Complete Solutions Manual

Precalculus with Limits
A Graphing Approach

TEXAS EDITION

SIXTH EDITION

Ron Larson
The Pennsylvania State University,
The Behrend College

CENGAGE Learning

Australia • Brazil • Mexico • Singapore • United Kingdom • United States

CENGAGE Learning

For product information and technology assistance, contact us at **Cengage Learning Customer & Sales Support, 1-800-354-9706**.

For permission to use material from this text or product, submit all requests online at **www.cengage.com/permissions**
Further permissions questions can be emailed to **permissionrequest@cengage.com**.

ISBN-13: 978-1-285-86773-1
ISBN-10: 1-285-86773-4

Cengage Learning
200 First Stamford Place, 4th Floor
Stamford, CT 06902
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at: **www.cengage.com/global**.

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

To learn more about Cengage Learning Solutions, visit **www.cengage.com**.

Purchase any of our products at your local college store or at our preferred online store **www.cengagebrain.com**.

Printed in the United States of America
1 2 3 4 5 6 7 18 17 16 15 14

Table of Contents

CHAPTER 1

Section 1.1

1. (a) iii (b) i (c) v (d) ii (e) iv

2. slope

3. parallel

4. They are perpendicular to each other.

5. Since $x = 3$ is a vertical line, all horizontal lines are perpendicular and have slope $m = 0$.

6. Since the line $y - (-2) = \frac{1}{2}(x - 5)$ is in point-slope form, the point $(5, -2)$ lies on the line.

7. (a) $m = \frac{2}{3}$. Since the slope is positive, the line rises. Matches L_2.

(b) m is undefined. The line is vertical. Matches L_3.

(c) $m = -2$. The line falls. Matches L_1.

8. (a) $m = 0$. The line is horizontal. Matches L_2.

(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.

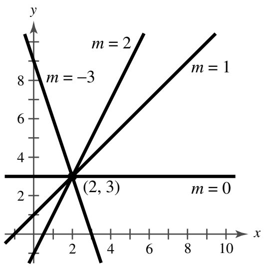
(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

9. $\text{Slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{2}$

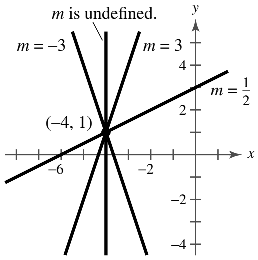
10. The line appears to go through $(0, 8)$ and $(2, 0)$.

$\text{Slope} = \dfrac{8 - 0}{0 - 2} = -4$

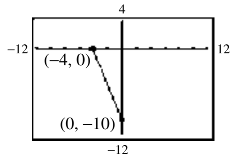
11.



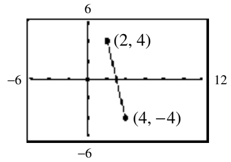
12.



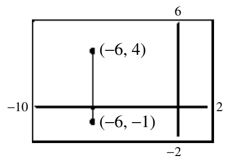
13. $\text{Slope} = \dfrac{0 - (-10)}{-4 - 0} = \dfrac{10}{-4} = -\dfrac{5}{2}$



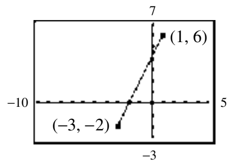
14. $\text{Slope} = \dfrac{-4 - 4}{4 - 2} = -4$



15. Slope is undefined.



16. $\text{Slope} = \dfrac{6 - (-2)}{1 - (-3)} = \dfrac{8}{4} = 2$



17. Since $m = 0$, y does not change. Three additional points are $(0, 1)$, $(3, 1)$, and $(-1, 1)$.

18. Since $m = 0$, y does not change. Three additional points are $(0, -2), (1, -2)$, and $(4, -2)$.

19. Since m is undefined, x does not change and the line is vertical. Three additional points are $(1, 1), (1, 2)$, and $(1, 3)$.

20. Because m is undefined, x does not change. Three additional points are $(-4, 0)$, $(-4, 3)$, and $(-4, 5)$.

21. Since $m = -2$, y decreases 2 for every unit increase in x. Three additional points are $(1, -11)$, $(2, -13)$, and $(3, -15)$.

22. Since $m = 2$, y increases 2 for every unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

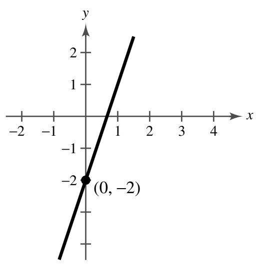
23. Since $m = \frac{1}{2}$, y increase 1 for every increase of 2 in x.

Three additional points are $(9, -1)$, $(11, 0)$, and $(13, 1)$.

24. Since $m = -\frac{1}{2}$, y decreases 1 for every increase of 2 units in x. Three additional points are $(1, -7)$, $(3, -8)$, and $(5, -9)$.

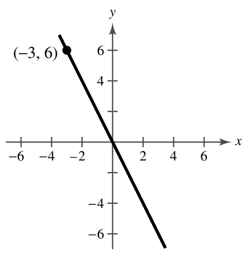
25. $m = 3,\ (0, -2)$

$$y + 2 = 3(x - 0)$$
$$y = 3x - 2 \Rightarrow 3x - y - 2 = 0$$



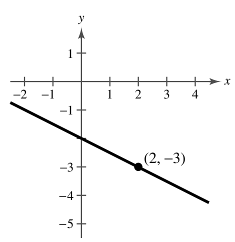
26. $m = -2,\ (-3, 6)$

$$y - 6 = -2(x + 3)$$
$$y = -2x$$
$$2x + y = 0$$



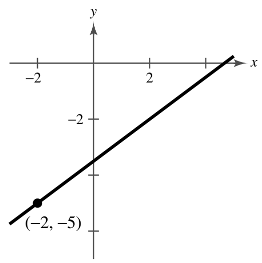
27. $m = -\frac{1}{2},\ (2, -3)$

$$y - (-3) = -\frac{1}{2}(x - 2)$$
$$y + 3 = -\frac{1}{2}x + 1$$
$$2y + 4 = -x$$
$$x + 2y + 4 = 0$$



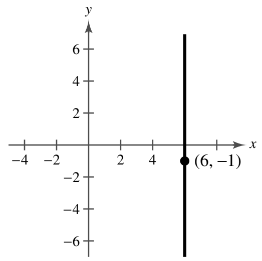
28. $m = \frac{3}{4},\ (-2, -5)$

$$y + 5 = \frac{3}{4}(x + 2)$$
$$4y + 20 = 3x + 6$$
$$0 = 3x - 4y - 14$$



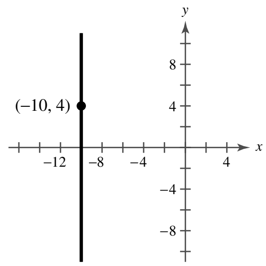
29. m is undefined, $(6, -1)$

$$x = 6$$
$$x - 6 = 0 \text{ vertical line}$$



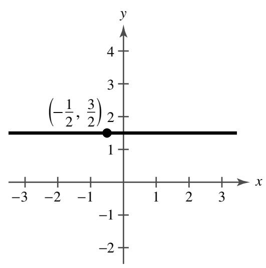
30. m is undefined, $(-10, 4)$

$$x = 10$$
$$x + 10 = 0 \text{ vertical line}$$



31. $m = 0,\ \left(-\frac{1}{2}, \frac{3}{2}\right)$

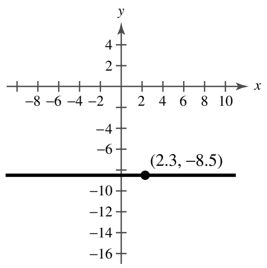
$$y - \frac{3}{2} = 0\left(x + \frac{1}{2}\right)$$
$$y - \frac{3}{2} = 0 \text{ horizontal line}$$



32. $m = 0, \ (2.3, -8.5)$

$y - (-8.5) = 0(x - 2.3)$

$y + 8.5 = 0$ horizontal line



33. Begin by letting $x = 1$ correspond to 2001. Then using the points $(1, 1.6)$ and $(9, 5.2)$, you have

$m = \frac{5.2 - 1.6}{9 - 1} = \frac{3.6}{8} = 0.45$

$y - 1.6 = 0.45(x - 1)$

$y = 0.45x + 1.15$

When $x = 17$:

$y = 0.45(17) + 1.15 = \$8.8$ million

34. Begin by letting $x = 0$ correspond to 2000. Then using the points $(0, 441{,}300)$ and $(8, 1{,}326{,}720)$, you have

$m = \frac{1{,}326{,}720 - 441{,}300}{8 - 0} = \frac{885{,}420}{8} = 110{,}677.5$

$y - 441{,}300 = 110{,}677.5(x - 0)$

$y = 110{,}677.5x + 441{,}300$

When $x = 16$:

$y = 110{,}677.5(16) + 441{,}300 = \$2{,}212{,}140$

35. $x - 2y = 4$

$-2y = -x + 4$

$y = \frac{1}{2}x - 2$

Slope: $\frac{1}{2}$

y-intercept: $(0, -2)$

The line passes through $(0, -2)$ and rises 1 unit for each horizontal increase of 2 units.

36. $3x + 4y = 1$

$4y = -3x + 1$

$y = \frac{-3}{4}x + \frac{1}{4}$

Slope: $-\frac{3}{4}$

y-intercept: $\left(0, \frac{1}{4}\right)$

The line passes through $\left(0, \frac{1}{4}\right)$ and falls 3 units for each horizontal increase of 4 units.

37. $2x - 5y + 10 = 0$

$-5y = -2x - 10$

$y = \frac{2}{5}x + 2$

Slope: $\frac{2}{5}$

y-intercept: $(0, 2)$

The line passes through $(0, 2)$ and rises 2 units for each horizontal increase of 5 units.

38. $4x - 3y - 9 = 0$

$-3y = -4x + 9$

$y = \frac{4}{3}x - 3$

Slope: $\frac{4}{3}$

y-intercept: $(0, -3)$

The line passes through $(0, -3)$ and rises 4 units for each horizontal increase of 3 units.

39. $x = -6$

Slope is undefined; no y-intercept.

The line is vertical and passes through $(-6, 0)$.

40. $y = 12$

Slope: 0

y-intercept: $(0, 12)$

The line is horizontal and passes through $(0, 12)$.

41. $3y + 2 = 0$

$3y = -2$

$y = -\frac{2}{3}$

Slope: 0

y-intercept: $\left(0, -\frac{2}{3}\right)$

The line is horizontal and passes through $\left(0, -\frac{2}{3}\right)$.

42. $2x - 5 = 0$

$2x = 5$

$x = \frac{5}{2}$

Slope is undefined; no y-intercept.

The line is vertical and passes through $\left(\frac{5}{2}, 0\right)$.

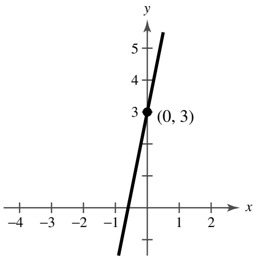
43. $5x - y + 3 = 0$

$$y = 5x + 3$$

(a) Slope: $m = 5$

y-intercept: $(0, 3)$

(b)



44. $2x + 3y - 9 = 0$

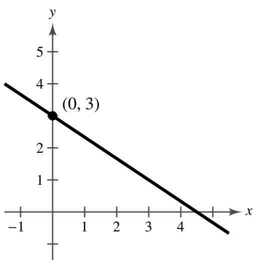
$$3y = -2x + 9$$

$$y = -\frac{2}{3}x + 3$$

(a) Slope: $m = -\frac{2}{3}$

y-intercept: $(0, 3)$

(b)



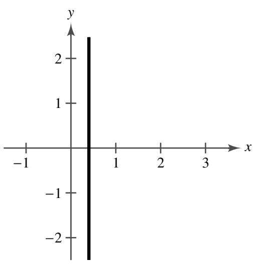
45. $5x - 2 = 0$

$$x = \frac{2}{5}$$

(a) Slope: undefined

No y-intercept

(b)



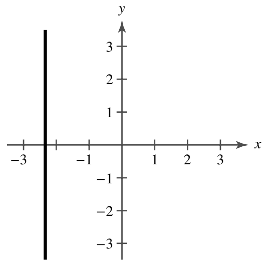
46. $3x + 7 = 0$

$$x = -\frac{7}{3}$$

(a) Slope: undefined

No y-intercept

(b)



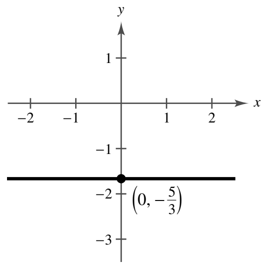
47. $3y + 5 = 0$

$$y = -\frac{5}{3}$$

(a) Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$

(b)



48. $-11 - 8y = 0$

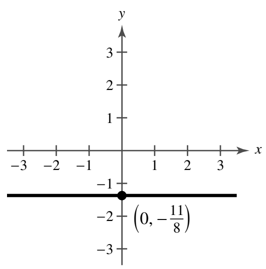
$$8y = -11$$

$$y = -\frac{11}{8}$$

(a) Slope: $m = 0$

y-intercept: $\left(0, -\frac{11}{8}\right)$

(b)



49. The slope is $\frac{-3 - (-7)}{1 - (-1)} = \frac{4}{2} = 2.$

$$y - (-3) = 2(x - 1)$$

$$y + 3 = 2x - 2$$

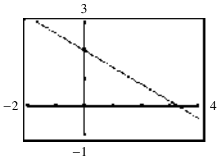
$$y = 2x - 5$$

50. The slope is $\dfrac{-1-\dfrac{3}{2}}{4-(-1)} = \dfrac{-\dfrac{5}{2}}{5} = -\dfrac{1}{2}$.

$$y-(-1) = -\frac{1}{2}(x-4)$$
$$y+1 = -\frac{1}{2}x+2$$
$$y = -\frac{1}{2}x+1$$

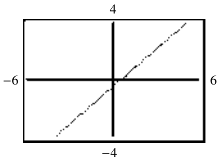
51. $(5, -1), (-5, 5)$

$$y+1 = \frac{5+1}{-5-5}(x-5)$$
$$y = -\frac{3}{5}(x-5)-1$$
$$y = -\frac{3}{5}x+2$$



52. $(4, 3), (-4, -4)$

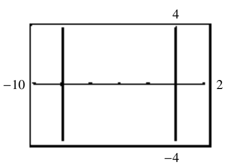
$$y-3 = \frac{-4-3}{-4-4}(x-4)$$
$$y-3 = \frac{7}{8}(x-4)$$
$$y = \frac{7}{8}x-\frac{1}{2}$$



53. $(-8, 1), (-8, 7)$

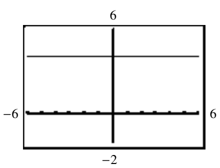
Since both points have an x-coordinate of −8, the slope is undefined and the line is vertical.

$$x+8=0$$



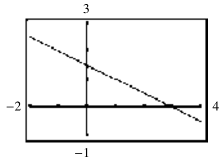
54. $(-1, 4), (6, 4)$

$$y-4 = \frac{4-4}{6-(-1)}(x+1)$$
$$y-4 = 0(x+1)$$
$$y-4 = 0$$
$$y = 4$$



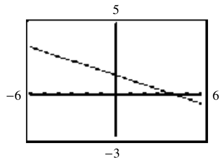
55. $\left(2, \dfrac{1}{2}\right), \left(\dfrac{1}{2}, \dfrac{5}{4}\right)$

$$y-\frac{1}{2} = \frac{\dfrac{5}{4}-\dfrac{1}{2}}{\dfrac{1}{2}-2}(x-2)$$
$$y = -\frac{1}{2}(x-2)+\frac{1}{2}$$
$$y = -\frac{1}{2}x+\frac{3}{2}$$



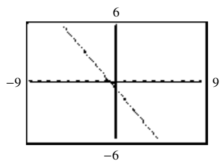
56. $(1, 1), \left(6, -\dfrac{2}{3}\right)$

$$y-1 = \frac{-\dfrac{2}{3}-1}{6-1}(x-1)$$
$$y-1 = -\frac{1}{3}(x-1)$$
$$y-1 = -\frac{1}{3}x+\frac{1}{3}$$
$$y = -\frac{1}{3}x+\frac{4}{3}$$



57. $\left(-\dfrac{1}{10}, -\dfrac{3}{5}\right), \left(\dfrac{9}{10}, -\dfrac{9}{5}\right)$

$$y+\frac{3}{5} = \frac{-\dfrac{9}{5}+\dfrac{3}{5}}{\dfrac{9}{10}+\dfrac{1}{10}}\left(x+\frac{1}{10}\right)$$
$$y+\frac{3}{5} = -\frac{6}{5}\left(x+\frac{1}{10}\right)$$
$$y = -\frac{6}{5}x-\frac{18}{25}$$



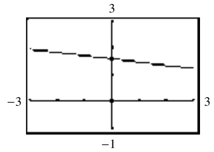
58. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

$$y - \frac{3}{2} = \frac{\frac{7}{4} - \frac{3}{2}}{-\frac{4}{3} - \frac{3}{4}}\left(x - \frac{3}{4}\right)$$

$$y - \frac{3}{2} = -\frac{3}{25}\left(x - \frac{3}{4}\right)$$

$$y - \frac{3}{2} = -\frac{3}{25}x + \frac{9}{100}$$

$$y = -\frac{3}{25}x + \frac{159}{100}$$

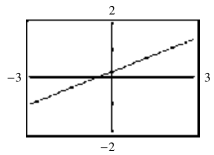


59. $(1, 0.6), (-2, -0.6)$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$

$$y = 0.4(x - 1) + 0.6$$

$$y = 0.4x + 0.2$$

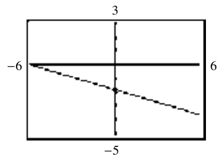


60. $(-8, 0.6), (2, -2.4)$

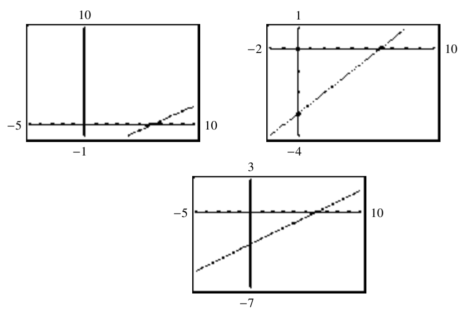
$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$

$$y - 0.6 = -0.3(x + 8)$$

$$y = -0.3x - 1.8$$

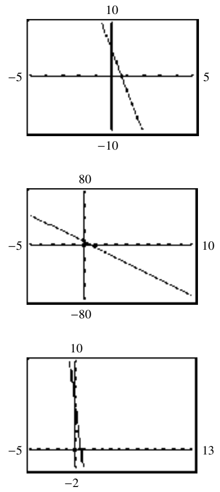


61.



The first graph does not show both intercepts. The third graph is best because it shows both intercepts and gives the most accurate view of the slope by using a square setting.

62.



The second graph does not give a good view of the intercepts. The third graph is best because it gives the most accurate view of the slope by using a square setting.

63. L_1: $(0, -1), (5, 9)$

$$m_1 = \frac{9 + 1}{5 - 0} = 2$$

L_2: $(0, 3), (4, 1)$

$$m_2 = \frac{1 - 3}{4 - 0} = -\frac{1}{2} = -\frac{1}{m_1}$$

L_1 and L_2 are perpendicular.

64. L_1: $(-2, -1), (1, 5)$

$$m_1 = \frac{5 - (-1)}{1 - (-2)} = \frac{6}{3} = 2$$

L_2: $(1, 3), (5, -5)$

$$m_2 = \frac{-5 - 3}{5 - 1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

65. L_1: $(3, 6), (-6, 0)$

$$m_1 = \frac{0 - 6}{-6 - 3} = \frac{2}{3}$$

L_2: $(0, -1), \left(5, \frac{7}{3}\right)$

$$m_2 = \frac{\frac{7}{3} + 1}{5 - 0} = \frac{2}{3} = m_1$$

L_1 and L_2 are parallel.

66. L_1: $(4, 8), (-4, 2)$

$$m_1 = \frac{2 - 8}{-4 - 4} = \frac{-6}{-8} = \frac{3}{4}$$

L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

$$m_2\colon \frac{(1/3) - (-5)}{-1 - 3} = \frac{16/3}{-4} = -\frac{4}{3}$$

The lines are perpendicular.

67. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

Slope: $m = 2$

(a) Parallel slope: $m = 2$

$y - 1 = 2(x - 2)$

$y = 2x - 3$

(b) Perpendicular slope: $m = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 2)$

$y = -\frac{1}{2}x + 2$

68. $x + y = 7$

$y = -x + 7$

Slope: $m = -1$

(a) Parallel slope: $m = -1$

$y - 2 = -1(x + 3)$

$y = -x - 1$

(b) Perpendicular slope: $m = 1$

$y - 2 = 1(x + 3)$

$y = x + 5$

69. $3x + 4y = 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

Slope: $m = -\frac{3}{4}$

(a) Parallel slope: $m = -\frac{3}{4}$

$y - \frac{7}{8} = -\frac{3}{4}\left(x + \frac{2}{3}\right)$

$y = -\frac{3}{4}x + \frac{3}{8}$

(b) Perpendicular slope: $m = \frac{4}{3}$

$y - \frac{7}{8} = \frac{4}{3}\left(x + \frac{2}{3}\right)$

$y = \frac{4}{3}x + \frac{127}{72}$

70. $3x - 2y = 6$

$y = \frac{3}{2}x - 6$

Slope: $m = \frac{3}{2}$

(a) Parallel slope: $m = \frac{3}{2}$

$y + 1 = \frac{3}{2}\left(x - \frac{2}{5}\right)$

$y = \frac{3}{2}x - \frac{8}{5}$

(b) Perpendicular slope: $m = -\frac{2}{3}$

$y + 1 = -\frac{2}{3}\left(x - \frac{2}{5}\right)$

$y = -\frac{2}{3}x - \frac{11}{15}$

71. $6x + 2y = 9$

$2y = -6x + 9$

$y = -3x + \frac{9}{2}$

Slope: $m = -3$

(a) Parallel slope: $m = -3$

$y + 1.4 = -3(x + 3.9)$

$y = -3x - 13.1$

(b) Perpendicular slope: $m = \frac{1}{3}$

$y + 1.4 = \frac{1}{3}(x + 3.9)$

$y = \frac{1}{3}x - \frac{1}{10}$

72. $5x + 4y = 1$

$y = -\frac{5}{4}x + \frac{1}{4}$

Slope: $m = -\frac{5}{4} = -1.25$

(a) Parallel slope: $m = -\frac{5}{4}$

$y - 2.4 = -\frac{5}{4}(x + 1.2)$

$y = -1.25x + 0.9$

(b) Perpendicular slope: $m = 0.8$

$y - 2.4 = 0.8(x + 1.2)$

$y = 0.8x + 3.36$

73. $x - 4 = 0$ vertical line

Slope is undefined.

(a) $x - 3 = 0$ passes through $(3, -2)$ and is vertical.

(b) $y = -2$ passes through $(3, -2)$ and is horizontal.

74. $y - 2 = 0$
$y = 2$ horizontal line

Slope: $m = 0$

(a) $y = -1$ passes through $(3, -1)$ and is horizontal.
(b) $x - 3 = 0$ passes through $(3, -1)$ and is vertical.

75. $y + 2 = 0$
$y = -2$ horizontal line

Slope: $m = 0$

(a) $y = 1$ passes through $(-4, 1)$ and is horizontal.
(b) $x + 4 = 0$ passes through $(-4, 1)$ and is vertical.

76. $x + 5 = 0$ vertical line

Slope is undefined.

(a) $x + 2 = 0$ passes through $(-2, 4)$ and is vertical.
(b) $y = 4$ passes through $(-2, 4)$ and is horizontal.

77. The slope is 2 and $(-1, -1)$ line on the line. Hence,

$$y - (-1) = 2(x - (-1))$$
$$y + 1 = 2(x + 1)$$
$$y = 2x + 1.$$

78. The slope is -2 and $(-1, 1)$ lines on the line. Hence,

$$y - 1 = -2(x - (-1))$$
$$y - 1 = -2(x + 1)$$
$$y = -2x - 1.$$

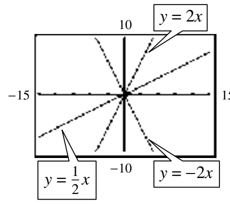
79. The slope of the given line is 2. Then y_2 has slope $-\frac{1}{2}$. Hence,

$$y - 2 = -\frac{1}{2}(x - (-2))$$
$$y - 2 = -\frac{1}{2}(x + 2)$$
$$y = -\frac{1}{2}x + 1.$$

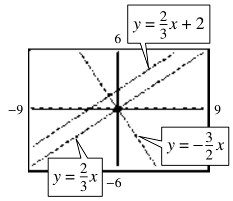
80. The slope of the given line is 3. Then y_2 has slope $-\frac{1}{3}$. Hence,

$$y - 5 = -\frac{1}{3}(x - (-3))$$
$$y - 5 = -\frac{1}{3}(x + 3)$$
$$y = -\frac{1}{3}x + 4.$$

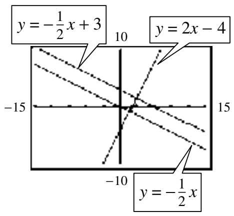
81. The lines $y = \frac{1}{2}x$ and $y = -2x$ are perpendicular.



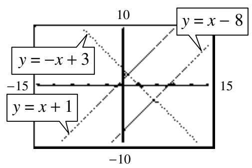
82. The lines $y = \frac{2}{3}x$ and $y = \frac{2}{3}x + 2$ are parallel. Both are perpendicular to $y = -\frac{3}{2}x$.



83. The lines $y = -\frac{1}{2}x$ and $y = -\frac{1}{2}x + 3$ are parallel. Both are perpendicular to $y = 2x - 4$.



84. The lines $y = x - 8$ and $y = x + 1$ are parallel. Both are perpendicular to $y = -x + 3$.



85. $$\frac{\text{rise}}{\text{run}} = \frac{3}{4} = \frac{x}{\frac{1}{2}(32)}$$
$$\frac{3}{4} = \frac{x}{16}$$
$$4x = 48$$
$$x = 12$$

The maximum height in the attic is 12 feet.

86. $$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
$$\frac{-12}{100} = \frac{-2000}{x}$$
$$-12x = (-2000)(100)$$
$$x = 16{,}666\frac{2}{3}\text{ ft} \approx 3.16 \text{ miles}$$

87.

(a)

Years	Slope
2000–2001	$1023 - 995 = 28$
2001–2002	$1247 - 1023 = 224$
2002–2003	$1211 - 1247 = -36$
2003–2004	$1257 - 1211 = 46$
2004–2005	$1380 - 1257 = 123$
2005–2006	$1431 - 1380 = 51$
2006–2007	$1436 - 1431 = 5$
2007–2008	$1464 - 1436 = 28$

The greatest increase was \$224 million from 2001 to 2002.
The greatest decrease was \$36 million from 2002 to 2003.

(b) Using the points $(0, 995)$ and $(8, 1464)$, the slope is $m = \dfrac{1464 - 995}{8 - 0} = 58.625$.

Then $y - 995 = 58.625(x - 0)$

$$y = 58.625x + 995$$

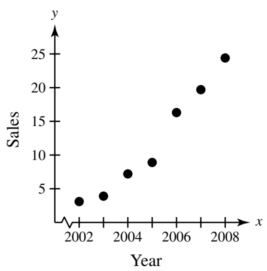
(c) There was an average increase in sales of about \$ 58.625 million per year from 2000 to 2008.

(d) When $x = 10$: $y = 58.625(10) + 995$

$$y = \$1581.25 \text{ million}$$

Answers will vary.

88. (a)



(b)

Years	Slopes
2002–2003	$3.9 - 3.1 = 0.8$
2003–2004	$7.2 - 3.9 = 3.3$
2004–2005	$8.9 - 7.2 = 1.7$
2005–2006	$16.3 - 8.9 = 7.4$
2006–2007	$19.7 - 16.3 = 3.4$
2007–2008	$24.4 - 19.7 = 4.7$

The greatest increase was \$7.4 million from 2005 to 2006.

The least increase was \$0.8 million from 2002 to 2003.

(c) Using the points $(2, 3.1)$ and $(8, 24.4)$, the slope is $m = \dfrac{24.4 - 3.1}{8 - 2} = 3.55$.

Then

$$y - 3.1 = 3.55(x - 2)$$
$$y = 3.55x - 4$$

(d) There was an average increase of approximately \$3.55 million in profit per year from 2002 to 2008.

(e) When $x = 10$, $y = 3.55(10) - 4 = \$31.5$ million. Answers will vary.

For Exercises 89 – 92, $t = 9$ corresponds to 2009.

89. $(9, 2540)$, $m = 125$

$$V - 2540 = 125(t - 9)$$
$$V = 125t + 1415$$

90. $(9, 156)$, $m = 4.50$

$$V - 156 = 4.50(t - 9)$$
$$V = 4.50t + 115.5$$

91. $(9, 20{,}400)$, $m = -2000$

$$V - 20{,}400 = -2000(t - 9)$$
$$V = -2000t + 38{,}400$$

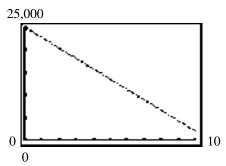
92. $(9, 245{,}000)$, $m = -5600$

$$V - 245{,}000 = -5600(t - 9)$$
$$V = -5600t + 295{,}400$$

93. (a) $(0, 25{,}000)$, $(10, 2000)$

$$V - 25{,}000 = \frac{2000 - 25{,}000}{10 - 0}(t - 0)$$
$$V - 25{,}000 = -2300t$$
$$V = -2300t + 25{,}000$$

(b)



t	0	1	2	3	4	5
V	25,000	22,700	20,400	18,100	15,800	13,500

6	7	8	9	10
11,200	8900	6600	4300	2000

(c) $t = 0$: $V = -2300(0) + 25{,}000 = 25{,}000$
$t = 1$: $V = -2300(1) + 25{,}000 = 22{,}700$
etc.

94. (a) Using the points $(0, 32)$ and $(100, 212)$, we have

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$
$$F - 32 = \frac{9}{5}(C - 0)$$
$$F = \frac{9}{5}C + 32.$$

(b) $F = \frac{9}{5}C + 32$

$F = 0°$: $0 = \frac{9}{5}C + 32$

$-32 = \frac{9}{5}C$

$-17.8 \approx C$

$C = 10°$: $F = \frac{9}{5}(10) + 32$

$F = 18 + 32$

$F = 50$

$F = 90°$: $90 = \frac{9}{5}C + 32$

$58 = \frac{9}{5}C$

$32.2 \approx C$

$C = -10°$: $F = \frac{9}{5}(-10) + 32$

$F = -18 + 32$

$F = 14$

$F = 68°$: $68 = \frac{9}{5}C + 32$

$36 = \frac{9}{5}C$

$20 = C$

$C = 177°$: $F = \frac{9}{5}(177) + 32$

$F = 318.6 + 32$

$F = 350.6$

C	−17.8°	−10°	10°	20°	32.2°	177°
F	0°	14°	50°	68°	90°	350.6°

95.

(a) $C = 36{,}500 + 9.25t + 18.50t$
$C = 36{,}500 + 27.75t$

(b) $R = tp$ (t hours at $\$p$ per hour)
$R = t(65)$
$R = 65t$

(c) $P = R - C$
$P = 65t - (36{,}500 + 27.75t)$
$P = 37.25t - 36{,}500$

(d) $P = 0$:
$37.25t - 36{,}500 = 0$
$37.25t = 36{,}500$
$t \approx 980$ hours

96.

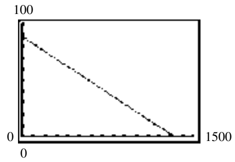
(a) $(580, 50), (625, 47)$

$x - 50 = \frac{47 - 50}{625 - 580}(p - 580)$

$x - 50 = \frac{-1}{15}(p - 580)$

$x = -\frac{1}{15}p + \frac{266}{3}$

(b)



If $p = 655$, $x = 45$ units.

Algebraically, $x = -\frac{1}{15}(655) + \frac{266}{3} = 45$.

(c) If $p = 595$, $x = 49$ units.

Algebraically, $x = -\frac{1}{15}(595) + \frac{266}{3} = 49$.

97.

(a) Using the points $(1990, 75{,}365)$ and $(2009, 87{,}163)$ the slope is

$m = \frac{87{,}163 - 75{,}365}{2009 - 1990} - \frac{11{,}798}{19}$.

The average annual increase in enrollment was about 621 students/year.

(b) 1995: $75{,}365 + 5(621) = 78{,}470$ students
2000: $75{,}365 + 10(621) = 81{,}575$ students
2005: $75{,}365 + 15(621) = 84{,}680$ students

(c) Using $m = 621$ and letting $x = 0$ correspond to 1990,

$y - 75{,}365 = 621(x - 0)$

$y = 621x + 75{,}365$

The slope is 621 and it determines the average increase in enrollment per year from 1990 to 2009.

98. Answers will vary. Sample answer: Slope is the rate of change over an interval; average rate of change is the slope of the line passing through the first and last points of a plot.

99. False. The slopes are different:

$\frac{4 - 2}{-1 + 8} = \frac{2}{7}$

$\frac{7 + 4}{-7 - 0} = -\frac{11}{7}$

100. False.

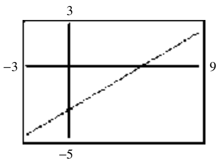
The equation of the line joining $(10, -3)$ and $(2, -9)$ is

$$y+3=\frac{-9+3}{2-10}(x-10)$$
$$y+3=\frac{3}{4}(x-10)$$
$$y=\frac{3}{4}x-\frac{21}{2}.$$

For $x=-12$, $y=\frac{3}{4}(-12)-\frac{21}{2}$
$$=-19.5$$
$$\neq\frac{-37}{2}$$
$$=-18.5$$

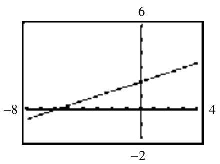
101. $\dfrac{x}{5}+\dfrac{y}{-3}=1$
$$-3x+5y+15=0$$

a and b are the x- and y- intercepts.



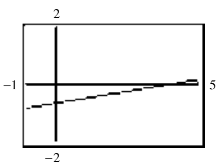
102. $\dfrac{x}{-6}+\dfrac{y}{2}=1$
$$y=2\left(1+\frac{x}{6}\right)$$
$$y=\frac{x}{3}+2$$

a and b are the x- and y-intercepts.



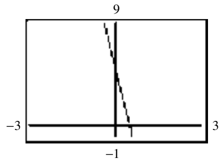
103. $\dfrac{x}{4}+\dfrac{y}{-\frac{2}{3}}=1$
$$-\frac{2}{3}x+4y=\frac{-8}{3}$$
$$-2x+12y=-8$$

a and b are the x- and y-intercepts.



104. $\dfrac{x}{\frac{1}{2}}+\dfrac{y}{5}=1$
$$5x+\frac{1}{2}y=\frac{5}{2}$$
$$10x+y=5$$

a and b are the x- and y-intercepts.



105. $\dfrac{x}{2}+\dfrac{y}{3}=1$
$$3x+2y-6=0$$

106. $\dfrac{x}{a}+\dfrac{y}{b}=1$
$$\frac{x}{-5}+\frac{y}{-4}=1$$
$$4x+5y+20=0$$

107. $\dfrac{x}{-1/6}+\dfrac{y}{-2/3}=1$
$$-6x-\frac{3}{2}y=1$$
$$12x+3y+2=0$$

108. $\dfrac{x}{a}+\dfrac{y}{b}=1$
$$\frac{x}{3/4}+\frac{y}{4/5}=1$$
$$\frac{4}{5}x+\frac{3}{4}y=\frac{3}{5}$$
$$16x+15y-12=0$$

109. The slope is positive and the y-intercept is positive. Matches (a).

110. The slope is negative and the y-intercept is negative. Matches (b).

111. Both lines have positive slope, but their y-intercepts differ in sign. Matches (c).

112. The lines intersect in the first quadrant at a point (x, y) where $x < y$. Matches (a).

113. No. The line $y = 2$ does not have and x-intercept.

114. No. $x = 1$ cannot be written in slope-intercept form because the slope is undefined.

115. Yes. Once a parallel line is established to the given line, there are an infinite number of distances away from that line, and thus an infinite number of parallel lines.

116.
(a) The slope is $m = -10$. This represents the decrease in the amount of the loan each week. Matches graph (ii).
(b) The y-intercept is 12.5 and the slope is 1.5, which represents the increase in hourly wage per unit produced. Matches graph (iii).
(c) The slope is $m = 0.35$. This represents the increase in travel cost for each mile driven. Matches graph (i).
(d) The y-intercept is 600 and the slope is -100, which represents the decrease in the value of the word processor each year. Matches graph (iv).

117. Yes. $x + 20$

118. Yes. $3x - 10x^2 + 1 = -10x^2 + 3x + 1$

119. No. The term $x^{-1} = \frac{1}{x}$ causes the expression to not be a polynomial.

120. Yes. $2x^2 - 2x^4 - x^3 + 2 = -2x^4 - x^3 + 2x^2 + 2$

121. No. This expression is not defined for $x = \pm 3$.

122. No.

123. $x^2 - 6x - 27 = (x - 9)(x + 3)$

124. $x^2 - 11x + 28 = (x - 4)(x - 7)$

125. $2x^2 + 11x - 40 = (2x - 5)(x + 8)$

126. $3x^2 - 16x + 5 = (3x - 1)(x - 5)$

127. Answers will vary.

Section 1.2

1. domain, range, function

2. independent, dependent

3. No. The input element $x = 3$ cannot be assigned to more than exactly one output element.

4. To find $g(x+1)$ for $g(x) = 3x - 2$, substitute x with the quantity $x + 1$.

$$\begin{aligned} g(x+1) &= 3(x+1) - 2 \\ &= 3x + 3 - 2 \\ &= 3x + 1 \end{aligned}$$

5. No. The domain of the function $f(x) = \sqrt{1+x}$ is $[-1, \infty)$ which does not include $x = -2$.

6. The domain of a piece-wise function must be explicitly described, so that it can determine which equation is used to evaluate the function.

7. Yes, it does represent a function. Each domain value is matched with only one range value.

8. No, it is not a function. The domain value of -1 is matched with two output values.

9. No, it does not represent a function. The domain values are each matched with three range values.

10. Yes. Each element, or state, in the domain is assigned to exactly one element, or electoral votes, in the range.

11. Yes, the relation represents y as a function of x. Each domain value is matched with only one range value.

12. No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.

13. (a) Each element of A is matched with exactly one element of B, so it does represent a function.
(b) The element 1 in A is matched with two elements, -2 and 1 of B, so it does not represent a function.
(c) Each element of A is matched with exactly one element of B, so it does represent a function.

14. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.
(b) Each element of A is matched with exactly one element of B, so it does represent a function.
(c) This is not a function from A to B (it represents a function from B to A instead).

15. Both are functions. For each year there is exactly one and only one average price of a name brand prescription and average price of a generic prescription.

16. Since $b(t)$ represents the average price of a name brand prescription, $b(2007) \approx \$119.50$.

Since $g(t)$ represents the average price of a generic prescription, $g(2000) \approx \$19.00$.

17. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

Thus, y *is not* a function of x. For instance, the values $y = 2$ and $y = -2$ both correspond to $x = 0$.

18. $x = y^2 + 1$

$y = \pm\sqrt{x - 1}$

This *is not* a function of x. For example, the values $y = 2$ and $y = -2$ both correspond to $x = 5$.

19. $y = \sqrt{x^2 - 1}$

This *is* a function of x.

20. $y = \sqrt{x + 5}$

This *is* a function of x.

21. $2x+3y=4 \Rightarrow y=\frac{1}{3}(4-2x)$

Thus, *y is* a function of *x*.

22. $x=-y+5 \Rightarrow y=-x+5.$

This *is* a function of *x*.

23. $y^2=x^2-1 \Rightarrow y=\pm\sqrt{x^2-1}$

Thus, *y is not* a function of *x*. For instance, the values $y=\sqrt{3}$ and $y=-\sqrt{3}$ both correspond to $x=2$.

24. $x+y^2=3 \Rightarrow y=\pm\sqrt{3-x}$

Thus, *y is not* a function of *x*.

25. $y=|4-x|$

This *is* a functions of *x*.

26. $|y|=4-x \Rightarrow y=4-x$ or $y=-(4-x)$

Thus, *y is not* a function of *x*.

27. $x=-7$ *does not* represent *y* as a function of *x*. All values of *y* correspond to $x=-7$.

28. $y=8$ *is* a function of *x*, a constant function.

29. $f(t)=3t+1$

(a) $f(2)=3(2)+1=7$
(b) $f(-4)=3(-4)+1=-11$
(c) $f(t+2)=3(t+2)+1=3t+7$

30. $g(y)=7-3y$

(a) $g(0)=7-3(0)=7$

(b) $g\left(\frac{7}{3}\right)=7-3\left(\frac{7}{3}\right)=0$

(c) $g(s+2)=7-3(s+2)$
$=7-3s-6=1-3s$

31. $h(t)=t^2-2t$

(a) $h(2)=2^2-2(2)=0$
(b) $h(1.5)=(1.5)^2-2(1.5)=-0.75$
(c) $h(x+2)=(x+2)^2-2(x+2)=x^2+2x$

32. $V(r)=\frac{4}{3}\pi r^3$

(a) $V(3)=\frac{4}{3}\pi(3)^3=36\pi$

(b) $V\left(\frac{3}{2}\right)=\frac{4}{3}\pi\left(\frac{3}{2}\right)^3=\frac{4}{3}\cdot\frac{27}{8}\pi=\frac{9\pi}{2}$

(c) $V(2r)=\frac{4}{3}\pi(2r)^3=\frac{32\pi r^3}{3}$

33. $f(y)=3-\sqrt{y}$

(a) $f(4)=3-\sqrt{4}=1$

(b) $f(0.25)=3-\sqrt{0.25}=2.5$

(c) $f(4x^2)=3-\sqrt{4x^2}=3-2|x|$

34. $f(x)=\sqrt{x+8}+2$

(a) $f(-4)=\sqrt{-4+8}+2=4$

(b) $f(8)=\sqrt{8+8}+2=6$

(c) $f(x-8)=\sqrt{x-8+8}+2=\sqrt{x}+2$

35. $q(x)=\frac{1}{x^2-9}$

(a) $q(-3)=\frac{1}{(-3)^2-9}=\frac{1}{9-9}=\frac{1}{0}$ undefined

(b) $q(2)=\frac{1}{(2)^2-9}=\frac{1}{4-9}=-\frac{1}{5}$

(c) $q(y+3)=\frac{1}{(y+3)^2-9}=\frac{1}{y^2+6y+9-9}=\frac{1}{y^2+6y}$

36. $q(t)=\frac{2t^2+3}{t^2}$

(a) $q(2)=\frac{2(2)^2+3}{(2)^2}=\frac{8+3}{4}=\frac{11}{4}$

(b) $q(0)=\frac{2(0)^2+3}{(0)^2}$ Division by zero is undefined.

(c) $q(-x)=\frac{2(-x)^2+3}{(-x)^2}=\frac{2x^2+3}{x^2}$

37. $f(x)=\frac{|x|}{x}$

(a) $f(9)=\frac{|9|}{9}=1$

(b) $f(-9)=\frac{|-9|}{-9}=-1$

(c) $f(t)=\frac{|t|}{t}=\begin{cases}1, & t>0\\ -1, & t<0\end{cases}$

$f(0)$ is a undefined.

38. $f(x)=|x|+4$

(a) $f(5)=|5|+4=9$

(b) $f(-5)=|-5|+4=9$

(c) $f(t)=|t|+4$

39. $f(x)=\begin{cases}2x+1, & x<0\\ 2x+2, & x\ge 0\end{cases}$

(a) $f(-1)=2(-1)+1=-1$
(b) $f(0)=2(0)+2=2$
(c) $f(2)=2(2)+2=6$

40. $f(x) = \begin{cases} 2x+5, & x \le 0 \\ 2-x^2, & x > 0 \end{cases}$

(a) $f(-2) = 2(-2)+5 = 1$

(b) $f(0) = 2(0)+5 = 5$

(c) $f(1) = 2-1^2 = 1$

41. $f(x) = \begin{cases} x^2+2, & x \le 1 \\ 2x^2+2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2+2 = 6$

(b) $f(1) = (1)^2+2 = 3$

(c) $f(2) = 2(2)^2+2 = 10$

42. $f(x) = \begin{cases} x^2-4, & x \le 0 \\ 1-2x^2, & x > 0 \end{cases}$

(a) $f(-2) = (-2)^2-4 = 4-4 = 0$

(b) $f(0) = 0^2-4 = -4$

(c) $f(1) = 1-2(1^2) = 1-2 = -1$

43. $f(x) = \begin{cases} x+2, & x < 0 \\ 4, & 0 \le x < 2 \\ x^2+1, & x \ge 2 \end{cases}$

(a) $f(-2) = (-2)+2 = 0$

(b) $f(1) = 4$

(c) $f(4) = 4^2+1 = 17$

44. $f(x) = \begin{cases} 5-2x, & x < 0 \\ 5, & 0 \le x < 1 \\ 4x+1, & x \ge 1 \end{cases}$

(a) $f(-2) = 5-2(-2) = 9$

(b) $f\left(\frac{1}{2}\right) = 5$

(c) $f(1) = 4(1)+1 = 5$

45. $f(x) = x^2$

$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

46. $f(x) = x^2 - 3$

$\{(-2, 1), (-1, -2), (0, -3), (1, -2), (2, 1)\}$

47. $f(x) = |x| + 2$

$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$

48. $f(x) = |x+1|$

$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$

49. $h(t) = \frac{1}{2}|t+3|$

$h(-5) = \frac{1}{2}|-5+3| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$

$h(-4) = \frac{1}{2}|-4+3| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$

$h(-3) = \frac{1}{2}|-3+3| = \frac{1}{2}|0| = 0$

$h(-2) = \frac{1}{2}|-2+3| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$

$h(-1) = \frac{1}{2}|-1+3| = \frac{1}{2}|2| = \frac{1}{2}(2) = 1$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

50. $f(s) = \dfrac{|s-2|}{s-2}$

$f(0) = \dfrac{|0-2|}{0-2} = \dfrac{2}{-2} = -1$

$f(1) = \dfrac{|1-2|}{1-2} = \dfrac{1}{-1} = -1$

$f\left(\frac{3}{2}\right) = \dfrac{\left|\frac{3}{2}-2\right|}{\frac{3}{2}-2} = \dfrac{\frac{1}{2}}{-\frac{1}{2}} = -1$

$f\left(\frac{5}{2}\right) = \dfrac{\left|\frac{5}{2}-2\right|}{\frac{5}{2}-2} = \dfrac{\frac{1}{2}}{\frac{1}{2}} = 1$

$f(4) = \dfrac{|4-2|}{4-2} = \dfrac{2}{2} = 1$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	-1	-1	-1	1	1

51. $f(x) = 15 - 3x = 0$

$3x = 15$

$x = 5$

52. $f(x) = 5x + 1 = 0$

$5x = -1$

$x = -\frac{1}{5}$

53. $f(x) = \dfrac{3x-4}{5} = 0$

$3x - 4 = 0$

$3x = 4$

$x = \frac{4}{3}$

54. $f(x) = \frac{2x-3}{7} = 0$

$$2x - 3 = 0$$
$$2x = 3$$
$$x = \frac{3}{2}$$

55. $f(x) = 5x^2 + 2x - 1$

Since $f(x)$ is a polynomial, the domain is all real numbers *x*.

56. $g(x) = 1 - 2x^2$

Because $g(x)$ is a polynomial, the domain is all real numbers *x*.

57. $h(t) = \frac{4}{t}$

Domain: All real numbers except $t = 0$

58. $s(y) = \frac{3y}{y+5}$

$$y + 5 \neq 0$$
$$y \neq -5$$

The domain is all real numbers $y \neq -5$.

59. $f(x) = \sqrt[3]{x-4}$

Domain: all real numbers *x*

60. $f(x) = \sqrt[4]{x^2 + 3x}$

$x^2 + 3x = x(x+3) \geq 0$

Domain: $x \leq -3$ or $x \geq 0$

61. $g(x) = \frac{1}{x} - \frac{3}{x+2}$

Domain: All real numbers except $x = 0,\ x = -2$

62. $h(x) = \frac{10}{x^2 - 2x}$

$$x^2 - 2x \neq 0$$
$$x(x-2) \neq 0$$

The domain is all real numbers except $x = 0, x = 2$.

63. $g(y) = \frac{y+2}{\sqrt{y-10}}$

$$y - 10 > 0$$
$$y > 10$$

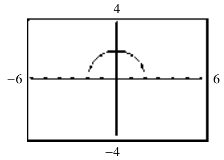
Domain: all $y > 10$

64. $f(x) = \frac{\sqrt{x+6}}{6+x}$

$x + 6 \geq 0$ for numerator, and $x \neq -6$ for denominator.

Domain: all $x > -6$

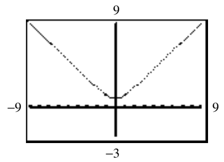
65. $f(x) = \sqrt{4 - x^2}$



Domain: $[-2,\ 2]$

Range: $[0,\ 2]$

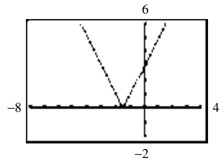
66. $f(x) = \sqrt{x^2 + 1}$



Domain: all real numbers

Range: $1 \leq y$

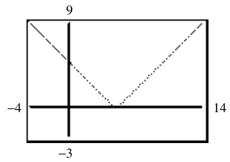
67. $g(x) = |2x + 3|$



Domain: $(-\infty,\ \infty)$

Range: $[0,\ \infty)$

68. $g(x) = |x - 5|$



Domain: all real numbers

Range: $y \geq 0$

69. $A = \pi r^2,\ C = 2\pi r$

$$r = \frac{C}{2\pi}$$
$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

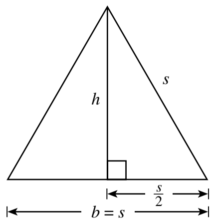
70. $A = \frac{1}{2}bh$, in an equilateral triangle $b = s$ and:

$$s^2 = h^2 + \left(\frac{s}{2}\right)^2$$

$$h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$$

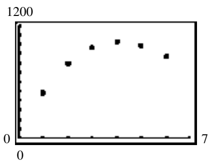
$$h = \sqrt{\frac{4s^2}{4} - \frac{s^2}{4}} = \frac{\sqrt{3}s}{2}$$

$$A = \frac{1}{2}s \cdot \frac{\sqrt{3}s}{2} = \frac{\sqrt{3}s^2}{4}$$



71. (a) From the table, the maximum volume seems to be 1024 cm^3, corresponding to $x = 4$.

(b)

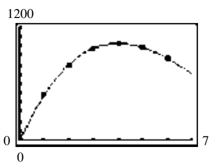


Yes, V is a function of x.

(c) $V = \text{length} \times \text{width} \times \text{height}$
$= (24 - 2x)(24 - 2x)x$
$= x(24 - 2x)^2 = 4x(12 - x)^2$

Domain: $0 < x < 12$

(d)



The function is a good fit. Answers will vary.

72. $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}xy.$

Since $(0, y)$, $(2, 1)$ and $(x, 0)$ all lie on the same line, the slopes between any pair of points are equal.

$$\frac{1-y}{2-0} = \frac{1-0}{2-x}$$

$$1 - y = \frac{2}{2-x}$$

$$y = 1 - \frac{2}{2-x} = \frac{x}{x-2}$$

Therefore, $A = \frac{1}{2}xy = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2x-4}.$

The domain is $x > 2$, since $A > 0$.

73. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$, $0 < x < 6$.

74. (a) $V = (\text{length})(\text{width})(\text{height}) = yx^2$

But, $y + 4x = 108$, or $y = 108 - 4x$.

Thus, $V = (108 - 4x)x^2$.

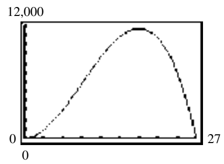
Since $y = 108 - 4x > 0$

$4x < 108$

$x < 27.$

Domain: $0 < x < 27$

(b)



(c) The highest point on the graph occurs at $x = 18$. The dimensions that maximize the volume are $18 \times 18 \times 36$ inches.

75. (a) Total Cost = Variable Costs + Fixed Costs

$$C = 68.20x + 248{,}000$$

(b) Revenue = Selling price × units sold
$R = 98.98x$

(c) Since $P = R - C$
$P = 98.98x - (68.20x + 248{,}000)$
$P = 30.78x - 248{,}000.$

76. (a) The independent variable is x and represents the month. The dependent variable is y and represents the monthly revenue.

(b) $f(x) = \begin{cases} -1.97x + 26.3, & 7 \le x \le 12 \\ 0.505x^2 - 1.47x + 6.3, & 1 \le x \le 6 \end{cases}$

Answers will vary.

(c) $f(5) = 11.575$, and represents the revenue in May: \$11,575.

(d) $f(11) = 4.63$, and represents the revenue in November: \$4630.

(e) The values obtained from the model are close approximations to the actual data.

77. (a) The independent variable is t and represents the year. The dependent variable is n and represents the numbers of miles traveled.

(b)

t	0	1	2	3	4	5	6
$n(t)$	581	645.26	699.04	742.34	775.16	797.5	809.36

t	7	8	9	10	11	12	13
$n(t)$	843.9	869.6	895.3	921	946.7	972.4	998.1

t	14	15	16	17
$n(t)$	1023.8	1049.5	1075.2	1100.9

(c) The model fits the data well.

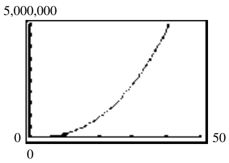
78. (a) $F(y) = 149.76\sqrt{10}y^{5/2}$

y	5	10	20	30	40
$F(y)$	26,474	149,760	847,170	2,334,527	4,792,320

(Answers will vary.)

F increases very rapidly as y increases.

(b)



$X_{min} = 0$
$X_{max} = 50$
$x_{scl} = 10$
$Y_{min} = 0$
$Y_{max} = 5{,}000{,}000$
$Y_{scl} = 500{,}000$

(c) From the table, $y \approx 22$ ft (slightly above 20). You could obtain a better approximation by completing the table for values of y between 20 and 30.

(d) By graphing $F(y)$ together with the horizontal line $y_2 = 1{,}000{,}000$, you obtain $y \approx 21.37$ feet.

79. No. If $x = 60$, $y = -0.004(60)^2 + 0.3(60) + 6$
$y = 9.6$ feet

Since the first baseman can only jump to a height of 8 feet, the throw will go over his head.

80. (a) $\dfrac{f(2008) - f(2000)}{2008 - 2000} \approx \$25 \text{ million/year}$

This represents the average increase in sales per year from 2000 to 2008.

(b)

t	0	1	2	3	4
$S(t)$	84	92.2	105.4	123.5	146.6

t	5	6	7	8
$S(t)$	174.7	207.7	245.7	288.7

The model approximates the data well.

81. $f(x) = 2x$

$$\frac{f(x+c) - f(x)}{c} = \frac{2(x+c) - 2x}{c} = \frac{2c}{c} = 2,\ c \neq 0$$

82. $g(x) = 3x - 1$

$$g(x+h) = 3(x+h) - 1 = 3x + 3h - 1$$
$$g(x+h) - g(x) = (3x + 3h - 1) - (3x - 1) = 3h$$
$$\frac{g(x+h) - g(x)}{h} = \frac{3h}{h} = 3,\ h \neq 0$$

83. $f(x) = x^2 - x + 1$, $f(2) = 3$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 - (2+h) + 1 - 3}{h} = \frac{4 + 4h + h^2 - 2 - h + 1 - 3}{h} = \frac{h^2 + 3h}{h} = h + 3,\ h \neq 0$$

84. $f(x) = x^3 + x$

$$f(x+h) = (x+h)^3 + (x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h$$
$$f(x+h) - f(x) = (x^3 + 3x^2h + 3xh^2 + h^3 + x + h) - (x^3 + x) = 3x^2h + 3xh^2 + h^3 + h = h(3x^2 + 3xh + h^2 + 1)$$
$$\frac{f(x+h) - f(x)}{h} = \frac{h(3x^2 + 3xh + h^2 + 1)}{h} = 3x^2 + 3xh + h^2 + 1,\ h \neq 0$$

85. $f(t) = \dfrac{1}{t}$, $f(1) = 1$

$$\frac{f(t) - f(1)}{t - 1} = \frac{\frac{1}{t} - 1}{t - 1} = \frac{1 - t}{t(t-1)} = -\frac{1}{t},\ t \neq 1$$

86. $f(x) = \dfrac{4}{x+1}$

$$f(7) = \frac{4}{7+1} = \frac{1}{2}$$
$$\frac{f(x) - f(7)}{x - 7} = \frac{\frac{4}{x+1} - \frac{1}{2}}{x - 7} = \frac{8 - (x+1)}{2(x+1)(x-7)} = \frac{7 - x}{2(x+1)(x-7)} = -\frac{1}{2(x+1)}, x \neq 7$$

87. False. The range of $f(x)$ is $(-1, \infty)$.

88. True. The first number in each ordered pair corresponds to exactly one second number.

89. $f(x) = \sqrt{x} + 2$

Domain: $[0, \infty)$ or $x \geq 0$

Range: $[2, \infty)$ or $y \geq 2$

90. $f(x) = \sqrt{x + 3}$

Domain: $[-3, \infty)$ or $x \geq -3$

Range: $[0, \infty)$ or $y \geq 0$

91. No. f is not the independent variable. Because the value of f depends on the value of x, x is the independent variable and f is the dependent variable.

92. (a) A relation is two quantities that are related to each other by some rule of correspondence. A function is a relation that matches each item from one set with exactly one item from a different set.

(b) The domain is the set of input values of a function. The range is the set of output values.

93. $12 - \dfrac{4}{x+2} = \dfrac{12(x+2)-4}{x+2} = \dfrac{12x+20}{x+2}$

94. $\dfrac{3}{x^2+x-20} + \dfrac{x}{x^2+4x-5} = \dfrac{3}{(x+5)(x-4)} + \dfrac{x}{(x+5)(x-1)}$

$= \dfrac{3(x-1)}{(x+5)(x-4)(x-1)} + \dfrac{x(x-4)}{(x+5)(x-1)(x-4)}$

$= \dfrac{3x-3+x^2-4x}{(x+5)(x-4)(x-1)} = \dfrac{x^2-x-3}{(x+5)(x-4)(x-1)}$

95. $\dfrac{2x^3+11x^2-6x}{5x} \cdot \dfrac{x+10}{2x^2+5x-3} = \dfrac{x(2x^2+11x-6)(x+10)}{5x(2x-1)(x+3)}$

$= \dfrac{(2x-1)(x+6)(x+10)}{5(2x-1)(x+3)}$

$= \dfrac{(x+6)(x+10)}{5(x+3)},\ x \neq 0,\ \dfrac{1}{2}$

96. $\dfrac{x+7}{2(x-9)} \div \dfrac{x-7}{2(x-9)} = \dfrac{x+7}{2(x-9)} \cdot \dfrac{2(x-9)}{x-7} = \dfrac{x+7}{x-7},\ x \neq 9$

Section 1.3

1. decreasing

2. even

3. Domain: $1 \le x \le 4$ or $[1, 4]$

4. No. If a vertical line intersects the graph more than once, then it does not represent y as a function of x.

5. If $f(2) \ge f(2)$ for all x in $(0, 3)$, then $(2, f(2))$ is a relative maximum of f.

6. Since $f(x) = [\![x]\!] = n$, where n is an integer and $n \le x$, the input value of x needs to be greater than or equal to 5 but less than 6 in order to produce an output value of 5. So the interval $[5, 6)$ would yield a function value of 5.

7. Domain: all real numbers, $(-\infty, \infty)$

Range: $(-\infty, 1]$

$f(0) = 1$

8. Domain: all real numbers, $(-\infty, \infty)$

Range: all real numbers, $(-\infty, \infty)$

$f(0) = 2$

9. Domain: $[-4, 4]$

Range: $[0, 4]$

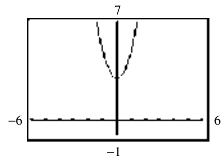
$f(0) = 4$

10. Domain: all real numbers, $(-\infty, \infty)$

Range: $[-3, \infty)$

$f(0) = -3$

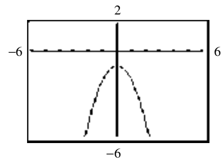
11. $f(x) = 2x^2 + 3$



Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

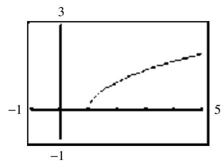
12. $f(x) = -x^2 - 1$



Domain: $(-\infty, \infty)$

Range: $(-\infty, -1]$

13. $f(x) = \sqrt{x-1}$

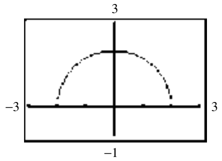


Domain: $x - 1 \ge 0 \Rightarrow x \ge 1$ or $[1, \infty)$

Range: $[0, \infty)$

14. $h(t) = \sqrt{4 - t^2}$

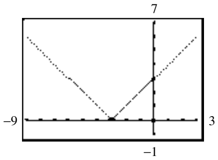
$4 - t^2 \geq 0 \Rightarrow t^2 \leq 4$



Domain: $[-2, 2]$

Range: $[0, 2]$

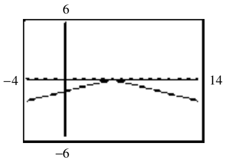
15. $f(x) = |x + 3|$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

16. $f(x) = -\frac{1}{4}|x - 5|$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

17. (a) Domain: $(-\infty, \infty)$

(b) Range: $[-2, \infty)$

(c) $f(x) = 0$ at $x = -1$ and $x = 3$.

(d) The values of $x = -1$ and $x = 3$ are the x-intercepts of the graph of *f*.

(e) $f(0) = -1$

(f) The value of $y = -1$ is the y-intercept of the graph of *f*.

(g) The value of *f* at $x = 1$ is $f(1) = -2$.

The coordinates of the point are $(1, -2)$.

(h) The value of *f* at $x = -1$ is $f(-1) = 0$.

The coordinates of the point are $(-1, 0)$.

(i) The coordinates of the point are $(-3, f(-3))$ or $(-3, 2)$.

18. (a) Domain: $(-\infty, \infty)$

(b) Range: $(-\infty, 4]$

(c) $f(x) = 0$ at $x = -4$ and $x = 2$.

(d) The values of $x = -4$ and $x = 2$ are the x-intercepts of the graph of *f*.

(e) $f(0) = 4$

(f) The value of $y = 4$ is the y-intercept of the graph of *f*.

(g) The value of *f* at $x = 1$ is $f(1) = 3$.

The coordinates of the point are $(1, 3)$.

(h) The value of f at $x = -1$ is $f(-1) = 3$.

The coordinates of the point are $(-1, 3)$.

(i) The coordinates of the point are $(-3, f(-3))$ or $(-3, 1)$.

19. $y = \frac{1}{2}x^2$

A vertical line intersects the graph just once, so *y* is a function of x. Graph $y_1 = \frac{1}{2}x^2$.

20. $x - y^2 = 1 \Rightarrow y = \pm\sqrt{x - 1}$

y is not a function of *x*. The vertical line $x = 2$ intersects the graph twice. Graph $y_1 = \sqrt{x - 1}$ and $y_2 = -\sqrt{x - 1}$.

21. $x^2 + y^2 = 25$

A vertical line intersects the graph more than once, so *y* is not a function of *x*. Graph the circle as $y_1 = \sqrt{25 - x^2}$ and $y_2 = -\sqrt{25 - x^2}$.

22. $x^2 = 2xy - 1$

A vertical line intersects the graph just once, so *y* is a function of *x*. Solve for *y* and graph $y_1 = \frac{x^2 + 1}{2x}$.

23. $f(x) = \frac{3}{2}x$

f is increasing on $(-\infty, \infty)$.

24. $f(x) = x^2 - 4x$

The graph is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

25. $f(x) = x^3 - 3x^2 + 2$

f is increasing on $(-\infty, 0)$ and $(2, \infty)$.

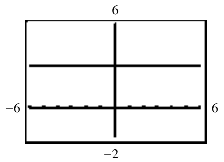
f is decreasing on $(0, 2)$.

26. $f(x) = \sqrt{x^2 - 1}$

The graph is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

27. $f(x) = 3$

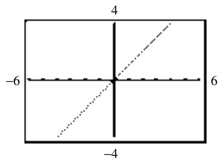
(a)



(b) f is constant on $(-\infty, \infty)$.

28. $f(x) = x$

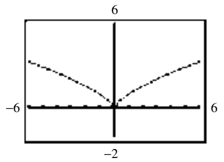
(a)



(b) Increasing on $(-\infty, \infty)$

29. $f(x) = x^{2/3}$

(a)

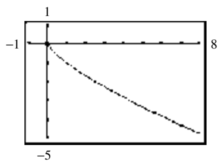


(b) Increasing on $(0, \infty)$

Decreasing on $(-\infty, 0)$

30. $f(x) = -x^{3/4}$

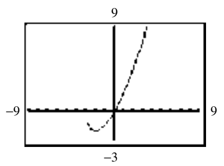
(a)



(b) Decreasing on $(0, \infty)$

31. $f(x) = x\sqrt{x+3}$

(a)

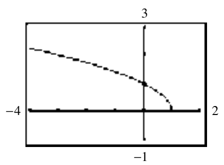


(b) Increasing on $(-2, \infty)$

Decreasing on $(-3, -2)$

32. $f(x) = \sqrt{1-x}$

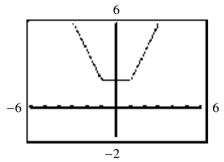
(a)



(b) Decreasing on $(-\infty, 1)$

33. $f(x) = |x+1| + |x-1|$

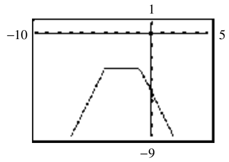
(a)



(b) Increasing on $(1, \infty)$, constant on $(-1, 1)$, decreasing on $(-\infty, -1)$

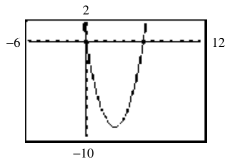
34. $f(x) = -|x+4| - |x+1|$

(a)



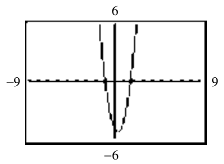
(b) Increasing on $(-\infty, -4)$, constant on $(-4, -1)$, decreasing on $(-1, \infty)$

35. $f(x) = x^2 - 6x$



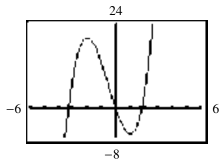
Relative minimum: $(3, -9)$

36. $f(x) = 3x^2 - 2x - 5$



Relative minimum: $(0.33, -5.33)$

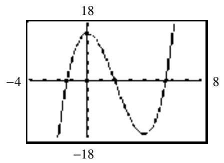
37. $y = 2x^3 + 3x^2 - 12x$



Relative minimum: $(1, -7)$

Relative maximum: $(-2, 20)$

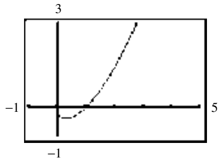
38. $y = x^3 - 6x^2 + 15$



Relative minimum: $(4, -17)$

Relative maximum: $(0, 15)$

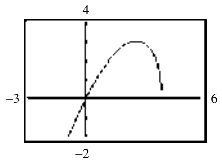
39. $h(x) = (x-1)\sqrt{x}$



Relative minimum: $(0.33, -0.38)$

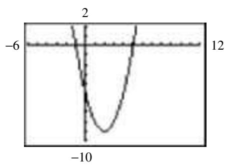
$(0, 0)$ is not a relative maximum because it occurs at the endpoint of the domain $[0, \infty)$.

40. $g(x) = x\sqrt{4-x}$



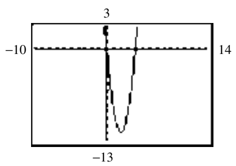
Relative maximum: $(2.67, 3.08)$

41. $f(x) = x^2 - 4x - 5$



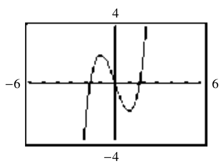
Relative minimum: $(2, -9)$

42. $f(x) = 3x^2 - 12x$



Relative minimum: $(2, -12)$

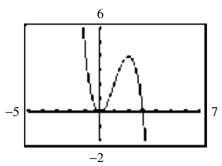
43. $f(x) = x^3 - 3x$



Relative minimum: $(1, -2)$

Relative maximum: $(-1, 2)$

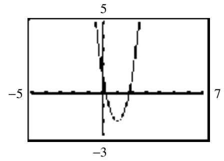
44. $f(x) = -x^3 + 3x^2$



Relative minimum: $(0, 0)$

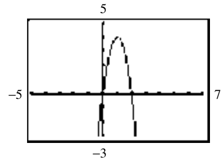
Relative maximum: $(2, 4)$

45. $f(x) = 3x^2 - 6x + 1$



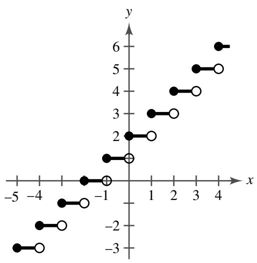
Relative minimum: $(1, -2)$

46. $f(x) = 8x - 4x^2$

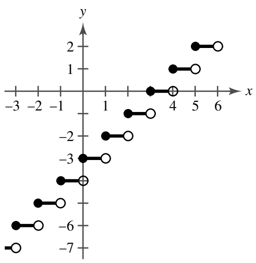


Relative maximum: $(1, 4)$

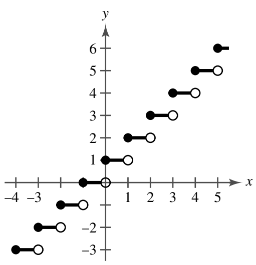
47. $f(x) = [\![x]\!] + 2$



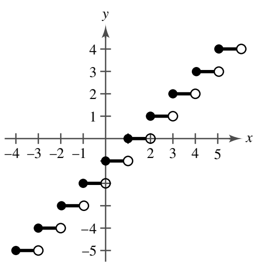
48. $f(x) = [\![x]\!] - 3$



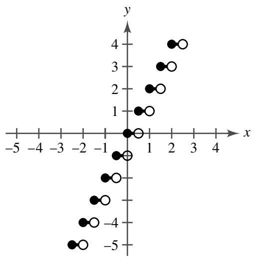
49. $f(x) = [\![x-1]\!] + 2$



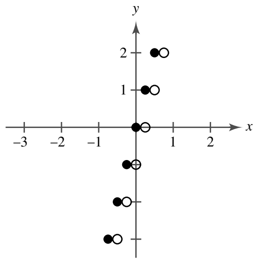
50. $f(x) = [\![x-2]\!] + 1$



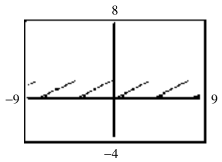
51. $f(x) = [\![2x]\!]$



52. $f(x) = [\![4x]\!]$



53. $s(x) = 2\left(\frac{1}{4}x - \left[\!\left[\frac{1}{4}x\right]\!\right]\right)$

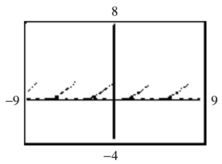


Domain: $(-\infty, \infty)$

Range: $[0, 2)$

Sawtooth pattern

54. $g(x) = 2\left(\frac{1}{4}x - \left[\!\left[\frac{1}{4}x\right]\!\right]\right)^2$

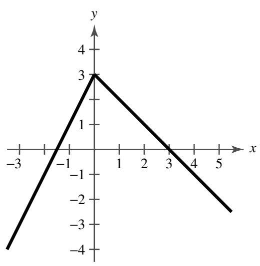


Domain: $(-\infty, \infty)$

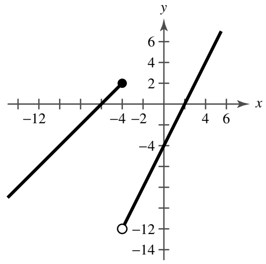
Range: $[0, 2)$

Sawtooth pattern

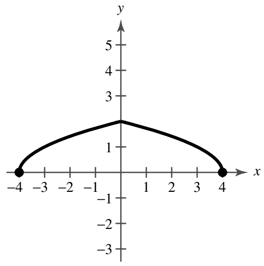
55. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$



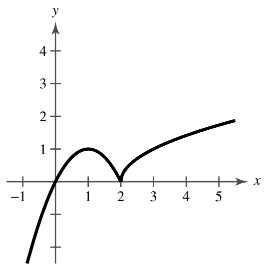
56. $f(x) = \begin{cases} x + 6, & x \leq -4 \\ 2x - 4, & x > -4 \end{cases}$



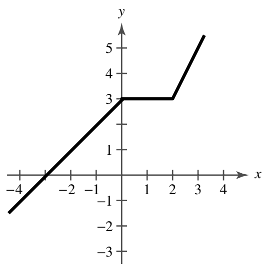
57. $f(x) = \begin{cases} \sqrt{x + 4}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$



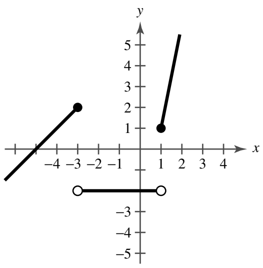
58. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$



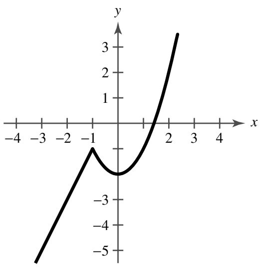
59. $f(x) = \begin{cases} x + 3, & x \leq 0 \\ 3, & 0 < x \leq 2 \\ 2x - 1, & x > 2 \end{cases}$



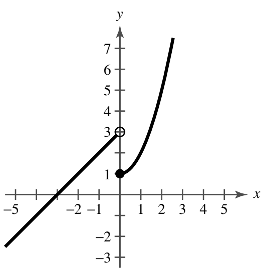
60. $g(x) = \begin{cases} x+5, & x \le -3 \\ -2, & -3 < x < 1 \\ 5x-4, & x \ge 1 \end{cases}$



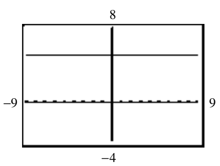
61. $f(x) = \begin{cases} 2x+1, & x \le -1 \\ x^2-2, & x > -1 \end{cases}$



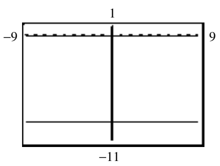
62. $h(x) = \begin{cases} 3+x, & x < 0 \\ x^2+1, & x \ge 0 \end{cases}$



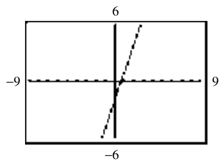
63. $f(x) = 5$ is even.



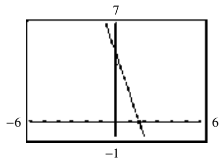
64. $f(x) = -9$ is even.



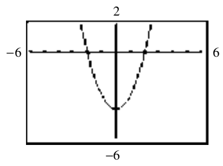
65. $f(x) = 3x - 2$ is neither even nor odd.



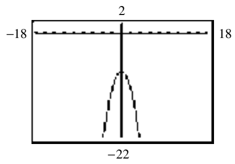
66. $f(x) = 5 - 3x$ is neither even nor odd.



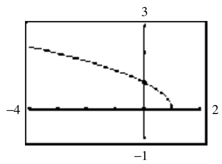
67. $h(x) = x^2 - 4$ is even.



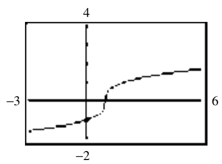
68. $f(x) = -x^2 - 8$ is even.



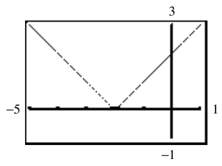
69. $f(x) = \sqrt{1-x}$ is neither even nor odd.



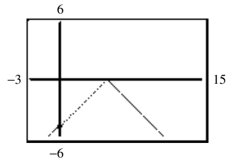
70. $g(t) = \sqrt[3]{t-1}$ is neither even nor odd.



71. $f(x) = |x+2|$ is neither even nor odd.



72. $f(x) = -|x - 5|$ is neither even nor odd.



73. $\left(-\frac{3}{2},\ 4\right)$

(a) If f is even, another point is $\left(\frac{3}{2},\ 4\right)$.

(b) If f is odd, another point is $\left(\frac{3}{2},\ -4\right)$.

74. $\left(-\frac{5}{3},\ -7\right)$

(a) If f is even, another point is $\left(\frac{5}{3},\ -7\right)$.

(b) If f is odd, another point is $\left(\frac{5}{3},\ 7\right)$.

75. $(4,\ 9)$

(a) If f is even, another point is $(-4,\ 9)$.

(b) If f is odd, another point is $(-4,\ -9)$.

76. $(5,\ -1)$

(a) If f is even, another point is $(-5,\ -1)$.

(b) If f is odd, another point is $(-5,\ 1)$.

77. $(x,\ -y)$

(a) If f is even, another point is $(-x,\ -y)$.

(b) If f is odd, another point is $(-x,\ y)$.

78. $(2a,\ 2c)$

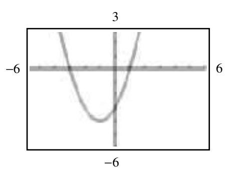
(a) If f is even, another point is $(-2a,\ 2c)$

(b) If f is odd, another point is $(-2a,\ -2c)$.

79. (a)
$$\begin{aligned} f(-t) &= (-t)^2 + 2(-t) - 3 \\ &= t^2 - 2t - 3 \\ &\neq f(t) \neq -f(t) \end{aligned}$$

f is neither even nor odd.

(b)



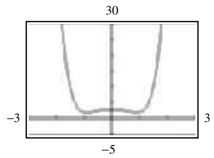
The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, f is neither even nor odd.

(c) Tables will vary. f is neither even nor odd.

80. (a)
$$\begin{aligned} f(-x) &= (-x)^6 - 2(-x)^2 + 3 \\ &= x^6 - 2x^2 + 3 \\ &= f(x) \end{aligned}$$

f is even.

(b)



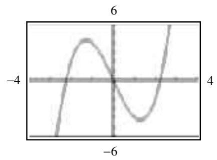
The graph is symmetric with respect to the y-axis. So, f is even.

(c) Tables will vary. f is even

81. (a)
$$\begin{aligned} g(-x) &= (-x)^3 - 5(-x) \\ &= -x^3 + 5x \\ &= -(x^3 - 5x) \\ &= -g(x) \end{aligned}$$

g is odd.

(b)



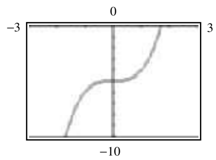
The graph is symmetric with respect to the origin. So, g is odd.

(c) Tables will vary. g is odd.

82. (a)
$$\begin{aligned} h(-x) &= (-x)^3 - 5 \\ &= -x^3 - 5 \\ &\neq h(x) \neq -h(x) \end{aligned}$$

h is neither even nor odd.

(b)



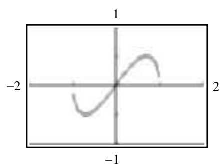
The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, h is neither even nor odd.

(c) Tables will vary. h is neither even nor odd.

83. (a)
$$\begin{aligned} f(-x) &= (-x)\sqrt{1 - (-x)^2} \\ &= -x\sqrt{1 - x^2} \\ &= -f(x) \end{aligned}$$

f is odd.

(b)



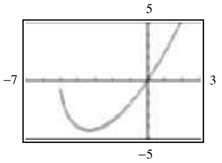
The graph is symmetric with respect to the orign. So, f is odd.

(c) Tables will vary. f is odd.

84. (a) $f(-x) = (-x)\sqrt{(-x)+5}$
$= -x\sqrt{-x+5}$
$\neq f(x) \neq -f(x)$

f is neither even nor odd.

(b)



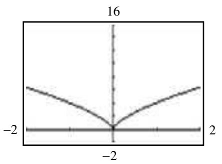
The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, f is neither even nor odd.

(c) Tables will vary. f is neither even nor odd.

85. (a) $g(-s) = 4(-s)^{\frac{2}{3}}$
$= 4\left(\sqrt[3]{-s}\right)^2$
$= 4s^2$
$= g(s)$

g is even.

(b)



The graph is symmetric with respect to the y-axis. So, g is even.

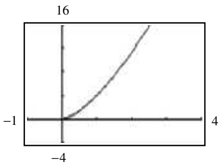
(c) Tables will vary. g is even.

86. (a) $g(-s) = 4(-s)^{\frac{3}{2}}$

g is not defined for $s < 0$.

g is neither even not odd.

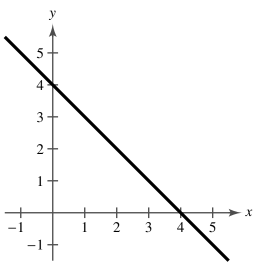
(b)



The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, g is neither even nor odd.

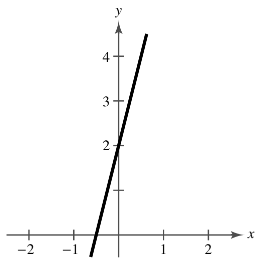
(c) Tables will vary. g is neither even nor odd.

87. $f(x) = 4 - x \geq 0$



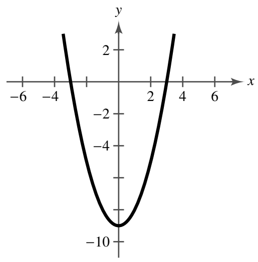
$f(x) \geq 0$
$4 - x \geq 0$
$4 \geq x$
$(-\infty, 4]$

88. $f(x) = 4x + 2$



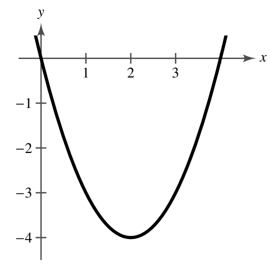
$f(x) \geq 0$
$4x + 2 \geq 0$
$4x \geq -2$
$x \geq -\dfrac{1}{2}$
$[-\frac{1}{2}, \infty)$

89. $f(x) = x^2 - 9 \geq 0$



$f(x) \geq 0$
$x^2 - 9 \geq 0$
$x^2 \geq 9$
$x \geq 3$ or $x \leq -3$
$(-\infty, -3], [3, \infty)$

90. $f(x) = x^2 - 4x$

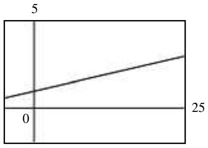


$f(x) \geq 0$
$x^2 - 4x \geq 0$
$x(x-4) \geq 0$
$(-\infty, 0], [4, \infty)$

91. (a) C_2 is the appropriate model.

The cost of the first minute is $1.05 and the cost increases $0.08 when the next minute begins, and so on.

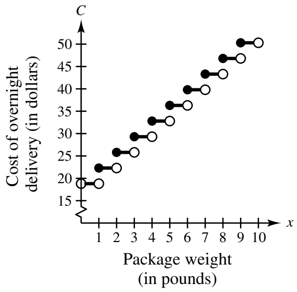
(b)



The cost of an 18-minute, 45-second call is

$$C_2(18.75) = 1.05 - 0.08[\![-(18.75-1)]\!]$$
$$= \$2.49.$$

92. $C = 18.80 + 3.50[\![x]\!],\ x > 0$



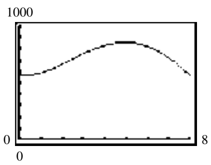
93. h = top − bottom

$$= (-x^2 + 4x - 1) - 2$$
$$= -x^2 + 4x - 3,\ 1 \le x \le 3$$

94. h = top − bottom

$$= 3 - (4x - x^2)$$
$$= 3 - 4x + x^2,\ 0 \le x \le 1$$

95. (a)



(b) The number of cooperative homes and condos was increasing from 2000 to 2005, and decreasing from 2005 to 2008.

(c) The maximum number of cooperative homes and condos was approximately 855 in 2005.

96.

Interval	*Intake Pipe*	*Drain Pipe 1*	*Drain Pipe 2*
[0, 5]	Open	Closed	Closed
[5, 10]	Open	Open	Closed
[10, 20]	Closed	Closed	Closed
[20, 30]	Closed	Closed	Open
[30, 40]	Open	Open	Open
[40, 45]	Open	Closed	Open
[45, 50]	Open	Open	Open
[50, 60]	Open	Open	Closed

97. False. The domain of $f(x) = \sqrt{x^2}$ is the set of all real numbers.

98. False. The domain must be symmetric about the y-axis.

99. c

100. d

101. b

102. e

103. a

104. f

105. Yes, $x = y^2 + 1$ defines x as a function of y. Any horizontal line can be drawn without intersecting the graph more than once.

106. No, $x^2 + y^2 = 25$ does not represent x as a function of y. For instance, $(-3, 4)$ and $(3, 4)$ both lie on the graph.

107. Yes, $f(x) = \begin{cases} \vdots & \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ \vdots & \end{cases}$

108. The graph of the greatest integer function is a series of steps with a closed circle at the left end and an open circle at the right end. The graph of a line with a slope of zero is one continuous horizontal line with no steps.

109. f is an even function.

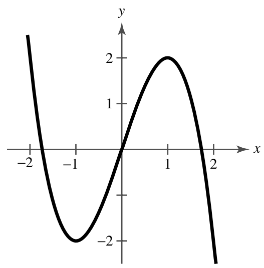
(a) $g(x) = -f(x)$ is even because $g(-x) = -f(-x) = -f(x) = g(x)$.

(b) $g(x) = f(-x)$ is even because $g(-x) = f(-(-x)) = f(x) = f(-x) = g(x)$.

(c) $g(x) = f(x) - 2$ is even because $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

(d) $g(x) = -f(x-2)$ is neither even nor odd because $g(-x) = -f(-x-2) = -f(x+2) \ne g(x)$ nor $-g(x)$.

110. (a)



(b) Using the graph from part (a), the domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

(c) Using the graph from part (a), you can see that the graph is increasing on $-1 < x < 1$, and the graph is decreasing on $x < -1$ and $x > 1$.

(d) Using the graph from part (a), there is a relative minimum at $(-1, -2)$ and a relative maximum at $(1, 2)$.

111. $f(x) = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$

$$f(-x) = a_{2n+1}(-x)^{2n+1} + a_{2n-1}(-x)^{2n-1} + \cdots + a_3(-x)^3 + a_1(-x)$$
$$= -a_{2n+1}x^{2n+1} - a_{2n-1}x^{2n-1} - \cdots - a_3x^3 - a_1x = -f(x)$$

Therefore, $f(x)$ is odd.

112. $f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$$
$$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x)$$

$f(-x) = f(x)$; thus, $f(x)$ is even.

113. $-2x^2 + 8x$

Terms: $-2x^2, 8x$

Coefficients: $-2, 8$

114. $10 + 3x$

Terms: $3x, 10$

Coefficient: 3

115. $\frac{x}{3} - 5x^2 + x^3$

Terms: $\frac{x}{3}, -5x^2, x^3$

Coefficients: $\frac{1}{3}, -5, 1$

116. $7x^4 + \sqrt{2}x^2$

Terms: $7x^4, \sqrt{2}x^2$

Coefficients: $7, \sqrt{2}$

117. $f(x) = -x^2 - x + 3$

(a) $f(4) = -(4)^2 - 4 + 3 = -17$

(b) $f(-2) = -(-2)^2 - (-2) + 3 = 1$

(c) $f(x-2) = -(x-2)^2 - (x-2) + 3$
$$= -(x^2 - 4x + 4) - x + 2 + 3$$
$$= -x^2 + 3x + 1$$

118. $f(x) = x\sqrt{x-3}$

(a) $f(3) = 3\sqrt{3-3} = 0$

(b) $f(12) = 12\sqrt{12-3}$
$$= 12\sqrt{9} = 12(3) = 36$$

(c) $f(6) = 6\sqrt{6-3} = 6\sqrt{3}$

119. $f(x) = x^2 - 2x + 9$

$$f(3+h) = (3+h)^2 - 2(3+h) + 9 = 9 + 6h + h^2 - 6 - 2h + 9$$
$$= h^2 + 4h + 12$$
$$f(3) = 3^2 - 2(3) + 9 = 12$$
$$\frac{f(3+h) - f(3)}{h} = \frac{(h^2 + 4h + 12) - 12}{h} = \frac{h(h+4)}{h} = h + 4, h \neq 0$$

120. $f(x) = 5 + 6x - x^2$

$$f(6+h) = 5 + 6(6+h) - (6+h)^2$$
$$= 5 + 36 + 6h - (36 + 12h + h^2)$$
$$= -h^2 - 6h + 5$$
$$f(6) = 5 + 6(6) - 6^2 = 5$$
$$\frac{f(h+6) - f(6)}{h} = \frac{(-h^2 - 6h + 5) - 5}{h} = \frac{h(-h-6)}{h}$$
$$= -h - 6, h \neq 0$$

Section 1.4

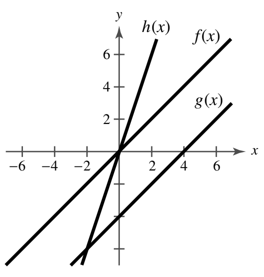
1. Horizontal shifts, vertical shifts, and reflections are rigid transformations.

2. (a) ii
(b) iv
(c) iii
(d) i

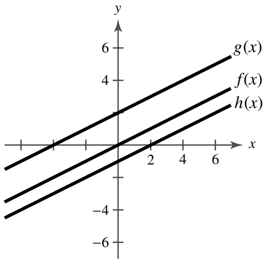
3. $-f(x), f(-x)$

4. $c > 1, 0 < c < 1$

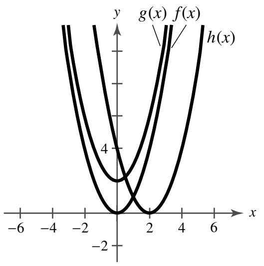
5.



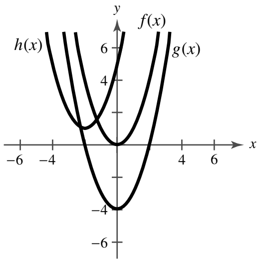
6.



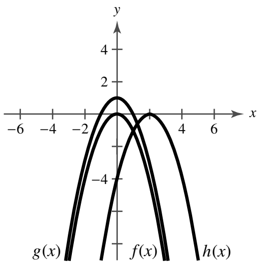
7.



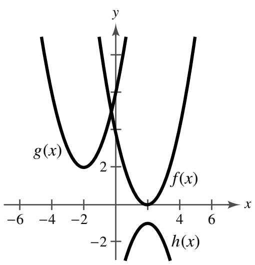
8.



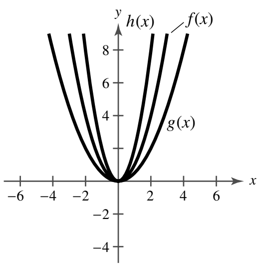
9.



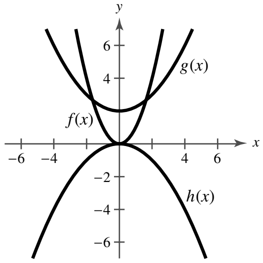
10.



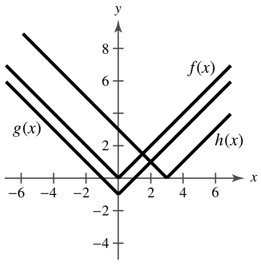
11.



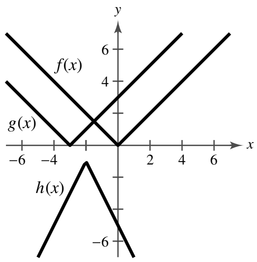
12.



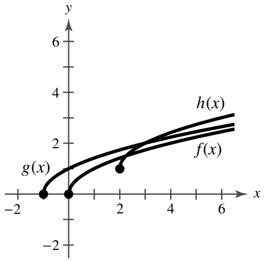
13.



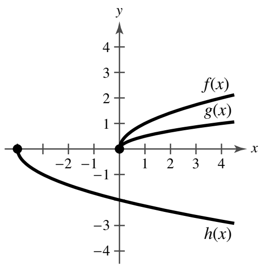
14.



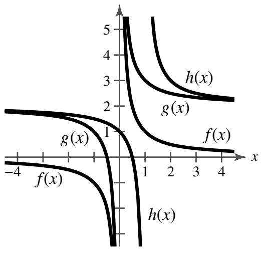
15.



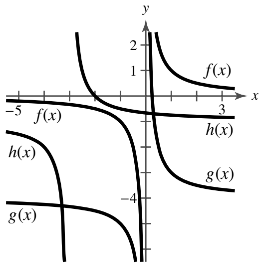
16.



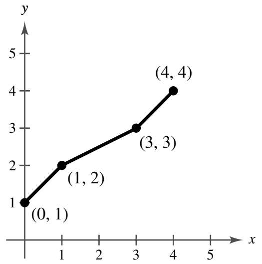
17.



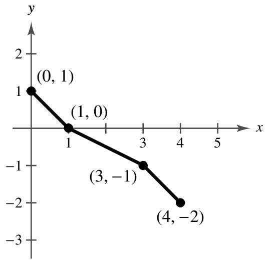
18.



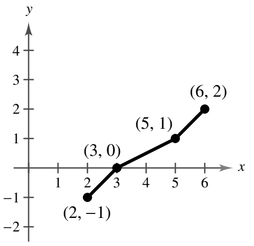
19. (a) $y = f(x) + 2$



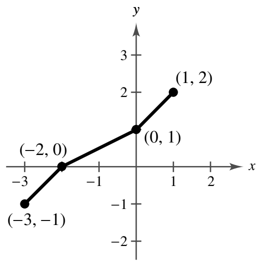
(b) $y = -f(x)$



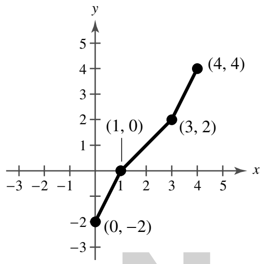
(c) $y = f(x-2)$



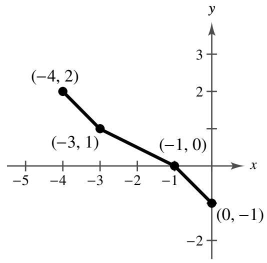
(d) $y = f(x+3)$



(e) $y = 2f(x)$

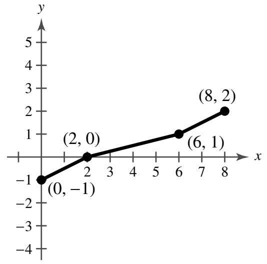


(f) $y = f(-x)$

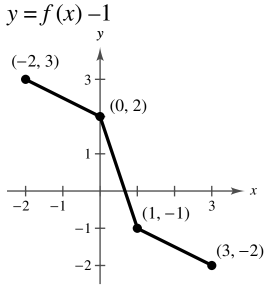


(g) Let $g(x) = f\left(\frac{1}{2}x\right)$. Then from the graph,

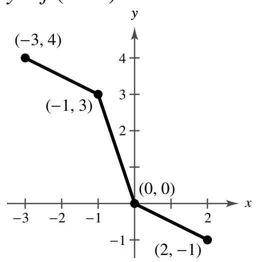
$$g(0) = f\left(\tfrac{1}{2}(0)\right) = f(0) = -1$$
$$g(2) = f\left(\tfrac{1}{2}(2)\right) = f(1) = 0$$
$$g(6) = f\left(\tfrac{1}{2}(6)\right) = f(3) = 1$$
$$g(8) = f\left(\tfrac{1}{2}(8)\right) = f(4) = 2$$



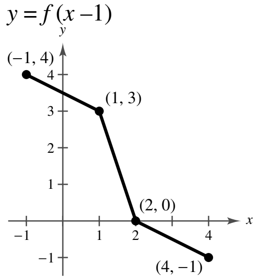
20. (a) $y = f(x) - 1$



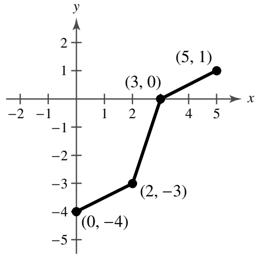
(b) $y = f(x+1)$



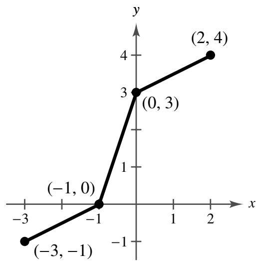
(c) $y = f(x-1)$



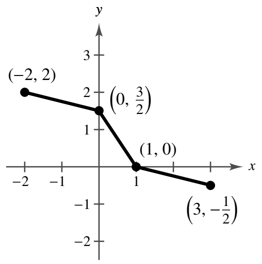
(d)



(e)

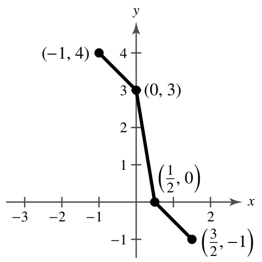


(f)



(g) Let $g(x) = f(2x)$. Then from the graph,

$$g(-1) = f(2(-1)) = f(-2) = 4$$
$$g(0) = f(2(0)) = f(0) = 3$$
$$g\left(\tfrac{1}{2}\right) = f\left(2\left(\tfrac{1}{2}\right)\right) = f(1) = 0$$
$$g\left(\tfrac{3}{2}\right) = f\left(2\left(\tfrac{3}{2}\right)\right) = f(3) = -1.$$



21. The graph of $f(x) = x^2$ should have been shifted one unit to the left instead of one unit to the right.

22. The graph of $f(x) = x^2$ should have been shifted one unit to the right instead of one unit downward.

23. $y = \sqrt{x} + 2$ is $f(x) = \sqrt{x}$ shifted vertically upward two units.

24. $y = \dfrac{1}{x} - 5$ is $f(x) = \dfrac{1}{x}$ shifted vertically five units downward.

25. $y = (x-4)^3$ is $f(x) = x^3$ shifted horizontally four units to the right.

26. $y = |x+5|$ is $f(x) = |x|$ shifted horizontally five units to the left.

27. $y = x^2 - 2$ is $f(x) = x^2$ shifted vertically two units downward.

28. $y = \sqrt{x-2}$ is $f(x) = \sqrt{x}$ shifted horizontally two units to the right.

29. Horizontal shift three units to left of $y = x$: $y = x + 3$ (or vertical shift three units upward)

30. Horizontal shift two units to the left of $y = \dfrac{1}{x}$: $y = \dfrac{1}{x+2}$

31. Vertical shift one unit downward of $y = x^2$: $y = x^2 - 1$

32. Horizontal shift two units to the left of $y = |x|$: $y = |x+2|$

33. Reflection in the x-axis and a vertical shift one unit upward of $y = \sqrt{x}$: $y = 1 - \sqrt{x}$

34. Reflection in the x-axis and a vertical shift one unit upward of $y = x^3$: $y = 1 - x^3$

35. $y = -|x|$ is $f(x)$ reflected in the x-axis.

36. $y = |-x|$ is a reflection in the y-axis. In fact $y = |-x| = |x|$, therefore $y = |-x|$ is identical to $y = |x|$.

37. $y = (-x)^2$ is a reflection in the y-axis. In fact, $y = (-x)^2 = x^2$, therefore $y = (-x)^2$ is identical to $y = x^2$.

38. $y = -x^3$ is a reflection of $f(x) = x^3$ in the x-axis. However, since $y = -x^3 = (-x)^3$, either a reflection in the x-axis or a reflection in the y-axis produces the same graph.

39. $y = \dfrac{1}{-x}$ is a reflection of $f(x) = \dfrac{1}{x}$ in the y-axis.

However, since $y = \dfrac{1}{-x} = -\dfrac{1}{x}$, either a reflection in the y-axis or a reflection in the x-axis produces the same graph.

40. $y = -\dfrac{1}{x}$ is a reflection of $f(x) = \dfrac{1}{x}$ in the x-axis.

However, since $y = -\dfrac{1}{x} = \dfrac{1}{-x}$, either a reflection in the x-axis or a reflection in the y-axis produces the same graph.

41. $y = 4|x|$ is a vertical stretch of $f(x) = |x|$.

42. $p(x) = \dfrac{1}{2}x^2$ is a vertical shrink of $f(x) = \dfrac{x-1}{4}$.

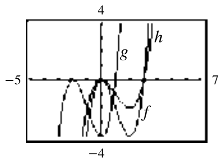
43. $g(x) = \dfrac{1}{4}x^3$ is a vertical shrink of $f(x) = x^3$.

44. $y = 2\sqrt{x}$ is a vertical stretch of $f(x) = \sqrt{x}$.

45. $f(x) = \sqrt{4x}$ is a horizontal shrink of $f(x) = \sqrt{x}$. However, since $f(x) = \sqrt{4x} = 2\sqrt{x}$, it also can be described as a vertical stretch of $f(x) = \sqrt{x}$.

46. $y = \left|\frac{1}{2}x\right| = \frac{1}{2}|x|$ is a vertical shrink of $f(x) = |x|$.

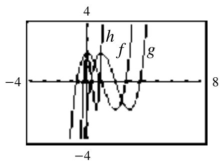
47. $f(x) = x^3 - 3x^2$



$g(x) = f(x+2) = (x+2)^3 - 3(x+2)^2$ is a horizontal shift two units to the left.

$h(x) = \frac{1}{2}f(x) = \frac{1}{2}(x^3 - 3x^2)$ is a vertical shrink.

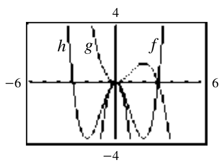
48. $f(x) = x^3 - 3x^2 + 2$



$g(x) = f(x-1) = (x-1)^3 - 3(x-1)^2 + 2$ is a horizontal shift one unit to the right.

$h(x) = f(3x) = (3x)^3 - 3(3x)^2 + 2$ is a horizontal shrink.

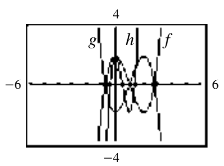
49. $f(x) = x^3 - 3x^2$



$g(x) = -\frac{1}{3}f(x) = -\frac{1}{3}(x^3 - 3x^2)$ is a reflection in the x-axis and a vertical shrink.

$h(x) = f(-x) = (-x)^3 - 3(-x)^2$ is a reflection in the y-axis.

50. $f(x) = x^3 - 3x^2 + 2$



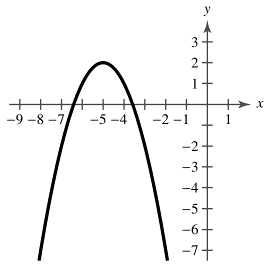
$g(x) = -f(x) = -(x^3 - 3x^2 + 2)$ is a reflection in the x-axis.

$h(x) = f(2x) = (2x)^3 - 3(2x)^2 + 2$ is a horizontal shrink.

51. (a) $f(x) = x^2$

(b) $g(x) = 2 - (x+5)^2$ is obtained from *f* by a horizontal shift to the left five units, a reflection in the x-axis, and a vertical shift upward two units.

(c)

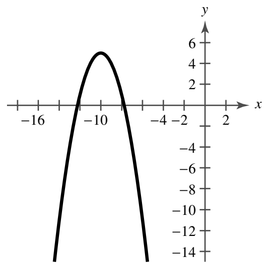


(d) $g(x) = 2 - f(x+5)$

52. (a) $f(x) = x^2$

(b) $g(x) = -(x+10)^2 + 5$ is obtained from *f* by a horizontal shift 10 units to the left, a reflection in the x-axis, and a vertical shift 5 units upward.

(c)

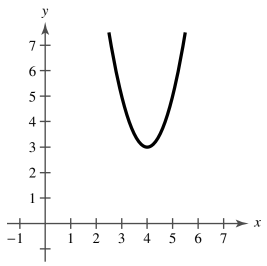


(d) $g(x) = -f(x+10) + 5$

53. (a) $f(x) = x^2$

(b) $g(x) = 3 + 2(x-4)^2$ is obtained from *f* by a horizontal shift four units to the right, a vertical stretch of 2, and a vertical shift upward three units.

(c)

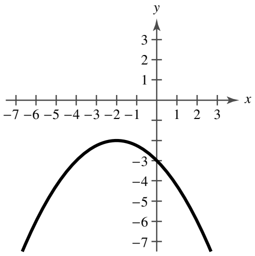


(d) $g(x) = 3 + 2f(x-4)$

54. (a) $f(x) = x^2$

(b) $g(x) = -\frac{1}{4}(x+2)^2 - 2$ is obtained from *f* by a horizontal shift two units to the left, a vertical shrink of $\frac{1}{4}$, a reflection in the x-axis, and a vertical shift two units downward.

(c)

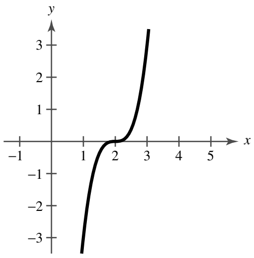


(d) $g(x) = -\dfrac{1}{4}f(x+2) - 2$

55. (a) $f(x) = x^3$

(b) $g(x) = 3(x-2)^3$ is obtained from f by a horizontal shift two units to the right followed by a vertical stretch of 3.

(c)

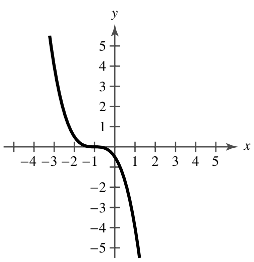


(d) $g(x) = 3f(x-2)$

56. (a) $f(x) = x^3$

(b) $g(x) = -\dfrac{1}{2}(x+1)^3$ is obtained from f by a horizontal shift one unit to the left, a vertical shrink of $\dfrac{1}{2}$, and a reflection in the x-axis.

(c)

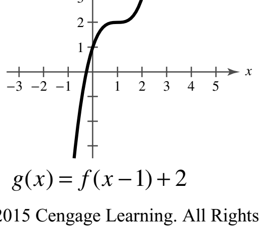


(d) $g(x) = -\dfrac{1}{2}f(x+1)$

57. (a) $f(x) = x^3$

(b) $g(x) = (x-1)^3 + 2$ is obtained from f by a horizontal shift one unit to the right and a vertical shift upward two units.

(c)

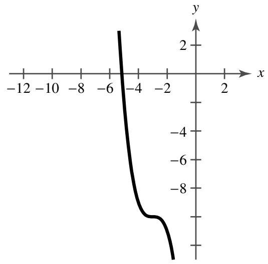


(d) $g(x) = f(x-1) + 2$

58. (a) $f(x) = x^3$

(b) $g(x) = -(x+3)^3 - 10$ is obtained from f by a horizontal shift 3 units to the left, a reflection in the x-axis, and a vertical shift 10 units downward.

(c)

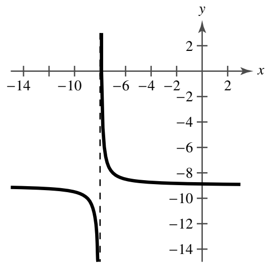


(d) $g(x) = -f(x+3) - 10$

59. (a) $f(x) = \dfrac{1}{x}$

(b) $g(x) = \dfrac{1}{x+8} - 9$ is obtained from f by a horizontal shift eight units to the left and a vertical shift nine units downward.

(c)

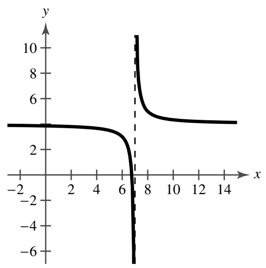


(d) $g(x) = f(x+8) - 9$

60. (a) $f(x) = \dfrac{1}{x}$

(b) $g(x) = \dfrac{1}{x-7} + 4$ is obtained from f by a horizontal shift seven units to the right and a vertical shift four units upward.

(c)

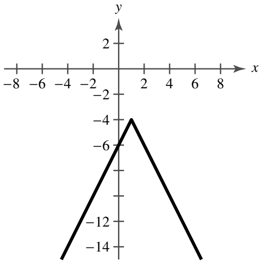


(d) $g(x) = f(x-7) + 4$

61. (a) $f(x) = |x|$

(b) $g(x) = -2|x-1| - 4$ is obtained from f by a horizontal shift one unit to the right, a vertical stretch of 2, a reflection in the x-axis, and a vertical shift downward four units.

(c)

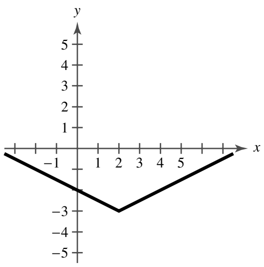


(d) $g(x) = -2f(x-1) - 4$

62. (a) $f(x) = |x|$

(b) $g(x) = \frac{1}{2}|x-2| - 3$ is obtained from *f* by a horizontal shift two units to the right, a vertical shrink, and a vertical shift three units downward.

(c)

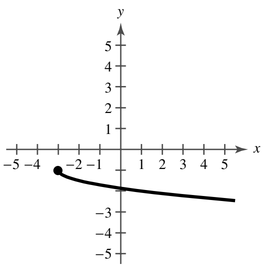


(d) $g(x) = -\frac{1}{2}f(x-2) - 3$

63. (a) $f(x) = \sqrt{x}$

(b) $g(x) = -\frac{1}{2}\sqrt{x+3} - 1$ is obtained from *f* by a horizontal shift three units to the left, a vertical shrink, a reflection in the *x*-axis, and a vertical shift one unit downward.

(c)

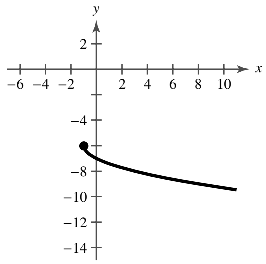


(d) $g(x) = -\frac{1}{2}f(x+3) - 1$

64. (a) $f(x) = \sqrt{x}$

(b) $g(x) = -\sqrt{x+1} - 6$ is obtained from *f* by a horizontal shift one unit to the left, a reflection in the *x*-axis, and a vertical shift six units downward.

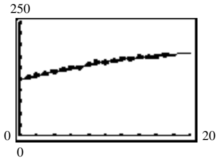
(c)



(d) $g(x) = -f(x+1) - 6$

65. (a) $f(t)$ is a horizontal shift of 24.7 units to the right, a vertical shift of 183.4 units upward, a reflection in the *t*-axis (horizontal axis), and a vertical shrink.

(b)



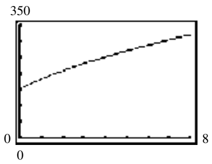
(c) $G(t) = F(t+10) = -0.099[(t+10) - 24.7]^2 + 183.4$

$= -0.099(t-14.7)^2 + 183.4$

To make a horizontal shift 10 years backward (10 units left), add 10 to *t*.

66. (a) $S(t)$ is a horizontal shift of 2.37 units to the left and a vertical stretch.

(b)



(c) Let $S(t) = 400$ and solve for *t*.

$$99\sqrt{t+2.37} = 400$$

$$\sqrt{t+2.37} = \frac{400}{99}$$

$$t + 2.37 = \frac{160{,}000}{9801}$$

$$t = \frac{160{,}000}{9801} - 2.37$$

$$t \approx 13.95 \text{ (or year 2014)}$$

In the year 2014, sales will be approximately $400 million.

(d) $G(t) = S(t+5) = 99\sqrt{(t+5)+2.37} = 99\sqrt{t+7.37}$

To make a horizontal shift 5 years backward (5 units left), add 5 to *t*.

67. False. $y = f(-x)$ is a reflection in the *y*-axis.

68. True. $y = |x| + 6$ and $y = |-x| + 6$ are identical because a reflection in the *y*-axis of $y = |x| + 6$ will be identical to itself. Additionally it is an even function.

69. $y = f(-x)$ is a reflection in the *y*-axis, so the *x*-intercepts are $x = -2$ and $x = 3$.

70. $y = 2f(x)$ is a vertical stretch, so the x-intercepts are the same: $x = 2, \ -3$.

71. $y = f(x) + 2$ is a vertical shift, so you cannot determine the x-intercepts.

72. $y = f(x-3)$ is a horizontal shift 3 units to the right, so the x-intercepts are $x = 5$ and $x = 0$.

73. The vertex is approximately at $(2, 1)$ and the graph opens upward. Matches (c).

74. The domain is $[0, -\infty)$ and $(0, -4)$ is approximately on the graph, and $f(x) < 0$. Matches (a) and (b).

75. The vertex is approximately $(2, -4)$ and the graph opens upward. Matches (c).

76. The graph of f is $y = x^3$ shifted to the left approximately four units, reflected in the x-axis, and shifted upward approximately two units. Matches (b) and (d).

77. Answers will vary.

78. Since $y = f(x+2) - 1$ is a horizontal shift of two units to the left and a vertical shift one unit downward, the point $(0, 1)$ will shift to $(-2, 0)$, $(1, 2)$ will shift to $(-1, 1)$, and $(2, 3)$ will shift to $(0, 2)$.

79. (a) Since $0 < a < 1$, $g(x) = ax^2$ will be a vertical shrink of $f(x) = x^2$.

(b) Since $a > 1$, $g(x) = ax^2$ will be a vertical stretch of $f(x) = x^2$.

80. (a) Since, $y = f(-x)$ is a reflection in the y-axis, it will be increasing on $(-\infty, -2)$ and decreasing on $(-2, \infty)$.

(b) Since, $y = -f(x)$ is a reflection in the x-axis, it will be increasing on $(2, \infty)$ and decreasing on $(-\infty, 2)$.

(c) Since $y = 2f(x)$ is a vertical stretch, it will remain increasing on $(-\infty, 2)$ and decreasing on $(2, \infty)$.

(d) Since $y = f(x) - 3$ is a vertical shift three unit downward, it will remain increasing on $(-\infty, 2)$ and decreasing on $(2, \infty)$.

(e) Since $y = f(x+1)$ is a horizontal shift one unit to the left, it will be increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$.

81. Slope $L_1: \dfrac{10+2}{2+2} = 3$

Slope $L_2: \dfrac{9-3}{3+1} = \dfrac{3}{2}$

Neither parallel nor perpendicular

82. Slope $L_1: \dfrac{3-(-7)}{4-(-1)} = \dfrac{10}{5} = 2$

Slope $L_2: \dfrac{-7-5}{-2-1} = \dfrac{-12}{-3} = 4$

Neither parallel nor perpendicular

83. Domain: All $x \neq 9$

84. $f(x) = \dfrac{\sqrt{x-5}}{x-7}$

Domain: $x \geq 5$ and $x \neq 7$

85. Domain:
$100 - x^2 \geq 0 \Rightarrow x^2 \leq 100 \Rightarrow -10 \leq x \leq 10$

86. $f(x) = \sqrt[3]{16 - x^2}$

Domain: all real numbers

Section 1.5

1. addition, subtraction, multiplication, division

2. composition

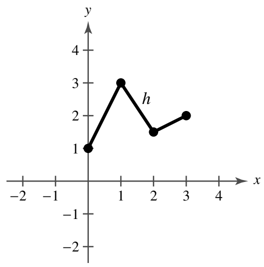
3. $g(x)$

4. inner, outer

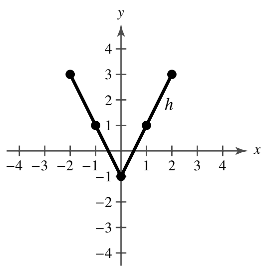
5. Since $(fg)(x) = 2x(x^2+1)$ and $f(x) = x^2 + 1$, $g(x) = 2x$, and $(fg)(x) = (gf)(x) = (2x)f(x)$.

6. Since $(f \circ g)(x) = f(g(x))$ and $(f \circ g)(x) = f(x^2+1)$, $g(x)$ must equal $x^2 + 1$.

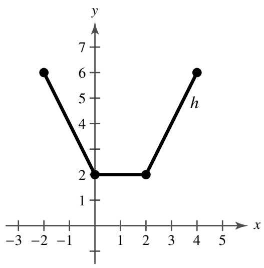
7.



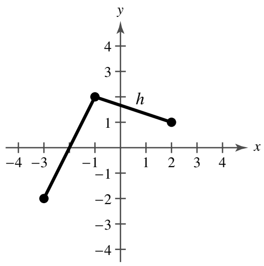
8.



9.



10.



11. $f(x) = x + 3,\ g(x) = x - 3$

(a) $(f + g)(x) = f(x) + g(x) = (x + 3) + (x - 3) = 2x$

(b) $(f - g)(x) = f(x) - g(x) = (x + 3) - (x - 3) = 6$

(c) $(fg)(x) = f(x)g(x) = (x + 3)(x - 3) = x^2 - 9$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x + 3}{x - 3}$

Domain: all $x \neq 3$

12. $f(x) = 2x - 5,\ g(x) = 1 - x$

(a) $(f + g)(x) = 2x - 5 + 1 - x = x - 4$

(b) $(f - g)(x) = 2x - 5 - (1 - x)$
$= 2x - 5 - 1 + x$
$= 3x - 6$

(c) $(fg)(x) = (2x - 5)(1 - x)$
$= 2x - 2x^2 - 5 + 5x$
$= -2x^2 + 7x - 5$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{1 - x}$

Domain: $1 - x \neq 0$
$x \neq 1$

13. $f(x) = x^2,\ g(x) = 1 - x$

(a) $(f + g)(x) = f(x) + g(x) = x^2 + (1 - x) = x^2 - x + 1$

(b) $(f - g)(x) = f(x) - g(x) = x^2 - (1 - x) = x^2 + x - 1$

(c) $(fg)(x) = f(x) \cdot g(x) = x^2(1 - x) = x^2 - x^3$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2}{1 - x}$

Domain: all $x \neq 1$

14. $f(x) = 2x - 5,\ g(x) = 5$

(a) $(f + g)(x) = 2x - 5 + 5 = 2x$

(b) $(f - g)(x) = 2x - 5 - 5 = 2x - 10$

(c) $(fg)(x) = (2x - 5)(5) = 10x - 25$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{5}$

Domain: $-\infty < x < \infty$

15. $f(x) = x^2 + 5,\ g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = x^2 + 5 + \sqrt{1 - x}$

(b) $(f - g)(x) = x^2 + 5 - \sqrt{1 - x}$

(c) $(fg)(x) = (x^2 + 5)\sqrt{1 - x}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + 5}{\sqrt{1 - x}}$

Domain: $x < 1$

16. $f(x) = \sqrt{x^2 - 4},\ g(x) = \dfrac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \dfrac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \dfrac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \left(\sqrt{x^2 - 4}\right)\left(\dfrac{x^2}{x^2 + 1}\right) = \dfrac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \dfrac{x^2}{x^2 + 1}$
$= \dfrac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

Domain: $x^2 - 4 \geq 0$ and $x \neq 0$
$x \geq 2$ or $x \leq -2$

17. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x^2}$

(a) $(f + g)(x) = \dfrac{1}{x} + \dfrac{1}{x^2} = \dfrac{x + 1}{x^2}$

(b) $(f - g)(x) = \dfrac{1}{x} - \dfrac{1}{x^2} = \dfrac{x - 1}{x^2}$

(c) $(fg)(x) = \dfrac{1}{x} \cdot \dfrac{1}{x^2} = \dfrac{1}{x^3}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\frac{1}{x}}{\frac{1}{x^2}} = x,\ x \neq 0$

Domain: $x \neq 0$

18. $f(x) = \dfrac{x}{x+1},\ g(x) = x^3$

(a) $(f+g)(x) = \dfrac{x}{x+1} + x^3 = \dfrac{x + x^4 + x^3}{x+1}$

(b) $(f-g)(x) = \dfrac{x}{x+1} - x^3 = \dfrac{x - x^4 - x^3}{x+1}$

(c) $(fg)(x) = \dfrac{x}{x+1} \cdot x^3 = \dfrac{x^4}{x+1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{x+1} \div x^3$

$= \dfrac{x}{x+1} \cdot \dfrac{1}{x^3} = \dfrac{1}{x^2(x+1)}$

Domain: $x \neq 0,\ x \neq -1$

19. $(f+g)(3) = f(3) + g(3)$
$= (3^2 - 1) + (3 - 2)$
$= 8 + 1 = 9$

20. $(f-g)(-2) = f(-2) - g(-2)$
$= ((-2)^2 - 1) - (-2 - 2)$
$= 3 - (-4) = 7$

21. $(f-g)(0) = f(0) - g(0)$
$= (0 - 1) - (0 - 2)$
$= 1$

22. $(f+g)(1) = f(1) + g(1)$
$= (1^2 - 1) + (1 - 2)$
$= 0 + (-1)$
$= -1$

23. $(fg)(6) = f(6)g(6)$
$= (6^2 - 1)(6 - 2)$
$= (35)(4)$
$= 140$

24. $(fg)(-4) = f(-4)g(-4)$
$= ((-4)^2 - 1)(-4 - 2)$
$= (15)(-6)$
$= -90$

25. $\left(\dfrac{f}{g}\right)(-5) = \dfrac{f(-5)}{g(-5)}$

$= \dfrac{(-5)^2 - 1}{-5 - 2}$

$= \dfrac{24}{-7}$

$= -\dfrac{24}{7}$

26. $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)}$

$= \dfrac{0 - 1}{0 - 2}$

$= \dfrac{1}{2}$

27. $(f-g)(2t) = f(2t) - g(2t)$
$= ((2t)^2 - 1) - (2t - 2)$
$= 4t^2 - 2t + 1$

28. $(f+g)(t-4) = f(t-4) + g(t-4)$
$= ((t-4)^2 - 1) + (t - 4 - 2)$
$= t^2 - 8t + 15 + t - 6$
$= t^2 - 7t + 9$

29. $(fg)(-5t) = f(-5t)g(-5t)$
$= ((-5t)^2 - 1)(-5t - 2)$
$= (25t^2 - 1)(-5t - 2)$
$= -125t^3 - 50t^2 + 5t + 2$

30. $(fg)(3t^2) = f(3t^2)g(3t^2)$
$= ((3t^2)^2 - 1)(3t^2 - 2)$
$= (9t^4 - 1)(3t^2 - 2)$
$= 27t^6 - 18t^4 - 3t^2 + 2$

31. $\left(\dfrac{f}{g}\right)(-t) = \dfrac{f(-t)}{g(-t)}$

$= \dfrac{(-t)^2 - 1}{-t - 2}$

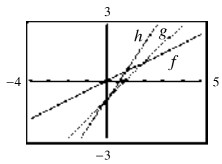
$= \dfrac{t^2 - 1}{-t - 2} = \dfrac{1 - t^2}{t + 2},\ t \neq -2$

32. $\left(\dfrac{f}{g}\right)(t+2) = \dfrac{f(t+2)}{g(t+2)}$

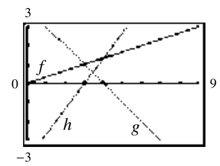
$= \dfrac{(t+2)^2 - 1}{(t+2) - 2}$

$= \dfrac{t^2 + 4t + 3}{t},\ t \neq 0$

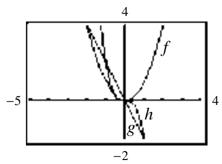
33.



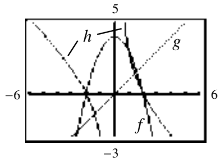
34.



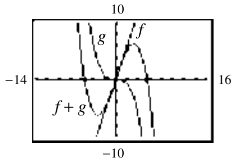
35.



36.



37. $f(x) = 3x,\ g(x) = -\dfrac{x^3}{10},\ (f+g)(x) = 3x - \dfrac{x^3}{10}$

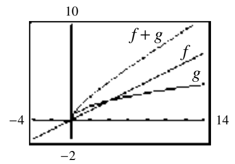


For $0 \le x \le 2,\ f(x)$ contributes more to the magnitude.

For $x > 6,\ g(x)$ contributes more to the magnitude.

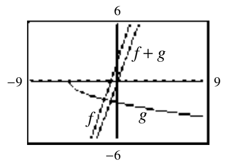
38. $f(x) = \dfrac{x}{2},\ g(x) = \sqrt{x},$

$(f+g)(x) = \dfrac{x}{2} + \sqrt{x}$



$g(x)$ contributes more to the magnitude of the sum for $0 \le x \le 2.$ $f(x)$ contributes more to the magnitude of the sum for $x > 6$.

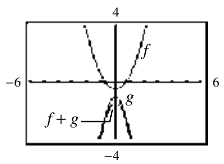
39. $f(x) = 3x + 2,\ g(x) = -\sqrt{x+5},$
$(f+g)(x) = 3x + 2 - \sqrt{x+5}$



$f(x)$ contributes more to the magnitude in both intervals.

40. $f(x) = x^2 - \dfrac{1}{2},\ g(x) = -3x^2 - 1,$

$(f+g)(x) = \left(x^2 - \dfrac{1}{2}\right) + (-3x^2 - 1) = -2x^2 - \dfrac{3}{2}$



$g(x)$ contributes more to the magnitude on both intervals.

41. $f(x) = x^2,\ g(x) = x - 1$

(a) $(f \circ g)(x) = f(g(x)) = f(x-1) = (x-1)^2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

(c) $(f \circ g)(0) = (0-1)^2 = 1$

42. $f(x) = \sqrt[3]{x-1},\ g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$

$= f(x^3+1)$

$= \sqrt[3]{(x^3+1) - 1}$

$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt[3]{x-1}\right)$

$= \left(\sqrt[3]{x-1}\right)^3 + 1$

$= (x-1) + 1 = x$

(c) $(f \circ g)(0) = 0$

43. $f(x) = 3x + 5,\ g(x) = 5 - x$

(a) $(f \circ g)(x) = f\big(g(x)\big) = f(5-x) = 3(5-x) + 5 = 20 - 3x$

(b) $(g \circ f)(x) = g\big(f(x)\big) = g(3x+5) = 5 - (3x+5) = -3x$

(c) $(f \circ g)(0) = 20$

44. $f(x) = x^3,\ g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f\big(g(x)\big) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

(b) $(g \circ f)(x) = g\big(f(x)\big) = g\left(x^3\right) = \dfrac{1}{x^3}$

(c) $(f \circ g)(0)$ is not defined.

45. (a) Domain of *f*: $x + 4 \ge 0$ or $x \ge -4$

(b) Domain of *g*: all real numbers

(c) $(f \circ g)(x) = f\big(g(x)\big) = f\left(x^2\right) = \sqrt{x^2 + 4}$

Domain: all real numbers

46. (a) Domain of f: $x + 3 \ge 0 \Rightarrow x \ge -3$
(b) Domain of *g*: all real numbers
(c) $(f \circ g)(x) = f\left(\dfrac{x}{2}\right) = \sqrt{\dfrac{x}{2} + 3}$

Domain: $\dfrac{x}{2} + 3 \ge 0 \Rightarrow x \ge -6$

47. (a) Domain of *f*: all real numbers
(b) Domain of *g*: all $x \ge 0$
(c) $(f \circ g)(x) = f\big(g(x)\big) = f\left(\sqrt{x}\right)$

$= \left(\sqrt{x}\right)^2 + 1 = x + 1,\ x \ge 0$

Domain: $x \ge 0$

48. (a) Domain of f: $x \geq 0$

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(g(x)) = f(x^4) = (x^4)^{1/4} = x$

Domain: all real numbers

49. (a) Domain of f: all $x \neq 0$

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(x+3) = \dfrac{1}{x+3}$

Domain: all $x \neq -3$

50. (a) Domain of f: all $x \neq 0$

(b) Domain of g: all $x \neq 0$

(c) $(f \circ g)(x) = f\left(\dfrac{1}{2x}\right) = 2x,\ x \neq 0$

Domain: all $x \neq 0$

51. (a) Domain of f: all real numbers

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(g(x)) = f(3-x)$

$$= |(3-x)-4| = |-x-1| = |x+1|$$

Domain: all real numbers

52. (a) Domain of f: all $x \neq 0$

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(g(x)) = f(x-1) = \dfrac{2}{|x-1|}$

Domain: all $x \neq 1$

53. (a) Domain of f: all real numbers

(b) Domain of g: all $x \neq \pm 2$

(c) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x^2-4}\right) = \dfrac{1}{x^2-4} + 2$

Domain: $x \neq \pm 2$

54. (a) Domain of f: all $x \neq \pm 1$

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(x+1) = \dfrac{3}{(x+1)^2 - 1}$

$$= \frac{3}{x^2+2x} = \frac{3}{x(x+2)}$$

Domain: all $x \neq 0, -2$.

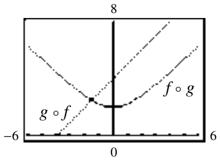
55. (a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2+4}$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x+4}\right) = \left(\sqrt{x+4}\right)^2$

$= x+4,\ x \geq -4$

(b)



They are not equal.

56. (a) $(f \circ g)(x) = f(g(x)) = f(x^3 - 1)$

$$= \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$$

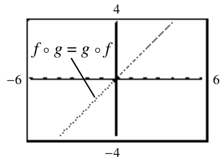
Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt[3]{x+1}\right)$

$$= \left[\sqrt[3]{x+1}\right]^3 - 1$$

$$= (x+1) - 1 = x$$

(b)



They are equal.

57. (a) $(f \circ g)(x) = f(g(x)) = f(3x+9)$

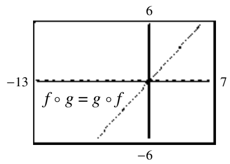
$$= \frac{1}{3}(3x+9) - 3 = x$$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{3}x - 3\right)$

$$= 3\left(\frac{1}{3}x - 3\right) + 9 = x$$

(b)

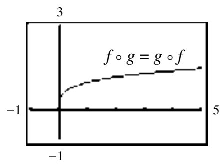


They are equal.

58. (a) $(f \circ g)(x) = (g \circ f)(x) = \sqrt{\sqrt{x}} = x^{1/4}$

Domain: all $x \geq 0$

(b)



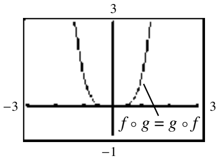
They are equal.

59. (a) $(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{2/3} = x^4$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g(x^{2/3}) = (x^{2/3})^6 = x^4$

(b)



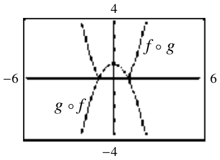
They are equal.

60. (a) $(f \circ g)(x) = f(g(x)) = f(-x^2 + 1) = |-x^2 + 1|$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g(|x|) = -|x|^2 + 1$
$= 1 - x^2$

(b)



They are not equal.

61. (a) $(f \circ g)(x) = f(g(x)) = f(4 - x) = 5(4 - x) + 4$
$= 24 - 5x$
$(g \circ f)(x) = g(f(x)) = g(5x + 4) = 4 - (5x + 4)$
$= -5x$

(b) They are not equal because $24 - 5x \neq -5x$.

(c)

x	$f(g(x))$	$g(f(x))$
0	24	0
1	19	–5
2	14	–10
3	9	–15

62. (a)

$(f \circ g)(x) = f(4x + 1) = \frac{1}{4}[(4x + 1) - 1] = \frac{1}{4}[4x] = x$

$(g \circ f)(x) = g\left(\frac{1}{4}(x - 1)\right) = 4\left[\frac{1}{4}(x - 1)\right] + 1 = (x - 1) + 1 = x$

(b) They are equal because $x = x$.

(c)

x	$f(g(x))$	$g(f(x))$
–1	–1	–1
0	0	0
1	1	1
2	2	2
3	3	3

63. (a)

$(f \circ g)(x) = f(g(x)) = f(x^2 - 5) = \sqrt{(x^2 - 5) + 6} = \sqrt{x^2 + 1}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x + 6}) = (\sqrt{x + 6})^2 - 5$
$= (x + 6) - 5 = x + 1,\ x \geq -6$

(b) They are not equal because $\sqrt{x^2 + 1} \neq x + 1$.

(c)

x	$f(g(x))$	$g(f(x))$
0	1	1
–2	$\sqrt{5}$	–1
3	$\sqrt{10}$	4

64. (a) $(f \circ g)(x) = f(\sqrt[3]{x + 10}) = [\sqrt[3]{x + 10}]^3 - 4$
$= (x + 10) - 4 = x + 6$
$(g \circ f)(x) = g(x^3 - 4) = \sqrt[3]{(x^3 - 4) + 10} = \sqrt[3]{x^3 + 6}$

(b) They are not equal because $x + 6 \neq \sqrt[3]{x^3 + 6}$.

(c)

x	$f(g(x))$	$g(f(x))$
–2	4	$\sqrt[3]{-2}$
0	6	$\sqrt[3]{6}$
1	7	$\sqrt[3]{7}$
2	8	$\sqrt[3]{14}$
3	9	$\sqrt[3]{33}$

65. (a) $(f \circ g)(x) = f(g(x)) = f(2x^3) = |2x^3|$
$(g \circ f)(x) = g(f(x)) = g(|x|) = 2|x|^3$

(b) They are equal because $|2x^3| = 2|x|^3$.

(c)

x	$f(g(x))$	$g(f(x))$
–1	2	2
0	0	0
1	2	2
2	16	16

66. (a)

$(f \circ g)(x) = f(g(x)) = f(-x) = \frac{6}{3(-x) - 5} = \frac{6}{-3x - 5}$

$(g \circ f)(x) = g\left(\frac{6}{3x - 5}\right) = -\left(\frac{6}{3x - 5}\right) = \frac{-6}{3x - 5}$

(b) They are not equal because $\frac{6}{-3x - 5} \neq \frac{-6}{3x - 5}$.

(c)

x	$f(g(x))$	$g(f(x))$
0	$-\frac{6}{5}$	$\frac{6}{5}$
1	$-\frac{3}{4}$	3
2	$-\frac{6}{11}$	–6
3	$-\frac{3}{7}$	$-\frac{3}{2}$

67. (a) $(f+g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{0}{2} = 0$

68. (a) $(f-g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

69. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

70. (a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

71. Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$. This is not a unique solution. Another possibility is $f(x) = (x+1)^2$ and $g(x) = 2x$.

72. Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$. This answer is not unique. Another possibility is $f(x) = (x+1)^3$ and $g(x) = -x$.

73. Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$. This answer is not unique. Other possibilities are

$f(x) = \sqrt[3]{x-4}$ and $g(x) = x^2$ or

$f(x) = \sqrt[3]{-x}$ and $g(x) = 4 - x^2$ or

$f(x) = \sqrt[9]{x}$ and $g(x) = (x^2 - 4)^3$.

74. Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$, then $\big(f \circ g\big)(x) = h(x)$. This answer is not unique. Another possibility is $f(x) = \sqrt{9+x}$ and $g(x) = -x$.

75. Let $f(x) = \frac{1}{x}$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$. This is not a unique solution. Other possibilities are

$f(x) = \frac{1}{x+2}$ and $g(x) = x$ or $f(x) = \frac{1}{x+1}$ and $g(x) = x + 1$.

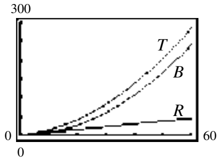
76. Let $g(x) = 5x + 2$ and $f(x) = \frac{4}{x^2}$, then $(f \circ g)(x) = h(x)$. This answer is not unique. Another possibility is $f(x) = \frac{4}{x}$ and $g(x) = (5x+2)^2$.

77. Let $f(x) = x^2 + 2x$ and $g(x) = x + 4$, then $(f \circ g)(x) = h(x)$. This answer is not unique. Another possibility is $f(x) = x$ and $g(x) = (x+4)^2 + 2(x+4)$.

78. Let $g(x) = x + 3$ and $f(x) = x^{3/2} + 4x^{1/2}$, then $(f \circ g)(x) = h(x)$. This answer is not unique. Another possibility is $f(x) = (x+1)^{3/2} + 4(x+1)^{1/2}$ and $g(x) = x + 2$.

79. (a) $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

(b)



(c) $B(x)$ contributes more to $T(x)$ at higher speeds.

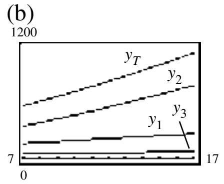
80. (a)

Year	1997	1998	1999	2000	2001	2002
y_1	161.5	172	182.5	193	203.5	214
y_2	348.54	386.04	424.86	465	506.46	549.24
y_3	54.27	54.72	55.63	57	58.83	61.12

Year	2003	2004	2005	2006	2007
y_1	224.5	235	245.5	256	266.5
y_2	593.34	638.76	685.5	733.56	782.94
y_3	63.87	67.08	70.75	74.88	79.47

The models are a good fit for the data. The variation of the model from the actual data is small in comparison to the sizes of the numbers.

(b)



y_T represents the total out-of-pocket payments, insurance premiums, and other types of payments in billions of dollars spent on health services and supplies in the United States and Puerto Rico for each year t.

81. (a) $r(x) = \frac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x))$

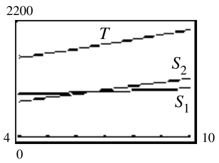
$$= A\left(\frac{x}{2}\right) = \pi\left(\frac{x}{2}\right)^2 = \frac{1}{4}\pi x^2$$

$A \circ r$ represents the area of the circular base of the tank with radius $\frac{x}{2}$.

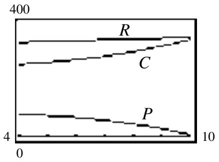
82. $(A \circ r)(t) = A(r(t))$
$= A(0.6t)$
$= \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ gives the area of the circle as a function of time.

83. (a) Since
$T = S_1 + S_2,\ T = (830 + 1.2t^2) + (390 + 75.4t)$
$T = 1.2t^2 + 75.4t + 1220.$

(b) 

84. (a) Since
$P = R - C,\ P = (320 + 2.8t) - (260 - 8t + 1.6t^2)$
$P = -1.6t^2 + 10.8t + 60$

(b) 

85. (a) $(N \circ T)(t) = N(T(t))$
$= N(2t + 1)$
$= 10(2t + 1)^2 - 20(2t + 1) + 600$
$= 40t^2 + 590$

$N \circ T$ represents the number of bacteria as a function of time.

(b) $(N \circ T)(6) = 10(13^2) - 20(13) + 600 = 2030$
At time $t = 6$, there are 2030 bacteria.

(c) $N = 800$ when $t \approx 2.3$ hours.

86. (a) Area $= \pi r^2,\ r(t) = 5.25\sqrt{t}.$ Hence
$(A \circ r)(t) = \pi\left[5.25\sqrt{t}\right]^2 = 27.5625\pi t, \quad t \geq 0$

(b) $(A \circ r)(36) = 27.5625\pi(36) = 992.25\pi$
≈ 3117 square meters

(c) $A = 6250 = 27.5625\pi t \Rightarrow t \approx 72.2$ hours

87. First, write the distance each plane is from point P. The plane that is 200 miles from point P is traveling at 450 miles per hour. Its distance is $200 - 450t$. Similarly, the other plane is $150 - 450t$ from point P.

So, the distance between the planes $s(t)$ can be found using the distance formula (or the Pythagorem Theorem): $s(t) = \sqrt{(200 - 450t)^2 + (150 - 450t)^2}$
$s(t) = 50\sqrt{162t^2 - 126t + 25}$

88. (a) $R(p) = p - 1200$

(b) $S(p) = 0.92p$

(c) $(R \circ S)(p) = 0.92p - 1200$. This is the cost if the discount is taken before the rebate.

$(S \circ R)(p) = 0.92(p - 1200)$. This is the cost if the rebate is taken before the discount.

(d) $(R \circ S)(18{,}400) = \$15{,}728$
$(S \circ R)(18{,}400) = \$15{,}824$

The discount first yields a lower cost because the discount is applied to the full amount and then the rebate is taken.

89. False. $g(x) = x - 3$

90. True. $(f \circ g)(x) = f(g(x))$ is only defined if $g(x)$ is in the domain of f.

91. Let A, B, and C be the three siblings, in decreasing age. Then $A = 2B$ and $B = \frac{1}{2}C + 6.$

(a) $A = 2B = 2\left(\frac{1}{2}C + 6\right) = C + 12$

(b) If $A = 16$, then $B = 8$ and $C = 4.$

92. From Exercise 91, $A = 2B$ and $B = \frac{1}{2}C + 6.$

(a) $2(B - 6) = C$ and $B = \frac{1}{2}A.$ Hence,
$C = 2\left(\frac{1}{2}A - 6\right) = A - 12.$

(b) If $C = 2$, then $B = 7$ and $A = 14.$

93. Let $f(x)$ and $g(x)$ be odd functions, and define $h(x) = f(x)g(x)$. Then
$h(-x) = f(-x)g(-x)$
$= [-f(x)][-g(x)]$ since f and g are both odd
$= f(x)g(x) = h(x).$

Thus, h is even.

Let $f(x)$ and $g(x)$ be even functions, and define $h(x) = f(x)g(x)$. Then
$h(-x) = f(-x)g(-x)$
$= f(x)g(x)$ since f and g are both even
$= h(x).$

Thus, h is even.

94. The product of an odd function and an even function is odd. Let f be odd and g even.
Then $(fg)(-x) = f(-x)g(-x) = -f(x)g(x) = -(fg)(x).$
Thus, fg is odd.

95. $g(-x) = \frac{1}{2}[f(-x) + f(-(-x))] = \frac{1}{2}[f(-x) + f(x)] = g(x),$
which shows that g is even.

$h(-x) = \frac{1}{2}[f(-x) - f(-(-x))] = \frac{1}{2}[f(-x) - f(x)]$
$= -\frac{1}{2}[f(x) - f(-x)] = -h(x),$
which shows that h is odd.

96. (a) $f(x)=\frac{1}{2}[f(x)+f(-x)]+\frac{1}{2}[f(x)-f(-x)]$
$=g(x)+h(x)$

where g is even and h is odd.

(b) $f(x)=\frac{1}{2}[(x^2-2x+1)+(x^2+2x+1)]$
$+\frac{1}{2}[(x^2-2x+1)-(x^2+2x+1)]$
$=\frac{1}{2}[2x^2+2]+\frac{1}{2}[-4x]=(x^2+1)+(-2x)$

$g(x)=\frac{1}{2}\left[\frac{1}{x+1}+\frac{1}{-x+1}\right]+\frac{1}{2}\left[\frac{1}{x+1}-\frac{1}{-x+1}\right]$
$=\frac{-1}{(x+1)(x-1)}+\frac{x}{(x+1)(x-1)}$

97. (a) If $f(x)=x^2$ and $g(x)=\frac{1}{x-2}$, then

$f(g(x))=\left(\frac{1}{x-2}\right)^2=\frac{1^2}{(x-2)^2}=\frac{1}{(x-2)^2}=h(x).$

(b) If $f(x)=\frac{1}{x-2}$ and $g(x)=x^2$, then

$f(g(x))=\frac{1}{x^2-2}\neq h(x).$

(c) If $f(x)=\frac{1}{x}$ and $g(x)=(x-2)^2$, then

$f(g(x))=\frac{1}{(x-2)^2}=h(x).$

98. The domain of $f(x)=x^2$ is $(-\infty,\infty)$ and the domain of $g(x)=\sqrt{x}$ is $[0,\infty)$. For $f(g(x))=g(f(x))=x$, the domain of f must be restricted to $[0,\infty)$ because the domain of g is already $[0,\infty)$.

99. Three points on the graph of $y=-x^2+x-5$ are $(0,-5)$, $(1,-5)$, and $(2,-7)$.

100. Three points on the graph of $y=\frac{1}{5}x^3-4x^2+1$ are $(0, 1)$, $(1, -2.8)$ and $(-1, -3.2)$.

101. Three points on the graph of $x^2+y^2=24$ are $(\sqrt{24}, 0)$, $(-\sqrt{24}, 0)$, and $(0, \sqrt{24})$.

102. Three points on the graph of $y=\frac{x}{x^2-5}$ are $(0, 0)$, $\left(1, -\frac{1}{4}\right)$ and $\left(-1, \frac{1}{4}\right)$.

103. First $m=\frac{8-(-2)}{-3-(-4)}=\frac{10}{1}=10$, and using the point $(-4,-2)$, $y-(-2)=10(x-(-4))$

$y+2=10x+40$
$y=10x+38.$

104. First $m=\frac{2-5}{-8-1}=\frac{-3}{-9}=\frac{1}{3}$, and using the point $(1, 5)$, $y-5=\frac{1}{3}(x-1)$

$y=\frac{1}{3}x+\frac{14}{3}.$

105. First $m=\frac{4-(-1)}{-\frac{1}{3}-\frac{3}{2}}=\frac{5}{-\frac{11}{6}}=-\frac{30}{11}$, and using the point $\left(\frac{3}{2}, -1\right)$, $y-(-1)=-\frac{30}{11}\left(x-\frac{3}{2}\right)$

$y+1=-\frac{30}{11}x+\frac{45}{11}$
$y=-\frac{30}{11}x+\frac{34}{11}.$

106. First $m=\frac{3.1-1.1}{-4-0}=\frac{2.0}{-4}=-0.5$, and using the point $(0, 1.1)$, $y-1.1=-0.5\,(x-0)$

$y=-0.5x+1.1$

Section 1.6

1. inverse, f^{-1}

2. range, domain

3. $y=x$

4. one-to-one

5. If a function is one-to-one, no two x-values in the domain can correspond to the same y-value in the range. Therefore, a horizontal line can intersect the graph at most once.

6. No. If both the points $(1, 4)$ and $(2, 4)$ lie on the graph of a function, then the function is not one-to-one; it would not pass the Horizontal Line Test.

7. $f(x)=6x$

$f^{-1}(x)=\frac{1}{6}x$

$f\left(f^{-1}(x)\right)=f\left(\frac{1}{6}x\right)=6\left(\frac{1}{6}x\right)=x$

$f^{-1}\left(f(x)\right)=f^{-1}(6x)=\frac{1}{6}(6x)=x$

8. $f(x)=\frac{1}{3}x$

$f^{-1}(x)=3x$

$f\left(f^{-1}(x)\right)=f(3x)=\frac{1}{3}(3x)=x$

$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{1}{3}x\right)=3\left(\frac{1}{3}x\right)=x$

9. $f(x) = x + 7$

$f^{-1}(x) = x - 7$

$f(f^{-1}(x)) = f(x-7) = (x-7) + 7 = x$

$f^{-1}(f(x)) = f^{-1}(x+7) = (x+7) - 7 = x$

10. $f(x) = x - 3$

$f^{-1}(x) = x + 3$

$f(f^{-1}(x)) = f(x+3) = (x+3) - 3 = x$

$f^{-1}(f(x)) = f^{-1}(x-3) = (x-3) + 3 = x$

11. $f(x) = 2x + 1$

$f^{-1}(x) = \frac{x-1}{2}$

$f(f^{-1}(x)) = f\left(\frac{x-1}{2}\right) = 2\left(\frac{x-1}{2}\right) + 1$

$= (x-1) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(2x+1) = \frac{(2x+1)-1}{2} = \frac{2x}{2} = x$

12. $f(x) = \frac{x-1}{4}$

$f^{-1}(x) = 4x + 1$

$f(f^{-1}(x)) = f(4x+1) = \frac{(4x+1)-1}{4} = \frac{4x}{4} = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{x-1}{4}\right)() = 4\left(\frac{x-1}{4}\right) + 1$

$= (x-1) + 1 = x$

13. $f(x) = \sqrt[3]{x}$

$f^{-1}(x) = x^3$

$f(f^{-1}(x)) = f(x^3) = \sqrt[3]{x^3} = x$

$f^{-1}(f(x)) = f^{-1}(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

14. $f(x) = x^5$

$f^{-1}(x) = \sqrt[5]{x}$

$f(f^{-1}(x)) = f(\sqrt[5]{x}) = (\sqrt[5]{x})^5 = x$

$f^{-1}(f(x)) = f^{-1}(x^5) = \sqrt[5]{x^5} = x$

15. The inverse is a line through $(-1, 0)$. Matches graph (c).

16. The inverse is a line through $(0, 6)$ and $(6, 0)$. Matches graph (b).

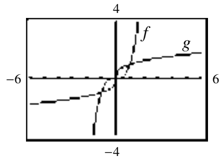
17. The inverse is half a parabola starting at $(1, 0)$. Matches graph (a).

18. The inverse is a reflection in $y = x$ of a third-degree equation through $(0, 0)$. Matches graph (d).

19. $f(x) = x^3,\ g(x) = \sqrt[3]{x}$

$f(g(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

$g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$

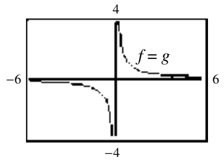


Reflections in the line $y = x$

20. $f(x) = \frac{1}{x},\ g(x) = \frac{1}{x}$

$f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = 1 \div \frac{1}{x} = 1 \cdot \frac{x}{1} = x$

$g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{1/x} = 1 \div \frac{1}{x} = 1 \cdot \frac{x}{1} = x$



The graphs are the same.

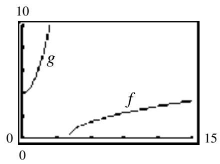
21. $f(x) = \sqrt{x-4};\ g(x) = x^2 + 4,\ x \ge 0$

$f(g(x)) = f(x^2 + 4),\ x \ge 0$

$= \sqrt{(x^2+4) - 4} = x$

$g(f(x)) = g(\sqrt{x-4})$

$= (\sqrt{x-4})^2 + 4 = x$



Reflections in the line $y = x$

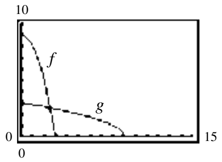
22. $f(x) = 9 - x^2,\ x \ge 0$

$g(x) = \sqrt{9-x},\ x \le 9$

$f(g(x)) = f(\sqrt{9-x}) = 9 - (\sqrt{9-x})^2$

$= 9 - (9 - x) = x$

$g(f(x)) = g(9 - x^2) = \sqrt{9 - (9 - x^2)} = \sqrt{x^2} = x$

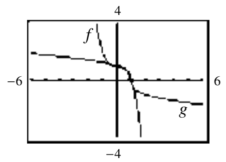


Reflections in the line $y = x$

23. $f(x) = 1 - x^3,\ g(x) = \sqrt[3]{1-x}$

$$f(g(x)) = f\left(\sqrt[3]{1-x}\right) = 1 - \left(\sqrt[3]{1-x}\right)^3 = 1 - (1-x) = x$$

$$g(f(x)) = g\left(1-x^3\right) = \sqrt[3]{1-\left(1-x^3\right)} = \sqrt[3]{x^3} = x$$

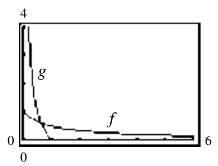


Reflections in the line $y = x$

24. $f(x) = \dfrac{1}{1+x},\ x \ge 0;\ g(x) = \dfrac{1-x}{x},\ 0 < x \le 1$

$$f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1+\left(\frac{1-x}{x}\right)} = \frac{1}{\frac{x}{x}+\frac{1-x}{x}} = \frac{1}{\frac{1}{x}} = x$$

$$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1-\left(\frac{1}{1+x}\right)}{\left(\frac{1}{1+x}\right)} = \frac{\frac{1+x}{1+x}-\frac{1}{1+x}}{\frac{1}{1+x}} = \frac{\frac{x}{1+x}}{\frac{1}{1+x}} = \frac{x}{1+x}\cdot\frac{x+1}{1} = x$$



Reflections in the line $y = x$

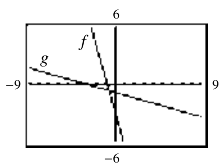
25. (a) $f(g(x)) = f\left(-\dfrac{2x+6}{7}\right)$

$$= -\frac{7}{2}\left(-\frac{2x+6}{7}\right) - 3 = x$$

$$g(f(x)) = g\left(-\frac{7}{2}x - 3\right)$$

$$= -\frac{2\left(-\frac{7}{2}x-3\right)+6}{7} = x$$

(b)



(c) $Y_1 = -\dfrac{7}{2}X - 3$

$Y_2 = -\dfrac{2X+6}{7}$

$Y_3 = Y_1(Y_2)$

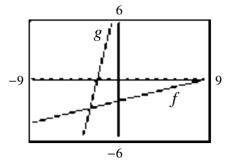
$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
−4	−4	−4
−2	−2	−2
0	0	0
2	2	2
4	4	4

26. (a) $f(g(x)) = f(4x+9) = \dfrac{(4x+9)-9}{4} = x$

$$g(f(x)) = g\left(\frac{x-9}{4}\right) = 4\left(\frac{x-9}{4}\right) + 9 = x$$

(b)



(c) $Y_1 = \dfrac{X-9}{4}$

$Y_2 = 4X + 9$

$Y_3 = Y_1(Y_2)$

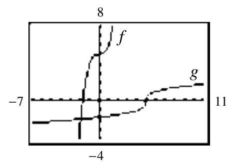
$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
−3	−3	−3
1	1	1
5	5	5
9	9	9

27. (a) $f(g(x)) = f\left(\sqrt[3]{x-5}\right) = \left(\sqrt[3]{x-5}\right)^3 + 5 = x$

$$g(f(x)) = g(x^3+5) = \sqrt[3]{(x^3+5)-5} = x$$

(b)



(c) $Y_1 = X^3 + 5$

$Y_2 = \sqrt[3]{X-5}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

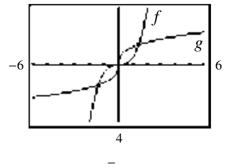
X	Y_3	Y_4
−2	−2	−2
−1	−1	−1
0	0	0
1	1	1
4	4	4

28. (a) $f(g(x)) = f\left(\sqrt[3]{2x}\right) = \dfrac{\left(\sqrt[3]{2x}\right)^3}{2} = x$

$$g(f(x)) = g\left(\frac{x^3}{2}\right) = \sqrt[3]{2\left(\frac{x^3}{2}\right)} = x$$

X	Y_3	Y_4
−2	−2	−2
−1	−1	−1
0	0	0
1	1	1
4	4	4

(b)



(c) $Y_1 = \dfrac{X^3}{2}$

$Y_2 = \sqrt[3]{2X}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–2	–2	–2
0	0	0
2	2	2
4	4	4
6	6	6

29. (a) $f(g(x)) = f(8 + x^2)$

$= -\sqrt{(8 + x^2) - 8}$

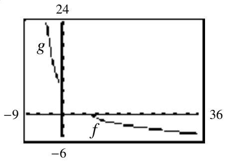
$= -\sqrt{x^2} = -(-x) = x, \ x \le 0$

$g(f(x)) = g(-\sqrt{x-8})$

$= 8 + (-\sqrt{x-8})^2$

$= 8 + (x - 8) = x, \ x \ge 8$

(b)



(c) $Y_1 = -\sqrt{x-8}$

$Y_2 = 8 + x^2, \ x = 0$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
8	8	8
9	9	9
12	12	12
15	15	15
20	20	20

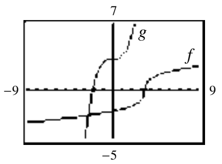
30. (a)

$$f(g(x)) = f\left(\frac{x^3+10}{3}\right) = \sqrt[3]{3\left(\frac{x^3+10}{3}\right) - 10} = x$$

$g(f(x)) = g(\sqrt[3]{3x-10})$

$$= \frac{(\sqrt[3]{3x-10})^3 + 10}{3} = \frac{3x - 10 + 10}{3} = x$$

(b)



(c) $Y_1 = \sqrt[3]{3x-10}$

$Y_2 = \dfrac{x^3+10}{3}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
−18	−18	−18
0	0	0
⅔	⅔	⅔
3	3	3
6	6	6

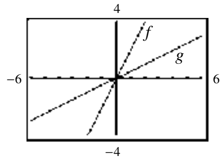
31. (a) $f(g(x)) = f\left(\dfrac{x}{2}\right)$

$= 2\left(\dfrac{x}{2}\right) = x$

$g(f(x)) = g(2x)$

$= \dfrac{2x}{2} = x$

(b)



(c) $Y_1 = 2x$

$Y_2 = \dfrac{x}{2}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–4	–4	–4
–2	–2	–2
0	0	0
2	2	2
4	4	4

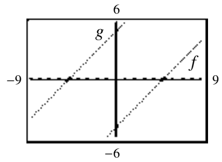
32. (a) $f(g(x)) = f(x + 5)$

$= (x + 5) - 5 = x$

$g(f(x)) = g(x - 5)$

$= (x - 5) + 5 = x$

(b)



(c) $Y_1 = x - 5$

$Y_2 = x + 5$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–2	–2	–2
–1	–1	–1
0	0	0
1	1	1
2	2	2

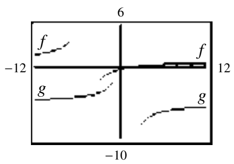
33. (a) $f(g(x)) = f\left(-\dfrac{5x+1}{x-1}\right)$

$$= \frac{\left(-\dfrac{5x+1}{x-1}\right) - 1}{\left(-\dfrac{5x+1}{x-1}\right) + 5} = \frac{\dfrac{6x}{x-1}}{\dfrac{6}{x-1}} = x, \ x \ne 1$$

$g(f(x)) = g\left(\dfrac{x-1}{x+5}\right)$

$$= -\frac{5\left(\dfrac{x-1}{x+5}\right) + 1}{\left(\dfrac{x-1}{x+5}\right) - 1} = \frac{\dfrac{6x}{x+5}}{\dfrac{6}{x+5}} = x, \ x \ne -5$$

(b)



(c) $Y_1 = \dfrac{x-1}{x+5}$

$Y_2 = -\dfrac{5x+1}{x-1}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–2	–2	–2
–1	–1	–1
0	0	0
1	1	1
2	2	2

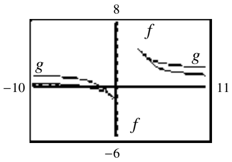
34. (a) $f(g(x)) = f\left(\dfrac{2x+3}{x-1}\right)$

$$= \frac{\left(\dfrac{2x+3}{x-1}\right)+3}{\left(\dfrac{2x+3}{x-1}\right)-2} = \frac{\dfrac{5x}{x-1}}{\dfrac{5}{x-1}} = x,\ x \neq 1$$

$g(f(x)) = g\left(\dfrac{x+3}{x-2}\right)$

$$= \frac{2\left(\dfrac{x+3}{x-2}\right)+3}{\left(\dfrac{x+3}{x-2}\right)-1} = \frac{\dfrac{5x}{x-2}}{\dfrac{5}{x-2}} = x,\ x \neq 2$$

(b)



(c) $Y_1 = \dfrac{x+3}{x-2}$

$Y_2 = \dfrac{2x+3}{x-1}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
−2	−2	−2
−1	−1	−1
0	0	0
1	1	1
3	3	3

35. Yes. No two elements, number of cans, in the domain correspond to the same element, price, in the range.

36. No. Both elements, ½ hour and 1 hour, in the domain correspond to the same element, $40, in the range.

37. No. Both x-values, –3 and 0, in the domain correspond to the y-value 6 in the range.

38. Yes. No two x-values in the domain correspond to the same y-value in the range.

39. Not a function.

40. It is the graph of a function, but not one-to-one.

41. It is the graph of a one-to-one function.

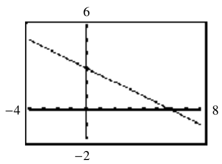
42. It is the graph of a one-to-one function.

43. It is the graph of a one-to-one function.

44. It is the graph of a one-to-one function.

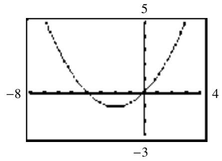
45. $f(x) = 3 - \dfrac{1}{2}x$

f is one-to-one because a horizontal line will intersect the graph at most once.



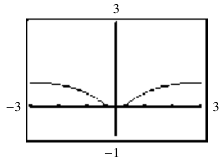
46. $f(x) = \dfrac{1}{4}(x+2)^2 - 1$

f does not pass the Horizontal Line Test, so f is not one-to-one.



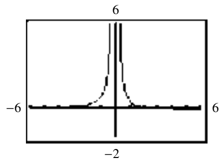
47. $h(x) = \dfrac{x^2}{x^2+1}$

h is not one-to-one because some horizontal lines intersect the graph twice.



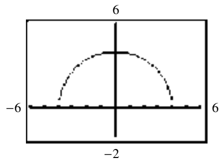
48. $g(x) = \dfrac{4-x}{6x^2}$

g does not pass the Horizontal Line Test, so g is not one-to-one.



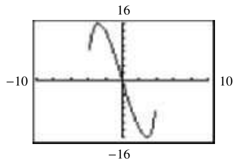
49. $h(x) = \sqrt{16-x^2}$

h is not one-to-one because some horizontal lines intersect the graph twice.



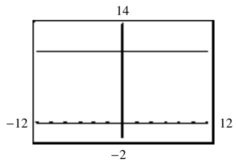
50. $f(x) = -2x\sqrt{16-x^2}$

f is not one-to-one because it does not pass the Horizontal Line Test.



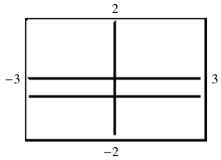
51. $f(x) = 10$

f is not one-to-one because the horizontal line $y = 10$ intersects the graph at every point on the graph.



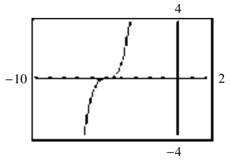
52. $f(x) = -0.65$

f is not one-to-one because it does not pass the Horizontal Line Test.



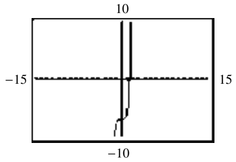
53. $g(x) = (x+5)^3$

g is one-to-one because a horizontal line will intersect the graph at most once.



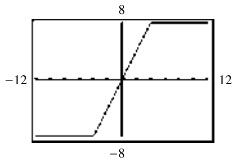
54. $f(x) = x^5 - 7$

f is one-to-one because it passes the Horizontal Line Test.



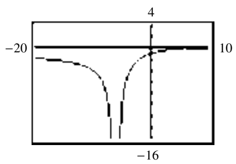
55. $h(x) = |x+4| - |x-4|$

h is not one-to-one because some horizontal lines intersect the graph more than once.

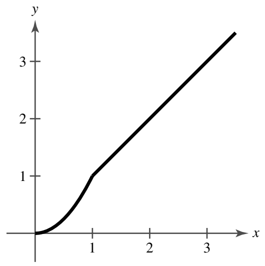


56. $f(x) = -\dfrac{|x-6|}{|x+6|}$

f is not one-to-one because it does not pass the Horizontal Line Test.

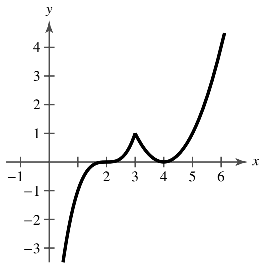


57.



The graph of the function passes the Horizontal Line Test and does have an inverse function.

58.

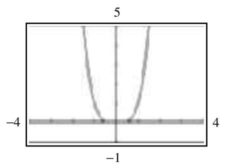


The graph of the function does not pass the Horizontal Line Test and does not have an inverse function.

59. $f(x) = x^4$

f is not one-to-one.

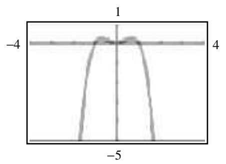
For example, $f(2) = f(-2) = 16$.



60. $g(x) = x^2 - x^4$

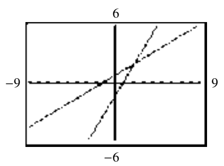
g is not one-to-one.

For example, $g(1) = g(-1) = 0$.



61.

$$f(x) = \frac{3x+4}{5}$$
$$y = \frac{3x+4}{5}$$
$$x = \frac{3y+4}{5}$$
$$5x = 3y + 4$$
$$5x - 4 = 3y$$
$$\frac{5x-4}{3} = y$$
$$f^{-1}(x) = \frac{5x-4}{3}$$



f is one-to-one and has an inverse function.

62. $f(x) = 3x + 5$

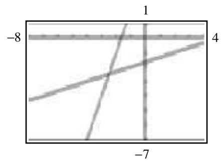
$y = 3x + 5$

$x = 3y + 5$

$x - 5 = 3y$

$\frac{x - 5}{3} = y$

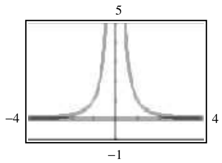
$f^{-1}(x) = \frac{x - 5}{3}$



f is one-to-one and has an inverse function.

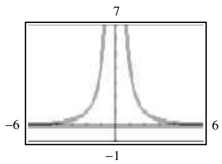
63. $f(x) = \frac{1}{x^2}$ is not one-to-one.

For example, $f(1) = f(-1) = 1$.



64. $h(x) = \frac{4}{x^2}$ is not one-to-one.

For example, $h(1) = h(-1) = 4$.



65. $f(x) = (x + 3)^2,\ x \ge -3,\ y \ge 0$

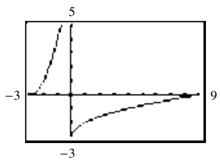
$y = (x + 3)^2,\ x \ge -3,\ y \ge 0$

$x = (y + 3)^2,\ y \ge -3,\ x \ge 0$

$\sqrt{x} = y + 3,\ y \ge -3,\ x \ge 0$

$y = \sqrt{x} - 3,\ x \ge 0,\ y \ge -3$

f is one-to-one and has an inverse function.



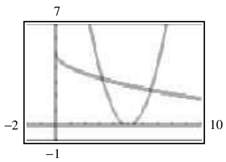
66. $q(x) = (x - 5)^2$

$y = (x - 5)^2,\ x \le 5$

$x = (y - 5)^2,\ y \le 5$

$-\sqrt{x} = y - 5,\ y \le 5$

$y = -\sqrt{x} + 5$



q is one-to-one and has an inverse function.

67. $f(x) = \sqrt{2x + 3} \Rightarrow x \ge -\frac{3}{2},\ y \ge 0$

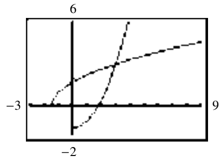
$y = \sqrt{2x + 3},\ x \ge -\frac{3}{2},\ y \ge 0$

$x = \sqrt{2y + 3},\ y \ge -\frac{3}{2},\ x \ge 0$

$x^2 = 2y + 3,\ x \ge 0,\ y \ge -\frac{3}{2}$

$y = \frac{x^2 - 3}{2},\ x \ge 0,\ y \ge -\frac{3}{2}$

f is one-to-one and has an inverse function.



68. $f(x) = \sqrt{x - 2} \Rightarrow x \ge 2,\ y \ge 0$

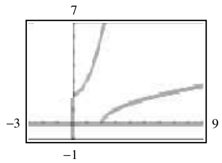
$y = \sqrt{x - 2},\ x \ge 2,\ y \ge 0$

$x = \sqrt{y - 2},\ y \ge 2,\ x \ge 0$

$x^2 = y - 2,\ x \ge 0,\ y \ge 2$

$x^2 + 2 = y,\ x \ge 0,\ y \ge 2$

f is one-to-one and has an inverse function.



69. $f(x) = |x - 2|,\ x \le 2,\ y \ge 0$

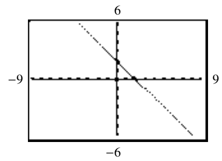
$y = |x - 2|$

$x = |y - 2|,\ y \le 2,\ x \ge 0$

$x = -(y - 2)$ since $y - 2 \le 0$.

$x = -y + 2$

$y = -x + 2,\ x \ge 0,\ y \le 2$

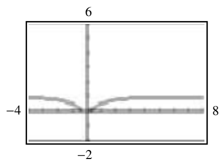


f is one-to-one and has an inverse function.

70. $f(x) = \frac{x^2}{x^2 + 1}$ is not one-to-one.

For instance, $f(1) = f(-1) = \frac{1}{2}$.

f does not have an inverse.



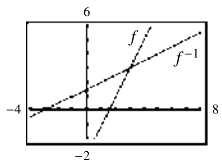
71. $f(x) = 2x - 3$

$y = 2x - 3$

$x = 2y - 3$

$y = \frac{x + 3}{2}$

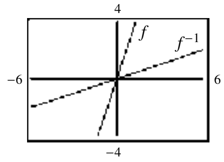
$f^{-1}(x) = \frac{x + 3}{2}$



Reflections in the line $y = x$

72. $f(x) = 3x$

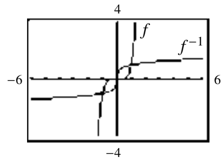
$$y = 3x$$
$$x = 3y$$
$$\frac{x}{3} = y$$
$$f^{-1}(x) = \frac{x}{3}$$



Reflections in the line $y = x$

73. $f(x) = x^5$

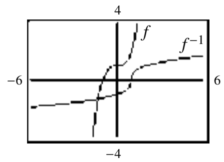
$$y = x^5$$
$$x = y^5$$
$$y = \sqrt[5]{x}$$
$$f^{-1}(x) = \sqrt[5]{x}$$



Reflections in the line $y = x$

74. $f(x) = x^3 + 1$

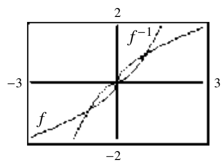
$$y = x^3 + 1$$
$$x = y^3 + 1$$
$$x - 1 = y^3$$
$$\sqrt[3]{x-1} = y$$
$$f^{-1}(x) = \sqrt[3]{x-1}$$



Reflections in the line $y = x$

75. $f(x) = x^{3/5}$

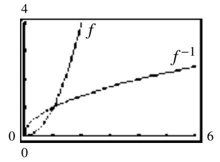
$$y = x^{3/5}$$
$$x = y^{3/5}$$
$$y = x^{5/3}$$
$$f^{-1}(x) = x^{5/3}$$



Reflections in the line $y = x$

76. $f(x) = x^2,\ x \ge 0$

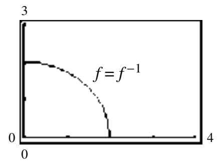
$$y = x^2$$
$$x = y^2$$
$$\sqrt{x} = y$$
$$f^{-1}(x) = \sqrt{x}$$



Reflections in the line $y = x$

77. $f(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$

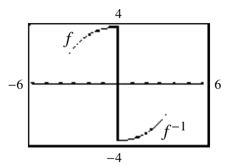
$$y = \sqrt{4 - x^2}$$
$$x = \sqrt{4 - y^2}$$
$$x^2 = 4 - y^2$$
$$y^2 = 4 - x^2$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$$



The graphs are the same.

78. $f(x) = \sqrt{16 - x^2},\ -4 \le x \le 0$

$$y = \sqrt{16 - x^2}$$
$$x = \sqrt{16 - y^2},\ -4 \le y \le 0$$
$$x^2 = 16 - y^2$$
$$y^2 = 16 - x^2$$
$$y = -\sqrt{16 - x^2}$$
$$f^{-1}(x) = -\sqrt{16 - x^2},\ 0 \le x \le 4$$

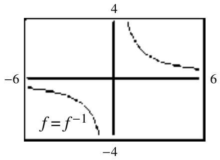


Reflections in the line $y = x$

79. $f(x) = \dfrac{4}{x}$

$$y = \frac{4}{x}$$
$$x = \frac{4}{y}$$
$$xy = 4$$

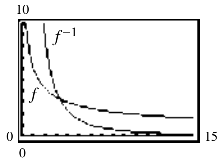
$$y = \frac{4}{x}$$
$$f^{-1}(x) = \frac{4}{x}$$



The graphs are the same.

80. $f(x) = \frac{6}{\sqrt{x}}$

$$y = \frac{6}{\sqrt{x}}$$
$$x = \frac{6}{\sqrt{y}}$$
$$x^2 = \frac{36}{y}$$
$$y = \frac{36}{x^2},\ x > 0$$
$$f^{-1}(x) = \frac{36}{x^2},\ x > 0$$



Reflections in the line $y = x$

81. If we let $f(x) = (x-2)^2,\ x \ge 2,$ then f has an inverse function. [**Note:** We could also let $x \le 2$.]

$$y = (x-2)^2$$
$$x = (y-2)^2$$
$$\sqrt{x} = y - 2$$
$$\sqrt{x} + 2 = y$$
$$f^{-1}(x) = \sqrt{x} + 2$$

Domain of f: $x \ge 2$ Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$ Range of f^{-1}: $y \ge 2$

82. If we let $f(x) = 1 - x^4,\ x \ge 0,$ then f has an inverse function. [**Note:** We could also let $x \le 0$.]

$$y = 1 - x^4$$
$$x = 1 - y^4$$
$$y^4 = 1 - x$$
$$y = \sqrt[4]{1-x}$$
$$f^{-1}(x) = \sqrt[4]{1-x}$$

Domain of f: $x \ge 0$ Range of f: $y \le 1$

Domain of f^{-1}: $x \le 1$ Range of f^{-1}: $y \ge 0$.

83. If we let $f(x) = |x+2|,\ x \ge -2,$ then f has an inverse function. [**Note:** We could also let $x \le -2$.]

$$y = x + 2$$
$$x = y + 2$$
$$x - 2 = y$$
$$f^{-1}(x) = x - 2$$

Domain of f: $x \ge -2$

Domain of f^{-1}: $x \ge 0$

Range of f: $y \ge 0$

Range of f^{-1}: $y \ge -2$

84. If we let $f(x) = |x-2|,\ x \ge 2,$ then f has an inverse function.
[**Note:** We could also let $x \le 2$.]

$$y = x - 2$$
$$x = y - 2$$
$$x + 2 = y$$
$$f^{-1}(x) = x + 2$$

Domain of f: $x \ge -2$ Range of f: $y \ge 0$

Domain of f^{-1}: $x \ge 0$ Range of f^{-1}: $y \ge -2$

85. If we let $f(x) = (x+3)^2,\ x \ge -3$ then f has an inverse function. [**Note:** We could also let $x \le -3$.]

$$y = (x+3)^2$$
$$x = (y+3)^2$$
$$\sqrt{x} = y + 3$$
$$y = \sqrt{x} - 3$$
$$f^{-1}(x) = \sqrt{x} - 3$$

Domain of f: $x \ge -3$ Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$ Range of f^{-1}: $y \ge -3$

86. If we let $f(x) = (x-4)^2,\ x \ge 4,$ then f has an inverse function. [**Note:** We could also let $x \le 4$.]

$$y = (x-4)^2$$
$$x = (y-4)^2$$
$$\sqrt{x} = y - 4$$
$$y = \sqrt{x} + 4$$
$$f^{-1}(x) = \sqrt{x} + 4$$

Domain of f: $x \ge 4$ Range of f: $y \ge 0$
Domain of f^{-1}: $x \ge 0$ Range of f^{-1}: $y \ge 4$

87. If we let $f(x) = -2x^2 + 5,\ x \ge 0,$ then f has an inverse function. [**Note:** We could also let $x \le -3$.]

$$y = -2x^2 + 5$$
$$x = -2y^2 + 5$$
$$x - 5 = -2y^2$$
$$y^2 = \frac{x-5}{-2} = \frac{5-x}{2}$$
$$y = \sqrt{\frac{(5-x)}{2}}$$
$$f^{-1}(x) = \sqrt{\frac{5-x}{2}}$$

Domain of $f: x \ge 0$ Range of $f: y \le 5$
Domain of $f^{-1}: x \le 5$ Range of $f^{-1}: y \ge 0$

88. If we let $f(x) = \frac{1}{2}x^2 - 1,\ x \ge 0,$ then f has an inverse function. [**Note:** We could also let $x \le 0$.]

$$y = \frac{1}{2}x^2 - 1$$
$$x = \frac{1}{2}y^2 - 1$$
$$2(x+1) = y^2$$
$$f^{-1}(x) = \sqrt{2x+2}$$

Domain of $f: x \ge 0$ Range of $f: y \ge -1$
Domain of $f^{-1}: x \ge -1$ Range of $f^{-1}: y \ge 0$

89. If we let $f(x) = |x-4| + 1,\ x \ge 4,$ then f has an inverse function. [**Note:** We could also let $x \le 4$.]

$$y = |x-4| + 1$$
$$y = x - 3 \text{ because } x \ge 4$$
$$x = y - 3$$
$$y = x + 3$$
$$f^{-1}(x) = x + 3$$

Domain of $f: x \ge 4$ Range of $f: y \ge 1$
Domain of $f^{-1}: x \ge 1$ Range of $f^{-1}: y \ge 4$

90. If we let $f(x) = -|x-1| - 2,\ x \ge 1,$ then f has an inverse function. [**Note:** We could also let $x \le 1$.]

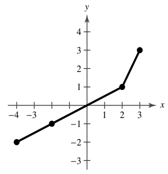
$$y = -|x-1| - 2 = -(x-1) - 2 \text{ because } x \ge 1$$
$$y = -x - 1$$
$$x = -y - 1$$
$$x + 1 = -y$$
$$f^{-1}(x) = -x - 1$$

Domain of $f: x \ge 1$ Range of $f: y \le -2$
Domain of $f^{-1}: x \le -2$ Range of $f^{-1}: y \ge 1$

91.

x	$f(x)$
-2	-4
-1	-2
1	2
3	3

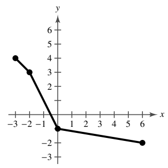
x	$f^{-1}(x)$
-4	-2
-2	-1
2	1
3	3



92.

x	$f(x)$
4	-3
3	-2
-1	0
-2	6

x	$f^{-1}(x)$
-3	4
-2	3
0	-1
6	-2



93. $f^{-1}(0) = \frac{1}{2}$ because $f\left(\frac{1}{2}\right) = 0$.

94. $g^{-1}(0) = -2$ because $g(-2) = 0$.

95. $(f \circ g)(2) = f(3) = -2$

96. $g(f(-4)) = g(4) = 6$

97. $f^{-1}(g(0)) = f^{-1}(2) = 0$

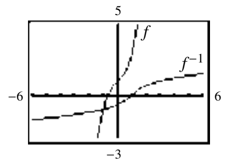
98. $(g^{-1} \circ f)(3) = g^{-1}(-2) = -3$

99. $(g \circ f^{-1})(2) = g(0) = 2$

100. $(f^{-1} \circ g^{-1})(6) = f^{-1}(g^{-1}(6)) = f^{-1}(4) = -4$

101. $f(x) = x^3 + x + 1$

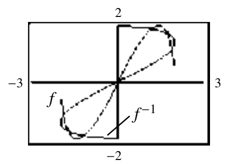
(a) and (b)



(c) The graph of the inverse relation is an inverse function since it satisfies the Vertical Line Test.

102. $f(x) = x\sqrt{4 - x^2}$

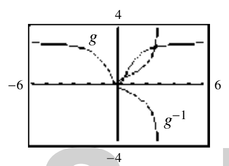
(a) and (b)



(c) Not an inverse function since it does not satisfy the Vertical Line Test.

103. $g(x) = \frac{3x^2}{x^2 + 1}$

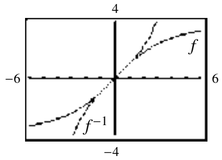
(a) and (b)



The graph of the inverse relation is not an inverse function since it does not satisfy the Vertical Line Test.

104. $f(x) = \dfrac{4x}{\sqrt{x^2+15}}$

(a) and (b)



(c) Inverse function since it satisfies the Vertical Line Test.

In Exercises 105 – 110, $f(x) = \frac{1}{8}x - 3,\ f^{-1}(x) = 8(x+3),$ $g(x) = x^3,\ g^{-1}(x) = \sqrt[3]{x}.$

105. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) = f^{-1}\left(\sqrt[3]{1}\right)$
$$= 8\left(\sqrt[3]{1}+3\right) = 8(1+3) = 32$$

106. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3)) = g^{-1}(8(-3+3))$
$$= g^{-1}(0) = \sqrt[3]{0} = 0$$

107. $(f^{-1} \circ f^{-1})(6) = f^{-1}(f^{-1}(6)) = f^{-1}(8[6+3])$
$$= f^{-1}(72) = 8(72+3) = 600$$

108. $(g^{-1} \circ g^{-1})(-4) = g^{-1}(g^{-1}(-4)) = g^{-1}\left(\sqrt[3]{-4}\right)$
$$= \sqrt[3]{\sqrt[3]{-4}} = -\sqrt[9]{4}$$

109. $(fg)(x) = f(g(x)) = f\left(x^3\right) = \frac{1}{8}x^3 - 3$

Now find the inverse of $(f \circ g)(x) = \frac{1}{8}x^3 - 3$:

$$y = \frac{1}{8}x^3 - 3$$
$$x = \frac{1}{8}y^3 - 3$$
$$x + 3 = \frac{1}{8}y^3$$
$$8(x+3) = y^3$$
$$\sqrt[3]{8(x+3)} = y$$
$$(f \circ g)^{-1}(x) = 2\sqrt[3]{x+3}$$

Note: $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

110. $(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$
$$= g^{-1}(8(x+3))$$
$$= \sqrt[3]{8(x+3)}$$
$$= 2\sqrt[3]{x+3}$$

In Exercises 111 to 114,
$f(x) = x+4,\ f^{-1}(x) = x-4,\ g(x) = 2x-5,\ g^{-1}(x) = \dfrac{x+5}{2}.$

111. $(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$
$$= g^{-1}(x-4)$$
$$= \frac{(x-4)+5}{2}$$
$$= \frac{x+1}{2}$$

112. $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x))$
$$= f^{-1}\left(\frac{x+5}{2}\right)$$
$$= \frac{x+5}{2} - 4$$
$$= \frac{x+5-8}{2}$$
$$= \frac{x-3}{2}$$

113. $(f \circ g)(x) = f(g(x)) = f(2x-5) = (2x-5)+4 = 2x-1.$
Now find the inverse function of $(f \circ g)(x) = 2x-1.$

$$y = 2x-1$$
$$x = 2y-1$$
$$x+1 = 2y$$
$$y = \frac{x+1}{2}$$
$$(f \circ g)^{-1}(x) = \frac{x+1}{2}$$

Note that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x);$ see Exercise 111.

114. $(g \circ f)(x) = g(f(x)) = g(x+4) = 2(x+4)-5$
$$= 2x+8-5 = 2x+3.$$

Now find the inverse function of $(g \circ f)(x) = 2x+3.$

$$y = 2x+3$$
$$x = 2y+3$$
$$x-3 = 2y$$
$$\frac{x-3}{2} = y$$
$$(g \circ f)^{-1}(x) = \frac{x-3}{2}$$

Note that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}.$

115. (a) Yes, *f* is one-to-one. For each European shoe size, there is exactly one U.S. shoe size.

(b) $f(11) = 45$

(c) $f^{-1}(43) = 10$ because $f(10) = 43.$

(d) $f(f^{-1}(41)) = f(8) = 41$

(e) $f^{-1}(f(13)) = f^{-1}(47) = 13$

116. If two functions are inverse functions of each other, then $g(g^{-1}(x)) = g^{-1}(g(x)) = x,$ so $g^{-1}(g(6)) = 6$

117. (a) CALL ME LATER corresponds to numerical values: 3 1 12 12 0 13 5 0 12 1 20 5 18. Using f to encode,

$$f(3) = 19$$
$$f(1) = 9$$
$$f(12) = 64$$
$$f(12) = 64$$
$$f(0) = 4$$
$$f(13) = 69$$
$$f(5) = 29$$
$$f(0) = 4$$
$$f(12) = 64$$
$$f(1) = 9$$
$$f(20) = 104$$
$$f(5) = 29$$
$$f(18) = 94$$

(b) For $f(x) = 5x + 4$, $f^{-1}(x) = \dfrac{x-4}{5}$.

Using f^{-1} to decode, $f^{-1}(119) = 23$

$$f^{-1}(44) = 8$$
$$f^{-1}(9) = 1$$
$$f^{-1}(104) = 20$$
$$f^{-1}(4) = 0$$
$$f^{-1}(104) = 20$$
$$f^{-1}(49) = 9$$
$$f^{-1}(69) = 13$$
$$f^{-1}(29) = 5$$

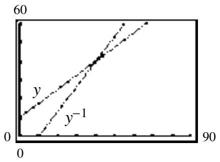
Converting from numerical values to letters, the message is WHAT TIME.

118. (a) $y = 10 + 0.75x$

$$x = 10 + 0.75y$$
$$x - 10 = 0.75y$$
$$\frac{x-10}{0.75} = y$$

So, $y^{-1} = \dfrac{x-10}{0.75}$. y^{-1} is the number of units produced while x is the hourly range.

(b)



(c) When $y^{-1} = 10$, $x = \$17.50$.

(d) When $x = \$21.25$, $y^{-1} = 15$ units.

119. False. $f(x) = x^2$ is even, but f^{-1} does not exist.

120. True. If $(0, b)$ is the y-intercept of f, then $(b, 0)$ is the x-intercept of f^{-1}.

121. Yes. The inverse would give the time it took to complete n miles.

122. This situation could be represented by a one-to-one function if the population continues to increase. The inverse function would represent the population in terms of the year.

123. No. The function oscillates.

124. This situation could not be represented by a one-to-one function because height remains constant after a certain age.

125. The graph of f^{-1} is a reflection of the graph of f in the line $y = x$.

126. If the domain of f is $[0, 9]$ and the range is $[-3, 3]$, and since the graphs of f and f^{-1} can be described as if the point (a, b) lies on the graph of f, then the point (b, a) lies on the graph of f^{-1}, then the domain of f^{-1} is $[-3, 3]$ and the range is $[0, 9]$.

127. (a) The function f will have an inverse function because no two temperatures in degrees Celsius will correspond to the same temperature in degrees Fahrenheit.

(b) $f^{-1}(50)$ would represent the temperature in degrees Celsius for a temperature of $50°$ Fahrenheit.

128. Yes. The function would pass the Horizontal Line Test and therefore have an inverse function.

129. The constant function $f(x) = c$, whose graph is a horizontal line, would never have an inverse function.

130. (a) No, the graphs are not reflections of each other in the line $y = x$.

(b) Yes, the graphs are reflections of each other in the line $y = x$.

(c) Yes, the graphs are reflections of each other in the line $y = x$.

(d) Yes, the graphs are reflections of each other in the line $y = x$.

131. We will show that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$ for all x in their domains.

Let $y = (f \circ g)^{-1}(x) \Rightarrow (f \circ g)(y) = x$ then $f(g(y)) = x \Rightarrow f^{-1}(x) = g(y)$.

Hence,

$$(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x)) = g^{-1}(g(y)) = y = (f \circ g)^{-1}(x).$$

Thus, $g^{-1} \circ f^{-1} = (f \circ g)^{-1}$.

132. If f is one-to-one, then f^{-1} exists. If f is odd, then $f(-x) = -f(x)$. Consider $f(x) = y \leftrightarrow f^{-1}(y) = x$. Then $f^{-1}(-y) = f^{-1}(-f(x)) = f^{-1}(f(-x)) = -x = -f^{-1}(y)$.

Thus, f^{-1} is odd.

133. $\dfrac{27x^3}{3x^2} = 9x,\ x \neq 0$

134. $\dfrac{5x^2y}{xy + 5x} = \dfrac{5x^2y}{x(y+5)} = \dfrac{5xy}{y+5},\ x \neq 0$

135. $\dfrac{x^2 - 36}{6 - x} = \dfrac{(x-6)(x+6)}{-(x-6)} = \dfrac{x+6}{-1} = -x - 6,\ x \neq 6$

136. $\dfrac{x^2+3x-40}{x^2-3x-10}=\dfrac{(x-5)(x+8)}{(x-5)(x+2)}=\dfrac{x+8}{x+2},\ x\neq 5$

137. $x = 5$. No, it does not pass the Vertical Line Test.

138. $y=\sqrt{x+2}$

Yes, y is a function of x.

139. $x^2+y^2=9$

$$y=\pm\sqrt{9-x^2}$$

No, y is not a function of x.

140. $x-y^2=0$

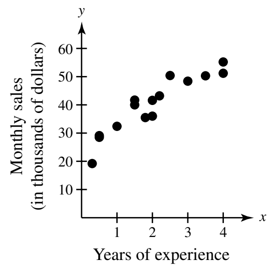
$$y^2=x$$

$$y=\pm\sqrt{x}$$

No, y is not a function of x.

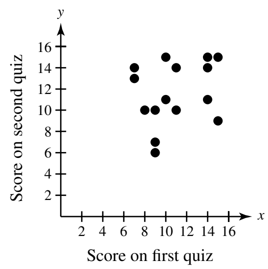
Section 1.7

1. positive
2. regression or linear regression
3. negative
4. No. The closer the correlation coefficient $|r|$ is to 1, the better the fit.
5. (a)



(b) Yes, the data appears somewhat linear. The more experience x corresponds to higher sales y.

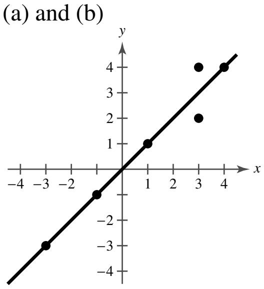
6. (a)



(b) No. Quiz scores are dependent on several variables, such as study time, class attendance, etc.

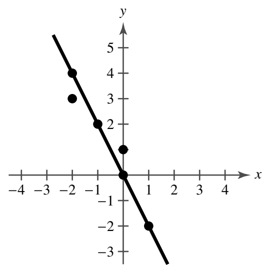
7. Negative correlation
8. No correlation
9. No correlation
10. Positive correlation

11. (a) and (b)



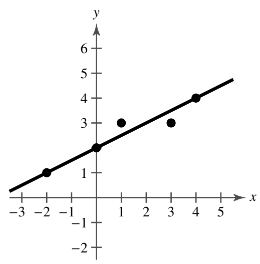
(c) $y = x$

12. (a) and (b)



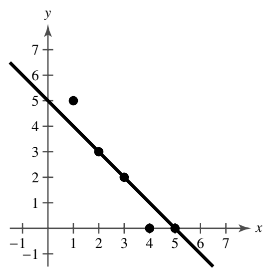
(c) $y = -2x$

13. (a) and (b)



(c) $y=\dfrac{1}{2}x+2$

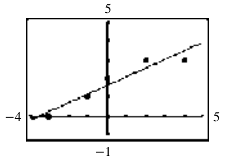
14. (a) and (b)



(c) $y = -x + 5$

15. $y = 0.46x + 1.6$

(a)



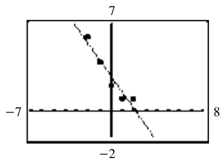
(b)

x	–3	–1	0	2	4
Linear equation	0.22	1.14	1.6	2.52	3.44
Given data	0	1	2	3	3

The model fits the data well.

16. $y = -1.3x + 2.8$

(a)

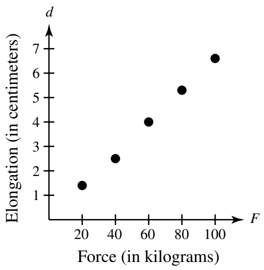


(b)

x	–2	–1	0	1	2
Linear equation	5.4	4.1	2.8	1.5	0.2
Given data	6	4	2	1	1

The model fits the data fairly well.

17. (a)

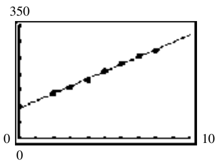


(b) $d = 0.07F - 0.3$

(c) $d = 0.066F$ or $F = 15.13d + 0.096$

(d) If $F = 55$, $d = 0.066(55) \approx 3.63$ cm.

18. (a) and (c)



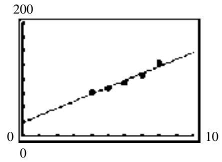
The model fits the data well.

(b) $S = 22.60t + 93.9$

(d) For 2015, $t = 15$ and $S = 22.60(15) + 93.9$
$= 432.9 \approx 433$ million subscribers.

Answers will vary.

19. (a) and (c)



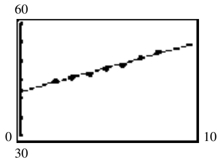
The model fits the data well.

(b) $T = 12.37t + 24.04$

(d) For 2010,
$t = 10$ and $T = 12.37(10) + 24.04 = \$147.74$ million.
For 2015,
$t = 15$ and $T = 12.37(15) + 24.04 = \$209.59$ million.

(e) 12.37; The slope represents the average annual increase in salaries (in millions of dollars).

20. (a) and (c)



The model fits the data well.

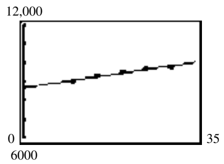
(b) $S = 1.26t + 41.8$

(d) For 2016, $t = 16$ and $S = 1.26(16) + 41.8 = \$62.0$ thousand.

For 2018, $t = 18$ and $S = 12.6(18) + 41.8 = \$64.5$ thousand.

Answers will vary.

21. (a) and (c)



(b) $P = 38.98t + 8655.4$

(d)

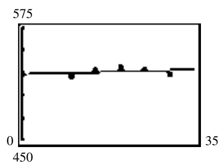
Year	2010	2015	2020	2025	2030
Actual	9018	9256	9462	9637	9802
Model	9045.2	9240.1	9435	9629.9	9824.8

The model fits the data well.

(e) For 2050,
$t = 50$ and $P = 38.98(50) + 8655.4 = 10{,}604{,}400$ people.

Answers will vary.

22. (a) and (c)



(b) $P = 0.14t + 523.4$

(d)

Year	2010	2015	2020	2025	2030
Actual	520	528	531	529	523
Model	524.8	525.5	526.2	526.9	527.6

The model fits the data well.

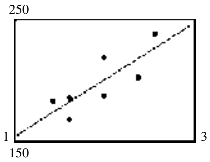
(e) For 2050,
$t = 50$ and $P = 0.14(50) + 523.4 = 530{,}400$ people.

Answers will vary.

23. (a) $y = 47.77x + 103.8$

Correlation coefficient: 0.81238

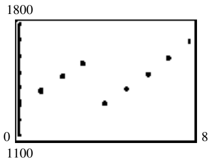
(b)



(c) The slope represents the increase in sales due to increased advertising.

(d) For \$1500, $x = 1.5$ and $y = 175.455$ or \$175,455.

24. (a)

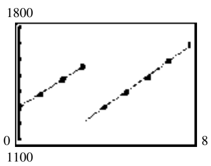


The first four points and the last five points are approximately linear.

(b) $T_1 = 83.2t + 1304,\ 0 < t \le 3$

$T_2 = 94.2t + 928,\ 3 < t \le 8$

(c) $T = \begin{cases} 83.2t + 1304, & 0 \le t \le 3 \\ 94.2t + 928, & 3 < t \le 8 \end{cases}$



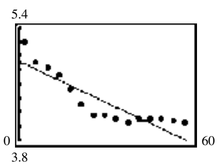
(d) Answers will vary.

25. (a) $T = -0.019t + 4.92$

$r \approx -0.886$

(b) The negative slope means that the winning times are generally decreasing over time.

(c)



(d)

Year	1952	1956	1960	1964	1968
Actual	5.20	4.91	4.84	4.72	4.53
Model	4.88	4.81	4.73	4.65	4.58

Year	1972	1976	1980	1984	1988
Actual	4.32	4.16	4.15	4.12	4.06
Model	4.50	4.43	4.35	4.27	4.20

Year	1992	1996	2000	2004	2008
Actual	4.12	4.12	4.10	4.09	4.05
Model	4.12	4.05	3.97	3.89	3.82

The model does not fit the data well.

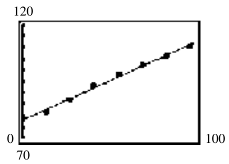
(e) The closer $|r|$ is to 1, the better the model fits the data.

(f) No. The winning times have leveled off in recent years, but the model values continue to decrease to unrealistic times.

26. (a) $l = 0.34d + 77.9;\ r \approx 0.993$

(b) Yes; Since $r \approx 0.993$ and $|r|$ is close to 1, the model fits the data well.

(c)



The data fits the model well.

(d) When $d = 112$,
$l = 0.34(112) + 77.9 = 115.98 \approx 116$ cm.

27. True. To have positive correlation, the y-values tend to increase as x increases.

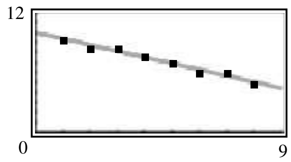
28. False. The closer the correlation coefficient is to –1 or 1, the better a line fits the data.

29. Answers will vary.

30. (a) (i) $y = -0.62x + 10.0$

$r = -0.986$

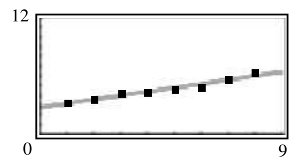
The data are decreasing, so the slope and correlation coefficient are negative.



(ii) $y = 0.41x + 2.7$

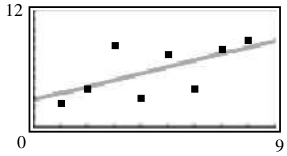
$r = 0.973$

The slope is less sleep than the slope of the line in (iii). The line is a better fit for the data than the line in (iii), so the correlation coefficient will be greater.



(iii) $y = 0.68x + 2.7$
$r = 0.62$

The slope is steeper than the slope of the line in (ii). The line is not a good fit for the data, so the correlation coefficient will not be close to 1.



(b) Model (i) is the best fit for its data because its r-value is -0.986, and therefore $|r|$ is closest to 1.

31. $f(x) = 2x^2 - 3x + 5$

(a) $f(-1) = 2 + 3 + 5 = 10$

(b) $f(w+2) = 2(w+2)^2 - 3(w+2) + 5$
$= 2w^2 + 5w + 7$

32. $g(x) = 5x^2 - 6x + 1$

(a) $g(-2) = 5(4) - 6(-2) + 1 = 33$

(b) $g(z-2) = 5(z-2)^2 - 6(z-2) + 1$
$= 5z^2 - 26z + 33$

33. $6x + 1 = -9x - 8$
$15x = -9$
$x = -\frac{9}{15} = -\frac{3}{5}$

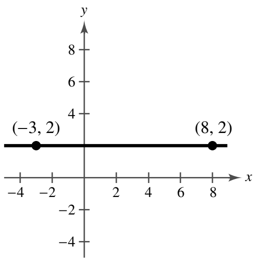
34. $3(x-3) = 7x + 2$
$-11 = 4x$
$x = -\frac{11}{4}$

35. $8x^2 - 10x - 3 = 0$
$(4x+1)(2x-3) = 0$
$x = -\frac{1}{4}, \frac{3}{2}$

36. $10x^2 - 23x - 5 = 0$
$(2x-5)(5x+1) = 0$
$x = \frac{5}{2}, -\frac{1}{5}$

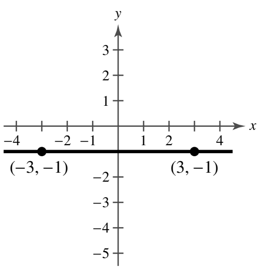
Chapter 1 Review

1.



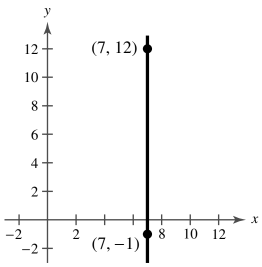
$m = \frac{2-2}{8-(-3)} = \frac{0}{11} = 0$

2.



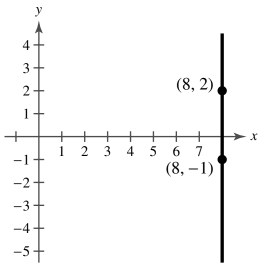
$m = \frac{-1-(-1)}{-3-3} = \frac{0}{-6} = 0$

3.



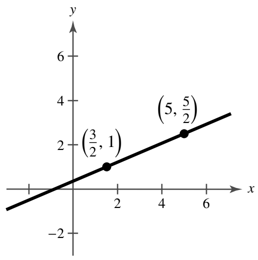
m is undefined.

4.



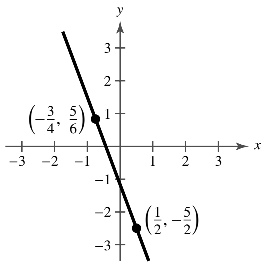
m is undefined.

5.



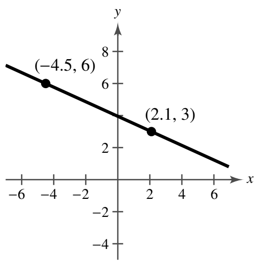
$$m = \frac{(5/2) - 1}{5 - (3/2)} = \frac{3/2}{7/2} = \frac{3}{7}$$

6.



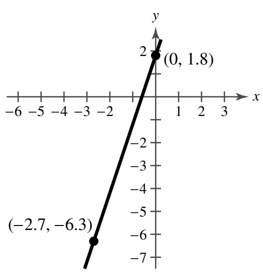
$$m = \frac{\frac{5}{6} - \left(-\frac{5}{2}\right)}{-\frac{3}{4} - \frac{1}{2}} = \frac{\frac{5}{6} + \frac{15}{6}}{-\frac{3}{4} - \frac{2}{4}} = \frac{\frac{10}{3}}{-\frac{5}{4}}$$
$$= -\frac{10}{3} \cdot \frac{4}{5} = -\frac{8}{3}$$

7.



$$m = \frac{3 - 6}{2.1 - (-4.5)} = \frac{-3}{6.6} = -\frac{30}{66} = -\frac{5}{11}$$

8.



$$m = \frac{1.8 - (-6.3)}{0 - (-2.7)} = \frac{8.1}{2.7} = 3$$

9. (a) $y + 1 = \frac{1}{4}(x - 2)$
$4y + 4 = x - 2$
$-x + 4y + 6 = 0$

(b) Three additional points:
$(2 + 4, -1 + 1) = (6, 0)$
$(6 + 4, 0 + 1) = (10, 1)$
$(10 + 4, 1 + 1) = (14, 2)$
(other answers possible)

10. (a) $y - 5 = -\frac{3}{2}(x + 3)$
$2y - 10 = -3x - 9$
$3x + 2y - 1 = 0$

(b) Three additional points:
$(-3 + 2, 5 - 3) = (-1, 2)$
$(-1 + 2, 2 - 3) = (1, -1)$
$(1 + 2, -1 - 3) = (3, -4)$

(other answers possible)

11. (a) $y + 5 = \frac{3}{2}(x - 0)$
$2y + 10 = 3x$
$-3x + 2y + 10 = 0$

(b) Three additional points:
$(0 + 2, -5 + 3) = (2, -2)$
$(2 + 2, -2 + 3) = (4, 1)$
$(4 + 2, 1 + 3) = (6, 4)$

(other answers possible)

12. (a) $y - \frac{7}{8} = -\frac{4}{5}(x - 0)$
$40y - 35 = -32x$
$32x + 40y - 35 = 0$

(b) Three additional points:
$\left(0 + 5, \frac{7}{8} - 4\right) = \left(5, -\frac{25}{8}\right)$
$\left(5 + 5, -\frac{25}{8} - 4\right) = \left(10, -\frac{57}{8}\right)$
$\left(10 + 5, -\frac{57}{8} - 4\right) = \left(15, -\frac{89}{8}\right)$

(other answers possible)

13. (a) $y - 6 = 0(x + 2) = 0$
$y = 6$ (horizontal line)
$y - 6 = 0$

(b) Three additional points:
(0, 6), (1, 6), (−1, 6)

(other answers possible)

14. (a) $y - 8 = 0(x + 8) = 0$
$y = 8$ (horizontal line)
$y - 8 = 0$

(b) Three additional points: (0, 8), (1, 8), (2, 8)

(other answers possible)

15. (a) *m* is undefined means that the line is vertical.
$x - 10 = 0$

(b) Three additional points: (10, 0), (10, 1), (10, 2)

(other answers possible)

16. (a) *m* is undefined means that the line is vertical.
$x - 5 = 0$

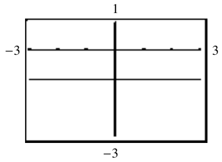
(b) Three additional points: (5, 0), (5, 1), (5, 2)
(other answers possible)

17. $(2, -1), (4, -1)$

$m = \frac{-1-(-1)}{4-2} = \frac{0}{2} = 0$ (The line is horizontal.)

$y-(-1) = 0(x-2)$

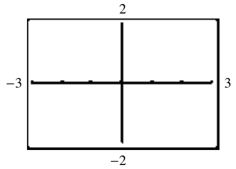
$y = -1$



18. $(0, 0), (0, 10)$

$m = \frac{10-0}{0-0} = \frac{10}{0}$, the slope is undefined and the line is vertical.

$x = 0$

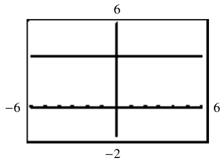


19. $\left(7, \frac{11}{3}\right), \left(9, \frac{11}{3}\right)$

$m = \frac{\frac{11}{3} - \frac{11}{3}}{9-7} = \frac{0}{2} = 0$ (The line is horizontal.)

$y - \frac{11}{3} = 0(x-7)$

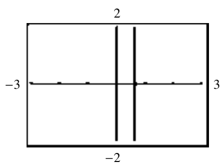
$y = \frac{11}{3}$



20. $\left(\frac{5}{8}, 4\right), \left(\frac{5}{8}, -6\right)$

$m = \frac{-6-4}{\frac{5}{8} - \frac{5}{8}} = \frac{-10}{0}$, the slope is undefined and the line is vertical.

$x = \frac{5}{8}$

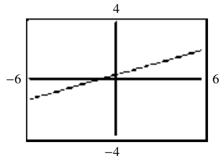


21. $(-1, 0), (6, 2)$

$m = \frac{2-0}{6-(-1)} = \frac{2}{7}$

$y - 0 = \frac{2}{7}(x+1)$

$y = \frac{2}{7}x + \frac{2}{7}$

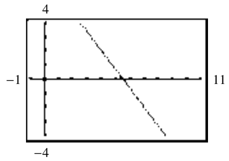


22. $(1, 6), (4, 2)$

$m = \frac{2-6}{4-1} = \frac{-4}{3}$

$y - 6 = -\frac{4}{3}(x-1)$

$y = -\frac{4}{3}x + \frac{22}{3}$

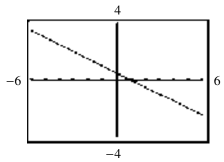


23. $(3, -1), (-3, 2)$

$m = \frac{2-(-1)}{-3-3} = \frac{3}{-6} = -\frac{1}{2}$

$y-(-1) = -\frac{1}{2}(x-3)$

$y = -\frac{1}{2}x + \frac{1}{2}$

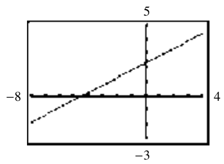


24. $\left(-\frac{5}{2}, 1\right), \left(-4, \frac{2}{9}\right)$

$m = \frac{\frac{2}{9} - 1}{-4 - (-\frac{5}{2})} = \frac{-\frac{7}{9}}{-\frac{3}{2}} = \frac{14}{27}$

$y - 1 = \frac{14}{27}\left(x - \left(-\frac{5}{2}\right)\right)$

$y = \frac{14}{27}x + \frac{62}{27}$



For Exercise 25–28, $t = 0$ corresponds to 2010.

25. $(0, 12{,}500)$, $m = 850$

$V - 12{,}500 = 850(t-0)$

$V = 850t + 12{,}500$

26. $(0,\ 3795),\ m = -115$

$$V - 3795 = -115(t - 0)$$
$$V = -115t + 3795$$

27. $(0,\ 625.50),\ m = 42.70$

$$V - 625.50 = 42.70(t - 0)$$
$$V = 42.70t + 625.50$$

28. $(0,\ 72.95),\ m = -5.15$

$$V - 72.95 = -5.15(t - 0)$$
$$V = -5.15t + 72.95$$

29. $(2,\ 160{,}000),\ (3,\ 185{,}000)$

$$m = \frac{185{,}000 - 160{,}000}{3 - 2} = 25{,}000$$
$$S - 160{,}000 = 25{,}000(t - 2)$$
$$S = 25{,}000t + 110{,}000$$

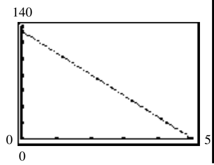
For the fourth quarter let $t = 4$. Then we have

$$S = 25{,}000(4) + 110{,}000 = \$210{,}000.$$

30. (a) Since $t = 0$ corresponds to 2010, the V-intercept is given by $(0,\ 134)$, and since the value is decreasing at a rate of \$26.80 per year, the slope is $m = -26.80$.

$$V = -26.80t + 134$$

(b)



$X_{min} = 0$
$X_{max} = 5$
$Xscl = 1$
$Y_{min} = 0$
$Y_{max} = 140$
$Y_{scl} = 20$

Explanations will vary.

(c) For 2014, $t = 14$ and
$V = -26.80(14) + 134 = \$26.80.$

(d) Set $V = 0$.

$$0 = -26.80t + 134$$
$$26.80t = 134$$
$$t = 5 \text{ or } 2015$$

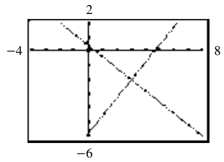
31. $5x - 4y = 8 \ \Rightarrow\ y = \frac{5}{4}x - 2$ and $m = \frac{5}{4}$

(a) Parallel slope: $m = \frac{5}{4}$

$$y - (-2) = \frac{5}{4}(x - 3)$$
$$4y + 8 = 5x - 15$$
$$0 = 5x - 4y - 23$$
$$y = \frac{5}{4}x - \frac{23}{4}$$

(b) Perpendicular slope: $m = -\frac{4}{5}$

$$y - (-2) = -\frac{4}{5}(x - 3)$$
$$5y + 10 = -4x + 12$$
$$4x + 5y - 2 = 0$$
$$y = -\frac{4}{5}x + \frac{2}{5}$$



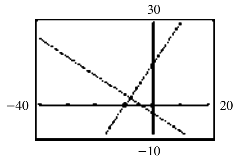
32. $2x + 3y = 5 \ \Rightarrow\ y = -\frac{2}{3}x + \frac{5}{3}$ and $m = -\frac{2}{3}$

(a) Parallel slope: $m = -\frac{2}{3}$

$$y - 3 = -\frac{2}{3}(x + 8)$$
$$3y - 9 = -2x - 16$$
$$2x + 3y + 7 = 0$$
$$y = -\frac{2}{3}x + \frac{7}{3}$$

(b) Perpendicular slope: $m = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x + 8)$$
$$2y - 6 = 3x + 24$$
$$3x - 2y + 30 = 0$$
$$y = \frac{3}{2}x + 15$$



33. (a) Not a function. 20 is assigned two different values.
(b) Function

34. (a) Function
(b) Not a function. w is assigned two different values and u is unassigned.

35. $16x^2 - y^2 = 0 \Rightarrow y = \pm 4x$

No, y is not a function of x. Some x-values correspond to two y-values. For example, $x = 1$ corresponds to $y = 4$ and $y = -4$.

36. $x^3 + y^2 = 64 \Rightarrow y = \pm\sqrt{64 - x^3}$

No, y is not a function of x. Some x-values correspond to two y-values. For example, $x = 0$, corresponds to $y = 8$ and $y = -8$.

37. $y = 2x - 3$

This is a function of x.

38. $y = -2x + 10$

This is a function of x.

39. $y = \sqrt{1 - x}$

This is a function of x.

40. $y = \sqrt{x^2 + 4}$

This is a function of x.

41. $|y| = x + 2 \Rightarrow y = x + 2$ or $y = -(x + 2)$

Thus, y is not a function of x. Some x-values correspond to two y-values. For example, $x = 1$ corresponds to $y = 3$ and $y = -3$.

42. $16 - |y| - 4x = 0 \Rightarrow |y| = 16 - 4x$

$\Rightarrow y = 16 - 4x$ or $y = -(16 - 4x)$

Thus, y is not a function of x. Some x-values correspond to two y-values. For example, $x = 0$ corresponds to $y = 16$ and $y = -16$.

43. $f(x) = x^2 + 1$

(a) $f(1) = 1^2 + 1 = 2$

(b) $f(-3) = (-3)^2 + 1 = 10$

(c) $f(b^3) = (b^3)^2 + 1 = b^6 + 1$

(d) $f(x-1) = (x-1)^2 + 1 = x^2 - 2x + 2$

44. $g(x) = \sqrt{x^2 + 1}$

(a) $g(-1) = \sqrt{(-1)^2 + 1} = \sqrt{1+1} = \sqrt{2}$

(b) $g(3) = \sqrt{3^2 + 1} = \sqrt{9+1} = \sqrt{10}$

(c) $g(3x) = \sqrt{(3x)^2 + 1} = \sqrt{9x^2 + 1}$

(d) $g(x+2) = \sqrt{(x+2)^2 + 1} = \sqrt{x^2 + 4x + 4 + 1}$

$= \sqrt{x^2 + 4x + 5}$

45. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$

(a) $h(-2) = 2(-2) + 1 = -3$

(b) $h(-1) = 2(-1) + 1 = -1$

(c) $h(0) = 0^2 + 2 = 2$

(d) $h(2) = 2^2 + 2 = 6$

46. $f(x) = \dfrac{3}{2x - 5}$

(a) $f(1) = \dfrac{3}{2(1) - 5} = -1$

(b) $f(-2) = \dfrac{3}{2(-2) - 5} = \dfrac{3}{-9} = -\dfrac{1}{3}$

(c) $f(t) = \dfrac{3}{2t - 5}$

(d) $f(10) = \dfrac{3}{2(10) - 5} = \dfrac{3}{15} = \dfrac{1}{5}$

47. The domain of $f(x) = \dfrac{x-1}{x+2}$ is all real numbers $x \ne -2$.

48. The domain of $f(x) = \dfrac{x^2}{x^2 + 1}$ is the set of all real numbers.

49. $f(x) = \sqrt{25 - x^2}$

$25 - x^2 \ge 0$

$(5 + x)(5 - x) \ge 0$

The domain is $[-5, 5]$.

50. $f(x) = \sqrt{x^2 - 16}$

$x^2 - 16 \ge 0$

$x^2 \ge 16$

The domain is $(-\infty, -4] \cup [4, \infty)$.

51. (a) $C(x) = 16{,}000 + 5.35x$

(b) $P(x) = R(x) - C(x)$

$= 8.20x - (16{,}000 + 5.35x)$

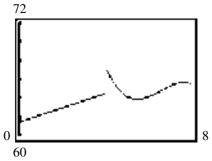
$= 2.85x - 16{,}000$

52. (a)

t	0	1	2	3	4
$n(t)$	61.40	62.16	62.92	63.68	64.44

t	5	6	7	8
$n(t)$	64.19	64.09	65.19	65.49

(b)



2009: 62.99 million; 2010: 55.7 million;

2011: 41.61 million; 2012: 18.72 million

The values seem unreasonable because there is a steep decline of enrollment over those years.

53. $f(x) = 2x^2 + 3x - 1$

$f(x+h) = 2(x+h)^2 + 3(x+h) - 1$

$= 2x^2 + 4xh + 2h^2 + 3x + 3h - 1$

$\dfrac{f(x+h) - f(x)}{h} =$

$\dfrac{(2x^2 + 4xh + 2h^2 + 3x + 3h - 1) - (2x^2 + 3x - 1)}{h}$

$= \dfrac{4xh + 2h^2 + 3h}{h}$

$= 4x + 2h + 3, \; h \ne 0$

54. $f(x) = x^2 - 3x + 5$

$f(x+h) = (x+h)^2 - 3(x+h) + 5$

$= x^2 + 2xh + h^2 - 3x - 3h + 5$

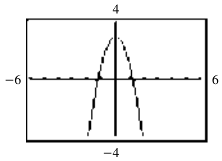
$\dfrac{f(x+h) - f(x)}{h} = \dfrac{x^2 + 2xh + h^2 - 3x - 3h + 5 - (x^2 - 3x + 5)}{h}$

$= \dfrac{2xh + h^2 - 3h}{h}$

$= \dfrac{h(2x + h - 3)}{h}$

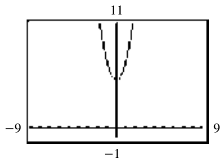
$= 2x + h - 3, \; h \ne 0$

55. Domain: all real numbers x



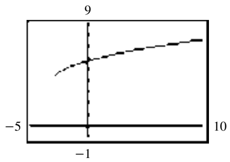
Range: $y \le 3$

56.



Domain: all real numbers x
Range: $[5, \infty)$

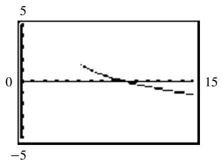
57.



Domain: $[-3, \infty)$

Range: $[4, \infty)$

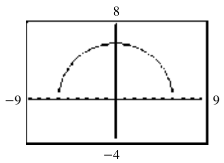
58.



Domain: $[5, \infty)$

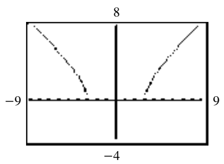
Range: $(-\infty, 2]$

59.



Domain: $36 - x^2 \ge 0 \Rightarrow x^2 \le 36 \Rightarrow -6 \le x \le 6$
Range: $0 \le y \le 6$

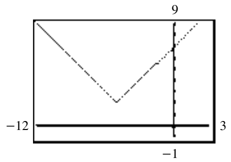
60.



Domain: $(-\infty, -3], [3, \infty)$

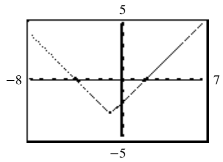
Range: $[0, \infty)$

61.



Domain: all real numbers x
Range: $[2, \infty)$

62.



Domain: all real numbers x
Range: $[-3, \infty)$

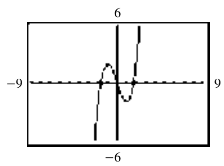
63. $y - 4x = x^2$
A vertical line intersects the graph just once, so y is a function of x. Solve for y and graph $y_1 = x^2 + 4x$.

64. $|x+5| - 2y = 0$
A vertical line intersects the graph just once, so y is a function of x. Solve for y and graph $y_1 = \frac{1}{2}|x+5|$.

65. $3x + y^2 - 2 = 0$
A vertical line intersects the graph more than once, so y is not a function of x. Solve for y and graph
$y_1 = \sqrt{-3x+2}$ and $y_2 = -\sqrt{-3x+2}$.

66. $x^2 + y^2 - 49 = 0$
A vertical line intersects the graph more than once, so y is not a function of x. Solve for y and graph
$y_1 = \sqrt{49 - x^2}$ and $y_2 = -\sqrt{49 - x^2}$.

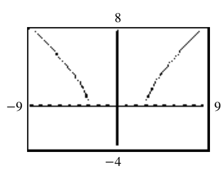
67. $f(x) = x^3 - 3x$
(a)



(b) Increasing on $(-\infty, -1)$ and $(1, \infty)$

Decreasing on $(-1, 1)$

68. $f(x) = \sqrt{x^2 - 9}$
(a)

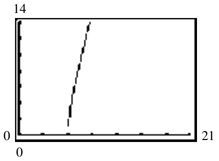


(b) Increasing on $(3, \infty)$

Decreasing on $(-\infty, -3)$

69. $f(x) = x\sqrt{x-6}$

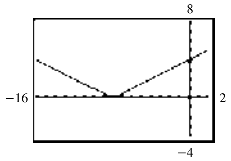
(a)



(b) Increasing on $(6, \infty)$

70. $f(x) = \dfrac{|x+8|}{2}$

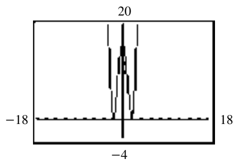
(a)



(b) Increasing on $(-8, \infty)$

(c) Decreasing on $(-\infty, -8)$

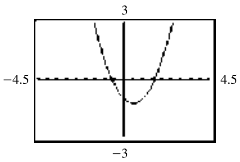
71. $f(x) = (x^2 - 4)^2$



Relative minima: $(-2, 0)$ and $(2, 0)$

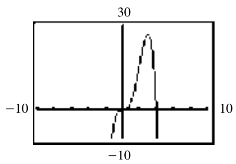
Relative maximum: $(0, 16)$

72. $f(x) = x^2 - x - 1$



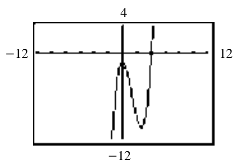
Relative minimum: $(0.5, -1.25)$

73. $h(x) = 4x^3 - x^4$



Relative maximum: $(3, 27)$

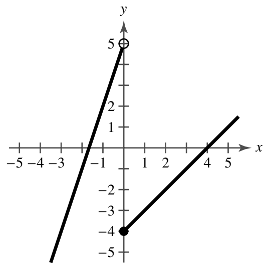
74. $f(x) = x^3 - 4x^2 - 1$



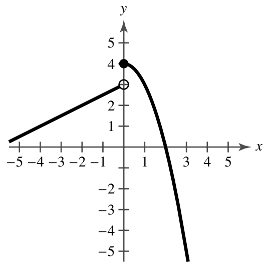
Relative maximum: $(0, -1)$

Relative minimum: $(2.67, -10.48)$

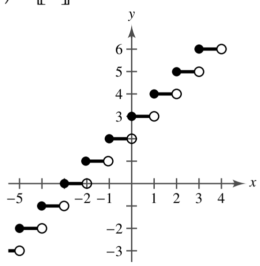
75. $f(x) = \begin{cases} 3x + 5, & x < 0 \\ x - 4, & x \geq 0 \end{cases}$



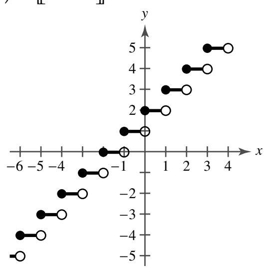
76. $f(x) = \begin{cases} \frac{1}{2}x + 3, & x < 0 \\ 4 - x^2, & x \geq 0 \end{cases}$



77. $f(x) = [\![x]\!] + 3$



78. $f(x) = [\![x + 2]\!]$

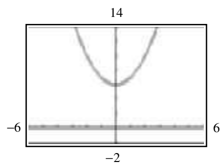


79. $f(-x) = (-x)^2 + 6$

$= x^2 + 6$

$= f(x)$

f is even.



The graph is symmetric with respect to the y-axis. So, f is even.

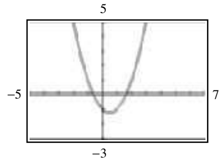
80. $f(-x) = (-x)^2 - (-x) - 1$

$= x^2 + x - 1$

$\neq f(x)$

and $f(-x) \neq -f(x)$

f is neither even nor odd.



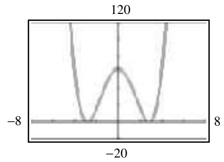
The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, f is neither even nor odd.

81. $f(-x) = \left((-x)^2 - 8\right)^2$

$= \left(x^2 - 8\right)^2$

$= f(x)$

f is even.



The graph is symmetric with respect to the y-axis. So, f is even.

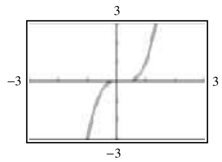
82. $f(-x) = 2(-x)^3 - (-x)^2$

$= -2x^3 - x^2$

$\neq f(x)$

and $f(-x) \neq -f(x)$

f is neither even nor odd.

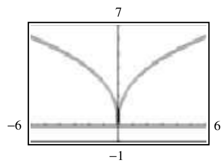


The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, f is neither even nor odd.

83. $f(-x) = 3(-x)^{\frac{5}{2}} \neq f(x)$ and $f(-x) \neq -f(x)$

f is neither even nor odd.

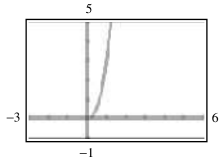
(Note that the domain of f is $x \geq 0$.)



The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, f is neither even nor odd.

84. $f(-x) = 3(-x)^{\frac{2}{5}} = 3x^{\frac{2}{5}} = f(x)$

f is even.



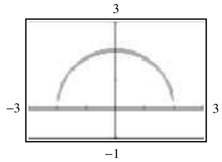
The graph is symmetric with respect to the y-axis. So, f is even.

85. $f(-x) = \sqrt{4 - (-x)^2}$

$= \sqrt{4 - x^2}$

$= f(x)$

f is even.



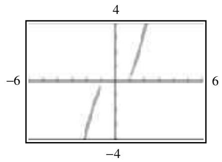
The graph is symmetric with respect to the y-axis. So, f is even.

86. $f(-x) = (-x)\sqrt{(-x)^2 - 1}$

$= -x\sqrt{x^2 - 1}$

$= -f(x)$

f is odd.



The graph is symmetric with respect to the origin. So, f is odd.

87. Horizontal shift three units to the right of $f(x) = \dfrac{1}{x}$: $y = \dfrac{1}{x - 3}$

88. Reflection in the x-axis, followed by a vertical shift five units upward of $f(x) = x$: $y = -x + 5$

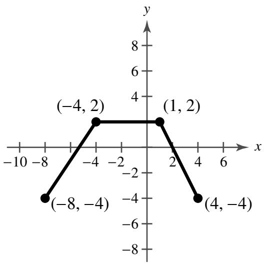
89. Horizontal shift two units to the right, followed by a vertical shift one unit upward of $f(x) = x^2$: $y = (x - 2)^2 + 1$

90. Reflection in the x-axis, followed by a vertical shift two units downward of $f(x) = x^3$: $y = -x^3 - 2$

91. Vertical shift three units upward of $f(x) = |x|$: $y = |x| + 3$

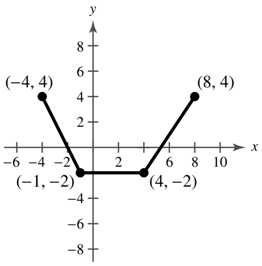
92. Horizontal shift three units to the right, followed by a reflection in the x-axis of $f(x) = \sqrt{x}$: $y = -\sqrt{x - 3}$

93.



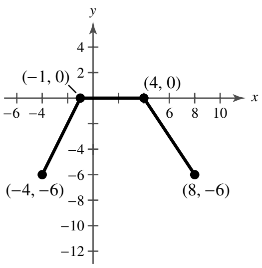
$y = f(-x)$ is a reflection in the y-axis.

94.



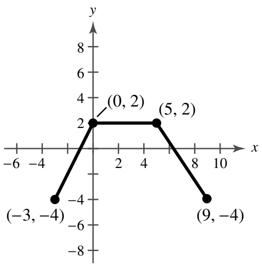
$y = -f(x)$ is a reflection in the x-axis.

95.



$y = f(x) - 2$ is a vertical shift two units downward.

96.



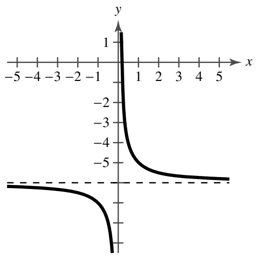
$y = f(x-1)$ is a horizontal shift one unit to the right.

97. $h(x) = \dfrac{1}{x} - 6$

(a) $f(x) = \dfrac{1}{x}$

(b) The graph of h is a vertical shift six units downward of f.

(c)



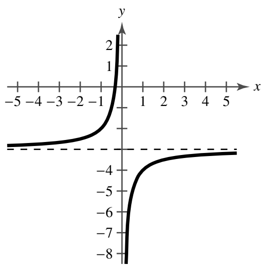
(d) $h(x) = f(x) - 6$

98. $h(x) = -\dfrac{1}{x} - 3$

(a) $f(x) = \dfrac{1}{x}$

(b) The graph of h is a reflection in the x-axis and a vertical shift three units downward of f.

(c)



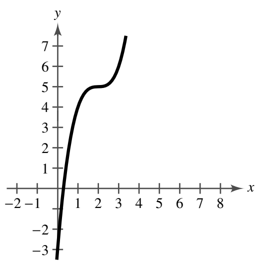
(d) $h(x) = -f(x) - 3$

99. $h(x) = (x-2)^3 + 5$

(a) $f(x) = x^3$

(b) The graph of h is a horizontal shift of f two units to the right, followed by a vertical shift five units upward of f.

(c)



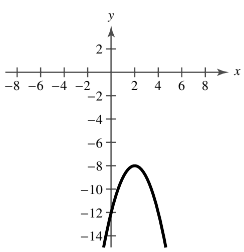
(d) $h(x) = (x-2)^3 + 5 = f(x-2) + 5$

100. $h(x) = -(x-2)^2 - 8$

(a) $f(x) = x^2$

(b) h is a horizontal shift two units to the right, a reflection in the x-axis, followed by a vertical shift eight units downward of f.

(c)



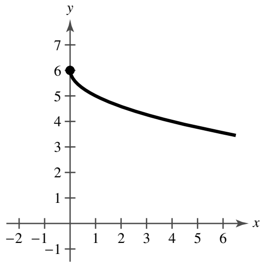
(d) $h(x) = -f(x-2) - 8$

101. $h(x) = -\sqrt{x} + 6$

(a) $f(x) = \sqrt{x}$

(b) The graph of h is a reflection in the x-axis and a vertical shift six units upward of f.

(c)



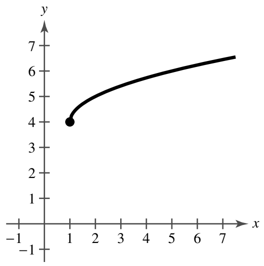
(d) $h(x) = -f(x) + 6$

102. $h(x) = \sqrt{x-1} + 4$

(a) $f(x) = \sqrt{x}$

(b) The graph of h is a horizontal shift one unit to the right and a vertical shift four units upward of f.

(c)



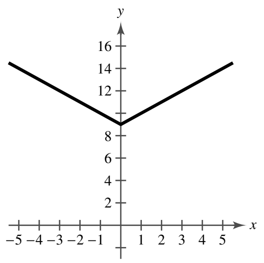
(d) $h(x) = f(x-1) + 4$

103. $h(x) = |x| + 9$

(a) $f(x) = |x|$

(b) The graph of h is a vertical shift of f nine units upward.

(c)



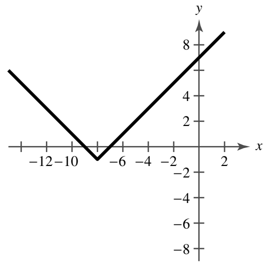
(d) $h(x) = |x| + 9$
$= f(x) + 9$

104. $h(x) = |x+8| - 1$

(a) $f(x) = |x|$

(b) h is a horizontal shift eight units to the left, followed by a vertical shift one unit downward of f.

(c)



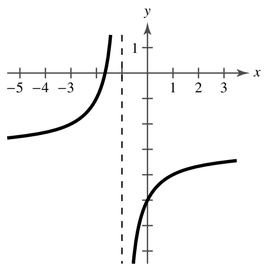
(d) $h(x) = f(x+8) - 1$

105. $h(x) = \dfrac{-2}{x+1} - 3$

(a) $f(x) = \dfrac{1}{x}$

(b) h is a horizontal shift one unit to the left, a reflection in the x-axis, a vertical stretch, followed by a vertical shift three units downward of f.

(c)



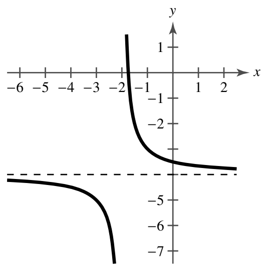
(d) $h(x) = -2f(x+1) - 3$

106. $h(x) = \dfrac{1}{x+2} - 4$

(a) $f(x) = \dfrac{1}{x}$

(b) h is a horizontal shift two units to the left, followed by a vertical shift four units downward of f.

(c)



(d) $h(x) = f(x+2) - 4$

107.
$$\begin{aligned}(f-g)(4) &= f(4) - g(4)\\ &= \left[3 - 2(4)\right] - \sqrt{4}\\ &= -5 - 2\\ &= -7\end{aligned}$$

108.
$$\begin{aligned}(f+h)(5) &= f(5) + h(5)\\ &= -7 + 77\\ &= 70\end{aligned}$$

109.
$$\begin{aligned}(f+g)(25) &= f(25) + g(25)\\ &= -47 + 5\\ &= -42\end{aligned}$$

110. $(g-h)(1) = g(1) - h(1) = 1 - 5 = -4$

111. $(fh)(1) = f(1)h(1) = (3-2(1))(3(1)^2+2) = (1)(5) = 5$

112. $\left(\frac{g}{h}\right)(1) = \frac{g(1)}{h(1)} = \frac{1}{5}$

113. $(h \circ g)(5) = h(g(5)) = h\left(\sqrt{5}\right) = 3\left(\sqrt{5}\right)^2 + 2 = 17$

114. $(g \circ f)(-3) = g(f(-3)) = g(9) = \sqrt{9} = 3$

115. $(f \circ h)(-4) = f(h(-4)) = f(50) = 3 - 2(50) = -97$

116. $(g \circ h)(6) = g(h(6)) = g(110) = \sqrt{110}$

117. $f(x) = x^2, g(x) = x + 3$

$(f \circ g)(x) = f(x+3) = (x+3)^2 = h(x)$

118. $f(x) = x^3, g(x) = 1 - 2x$

$(f \circ g)(x) = f(1-2x) = (1-2x)^3 = h(x)$

119. $f(x) = \sqrt{x}, g(x) = 4x + 2$

$(f \circ g)(x) = f(4x+2) = \sqrt{4x+2} = h(x)$

120. $f(x) = \sqrt[3]{x}, g(x) = (x+2)^2$

$(f \circ g)(x) = f\left((x+2)^2\right) = \sqrt[3]{(x+2)^2} = h(x)$

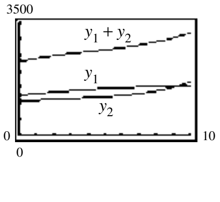
121. $f(x) = \frac{4}{x}, g(x) = x + 2$

$(f \circ g)(x) = f(x+2) = \frac{4}{x+2} = h(x)$

122. $f(x) = \frac{6}{x^3}, g(x) = 3x + 1$

$(f \circ g)(x) = f(3x+1) = \frac{6}{(3x+1)^3} = h(x)$

123.



124. For 2010, $t = 10$.

$y_1 + y_2 = \left[-2.61(10)^2 + 55.0(10) + 1244\right] + \left[0.949(10)^3 - 8.02(10)^2 + 44.4(10) + 1056\right]$
$= 1533 + 1647$
$= 3180$ thousand or 3,180,000 students

125. $f(x) = 6x$

$f^{-1}(x) = \frac{1}{6}x$

$f(f^{-1}(x)) = f\left(\frac{1}{6}x\right) = 6\left(\frac{1}{6}x\right) = x$

$f^{-1}(f(x)) = f^{-1}(6x) = \frac{1}{6}(6x) = x$

126. $f(x) = x + 5$

$f^{-1}(x) = x - 5$

$f(f^{-1}(x)) = f(x-5) = (x-5) + 5 = x$

$f^{-1}(f(x)) = f^{-1}(x+5) = (x+5) - 5 = x$

127. $f(x) = \frac{1}{2}x + 3$

$f^{-1}(x) = 2(x-3) = 2x - 6$

$f(f^{-1}(x)) = f(2(x-3)) = \frac{1}{2}(2(x-3)) + 3 = x - 3 + 3 = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{2}x + 3\right) = 2\left(\frac{1}{2}x + 3 - 3\right) = 2\left(\frac{1}{2}x\right) = x$

128. $f(x) = \frac{x-4}{5}$

$f^{-1}(x) = 5x + 4$

$f(f^{-1}(x)) = f(5x+4) = \frac{5x+4-4}{5} = \frac{5x}{5} = x$

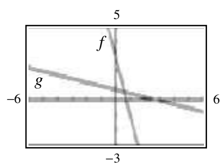
$f^{-1}(f(x)) = f^{-1}\left(\frac{x-4}{5}\right) = 5\left(\frac{x-4}{5}\right) + 4 = x - 4 + 4 = x$

129. $f(x) = 3 - 4x, g(x) = \frac{3-x}{4}$

(a) $f(g(x)) = 3 - 4\left(\frac{3-x}{4}\right) = 3 - (3-x) = x$

$g(f(x)) = \frac{3-(3-4x)}{4} = \frac{4x}{4} = x$

(b)



(c)

$Y_1 = 3 - 4X$

$Y_2 = \dfrac{3 - X}{4}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

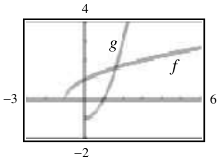
X	Y_3	Y_4
−2	−2	−2
−1	−1	−1
0	0	0
1	1	1
2	2	2

130. $f(x) = \sqrt{x+1},\ g(x) = x^2 - 1,\ x \ge 0$

(a) $f(g(x)) = \sqrt{(x^2 - 1) + 1} = \sqrt{x^2} = x$

$g(f(x)) = \left(\sqrt{x+1}\right)^2 - 1 = x + 1 - 1 = x$

(b)



(c)

$Y_1 = \sqrt{x} + 1$

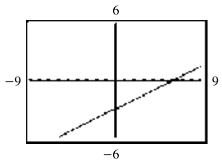
$Y_2 = x^2 - 1,\ X \ge 0$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

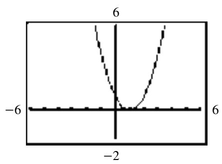
X	Y_3	Y_4
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4

131.



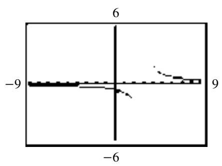
$f(x) = \dfrac{1}{2}x - 3$ passes the Horizontal Line Test, and hence is one-to-one and has an inverse function.

132.



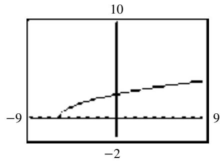
$f(x) = (x-1)^2$ does not pass the Horizontal Line Test, so f is not one-to-one.

133.



$h(t) = \dfrac{2}{t-3}$ passes the Horizontal Line Test, and hence is one-to-one and has an inverse function.

134.



$g(x) = \sqrt{x+6}$ passes the Horizontal Line Test, and hence is one-to-one and has an inverse function.

135.

$$y = \frac{1}{2}x - 5$$
$$x = \frac{1}{2}y - 5$$
$$x + 5 = \frac{1}{2}y$$
$$y = 2(x+5)$$
$$f^{-1}(x) = 2x + 10$$

136.

$$f(x) = \frac{7x+3}{8}$$
$$y = \frac{1}{8}(7x+3)$$
$$x = \frac{1}{8}(7y+3)$$
$$8x = 7y + 3$$
$$8x - 3 = 7y$$
$$f^{-1}(x) = \frac{1}{7}(8x-3)$$

137.

$$f(x) = 4x^3 - 3$$
$$y = 4x^3 - 3$$
$$x = 4y^3 - 3$$
$$x + 3 = 4y^3$$
$$\frac{x+3}{4} = y^3$$
$$f^{-1}(x) = \sqrt[3]{\frac{x+3}{4}}$$

138.

$$y = 5x^3 + 2$$
$$x = 5y^3 + 2$$
$$x - 2 = 5y^3$$
$$\frac{x-2}{5} = y^3$$
$$f^{-1}(x) = \sqrt[3]{\frac{x-2}{5}}$$

139.

$$f(x) = \sqrt{x+10}$$
$$y = \sqrt{x+10},\ x \ge -10,\ y \ge 0$$
$$x = \sqrt{y+10},\ y \ge -10,\ x \ge 0$$
$$x^2 = y + 10$$
$$x^2 - 10 = y$$
$$f^{-1}(x) = x^2 - 10,\ x \ge 0$$

140. $f(x) = 4\sqrt{6-x},\ x \le 6,\ y \ge 0$

$$y = 4\sqrt{6-x}$$
$$x = 4\sqrt{6-y},\ y \le 6,\ x \ge 0$$
$$x^2 = 16(6-y) = 96 - 16y$$
$$16y = 96 - x^2$$
$$y = \frac{96-x^2}{16}$$
$$f^{-1}(x) = \frac{96-x^2}{16},\ x \ge 0$$

141. $f(x) = \frac{1}{4}x^2 + 1,\ x \ge 0$

$$y = \frac{1}{4}x^2 + 1$$
$$x = \frac{1}{4}y^2 + 1$$
$$x - 1 = \frac{1}{4}y^2$$
$$4(x-1) = y^2$$
$$f^{-1}(x) = \sqrt{4(x-1)} = 2\sqrt{x-1}$$

The positive square root is chosen as f^{-1} since the domain of f is $[0, \infty)$.

142. $f(x) = 5 - \frac{1}{9}x^2,\ x \ge 0$

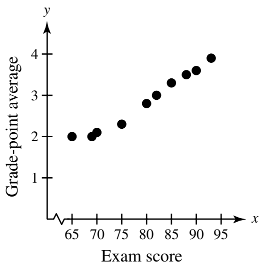
$$y = 5 - \frac{1}{9}x^2$$
$$x = 5 - \frac{1}{9}y^2$$
$$x - 5 = -\frac{1}{9}y^2$$
$$-9(x-5) = y^2$$
$$f^{-1}(x) = \sqrt{-9(x-5)} = 3\sqrt{5-x}$$

The positive square root is chosen as f^{-1} since the domain of f is $[0, \infty)$.

143. Negative correlation

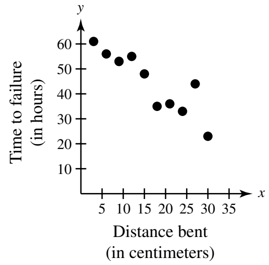
144. No correlation

145. (a)



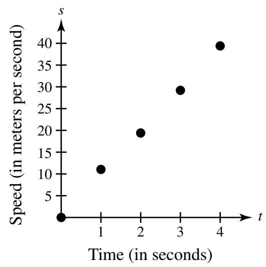
(b) Yes, the relationship is approximately linear. Higher entrance exam scores, x, are associated with higher grade-point averages, y.

146. (a)



(b) Yes, the relationship is approximately linear. The time to failure, y, decreases as the distance bent, x, increases.

147. (a)



(b) $s \approx 10t$ (Approximations will vary.)

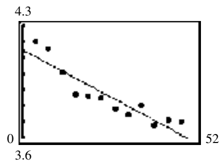
(c) $s = 9.7t + 0.4;\ r \approx 0.99933$

(d) For $t = 2.5$:

$$s = 9.7(2.5) + 0.4$$
$$= 24.25 + 0.4$$
$$= 24.65 \text{ m/sec}$$

148. (a) $y = -0.0109x + 4.146$

(b)



(c) and (d) Answers will vary.

149. False. $g(x) = -[(x-6)^2 + 3] = -(x-6)^2 - 3$ and $g(-1) = -52 \ne 28$

150. True. $f^{-1}(x) = x^{1/n}$, n odd

151. False. $f(x) = \frac{1}{x}$ or $f(x) = x$ satisfies $f = f^{-1}$.

Chapter 1 Test

1. $5x + 2y = 3$

$$2y = -5x + 3$$

$$y = -\frac{5}{2}x + \frac{3}{2}$$

$$\text{Slope} = -\frac{5}{2}$$

(a) Parallel line slope: $-\frac{5}{2}$

$$y - 4 = -\frac{5}{2}(x - 0)$$

$$y = -\frac{5}{2}x + 4$$

$$5x + 2y - 8 = 0$$

(b) Perpendicular line slope: $\frac{2}{5}$

$$y - 4 = \frac{2}{5}(x - 0)$$

$$y = \frac{2}{5}x + 4$$

$$2x - 5y + 20 = 0$$

2. $\text{Slope} = \frac{4 - (-1)}{-3 - 2} = \frac{5}{-5} = -1$

$$y + 1 = -1(x - 2)$$

$$y = -x + 1$$

3. No. For some x there corresponds more than one value of y. For instance, if $x = 1$, $y = \pm\frac{1}{\sqrt{3}}$.

4. $f(x) = |x + 2| - 15$

(a) $f(-8) = |-8 + 2| - 15 = 6 - 15 = -9$

(b) $f(14) = |14 + 2| - 15 = 16 - 15 = 1$

(c) $f(t - 6) = |t - 6 + 2| - 15 = |t - 4| - 15$

5. $3 - x \ge 0 \Rightarrow$ domain is all $x \le 3$.

6. Total Cost = Variable Costs + Fixed Costs

$$C = 25.60x + 24{,}000$$

Revenue = Price per unit × number of units

$$R = 99.50x$$

Profit = Revenue − Cost

$$P = 99.50x - (25.60x + 24{,}000)$$

$$= 73.90x - 24{,}000$$

7. $f(-x) = 2(-x)^3 - 3(-x)$

$$= -2x^3 + 3x = -f(x)$$

Odd

8. $f(-x) = 3(-x)^4 + 5(-x)^2$

$$= 3x^4 + 5x^2 = f(x)$$

Even

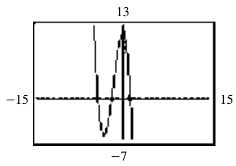
9. $h(x) = \frac{1}{4}x^4 - 2x^2 = \frac{1}{4}x^2(x^2 - 8)$

By graphing h, you see that the graph is increasing on $(-2, 0)$ and $(2, \infty)$ and decreasing on $(-\infty, -2)$ and $(0, 2)$.

10. $g(t) = |t + 2| - |t - 2|$

By graphing g, you see that the graph is increasing on $(-2, 2)$, and constant on $(-\infty, -2)$ and $(2, \infty)$.

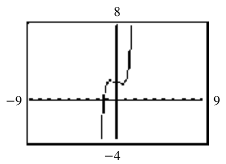
11.



Relative minimum: $(-3.33, -6.52)$

Relative maximum: $(0, 12)$

12.



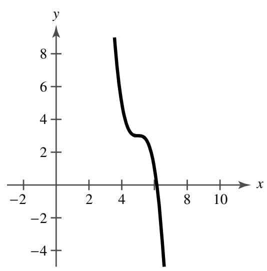
Relative minimum: $(0.77, 1.81)$

Relative maximum: $(-0.77, 2.19)$

13. (a) $f(x) = x^3$

(b) g is obtained from f by a horizontal shift five units to the right, a vertical stretch of 2, a reflection in the x-axis, and a vertical shift three units upward.

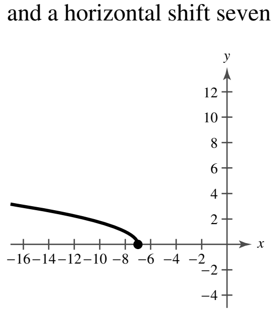
(c)



14. (a) $f(x) = \sqrt{x}$

(b) g is obtained from f by a reflection in the y-axis, and a horizontal shift seven units to the left.

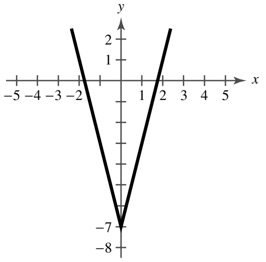
(c)



15. (a) $f(x) = |x|$

(b) g is obtained from f by a vertical stretch of 4 followed by a vertical shift seven units downward.

(c)



16. (a) $(f - g)(x) = x^2 - \sqrt{2 - x}$

Domain: $x \le 2$

(b) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2}{\sqrt{2 - x}}$

Domain: $x < 2$

(c) $(f \circ g)(x) = f\left(\sqrt{2 - x}\right) = 2 - x$

Domain: $x \le 2$

(d) $(g \circ f)x = g\left(x^2\right) = \sqrt{2 - x^2}$

Domain: $-\sqrt{2} \le x \le \sqrt{2}$

17. $f(x) = x^3 + 8$

Yes, f is one-to-one and has an inverse function.

$$y = x^3 + 8$$
$$x = y^3 + 8$$
$$x - 8 = y^3$$
$$\sqrt[3]{x - 8} = y$$
$$f^{-1}(x) = \sqrt[3]{x - 8}$$

18. $f(x) = x^2 + 6$

No, f is not one-to-one, and does not have an inverse function.

19. $f(x) = \dfrac{3x\sqrt{x}}{8}$

Yes, f is one-to-one and has an inverse function.

$$y = \frac{3}{8}x^{3/2},\ x \ge 0,\ y \ge 0$$
$$x = \frac{3}{8}y^{3/2},\ y \ge 0,\ x \ge 0$$
$$\frac{8}{3}x = y^{3/2}$$
$$\left(\frac{8}{3}x\right)^{2/3} = y$$
$$f^{-1}(x) = \left(\frac{8}{3}x\right)^{2/3},\ x \ge 0$$

20. $C = 1.686t + 31.09$

Let $C = 50$ and solve for t.

$$50 = 1.686t + 31.09$$
$$18.91 = 1.686t$$
$$t \approx 11.21 \text{ or approximately } 2012$$

CHAPTER 2

Section 2.1

1. nonnegative integer, real

2. quadratic, parabola

3. Yes, $f(x) = (x - 2)^2 + 3$ is in the form $f(x) = a(x - h)^2 + k$. The vertex is $(2, 3)$.

4. No, the graph of $f(x) = -3x^2 + 5x + 2$ is a parabola opening downward, therefore there is a relative maximum at its vertex.

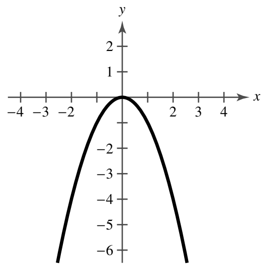
5. $f(x) = (x - 2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (c).

6. $f(x) = 3 - x^2$ opens downward and has vertex $(0, 3)$. Matches graph (d).

7. $f(x) = x^2 + 3$ opens upward and has vertex $(0, 3)$. Matches graph (b).

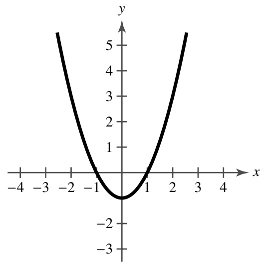
8. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (a).

9.



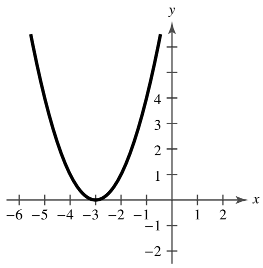
The graph of $y = -x^2$ is a reflection of $y = x^2$ in the x-axis.

10.



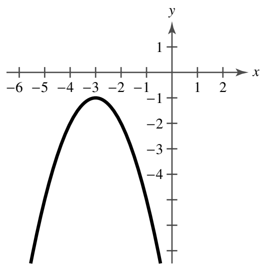
The graph of $y = x^2 - 1$ is a vertical shift downward one unit of $y = x^2$.

11.



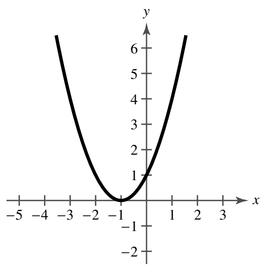
The graph of $y = (x + 3)^2$ is a horizontal shift three units to the left of $y = x^2$.

12.



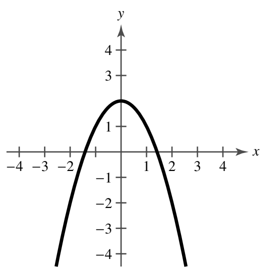
The graph of $y = -(x + 3)^2 - 1$ is a reflection in the x-axis, a horizontal shift three units to the left, and a vertical shift one unit downward of $y = x^2$.

13.



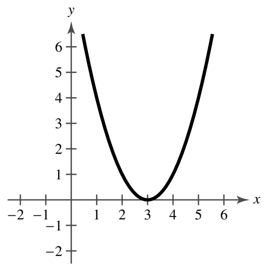
The graph of $y = (x + 1)^2$ is a horizontal shift one unit to the left of $y = x^2$.

14.



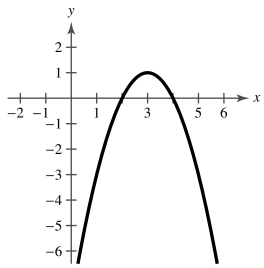
The graph of $y = -x^2 + 2$ is a reflection in the x-axis and a vertical shift two units upward of $y = x^2$.

15.



The graph of $y=(x-3)^2$ is a horizontal shift three units to the right of $y=x^2$.

16.



The graph of $y=-(x-3)^2+1$ is a horizontal shift three units to the right, a reflection in the x-axis, and a vertical shift one unit upward of $y=x^2$.

17. $f(x)=25-x^2$

$=-x^2+25$

A parabola opening downward with vertex $(0,\ 25)$

18. $f(x)=x^2-7$

A parabola opening upward with vertex $(0,\ -7)$

19. $f(x)=\frac{1}{2}x^2-4$

A parabola opening upward with vertex $(0,\ -4)$

20. $f(x)=16-\frac{1}{4}x^2$

$=-\frac{1}{4}x^2+16$

A parabola opening downward with vertex $(0,\ 16)$

21. $f(x)=(x+4)^2-3$

A parabola opening upward with vertex $(-4,\ -3)$

22. $f(x)=(x-6)^2+3$

A parabola opening upward with vertex $(6,\ 3)$

23. $h(x)=x^2-8x+16$

$=(x-4)^2$

A parabola opening upward with vertex $(4,\ 0)$

24. $g(x)=x^2+2x+1$

$=(x+1)^2$

A parabola opening upward with vertex $(-1,\ 0)$

25. $f(x)=x^2-x+\frac{5}{4}$

$=(x^2-x)+\frac{5}{4}$

$=\left(x^2-x+\frac{1}{4}\right)+\frac{5}{4}-\frac{1}{4}$

$=\left(x-\frac{1}{2}\right)^2+1$

A parabola opening upward with vertex $\left(\frac{1}{2},\ 1\right)$

26. $f(x)=x^2+3x+\frac{1}{4}$

$=(x^2+3x)+\frac{1}{4}$

$=\left(x^2+3x+\frac{9}{4}\right)+\frac{1}{4}-\frac{9}{4}$

$=\left(x+\frac{3}{2}\right)^2-2$

A parabola opening upward with vertex $\left(-\frac{3}{2},\ -2\right)$

27. $f(x)=-x^2+2x+5$

$=-(x^2-2x)+5$

$=-(x^2-2x+1)+5+1$

$=-(x-1)^2+6$

A parabola opening downward with vertex $(1,6)$

28. $f(x)=-x^2-4x+1$

$=-(x^2+4x)+1$

$=-(x^2+4x+4)+1+4$

$=-(x+2)^2+5$

A parabola opening downward with vertex $(-2,\ 5)$

29. $h(x)=4x^2-4x+21$

$=4(x^2-x)+21$

$=4\left(x^2-x+\frac{1}{4}\right)+21-4\left(\frac{1}{4}\right)$

$=4\left(x-\frac{1}{2}\right)^2+20$

A parabola opening upward with vertex $\left(\frac{1}{2},\ 20\right)$

30. $f(x)=2x^2-x+1$

$=2\left(x^2-\frac{1}{2}x\right)+1$

$=2\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)+1-2\left(\frac{1}{16}\right)$

$=2\left(x-\frac{1}{4}\right)^2+\frac{7}{8}$

A parabola opening upward with vertex $\left(\frac{1}{4},\ \frac{7}{8}\right)$

31. $$f(x) = -(x^2 + 2x - 3)$$
$$= -(x^2 + 2x) + 3$$
$$= -(x^2 + 2x + 1) + 3 + 1$$
$$= -(x + 1)^2 + 4$$
$$-(x^2 + 2x - 3) = 0$$
$$-(x + 3)(x - 1) = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x - 1 = 0 \Rightarrow x = 1$$

A parabola opening downward with vertex $(-1,\ 4)$ and x-intercepts $(-3,\ 0)$ and $(1,\ 0)$.

32. $$f(x) = -(x^2 + x - 30)$$
$$= -(x^2 + x) + 30$$
$$= -\left(x^2 + x + \frac{1}{4}\right) + 30 + \frac{1}{4}$$
$$= -\left(x + \frac{1}{2}\right)^2 + \frac{121}{4}$$
$$-(x^2 + x - 30) = 0$$
$$-(x - 5)(x + 6) = 0$$
$$x - 5 = 0 \Rightarrow x = 5$$
$$x + 6 = 0 \Rightarrow x = -6$$

A parabola opening downward with vertex $\left(-\frac{1}{2}, \frac{121}{4}\right)$ and x-intercepts $(5,\ 0)$ and $(-6,\ 0)$.

33. $$g(x) = x^2 + 8x + 11$$
$$= (x^2 + 8x) + 11$$
$$= (x^2 + 8x + 16) + 11 - 16$$
$$= (x + 4)^2 - 5$$
$$x^2 + 8x + 11 = 0$$
$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(11)}}{2(1)}$$
$$= \frac{-8 \pm \sqrt{64 - 44}}{2}$$
$$= \frac{-8 \pm \sqrt{20}}{2}$$
$$= \frac{-8 \pm 2\sqrt{5}}{2}$$
$$= -4 \pm \sqrt{5}$$

A parabola opening upward with vertex $(-4,\ -5)$ and x-intercepts $(-4 \pm \sqrt{5},\ 0)$.

34. $$f(x) = x^2 + 10x + 14$$
$$= (x^2 + 10x) + 14$$
$$= (x^2 + 10x + 25) + 14 - 25$$
$$= (x + 5)^2 - 11$$
$$x^2 + 10x + 14 = 0$$
$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(14)}}{2(1)}$$
$$= \frac{-10 \pm \sqrt{100 - 56}}{2}$$
$$= \frac{-10 \pm \sqrt{44}}{2}$$
$$= \frac{-10 \pm 2\sqrt{11}}{2}$$
$$= -5 \pm \sqrt{11}$$

A parabola opening upward with vertex $(-5,\ -11)$ and x-intercepts $(-5 \pm \sqrt{11},\ 0)$

35. $$f(x) = -2x^2 + 16x - 31$$
$$= -2(x^2 - 8x) - 31$$
$$= -2(x^2 - 8x + 16) - 31 + 32$$
$$= -2(x - 4)^2 + 1$$
$$-2x^2 + 16x - 31 = 0$$
$$x = \frac{-16 \pm \sqrt{16^2 - 4(-2)(-31)}}{2(-2)}$$
$$= \frac{-16 \pm \sqrt{256 - 248}}{-4}$$
$$= \frac{-16 \pm \sqrt{8}}{-4}$$
$$= \frac{-16 \pm 2\sqrt{2}}{-4}$$
$$= 4 \pm \frac{1}{2}\sqrt{2}$$

A parabola opening downward with vertex (4, 1) and x-intercepts $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$

36. $$f(x) = -4x^2 + 24x - 41$$
$$= -4(x^2 - 6x) - 41$$
$$= -4(x^2 - 6x + 9) - 41 + 36$$
$$= -4(x - 3)^2 - 5$$
$$-4x^2 + 24x - 41 = 0$$
$$x = \frac{-24 \pm \sqrt{24^2 - 4(-4)(-41)}}{2(-4)}$$
$$= \frac{-24 \pm \sqrt{576 - 656}}{-8}$$
$$= \frac{-24 \pm \sqrt{-80}}{-8}$$

No real solution

A parabola opening downward with vertex $(3,\ -5)$ and no x-intercepts

37. $(-1,\ 4)$ is the vertex.

$f(x) = a(x+1)^2 + 4$

Since the graph passes through the point $(1,\ 0)$, we have:

$$\begin{aligned} 0 &= a(1+1)^2 + 4 \\ 0 &= 4a + 4 \\ -1 &= a \end{aligned}$$

Thus, $f(x) = -(x+1)^2 + 4$. Note that $(-3,\ 0)$ is on the parabola.

38. $(-2,\ -1)$ is the vertex.

$f(x) = a(x+2)^2 - 1$

Since the graph passes through $(0,\ 3)$, we have:

$$\begin{aligned} 3 &= a(0+2)^2 - 1 \\ 3 &= 4a - 1 \\ 4 &= 4a \\ 1 &= a \end{aligned}$$

Thus, $y = (x+2)^2 - 1$.

39. $(-2,5)$ is the vertex.

$f(x) = a(x+2)^2 + 5$

Since the graph passes through the point $(0,\ 9)$, we have:

$$\begin{aligned} 9 &= a(0+2)^2 + 5 \\ 4 &= 4a \\ 1 &= a \end{aligned}$$

Thus, $f(x) = (x+2)^2 + 5$.

40. $(4,\ 1)$ is the vertex.

$f(x) = a(x-4)^2 + 1$

Since the graph passes through the point $(6,\ -7)$, we have:

$$\begin{aligned} -7 &= a(6-4)^2 + 1 \\ -7 &= 4a + 1 \\ -8 &= 4a \\ -2 &= a \end{aligned}$$

Thus, $f(x) = -2(x-4)^2 + 1$.

41. $(1,\ -2)$ is the vertex.

$f(x) = a(x-1)^2 - 2$

Since the graph passes through the point $(-1,\ 14)$, we have:

$$\begin{aligned} 14 &= a(-1-1)^2 - 2 \\ 14 &= 4a - 2 \\ 16 &= 4a \\ 4 &= a \end{aligned}$$

Thus, $f(x) = 4(x-1)^2 - 2$.

42. $(-4,\ -1)$ is the vertex.

$f(x) = a(x+4)^2 - 1$

Since the graph passes through the point $(-2,\ 4)$, we have:

$$\begin{aligned} 4 &= a(-2+4)^2 - 1 \\ 5 &= 4a \\ a &= \frac{5}{4} \end{aligned}$$

Thus, $f(x) = \frac{5}{4}(x+4)^2 - 1$.

43. $\left(\frac{1}{2},\ 1\right)$ is the vertex.

$f(x) = a\left(x - \frac{1}{2}\right)^2 + 1$

Since the graph passes through the point $\left(-2,\ -\frac{21}{5}\right)$, we have:

$$\begin{aligned} -\frac{21}{5} &= a\left(-2 - \frac{1}{2}\right)^2 + 1 \\ -\frac{21}{5} &= \frac{25}{4}a + 1 \\ -\frac{26}{5} &= \frac{25}{4}a \\ -\frac{104}{125} &= a \end{aligned}$$

Thus, $f(x) = -\frac{104}{125}\left(x - \frac{1}{2}\right)^2 + 1$.

44. $\left(-\frac{1}{4},\ -1\right)$ is the vertex.

$f(x) = a\left(x + \frac{1}{4}\right)^2 - 1$

Since the graph passes through the point $\left(0, -\frac{17}{16}\right)$, we have:

$$\begin{aligned} -\frac{17}{16} &= a\left(0 + \frac{1}{4}\right)^2 - 1 \\ -\frac{17}{16} &= \frac{1}{16}a - 1 \\ -\frac{1}{16} &= \frac{1}{16}a \\ a &= -1 \end{aligned}$$

Thus, $f(x) = -\left(x + \frac{1}{4}\right)^2 - 1$.

45. $y = x^2 - 4x - 5$

x-intercepts: $(5,\ 0)$, $(-1,\ 0)$

$$\begin{aligned} 0 &= x^2 - 4x - 5 \\ 0 &= (x-5)(x+1) \\ x &= 5 \text{ or } x = -1 \end{aligned}$$

46. $y = 2x^2 + 5x - 3$

x-intercepts: $\left(\frac{1}{2},\ 0\right),\ (-3,\ 0)$

$0 = 2x^2 + 5x - 3$

$0 = (2x - 1)(x + 3)$

$x = \frac{1}{2},\ -3$

47. $y = x^2 + 8x + 16$

x-intercept: $(-4,\ 0)$

$0 = x^2 + 8x + 16$

$0 = (x + 4)^2$

$x = -4$

48. $y = x^2 - 6x + 9$

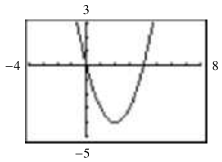
x-intercept: $(3,\ 0)$

$0 = x^2 - 6x + 9$

$0 = (x - 3)^2$

$x = 3$

49. $y = x^2 - 4x$



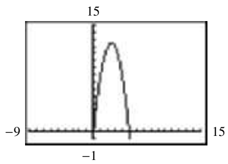
x-intercepts: $(0,\ 0),\ (4,\ 0)$

$0 = x^2 - 4x$

$0 = x(x - 4)$

$x = 0$ or $x = 4$

50. $y = -2x^2 + 10x$



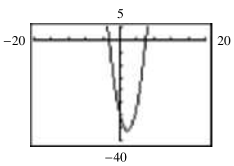
x-intercepts: $(0,\ 0),\ (5,\ 0)$

$0 = -2x^2 + 10x$

$0 = x(-2x + 10)$

$x = 0,\ x = 5$

51. $y = 2x^2 - 7x - 30$



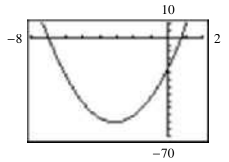
x-intercepts: $\left(-\frac{5}{2},\ 0\right),\ (6,\ 0)$

$0 = 2x^2 - 7x - 30$

$0 = (2x + 5)(x - 6)$

$x = -\frac{5}{2}$ or $x = 6$

52. $y = 4x^2 + 25x - 21$



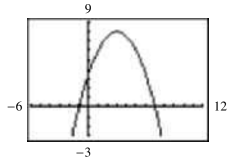
x-intercepts: $(-7,\ 0),\ (0.75,\ 0)$

$0 = 4x^2 + 25x - 21$

$\quad = (x + 7)(4x - 3)$

$x = -7,\ \frac{3}{4}$

53. $y = -\frac{1}{2}(x^2 - 6x - 7)$



x-intercepts: $(-1,\ 0),\ (7,\ 0)$

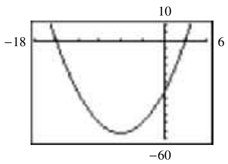
$0 = -\frac{1}{2}(x^2 - 6x - 7)$

$0 = x^2 - 6x - 7$

$0 = (x + 1)(x - 7)$

$x = -1,\ 7$

54. $y = \frac{7}{10}(x^2 + 12x - 45)$



x-intercepts: $(3,\ 0),\ (-15,\ 0)$

$0 = \frac{7}{10}(x^2 + 12x - 45)$

$0 = x^2 + 12x - 45$

$\quad = (x - 3)(x + 15)$

$x = 3,\ -15$

55. $f(x) = [x - (-1)](x - 3)$, opens upward

$\quad = (x + 1)(x - 3)$

$\quad = x^2 - 2x - 3$

$g(x) = -[x - (-1)](x - 3)$, opens downward

$\quad = -(x + 1)(x - 3)$

$\quad = -(x^2 - 2x - 3)$

$\quad = -x^2 + 2x + 3$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1,\ 0)$ and $(3,\ 0)$ for all real numbers $a \neq 0$.

56. $f(x) = x(x - 10) = x^2 - 10x$, opens upward.

$g(x) = -x(x - 10) = -x^2 + 10x$, opens downward.

Note: $f(x) = ax(x - 10)$ has x-intercepts $(0,\ 0)$ and $(10,\ 0)$ for all real numbers $a \neq 0$.

57. $f(x) = [x-(-3)]\left[x-\left(-\frac{1}{2}\right)\right](2)$, opens upward

$$= (x+3)\left(x+\frac{1}{2}\right)(2)$$
$$= (x+3)(2x+1)$$
$$= 2x^2+7x+3$$

$g(x) = -(2x^2+7x+3)$, opens downward

$$= -2x^2-7x-3$$

Note: $f(x) = a(x+3)(2x+1)$ has x-intercepts $(-3, 0)$ and $\left(-\frac{1}{2}, 0\right)$ for all real numbers $a \neq 0$.

58. $f(x) = 2\left[x-\left(-\frac{5}{2}\right)\right](x-2)$

$$= 2\left(x+\frac{5}{2}\right)(x-2)$$

$= 2x^2+x-10$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -2x^2-x+10$

Note: $f(x) = a\left(x+\frac{5}{2}\right)(x-2)$ has x-intercepts $\left(-\frac{5}{2}, 0\right)$ and $(2, 0)$ for all real numbers $a \neq 0$.

59. Let $x =$ the first number and $y =$ the second number.

Then the sum is $x+y=110 \Rightarrow y = 110-x$.

The product is

$$P(x) = xy = x(110-x) = 110x - x^2.$$
$$P(x) = -x^2+110x$$
$$= -(x^2-110x+3025-3025)$$
$$= -[(x-55)^2-3025]$$
$$= -(x-55)^2+3025$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

60. Let $x =$ first number and $y =$ second number. Then $x+y=66$ or $y = 66-x$. The product P is given by $P(x) = xy$.

$$P(x) = x(66-x)$$
$$= 66x - x^2$$
$$= -(x^2-66x)$$
$$= -(x^2-66x+33^2)+33^2$$
$$= -(x-33)^2+1089$$

The maximum P occurs at the vertex where $x = 33$, so $y = 66-33 = 33$. Therefore, the two numbers are 33 and 33.

61. Let x be the first number and y be the second number. Then $x+2y=24 \Rightarrow x = 24-2y$. The product is

$P = xy = (24-2y)y = 24y-2y^2$.

Completing the square,

$$P = -2y^2+24y$$
$$= -2(y^2-12y+36)+72$$
$$= -2(y-6)^2+72.$$

The maximum value of the product P occurs at the vertex of the parabola and equals 72. This happens when $y = 6$ and $x = 24-2(6) = 12$.

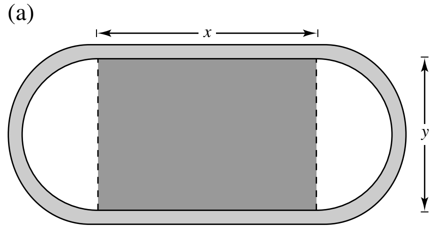
62. Let $x =$ first number and $y =$ second number.

Then $x+3y=42$, $y = \frac{1}{3}(42-x)$. The product is

$$P(x) = xy = x\frac{1}{3}(42-x) = 14x - \frac{1}{3}x^2.$$
$$P(x) = -\frac{1}{3}x^2+14x$$
$$= -\frac{1}{3}(x^2-42x)$$
$$= -\frac{1}{3}(x^2-42x+441)+147$$
$$= -\frac{1}{3}(x-21)^2+147.$$

The maximum value of the product is 147, and occurs when $x = 21$ and $y = \frac{1}{3}(42-21) = 7$.

63. (a)



(b) Radius of semicircular ends of track: $r = \frac{1}{2}y$

Distance around two semicircular parts of track:

$$d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$$

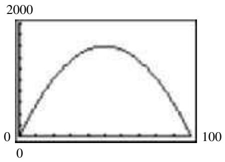
(c) Distance traveled around track in one lap:

$$d = \pi y + 2x = 200$$
$$\pi y = 200-2x$$
$$y = \frac{200-2x}{\pi}$$

(d) Area of rectangular region: $A = xy = x\left(\frac{200-2x}{\pi}\right)$

(e) The area is maximum when $x = 50$ and

$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}$.



64. (a) $4x + 3y = 200 \Rightarrow y = \frac{1}{3}(200 - 4x)$

$\Rightarrow A = 2xy = 2x\frac{1}{3}(200 - 4x) = \frac{8x}{3}(50 - x)$

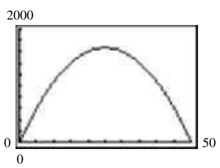
(b)

x	y	Area
2	$\frac{1}{3}[200 - 4(2)]$	$2xy = 256$
4	$\frac{1}{3}[200 - 4(4)]$	$2xy \approx 491$
6	$\frac{1}{3}[200 - 4(6)]$	$2xy = 704$
8	$\frac{1}{3}[200 - 4(8)]$	$2xy = 896$
10	$\frac{1}{3}[200 - 4(10)]$	$2xy \approx 1067$
12	$\frac{1}{3}[200 - 4(12)]$	$2xy = 1216$

x	y	Area
20	$\frac{1}{3}[200 - 4(20)]$	$2xy = 1600$
22	$\frac{1}{3}[200 - 4(22)]$	$2xy \approx 1643$
24	$\frac{1}{3}[200 - 4(24)]$	$2xy = 1664$
26	$\frac{1}{3}[200 - 4(26)]$	$2xy = 1664$
28	$\frac{1}{3}[200 - 4(28)]$	$2xy \approx 1643$
30	$\frac{1}{3}[200 - 4(30)]$	$2xy = 1600$

Maximum area when $x = 25$, $y = 33\frac{1}{3}$

(c) $A = \frac{8x(50 - x)}{3}$



Maximum when $x = 25$, $y = 33\frac{1}{3}$

(d) $A = \frac{8}{3}x(50 - x)$

$= -\frac{8}{3}(x^2 - 50x)$

$= -\frac{8}{3}(x^2 - 50x + 625 - 625)$

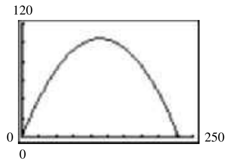
$= -\frac{8}{3}[(x - 25)^2 - 625]$

$= -\frac{8}{3}(x - 25)^2 + \frac{5000}{3}$

The maximum area occurs at the vertex and is $\frac{5000}{3}$ square feet. This happens when $x = 25$ feet and $y = \frac{(200 - 4(25))}{3} = \frac{100}{3}$ feet. The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet.

(e) The result are the same.

65. (a)



(b) When $x = 0$, $y = \frac{3}{2}$ feet.

(c) The vertex occurs at

$x = \frac{-b}{2a} = \frac{-9/5}{2(-16/2025)} = \frac{3645}{32} \approx 113.9.$

The maximum height is

$$y = \frac{-16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + \frac{3}{2}$$

≈ 104.0 feet.

(d) Using a graphing utility, the zero of y occurs at $x \approx 228.6$, or 228.6 feet from the punter.

66. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

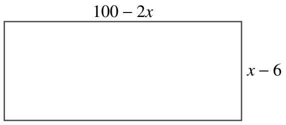
The maximum height of the dive occurs at the vertex,

$$x = \frac{-b}{2a} = -\frac{\frac{24}{9}}{2\left(\frac{-4}{9}\right)} = 3.$$

The height at $x = 3$ is $-\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 = 16.$

The maximum height of the dive is 16 feet.

67. (a)



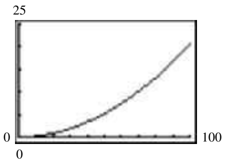
$A = lw$
$A = (100 - 2x)(x - 6)$
$A = -2x^2 + 112x - 600$

(b) $Y_1 = -2x^2 + 112x - 600$

X	Y
25	950
26	960
27	966
28	968
29	966
30	960

The area is maximum when $x = 28$ inches.

68. (a)



(b) The parabola intersects $y = 10$ at $s \approx 59.4$. Thus, the maximum speed is 59.4 mph. Analytically,

$$0.002s^2 + 0.05s - 0.029 = 10$$
$$2s^2 + 50s - 29 = 10{,}000$$
$$2s^2 + 50s - 10{,}029 = 0.$$

Using the Quadratic Formula,

$$s = \frac{-50 \pm \sqrt{50^2 - 4(2)(-10{,}029)}}{2(2)}$$
$$= \frac{-50 \pm \sqrt{82{,}732}}{4} \approx -84.4,\ 59.4.$$

The maximum speed is the positive root, 59.4 mph.

69. $R(p) = -10p^2 + 1580p$

(a) When $p = \$50$, $R(50) = \$54{,}000$.
When $p = \$70$, $R(70) = \$61{,}600$.
When $p = \$90$, $R(90) = \$61{,}200$.

(b) The maximum R occurs at the vertex,

$$p = \frac{-b}{2a}$$
$$p = \frac{-1580}{2(-10)} = \$79$$

(c) When $p = \$79$, $R(79) = \$62{,}410$.

(d) Answers will vary.

70. $R(p) = -12p^2 + 372p$

(a) When $p = \$12$, $R(12) = \$2736$.
When $p = \$16$, $R(16) = \$2880$.
When $p = \$20$, $R(20) = \$2640$.

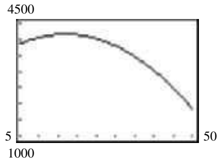
(b) The maximum R occurs at the vertex:

$$p = \frac{-b}{2a}$$
$$p = \frac{-372}{2(-12)} = \$15.50$$

(c) When $p = \$15.50$, $R(15.50) = \$2883$.

(d) Answers will vary.

71. (a)



(b) Using the graph, during 1966 the maximum average annual consumption of cigarettes appears to have occurred and was 4155 cigarettes per person.

Yes, the warning had an effect because the maximum consumption occurred in 1966 and consumption decreased from then on.

(c) In 2000, $C(50) = 1852$ cigarettes per person.

$\frac{1852}{365} \approx 5$ cigarettes per day

72. (a) According to the model, $t = \frac{-b}{2a}$ or

$$t = \frac{-271.4}{2(-8.87)} \approx 15 \text{ or } 2005.$$
$P(15) \approx 82{,}437{,}000$

(b) When $t = 110$, $P(110) = 2{,}889{,}000$.

No, the population of Germany would not be expected to decrease this much

73. True.

$-12x^2 - 1 = 0$
$12x^2 = -1$, impossible

74. True. For $f(x)$, $\frac{-b}{2a} = -\frac{-10}{2(-4)} = -\frac{10}{8} = -\frac{5}{4}$.

For $g(x)$, $\frac{-b}{2a} = \frac{-30}{2(12)} = \frac{-30}{24} = \frac{-5}{4}$.

In both cases, $x = -\frac{5}{4}$ is the axis of symmetry.

75. The parabola opens downward and the vertex is $(-2, -4)$. Matches (c) and (d).

76. The parabola opens upward and the vertex is $(1, 3)$. Matches (a).

77. The graph of $f(x) = (x - z)^2$ would be a horizontal shift z units to the right of $g(x) = x^2$.

78. The graph of $f(x) = x^2 - z$ would be a vertical shift z units downward of $g(x) = x^2$.

79. The graph of $f(x) = z(x-3)^2$ would be a vertical stretch $(z > 1)$ and horizontal shift three units to the right of $g(x) = x^2$. The graph of $f(x) = z(x-3)^2$ would be a vertical shrink $(0 < z < 1)$ and horizontal shift three units to the right of $g(x) = x^2$.

80. The graph of $f(x) = zx^2 + 4$ would be a vertical stretch $(z > 1)$ and vertical shift four units upward of $g(x) = x^2$. The graph of $f(x) = zx^2 + 4$ would be a vertical shrink $(0 < z < 1)$ and vertical shift four units upward of $g(x) = x^2$.

81. For $a < 0$, $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is a maximum when $x = \frac{-b}{2a}$. In this case, the maximum value is $c - \frac{b^2}{4a}$. Hence,

$$25 = -75 - \frac{b^2}{4(-1)}$$
$$-100 = 300 - b^2$$
$$400 = b^2$$
$$b = \pm 20.$$

82. For $a < 0$, $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is a maximum when $x = \frac{-b}{2a}$. In this case, the maximum value is $c - \frac{b^2}{4a}$. Hence,

$$48 = -16 - \frac{b^2}{4(-1)}$$
$$-192 = 64 - b^2$$
$$b^2 = 256$$
$$b = \pm 16.$$

83. For $a > 0$, $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is a minimum when $x = \frac{-b}{2a}$. In this case, the minimum value is $c - \frac{b^2}{4a}$. Hence,

$$10 = 26 - \frac{b^2}{4}$$
$$40 = 104 - b^2$$
$$b^2 = 64$$
$$b = \pm 8.$$

84. For $a > 0$, $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is a minimum when $x = \frac{-b}{2a}$. In this case, the minimum value is $c - \frac{b^2}{4a}$. Hence,

$$-50 = -25 - \frac{b^2}{4}$$
$$-200 = -100 - b^2$$
$$b^2 = 100$$
$$b = \pm 10.$$

85. Let x = first number and y = second number.
Then $x + y = s$ or $y = s - x$.
The product is given by $P = xy$ or $P = x(s - x)$.

$$P = x(s - x)$$
$$P = sx - x^2$$

The maximum P occurs at the vertex when $x = \frac{-b}{2a}$.

$$x = \frac{-s}{2(-1)} = \frac{s}{2}$$

When $x = \frac{s}{2}$, $y = s - \frac{s}{2} = \frac{s}{2}$.

So, the numbers x and y are both $\frac{s}{2}$.

86. If $f(x) = ax^2 + bx + c$ has two real zeros, then by the Quadratic Formula they are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The average of the zeros of f is

$$\frac{\frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}}{2} = \frac{\frac{-2b}{2a}}{2} = -\frac{b}{2a}.$$

This is the x-coordinate of the vertex of the graph.

87. $y = ax^2 + bx - 4$
$(1, 0)$ on graph: $0 = a + b - 4$
$(4, 0)$ on graph: $0 = 16a + 4b - 4$
From the first equation, $b = 4 - a$.
Thus, $0 = 16a + 4(4 - a) - 4 = 12a + 12 \Rightarrow a = -1$ and hence $b = 5$, and $y = -x^2 + 5x - 4$.

88. Model (a) is preferable. $a > 0$ means the parabola opens upward and profits are increasing for t to the right of the vertex

$$t \geq -\frac{b}{(2a)}.$$

89. $x + y = 8 \Rightarrow y = 8 - x$

$$-\frac{2}{3}x + 8 - x = 6$$
$$-\frac{5}{3}x + 8 = 6$$
$$-\frac{5}{3}x = -2$$
$$x = 1.2$$
$$y = 8 - 1.2 = 6.8$$

The point of intersection is $(1.2, 6.8)$.

90. $y = 3x - 10 = \frac{1}{4}x + 1$

$$12x - 40 = x + 4$$
$$11x = 44$$
$$x = 4$$
$$y = 3(4) - 10$$
$$y = 12 - 10 = 2$$

The graphs intersect at $(4, 2)$.

91. $y = x + 3 = 9 - x^2$

$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3, \quad x = 2$$
$$y = -3 + 3 = 0$$
$$y = 2 + 3 = 5$$

Thus, $(-3, 0)$ and $(2, 5)$ are the points of intersection.

92. $y = x^3 + 2x - 1 = -2x + 15$

$$x^3 + 4x - 16 = 0$$
$$(x - 2)\left(x^2 + 2x + 8\right) = 0$$
$$x = 2$$
$$y = -2(2) + 15 = -4 + 15 = 11$$

The graphs intersect at $(2, 11)$.

93. Answers will vary. (Make a Decision)

Section 2.2

1. continuous

2. $n, \; n - 1$

3. (a) solution
(b) $(x - a)$
(c) $(a, 0)$

4. touches, crosses

5. No. If f is an even-degree fourth-degree polynomial function, its left and right end behavior is either that it rises left and right or falls left and right.

6. No. Assuming f is an odd-degree polynomial function, if its leading coefficient is negative, it must rise to the left and fall to the right.

7. Because f is a polynomial, it is a continuous on $[x_1, x_2]$ and $f(x_1) < 0$ and $f(x_2) > 0$. Then $f(x) = 0$ for some value of x in $[x_1, x_2]$.

8. The real zero in $[x_3, x_4]$ is of even multiplicity, since the graph touches the x-axis but does not cross the x-axis.

9. $f(x) = -2x + 3$ is a line with y-intercept $(0, 3)$. Matches graph (f).

10. $f(x) = x^2 - 4x$ is a parabola with intercepts $(0, 0)$ and $(4, 0)$ and opens upward. Matches graph (h).

11. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (c).

12. $f(x) = 2x^3 - 3x + 1$ has intercepts $(0, 1)$, $(1, 0)$, $\left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}, 0\right)$ and $\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}, 0\right)$. Matches graph (a).

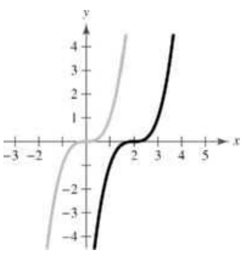
13. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and $\left(\pm 2\sqrt{3}, 0\right)$. Matches graph (e).

14. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ has y-intercept $\left(0, -\frac{4}{3}\right)$. Matches graph (d).

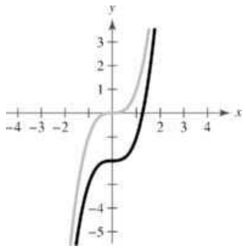
15. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (g).

16. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ has intercepts $(0, 0)$, $(1, 0)$, $(-1, 0)$, $(3, 0)$, and $(-3, 0)$. Matches (b).

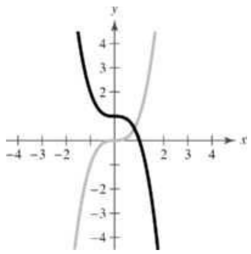
17. The graph of $f(x) = (x-2)^3$ is a horizontal shift two units to the right of $y = x^3$.



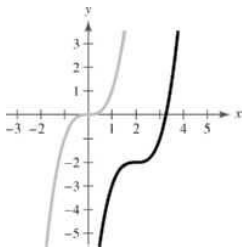
18. The graph of $f(x) = x^3 - 2$ is a vertical shift two units downward of $y = x^3$.



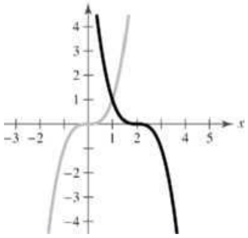
19. The graph of $f(x) = -x^3 + 1$ is a reflection in the x-axis and a vertical shift one unit upward of $y = x^3$.



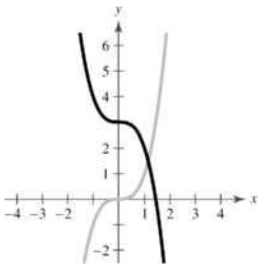
20. The graph of $f(x) = (x-2)^3 - 2$ is a horizontal shift two units to the right and a vertical shift two units downward of $y = x^3$.



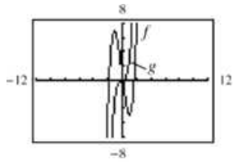
21. The graph of $f(x) = -(x-2)^3$ is a horizontal shift two units to the right and a reflection in the x-axis of $y = x^3$.



22. The graph of $f(x) = -x^3 + 3$ is a reflection in the x-axis and a vertical shift three units upward of $y = x^3$.

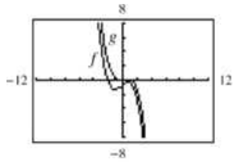


23.



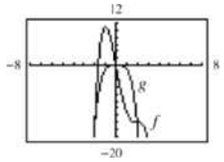
Yes, because both graphs have the same leading coefficient.

24.



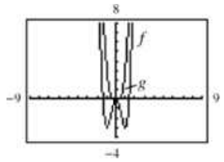
Yes, because both graphs have the same leading coefficient.

25.



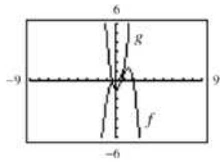
Yes, because both graphs have the same leading coefficient.

26.



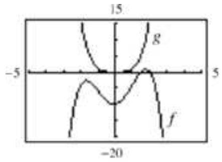
Yes, because both graphs have the same leading coefficient.

27.



No, because the graphs have different leading coefficients.

28.



No, because the graphs have different leading coefficients.

29. $f(x) = 2x^4 - 3x + 1$

Degree: 4

Leading coefficient: 2

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

30. $h(x) = 1 - x^6$

Degree: 6

Leading coefficient: –1

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

31. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: –3

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

32. $f(x) = \frac{1}{3}x^3 + 5x$

Degree: 3

Leading coefficient: $\frac{1}{3}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

33. $f(x) = \dfrac{6x^5 - 2x^4 + 4x^2 - 5x}{3}$

Degree: 5

Leading coefficient: $\frac{6}{3} = 2$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

34. $f(x) = \dfrac{3x^7 - 2x^5 + 5x^3 + 6x^2}{4}$

Degree: 7

Leading coefficient: $\frac{3}{4}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

35. $h(t) = -\frac{2}{3}\left(t^2 - 5t + 3\right)$

Degree: 2

Leading coefficient: $-\frac{2}{3}$

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

36. $f(s) = -\frac{7}{8}\left(s^3 + 5s^2 - 7s + 1\right)$

Degree: 3

Leading coefficient: $-\frac{7}{8}$

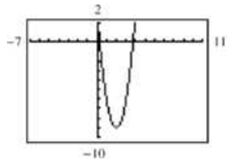
The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

37. (a)

$$f(x) = 3x^2 - 12x + 3 = 3\left(x^2 - 4x + 1\right) = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

(b)



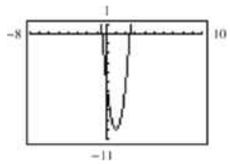
(c) $x \approx 3.732, 0.268$; the answers are approximately the same.

38. (a)

$$g(x) = 5\left(x^2 - 2x - 1\right) = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(-1)}}{2} = 1 \pm \sqrt{2}$$

(b)



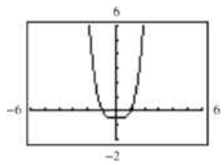
(c) $x \approx -0.414, 2.414$; the answers are approximately the same.

39. (a)

$$g(t) = \frac{1}{2}t^4 - \frac{1}{2} = \frac{1}{2}\left(t + 1\right)(t - 1)(t^2 + 1) = 0$$

$$t = \pm 1$$

(b)

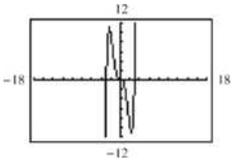


(c) $t = \pm 1$; the answers are the same.

40. (a) $0 = \frac{1}{4}x^3(x^2 - 9)$

$x = 0,\ \pm 3$

(b)



(c) $x = 0,\ \pm 3;$ the answers are the same.

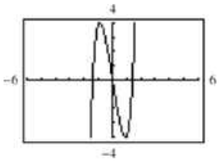
41. (a) $f(x) = x^5 + x^3 - 6x$

$= x\left(x^4 + x^2 - 6\right)$

$= x\left(x^2 + 3\right)(x^2 - 2) = 0$

$x = 0,\ \pm\sqrt{2}$

(b)



(c) $x = 0, 1.414, -1.414;$ the answers are approximately the same.

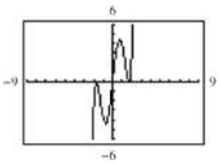
42. (a) $g(t) = t^5 - 6t^3 + 9t$

$= t\left(t^4 - 6t^2 + 9\right)$

$= t(t^2 - 3)^2 = 0$

$t = 0,\ \pm\sqrt{3}$

(b)



(c) $x = 0,\ \pm 1.732;$ the answers are approximately the same.

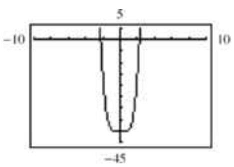
43. (a) $f(x) = 2x^4 - 2x^2 - 40$

$= 2\left(x^4 - x^2 - 20\right)$

$= 2\left(x^2 + 4\right)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right) = 0$

$x = \pm\sqrt{5}$

(b)



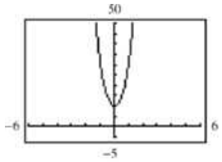
(c) $x = 2.236, -2.236;$ the answers are approximately the same.

44. (a) $f(x) = 5(x^4 + 3x^2 + 2)$

$= 5(x^2 + 1)(x^2 + 2) > 0$

No real zeros

(b)



(c) No real zeros

45. (a) $f(x) = x^3 - 4x^2 - 25x + 100$

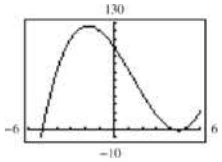
$= x^2(x - 4) - 25(x - 4)$

$= (x^2 - 25)(x - 4)$

$= (x - 5)(x + 5)(x - 4) = 0$

$x = \pm 5,\ 4$

(b)



(c) $x = 4, 5, -5;$ the answers are the same.

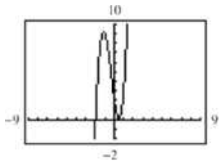
46. (a) $0 = 4x^3 + 4x^2 - 7x + 2$

$= (2x - 1)(2x^2 + 3x - 2)$

$= (2x - 1)(2x - 1)(x + 2) = 0$

$x = -2,\ \frac{1}{2}$

(b)



(c) $x = -2, \frac{1}{2};$ the answers are the same.

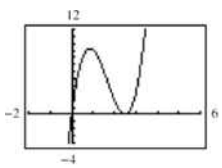
47. (a) $y = 4x^3 - 20x^2 + 25x$

$0 = 4x^3 - 20x^2 + 25x$

$0 = x(2x - 5)^2$

$x = 0,\ \frac{5}{2}$

(b)



(c) $x = 0,\ \frac{5}{2};$ the answers are the same.

48. (a) $y = x^5 - 5x^3 + 4x$

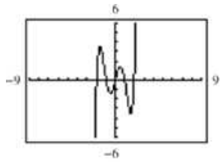
$= x(x^4 - 5x^2 + 4)$

$= x(x^2 - 4)(x^2 - 1)$

$= x(x - 2)(x + 2)(x - 1)(x + 1) = 0$

$x = 0, \ \pm 1, \ \pm 2$

(b)



(c) $x = 0, \ \pm 1, \ \pm 2$; the answers are the same.

49. $f(x) = x^2 - 25$

$= (x + 5)(x - 5)$

$x = \pm 5$ (multiplicity 1)

50. $f(x) = 49 - x^2$

$= (7 - x)(7 + x)$

$x = \pm 7$ (multiplicity 1)

51. $h(t) = t^2 - 6t + 9$

$= (t - 3)^2$

$t = 3$ (multiplicity 2)

52. $f(x) = x^2 + 10x + 25$

$= (x + 5)^2$

$x = -5$ (multiplicity 2)

53. $f(x) = x^2 + x - 2$

$= (x + 2)(x - 1)$

$x = -2, 1$ (multiplicity 1)

54. $f(x) = 2x^2 - 14x + 24$

$= 2(x^2 - 7x + 12)$

$= 2(x - 3)(x - 4)$

$x = 3, 4$ (multiplicity 1)

55. $f(t) = t^3 - 4t^2 + 4t$

$= t(t - 2)^2$

$t = 0$ (multiplicity 1), 2 (multiplicity 2)

56. $f(x) = x^4 - x^3 - 20x^2$

$= x^2(x^2 - x - 20)$

$= x^2(x + 4)(x - 5)$

$x = -4$ (multiplicity 1), 5 (multiplicity 1), 0 (multiplicity 2)

57. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$

$= \frac{1}{2}(x^2 + 5x - 3)$

$$x = \frac{-5 \pm \sqrt{25 - 4(-3)}}{2} = -\frac{5}{2} \pm \frac{\sqrt{37}}{2}$$

$\approx 0.5414, \ -5.5414$ (multiplicity 1)

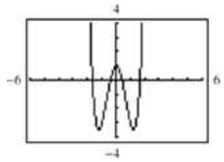
58. $f(x) = \frac{5}{3}x^2 + \frac{8}{3}x - \frac{4}{3}$

$= \frac{1}{3}(5x^2 + 8x - 4)$

$= \frac{1}{3}(5x - 2)(x + 2)$

$x = \frac{2}{5}, \ -2$ (multiplicity 1)

59. $f(x) = 2x^4 - 6x^2 + 1$

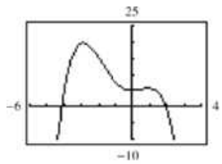


Zeros: $x \approx \pm 0.421, \pm 1.680$

Relative maximum: (0, 1)

Relative minima: (1.225, −3.5), (−1.225, −3.5)

60. $f(x) = -\frac{3}{8}x^4 - x^3 + 2x^2 + 5$

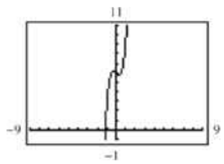


Zeros: −4.142, 1.934

Relative maxima: (0.915, 5.646), (−2.915, 19.688)

Relative minimum: (0, 5)

61. $f(x) = x^5 + 3x^3 - x + 6$

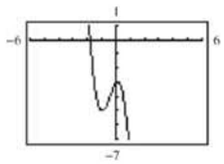


Zero: $x \approx -1.178$

Relative maximum: (−0.324, 6.218)

Relative minimum: (0.324, 5.782)

62. $f(x) = -3x^3 - 4x^2 + x - 3$

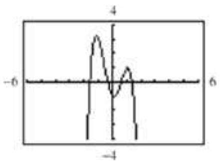


Zero: −1.819

Relative maximum: (0.111, −2.942)

Relative minimum: (−1, −5)

63. $f(x) = -2x^4 + 5x^2 - x - 1$

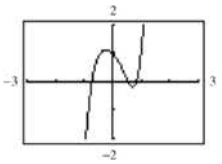


Zeros: $-1.618, \ -0.366, \ 0.618, 1.366$

Relative minimum: $(0.101, \ -1.050)$

Relative maxima: $(-1.165, \ 3.267), (1.064, \ 1.033)$

64. $f(x) = 3x^5 - 2x^2 - x + 1$



Zeros: $-0.737, 0.548, 0.839$

Relative minimum: $(0.712, \ -0.177)$

Relative maximum: $(-0.238, \ 1.122)$

65. $f(x) = (x-0)(x-4) = x^2 - 4x$

Note: $f(x) = a(x-0)(x-4) = ax(x-4)$ has zeros 0 and 4 for all nonzero real numbers a.

66. $f(x) = (x+7)(x-2) = x^2 + 5x - 14$

Note: $f(x) = a(x+7)(x-2)$ has zeros -7 and 2 for all nonzero real numbers a.

67. $f(x) = (x-0)(x+2)(x+3) = x^3 + 5x^2 + 6x$

Note: $f(x) = ax(x+2)(x+3)$ has zeros $0, \ -2,$ and -3 for all nonzero real numbers a.

68. $f(x) = (x-0)(x-2)(x-5) = x^3 - 7x^2 + 10x$

Note: $f(x) = ax(x-2)(x-5)$ has zeros $0, \ 2,$ and 5 for all nonzero real numbers a.

69. $f(x) = (x-4)(x+3)(x-3)(x-0)$

$= (x-4)(x^2-9)x$

$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros $4, \ -3, \ 3,$ and 0 for all nonzero real numbers a.

70. $f(x) = (x-(-2))(x-(-1))(x-0)(x-1)(x-2)$

$= x(x+2)(x+1)(x-1)(x-2)$

$= x(x^2-4)(x^2-1)$

$= x(x^4 - 5x^2 + 4)$

$= x^5 - 5x^3 + 4x$

Note: $f(x) = ax(x+2)(x+1)(x-1)(x-2)$ has zeros $-2, \ -1, \ 0, \ 1, \ 2,$ for all nonzero real numbers a.

71. $f(x) = \left[x - \left(1+\sqrt{3}\right)\right]\left[x - \left(1-\sqrt{3}\right)\right]$

$= \left[\left(x-1\right) - \sqrt{3}\right]\left[\left(x-1\right) + \sqrt{3}\right]$

$= (x-1)^2 - \left(\sqrt{3}\right)^2$

$= x^2 - 2x + 1 - 3$

$= x^2 - 2x - 2$

Note: $f(x) = a(x^2 - 2x - 2)$ has zeros $1+\sqrt{3}$ and $1-\sqrt{3}$ for all nonzero real numbers a.

72. $f(x) = \left(x - \left(6+\sqrt{3}\right)\right)\left(x - \left(6-\sqrt{3}\right)\right)$

$= \left(\left(x-6\right) - \sqrt{3}\right)\left(\left(x-6\right) + \sqrt{3}\right)$

$= (x-6)^2 - 3$

$= x^2 - 12x + 36 - 3$

$= x^2 - 12x + 33$

Note: $f(x) = a(x^2 - 12x + 33)$ has zeros $6+\sqrt{3}$ and $6-\sqrt{3}$ for all nonzero real numbers a.

73. $f(x) = (x-2)\left[x - \left(4+\sqrt{5}\right)\right]\left[x - \left(4-\sqrt{5}\right)\right]$

$= (x-2)\left[\left(x-4\right) - \sqrt{5}\right]\left[\left(x-4\right) + \sqrt{5}\right]$

$= (x-2)[(x-4)^2 - 5]$

$= x^3 - 10x^2 + 27x - 22$

Note: $f(x) = a(x-2)[(x-4)^2 - 5]$ has zeros $2, \ 4+\sqrt{5},$ and $4-\sqrt{5}$ for all nonzero real numbers a.

74. $f(x) = (x-4)\left(x - \left(2+\sqrt{7}\right)\right)\left(x - \left(2-\sqrt{7}\right)\right)$

$= (x-4)\left(\left(x-2\right) - \sqrt{7}\right)\left(\left(x-2\right) + \sqrt{7}\right)$

$= (x-4)((x-2)^2 - 7)$

$= (x-4)(x^2 - 4x - 3)$

$= x^3 - 8x^2 + 13x + 12$

Note: $f(x) = a(x-4)(x^2 - 4x - 3)$ has zeros $4, \ 2 \pm \sqrt{7}$ for all nonzero real numbers a.

75. $f(x) = (x+2)^2(x+1) = x^3 + 5x^2 + 8x + 4$

Note: $f(x) = a(x+2)^2(x+1)$ has zeros $-2, \ -2,$ and -1 for all nonzero real numbers a.

76. $f(x) = (x-3)(x-2)^3$

$= x^4 - 9x^3 + 30x^2 - 44x + 24$

Note: $f(x) = a(x-3)(x-2)^3$ has zeros $3, \ 2, \ 2, \ 2$ for all nonzero real numbers a.

77. $f(x) = (x+4)^2(x-3)^2$

$= x^4 + 2x^3 - 23x^2 - 24x + 144$

Note: $f(x) = a(x+4)^2(x-3)^2$ has zeros $-4, \ -4, \ 3, \ 3$ for all nonzero real numbers a

78. $f(x) = (x-5)^3(x-0)^2$
$= x^5 - 15x^4 + 75x^3 - 125x^2$

Note: $f(x) = a(x-5)^3x^2$ has zeros $5, 5, 5, 0, 0$ for all nonzero real numbers a.

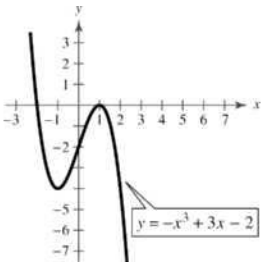
79. $f(x) = -(x+1)^2(x+2)$
$= -x^3 - 4x^2 - 5x - 2$

Note: $f(x) = a(x+1)^2(x+2)^2$, $a < 0$, has zeros $-1, -1, -2$, rises to the left, and falls to the right.

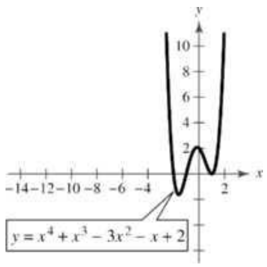
80. $f(x) = -(x-1)^2(x-4)^2$
$= -x^4 + 10x^3 - 33x^2 + 40x - 16$

Note: $f(x) = a(x-1)^2(x-4)^2$, $a < 0$, has zeros $1, 1, 4, 4$, falls to the left, and falls to the right.

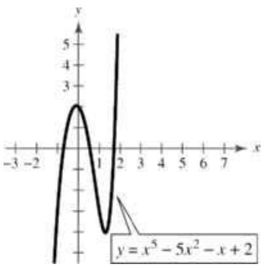
81.



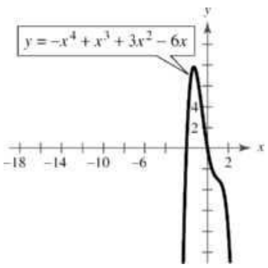
82.



83.



84.

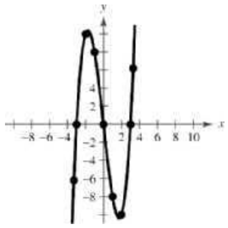


85. (a) The degree of *f* is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 9x = x(x^2 - 9) = x(x-3)(x+3)$

Zeros: $0, 3, -3$

(c) and (d)

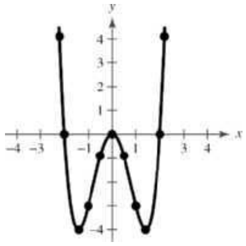


86. (a) The degree of *g* is even and the leading coefficient is 1. The graph rises to the left and rises to the right.

(b) $g(x) = x^4 - 4x^2 = x^2(x^2 - 4)$
$= x^2(x-2)(x+2)$

Zeros: $0, 2, -2$

(c) and (d)

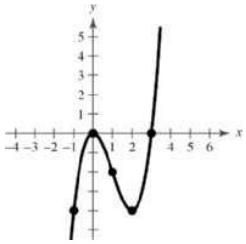


87. (a) The degree of *f* is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 3x^2 = x^2(x-3)$

Zeros: $0, 3$

(c) and (d)

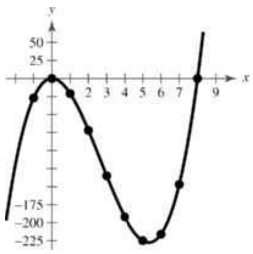


88. (a) The degree of *f* is odd and the leading coefficient is 3. The graph falls to the left and rises to the right.

(b) $f(x) = 3x^3 - 24x^2 = 3x^2(x-8)$

Zeros: $0, 8$

(c) and (d)

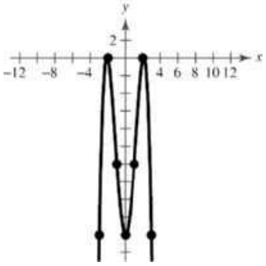


89. (a) The degree of f is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f(x) = -x^4 + 9x^2 - 20 = -(x^2 - 4)(x^2 - 5)$

Zeros: $\pm 2,\ \pm\sqrt{5}$

(c) and (d)

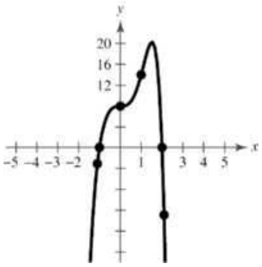


90. (a) The degree of f is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f(x) = -x^6 + 7x^3 + 8 = -(x^3 + 1)(x^3 - 8)$

Zeros: $-1,\ 2$

(c) and (d)



91. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

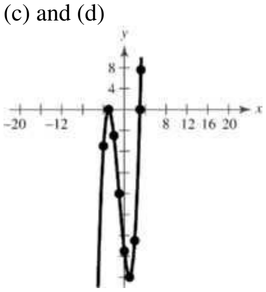
(b) $f(x) = x^3 + 3x^2 - 9x - 27 = x^2(x+3) - 9(x+3)$

$= (x^2 - 9)(x + 3)$

$= (x - 3)(x + 3)^2$

Zeros: $3,\ -3$

(c) and (d)



92. (a) The degree of h is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

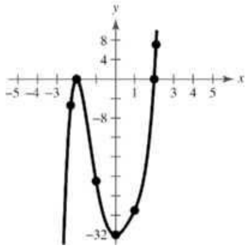
(b) $h(x) = x^5 - 4x^3 + 8x^2 - 32 = x^3(x^2 - 4) + 8(x^2 - 4)$

$= (x^3 + 8)(x^2 - 4)$

$= (x + 2)(x^2 - 2x + 4)(x - 2)(x + 2)$

Zeros: $-2,\ 2$

(c) and (d)

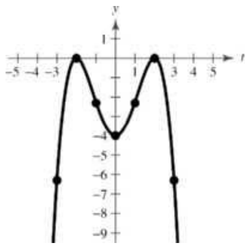


93. (a) The degree of g is even and the leading coefficient is $-\dfrac{1}{4}$. The graph falls to the left and falls to the right.

(b) $g(t) = -\dfrac{1}{4}(t^4 - 8t^2 + 16) = -\dfrac{1}{4}(t^2 - 4)^2$

Zeros: $-2,\ -2,\ 2,\ 2$

(c) and (d)

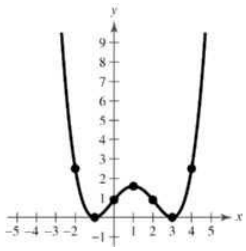


94. (a) The degree of g is even and the leading coefficient is $\dfrac{1}{10}$. The graph rises to the right and rises to the left.

(b) $g(x) = \dfrac{1}{10}(x + 1)^2(x - 3)^2$

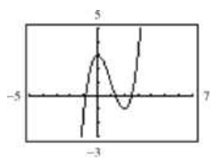
Zeros: $-1,\ 3$

(c) and (d)



95. $f(x) = x^3 - 3x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-1, 0)$, $(1, 2)$ and $(2, 3)$.

(b) Zeros: –0.879, 1.347, 2.532

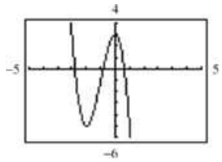
x	y
–0.9	–0.159
–0.89	–0.0813
–0.88	–0.0047
–0.87	0.0708
–0.86	0.14514
–0.85	0.21838
–0.84	0.2905

x	y
1.3	0.127
1.31	0.09979
1.32	0.07277
1.33	0.04594
1.34	0.0193
1.35	–0.0071
1.36	–0.0333

x	y
2.5	–0.125
2.51	–0.087
2.52	–0.0482
2.53	–0.0084
2.54	0.03226
2.55	0.07388
2.56	0.11642

96. $f(x) = -2x^3 - 6x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-3, -2)$, $(-1, 0)$, and $(0, 1)$.

(b) Zeros: –2.810, –0.832, 0.642

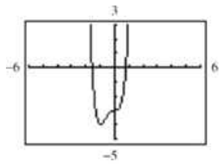
x	y_1
–2.83	–0.277
–2.82	–0.137
–2.81	≈ 0
–2.80	–0.136
–2.79	–0.269

x	y_1
–0.86	–0.166
–0.85	–0.0107
–0.84	–0.048
–0.83	0.010
–0.82	0.068

x	y_1
0.62	0.217
0.63	0.119
0.64	0.018
0.65	–0.084
0.66	–0.189

97. $g(x) = 3x^4 + 4x^3 - 3$

(a)



The function has two zeros. They are in the intervals $(-2, -1)$ and $(0, 1)$.

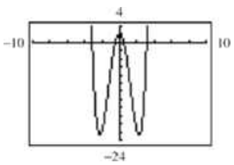
(b) Zeros: –1.585, 0.779

x	y_1
–1.6	0.2768
–1.59	0.09515
–1.58	–0.0812
–1.57	–0.2524
–1.56	–0.4184
–1.55	–0.5795
–1.54	–0.7356

x	y_1
0.75	–0.3633
0.76	–0.2432
0.77	–0.1193
0.78	0.00866
0.79	0.14066
0.80	0.2768
0.81	0.41717

98. $h(x) = x^4 - 10x^2 + 2$

(a)



The function has four zeros. They are in the intervals $(0, 1)$, $(3, 4)$, $(-1, 0)$, and $(-4, -3)$.

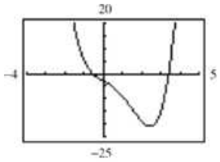
(b) Notice that h is even. Hence, the zeros come in symmetric pairs. Zeros: ±0.452, ±3.130.
Because the function is even, we only need to verify the positive zeros.

x	y_1
0.42	0.26712
0.43	0.18519
0.44	0.10148
0.45	0.01601
0.46	–0.0712
0.47	–0.1602
0.48	–0.2509

x	y_1
3.09	–2.315
3.10	–1.748
3.11	–1.171
3.12	–0.5855
3.13	0.01025
3.14	0.61571
3.15	1.231

99. $f(x) = x^4 - 3x^3 - 4x - 3$

(a)



The function has two zeros. They are in the intervals $(-1, 0)$ and $(3, 4)$.

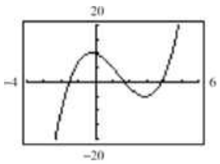
(b) Zeros: –0.578, 3.418

x	y_1
–0.61	0.2594
–0.60	0.1776
–0.59	0.09731
–0.58	0.0185
–0.57	–0.0589
–0.56	–0.1348
–0.55	–0.2094

x	y_1
3.39	–1.366
3.40	–0.8784
3.41	–0.3828
3.42	0.12071
3.43	0.63205
3.44	1.1513
3.45	1.6786

100. $f(x) = x^3 - 4x^2 - 2x + 10$

(a)



The function has three zeros. They are in the intervals $(-2, -1)$, $(1, 2)$, and $(3, 4)$

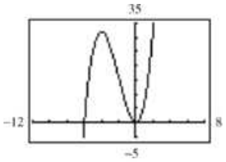
(b) Zeros: –1.537, 1.693, 3.843

x	y_1
–1.56	–0.4108
–1.55	–0.2339
–1.54	–0.0587
–1.53	0.11482
–1.52	0.28659
–1.51	0.45665
–1.50	0.625

x	y_1
1.66	0.2319
1.67	0.16186
1.68	0.09203
1.69	0.02241
1.70	–0.047
1.71	–0.1162
1.72	–0.1852

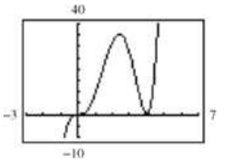
x	y_1
3.82	–0.2666
3.83	–0.1537
3.84	–0.0393
3.85	0.07663
3.86	0.19406
3.87	0.313
3.88	0.43347

101. $f(x) = x^2(x+6)$



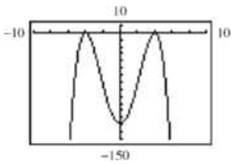
No symmetry
Two x-intercepts

102. $h(x) = x^3(x-4)^2$



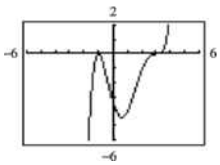
No symmetry
Two x-intercepts

103. $g(t) = -\frac{1}{2}(t-4)^2(t+4)^2$



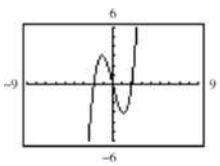
Symmetric with respect to the y-axis
Two x-intercepts

104. $g(x) = \frac{1}{8}(x+1)^2(x-3)^3$



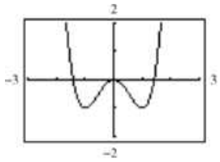
No symmetry
Two x-intercepts

105. $f(x) = x^3 - 4x = x(x+2)(x-2)$



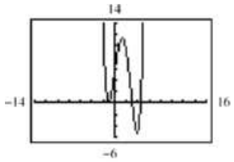
Symmetric with respect to the origin
Three x-intercepts

106. $f(x) = x^4 - 2x^2$



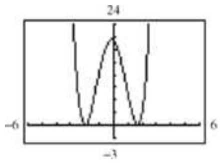
Symmetric with respect to the y-axis
Three x-intercepts

107. $g(x) = \frac{1}{5}(x+1)^2(x-3)(2x-9)$



No symmetry
Three x-intercepts

108. $h(x) = \frac{1}{5}(x+2)^2(3x-5)^2$



No symmetry
Two x-intercepts

109. (a) Volume = length × width × height

Because the box is made from a square, length = width.

Thus: Volume = $(\text{length})^2 \times$ height = $(36-2x)^2 x$

(b) Domain: $0 < 36 - 2x < 36$

$-36 < -2x < 0$

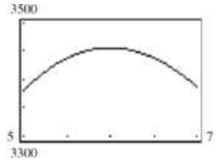
$18 > x > 0$

(c)

Height, x	Length and Width	Volume, V
1	$36-2(1)$	$1[36-2(1)]^2 = 1156$
2	$36-2(2)$	$2[36-2(2)]^2 = 2048$
3	$36-2(3)$	$3[36-2(3)]^2 = 2700$
4	$36-2(4)$	$4[36-2(4)]^2 = 3136$
5	$36-2(5)$	$5[36-2(5)]^2 = 3380$
6	$36-2(6)$	$6[36-2(6)]^2 = 3456$
7	$36-2(7)$	$7[36-2(7)]^2 = 3388$

Maximum volume is in the interval $5 < x < 7$.

(d)



$x = 6$ when $V(x)$ is maximum.

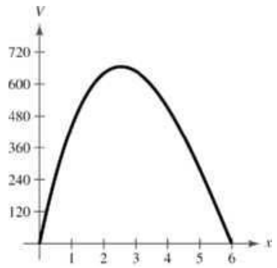
110. (a) $V(x) = \text{length} \times \text{width} \times \text{height}$

$= (24 - 2x)(24 - 4x)x$

$= 8x(12 - x)(6 - x)$

(b) Domain: $0 < x < 6$

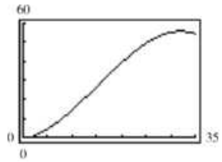
(c)



Maximum occurs at $x = 2.54$.

111. The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$ The point is (200, 160) which corresponds to spending \$2,000,000 on advertising to obtain a revenue of \$160 million.

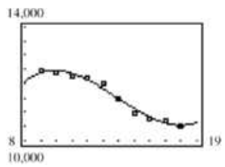
112.



Point of diminishing returns: $(15.2, 27.3)$

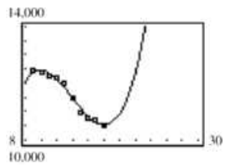
15.2 years

113. (a)



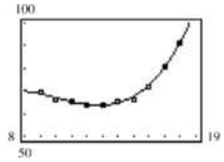
The model fits the data well.

(b)

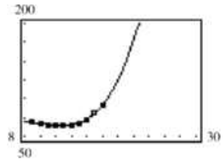


Answers will vary. Sample answer: You could use the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

(c)



The model fits the data well.



Answers will vary. Sample answer: You could use the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

114. True. $f(x) = x^6$ has only one zero, 0.

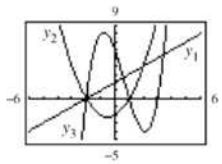
115. True. The degree is odd and the leading coefficient is –1.

116. False. The graph touches at $x = 1$, but does not cross the *x*-axis there.

117. False. The graph crosses the *x*-axis at $x = -3$ and $x = 0$.

118. False. The graph rises to the left, and rises to the right.

119.



The graph of y_3 will fall to the left and rise to the right. It will have another *x*-intercept at $(3, 0)$ of odd multiplicity (crossing the *x*-axis).

120. (a) Degree: 3
Leading coefficient: Positive

(b) Degree: 2
Leading coefficient: Positive

(c) Degree: 4
Leading coefficient: Positive

(d) Degree: 5
Leading coefficient: Positive

121. $(f + g)(-4) = f(-4) + g(-4)$

$= -59 + 128 = 69$

122. $(g - f)(3) = g(3) - f(3) = 8(3)^2 - [14(3) - 3]$

$= 72 - 39 = 33$

123. $(f \circ g)\left(-\frac{4}{7}\right) = f\left(-\frac{4}{7}\right) g\left(-\frac{4}{7}\right) = (-11)\left(\frac{8 \cdot 16}{49}\right)$

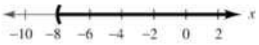
$= -\frac{1408}{49} \approx -28.7347$

124. $\left(\frac{f}{g}\right)(-1.5) = \frac{f(-1.5)}{g(-1.5)} = \frac{-24}{18} = -\frac{4}{3}$

125. $(f \circ g)(-1) = f(g(-1)) = f(8) = 109$

126. $(g \circ f)(0) = g(f(0)) = g(-3) = 8(-3)^2 = 72$

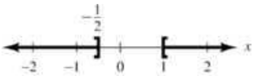
127. $3(x-5) < 4x - 7$



$3x - 15 < 4x - 7$

$-8 < x$

128. $2x^2 - x \geq 1$



$2x^2 - x - 1 \geq 0$

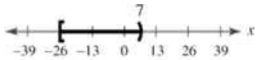
$(2x+1)(x-1) \geq 0$

$[2x+1 \geq 0 \text{ and } x-1 \geq 0]$ or $[2x+1 \leq 0 \text{ and } x-1 \leq 0]$

$\left[x \geq -\frac{1}{2} \text{ and } x \geq 1\right]$ or $\left[x \leq -\frac{1}{2} \text{ and } x \leq 1\right]$

$x \geq 1$ or $x \leq -\frac{1}{2}$

129. $\frac{5x-2}{x-7} \leq 4$



$\frac{5x-2}{x-7} - 4 \leq 0$

$\frac{5x - 2 - 4(x-7)}{x-7} \leq 0$

$\frac{x+26}{x-7} \leq 0$

$[x+26 \geq 0 \text{ and } x-7 < 0]$ or $[x+26 \leq 0 \text{ and } x-7 > 0]$

$[x \geq -26 \text{ and } x < 7]$ or $[x \leq -26 \text{ and } x > 7]$

$-26 \leq x < 7$ impossible

130. $|x+8| - 1 \geq 15$

$|x+8| \geq 16$

$x+8 \geq 16$ or $x+8 \leq -16$

$x \geq 8$ or $x \leq -24$

Section 2.3

1. $f(x)$ is the dividend, $d(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder.

2. improper

3. constant term, leading coefficient

4. Descartes's Rule, Signs

5. upper, lower

6. According to Descartes's Rule of Signs, given that $f(-x)$ has 5 variations in sign, there are either 5, 3, or 1 negative real zeros.

7. According to the Remainder Theorem, if you divide $f(x)$ by $x-4$ and the remainder is 7, then $f(4) = 7$.

8. To check whether $x-3$ is a factor of $f(x) = x^3 - 2x^2 + 3x + 4$, the synthetic division format should appear as $3\,\underline{|\,1\ -2\ 3\ 4}$ and if it is a factor, the remainder should be 0.

9.

$$\begin{array}{r} 2x+4 \\ x+3\overline{)2x^2+10x+12} \\ \underline{2x^2+6x} \\ 4x+12 \\ \underline{4x+12} \\ 0 \end{array}$$

$\frac{2x^2+10x+12}{x+3} = 2x+4,\ x \neq -3$

10.

$$\begin{array}{r} 5x+3 \\ x-4\overline{)5x^2-17x-12} \\ \underline{5x^2-20x} \\ 3x-12 \\ \underline{3x-12} \\ 0 \end{array}$$

$\frac{5x^2-17x-12}{x-4} = 5x+3,\ x \neq 4$

11.
$$\begin{array}{r} x^3+3x^2\qquad -1 \\ x+2\overline{\big)x^4+5x^3+6x^2-x-2} \\ \underline{x^4+2x^3}\qquad\qquad\quad \\ 3x^3+6x^2\qquad\quad \\ \underline{3x^3+6x^2}\qquad\quad \\ -x-2 \\ \underline{-x-2} \\ 0 \end{array}$$

$$\frac{x^4+5x^3+6x^2-x-2}{x+2}=x^3+3x^2-1,\ x\neq -2$$

12.
$$\begin{array}{r} x^2\ -x-20 \\ x-3\overline{\big)x^3-4x^2-17x+6} \\ \underline{x^3-3x^2}\qquad\qquad\quad \\ -x^2-17x\qquad \\ \underline{-x^2+\ 3x}\qquad \\ -20x+6 \\ \underline{-20x+60} \\ -54 \end{array}$$

$$\frac{x^3-4x^2-17x+6}{x-3}=x^2-x-20-\frac{54}{x-3}$$

13.
$$\begin{array}{r} x^2-\ 3x+1 \\ 4x+5\overline{\big)4x^3-7x^2-11x+5} \\ \underline{-4x^3+5x^2}\qquad\qquad\quad \\ -12x^2-11x\qquad \\ \underline{-12x^2-15x}\qquad \\ 4x+5 \\ -\underline{4x+5} \\ 0 \end{array}$$

$$\frac{4x^3-7x^2-11x+5}{4x+5}=x^2-3x+1,\ x\neq -\frac{5}{4}$$

14.
$$\begin{array}{r} x^2-25 \\ 2x-3\overline{\big)2x^3-3x^2-50x+75} \\ \underline{2x^3-3x^2}\qquad\qquad\quad \\ -50x+75 \\ \underline{-50x+75} \\ 0 \end{array}$$

$$\frac{2x^3-3x^2-50x+75}{2x-3}=x^2-25,\ x\neq\frac{3}{2}$$

15.
$$\begin{array}{r} 7x^2\ -14x+28 \\ x+2\overline{\big)7x^3+\ 0x^2\ +0x\ +\ 3} \\ \underline{7x^3+14x^2}\qquad\qquad\quad \\ -14x^2\qquad\qquad \\ \underline{-14x^2-28x}\qquad \\ 28x+3 \\ \underline{28x+56} \\ -53 \end{array}$$

$$\frac{7x^3+3}{x+2}=7x^2-14x+28-\frac{53}{x+2}$$

16.
$$\begin{array}{r} 4x^3-2x^2+\ x-\frac{1}{2} \\ 2x+1\overline{\big)8x^4+0x^3+0x^2+0x-5} \\ \underline{8x^4+4x^3}\qquad\qquad\qquad\quad \\ -4x^3\qquad\qquad\qquad \\ \underline{-4x^3-2x^2}\qquad\qquad \\ 2x^2\qquad\qquad \\ \underline{2x^2+x}\qquad \\ -x-5\qquad \\ \underline{-x-\frac{1}{2}}\qquad \\ -\frac{9}{2}\qquad \end{array}$$

$$\frac{8x^4-5}{2x+1}=4x^3-2x^2+x-\frac{1}{2}-\frac{\frac{9}{2}}{2x+1}$$

17.
$$\begin{array}{r} 3x+5 \\ 2x^2+0x+1\overline{\big)6x^3+10x^2+\ x+8} \\ \underline{6x^3+\ 0x^2\ +3x}\quad \\ 10x^2-2x\ +8 \\ \underline{10x^2+0x\ +5} \\ -2x+3 \end{array}$$

$$\frac{6x^3+10x^2+x+8}{2x^2+1}=3x+5-\frac{2x-3}{2x^2+1}$$

18.
$$\begin{array}{r} x^2+2x+4 \\ x^2-2x+3\overline{\big)x^4+0x^3+3x^2+0x+1} \\ \underline{x^4-2x^3+3x^2}\qquad\qquad\quad \\ 2x^3\qquad\ +\ 0x\qquad \\ \underline{2x^3-\ 4x^2+6x}\qquad \\ 4x^2-6x+1 \\ \underline{4x^2-8x+12} \\ 2x-11 \end{array}$$

$$\frac{x^4+3x^2+1}{x^2-2x+3}=x^2+2x+4+\frac{2x-11}{x^2-2x+3}$$

19.
$$\begin{array}{r} x \\ x^2+1\overline{\big)x^3+0x^2+0x-9} \\ \underline{x^3\qquad\ +\ x}\quad \\ -x-9 \end{array}$$

$$\frac{x^3-9}{x^2+1}=x-\frac{x+9}{x^2+1}$$

20.
$$\begin{array}{r} x^2 \\ x^3-1\overline{\big)x^5+0x^4+0x^3+0x^2+0x+7} \\ \underline{x^5\qquad\qquad\qquad -x^2\qquad\qquad} \\ x^2\qquad +7 \end{array}$$

$$\frac{x^5+7}{x^3-1}=x^2+\frac{x^2+7}{x^3-1}$$

21.
$$\begin{array}{r} 2x \\ x^2-2x+1\overline{\big)\,2x^3-4x^2-15x+5} \\ \underline{2x^3-4x^2+\ 2x} \\ -17x+5 \end{array}$$
$$\frac{2x^3-4x^2-15x+5}{(x-1)^2}=2x-\frac{17x-5}{(x-1)^2}$$

22. $(x-1)^3=x^3-3x^2+3x-1$
$$\begin{array}{r} x+3 \\ x^3-3x^2+3x-1\overline{\big)\,x^4 } \\ \underline{x^4-3x^3+3x^2-x} \\ 3x^3-3x^2+x \\ \underline{3x^3-9x^2+\ 9x-\ 3} \\ 6x^2+8x+3 \end{array}$$
$$\frac{x^4}{(x-1)^3}=x+3+\frac{6x^2-8x+3}{(x-1)^3}$$

23.
$$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$
$$\frac{3x^3-17x^2+15x-25}{x-5}=3x^2-2x+5,\ x\neq 5$$

24.
$$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & 0 \end{array}$$
$$\frac{5x^3+18x^2+7x-6}{x+3}=5x^2+3x-2,\ x\neq -3$$

25.
$$\begin{array}{r|rrrr} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$
$$\frac{6x^3+7x^2-x+26}{x-3}=6x^2+25x+74+\frac{248}{x-3}$$

26.
$$\begin{array}{r|rrrr} -6 & 2 & 14 & -20 & 7 \\ & & -12 & -12 & 192 \\ \hline & 2 & 2 & -32 & 199 \end{array}$$
$$\frac{2x^3+14x^2-20x+7}{x+6}=2x^2+2x-32+\frac{199}{x+6}$$

27.
$$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$
$$\frac{9x^3-18x^2-16x+32}{x-2}=9x^2-16,\ x\neq 2$$

28.
$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$
$$\frac{5x^3+6x+8}{x+2}=5x^2-10x+26-\frac{44}{x+2}$$

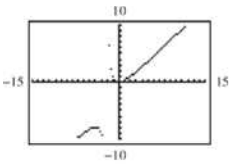
29.
$$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$
$$\frac{x^3+512}{x+8}=x^2-8x+64,\ x\neq -8$$

30.
$$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$
$$\frac{x^3-729}{x-9}=x^2+9x+81,\ x\neq 9$$

31.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$
$$\frac{4x^3+16x^2-23x-15}{x+\frac{1}{2}}=4x^2+14x-30,\ x\neq -\frac{1}{2}$$

32.
$$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$
$$\frac{3x^3-4x^2+5}{x-\frac{3}{2}}=3x^2+\frac{1}{2}x+\frac{3}{4}+\frac{49}{8x-12}$$

33.
$$\begin{aligned} y_2 &= x-2+\frac{4}{x+2} \\ &= \frac{(x-2)(x+2)+4}{x+2} \\ &= \frac{x^2-4+4}{x+2} \\ &= \frac{x^2}{x+2} \\ &= y_1 \end{aligned}$$



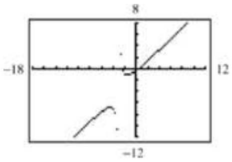
34. $y_2 = x - 1 + \dfrac{2}{x+3}$

$$= \frac{(x-1)(x+3)+2}{x+3}$$

$$= \frac{x^2+2x-3+2}{x+3}$$

$$= \frac{x^2+2x-1}{x+3}$$

$$= y_1$$



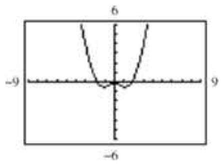
35. $y_2 = x^2 - 8 + \dfrac{39}{x^2+5}$

$$= \frac{(x^2-8)(x^2+5)+39}{x^2+5}$$

$$= \frac{x^4-8x^2+5x^2-40+39}{x^2+5}$$

$$= \frac{x^4-3x^2-1}{x^2+5}$$

$$= y_1$$

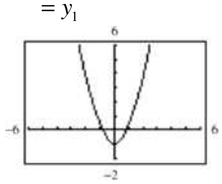


36. $y_2 = x^2 - \dfrac{1}{x^2+1}$

$$= \frac{x^2(x^2+1)-1}{x^2+1}$$

$$= \frac{x^4+x^2-1}{x^2+1}$$

$$= y_1$$



37. $f(x) = x^3 - x^2 - 14x + 11,\ k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x-4)(x^2+3x-2)+3$

$f(4) = (0)(26)+3 = 3$

38. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14,\ k = -\dfrac{2}{3}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$f(x) = \left(x + \dfrac{2}{3}\right)(15x^3 - 6x + 4) + \dfrac{34}{3}$

$f\left(-\dfrac{2}{3}\right) = \dfrac{34}{3}$

39.

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2+3\sqrt{2} & 6 \\ \hline & 1 & 3+\sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = (x-\sqrt{2})\left(x^2+(3+\sqrt{2})x+3\sqrt{2}\right)-8$

$f(\sqrt{2}) = 0(4+6\sqrt{2}) - 8 = -8$

40.

$$\begin{array}{r|rrrr} -\sqrt{5} & 1 & 2 & -5 & -4 \\ & & -\sqrt{5} & 5-2\sqrt{5} & 10 \\ \hline & 1 & 2-\sqrt{5} & -2\sqrt{5} & 6 \end{array}$$

$f(x) = (x+\sqrt{5})(x^2+(2-\sqrt{5})x-2\sqrt{5})+6$

$f(-\sqrt{5}) = 6$

41.

$$\begin{array}{r|rrrr} 1-\sqrt{3} & 4 & -6 & -12 & -4 \\ & & 4-4\sqrt{3} & 10-2\sqrt{3} & 4 \\ \hline & 4 & -2-4\sqrt{3} & -2-2\sqrt{3} & 0 \end{array}$$

$f(x) = (x-1+\sqrt{3})[4x^2-(2+4\sqrt{3})x-(2+2\sqrt{3})]$

$f(1-\sqrt{3}) = 0$

42.

$$\begin{array}{r|rrrr} 2+\sqrt{2} & -3 & 8 & 10 & -8 \\ & & -6-3\sqrt{2} & -2-4\sqrt{2} & 8 \\ \hline & -3 & 2-3\sqrt{2} & 8-4\sqrt{2} & 0 \end{array}$$

$f(x) = [x-(2+\sqrt{2})][-3x^2+(2-3\sqrt{2})x+8-4\sqrt{2}]$

$f(2+\sqrt{2}) = 0$

43. $f(x) = 2x^3 - 7x + 3$

(a)

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -7 & 3 \\ & & 2 & 2 & -5 \\ \hline & 2 & 2 & -5 & -2 \end{array} \quad = f(1)$$

(b)

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 3 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 1 \end{array} \quad = f(-2)$$

(c)

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 0 & -7 & 3 \\ & & 1 & \frac{1}{2} & -\frac{13}{4} \\ \hline & 2 & 1 & -\frac{13}{2} & -\frac{1}{4} \end{array} \quad = f\left(\frac{1}{2}\right)$$

(d)

$$\begin{array}{r|rrrr} 2 & 2 & 0 & -7 & 3 \\ & & 4 & 8 & 2 \\ \hline & 2 & 4 & 1 & 5 \end{array} \quad = f(2)$$

44. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

(a) $$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & 175 \end{array} = g(2)$$

(b) $$\begin{array}{r|rrrrrrr} 1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 2 & 2 & 5 & 5 & 4 & 4 \\ \hline & 2 & 2 & 5 & 5 & 4 & 4 & 7 \end{array} = g(1)$$

(c) $$\begin{array}{r|rrrrrrr} 3 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 6 & 18 & 63 & 189 & 564 & 1692 \\ \hline & 2 & 6 & 21 & 63 & 188 & 564 & 1695 \end{array} = g(3)$$

(d) $$\begin{array}{r|rrrrrrr} -1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & -2 & 2 & -5 & 5 & -4 & 4 \\ \hline & 2 & -2 & 5 & -5 & 4 & -4 & 7 \end{array} = g(-1)$$

45. $h(x) = x^3 - 5x^2 - 7x + 4$

(a) $$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array} = h(3)$$

(b) $$\begin{array}{r|rrrr} 2 & 1 & -5 & -7 & 4 \\ & & 2 & -6 & -26 \\ \hline & 1 & -3 & -13 & -22 \end{array} = h(2)$$

(c) $$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array} = h(-2)$$

(d) $$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array} = h(-5)$$

46. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

(a) $$\begin{array}{r|rrrrr} 1 & 4 & -16 & 7 & 0 & 20 \\ & & 4 & -12 & -5 & -5 \\ \hline & 4 & -12 & -5 & -5 & 15 \end{array} = f(1)$$

(b) $$\begin{array}{r|rrrrr} -2 & 4 & -16 & 7 & 0 & 20 \\ & & -8 & 48 & -110 & 220 \\ \hline & 4 & -24 & 55 & -110 & 240 \end{array} = f(-2)$$

(c) $$\begin{array}{r|rrrrr} 5 & 4 & -16 & 7 & 0 & 20 \\ & & 20 & 20 & 135 & 675 \\ \hline & 4 & 4 & 27 & 135 & 695 \end{array} = f(5)$$

(d) $$\begin{array}{r|rrrrr} -10 & 4 & -16 & 7 & 0 & 20 \\ & & -40 & 560 & -5670 & 56{,}700 \\ \hline & 4 & -56 & 567 & -5670 & 56{,}720 \end{array} = f(-10)$$

47. $$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^3 - 7x + 6 = (x-2)(x^2 + 2x - 3)$

$= (x-2)(x+3)(x-1)$

Zeros: $2, -3, 1$

48. $$\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$x^3 - 28x - 48 = (x+4)(x^2 - 4x - 12)$

$= (x+4)(x-6)(x+2)$

Zeros: $-4, -2, 6$

49. $$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$2x^3 - 15x^2 + 27x - 10$

$= (x - \frac{1}{2})(2x^2 - 14x + 20)$

$= (2x-1)(x-2)(x-5)$

Zeros: $\frac{1}{2}, 2, 5$

50. $$\begin{array}{r|rrrr} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$$

$48x^3 - 80x^2 + 41x - 6 = (x - \frac{2}{3})(48x^2 - 48x + 9)$

$= (3x-2)(4x-3)(4x-1)$

Zeros: $\frac{2}{3}, \frac{3}{4}, \frac{1}{4}$

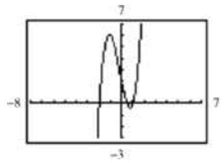
51. (a) $$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

(b) $2x^2 - 3x + 1 = (2x-1)(x-1)$

Remaining factors: $(2x-1), (x-1)$

(c) $f(x) = (x+2)(2x-1)(x-1)$

(d) Real zeros: $-2, \frac{1}{2}, 1$



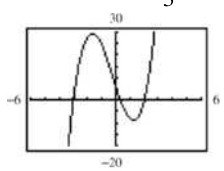
52. (a) $$\begin{array}{r|rrrr} -3 & 3 & 2 & -19 & 6 \\ & & -9 & 21 & -6 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

(b) $3x^2 - 7x + 2 = (3x-1)(x-2)$

Remaining factors: $(3x-1), (x-2)$

(c) $f(x) = (x+3)(3x-1)(x-2)$

(d) Real zeros: $-3, \frac{1}{3}, 2$



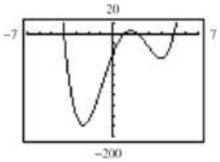
53. (a)
$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \end{array}$$
$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

(b) $x^2 - 3x + 2 = (x-2)(x-1)$

Remaining factors: $(x-2)$, $(x-1)$

(c) $f(x) = (x-5)(x+4)(x-2)(x-1)$

(d) Real zeros: $5, \ -4, \ 2, \ 1$



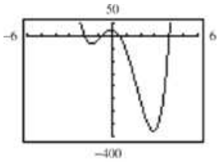
54. (a)
$$\begin{array}{r|rrrrr} -2 & 8 & -14 & -71 & -10 & 24 \\ & & -16 & 60 & 22 & -24 \\ \hline & 8 & -30 & -11 & 12 & 0 \end{array}$$
$$\begin{array}{r|rrrr} 4 & 8 & -30 & -11 & 12 \\ & & 32 & 8 & -12 \\ \hline & 8 & 2 & -3 & 0 \end{array}$$

(b) $8x^2 + 2x - 3 = (4x+3)(2x-1)$

Remaining factors: $(4x+3)$, $(2x-1)$

(c) $f(x) = (x+2)(x-4)(4x+3)(2x-1)$

(d) Real zeros: $-2, \ 4, \ -\frac{3}{4}, \ \frac{1}{2}$



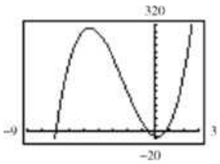
55. (a)
$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$

(b) $6x^2 + 38x - 28 = (3x-2)(2x+14)$

Remaining factors: $(3x-2)$, $(x+7)$

(c) $f(x) = (2x+1)(3x-2)(x+7)$

(d) Real zeros: $-\frac{1}{2}, \ \frac{2}{3}, \ -7$



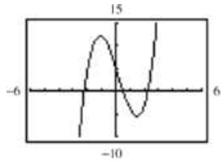
56. (a)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

(b) $2x^2 - 10 = 2(x - \sqrt{5})(x + \sqrt{5})$

Remaining factors: $(x - \sqrt{5})$, $(x + \sqrt{5})$

(c) $f(x) = (2x-1)(x+\sqrt{5})(x-\sqrt{5})$

(d) Real zeros: $\frac{1}{2}, \ \pm\sqrt{5}$



57. $f(x) = x^3 + 3x^2 - x - 3$

$p =$ factor of -3
$q =$ factor of 1

Possible rational zeros: $\pm 1, \ \pm 3$

$f(x) = x^2(x+3) - (x+3) = (x+3)(x^2-1)$

Rational zeros: $\pm 1, \ -3$

58. $f(x) = x^3 - 4x^2 - 4x + 16$

$p =$ factor of 16
$q =$ factor of 1

Possible rational zeros: $\pm 1, \ \pm 2, \ \pm 4, \ \pm 8, \ \pm 16$

$f(x) = x^2(x-4) - 4(x-4) = (x-4)(x^2-4)$

Rational zeros: $4, \ \pm 2$

59. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

$p =$ factor of -45
$q =$ factor of 2

Possible rational zeros: $\pm 1, \ \pm 3, \ \pm 5, \ \pm 9, \ \pm 15, \ \pm 45,$
$\pm\frac{1}{2}, \ \pm\frac{3}{2}, \ \pm\frac{5}{2}, \ \pm\frac{9}{2}, \ \pm\frac{15}{2}, \ \pm\frac{45}{2}$

Using synthetic division, -1, 3, and 5 are zeros.

$f(x) = (x+1)(x-3)(x-5)(2x-3)$

Rational zeros: $-1, \ 3, \ 5, \ \frac{3}{2}$

60. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

$p =$ factor of -2
$q =$ factor of 4

Possible rational zeros: $\pm 2, \ \pm 1, \ \pm\frac{1}{2}, \ \pm\frac{1}{4}$

Using synthetic division, -1, 1, and 2 are zeros.

$f(x) = (x+1)(x-1)(x-2)(2x-1)(2x+1)$

Rational zeros: $\pm 1, \ \pm\frac{1}{2}, \ 2$

61. $f(x) = 2x^4 - x^3 + 6x^2 - x + 5$

4 variations in sign $\Rightarrow$ 4, 2, or 0 positive real zeros
$f(-x) = 2x^4 + x^3 + 6x^2 + x + 5$
0 variations in sign $\Rightarrow$ 0 negative real zeros

62. $f(x) = 3x^4 + 5x^3 - 6x^2 + 8x - 3$

3 sign changes $\Rightarrow$ 3 or 1 positive real zeros
$f(-x) = 3x^4 - 5x^3 - 6x^2 - 8x - 3$
1 sign change $\Rightarrow$ 1 negative real zero

63. $g(x) = 4x^3 - 5x + 8$

2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros
$g(-x) = -4x^3 + 5x + 8$
1 variation in sign $\Rightarrow$ 1 negative real zero

64. $g(x) = 2x^3 - 4x^2 - 5$

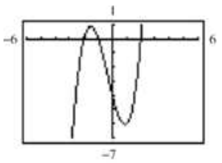
1 sign change $\Rightarrow$ 1 positive real zero
$g(-x) = -2x^3 - 4x^2 - 5$
No sign change $\Rightarrow$ no negative real zeros

65. $f(x) = x^3 + x^2 - 4x - 4$

(a) $f(x)$ has 1 variation in sign $\Rightarrow$ 1 positive real zero.
$f(-x) = -x^3 + x^2 + 4x - 4$ has 2 variations in sign $\Rightarrow$ 2 or 0 negative real zeros.

(b) Possible rational zeros: $\pm 1, \pm 2, \pm 4$

(c)



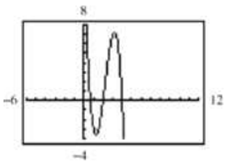
(d) Real zeros: $-2, -1, 2$

66. (a) $f(x) = -3x^3 + 20x^2 - 36x + 16$

3 sign changes $\Rightarrow$ 3 or 1 positive real zeros
$f(-x) = 3x^3 + 20x^2 + 36x + 16$
0 sign changes $\Rightarrow$ No negative real zeros

(b) Possible rational zeros:
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}, \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

(c)



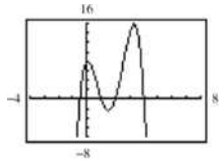
(d) Zeros: $\frac{2}{3}, 2, 4$

67. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) $f(x)$ has variations in sign $\Rightarrow$ 3 or 1 positive real zeros.
$f(-x) = -2x^4 - 13x^3 - 21x^2 - 2x + 8$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros: $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

(c)



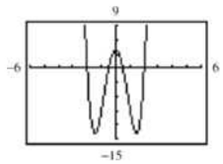
(d) Real zeros: $-\frac{1}{2}, 1, 2, 4$

68. (a) $f(x) = 4x^4 - 17x^2 + 4$

2 sign changes $\Rightarrow$ 0 or 2 positive real zeros
$f(-x) = 4x^4 - 17x^2 + 4$
2 sign changes $\Rightarrow$ 0 or 2 negative real zeros

(b) Possible rational zeros: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

(c)



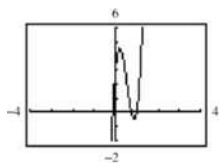
(d) Zeros: $\pm 2, \pm\frac{1}{2}$

69. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) $f(x)$ has 2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros.
$f(-x) = -32x^3 - 52x^2 - 17x + 3$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros:
$\pm\frac{1}{32}, \pm\frac{1}{16}, \pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{32}, \pm\frac{3}{16}, \pm\frac{3}{8}, \pm\frac{3}{4}, \pm\frac{3}{2}, \pm 3$

(c)



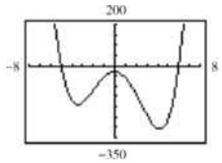
(d) Real zeros: $1, \frac{3}{4}, -\frac{1}{8}$

70. $f(x) = x^4 - x^3 - 29x^2 - x - 30$

(a) $f(x)$ has 1 variation in sign $\Rightarrow$ 1 positive real zero.
$f(-x) = x^4 + x^3 - 29x^2 + x - 30$ has 3 variations in signs $\Rightarrow$ 3 or 1 negative real zeros.

(b) Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

(c)



(d) Real zeros: −5, 6

71. $f(x) = x^4 - 4x^3 + 15$

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 0 & 15 \\ & & 4 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 15 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 0 & 15 \\ & & -1 & 5 & -5 & 5 \\ \hline & 1 & -5 & 5 & -5 & 20 \end{array}$$

−1 is a lower bound.

Real zeros: 1.937, 3.705

72. $f(x) = 2x^3 - 3x^2 - 12x + 8$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -12 & 8 \\ & & 8 & 20 & 32 \\ \hline & 2 & 5 & 8 & 40 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -12 & 8 \\ & & -6 & 27 & -45 \\ \hline & 2 & -9 & 15 & -37 \end{array}$$

−3 is a lower bound.

Real zeros: −2.152, 0.611, 3.041

73. $f(x) = x^4 - 4x^3 + 16x - 16$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 25 & 105 & 525 & 2705 \\ \hline & 5 & 21 & 105 & 541 & 2689 \end{array}$$

5 is an upper bound.

$$\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$$

−3 is a lower bound.

Real zeros: −2, 2

74. $f(x) = 2x^4 - 8x + 3$

$$\begin{array}{r|rrrrr} 3 & 2 & 0 & 0 & -8 & 3 \\ & & 6 & 18 & 54 & 138 \\ \hline & 2 & 6 & 18 & 46 & 141 \end{array}$$

3 is an upper bound.

$$\begin{array}{r|rrrrr} -4 & 2 & 0 & 0 & -8 & 3 \\ & & -8 & 32 & -128 & 544 \\ \hline & 2 & -8 & 32 & -136 & 547 \end{array}$$

−4 is lower bound.

Real zeros: 0.380, 1.435

75. $P(x) = x^4 - \frac{25}{4}x^2 + 9$

$$= \frac{1}{4}(4x^4 - 25x^2 + 36)$$

$$= \frac{1}{4}(4x^2 - 9)(x^2 - 4)$$

$$= \frac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2)$$

The rational zeros are $\pm\frac{3}{2}$ and ± 2.

76. $f(x) = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$f(x) = \frac{1}{2}(x - 4)(2x^2 + 5x - 3)$$

$$= \frac{1}{2}(x - 4)(2x - 1)(x + 3)$$

Rational zeros: $-3, \frac{1}{2}, 4$

77. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$$= \frac{1}{4}(4x^3 - x^2 - 4x + 1)$$

$$= \frac{1}{4}[x^2(4x - 1) - (4x - 1)]$$

$$= \frac{1}{4}(4x - 1)(x^2 - 1)$$

$$= \frac{1}{4}(4x - 1)(x + 1)(x - 1)$$

The rational zeros are $\frac{1}{4}$ and ± 1.

78. $f(z) = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

Possible rational zeros: $\pm 1, \ \pm 2, \ \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \ \pm\frac{1}{6}$

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array}$$

$$\begin{aligned} f(x) &= \frac{1}{6}(z+2)(6z^2 - z - 1) \\ &= \frac{1}{6}(z+2)(3z+1)(2z-1) \end{aligned}$$

Rational zeros: $-2, \ -\frac{1}{3}, \ \frac{1}{2}$

79. $\begin{aligned} f(x) &= x^3 - 1 \\ &= (x-1)(x^2 + x + 1) \end{aligned}$

Rational zeros: $1 \ (x = 1)$

Irrational zeros: 0

Matches (d).

80. $\begin{aligned} f(x) &= x^3 - 2 \\ &= (x - \sqrt[3]{2})\left(x^2 + \sqrt[3]{2}\,x + \sqrt[3]{4}\right) \end{aligned}$

Rational zeros: 0

Irrational zeros: $1 \ \left(x = \sqrt[3]{2}\right)$

Matches (a).

81. $f(x) = x^3 - x = x(x+1)(x-1)$

Rational zeros: $3 \ (x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

82. $\begin{aligned} f(x) &= x^3 - 2x \\ &= x(x^2 - 2) \\ &= x\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) \end{aligned}$

Rational zeros: $1 \ (x = 0)$

Irrational zeros: $2 \ \left(x = \pm\sqrt{2}\right)$

Matches (c).

83. $y = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

Using the graph and synthetic division, $-\frac{1}{2}$ is a zero.

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -9 & 5 & 3 & -1 \\ & & -1 & 5 & -5 & 1 \\ \hline & 2 & -10 & 10 & -2 & 0 \end{array}$$

$y = \left(x + \frac{1}{2}\right)(2x^3 - 10x^2 + 10x - 2)$

$x = 1$ is a zero of the cubic, so

$y = (2x+1)(x-1)(x^2 - 4x + 1).$

For the quadratic term, use the Quadratic Formula.

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

The real zeros are $-\frac{1}{2}, \ 1, \ 2 \pm \sqrt{3}.$

84. $y = x^4 - 5x^3 - 7x^2 + 13x - 2$

Using the graph and synthetic division, 1 and –2 are zeros.

$y = (x-1)(x+2)(x^2 - 6x + 1)$

For the quadratic term, use the Quadratic Formula.

$$x = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}$$

The real zeros are $1, \ -2, \ 3 \pm 2\sqrt{2}.$

85. $y = -2x^4 + 17x^3 - 3x^2 - 25x - 3$

Using the graph and synthetic division, -1 and $\frac{3}{2}$ are zeros.

$y = -(x+1)(2x-3)(x^2 - 8x - 1)$

For the quadratic term, use the Quadratic Formula.

$$x = \frac{8 \pm \sqrt{64 + 4}}{2} = 4 \pm \sqrt{17}$$

The real zeros are $-1, \ \frac{3}{2}, 4 \pm \sqrt{17}.$

86. $y = -x^4 + 5x^3 - 10x - 4$

Using the graph and synthetic division, 2 and –1 are zeros.

$y = -(x-2)(x+1)(x^2 - 4x - 2)$

For the quadratic term, use the Quadratic Formula.

$$x = \frac{4 \pm \sqrt{16 + 8}}{2} = 2 \pm \sqrt{6}$$

The real zeros are $2, \ -1, \ 2 \pm \sqrt{6}.$

87. $3x^4 - 14x^2 - 4x = 0$

$x(3x^3 - 14x - 4) = 0$

$x = 0$ is a real zero.

Possible ratoinal zeros: $\pm 1,\ \pm 2,\ \pm 4,\ \pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm\frac{4}{3}$

$$\begin{array}{r|rrrr} -2 & 3 & 0 & -14 & -4 \\ & & -6 & 12 & 4 \\ \hline & 3 & -6 & -2 & 0 \end{array}$$

$x = -2$ is a real zero.

Use the Quadratic Formula. $3x^2 - 6x - 2 = 0$

$$x = \frac{3 \pm \sqrt{15}}{3}$$

Real zeros: $x = 0,\ -2,\ \frac{3 \pm \sqrt{15}}{3}$

88. $4x^4 - 11x^3 - 22x^2 + 8x = 0$

$x(4x^3 - 11x^2 - 22x + 8) = 0$

$x = 0$ is a real zero.

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{4}, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} 4 & 4 & -11 & -22 & 8 \\ & & 16 & 20 & -8 \\ \hline & 4 & 5 & -2 & 0 \end{array}$$

$x = 4$ is a real zero.

Using the Quadratic Formula.

$4x^2 + 5x - 2 = 0$ $\qquad x = \frac{-5 \pm \sqrt{57}}{8}$

Real zeros: $x = 0,\ 4,\ \frac{-5 \pm \sqrt{57}}{8}$

89. $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1,\ \pm 2,\ \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$z = -1$ and $z = 2$ are real zeros.

$z^4 - z^3 - 2z - 4 = (z + 1)(z - 2)(z^2 + 2) = 0$

The only real zeros are –1 and 2. You can verify this by graphing the function $f(z) = z^4 - z^3 - 2z - 4$.

90. $4x^3 + 7x^2 - 11x - 18 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18,$ $\pm\frac{1}{4},\ \pm\frac{1}{2},\ \pm\frac{3}{4},\ \pm\frac{3}{2},\ \pm\frac{9}{4},\ \pm\frac{9}{2}$

$$\begin{array}{r|rrrr} -2 & 4 & 7 & -11 & -18 \\ & & -8 & 2 & 18 \\ \hline & 4 & -1 & -9 & 0 \end{array}$$

$x = -2$ is a real zero.

Use the Quadratic Formula. $4x^2 - x - 9 = 0$

$$x = \frac{1 \pm \sqrt{145}}{8}$$

Real zeros: $x = -2,\ \frac{1 \pm \sqrt{145}}{8}$

91. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Possible rational zeros:

$\pm\frac{1}{2},\ \pm 1,\ \pm\frac{3}{2},\ \pm 2,\ \pm 3,\ \pm 6$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & 7 & -26 & 23 & -6 \\ & & 1 & 4 & -11 & 6 \\ \hline & 2 & 8 & -22 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & 8 & -22 & 12 \\ & & 2 & 10 & -12 \\ \hline & 2 & 10 & -12 & 0 \end{array}$$

$$\begin{array}{r|rrr} -6 & 2 & 10 & -12 \\ & & -12 & 12 \\ \hline & 2 & -2 & 0 \end{array}$$

$$\begin{array}{r|rr} 1 & 2 & -2 \\ & & 2 \\ \hline & 2 & 0 \end{array}$$

$x = \frac{1}{2},\ x = 1, x = -6,$ and $x = 1$ are real zeros.

$(y + 6)(y - 1)^2(2y - 1) = 0$

Real zeros: $x = -6,\ 1,\ \frac{1}{2}$

92. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

$x(x^4 - x^3 - 3x^2 + 5x - 2) = 0$

$x = 0$ is a real zero.

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -3 & 5 & -2 \\ & & 1 & 0 & -3 & 2 \\ \hline & 1 & 0 & -3 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x=1$ and $x=-2$ are real zeros.

$$x(x-1)(x+2)(x^2-2x+1)=0$$
$$x(x-1)(x+2)(x-1)(x-1)=0$$

Real zeros: $-2,\ 0,\ 1$

93. $4x^4-55x^2-45x+36=0$

Possible rational zeros:
$\pm\frac{1}{2},\ \pm\frac{3}{4},\ \pm1,\ \pm\frac{3}{2},\ \pm2,\ \pm\frac{9}{4},\ \pm3,\ \pm4,\ \pm\frac{9}{2},\ \pm6,\ \pm9,\ \pm18$

$$\begin{array}{r|rrrrr} 4 & 4 & 0 & -55 & -45 & 36 \\ & & 16 & 64 & 36 & -36 \\ \hline & 4 & 16 & 9 & -9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 4 & 16 & 9 & -9 \\ & & -12 & -12 & 9 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2} & 4 & 4 & -3 \\ & & 2 & 3 \\ \hline & 4 & 6 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{3}{2} & 4 & 6 \\ & & -6 \\ \hline & 4 & 0 \end{array}$$

$x=4,\ x=-3,\ x=\frac{1}{2},$ and $x=-\frac{3}{2}$ are real zeros.

$$(x-4)(x+3)(2x-1)(2x+3)=0$$

Real zeros: $x=4,-3,\frac{1}{2},-\frac{3}{2}$

94. $4x^4-43x^2-9x+90=0$

Possible rational zeros:

$\pm1, \pm2, \pm3, \pm5, \pm6, \pm9, \pm10, \pm15, \pm18,$

$\pm30, \pm45, \pm90, \pm\frac{1}{2},$

$\pm\frac{3}{2},\ \pm\frac{3}{4},\ \pm\frac{5}{2},\ \pm\frac{5}{4},\ \pm\frac{9}{2},\ \pm\frac{9}{4},\ \pm\frac{15}{2},\ \pm\frac{15}{4},\ \pm\frac{45}{2},\ \pm\frac{45}{4}$

$$\begin{array}{r|rrrrr} -\frac{5}{2} & 4 & 0 & -43 & -9 & 90 \\ & & -10 & 25 & 45 & -90 \\ \hline & 4 & -10 & -18 & 36 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 4 & -10 & -18 & 36 \\ & & -8 & 36 & -36 \\ \hline & 4 & -18 & 18 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & -18 & 18 \\ & & 6 & -18 \\ \hline & 4 & -12 & 0 \end{array}$$

$$\begin{array}{r|rr} 3 & 4 & -12 \\ & & 12 \\ \hline & 4 & 0 \end{array}$$

$x=-\frac{5}{2},\ x=-2,\ x=\frac{3}{2},$ and $x=3$ are real zeros.

$$(2x+5)(x+2)(2x-3)(x-3)=0$$

Real zeros: $x=-\frac{5}{2},\ -2,\ \frac{3}{2},\ 3$

95. $8x^4+28x^3+9x^2-9x=0$
$x(8x^3+28x^2+9x-9)=0$

$x=0$ is a real zero.

Possible rational zeros:

$\pm1,\ \pm3,\ \pm9,\ \pm\frac{1}{8},\ \pm\frac{3}{8},\ \pm\frac{9}{8},\ \pm\frac{1}{2},\ \pm\frac{3}{2},\ \pm\frac{9}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{4},\ \pm\frac{9}{4}$

$$\begin{array}{r|rrrr} -3 & 8 & 28 & 9 & -9 \\ & & -24 & -12 & 9 \\ \hline & 8 & 4 & -3 & 0 \end{array}$$

$x=-3$ is a real zeros

Use the Quadratic Formula.

$8x^2+4x-3=0 \qquad x=\dfrac{-\pm\sqrt{7}}{4}$

Real zeros: $x=0,\ -3,\ \dfrac{-1\pm\sqrt{7}}{4}$

96. $x^5+5x^4-5x^3-15x^2-6x=0$
$x(x^4+5x^3-5x^2-15x-6)=0$

$x=0$ is a real zero.

Possible rational zeros: $\pm1,\ \pm2,\ \pm3,\ \pm6$

$$\begin{array}{r|rrrrr} -1 & 1 & 5 & -5 & -15 & -6 \\ & & -1 & -4 & 9 & 6 \\ \hline & 1 & 4 & -9 & -6 & 0 \end{array}$$

$x=-1$ is a real zero.

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -9 & -6 \\ & & 2 & 12 & 6 \\ \hline & 1 & 6 & 3 & 0 \end{array}$$

$x=2$ is a real zero.

Use the Quadratic Formula.

$x^2+6x+3=0 \qquad x=-3\pm\sqrt{6}$

Real zeros: $x=0,-1,\ 2,-3\pm\sqrt{6}$

97. $4x^5+12x^4-11x^3-42x^2+7x+30=0$

Possible rational zeros:
$\pm1,\ \pm2,\ \pm3,\ \pm5,\ \pm6,\ \pm10,\ \pm15,\ \pm30,$
$\pm\frac{1}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{2},\ \pm\frac{3}{4},\ \pm\frac{5}{2},\ \pm\frac{5}{4},\ \pm\frac{15}{2},\ \pm\frac{15}{4}$

$$\begin{array}{r|rrrrrr} 1 & 4 & 12 & -11 & -42 & 7 & 30 \\ & & 4 & 16 & 5 & -37 & -30 \\ \hline & 4 & 16 & 5 & -37 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 4 & 16 & 5 & -37 & -30 \\ & & -4 & -12 & 7 & 30 \\ \hline & 4 & 12 & -7 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 4 & 12 & -7 & -30 \\ & & -8 & -8 & 30 \\ \hline & 4 & 4 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & 4 & -15 \\ & & 6 & 15 \\ \hline & 4 & 10 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{5}{2} & 4 & 10 \\ & & -10 \\ \hline & 4 & 0 \end{array}$$

$x = 1,\ x = -1,\ x = -2,\ x = \dfrac{3}{2}$, and $x = -\dfrac{5}{2}$ are real zeros.

$(x-1)(x+1)(x+2)(2x-3)(2x+5) = 0$

Real zeros: $x = 1,\ -1,\ -2,\ \dfrac{3}{2},\ -\dfrac{5}{2}.$

98. $4x^5 + 8x^4 - 15x^3 - 23x^2 + 11x + 15 = 0$

Possible rational zeros:

$\pm 1,\ \pm 3,\ \pm 5,\ \pm 15,\ \pm\dfrac{1}{2},\ \pm\dfrac{1}{4},\ \pm\dfrac{3}{2},\ \pm\dfrac{3}{4},\ \pm\dfrac{5}{2},\ \pm\dfrac{5}{4},\ \pm\dfrac{15}{2},\ \pm\dfrac{15}{4}$

$$\begin{array}{r|rrrrrr} 1 & 4 & 8 & -15 & -23 & 11 & 15 \\ & & 4 & 12 & -3 & -26 & -15 \\ \hline & 4 & 12 & -3 & -26 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 4 & 12 & -3 & -26 & -15 \\ & & -4 & -8 & 11 & 15 \\ \hline & 4 & 8 & -11 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 4 & 8 & -11 & -15 \\ & & -4 & -4 & 15 \\ \hline & 4 & 4 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & 4 & -15 \\ & & 6 & 15 \\ \hline & 4 & 10 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{5}{2} & 4 & 10 \\ & & -10 \\ \hline & 4 & 0 \end{array}$$

$x = 1,\ x = -1,\ x = -1,\ x = \dfrac{3}{2}$, and $x = \dfrac{5}{2}$ are real zeros.

Real zeros: $x = 1,\ -1,\ -1,\ \dfrac{3}{2},\ -\dfrac{5}{2}.$

99. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) Zeros: $-2,\ 3.732,\ 0.268$

(b) $$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$ $t = -2$ is a zero.

(c) $h(t) = (t+2)(t^2 - 4t + 1)$

$= (t+2)\left[t - (\sqrt{3}+2)\right]\left[t + (\sqrt{3}-2)\right]$

100. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) Zeros $6,\ 5.236,\ 0.764$

(b) $$\begin{array}{r|rrrr} 6 & 1 & -12 & 40 & -24 \\ & & 6 & -36 & 24 \\ \hline & 1 & -6 & 4 & 0 \end{array}$$ $s = 6$ is a zeros.

(c) $f(s) = (s-6)(s^2 - 6s + 4)$

$= (s-6)(s-3-\sqrt{5})(s-3-\sqrt{5})$

101. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $x = 0,\ 3,\ 4,\ \pm 1.414$

(b) $$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

$x = 3$ is a zero.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$ $x = 4$ is a zero.

(c) $h(x) = x(x-3)(x-4)(x^2 - 2)$

$= x(x-3)(x-4)(x-\sqrt{2})(x+\sqrt{2})$

102. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) $x = \pm 3.0,\ 1.5,\ 0.333$

(b) $$\begin{array}{r|rrrrr} 3 & 6 & -11 & -51 & 99 & -27 \\ & & 18 & 21 & -90 & 27 \\ \hline & 6 & 7 & -30 & 9 & 0 \end{array}$$

$x = 3$ is a zero.

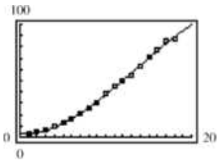
$$\begin{array}{r|rrrr} -3 & 6 & 7 & -30 & 9 \\ & & -18 & 33 & -9 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

$x = -3$ is a zero.

(c) $g(x) = (x-3)(x+3)(6x^2 - 11x + 3)$

$= (x-3)(x+3)(3x-1)(2x-3)$

103. (a)



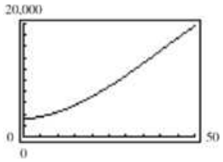
(b) The model fits the data well.

(c) $S = -0.0135t^3 + 0.545t^2 - 0.71t + 3.6$

25	–0.0135	0.545	–0.71	3.6
		–0.3375	5.1875	111.9375
	– 0.0135	0.2075	4.4775	115.5375

In 2015 (t = 25), the model predicts approximately 116 subscriptions per 100 people, obviously not a reasonable prediction because you cannot have more subscriptions than people.

104. (a)



(b) In 1960 $(t = 0)$, there were approximately 3,167,000 employees.

40	–0.084	10.32	–23.5	3167
		–3.36	278.4	10,196
	– 0.084	6.96	254.9	13,363

In 2000 (t = 40), the model predicts approximately 13,363,000 employees.

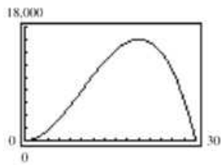
(c) Answers will vary.

105. (a) Combined length and width:

$$4x + y = 120 \Rightarrow y = 120 - 4x$$

$$\begin{aligned} \text{Volume} = l \cdot w \cdot h &= x^2 y \\ &= x^2(120 - 4x) \\ &= 4x^2(30 - x) \end{aligned}$$

(b)



Dimension with maximum volume: $20 \times 20 \times 40$

(c) $$13{,}500 = 4x^2(30 - x)$$

$$4x^3 - 120x^2 + 13{,}500 = 0$$

$$x^3 - 30x^2 + 3375 = 0$$

15	1	–30	0	3375
		15	–225	–3375
	1	–15	–225	0

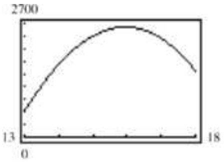
$$(x - 15)(x^2 - 15x - 225) = 0$$

Using the Quadratic Formula, $x = 15$ or $\dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

106. $y = -5.05x^3 + 3857x - 38{,}411.25, \ 13 \le x \le 18$

(a)



(b) The second air-fuel ratio of 16.89 can be obtained by finding the second point where the curves y and $y_1 = 2400$ intersect.

(c) Solve $-5.05x^3 + 3857x - 38{,}411.25 = 2400$ or $-5.05x^3 + 3857x - 40{,}811.25 = 0$.

By synthetic division:

15	–5.05	0	3857	–40,811.25
		–75.75	–1136.25	40,811.25
	–5.05	–75.75	2720.75	0

The positive zero of the quadratic $-5.05x^2 - 75.75x + 2720.75$ can be found using the Quadratic Formula.

$$x = \frac{75.75 - \sqrt{(-75.75)^2 - 4(-5.05)(2720.75)}}{2(-5.05)} \approx 16.89$$

107. False, $-\dfrac{4}{7}$ is a zero of f.

108. False, $f\left(\dfrac{1}{7}\right) \approx -7.896 \ne 0$.

109. The zeros are 1, 1, and –2. The graph falls to the right.

$y = a(x - 1)^2(x + 2), \ a < 0$

Since $f(0) = -4, \ a = -2$.

$y = -2(x - 1)^2(x + 2)$

110. The zeros are 1, –1, and –2. The graph rises to the right.

$y = a(x - 1)(x + 1)(x + 2), \ a > 0$

Since $f(0) = -4, \ a = 2$.

$y = 2(x - 1)(x + 1)(x + 2)$

111. $f(x) = -(x + 1)(x - 1)(x + 2)(x - 2)$

112. Use synthetic division.

3	1	$-k$	$2k$	–12
		3	$9 - 3k$	$27 - 3k$
	1	$3 - k$	$9 - k$	$15 - 3k$

Since the remainder $15 - 3k$ should be 0, $k = 5$.

113. (a) $\dfrac{x^2-1}{x-1} = x+1,\ x \neq 1$

(b) $\dfrac{x^3-1}{x-1} = x^2+x+1,\ x \neq 1$

(c) $\dfrac{x^4-1}{x-1} = x^3+x^2+x+1,\ x \neq 1$

In general,

$$\frac{x^n-1}{x-1} = x^{n-1} + x^{n-2} + \cdots + x + 1,\ x \neq 1.$$

114. (a) f has 1 negative real zero. $f(-x)$ has only 1 sign change.

(b) Because f has 4 sign changes, f has either 4 or 2 positive real zeros. The graph either turns upward and rises to the right or it turns upward and crosses the x-axis, then turns downward and crosses the x-axis and then turns back upward and rises to the right.

(c) No. No factor of 3 divided by a factor of 2 is equal to $-\dfrac{1}{3}$.

(d) Use synthetic division. If $r = 0$, then $x - \dfrac{3}{2}$ is a factor of f. Otherwise, if each number in the last row is either positive or 0, then $x = \dfrac{3}{2}$ is an upper bound.

115. $9x^2 - 25 = 0$

$$(3x+5)(3x-5) = 0$$
$$x = -\frac{5}{3},\ \frac{5}{3}$$

116. $16x^2 - 21 = 0$

$$x^2 = \frac{21}{16}$$
$$x = \pm\frac{\sqrt{21}}{4}$$

117. $2x^2 + 6x + 3 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-6 \pm \sqrt{12}}{4}$$
$$= \frac{-3 \pm \sqrt{3}}{2}$$
$$x = -\frac{3}{2} + \frac{\sqrt{3}}{2}, -\frac{3}{2} - \frac{\sqrt{3}}{2}$$

118. $8x^2 - 22x + 15 = 0$

$$(4x-5)(2x-3) = 0$$
$$x = \frac{5}{4},\ \frac{3}{2}$$

Section 2.4

1. (a) ii
(b) iii
(c) i

2. $\sqrt{-1},\ -1$

3. $(7+6i)+(8+5i) = (7+8)+(6+5)i$
$= 15 + 11i$

The real part is 15 and the imaginary part is $11i$.

4. When multiplying complex numbers, the FOIL Method can be used.

$$(a+bi)(c+di) = ac + adi + bci + bdi^2$$
$$= (ac - bd) + (ad + bc)i$$

5. The additive inverse of $2 - 4i$ is $-2 + 4i$ so that $(2-4i)+(-2+4i) = 0$.

6. The complex conjugate of $2-4i$ is $2+4i$ so that $(2-4i)(2+4i) = 4 - 16i^2 = 4 + 16 = 20$.

7. $a + bi = -9 + 4i$
$a = -9$
$b = 4$

8. $a + bi = 12 + 5i$
$a = 12$
$b = 5$

9. $3a + (b+3)i = 9 + 8i$

$3a = 9 \qquad b + 3 = 8$

$a = 3 \qquad b = 5$

10. $(a+6) + 26i = 6 - i$

$a + 6 = 6 \qquad 2b = -1$

$-a = 0 \qquad b = -\dfrac{1}{2}$

11. $5 + \sqrt{-16} = 5 + \sqrt{16(-1)}$
$= 5 + 4i$

12. $2 - \sqrt{-9} = 2 - \sqrt{9(-1)}$
$= 2 - 3i$

13. $-6 = -6 + 0i$

14. $8 = 8 + 0i$

15. $-5i + i^2 = -5i - 1 = -1 - 5i$

16. $-3i^2 + i = -3(-1) + i$
$= 3 + i$

17. $\left(\sqrt{-75}\right)^2 = -75$

18. $\left(\sqrt{-4}\right)^2 - 7 = -4 - 7$
$= -11$

19. $\sqrt{-0.09} = \sqrt{0.09}i = 0.3i$

20. $\sqrt{-0.0004} = 0.02i$

21. $(4+i)-(7-2i) = (4-7)+(1+2)i$
$= -3+3i$

22. $(11-2i)-(-3+6i) = (11+3)+(-2-6)i$
$= 14-8i$

23. $(-1+8i)+(8-5i) = (-1+8)+(8-5)i$
$= 7+3i$

24. $(7+6i)+(3+12i) = (7+3)+(6+12)i$
$= 10+18i$

25. $13i-(14-7i) = 13i-14+7i = -14+20i$

26. $22+(-5+8i)-9i = (22+(-5))+(8-9)i$
$= 17-i$

27. $\left(\frac{3}{2}+\frac{5}{2}i\right)+\left(\frac{5}{3}+\frac{11}{3}i\right) = \left(\frac{3}{2}+\frac{5}{3}\right)+\left(\frac{5}{2}+\frac{11}{3}\right)i$
$= \frac{9+10}{6}+\frac{15+22}{6}i$
$= \frac{19}{6}+\frac{37}{6}i$

28. $\left(\frac{3}{4}+\frac{7}{5}i\right)-\left(\frac{5}{6}-\frac{1}{6}i\right) = \left(\frac{3}{4}-\frac{5}{6}\right)+\left(\frac{7}{5}+\frac{1}{6}\right)i$
$= -\frac{1}{12}+\frac{47}{30}i$

29. $(1.6+3.2i)+(-5.8+4.3i) = -4.2+7.5i$

30. $-(-3.7-12.8i)-(6.1-16.3i)$
$= (3.7+12.8i)+(-6.1+16.3i)$
$= (3.7-6.1)+(12.8+16.3)i$
$= 2.4+29.1i$

31. $4(3+5i) = 12+20i$

32. $-6(5-3i) = -30+18i$

33. $(1+i)(3-2i) = 3-2i+3i-2i^2$
$= 3+i+2$
$= 5+i$

34. $(6-2i)(2-3i) = 12-18i-4i+6i^2$
$= 12-22i-6$
$= 6-22i$

35. $4i(8+5i) = 32i+20i^2$
$= 32i+20(-1)$
$= -20+32i$

36. $-3i(6-i) = -18i-3 = -3-18i$

37. $\left(\sqrt{14}+\sqrt{10}\,i\right)\left(\sqrt{14}-\sqrt{10}\,i\right) = 14-10i^2 = 14+10 = 24$

38. $\left(\sqrt{3}+\sqrt{15}i\right)\left(\sqrt{3}-\sqrt{15}i\right) = 3-15i^2$
$= 3-15(-1)$
$= 3+15$
$= 18$

39. $(6+7i)^2 = 36+42i+42i+49i^2$
$= 36+84i-49$
$= -13+84i$

40. $(5-4i)^2 = 25-20i-20i+16i^2$
$= 25-40i-16$
$= 9-40i$

41. $(4+5i)^2-(4-5i)^2$
$= \left[(4+5i)+(4-5i)\right]\left[(4+5i)-(4-5i)\right] = 8(10i) = 80i$

42. $(1-2i)^2-(1+2i)^2 = 1-4i+4i^2-(1+4i+4i^2)$
$= 1-4i+4i^2-1-4i-4i^2$
$= -8i$

43. $4-3i$ is the complex conjugate of $4+3i$.

$(4+3i)(4-3i) = 16+9 = 25$

44. The complex conjugate of $7-5i$ is $7+5i$.

$(7-5i)(7+5i) = 49+25 = 74$

45. $-6+\sqrt{5}i$ is the complex conjugate of $-6-\sqrt{5}i$.

$\left(-6-\sqrt{5}i\right)\left(-6+\sqrt{5}i\right) = 36+5 = 41$

46. The complex conjugate of $-3+\sqrt{2}i$ is $-3-\sqrt{2}i$.

$\left(-3+\sqrt{2}i\right)\left(-3-\sqrt{2}i\right) = 9+2 = 11$

47. $-\sqrt{20}i$ is the complex conjugate of $\sqrt{-20} = \sqrt{20}i$.

$\left(\sqrt{20}i\right)\left(-\sqrt{20}i\right) = 20$

48. The complex conjugate of $\sqrt{-13} = \sqrt{13}i$ is $-\sqrt{13}i$.

$\sqrt{-13}\left(-\sqrt{13}i\right) = \left(\sqrt{13}i\right)\left(-\sqrt{13}i\right) = 13$

49. $3+\sqrt{2}i$ is the complex conjugate of
$3-\sqrt{-2} = 3-\sqrt{2}\,i$

$(3-\sqrt{2}i)(3+\sqrt{2}i) = 9+2 = 11$

50. The complex conjugate of $1+\sqrt{-8}=1+2\sqrt{2}i$ is $1-2\sqrt{2}i$.

$$\left(1+2\sqrt{2}i\right)\left(1-2\sqrt{2}i\right)=1+8=9$$

51. $\dfrac{6}{i}=\dfrac{6}{i}\cdot\dfrac{-i}{-i}=\dfrac{-6i}{-i^2}=\dfrac{-6i}{1}=-6i$

52. $\dfrac{-5}{2i}\cdot\dfrac{i}{i}=\dfrac{-5i}{-2}=\dfrac{5}{2}i$

53. $\dfrac{2}{4-5i}=\dfrac{2}{4-5i}\cdot\dfrac{4+5i}{4+5i}=\dfrac{8+10i}{16+25}=\dfrac{8}{41}+\dfrac{10}{41}i$

54. $\dfrac{3}{1-i}\cdot\dfrac{1+i}{1+i}=\dfrac{3+3i}{1-i^2}=\dfrac{3+3i}{2}=\dfrac{3}{2}+\dfrac{3}{2}i$

55.
$$\begin{aligned}\frac{2+i}{2-i}&=\frac{2+i}{2-i}\cdot\frac{2+i}{2+i}\\&=\frac{4+4i+i^2}{4+1}\\&=\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i\end{aligned}$$

56.
$$\begin{aligned}\frac{8-7i}{1-2i}\cdot\frac{1+2i}{1+2i}&=\frac{8+16i-7i-14i^2}{1-4i^2}\\&=\frac{22+9i}{5}=\frac{22}{5}+\frac{9}{5}i\end{aligned}$$

57.
$$\begin{aligned}\frac{i}{(4-5i)^2}&=\frac{i}{16-25-40i}\\&=\frac{i}{-9-40i}\cdot\frac{-9+40i}{-9+40i}\\&=\frac{-40-9i}{81+40^2}\\&=-\frac{40}{1681}-\frac{9}{1681}i\end{aligned}$$

58.
$$\begin{aligned}\frac{5i}{(2+3i)^2}&=\frac{5i}{-5+12i}\cdot\frac{-5-12i}{-5-12i}\\&=\frac{-25i+60}{25+144}\\&=\frac{60}{169}-\frac{25}{169}i\end{aligned}$$

59.
$$\begin{aligned}\frac{2}{1+i}-\frac{3}{1-i}&=\frac{2(1-i)-3(1+i)}{(1+i)(1-i)}\\&=\frac{2-2i-3-3i}{1+1}\\&=\frac{-1-5i}{2}\\&=-\frac{1}{2}-\frac{5}{2}i\end{aligned}$$

60.
$$\begin{aligned}\frac{2i}{2+i}+\frac{5}{2-i}&=\frac{2i(2-i)}{(2+i)(2-i)}+\frac{5(2+i)}{(2+i)(2-i)}\\&=\frac{4i-2i^2+10+5i}{4-i^2}\\&=\frac{12+9i}{5}\\&=\frac{12}{5}+\frac{9}{5}i\end{aligned}$$

61.
$$\begin{aligned}\frac{i}{3-2i}+\frac{2i}{3+8i}&=\frac{3i+8i^2+6i-4i^2}{(3-2i)(3+8i)}\\&=\frac{-4+9i}{9+18i+16}\\&=\frac{-4+9i}{25+18i}\cdot\frac{25-18i}{25-18i}\\&=\frac{-100+72i+225i+162}{25^2+18^2}\\&=\frac{62+297i}{949}\\&=\frac{62}{949}+\frac{297}{949}i\end{aligned}$$

62.
$$\begin{aligned}\frac{1+i}{i}-\frac{3}{4-i}&=\frac{1+i}{i}\cdot\frac{-i}{-i}-\frac{3}{4-i}\cdot\frac{4+i}{4+i}\\&=\frac{-i+1}{1}-\frac{12+3i}{16+1}\\&=\frac{5}{17}-\frac{20}{17}i\end{aligned}$$

63.
$$\begin{aligned}\sqrt{-18}-\sqrt{-54}&=3\sqrt{2}i-3\sqrt{6}i\\&=3\left(\sqrt{2}-\sqrt{6}\right)i\end{aligned}$$

64.
$$\begin{aligned}\sqrt{-50}+\sqrt{-275}&=5\sqrt{2}i+5\sqrt{11}i\\&=5\left(\sqrt{2}+\sqrt{11}\right)i\end{aligned}$$

65.
$$\begin{aligned}\left(-3+\sqrt{-24}\right)+\left(7-\sqrt{-44}\right)&=\left(-3+2\sqrt{6}i\right)+\left(7-2\sqrt{11}i\right)\\&=4+\left(2\sqrt{6}-2\sqrt{11}\right)i\\&=4+2\left(\sqrt{6}-\sqrt{11}\right)i\end{aligned}$$

66.
$$\begin{aligned}\left(-12-\sqrt{-72}\right)+\left(9+\sqrt{-108}\right)&=\left(-12-6\sqrt{2}i\right)+\left(9+6\sqrt{3}i\right)\\&=-3+\left(-6\sqrt{2}+6\sqrt{3}\right)i\\&=-3+6\left(\sqrt{3}-\sqrt{2}\right)i\end{aligned}$$

67.
$$\begin{aligned}\sqrt{-6}\cdot\sqrt{-2}&=\left(\sqrt{6}i\right)\left(\sqrt{2}i\right)\\&=\sqrt{12}\,i^2=\left(2\sqrt{3}\right)(-1)=-2\sqrt{3}\end{aligned}$$

68.
$$\begin{aligned}\sqrt{-5}\cdot\sqrt{-10}&=\left(\sqrt{5}i\right)\left(\sqrt{10}i\right)\\&=\sqrt{50}i^2=5\sqrt{2}(-1)=-5\sqrt{2}\end{aligned}$$

69. $\left(\sqrt{-10}\right)^2=\left(\sqrt{10}i\right)^2=10i^2=-10$

70. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

71. $\left(2-\sqrt{-6}\right)^2 = \left(2-\sqrt{6}i\right)\left(2-\sqrt{6}i\right)$

$$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2$$

$$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)$$

$$= 4 - 6 - 4\sqrt{6}i$$

$$= -2 - 4\sqrt{6}i$$

72. $\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right) = \left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)$

$$= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$$

$$= 21 + \sqrt{50} + 7\sqrt{5}i - 3\sqrt{10}i$$

$$= \left(21 + 5\sqrt{2}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$$

73. $x^2 + 25 = 0$

$$x^2 = -25$$

$$x = \pm 5i$$

74. $x^2 + 32 = 0$

$$x^2 = -32$$

$$x = \pm\sqrt{32}i = \pm 4\sqrt{2}i$$

75. $x^2 - 2x + 2 = 0;\ a = 1,\ b = -2,\ c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= 1 \pm i$$

76. $x^2 + 6x + 10 = 0;\ a = 1,\ b = 6,\ c = 10$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{-4}}{2}$$

$$= \frac{-6 + 2i}{2}$$

$$= -3 \pm i$$

77. $4x^2 + 16x + 17 = 0;\ a = 4,\ b = 16,\ c = 17$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)}$$

$$= \frac{-16 \pm \sqrt{-16}}{8}$$

$$= \frac{-16 \pm 4i}{8}$$

$$= -2 \pm \frac{1}{2}i$$

78. $9x^2 - 6x + 37 = 0;\ a = 9,\ b = -6,\ c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)}$$

$$= \frac{6 \pm \sqrt{-1296}}{18}$$

$$= \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i$$

79. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$

$$= \frac{4 \pm \sqrt{-176}}{32}$$

$$= \frac{4 \pm 4\sqrt{11}\,i}{32}$$

$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

80. $4x^2 + 16x + 15 = 0;\ a = 4,\ b = 16,\ c = 15$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)}$$

$$= \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}$$

$$x = -\frac{12}{8} = -\frac{3}{2} \text{ or } x = -\frac{20}{8} = -\frac{5}{2}$$

81. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{-72}}{6}$$

$$= \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

82. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$ Multiply both sides by 16.

$14x^2 - 12x + 5 = 0;\ a = 14,\ b = -12,\ c = 5$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)}$$

$$= \frac{12 \pm \sqrt{-136}}{28}$$

$$= \frac{12 \pm 2\sqrt{34}i}{28}$$

$$= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i$$

83. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$$7x^2 - 10x - 50 = 0;\ a = 7,\ b = -10,\ c = -50$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)}$$

$$= \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14}$$

$$= \frac{5}{7} \pm \frac{5\sqrt{15}}{7}$$

84. $4.5x^2 - 3x + 12 = 0;\ a = 4.5,\ b = -3,\ c = 12$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)}$$

$$= \frac{3 \pm \sqrt{-207}}{9} = \frac{3 \pm 3\sqrt{23}i}{9} = \frac{1}{3} \pm \frac{\sqrt{23}}{3}i$$

85. $-6i^3 + i^2 = -6i^2 i + i^2 = -6(-1)i + (-1) = 6i - 1 = -1 + 6i$

86. $4i^2 - 2i^3 = -4 + 2i$

87. $\left(\sqrt{-75}\right)^3 = \left(5\sqrt{3}i\right)^3 = 5^3\left(\sqrt{3}\right)^3 i^3 = 125\left(3\sqrt{3}\right)(-i)$

$$= -375\sqrt{3}i$$

88. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6 = 8i^6 = 8i^4 i^2 = -8$

89. $\frac{1}{i^3} = \frac{1}{i^3} \cdot \frac{i}{i} = \frac{i}{i^4} = \frac{i}{1} = i$

90. $\frac{1}{(2i)^3} = \frac{1}{8i^3} = \frac{1}{-8i} \cdot \frac{8i}{8i} = \frac{8i}{-64i^2} = \frac{1}{8}i$

91. (a) $(2)^3 = 8$

(b) $\left(-1+\sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$

$$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3$$

$$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i$$

$$= 8$$

(c) $\left(-1-\sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3$

$$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3$$

$$= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i$$

$$= 8$$

The three numbers are cube roots of 8.

92. (a) $2^4 = 16$

(b) $(-2)^4 = 16$

(c) $(2i)^4 = 2^4 i^4 = 16(1) = 16$

(d) $(-2i)^4 = (-2)^4 i^4 = 16(1) = 16$

93. (a) $i^{20} = (i^4)^5 = (1)^5 = 1$

(b) $i^{45} = (i^4)^{11} i = (1)^{11} i = i$

(c) $i^{67} = (i^4)^{16} i^3 = (1)^{16}(-i) = -i$

(d) $i^{114} = (i^4)^{28} i^2 = (1)^{28}(-1) = -1$

94. (a) $z_1 = 5 + 2i$

$z_2 = 3 - 4i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{5+2i} + \frac{1}{3-4i}$$

$$= \frac{(3-4i) + (5+2i)}{(5+2i)(3-4i)}$$

$$= \frac{8-2i}{23-14i}$$

$$z = \frac{23-14i}{8-2i}\left(\frac{8+2i}{8+2i}\right)$$

$$= \frac{212-66i}{68} \approx 3.118 - 0.971i$$

(b) $z_1 = 16i + 9$

$z_2 = 20 - 10i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9+16i} + \frac{1}{20-10i}$$

$$= \frac{(20-10i) + (9+16i)}{(9+16i)(20-10i)}$$

$$= \frac{29+6i}{340+230i}$$

$$z = \frac{340+230i}{29+6i}\left(\frac{29-6i}{29-6i}\right) = \frac{11240+4630i}{877} \approx 12.816 + 5.279i$$

95. False. A real number $a + 0i = a$ is equal to its conjugate.

96. False. $i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = 1 - 1 + 1 - i + i = 1$

97. False. For example, $(1+2i) + (1-2i) = 2$, which is not an imaginary number.

98. False. For example, $(i)(i) = -1$, which is not an imaginary number.

99. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$$\overline{z_1 z_2} = \overline{(a_1 + b_1 i)(a_2 + b_2 i)}$$

$$= \overline{(a_1 a_2 - b_1 b_2) + (a_1 b_2 + b_1 a_2)i}$$

$$= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + b_1 a_2)i$$

$$= (a_1 - b_1 i)\ (a_2 - b_2 i)$$

$$= \overline{a_1 + b_1 i}\ \overline{a_2 + b_2 i}$$

$$= \overline{z_1}\,\overline{z_2}.$$

100. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$$\overline{z_1 + z_2} = \overline{(a_1 + b_1 i) + (a_2 + b_2 i)}$$

$$= \overline{(a_1 + a_2) + (b_1 + b_2)i}$$

$$= (a_1 + a_2) - (b_1 + b_2)i$$

$$= (a_1 - b_1 i) + (a_2 - b_2 i)$$

$$= \overline{(a_1 + b_1 i)} + \overline{(a_2 + b_2 i)}$$

$$= \overline{z_1} + \overline{z_2}.$$

101. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}\,i = 6i^2 = -6$

102. $f(x) = 2(x-3)^2 - 4, g(x) = -2(x-3)^2 - 4$

(a) The graph of *f* is a parabola with vertex at the point $(3, -4)$. The *a* value is positive, so the graph opens upward.

The graph of *g* is also a parabola with vertex at the point $(3, -4)$. The *a* value is negative, so the graph opens downward.

f has an *x*-intercept and *g* does not because when $g(x) = 0$, *x* is a complex number.

(b)
$$\begin{aligned} f(x) &= 2(x-3)^2 - 4 \\ 0 &= 2(x-3)^2 - 4 \\ 4 &= 2(x-3)^2 \\ 2 &= (x-3)^2 \\ \pm\sqrt{2} &= x - 3 \\ 3 \pm \sqrt{2} &= x \end{aligned}$$

$$\begin{aligned} g(x) &= -2(x-3)^2 - 4 \\ 0 &= -2(x-3)^2 - 4 \\ 4 &= -2(x-3)^2 \\ -2 &= (x-3)^2 \\ \pm\sqrt{-2} &= x - 3 \\ 3 \pm \sqrt{2}i &= x \end{aligned}$$

(c) If all the zeros contain *i*, then the graph has no *x*-intercepts.

(d) If *a* and *k* have the same sign (both positive or both negative), then the graph of *f* has no *x*-intercepts and the zeros are complex. Otherwise, the graph of *f* has *x*-intercepts and the zeros are real.

103. $(4x-5)(4x+5) = 16x^2 - 20x + 20x - 25 = 16x^2 - 25$

104. $(x+2)^3 = x^3 + 3x^2 2 + 3x(2)^2 + 2^3$
$= x^3 + 6x^2 + 12x + 8$

105. $\left(3x - \frac{1}{2}\right)(x+4) = 3x^2 - \frac{1}{2}x + 12x - 2 = 3x^2 + \frac{23}{2}x - 2$

106. $(2x-5)^2 = 4x^2 - 20x + 25$

Section 2.5

1. Fundamental Theorem, Algebra

2. irreducible, reals

3. The Linear Factorization Theorem states that a polynomial function *f* of degree *n*, $n > 0$, has exactly *n* linear factors

$$f(x) = a(x-c_1)(x-c_2)\ldots(x-c_n).$$

4. Since complex zeros occur in conjugate pairs, if a fourth-degree polynomial function has zeros –1, 3, and 2*i*, then –2*i* is also a zero.

5. $f(x) = x^3 + x$ has exactly 3 zeros. Matches (c).

6. $f(x) = -x + 7$ has exactly 1 zero. Matches (a).

7. $f(x) = x^5 + 9x^3$ has exactly 5 zeros. Matches (d).

8. $f(x) = x^2 - 14x + 49$ has exactly 2 zeros. Matches (b).

9. $f(x) = x^2 + 25$
$$\begin{aligned} x^2 + 25 &= 0 \\ x^2 &= -25 \\ x &= \pm\sqrt{-25} \\ x &= \pm 5i \end{aligned}$$

10. $f(x) = x^2 + 2$
$$\begin{aligned} x^2 + 2 &= 0 \\ x^2 &= -2 \\ x &= \pm\sqrt{-2} \\ x &= \pm\sqrt{2}i \end{aligned}$$

11. $f(x) = x^3 + 9x$
$$\begin{aligned} x^3 + 9x &= 0 \\ x(x^2 + 9) &= 0 \\ x = 0 \quad x^2 + 9 &= 0 \\ x^2 &= -9 \\ x &= \pm\sqrt{-9} \\ x &= \pm 3i \end{aligned}$$

12. $f(x) = x^3 + 49x$
$$\begin{aligned} x^3 + 49x &= 0 \\ x(x^2 + 49) &= 0 \\ x = 0 \quad x^2 + 49 &= 0 \\ x^2 &= -49 \\ x &= \pm\sqrt{-49} \\ x &= \pm 7i \end{aligned}$$

13. $f(x) = x^3 - 4x^2 + x - 4 = x^2(x-4) + 1(x-4) = (x-4)(x^2+1)$
Zeros: $4, \pm i$
The only real zero of *f(x)* is $x = 4$. This corresponds to the *x*-intercept of $(4, 0)$ on the graph.

14. $f(x) = x^3 - 4x^2 - 4x + 16$
$$\begin{aligned} &= x^2(x-4) - 4(x-4) \\ &= (x^2 - 4)(x-4) \\ &= (x+2)(x-2)(x-4) \end{aligned}$$

The zeros are $x = 2, -2$, and 4. This corresponds to the *x*-intercepts of $(-2, 0)$, $(2, 0)$, and $(4, 0)$ on the graph.

15. $f(x) = x^4 + 4x^2 + 4 = (x^2 + 2)^2$

Zeros: $\pm\sqrt{2}i,\ \pm\sqrt{2}i$

$f(x)$ has no real zeros and the graph of $f(x)$ has no x-intercepts.

16. $f(x) = x^4 - 3x^2 - 4$
$= (x^2 - 4)(x^2 + 1)$
$= (x + 2)(x - 2)(x^2 + 1)$

Zeros: $\pm 2, \pm i$

The only real zeros are $x = -2,\ 2$. This corresponds to the x-intercepts of $(-2,\ 0)$ and $(2,\ 0)$ on the graph.

17. $h(x) = x^2 - 4x + 1$

h has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}.$$

$$h(x) = \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right]$$
$$= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$

18. $g(x) = x^2 + 10x + 23$

g has no rational zeros. By the Quadratic Formula, the zeros are $x = \dfrac{-10 \pm \sqrt{8}}{2} = -5 \pm \sqrt{2}$.

$$g(x) = \left[x - (-5 + \sqrt{2})\right]\left[x - (-5 - \sqrt{2})\right]$$
$$= (x + 5 + \sqrt{2})(x + 5 - \sqrt{2})$$

19. $f(x) = x^2 - 12x + 26$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(26)}}{2} = 6 \pm \sqrt{10}.$$

$$f(x) = \left[x - (6 + \sqrt{10})\right]\left[x - (6 - \sqrt{10})\right]$$
$$= (x - 6 - \sqrt{10})(x - 6 + \sqrt{10})$$

20. $f(x) = x^2 + 6x - 2$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-2)}}{2} = -3 \pm \sqrt{11}.$$

$$f(x) = \left[x - (-3 + \sqrt{11})\right]\left[x - (-3 - \sqrt{11})\right]$$
$$= (x + 3 - \sqrt{11})(x + 3 + \sqrt{11})$$

21. $f(x) = x^2 + 25$
Zeros: $\pm 5i$
$f(x) = (x + 5i)(x - 5i)$

22. $f(x) = x^2 + 36$
Zeros: $\pm 6i$
$f(x) = (x + 6i)(x - 6i)$

23. $f(x) = 16x^4 - 81$
$= (4x^2 - 9)(4x^2 + 9)$
$= (2x - 3)(2x + 3)(2x + 3i)(2x - 3i)$

Zeros: $\pm\dfrac{3}{2},\ \pm\dfrac{3}{2}i$

24. $f(y) = 81y^4 - 625$
$= (9y^2 + 25)(9y^2 - 25)$
$= (3y + 5i)(3y - 5i)(3y + 5)(3y - 5)$

Zeros: $\pm\dfrac{5}{3}, \pm\dfrac{5}{3}i$

25. $f(z) = z^2 - z + 56$

$$z = \frac{1 \pm \sqrt{1 - 4(56)}}{2}$$
$$= \frac{1 \pm \sqrt{-223}}{2}$$
$$= \frac{1}{2} \pm \frac{\sqrt{223}}{2}i$$

Zeros: $\dfrac{1}{2} \pm \dfrac{\sqrt{223}}{2}i$

$$f(z) = \left(z - \frac{1}{2} + \frac{\sqrt{223}i}{2}\right)\left(z - \frac{1}{2} - \frac{\sqrt{223}i}{2}\right)$$

26. $h(x) = x^2 - 4x - 3$

$$x = \frac{4 \pm \sqrt{16 + 12}}{2} = 2 \pm \sqrt{7}$$

Zeros: $2 \pm \sqrt{7}$

$$h(x) = \left(x - 2 + \sqrt{7}\right)\left(x - 2 - \sqrt{7}\right)$$

27. $f(x) = x^4 + 10x^2 + 9$
$= (x^2 + 1)(x^2 + 9)$
$= (x + i)(x - i)(x + 3i)(x - 3i)$

The zeros of $f(x)$ are $x = \pm i$ and $x = \pm 3i$.

28. $f(x) = x^4 + 29x^2 + 100$
$= (x^2 + 25)(x^2 + 4)$
Zeros: $x = \pm 2i, \pm 5i$

$f(x) = (x + 2i)(x - 2i)(x + 5i)(x - 5i)$

29. $f(x) = 3x^3 - 5x^2 + 48x - 80$

Using synthetic division, $\dfrac{5}{3}$ is a zero:

$\frac{5}{3}$	3	−5	48	−80
		5	0	80
	3	0	48	0

$$f(x) = \left(x - \frac{5}{3}\right)\left(3x^2 + 48\right)$$
$$= (3x-5)(x^2+16)$$
$$= (3x-5)(x+4i)(x-4i)$$

The zeros are $\frac{5}{3}$, $4i$, $-4i$.

30. $f(x) = 3x^3 - 2x^2 + 75x - 50$

Using synthetic division, $\frac{2}{3}$ is a zero:

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 75 & -50 \\ & & 2 & 0 & 50 \\ \hline & 3 & 0 & 75 & 0 \end{array}$$

$$f(x) = \left(x - \frac{2}{3}\right)\left(3x^3 + 75\right)$$
$$= (3x-2)(x^2+25)$$
$$= (3x-2)(x+5i)(x-5i)$$

The zeros are $\frac{2}{3}$, $5i$, $-5i$.

31. $f(t) = t^3 - 3t^2 - 15t + 125$

Possible rational zeros: ± 1, ± 5, ± 25, ± 125

$$\begin{array}{r|rrrr} -5 & 1 & -3 & -15 & 125 \\ & & -5 & 40 & -125 \\ \hline & 1 & -8 & 25 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $t^2 - 8t + 25$ are

$$t = \frac{8 \pm \sqrt{64-100}}{2} = 4 \pm 3i.$$

The zeros of $f(t)$ are $t = -5$ and $t = 4 \pm 3i$.

$$f(t) = \left[t - (-5)\right]\left[t - (4+3i)\right]\left[t - (4-3i)\right]$$
$$= (t+5)(t-4-3i)(t-4+3i)$$

32. $f(x) = x^3 + 11x^2 + 39x + 29$

$$\begin{array}{r|rrrr} -1 & 1 & 11 & 39 & 29 \\ & & -1 & -10 & -29 \\ \hline & 1 & 10 & 29 & 0 \end{array}$$

Zeros: $x = -1$, $\dfrac{-10 \pm \sqrt{16}i}{2} = -5 \pm 2i$

$$f(x) = (x+1)(x+5+2i)(x+5-2i)$$

33. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros: $\pm 6, \pm\frac{6}{5}, \pm 3, \pm\frac{3}{5}, \pm 2, \pm\frac{2}{5}, \pm 1, \pm\frac{1}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $5x^2 - 10x + 30$ are those of $x^2 - 2x + 6$:

$$x = \frac{2 \pm \sqrt{4 - 4(6)}}{2} = 1 \pm \sqrt{5i}$$

Zeros: $-\frac{1}{5}$, $1 \pm \sqrt{5i}$

$$f(x) = 5\left(x + \frac{1}{5}\right)\left[x - \left(1 + \sqrt{5i}\right)\right]\left[x - \left(1 - \sqrt{5i}\right)\right]$$
$$= (5x+1)\left(x - 1 - \sqrt{5i}\right)\left(x - 1 + \sqrt{5i}\right)$$

34. $f(s) = 3s^3 - 4s^2 + 8s + 8$
$$= (3s+2)(s^2 - 2s + 4)$$

Factoring the quadratic,

$$s = \frac{2 \pm \sqrt{4-16}}{2} = 1 \pm \sqrt{3}i.$$

Zeros: $-\frac{2}{3}, 1 \pm \sqrt{3}i$

$$f(s) = (3s+2)(s - 1 + \sqrt{3i})(s - 1 - \sqrt{3i})$$

35. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: ± 1, ± 2, ± 4, ± 8, ± 16

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline 2 & 1 & -2 & 4 & -8 & 0 \\ & & 2 & 0 & 8 & \\ \hline & 1 & 0 & 4 & 0 & \end{array}$$

$$g(x) = (x-2)(x-2)\left(x^2+4\right)$$
$$= (x-2)^2(x+2i)(x-2i)$$

The zeros of g are $2, 2,$ and $\pm 2i$.

36. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline -3 & 1 & 3 & 1 & 3 & 0 \\ & & -3 & 0 & -3 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

Zeros: $x = -3, -3, \pm i$

$$h(x) = (x+3)^2(x+i)(x-i)$$

37. (a) $f(x) = x^2 - 14x + 46.$

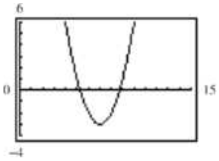
By the Quadric Formula,

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(46)}}{2} = 7 \pm \sqrt{3}.$$

The zeros are $7 + \sqrt{3}$ and $7 - \sqrt{3}$.

(b) $f(x) = \left[x - (7 + \sqrt{3})\right]\left[x - (7 - \sqrt{3})\right]$
$$= (x - 7 - \sqrt{3})(x - 7 + \sqrt{3})$$

(c) x-intercepts: $\left(7+\sqrt{3},\ 0\right)$ and $\left(7-\sqrt{3},\ 0\right)$



38. (a) $f(x)=x^2-12x+34$

By the Quadric Formula,

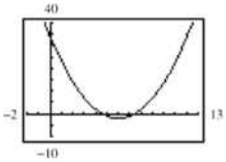
$$x=\frac{12\pm\sqrt{(-12)^2-4(34)}}{2}=6\pm\sqrt{2}.$$

The zeros are $6+\sqrt{2}$ and $6-\sqrt{2}$.

(b) $f(x)=\left(x-\left(6+\sqrt{2}\right)\right)\left(x-\left(6-\sqrt{2}\right)\right)$

$=\left(x-6-\sqrt{2}\right)\left(x-6+\sqrt{2}\right)$

(c) x-intercepts: $\left(6+\sqrt{2},0\right)$ $\left(6-\sqrt{2},0\right)$



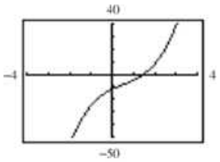
39. (a) $f(x)=2x^3-3x^2+8x-12$

$=\left(2x-3\right)\left(x^2+4\right)$

The zeros are $\dfrac{3}{2}$ and $\pm 2i$.

(b) $f(x)=(2x-3)(x+2i)(x-2i)$

(c) x-intercept: $\left(\dfrac{3}{2},\ 0\right)$



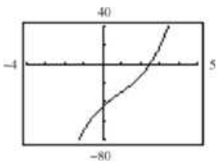
40. (a) $f(x)=2x^3-5x^2+18x-45$

$=(2x-5)(x^2+9)$

The zeros are $\dfrac{5}{2}$ and $\pm 3i$.

(b) $f(x)=\left(2x-5\right)\left(x+3i\right)\left(x-3i\right)$

(c) x-intercept: $\left(\dfrac{5}{2},\ 0\right)$



41. (a) $f(x)=x^3-11x+150$

$=\left(x+6\right)\left(x^2-6x+25\right)$

Use the Quadratic Formula to find the zeros of $x^2-6x+25$.

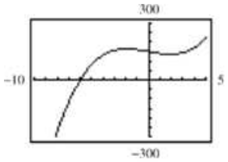
$$x=\frac{6\pm\sqrt{(-6)^2-4(25)}}{2}=3\pm 4i.$$

The zeros are $-6, 3+4i,$ and $3-4i$.

(b)

$f(x)=\left(x+6\right)\left(x-3+4i\right)\left(x-3-4i\right)$

(c) x-intercept: $(-6,\ 0)$



42. (a) $f(x)=x^3+10x^2+33x+34$

$=\left(x+2\right)\left(x^2+8x+17\right)$

Use the Quadratic Formula to find the zeros of $x^2+8x+17$.

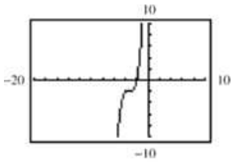
$$x=\frac{-8\pm\sqrt{8^2-4(17)}}{2}$$

$$=\frac{-8\pm\sqrt{-4}}{2}=-4+i$$

The zeros are $-2,\ -4+i,$ and $-4-i$.

(b) $f(x)=\left(x+2\right)\left(x+4+i\right)\left(x+4-i\right)$

(c) x-intercept: $(-2,\ 0)$



43. (a) $f(x)=x^4+25x^2+144$

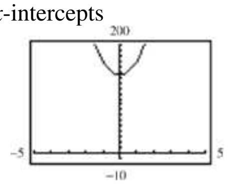
$=\left(x^2+9\right)\left(x^2+16\right)$

The zeros are $\pm 3i,\ \pm 4i$.

(b) $f(x)=\left(x^2+9\right)\left(x^2+16\right)$

$=(x+3i)(x-3i)(x+4i)(x-4i)$

(c) No x-intercepts



44. (a) $f(x) = x^4 - 8x^3 + 17x^2 - 8x + 16$

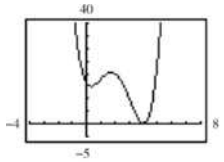
$= (x^2+1)(x^2-8x+16)$

$= (x^2+1)(x-4)^2$

The zeros are $i, -i, 4,$ and 4.

(b) $f(x) = (x^2+1)(x-4)^2$

(c) x-intercept: (4, 0)



45. $f(x) = (x-2)(x-i)(x+i)$

$= (x-2)(x^2+1)$

Note that $f(x) = a(x^3 - 2x^2 + x - 2)$, where a is any nonzero real number, has zeros $2, \pm i$.

46. $f(x) = (x-3)(x-4i)(x+4i)$

$= (x-3)(x^2+16)$

$= x^3 - 3x^2 + 16x - 48$

Note that $f(x) = a(x^3 - 3x^2 + 16x - 48)$, where a is any nonzero real number, has zeros $3, \pm 4i$.

47. $f(x) = (x-2)^2(x-4-i)(x-4+i)$

$= (x-2)^2(x-8x+16+1)$

$= (x^2-4x+4)(x^2-8x+17)$

$= x^4 - 12x^3 + 53x^2 - 100x + 68$

Note that $f(x) = a(x^4 - 12x^3 + 53x^2 - 100x + 68)$, where a is any nonzero real number, has zeros $2, 2, 4 \pm i$.

48. Because $2+5i$ is a zero, so is $2-5i$.

$f(x) = (x+1)^2(x-2-5i)(x-2+5i)$

$= (x+1)^2(x^2-4x+4+25)$

$= (x^2+2x+1)(x^2-4x+29)$

$= x^4 - 2x^3 + 22x^2 + 54x + 29$

Note that $f(x) = a(x^4 - 2x^3 + 22x^2 + 54x + 29)$, where a is any nonzero real number, has zeros $-1, -1, 2 \pm 5i$.

49. Because $1+\sqrt{2}i$ is a zero, so is $1-\sqrt{2}i$.

$f(x) = (x-0)(x+5)(x-1-\sqrt{2}i)(x-1+\sqrt{2}i)$

$= (x^2+5x)(x^2-2x+1+2)$

$= (x^2+5x)(x^2-2x+3)$

$= x^4 + 3x^3 - 7x^2 - 15x$

Note that $f(x) = a(x^4 + 3x^3 - 7x^2 + 15x)$, where a is any nonzero real number, has zeros $0, -5, 1 \pm \sqrt{2}i$.

50. Because $1+\sqrt{2}i$ is a zero, so is $1 \pm \sqrt{2}i$.

$f(x) = (x-0)(x-4)(x-1-\sqrt{2}i)(x-1+\sqrt{2}i)$

$= (x^2-4x)(x^2-2x+1+2)$

$= x^4 - 6x^3 + 11x^2 - 12x$

Note that $f(x) = a(x^4 - 6x^3 + 11x^2 - 12x)$, where a is any nonzero real number, has zeros $0, 4, 1 \pm \sqrt{2}i$.

51. (a) $f(x) = a(x-1)(x+2)(x-2i)(x+2i)$

$= a(x-1)(x+2)(x^2+4)$

$f(1) = 10 = a(-2)(1)(5) \Rightarrow a = -1$

$f(x) = -(x-1)(x+2)(x-2i)(x+2i)$

(b) $f(x) = -(x-1)(x+2)(x^2+4)$

$= -(x^2+x-2)(x^2+4)$

$= -x^4 - x^3 - 2x^2 - 4x + 8$

52. (a) $f(x) = a(x+1)(-2)(x-i)(x+i)$

$= a(x+1)(x-2)(x^2+1)$

$f(1) = 8 = a(2)(-1)(2) \Rightarrow a = -2$

$f(x) = -2(x+1)(x-2)(x-i)(x+i)$

(b) $f(x) = -2(x^2-x-2)(x^2+1)$

$= -2x^4 + 2x^3 + 2x^2 + 2x + 4$

53. (a) $f(x) = a(x+1)(x-2-\sqrt{5}i)(x-2+\sqrt{5}i)$

$= a(x+1)(x^2-4x+4+5)$

$= a(x+1)(x^2-4x+9)$

$f(-2) = 42 = a(-1)(4+8+9) \Rightarrow a = -2$

$f(x) = -2(x+1)(x-2-\sqrt{5}i)(x-2+\sqrt{5}i)$

(b) $f(x) = -2(x+1)(x^2-4x+9)$

$= -2x^3 + 6x^2 - 10x - 18$

54. (a) $f(x) = a(x+2)(x-2-2\sqrt{2}i)(x-2+2\sqrt{2}i)$

$= a(x+2)(x^2-4x+4+8)$

$= a(x+2)(x^2-4x+12)$

$f(-1) = -34 = a(1)(17) \Rightarrow a = -2$

$f(x) = -2(x+2)(x-2-2\sqrt{2}i)(x-2+2\sqrt{2}i)$

(b) $f(x) = -2(x+2)(x^2-4x+12)$

$= -2x^3 + 4x^2 - 8x - 48$

55. $f(x) = x^4 - 6x^2 - 7$

(a) $f(x) = (x^2-7)(x^2+1)$

(b) $f(x) = (x-\sqrt{7})(x+\sqrt{7})(x^2+1)$

(c) $f(x) = (x-\sqrt{7})(x+\sqrt{7})(x+i)(x-i)$

56. $f(x) = x^4 + 6x^2 - 27$

(a) $f(x) = \left(x^2 + 9\right)\left(x^2 - 3\right)$

(b) $f(x) = \left(x^2 + 9\right)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

(c) $f(x) = \left(x + 3i\right)\left(x - 3i\right)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

57. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

(a) $f(x) = \left(x^2 - 6\right)\left(x^2 - 2x + 3\right)$

(b) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x^2 - 2x + 3\right)$

(c) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$

58. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$

(a) $f(x) = \left(x^2 + 4\right)\left(x^2 - 3x - 5\right)$

(b) $f(x) = \left(x^2 + 4\right)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

(c) $f(x) = \left(x + 2i\right)\left(x - 2i\right)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

59. $f(x) = 2x^3 + 3x^2 + 50x + 75$

Since is $5i$ a zero, so is $-5i$.

$$\begin{array}{r|rrrr} 5i & 2 & 3 & 50 & 75 \\ & & 10i & -50+15i & -75 \\ \hline & 2 & 3+10i & 15i & 0 \end{array}$$

$$\begin{array}{r|rrr} -5i & 2 & 3+10i & 15i \\ & & -10i & -15i \\ \hline & 2 & 3 & 0 \end{array}$$

The zero of $2x + 3$ is $x = -\dfrac{3}{2}.$ The zeros of *f* are $x = -\dfrac{3}{2}$ and $x = \pm 5i.$

Alternate solution

Since $x = \pm 5i$ are zeros of $f(x)$, $(x + 5i)(x - 5i) = x^2 + 25$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r} 2x+3 \\ x^2+0x+25\ \overline{)\,2x^3+3x^2+50x+75} \\ \underline{2x^3+0x^2+50x} \\ 3x^2+0x+75 \\ \underline{3x^2+0x+75} \\ 0 \end{array}$$

Thus, $f(x) = \left(x^2 + 25\right)\left(2x + 3\right)$ and the zeros of *f* are $x = \pm 5i$ and $x = -\dfrac{3}{2}.$

60. $f(x) = x^3 + x^2 + 9x + 9$

Since is $3i$ is a zero, so is $-3i$.

$$\begin{array}{r|rrrr} 3i & 1 & 1 & 9 & 9 \\ & & 3i & -9+3i & -9 \\ \hline & 1 & 1+3i & 3i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3i & 1 & 1+3i & 3i \\ & & -3i & -3i \\ \hline & 1 & 1 & 0 \end{array}$$

The zeros of *f* are $3i, -3i,$ and -1 .

61. $g(x) = x^3 - 7x^2 - x + 87.$ Since $5 + 2i$ is a zero, so is $5 - 2i.$

$$\begin{array}{r|rrrr} 5+2i & 1 & -7 & -1 & 87 \\ & & 5+2i & -14+6i & -87 \\ \hline & 1 & -2+2i & -15+6i & 0 \end{array}$$

$$\begin{array}{r|rrr} 5-2i & 1 & -2+2i & -15+6i \\ & & 5-2i & 15-6i \\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x + 3$ is $x = -3.$
The zeros of *f* are $-3, 5 \pm 2i.$.

62. $g(x) = 4x^3 + 23x^2 + 34x - 10$

Since $-3 + i$ is a zero, so is $-3 - i$

$$\begin{array}{r|rrrr} -3+i & 4 & 23 & 34 & -10 \\ & & -12+4i & -37-i & 10 \\ \hline & 4 & 11+4i & -3-i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3-i & 4 & 11+4i & -3-i \\ & & -12-4i & 3+i \\ \hline & 4 & -1 & 0 \end{array}$$

The zero of $4x - 1$ is $x = \dfrac{1}{4}.$ The zeros of $g(x)$ are $x = -3 \pm i$ and $x = \dfrac{1}{4}.$

Alternate solution

Since $-3 \pm i$ are zeros of $g(x)$,

$$[x - (-3 + i)][x - (-3 - i)] = [(x - 3) - i][(x + 3) + i]$$
$$= (x + 3)^2 - i^2 = x^2 + 6x + 10$$

is a factor of $g(x)$. By long division we have:

$$\begin{array}{r} 4x-1 \\ x^2+6x+10\ \overline{)\,4x^3+23x^2+34x-10} \\ \underline{4x^3+24x^2+40x} \\ -x^2-6x-10 \\ \underline{-x^2-6x-10} \\ 0 \end{array}$$

Thus, $g(x) = (x^2 + 6x + 10)(4x - 1)$ and the zeros of g are $x = -3 \pm i$ and $x = \frac{1}{4}$.

63. $h(x) = 3x^3 - 4x^2 + 8x + 8$ Since $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$. The zeros of h are $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$.

64. $f(x) = x^3 + 4x^2 + 14x + 20$

Since $-1 - 3i$ is a zero, so is $-1 + 3i$.

$$\begin{array}{r|rrrr} -1-3i & 1 & 4 & 14 & 20 \\ & & -1-3i & -12-6i & -20 \\ \hline & 1 & 3-3i & 2-6i & 0 \end{array}$$

$$\begin{array}{r|rrr} -1+3i & 1 & 3-3i & 2-6i \\ & & -1+3i & -2+6i \\ \hline & 1 & 2 & 0 \end{array}$$

The zero of $x + 2$ is $x = -2$. The zeros of f are $x = -2, -1 \pm 3i$.

65. $h(x) = 8x^3 - 14x^2 + 18x - 9$. Since $\frac{1}{2}(1 - \sqrt{5}i)$ is a zero, so is $\frac{1}{2}(1 + \sqrt{5}i)$.

$$\begin{array}{r|rrrr} \frac{1}{2}\left(1-\sqrt{5}i\right) & 8 & -14 & 18 & -9 \\ & & 4-4\sqrt{5}i & -15+3\sqrt{5}i & 9 \\ \hline & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2}\left(1+\sqrt{5}i\right) & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i \\ & & 4+4\sqrt{5}i & -3-3\sqrt{5}i \\ \hline & 8 & -6 & 0 \end{array}$$

The zero of $8x - 6$ is $x = \frac{3}{4}$. The zeros of h are $x = \frac{3}{4}, \frac{1}{2}(1 \pm \sqrt{5}i)$.

66. $f(x) = 25x^3 - 55x^2 - 54x - 18$

Since $\frac{1}{5}(-2 + \sqrt{2}i) = \frac{-2 + \sqrt{2}i}{5}$ is a zero, so is $\frac{-2 - \sqrt{2}i}{5}$.

$$\begin{array}{r|rrrr} \frac{-2+\sqrt{2}i}{5} & 25 & -55 & -54 & -18 \\ & & -10+5\sqrt{2}i & 24-15\sqrt{2}i & 18 \\ \hline \end{array}$$

$$\begin{array}{r|rrr} \frac{-2-\sqrt{2}i}{5} & 25 & -65+5\sqrt{2}i & -30-15\sqrt{2}i \quad 0 \\ & & -10-5\sqrt{2}i & 30+15\sqrt{2}i \\ \hline & 25 & -75 & 0 \end{array}$$

The zero of $25x - 75$ is $x = 3$. The zeros of f are $x = 3, \frac{-2 \pm \sqrt{2}i}{5}$.

67. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$

(a) The *root* feature yields the real roots 1 and 2, and the complex roots $-3 \pm 1.414i$.

(b) By synthetic division:

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -5 & -21 & 22 \\ & & 1 & 4 & -1 & -22 \\ \hline & 1 & 4 & -1 & -22 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -1 & -22 \\ & & 2 & 12 & 22 \\ \hline & 1 & 6 & 11 & 0 \end{array}$$

The complex roots of $x^2 + 6x + 11$ are

$$x = \frac{-6 \pm \sqrt{6^2 - 4(11)}}{2} = -3 \pm \sqrt{2}i.$$

68. $f(x) = x^3 + 4x^2 + 14x + 20$

(a) The *root* feature yields the real root – 2 and the complex roots $-1 \pm 3i$.

(b) By synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 14 & 20 \\ & & -2 & -4 & -20 \\ \hline & 1 & 2 & 10 & 0 \end{array}$$

The complex roots of $x^2 + 2x + 10$ are

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(10)}}{2} = -1 \pm 3i.$$

69. $h(x) = 8x^3 - 14x^2 + 18x - 9$

(a) The *root* feature yields the real root 0.75, and the complex roots $0.5 \pm 1.118i$.

(b) By synthetic division:

$$\begin{array}{r|rrrr} \frac{3}{4} & 8 & -14 & 18 & -9 \\ & & 6 & -6 & 9 \\ \hline & 8 & -8 & 12 & 0 \end{array}$$

The complex roots of $8x^2 - 8x + 12$ are

$$x = \frac{8 \pm \sqrt{64 - 4(8)(12)}}{2(8)} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}i.$$

70. $f(x) = 25x^3 - 55x^2 - 54x - 18$

(a) The *root* feature yields the real root 3 and the complex roots $-0.4 \pm 0.2828i$.

(b) By synthetic division:

$$\begin{array}{r|rrrr} 3 & 25 & -55 & -54 & -18 \\ & & 75 & 60 & 18 \\ \hline & 25 & 20 & 6 & 0 \end{array}$$

The complex roots of $25x^2 + 20x + 6$ are

$$x = \frac{-20 \pm \sqrt{400 - 4(25)6}}{2(25)} = \frac{-2 \pm \sqrt{2}i}{5}.$$

71. To determine if the football reaches a height of 50 feet, set $h(t) = 50$ and solve for t.

$$-16t^2 + 48t = 50$$
$$-16t^2 + 48t - 50 = 0$$

Using the Quadratic formula:

$$t = \frac{-(48) \pm \sqrt{48^2 - 4(-16)(-50)}}{2(-16)}$$
$$t = \frac{-(48) \pm \sqrt{-896}}{-32}$$

Because the discriminant is negative, the solutions are not real, therefore the football does not reach a height of 50 feet.

72. First find the revenue quation, $R = xP$.

$$R = x(140 - 0.001x)$$
$$R = 140x - 0.001x^2$$

Then $P = R - C$

$$P = (140x - 0.001x^2) - (40x + 150{,}000)$$
$$P = -0.001x^2 + 100x - 150{,}000$$

Next, set $P = 3{,}000{,}000$ and solve for x.

$$-0.001x^2 + 100x - 150{,}000 = 3{,}000{,}000$$
$$-0.001x^2 + 100x - 3{,}150{,}000 = 0$$

Using the Quadratic Formula

$$x = \frac{-(100) \pm \sqrt{(100)^2 - 4(-0.001)(-3{,}150{,}000)}}{2(-0.001)}$$
$$x = \frac{-100 \pm \sqrt{-260}}{-0.002}$$

Because the discriminant is negative, the solutions are not real. Therefore, there is no price P that will yield a profit of \$3 million.

73. False, a third-degree polynomial must have at least one real zero.

74. False. Because complex zeros occur in conjugate pairs, $[x + (4 - 3i)]$ is also a factor of f.

75. Answers will vary.

76. $f(x) = x^4 - 4x^2 + k$

$$x^2 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(k)}}{2(1)} = \frac{4 \pm 2\sqrt{4-k}}{2} = 2 \pm \sqrt{4-k}$$
$$x = \pm\sqrt{2 \pm \sqrt{4-k}}$$

(a) For there to be four distinct real roots, both $4 - k$ and $2 \pm \sqrt{4-k}$ must be positive. This occurs when $0 < k < 4$. So, some possible k-values are $k = 1,\ k = 2,\ k = 3,\ k\frac{1}{2},\ k = \sqrt{2}$, etc.

(b) For there to be two real roots, each of multiplicity 2, $4 - k$ must equal zero. So, $k = 4$.

(c) For there to be two real zeros and two complex zeros, $2 + \sqrt{4-k}$ must be positive and $2 - \sqrt{4-k}$ must be negative. This occurs when $k < 0$. So, some possible k-values are $k = -1,\ k = -2,\ k = -\frac{1}{2}$, etc.

(d) For there to be four complex zeros, $2 \pm \sqrt{4-k}$ must be nonreal. This occurs when $k > 4$. Some possible k-values are $k = 5,\ k = 6,\ k = 7.4$, etc.

77. $f(x) = x^2 - 7x - 8$

$$= \left(x - \frac{7}{2}\right)^2 - \frac{81}{4}$$

A parabola opening upward with vertex $\left(\frac{7}{2},\ -\frac{81}{4}\right)$

78. $f(x) = -x^2 + x + 6$

$$= -\left(x - \frac{1}{2}\right)^2 + \frac{25}{4}$$

A parabola opening downward with vertex $\left(\frac{1}{2},\ \frac{25}{4}\right)$

79. $f(x) = 6x^2 + 5x - 6$

$$= 6\left(x + \frac{5}{12}\right)^2 - \frac{169}{24}$$

A parabola opening upward with vertex $\left(-\frac{5}{12},\ -\frac{169}{24}\right)$

80. $f(x) = 4x^2 + 2x - 12$

$$= 4\left(x + \frac{1}{4}\right)^2 - \frac{49}{4}$$

A parabola opening upward with vertex $\left(-\frac{1}{4},\ -\frac{49}{4}\right)$

Section 2.6

1. rational functions

2. vertical asymptote

3. To determine the vertical asymptote(s) of the graph of $y = \frac{9}{x-3}$, find the real zeros of the denominator of the equation. (Assuming no common factors in the numerator and denominator)

4. No, the x-axis, $y = 0$, is the horizontal asymptote of the graph of $y = \frac{2x}{3x^2 - 5}$ because the numerator's degree is less than the denominator's degree

5. $f(x) = \frac{1}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	–2
0.9	–10
0.99	–100
0.999	–1000

x	$f(x)$
1.5	2
1.1	10
1.01	100
1.001	1000

x	$f(x)$
5	0.25
10	$0.\overline{1}$
100	$0.\overline{01}$
1000	$0.\overline{001}$

x	$f(x)$
–5	$-0.1\overline{6}$
–10	$-0.\overline{09}$
–100	$-0.\overline{0099}$
–1000	$-0.\overline{000999}$

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

6. $f(x) = \frac{5x}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	–5
0.9	–45
0.99	–495
0.999	–4995

x	$f(x)$
1.5	15
1.1	55
1.01	505
1.001	5005

x	$f(x)$
5	6.25
10	$5.\overline{55}$
100	$5.\overline{05}$
1000	$5.\overline{005}$

x	$f(x)$
–5	$4.1\overline{6}$
–10	$4.\overline{54}$
–100	$4.95\overline{0495}$
–1000	4.995

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

7. $f(x) = \frac{3x}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	3
0.9	27
0.99	297
0.999	2997

x	$f(x)$
1.5	9
1.1	33
1.01	303
1.001	3003

x	$f(x)$
5	3.75
10	$3.\overline{33}$
100	$3.\overline{03}$
1000	$3.\overline{003}$

x	$f(x)$
–5	–2.5
–10	–2.727
–100	–2.970
–1000	–2.997

(c) f approaches ∞ from both the left and the right of 1.

8. $f(x) = \frac{3}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	6
0.9	30
0.99	300
0.999	3000

x	$f(x)$
1.5	6
1.1	30
1.01	300
1.001	3000

x	$f(x)$
5	0.75
10	$0.\overline{33}$
100	$0.\overline{03}$
1000	$0.\overline{003}$

x	$f(x)$
–5	0.5
–10	$0.\overline{27}$
–100	0.0297
–1000	0.003

(c) f approaches ∞ from both the left and the right of 1.

9. $f(x) = \frac{3x^2}{x^2 - 1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	–1
0.9	–12.79
0.99	–147.8
0.999	–1498

x	$f(x)$
1.5	5.4
1.1	17.29
1.01	152.3
1.001	1502.3

x	$f(x)$
5	3.125
10	$3.0\overline{3}$
100	$3.\overline{0003}$
1000	3

x	$f(x)$
−5	3.125
−10	$3.0\overline{3}$
−100	$3.\overline{0003}$
−1000	3

(c) f approaches $-\infty$ from the left of 1, and ∞ from the right of 1. f approaches ∞ from the left of –1, and $-\infty$ from the right of –1.

10. $f(x) = \dfrac{4x}{x^2 - 1}$

(a) Domain: all $x \neq \pm 1$

(b)

x	$f(x)$
0.5	$-2.\overline{66}$
0.9	−18.95
0.99	−199
0.999	−1999

x	$f(x)$
1.5	4.8
1.1	20.95
1.01	201
1.001	2001

x	$f(x)$
5	$-0.8\overline{33}$
10	$0.\overline{40}$
100	0.04
1000	0.004

x	$f(x)$
−5	$-0.8\overline{33}$
−10	$-0.\overline{40}$
−100	−0.04
−1000	−0.004

(c) f approaches $-0.8\overline{33}$ ∞ from the left of 1, and ∞ from the right of 1. f approaches $-\infty$ from the left of
–1, and ∞ from the right of –1.

11. $f(x) = \dfrac{2}{x+2}$

Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$
Matches graph (a).

12. $f(x) = \dfrac{1}{x-3}$

Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$
Matches graph (d).

13. $f(x) = \dfrac{4x+1}{x}$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 4$
Matches graph (c).

14. $f(x) = \dfrac{1-x}{x}$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = -1$
Matches graph (e).

15. $f(x) = \dfrac{x-2}{x-4}$

Vertical asymptote: $x = 4$
Horizontal asymptote: $y = 1$
Matches graph (b).

16. $f(x) = -\dfrac{x+2}{x+4}$

Vertical asymptote: $x = -4$
Horizontal asymptote: $y = -1$
Matches graph (f).

17. $f(x) = \dfrac{1}{x^2}$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$ or x-axis

18. $f(x) = \dfrac{3}{(x-2)^2} = \dfrac{3}{x^2 - 4x + 4}$

Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 0$ or x-axis

19. $f(x) = \dfrac{2x^2}{x^2 + x - 6} = \dfrac{2x^2}{(x+3)(x-2)}$

Vertical asymptotes: $x = -3,\ x = 2$
Horizontal asymptote: $y = 2$

20. $f(x) = \dfrac{x^2 - 4x}{x^2 - 4} = \dfrac{x(x-4)}{(x+2)(x-2)}$

Vertical asymptotes: $x = -2,\ x = 2$
Horizontal asymptote: $y = 1$

21. $f(x) = \dfrac{x(2+x)}{2x - x^2} = \dfrac{\cancel{x}(x+2)}{-\cancel{x}(x-2)} = -\dfrac{x+2}{x-2},\ x \neq 0$

Vertical asymptote: $x = 2$
Horizontal asymptote: $y = -1$
Hole at $x = 0$

22. $f(x) = \dfrac{x^2 + 2x + 1}{2x^2 - x - 3} = \dfrac{(x+1)\cancel{(x+1)}}{(2x-3)\cancel{(x+1)}} = \dfrac{x+1}{2x-3},\ x \neq -1$

Vertical asymptote: $x = 3/2$
Horizontal asymptote: $y = 1/2$
Hole at $x = -1$

23. $f(x) = \dfrac{x^2 - 25}{x^2 + 5x} = \dfrac{\cancel{(x+5)}(x-5)}{x\cancel{(x+5)}} = \dfrac{x-5}{x},\ x \neq -5$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 1$
Hole at $x = -5$

24. $f(x) = \dfrac{3 - 14x - 5x^2}{3 + 7x + 2x^2} = \dfrac{-\cancel{(x+3)}(5x-1)}{\cancel{(x+3)}(2x+1)} = \dfrac{-(5x-1)}{2x+1}$

Vertical asymptote: $x = -1/2$
Horizontal asymptote: $y = -5/2$
Hole at $x = -3$

25. $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$

(a) Domain: all real numbers
(b) Continuous
(c) Vertical asymptote: none
Horizontal asymptote: $y = 3$

26. $f(x) = \dfrac{3x^2 + 1}{x^2 + x + 9}$

(a) Domain: all real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]
(b) Continuous
(c) Vertical asymptote: none
Horizontal asymptote: $y = 3$

27. $f(x) = \dfrac{x^2 + 3x - 4}{-x^3 + 27} = \dfrac{(x+4)(x-1)}{-(x-3)(x^2 + 3x + 9)}$

(a) Domain: all real numbers x except $x = 3$
(b) Not continuous at $x = 3$
(c) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$ or x-axis

28. $f(x) = \dfrac{4x^3 - x^2 + 3}{3x^3 + 24} = \dfrac{4x^3 - x^2 + 3}{3(x+2)(x^2 - 2x + 4)}$

(a) Domain: all real numbers x except $x = -2$
(b) Not continuous at $x = -2$
(c) Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 4/3$

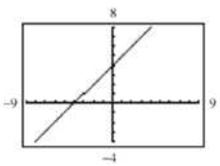
29. $f(x) = \dfrac{x^2 - 16}{x - 4} = \dfrac{(x+4)\cancel{(x-4)}}{\cancel{x-4}} = x + 4,\ x \neq 4$

$g(x) = x + 4$

(a) Domain of f: all real x except $x = 4$
Domain of g: all real x
(b) Vertical asymptote:
f has none. g has none.
Hole: f has a hole at $x = 4$; g has none.
(c)

x	1	2	3	4	5	6	7
$f(x)$	5	6	7	Undef.	9	10	11
$g(x)$	5	6	7	8	9	10	11

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

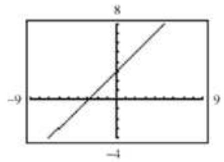
30. $f(x) = \dfrac{x^2 - 9}{x - 3} = \dfrac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = x + 3,\ x \neq 3$

$g(x) = x + 3$

(a) Domain of f: all real x except $x = 3$
Domain of g: all real x
(b) Vertical asymptote: f has none.
g has none.
Hole: f has a hole at $x = 3$; g has none.
(c)

x	0	1	2	3	4	5	6
$f(x)$	3	4	5	Undef.	7	8	9
$g(x)$	3	4	5	6	7	8	9

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

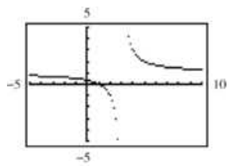
31. $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3} = \dfrac{\cancel{(x+1)}(x-1)}{(x-3)\cancel{(x+1)}} = \dfrac{x-1}{x-3},\ x \neq -1$

$g(x) = \dfrac{x-1}{x-3}$

(a) Domain of f: all real x except $x = 3$ and $x = -1$
Domain of g: all real x except $x = 3$
(b) Vertical asymptote: f has a vertical asymptote at $x = 3$.
g has a vertical asymptote at $x = 3$.
Hole: f has a hole at $x = -1$; g has none.
(c)

x	−2	−1	0	1	2	3	4
$f(x)$	$\frac{3}{5}$	Undef.	$\frac{1}{3}$	0	−1	Undef.	3
$g(x)$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	0	−1	Undef.	3

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

32. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2} = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x-1)} = \dfrac{x+2}{x-1},\ x \neq 2$

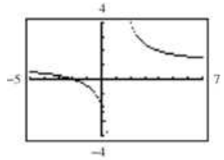
$g(x) = \dfrac{x+2}{x-1}$

(a) Domain of f: all real x except $x = 1$ and $x = 2$
Domain of g: all real x except $x = 1$
(b) Vertical asymptote: f has a vertical asymptote at $x = 1$.
g has a vertical asymptote at $x = 1$.
Hole: f has a hole at $x = 2$; g has none.

(c)

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	Undef.	Undef.	$\frac{5}{2}$
$g(x)$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	Undef.	4	$\frac{5}{2}$

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

33. $f(x) = 4 - \frac{1}{x}$

As $x \to \pm\infty$, $f(x) \to 4$.

As $x \to \infty$, $f(x) \to 4$ but is less than 4.

As $x \to -\infty$, $f(x) \to 4$ but is greater than 4.

34. $f(x) = 2 + \frac{1}{x-3}$

As $x \to \pm\infty$, $f(x) \to 2$.

As $x \to \infty$, $f(x) \to 2$ but is greater than 2.

As $x \to -\infty$, $f(x) \to 2$ but is less than 2.

35. $f(x) = \frac{2x-1}{x-3}$

As $x \to \pm\infty$, $f(x) \to 2$.

As $x \to \infty$, $f(x) \to 2$ but is greater than 2.

As $x \to -\infty$, $f(x) \to 2$ but is less than 2.

36. $f(x) = \frac{2x-1}{x^2+1}$

As $x \to \pm\infty$, $f(x) \to 0$. .

As $x \to \infty$, $f(x) \to 0$ but is greater than 0.

As $x \to -\infty$, $f(x) \to 0$ but is less than 0.

37. $g(x) = \frac{x^2-4}{x+3} = \frac{(x-2)(x+2)}{x+3}$

The zeros of g correspond to the zeros of the numerator and are $x = \pm 2$.

38. $g(x) = \frac{x^3-8}{x^2+4}$

The zero of g corresponds to the zero of the numerator and is $x = 2$.

39. $f(x) = 1 - \frac{2}{x-5} = \frac{x-7}{x-5}$

The zero of f corresponds to the zero of the numerator and is $x = 7$.

40. $h(x) = 5 + \frac{3}{x^2+1}$

There are no real zeros.

41. $g(x) = \frac{x^2-2x-3}{x^2+1} = \frac{(x-3)(x+1)}{x^2+1} = 0$

Zeros: $x = -1, 3$

42. $g(x) = \frac{x^2-5x+6}{x^2+4} = \frac{(x-3)(x-2)}{x^2+4} = 0$

Zeros: $x = 2, 3$

43. $f(x) = \frac{2x^2-5x+2}{2x^2-7x+3} = \frac{(2x-1)(x-2)}{(2x-1)(x-3)} = \frac{x-2}{x-3},\ x \neq \frac{1}{2}$

Zero: $x = 2$ ($x = \frac{1}{2}$ is not in the domain.)

44. $f(x) = \frac{2x^2+3x-2}{x^2+x-2} = \frac{(x+2)(2x-1)}{(x+2)(x+1)} = \frac{2x-1}{x+1},\ x = -2$

Zero: $x = \frac{1}{2}$ ($x = -2$ is not in the domain.)

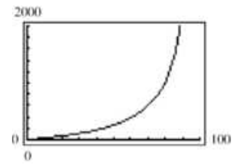
45. $C = \frac{255p}{100-p},\ 0 \le p < 100$

(a) find $C(10) = \frac{255(10)}{100-10} = \$28.3$ million

$C(40) = \frac{255(40)}{100-40} = \$170$ million

$C(75) = \frac{255(75)}{100-75} = \$765$ million

(b)



(c) No. The function is undefined at $p = 100\,\%$.

46.

(a) Use data $\left(16, \frac{1}{3}\right), \left(32, \frac{1}{4.7}\right), \left(44, \frac{1}{9.8}\right), \left(50, \frac{1}{19.7}\right), \left(60, \frac{1}{39.4}\right)$.

$$\frac{1}{y} = -0.007x + 0.445$$

$$y = \frac{1}{0.445 - 0.007x}$$

(b)

x	16	32	44	50	60
y	3.0	4.5	7.3	10.5	40

(Answers will vary.)

(c) No, the function is negative for $x = 70$.

47.

(a)

M	200	400	600	800	1000	1200	1400	1600	1800	2000
t	0.472	0.596	0.710	0.817	0.916	1.009	1.096	1.178	1.255	1.328

The greater the mass, the more time required per oscillation. The model is a good fit to the actual data.

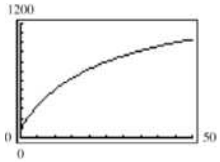
(b) You can find M corresponding to $t = 1.056$ by finding the point of intersection of

$$t = \frac{38M + 16{,}965}{10(M + 500)} \text{ and } t = 1.056.$$

If you do this, you obtain $M \approx 1306$ grams.

48. $N = \dfrac{20(5 + 3t)}{1 + 0.04t}$, $0 \le t$

(a)



(b) $N(5) \approx 333$ deer

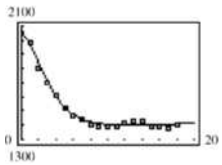
$N(10) = 500$ deer

$N(25) = 800$ deer

(c) The herd is limited by the horizontal asymptote:

$$N = \frac{60}{0.04} = 1500 \text{ deer}$$

49. (a) The model fits the data well.



(a) $N = \dfrac{77.095t^2 - 216.04t + 2050}{0.052t^2 - 0.8t + 1}$

$2009: N(19) \approx 1{,}412{,}000$

$2010: N(20) \approx 1{,}414{,}000$

$2011: N(11) \approx 1{,}416{,}000$

Answers will vary.

(b) Horizontal asymptote: $N = 1482.6$ (approximately) Answers will vary.

50. False. A rational function can have at most n vertical asymptotes, where n is the degree of the denominator.

51. False. For example, $f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

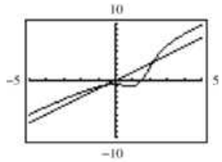
52. If the domain of f (a rational function) is all real x except $x = c$, then either the graph of f has a vertical asymptote at $x = c$ (numerator and denominator do not have the like factor $x - c$) or the graph of f has a hole at $x = c$ (numerator and denominator do have like factor $x - c$).

53. No. If $x = c$ is also a zero of the denominator of f, then f is undefined at $x = c$, and the graph of f may have a hole of vertical asymptote at $x = c$.

54. Yes. If the graph of f, a rational function, has a vertical asymptote at $x = 4$, then f may have a common factor of $x - 4$. For example,

$$f(x) = \frac{(x - 4)(x + 1)}{(x - 4)(x - 4)}.$$

55.



The graphs of $y_1 = \dfrac{3x^3 - 5x^2 + 4x - 5}{2x^2 - 6x + 7}$ and $y_2 = \dfrac{3x^3}{2x^2}$ are approximately the same graph as $x \to \infty$ and $x \to -\infty$.

Therefore as $x \to \pm\infty$, the graph of a rational function $y = \dfrac{a_n x^n + \dots a_1 x + a_0}{b_m x^m + \dots b_1 x + b_0}$ appears to be very close to the graph of $y = \dfrac{a_n x^n}{b_m x^m}$.

56.

(a) $f(x) = \dfrac{x - 1}{x^3 - 8}$

(b) $f(x) = \dfrac{x - 2}{(x + 1)^2}$

(c) $f(x) = \dfrac{2(x + 3)(x - 3)}{(x + 2)(x - 1)} = \dfrac{2x^2 - 18}{x^2 + x - 2}$

(d) $f(x) = \dfrac{-2(x + 2)(x - 3)}{(x + 1)(x - 2)} = \dfrac{-2x^2 + 2x + 12}{x^2 - x - 2}$

57. $y - 2 = \dfrac{-1 - 2}{0 - 3}(x - 3) = 1(x - 3)$

$y = x - 1$

$y - x + 1 = 0$

58. $y - 1 = \dfrac{1 + 5}{-6 - 4}(x + 6)$

$-10y + 10 = 6x + 36$

$3x + 5y + 13 = 0$

59. $y - 7 = \dfrac{10 - 7}{3 - 2}(x - 2) = 3(x - 2)$

$y = 3x + 1$

$3x - y + 1 = 0$

60. $y - 0 = \dfrac{4 - 0}{-9 - 0}(x - 0)$

$-9y = 4x$

$4x + 9y = 0$

61.

$$\begin{array}{r} x + 9 \\ x - 4 \overline{\big)\, x^2 + 5x + 6} \\ \underline{x^2 - 4x} \\ 9x + 6 \\ \underline{9x - 36} \\ 42 \end{array}$$

$$\frac{x^2 + 5x + 6}{x - 4} = x + 9 + \frac{42}{x - 4}$$

62.

$$\begin{array}{r|rrr} 3 & 1 & -10 & 15 \\ & & 3 & -21 \\ \hline & 1 & -7 & -6 \end{array}$$

$$\frac{x^2-10x+15}{x-3} = x-7+\frac{-6}{x-3}$$

63.

$$\begin{array}{rl} & 2x^2-9 \\ x^2+5 & \overline{)2x^4+0x^3+x^2+0x-11} \\ & \underline{2x^4+\quad 10x^2} \\ & -9x^2-\quad 11 \\ & \underline{-9x^2-\quad 45} \\ & 34 \end{array}$$

$$\frac{2x^4+x^2-11}{x^2+5} = 2x^2-9+\frac{34}{x^2+5}$$

64.

$$\begin{array}{rl} & 2x^4-3x^3+6x^2-9x+\frac{27}{2} \\ 2x+3 & \overline{)4x^5+0x^4+3x^3+0x^2+0x-10} \\ & \underline{4x^5+6x^4} \\ & -6x^4+3x^3 \\ & \underline{-6x^4-9x^3} \\ & 12x^3 \\ & \underline{12x^3+18x^2} \\ & -18x^2 \\ & \underline{-18x^2-27x} \\ & 27x-10 \\ & \underline{27x+\frac{81}{2}} \\ & -\frac{101}{2} \end{array}$$

$$\frac{4x^5+3x^3-10}{2x+3} = 2x^4-3x^3+6x^2-9x+\frac{27}{2}-\frac{101}{4x+6}$$

Section 2.7

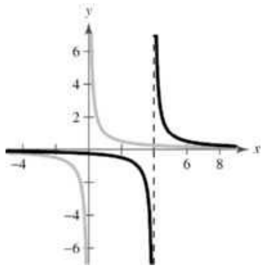
1. slant, asymptote

2. vertical

3. Yes. Because the numerator's degree is exactly 1 greater than that of the denominator, the graph of *f* has a slant asymptote.

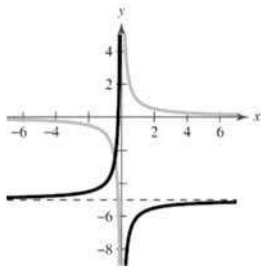
4. The slant asymptote's equation is that of the quotient**,** $y = x-1.$**.**

5.



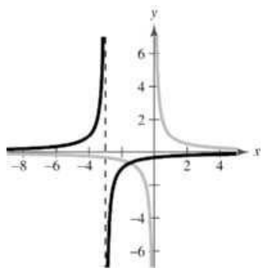
The graph of $g = \frac{1}{x-4}$ is a horizontal shift four units to the right of the graph of $f(x) = \frac{1}{x}$.

6.



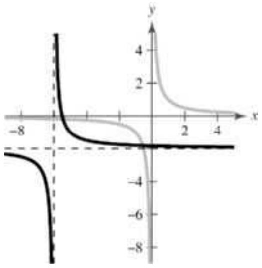
The graph of $g = \frac{-1}{x} - 5$ is a reflection in the *x*-axis and a vertical shift five units downward of the graph of $f(x) = \frac{1}{x}$.

7.



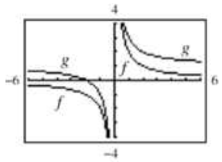
The graph of $g(x) = \frac{-1}{x+3}$ is a reflection in the *x*-axis and a horizontal shift three units to the left of the graph of $f(x) = \frac{1}{x}$.

8.



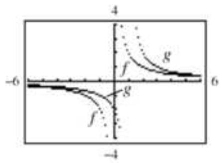
The graph of $g(x) = \frac{1}{x+6} - 2$ is a horizontal shift six units to the left and a vertical shift two units downward of the graph of $f(x) = \frac{1}{x}$.

9. $g(x) = \frac{2}{x} + 1$



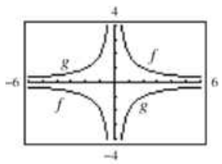
Vertical shift one unit upward

10. $g(x) = \frac{2}{x-1}$



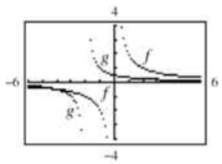
Horizontal shift one unit to the right

11. $g(x) = -\frac{2}{x}$



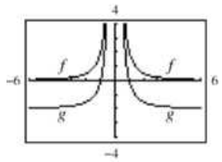
Reflection in the x-axis

12. $g(x) = \frac{1}{x+2}$



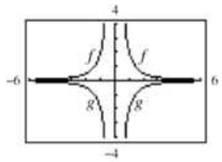
Horizontal shift two units to the left, and vertical shrink

13. $g(x) = \frac{2}{x^2} - 2$



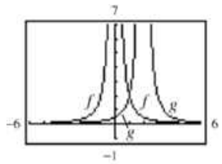
Vertical shift two units downward

14. $g(x) = -\frac{2}{x^2}$



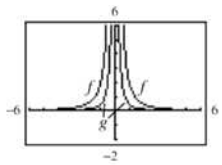
Reflection in the x–axis

15. $g(x) = \frac{2}{(x-2)^2}$



Horizontal shift two units to the right

16. $g(x) = \frac{1}{2x^2}$

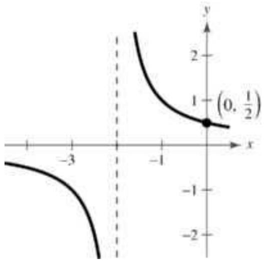


Vertical shrink

17. $f(x) = \frac{1}{x+2}$

y-intercept: $\left(0, \frac{1}{2}\right)$

Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$

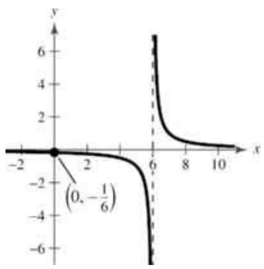


x	-4	-3	-1	0	1
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$

18. $f(x) = \dfrac{1}{x-6}$

y-intercept: $\left(0, -\dfrac{1}{6}\right)$

Vertical asymptote: $x = 6$
Horizontal asymptote: $y = 0$

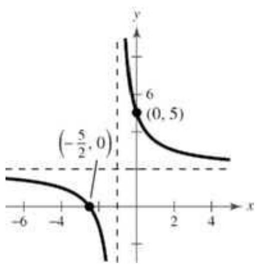


x	-1	0	2	4	8	10
y	$-\frac{1}{7}$	$-\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$

19. $C(x) = \dfrac{5+2x}{1+x} = \dfrac{2x+5}{x+1}$

x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

y-intercept: $(0, 5)$
Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 2$

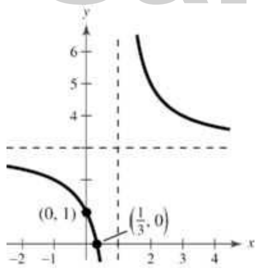


x	-4	-3	-2	0	1	2
$C(x)$	1	$\frac{1}{2}$	-1	5	$\frac{7}{2}$	3

20. $P(x) = \dfrac{1-3x}{1-x} = \dfrac{3x-1}{x-1}$

x-intercept: $\left(\dfrac{1}{3}, 0\right)$

y-intercept: $(0, 1)$
Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 3$

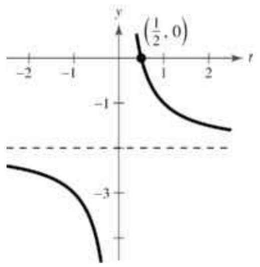


x	-1	0	2	3
y	2	1	5	4

21. $f(t) = \dfrac{1-2t}{t} = -\dfrac{2t-1}{t}$

t-intercept: $\left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $t = 0$
Horizontal asymptote: $y = -2$



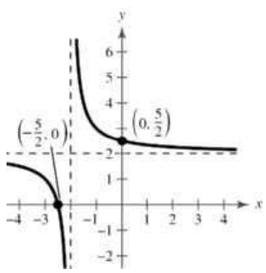
x	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$

22. $g(x) = \dfrac{1}{x+2} + 2 = \dfrac{2x+5}{x+2}$

y-intercept: $\left(0, \dfrac{5}{2}\right)$

x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 2$



x	-4	$-\frac{5}{2}$	-1	0	2
y	$\frac{3}{2}$	0	3	$\frac{5}{2}$	$\frac{9}{4}$

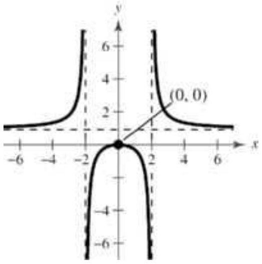
23. $f(x) = \dfrac{x^2}{x^2 - 4}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = 2,\ x = -2$

Horizontal asymptote: $y = 1$

y-axis symmetry



x	-4	-1	0	-1	4
y	$\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{4}{3}$

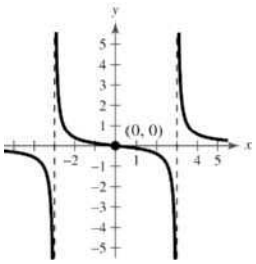
24. $g(x) = \dfrac{x}{x^2 - 9}$

Intercepts: $(0, 0)$

Vertical asymptotes: $x = \pm 3$

Horizontal asymptote: $y = 0$

Origin symmetry



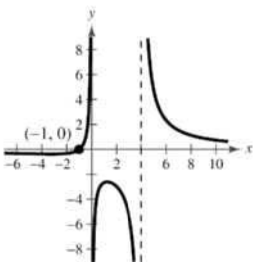
x	-5	-4	-2	0	2	4	5
y	$-\frac{5}{16}$	$-\frac{4}{7}$	$\frac{2}{5}$	0	$-\frac{2}{5}$	$\frac{4}{7}$	$\frac{5}{16}$

25. $g(x) = \dfrac{4(x+1)}{x(x-4)}$

Intercept: $(-1, 0)$

Vertical asymptotes: $x = 0$ and $x = 4$

Horizontal asymptote: $y = 0$

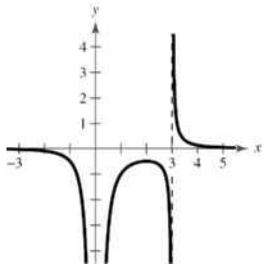


x	-2	-1	1	2	3	5	6
y	$-\frac{1}{3}$	0	$-\frac{8}{3}$	-3	$-\frac{16}{3}$	$\frac{24}{5}$	$\frac{7}{3}$

26. $h(x) = \dfrac{2}{x^2(x-3)}$

Vertical asymptotes: $x = 0,\ x = 3$

Horizontal asymptote: $y = 0$



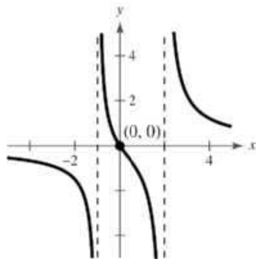
x	-2	0	1	2	3	4
y	$-\frac{1}{10}$	Undef.	-1	$-\frac{1}{2}$	Undef.	$\frac{1}{8}$

27. $f(x) = \dfrac{3x}{x^2 - x - 2} = \dfrac{3x}{(x+1)(x-2)}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = -1,\ 2$

Horizontal asymptote: $y = 0$



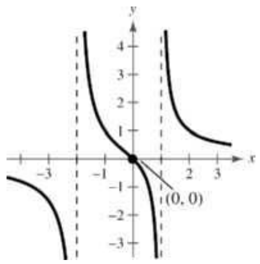
x	-3	0	1	3	4
y	$-\frac{9}{10}$	0	$-\frac{3}{2}$	$\frac{9}{4}$	$\frac{6}{5}$

28. $f(x) = \dfrac{2x}{x^2 + x - 2} = \dfrac{2x}{(x+2)(x-1)}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = -2,\ 1$

Horizontal asymptote: $y = 0$



x	-4	-3	-1	0	$\frac{1}{2}$	2	3
y	$-\frac{4}{5}$	$-\frac{3}{2}$	1	0	$-\frac{4}{5}$	1	$\frac{3}{5}$

29. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x+3)}{(x-2)(x+3)} = \dfrac{x}{x-2},$

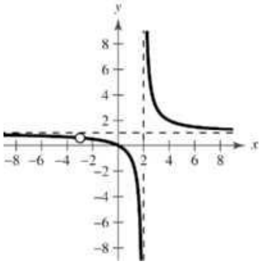
$x \neq -3$

Intercept: $(0, 0)$

Vertical asymptote: $x = 2$

(There is a hole at $x = -3$.)

Horizontal asymptote: $y = 1$



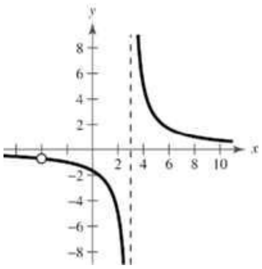
x	-2	-1	0	1	2	3
y	$\dfrac{1}{2}$	$\dfrac{1}{3}$	0	-1	Undef.	3

30. $g(x) = \dfrac{5(x+4)}{x^2 + x - 12} = \dfrac{5(x+4)}{(x+4)(x-3)} = \dfrac{5}{x-3},$

$x \neq -4$

Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$



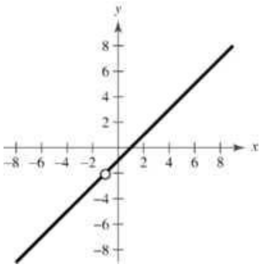
Hole at $x = -4$

x	-4	0	1	3	4
y	Undef.	$-\dfrac{5}{3}$	$-\dfrac{5}{2}$	Undef.	5

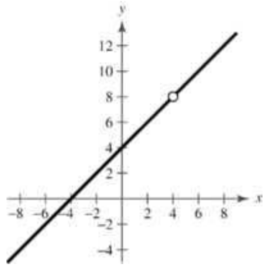
31. $f(x) = \dfrac{x^2 - 1}{x+1} = \dfrac{(x+1)(x-1)}{x+1} = x - 1,$

$x \neq -1$

The graph is a line, with a hole at $x = -1$.



32. $f(x) = \dfrac{x^2 - 16}{x-4} = x + 4,\ x \neq 4$

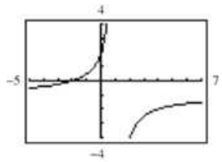


Hole at $x = 4$

33. $f(x) = \dfrac{2+x}{1-x} = -\dfrac{x+2}{x-1}$

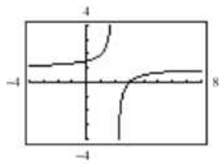
Vertical asymptote: $x = 1$

Horizontal asymptote: $y = -1$



Domain: $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

34. $f(x) = \dfrac{3-x}{2-x} = \dfrac{x-3}{x-2}$



x-intercept: $(3, 0)$

y-intercept: $\left(0, \dfrac{3}{2}\right)$

Vertical asymptote: $x = 2$

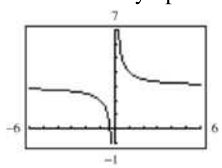
Horizontal asymptote: $y = 1$

Domain: all $x \neq 2$

35. $f(t) = \dfrac{3t+1}{t}$

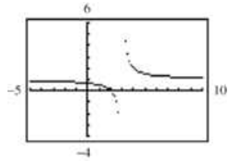
Vertical asymptote: $t = 0$

Horizontal asymptote: $y = 3$



Domain: $t \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

36. $h(x) = \dfrac{x-2}{x-3}$



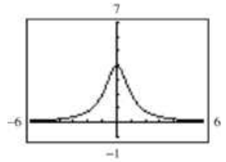
x-intercept: $(2, 0)$

y-intercept: $\left(0, \dfrac{2}{3}\right)$

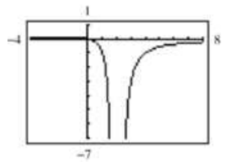
Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 1$
Domain: all $x \neq 3$

37. $h(t) = \dfrac{4}{t^2 + 1}$

Domain: all real numbers or $(-\infty, \infty)$
Horizontal asymptote: $y = 0$



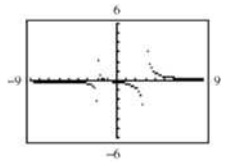
38. $g(x) = -\dfrac{x}{(x-2)^2}$



Domain: all real numbers except 2 or $(-\infty, 2) \cup (2, \infty)$
Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 0$

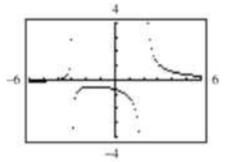
39. $f(x) = \dfrac{x+1}{x^2 - x - 6} = \dfrac{x+1}{(x-3)(x+2)}$

Domain: all real numbers except $x = 3, \ -2$
Vertical asymptotes: $x = 3, x = -2$
Horizontal asymptote: $y = 0$



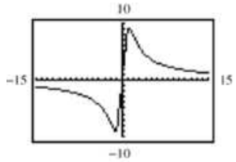
40. $f(x) = \dfrac{x+4}{x^2 + x - 6}$

Domain: all real numbers except –3 and 2 or $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$



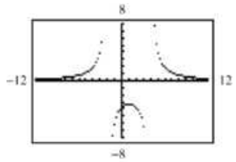
Vertical asymptotes: $x = -3, x = 2$
Horizontal asymptote: $y = 0$

41. $f(x) = \dfrac{20x}{x^2+1} - \dfrac{1}{x} = \dfrac{19x^2 - 1}{x(x^2+1)}$



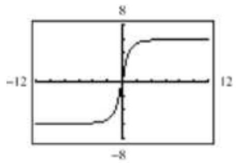
Domain: all real numbers except 0, or $(-\infty, 0) \cup (0, \infty)$
Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$

42. $f(x) = 5\left(\dfrac{1}{x-4} - \dfrac{1}{x-2}\right) = \dfrac{30}{(x-4)(x+2)}$



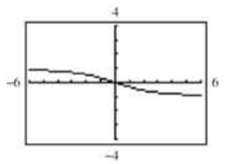
Domain: all real numbers except -2 and 4
Vertical asymptotes: $x = -2$, $x = 4$
Horizontal asymptote: $y = 0$

43. $h(x) = \dfrac{6x}{\sqrt{x^2+1}}$



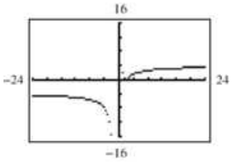
There are two horizontal asymptotes, $y = \pm 6$.

44. $f(x) = \dfrac{-x}{\sqrt{9 + x^2}}$



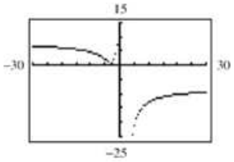
There are two horizontal asymptotes, $y = \pm 1$.

45. $g(x) = \dfrac{4|x-2|}{x+1}$



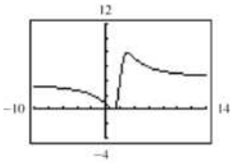
There are two horizontal asymptotes, $y = \pm 4$ and one vertical asymptote, $x = -1$.

46. $f(x) = \dfrac{-8|3+x|}{x-2} = \dfrac{8|3+x|}{2-x}$



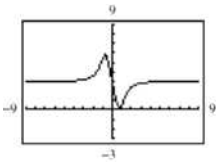
There are two horizontal asymptotes, $y = -8$ and $y = 8$, and one vertical asymptote, $x = 2$.

47. $f(x) = \dfrac{4(x-1)^2}{x^2-4x+5}$



The graph crosses its horizontal asymptote, $y = 4$.

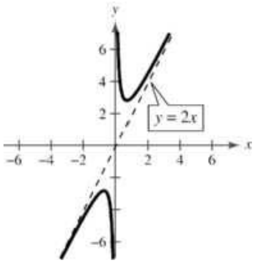
48. $g(x) = \dfrac{3x^4-5x+3}{x^4+1}$



The graph crosses its horizontal asymptote, $y = 3$.

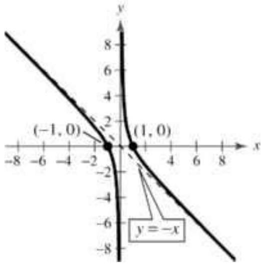
49. $f(x) = \dfrac{2x^2+1}{x} = 2x + \dfrac{1}{x}$

Vertical asymptote: $x = 0$
Slant asymptote: $y = 2x$
Origin symmetry



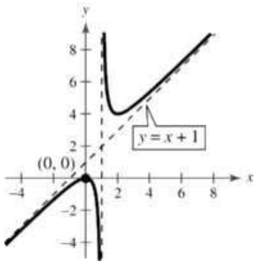
50. $g(x) = \dfrac{1-x^2}{x}$

Intercepts: $(1, 0), (-1, 0)$
Vertical asymptote: $x = 0$
Slant asymptote: $y = -x$



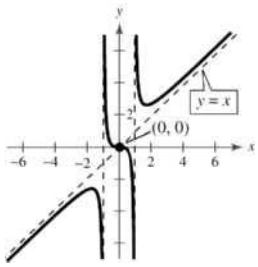
51. $h(x) = \dfrac{x^2}{x-1} = x + 1 + \dfrac{1}{x-1}$

Intercept: $(0, 0)$
Vertical asymptote: $x = 1$
Slant asymptote: $y = x + 1$



52. $f(x) = \dfrac{x^3}{x^2-1} = x + \dfrac{x}{x^2-1}$

Intercept: $(0, 0)$
Vertical asymptotes: $x = \pm 1$
Slant asymptote: $y = x$
Origin symmetry



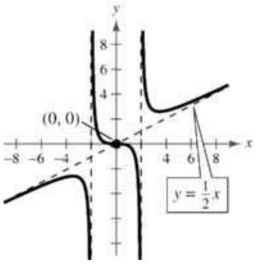
53. $g(x) = \frac{x^3}{2x^2 - 8} = \frac{1}{2}x + \frac{4x}{2x^2 - 8}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = \frac{1}{2}x$

Origin symmetry



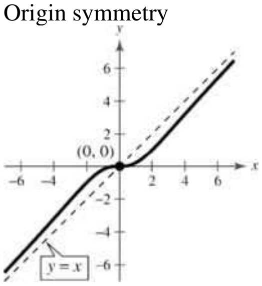
54. $f(x) = \frac{x^3}{x^2 + 4} = x - \frac{4x}{x^2 + 4}$

Intercept: $(0, 0)$

Vertical asymptotes: None

Slant asymptote: $y = x$

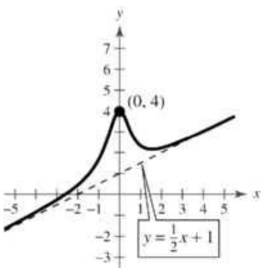
Origin symmetry



55. $f(x) = \frac{x^3 + 2x^2 + 4}{2x^2 + 1} = \frac{x}{2} + 1 + \frac{3 - \frac{x}{2}}{2x^2 + 1}$

Intercepts: $(-2.594, 0)$, $(0, 4)$

Slant asymptote: $y = \frac{x}{2} + 1$

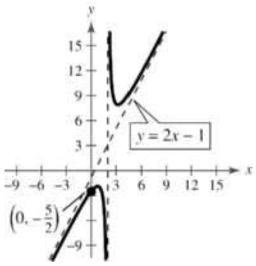


56. $f(x)\frac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \frac{3}{x - 2}$

y-intercept: $\left(0, -\frac{5}{2}\right)$

Vertical asymptote: $x = 2$

Slant asymptote: $y = 2x - 1$



57. $y = \frac{x + 1}{x - 3}$

x-intercept: $(-1, 0)$

$0 = \frac{x + 1}{x - 3}$

$0 = x + 1$

$-1 = x$

58. $y = \frac{2x}{x - 3}$

x-intercept: $(0, 0)$

$0 = \frac{2x}{x - 3}$

$0 = 2x$

$0 = x$

59. $y = \frac{1}{x} - x$

x-intercepts: $(\pm 1, 0)$

$0 = \frac{1}{x} - x$

$x = \frac{1}{x}$

$x^2 = 1$

$x = \pm 1$

60. $y = x - 3 + \frac{2}{x}$

x-intercepts: $(1, 0)$, $(2, 0)$

$0 = x - 3 + \frac{2}{x}$

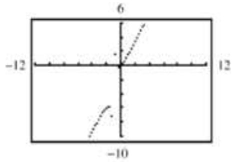
$0 = x^2 - 3x + 2$

$0 = (x - 1)(x - 2)$

$x = 1, 2$

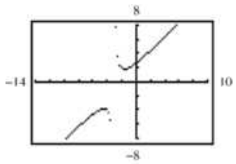
61. $y = \dfrac{2x^2 + x}{x+1} = 2x - 1 + \dfrac{1}{x+1}$

Domain: all real numbers except $x = -1$
Vertical asymptote: $x = -1$
Slant asymptote: $y = 2x - 1$



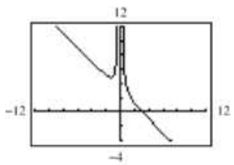
62. $y = \dfrac{x^2 + 5x + 8}{x+3} = x + 2 + \dfrac{2}{x+3}$

Domain: all $x \neq -3$
Vertical asymptote: $x = -3$
Slant asymptote: $y = x + 2$



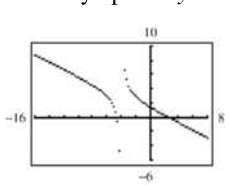
63. $y = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers except 0
or $(-\infty, 0) \cup (0, \infty)$
Vertical asymptote: $x = 0$
Slant asymptote: $y = -x + 3$



64. $y = \dfrac{12 - 2x - x^2}{2(4+x)} = -\dfrac{1}{2}x + 1 + \dfrac{2}{4+x}$

Domain: all real numbers except –4 or
$(-\infty, -4) \cup (-4, \infty)$
x-intercepts: $(-4.61, 0), (2.61, 0)$
y-intercept: $\left(0, \dfrac{3}{2}\right)$
Vertical asymptote: $x = -4$
Slant asymptote: $y = -\dfrac{1}{2}x + 1$



65. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4} = \dfrac{(x-4)(x-1)}{(x-2)(x+2)}$

Vertical asymptotes: $x = 2, x = -2$
Horizontal asymptote: $y = 1$
No slant asymptotes, no holes

66. $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - 9} = \dfrac{(x-4)(x+2)}{(x-3)(x+3)}$

Vertical asymptotes: $x = 3, x = -3$
Horizontal asymptote: $y = 1$
No slant asymptotes, no holes

67. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - x - 6} = \dfrac{(2x-1)(x-2)}{(2x+3)(x-2)} = \dfrac{2x-1}{2x+3}$,

$x \neq 2$

Vertical asymptote: $x = -\dfrac{3}{2}$
Horizontal asymptote: $y = 1$
No slant asymptotes
Hole at $x = 2$, $\left(2, \dfrac{3}{7}\right)$

68. $f(x) = \dfrac{3x^2 - 8x + 4}{2x^2 - 3x - 2} = \dfrac{(3x-2)(x-2)}{(2x+1)(x-2)} = \dfrac{3x-2}{2x+1}$,

$x \neq 2$

Vertical asymptote: $x = -\dfrac{1}{2}$
Horizontal asymptote: $y = \dfrac{3}{2}$
No slant asymptotes
Hole at $x = 2$, $\left(2, \dfrac{4}{5}\right)$

69. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$= \dfrac{(x-1)(x+1)(2x-1)}{(x+1)(x+2)}$

$= \dfrac{(x-1)(2x-1)}{x+2}, \; x \neq -1$

Long division gives

$f(x) = \dfrac{2x^2 - 3x + 1}{x+2} = 2x - 7 + \dfrac{15}{x+2}.$

Vertical asymptote: $x = -2$
No horizontal asymptote
Slant asymptote: $y = 2x - 7$
Hole at $x = -1$, $(-1, 6)$

70. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$

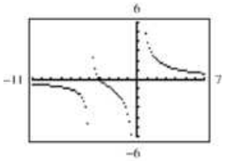
$= \dfrac{(x-2)(x+2)(2x+1)}{(x-2)(x-1)}$

$= \dfrac{(x+2)(2x+1)}{x-1},\ x \neq 2$

Long division gives $f(x) = 2x + 7 + \dfrac{9}{x-1}$.

Vertical asymptote: $x = 1$
No horizontal asymptote
Slant asymptote: $y = 2x + 7$
Hole at $x = 2,\ (2,\ 20)$

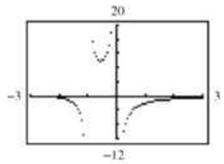
71. $y = \dfrac{1}{x+5} + \dfrac{4}{x}$



x-intercept: $(-4,\ 0)$

$$0 = \frac{1}{x+5} + \frac{4}{x}$$
$$-\frac{4}{x} = \frac{1}{x+5}$$
$$-4(x+5) = x$$
$$-4x - 20 = x$$
$$-5x = 20$$
$$x = -4$$

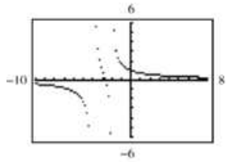
72. $y = \dfrac{2}{x+1} - \dfrac{3}{x}$



x-intercept: $(-3,\ 0)$

$$\frac{2}{x+1} - \frac{3}{x} = 0$$
$$\frac{2}{x+1} = \frac{3}{x}$$
$$2x = 3x + 3$$
$$-3 = x$$

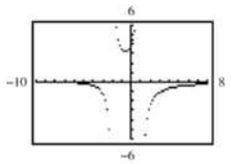
73. $y = \dfrac{1}{x+2} + \dfrac{2}{x+4}$



x-intercept: $\left(-\dfrac{8}{3},\ 0\right)$

$$\frac{1}{x+2} + \frac{2}{x+4} = 0$$
$$\frac{1}{x+2} = \frac{-2}{x+4}$$
$$x + 4 = -2x - 4$$
$$3x = -8$$
$$x = -\frac{8}{3}$$

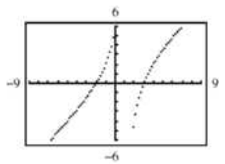
74. $y = \dfrac{2}{x+2} - \dfrac{3}{x-1}$



x-intercept: $(-8,\ 0)$

$$\frac{2}{x+2} = \frac{3}{x-1}$$
$$2x - 2 = 3x + 6$$
$$-8 = x$$

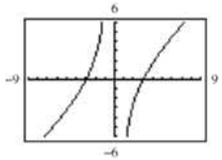
75. $y = x - \dfrac{6}{x-1}$



x-intercepts: $(-2, 0),\ (3, 0)$

$$0 = x - \frac{6}{x-1}$$
$$\frac{6}{x-1} = x$$
$$6 = x(x-1)$$
$$0 = x^2 - x - 6$$
$$0 = (x+2)(x-3)$$
$$x = -2,\ x = 3$$

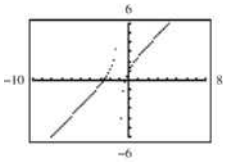
76. $y = x - \dfrac{9}{x}$



x-intercepts: $(-3, 0)$, $(3, 0)$

$$0 = x - \frac{9}{x}$$
$$\frac{9}{x} = x$$
$$9 = x^2$$
$$\pm 3 = x$$

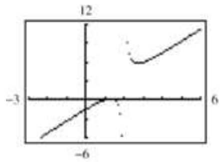
77. $y = x + 2 - \dfrac{1}{x+1}$



x-intercepts: $(-2.618, 0)$, $(-0.382, 0)$

$$x + 2 = \frac{1}{x+2}$$
$$x^2 + 3x + 2 = 1$$
$$x^2 + 3x + 1 = 0$$
$$x = \frac{-3 \pm \sqrt{9-4}}{2}$$
$$= -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$$
$$\approx -2.618, -0.382$$

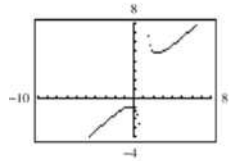
78. $y = 2x - 1 + \dfrac{1}{x-2}$



x-intercepts: $(1, 0)$, $\left(\dfrac{3}{2}, 0\right)$

$$2x - 1 + \frac{1}{x-2} = 0$$
$$\frac{1}{x-2} = 1 - 2x$$
$$1 = -2x^2 + 5x - 2$$
$$2x^2 - 5x + 3 = 0$$
$$(x-1)(2x-3) = 0$$
$$x = 1, \frac{3}{2}$$

79. $y = x + 1 + \dfrac{2}{x-1}$

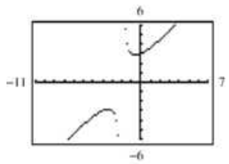


No x-intercepts

$$x + 1 + \frac{2}{x-1} = 0$$
$$\frac{2}{x-1} = -x - 1$$
$$2 = -x^2 + 1$$
$$x^2 + 1 = 0$$

No real zeros

80. $y = x + 2 + \dfrac{2}{x+2}$

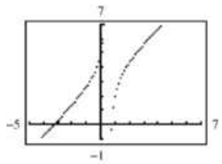


No x-intercepts

$$x + 2 + \frac{2}{x+2} = 0$$
$$\frac{2}{x+2} = -x - 2$$
$$2 = -x^2 - 4x - 4$$
$$x^2 + 4x + 6 = 0$$

Because $b^2 - 4ac = 16 - 24 < 0$, there are no real zeros.

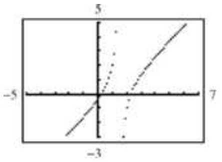
81. $y = x + 3 - \dfrac{2}{2x-1}$



x-intercepts: $(0.766, 0)$, $(-3.266, 0)$

$$x + 3 - \frac{2}{2x-1} = 0$$
$$x + 3 = \frac{2}{2x-1}$$
$$2x^2 + 5x - 3 = 2$$
$$2x^2 + 5x - 5 = 0$$
$$x = \frac{-5 \pm \sqrt{25 - 4(2)(-5)}}{4}$$
$$= \frac{-5 \pm \sqrt{65}}{4}$$
$$\approx 0.766, -3.266$$

82. $y = x - 1 - \dfrac{2}{2x-3}$



x-intercepts: $(0.219,\ 0),\ (2.281,\ 0)$

$$x - 1 - \frac{2}{2x-3} = 0$$
$$x - 1 = \frac{2}{2x-3}$$
$$2x^2 - 5x + 3 = 2$$
$$2x^2 - 5x + 1 = 0$$
$$x = \frac{5 \pm \sqrt{25-8}}{4}$$
$$= \frac{5 \pm \sqrt{17}}{4} \approx 0.219,\ 2.281$$

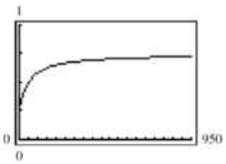
83.

(a) $0.25(50) + 0.75(x) = C(50 + x)$

$$\frac{12.5 + 0.75x}{50 + x} = C$$
$$\frac{50 + 3x}{200 + 4x} = C$$
$$C = \frac{3x + 50}{4(x + 50)}$$

(b) Domain: $x \ge 0$ and $x \le 1000 - 50 = 950$
Thus, $0 \le x \le 950$.

(c)



As the tank fills, the rate that the concentration is increasing slows down. It approaches the horizontal asymptote $C = \dfrac{3}{4} = 0.75$. When the tank is full $(x = 950)$, the concentration is $C = 0.725$.

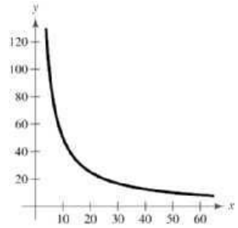
84.

(a) Area $= xy = 500$

$$y = \frac{500}{x}$$

(b) Domain: $x > 0$

(c)



For $x = 30$, $y = \dfrac{500}{30} = 16\dfrac{2}{3}$ meters.

85.

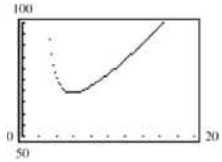
(a) $A = xy$ and

$$(x-2)(y-4) = 30$$
$$y - 4 = \frac{30}{x-2}$$
$$y = 4 + \frac{30}{x-2} = \frac{4x+22}{x-2}$$

Thus, $A = xy = x\left(\dfrac{4x+22}{x-2}\right) = \dfrac{2x(2x+11)}{x-2}$.

(b) Domain: Since the margins on the left and right are each 1 inch, $x > 2$, or $(2, \infty)$.

(c)



The area is minimum when $x \approx 5.87$ in. and $y \approx 11.75$ in.

86.

(a) The line passes through the points $(a,\ 0)$ and $(3,\ 2)$ and has a slope of

$$m = \frac{2-0}{3-a} = \frac{2}{3-a}.$$
$$y - 0 = \frac{2}{3-a}(x - a)$$
$$y = \frac{2(x-a)}{3-a} = \frac{-2(a-x)}{-1(a-3)} \text{ by the point-slope form}$$
$$= \frac{2(a-x)}{a-3},\ 0 \le x \le a$$

(b)

The area of a triangle is $A = \dfrac{1}{2}bh$.

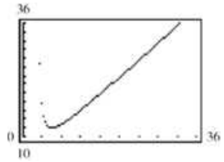
$b = a$

$h = y$ when $x = 0$, so $h = \dfrac{2(a-0)}{a-3} = \dfrac{2a}{a-3}$.

$$A = \frac{1}{2}a\left(\frac{2a}{a-3}\right) = \frac{a^2}{a-3}$$

(c) $A = \dfrac{a^2}{a-3} = a + 3 + \dfrac{9}{a-3}$

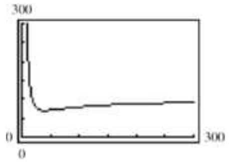
Vertical asymptote: $a = 3$



Slant asymptote: $A = a + 3$

A is a minimum when $a = 6$ and $A = 12$.

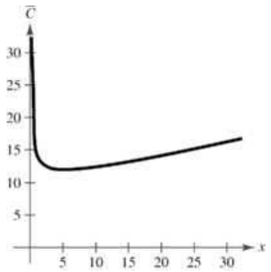
87. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right),\ 1 \le x$



The minimum occurs when $x = 40.4 \approx 40$.

88. $\overline{C} = \dfrac{C}{x} = \dfrac{0.2x^2 + 10x + 5}{x},\ x > 0$

x	0.5	1	2	3	4	5	6	7
$\overline{C}$	20.1	15.2	12.9	12.3				



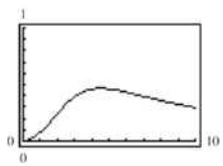
The minimum average cost occurs when $x = 5$.

89.

$C = \dfrac{3t^2 + t}{t^3 + 50},\ 0 \le t$

(a) The horizontal asymptote is the t-axis, or $C = 0$. This indicates that the chemical eventually dissipates.

(b)



The maximum occurs when $t \approx 4.5$.

(c) Graph C together with $y = 0.345$. The graphs intersect at $t \approx 2.65$ and $t \approx 8.32$. $C < 0.345$ when $0 \le t \le 2.65$ hours and when $t > 8.32$ hours.

90.

(a) $\text{Rate} \times \text{Time} = \text{Distance}$ or $\dfrac{\text{Distance}}{\text{Rate}} = \text{Time}$

$$\frac{100}{x} + \frac{100}{y} = \frac{200}{50} = 4$$

$$\frac{25}{x} + \frac{25}{y} = 1$$

$$25y + 25x = xy$$

$$25x = xy - 25y$$

$$25x = y(x - 25)$$

$$y = \frac{25x}{x - 25}$$

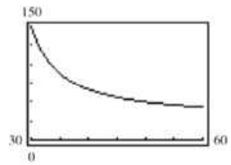
(b) Vertical asymptote: $x = 25$
Horizontal asymptote: $y = 25$

(c)

x	30	35	40	45	50	55	60
y	150	87.5	66.7	56.3	50	45.8	42.9

The results in the table are unexpected. You would expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

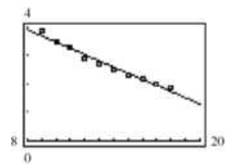
(d)



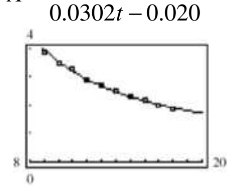
(e) No, it is not possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

91.

(a) $A = -0.2182t + 5.665$



(b) $A = \dfrac{1}{0.0302t - 0.020}$



(c)

Year	1999	2000	2001	2002
Original data, A	3.9	3.5	3.3	2.9
Model from (a), A	3.7	3.5	3.3	3.0
Model from (b), A	4.0	3.5	3.2	2.9

Year	2003	2004	2005	2006
Original data, A	2.7	2.5	2.3	2.2
Model from (a), A	2.8	2.6	2.4	2.2
Model from (b), A	2.7	2.5	2.3	2.2

Year	2007	2008
Original data, A	2.0	1.9
Model from (a), A	2.0	1.7
Model from (b), A	2.0	1.9

Answers will vary.

92.

(a) Domain: $t \ge 0$; the model is valid only after the elk have been released.

(b) At $t = 0$, $P = 10$.

(c) $P(25) \approx 22$ elk

$P(50) \approx 24$ elk

$P(100) \approx 25$ elk

(d) Yes, the horizontal asymptote $y = \dfrac{2.7}{0.1} \approx 27$ is the limit.

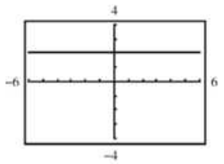
93. False. The graph of a rational function is continuous when the polynomial in the denominator has no real zeros.

94. False. $f(x) = \dfrac{x}{x^3+1}$ crosses its horizontal asymptote $y = 0$ at $x = 0$.

95. $h(x) = \dfrac{6-2x}{3-x} = \dfrac{2(3-x)}{3-x} = 2,\ x \ne 3$

Since *h(x)* is not reduced and $(3-x)$ is a factor of both the numerator and the denominator, $x = 3$ is not a horizontal asymptote.

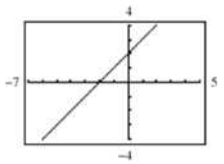
There is a hole in the graph at $x = 3$.



96. $g(x) = \dfrac{x^2+x-2}{x-1}$

$= \dfrac{(x+2)(x-1)}{x-1} = x+2,\ x \ne 1$

Since *g(x)* is not reduced $(x-1)$ is a factor of both the numerator and the denominator, $x = 1$ is not a horizontal asymptote.



There is a hole at $x = 1$.

97. *Horizontal asymptotes*:

If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote. If the degree of the numerator is less than the degree of the denominator, then there is a horizontal asymptote at $y = 0$.

If the degree of the numerator is equal to the degree of the denominator, then there is a horizontal asymptote at the line given by the ratio of the leading coefficients.

Vertical asymptotes:

Set the denominator equal to zero and solve.

Slant asymptotes:

If there is no horizontal asymptote and the degree of the numerator is exactly one greater than the degree of the denominator, then divide the numerator by the denominator. The slant asymptote is the result, not including the remainder.

98.

(a) $y = x+1+\dfrac{a}{x+2}$ has a slant asymptote $y = x+1$ and a vertical asymptote $x = -2$.

$0 = 2+1+\dfrac{a}{2+2}$

$0 = 3+\dfrac{a}{4}$

$\dfrac{a}{4} = -3$

$a = -12$

Hence, $y = x+1-\dfrac{12}{x+2} = \dfrac{x^2+3x-10}{x+2}$.

(b) $y = x-2+\dfrac{a}{x+4}$ has slant asymptote $y = x-2$ and vertical asymptote at $x = -4$. We determine a so that y has a zero at $x = 3$:

$0 = 3-2+\dfrac{a}{3+4} = 1+\dfrac{a}{7} \Rightarrow a = -7$

Hence, $y = x-2+\dfrac{-7}{x+4} = \dfrac{x^2+2x-15}{x+4}$.

99. $\left(\dfrac{x}{8}\right)^{-3} = \left(\dfrac{8}{x}\right)^3 = \dfrac{512}{x^3}$

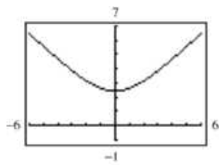
100. $\left(4x^2\right)^{-2} = \dfrac{1}{\left(4x^2\right)^2} = \dfrac{1}{16x^4}$

101. $\dfrac{3^{7/6}}{3^{1/6}} = 3^{6/6} = 3$

102. $\dfrac{x^{-2}\cdot x^{1/2}}{x^{-1}\cdot x^{5/2}} = \dfrac{x\cdot x^{1/2}}{x^2\cdot x^{5/2}} = \dfrac{x^{3/2}}{x^{9/2}}$

$= \dfrac{1}{x^3}$

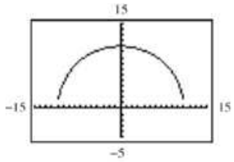
103.



Domain: all x

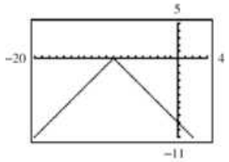
Range: $y \ge \sqrt{6}$

104.



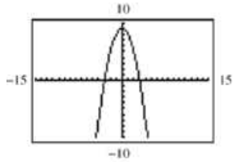
Domain: $-11 \le x \le 11$
Range: $0 \le y \le 11$

105.



Domain: all x
Range: $y \le 0$

106.



Domain: all x
Range: $y \le 9$

107. Answers will vary.

Section 2.8

1. quadratic

2. $S^2 = 0.9688$, the closer the value of S^2 is to 1, the better fit of the model.

3. Quadratic

4. Linear

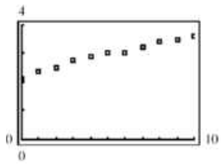
5. Linear

6. Neither

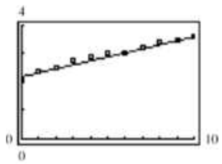
7. Neither

8. Quadratic

9. (a)



(b) Linear model is better.
(c) $y = 0.14x + 2.2$, linear
(d)

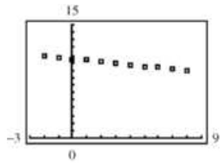


(e)

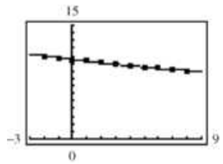
x	0	1	2	3	4
y	2.1	2.4	2.5	2.8	2.9
Model	2.2	2.4	2.5	2.6	2.8

x	5	6	7	8	9	10
y	3.0	3.0	3.2	3.4	3.5	3.6
Model	2.9	3.0	3.2	3.4	3.5	3.6

10. (a)



(b) Quadratic model is better.
(c) $= 0.006x^2 - 0.23x + 10.5$, quadratic
(d)

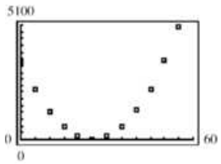


(e)

x	–2	–1	0	1	2
y	11	10.7	10.4	10.3	10.1
Model	11	10.7	10.4	10.3	10.1

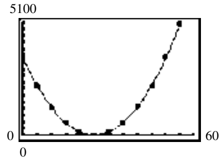
x	3	4	5	6	7	8
y	9.9	9.6	9.4	9.4	9.2	9.0
Model	9.9	9.7	9.5	9.3	9.2	9.0

11. (a)



(b) Quadratic model is better.
(c) $y = 5.55x^2 - 277.5x + 3478$

(d)

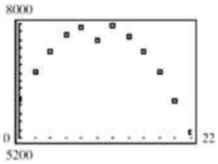


(e)

x	0	5	10	15	20	25
y	3480	2235	1250	565	150	12
Model	3478	2229	1258	564	148	9

x	30	35	40	45	50	55
y	145	575	1275	2225	3500	5010
Model	148	564	1258	2229	3478	5004

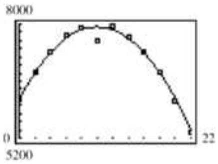
12. (a)



(b) Quadratic model is better.

(c) $y = -17.79x^2 + 354.8x + 6163$

(d)

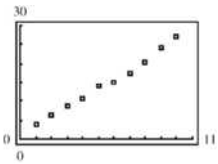


(e)

x	0	2	4	6	8	10
y	6140	6815	7335	7710	7915	7590
Model	6163	6801	7298	7651	7863	7932

x	12	14	16	18	20	22
y	7975	7700	7325	6820	6125	5325
Model	7859	7643	7286	6785	6143	5358

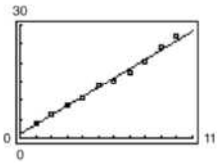
13. (a)



(b) Linear model is better.

(c) $y = 2.48x + 1.1$

(d)

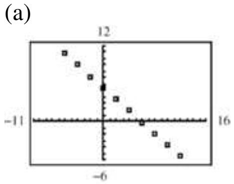


(e)

x	1	2	3	4	5
Actual, y	4.0	6.5	8.8	10.6	13.9
Model, y	3.6	6.1	8.5	11.0	13.5

x	6	7	8	9	10
Actual, y	15.0	17.5	20.1	24.0	27.1
Model, y	16.0	18.5	20.9	23.4	25.9

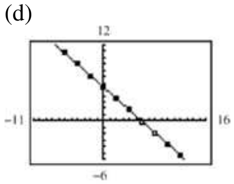
14. (a)



(b) Linear model is better.

(c) $y = -0.89x + 5.3$

(d)

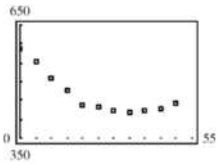


(e)

x	−6	−4	−2	0	2
Actual, y	10.7	9.0	7.0	5.4	3.5
Model, y	10.6	8.9	7.1	5.3	3.5

x	4	6	8	10	12
Actual, y	1.7	−0.1	−1.8	−3.6	−5.3
Model, y	1.7	0.0	−1.8	−3.6	−5.4

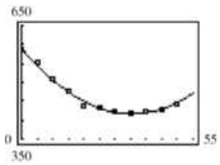
15. (a)



(b) Quadratic is better.

(c) $y = 0.14x^2 - 9.9x + 591$

(d)

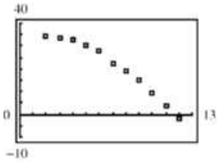


(e)

x	0	5	10	15	20	25
Actual, y	587	551	512	478	436	430
Model, y	591	545	506	474	449	431

x	30	35	40	45	50
Actual, y	424	420	423	429	444
Model, y	420	416	419	429	446

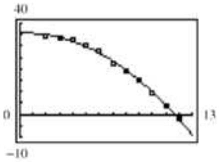
16. (a)



(b) Quadratic model is better.

(c) $y = -0.283x^2 + 0.25x + 35.6$

(d)

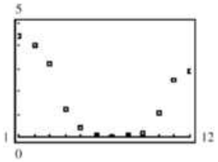


(e)

x	2	3	4	5	6	7
Actual, y	34.3	33.8	32.6	30.1	27.8	22.5
Model, y	35.0	33.8	32.1	29.8	26.9	23.5

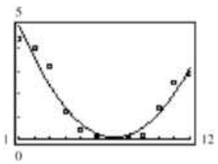
x	8	9	10	11	12
Actual, y	19.1	14,8	9.4	3.7	–1.6
Model, y	19.5	14.9	9.8	4.1	–2.2

17. (a)



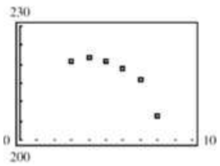
(b) $P = 0.1323t^2 - 1.893t + 6.85$

(c)



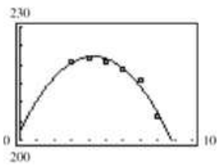
(d) The model's minimum is $H \approx 0.1$ at $t = 7.4$. This corresponds to July.

18. (a)



(b) $S = -1.093t^2 + 9.32t + 202.4$

(c)



(d) To determine when sales will be less than $180 billion, set $S = 180$. Using Quadratic Formula, solve for t.

$-1.093t^2 + 9.32t + 202.4 = 180$

$-1.093t^2 + 9.32t + 22.4 = 0$

$$t = \frac{-9.32 \pm \sqrt{(9.32)^2 - 4(-1.093)(22.4)}}{2(-1.093)}$$

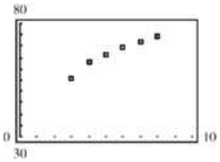
$$t = \frac{-9.32 \pm \sqrt{184.7952}}{-2.186}$$

$t \approx 10.48$ or 2010

Therefore after 2010, sales are expected to be less than $180 billion.

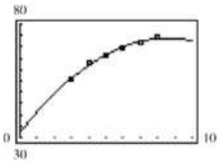
This is not a good model for predicting future years. In fact, by 2019 the model gives negative values for sales.

19. (a)



(b) $P = -0.5638t^2 + 9.690t + 32.17$

(c)



(d) To determine when the percent P of the U.S. population who used the Internet falls below 60%, set P = 60. Using the Quadratic Formula, solve for t.

$-0.5638t^2 + 9.690t + 32.17 = 60$

$-0.5638t^2 + 9.690t - 27.83 = 0$

$$t = \frac{-(9.690) \pm \sqrt{(9.690)^2 - 4(-0.5638)(-27.83)}}{2(-0.5638)}$$

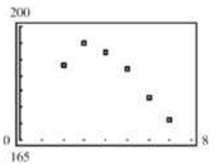
$$t = \frac{-9.690 \pm \sqrt{156.658316}}{-1.1276}$$

$t \approx 13.54$ or 2014

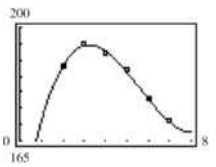
Therefore after 2014, the percent of the U.S. population who use the Internet will fall below 60%.

This is not a good model for predicting future years. In fact, by 2021, the model gives negative values for the percentage.

20. (a)



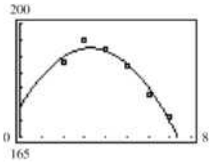
(b)



(c) $H = -1.68t^2 + 11.1t + 174$

$r^2 \approx 0.9567$

(d)



The model fits the data fairly well.

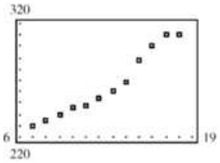
(e) The cubic model is a better fit because its coefficient of determination is closer to 1.

(f)

Year	2008	2009	2010
$H*$	164	159	155
Cubic model	168	172	186
Quadratic model	155	138	117

Answers will vary.

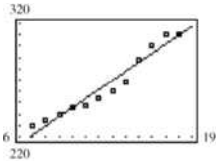
21. (a)



(b) $T = 7.97t + 166.1$

$r^2 \approx 0.9469$

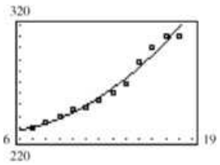
(c)



(d) $T = 0.459t^2 - 3.51t + 232.4$

$r^2 \approx 0.9763$

(e)



(f) The quadratic model is a better fit. Answers will vary.

(g) To determine when the number of televisions, T, in homes reaches 350 million, set T for the model equal to 350 and solve for t.

Linear model:

$$7.97t + 166.1 = 350$$
$$7.97t = 183.9$$
$$t \approx 23.1 \text{ or } 2014$$

Quadratic model:

$$0.459t^2 - 3.51t + 232.4 = 350$$
$$0.459t^2 - 3.51t - 117.6 = 0$$

Using the Quadratic Formula,

$$t = \frac{-(-3.51) \pm \sqrt{(-3.51)^2 - 4(0.459)(-117.6)}}{2(0.459)}$$
$$t = \frac{3.51 \pm \sqrt{228.2337}}{0.918}$$
$$t \approx 20.3 \text{ or } 2011$$

22. True. A quadratic model with a negative leading coefficient opens downward. So, its vertex is at its highest point.

23. True. A quadratic model with a positive leading coefficient opens upward. So, its vertex is at its lowest point.

24. False. A quadratic model could be a better fit for data that are positively correlated.

25. The model is above all data points.

26. Because the data appears more quadratic than linear, a quadratic model would be the better fit. Therefore the S^2-value of 0.9995 must be that of the quadratic model.

27. (a) $(f \circ g)(x) = f(x^2 + 3) = 2(x^2 + 3) - 1 = 2x^2 + 5$

(b) $(g \circ f)(x) = g(2x - 1) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4$

28. (a) $(f \circ g)(x) = f(2x^2 - 1) = 5(2x^2 - 1) + 8 = 10x^2 + 3$

(b) $(g \circ f)(x) = g(5x + 8) = 2(5x + 8)^2 - 1 = 50x^2 + 160x + 127$

29. (a) $(f \circ g)(x) = f(\sqrt[3]{x + 1}) = x + 1 - 1 = x$

(b) $(g \circ f)(x) = g(x^3 - 1) = \sqrt[3]{x^3 - 1 + 1} = x$

30. (a) $(f \circ g)(x) = f(x^3 - 5) = \sqrt[3]{x^3 - 5 + 5} = x$

(b) $(g \circ f)(x) = g(\sqrt[3]{x + 5}) = \left[\sqrt[3]{x + 5}\right]^3 - 5 = x$

31. f is one-to-one.

$$y = 2x + 5$$
$$x = 2y + 5$$
$$2y = x - 5$$
$$y = \frac{(x - 5)}{2} \Rightarrow f^{-1}(x) = \frac{x - 5}{2}$$

32. f is one-to-one.

$$y = \frac{x - 4}{5}$$
$$x = \frac{y - 4}{5}$$
$$5x + 4 = y \Rightarrow f^{-1}(x) = 5x + 4$$

33. f is one-to-one on $[0, \infty)$.

$$y = x^2 + 5,\ x \ge 0$$
$$x = y^2 + 5,\ y \ge 0$$
$$y^2 = x - 5$$
$$y = \sqrt{x - 5} \Rightarrow f^{-1}(x) = \sqrt{x - 5},\ x \ge 5$$

34. f is one-to-one.

$$y = 2x^2 - 3,\ x \ge 0$$
$$x = 2y^2 - 3,\ y \ge 0$$
$$y^2 = \frac{(x+3)}{2}$$
$$y = \sqrt{\frac{x+3}{2}} \Rightarrow f^{-1}(x) = \sqrt{\frac{x+3}{2}} = \frac{\sqrt{2x+6}}{2},\ x \ge -3$$

35. For $1 - 3i$, the complex conjugate is $1 + 3i$.

$$(1-3i)(1+3i) = 1 - 9i^2 = 1 + 9 = 10$$

36. For $-2 + 4i$, the complex conjugate is $-2 - 4i$.

$$(-2+4i)(-2-4i) = 4 - 16i^2 = 4 + 16 = 20$$

37. For $-5i$, the complex conjugate is $5i$.

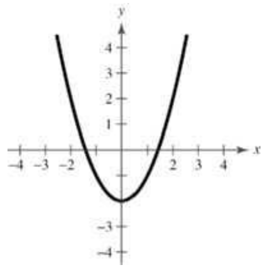
$$(-5i)(5i) = -25i^2 = 25$$

38. For $8i$, the complex conjugate is $-8i$.

$$(8i)(-8i) = -64i^2 = 64$$

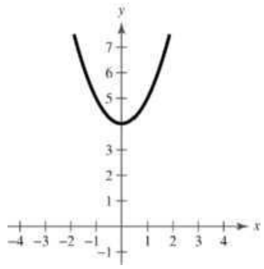
Chapter 2 Review Exercises

1.



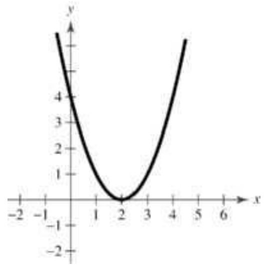
The graph of $y = x^2 - 2$ is a vertical shift two units downward of $y = x^2$.

2.



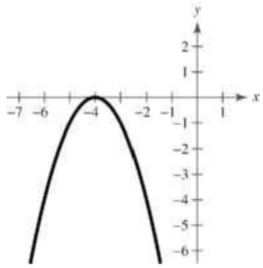
The graph of $y = x^2 + 4$ is a vertical shift four units upward of $y = x^2$.

3.



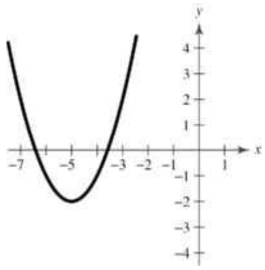
The graph of $y = (x-2)^2$ is a horizontal shift two units to the right of $y = x^2$.

4.



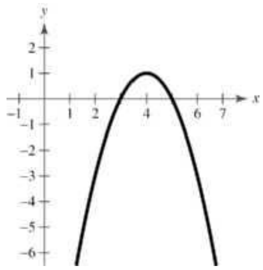
The graph of $y = -(x+4)^2$ is a horizontal shift four units to the left and a reflection in the x-axis of $y = x^2$.

5.



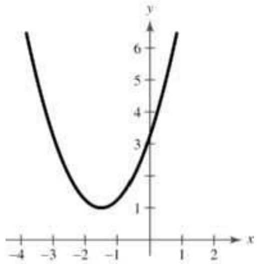
The graph of $y = (x+5)^2 - 2$ is a horizontal shift five units to the left and a vertical shift two units downward of $y = x^2$.

6.



The graph of $y = -(x-4)^2 + 1$ is a horizontal shift four units to the right, a reflection in the x-axis, and a vertical shift one unit upward of $y = x^2$.

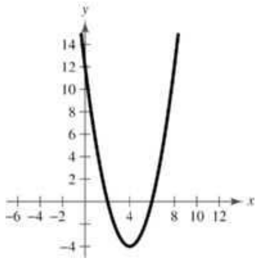
7. The graph of $f(x)=\left(x+3/2\right)^2+1$ is a parabola opening upward with vertex $\left(-\frac{3}{2},\ 1\right)$, and no x-intercepts.



8. The graph of $f(x)=(x-4)^2-4$ is a parabola opening upward with vertex $(4,\ -4)$.

x-intercepts: $(x-4)^2-4=0$

$$(x-4)^2=4$$
$$x-4=\pm 2$$
$$x=6,\ 2$$
$$(6,\ 0), (2,\ 0)$$



9. $f(x)=\frac{1}{3}(x^2+5x-4)$

$$=\frac{1}{3}\left(x^2+5x+\frac{25}{4}-\frac{25}{4}\right)-\frac{4}{3}$$
$$=\frac{1}{3}\left(x+\frac{5}{2}\right)^2-\frac{41}{12}$$

The graph of f is a parabola opening upward with vertex $\left(-\frac{5}{2},\ -\frac{41}{12}\right)$.

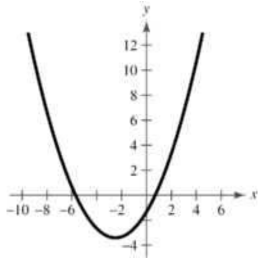
x-intercepts: $\frac{1}{3}\left(x+\frac{5}{2}\right)^2-\frac{41}{12}=0.$

or

$$\frac{1}{3}\left(x^2+5x-4\right)=0$$
$$x^2+5x-4=0$$

Use Quadratic formula.

$$x=\frac{-5\pm\sqrt{41}}{2}$$
$$\left(\frac{-5+\sqrt{41}}{2},\ 0\right), \left(\frac{-5-\sqrt{41}}{2},\ 0\right)$$



10. $f(x)=3x^2-12x+11$

$$=3(x^2-4x)+11$$
$$=3(x^2-4x+4-4)+11$$
$$=3(x-2)^2-1$$

The graph of f is a parabola opening upward with vertex $(2,-1)$.

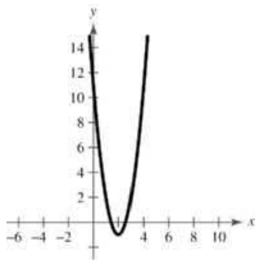
x-intercepts: $3(x-2)^2-1=0$

or

$$3x^2-12x+11=0$$

Use Quadratic Formula.

$$x=\frac{6\pm\sqrt{3}}{3}$$
$$\left(\frac{6+\sqrt{3}}{3},\ 0\right), \left(\frac{6-\sqrt{3}}{3}\right)$$



11. Vertex : $(1,-4)\Rightarrow f(x)=a(x-1)^2-4$

Point: $(2,-3)\Rightarrow -3=a(2-1)^2-4$

$$1=a$$

Thus, $f(x)=(x-1)^2-4.$

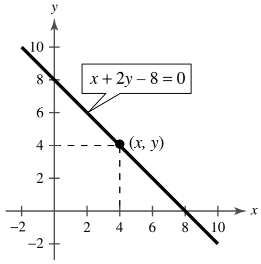
12. Vertex: $(2,\ 3)\Rightarrow y=a(x-2)^2+3$

Point: $(0,\ 2)\Rightarrow 2=a(0-2)^2+3$

$$=4a+3\Rightarrow a=-\frac{1}{4}$$

Thus, $f(x)=-\frac{1}{4}(x-2)^2+3.$

13.



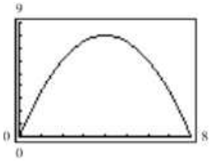
(a) $A = xy$

If $x + 2y - 8 = 0$, then $y = \frac{8 - x}{2}$.

$$A = x\left(\frac{8 - x}{2}\right)$$

$$A = 4x - \frac{1}{2}x^2,\ 0 < x < 8$$

(b)



Using the graph, when $x = 4$, the area is a maximum.

When $x = 4$, $y = \frac{8 - 4}{2} = 2$.

(c) $$A = -\frac{1}{2}x^2 + 4x$$

$$= -\frac{1}{2}(x^2 - 8x)$$

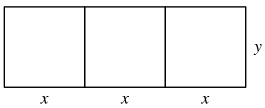
$$= -\frac{1}{2}(x^2 - 8x + 16 - 16)$$

$$= -\frac{1}{2}(x - 4)^2 + 8$$

The graph of A is a parabola opening downward, with vertex $(4,\ 8)$. Therefore, when $x = 4$, the maximum area is 8 square units.

Yes, graphically and algebraically the same dimensions result.

14.



$6x + 4y = 1500$	Total amount of fencing
$A = 3xy$	Area enclosed

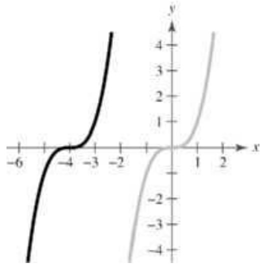
Because $y = \frac{1}{4}(1500 - 6x)$,

$$A = 3x\left(\frac{1}{4}\right)(1500 - 6x)$$

$$= -\frac{9}{2}x^2 + 1125x.$$

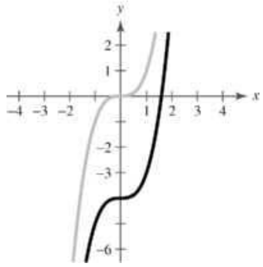
The vertex is at $x = \frac{-b}{2a} = \frac{-1125}{2\left(-\frac{9}{2}\right)} = 125$. Thus $x = 125$ feet, $y = \frac{1}{4}(1500 - 6(125)) = 187.5$, and the dimensions are 375 feet by 187.5 feet.

15.



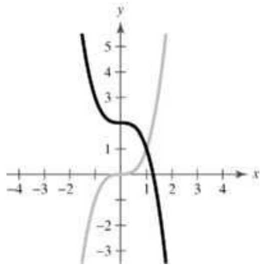
The graph of $f(x) = (x + 4)^3$ is a horizontal shift four units to the left of $y = x^3$.

16.



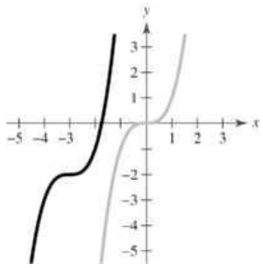
The graph of $f(x) = x^3 - 4$ is a vertical shift four units downward of $y = x^3$.

17.



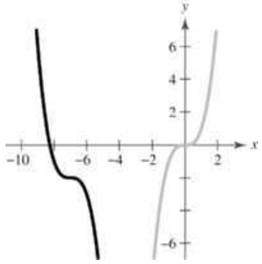
The graph if $f(x) = -x^3 + 2$ is a reflection in the x-axis and a vertical shift two units upward of $y = x^3$.

18.



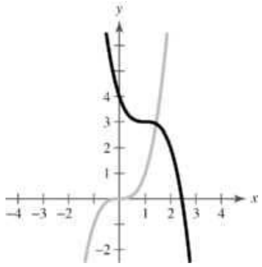
The graph of $f(x) = (x + 3)^3 - 2$ is a horizontal shift three units to the left and a vertical shift two units downward of $y = x^3$.

19.



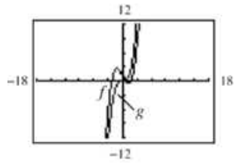
The graph of $f(x) = -(x+7)^3 - 2$ is a horizontal shift seven units to the left, a reflection in the x-axis, and a vertical shift two units downward of $y = x^3$.

20.



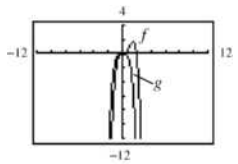
The graph of $f(x) = -(x-1)^3 + 3$ is a horizontal shift one units to the right, a reflection in the x-axis, and a vertical shift three units upward of $y = x^3$.

21. $f(x) = \frac{1}{2}x^3 - 2x + 1;\quad g(x) = \frac{1}{2}x^3$



The graphs have the same end behavior. Both functions are of the same degree and have positive leading coefficients.

22. $f(x) = -x^4 + 2x^3;\quad g(x) = -x^4$



The graphs have the same end behavior. Both functions are of the same degree and have negative leading coefficients.

23. $f(x) = -x^2 + 6x + 9$
The degree is even and the leading coefficient is negative. The graph falls to the left and right.

24. $f(x) = \frac{1}{2}x^3 + 2x$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

25. $f(x) = \frac{3}{4}\left(x^4 + 3x^2 + 2\right)$

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

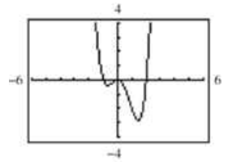
26. $h(x) = -x^5 - 7x^2 + 10x$
The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

27. (a) $x^4 - x^3 - 2x^2 = x^2\left(x^2 - x - 2\right)$

$= x^2(x-2)(x+1) = 0$

Zeros: $x = -1,\ 0,\ 0,\ 2$

(b)



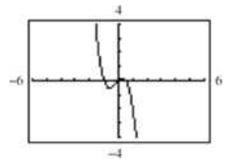
(c) Zeros: $x = -1,\ 0,\ 0,\ 2$; the same

28. (a) $-2x^3 - x^2 + x = -x\left(2x^2 + x - 1\right)$

$= -x(2x-1)(x+1) = 0$

Zeros: $x = -1,\ 0,\ \frac{1}{2}$

(b)

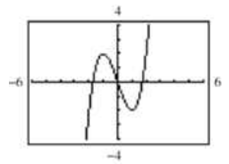


(c) Zeros: $x = -1,\ 0,\ 0.5$; the same

29. (a) $t^3 - 3t = t\left(t^2 - 3\right) = t\left(t + \sqrt{3}\right)\left(t - \sqrt{3}\right) = 0$

Zeros: $t = 0, \pm\sqrt{3}$

(b)



(c) Zeros: $t = 0, \pm 1.732$; the same

30. (a) For the quadratic, $x = \dfrac{-10 \pm \sqrt{100 - 112}}{2}$

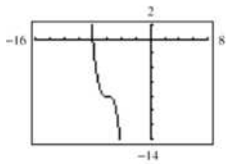
$= -5 \pm \sqrt{3}i.$

Zeros: $-8,\ -5 \pm \sqrt{3}i$

$-(x+6)^3 - 8 = x^3 + 18x^2 + 108x + 224$

$= (x+8)\left(x^2 + 10x + 28\right) = 0$

(b)

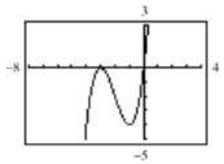


(c) Real zero: $x = -8$

31. (a) $x(x+3)^2 = 0$

Zeros: $x = 0, -3, -3$

(b)

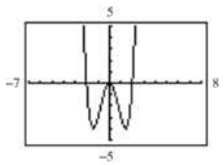


(c) Zeros: $x = -3, \ -3, \ 0$; the same

32. (a)
$$t^4 - 4t^2 = 0$$
$$t^2(t^2 - 4) = 0$$
$$t^2(t+2)(t-2) = 0$$

Zeros: $t = 0, \ 0, \ \pm 2$

(b)



(c) Zeros: $t = 0, \ 0, \ \pm 2$; the same

33. $f(x) = (x+2)(x-1)^2(x-5)$

$= x^4 - 5x^3 - 3x^2 + 17x - 10$

34. $f(x) = (x+3)x(x-1)(x-4)$

$= x^4 - 2x^3 - 11x^2 + 12x$

35. $f(x) = (x-3)(x-2+\sqrt{3})(x-2-\sqrt{3})$

$= x^3 - 7x^2 + 13x - 3$

36. $f(x) = (x+7)(x-4+\sqrt{6})(x-4-\sqrt{6})$

$= x^3 - x^2 - 46x + 70$

37. (a) The degree of *f* is even and the leading coefficient is 1. The graph rises to the left and rises to the right.

(b) $f(x) = x^4 - 2x^3 - 12x^2 + 18x + 27$

$= (x^4 - 12x^2 + 27) - (2x^3 - 18x)$

$= (x^2 - 9)(x^2 - 3) - 2x(x^2 - 9)$

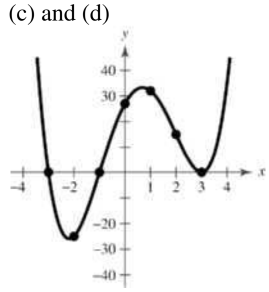
$= (x^2 - 9)(x^2 - 3 - 2x)$

$= (x+3)(x-3)(x^2 - 2x - 3)$

$= (x+3)(x-3)(x-3)(x+1)$

Zeros: $-3, \ 3, \ 3, \ -1$

(c) and (d)

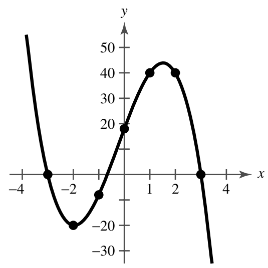


38. (a) The degree of *f* is odd and the leading coefficient is -3. The graph rises to the left and falls to the right.

(b) $f(x) = -3x^3 - 2x^2 + 27x + 18 = -(x-3)(x+3)(3x+2)$

Zeros: $x = \pm 3, \ -\dfrac{2}{3}$

(c) and (d)

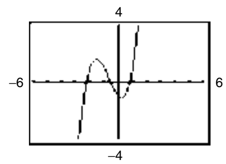


39. $f(x) = x^3 + 2x^2 - x - 1$

(a) $f(-3) < 0, f(-2) > 0 \Rightarrow$ zero in $(-3, \ -2)$

$f(-1) > 0, f(0) < 0 \Rightarrow$ zero in $(-1, \ 0)$

$f(0) < 0, f(1) > 0 \Rightarrow$ zero in $(0, \ 1)$



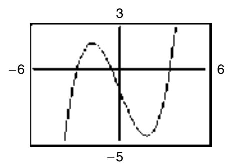
(b) Zeros: $-2.247, \ -0.555, \ 0.802$

40. (a) $f(x) = 0.24x^3 - 2.6x - 1.4$

(b) $f(-3) < 0, f(-2) > 0 \Rightarrow$ zero in $(-3, \ -2)$

$f(-1) > 0, f(0) < 0 \Rightarrow$ zero in $(-1, \ 0)$

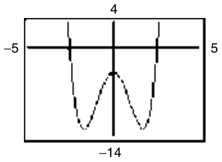
$f(3) < 0, f(4) > 0 \Rightarrow$ zero in $(3, \ 4)$



(b) Zeros: $-2.979, \ -0.554, \ 3.533$

41. $f(x) = x^4 - 6x^2 - 4$

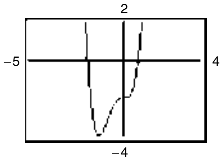
(a) $f(-3) > 0, f(-2) < 0 \Rightarrow$ zero in $(-3, -2)$
$f(2) < 0, f(3) > 0 \Rightarrow$ zero in $(2, 3)$



(b) Zeros: ± 2.570

42. $f(x) = 2x^4 + \frac{7}{2}x^3 - 2$

(a) $f(-2) > 0, f(-1) < 0 \Rightarrow$ zero in $(-2, -1)$
$f(0) < 0, f(1) > 0 \Rightarrow$ zero in $(0, 1)$



(b) Zeros: $-1.897, 0.738$

43.

$$\begin{array}{r|l} & 8x + 5 \\ 3x - 2 & 24x^2 - x - 8 \\ & \underline{24x^2 - 16x} \\ & \quad 15x - 8 \\ & \quad \underline{15x - 10} \\ & \qquad 2 \end{array}$$

Thus, $\frac{24x^2 - x - 8}{3x - 2} = 8x + 5 + \frac{2}{3x - 2}$.

44.

$$\begin{array}{r|l} & \frac{4}{3}x + \frac{8}{9} \\ 3x - 2 & 4x^2 + 0x + 7 \\ & \underline{4x^2 - \frac{8}{3}x} \\ & \quad \frac{8}{3}x + 7 \\ & \quad \underline{\frac{8}{3}x - \frac{16}{9}} \\ & \qquad \frac{79}{9} \end{array}$$

$$\frac{4x^2 + 7}{3x - 2} = \frac{4}{3}x + \frac{8}{9} + \frac{\frac{79}{9}}{3x - 2} = \frac{4}{3}x + \frac{8}{9} + \frac{79}{27x - 18}$$

45.

$$\begin{array}{r|l} & x^2 - 2 \\ x^2 - 1 & x^4 - 3x^2 + 2 \\ & \underline{x^4 - x^2} \\ & \quad -2x^2 + 2 \\ & \quad \underline{-2x^2 + 2} \\ & \qquad 0 \end{array}$$

Thus, $\frac{x^4 - 3x^2 + 2}{x^2 - 1} = x^2 - 2, (x \pm 1)$.

46.

$$\begin{array}{r|l} & 3x^2 + 4 \\ x^2 - 1 & 3x^4 - 3x^2 - 1 \\ & \underline{3x^4 - 3x^2} \\ & \quad 4x^2 - 1 \\ & \quad \underline{4x^2 - 4} \\ & \qquad 3 \end{array}$$

Thus, $\frac{3x^4 + x^2 - 1}{x^2 - 1} = 3x^2 + 4 + \frac{3}{x^2 - 1}$.

47.

$$\begin{array}{r|l} & 5x + 2 \\ x^2 - 3x + 1 & 5x^3 - 13x^2 - x + 2 \\ & \underline{5x^3 - 15x^2 + 5x} \\ & \quad 2x^2 - 6x + 2 \\ & \quad \underline{2x^2 - 6x + 2} \\ & \qquad 0 \end{array}$$

Thus, $\frac{5x^3 - 13x^2 - x + 2}{x^2 - 3x + 1} = 5x + 2$,
$\left(x \neq \frac{1}{2}(3 \pm \sqrt{5})\right)$.

48.

$$\begin{array}{r|l} & x^2 - x + 1 \\ x^2 + 2x & x^4 + x^3 - x^2 + 2x \\ & \underline{x^4 + 2x^3} \\ & \quad -x^3 - x^2 \\ & \quad \underline{-x^3 - 2x^2} \\ & \qquad x^2 + 2x \\ & \qquad \underline{x^2 + 2x} \\ & \qquad\quad 0 \end{array}$$

Thus, $\frac{x^4 + x^3 - x^2 + 2x}{x^2 + 2x} = x^2 - x + 1, (x \neq 0, -2)$.

49.

$$\begin{array}{r@{\,}l}
 & 3x^2 + 5x + 8 \\
2x^2 + 0x - 1 \,\big) & 6x^4 + 10x^3 + 13x^2 - 5x + 2 \\
 & \underline{6x^4 + 0x^3 - 3x^2} \\
 & \quad 10x^3 + 16x^2 - 5x \\
 & \quad \underline{10x^3 + 0x^2 - 5x} \\
 & \qquad 16x^2 - 0 + 2 \\
 & \qquad \underline{16x^2 + 0 - 8} \\
 & \qquad\qquad 10
\end{array}$$

Thus,
$\dfrac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1} = 3x^2 + 5x + 8 + \dfrac{10}{2x^2 - 1}$.

50.

$$\begin{array}{r@{\,}l}
 & x^2 - 3x + 2 \\
x^2 + 2 \,\big) & x^4 - 3x^3 + 4x^2 - 6x + 3 \\
 & \underline{x^4 \quad + 2x^2} \\
 & \quad -3x^3 + 2x^2 - 6x \\
 & \quad \underline{-3x^3 \qquad - 6x} \\
 & \qquad 2x^2 \qquad + 3 \\
 & \qquad \underline{2x^2 \qquad + 4} \\
 & \qquad\qquad -1
\end{array}$$

Thus, $\dfrac{x^4 - 3x^3 + 4x^2 - 6x + 3}{x^2 + 2} = x^2 - 3x + 2 + \dfrac{-1}{x^2 + 2}$.

51.

$$\begin{array}{r|rrrrr}
-2 & 0.25 & -4 & 0 & 0 & 0 \\
 & & -\frac{1}{2} & 9 & -18 & 36 \\
\hline
 & \frac{1}{4} & -\frac{9}{2} & 9 & -18 & 36
\end{array}$$

Thus, $\dfrac{0.25x^4 - 4x^3}{x + 2} = \dfrac{1}{4}x^3 - \dfrac{9}{2}x^2 + 9x - 18 + \dfrac{36}{x + 2}$.

52.

$$\begin{array}{r|rrrr}
5 & 0.1 & 0.3 & 0 & -0.5 \\
 & & 0.5 & 4 & 20 \\
\hline
 & 0.1 & 0.8 & 4 & 19.5
\end{array}$$

Thus, $\dfrac{0.1x^3 + 0.3x^2 - 0.5}{x - 5} = 0.1x^2 + 0.8x + 4 + \dfrac{19.5}{x - 5}$.

53.

$$\begin{array}{r|rrrrr}
\frac{2}{3} & 6 & -4 & -27 & 18 & 0 \\
 & & 4 & 0 & -18 & 0 \\
\hline
 & 6 & 0 & -27 & 0 & 0
\end{array}$$

Thus, $\dfrac{6x^4 - 4x^3 - 27x^2 + 18x}{x - (2/3)} = 6x^3 - 27x,\ x \neq \dfrac{2}{3}$.

54.

$$\begin{array}{r|rrrr}
\frac{1}{2} & 2 & 2 & -1 & 2 \\
 & & 1 & \frac{3}{2} & \frac{1}{4} \\
\hline
 & 2 & 3 & \frac{1}{2} & \frac{9}{4}
\end{array}$$

Thus, $\dfrac{2x^3 + 2x^2 - x + 2}{x - (1/2)} = 2x^2 + 3x + \dfrac{1}{2} + \dfrac{9/4}{x - (1/2)}$.

55.

$$\begin{array}{r|rrrr}
4 & 3 & -10 & 12 & -22 \\
 & & 12 & 8 & 80 \\
\hline
 & 3 & 2 & 20 & 58
\end{array}$$

Thus, $\dfrac{3x^3 - 10x^2 + 12x - 22}{x - 4} = 3x^2 + 2x + 20 + \dfrac{58}{x - 4}$.

56.

$$\begin{array}{r|rrrr}
1 & 2 & 6 & -14 & 9 \\
 & & 2 & 8 & -6 \\
\hline
 & 2 & 8 & -6 & 3
\end{array}$$

Thus, $\dfrac{2x^3 + 6x^2 - 14x + 9}{x - 1} = 2x^2 + 8x - 6 + \dfrac{3}{x - 1}$.

57. (a)

$$\begin{array}{r|rrrrr}
-3 & 1 & 10 & -24 & 20 & 44 \\
 & & -3 & -21 & 135 & -465 \\
\hline
 & 1 & 7 & -45 & 155 & -421 = f(-3)
\end{array}$$

(b)

$$\begin{array}{r|rrrrr}
-2 & 1 & 10 & -24 & 20 & 44 \\
 & & -2 & -16 & 80 & -200 \\
\hline
 & 1 & 8 & -40 & 100 & -156 = f(-2)
\end{array}$$

58. $g(t) = 2t^5 - 5t^4 - 8t + 20$

(a)

$$\begin{array}{r|rrrrrr}
-4 & 2 & -5 & 0 & 0 & -8 & 20 \\
 & & -8 & 52 & -208 & 832 & -3296 \\
\hline
 & 2 & -13 & 52 & -208 & 824 & -3276 = g(-4)
\end{array}$$

(b)

$$\begin{array}{r|rrrrrr}
\sqrt{2} & 2 & -5 & 0 & 0 & -8 & 20 \\
 & & 2\sqrt{2} & 4 - 5\sqrt{2} & 4\sqrt{2} - 10 & 8 - 10\sqrt{2} & -20 \\
\hline
 & 2 & 2\sqrt{2} - 5 & 4 - 5\sqrt{2} & 4\sqrt{2} - 10 & -10\sqrt{2} & 0
\end{array}$$

$= g\left(\sqrt{2}\right)$

59. $f(x) = x^3 + 4x^2 - 25x - 28$

(a)

$$\begin{array}{r|rrrr}
4 & 1 & 4 & -25 & -28 \\
 & & 4 & 32 & 28 \\
\hline
 & 1 & 8 & 7 & 0
\end{array}$$

$(x - 4)$ is a factor.

(b) $x^2 + 8x + 7 = (x + 1)(x + 7)$
Remaining factors: $(x + 1),\ (x + 7)$

(c) $f(x) = (x - 4)(x + 1)(x + 7)$

(d) Zeros: $4,\ -1,\ -7$

60. $f(x) = 2x^3 + 11x^2 - 21x - 90$

(a)

$$\begin{array}{r|rrrr}
-6 & 2 & 11 & -21 & -90 \\
 & & -12 & 6 & 90 \\
\hline
 & 2 & -1 & -15 & 0
\end{array}$$

$(x + 6)$ is a factor.

(b) Remaining factors of
$2x^2 - x - 15$ are $(2x + 5),\ (x - 3)$.

(c) $f(x) = (x + 6)(2x + 5)(x - 3)$

(d) Zeros: $-6,\ -\dfrac{5}{2},\ 3$

61. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

(a)
$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$(x+2)$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$(x-3)$ is a factor.

(b) $x^2 - 3x - 4 = (x-4)(x+1)$

Remaining factors: $(x-4)$, $(x+1)$

(c) $f(x) = (x+2)(x-3)(x-4)(x+1)$

(d) Zeros: $-2, 3, 4, -1$

62. $f(x) = x^4 - 11x^3 + 41x^2 - 61x + 30$

(a)
$$\begin{array}{r|rrrrr} 2 & 1 & -11 & 41 & -61 & 30 \\ & & 2 & -18 & 46 & -30 \\ \hline & 1 & -9 & 23 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -9 & 23 & -15 \\ & & 5 & -20 & 15 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$(x-2)$ and $(x-5)$ are factors.

(b) Remaining factors of $x^2 - 4x + 3$ are $(x-3), (x-1)$.

(c) $f(x) = (x-2)(x-5)(x-3)(x-1)$

(d) Zeros: $1, 2, 3, 5$

63. $f(x) = 4x^3 - 11x^2 + 10x - 3$

Possible rational zeros: $\pm 1, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{1}{2}, \pm \frac{1}{4}$

Zeros: $1, 1, \dfrac{3}{4}$

64. $f(x) = 10x^3 + 21x^2 - x - 6$

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm \frac{1}{2}, \pm \frac{3}{10}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{6}{5}, \pm \frac{3}{2}$

Zeros: $-2, \frac{1}{2}, -\frac{3}{5}$

65. $g(x) = 5x^3 - 6x + 9$ has two variation in sign $\Rightarrow 0$ or 2 positive real zeros.

$g(-x) = -5x^3 + 6x + 9$ has one variation in sign

$\Rightarrow 1$ negative real zero.

66. $f(x) = 2x^5 - 3x^2 + 2x - 1$ has three variations in sign

$\Rightarrow 1$ or 3 positive real zeros.

$f(-x) = -2x^5 - 3x^2 - 2x - 1$ has no variations in sign

$\Rightarrow 0$ negative real zeros.

67.
$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

All entries positive; $x = 1$ is upper bound.

$$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$$

Alternating signs; $x = -\frac{1}{4}$ is lower bound.

Real zero: $x = \dfrac{3}{4}$

68.
$$\begin{array}{r|rrrr} 8 & 2 & -5 & -14 & 8 \\ & & 16 & 88 & 592 \\ \hline & 2 & 11 & 74 & 600 \end{array}$$

All positive $\Rightarrow x = 8$ is upper bound.

$$\begin{array}{r|rrrr} -4 & 2 & -5 & -14 & 8 \\ & & -8 & 52 & -152 \\ \hline & 2 & -13 & 38 & -144 \end{array}$$

Alternating signs $\Rightarrow x = -4$ is lower bound.

Real zeros: $x = -2, \dfrac{1}{2}, 4$

69. $f(x) = 6x^3 + 31x^2 - 18x - 10$

Possible rational zeros:

$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$

Using synthetic division and a graph check $x = \frac{5}{6}$. .

$$\begin{array}{r|rrrr} \frac{5}{6} & 6 & 31 & -18 & -10 \\ & & 5 & 30 & 10 \\ \hline & 6 & 36 & 12 & 0 \end{array}$$

$x = \frac{5}{6}$ is a real zero.

Rewrite in polynomial form and use the Quadratic Formula:

$$6x^2 + 36x + 12 = 0$$
$$6(x^2 + 6x + 2) = 0$$
$$x^2 + 6x + 2 = 0$$
$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{28}}{2} = \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

Real zeros: $x = \frac{5}{6}, -3 \pm \sqrt{7}$

70. $f(x) = x^3 - 1.3x^2 - 1.7x + 0.6$

$= \frac{1}{10}(x-2)(x+1)(10x-3)$

Zeros: $x = -1, 2, \frac{3}{10}$

71. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$

Possible rational zeros:
$\pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

Use a graphing utility to see that $x = -1$ and $x = 3$ are probably zeros.

$$\begin{array}{r|rrrrr} -1 & 6 & -25 & 14 & 27 & -18 \\ & & -6 & 31 & -45 & 18 \\ \hline & 6 & -31 & 45 & -18 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 6 & -31 & 45 & -18 \\ & & 18 & -39 & 18 \\ \hline & 6 & -13 & 6 & 0 \end{array}$$

$$6x^4 - 25x^3 + 14x^2 + 27x - 18 = (x+1)(x-3)(6x^2 - 13x + 6) = (x+1)(x-3)(3x-2)(2x-3)$$

Zeros: $x = -1, 3, \frac{2}{3}, \frac{3}{2}$

72. $f(x) = 5x^4 + 126x^2 + 25 = (5x^2 + 1)(x^2 + 25)$

No real zeros

73. $6 + \sqrt{-25} = 6 + 5i$

74. $-\sqrt{-12} + 3 = -2\sqrt{3}i + 3 = 3 - 2\sqrt{3}i$

75. $-2i^2 + 7i = 2 + 7i$

76. $-i^2 - 4i = 1 - 4i$

77. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i) = 3 + 7i$

78. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\sqrt{2}i$

79. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

80. $(1 + 6i)(5 - 2i) = 5 - 2i + 30i + 12 = 17 + 28i$

81. $(10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2 = -4 - 46i$

82. $i(6 + i)(3 - 2i) = i(18 + 3i - 12i + 2) = i(20 - 9i) = 9 + 20i$

83. $(3 + 7i)^2 + (3 - 7i)^2 = (9 + 42i - 49) + (9 - 42i - 49) = -80$

84. $(4 - i)^2 - (4 + i)^2 = (16 - 8i - 1) - (16 + 8i - 1) = -16i$

85. $\left(\sqrt{-16} + 3\right)\left(\sqrt{-25} - 2\right) = (4i + 3)(5i - 2) = -20 - 8i + 15i - 6 = -26 + 7i$

86. $\left(5 - \sqrt{-4}\right)\left(5 + \sqrt{-4}\right) = (5 - 2i)(5 + 2i) = 25 + 4 = 29$

87. $\sqrt{-9} + 3 + \sqrt{-36} = 3i + 3 + 6i = 3 + 9i$

88. $7 - \sqrt{-81} + \sqrt{-49} = 7 - 9i + 7i = 7 - 2i$

89. $\frac{6+i}{i} = \frac{6+i}{i} \cdot \frac{-i}{-i} = \frac{-6i - i^2}{-i^2} = \frac{-6i + 1}{1} = 1 - 6i$

90. $\frac{4}{-3i} = \frac{-4}{3i} \cdot \frac{-i}{-i} = \frac{4i}{3} = \frac{4}{3}i$

91. $\frac{3+2i}{5+i} \cdot \frac{5-i}{5-i} = \frac{15 + 10i - 3i + 2}{25 + 1} = \frac{17}{26} + \frac{7}{26}i$

92. $\frac{1-7i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2 - 21 - 17i}{4 + 9} = -\frac{19}{13} - \frac{17}{13}i$

93. $x^2 + 16 = 0$
$x^2 = -16$
$x = \pm\sqrt{-16}$
$x = \pm 4i$

94. $x^2 + 48 = 0$
$x^2 = -48$
$x = \pm\sqrt{-48}$
$x = \pm 4\sqrt{3}i$

95. $x^2 + 3x + 6 = 0$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(6)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{-15}}{2}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}i$$

96. $x^2 + 4x + 8 = 0$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(8)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{-16}}{2}$$
$$x = \frac{-4 \pm 4i}{2}$$
$$x = -2 \pm 2i$$

97. $3x^2 - 5x + 6 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-47}}{6}$$

$$x = \frac{5}{6} \pm \frac{\sqrt{47}}{6}i$$

98. $5x^2 - 2x + 4 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(4)}}{2(5)}$$

$$x = \frac{2 \pm \sqrt{-76}}{10}$$

$$x = \frac{2 \pm 2\sqrt{19}i}{10}$$

$$x = \frac{1}{5} \pm \frac{\sqrt{19}}{5}i$$

99. $x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$x + 3 = 0$

$x = -3$

100. $x^2 - 10x + 25 = 0$

$(x-5)^2 = 0$

$x - 5 = 0$

$x = 5$

101. $x^3 + 16x = 0$

$x(x^2 + 16) = 0$

$x = 0 \quad x^2 + 16 = 0$

$x^2 = -16$

$x = \pm 4i$

102. $x^3 + 144x = 0$

$x(x^2 + 144) = 0$

$x = 0 \quad x^2 + 144 = 0$

$x^2 = -144$

$x = \pm 12i$

103. $f(x) = 3x(x-2)^2$

Zeros: 0, 2, 2

104. $f(x) = (x-4)(x+9)^2$

Zeros: 4, −9, −9

105. $h(x) = x^3 - 7x^2 + 18x - 24$

4	1	−7	18	−24
		4	−12	24
	1	−3	6	0

$x = 4$ is a zero. Applying the Quadratic Formula on $x^2 - 3x + 6$,

$$x = \frac{3 \pm \sqrt{9 - 4(6)}}{2} = \frac{3}{2} \pm \frac{\sqrt{15}}{2}i.$$

Zeros: $4, \frac{3}{2} + \frac{\sqrt{15}}{2}i, \ \frac{3}{2} - \frac{\sqrt{15}}{2}i$

$$h(x) = (x-4)\left(x - \frac{3 + \sqrt{15}i}{2}\right)\left(x - \frac{3 - \sqrt{15}i}{2}\right)$$

106. $f(x) = 2x^3 - 5x^2 + 9x + 40$

$= (2x+5)(x^2 - 5x + 8)$

Quadratic Formula: $x = \frac{5 \pm \sqrt{25 - 32}}{2} = \frac{5 \pm \sqrt{7}i}{2}$

Zeros: $-\frac{5}{2}, \ \frac{5}{2} \pm \frac{\sqrt{7}}{2}i$

$$f(x) = (2x+5)\left(x - \frac{5}{2} + \frac{\sqrt{7}}{2}i\right)\left(x - \frac{5}{2} - \frac{\sqrt{7}}{2}i\right)$$

107. $f(x) = 2x^4 - 5x^3 + 10x - 12$

2	2	−5	0	10	−12
		4	−2	−4	12
	2	−1	−2	6	0

$x = 2$ is a zero.

$-\frac{3}{2}$	2	−1	−2	6
		−3	6	−6
	2	−4	4	0

$x = -\frac{3}{2}$ is a zero.

$$f(x) = (x-2)\left(x + \frac{3}{2}\right)(2x^2 - 4x + 4)$$

$$= (x-2)(2x+3)(x^2 - 2x + 2)$$

By the Quadratic Formula, applied to $x^2 - 2x + 2$,

$$x = \frac{2 \pm \sqrt{4 - 4(2)}}{2} = 1 \pm i.$$

Zeros: $2, -\frac{3}{2}, \ 1 \pm i$

$$f(x) = (x-2)(2x+3)(x-1+i)(x-1-i)$$

108. $g(x) = 3x^4 - 4x^3 + 7x^2 + 10x - 4$

$= (x+1)(3x-1)(x^2 - 2x + 4)$

Quadratic Formula: $x = \dfrac{2 \pm \sqrt{4-16}}{2} = 1 \pm \sqrt{3}i$

Zeros: $-1, \dfrac{1}{3}, 1 \pm \sqrt{3}i$

$g(x) = (x+1)(3x-1)(x - 1 + \sqrt{3}i)(x - 1 - \sqrt{3}i)$

109. $f(x) = x^5 + x^4 + 5x^3 + 5x^2$

$= x^2(x^3 + x^2 + 5x + 5)$

$= x^2[x^2(x+1) + 5(x+1)]$

$= x^2(x+1)(x^2+5)$

$= x^2(x+1)(x+\sqrt{5}i)(x-\sqrt{5}i)$

Zeros: $0, 0, -1, \pm\sqrt{5}i$

110. $f(x) = x^5 - 5x^3 + 4x$

$= x(x^4 - 5x^2 + 4)$

$= x(x^2-4)(x^2-1)$

$f(x) = x(x-2)(x+2)(x-1)(x+1)$

Zeros : $0, \pm 1, \pm 2$

111. $f(x) = x^3 - 4x^2 + 6x - 4$

(a) $x^3 - 4x^2 + 6x - 4 = (x-2)(x^2 - 2x + 2)$

By the Quadratic Formula for $x^2 - 2x + 2$,

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(2)}}{2} = 1 \pm i$

Zeros: $2, 1+i, 1-i$

(b) $f(x) = (x-2)(x-1-i)(x-1+i)$

(c) x-intercept: $(2, 0)$

112. (a) $f(x) = x^3 - 5x^2 - 7x + 51$

$= (x+3)(x^2 - 8x + 17)$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4(17)}}{2} = \dfrac{8 \pm \sqrt{-4}}{2}$

$= 4 \pm i$

Zeros: $-3, 4+i, 4-i$

(b) $f(x) = (x+3)(x-4-i)(x-4+i)$

(c) x-intercept: $(-3, 0)$

113. (a) $f(x) = -3x^3 - 19x^2 - 4x + 12$

$= -(x+1)(3x^2 + 16x - 12)$

-1	-3	-19	-4	12
		3	16	-12
	-3	-16	12	0

$3x^2 + 16x - 12 = 0$

$(3x-2)(x+6) = 0$

$3x - 2 = 0 \Rightarrow x = \dfrac{2}{3}$

$x + 6 = 0 \Rightarrow x = -6$

Zeros: $-1, \dfrac{2}{3}, -6$

(b) $f(x) = -(x+1)(3x-2)(x+6)$

(c) x-intercepts: $(-1, 0), (-6, 0), \left(\dfrac{2}{3}, 0\right)$

114. (a) $f(x) = 2x^3 - 9x^2 + 22x - 30$

$= (2x-5)(x^2 - 2x + 6)$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(6)}}{2} = 1 \pm \sqrt{5}i$

Zeros: $\dfrac{5}{2}, 1+\sqrt{5}i, 1-\sqrt{5}i$

(b) $f(x) = (2x-5)(x-1-\sqrt{5}i)(x-1+\sqrt{5}i)$

(c) x-intercepts: $\left(\dfrac{5}{2}, 0\right)$

115. $f(x) = x^4 + 34x^2 + 225$

(a) $x^4 + 34x^2 + 225 = (x^2+9)(x^2+25)$

Zeros: $\pm 3i, \pm 5i$

(b) $(x+3i)(x-3i)(x+5i)(x-5i)$

(c) No x-intercepts

116. (a), (b)

$f(x) = x^4 + 10x^3 + 26x^2 + 10x + 25$

$= (x^2+1)(x^2+10x+25)$

$= (x^2+1)(x+5)^2 = (x+i)(x-i)(x+5)^2$

Zeros: $\pm i, -5, -5$

(c) x-intercept: $(-5, 0)$

117. Since $5i$ is a zero, so is –5i.

$f(x) = (x-4)(x+2)(x-5i)(x+5i)$

$= (x^2 - 2x - 8)(x^2 + 25)$

$= x^4 - 2x^3 + 17x^2 - 50x - 200$

118. Since $2i$ is a zero, so is $-2i$.

$f(x) = (x-2)(x+2)(x-2i)(x+2i)$

$= (x^2-4)(x^2+4)$

$= x^4 - 16$

119. Since $-3+5i$ is a zero, so is $-3-5i$.

$$\begin{aligned} f(x) &= (x-1)(x+4)(x+3-5i)(x+3+5i) \\ &= (x^2+3x-4)((x+3)^2+25) \\ &= (x^2+3x-4)(x^2+6x+34) \\ &= x^4+9x^3+48x^2+78x-136 \end{aligned}$$

120. Since $1+\sqrt{3}i$ is a zero, so is $1-\sqrt{3}i$.

$$\begin{aligned} f(x) &= (x+4)(x+4)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i) \\ &= (x^2+8x+16)((x-1)^2+3) \\ &= (x^2+8x+16)(x^2-2x+4) \\ &= x^4+6x^3+4x^2+64 \end{aligned}$$

121. $f(x) = x^4-2x^3+8x^2-18x-9$

(a) $f(x) = (x^2+9)(x^2-2x-1)$

(b) For the quadratic

$$x^2-2x-1, x = \frac{2\pm\sqrt{(-2)^2-4(-1)}}{2} = 1\pm\sqrt{2}.$$

$$f(x) = \left(x^2+9\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$$

(c) $f(x) = \left(x+3i\right)\left(x-3i\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

122. $f(x) = x^4-4x^3+3x^2+8x-16$

(a) $f(x) = (x^2-x-4)(x^2-3x+4)$

(b) $x = \dfrac{1\pm\sqrt{(-1)^2-4(-4)}}{2} = \dfrac{1}{2}\pm\dfrac{\sqrt{17}}{2}$

$$f(x) = \left(x-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)\left(x^2-3x+4\right)$$

(c) $x = \dfrac{3\pm\sqrt{(-3)^2-4(4)}}{2} = \dfrac{3}{2}\pm\dfrac{\sqrt{7}}{2}i$

$$f(x) = \left(x-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)\left(x-\frac{3}{2}+\frac{\sqrt{7}}{2}i\right)\left(x-\frac{3}{2}-\frac{\sqrt{7}}{2}i\right)$$

123. Zeros: $-2i$, 2i

$(x+2i)(x-2i) = x^2+4$ is a factor.

$f(x) = (x^2+4)(x+3)$

Zeros: $\pm 2i, -3$

124. Zeros: $2+\sqrt{5}i$, $2-\sqrt{5}i$

$\left(x-2-\sqrt{5}i\right)\left(x-2-\sqrt{5}i\right) = \left(x-2\right)^2+5 = x^2-4x+9$ is a factor.

$f(x) = \left(x^2-4x+9\right)\left(2x+1\right)$

Zeros: $x = -\dfrac{1}{2}$, $2\pm\sqrt{5}i$

125. $f(x) = \dfrac{2-x}{x+3}$

(a) Domain: all $x \neq -3$
(b) Not continuous
(c) Horizontal asymptote: $y = -1$
Vertical asymptote: $x = -3$

126. $f(x) = \dfrac{4x}{x-8}$

(a) Domain: all $x \neq 8$
(b) Not continuous
(c) Horizontal asymptote: $y = 4$
Vertical asymptote: $x = 8$

127. $f(x) = \dfrac{2}{x^2-3x-18} = \dfrac{2}{(x-6)(x+3)}$

(a) Domain: all $x \neq 6, \ -3$
(b) Not continuous
(c) Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = 6, \ x = -3$

128. $f(x) = \dfrac{2x^2+3}{x^2+x+3}$

The denominator x^2+x+3 has no real zeros.

(a) Domain: all x
(b) Continuous
(c) Horizontal asymptote: $y = 2$
Vertical asymptote: none

129. $f(x) = \dfrac{7+x}{7-x}$

(a) Domain: all $x \neq 7$
(b) Not continuous
(c) Horizontal asymptote: $y = -1$
Vertical asymptote: $x = 7$

130. $f(x) = \dfrac{6x}{x^2-1} = \dfrac{6x}{(x+1)(x-1)}$

(a) Domain: all $x \neq \pm 1$
(b) Not continuous
(c) Horizontal asymptote: $y = 0$
Vertical asymptote: $x = \pm 1$

131. $f(x) = \dfrac{4x^2}{2x^2-3}$

(a) Domain: all $x \neq \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$
(b) Not continuous
(c) Horizontal asymptote: $y = 2$
Vertical asymptote: $x = \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

132. $f(x) = \dfrac{3x^2-11x-4}{x^2+2}$

(a) Domain: all x
(b) Continuous
(c) Horizontal asymptote: $y = 3$
No vertical asymptote

133. $f(x) = \dfrac{2x-10}{x^2-2x-15} = \dfrac{2(x-5)}{(x-5)(x+3)} = \dfrac{2}{x+3}, \ x \neq 5$

(a) Domain: all $x \neq 5, \ -3$
(b) Not continuous
(c) Vertical asymptote: $x = -3$
(There is a hole at $x = 5.$)
Horizontal asymptote: $y = 0$

134. $f(x) = \dfrac{x^3 - 4x^2}{x^2 + 3x + 2} = \dfrac{x^2(x-4)}{(x+2)(x+1)}$

(a) Domain: all $x \neq -1, -2$
(b) Not continuous
(c) Vertical asymptote: $x = -2, x = -1$
No horizontal asymptotes

135. $f(x) = \dfrac{x-2}{|x|+2}$

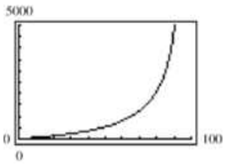
(a) Domain: all real numbers
(b) Continuous
(c) No vertical asymptotes
Horizontal asymptotes: $y = 1, y = -1$

136. $f(x) = \dfrac{2x}{|2x-1|}$

(a) Domain: all $x \neq \dfrac{1}{2}$
(b) Not continuous
(c) Vertical asymptote: $x = \dfrac{1}{2}$
Horizontal asymptotes: $y = 1$ (to the right)
$y = -1$ (to the left)

137. $C = \dfrac{528p}{100-p}, \ 0 \le p < 100$

(a) When $p = 25$, $C = \dfrac{528(25)}{100-25} = \$176$ million.
When $p = 50$, $C = \dfrac{528(50)}{100-50} = \$528$ million.
When $p = 75$, $C = \dfrac{528(75)}{100-75} = \$1584$ million.

(b)


Answers will vary.

(c) No. As $p \to 100$, C approaches infinity.

138. $y = \dfrac{1.568x - 0.001}{6.360x + 1}, \ x > 0$

The moth will be satiated at the horizontal asymptote,
$y = \dfrac{1.568}{6.360} \approx 0.247$ mg.

139. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 1}$
$= \dfrac{(x-4)(x-1)}{(x-1)(x+1)}$
$= \dfrac{x-4}{x+1}, \ x \neq 1$

Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 1$
No slant asymptotes
Hole at $x = 1$

140. $f(x) = \dfrac{2x^2 - 7x + 3}{2x^2 - 3x - 9}$
$= \dfrac{(x-3)(2x-1)}{(x-3)(2x+3)}$
$= \dfrac{2x-1}{2x+3}, \ x \neq 3$

Vertical asymptote: $x = -\dfrac{3}{2}$
Horizontal asymptote: $y = 1$
No slant asymptotes
Hole at $x = 3$

141. $f(x) = \dfrac{3x^2 + 5x - 2}{x + 1}$
$= \dfrac{(3x-1)(x+2)}{x+1}$

Vertical asymptote: $x = -1$
Horizontal asymptote: none
Long division gives:

$$\begin{array}{r} 3x + 2 \\ x+1 \overline{\big) 3x^2 + 5x - 2} \\ \underline{3x^2 + 3x} \\ 2x - 2 \\ \underline{2x + 2} \\ -4 \end{array}$$

Slant asymptote: $y = 3x + 2$

142. $f(x) = \dfrac{2x^2 + 5x + 3}{x - 2}$
$= \dfrac{(2x+3)(x+1)}{x-2}$

Vertical asymptote: $x = 2$
Horizontal asymptote: none
Long division gives:

$$\begin{array}{r} 2x + 9 \\ x-2 \overline{\big) 2x^2 + 5x + 3} \\ \underline{2x^2 - 4x} \\ 9x + 3 \\ \underline{9x - 18} \\ 21 \end{array}$$

Slant asymptote: $y = 2x + 9$

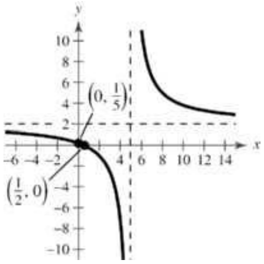
143. $f(x) = \dfrac{2x-1}{x-5}$

Intercepts: $\left(0, \dfrac{1}{5}\right), \left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 2$

x	-4	-1	0	$\frac{1}{2}$	1	6	8
y	1	$\frac{1}{2}$	$\frac{1}{5}$	0	$-\frac{1}{4}$	11	5



144. $f(x) = \dfrac{x-3}{x-2}$

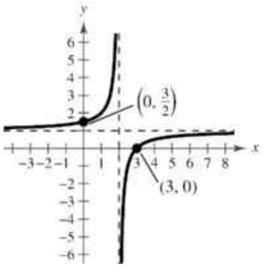
x-intercept: $(3, 0)$

y-intercept: $\left(0, \dfrac{3}{2}\right)$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

x	-1	0	1	3	4	5
y	$\frac{4}{3}$	$\frac{3}{2}$	2	0	$\frac{1}{2}$	$\frac{2}{3}$



145. $f(x) = \dfrac{2x^2}{x^2-4}$

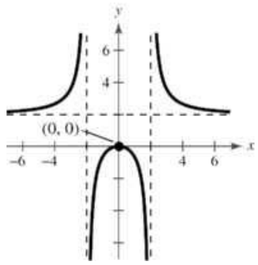
Intercept: $(0, 0)$

y-axis symmetry

Vertical asymptotes: $x = \pm 2$

Horizontal asymptote: $y = 2$

	-6	-4	-1	0	1	4	6
y	$\frac{9}{4}$	$\frac{8}{3}$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{8}{3}$	$\frac{9}{4}$



146. $f(x) = \dfrac{5x}{x^2+1}$

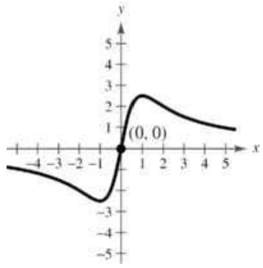
Intercept: $(0, 0)$

Symmetry: origin

Horizontal asymptote: $y = 0$

No vertical asymptotes

x	-3	-2	-1	0	1	2	3
y	$-\frac{3}{2}$	-2	$-\frac{5}{2}$	0	$\frac{5}{2}$	2	$\frac{3}{2}$



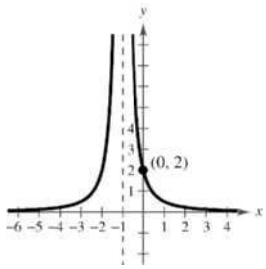
147. $f(x) = \dfrac{2}{(x+1)^2}$

Intercept: $(0, 2)$

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = -1$

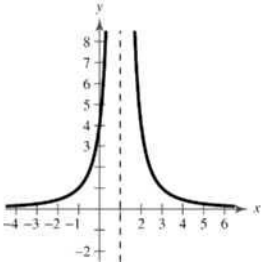
x	-4	-3	-2	0	1	2
y	$\frac{2}{9}$	$\frac{1}{2}$	2	2	$\frac{1}{2}$	$\frac{2}{9}$



148. $h(x) = \dfrac{4}{(x-1)^2}$

y-intercept: (0, 4)
Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 0$

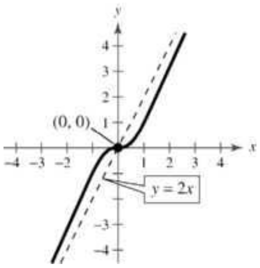
x	-2	-1	0	2	3	4
y	$\frac{4}{9}$	1	4	4	1	$\frac{4}{9}$



149. $f(x) = \dfrac{2x^3}{x^2+1} = 2x - \dfrac{2x}{x^2+1}$

Intercept: (0, 0)
Origin symmetry
Slant asymptote: $y = 2x$

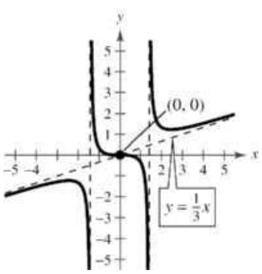
x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



150. $f(x) = \dfrac{x^3}{3x^2-6} = \dfrac{1}{3}x + \dfrac{2x}{3x^2-6} = \dfrac{1}{3}\left[x + \dfrac{2x}{x^2-2}\right]$

Intercepts: (0, 0)
Vertical asymptotes: $x = \pm\sqrt{2}$
Slant asymptote: $y = \dfrac{1}{3}x$

x	-4	-2	-1	0	1	2	4
y	-1.52	-1.33	0.33	0	-0.33	1.33	1.52

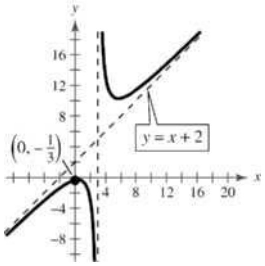


151. $f(x) = \dfrac{x^2-x+1}{x-3} = x + 2 + \dfrac{7}{x-3}$

Intercept: $\left(0, -\dfrac{1}{3}\right)$

Vertical asymptote: $x = 3$
Slant asymptote: $y = x + 2$

x	-4	0	2	4	5
y	-3	$-\frac{1}{3}$	-3	13	10.5

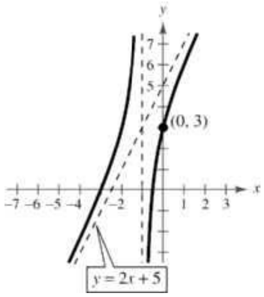


152. $f(x) = \dfrac{2x^2+7x+3}{x+1} = 2x + 5 - \dfrac{2}{x+1}$

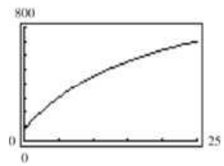
Intercepts: $(0, 3)$, $(-3, 0)$, $\left(-\dfrac{1}{2}, 0\right)$

Vertical asymptote: $x = -1$
Slant asymptote: $y = 2x + 5$

x	-5	-4	-3	-2	0	1
y	$-\frac{9}{2}$	$-\frac{7}{3}$	0	3	3	6



153. (a)



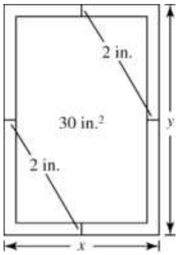
5 years: $N(5) = \dfrac{20(4+3(5))}{1+0.05(5)} = 304$ thousand fish

10 years: $N(10) = \dfrac{20(4+3(10))}{1+0.05(10)} = 453.\overline{3}$ thousand fish

25 years: $N(25) = \dfrac{20(4+3(25))}{1+0.05(25)} = 702.\overline{2}$ thousand fish

(b) The maximum number of fish is $N = 1{,}200{,}000$. The graph of N has a horizontal asymptote at $N = 1200$ or 1,200,000 fish.

154. (a)



(b) $(x-4)(y-4) = 30 \Rightarrow y = 4 + \dfrac{30}{x-4}$

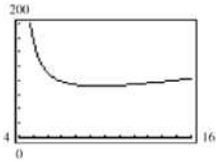
$$\text{Area} = A = xy = x\left[4 + \frac{30}{x-4}\right]$$

$$= x\left[\frac{4x - 16 + 30}{x-4}\right]$$

$$= \frac{2x(2x+7)}{x-4}$$

(c) Domain: $x > 4$

(d)



9.48 by 9.48

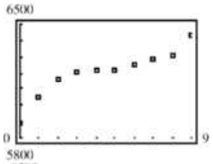
155. Quadratic model

156. Neither

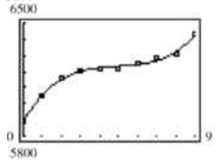
157. Linear model

158. Quadratic model

159. (a)

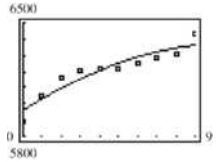


(b)



(c) $S = -3.49t^2 + 76.3t + 5958;\ R^2 \approx 0.8915$

(d)



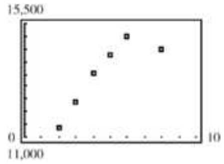
(e) The cubic model is a better fit because it more closely follows the pattern of the data.

(f) For 2012, let $t = 12$.

$$S(12) = 2.520(12)^3 - 37.51(12)^2 + 192.4(12) + 5895$$
$$= 7157 \text{ stations}$$

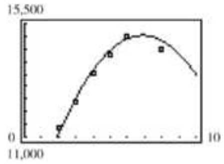
160.

(a)



(b) $S = -161.65t^2 + 2230.9t + 7333$

(c)



The models fits the data well.

(d) Set $S = 11$, and solve for t.

$$-161.65t^2 + 2230.9t + 7333 = 11000$$

$$-161.65t^2 + 2230.9t - 3667 = 0$$

Using the Quadratic Formula,

$t \approx 11.9$ or 2012.

(e) Answers will vary.

161. False. The degree of the numerator is two more than the degree of the denominator.

162. False. A fourth degree polynomial with real coefficients can have at most four zeros. Since $-8i$ and $4i$ are zeros, so are $8i$ and $-4i$.

163. False. $(1+i)+(1-i) = 2$, a real number

164. The domain will still restrict any value that makes the denominator equal to zero.

165. Not every rational function has a vertical asymptote. For example,

$$y = \frac{x}{x^2+1}.$$

166. $\sqrt{-6}\sqrt{-6} \neq \sqrt{(-6)(-6)}$

In fact, $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = -6$.

167. The error is $\sqrt{-4} \neq 4i$. In fact,

$-i\left(\sqrt{-4} - 1\right) = -i(2i-1) = 2 + i$.

168.

(a) $i^{40} = (i^4)^{10} = 1^{10} = 1$

(b) $i^{25} = i(i^{24}) = i(1) = i$

(c) $i^{50} = i^2(i^{48}) = (-1)(1) = -1$

(d) $i^{67} = i^3(i^{64}) = -i(1) = -i$

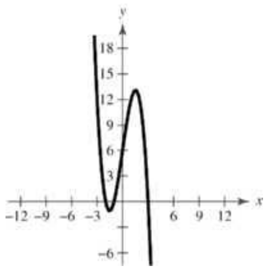
Chapter 2 Test

1. $y = x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x+2)^2 - 1$
Vertex: $(-2, -1)$
$x = 0 \Rightarrow y = 3$
$y = 0 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow (x+3)(x+1) = 0 \Rightarrow x = -1, -3$
Intercepts: $(0, 3)$, $(-1, 0)$, $(-3, 0)$

2. Let $y = a(x-h)^2 + k$. The vertex $(3, -6)$ implies that $y = a(x-3)^2 - 6$. For $(0, 3)$ you obtain
$3 = a(0-3)^2 - 6 = 9a - 6 \Rightarrow a = 1.$
Thus, $y = (x-3)^2 - 6 = x^2 - 6x + 3.$

3. $f(x) = 4x^3 + 4x^2 + x = x(4x^2 + 4x + 1) = x(2x+1)^2$
Zeros: 0 (multiplicity 1)
$-\frac{1}{2}$ (multiplicity 2)

4. $f(x) = -x^3 + 7x + 6$



5.
$$\begin{array}{r} 3x \\ x^2+1 \overline{)\,3x^3 + 0x^2 + 4x - 1} \\ \underline{3x^3 \quad + 3x} \\ x - 1 \end{array}$$

$3x + \frac{x-1}{x^2+1}$

6.

2	2	0	−5	0	−3
		4	8	6	12
	2	4	3	6	9

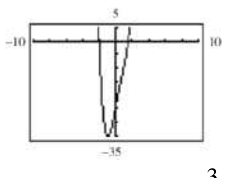
$2x^3 + 4x^2 + 3x + 6 + \frac{9}{x-2}$

7.

−2	3	0	−6	5	−1
		−6	12	−12	14
	3	−6	6	−7	13

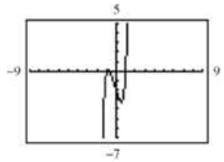
$f(-2) = 13$

8. Possible rational zeros:
$\pm 24, \pm 12, \pm 8, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1, \pm \frac{3}{2}, \pm \frac{1}{2}$



Rational zeros: $-2, \frac{3}{2}$

9. Possible rational zeros: $\pm 2, \pm 1, \pm \frac{2}{3}, \pm \frac{1}{3}$



Rational zeros: $\pm 1, -\frac{2}{3}$

10. $f(x) = x^3 - 7x^2 + 11x + 19$
$= (x+1)(x^2 - 8x + 19)$

For the quadratic,
$x = \frac{8 \pm \sqrt{64 - 4(19)}}{2} = 4 \pm \sqrt{3}i.$
Zeros: $-1, 4 \pm \sqrt{3}\,i$
$f(x) = (x+1)(x - 4 + \sqrt{3}i)(x - 4 - \sqrt{3}i)$

11. $(-8 - 3i) + (-1 - 15i) = -9 - 18i$

12. $\left(10 + \sqrt{-20}\right) - \left(4 - \sqrt{-14}\right) = 6 + 2\sqrt{5}i + \sqrt{14}i = 6 + \left(2\sqrt{5} + \sqrt{14}\right)i$

13. $(2+i)(6-i) = 12 + 6i - 2i + 1 = 13 + 4i$

14. $(4+3i)^2 - (5+i)^2 = (16 + 24i - 9) - (25 + 10i - 1) = -17 + 14i$

15. $\frac{8+5i}{6-i} \cdot \frac{6+i}{6+i} = \frac{48 + 30i + 8i - 5}{36+1} = \frac{43}{37} + \frac{38}{37}i$

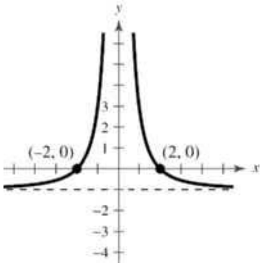
16. $\frac{5i}{2+i} \cdot \frac{2-i}{2-i} = \frac{10i + 5}{4+1} = 1 + 2i$

17. $\frac{(2i-1)}{(3i+2)} \cdot \frac{2-3i}{2-3i} = \frac{6 - 2 + 4i + 3i}{4+9}$
$= \frac{4}{13} + \frac{7}{13}i$

18. $x^2 + 75 = 0$
$x^2 = -75$
$x = \pm\sqrt{-75}$
$= \pm 5\sqrt{3}i$

19. $x^2 - 2x + 8 = 0$
$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(8)}}{2(1)}$
$x = \frac{2 \pm \sqrt{-28}}{2}$
$x = \frac{2 \pm 2\sqrt{7}i}{2}$
$x = 1 \pm \sqrt{7}i$

20.



Vertical asymptote: $x = 0$

Intercepts: $(2, 0)$, $(-2, 0)$

Symmetry: y-axis

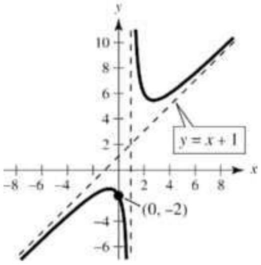
Horizontal asymptote: $y = -1$

21. $g(x) = \dfrac{x^2 + 2}{x - 1} = x + 1 + \dfrac{3}{x - 1}$

Vertical asymptote: $x = 1$

Intercept: $(0, -2)$

Slant asymptote: $y = x + 1$

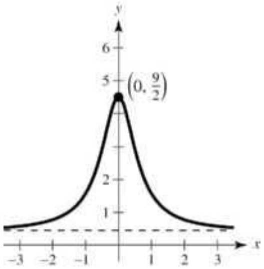


22. $f(x) = \dfrac{2x^2 + 9}{5x^2 + 2}$

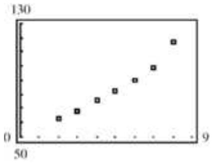
Horizontal asymptote: $y = \dfrac{2}{5}$

y-axis symmetry

Intercept: $\left(0, \dfrac{9}{2}\right)$

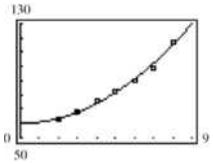


23. (a)



(b) $A = 0.861t^2 + 0.03t + 60.0$

(c)



The model fits the data well.

(d) For 2010, let $t = 10$.

$A(10) = 0.861(10)^2 + 0.03(10) + 60.0$

$= \$146.4$ million

For 2012, let $t = 12$.

$A(12) = 0.861(12)^2 + 0.03(12) + 60.0$

$\approx \$184.3$ billion

(e) Answers will vary.