SOLUTIONS MANUAL FOR

SENSORS AND ACTUATORS

Engineering System Instrumentation

SECOND EDITION

by

Clarence W. de Silva

CRC Press
Taylor & Francis Group

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PREFACE

This manual is prepared primarily to assist the instructors who use the book *SENSORS AND ACTUATORS—Engineering System Instrumentation, 2nd Edition*. It includes hints for structuring the material for a course in the subject and provides complete solutions to the end of chapter problems of the textbook.

The book *SENSORS AND ACTUATORS—Engineering System Instrumentation, 2nd Edition*, introduces the subject of Engineering System Instrumentation, with an emphasis on sensors, transducers, actuators, and signal modification devices. Specifically, it deals with "instrumenting" an engineering system through the incorporation of suitable sensors, actuators, and associated interface hardware. It will serve as both a textbook for engineering students and a reference book for practicing professionals. As a textbook, it is suitable for courses in control system instrumentation; sensors and actuators; instrumentation of engineering systems; and mechatronics. There is adequate material in the book for two fourteen-week courses, one at the junior (third-year undergraduate) or senior (fourth-year undergraduate) level and the other at the first-year graduate level. In view of the practical considerations, design issues, and industrial techniques that are presented throughout the book, and in view of the simplified and snap-shot style presentation of more advanced theory and concepts, the book will serve as a useful reference tool for engineers, technicians, project managers, and other practicing professionals in industry and in research laboratories, in the fields of control engineering, mechanical engineering, electrical and computer engineering, manufacturing engineering, and mechatronics.

The material presented in the book serves as a firm foundation, for subsequent building up of expertise in the subject—perhaps in an industrial setting or in an academic research laboratory—with further knowledge of hardware, software, and analytical skills (along with the essential hands-on experience) gained during the process. Undoubtedly, for best results, a course in sensors and actuators, mechatronics, or engineering system instrumentation should be accompanied by a laboratory component and class projects.

Sensors are needed to measure (sense) unknown signals and parameters of an engineering system and its environment. This knowledge will be useful not only in operating or controlling the system but also for many other purposes such as process monitoring; experimental modeling (i.e., model identification); product testing and qualification; product quality assessment; fault prediction, detection and diagnosis; warning generation; and surveillance. Actuators are needed to "drive" a plant. As another category of actuators, *control actuators* perform control actions, and in particular they drive control devices. Since many different types and levels of signals are present in a dynamic system, *signal modification* (including signal conditioning and signal conversion) is indeed a crucial function associated with sensing and actuation. In particular, signal modification is an important consideration in component interfacing. It is clear that the subject of system instrumentation should deal with sensors, transducers, actuators, signal modification, and component interconnection. In particular, the subject should address the identification of the necessary system components with respect to type, functions, operation and interaction, and proper selection and interfacing of these components for various applications. Parameter selection (including component sizing and system tuning) is an important step as well. Design is a necessary part of system instrumentation, for it is design that enables us to build a system that meets the performance requirements—starting, perhaps, with a few basic components such as sensors, actuators, controllers, compensators, and signal modification devices. The main objective of the book is to provide a foundation in all these important topics of engineering system instrumentation.

A Note to the Instructors

A syllabus for a fourth year undergraduate course or a first year graduate course in the subject is given below.

CONTROL SENSORS AND ACTUATORS

Prerequisites

- For engineering graduate students: motivation
- For undergraduate students: A course in feedback controls
  + consent of the instructor

Introduction

Actuators are needed to perform control "actions" as well as to directly "drive" a plant (process, machine, engine). Sensors and transducers are necessary to "measure" output signals for *feedback control*, to "measure" input signals for *feedforward control*, to "measure" process variables for system monitoring, diagnosis and supervisory control, and for a variety of other purposes of measurement.

The course will study a selected set of sensors, actuators, and signal modification devices as employed in robotic and mechatronic systems. General and practical issues of sensors and actuators in an engineering system will be discussed. Operating principles, modelling, design considerations, ratings, specifications, selection, and applications of typical sensors and actuators will be studied. Filtering amplification, error analysis, and estimation from measured data will be covered as complementary topics.

Textbook

De Silva, C.W., *SENSORS AND ACTUATORS—Engineering System Instrumentation*, Taylor & Francis, 2nd Edition, Taylor & Francis/CRC Press, Boca Raton, FL, 2015.

Course Plan

Week	Starts on	Topic	Read
1	Jan. 06	Introduction	Chapter 1
2	Jan. 13	Performance Specification, Instrumentation of Engineering Systems	Chapter 3
3	Jan. 20	Component Matching, Amplifiers, Filters, and Other Interface Hardware	Chapter 2
4	Jan. 27	Estimation from Measured Data	Chapter 4
5	Feb. 03	Analog Motion Sensors	Chapter 5
6	Feb. 10 **Project proposals due.**	Torque and Force Sensors	Chapter 5
7	Feb. 17	Digital Motion Sensors, Tactile Sensors, and Innovative Sensors	Chapter 6
8	Feb. 24	Mechanical Transmission Devices	Chapter 7
9	Mar. 02	Stepper Motors	Chapter 8
10	Mar. 09	DC and AC Motors	Chapter 9
11	Mar. 16 **(Exam on Mar. 16)**	Hydraulic Actuators	Chapter 9
12	Mar. 23	Review	Chapters 1-9
13	Mar. 30	**Project presentations.**	
14	Apr. 6	**Project presentations.**	

***Note*: Final Take-Home Exam/Project Report due on April 12th.**

Grade Composition

Intermediate exam	=	30%
Project proposal	=	10%
Attendance/Participation	=	10%
Final Take-Home Exam/Project	=	50%
		100%
		====

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Chapter 1 Instrumentation of an Engineering System

Solution 1.1

Open-Loop Control System

This does not use information on current response of the "plant" to establish the control action. E.g., so-called feed-forward control of a robot arm. The joint torques (or motor input signals) are computed using a dynamic model of the robot (inverse plant) with desired angles of rotation as inputs. These signals drive the joint motors, which in turn produce the actual joint angles. In the open-loop case these are not measured and feedback.

Another example would be a household stove (gas or electric). The heat setting is manually selected. The actual heat flow is not measured.

Feedback Control System

This uses information on plant response to establish the control input. E.g., in feedback control of robot arms, joint angles (and angular velocities) are measured using suitable sensors (optical encoders, resolvers, pots, tachometers, RVDT's, etc.). This information is used in feedback to compute the control action.

In thermostatic control of temperature in a building, the temperature is measured, compared with the set point value (reference input) and the sign of the difference is used to turn on or shut off the heat source.

Simple Oscillator:

The oscillator (mass-spring-damper) is considered the plant in this case. The "apparent" feedback path (through k) is a "natural" feedback within the plant. The response y is not sensed and used to determine $f(t)$ to control the oscillator. Hence the system configuration is not a feedback control system.

If, however, mass m is considered the plant, then the spring can be interpreted as a "passive" feedback element. The spring "senses" the position of the mass and feeds back a force to restore the position of the mass. In this sense it is a (passive) feedback control system.

Solution 1.2

Lights On-off System for an Art Gallery

There are two essential measurements in this system

(a) Light intensity detection
(b) People count.

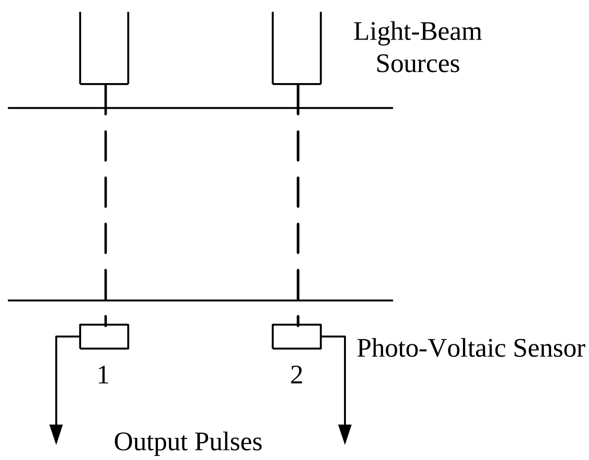
We should not measure the light intensity inside the gallery because there will be ambiguity as to the control action. Specifically, when the lights are on at night, the sensor would probably instruct the lights to be turned off thinking it is the day time because it could not differentiate between daylight and artificial light. To avoid this, a simple timer to indicate a rational time interval as the night time (e.g., 7:00 p.m. - 12:00

midnight) could be used. Alternatively a photo-voltaic sensor could be installed outside the windows of the gallery or on top of a sunroof.

People count has to be made directionally (i.e., entering or leaving) at each door. Hence a pair of probes is needed. Force sensors on the floor, turnstile counters, or light-pulse sensors may be used for this purpose. For example, consider the following arrangement:
The light beams are generated by laser or LED visible-light sources. They are received by a pair of photo-voltaic cells. When the beam is intercepted for a short period of time, an output pulse is generated at the corresponding photo cell (see Figure S1.2(a). The order of the pulses determines the direction (entrance or exit) of travel.

Even though measurements are made in the system, this is essentially an "open-loop" control system, as clear from the system schematic diagram shown in Figure S1.2(b).

(a)



(b)

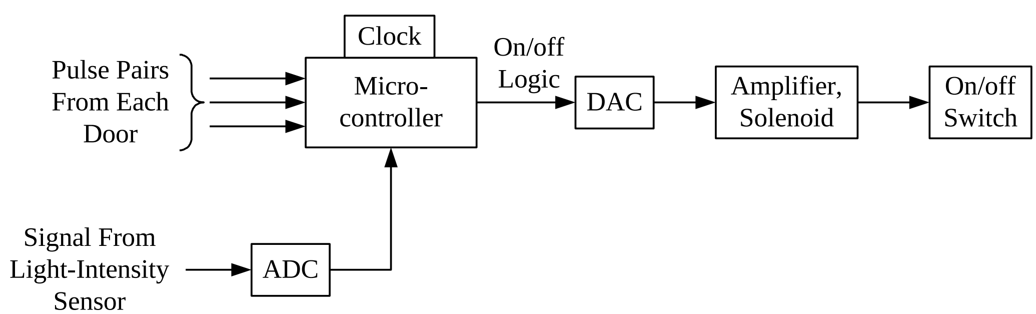


Figure S1.2: (a) People counting device; (b) Control system.

The system output is the on/off status of the switch controlling the gallery lights. Even though the number of people in the gallery is counted and the light intensity is measured to control the switch, the status of the switch is not used to control the number of people in the gallery, or the day-light intensity. Hence there is no feedback path.

The operation of the control system is straightforward. The pulse signals from each door are detected and timed. This determines the people entering and leaving. A count (COUNT) is kept. Furthermore, the light intensity (INT) is measured and compared with a desired level (INTD). A logic circuit can be developed to realize the following logic:

$$LOGIC = (COUNT.GT.0).AND.(INT.LT.INTD)$$

If this function is TRUE, the lights are turned on using a suitable actuator (e.g., a solenoid actuated by a current). Otherwise the lights are turned off.

Solution 1.3

Component	**Component Type**
Stepper Motor	Actuator
PID Circuit	Controller
Power Amp	Signal Modifier
ADC	Signal Modifier
DAC	Signal Modifier
Optical Encoder	Sensor/transducer
Process Computer	Controller
FFT Analyzer	Signal Modifier
DSP	Signal Modifier/Controller

Solution 1.4

(a) Modeling errors, system parameter variations, random disturbances
(b) Use feedback control.

Solution 1.5

Advantages of Analog Control:
Simple, extensive past experience is available, relatively easy to troubleshoot.

Disadvantages of Analog Control:
Assumes linear behavior (Coriolis and centrifugal forces, nonlinear damping, payload changes may be present, which are nonlinear)
Bulky and costly.
Difficult to implement complex control schemes.

Tuning and adaptation cannot be carried out in real time.
Not very flexible (not adaptable to different processes and process conditions).

Solution 1.6

The schematic diagram of an automated bottle-filling system is shown in Figure S1.6.

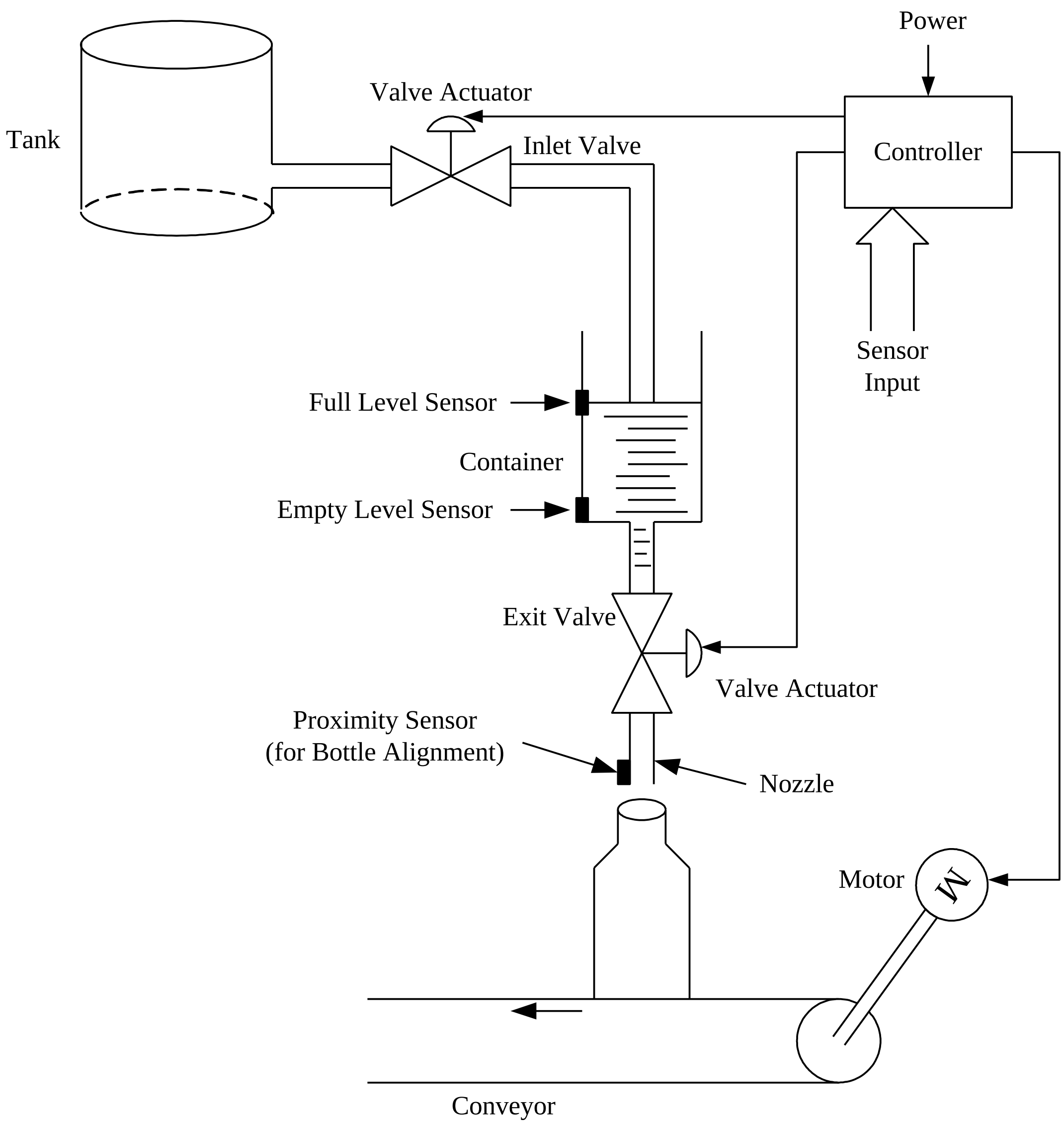


Figure S1.6: Schematic diagram of an automated bottle-filling system.

The operation of the automated bottle-filling system can be described by the following series of steps:

1. When the power is on, the controller checks the sensor input to see (1) is the filling container full, and (2) is there an empty bottle under the nozzle?
2. If the first condition is not satisfied, the inlet valve is opened to fill the container until "full container" signal from the corresponding sensor is received.
3. If only the first condition is satisfied, the motor is activated to move an empty bottle under the nozzle.
4. The motor is stopped when "bottle in position" is detected (from the proximity sensor).
5. The exit valve is opened to fill the bottle.
6. When "container is empty" signal is received (from empty level sensor) the exit valve is closed. The motor is turned on again and the conveyor moves away the filled bottle.
7. Go to Step 1. The whole process is repeated again and again until either power is off or the "process stop" command is received by the controller.

Solution 1.7

Note that one component may perform several functions.
Controller: Thermostat
Actuator: Valve actuator
Sensor: Thermocouple, pilot flame detector
Signal Modification: Transmitters and signal conditioning devices for thermostat signal to the valve, thermocouple signal, and pilot flame detector signal.

Operation: The thermocouple measures the room temperature, compares it with the set point, and determines the error (= set point - actual temperature). If the error is positive, a signal is transmitted to turn on the natural gas valve. If negative, the valve is turned off. The pilot flame detector checks if the pilot flame is off. If so it overrides the actuator signal and turns off the valve.

For better performance, measure the water flow rate, the inlet water temperature, and the outside temperature and incorporate a feedforward control as well as the original feedback scheme. In particular, the time delay in the process reaction can be considerably reduced by this method. Also a more sophisticated control scheme may be able to produce an improved temperature regulation, but it is not necessary in typical situations.

Solution 1.8

(a) Load torque (using a dynamometer), or armature current of the dc motor
(b) Input temperature of the liquid (using a hot-wire device)
(c) Flow rate of the liquid (using a flow meter); Temperature outside the room (using a thermocouple); Temperature of steam at radiator input
(d) Tactile forces at the gripper (using piezoelectric, capacitive or strain gauge sensors); Weight of the part to be picked up
(e) Torque transmitted at manipulator joints (using strain gauge torque sensor); Curvature of the seam contour (using image processing).

Solution 1.9

(a) Muscle contraction, body movements, body temperature, heart rate
(b) Decisions, profits, finished products
(c) Electric power, pollution rate.
(d) Front wheel turn, direction of heading, noise level, pollution level.
(e) Joint motions, position, velocity, acceleration, torque, end-effector motion.

Solution 1.10

Lowest level: 1ms
Highest level: $1 \text{ day} = 24 \times 60 \times 60 \text{sec}$

$$\Rightarrow \frac{1}{24 \times 60 \times 60} \text{Hz} = 11 \times 10^{-6} \text{ Hz}$$

By Shannon's sampling theorem, control bandwidth may be taken as half this value.

Solution 1.11

The key features of a modern day cost effective process controller are:

(i) Programmability	- This increases the flexibility of control by allowing different control algorithms to be implemented without the need to having to change any hardware.
(ii) Modularity	- Extensions or modifications to existing hardware is made least expensive by employing different modules of control units to carry out different tasks, rather than using an all-in-one approach. It also increases the reliability since the failure of one module does not affect the operation of others. Maintenance and repair become easier and faster.
(iii) General Purpose Hardware	- Use of such components allows replacement easier and inexpensive.

Solution 1.12

The programmable logic controller is an electronic device, which can switch on or off its outputs depending on the status of its inputs. The switching characteristics can be programmed to respond to almost any combination of input states. In Figure S1.12, a PLC is employed to sort fruits on a conveyor into various categories depending on their size and quality. At the feeding end of the conveyor is a camera, which captures images of the incoming fruits and sends them to the image processing station for analysis. The output of the image processor is a series of two state signals, each of which has a single attribute of either size or quality, associated with it. That is, for each sample of fruit, only one size attribute signal and only one quality attribute signal can be in the ON state, and all other attribute signals must be in the OFF state. The PLC is programmed to switch ON one of its outputs for a particular combination of its inputs. A series of such outputs drive, through amplifiers (not shown), separate solenoids, which control the output ports for fruits. The whole arrangement is synchronized by the PLC through a signal derived from an object sensing device on the conveyor.

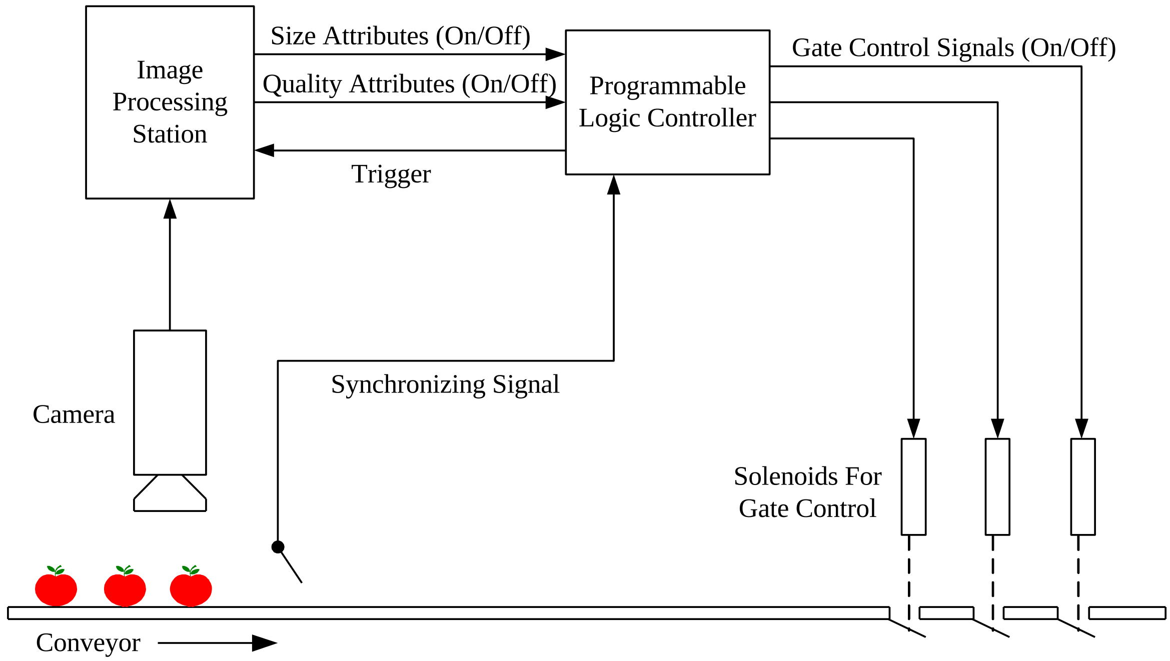


Figure S1.12: An automated grading system for fruit.

<u>Solution 1.13</u>

Measure inputs for feedforward control.

Measure outputs for system monitoring, failure detection, and diagnosis.

Measure signals for security (safety) reasons and to sound an alarm.

Measure outputs during the teach mode and store for use in the repeat mode, in teach/repeat applications.

1. Detonation sensor
2. Hot film mass-flow sensor
3. Crack sensor
4. Throttle position sensor
5. Cam sensor

6. Temperature sensor
7. Pressure sensor

Solution 1.14

A graphical representation of controller classification is given in Figure S1.14.

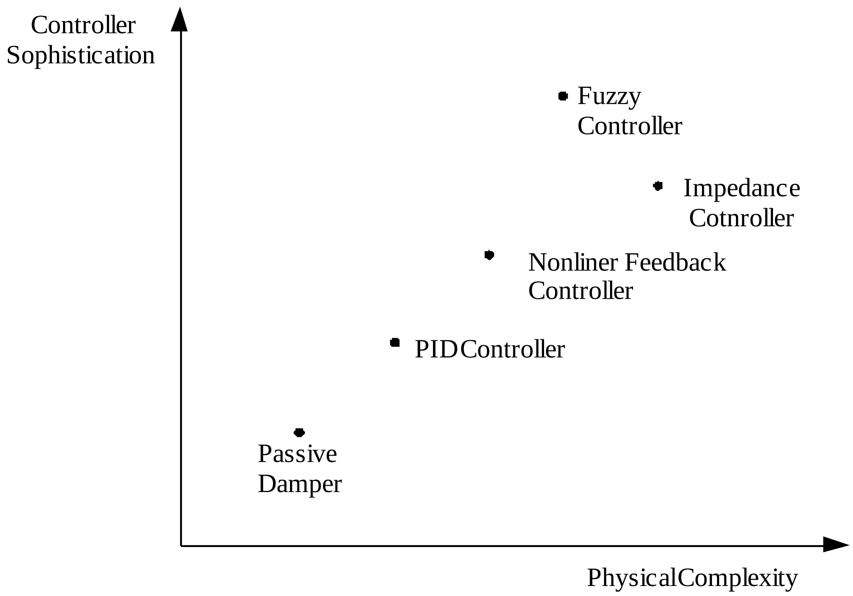


Figure S1.14: A graphical method of controller classification.

Solution 1.15

By digital it does not mean that X-ray is not used. It implies that since the X-ray images are digitized and enhanced, lower X-ray levels can be used to obtain the images. So, the 'digital" aspect enters not in the sensor but rather in the image representation and processing.

Solution 1.16

Plant: Wood Drying Kiln

Drying is the final process before the wood is available for general use, and to achieve the required serviceability in furniture manufacture, building, millwork and other wood product processes. The drying process is used to remove the moisture content of wood to assure high product quality, and is essential for imparting desirable properties to wood, including dimensional stability, workability, and hardening (e.g., as is required for tools), and promoting better absorption of treatments or adhesives. Properly dried wood provides a desirable surface texture as compared to wood that has not been dried, and can be machined or glued relatively easily. Moreover, drying of wood increases the strength, kills infestation, hardens pitch, preserves color, reduces weight (advantages in shipping and storing), and controls shrinkage. Fresh cut wood is dried in many different ways. The common commercial method is uses a wood-drying kiln, for accelerated drying.

Kilns are perhaps the only practical means of rapid and high-volume drying of fresh forest lumber. Kilns are controlled enclosures used to dry products like lumber, poles, and raw materials such as the veneered wood and core fiber used in plywood panels. Stacks of wood are placed in the drying chamber (kiln) and the heated air is circulated through them. Typically, rail-mounted platforms carry the wood material in and out of a kiln. The kiln chamber is then sealed and heat is applied by steam or direct-fired air. Sometimes pressure or a vacuum is introduced into the chamber, depending on the product. The flow, temperature and humidity of the air have to be properly controlled in order to produce good drying results.

Performance Requirements

Typical kiln temperatures range between 200 and 230 °F. While absolute estimates of the energy used in kiln drying are highly specific to the conditions of a given operation, engineering data indicate that steam applied and maintained at a temperature of near the 230 °F limit permitted by the American National Standards Institute standard will apply heat to a product surface at a potential rate of roughly 22,000 Btu per square inch. Drying times generally vary from 1 to 6 days. Longer drying times are required for wood that receives oilborne or preservative treatments. Subjective anecdotal information indicates that the energy required to dry about 500 cubic feet of lumber from an as-received condition to a 20 % wet basis moisture content is approximately 10 million Btu.

The specific application of wood is mainly determined by its final moisture content (m.c.) after drying. For example, an application like furniture making requires a final m.c. of 12% or lower. Quality of the dried lumber product is unpredictable, unreliable and non-repeatable. Kiln operators should frequently monitor the kiln operation and should make parameter adjustments as appropriate. Many years of experience would be required before an operator is given charge of carrying out these tasks. Problems can arise due to unattended operation during off-hours. The common practice of lowering the desired operating temperature during off-hours would lead to energy inefficiency. Also, an unexpected situation may occur during the unattended period, and may lead to undesirable defects in the drying boards. Furthermore, in view of the complexity, nonlinearity, and time-variant and distributed nature of the drying process, the quality of the dried wood may not be uniformly satisfactory in general. The fact that the drying results are unpredictable and that the entire process requires humans to close the control loop, provide an opportunity to use advanced technologies of sensing, actuation and control industrial kilns, with the goals of reducing the energy consumption and improving the quality of dried product.

About 65% of the $250 billion/year forest product sales is attributed to lumber and various wood products. Innovations in sensing, actuation, and control can result in significant reductions in energy usage in kilns. The study summarized here provides an indication of the technologies that are appropriate and the energy savings that are possible.

Constraints

A conventional wood-drying kiln basically consists of electric coils for heating the air, which is circulated by rows of fans along the upper deck of the kiln. The heated, dry air is directed through the stacked lumber by a plenum chamber. The water removed from the wood is turned into water vapor by evaporation, and the saturated air is then released through air vents. A conventional kiln operates in an open-loop manner based on a pre-specified drying schedule. This process requires a full-time operator to frequently monitor and manually adjust all parameters according to the preset schedule. Due to the complex and distributed nature of the wood drying process, the end product is usually unpredictable, unreliable and unrepeatable. Energy efficient and automated lumber drying facilities are desirable. As well, the quality of the dried end product has to be acceptable, uniform, and repeatable. In summary, the following problems are faced by the existing conventional wood-drying kilns:

- They operate according to a predetermined drying schedule;
- They rely too heavily on experienced kiln operators for kiln configuration setting and for modification of the drying schedules;
- They require dedicated attention of on-site operators;
- They are left unattended during off-hours;
- They are subjected to lower operating temperatures during off-hours in order to reduce the energy consumption; and are not monitored during unattended periods – possibly resulting in product defects; e.g., splits or cracks.

Kiln drying is an energy-intensive process. In addition to the energy that is used for the drying process itself, some energy (electrical) is used for operating the fans in a kiln and for product repositioning during drying. The United States Department of Agriculture's Forest Product Laboratory research indicates that drying operations more commonly burn wood wastes rather than fossil fuels for their energy source. Proper air circulation and optimum temperature and residence schedules can result in significant reductions in kiln drying energy. In addition, Environmental concerns involve emissions from kilns, combustion systems, and treating agents. Waste heat from kilns can be recovered by means of heat exchangers. Wood-drying kilns have been suggested as a candidate technology using ground-source heat pumps for supplemental energy. These observations indicate that wood drying kilns provide a major opportunity for achieving significant benefits in energy efficiency through the use of advanced technology.

Sensors

Consider the prototype wood-drying kiln shown in Figure S1.16(a), which is a downscaled version of a conventional kiln that is used in industry, and has the dimensions of approximately $9 \times 4 \times 3$. The kiln has 12 thermocouples strategically positioned within it, to measure the kiln temperature; 2 relative humidity (RH%) sensors (wet-bulb/dry-bulb type), to measure the RH inside the kiln; one air velocity transmitter (hot-wire anemometer) to measure the air flow rate in the plenum; and 8 pairs of wood moisture content (MC) sensors that are nailed into the wood.

Actuators

The prototype kiln is equipped with a pulse-width-modulated (PWM) filament heater and a variable speed fan as the actuators for heating and air circulation.

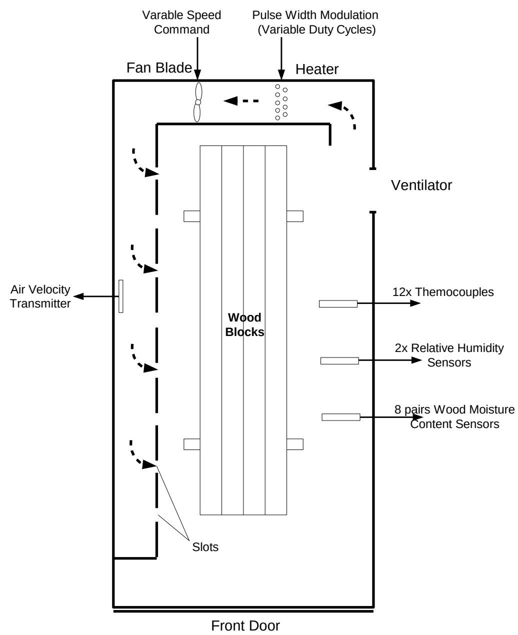


Figure S1.16(a): Schematic diagram of a prototype wood-drying kiln (top view).

Signal Modification and Interfacing Hardware

The thermocouple output is analog, and in the mV range. Hence a voltage amplifier is needed.

The flow sensor is equipped with signal conditioning, and provides an analog output in the range 4-20 mA DC.

The RH sensors have signal conditioning circuitry, providing an analog output in the range 1-5 V for an RH range of 0-100%

The Wagner L612 moisture-content sensor has signal conditioning hardware, and provides an analog output of several volts.

The fan control requires an analog signal in the range 0-5 V.

The heater control requires a pulse-width modulation signal based on the heat demand.

Control System

The computer control system of the experimental dryer is shown in Figure S1.16(b).

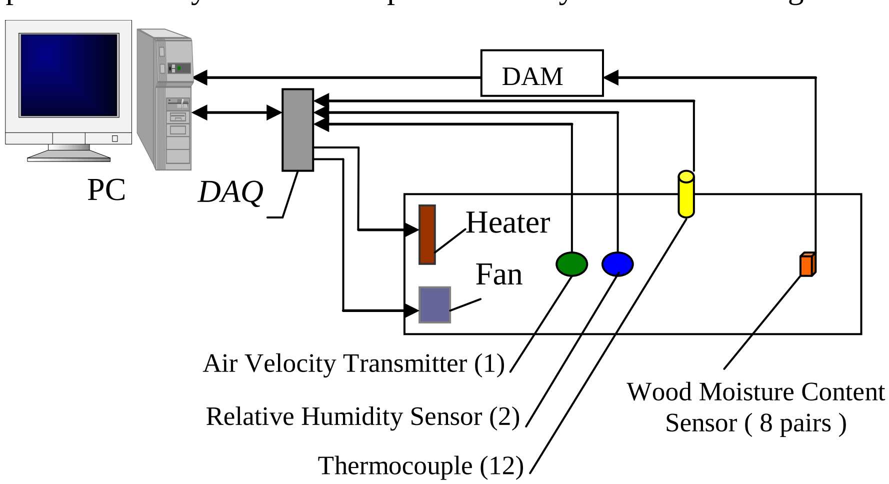


Figure S1.16(b): The kiln control system.

The electrical heater is an on/off type. The variable speed fan draws air through the heater filament and then blows it out of the plenum, located on one side of the kiln, through the slots that are equally spaced within the plenum. These two actuators are located in a small chamber situated at the back of the kiln. The system outputs are all connected to a data acquisition and analogue output board (DAQ) and then to a personal computer (PC). The wood moisture content sensors are connected to a data acquisition module (DAM), which separately communicates with the PC. The PC, on monitoring the system variables, generates control signals to both actuators, according to a programmed control scheme. All interactions are supported by the software programming and interface developed in Delphi.

Control Scheme

A conventional proportional-integral-derivative (PID) controller along with a knowledge-based (fuzzy) controller are implemented. Conventional model-based control technique may not be able to provide good drying result since the controller performance is mainly determined by the accuracy of the system model. Intelligent control using fuzzy logic, on the other hand, is capable of handling complex nonlinear processes, and it can provide the flexibility that conventional crisp control does not provide.

A double loop control system consists of a *Fuzzy Moisture controller* in the outer loop for correcting the moisture error, and a *Temperature PID controller* (T-PID) in the inner loop with a *pulse-width-modulated* (PWM) heater controller. The block diagram of the Fuzzy-PID control system is given in Figure S1.16(c).

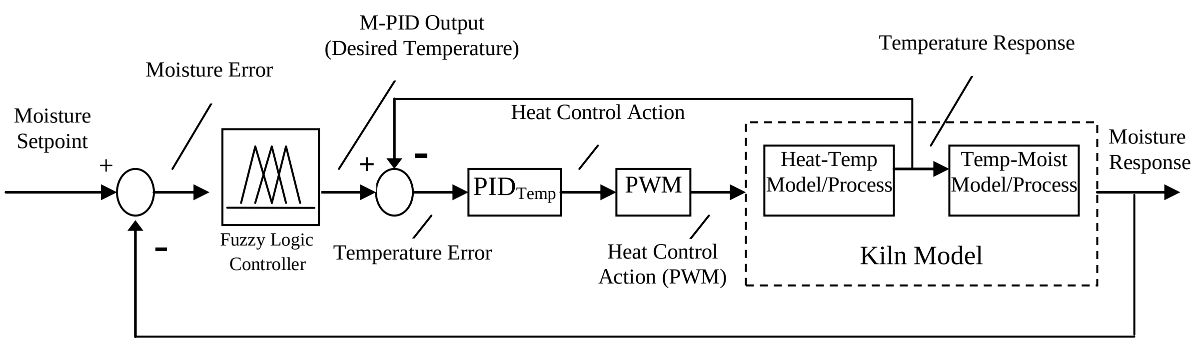


Figure S1.16(c): Block diagram of the Fuzzy-PID control system.

Graphic User Interface (GUI)

Since the kiln operators are normally not knowledgeable in control system instrumentation and control techniques, a user-friendly interface is desirable. Since system monitoring, data recording, and control are automated, the interface should primarily provide for emergency shut down and start up, and for accessing logged data and plant conditions.

Chapter 2 Component Interconnection and Signal Conditioning

Solution 2.1

(a):

$$\text{Electrical Impedance } = \frac{\text{Voltage Output}}{\text{Current Input}}$$

$$\text{Mechanical Impedance } = \frac{\text{Force Output}}{\text{Velocity Input}}$$

(b): Both these impedances are frequency response functions (defined in the frequency domain). Both define the resistance provided by the load against the driving force. High-impedance devices need high levels of effort (voltage or force) to drive them (i.e., to pass current through electrical impedances, or to cause movement of mechanical loads). Note that voltage is an across-variable whereas force is a through-variable. Hence, there is an inconsistency in the definitions of "impedance," with respect to the force-current analogy.

(c): To avoid this inconsistency, we may use the force-voltage analogy, in which voltage and force are termed "effort" variables and velocity and current are "flow" variables, as in the "bond graph" nomenclature.

Note, however, that in order to use general relations for interconnecting basic elements (in forming multicomponent devices or circuits), it is the across-variable and through-variable nomenclature that is applicable. Specifically, when two elements are connected in series, the through-variable is common and the across-variables add; when two elements are connected in parallel, the across-variable is common and the through-variables add. Hence, it is the through- and across-variable nomenclature that is natural with regard to component interconnection. In this context we may define a generalized series element or generalized impedance (to include electrical impedance or mechanical mobility) and a generalized parallel element (to include electrical admittance or mechanical impedance).

(d): The input impedance has to be comparatively high for a measuring device that is connected in parallel, to measure an across variable, whereas the input impedance has to be quite low for a device that connected in series, to measures a through variable. This is essential to reduce loading errors. The output impedance of a measuring device has to be low in order to maintain a high *sensitivity*, and get acceptable signal levels for processing, actuating or recording.

When cascading two devices, in order to reduce the "loading" of one device by the other, and to maintain good frequency characteristics, the output impedance of the first device (which provides the signal) has to be smaller in comparison to the input impedance of the second device (which receives the signal). Otherwise, the signal will be distorted by the second device (the load). If power transfer characteristics are important, however, one impedance should be the complex conjugate of the other. Different matching criteria are used depending on the applications.

Solution 2.2

1. Maximum power transfer
2. Power transfer at maximum efficiency
3. Reflection prevention in signal transmission
4. Loading reduction

Solution 2.3

When a measuring device is connected to a system, the conditions in the system itself will change, as the measured signal flows through the measuring device. For example, in electrical measurements, a current may pass through the measuring device, thereby altering the voltages and currents in the original system. This is called "electrical loading," and will introduce an error, as the measurand itself is distorted. Similarly, in mechanical measurements, due to the mass of the measuring device, the mechanical condition (forces, motions) of the original system will change, thereby affecting the measurand and causing an error. This is called mechanical loading.

Now consider the system shown in Figure P2.3. We have: $v_o = K\left[\dfrac{Z_i}{Z_s + Z_i}\right] v_i$

For a voltage follower, $K = 1$ and $Z_o << Z_i$. Hence, $v_o = \left[\dfrac{Z_i}{Z_s + Z_i}\right] v_i$, or $\dfrac{v_o}{v_i} = \dfrac{(Z_i / Z_s)}{1 + (Z_i / Z_s)}$.

This relationship is sketched in Figure S2.3.

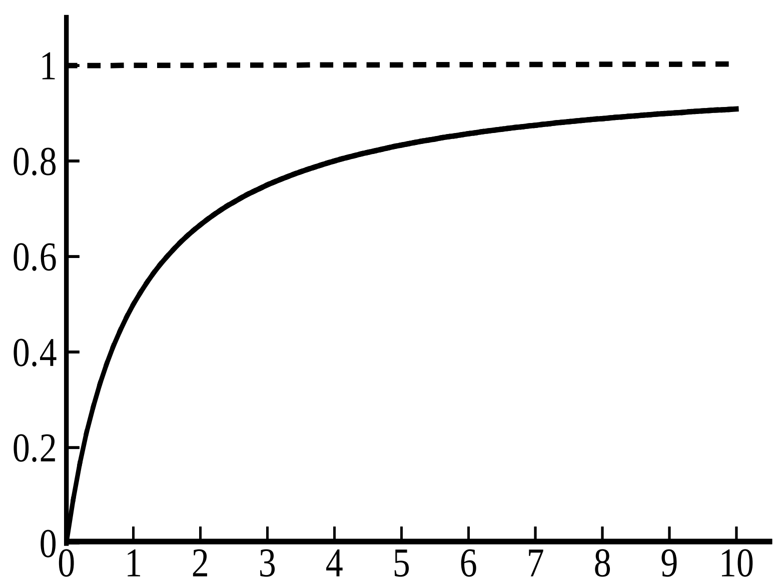


Figure S2.3: Non-dimensional curve of loading performance.

Some representative values of the curve are tabulated below.

Z_i / Z_s	v_o / v_i
0.1	0.091
0.5	0.55
1	0.5
2	0.667
5	0.855
7	0.875
10	0.909

***Note*:** Performance improves with the impedance ratio Z_i / Z_s.

Solution 2.4

Open-circuit voltage at the output port is (in the frequency domain)

$$v_{oc} = \frac{\left[R_2 + \dfrac{1}{j\omega C}\right]}{\left[R_1 + R_2 + j\omega L + \dfrac{1}{j\omega C}\right]} = v_{eq} \qquad \text{(i)}$$

Note: Equivalent source v_{eq} is expressed here as a function of frequency. Its corresponding time function $v_{eq}(t)$ is obtained by using inverse Fourier transform. Alternatively, first replace $j\omega$ by the Laplace variable *s*: $v_{eq}(s) = \dfrac{\left[R_2 + \dfrac{1}{sC}\right]}{\left[R_1 + R_2 + sL + \dfrac{1}{sC}\right]} v(s)$. Then obtain the inverse Laplace transform, for a given *v*(*s*), using Laplace transform tables.

Now, in order to determine Z_{eq}, note from Figure P2.4(b) that if the output port is shorted, the resulting short circuit current i_{SC} is given by: $i_{SC} = \dfrac{v_{eq}}{Z_{eq}}$. Hence,

$$Z_{eq} = \frac{v_{eq}}{i_{sc}} = \frac{v_{oc}}{i_{sc}} \qquad \text{(ii)}$$

Since we know v_{oc} (or v_{eq}) from equation (i) we only have to determine i_{sc}. Using the actual circuit with shorted output, we see that there is no current through the parallel impedance $R_2 + \dfrac{1}{j\omega C}$ because the potential difference across it is zero. Thus,

$$i_{sc} = \frac{v}{(R_1 + j\omega L)} \qquad \text{(iii)}$$

Now substituting Equations (i) and (iii) in (ii) we have:

$$Z_{eq} = \left[\frac{\left[R_2 + \frac{1}{j\omega C} \right] [R_1 + j\omega L]}{\left[R_1 + R_2 + j\omega L + \frac{1}{j\omega C} \right]} \right]$$

Solution 2.5

(a) Load power efficiency $\eta = \frac{R_l}{(R_l + R_s)} = \frac{R_l / R_s}{(R_l / R_s + 1)}$

(b) Load power $p_l = \frac{{v_s}^2 R_l}{[R_l + R_s]^2}$; Maximum load power (occurs at $R_l = R_s$) $p_{\max} = \frac{{v_s}^2}{4R_s}$

➔ $p_l / p_{\max} = \frac{4R_l / R_s}{[R_l / R_s + 1]^2}$

We use the following MATLAB script (.m file) to generate the two curves:

```
% Efficiency and load power curves
lr=[]; eff=[]; pw=[];  % declare vectors
lr=0; eff=0; pw=0; %initialize variables
for i=1:100
a=0.1*i; %load resistance ratio
lr(end+1)=a; % store load resistance
eff(end+1)=a/(a+1); % efficiency
pw(end+1)=4*a/(a+1)^2;  % load power
end
plot(lr,eff,'-',lr,pw,'-',lr,pw,'x')
```

The two curves are plotted in Figure S2.5.

It is seen that maximum efficiency does not correspond to maximum power. In particular, the efficiency increases monotonically with the load resistance while the maximum power occurs when $R_l = R_s$. Hence, a reasonable trade-off in matching the resistances would be needed when both considerations are important.

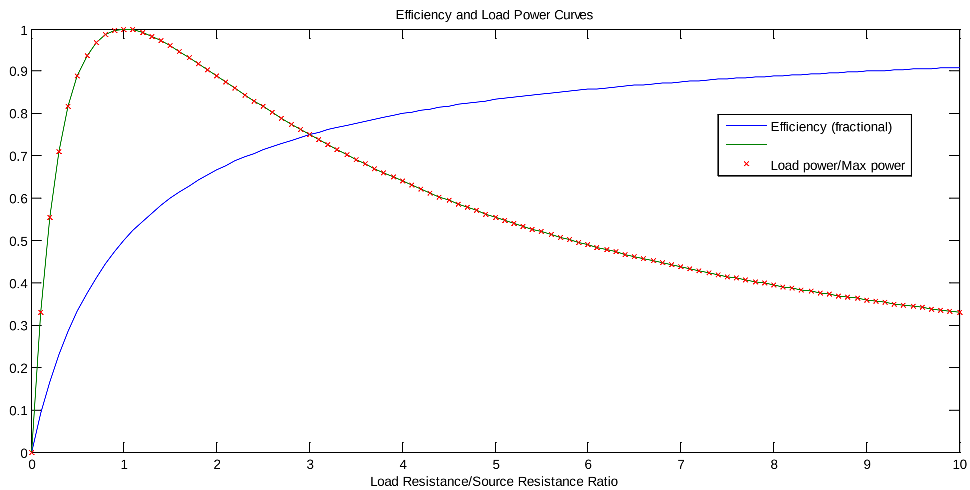


Figure S2.5: Variation of efficiency and maximum power with load resistance.

Solution 2.6

Voltage is an across variable. In order to reduce loading effects, the resistance of a voltmeter should be much larger than the output impedance of system or load impedance. Then, the voltmeter will not draw a significant part of the signal current (and will not distort the signal). Current is a through variable. The resistance of an ammeter should be much smaller than the output impedance of system or load impedance. Then, the ammeter will not provide a significant voltage drop (and will not distort the signal).

Voltmeter should be able to operate with a low current (due to its high resistance) and associated low torque, in conventional electromagnetic deflection type meters. Low torque means, a torsional spring having low stiffness has to be used to get an adequate meter reading. This makes the meter slow, less robust, and more nonlinear, even though high sensitivity is realized.

Ammeter should be able to carry a large current because of its low resistance. Hence meter torque would be high in conventional designs. This can create thermal problems, magnetic hysteresis, and other nonlinearities. The device can be made fast, robust, and mechanically linear, however, while obtaining sufficient sensitivity.

Note: The torque is not a factor in modern digital multi-meters.

Solution 2.7

(a) The input impedance of the amplifier = 500 MΩ.

$$\text{Estimated error } = \frac{10}{(500+10)} \times 100\% = 2\%$$

(b) Impedance of the speaker = 4 Ω.

$$\text{Estimated error } = \frac{0.1}{(4+0.1)} \times 100\% = 2.4\%$$

Solution 2.8

$$v_o = F_1(f_o, f_i) \quad \Rightarrow \quad \delta v_o = \frac{\partial F_1}{\partial f_o}\ \delta f_o + \frac{\partial F_1}{\partial f_i}\ \delta f_i \qquad \text{(i)}$$

$$v_i = F_2(f_o, f_i) \quad \Rightarrow \quad \delta v_i = \frac{\partial F_2}{\partial f_o}\ \delta f_o + \frac{\partial F_2}{\partial f_i}\ \delta f_i \qquad \text{(ii)}$$

In terms of incremental variables about an operating point, we can define the input impedance Z_i and the output impedance Z_o as

$$Z_i = \frac{\delta v_i}{\delta f_i} \qquad \text{with} \quad \delta f_o = 0 \qquad \text{(iii)}$$

$$Z_o = \frac{\delta v_o \text{ with } \delta f_o = 0}{\delta f_o \text{ with } \delta v_o = 0} \qquad \text{(iv)}$$

Note: $\delta f_o = 0$ corresponds to incremental open-circuit condition and $\delta v_o = 0$ corresponds to incremental short-circuit condition.

From (ii) with $\delta f_o = 0$ (i.e., open circuit at output) we get $Z_i = \frac{\partial F_2}{\partial f_i}$.

Now using the open-circuit by subscript "*oc*" and the short-circuit by subscript "*sc*" we have:
From (i):

$$\delta v_o \Big|_{oc} = \frac{\partial F_1}{\partial f_i} \delta f_i \Big|_{oc} \qquad \text{(v)}$$

From (ii):

$$\delta v_i \Big|_{oc} = \frac{\partial F_2}{\partial f_i} \delta f_i \Big|_{oc} \qquad \text{(vi)}$$

Note: δv_i is an independent increment, which does not depend on whether *oc* or *sc* condition exists at the output. But δf_i will change depending on the output condition.
From (v) and (vi):

$$\delta v_o \Big|_{oc} = \frac{\partial F_1}{\partial f_i} \Big/ \frac{\partial F_2}{\partial f_i} \qquad \delta v_i \qquad \text{(vii)}$$

From (i):

$$0 = \frac{\partial F_1}{\partial f_o}\ \delta f_o \Big|_{sc} + \frac{\partial F_1}{\partial f_i}\ \delta f_i \Big|_{sc} \qquad \text{(viii)}$$

From (ii):

$$\delta v_i = \frac{\partial F_2}{\partial f_o}\ \delta f_o\Big|_{sc} + \frac{\partial F_2}{\partial f_i}\ \delta f_i\ \Big|_{sc} \qquad \text{(ix)}$$

Eliminating $\delta f_i\ \Big|_{sc}$ from Equations(viii) and (ix) we get,

$$\delta f_o\Big|_{sc} = \frac{1}{\left[\dfrac{\partial F_2}{\partial f_o} - \dfrac{\partial F_1}{\partial f_o}\cdot\dfrac{\partial F_2}{\partial f_i}\Big/\dfrac{\partial F_1}{\partial f_i}\right]}\ \delta v_i \qquad \text{(x)}$$

Substitute (vii) and (x) in (iv):

$$Z_o = \left[\frac{\partial F_1}{\partial f_i}\Big/\frac{\partial F_2}{\partial f_i}\right]\left[\frac{\partial F_2}{\partial f_o} - \frac{\partial F_1}{\partial f_o}\cdot\frac{\partial F_2}{\partial f_i}\Big/\frac{\partial F_1}{\partial f_i}\right] = \frac{\partial F_1}{\partial f_i}.\frac{\partial F_2}{\partial f_o}\Big/\frac{\partial F_1}{\partial f_i} - \frac{\partial F_1}{\partial f_o}$$

One way to experimentally determine Z_i and Z_o (under static conditions) is to first experimentally determine the two sets of operating curves given by $v_o = F_1(f_o, f_i)$ and $v_i = F_2(f_o, f_i)$ under steady-state conditions. For example f_o is kept constant and f_i is changed in increments to measure v_o and v_i once the steady state is reached. This will give two curves f_i versus v_o and v_i versus v_i for a particular value of f_o. Next f_o is incremented and another pair of curves is obtained. Once these two sets of curves are obtained for the required range for f_i and f_o, the particular derivatives are determined from using the general method shown in Figure S2.8, for the case $z = F(x,y)$ with: $\frac{\partial z}{\partial x} \cong \alpha$ and $\frac{\partial z}{\partial y} \cong \frac{\Delta z}{\Delta y}$.

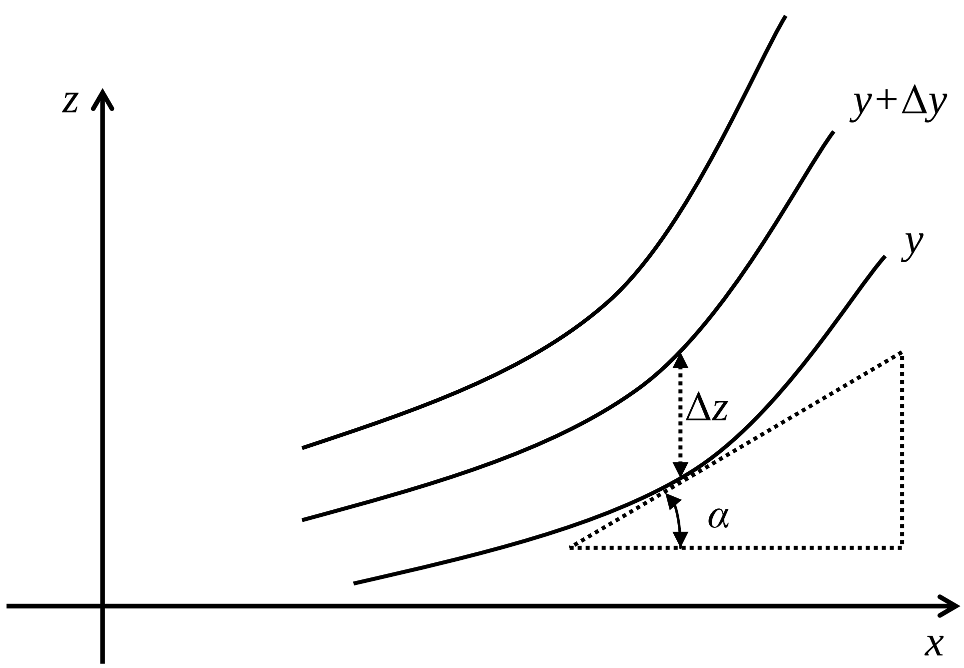


Figure S2.8: Computation of local slopes.

Solution 2.9

(a)
We have: $v_i = Z_c i_i$, $v_t = Z_l i_t$, $v_r = -Z_c i_r$, $v_t = v_i + v_r$, and $i_t = i_i + i_r$, where "i" denotes current and the subscript "r" denotes "reflected."

Substitute: $v_t = Z_l i_t = Z_l(i_i + i_r) = Z_l(\frac{v_i}{Z_c} - \frac{v_r}{Z_c}) = \frac{Z_l}{Z_c}(v_i - v_r) = \frac{Z_l}{Z_c}(v_i - (v_t - v_i)) = \frac{Z_l}{Z_c}(2v_i - v_t)$

➔ $(1+\frac{Z_l}{Z_c})v_t = 2\frac{Z_l}{Z_c}v_i$ ➔ $v_t = \frac{2Z_l}{(Z_l + Z_c)}v_i$

(b) We need $v_t = v_i$ ➔ $\frac{2Z_l}{(Z_l + Z_c)} = 1$ ➔ $Z_l = Z_c$

(c) Use a transformer with the required impedance ratio = (turns ratio)^2

Solution 2.10

For the given system, $\omega_n = \sqrt{\frac{1\times 10^6}{100}}$ rad/s = 100 rad/s and $\omega \geq 200$ rad/s. Hence, we have the frequency ratio $r \geq 2.0$.

For r = 2.0 and $\left|T_f\right|$ = 0.5 we have $0.5 = \sqrt{\frac{1+16\zeta^2}{9+16\zeta^2}}$ or, $\zeta = \sqrt{\frac{5}{48}}$. Hence,

$$b = 2\zeta\omega_n m = 2\sqrt{\frac{5}{48}}\times 100\times 100 \text{ N.s/m} \rightarrow b = 6.455\times 10^3 \text{ N.s/m}.$$

With this damping constant, for $r \geq 2$, we will have $\left|T_f\right| \leq 0.5$. Decreasing b will decrease $\left|T_f\right|$ in this frequency range.

To plot the Bode diagram using MATLAB, first note that:

$$2\zeta\omega_n = b/m = 6.455\times 10^3/100 = 64.55 \text{ rad/s and } \omega_n^2 = 10^4 \text{ (rad/s)}^2$$

The corresponding transmissibility function is $T_f = \frac{64.55s + 10^4}{s^2 + 64.55s + 10^4}$ with $s = j\omega$

The following MATLAB script will plot the required Bode diagram:

```
% Plotting of transmissibility function
clear;
m=100.0;
k=1.0e6;
b=6.455e3;
sys=tf([b/m k/m],[1 b/m k/m]);
bode(sys);
```

The resulting Bode diagram is shown in Figure S2.10. A transmissibility magnitude of 0.5 corresponds to $20\log_{10} 0.5\ \text{dB} = \text{-6.02 dB}$.

Note from the Bode magnitude curve in Figure S2.10.4 that at the frequency 200 rad/s the transmissibility magnitude is less than -6 dB and it decreases continuously for higher frequencies. This confirms that the designed system meets the design specification.

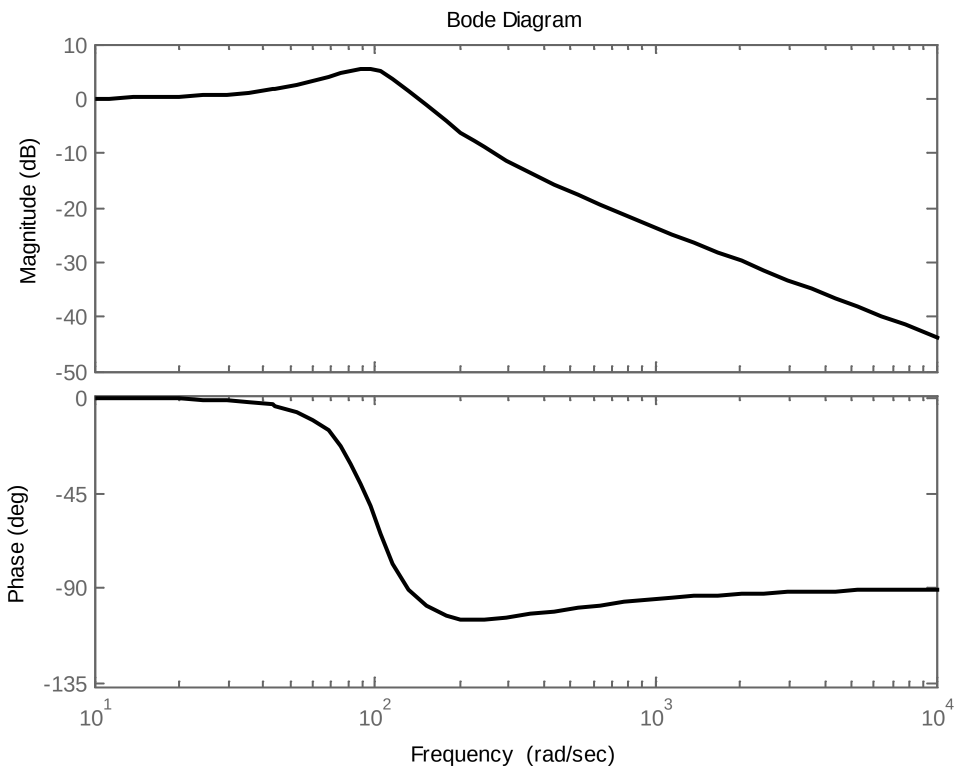


Figure S2.10: Transmissibility magnitude and phase curves of the designed system.

Solution 2.11

(a) Mechanical Loading

A motion variable that is being measured is modified due to forces (inertia, friction, etc.) of the measuring device.

(b) Electrical Loading

The output voltage signal of the sensor is modified from the open circuit value due to the current flowing through external circuitry (load).

Mechanical loading can be reduced by using noncontact sensors, reducing inertia and friction, etc.

Electrical loading can be reduced by using a low-output-impedance sensor, high-impedance load, impedance transformer, etc. Some typical values of the listed parameters are given in the following table:

Parameter	Ideal Value	Typical Value
Input Impedance	Infinity	1 M Ω
Output Impedance	Zero	50 Ω
Gain	Infinity	10^6
Bandwidth	Infinity	10 kHz

Solution 2.12

The differential signal from the secondary windings is amplified by the ac amplifier and is supplied to the demodulator. A carrier signal is used by the demodulator to demodulate the differential ac signal. The modulating signal that is extracted in this manner is proportional to the machine displacement. This signal is filtered to remove high-frequency noise (and perhaps the carrier component left by the demodulator), and then amplified and digitized (using an ADC) to be fed into the machine control computer.

The compensating resistor R_c may be connected between the points *A* and *B* or *A* and *C*, as shown in Figure S2.12.

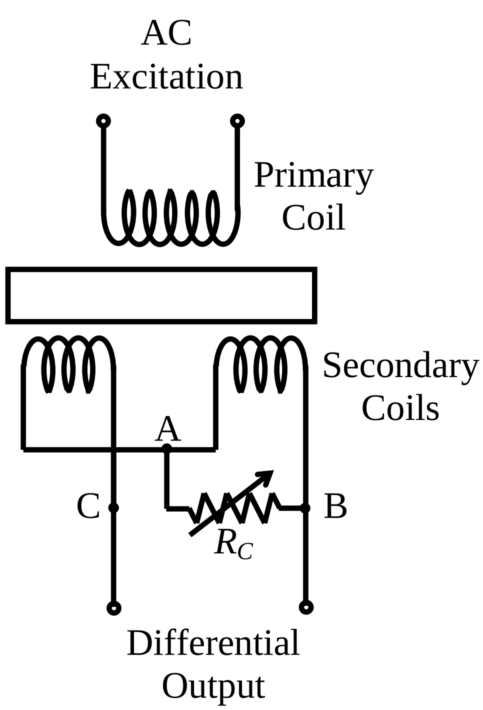


Figure S2.12: Null compensation for an LVDT.

Solution 2.13

(a)

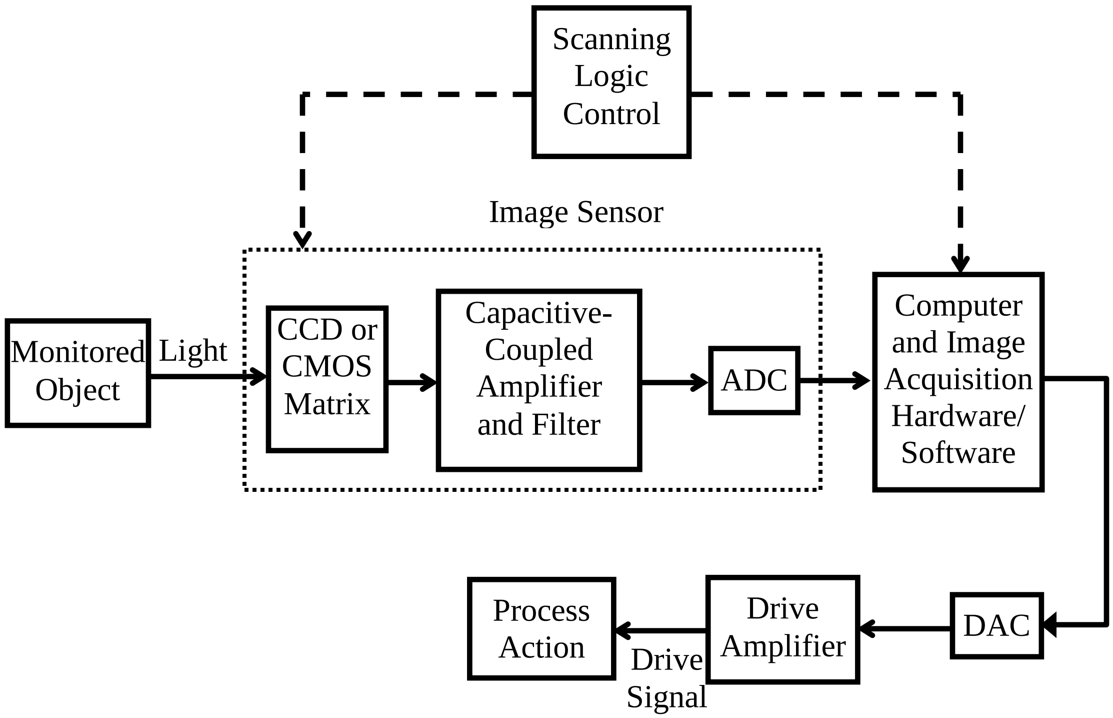


Figure S2.13: Monitoring of an industrial process using image processing.

(b) Data rate = $488 \times 380 \times 8 \times 30$ bits/s = 44.5 Megabits/s

(c) Since hardware processors are faster, we prefer them for this level of high data rates for real-time action. Also, they are cheaper when mass produced. Disadvantages include limitations on algorithm complexity in image processing and memory size.

Solution 2.14

Since the open-loop gain K of an op-amp is very high (10^5 to 10^9) and the output voltage cannot exceed the saturation voltage (which is of the order of 10 V) the input voltage $v_i = v_{ip} - v_{in}$ is of the order of a few μV, which can be assumed zero (when compared with the operating voltages) for most practical purposes. Hence, $v_{ip} = v_{in}$. Next since the input impedance Z_i is very high (MΩ), the current through the input leads has to be very small for this very small v_i under unsaturated conditions.

(a) The saturated output of the op-amp must be 14 V in this example. The ac noise (line noise, ground loops, etc.) in the circuit can easily exceed the saturation input (on the order of 10 μV) of the op-amp, under open-loop conditions. Hence, $v_i = v_{ip} - v_{in}$ can oscillate between + and - values of the saturation input. This provides an output, which switches between the *+ve* saturated output $+v_{sat}$ and the *-ve* saturated output $-v_{sat}$ of the op-amp.

(b)

Case 1: $v_{ip} = -1\ \mu\text{V}$, $v_{in} = +0.5\ \mu\text{V}$ ➔ $v_i = v_{ip} - v_{in} = -1 - 0.5\ \mu\text{V} = -1.5\ \mu\text{V}$

➔ $$v_o = -1.5 \times 5 \times 10^6\ \mu\text{V} = -1.5 \times 5\ \text{V} = -7.5\ \text{V}$$

This is valid since the output is not saturated.

Case 2: $v_{ip} = 0$, $v_{in} = 5\ \mu\text{V}$ ➔ $v_i = 0 - 5\ \mu\text{V} = -5\ \mu\text{V}$

➔ $$v_o = -5 \times 5 \times 10^6\ \mu\text{V} = -25\ \text{V}$$

➔ Op-amp is saturated ➔ The actual output would be $v_o = -14\ \text{V}$

Solution 2.15

(a) Offset Current (Typically in nA)
Bias currents are needed to operate the transistor elements in an op-amp IC. These currents i_+ and i_- flow through the input leads of an op-amp. The offset current is the difference $i_+ - i_-$. Ideally, the offset current is zero.

(b) Offset Voltage (Typically in mV or less)
Due to internal circuitry (IC) in an op-amp, the output voltage might not be zero even when the two inputs are maintained at the same potential (say, ground). This is known as the offset voltage at output. Furthermore, due to unbalances in the internal circuitry, the potentials at the two input leads of an op-amp will not be equal even when the output is zero. This potential difference at the input leads is known as the input offset voltage. This is usually modeled as a small voltage source connected to one of the input leads

(c) Unequal Gains (Can range over 10^5 to 10^9)
The open-loop gain of an op-amp with respect to the "+" input lead may be different from that with respect to the "-" input lead. This is known as unequal gains.

(d) Slew Rate (Typically about 0.5 V/μs)
When the input voltage is instantaneously changed, the op-amp output will not change instantaneously. The maximum rate at which the output voltage can change (usually expressed in V/μs) is known as the slew rate of an op-amp.

Even though K and Z_i are not precisely known and can vary with time and frequency, their magnitudes are large. Hence, we can make the basic assumptions: equal potential at the two input leads and zero current through the input leads, under unsaturated conditions. Then, these parameters do not enter the output equations of an op-amp circuit.

Solution 2.16

(a)
A voltage follower is an amplifier having a unity voltage gain, a very high input impedance, and a very low output impedance. A simple model for a voltage follower is obtained by connecting the "-" lead of an op-amp to the output (feedback path) and using the "+" lead as the input lead. Under unsaturated conditions we have $v_o = v_i$. It is known that the input impedance of a voltage follower is much larger than that of the original op-amp (which itself is quite large—megohm range) and the output impedance of a voltage follower is much smaller than that of the original op-amp (which is also small). Hence, a voltage follower functions primarily as an impedance transformer that provides the ability to acquire a voltage from a high-impedance device, where the current is rather low (e.g., a high-impedance sensor) and transmitting that voltage signal into a low-impedance device, without distorting the acquired voltage.

(b)
Consider circuit in Figure P2.16. Since $v_B = 0$, we have $v_A = 0$.

Hence, current summation at node A gives: $\frac{v_i}{R} + \frac{v_o}{R_f} = 0$

Note: The current through an input lead of an op-amp has to be zero.

Hence, $\frac{v_o}{v_i} = -\frac{R_f}{R}$ and $K_v = -\frac{R_f}{R}$ ➔ This is an inverting amplifier.

Solution 2.17

Slew rate: $s = 2\pi f_b a$ (i)
where, a = output amplitude, f_b = bandwidth (Hz).

The rise time T_r is inversely proportional to f_b. Hence, $f_b = \frac{k}{T_r}$ where, k = constant.

Substitution gives: $s = \frac{2\pi ka}{T_r}$ (ii)

From (i): For constant s, bandwidth decreases as a is increased.

For a sine signal, substitute the given values in (i): $f_b = \frac{0.5}{2\pi \times 2.5}$ MHz $= 31.8$ kHz

Next, for a step input, use $s = \frac{\Delta y}{\Delta t}$ where, Δy = step size, $= \Delta t$ rise time

Substitute numerical values: $\Delta t = \frac{\Delta y}{s} = \frac{2.5}{0.5}\ \mu\text{s} = 5\ \mu\text{s}$.

Solution 2.18

(a) Common-mode voltage v_{cm} = voltage common to the two input leads of a differential amplifier = average of the two inputs.

Common-mode output voltage v_{ocm} = output voltage of the amplifier due to v_{cm} (i.e., in the absence of any voltage differential at the input.)

(b) Common-mode gain = $\dfrac{v_{ocm}}{v_{cm}}$

(c) $$CMRR = K\frac{v_{cm}}{v_{ocm}} = \frac{K}{\text{common-mode gain}}$$

where, K = amplifier gain (i.e., differential gain or gain at the output for the inferential input).

Specifically: $v_o = K(v_{ip} - v_{in}) + K_{cm} \times \frac{1}{2}(v_{ip} + v_{in})$

Typically $CMRR \approx 20{,}000$.

When A is closed and B is open, the flying capacitor C gets charged to the differential voltage $v_{i1} - v_{i2}$ and hence the common-mode voltage does not enter. When A is open and B is closed, the capacitor voltage, which does not contain the common-mode signal, is applied to the differential amplifier.

Solution 2.19

The textbook definition of stability relates to the dynamic model (linear or nonlinear) of a system and hence to its natural response. In particular, in a linear system, if at least one pole (eigenvalue) has a positive real part, the natural response of the system will diverge, and the system is unstable.

Instrumentation stability is linked to the drift associated with change in parameters of the instrument or change in the environmental conditions.

$$\text{Temperature drift} = \frac{\text{Change in output}}{\text{Change in temperature}}$$

assuming that the other conditions and the input are maintained constant.

$$\text{Long term drift} = \frac{\text{Change in output}}{\text{Duration}}$$

assuming that the other conditions and input are the same.

Ways to Reduce Drift: Regulate the power supply; Use feedback**;** Keep the environment uniform**;** Use compensating elements and circuitry**;** Recalibrate the device before each use.

Solution 2.20

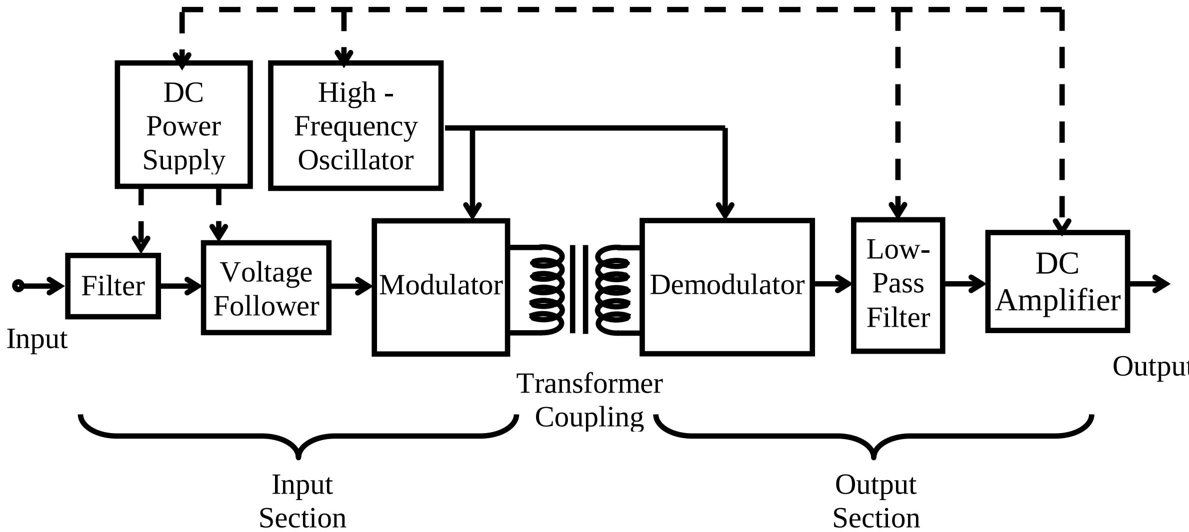


Figure S2.20: An isolation amplifier.

Solution 2.21

Possible causes:
1. Faulty cellphone charger and it not having a ground lead and pin
2. Faulty laptop charger and it not having a ground lead and pin

Faulty Cellphone Charger: Due to a short-circuit, the high voltage (110-240 VAC) will leak into its cable and reach the cellphone. If the cellphone is not properly grounded/isolated, the voltage will form a path through the user's body. According to the burns, this path has to include the chest and the ears (possibly through the headphone cable).

Faulty laptop Charger: Due to a short, the high voltage (110-240 VAC) will leak from the charger into the DC cable segment that is connected to the laptop. If the laptop is not properly grounded/isolated, the voltage will form a path through the user's body. According to the burns, this path has to include the chest and the ears (possibly through the headphone cable).

On the one hand, the newspaper report indicated that there were inexpensive and non-compliant cellphone chargers in the market. However, since the power consumption of the cellphone charger is relatively low and since the electricity path through the body included the ears (*Note*: The headphones were connected to the laptop, not to the cellphone) the other possibilities of fault need to be investigated as well. Typically, however, the laptop chargers (particularly those provided by reputed laptop manufacturers) are subjected to rigorous standards, inspection, and quality control (so are cellphone chargers from reputed manufacturers).

Solution 2.22

Passive filters are circuits made of passive elements, which do not require an external power supply to operate. These circuits allow through those signal components in a certain frequency range and block off the remaining frequency components.

Advantages and disadvantages of passive filters: See disadvantages and advantages of active filters.

The voltage follower is an impedance transformer. It reduces loading problems by providing a very high input impedance and very low output impedance. Furthermore, it does not change the voltage gain.

Solution 2.23

Applications:

(a) Anti-aliasing filters in digital signal processing
(b) To remove dc components in ac signals
(c) As tracking filters
(d) To remove line noise in signals.

Each single-pole stage will have a transfer function of the form: $G_i(s) = \dfrac{k_i(\tau_{zi}s+1)}{(\tau_{pi}s+1)}$

Hence, the cascaded filter will have the transfer function:

$$G(s) = \Pi G_i(s) = \Pi\ k_i \frac{(\tau_{zi}s+1)}{(\tau_{pi}s+1)},$$ where "Π" denotes the product operation.

Note that the poles are at $-\dfrac{1}{\tau_{pi}}$ and these are all real; there are no complex poles. Hence, there cannot be resonant peaks.

Solution 2.24

It provides the flattest magnitude over the pass band among all filters of the same order (same pole count).
Also, we prefer a very sharp cutoff (i.e., steep roll-up and roll-down).

Solution 2.25

(a)

Op-amp properties: 1. Voltages at input leads are equal; 2. Currents through input leads = 0

Op-amp property: $v_B = v_P = v_o$ (i)

Current Balance at Node A: $\dfrac{(v_i - v_A)}{Z_c} = \dfrac{(v_A - v_B)}{Z_c} + \dfrac{(v_A - v_P)}{R}$ (ii)

Current Balance at Node B: $\dfrac{(v_A - v_B)}{Z_c} = \dfrac{v_B}{R}$ (iii)

Note: $Z_c = \dfrac{1}{Cs}$ = impedance of capacitor

Substitute (i) and (iii) in (ii): $\dfrac{(v_i - v_A)}{Z_c} = \dfrac{v_o}{R} + \dfrac{(v_A - v_o)}{R} = \dfrac{v_A}{R} \rightarrow v_i = (1 + \dfrac{1}{\tau s})v_A$ (iv)

Substitute (i) in (iii): $\dfrac{(v_A - v_o)}{Z_c} = \dfrac{v_o}{R} \rightarrow v_A = (1 + \dfrac{1}{\tau s})v_o$ (v)

Note: $\tau = RC$ = time constant

Substitute (iv) in (v): $G(s) = \dfrac{v_o}{v_i} = \dfrac{(\tau s)^2}{(\tau s + 1)^2}$

This is a 2nd order transfer function ➔ 2-pole filter

(b)

With $s = j\omega$ in $G(s)$, we have $G(j\omega) = \dfrac{-\tau^2\omega^2}{(1 + \tau j\omega)}$

Filter magnitude $|G(j\omega)| = \dfrac{\tau^2\omega^2}{(1 + \tau^2\omega^2)}$

The magnitude of the filter transfer function is sketched in Figure S2.25. This represents a high-pass filter.

(c)

When, $\omega << \dfrac{1}{\tau}$: $|G(j\omega)| \cong \tau^2\omega^2$

When, $\omega >> \dfrac{1}{\tau}$: $|G(j\omega)| \cong \dfrac{\tau^2\omega^2}{\tau^2\omega^2} = 1$

Hence, we may use $\omega_c = \dfrac{1}{\tau}$ as the cutoff frequency.

Note: $|G(j\omega)| \rightarrow 1$ as $\omega \rightarrow \infty$

For small ω: Roll-up slope of $|G(j\omega)|$ curve is 40 dB/decade.

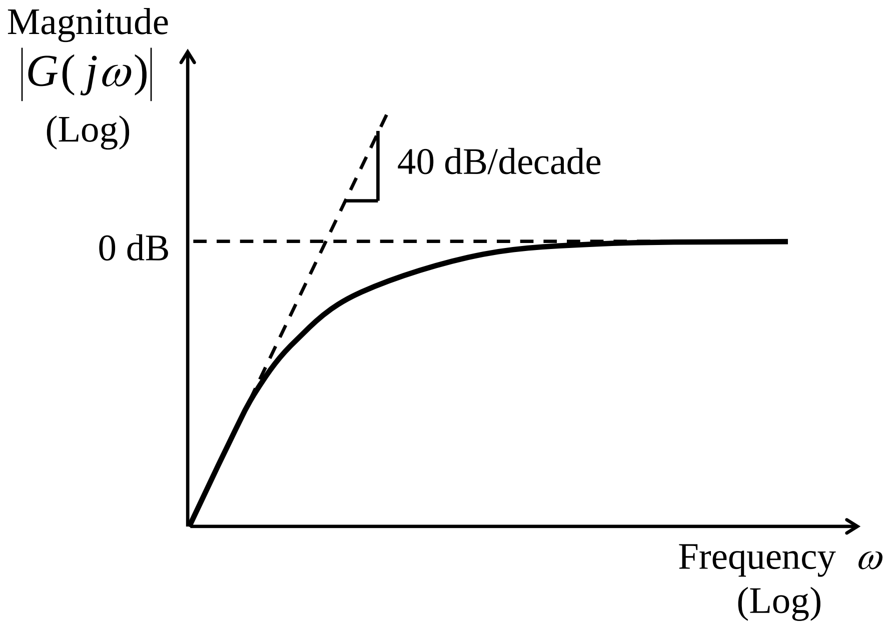


Figure S2.25: Filter transfer function magnitude.

Solution 2.26

Strain Gauge for force Sensing: Low-frequency noise due to ambient temperature fluctuations. These may be compensated for (using abridge circuit) and also through high-pass filtering

Wearable Ambulatory Monitoring (WAM): In human mobility monitoring (e.g., in telehealth applications) a popular WAM sensor is a combined accelerometer and gyroscope. Both sensors will be affected by bias, removal of which would need high-pass filtering). High-frequency artifacts may be generated in the sensed signal due to muscle tremor and low-frequency artifacts may be formed due to respiration. These may be removed using band-pass filtering.

Microphone (Robotic Voice Commands): A band-pass filter for the human vocal range (80Hz to 1100Hz).

AC-powered Tachometer for Speed Sensing: Line noise (60 Hz) may be removed using a notch filter.

Solution 2.27

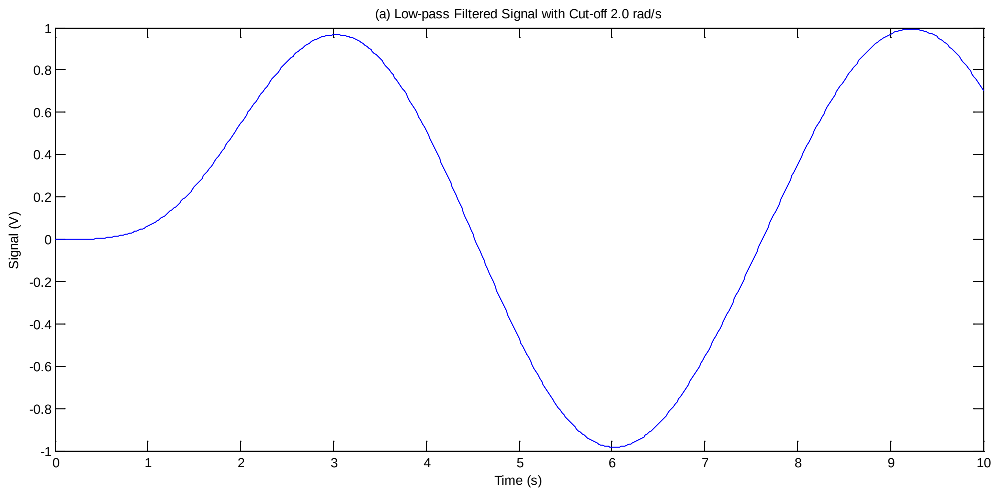
(a)
The main signal component appears sinusoidal with frequency ~ 1 rad/s (period ~ 6.3 s). From the figure it is not clear whether there is a superimposed sinusoidal signal of high frequency and/or high-frequency noise, even though some oscillations are observed in the noise.

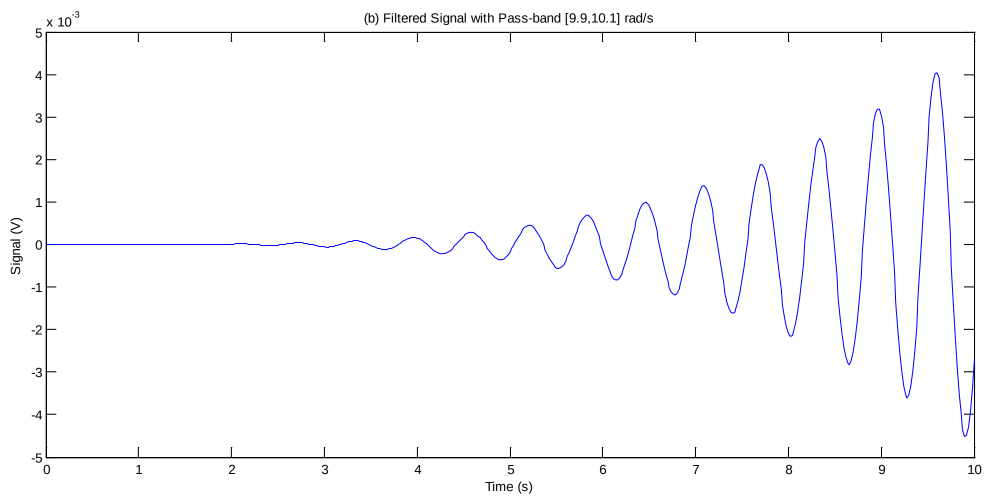
(b)
We use the following MATLAB command to obtain the four-pole Butterworth low-pass filter with cut-off frequency at 2.0 rad/s:

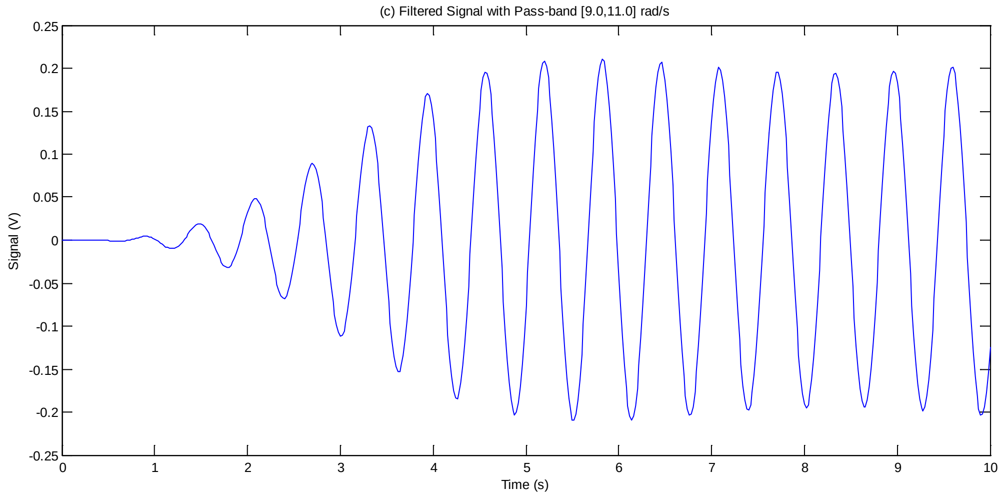
```
>> [b,a]=butter(4,2.0,'s')
b =
          0          0          0          0    16.0000
a =
     1.0000     5.2263    13.6569    20.9050    16.0000
```

Then, we use the following MATLAB commands to filter the data signal using this filter, and plot the result shown in Figure S2.27(a):

```
>>y1=lsim(b,a,u,t);
>> plot(t,y1,'-')
```







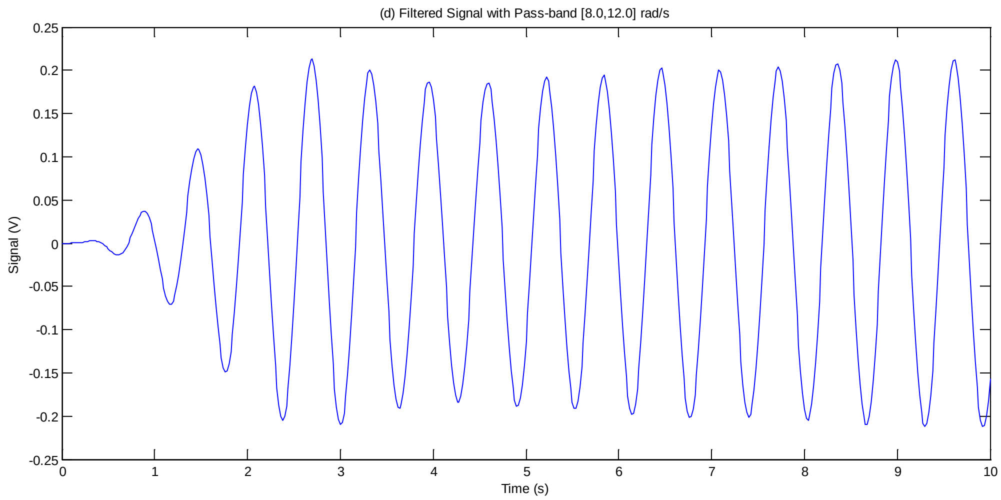


Figure S2.27: Filtered signals. (a) Low-pass at 2.0 rad/s; (b) Band-pass over [9.9, 10.1]; (c) Band-pass over [9.0, 11.0]; (d) Band-pass over [8.0, 12.0].

It is seen that the filtered signal has a frequency of 1.0 rad/s with the correct amplitude (1.0) and negligible phase shift. Initially some signal distortion is seen due to the transient nature of the output. However, the steady-state value is reached in half a period of the signal.

(c)
Band-pass filtering for the three cases are obtained using the following MATLAB commands:

```
>> Wn=[9.9,10.1];
>> [b2,a2] = butter(4,Wn,'bandpass','s');
>> y2=lsim(b2,a2,u,t);
>> plot(t,y2,'-')
```

```
>> Wn=[9.0,11.0];
>> [b2,a2] = butter(4,Wn,'bandpass','s');
>> y2=lsim(b2,a2,u,t);
>> plot(t,y2,'-')

>> Wn=[8.0,12.0];
>> [b2,a2] = butter(4,Wn,'bandpass','s');
>> y2=lsim(b2,a2,u,t);
>> plot(t,y2,'-')
```

The results are shown in Figures S2.27 (b)-(d). The very narrow pass-band produced a filtered result that took a rather long time to reach the steady state of amplitude 0.2 (i.e., the filter had a larger time constant). When the pass-band was increased, the steady state was reached quicker (i.e., smaller filter time constant). However, the amplitude distortion of the filtered signal was noticeable as a result.

Solution 2.28

If a characteristic of a signal "*B*" is changed with respect to time, depending on some characteristic parameter of another signal "*A*," this process is termed *modulation*. The *modulating signal* (or data signal) is the signal *A*. The *carrier signal* is the signal *B*. The output signal of the modulation process is the *modulated signal*. The process of recovering the data signal (*A*) from the modulated signal is known as *demodulation*.

(a) Amplitude Modulation (AM)
The carrier is a periodic signal (typically a sine wave). The amplitude of the carrier signal is varied in proportion to the magnitude of the data signal. Specifically, the carrier signal is multiplied by the data signal. In one form of AM, the carried signal is added again to the resulting product signal. The AM technique is used in radio transmission and in sensing (e.g., differential transformer). The sign of the data signal is represented by a 180° phase change in the carrier signal.

(b) Frequency Modulation (FM)
The carrier is typically a sine wave signal. The frequency of the carrier signal is varied in proportion to the magnitude of the data signal. This process is commonly used in radio transmission and data storage. Sign of the data signal is represented by changing the carrier phase angle by 180°.

(c) Phase Modulation (PM)
The carrier signal is typically a sine wave. The phase angle of the carrier signal is varied in proportion to the magnitude of the data signal. Used in signal transmission. Sign of the data signal is represented by positive or negative phase change in the carrier.

(d) Pulse-width Modulation (PWM)

The carrier is a pulse signal. The pulse width of the carrier is changed in proportion to the magnitude of the data signal. Both the spacing between the pulses (pulse period) and the pulse amplitude are kept constant. Used in dc motor speed control, other control applications, and digital-to-analog conversion (DAC). Sign of data is accounted for by using both +*ve* and -*ve* pulses.

(e) Pulse-Frequency Modulation (PFM)
The carrier is a pulse signal. The frequency of the pulses is changed in proportion to the magnitude of the data signal. Pulse width and pulse amplitude are maintained constant (and the pulse period is varied). Used in dc motor speed control. Sign of data is accounted for by using both +*ve* and -*ve* pulses.

(f) Pulse-Code Modulation (PCM)
Carrier signal is a pulse sequence. The value of the data signal at a given time instant is represented in the binary form and this value is represented in the carrier (of by equally spaced pulses) using the fact that the presence of a pulse can be used to represent binary 0. Then for a given word size, say n bits, a maximum of n pulses have to be transmitted. The sign of the data word may be represented by an additional bit, known as the sign bit (using, say 1 to represent "+" and 0 to represent "-"). Separation between one data word and the next may be detected through "framing" a data word using "start bits" and "end bits." Used in digital communication.

Solution 2.29

Intentional AM

- Radio broadcast

  AM will improve signal communication with reduced distortion by noise and transmission loss. It will also facilitate making several broadcasts simultaneously in the same geographic area (due to the frequency-shifting property of AM)
- Signal conditioning

  AM enables us to exploit advantages of ac signal conditioning hardware (improved stability, reduced drift, etc.). Also, the AM process will improve the signal level and noise immunity as a result of the use of the original signal (to be conditioned) to modulate a high-frequency, high-power carrier signal.

Natural AM

- Any device that uses the transformer action (primary winding and secondary winding with the primary coil being excited by an AC; e.g., linear variable differential transducer or LVDT, ac tachometer).
- A rotating machine with a fault; e.g., a gearbox with a fault on a tooth, a turbine rotor with eccentricity or damaged blade.

Yes. In the first device AM provides the advantages of ac signal conditioning, and improves noise immunity and signal level.

In the second device, the AM principle is useful for fault detection and diagnosis (through identification of the associated frequencies).

Solution 2.30

(a) Ball passing frequency = carrier frequency $f_c = \dfrac{840+960}{2}$ = 900 Hz

Hence, Estimated shaft speed = $f_b = 900-840$ rev/s $= 60$ rev/s $= 3600$ r.p.m.

Number of balls = $\dfrac{\text{ball passing frequency}}{\text{shaft speed}} = \dfrac{900}{60} = 15$

Peak frequencies = 900 ± 60 Hz $= [840 \text{ and } 960]$ Hz, (see Figure P2.30).

(b) Yes.

Solution 2.31

(a) Phase-Sensitive Demodulation

If the sign of the data signal (modulating signal) is preserved at all times during modulation and is properly recovered in the demodulation process, we have a phase-sensitive demodulation. *Note*: A sign change corresponds to a 180° phase change of the modulated signal; hence, the name.

(b) Half-Wave Demodulation

In half-wave demodulation, an output is generated during every other half-period of the carrier signal.

(c) Full-Wave Demodulation

Here, an output (demodulated value) is generated continuously for every point of the carrier signal.

The rotating frequency (rev/s) f_o modulates the responses produced by the forcing functions such as gear meshing, ball or roller hammer, and blade passing in rotating machinery. Hence, if the forcing frequency is f_c, according to the modulation theorem, peaks occur at $f_c \pm f_o$ instead of at f_c.

Solution 2.32

Resolution corresponds to 1 LSB ➔ 10/ 2^8 = 10/256 = 0.04 V
Quantization error 9rounding off) corresponds to half this value → 0.02 V

Solution 2.33

(a)

The decimal value of the input binary word is given by

$$D = 2^{n-1}b_{n-1} + 2^{n-2}b_{n-2} + \ldots + 2^{0}b_{0}. \qquad \text{(i)}$$

The least significant bit (LSB) is b_0 and the most significant bit (MSB) is b_{n-1}. The analog output voltage v of the DAC has to be proportional to D.

Each bit b_i in the digital word w will activate a solid-state microswitch in the switching circuit, typically by sending a switching voltage pulse. If b_i = 1, the circuit lead will be connected to the $-v_{ref}$ supply, providing an input voltage $v_i = -v_{ref}$ to the corresponding weighting resistor $2^{n-i-1}R$. If, on the other hand, b_i = 0, then the circuit lead will be connected to ground, thereby providing an input voltage v_i = 0 to the same resistor. Note that the MSB is connected to the smallest resistor (R) and the LSB is connected to the largest resistor ($2^{n-1}R$). By writing the summation of currents at node A of the output op-amp, we get $\dfrac{v_{n-1}}{R} + \dfrac{v_{n-2}}{2R} + \ldots \dfrac{v_0}{2^{n-1}R} + \dfrac{v}{R/2} = 0$.

In writing this equation, we have used the two principal facts for an op-amp: the voltage is the same at both input leads and the current through each lead is zero. Note that the positive lead is grounded and hence node A should have zero voltage. Now, since $v_i = -b_i v_{ref}$, where b_i = 0 or 1 depending on the bit value (state of the corresponding switch), we have

$$v = \left[b_{n-1} + \frac{b_{n-2}}{2} + \ldots + \frac{b_0}{2^{n-1}}\right]\frac{v_{ref}}{2} \qquad \text{(ii)}$$

Clearly, as required, the output voltage v is proportional to the value D of the digital word w.

(b)

The full-scale value (FSV) of the analog output occurs when all b_i are equal to 1. Hence,

$$\text{FSV} = \left[1 + \frac{1}{2} + \ldots + \frac{1}{2^{n-1}}\right]\frac{v_{ref}}{2}$$

Using the commonly known formula for the sum of a geometric series

$$1 + r + r^2 + \ldots + r^{n-1} = \frac{(1-r^n)}{(1-r)} \qquad \text{(iii)}$$

we get

$$\text{FSV} = \left(1 - \frac{1}{2^n}\right)v_{ref} \qquad \text{(iv)}$$

Note: This value is slightly smaller than the reference voltage v_{ref}.

(c)

A major drawback of the weighted-resistor DAC is that the range of the resistance value in the weighting circuit is very wide. This presents a practical difficulty, particularly when the size (number of bits n) of the DAC is large. Use of resistors with widely different magnitudes in the same circuit can create accuracy problems. For example, since the MSB corresponds to the smallest weighting resistor, it follows that the resistors must have a very high precision.

Solution 2.34

For an ADC,
Resolution: This is the change in analog input that corresponds to one LSB change in the digital output.

Dynamic Range: This is the ratio of the span (maximum value - minimum value) to the resolution, expressed in dB.

For an n-bit ADC, DR = 2^n-1 as a ratio $= 20\log_{10}(2^n - 1)$ dB

Full-scale value: The value of the analog input corresponding to the maximum digital output (i.e., when all the bits are at 1).

Quantization Error: Error introduced due to digitization of a sampled data value. This is equal to: (Signal value corresponding to the digital output)-(Actual sampled value just before conversion).

This error is less than one LSB. With rounding-off, this error is less than 1/2 LSB.

Solution 2.35

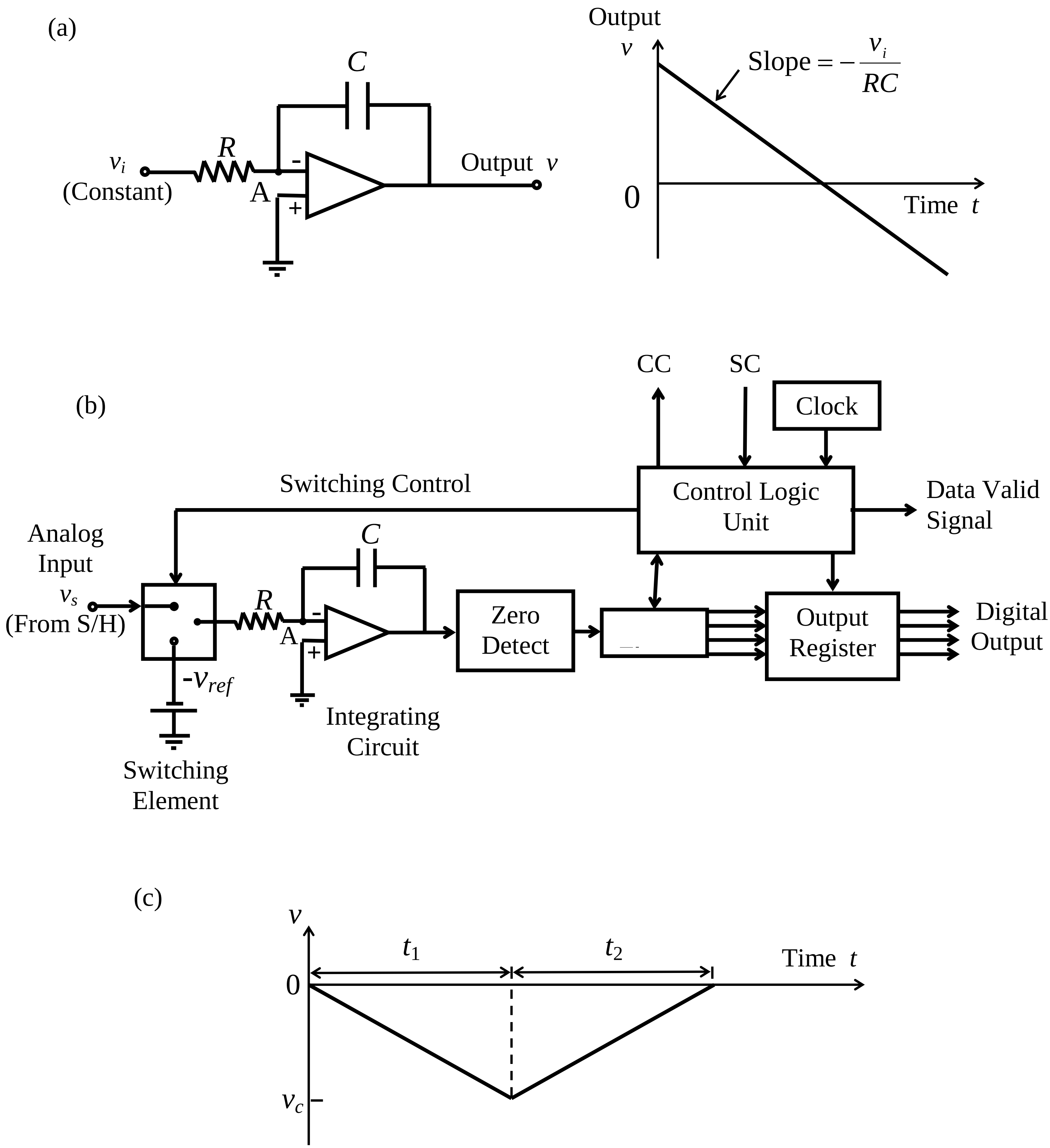
(a) Dual-Slope ADC

A dual-slope ADC is based on timing (i.e., counting the number of clock pulses during) a capacitor-charging process. It uses an RC integrating circuit. Hence, it is also known as an *integrating ADC*. It is simple and inexpensive. In particular, an internal DAC is not utilized and hence, DAC errors will not enter the ADC output. Furthermore, the parameters R and C in the integrating circuit do not enter the ADC output. As a result, the device is self-compensating in terms of circuit-parameter variations due to temperature, aging, so on. A shortcoming of this ADC is its slow conversion rate because, for accurate results, the signal integration has to proceed for a longer time in comparison to the conversion time for a successive approximation ADC.

The principle of operation can be explained with reference to the integrating circuit shown in Figure S2.35(a). Here, v_i is a constant input voltage to the circuit and v is the output voltage. Since the positive lead of the op-amp is grounded, the negative lead (and node A) also will have zero voltage. In addition, the currents through the op-amp leads are negligible. Hence, the current balance at node A gives $\frac{v_i}{R} + C\frac{dv}{dt} = 0$. Integrating this equation for constant v_i, we get

$$v(t) = v(0) - \frac{v_i t}{RC} \quad \text{(i)}$$

Equation (i) is used in obtaining a principal result for the dual-slope ADC.


(a)
C
R
v_i
(Constant)
A
Output v
Output
v
Slope $= -\frac{v_i}{RC}$
0
Time t
(b)
CC
SC
Clock
Switching Control
Control Logic Unit
Data Valid Signal
Analog Input
v_s
(From S/H)
C
R
A
$-v_{ref}$
Zero Detect
Output Register
Digital Output
Integrating Circuit
Switching Element
(c)
v
t_1
t_2
Time t
0
v_c

(d)

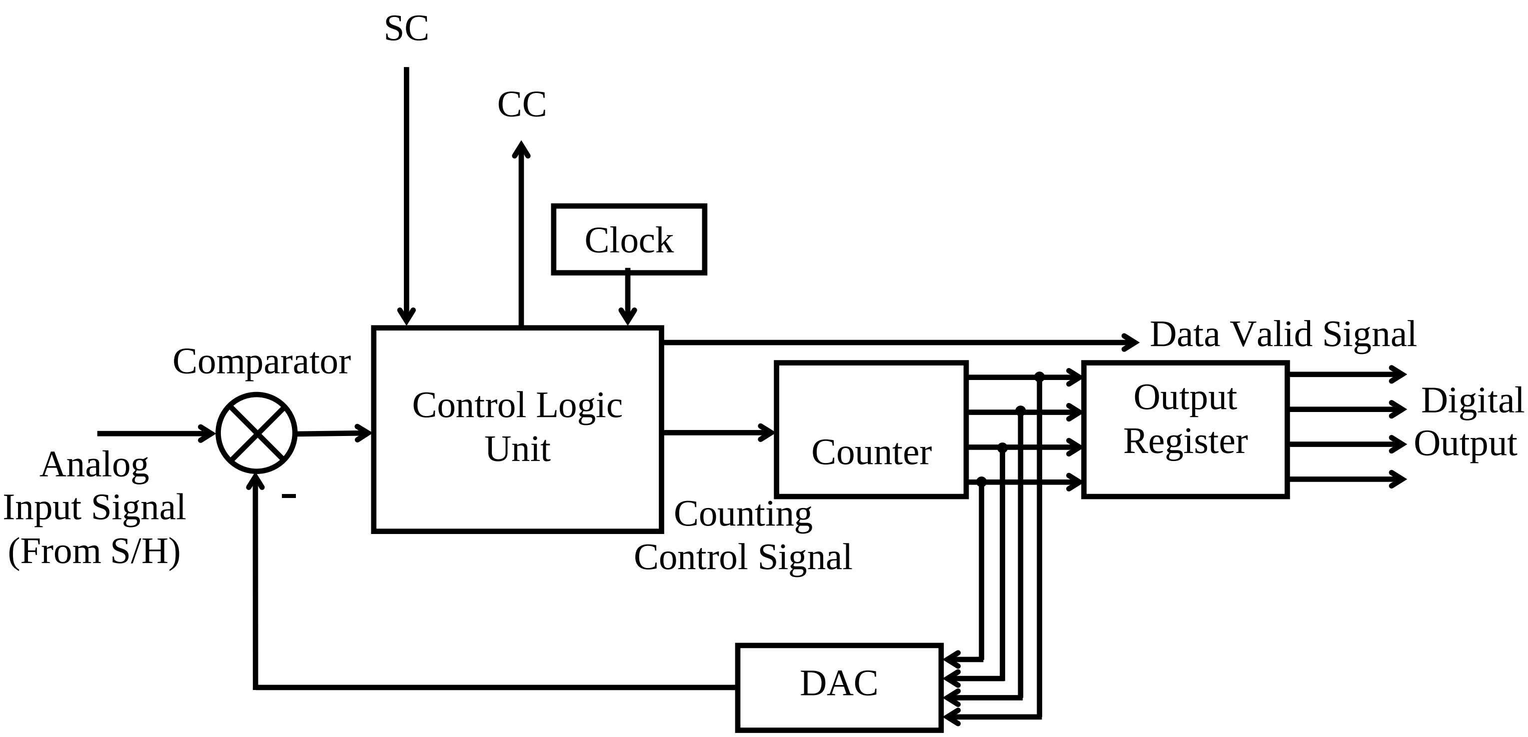


Figure S2.35: (a) RC integrating circuit; (b) Dual-slope ADC; (c) Dual-slope charging–discharging curve; (d) Counter ADC.

A schematic diagram for a dual-slope ADC is shown in Figure S2.35(b). Initially, the capacitor C in the integrating circuit is discharged (zero voltage). Then, the analog signal v_s is supplied to the switching element and held constant by the S/H. Simultaneously, a control signal "conversion start" is sent to the control logic unit. This clears the timer and the output register (i.e., all bits are set to zero) and sends a pulse to the switching element to connect the input v_s to the integrating circuit. Also, a signal is sent to the timer to initiate timing (counting). The capacitor C will begin to charge. Equation (i) is now applicable with input $v_i = v_s$ and the initial state $v(0) = 0$. Suppose that the integrator output v becomes $-v_c$ at time $t = t_1$. Hence, from Equation (i), we get

$$v_c = \frac{v_s t_1}{RC} \tag{ii}$$

The timer will keep track of the capacitor charging time (as a clock pulse count n) and will inform the control logic unit when the elapsed time is t_1 (i.e., when the count is n_1). *Note*: t_1 and n_1 are fixed (and known) parameters, but voltage v_c depends on the value of v_s, and is unknown.

At this point, the control logic unit sends a signal to the switching unit, which will connect the input lead of the integrator to a reference voltage of opposite polarity (a negative supply voltage $-v_{ref}$). Simultaneously, a signal is sent to the timer to clear its contents and start timing (counting) again. Now the capacitor begins to discharge. The output of the integrating circuit is monitored by the zero-detect unit. When this output becomes zero, the zero-detect unit sends a signal to the timer to stop counting. A comparator (differential amplifier) with one of the two input leads set at zero potential may serve as the zero-detect unit.

Now suppose that the elapsed time is t_2 (with a corresponding count of n_2). It should be clear that Equation (i) is valid for the capacitor discharging process as well. *Note*: $v_i = -v_{ref}$ and $v(0) = -v_c$ in this case. Also, $v(t) = 0$ at $t = t_2$. Hence, from Equation (i), we have $0 = -v_c + \frac{v_{ref} t_2}{RC}$, or

$$v_c = \frac{v_{ref} t_2}{RC} \tag{iii}$$

On dividing Equation (ii) by Equation (iii), we get $v_s = v_{ref} \frac{t_2}{t_1}$. However, the timer pulse count is proportional to the elapsed time. Hence, $\frac{t_2}{t_1} = \frac{n_2}{n_1}$. Now we have

$$v_s = \frac{v_{ref}}{n_1} n_2 \tag{iv}$$

Since v_{ref} and n_1 are fixed quantities, v_{ref}/n_1 can be interpreted as a scaling factor for the analog input. Then, it follows from Equation (iv) that the second count n_2 is proportional to the analog signal sample v_s. *Note*: The timer output is available in the digital form. Accordingly, the count n_2 is used as the digital output of the ADC.

At the end of the capacitor discharge period, the count n_2 in the timer is transferred to the output register of the ADC, and the "data valid" signal is set. The contents of the output register are now ready to be read by the interfaced digital system, and the ADC is ready to convert a new sample.

The charging–discharging curve for the capacitor during the conversion process is shown in Figure S2.35(c). The slope of the curve during charging is $-v_s/RC$, and the slope during discharging is $+v_{ref}/RC$. The reason for the use of the term "dual slope" to denote this ADC is therefore clear.

As mentioned before, any variations in R and C do not affect the accuracy of the output. But, it should be clear from the foregoing discussion that the conversion time depends on the capacitor discharging time t_2 (*Note*: t_1 is fixed), which in turn depends on v_c and hence on the input signal value v_s (see Equation (ii)). It follows that, unlike the successive approximation ADC, the dual-slope ADC has a conversion time that directly depends on the magnitude of the input data sample. This may be considered as a disadvantage because in many applications we prefer to have a constant conversion rate.

The foregoing discussion assumes that the input signal is positive. For a negative signal, the polarity of the supply voltage v_{ref} has to be changed. Furthermore, the sign has to be properly represented in the contents of the output register as, for example, in the case of successive approximation ADC.

(b) Counter ADC

The counter-type ADC has several aspects in common with the successive approximation ADC. Both are comparison-type (or closed loop) ADCs. Both use a DAC unit internally to compare the input signal with the converted signal. The main difference is that in a counter ADC the comparison starts with the LSB and proceeds down. It follows that, in a counter ADC, the conversion time depends on the signal level, because the counting (comparison) stops when a match is made, resulting in shorter conversion times for smaller signal values.

A schematic diagram for a counter ADC is shown in Figure S2.35(d). Note that this is quite similar to Figure 2.38 for the Successive approximation ADC. Initially, all registers are cleared (i.e., all bits and counts are set to zero). As an analog data signal (from the S/H) arrives at the comparator, an SC pulse is sent to the control logic unit. When the ADC is ready for conversion (i.e., when data valid signal is on), the control logic unit initiates the counter. Now, the counter sets its count to 1, and the LSB of the DAC register is set to 1 as well. The resulting DAC output is subtracted from the analog input, by means of the comparator. If the comparator output is positive, the count is incremented by one, and this causes the binary number in the DAC register to be incremented by one LSB. The new (increased) output of the DAC is now compared with the input signal. This cycle of count incrementing and comparison is repeated until the comparator output becomes less than or equal to zero. At that point, the control logic unit sends out a CC signal and transfers the contents of the counter to the output register. Finally, the data valid signal is turned on, indicating that the ADC is ready for a new conversion cycle, and the contents of the output register (the digital output) is available to be read by the interfaced digital system.

The count of the counter is available in the binary form, which is compatible with the output register as well as the DAC register. Hence, the count can be transferred directly to these registers. The count when the analog signal is equal to (or slightly less than) the output of the DAC, is proportional to the analog signal value. Hence, this count represents the digital output. In bipolar operation, the sign of the input signal has to be properly accounted for.

Solution 2.36

Dual-slope (integrating) ADC: In this case, the conversion time is the total time needed to generate the two counts n_1 and n_2 (see Figure S2.35(c). Hence,

$$t_c = (n_1 + n_2)\Delta t \tag{i}$$

Note that n_1 is a fixed count. However, n_2 is a variable count, which represents the digital output, and is proportional to the analog input (signal level). Hence, in this type of ADC, conversion time depends on the analog input level. The largest output for an n bit converter is $2n-1$. Hence, the largest conversion time may be given by

$$t_{c\max} = (n_1 + 2^n - 1)\Delta T \tag{ii}$$

Counter ADC: For a counter ADC, the conversion time is proportional to the number of bit transitions (1 LSB/step) from zero to the digital output n_o. Hence, the conversion time is given by

$$t_c = n_o \Delta t \tag{iii}$$

where n_o is the digital output value (in decimal). Note that for this ADC as well, t_c depends on the magnitude of the input data sample. For an n bit ADC, since the maximum value of n_o is $2^n - 1$, we have the maximum conversion time

$$t_{c\max} = (2^n - 1)\Delta t \quad \text{(iv)}$$

By comparing these results with that obtained for a successive-approximation ADC, it can be concluded that the successive-approximation ADC is the fastest of the three types.

Solution 2.37

(a) **Direct-conversion ADC (Flash ADC):** An n-bit digital word can represent $2^n - 1$ digital values, and each additional bit in the word multiplies this by a factor of 2. This ADC, uses a bank of $2^n - 1$ comparators, each having a reference value corresponding to one of these digital values. The analog signal is sampled into the comparators, in parallel. A comparator fires its digital value if the sampled signal value matches it (to less than half-bit accuracy). This is very fast and capable of gigahertz sampling rates (*Note*: Comparison is done in parallel). Since the bit size increases the number of comparators in an exponential manner, typically, this ADC is limited to 8 bits (to reduce hardware costs and error).

(b) **Ramp-compare ADC:** This is similar to the dual-slope ADC. The main difference of the present ADC is the use of a ramping signal instead of an integrator. Specifically, it uses a saw-tooth signal that ramps and then quickly returns to zero. Clock counts start as the ramping starts, and stops when the ramp voltage matches (as detected by a comparator) the input signal (analog) value. The clock count gives a digital value, which represents the analog data value. *Note*: The full-scale value (FSV) of the ADC register corresponds to the reference value of the comparator voltage. can be properly scaled using , a comparator fires, and the timer's value is recorded. An oscillator can be used to generate the ramp signal. This ADC is relatively simple with regard to hardware.

(c) **Wilkinson ADC:** In this ADC, the input voltage (analog data) is compared with that produced by a charging capacitor. The capacitor is allowed to charge until its voltage is equal to the amplitude of the input pulse (a comparator determines when this condition has been reached), and it is discharged to zero. The charge-discharge times are counted using a clock, and used to determine the corresponding digital value. Hence, this approach is basically the same as that of a dual-slope (integrating) ADC.

(d) **Delta-encoded ADC:** It uses a counter, DAC, and a comparator (compare with other ADCs that use similar hardware). The count is incremented and the resulting digital value is fed into the DAC. The DAC output is compared with the incoming analog data value. The count is stopped when the two values match (up to 1 count increment). The corresponding digital input to the DAC is the output of the ADC. *Note*: Here, "delta" denotes the count "increment," which is essentially one LSB bit value of the ADC (or DAC). The method is also called "delta-comparison."

(e) **Pipeline ADC:** This uses an internal DAC (as with many other ADC methods) and is somewhat similar to the successive-approximation ADC, but is faster. It uses two or more steps of conversion, starting with a "coarse" conversion and ending with a "fine" conversion. After the coarse conversion in the first step, the difference between the analog input value and the DAC output value is then converted using a finer ADC, in the next step. The digital results are combined in a last step.