Chapter 1: Critical Thinking

1.1 Public policy and Simpson's Paradox: Is "average" always average?

1.1.1 Simpson's paradox occurs when data are improperly aggregated.

1.1.2 True

1.1.3 False

1.1.4 True

1.1.5 $\frac{30}{40} = 0.75 = 75\%$

1.1.6 $\frac{50}{200} = 0.25 = 25\%$

1.1.7 $0.32 * 5000 = 1600$

1.1.8 $0.06 * 4500 = 270$

1.1.9 No, this scenario is not possible.

1.1.10 No, the same number of tests with the same number of points make this scenario impossible.

1.1.11 Yes this is possible and this is an example of Simpson's Paradox.

1.1.12 No, because there are the same total number of men and women, this scenario is impossible.

1.1.13 Yes, this scenario is possible but this is not an example of Simpson's Paradox.

1.1.14 Yes, this scenario is possible but this is not an example of Simpson's Paradox.

1.1.15 Yes, this scenario is possible but this is not an example of Simpson's Paradox.

1.1.16 (a) To determine the percentage of those who passed, compute $\frac{\#\text{passed}}{\#\text{tested}} \times 100\%$. For example, for the local school low income students compute: $\frac{300}{500} \times 100\% = 60.0\%$. The rest are computed in a similar manner.

	Local			Statewide		
	Tested	Passed	%passed	Tested	Passed	%passed
Low	500	300	60.0%	100,000	61,000	61.0%
High	7000	4725	67.5%	400,000	272,000	68.0%
Total	7500	5025	67.0%	500,000	333,000	66.6%

(b) For the low income students the pass rate in the local school was 60.0% vs. the statewide rate of 61.0%. Hence the rate was lower in the local school. The high income students had a local pass rate of 67.5% and a statewide pass rate of 68.0%; so it was also lower in the local school.
(c) The aggregate of all students in the local school had a 67% pass rate vs. a 66.6% statewide. Therefore, when considering all students combined, the local school had a higher pass rate.

1.1.17 (a) To compute the batting average we find $\frac{\#\text{hits}}{\#\text{at bats}}$. For Batter 1 in 2013, we compute $\frac{150}{500} = 0.300$. In 2014 we compute $\frac{3}{12} = 0.250$. To find batter 1's average over the two-year period, we take the total hits over that period, 150+3, and divide by the total at-bats, 500+12, to get $\frac{153}{512} = 0.299$. For Batter 2 in 2013 we have $\frac{8}{20} = 0.400$; and in 2014, $\frac{78}{300} = 0.260$. To find batter 2's average over the two-year period we compute $\frac{8+78}{20+300} = \frac{86}{320} = 0.269$.
(b) In 2013 Batter 2 had a higher average (0.400 vs. 0.300).
(c) In 2014 Batter 2 had a higher average (0.260 vs. 0.250).
(d) Over the two-year period Batter 1 had a higher average, 0.299 vs. 0.269.

1.1.18a The percentage of Mercy Hospital patients who were in *fair (or better)* condition and who lived is equal to the number of patients who lived divided by the total patients treated multiplied by 100%:

$$\frac{\text{Lived}}{\text{Total}} \times 100\% = \frac{580}{590} \times 100\% = 98.3\%$$

A similar computation is done for County Hospital:

$$\frac{\text{Lived}}{\text{Total}} \times 100\% = \frac{860}{890} \times 100\% = 96.6\%$$

1.1.18b The percentage of Mercy Hospital patients who were in *worse-than-fair* condition and who lived is equal to the number of patients who lived divided by the total patients treated multiplied by 100%:

$$\frac{\text{Lived}}{\text{Total}} \times 100\% = \frac{210}{410} \times 100\% = 51.2\%$$

A similar computation is done for County Hospital:

$$\frac{\text{Lived}}{\text{Total}} \times 100\% = \frac{40}{110} \times 100\% = 36.4\%$$

1.1.18c Mercy Hospital has a lower overall survival rate than County Hospital. However, Mercy Hospital has a larger number of individuals with *worse-than-fair* condition compared to County Hospital. An appropriate argument would be that for each condition *fair (or better)*, and *worse-than-fair*, the survival rates are greater at Mercy Hospital than at County Hospital. The reverse result for the combined table is an example of Simpson's Paradox.

1.1.18d Mercy Hospital is the wise choice for the child, although the survival rate of patients in *fair (or better)* condition is nearly identical between the hospitals. If the child's condition were *less than favorable*, Mercy Hospital has the higher survival rate.

1.1.19a The percentage of men hired in the hardware department is equal to the number hired divided by the total number of applications multiplied by 100%:

$$\frac{\text{Hired}}{\text{Applied}} \times 100\% = \frac{60}{80} \times 100\% = 75\%$$

A similar computation is done for men hired in the women's apparel department:

$$\frac{\text{Hired}}{\text{Applied}} \times 100\% = \frac{2}{20} \times 100\% = 10\%$$

A similar computation is done for women hired in the hardware department:

$$\frac{\text{Hired}}{\text{Applied}} \times 100\% = \frac{15}{20} \times 100\% = 75\%$$

A similar computation is done for women hired in the women's apparel department:

$$\frac{\text{Hired}}{\text{Applied}} \times 100\% = \frac{30}{300} \times 100\% = 10\%$$

1.1.19b An argument for gender discrimination could be made that by comparing the proportion of women hired in hardware and women's apparel (14.1% combined) at U-Mart versus the proportion of men in both departments (62% combined). The hiring rate for men appears to be larger.

1.1.19c An argument against gender discrimination could by made by comparing the hiring rates for any particular department in the store. Both men and women had a 75% chance of being hired in hardware, while both sexes had only a 10% chance of being hired in women's apparel.

1.1.19d Answers will vary.

1.1.20a The pass rate is found as the ratio of students passing the course to the total number of students multiplied by 100%:

School A

$$\frac{\text{Passed}}{\text{Total}} \times 100\% = \frac{400}{477} \times 100\% = 83.9\%$$

School B

$$\frac{\text{Passed}}{\text{Total}} \times 100\% = \frac{811}{1001} \times 100\% = 81.0\%$$

1.1.20b The pass rate is found as the ratio of students passing the course to the total number of students multiplied by 100%:

School A

$$\frac{\text{Passed}}{\text{Total}} \times 100\% = \frac{199}{379} \times 100\% = 52.5\%$$

School B

$$\frac{\text{Passed}}{\text{Total}} \times 100\% = \frac{38}{105} \times 100\% = 36.2\%$$

1.1.20c The percentage of students in School A above the poverty line is found as the ratio of the total number of students above the poverty line to the total number of students for School A multiplied by 100%:

$$\frac{\text{Above poverty line}}{\text{Total students}} \times 100\% = \frac{477}{379 + 477} \times 100\% = 55.7\%$$

1.1.20d The percentage of students in School B above the poverty line is found as the ratio of the total number of students above the poverty line to the total number of students for School B multiplied by 100%:

$$\frac{\text{Above poverty line}}{\text{Total students}} \times 100\% = \frac{1001}{1001 + 105} \times 100\% = 90.5\%$$

1.1.20.e

	Passed	Failed	Total	% passing
School A	599	257	856	$599/856 = 0.700 = 70.0\%$
School B	849	257	1106	$849/1106 = 0.768 = 76.8\%$

1.1.20f The appropriate response would be that the overall percentage of students passing is higher at School B, but this is an artifact of Simpson's Paradox. For both students above and below the poverty line, School B has a lower success rate than School A.

1.1.20g A parent would be wise to choose School A regardless of economic status as the child's chance of achieving a pass are greater than at School B.

1.1.21a

	Had insurance	No insurance	Total	% with insurance
Children not traced	195	979	1174	$195/1174 = 0.17 = 17\%$
Five-year group	46	370	416	$46/416 = 0.11 = 11\%$

1.1.21b The two percentages are different with the proportion of children with insurance in the *five-year* group being slightly lower. This indicates that the children who were able to be followed five years later, tended to have a lower insurance coverage.

1.1.21c

White Participants

	Had insurance	No insurance	Total	% with insurance
Children not traced	104	22	126	$104/126 \times 100\% = 83\%$
Five-year group	10	2	12	$10/12 \times 100\% = 83\%$

1.1.21d

Black Participants

	Had insurance	No insurance	Total	% with insurance
Children not traced	91	957	1048	$91/1048 \times 100\% = 9\%$
Five-year group	36	368	404	$36/404 \times 100\% = 9\%$

1.1.21e The percentage of students with insurance between the *five-year* group and *children not traced* are approximately identical (small difference from rounding) for both white and black participants.

1.1.21f Because the results for the two groups are identical when broken by race, there is no evidence of a difference in insurance coverage between the *five-year* and *not-traced* groups.

1.1.22a In 1960, 80% of whites ($0.8 * 90 = 72$) graduated, whereas 60% of Hispanics ($0.6 * 10 = 6$) graduated. Therefore, $\frac{78}{100} = 78\%$ was the overall graduation rate in 1960.

1.1.22b Now, both graduation rates improved as 88% of whites graduated ($0.88 * 50 = 44$) compared to 66% of Hispanics ($0.66 * 50 = 33$). This leads to an overall graduation rate of $\frac{77}{100} = 77\%$.

1.1.22c Both whites and Hispanics have a higher graduation rate now when compared to 1960. However, the overall graduation rate dropped 1% over that same time period.

1.1.23 Answers may vary.

1.1.24 Answers may vary.

1.1.25 Answers may vary.

1.1.26 Answers may vary.

1.1.27 Answers may vary.

1.1.28 Correct: 1% is the absolute change when talking about percentage points.
Incorrect: 5% to 6% is a 20% increase when using relative change.

1.1.29 Yes, because the same number of students have a 90% average as a 70% average. Therefore, the numbers would work out to an overall average of 80%.

1.1.30 No, this would depend on the number of men and women that make up each group. The overall average is only 70 if the number of men and women is the same.

1.1.31 Answers may vary.

1.1.32 Answers may vary.

1.1.33 The increase in total paid is $100 dollars. This is a 10% increase. Adding 10% to 10% to get 20% is not accurate.

1.1.34 The increase in total paid is 50.50. This is a 5.05% increase, which is closer to 5%.
How this is related to Simpson's paradox: answers may vary.

1.2 Logic and informal fallacies: Does that argument hold water?

1.2.1 Answers will vary.

1.2.2 a. A distorted or different position

1.2.3 An inaccurate or incomplete list of alternatives.

1.2.4 In a fallacy of relevance, the premises are logically irrelevant to the conclusion. In a fallacy or presumption, false or misleading assumptions are made.

1.2.5 Deductive Reasoning

1.2.6 Inductive Reasoning

1.2.7 **False Authority.** Being an accountant does not make her an expert in biology.

1.2.8 Looking at the table one sees that for each additional bag, another 200 square feet are covered. Therefore if 5 bags are purchased, using inductive reasoning, we can say that 1000 square feet are covered.

1.2.9 **Premise.** All dogs go to heaven and my terrier is a dog.
Conclusion. My dog will go to heaven.

1.2.10 **Premise.** Take this new drug.
Conclusion. You will lose weight.

1.2.11 **Premise.** The mayor was caught in a lie.
Conclusion. Nobody believes the mayor anymore.

1.2.12 **Premise.** Today's children do not respect authority. A disdain for authority leads to social unrest.
Conclusion. The social fabric is at risk.

1.2.13 **Premise.** People have freedom of choice.
Conclusion. They will always choose peace.

1.2.14 **Premise.** Do not speak all that you know or all that you see.
Conclusion. You will live in peace and at ease.

1.2.15 **Premise.** Bad times have a scientific value.
Conclusion. These are occasions a good learner would not miss.

1.2.16 **Valid argument.**

1.2.17 **Not a valid argument.** Some senators are dishonest, but being a senator does not necessarily imply being dishonest.

1.2.18 **Valid argument.** All diabetic patients have abnormal blood-glucose levels.

1.2.19 **Not a valid argument.** There may be other reasons for getting a speeding ticket other than driving too fast.

1.2.20 **Not a valid argument.** There may be other ways to arrive at Oz other than following the yellow brick road.

1.2.21 **Not a valid argument.** Other public servants (e.g., fire fighters) also serve and protect the public.

1.2.22 **Confusing cause and effect.** The two events occur together, but may not be causal.

1.2.23 **Dismissal based on personal attack.** What relevance is the fact that he dropped out of school?

1.2.24 **False authority.** Why should a math teacher be an expert on nutrition?

1.2.25 **False cause.** The two events occur together, but may not be causal.

1.2.26 **Circular reasoning.** Conclusion (it will cost more) is a restatement of the premise (more expensive car).

1.2.27 **False dilemma.** Just because you don't support the war, doesn't imply that you are our enemy.

1.2.28 **Appeal to common practice.** Just because many congressmen accept money, doesn't make it right.

1.2.29 **Appeal to common practice.** The fallacy here is thinking that Crest must be the best product if everyone uses it.

1.2.30 **Hasty generalization.** Three parks is an inadequate number to visit to make a complete generalization for all of California.

1.2.31 **Appeal to ignorance.** Some members of your family who were tested were found to be free of HIV. But what about the members of your family who were not tested?

1.2.32 **Dismissal based on personal attack.** Disregarding his facts because of his opinions on a separate issue.

1.2.33 **Straw man.** How do you know that studying will not result in an A grade?

1.2.34 **False cause.** Just because everybody wants to enroll in Professor Smith's section does not mean it's because of the Professor's teaching ability.

1.2.35 **False dilemma.** How do you know this is the biggest party of the year?

1.2.36 **Hasty generalization.** Just because the product worked well does not mean it will show better results than other products.

1.2.37 **Circular reasoning.** The same statement was repeated twice.

1.2.38 **Hasty generalization.** You can't apply the traits of only three individuals to the entire population.

1.2.39 **Straw man.** Money going overseas is an idea falsely associated with foreign aid.

1.2.40 **Appeal to fear.** The fact that the antagonist is having dinner with your boss does not affect the answer to a debate.

1.2.41 **Fallacy of accident.** While all men are created equal, subsequent events and training are not the same for all individuals.

1.2.42 **Slippery slope.** Thinking that if taxes are continually cut, then there won't be enough money to repair the roads is a fallacy, because it ignores the possibility that spending can be shifted and other programs can be eliminated.

1.2.43 **Gambler's fallacy.** The fallacy is that, if you lose 20 times in a row, you are more likely to win on subsequent attempts. However, each time you buy a lottery ticket, the chances of winning do not depend on previous outcomes.

1.2.44 **Currency conversion.** Because 8 euros = \$9.04, then 16 euros = \$9.04 × 2 = \$18.08.

1.2.45 **Children's blocks.** The number of blocks is found by multiplying all sides together or n^3. Therefore, a 50-inch cube will contain $50 \times 50 \times 50 = 125,000$ blocks.

1.2.46 Consider the following table:

n	1	2	3	4	5	...
Sequence	2	4	6	8	10	
Pattern	2×1	2×2	2×3	2×4	2×5	

The general pattern for the nth sequence value is $2 \times n$.

1.2.47 Consider the following table:

n	1	2	3	4	5	...
Sequence	1	3	5	7	9	
Pattern	$(2 \times 1) - 1$	$(2 \times 2) - 1$	$(2 \times 3) - 1$	$(2 \times 4) - 1$	$(2 \times 5) - 1$	

The general pattern for the nth sequence value is $(2 \times n) - 1 = 2n - 1$.

1.2.48 Consider the following table:

n	1	2	3	4	5	...
Sequence	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	
Pattern	$\frac{1}{2^0}$	$\frac{1}{2^1}$	$\frac{1}{2^2}$	$\frac{1}{2^3}$	$\frac{1}{2^4}$	

The general pattern for the nth sequence value is $\frac{1}{2^{n-1}}$.

1.2.49 **Research.** Answers will vary.

1.2.50 **Irving Copi.** Answers may vary.

1.2.51 **Aristotle.** Answers may vary.

1.2.52 **More fallacies.** Answers may vary.

1.2.53 **Arguments.** Answers may vary.

1.2.54 **Appeal to Common Practice.** Answers may vary.

1.2.55 **False Dilemma.** Answers may vary.

1.2.56 **City Councilman.** Answers may vary.

1.2.57 **Authority.** Answers may vary.

1.2.58 **Astrophysics.** Answers may vary.

1.2.59 **Warming.** Answers may vary.

1.2.60 **Terrorists.** Answers may vary.

1.2.61 **Hasty Induction.** Answers may vary.

1.3 Formal logic and truth tables: Do computers think?

1.3.1 1-b, 2-a, 3-d, 4-c

1.3.2 c

1.3.3 True

1.3.4 True

1.3.5 p is true and q is false

1.3.6 $q \to p$

1.3.7 **Representing statements.** (NOT p) $\to$ (NOT q)

1.3.8 **Conditional.** p : If you don't stop talking. q : I will leave.
Conditional $p \to q$ (i.e., if you don't stop talking, then I will leave).

1.3.9 **A research project using the Internet.** Answers will vary, but along the lines of (France OR England) AND (18*th* century OR 19*th* century).

1.3.10 **Converse.** If I carry an umbrella, then it rains. This is not reasonable because what you carry does not affect the weather.

1.3.11 **Inverse.** If I don't buy this MP3 player, then I can't listen to my favorite songs. This statement is less reasonable than the original because the MP3 player is not required to listen to your favorite songs.

1.3.12 **Politicians.** There were probably more factors affecting the health of the economy than just the proposed bill.

1.3.13 **Taxing implications.** There are several types of penalties, not all of which are related to lateness (e.g., failure to declare income is penalized, even if you file on time).

1.3.14 (NOT p) and q

1.3.15 $q \to$ (NOT p)

1.3.16 (NOT p) OR (NOT q)

1.3.17 $q \to$ (NOT p)

1.3.18 (NOT p) $\to q$

1.3.19 p: Pass my bill. r: Economy will recover. $p \to r$

1.3.20 p: Pass your bill. q: Economy recovered. (NOT p) and q

1.3.21 c: I want cereal for breakfast. e: I want eggs for breakfast. c and e

1.3.22 p: I want cereal for breakfast. q: I want eggs for breakfast. p OR q

1.3.23 c: You clean your room. f: I will tell your father. (NOT c) $\to f$

1.3.24 p_1: Clean your room. p_2: Take out the trash. q: I will give you a dollar. (p_1 and p_2) $\to q$

1.3.25 m: We can go to a movie. r: We can go to a restaurant. d: I'll go downtown with you.
(m OR r) $\to d$

1.3.26 (NOT p) $\to q$: If he is not American, then he is Canadian.
p OR q: He is American or he is Canadian.

1.3.27 NOT(p OR q): He is neither American nor Canadian.
(NOT p) and (NOT q): He is not American and he is not Canadian (i.e., same as previous statement).

1.3.28 NOT(p and q): He is not both an American and a Canadian.
(NOT p) OR (NOT q): He is not American or he is not Canadian, which is logically equivalent to the previous statement as well.

1.3.29 p: $3 + 3 = 7$ is FALSE.
q: $2 + 2 = 5$ is FALSE.
p OR q = FALSE OR FALSE = FALSE.

1.3.30 p: $3 + 3 = 6$ is TRUE.
q: $2 + 2 = 4$ is TRUE.
p OR q = TRUE OR TRUE = TRUE.

1.3.31 p: Pigs fly is FALSE.
q: Mickey Mouse is President is FALSE.
$p \to q$ = FALSE $\to$ FALSE is TRUE (see definition of conditional statements).

1.3.32 p: George Washington was the first U.S. President is TRUE.
q: Mickey Mouse is the current U.S. President is FALSE.
$p \to q$ = TRUE $\to$ FALSE IS FALSE (see definition of conditional statements).

1.3.33 p: Pigs fly is FALSE.
q: I will be President.
$p \to q$ = FALSE $\to$ TRUE is TRUE (see definition of conditional statements).

1.3.34 p: Pigs fly is FALSE.
q: Earth is flat is FALSE.
$p \to q$ = FALSE $\to$ FALSE is TRUE (see definition of conditional statements).

1.3.35 p: Earth is flat is FALSE.
q: Pigs fly is FALSE.
$p \to q$ = FALSE $\to$ FALSE is TRUE (see definition of conditional statements).

1.3.36 p: Pigs fly is FALSE.
q: Oceans are water is TRUE.
$p \to q$ = FALSE $\to$ TRUE is TRUE (see definition of conditional statements).

1.3.37 p: Oceans are water is TRUE.
$p \to q$ = TRUE $\to$ FALSE is FALSE (see definition of conditional statements).

1.3.38 q: Pigs fly is FALSE.
FALSE

1.3.39 TRUE

1.3.40 TRUE

1.3.41 TRUE

1.3.42 FALSE

1.3.43 TRUE

1.3.44 **Premise**: It is a dog. **Conclusion**: It chases cats.
Inverse: If it is not a dog, then it does not chase cats.

1.3.45 **Premise**: You don't clean your room. **Conclusion**: I'll tell your father.
Inverse: If you clean your room, then I won't tell your father.

1.3.46 **Premise**: If you clean your room and take out the trash. **Conclusion**: I'll give you a dollar.
Inverse: If it is not the case that you clean your room and take out the trash, then I won't give you a dollar. (If you don't clean your room and take out the trash, then I won't give you a dollar.)

1.3.47 **Premise**: You agree to go to a movie or restaurant. **Conclusion**: I'll go downtown with you.
Inverse: If it is not the case that you agree to go to a movie or restaurant, then I won't go downtown with you. (If you don't agree to go to a movie or restaurant, then I won't go downtown with you.)

1.3.48 **Premise**: I am elected. **Conclusion**: Taxes will be cut.
Inverse: If I am not elected, then taxes will not be cut.

1.3.49 **Premise**: It is a math course. **Conclusion**: It is important.
Inverse: If it is not a math course, then it is not important.

1.3.50 **Premise**: You don't exercise regularly. **Conclusion**: Your health will decline.
Inverse: If you exercise regularly, then your health will not decline.

1.3.51 **Premise**: You take medicine. **Conclusion**: You will get well.
Inverse: If you don't take your medicine then you won't get well.

1.3.52 **Premise**: It is a dog. **Conclusion**: It chases cats.
Converse: If it chases cats, then it is a dog.

1.3.53 **Premise**: You don't clean your room. **Conclusion**: I will tell your father.
Converse: If I told your father, then you didn't clean your room.

1.3.54 **Premise**: You clean your room and take out the trash. **Conclusion**: I will give you a dollar.
Converse: If I gave you a dollar, then you cleaned your room and took out the trash.

1.3.55 **Premise**: You agree to go to a movie or a restaurant. **Conclusion**: I will go downtown with you.
Converse: If I went downtown with you, then you agreed to go to a movie or restaurant.

1.3.56 **Premise**: I am elected. **Conclusion**: Taxes will be cut.
Converse: If taxes were cut, then I was elected.

1.3.57 **Premise**: It is a math course. **Conclusion**: It is important.
Converse: If a course is important, then it is a math course.

1.3.58 **Premise**: You don't exercise regularly. **Conclusion**: Your health will decline.
Converse: If your health declines, then you don't exercise regularly.

1.3.59 **Premise**: You take the medicine. **Conclusion**: You will get well.
Converse: If you got well, then you took your medicine.

1.3.60 If there are no traces of water on Mars, then there is no life there.

1.3.61 If you do not choose the side of the oppressor, you are not neutral in situations of injustice.

1.3.62 If you wish to maintain respect for the law, then do not have ten thousand regulations.

1.3.63 **Premise**: You drink and drive. **Conclusion**: You will get arrested.
Contrapositive: If you didn't get arrested, then it is not the case that you were drinking and driving.

1.3.64 **Premise**: Aliens have visited Earth. **Conclusion**: We have found concrete evidence by now.
Contrapositive: If we have not found concrete evidence of aliens, then they have not visited Earth.

1.3.65 **Premise**: You don't clean your room. **Conclusion**: I will tell your father.
Contrapositive: If I did not tell your father, then you cleaned your room.

1.3.66 **Premise**: You clean your room and take out the trash. **Conclusion**: I'll give you a dollar.
Contrapositive: If I didn't give you a dollar, then you neither cleaned your room nor took out the trash.

1.3.67 **Premise**: You agree to go to a movie or a restaurant. **Conclusion**: I will go downtown with you.
Contrapositive: If I didn't go downtown with you, then we didn't go to a movie or restaurant.

1.3.68 **Premise**: Produce greenhouse gases. **Conclusion**: Global warming will occur.
Contrapositive: If global warming does not occur, then we have stopped producing greenhouse gases.

1.3.69 **Premise**: You are my friend. **Conclusion**: You drive a Porsche.
Contrapositive: If you do not drive a Porsche, then you are not my friend.

1.3.70 **Premise**: Pneumonia. **Conclusion**: Severe respiratory distress.
Contrapositive: If you do not have severe respiratory distress, then you do not have pneumonia.

1.3.71 **Premise**: Thunderstorms are approaching. **Conclusion**: They will be detected on radar.
Contrapositive: If the radar does not detect thunderstorms, then thunderstorms are not approaching.

1.3.72

p	q	p	(NOT q)	p AND (NOT q)
T	T	T	F	F
T	F	T	T	T
F	T	F	F	F
F	F	F	T	F

1.3.73

p	q	p	(p OR q)	$p \to$ (p OR q)
T	T	T	T	T
T	F	T	T	T
F	T	F	T	T
F	F	F	F	T

1.3.74

p	q	p	(p AND q)	$p \to$ (p AND q)
T	T	T	T	T
T	F	T	F	F
F	T	F	F	T
F	F	F	F	T

1.3.75

p	q	p	(p OR q)	p AND (p OR q)
T	T	T	T	T
T	F	T	T	T
F	T	F	T	F
F	F	F	F	F

1.3.76

p	q	(NOT p)	(NOT p) $\to q$	(p OR q)
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

Both expression have the same results in the truth table.

1.3.77

p	q	p AND q	NOT (p AND q)	NOT p	NOT q	(NOT p) OR (NOT q)
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Both expression have the same results in the truth table.

1.3.78

p	q	NOT p	(NOT p) AND q	NOT q	p OR (NOT q)	NOT (p OR (NOT q))
T	T	F	F	F	T	F
T	F	F	F	T	T	F
F	T	T	T	F	F	T
F	F	T	F	T	T	F

Both expression have the same results in the truth table.

1.3.79 **Logic gate AND.** Output is 0.

1.3.80 **Logic gate NAND.** Output is 1.

1.3.81 **Logic gate NOR.** Output is 1.

1.3.82 **History of truth tables.** Answers will vary.

1.3.83 **More research.** Answers will vary.

1.3.84 **Artificial intelligence.** Answers will vary.

1.3.85 **The jury is out.** No, reasoning may vary.

1.3.86 **It never happens.** Yes; We always beat State U. at football.

1.3.87 **A complicated situation.** Answers may vary.

1.3.88 **Martian.** The statement is not necessarily true; answers may vary.

1.3.89 **Iowa.** Answers may vary.

1.3.90 **NOR.** Yes, answers may vary.

1.3.91 **Conditional statements.** Answers may vary. The given statement is based off of *IF* Mickey Mouse was the first president.

1.3.92 **Contrapositive.** Answers may vary.

1.3.93 **Converse confusion.** Answers may vary.

1.4 Sets and Venn diagrams: Pictorial logic

1.4.1 they have no elements in common

1.4.2 B

1.4.3 common elements to both circles

1.4.4 Set A is a subset of set B.

1.4.5 {5,6,7,8,9}

1.4.6 {6,8}

1.4.7 $\{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}$

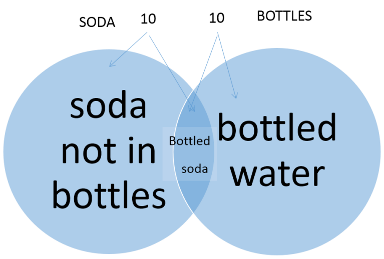
1.4.8 Answers will vary.

1.4.9 B and C are both subsets of A because every element in B is in A; likewise every element in C is in A. B and C are disjoint because they do not have a common element.

1.4.10 A and B are disjoint since A is the set of oceans and B is the set of continents and oceans are not continents and continents are not oceans. C contains Asia which is a continent and so in A (but not in B); and also Pacific, which is an ocean and so in B (but not in A). Hence C is not a subset of either A or B.

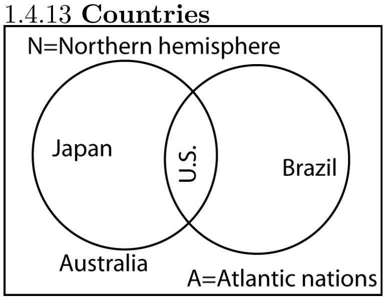
1.4.11 A is a subset of B because every Republican governor would be in the set of all governors. Both A and B are also subsets of C. All governors, including those that are Republican, can be classified as an elected state official.

1.4.12 **Miscounting.**

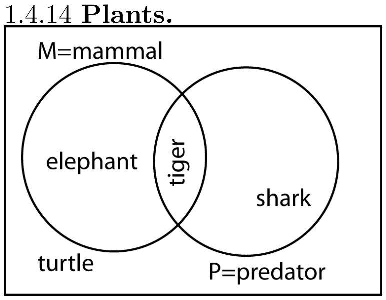


The Clerk can not necessarily conclude that there are 20 items altogether, nor can she/he determine the number of items. The only way the clerk could know the total is if she/he knew how many bottles of soda (the overlapping area) the customer plans to buy.

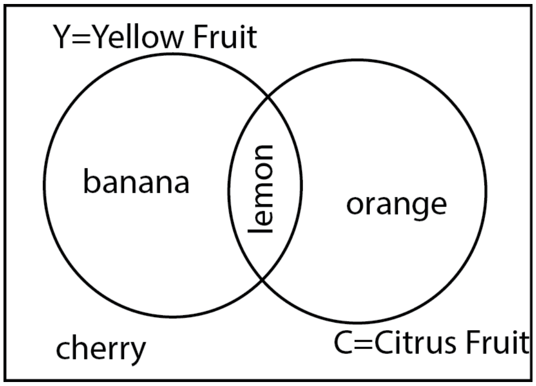
1.4.13 **Countries**



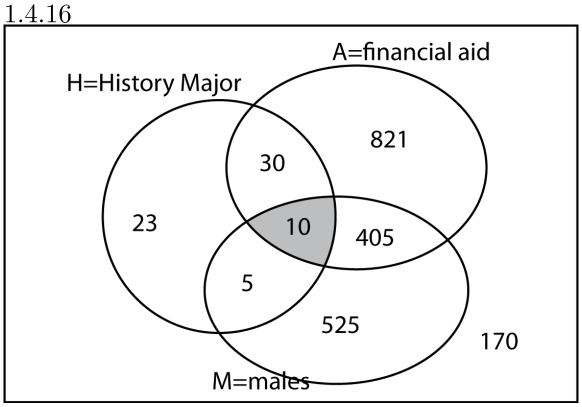
1.4.14 **Plants.**



1.4.15 **Fruit.**

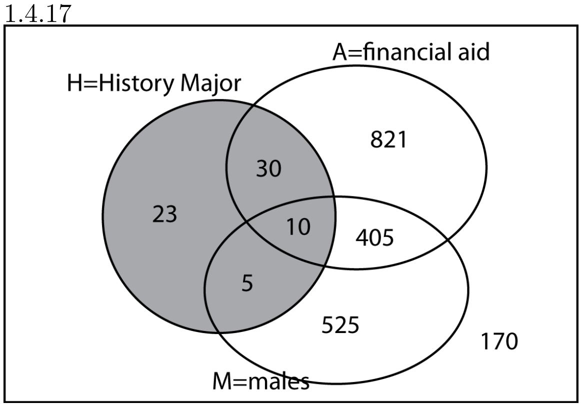


1.4.16



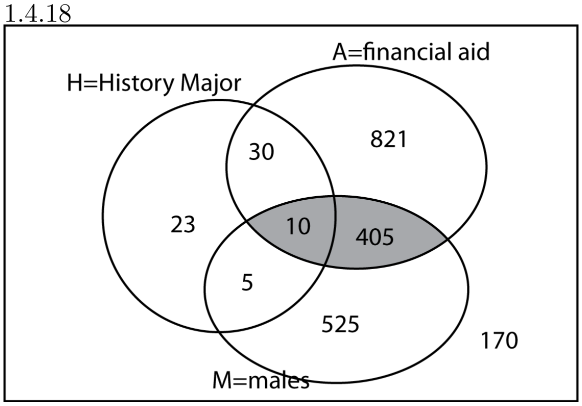
10 students.

1.4.17



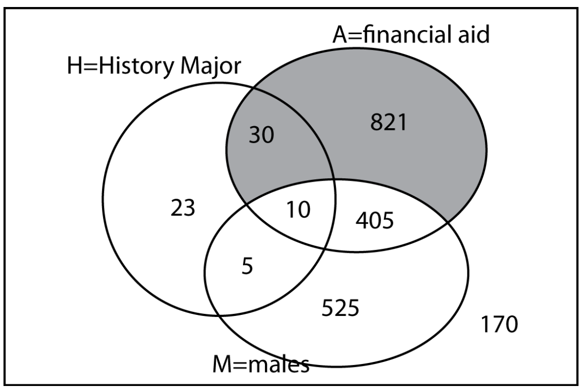
Total history majors include all numbers within the larger circle including central ones. $23+30+10+5 = 68$ history majors.

1.4.18



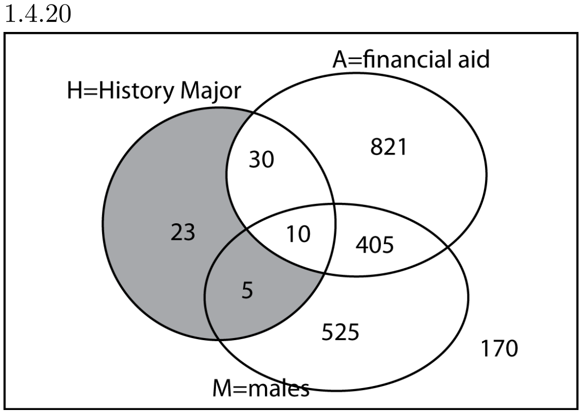
415 students.

1.4.19



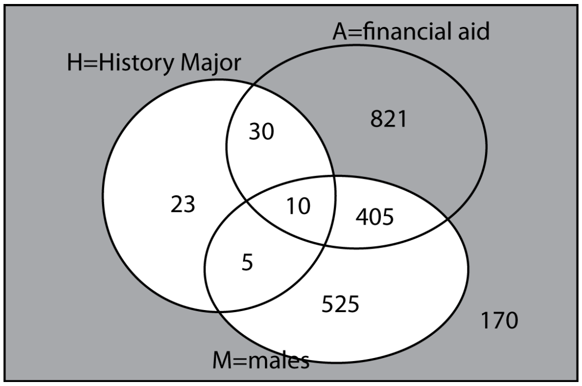
Any values not inside the circle for males have to be female. $821 + 30 = 851$ students.

1.4.20



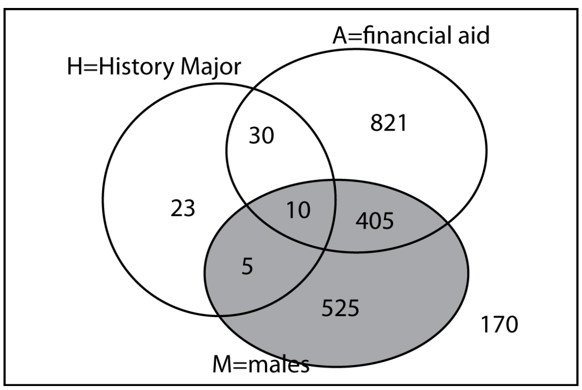
28 students.

1.4.21



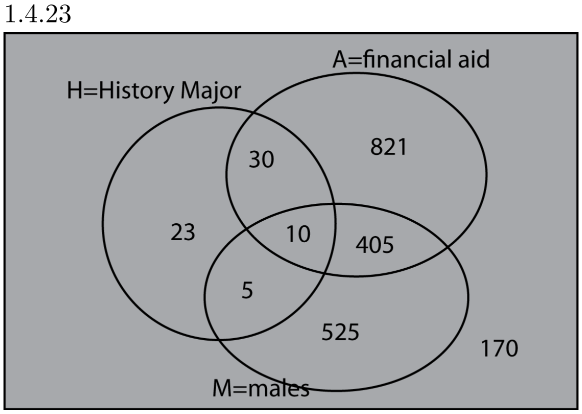
$170 + 821 = 991$ students.

1.4.22



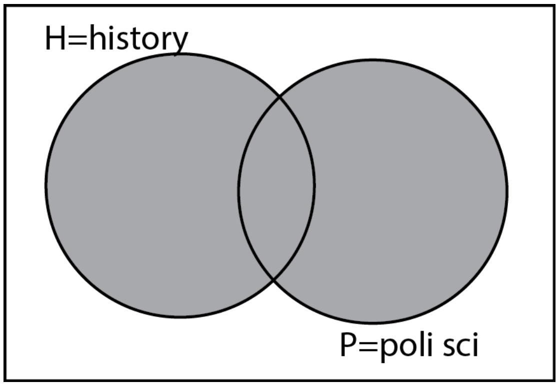
$525 + 405 + 5 + 10 = 945$ students.

1.4.23

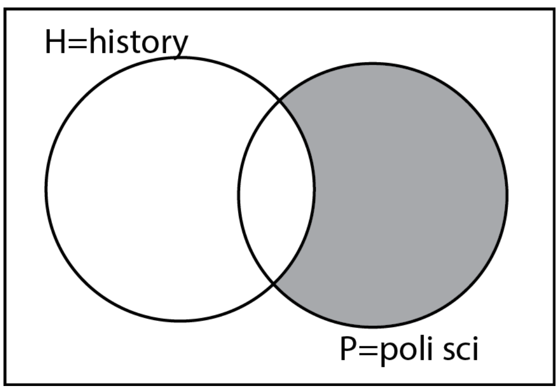


$23 + 5 + 10 + 30 + 821 + 405 + 525 + 170 = 1989$ students.

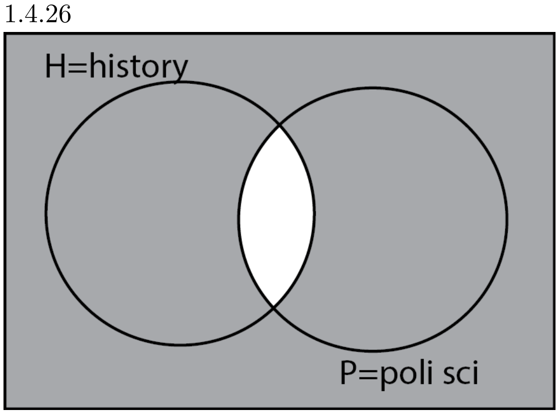
1.4.24



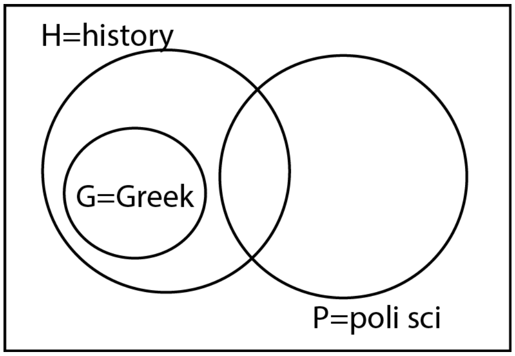
1.4.25



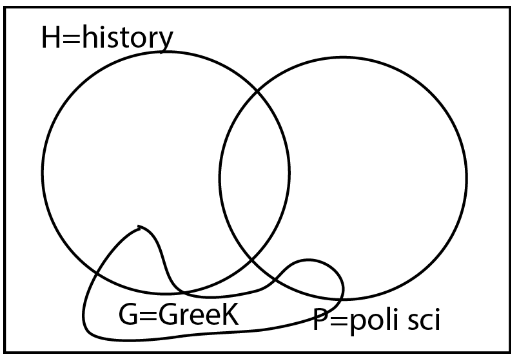
1.4.26



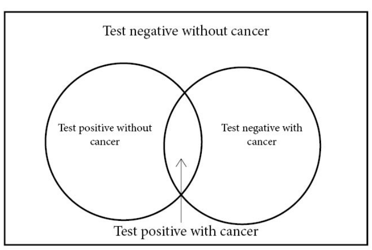
1.4.27



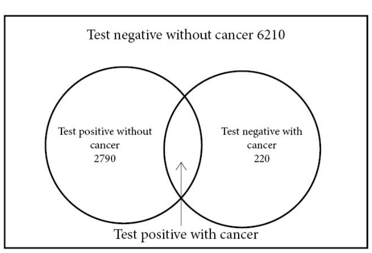
1.4.28



1.4.29a



1.4.29b

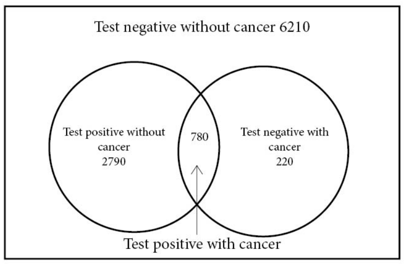


The number who test positive with cancer cannot be determined.

1.4.29c

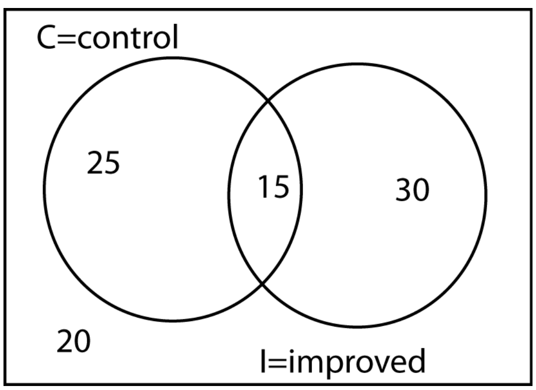
The total number tested is 10,000. Hence, true positives + 220+2790+6210=10,000.

	With prostrate cancer	Without prostrate cancer
Test Positive	780	2,790
Test Negative	220	6,210

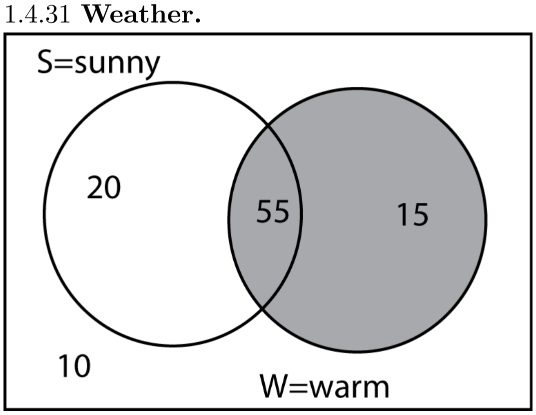


1.4.29d The sensitivity is $780/(780 + 220) = 780/1000 = 0.78 = 78\%$.

1.4.30 **A drug trial.**

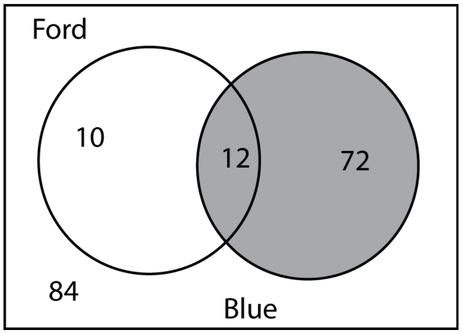


1.4.31 **Weather.**



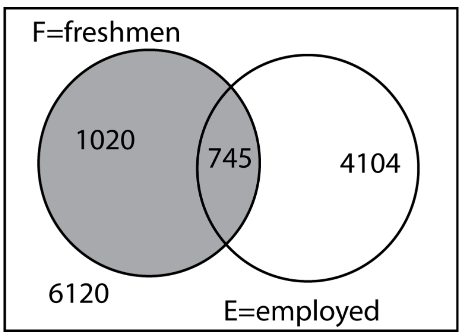
70 warm days.

1.4.32 **Cars.**



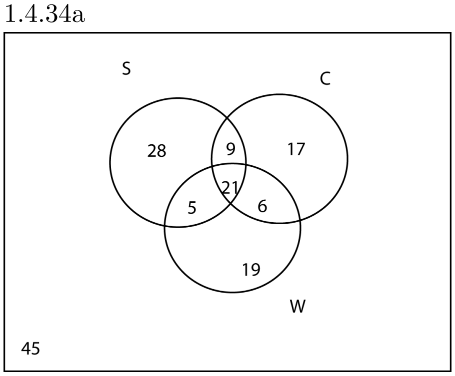
84 blue cars.

1.4.33 **Employed freshmen.**

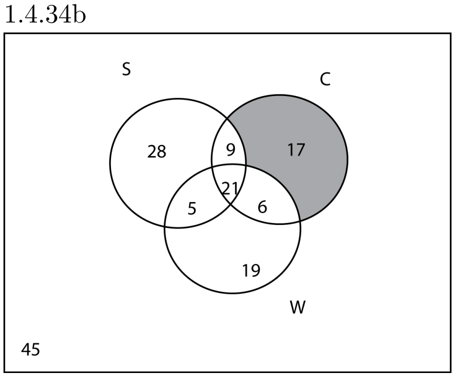


1765 freshmen.

1.4.34a

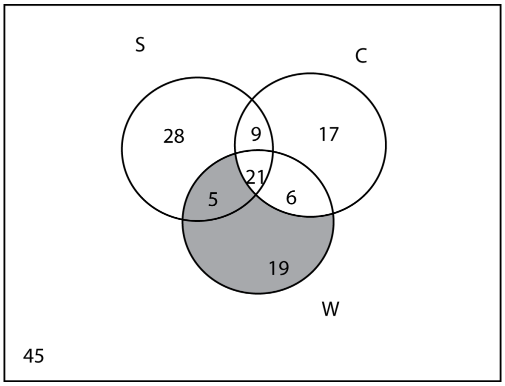


1.4.34b



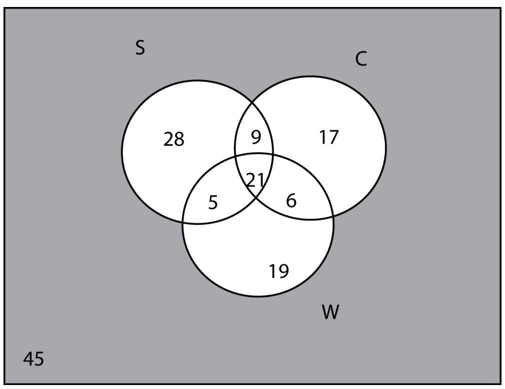
17 liked coffee only.

1.4.34c



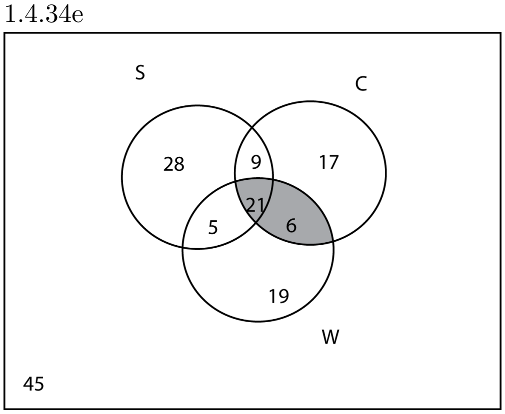
$19 + 5 = 24$ liked water but not coffee.

1.4.34d



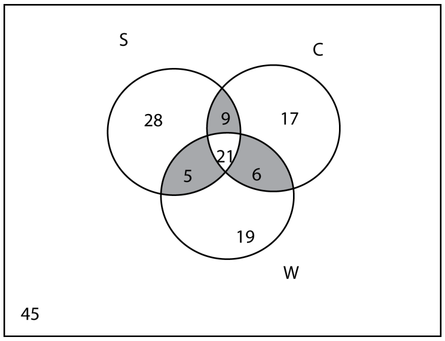
45 liked none of the three beverages.

1.4.34e



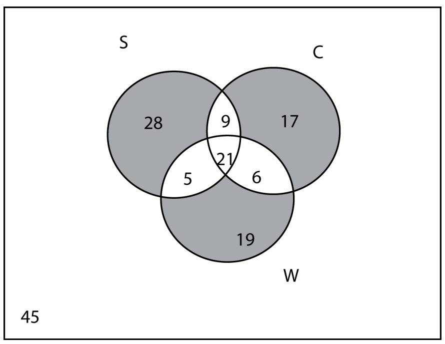
27 liked (both) coffee and water.

1.4.34f



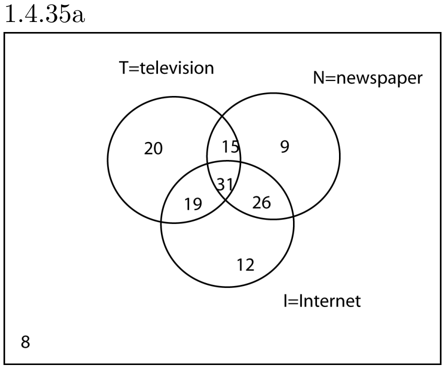
20 liked exactly two of the three beverages.

1.4.34g

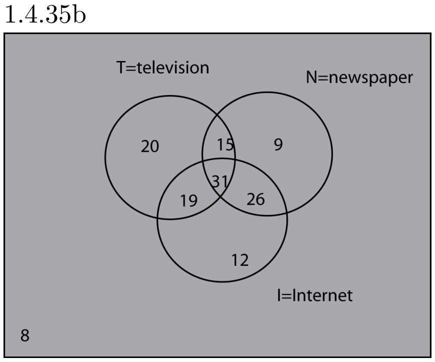


64 liked exactly one of the three beverages.

1.4.35a

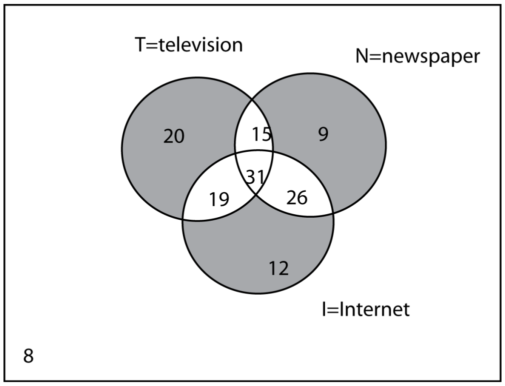


1.4.35b



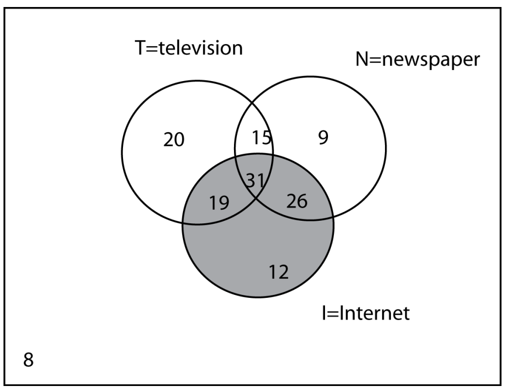
$9 + 12 + 8 + 26 + 31 + 15 + 19 + 20 = 140$ were surveyed.

1.4.35c



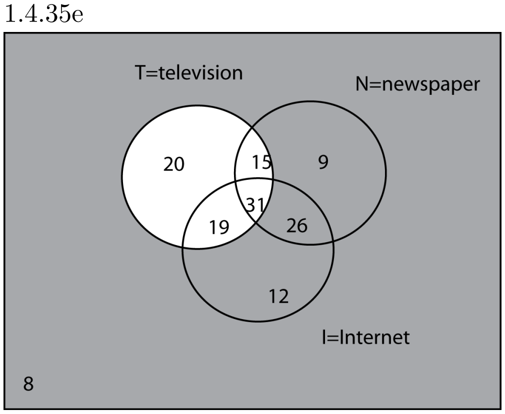
Twelve received information just from the Internet, 9 just from newspapers, and 20 just from television, making 41 who received information from one source.

1.4.35d



Nineteen received news from television and the Internet, 31 from all three, 26 from newspapers and the Internet, and 12 just from the Internet. $19+31+26+12=88$ got some of their news from the Internet.

1.4.35e



Twelve received news from the Internet alone, 26 from newspapers and the Internet, 9 from newspapers alone, and 8 from none of these. $12+26+9+8=55$ got none of their news from TV.

1.4.36 Answers may vary.

1.4.37 Answers may vary.

1.4.38 Answers may vary.

1.4.39 **What does it mean?** Answers may vary ... if the circles do not meet, there are no African American State Senators.

1.4.40 **Make an example.** Answers may vary.

1.4.41 **Four circles.** Answers may vary.

1.4.42 **Four categories.** 16

1.4.43 **Squares.** Answers may vary.

1.4.44 **Advantages.** Answers may vary.

1.4.45 **More advantages.** Answers may vary.

1.4.46 **Counting too many.** Answers may vary ... some students may play more than 1 sport.

1.4.47 **Counting.**
The smallest number is 40. All 40 students play all 3 sports.
The largest number is 120. No student plays more than 1 sport.

1.5 Critical thinking and number sense: What do these figures mean?

1.5.1 Answers may vary.

1.5.2 Answers may vary ... if using scientific notation, then 1×10^9 divided by 1×10^7 is approximately 100.

1.5.3 B

1.5.4 C

1.5.5 C

1.5.6 Answers may vary ... divide the total payment amount by the number of months you will be making payments.

1.5.7 **Rhinovirus.** It is $\frac{1 \times 10^{-6}}{20 \times 10^{-9}} = \frac{1000 \times 10^{-9}}{20 \times 10^{-9}} = 50$. So the virus is 50 times as large as the bacterium.

1.5.8 **Trillion.** There are 1000 millions in 1 billion and 1000 billions in a trillion. $1000 \times 1000 = 1,000,000$. There are 1 million millions in a trillion.

1.5.9 **Pennies.** There are 100 pennies in a dollar. So a million pennies is $\frac{1,000,000}{100} = \$10,000$.

1.5.10 **Oklahoma.** To compute how much each person would have to pay to reduce the deficit in Oklahoma, we compute $\frac{\$1.3 \times 10^9}{3.9 \times 10^6} = \$333$ per person.

1.5.11 $\frac{\$1,000,000,000}{\$100/\text{bill}} = 10$ million bills.

1.5.12 $\$1,000,000,000 \times 2.61$ in/bill $= 2.61$ billion inches wide. Now, 1 inch $= 1/12$ of a foot and 1 foot $= 1/5280$ of a mile, so this is $2,610,000,000 \times \frac{1}{12} \times \frac{1}{5280} = 41,193$ miles.

1.5.13 \$1,000,000,000 × 6.14 in/bill = 6.14 billion inches long. Now, 1 inch = 1/12 of a foot and 1 foot =1/5280 of a mile, so this is $6,140,000,000 \times \frac{1}{12} \times \frac{1}{5280} = 96,907$ miles.

This would stretch $\frac{96{,}907 \text{ miles}}{25{,}000 \text{ miles}} = 3.88$ times around the world!

1.5.14 Area = *length* × *width*. A single bill has an area of $6.14 \times 2.61 = 16.0254$ in^2. A billion dollars would cover 16.0254 billion in^2 1 square mile = $5280 \times 12 \times 5280 \times 12 = 4,014,489,600$ in^2 16.0254 billion in^2 $\times \frac{1}{4{,}014{,}489{,}600} = 3.99$, so 4 square miles.

1.5.15 volume = $l \times w \times h$. One bill has a volume of $6.14 \times 2.61 \times 0.0043 = 0.06890922$ in^3. A billion dollar bills would have volume of 68,909,220 in^3. 1 ft^3 = 12 inch ×12 inch ×12 inch = 1728 in^3. $68,909,200\ \cancel{in^3} \times \frac{1 ft^3}{1728\ \cancel{in^3}} = 39,878.02$ cubic ft, so 39,878 cubic ft.

1.5.16 This depends on the room of course. A typical room that is 27 by 24 by 14 ft has a volume of $27 \times 24 \times 14 = 9072\ ft^3$. A billion bills would fit in 4.4 such rooms.

1.5.17 **Counting to a billion.** If it takes 3 seconds to count to 10, then it would take $\frac{1{,}000{,}000{,}000}{10} \times 3 =$ 300,000,000 seconds.
$300,000,000\ \cancel{seconds} \times \frac{1\ \cancel{min}}{60\ \cancel{seconds}} \times \frac{1\ \cancel{hour}}{60\ \cancel{min}} \times \frac{1\ \cancel{day}}{24\ \cancel{hours}} \times \frac{1\ year}{365\ \cancel{days}} = 9.5$ years!

1.5.18 **Queue.** 1 foot is 1/5280 of a mile. $7,400,000,000\ \cancel{people} \times 12\ \cancel{inch}/\cancel{person} \times \frac{1\ \cancel{ft}}{12\ \cancel{inch}} \times \frac{1\ \cancel{mile}}{5280\ \cancel{ft}} \times \frac{1\ circumference}{25{,}000\ \cancel{miles}} = 56$ times around the world.

1.5.19 **Use of Social Media.** $\frac{1{,}000{,}000\ user}{1\ \cancel{day}} \times \frac{1\ \cancel{day}}{86{,}400\ second} = 11.6\ users/second$

1.5.20 **Lightyear.** $1\ \cancel{year} \times \frac{60\ \cancel{seconds}}{1\ \cancel{min}} \times \frac{60\ \cancel{mins}}{1\ \cancel{hour}} \times \frac{24\ \cancel{hours}}{1\ \cancel{day}} \times \frac{365\ \cancel{days}}{1\ \cancel{year}} \times 186,000\ miles/\cancel{second} = 5.87 \times 10^{12}$ = 5.9 trillion miles.

1.5.21 **Mars rovers.** $140,000,000\ \cancel{miles} \times \frac{1\ second}{186{,}000\ \cancel{miles}} = 752.7$ seconds each way. $752.7\ \cancel{seconds} \times \frac{1\ min}{60\ \cancel{seconds}} = 12.5$ minutes each way or 25 minutes for a round trip.

1.5.22 **Light.** 1 nano second is 1×10^{-9} seconds. $186,000\ miles/\cancel{second} \times 1 \times 10^{-9}\ \cancel{second} = 1.86 \times 10^{-4}$ mile. $1.86 \times 10^{-4} \cancel{mile} \times \frac{5280\ ft}{1\ \cancel{mile}} = 0.98$ ft, so 1 foot.

1.5.23 Answers may vary.

1.5.24 **Loan I.** Round \$7250 to \$7200 and note that there are 36 months in 3 years. Hence 7200/36 = \$200 per month.

1.5.25 **Loan II.** 2.5 years × 12 months/year = 30 months. $\frac{\$3000}{30\ months} = \$100.00$ per month.

1.5.26 **Gas.** 1200 miles at 30 miles per gallon and at \$3 per gallon gives $\frac{1200\ miles}{30\ miles/gallon} = 40$ gallons. Then $40\ gallons \times \$3/gallon = \$120$ for the trip. The exact cost is $\frac{1211\ miles}{29\ miles/gallon} = 41.76$ gallons. Then $\$41.76\ gallons \times \$3.10/gallon = \$129.46$ for the trip, so the answer without estimation is just \$9.46 more than the estimate.

1.5.27 **State population.** The estimated percentage is $\frac{40\ million}{320\ million} = 0.125 \times 100\% = 12.5\%$.
The exact percentage is $\frac{39\ million}{321\ million} = 0.121 \times 100\% = 12.1\%$. The estimated percentages is 0.4% more than the exact percentage.

1.5.28 **Oil.** Rounding down each of the numbers to 19,000,000 barrels with 40 gallons per barrel, and 115,000,000 households we approximate the value to be $\frac{19{,}000{,}000\ barrels \times 40\ gallons/barrel}{115{,}000{,}000\ households} = 6.61$ gallons/households/day. The exact value is $\frac{19{,}400{,}000\ barrels \times 42\ gallons/barrel}{117{,}000{,}000\ households} = 6.96$ gallons/households/day. So the answer without estimation is just 0.35 gallons/household/day more than the estimate.

1.5.29 **Carpet.** Convert the price of square feet into square yards. 1 square yard is 9 square feet; therefore, the price at the first store is $\$1.50 \times 9 = \$13.50$/square yard. The second store has a better value at $12.00 per square yard.

1.5.30 $\frac{324{,}000{,}000}{7{,}400{,}000{,}000} \times 100\% = 4.4\%$.

1.5.31 $\frac{7{,}400{,}000{,}000}{261{,}797} \frac{people}{sq.\ miles} = 28,266$ people per square mile.

1.5.32 There are $5280 \times 5280 = 27,878,400$ square feet in a square mile. There are 305 square miles in New York City which is $305 \times 27,878,400 = 8,502,912,000$ square feet in New York City. There were approximately $7,400,000,000$ people in the world in 2016. Each person would get $8,502,912,000/7,400,000,000 = 1.1$ square feet of room.

1.5.33 **A zettabyte.** 1 trillion gigabytes $= 1 \times 10^{12} gigabytes \times \frac{1 \times 10^9\ bytes}{1\ gigabyte} = 1 \times 10^{21}$.

1.5.34 Answers may vary.

1.5.35 Answers may vary.

1.5.36 Answers may vary.

1.5.37 Answers may vary.

1.5.38 **That's a lot.** $\frac{10^{82}}{10^{78}} = 10^4$. Therefore, 10^{82} is 10,000 times larger than 10^{78}. So, 10,000 of the smaller universe could fit inside the larger universe.

1.5.39 **The iPad.** Answers will vary based on length and settings of movies. Approximately 20 movies can be stored on a 16 gigabyte iPad.

1.5.40 **A grain of rice.** A grain of rice is 8 millimeters in length. Therefore, a grain of rice is 0.008 meters in length.

1.5.41 **Deficit.** If the decimal is moved two places to the left, that is 0.015 trillion or 15 billion dollar deficit.

1.5.42 **Days?** There are 365 days in a lightyear because a lightyear refers to the distance light travels in a year and there are 365 days in a year.

1.5.43 **Millionaires.** $10.4\ million \times 1\ million = 10.4 \times 10^6 \times 1 \times 10^6 = 10.4 \times 10^{12}$ or 10.4 trillion dollars.

1.5.44 **Powers of 10.** Answers may vary. Powers of 10 move decimal places and no other number (like 2) would be as convenient.

1.5.45 **Flooring.** How much area does that cover?

1.5.46 **Debt.** Answers may vary.