

Solutions Manual
to accompany
Principles of Highway Engineering and Traffic Analysis, 4e
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Chapter 2
Road Vehicle Performance

Metric Units

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Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad¹ software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ‘:=’ (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ‘:=’ is set equal to the value of the expression on the right side. For example, in the statement, $L := 1234$, the variable ‘L’ is assigned (i.e., set equal to) the value of 1234. Another example is $x := y + z$. In this case, x is assigned the value of $y + z$.
- The ‘=’ (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 = 18.568 + 10 \cdot (t - 12.792)$, the = is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable ‘t’ that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time ‘t’ is set equal to the function for departures at some time ‘t’ to find the time to queue clearance.
- The ‘=’ (standard equals) is used for a simple numeric evaluation. For example, referring to the $x := y + z$ assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of =)] and the value of z was 15, then the expression ‘x =’ would yield 25. Another example would be as follows: $s := 1800/3600$, with $s = 0.5$. That is, ‘s’ was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is ‘→’. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, $Q(t)$ is assigned the value of $\text{Arrivals}(t) - \text{Departures}(t)$, and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

¹ www.mathcad.com

Problem 2.1

Determine the power required to overcome aerodynamic drag.

$$\rho := 1.2256 \quad C_D := 0.29 \quad A_f := 1.9$$

(given)

$$V := 160 \cdot \frac{1000}{3600} \quad \text{m/s} \quad V = 44.4$$

solve for power

$$P_{Ra} := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^3 \quad (\text{Eq. 2.4})$$

$$P_{Ra} = 29643 \quad \text{kW}$$

Problem 2.2

Determine the final weight of the car.

$$\rho := 1.2256 \quad C_D := 0.30 \quad A_f := 2 \quad W_o := 9300 \quad (\text{given})$$

$$V_{\max} := 160 \cdot 0.2778$$

$$\text{At } V_{\max}, \text{ Power} = R_a V_{\max} + R_{rl} V_{\max}$$

Add one kW per 9 N. additional vehicle weight

$$\text{additional power} = W_a \times 1000/9$$

Solve for additional weight added to the vehicle, set resistance forces equal to additional kW

$$\frac{1000 W_a}{9} = \frac{\rho}{2} \cdot C_D \cdot A_f \cdot (V_{\max})^3 + 0.01 \cdot \left(1 + \frac{V_{\max}}{44.73} \right) \cdot (W_o + W_a) \cdot V_{\max}$$

$$W_a = 367.69$$

$$\text{Total} := W_o + W_a \quad \text{Total} = 9667.69 \quad \text{N}$$

Problem 2.3

Determine the distance from the vehicle's center of gravity to the front axle.

$$W := 11000 \quad \mu := 0.6 \quad L := 3.05 \quad h := 0.55 \quad f_{fl} := 0.01 \quad (\text{given})$$

$$FWD F_{\max} = RWD F_{\max}$$

$$\frac{\frac{\mu W (l_f - f_{fl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} = \frac{\frac{\mu W (l_r + f_{fl} \cdot h)}{L}}{1 + \frac{\mu \cdot h}{L}} \quad (\text{Eq. 2.14})$$

solve for l_r in terms of L and l_f left with one unknown (l_f)

$$l_r := L - l_f$$

$$\frac{\frac{\mu \cdot W \cdot (l_f - f_{fl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} = \frac{\frac{\mu \cdot W \cdot [(L - l_f) + f_{fl} \cdot h]}{L}}{1 + \frac{\mu \cdot h}{L}}$$

$$l_f = 1.365 \quad \text{m}$$

Problem 2.4

Determine the minimum coefficient of road adhesion.

$$W := 13300 \quad g := 9.807 \quad L := 5.10 \quad a := 4.6 \quad (\text{given})$$

$$h := .50 \quad f_{fl} := 0.01 \quad l_f := 3.55$$

$$F = ma + R_{fl} = \text{rear } F_{\max} = \frac{\mu W (l_f - f_{fl} h)}{1 - \mu h / L} \quad (\text{Eq. 2.14})$$

substitute for m , R_{fl}

$$m := \frac{W}{g} \quad R_{fl} := f_{fl} \cdot W \quad (\text{Eq. 2.6})$$

$$\frac{W}{g} \cdot a + f_{fl} \cdot W = \frac{\frac{\mu \cdot W \cdot (l_f - f_{fl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$$\mu = 0.646$$

Problem 2.5

Determine the distance from the vehicle's center of gravity to the rear axle.

$$W := 3013000 \quad g := 9.807 \quad L := 5.10 \quad a := 11.77$$

(given)

$$h := 1.78 \quad f_{rl} := 0.01 \quad \mu := 0.95$$

$$F = ma + R_{rl} = \text{rear } F_{\max} = \frac{\mu W (l_f - f_{rl} h)}{1 - \mu h / L} \quad (\text{Eq. 2.14})$$

substitute for m, R_{rl}

$$m := \frac{W}{g} \quad R_{rl} := f_{rl} \cdot W \quad (\text{Eq. 2.6})$$

$$\frac{W}{g} \cdot a + f_{rl} \cdot W = \frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{1 - \frac{\mu \cdot h}{L}}$$

$$l_f = 4.36 \quad \text{m}$$

Problem 2.6

Determine the lowest gear reduction ratio.

$$W := 12000 \quad r := 0.355 \quad L := 2.5$$

(given)

$$\mu := 1.0 \quad h := 0.450 \quad f_{fl} := 0.01 \quad l_f := 1.0$$

$$F = \text{rear } F_{\max} = \frac{\mu W (l_f - f_{fl} h) / L}{1 - \mu h / L} \quad (\text{Eq. 2.14})$$

"highest possible acceleration" means F_e is equal to F_{\max}

$$F_{\max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{fl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$$F_{\max} = 5827.317$$

$$M_e := 730 \quad \eta_d := 0.95$$

(given)

solve for ε_0

$$F_{\max} = \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r} \quad \varepsilon_0 := \frac{F_{\max} \cdot r}{M_e \cdot \eta_d} \quad (\text{Eq. 2.17})$$

$$\varepsilon_0 = 3$$

Determine the maximum acceleration from rest.

Problem 2.7

$$e_0 := 9 \quad r := 0.355 \quad g := 9.807 \quad \mu := 1.0 \quad f_{rl} := 0.01$$

(given)

$$h := 0.45 \quad l_f := 1.3 \quad L := 2.8 \quad W := 10900$$

$$R_{rl} := W \cdot f_{rl} \quad R_{rl} = 109 \quad (\text{Eq. 2.6})$$

$$M_{\text{ebase}} := 250 \quad M_{\text{emod}} := 290 \quad \eta_d := 0.90$$

solve for mass factor

$$\gamma_m := 1.04 + 0.0025 \cdot e_0^2 \quad \gamma_m = 1.24 \quad (\text{Eq. 2.20})$$

$$F_{\text{ebase}} := \frac{M_{\text{ebase}} \cdot e_0 \cdot \eta_d}{r} \quad F_{\text{ebase}} = 5704.23 \quad (\text{Eq. 2.17})$$

$$F_{\text{max}} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \quad F_{\text{max}} = 6008.91 \quad (\text{Eq. 2.14})$$

since $F_{\text{ebase}} < F_{\text{max}}$, use F_{ebase} for calculating acceleration with original engine

$$a_{\text{base}} := \frac{F_{\text{ebase}}}{\gamma_m \cdot \left(\frac{W}{g} \right)} \quad a_{\text{base}} = 4.13 \quad \frac{\text{m}}{\text{s}^2} \quad (\text{Eq. 2.19})$$

$$F_{\text{emod}} := \frac{M_{\text{emod}} \cdot e_0 \cdot \eta_d}{r} \quad F_{\text{emod}} = 6616.9 \quad (\text{Eq. 2.17})$$

since $F_{\text{emod}} > F_{\text{max}}$, use F_{max} for calculating acceleration with modified engine

$$a_{\text{mod}} := \frac{F_{\text{max}}}{\gamma_m \cdot \left(\frac{W}{g} \right)} \quad a_{\text{mod}} = 4.35 \quad \frac{\text{m}}{\text{s}^2} \quad (\text{Eq. 2.19})$$

Determine the maximum acceleration rate.

Problem 2.8

3000 rev/min = 50 rev/sec

$$i := 0.035 \quad n_e := 50 \quad z_o := 3.5 \quad r := 0.38 \quad g := 9.807 \quad (\text{given})$$

solve for velocity

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{z_o} \quad V = 32.9 \quad (\text{Eq. 2.18})$$

calculate aerodynamic resistance

$$\rho := 1.2256 \quad C_D := 0.35 \quad A_f := 2$$
$$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \quad R_a = 464.73 \quad (\text{Eq. 2.3})$$

calculate rolling resistance

$$f_{rl} := 0.01 \left(1 + \frac{V}{44.73} \right) \quad W := 13300 \quad (\text{Eq. 2.5})$$

$$R_{rl} := f_{rl} \cdot W \quad R_{rl} = 230.87$$

calculate mass factor

$$\gamma_m := 1.04 + 0.0025 \cdot z_o^2 \quad \gamma_m = 1.07 \quad (\text{Eq. 2.20})$$

$$M_e := 340 \quad n_d := 0.90$$

calculate engine-generated tractive effort

$$F_e := \frac{M_e \cdot z_o \cdot n_d}{r} \quad F_e = 2818.42 \quad (\text{Eq. 2.17})$$

$$F_{\text{net}} = F - \sum_n R = \gamma_m \cdot m \cdot a$$

$$\text{so} \quad a := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g} \right)} \quad a = 1.46 \frac{\text{m}}{\text{sec}^2} \quad (\text{Eq. 2.19})$$

Problem 2.9

Determine the drag coefficient.

$$M_e := 200 \quad \varepsilon_0 := 3.0 \quad \eta_d := 0.90 \quad r := 0.38 \quad (\text{given})$$

$$i := 0.02 \quad W := 9500 \quad n_e := \frac{4500}{60}$$

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_0} \quad V = 58.496 \quad (\text{Eq. 2.18})$$

$$F_e := \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r} \quad F_e = 1.421 \times 10^3 \quad (\text{Eq. 2.17})$$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{44.73} \right) \quad (\text{Eq. 2.5})$$

$$R_{rl} := f_{rl} \cdot W \quad R_{rl} = 219.238 \quad (\text{Eq. 2.6})$$

$$A_F := 1.8 \quad \rho := 1.2256$$

$$F_e = R_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_F \cdot V^2 \quad C_D := \frac{2(F_e - R_{rl})}{A_F \cdot V^2 \cdot \rho}$$

$$C_D = 0.318$$

Problem 2.10

Determine the drag coefficient.

$$M_e := 270 \quad \varepsilon_o := 3.0 \quad n_d := 0.90 \quad r := 0.355$$

(given)

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \quad F_e = 2053.5 \quad (\text{Eq. 2.17})$$

$$V := 240 \cdot \frac{1000}{3600} \quad W := 11000 \quad f_{rl} := 0.01 \cdot \left(1 + \frac{V}{44.7} \right) \quad (\text{Eq. 2.5})$$

$$R_{rl} := f_{rl} \cdot W \quad R_{rl} = 274.057 \quad (\text{Eq. 2.6})$$

$$\rho := 1.2256 \quad A_f := 2.3$$

set F_e equal to the sum of the resistance forces and solve for C_D

$$F_e = R_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \quad C_D := \frac{2(F_e - R_{rl})}{A_f \cdot V^2 \cdot \rho}$$

$$C_D = 0.284$$

Problem 2.11

Determine the maximum grade.

$$i := 0.035 \quad n_e := \frac{3500}{60} \quad \varepsilon_o := 3.2 \quad r := 0.355 \quad g := 9.807 \quad (\text{given})$$

calculate velocity

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \quad V = 39.2 \quad \text{m/s} \quad (\text{Eq. 2.18})$$

calculate aerodynamic resistance

$$\rho := 1.2256 \quad C_D := 0.35 \quad A_F := 2.3$$

$$R_a := \frac{\rho}{2} \cdot C_D \cdot A_F \cdot V^2 \quad R_a = 759.49 \quad \text{N} \quad (\text{Eq. 2.3})$$

calculate rolling resistance

$$f_{rl} := 0.01 \left(1 + \frac{V}{44.73} \right) \quad W := 11000 \quad \text{N} \quad (\text{Eq. 2.5})$$

$$R_{rl} := f_{rl} \cdot W \quad R_{rl} = 206.49 \quad \text{N}$$

calculate engine-generated tractive effort

$$M_e := 270 \quad n_d := 0.90 \quad (\text{Eq. 2.17})$$

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \quad F_e = 2190.42 \quad \text{N}$$

$$R_g := F_e - R_a - R_{rl} \quad (\text{Eq. 2.2})$$

$$R_g = 1224.44$$

$$G := \frac{R_g}{W} \quad (\text{Eq. 2.9})$$

$$G = 0.1113 \quad \text{therefore } G = 11.1\%$$

Alternative calculation for grade, using trig relationships

$$\theta_g := \arcsin\left(\frac{R_g}{W}\right)$$

$$\theta_g = 0.1115 \quad \text{radians}$$

$$\text{deg}\theta_g := \theta_g \cdot \frac{180}{\pi} \quad \text{convert from radians to degrees}$$

$$\text{deg}\theta_g = 6.391$$

$\tan \text{deg} = \text{opposite side} / \text{adjacent side}$

$$G := \tan(\theta_g) \cdot 100 \quad G = 11.2 \quad \%$$

Thus, error is minimal when assuming $G = \sin \theta_g$ for small to medium grades

Problem 2.12

Determine the torque the engine is producing and the engine speed.

$$F_e - \Sigma R = \gamma_m \cdot m \cdot a$$

at top speed, acceleration = 0; thus, $F_e - \Sigma R = 0$

$$\rho := 1.06 \quad C_D := 0.28 \quad A_F := 1.8 \quad V := 200 \cdot \frac{1000}{3600} \quad V = 55.556 \quad (\text{given})$$

calculate aerodynamic resistance

$$R_a := \frac{\rho}{2} \cdot C_D \cdot A_F \cdot V^2 \quad R_a = 824.444 \quad (\text{Eq. 2.3})$$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{44.73} \right) \quad f_{rl} = 0.022 \quad (\text{Eq. 2.5})$$

$$W := 12000 \quad i := 0.03 \quad \eta_d := 0.90 \quad \varepsilon_0 := 2.5 \quad r := 0.320 \quad (\text{given})$$

calculate rolling resistance

$$R_{rl} := f_{rl} \cdot W \quad R_{rl} = 269.042 \quad (\text{Eq. 2.6})$$

$$R_g := 0$$

sum of resistances is equal to engine-generated tractive effort, solve for M_e

$$F_e := R_a + R_{rl} + R_g \quad F_e = 1093.487 \quad (\text{Eq. 2.2})$$

$$F_e = \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r} \quad M_e := \frac{F_e \cdot r}{\varepsilon_0 \cdot \eta_d} \quad (\text{Eq. 2.17})$$

$$M_e = 155.518 \text{ N}\cdot\text{m}$$

Knowing velocity, solve for n_e

$$V = \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_0} \quad n_e := \frac{V \cdot \varepsilon_0}{2 \cdot \pi \cdot r \cdot (1 - i)} \quad (\text{Eq. 2.18})$$

$$n_e = 71.214 \quad \frac{\text{rev}}{\text{s}}$$

$$n_e \cdot 60 = 4273 \quad \frac{\text{rev}}{\text{min}}$$

Problem 2.13

Determine the maximum acceleration from rest.

$$W := 11000 \quad L := 2.03 \quad h := 0.50 \quad l_f := 0.76 \quad \mu := 0.75$$

$$f_{rl} := 0.01 \quad g := 9.807 \quad \text{(given)}$$

$$F_{\max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \quad F_{\max} = 3763.6 \quad \text{(Eq. 2.14)}$$

For maximum torque, derivative of torque equation equals zero

$$\frac{dM_e}{dn_e} = 6 - 0.09n_e = 0 \quad n_e := \frac{6}{0.09} \quad n_e = 66.67$$

plug this value into torque equation to get value of maximum torque

$$M_e := 6 \cdot n_e - 0.045 \cdot n_e^2 \quad M_e = 200$$

$$\varepsilon_o := 11 \quad n_d := 0.75 \quad r := 0.355$$

$$\varepsilon_o := 11 \quad n_d := 0.75 \quad r := 0.355$$

find engine-generated tractive effort from maximum torque

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \quad F_e = 4647.89 \quad \text{(Eq. 2.17)}$$

calculate rolling resistance and mass factor

$$R_{rl} := f_{rl} \cdot W \quad R_{rl} = 110 \quad \text{(Eq. 2.6)}$$

$$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_o^2 \quad \gamma_m = 1.34 \quad \text{(Eq. 2.20)}$$

$$F_{\max} < F_e \quad \text{so } F_{\max} \text{ is used}$$

calculate acceleration from maximum available tractive effort

$$a := \frac{F_{\max} - f_{rl} \cdot W}{\gamma_m \cdot \left(\frac{W}{g} \right)} \quad a = 2.43 \frac{m}{s^2} \quad \text{(Eq. 2.19)}$$

Problem 2.14

Determine speed of car.

$$\text{Power} = (2\pi M_e \cdot n_e) = 6.28 \cdot (6n_e^2 - 0.045n_e^3) = 37.68n_e^2 - 0.2826n_e^3 \quad (\text{Eq. 2.16})$$

$$P(n_e) := 37.68n_e^2 - 0.2826n_e^3$$

To find maximum power take derivative of power equation

$$\frac{d}{dn_e} P(n_e) \rightarrow 75.36 \cdot n_e - 0.8478 \cdot n_e^2 = 0$$

$$n_e := \frac{75.36}{0.8478} \quad n_e = 88.89$$

$$i := 0.035 \quad \varepsilon_o := 2 \quad r := 0.355$$

Calculate maximum velocity at maximum engine power

$$\underline{\underline{V}} := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \quad (\text{Eq. 2.18})$$

$$V = 95.66 \quad \frac{\text{m}}{\text{s}} \quad \frac{V}{0.2778} = 344.4 \quad \frac{\text{km}}{\text{h}}$$

Problem 2.15

Determine the acceleration for front- and rear-wheel-drive options.

$$M_e := 130 \quad \varepsilon_o := 4.5 \quad n_d := 0.80 \quad r := 0.33 \quad (\text{given})$$

R_a , R_{rl} , and γ_m are as before

calculate engine-generated tractive effort

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \quad F_e = 1418.182 \quad (\text{Eq. 2.17})$$

$$\gamma_m := 1.091 \quad W := 13300 \quad R_a := 6.26 \quad (\text{given})$$

$$R_{rl} := 146.22 \quad g := 9.807$$

calculate maximum acceleration

$$a_{\max} := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g} \right)} \quad a_{\max} = 0.855 \quad (\text{Eq. 2.19})$$

Rear - wheel drive

$$\mu := 0.2 \quad f_{rl} := 0.011 \quad h := 0.50 \quad L := 3.05 \quad l_f := \frac{3.05}{2}$$

$$F_{\max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \quad F_{\max} = 1370.125 \quad (\text{Eq. 2.14})$$

$$a_{\max} := \frac{F_{\max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g} \right)} \quad a_{\max} = 0.823 \quad \frac{m}{s^2} \quad 0.823 < 0.855 \quad (\text{Eq. 2.19})$$

Front - wheel drive

$$F_{\max} := \frac{\frac{\mu \cdot W \cdot (l_f + f_{rl} \cdot h)}{L}}{1 + \frac{\mu \cdot h}{L}} \quad F_{\max} = 1292.422 \quad (\text{Eq. 2.15})$$

$$a_{\max} := \frac{F_{\max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g} \right)} \quad a_{\max} = 0.77 \quad \frac{m}{s^2} \quad 0.77 < 0.855 \quad (\text{Eq. 2.19})$$

Problem 2.16

Determine weight and torque.

$$\mu := 0.8 \quad W_o := 8900 \quad \varepsilon_o := 10 \quad n_d := 0.8 \quad r := .355 \quad (\text{given})$$

$$l_f := 1.40 \quad f_{rl} := 0.01 \quad h := 0.55 \quad L := 2.55$$

$$F_e = \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \quad (\text{Eq. 2.17})$$

$$F_{\max} = \frac{\frac{\mu \cdot W_a \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \quad (\text{Eq. 2.14})$$

$$l_{f\text{new}} = l_f - \frac{2.5 \cdot (13 \cdot M_e)}{89} \quad W_a = W_o + 13 \cdot M_e$$

setting $F_e = F_{\max}$ and solving for M_e gives

$$\frac{M_e \cdot \varepsilon_o \cdot n_d}{r} = \frac{\frac{\mu \cdot (W_o + 13 \cdot M_e) \cdot \left[l_f - \frac{2.5 \cdot (13 \cdot M_e)}{89} \right] - f_{rl} \cdot h}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$$M_e = 3.751 \quad \text{N-m}$$

$$W_a := W_o + 13 \cdot M_e \quad \text{N} \quad W_a = 8948.8$$

Problem 2.17

Determine the difference in minimum theoretical stopping distances with and without aerodynamic resistance considered.

$$\rho := 1.2256 \quad C_D := 0.45 \quad A_F := 2.3 \quad V := 145 \cdot 0.2778 \quad (\text{given})$$

$$g := 9.807 \quad \gamma_b := 1.04 \quad W := 11000 \quad \eta_b := 1.0$$

$$f_{fl} := 0.0145 \quad \mu := 0.7 \quad \theta := 5.71$$

$$K_a := \frac{\rho}{2} \cdot C_D \cdot A_F \quad K_a = 0.634 \quad (\text{Eq. 2.37})$$

$$S := \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln \left(1 + \frac{K_a \cdot V^2}{\eta_b \cdot \mu \cdot W + f_{fl} \cdot W - W \cdot \sin(\theta \cdot \text{deg})} \right) \quad S = 130.22 \quad (\text{Eq. 2.42})$$

compared to $S = 134.97 \text{ m}$ and $S = 139.87 \text{ m}$

$$134.97 - 130.22 = 4.75 \text{ m}$$

$$139.87 - 130.22 = 9.65 \text{ m}$$

Problem 2.18

Determine the initial speed with and without aerodynamic resistance.

$$C_D := 0.40 \quad A_f := 2.4 \quad \rho := 1.2256 \quad W := 15600$$

$$\mu := 0.5 \quad f_{fl} := 0.015 \quad \eta_b := 0.78 \quad (\text{given})$$

$$S := 76 \quad g := 9.807 \quad \gamma_b := 1.04$$

$$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \quad K_a = 0.588 \quad (\text{Eq. 2.37})$$

with aerodynamic resistance considered

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln \left(1 + \frac{K_a \cdot V^2}{\mu \cdot \eta_b \cdot W + f_{fl} \cdot W} \right) \quad (\text{Eq. 2.43})$$

$$V = 24.423 \quad \frac{V}{0.2778} = 87.915 \quad \frac{\text{km}}{\text{h}}$$

with aerodynamic resistance ignored

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{fl})} \quad V := \sqrt{\frac{S \cdot 2 \cdot g \cdot (\eta_b \cdot \mu + f_{fl})}{\gamma_b}} \quad (\text{Eq. 2.42})$$

$$V = 24.094 \quad \frac{V}{0.2778} = 86.73 \quad \frac{\text{km}}{\text{h}}$$

Problem 2.19

Determine the unloaded braking efficiency, ignoring aerodynamic resistance.

$$\mu := 0.75 \quad f_{rl} := 0.018 \quad \gamma_b := 1.04 \quad (\text{given})$$

$$g := 9.807 \quad S := 61 \quad V := 100 \cdot 0.2778$$

solve Eq. 2.43 for braking efficiency

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})} \quad \eta_b := \frac{\gamma_b \cdot V^2}{S \cdot 2 \cdot g \cdot \mu} - f_{rl} \quad (\text{Eq. 2.43})$$

$$\eta_b = 0.8704$$

$$\eta_b \cdot 100 = 87.04 \quad \%$$

Problem 2.20

Determine the braking efficiency.

$$\mu := 0.60 \quad \gamma_b := 1.04 \quad g := 9.807 \quad S := 180 \quad G := 0.03 \quad (\text{given})$$

$$V_1 := 180 \cdot \frac{1000}{3600} \quad V_1 = 50$$

$$V_2 := 90 \cdot \frac{1000}{3600} \quad V_2 = 25$$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V_1 + V_2}{44.73} \right) \quad f_{rl} = 0.018 \quad (\text{Eq. 2.5})$$

solve for braking efficiency using theoretical stopping distance equation

$$S = \frac{\gamma_b \cdot (V_1^2 - V_2^2)}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl} - G)} \quad \eta_b := \frac{\gamma_b \cdot (V_1^2 - V_2^2)}{2 \cdot S \cdot g \cdot \mu} - f_{rl} + G \quad (\text{Eq. 2.43})$$

$$\eta_b = 0.9399 \quad \eta_b \cdot 100 = 93.99 \quad \%$$

Problem 2.21

Determine the maximum amount of cargo that can be carried.

$$V_1 := 120 \cdot 0.2778 \quad V_1 = 33.336 \quad \text{m/s} \quad V_2 := 0 \quad (\text{vehicle is assumed to stop})$$

$$\mu := 0.95 \quad \underline{G} := -0.04 \quad \underline{g} := 9.807 \quad (\text{given})$$

$$\gamma_b := 1.04 \quad \eta_b := 0.80 \quad \underline{S} := 90$$

$$V_{\text{avg}} := \frac{V_1 + V_2}{2} \quad V_{\text{avg}} = 16.668$$

$$f_{r1} := 0.01 \cdot \left(1 + \frac{V_{\text{avg}}}{44.73} \right) \quad f_{r1} = 0.0137 \quad (\text{Eq. 2.5})$$

solve for additional vehicle weight using theoretical stopping distance equation,
ignoring aerodynamic resistance

$$S = \frac{\gamma_b \cdot V_1^2}{2 \cdot g \cdot \left[\left(\eta_b - \frac{W}{100 \cdot 445} \right) \cdot \mu + f_{r1} + G \right]} \quad (\text{Eq. 2.43})$$

$$W = 3701.1 \quad \text{N}$$

Problem 2.22

Determine the speed of the car when it strikes the object.

$$C_D := 0.5 \quad A_f := 2.3 \quad W := 15600 \quad \rho := 1.2256 \quad (\text{given})$$

$$S := 45 \quad \mu := 0.85 \quad g := 9.807 \quad \gamma_b := 1.04$$

$$f_{fl} := 0.018 \quad \eta_b := 0.80$$

$$V_1 := 130 \cdot 0.2778 \quad V_1 = 36.114$$

$$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \quad K_a = 0.705 \quad (\text{Eq. 2.37})$$

How fast will the car be travelling when it strikes the object on a level surface?

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln \left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{fl} \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot (V_2^2) + f_{fl} \cdot W} \right] \quad (\text{Eq. 2.39})$$

$$V_2 = 25.961 \quad \frac{V_2}{0.2778} = 93.45 \quad \frac{\text{km}}{\text{h}}$$

How fast will the car be travelling when it strikes the object on a 5% grade?

$$G := 0.05$$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln \left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{fl} \cdot W + G \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot (V_2^2) + f_{fl} \cdot W + G \cdot W} \right] \quad (\text{Eq. 2.39})$$

$$V_2 = 25.147 \quad \frac{V_2}{0.2778} = 90.52 \quad \frac{\text{km}}{\text{h}}$$

Problem 2.23

Determine the speed of the car just before it impacted the object.

$$V_1 := 120 \cdot \frac{1000}{3600} \quad V_1 = 33.333 \quad (\text{given})$$

$$\gamma_b := 1.04 \quad g := 9.807$$

$$\eta_b := 0.90 \quad f_{fl} := 0.015$$

$$\mu_m := 0.6 \quad \mu_s := 0.3 \quad (\text{Table 2.4})$$

$$S_{al} := 60 \quad S_{skid} := 30 \quad (\text{given})$$

Find velocity of the car when it starts to skid

$$S_{al} = \frac{\gamma_b \cdot (V_1^2 - V_2^2)}{2 \cdot g \cdot (\eta_b \cdot \mu_m + f_{fl} - 0.03)} \quad (\text{Eq. 2.43})$$

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{al} \cdot g \cdot (\eta_b \cdot \mu_m + f_{fl} - 0.03)}{\gamma_b}}$$

$$V_2 = 22.738 \quad V_2 \cdot \frac{3600}{1000} = 81.858 \quad \frac{\text{km}}{\text{h}}$$

Vehicle's velocity at start of skid is $V_1 := 22.738$

Find velocity when the vehicle strikes the object

$$S_{skid} = \frac{\gamma_b \cdot (V_1^2 - V_2^2)}{2 \cdot g \cdot (\eta_b \cdot \mu_s + f_{fl} - 0.03)} \quad (\text{Eq. 2.43})$$

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{skid} \cdot g \cdot (\eta_b \cdot \mu_s + f_{fl} - 0.03)}{\gamma_b}}$$

$$V_2 = 19.306$$

$$V_2 \cdot \frac{3600}{1000} = 69.5 \quad \frac{\text{km}}{\text{h}}$$

Problem 2.24

Determine if the driver should appeal the ticket.

$\mu := 0.6$ (for good, wet pavement, and slide value because of skidding)

$\gamma_b := 1.04$ $g := 9.807$

$V_2 := 65 \cdot \frac{1000}{3600}$ $V_2 = 18.056$ (given)

$\eta_b := 0.95$ $S := 61$ $f_{rl} := 0.015$

Solve for the initial velocity of the car using theoretical stopping distance

$$S = \frac{\gamma_b \cdot (V_1^2 - V_2^2)}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl} - 0.04)} \quad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot (\eta_b \cdot \mu + f_{rl} - 0.04)}{\gamma_b} + V_2^2} \quad (\text{Eq. 2.43})$$



$V_1 = 30.871$

$$V_1 \cdot \frac{3600}{1000} = 111.13 \quad \frac{\text{km}}{\text{h}}$$

No, the driver should not appeal the ticket as the initial velocity was higher than the speed limit, in addition to the road being wet.

Problem 2.25

Determine the shortest distance from the stalled car that the driver could apply the brakes and stop before hitting it.

$$\eta_b := 0.90 \quad \gamma_b := 1.04 \quad f_{fl} := 0.013 \quad (\text{given})$$

$$V := 110 \cdot \frac{1000}{3600} \quad V = 30.556$$

$$S := 45 \quad g := 9.807$$

$$\mu_{\text{dry}} := 1.0$$

$$\mu_{\text{wet}} := 0.9 \quad (\text{Table 2.4})$$

Solve for velocity when entering wet section of pavement

$$S = \frac{\gamma_b \cdot (V_1^2 - 0)}{2 \cdot g \cdot (\eta_b \cdot \mu_{\text{wet}} + f_{fl} - 0.03)} \quad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot (\eta_b \cdot \mu_{\text{wet}} + f_{fl} - 0.03)}{\gamma_b}} \quad (\text{Eq. 2.43})$$

$$V_1 = 25.942 \quad V_1 \cdot \frac{3600}{1000} = 93.392$$

Using this as final velocity, solve for distance to slow to this velocity

$$S = \frac{\gamma_b \cdot (V^2 - V_1^2)}{2 \cdot g \cdot (\eta_b \cdot \mu_{\text{dry}} + f_{fl} - 0.03)} \quad (\text{Eq. 2.43})$$

$$S = 15.651$$

Add this distance to the 45 m of wet pavement

$$45 + S = 60.65 \quad \text{m}$$

Problem 2.26

Determine the braking efficiency of car 1.

$$\gamma_b := 1.04 \quad g := 9.807 \quad V := 100 \cdot 0.2778 \quad V = 27.78 \quad (\text{given})$$

$$t_{r1} := 2.5 \quad t_{r2} := 2.0 \quad \eta_{b2} := 0.75 \quad \mu := 0.80$$

$$f_{d1} := 0.01 \left(1 + \frac{V}{2 \cdot 44.73} \right) \quad f_{d1} = 0.013 \quad (\text{Eq. 2.5})$$

Total stopping distance is perception/reaction distance plus braking distance

Set stopping distance of two cars equal to each other and solve for η_{b1}

$$V \cdot t_{r1} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b1} \cdot \mu + f_{d1}} \right) = V \cdot t_{r2} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b2} \cdot \mu + f_{d1}} \right)$$

$$\eta_{b1} = 0.9514 \quad \eta_{b1} \cdot 100 = 95.14 \quad \%$$

Problem 2.27

Determine the student's associated perception reaction time.

$$V_1 := 90 \cdot 0.2778 \quad V_1 = 25.002 \quad \text{m/s} \quad (\text{given})$$

$$V_2 := 55 \cdot 0.2778 \quad V_2 = 15.28 \quad \text{m/s}$$

$$g := 9.807 \quad G := 0 \quad a := 3.4 \quad \text{ft/s}^2$$

Solve for distance to slow from 90 km/h to 55 km/h

$$d := \frac{(V_1)^2 - (V_2)^2}{2 \cdot a} \quad (\text{Eq. 2.45})$$

$$d = 57.6 \quad \text{m}$$

Subtract this distance from total distance to sign (185 m) to find perception/reaction time

$$d_s := 185 \quad (\text{given})$$

$$d_r := d_s - d \quad d_r := 185 - d \quad d_r = 127.4$$

$$t_r := \frac{d_r}{V_1} \quad t_r = 5.1 \quad \text{sec} \quad (\text{Eq. 2.49})$$

Problem 2.28

Comment on the student's claim.

Method 1:

$$V_1 := 110 - 0.2778 \quad V_1 = 30.558 \quad (\text{given})$$

$$g := 9.807 \quad a := 3.4$$

Find practical stopping distance

$$d := \frac{V_1^2}{2 \cdot a} \quad d = 137.322 \quad (\text{Eq. 2.46})$$

Subtract this distance from the total sight distance and solve for perception/reaction time

$$d_s := 180$$

$$d_r := d_s - d \quad d_r = 42.678$$

$$t_r := \frac{d_r}{V_1} \quad t_r = 1.397 \text{ sec} \quad (\text{Eq. 2.49})$$

This reaction time is well below the design value; therefore, the student's claim is unlikely.

Method 2:

$$V_1 := 110 - 0.2778 \quad V_1 = 30.558$$

$$g := 9.807 \quad a := 3.4 \quad t_r := 2.5 \quad G := 0$$

Stopping sight distance = practical stopping distance plus perception/reaction distance

$$\text{SSD} := \frac{V_1^2}{2 \cdot g \cdot \left[\left(\frac{a}{g} \right) + G \right]} + V_1 \cdot t_r \quad \text{SSD} = 213.717 \text{ m} \quad (\text{Eq. 2.47})$$

180 ft < 214 m (required from Table 3.1) therefore 180 m is not enough for 110 km/h design speed.

Problem 2.29

Determine the grade of the road.

$$V_1 := 90 \cdot 0.2778 \quad V_1 = 25.002 \quad (\text{given})$$

$$t_r := 2.5 \quad d_s := 140$$

Solve for distance travelled during braking (total distance minus perception/reaction)

$$d_r := V_1 \cdot t_r \quad d_r = 62.505 \quad (\text{Eq. 2.50})$$

$$d := d_s - d_r \quad d = 77.495 \quad (\text{Eq. 2.49})$$

$$a := 3.4 \quad g := 9.807 \quad (\text{given})$$

$$d = \frac{(V_1)^2}{2 \cdot g \cdot \left(\frac{a}{g} + G \right)} \quad G := \frac{V_1^2}{2 \cdot d \cdot g} - \frac{a}{g} \quad (\text{Eq. 2.47})$$

$$G = 0.0646 \quad G \cdot 100 = 6.46 \quad \%$$

Problem 2.30

Determine the driver's perception/reaction time before and after drinking.

$$V_1 := 90 \cdot 0.2778 \quad V_1 = 25.002 \quad \text{m/s} \quad (\text{given})$$

$$g := 9.807 \quad a := 3.4$$

$$\text{while sober,} \quad d_s := 160$$

solve for perception/reaction time using total stopping distance formula

$$d := \frac{V_1^2}{2 \cdot a} \quad (\text{Eq. 2.46})$$

$$d_s := d_r + d \quad d_r := d_s - d \quad (\text{Eq. 2.50})$$

$$d_r := V_1 \cdot t_r \quad t_r := \frac{d_r}{V_1} \quad (\text{Eq. 2.49})$$

substituting Eqs. 2.46 and 2.50 into Eq. 2.49 gives

$$t_r := \frac{d_s}{V_1} - \frac{V_1}{2a} \quad t_r = 2.72 \quad \text{sec}$$

$$\text{after drinking, driver strikes the object at} \quad V_2 := 55 \cdot 0.2778 \quad \text{m/s}$$

solve for perception/reaction time using total stopping distance formula

$$t_r := \frac{d_s}{V_1} - \frac{V_1^2 - V_2^2}{2a \cdot V_1} \quad t_r = 4.1 \quad \text{sec} \quad (\text{Eq. 2.50})$$