Solar Energy Engineering

Processes and Systems

Second Edition

By:

Soteris A. Kalogirou

SOLUTIONS MANUAL

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for

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CONTENTS

CHAPTER 2

2.3 Calculate the solar declination for the spring and fall equinoxes and the summer and winter solstices.

Using Table 2.1 to estimate day number and Eq. (2.5) the following can be estimated: Spring equinox is on March 22, day number is 81 and from Eq. (2.5):

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + N)\right] = 23.45 \sin\left[\frac{360}{365}(284 + 81)\right] = 0°$$

Similarly:
Fall equinox is on September 21, day number is 264 and $\delta=0°$
Summer solstice is on June 21, day number is 172 and $\delta=23.45°$
Winter solstice is on December 21, day number is 355 and $\delta= -23.45°$

2.4 Calculate the sunrise and sunset times and day length for the spring and fall equinoxes and the summer and winter solstices at 45°N latitude and 35°E longitude.

The standard longitude is 30°, therefore longitude correction from Eq. (2.3) is -4(30-35) = 20 min.

Spring equinox is on March 22, therefore from Table 2.1, day number is 81. Using Eq. (2.2):

$$B = (N - 81)\frac{360}{364} = (81 - 81)\frac{360}{364} = 0$$

From Eq. (2.1): ET=9.87 sin(2B)-7.53 cos(B)-1.5 sin(B) = 9.87 sin(0)-7.53 cos(0)-1.5 sin(0) = -7.53≈-8 min.

From Exercise 2.3, δ = 0° and from Eq. (2.17): Day Length = 2/15 $\cos^{-1}$[-tan(L) tan(δ)]
Or Day Length = 2/15 $\cos^{-1}$[-tan(45) tan(0)] = 12 hours.

Since solar noon is at the middle of the sunrise and sunset hours, it means that they are 12/2 = 6 hours away from local solar noon. Therefore, sunrise is at 12:00-6:00+0:08-0:20 = 5:48, sunset time is 12:00+6:00+0:08-0:20 = 17:48.

Similarly:

For fall equinox ET =+7.9≈8 min. Day length =12 hours. Therefore, sunrise is at 12:00-6:00-0:08-0:20 = 5:32, sunset time is 12:00+6:00-0:08-0:20 = 17:32.

For summer solstice ET =-1.5≈-2 min. Day length = 15.43 hours (15:26). Therefore, sunrise is at 12:00-7:43+0:02-0:20 = 3:59, sunset time is 12:00+7:43+0:02-0:20 = 19:25.

Winter solstice ET =+1 min. Day length = 8.57 hours (8:34). Therefore, sunrise is at 12:00-4:17-0:01-0:20 = 7:22, sunset time is 12:00+4:17-0:01-0:20 = 15:56.

2.5 Determine the solar altitude and azimuth angles at 10:00 am local time for Rome, Italy on June 10.

The geographic coordinates (latitude and longitude) of Rome can be obtained from http://www.infoplease.com/ipa/A0001769.html. Therefore, Rome is at latitude of 41.54° and longitude of 12.27°E. From Table 2.1, on June 10, day number is 161. Therefore, from Eq. (2.5) $\delta=23°$. From Eq. (2.1) on June 10, ET = 0.76 min ≈ 1 min. From Eq. (2.3), AST = LST+ET±4 (SL - LL) = 10:00+0:01-4(15-12.27) = 9:50 am (9.83 hr).

At 9:50 am the hour angle from Eq. (2.9), is: h=(9.83-12)*15 = -32.55°.

From Eq. (2.12): $\sin(\alpha) = \sin(L)\sin(\delta)+\cos(L)\cos(\delta)\cos(h) = \sin(41.54)\sin(23)+\cos(41.54)\cos(23)\cos(-32.55) = 0.8398 \rightarrow \alpha = 57.1°$.

From Eq. (2.13): $\sin(z) = \frac{\cos(\delta)\sin(h)}{\cos(\alpha)} = \frac{\cos(23)\sin(-32.55)}{\cos(57.1)} = -0.912 \rightarrow z = -65.8°$ (negative sign shows that the angle is eastward, which is obvious as 10:00 am is before noon).

2.6 Calculate the solar zenith and azimuth angles, the sunrise and sunset times and the day length for Cairo, Egypt, at 10:30 am solar time on April 10.

From the web site given in Exercise 2.5, Cairo is at latitude of 30° and longitude of 31°E. From Table 2.1, on April 10, day number is 100. Therefore from Eq. (2.5) $\delta=7.5°$. At 10:30 am hour angle is from Eq. (2.9), h=(10.5-12)*15 = -22.5°. From Eq. (2.12):
$\cos(\Phi) = \sin(30)\sin(7.5)+\cos(30)\cos(7.5)\cos(-22.5) = 0.858 \rightarrow \Phi = 30.8°$ and from Eq. (2.11) $\alpha = 59.2°$.

From Eq. (2.13): $\sin(z) = \frac{\cos(7.5)\sin(-22.5)}{\cos(59.2)} = -0.741 \rightarrow z = -47.8°$.

From Eq. (2.1) ET = -1.6 min ≈ -2 min. For Cairo the standard longitude is 30°, therefore longitude correction from Eq. (2.3) is -4(30-31) = 4 min.

From Eq. (2.17) day length = 12.58 hours (12:35). Therefore, sunrise is at 12:00-6:18-0:02+0:04 = 5:44, sunset time is 12:00+6:18-0:02+0:04 = 18:20.

2.7 Calculate the sunrise and sunset times and altitude and azimuth angles for London, England, on March 15 and September 15 at 10:00 am and 3:30 pm solar times.

From the web site given in Exercise 2.5, London is at latitude of 51° and longitude at 0°. From Table 2.1, on March 15, day number is 74 and on September 15 is 258. Therefore from Eq. (2.5) on March 15, $\delta=-2.8°$ and on September 15, $\delta=2.2°$. At 3:30 pm hour angle is from Eq. (2.9), h=(15.5-12)*15 = 52.5°. Similarly at 10:00 am h = -30°.

From Eq. (2.12) at 10:00 am on March 15, $\alpha = 30.4°$ and on September 15, $\alpha = 35.1°$. At 3:30 pm on March 15, $\alpha = 20.1°$ and on September 15, $\alpha = 24.4°$.

From Eq. (2.13) at 10:00 am on March 15, z = -35.4° and on September 15, z = -37.6°. At 3:30 pm on March 15, z = 57.6° and on September 15, z = 60.5°.

From Eq. (2.1) on March 15, ET = -9.7 min ≈ -10 min and on September 15, ET = 5.7 min ≈ 6 min. London is at longitude of 0°, therefore no longitude correction is required.

From Eq. (2.17) on March 15 day length = 11.54 hours (11:32). Therefore, sunrise is at 12:00-5:46+0:10 = 6:24, sunset time is 12:00+5:46+0:10 = 17:56.

From Eq. (2.17) on September 15 day length = 12.37 hours (12:22). Therefore, sunrise is at 12:00-6:11-0:06 = 5:43, sunset time is 12:00+6:11-0:06 = 18:05.

2.8 What is the solar time in Denver, Colorado, on June 10 at 10:00 am Mountain Standard Time?

The standard meridian for Mountain Standard Time is 105°. From the web site given in Exercise 2.5, Denver is at latitude of 39° and longitude at 105°. So, no longitude correction is required. From Table 2.1, on June 10, day number is 161. From Eq. (2.1) on June 10, ET = 0.76 min ≈ 1 min. Therefore, from Eq. (2.3) AST = LST+ET = 10:00+0:1=10:01 am.

2.9 A flat-plate collector in Nicosia, Cyprus, is tilted at 40° from horizontal and pointed 10° east of south. Calculate the solar incidence angle on the collector at 10:30 am and 2:30 pm solar times on March 10 and September 10.

Latitude of Nicosia, Cyprus is 35°N. From Table 2.1, on March 10, day number is 69 and on September 10 is 253. Therefore, from Eq. (2.5) on March 10, δ=-4.8° and on September 10, δ=4.2°. At 2:30 pm hour angle is from Eq. (2.9), h=(14.5-12)*15 = 37.5°. Similarly at 10:30 am h = -22.5°. Using Eq. (2.18):

On March 10 at 10:30 am, θ = 16°. At 2:30 pm θ = 43.8°
On September 10 at 10:30 am, θ = 18.3°. At 2:30 pm θ = 44.8°

2.10 A vertical surface in Athens, Greece, faces 15° west of due south. Calculate the solar incidence angle at 10:00 am and 3:00 pm solar times on January 15 and November 10.

Latitude of Athens, Greece is 38°. From Table 2.1, on January 15, day number is 15 and on November 10 is 314. Therefore, from Eq. (2.5) on January 15, δ=-21.3° and on November 10, δ = -17.9°. At 3:00 pm hour angle is from Eq. (2.9), h = (15-12)*15 = 45°. Similarly at 10:00 am, h = -30°. Using Eq. (2.19) for vertical surfaces:

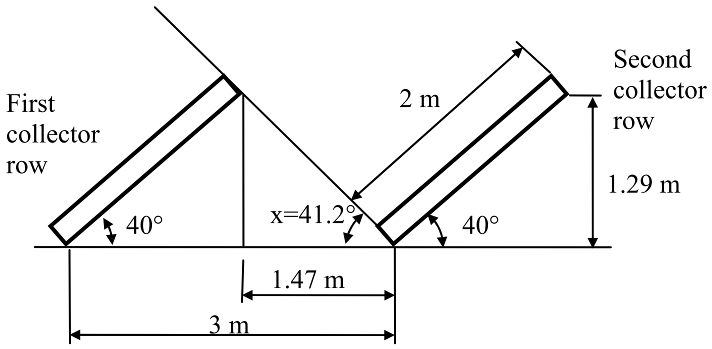
On January 15 at 10:00 am, θ = 50.5°. At 3:00 pm θ = 33.0°
On November 10 at 10:00 am, θ = 53.1°. At 3:00 pm θ = 36.1°

2.11 By using the sun path diagram find the solar altitude and azimuth angles for Athens, Greece on January 20 at 10:00 am.

To use the sun path diagram the declination is required. For January 20 (day number 20) δ=-20.3°. Latitude of Athens, Greece is 38° so can use the sun path diagram for 40°N latitude (see Appendix 3, Fig. A3.3) or interpolate with that at latitude 35°N shown in Fig. 2.17. The former is selected here. Therefore, α = 24° and z = -30° (actual values α = 25° and z = -31°).

2.12 Two rows of 6 m wide by 2 m high flat-plate collector arrays titled at 40° are facing due south. If these collectors are located in 35°N latitude, using the sun path diagram find the months of the year and the hours of day at which the front row will cast a shadow on the second row when the distance between the rows is 3 m. What should be the distance so there will be no shading?

For a collector length of 2 m and inclination of 40 degrees the actual height is 2xsin(40) = 1.29 m. The actual distance between the collectors is 3-2xcos(40) = 1.47 m. The angle x shown in Figure below is 41.2°. Using this value on Fig. 2.17 corresponds to a declination of -13° which form Fig. 2.7 corresponds from end of October to middle of February. From sun path diagram the minimum solar altitude at noon is 31.5°. Therefore the distance required to avoid shading completely is 1.29/tan(31.5) = 2.1 m or actual distance between collector rows is (3-1.47)+2.1 = 3.63 m.



2.13 Find the blackbody spectral emissive power at $\lambda = 8$ μm for a source at 400 K, 1000 K, and 6000 K.

Using Eq. (2.34) and the given data, we have:

At T=400K, $E_{b\lambda} = 128.22$ W/m^2-μm
At T=1000K, $E_{b\lambda} = 2260.27$ W/m^2-μm
At T=6000K, $E_{b\lambda} = 32623.37$ W/m^2-μm

2.14 Assuming that the sun is a blackbody at 5777 K. At what wavelength does the maximum monochromatic emissive power occur? What fraction of energy from this source is in the visible part of the spectrum in the range 0.38–0.78 μm?

As is given in Chapter 2 by differentiating Eq. (2.34) and equating to zero, the maximum distribution is equal to $\lambda_{max}T = 2897.8$ μm-K. Therefore as T = 5777K, $\lambda_{max} = 0.502$ μm. This agrees with the maximum shown in Fig. 2.26.

$\lambda_1 T = 0.38x5777 = 2195.3$ μm-K
$\lambda_2 T = 0.78x5777 = 4506.1$ μm-K

From Table 2.4 the fraction of energy between zero and 2195.3 μm-K is 10.0% and the fraction from zero to 4506.1 μm-K is 56.5%. Therefore, the fraction of energy in the visible part of the spectrum is 56.1-10 = 46.1%.

2.15 What percentage of blackbody radiation for a source at 323K is in the wavelength region 6–15 μm?

From the given data we have:
$\lambda_1 T = 6x323 = 1938$ μm-K
$\lambda_2 T = 15x323 = 4845$ μm-K
From Table 2.4 the fraction of energy between zero and 1938 μm-K is 5.8% and the fraction from zero to 4845 μm-K is 61.4%. Therefore, the percentage of the blackbody radiation in the range 6 to 15 μm is 61.4-5.8 = 55.6%.

2.16 A 2 mm thick glass sheet has a refraction index of 1.526 and an extinction coefficient of 0.2 cm^{-1}. Calculate the reflectivity, transmissivity, and absorptivity of the glass sheet at 0°, 20°, 40°, and 60° incidence angles.

At <u>normal incidence</u>, $\theta_1 = 0°$ and $\theta_2 = 0°$. From Eq. (2.51) the transmittance can be obtained as

$$\tau_a = e^{\left(-\frac{KL}{\cos(\theta_2)}\right)} = e^{\left(-\frac{0.2(0.2)}{\cos(0)}\right)} = 0.961$$

There is no polarization at normal incidence therefore from Eq. (2.49):

$$r_{(0)} = r_{\perp} = r_{\parallel} = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{1.526-1}{1.526+1}\right)^2 = 0.043$$

Therefore from Eqs. (2.52a)-(2.52c) we have:

$$\tau = \tau_\alpha \frac{1-r_{(0)}}{1+r_{(0)}}\left(\frac{1-r_{(0)}{}^2}{1-\left(r_{(0)}\tau_\alpha\right)^2}\right) = 0.961\left[\frac{1-0.043}{1+0.043}\left(\frac{1-0.043^2}{1-(0.043x0.961)^2}\right)\right] = 0.882$$

$$\rho = r_{(0)}\left(1+\tau_\alpha\tau_{(0)}\right) = 0.043(1+0.961x0.882) = 0.079$$

$$\alpha = \left(1-\tau_\alpha\right)\left(\frac{1-r_{(0)}}{1-r_{(0)}\tau_\alpha}\right) = \left(1-0.961\right)\left(\frac{1-0.043}{1-0.043x0.961}\right) = 0.039$$

At <u>angle of incidence = 20°</u>:

From Eq. (2.44) the refraction angle θ_2 is calculated as:

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right) = \sin^{-1}\left(\frac{\sin(20)}{1.526}\right) = 13°$$

From Eq. (2.51) the transmittance can be obtained as:

$$\tau_a = e^{\left(-\frac{KL}{\cos\theta_2}\right)} = e^{\left(-\frac{0.2(0.2)}{\cos(13)}\right)} = 0.960$$

From Eqs. (2.45) and (2.46):

$$r_{\perp} = \frac{\sin^2(\theta_2-\theta_1)}{\sin^2(\theta_2+\theta_1)} = \frac{\sin^2(13-20)}{\sin^2(13+20)} = 0.050$$

$$r_{\parallel} = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} = \frac{\tan^2(13-20)}{\tan^2(13+20)} = 0.036$$

From Eqs. (2.52a)-(2.52c) we have:

$$\tau = \frac{0.960}{2}\left[\frac{1-0.050}{1+0.050}\left(\frac{1-0.050^2}{1-(0.050\text{x}0.960)^2}\right) + \frac{1-0.036}{1+0.036}\left(\frac{1-0.036^2}{1-(0.036\text{x}0.960)^2}\right)\right]$$

$$= 0.480(0.905 + 0.930) = 0.881$$

$$\rho = 0.5\left[0.050(1+0.960\text{x}0.905) + 0.036(1+0.960\text{x}0.930)\right] = 0.081$$

$$\alpha = \frac{(1-0.960)}{2}\left(\frac{1-0.050}{1-0.050\text{x}0.960} + \frac{1-0.036}{1-0.036\text{x}0.960}\right) = 0.04$$

To avoid showing all the numerical calculation only the answers are given for the other two cases. For θ=40°; θ_2=24.9°, τ_a=0.957, $r_{\perp}$ =0.083, $r_{\parallel}$ =0.016, τ=0.868, ρ=0.091 and α=0.043. For θ=60°; θ_2=34.6°, τ_a=0.953, $r_{\perp}$ =0.185, $r_{\parallel}$ =0.001, τ=0.802, ρ=0.154 and α=0.047.

2.17 A flat-plate collector has an outer glass cover of 4 mm thick K = 23 m^{-1} and refractive index of 1.526, and a tedlar inner cover with refractive index of 1.45. Calculate the reflectivity, transmissivity, and absorptivity of the glass sheet at a 40° incidence angle by considering tedlar to be of a very small thickness; i.e., absorption within the material can be neglected.

From Eq. (2.44) the refraction angle θ_2 is calculated as:

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right) = \sin^{-1}\left(\frac{\sin(40)}{1.526}\right) = 24.9°$$

From Eq. (2.51) the transmittance can be obtained as:

$$\tau_a = e^{\left(-\frac{KL}{\cos\theta_2}\right)} = e^{\left(-\frac{23(0.004)}{\cos(24.9)}\right)} = 0.904$$

From Eqs. (2.45) and (2.46):

$$r_{\perp} = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} = \frac{\sin^2(24.9-40)}{\sin^2(24.9+40)} = 0.083$$

$$r_{\parallel} = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} = \frac{\tan^2(24.9-40)}{\tan^2(24.9+40)} = 0.016$$

From Eqs. (2.52a)-(2.52c) we have:

$$\tau_{\perp} = \tau_{\alpha}\frac{1-r_{\perp}}{1+r_{\perp}}\left(\frac{1-r_{\perp}^2}{1-(r_{\perp}\tau_{\alpha})^2}\right) = 0.904\left[\frac{1-0.083}{1+0.083}\left(\frac{1-0.083^2}{1-(0.083\text{x}0.904)^2}\right)\right] = 0.765$$

$$\tau_{\parallel} = \tau_{\alpha}\frac{1-r_{\parallel}}{1+r_{\parallel}}\left(\frac{1-r_{\parallel}^2}{1-(r_{\parallel}\tau_{\alpha})^2}\right) = 0.904\left[\frac{1-0.016}{1+0.016}\left(\frac{1-0.016^2}{1-(0.016\text{x}0.904)^2}\right)\right] = 0.875$$

$$\rho_\perp = r_\perp(1+\tau_\alpha\tau_\perp) = 0.083(1+0.904x0.765) = 0.140$$
$$\rho_\| = r_\|(1+\tau_\alpha\tau_\|) = 0.016(1+0.904x0.875) = 0.029$$

$$\alpha_\perp = (1-\tau_\alpha)\left(\frac{1-r_\perp}{1-r_\perp\tau_\alpha}\right) = (1-0.904)\left(\frac{1-0.083}{1-0.083x0.904}\right) = 0.095$$
$$\alpha_\| = (1-\tau_\alpha)\left(\frac{1-r_\|}{1-r_\|\tau_\alpha}\right) = (1-0.904)\left(\frac{1-0.016}{1-0.016x0.904}\right) = 0.096$$

To avoid repeating the same calculations the results for the tedlar cover are: $r_\perp = 0.067$, $r_\| = 0.011$, $\tau_\perp = 0.875, \tau_\| = 0.977$, $\rho_\perp = 0.126, \rho_\| = 0.022$, $\alpha_\perp = 0$, $\alpha_\| = 0$. These were obtained by considering $\tau_\alpha = 1.0$ as thickness is very small. The angle θ_2 is 26.3° estimated from Eq. (2.44) using n = 1.45 (given).

Finally, Eqs. (2.56) and (2.57) can be used as follows:

$$\tau = \frac{1}{2}\left[\left(\frac{\tau_1\tau_2}{1-\rho_1\rho_2}\right)_\perp + \left(\frac{\tau_1\tau_2}{1-\rho_1\rho_2}\right)_\|\right] = \frac{1}{2}\left[\left(\frac{0.765x0.875}{1-0.14x0.126}\right) + \left(\frac{0.875x0.977}{1-0.029x0.022}\right)\right] = 0.768$$

$$\rho = \frac{1}{2}\left[\left(\rho_1 + \frac{\tau\rho_2\tau_1}{\tau_2}\right)_\perp + \left(\rho_1 + \frac{\tau\rho_2\tau_1}{\tau_2}\right)_\|\right] = \frac{1}{2}\left[\begin{array}{l}\left(0.14 + \frac{0.768x0.126x0.765}{0.875}\right) \\ + \left(0.029 + \frac{0.768x0.022x0.875}{0.977}\right)\end{array}\right] = 0.134$$

$\alpha = 1 - \tau - \rho$ = 1-0.768-0.134 = 0.098

2.18 The glass plate of a solar greenhouse has a transmissivity of 0.90 for wavelengths between 0.32 and 2.8 μm and is completely opaque at shorter and longer wavelengths. If the sun is a blackbody radiating energy to the earth's surface at an effective temperature of 5770 K and the interior of the greenhouse is at 300 K, calculate the percent of incident solar radiation transmitted through the glass and the percent of thermal radiation emitted by the interior objects that is transmitted out.

The incoming solar radiation at 5770K we have:

$\lambda_1 T$ = 0.32x5770 = 1846 μm-K
$\lambda_2 T$ = 2.8x5770 = 16156 μm-K

From Table 2.4 we get:

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.453 = 45.3\%$$
$$\frac{E_b(0 \to \lambda_2 T)}{\sigma T^4} = 0.974 = 97.4\%$$

Therefore the percent of solar radiation incident on the glass in the wavelength range 0.32 to 2.8 μm is:

$$\frac{E_b(\lambda_1 T \to \lambda_2 T)}{\sigma T^4} = 97.4 - 45.3 = 52.1\%$$

The percentage of radiation transmitted through the glass is 0.9x52.1 = 46.9%.

For the outgoing infrared radiation at 300K we have:

$\lambda_1 T = 0.32x300 = 96$ μm-K
$\lambda_2 T = 2.8x300 = 840$ μm-K

From Table 2.4 we get:

$$\frac{E_b(0 \rightarrow \lambda_1 T)}{\sigma T^4} = 0.0 = 0\%$$

$$\frac{E_b(0 \rightarrow \lambda_2 T)}{\sigma T^4} = 0.0001 = 0.01\%$$

The percentage of radiation emitted is 0.9x0.01=0.009%

2.19 A 30 m^2 flat plate solar collector is absorbing radiation at a rate of 900 W/m^2. The environment temperature is 25°C and the collector emissivity is 0.85. Neglecting conduction and convection losses, calculate the equilibrium temperature of the collector and the net radiation exchange with the surroundings.

To be in equilibrium the collector losses energy at the rate it absorbs energy. The energy emitted from the collector can be obtained from Eq. (2.71) as: $Q_{12} = A_1 h_r (T_1 - T_2)$. The radiation heat transfer coefficient is given by Eq. (2.75): $h_r = \varepsilon_1 \sigma (T_1 + T_2)(T_1^2 + T_2^2)$. So we have two equations with two unknowns T_1 and h_r. Using the data given T_1=403.7K =130.7°C and h_r=8.515 W/m^2-K. The net radiation exchanged with the surroundings is 900x30=27 kW, estimated also from $Q = A\varepsilon\sigma(T_1^4 - T_2^4)$.

2.20 Two large parallel plates are maintained at 500 K and 350 K, respectively. The hotter plate has an emissivity of 0.6 and the colder one 0.3. Calculate the net radiation heat transfer between the plates.

Using Eq. (2.67):

$$Q/A = \frac{\sigma(T_1^4 - T_2^4)}{(1/\varepsilon_1) + (1/\varepsilon_2) - 1} = \frac{5.67x10^{-8}(500^4 - 350^4)}{(1/0.6) + (1/0.3) - 1} = 673.2 \ W/m^2$$

2.21 Find the direct normal and horizontal extraterrestrial radiation at 2:00 pm solar time on February 21 for 40°N latitude and the total solar radiation on an extraterrestrial horizontal surface for the day.

From Table 2.1 for February 21, day number = 52. From Eq. (2.5) on February 21, δ=-11.2°. At 2:00 pm h=30° and from Eq. (2.15) h_{ss}=80.4°. From Eq. (2.77):

$$G_{on} = G_{sc}\left[1 + 0.033\cos\left(\frac{360N}{365}\right)\right] = 1366\left[1 + 0.033\cos\left(\frac{360x52}{365}\right)\right] = 1394.2 \, W/m^2$$

From Eq. (2.12), cos(Φ)= sin(L) sin(δ)+cos(L) cos(δ) cos(h)
= sin(40) sin(-11.2)+cos(40) cos(-11.2) cos(30) = 0.526 or Φ=58.3°.

From Eq. (2.78): $G_{oH}=G_{on}x\cos(\Phi) = 1394.2x0.526 = 733.3$ W/m^2.

From Eq. (2.79):

$$H_o = \frac{24x3600x1366}{\pi}\left[1+0.033\cos\left(\frac{360x52}{365}\right)\right]\left(\begin{array}{l}\cos(40)\cos(-11.2)\sin(80.4)+\\ \left(\frac{\pi x80.4}{180}\right)\sin(40)\ \sin(-11.2)\end{array}\right) = 21.69\frac{MJ}{m^2}$$

2.22 Estimate the average hourly diffuse and total solar radiation incident on a horizontal surface for Rome, Italy, on March 10 at 10:00 am and 1:00 pm solar times if the monthly average daily total radiation is 18.1 MJ/m^2.

From Table 2.1 at March 10, N=69. At 10:00 am h=-30° and at 1:00 pm h=15°. From Eq. (2.5) δ=-4.8°. From Exercise 2.5, Rome is at latitude of 41.54°N. From Eq. (2.15) h_{ss}=85.7°. From Table 2.5 at 40°N latitude $\bar{H}_o$ = 27.4 MJ/m^2. From Eq. (2.82):

$\bar{K}_T = \frac{\bar{H}}{\bar{H}_o} = \frac{18.1}{27.4} = 0.661$. Using Eq. (2.105a):

$$\frac{\bar{H}_D}{\bar{H}} = 1.390 - 4.027(0.661) + 5.531(0.661)^2 - 3.108(0.661)^3 = 0.247$$

Therefore: $\bar{H}_D = 0.247\bar{H} = 0.247(18.1) = 4.47$ MJ/m^2-day

From Eq. (2.83) at 10:00 am:

$$r_d = \left(\frac{\pi}{24}\right)\frac{\cos(h)-\cos(h_{ss})}{\sin(h_{ss})-\left(\frac{2\pi h_{ss}}{360}\right)\cos(h_{ss})} = \left(\frac{\pi}{24}\right)\frac{\cos(-30)-\cos(85.7)}{\sin(85.7)-\left(\frac{2\pi 85.7}{360}\right)\cos(85.7)} = 0.117$$

Similarly at 1:00 pm. r_d = 0.132

From Eqs. (2.84b) and (2.84c):

α = 0.409 + 0.5016 sin(h_{ss}-60) = 0.409 + 0.5016 sin(85.7-60) = 0.626
β = 0.6609 - 0.4767 sin(h_{ss}-60) = 0.6609 - 0.4767 sin(85.7-60) = 0.454

From Eq. (2.84a) at 10:00 am:

$$r = \frac{\pi}{24}(\alpha+\beta\cos(h))\frac{\cos(h)-\cos(h_{ss})}{\sin(h_{ss})-\left(\frac{2\pi h_{ss}}{360}\right)\cos(h_{ss})}$$

$$= \frac{\pi}{24}(0.626+0.454\cos(-30))\frac{\cos(-30)-\cos(85.7)}{\sin(85.7)-\left(\frac{2\pi x85.7}{360}\right)\cos(85.7)} = 0.119$$

Similarly at 1:00 pm. r = 0.141

Therefore, at 10:00 am average hourly total radiation is 0.119x18.1 = 2.1539 MJ/m^2 = 2153.9 kJ/m^2 and average hourly diffuse radiation is 0.117x4.47 = 0.5230 MJ/m^2 = 523 kJ/m^2. At 1:00 pm average hourly total radiation is 0.141x18.1 = 2.5521 MJ/m^2 = 2552.1 kJ/m^2 and average hourly diffuse radiation is 0.132x4.47 = 0.590 MJ/m^2 = 590 kJ/m^2.

2.23 Calculate the beam and total radiation tilt factors and the beam and total radiation incident on a surface titled at 45° toward the equator one hour after local solar noon on

April 15. The surface is located at 40°N latitude and the ground reflectance is 0.25. For that day, the beam radiation at normal incidence is G_B = 710 W/m^2 and diffuse radiation on horizontal is G_D = 250 W/m^2.

From Table 2.1 for April 15, N=105 and δ=9.41°. For one hour after local solar noon h=15°. From Eq. (2.90a):

$$R_B = \frac{\sin(L-\beta)\sin(\delta)+\cos(L-\beta)\cos(\delta)\cos(h)}{\sin(L)\sin(\delta)+\cos(L)\cos(\delta)\cos(h)}$$
$$= \frac{\sin(40-45)\sin(9.41)+\cos(40-45)\cos(9.41)\cos(15)}{\sin(40)\sin(9.41)+\cos(40)\cos(9.41)\cos(15)} = 1.12$$

From the data given G = G_B+G_D = 710+250 = 960 W/m^2. From Eq. (2.99):

$$R = \frac{G_B}{G}R_B + \frac{G_D}{G}\left(\frac{1+\cos(\beta)}{2}\right)+\rho_G\left(\frac{1-\cos(\beta)}{2}\right)$$
$$= \frac{710}{960}1.12+\frac{250}{960}\left(\frac{1+\cos(45)}{2}\right)+0.25\left(\frac{1-\cos(45)}{2}\right)=1.09$$

Therefore, from Eq. (2.88) the beam radiation on tilted surface is G_{Bt} = $G_B R_B$ = 710x1.12 = 795 W/m^2. Similarly from Eq. (2.99) the total radiation on tilted surface is G_t = G.R = 960x1.09 = 1046 W/m^2.

2.24 For a south-facing surface located at 45°N latitude and tilted at 30° from horizontal, calculate the hourly values of the beam radiation tilt factor on September 10.

From Table 2.1 for September 10, N=253 and δ=4.22°. The estimation can be done using Eq. (2.90a) for each hour of the day as shown in previous Exercise. The actual estimation is done at the middle of each hour considered and the hour angle is estimated using Eq. (2.9). The results are shown in the following table.

Time	Actual time used in Eq. (2.90a)	h (°)	R_B
7-8	7:30	-67.5	1.204
8-9	8:30	-52.5	1.258
9-10	9:30	-37.5	1.281
10-11	10:30	-22.5	1.292
11-12	11:30	-7.5	1.297
12-13	12:30	7.5	1.297
13-14	13:30	22.5	1.292
14-15	14:30	37.5	1.281
15-16	15:30	52.5	1.258
16-17	16:30	67.5	1.204
17-18	17:30	82.5	1.005
18-19	18:30	97.5	2.666

2.25 A collector located in Berlin, Germany, is tilted at 50° and receives a monthly average daily total radiation $\overline{H}$ equal to 17 MJ/m^2-day. Determine the monthly mean beam and total radiation tilt factors for October for an area where the ground reflectance is 0.2. Estimate also the monthly average daily total solar radiation on the surface.

From the web site shown in Exercise 2.5 the latitude of Berlin Germany is 52.5°N. From Table 2.5 the monthly average daily total insolation on an extraterrestrial horizontal surface $\overline{H}_o$ is equal (by interpolation) to 15.4 MJ/m^2. Therefore from Eq. (2.82):

$$\overline{K}_T = \frac{\overline{H}}{\overline{H}_o} = \frac{17}{15.4} = 1.104$$

By considering the average day of October 15 (N=288) shown in Table 2.5 the declination is δ = -9.6°. Finally, the sunset hour angle from Eq. (2.15) is:

$$h_{ss} = \cos^{-1}[-\tan(L)\tan(\delta)] = \cos^{-1}[-\tan(52.5)\tan(-9.6)] = 77.3°$$

From Eq. (2.105b):

$$\frac{\overline{H}_D}{\overline{H}} = 0.775 + 0.00653(h_{ss} - 90) - \left[0.505 + 0.00455(h_{ss} - 90)\right]\cos(115\overline{K}_T - 103)$$
$$= 0.775 + 0.00653(77.3 - 90) - \left[0.505 + 0.00455(77.3 - 90)\right]\cos(115\text{x}1.104 - 103) = 0.283$$

From Eq. (2.109): $h'_{ss} = \min\{h_{ss}, \cos^{-1}[-\tan(L-\beta)\tan(\delta)]\}$
$= \min\{77.3, \cos^{-1}[-\tan(52.5-50)\tan(-9.6)]\} = \min\{77.3, 89.5\} = 77.3°$

From Eq. (1.108):

$$\overline{R}_B = \frac{\cos(L-\beta)\cos(\delta)\sin(h'_{ss}) + (\pi/180)h'_{ss}\sin(L-\beta)\sin(\delta)}{\cos(L)\cos(\delta)\sin(h_{ss}) + (\pi/180)h_{ss}\sin(L)\sin(\delta)}$$
$$= \frac{\cos(52.5-50)\cos(-9.6)\sin(77.3) + (\pi/180)77.3\sin(52.5-50)\sin(-9.6)}{\cos(52.5)\cos(-9.6)\sin(77.3) + (\pi/180)77.3\sin(52.5)\sin(-9.6)} = 2.337$$

Finally, from Eq. (2.107):

$$\overline{R} = \left(1 - \frac{\overline{H}_D}{\overline{H}}\right)\overline{R}_B + \frac{\overline{H}_D}{\overline{H}}\left(\frac{1+\cos(\beta)}{2}\right) + \rho_G\left(\frac{1-\cos(\beta)}{2}\right)$$
$$= (1-0.283)2.337 + 0.283\left(\frac{1+\cos(50)}{2}\right) + 0.2\left(\frac{1-\cos(50)}{2}\right) = 1.944$$

The average daily total radiation on the tilted surface for October is:
$\overline{H}_t = \overline{R}\,\overline{H} =$ 1.944(17) = 33 MJ/m^2-day