4^\circ$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–132.

The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \sin 35^\circ \cos 20^\circ \mathbf{i} - \sin 35^\circ \sin 20^\circ \mathbf{j} + \cos 35^\circ \mathbf{k}$$

$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{F}_1 = F_1 \mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k})\text{ N}$$

$$= \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\}\text{ N}$$

Unit Vector: The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k}$$

$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

Projected Component of $\mathbf{F}_1$ Along Line of Action of $\mathbf{F}_2$:

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$

$$= (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5)$$

$$= -50.6\text{ N}$$

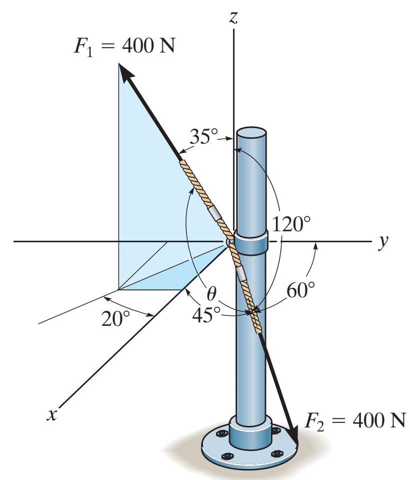
Negative sign indicates that the force component $(\mathbf{F}_1)F_2$ acts in the opposite sense of direction to that of $\mathbf{u}_{F2}$.

Thus the magnitude is $(F_1)_{F_2} = 50.6\text{ N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–133.

Determine the angle θ between the two cables attached to the post.



SOLUTION

Unit Vector:

$$\mathbf{u}_{F_1} = \sin 35° \cos 20° \mathbf{i} - \sin 35° \sin 20° \mathbf{j} + \cos 35° \mathbf{k}$$

$$= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 45° \mathbf{i} + \cos 60° \mathbf{j} + \cos 120° \mathbf{k}$$

$$= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}$$

The Angle Between Two Vectors θ: The dot product of two unit vectors must be determined first.

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k})$$

$$= 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5)$$

$$= -0.1265$$

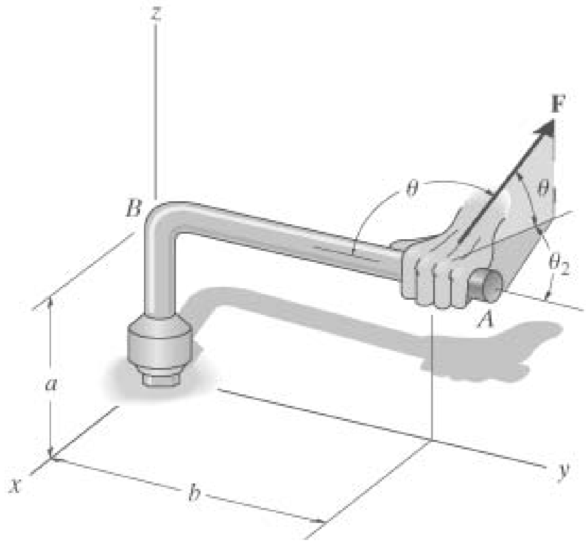
Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1265) = 97.3°$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–134.

Force **F** is applied to the handle of the wrench. Determine the angle θ between the tail of the force and the handle AB.



Given:

$a = 300\ \text{mm}$

$b = 500\ \text{mm}$

$F = 80\ \text{N}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 45\ \text{deg}$

Solution:

$$\mathbf{F_v} = F\begin{pmatrix} -\cos(\theta_1)\sin(\theta_2) \\ \cos(\theta_1)\cos(\theta_2) \\ \sin(\theta_1) \end{pmatrix} \qquad \mathbf{u_{ab}} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\theta = \text{acos}\left(\frac{\mathbf{F_v}\cdot\mathbf{u_{ab}}}{F}\right) \qquad \theta = 127.8\ \text{deg}$$

Ans.

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–135.

Determine the projected component of the force **F** acting along the axis *AB* of the pipe.

Given:

$F = 80\ \text{N}$

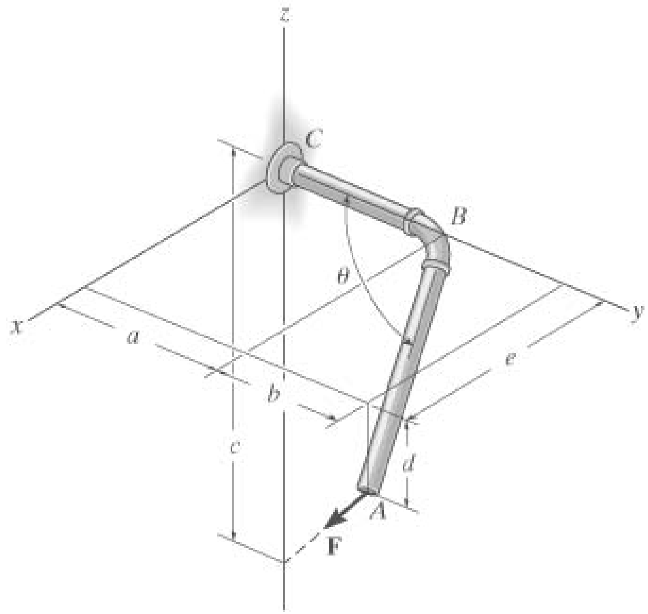
$a = 4\ \text{m}$

$b = 3\ \text{m}$

$c = 12\ \text{m}$

$d = 2\ \text{m}$

$e = 6\ \text{m}$



Solution:

Find the force and the unit vector

$$\mathbf{r_A} = \begin{pmatrix} -e \\ -a-b \\ d-c \end{pmatrix} \qquad \mathbf{r_A} = \begin{pmatrix} -6 \\ -7 \\ -10 \end{pmatrix}\ \text{m} \qquad \mathbf{F_v} = F\frac{\mathbf{r_A}}{|\mathbf{r_A}|} \qquad \mathbf{F_v} = \begin{pmatrix} -35.3 \\ -41.2 \\ -58.8 \end{pmatrix}\ \text{N}$$

$$\mathbf{r_{AB}} = \begin{pmatrix} -e \\ -b \\ d \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -6 \\ -3 \\ 2 \end{pmatrix}\ \text{m} \qquad \mathbf{u_{AB}} = \frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{u_{AB}} = \begin{pmatrix} -0.9 \\ -0.4 \\ 0.3 \end{pmatrix}$$

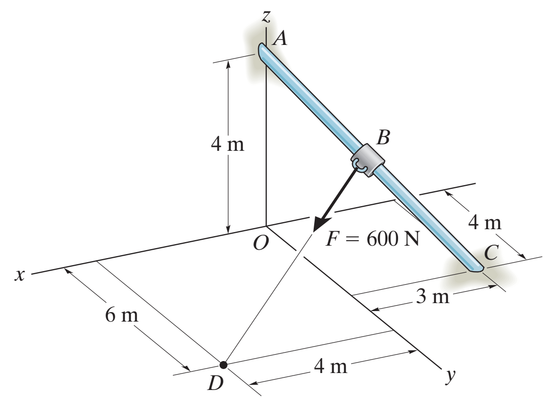
Now find the projection using the Dot product.

$F_{AB} = \mathbf{F_v} \cdot \mathbf{u_{AB}}$ $\qquad F_{AB} = 31.1\ \text{N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–136.

Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located at the midpoint of the rod.



SOLUTION

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \sqrt{41}\text{ m}$

$\mathbf{r}_{AB} = \dfrac{\mathbf{r}_{AC}}{2} = \dfrac{-3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$

$= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

$= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ m}$

$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887\text{ m}$

$\mathbf{F} = 600\left(\dfrac{\mathbf{r}_{BD}}{r_{BD}}\right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$F_{||} = \dfrac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \dfrac{(465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$

$F_{||} = 99.1408 = 99.1\text{ N}$ **Ans.**

Component of F perpendicular to $\mathbf{r}_{AC}$ is $F_{\perp}$

$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

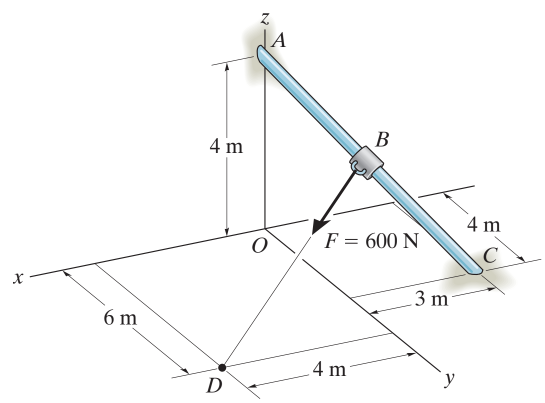
$F_{\perp}^2 = 600^2 - 99.1408^2$

$F_{\perp} = 591.75 = 592\text{ N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–137.

Determine the components of **F** that act along rod *AC* and perpendicular to it. Point *B* is located 3 m along the rod from end *C*.



SOLUTION

$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

$r_{CA} = 6.403124$

$\mathbf{r}_{CB} = \dfrac{3}{6.403124}(\mathbf{r}_{CA}) = 1.40556\mathbf{i} - 1.874085\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$

$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$

$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$

$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$

$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$

$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$

$r_{BD} = \sqrt{(5.5944)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$

$\mathbf{F} = 600\left(\dfrac{\mathbf{r}_{BD}}{r_{BD}}\right) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$

$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \qquad r_{AC} = \sqrt{41}$

Component of **F** along $\mathbf{r}_{AC}$ is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F}\cdot\mathbf{r}_{AC}}{r_{AC}} = \frac{(475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k})\cdot(-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$F_{||} = 82.4351 = 82.4 \text{ N}$ **Ans.**

Component of **F** perpendicular to $\mathbf{r}_{AC}$ is $\mathbf{F}_{\perp}$

$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$

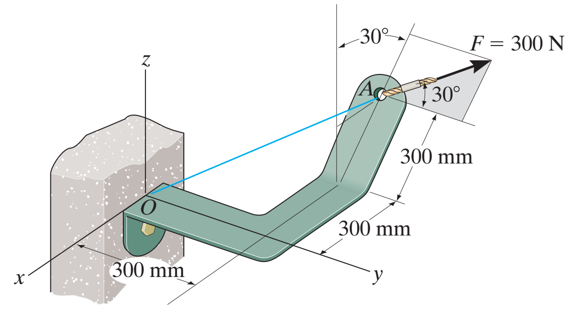
$F_{\perp}^2 = 600^2 - 82.4351^2$

$F_{\perp} = 594 \text{ N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–138.

Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{j}$$

$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260 \text{ N}$$

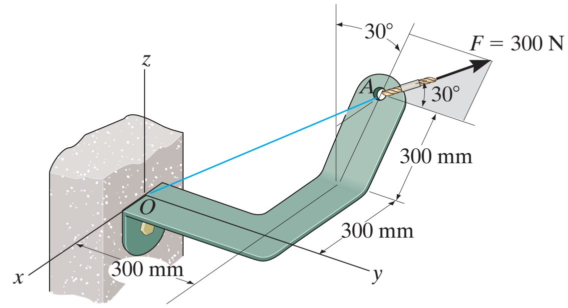
The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

$$F_x = 75 \text{ N}, \qquad F_y = 260 \text{ N}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–139.

Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

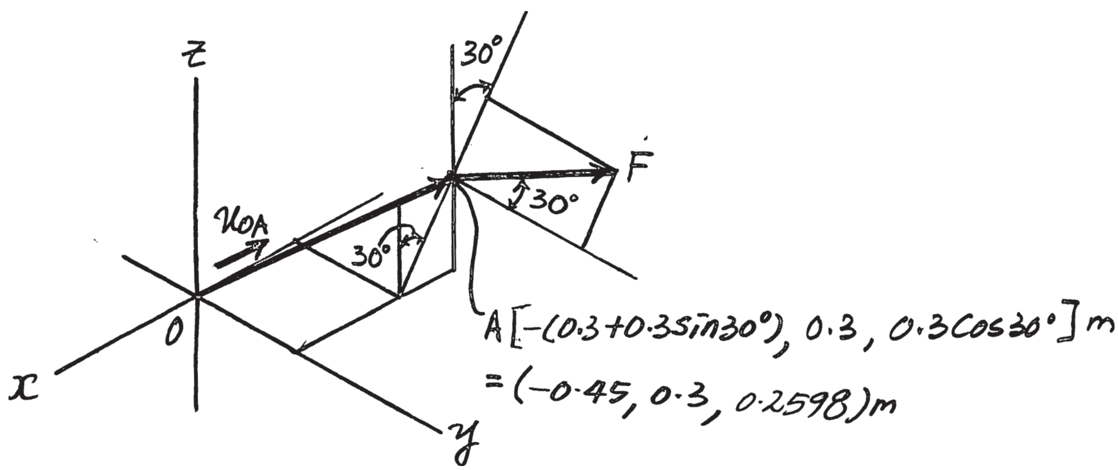
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line OA is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \left(-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}\right)$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

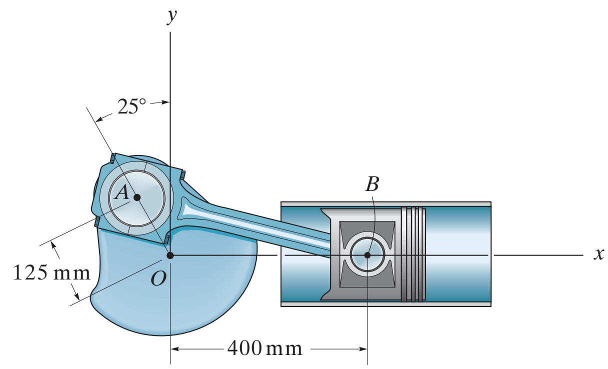
$$= 242 \text{ N}$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–140.

Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



SOLUTION

$$\mathbf{r}_{AB} = [400 - (-125 \sin 30°)]\mathbf{i} + (0 - 125 \cos 30°)\,\mathbf{j}$$

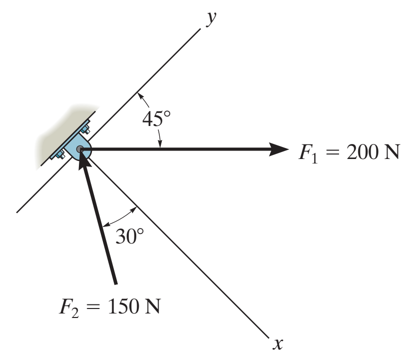
$$= \{462.5\,\mathbf{i} - 108.25\,\mathbf{j}\}\ \text{mm}$$

$$r_{AB} = \sqrt{(462.5)^2 + (108.25)^2} = 475\ \text{mm}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–141.

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

$F_{1x} = 200 \sin 45° = 141 \text{ N}$ **Ans.**

$F_{1y} = 200 \cos 45° = 141 \text{ N}$ **Ans.**

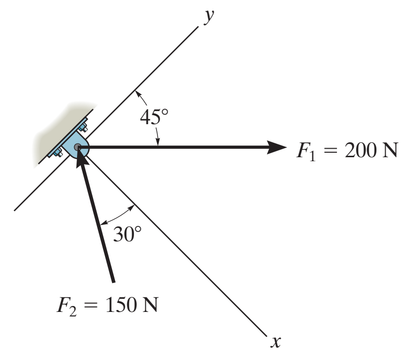
$F_{2x} = -150 \cos 30° = -130 \text{ N}$ **Ans.**

$F_{2y} = 150 \sin 30° = 75 \text{ N}$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–142.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$+\searrow F_{Rx} = \Sigma F_x;\qquad F_{Rx} = -150\cos 30° + 200\sin 45° = 11.518\text{ N}$

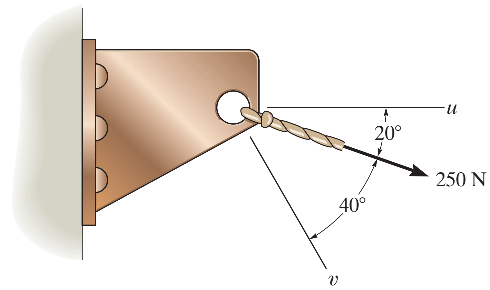
$\nearrow + F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 150\sin 30° + 200\cos 45° = 216.421\text{ N}$

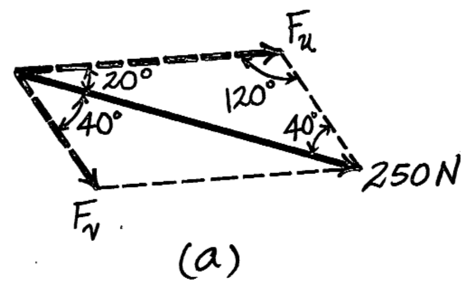
$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217\text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

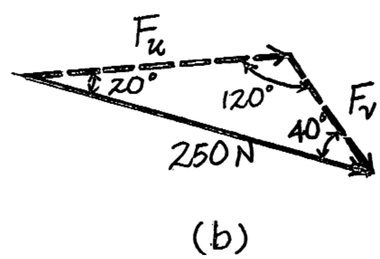
2–143. Resolve the 250-N force into components acting along the u and v axes and determine the magnitudes of these components.





$$\frac{250}{\sin 120°} = \frac{F_u}{\sin 40°}; \quad F_u = 186 \text{ N} \quad \textbf{Ans.}$$

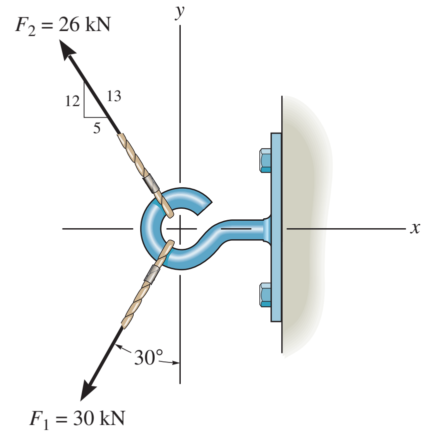
$$\frac{250}{\sin 120°} = \frac{F_v}{\sin 20°}; \quad F_v = 98.7 \text{ N} \quad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–144 .

Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = -30 \sin 30° \,\mathbf{i} - 30 \cos 30° \,\mathbf{j}$$

$$= \{-15.0\,\mathbf{i} - 26.0\,\mathbf{j}\} \text{ kN}$$ **Ans.**

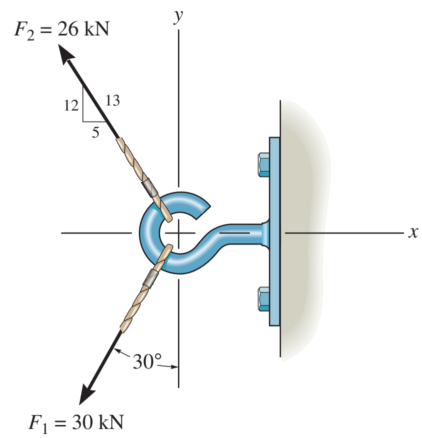
$$\mathbf{F}_2 = -\frac{5}{13}(26)\,\mathbf{i} + \frac{12}{13}(26)\,\mathbf{j}$$

$$= \{-10.0\,\mathbf{i} + 24.0\,\mathbf{j}\} \text{ kN}$$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–145.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

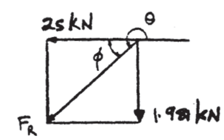
$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \quad F_{Rx} = -30 \sin 30° - \frac{5}{13}(26) = -25 \text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -30 \cos 30° + \frac{12}{13}(26) = -1.981 \text{ kN}$

$$F_R = \sqrt{(-25)^2 + (-1.981)^2} = 25.1 \text{ kN}$$ **Ans.**

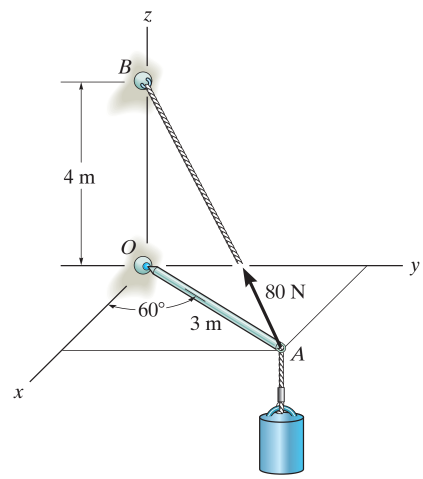
$$\phi = \tan^{-1}\left(\frac{1.981}{25}\right) = 4.53°$$

$$\theta = 180° + 4.53° = 185°$$ **Ans.**



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–146. Cable AB exerts a force of 80 N on the end of the 3-m-long boom OA. Determine the magnitude of the projection of this force along the boom.



Vector Analysis :

$$\mathbf{F} = 80\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 80\left(-\frac{3\cos 60^\circ}{5}\mathbf{i} - \frac{3\sin 60^\circ}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}\right)$$

$$= -24\mathbf{i} - 41.57\mathbf{j} + 64\mathbf{k}$$

$$\mathbf{u}_{AO} = -\cos 60^\circ\mathbf{i} - \sin 60^\circ\mathbf{j} = -0.5\mathbf{i} - 0.866\mathbf{j}$$

$$\text{Proj } F = \mathbf{F}\cdot\mathbf{u}_{AO} = (-24)(-0.5) + (-41.57)(-0.866) + (64)0 = 48.0\text{ N} \qquad \textbf{Ans.}$$

Scalar Anaysis :

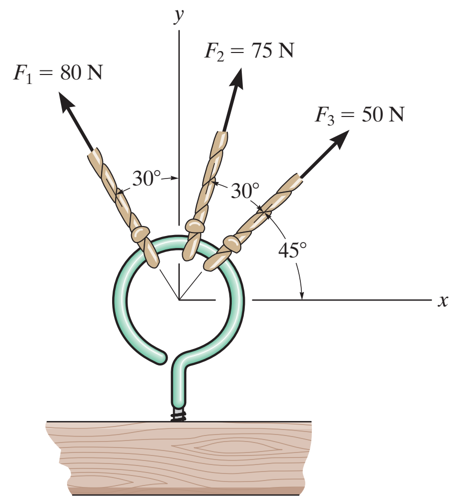
$$\text{Angle } OAB = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$$\text{Proj } F = 80\cos 53.13^\circ = 48.0\text{ N} \qquad \textbf{Ans.}$$

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–147.

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.



SOLUTION

$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50)\cos 105^\circ} = 104.7 \text{ N}$$

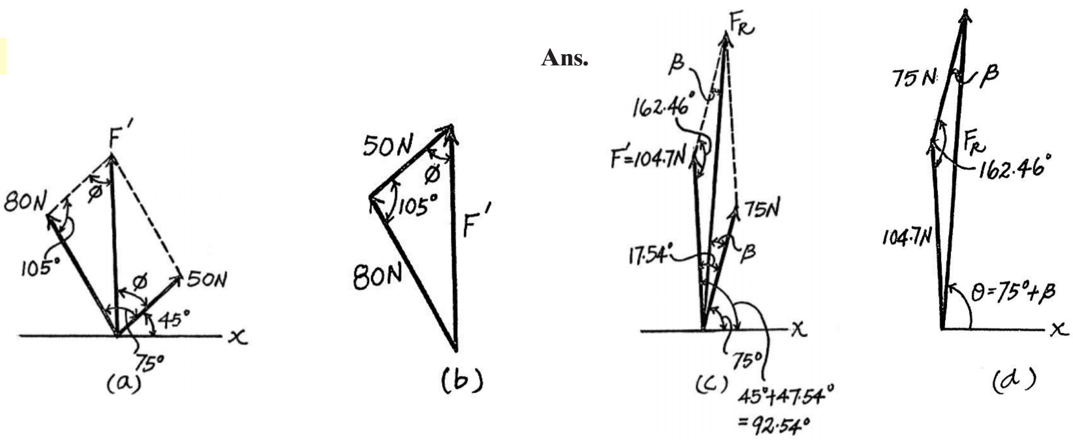
$$\frac{\sin\phi}{80} = \frac{\sin 105^\circ}{104.7}; \qquad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75)\cos 162.46^\circ}$$

$$F_R = 177.7 = 178 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{\sin\beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \qquad \beta = 10.23^\circ$$

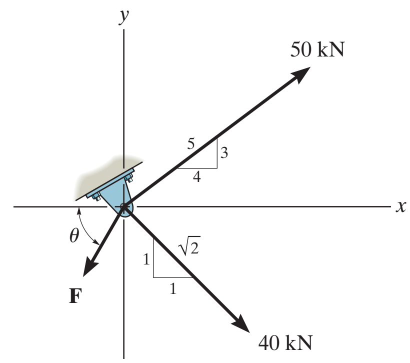
$$\theta = 75^\circ + 10.23^\circ = 85.2^\circ \qquad \textbf{Ans.}$$



© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–148.

If $\theta = 60°$ and $F = 20$ kN, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 50\left(\frac{4}{5}\right) + \frac{1}{\sqrt{2}}(40) - 20\cos 60° = 58.28 \text{ kN}$

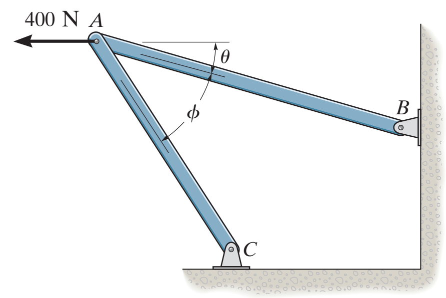
$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 50\left(\frac{3}{5}\right) - \frac{1}{\sqrt{2}}(40) - 20\sin 60° = -15.60 \text{ kN}$

$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3 \text{ kN}$ **Ans.**

$\phi = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0°$ **Ans.**

© Pearson Education South Asia Pte Ltd 2014. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

2–149. Determine the design angle θ $(0° \le \theta \le 90°)$ for strut AB so that the 400-N horizontal force has a component of 500 N directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.



Parallelogram Law **:** The parallelogram law of addition is shown in Fig. (a).

Trigonometry **:** Using law of sines [Fig. (b)], we have

$$\frac{\sin\theta}{500} = \frac{\sin 40°}{400}$$

$$\sin\theta = 0.8035$$

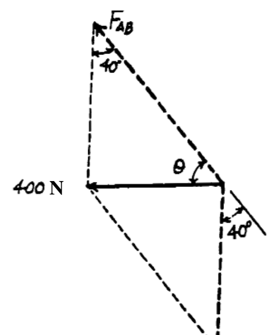
$$\theta = 53.46° = 53.5° \qquad \textbf{Ans.}$$

Thus, $\psi = 180° - 40° - 53.46° = 86.54°$

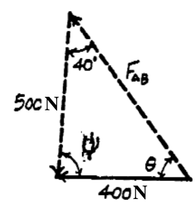
Using law of sines [Fig. (b)]

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

$$F_{AB} = 621 \text{ N} \qquad \textbf{Ans.}$$



(a)



(b)