Solutions Manual for
RF Microelectronics

Second Edition

Behzad Razavi

PRENTICE
HALL

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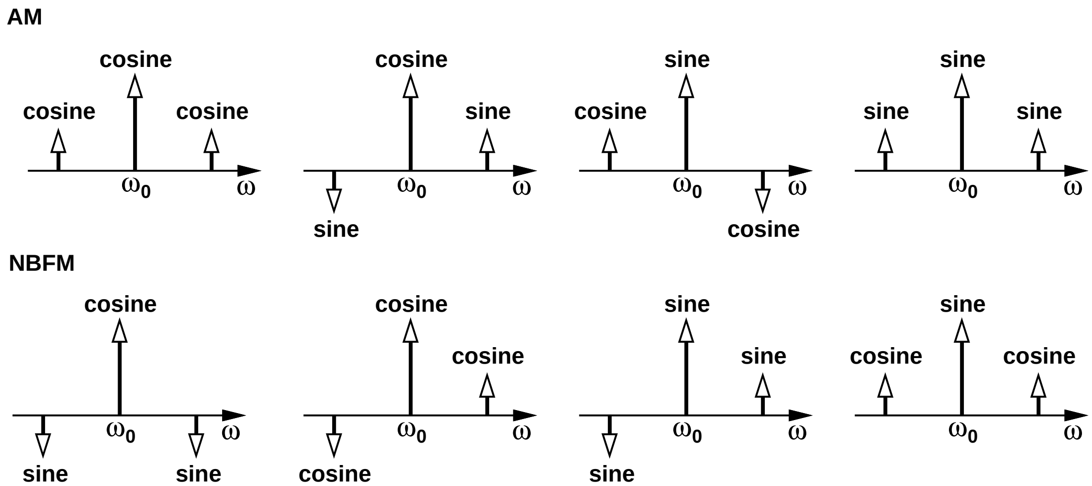
RF Microelectronics, Second Edition

Errata

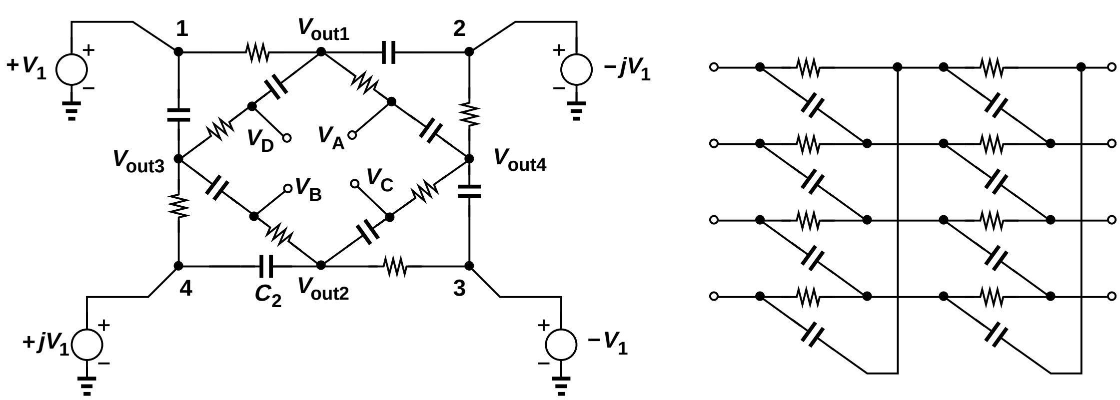
Behzad Razavi

- Prob. 2.3, second line should read: consider the cascade of identical ...

- Fig. 3.10 should be changed as shown below:



- Fig. 4.81(a) should be changed as shown below:



(a) (b)

- Example 4.36, the first sentence in solution should read: We have $V_{out1} = (1/2)(1 - j)V_1$ and ...

- Example 5.5, third line in solution: Since it is desired that $R_{in} = R_S$,

- Example 6.21, last three lines of solution: Note that $V_{n2}(f)$ is typically very large because M_2 and M_3 are relatively small.

- Example 7.6, Eq. (7.33) should read:

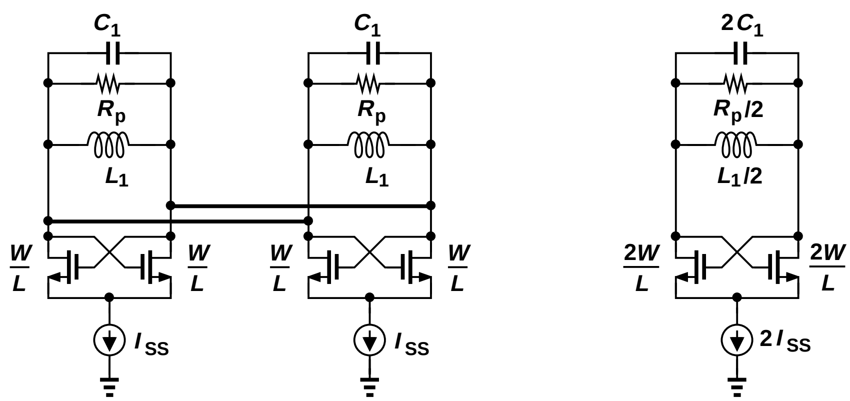
$$C_{eq} = \frac{C_1 + \cdots + C_{4(N-1)}}{[4(N-1)]^2} \tag{1}$$

  Eq. (7.125) in Problem 7.3 must also be corrected as above.

- p. 488, the sentence below Eq. (7.114) should read $Z_1 d = R_{tot}/2$ and $Y_1 d = C_{tot}s/2$.

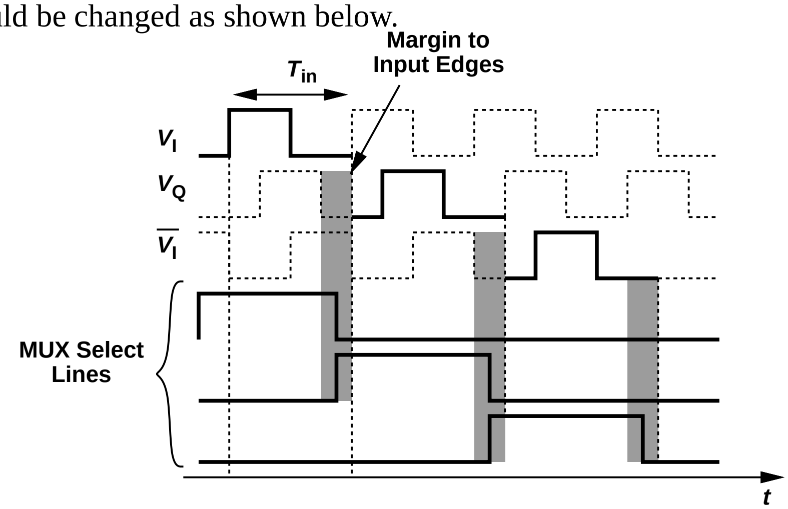
- Prob. 7.10, Assume the inductance is about 9 times that of one spiral.

- Fig. 8.84 (b) should be changed as follows:

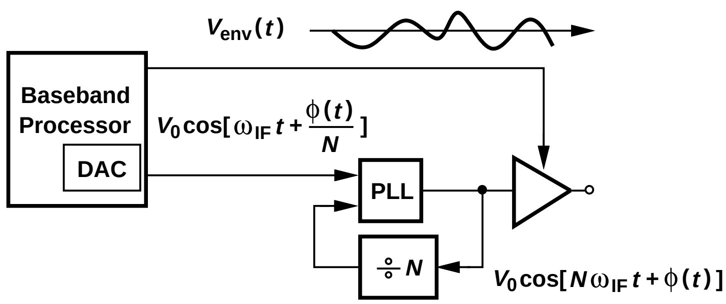


(a) (b)

- Fig. 11.45 should be changed as shown below.

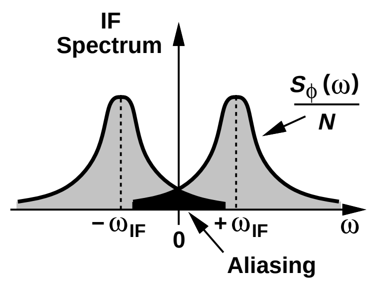


- Fig. 12.53(b) should be changed as shown below.



$V_{env}(t)$
Baseband Processor
DAC
$V_0 \cos[\omega_{IF} t + \frac{\phi(t)}{N}]$
PLL
$\div N$
$V_0 \cos[N\omega_{IF} t + \phi(t)]$

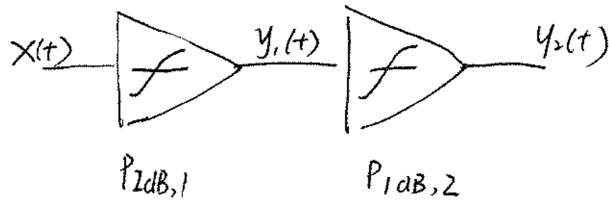
(a)



IF Spectrum
$\frac{S_\phi(\omega)}{N}$
$-\omega_{IF}$
0
$+\omega_{IF}$
ω
Aliasing

(b)

2.1 Solu:



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_2^2(t) + \beta_3 y_3^2(t)$$

then.
$$y_2(t) = \beta_1 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2$$
$$+ \beta_3 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3$$

Considering only the first- and third-order terms,

$$y_2(t) = \alpha_1 \beta_1 x(t) + (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) x^3(t) + \cdots$$
$$= [\alpha_1 \beta_1 A + \frac{3}{4} (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) A^3] \cos(t) + \cdots$$

$P_{1dB,1}$: $A_{in1,1dB} = \sqrt{0.145 \left|\frac{\alpha_1}{\alpha_3}\right|}$; $P_{1dB,2}$: $A_{in2,1dB} = \sqrt{0.145 \left|\frac{\beta_1}{\beta_3}\right|}$

$$P_{1dB} \Rightarrow \quad 20 \log \left| \alpha_1 \beta_1 + \frac{3}{4} (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) \cdot A^2_{in,1dB} \right| = 20 \log |\alpha_1 \beta_1| - 1dB$$

$$A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|}$$

Represented by the P_{1dB} of first and second stage.

$$\frac{1}{A^2_{in,1dB}} = \frac{1}{0.145} \left| \frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2 \beta_3}{\beta_1} \right|$$
$$= \left| \frac{1}{A^2_{in1,1dB}} + \frac{2}{0.145} \frac{\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2}{A^2_{in2,1dB}} \right|$$

2.2 Solu:

assuming $-3\,dBm$ A_1 at $2.42\,G$

$-35\,dBm$ A_2 at $2.43\,G$.

IM product : $\frac{3}{4}\gamma_3 A_1^2 \cdot A_2$

$-3\,dBm \Rightarrow A_1 = \sqrt{2 \cdot 50 \cdot 10^{-0.3} \times 10^{-3}} = 223.9\,mV$

$-35\,dBm \Rightarrow A_2 = \sqrt{2 \cdot 50 \times 10^{-3.5} \times 10^{-3}} = 5.6\,mV.$

We can write at LNA output :

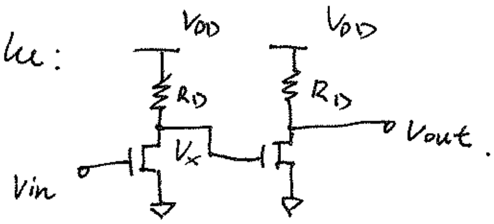
$$20 \lg |\gamma_1 \cdot A_{sig}| - 20\,dB = 20 \lg \left| \frac{3}{4} \gamma_3 A_1^2 \cdot A_2 \right|.$$

$$\Rightarrow \lg |\gamma_1 \cdot A_{sig}| = \lg \left| \frac{30}{4} \gamma_3 A_1^2 \cdot A_2 \right|$$

$$IIP_3 = \sqrt{\frac{4}{3} \left| \frac{\gamma_1}{\gamma_3} \right|} = \sqrt{\frac{4}{3} \cdot \frac{30}{4} \cdot \frac{A_1^2 \cdot A_2}{A_{sig}}} = 9.43\,V_p$$

$$= 29.5\,dBm.$$

2.3 solu:



$$I_D = K \cdot (V_{GS} - V_T)^2$$

$$V_{out} = V_{DD} - K \cdot R_D (V_x - V_T)^2$$

$$V_x = V_{DD} - K \cdot R_D (V_{in} - V_T)^2$$

$$V_{out} = V_{DD} - K \cdot R_D [V_{DD} - K \cdot R_D (V_{in} - V_T)^2 - V_T]^2$$

$$= V_{DD} - K \cdot R_D [(V_{DD} - V_T) - K \cdot R_D (V_{in} - V_T)^2]^2$$

$$= V_{DD} - K \cdot R_D [(V_{DD} - V_T)^2 + K^2 R_D^2 (V_{in} - V_T)^4 - 2 K R_D (V_{DD} - V_T)(V_{in} - V_T)^2]$$

1st order of Vin

$$\Rightarrow [4 \cdot (V_{DD} - V_T) \cdot K \cdot R_D V_T - 4 K^2 R_D^2 \cdot V_T^3] \cdot V_{in}.$$

3rd order of Vin

$$\Rightarrow [- 4 K^2 R_D^2 V_T] \cdot V_{in}^3$$

$$A_{IP3} = \sqrt{\frac{4}{3} \cdot \frac{4 \cdot (V_{DD} - V_T) K \cdot R_D V_T - 4 K^2 R_D^2 V_T^3}{4 K^2 R_D^2 V_T}}.$$

2.4. Solu.

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \alpha_4 x^4(t) + \alpha_5 x^5(t).$$

1° $\cos^3 \omega t = \frac{3}{4}\cos\omega t + \frac{1}{4}\cos 3\omega t.$

2° $\cos^2 \omega t = \dfrac{1+\cos 2\omega t}{2}$

3° $\cos^4 \omega t = \dfrac{1+\cos^2 2x + 2\cos 2x}{2^2}$

4° $\cos^5 \omega t = \left(\frac{3}{4}\cos\omega t + \frac{1}{4}\cos 3\omega t\right)\cdot\left(\dfrac{1+\cos 2\omega t}{2}\right)$

$$= \Big(\frac{3}{8}\cos\omega t + \frac{1}{8}\cos 3\omega t + \underbrace{\frac{3}{8}\cos\omega t\cdot\cos 2\omega t}_{=\frac{3}{16}[\cos\omega t + \cos 3\omega t]} + \underbrace{\frac{1}{8}\cos 3\omega t\cdot\cos 2\omega t}_{=\frac{1}{16}[\cos\omega t + \cos 5\omega t]}\Big)$$

1st order $\Rightarrow$. $\alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \left(\frac{3}{8}+\frac{3}{16}+\frac{1}{16}\right)\cdot\alpha_5 A^5.$

3rd order

$\Rightarrow$ $\frac{1}{4}\alpha_3 A^3 + \left(\frac{1}{8}+\frac{3}{16}\right)\alpha_5 \cdot A^5.$

(1) $P_{1dB} \Rightarrow$. $20\lg\left|\alpha_1 + \frac{3}{4}\alpha_3 A^2 + \frac{5}{8}\alpha_5 A^4\right| = 20\lg|\alpha_1| - 1\,dB.$

$$\Rightarrow \quad A_{in,1dB} = \sqrt{\frac{0.8\cdot(0.5625\,\alpha_3^2 - 0.27175\,\alpha_1\cdot\alpha_5)^{\frac{1}{2}} - 0.6\,\alpha_3}{\alpha_5}}.$$

(2) IIP_3 doesn't change.

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

2.5 (a) Solu: $A_{sig} = 0.1mV \times 0.7943 \Leftarrow -2dB = 10^{-0.1} = 0.7943$

$A_2 = 10mV \times 0.1413 \Leftarrow -17dB = 10^{-0.85} = 0.1413$

$A_3 = 10mV \times 0.0141 \Leftarrow -37dB = 10^{-1.85} = 0.0141$

at the output of amplifier:

$$20\lg|\gamma_1 \cdot A_{sig}| - 20dB = 20\lg\left|\frac{3}{4}\gamma_3 \cdot A_2^{3} \cdot A_3\right|$$

$$\Rightarrow |\gamma_1 \cdot 0.07943m| = \left|\frac{30}{4}\gamma_3\ 1.413m^2\ .0.141m\right|$$

$$\therefore A_{IIP3} = \sqrt{\frac{30}{4}\frac{1.413m^2 \cdot 0.141}{0.07943} \cdot \frac{4}{3}} = 5.95mV_p = -34.5dBm,$$

Neglect the nonlinearity of BPF.

(b). $y_1(t) = \gamma_1 x(t) + \gamma_3 x^3(t)$

$y_2(t) = \beta_1 y_1(t) + \beta_3 y_1^3(t)$.

Only considering the first and third order:

$$y_2(t) = \gamma_1\beta_1 x(t) + (\gamma_3\beta_1 + \gamma_1^3\beta_3)x^3(t) + \cdots$$

$$\therefore A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\gamma_1\beta_1}{\gamma_3\beta_1 + \gamma_1^3\beta_3}\right|}$$

$$\frac{1}{A_{IP3}^2} = \frac{1}{A_{IP3,1}^2} + \frac{\gamma_1^2}{A_{IP3,2}^2}$$

$$\Rightarrow \frac{1}{A_{IP3}^2} = \frac{1}{500m^2} + \frac{10}{5.95m^2}$$

$$\Rightarrow A_{IP3} = 1.875mV_p$$

2.6 Solu:

Let $x(t)$ be a random signal (Wide-sense stationary process)

Auto correlation function: $R_x(\tau) = E[x(t) \cdot x(t+\tau)]$

Let me prove that: $S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$.

Proof: $X_T(f) \triangleq \int_{-T/2}^{T/2} x(t) e^{-j2\pi f t} dt$.

$S_T(f) \triangleq E[\frac{1}{T} |X_T(f)|^2]$.

$S_x(f) = \lim_{T\to\infty} S_T(f)$.

$$E[|X_T(f)|^2] = E\left[\int_{-T/2}^{T/2} x(t) e^{-j2\pi f t} dt\right]^2$$

$$= E\left[\int_{-T/2}^{T/2} x(t) e^{-j2\pi f t} dt \cdot \int_{-T/2}^{T/2} x(\tau) e^{-j2\pi f \tau} d\tau\right]$$

$$= E\left[\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} x(t) x(\tau) e^{-j2\pi f(t-\tau)} dt\, d\tau\right]$$

$$= \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} E[x(t)\cdot x(\tau)] e^{-j2\pi f(t-\tau)} dt\, d\tau.$$

$$= \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} R_x(t-\tau) e^{-j2\pi f(t-\tau)} dt\, d\tau.$$

$$= \int_{-T}^{T} (T-|\tau|) R_x(\tau) e^{-j\pi f\tau} d\tau.$$

$$E[\frac{1}{T}|X_T(f)|^2] = \int_{-T}^{T} (1-\frac{|\tau|}{T}) R_x(\tau) e^{-j2\pi f\tau} d\tau.$$

Therefore: $S_x(f) = \lim_{T\to+\infty} E[\frac{1}{T}|X_T(f)|^2] = \int_{-\infty}^{+\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$.

2.7. Solu:

Assume $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$.

$x(t) = V_0 \cos \omega_0 t$.

3rd - harmonic : $\frac{\alpha_3 V_0^3}{4} = V_3$

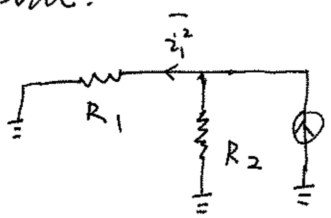
$$\Rightarrow \alpha_3 = \frac{4V_3}{V_0^3}$$

$$A_{in,1dB} = \sqrt{0.145 \left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$= \sqrt{0.145 \cdot \left|\alpha_1 / 4V_3/V_0^3\right|}$$

$$= \sqrt{\frac{0.145}{4}\left|\frac{\alpha_1 V_0^3}{V_3}\right|}.$$

12.8 soln:



$\overline{i_n^2} = \frac{4kT}{R_2}$

$$P_{R_1} = \overline{i_1^2}^{\,2} \cdot R_1$$

$$= \left(\sqrt{\frac{4kT}{R_2}} \cdot \frac{R_2}{R_1+R_2}\right)^2 \cdot R_1$$

$$= \frac{4kT}{(R_1+R_2)^2} \cdot R_1 R_2$$

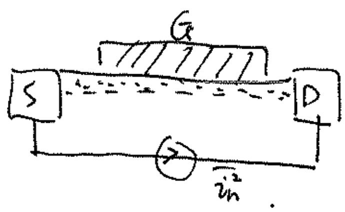
$\therefore P_{R_1} = P_{R_2}$

So, it proves that the noise power delivered by R_1 to R_2 is equal to that delivered by R_2 to R_2 at the same temperature.

$\therefore$ If it is not the truth, the energy would not be conserved.

2.9 Solu:

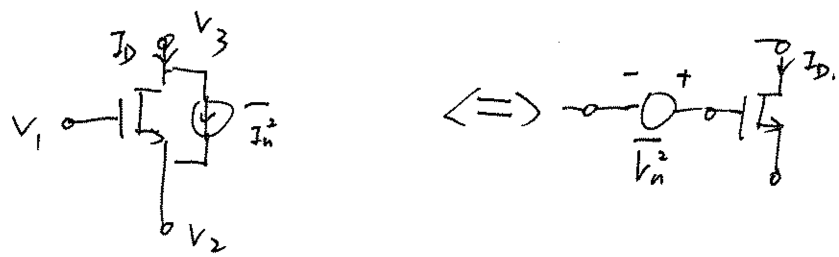
Why the channel thermal noise of a MOSFET is model by a current sourse b/w. S & D. rather than G & D.



Firstly, from the figure we can find the channel resistor is between source & drain. As a result, it is reasonable to model the noise by a current source between source and drain.

Secondly, MOSFET has the function of. transconductance. It's easy to transfer the current source ~~from~~ between S & D to the voltage source at the gate.

2.10 Solu:



Proof: transconductance : g_m.

Assume the transistor is in saturation region.

For small-signal analysis, $V_1 = 0$

$$I_D = \sqrt{\overline{I_n^2}} = \sqrt{4KT\gamma g_m}$$

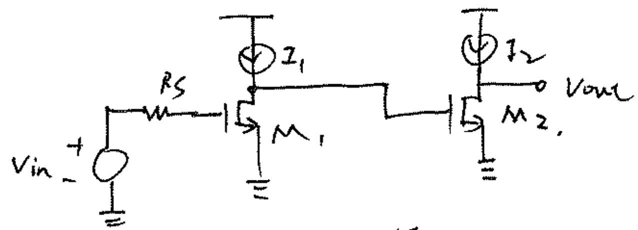
At the same time, for voltage source model.

$$I_D = V_{in} \cdot g_m = \sqrt{4KT\gamma \cdot g_m}$$

$$\Rightarrow \quad V_{in} = \sqrt{\frac{4KT\gamma}{g_m}}$$

$$\Rightarrow \quad \overline{V_n^2} = \frac{4KT\gamma}{g_m}.$$

2.11 Solu:



$$NF_1 = 1 + \frac{\gamma}{g_{m1} R_S} \qquad (2.122)$$

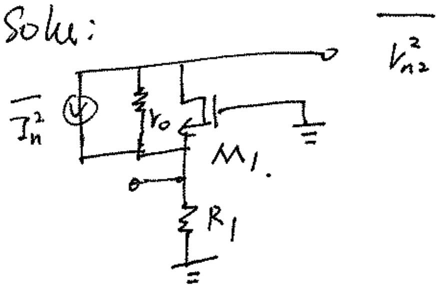
$$NF_2 = 1 + \frac{\gamma}{g_{m2} r_{o1}} \quad ; \quad A_{P1} = \frac{P_{out,av,1}}{P_{in,av,1}} = \frac{V_{in}^2 \cdot A_{V1}^2 \cdot \frac{1}{4r_{o1}}}{V_{in}^2 \cdot \frac{1}{4R_S}} = g_{m1}^2 \cdot r_{o1} \cdot R_S$$

$$\therefore NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}}$$

$$= 1 + \frac{\gamma}{g_{m1} R_S} + \frac{\gamma}{g_{m2} \cdot r_{o1}} \Big/ g_{m1}^2 r_{o1} R_S$$

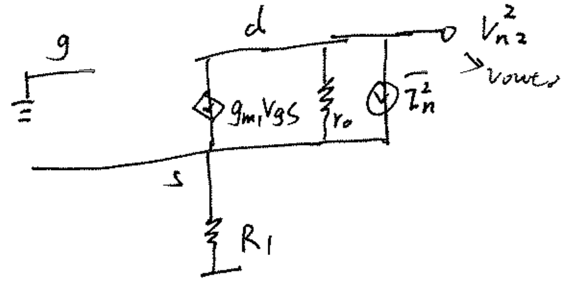
$$= 1 + \frac{\gamma}{g_{m1} R_S} + \frac{\gamma}{g_{m1}^2 r_{o1}^2 g_{m2} \cdot R_S}$$

12.12. Solu:



assume I_1 is ideal, and neglect the noise of R_1.

Proof:



For small-signal analysis,

$$g_m(-V_s) + \frac{V_{out} - V_s}{r_o} + I_n = \frac{V_s}{R_1}.$$

Because we cannot find any loop for the current through R_1

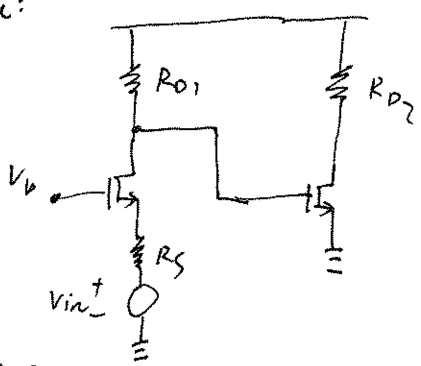
$V_s = 0$

$$\Rightarrow \quad \frac{V_{out}}{r_o} = -I_n$$

$$V_{out} = -I_n \cdot r_o$$

$$\therefore \quad \overline{V_{n2}^2} = \overline{I_{n2}^2} \cdot r_o^2.$$

2.13. Solu:

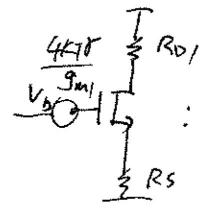


Neglect transistor cap.
flicker noise.
CLM.
body effect.

For 1st stage:

$R_{in1} = \frac{1}{g_{m1}}$, $R_{in2} = \infty$,

$$\overline{V_{n1}^2} = 4KTR_{D1} + \frac{4KT\gamma}{g_{m1}}\left(\frac{R_{D1}}{\frac{1}{g_{m1}}+R_S}\right)^2 \Leftarrow$$



: unloaded output noise.

For 2nd stage:

$$\overline{V_{n2}^2} = 4KTR_{D2} + 4KT\gamma \cdot g_{m2} \cdot R_{D2}^2 .$$

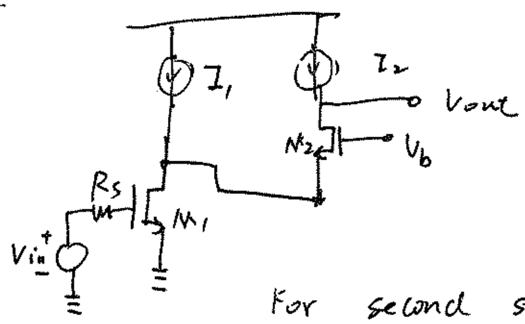
We now substitute these values in Eq. (2.126)

$$NF_{tot} = 1 + \frac{4KTR_{D1} + \frac{4KT\gamma}{g_{m1}}\left(\frac{R_{D1}}{\frac{1}{g_m}+R_S}\right)^2}{\left(\frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}}+R_S}\right)^2 (g_{m1} \cdot R_{D1})^2} \cdot \frac{1}{4KTR_S}$$

$$+ \frac{4KTR_D + 4KT\gamma \cdot g_{m2} \cdot R_{D2}^2}{\left(\frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}}+R_S}\right)^2 (g_{m1} \cdot R_{D1})^2 \cdot (g_{m2} \cdot R_{D2})^2} \cdot \frac{1}{4KTR_S} .$$

This result is different from the CS + CG configuration because the first stage's NF and input impedance are different, which affect the NF_{tot}.

2. 14. Solu:

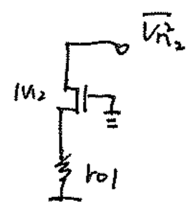


Consider CLM.

$$\overline{V_{n_1}^2} = \frac{4KT\gamma}{g_{m1}} \cdot (g_{m1} r_{o1})^2$$

$$= 4KT\gamma \cdot g_{m1} r_{o1}^2$$

For second stage:



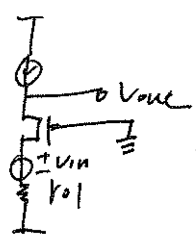
$$\overline{V_{n_2}^2} = 4KT\gamma \cdot g_{m2} \cdot r_{o2}^2$$

We now substitute these values in Eq. (2.126).

$$NF_{tot} = 1 + \frac{4KT\gamma \cdot g_{m1} r_{o1}^2}{(g_{m1} r_{o1})^2} \cdot \frac{1}{4KTR_S}$$

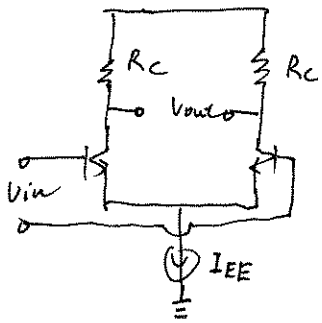
$$+ \frac{4KT\gamma g_{m2} r_{o2}^2}{(g_{m1} r_{o1})^2 \cdot \left(\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + r_{o1}}\right)^2 \cdot (g_{m2} r_{o2} + 1)^2} \cdot \frac{1}{4KTR_S}.$$

Note:



$\Rightarrow$ gain: $g_{m2} r_{o2} + 1$.

2.15 Solu:



IP3.

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \cdots$$

$$V_{out} = -2R_C I_{EE} \tanh\left[\frac{V_{in}}{2V_T}\right].$$

Only consider the first and third order.

$$V_{out} = -2R_C I_{EE}.\left(\frac{V_{in}}{2V_T} - \frac{1}{3}\left(\frac{V_{in}}{2V_T}\right)^3\right)$$

$$A_{in,IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$= \sqrt{\frac{4}{3} \cdot \frac{\frac{1}{2V_T}}{\frac{1}{3}\frac{1}{(2V_T)^3}}}$$

$$= 4V_T = 4\frac{KT}{q} = 4 \times 26mV = 104mV$$