2.1 Major Concepts

Objective Students will be able to recognize and graph linear and quadratic functions and use these functions to model situations and solve problems.

1. Fill in each blank.

a. $f(x) = 4x^3 - 7x^2 + 3x - 11$ is a polynomial of degree ______________________.

b. $f(x) = 4x - 6$ is a ______________________ function.

c. The rate of change of $f(x) = -13x + 9$ is ______________________.

d. The y-intercept of the graph of $y = \frac{2}{3}x + 4$ is the point ______________________.

e. The equation of a parabola is a ______________________ of degree 2.

f. If $f(x) = mx + b$ is decreasing, m is ______________________ than zero.

2. Fill in each blank.

a. The parabola $f(x) = ax^2 + bx + c$ opens upward if ______________________.

b. The vertex of the parabola $f(x) = a(x - h)^2 + k$ is located at the point ______________________.

c. The vertex of the parabola $f(x) = ax^2 + bx + c$ is located at the point ______________________.

d. The line of symmetry of the parabola $f(x) = ax^2 + bx + c$ is ______________________.

3. Fill in each blank with the phrase "moves left," "moves right," "moves upward," "moves downward," "becomes narrower," "becomes wider," or "remains the same."

a. For values of $a > 0$: As a increases, the parabola $f(x) = ax^2 + c$ ______________________.

b. For values of $b > 0$: As b increases, the vertex of the parabola $f(x) = x^2 + bx$ ______________________ and ______________________.

c. For values of $b > 0$: As b increases, the vertex of the parabola $f(x) = x^2 - bx$ ______________________ and ______________________.

d. For values of $c > 0$: As c increases, the parabola $f(x) = ax^2 + bx + c$ ______________________.

e. For values of $a > 0$: As a increases, the vertex of the parabola $f(x) = a(x - h)^2 + k$ ______________________.

2.1 Group Activity Worksheet

For use with Exercise 59 on page 171. Write the name of the group member assigned to each role.

Calculator operator Recorder

Facilitator Reporter

Beverage Business The Sweet Drip Beverage Co. sells cans of soda pop in machines. It finds that sales average 26,000 cans per month when the cans sell for 50¢ each. For each nickel increase in the price, the sales per month drop by 1000 cans.

a. Determine a function $R(x)$ that models the total revenue realized by Sweet Drip, where x is the number of \$0.05 increases in the price of a can.

b. Find a graph of $R(x)$ that clearly shows a maximum for $R(x)$. Sketch the graph using an appropriate scale and label its maximum point.

c. How much should Sweet Drip charge per can to realize the maximum revenue? What is the maximum revenue?

2.2 Major Concepts

Objective Students will be able to sketch power functions in the form of $f(x) = kx^a$ (where k and a are rational numbers).

1. How do each of the following restrictions affect the graph of $f(x) = k \cdot x^n$ where k is any real number and n is a positive integer?

a. $k < 0$ ______________________

b. $k > 0$ ______________________

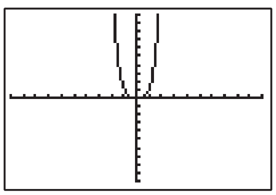
c. n is odd ______________________

d. n is even ______________________

e. $0 < k < 1$ ______________________

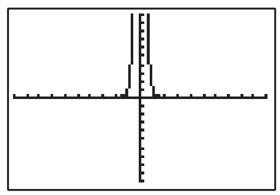
2. Without using a grapher, match the graphs with their functions listed at right.

a. ______________



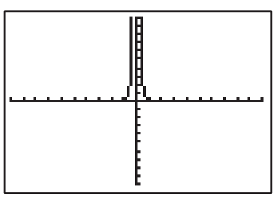
$[-10, 10]$ by $[-10, 10]$

b. ______________



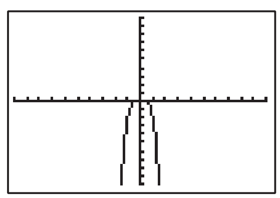
$[-10, 10]$ by $[-10, 10]$

c. ______________



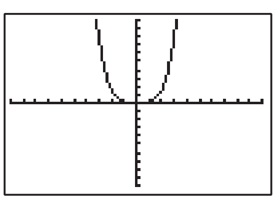
$[-10, 10]$ by $[-10, 10]$

d. ______________



$[-10, 10]$ by $[-10, 10]$

e. ______________



$[-10, 10]$ by $[-10, 10]$

i. $f(x) = x^{-4}$

ii. $f(x) = -2x^4$

iii. $f(x) = \left(\frac{1}{10}\right)x^{-4}$

iv. $f(x) = x^4$

v. $f(x) = \left(\frac{1}{10}\right)x^4$

2.2 Group Activity Worksheet

For use with Exercise 64 on page 184. Write the name of the group member assigned to each role.

n even ____________________ *n* odd, *m* odd ____________________

n odd, *m* even ____________________

Working in a group of three students, investigate the behavior of power functions of the form $f(x) = k \cdot x^{m/n}$, where m and n are positive with no factors in common. Have one group member investigate each of the following cases:

- n is even

- n is odd and m is even

- n is odd and m is odd

For each case decide whether f is even, f is odd, or f is undefined for $x < 0$. Solve graphically and confirm algebraically in a way to convince the rest of your group and your entire class.

2.3 Major Concepts

Objective Students will be able to graph polynomial functions, predict their end behavior, and find their real zeros using a grapher or an algebraic method.

1. Complete the table for polynomial functions $f(x)$.
(*Hint:* You may find it helpful to draw a possible sketch of each function.)

Degree of $f(x)$	Sign of leading coefficient	$f(x) \to$ ____ as $x \to \infty$	$f(x) \to$ ____ as $x \to -\infty$	Maximum number of real zeros	Maximum number of local minimums	Maximum number of local maximums
1	+					0
2	+					
	−			2	0	1
3	+					
4		∞				
	−			5		
6	−					
					3	4
			$-\infty$		5	5
		∞	$-\infty$		3	
	+		∞		4	

2. Let $f(x)$ be a polynomial function where $f(a) = 0$. Fill in each blank with the word "factor," "x-intercept," "zero," or "solution."

a. $x = a$ is a(n) ____________________ of the function $y = f(x)$.

b. $(a, 0)$ is a(n) ____________________ of the graph $y = f(x)$.

c. $(x - a)$ is a(n) ____________________ of $y = f(x)$.

d. $x = a$ is a(n) ____________________ of the equation $f(x) = 0$.

2.3 Group Activity Worksheet

For use with Exercise 86 on page 196. Write the name of the group member assigned to each role.

Calculator Operator	Recorder
Facilitator	Reporter

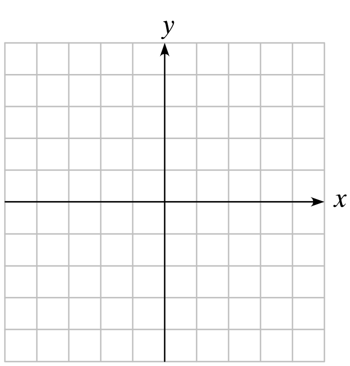
Consider functions of the form $f(x) = x^3 + bx^2 + x + 1$ where b is a nonzero real number.

a. Choose different values for b. Discuss as a group how the value of b affects the graph of the function. Write down your conclusions and explain how you arrived at them.

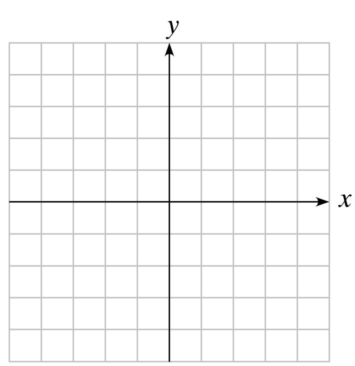
b. After completing (a), have each member of the group (individually) predict what the graphs of $f(x) = x^3 + 15x^2 + x + 1$ and $g(x) = x^3 - 15x^2 + x + 1$ will look like.

c. Compare your predictions with each other. Confirm whether they are correct. Sketch a graph of each function below.

$f(x)$



$g(x)$



2.4 Major Concepts

Objective Students will be able to divide polynomials using long division or synthetic division; apply the Remainder Theorem, the Factor Theorem, and the Rational Zeros Theorem; and find upper and lower bounds for zeros of polynomials.

1. Fill in the blanks to complete the statement of the division algorithm for polynomials.

Let $f(x)$ and $d(x)$ be polynomials such that the degree of f is ____________________

the degree of d, and $d(x) \neq 0$. Then there are unique polynomials $q(x)$ and $r(x)$,

called the ____________________ and ____________________, such that
$f(x) =$ ____________ $\cdot\, q(x) +$ ____________, where either $r(x) =$ ____________

or the degree of r is less than the degree of ____________.

2. Julian used synthetic division to divide $3x^5 - 3x^4 + 4x^2 - 3x + 10$ by $x + 2$ as shown:

$$\begin{array}{r|rrrrr} -2 & 3 & -3 & 4 & -3 & 10 \\ & & -6 & 18 & -44 & 94 \\ \hline & 3 & -9 & 22 & -47 & 104 \end{array}$$

Do you agree with his work? Explain.

3. In order to use synthetic division, the divisor must be a polynomial whose degree is ____________

and leading coefficient is ____________.

4. Suppose you want to divide $3x^4 - 2x^3 + x^2 - 2x + 5$ by $2x + 3$. How might you change this problem so that synthetic division can be used to get the correct answer?

5. Suppose $f(x)$ is a polynomial function. To evaluate $f(15)$ using the Remainder Theorem, you

would divide $f(x)$ by ____________________.

6. Suppose $f(x)$ is a polynomial function. According to the Factor Theorem, if $f(-3) = 0$, then

____________________ is a factor of $f(x)$.

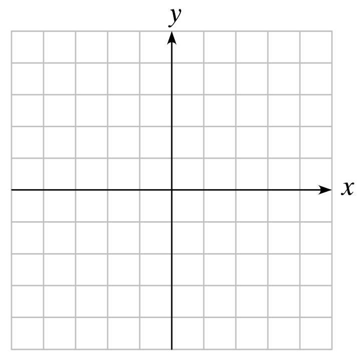
7. Use the division algorithm to explain why the reminder is always a constant when a polynomial in x is divided by $x - c$.

2.4 Group Activity Worksheet

For use with Exercise 79 on page 209. Write the name of the group member assigned to each role.

Calculator Operator	Recorder
Facilitator	Reporter

Use a grapher to graph $f(x) = x^4 + x^3 - 8x^2 - 2x + 7$.
Sketch the graph, showing window dimensions.



a. Use grapher methods to find approximate real number zeros.

b. Identify a list of four linear factors whose product could be called an *approximate factorization of* $f(x)$.

c. Discuss as a group what graphical and numerical methods you could use to show that the factorization from part (b) is reasonable. Write a summary of your conclusion.

2.5 Major Concepts

Objective Students will be able to factor polynomials with real coefficients using factors with complex coefficients.

1. Find the number of complex roots of the equation $x^{99} - 5 = 0$.

2. Find the number of complex zeros of the polynomial function $x^5 + 4x^3 - x^2 + 8x - 3$.

3. Let $f(x)$ be a polynomial function with real coefficients.

If $4 - 2i$ is a zero of $f(x)$, then find another zero of $f(x)$. ______________________

Then name a quadratic factor of $f(x)$. ______________________

4. The number $8 - 5i$ is a zero of multiplicity 4 of the polynomial function $g(x)$, which has real coefficients. What is the minimum possible degree of $g(x)$?

5. The numbers $5, 7, 2 + 3i, 3 + 5i$, and $3 - 5i$ are zeros of the polynomial function $h(x)$, which has real coefficients. What is the minimum possible degree of $h(x)$?

6. Fill in each blank with the word "odd" or "even."

The graph of a polynomial function crosses the x-axis at a zero of __________ multiplicity and is tangent to the x-axis at a zero of __________ multiplicity.

7. Explain how to use the discriminant $b^2 - 4ac$ to determine whether or not a quadratic factor $ax^2 + bx + c$ is irreducible over the reals.

8. Is it possible for a polynomial function of odd degree to have an even number of real number zeros? Explain.

9. Is it possible for a polynomial function of even degree to have an even number of real number zeros? Explain.

2.5 Group Activity Worksheet

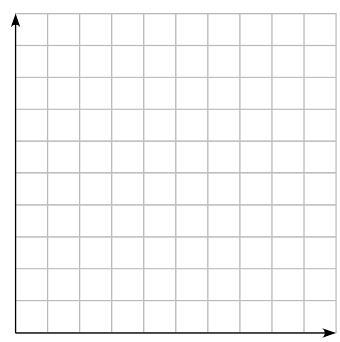
For use with Exercise 51 on page 216. Write the name of the group member assigned to each role.

Calculator Operator	Recorder
Facilitator	Reporter

Sally's distance D from a motion detector is given by the data in the table.

t(sec)	$D(m)$	t(sec)	$D(m)$
0.0	3.36	4.5	3.59
0.5	2.61	5.0	4.15
1.0	1.86	5.5	3.99
1.5	1.27	6.0	3.37
2.0	0.91	6.5	2.58
2.5	1.14	7.0	1.93
3.0	1.69	7.5	1.25
3.5	2.37	8.0	0.67
4.0	3.01		

a. Graph a scatter plot of the data and label the axes. Find a cubic regression equation, and graph it with the scatter plot.



b. As a group, discuss Sally's motion. Write a description of her motion.

c. Use the cubic regression equation to estimate when Sally changes direction. How far is she from the motion detector when she changes direction?

2.6 Major Concepts

Objective Students will be able to describe the graphs of rational functions, identify horizontal and vertical asymptotes, and predict the end behavior of rational functions.

1. Tell whether or not the function is a rational function. Explain.

a. $f(x) = \sqrt{\dfrac{x^2 + 2x - 5}{2x^2 - 3x + 4}}$

b. $g(x) = \dfrac{\dfrac{2x - 3}{x - 3} + \dfrac{5x + 4}{x + 2}}{\dfrac{3x - 8}{x - 3} - \dfrac{x + 10}{x + 2}}$

c. $h(x) = \dfrac{x^3 - 2^x}{3x - 5}$

d. $k(x) = \dfrac{x^3 + \sqrt{5}x + 1}{2x^2 - 3x + \sqrt{17}}$

2. Give an example of a rational function $g(x) = \dfrac{p(x)}{h(x)}$ in which the zeros of $p(x)$ are *not* zeros of $g(x)$.

3. Give an example of a rational function $k(x) = \dfrac{p(x)}{h(x)}$ in which the zeros of $h(x)$ are *not* asymptotes of $k(x)$.

4. Let $f(x) = \dfrac{p(x)}{h(x)}$, where $p(x)$ is a polynomial of degree a and $h(x)$ is a polynomial of degree b. Complete each statement with an equal sign or an inequality sign.

a. $f(x)$ has a horizontal asymptote if a ______ b.

b. The x-axis is an asymptote of $f(x)$ if a ______ b.

c. Polynomial division should be used to find the end behavior asymptote if a ______ b.

2.6 Group Activity Worksheet

For use with Exercise 69 on page 227. Write the name of the group member assigned to each role.

Parts a, c

Parts b, d

Compare algebraically and graphically the functions $f(x) = \dfrac{x^2 - 9}{x - 3}$ and $g(x) = x + 3$.

Answer the following questions and verify that your partner's answers are correct.

a. Are the domains equal?

b. Does f have a vertical asymptote? Explain.

c. Explain why the graphs appear to be identical. Describe their differences.

d. Are the functions identical? Explain why.

2.7 Major Concepts

Objective Students will be able to solve equations involving fractions using both algebraic and graphical techniques and identify extraneous solutions.

1. Clear the equation $\frac{2}{x - 3} - \frac{5}{x} = 1$ of fractions and simplify.

2. What is an *extraneous solution*?

3. Solve the equation $x - 7 = -\frac{10}{x}$ algebraically. Check for extraneous solutions.

4. Solve the equation $\frac{2x}{x + 2} = \frac{5}{x^2 - x - 6} - \frac{1}{x - 3}$ algebraically. Check for extraneous solutions. Confirm your answer graphically.

2.7 Group Activity Worksheet

For use with Exercise 38 on page 234.

Facilitator Recorder

Other Group Members

Designing a Swimming Pool Thompson Recreation, Inc., wants to build a rectangular swimming pool with the top of the pool having surface area 1000 ft^2. The pool is required to have a walk of uniform width 2 ft surrounding it. Let x be the length of one side of the pool.

a. Express the area of the plot of land needed for the pool and surrounding sidewalk as a function of x.

b. Find the dimensions of the plot of land that has the least area. What is the least area?

2.8 Major Concepts

Objective Students will be able to solve inequalities involving polynomials and rational functions by using both algebraic and graphical techniques.

1. Determine four possible polynomial inequalities that could be solved using the graph of $(x - 2)(x^2 + 15)(x + 3)^2$. Give the solution of each.

2. If a 4th degree polynomial $f(x)$ has 2 zeros, each of multiplicity 2, and $\lim_{x\to\infty} f(x) = \infty$, what is the solution of $f(x) < 0$?

3. A student proposed the following method of solving an inequality of the form $\dfrac{p(x)}{h(x)} > 0$:

   Find all real zeros of $p(x)$ and $h(x)$ and arrange them in order: $a_1 < a_2 < \cdots < a_n$. Test a value of x in the first interval, $(-\infty, a_1)$. If this value of x solves the equation, then the solution consists of every second interval beginning with the first one: $(-\infty, a_1)$, (a_2, a_3), (a_4, a_5), and so on. Otherwise, the solution consists of every second interval beginning with the second one: (a_1, a_2), (a_3, a_4) (a_5, a_6), and so on. Do you agree with this method? Explain.

4. Suppose you wish to solve an inequality of the form $\dfrac{p(x)}{h(x)} \geq 0$ or $\dfrac{p(x)}{h(x)} \leq 0$. The first step is to find all real zeros of $p(x)$ and $h(x)$ and arrange them in order: $a_1 < a_2 < \cdots < a_n$. Which of these zeros should be included in the final solution?

2.8 Group Activity Worksheet

For use with Exercise 61 on page 243. Write the name of the group member assigned to each role.

Recorder	Facilitator
Reporter	Calculator Operator

Design a Juice Can Flannery Cannery packs peaches in 0.5-L cylindrical cans.

a. Express the surface area S of the can as a function of the radius x (in cm).

b. Find the dimensions of the can if the surface is less than 900 cm^2.

c. Find the least possible surface area of the can. (*Hint:* Graph the function found in (a) on a grapher and use the trace feature to help find the minimum area.)

Sample Chapter Test

1. Use transformations to explain how to obtain the graph of $y = 2(x - 3)^2 - 5$ from the graph of $y = x^2$.

1. ____________________

____

____

2. Identify the vertex and the line of symmetry of the function $y = 5x^2 - 2x + 7$.

2. Vertex: ____________

Line of symmetry: ____________

3. The Minor Diner serves an all-you-can-eat lunch for \$6. An average of 90 lunches are sold per week. A market survey indicates that for each 25-cent increase in the price, the number of customers per week will decrease by 3.

a. Determine a function $R(x)$ that models the weekly revenue after x 25-cent increases in the price.

b. What price should the Minor Diner charge to maximize the weekly revenue? What is the maximum revenue?

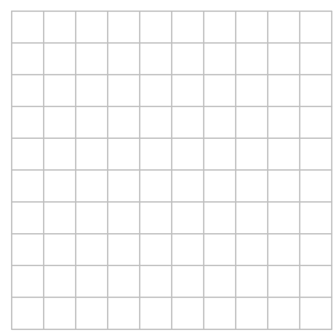
3. a. $R(x) =$ ____________

b. Price: ____________

Revenue: ____________

4. Graph the function $y = x^3 - 9x^2 + 23x + 1$. Choose a viewing window that shows a local maximum, a local minimum, and all the x-intercepts. Make a sketch of the grapher window, and show the viewing-window dimensions.

4.



5. Find all the zeros of $f(x) = 2x^4 - x^3 - 6x^2$ algebraically. Then, describe the end behavior of $f(x)$.

5. ____________________

____

6. Divide the polynomial $3x^4 - 10x^2 + 3x + 5$ by $x - 4$.

6. Quotient: ____________

Remainder: ____________

7. Find a degree 3 polynomial with real coefficients whose leading coefficient is 5 that has -2, 1, and 4 as zeros.

7. ____________________

8. Use synthetic division to show that $c = 3$ is an upper bound for the zeros of $f(x) = 3x^3 - 8x^2 + 4x - 6$. Explain.

8. ______________

9. The table shows the number of employees of the Gizmo Company.

Year	1972	1975	1978	1980	1983	1989
Number of Employees	247	475	658	546	493	605

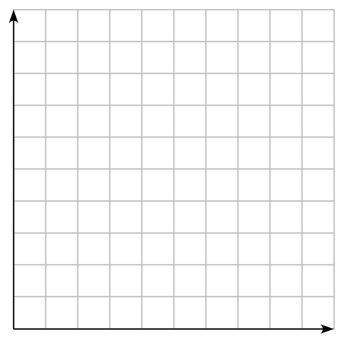
a. Find a cubic regression equation, using $x = 0$ to represent 1970. Round coefficients to the nearest 0.001.

b. Sketch a graph of the cubic regression equation together with a scatter plot of the data.

c. Use the regression equation to predict the number of employees in 1995.

9. a. ______________

b.



c. ______________

10. Use a grapher to approximate all of the function's real zeros. Round to the nearest 0.01.
$f(x) = 3x^6 - 5x^5 - 4x^3 + x^2 + x + 1$.

10. ______________

11. Use the rational zeros test to list *all* the possible candidates for rational zeros of the function $f(x) = 7x^6 - 3x^4 + 5x^2 - 10$.

11. ______________

12. Write a polynomial function with real coefficients in standard form whose zeros include -2, 3, and $4 - 5i$.

12. ______________

13. Find all zeros of $f(x) = x^3 + 9x^2 + 33x + 65$ and write a linear factorization of $f(x)$.

13. ______________

Standardized Test Preparation Questions

1. Write the equation for the parabola with vertex at $(1, 10)$ going through the point $(-1, 0)$.

A. $y = -\frac{5}{2}(x - 1)^2 + 10$ **B.** $y = -\frac{5}{2}(x + 1)^2 + 10$

C. $y = \frac{5}{2}(x + 1)^2$ **D.** $y = \frac{5}{2}(x - 1)^2$

2. Determine the relations between $y = x$, $y = x^2$ and $y = x^{-2}$ on the interval $[0, 1]$.

A. $x^{-2} \leq x \leq x^2$ **B.** $x \leq x^2 \leq x^{-2}$

C. $x^2 \leq x \leq x^{-2}$ **D.** $x^{-2} \leq x^2 \leq x$

3. Find a cubic function with zeros at $x = 2$, $x = 3$, and $x = 0$.

A. $y = x^3 + 2x^2 + 3x + 0$ **B.** $y = (x - 2)^3 + (x - 3)^3 + x^3$

C. $y = x(x - 2)(x - 3)$ **D.** $y = x(x + 2)(x + 3)$

4. Determine the asymptotes of $f(x) = \frac{x + 3}{x^2 + 1}$.

A. Vertical: $x = \pm 1$ Horizontal: $x = -3$

B. Vertical: $x = -3$ Horizontal: $x = \pm 1$

C. No vertical Horizontal: $x = 0$

D. Vertical: $x = -3$ No horizontal

5. Let $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$ and let $k = -2$. Synthetic division tells us:

A. -2 is an upper bound for the real zeros of f

B. -2 is a lower bound for the real zeros of f

C. -2 is neither an upper nor a lower bound for the real zeros of f

D. -2 is a zero of f

6. Find all of the real roots and their multiplicities of the function $f(x) = x^4 - 10x^3 + 23x^2$.

A. $x = 0$, multiplicity 1

B. $x = 0$, multiplicity 2

C. $x = 0$ multiplicity 1, $x = 5 \pm \sqrt{2}$, both with multiplicity 1

D. $x = 0$ multiplicity 2, $x = 5 \pm \sqrt{2}$, both with multiplicity 1

7. A square of side length s is inscribed in a circle. Write the area of the circle as a function of s.

A. $A = \dfrac{\pi s^2}{2}$ **B.** $A = \pi s^2$ **C.** $A = \dfrac{\pi s}{\sqrt{2}}$ **D.** $A = \dfrac{1}{2} s^2$

8. Find average rate of change of $f(x) = e^{x-1}$ between $x = -1$ and $x = 4$, to two decimal places.

A. 19.95 **B.** 3.99 **C.** 10.11 **D.** 7.39

9. The domain of $y = \sqrt{e^{-x}}$ is

A. $(0, \infty)$ **B.** $(-\infty, 0]$ **C.** $[0, \infty)$ **D.** All real numbers

10. Find the equation of the line through $(-1, 3)$ and parallel to the line $2x + 5y = -3$.

A. $y = -\dfrac{2}{5}x + \dfrac{13}{5}$ **B.** $y = 2x + 5$ **C.** $y = -\dfrac{5}{2}x + \dfrac{1}{2}$ **D.** $y = -\dfrac{2}{5}x + \dfrac{17}{5}$

11. Find the vertex of the parabola $y = x^2 + 2cx - 5$ where c is an unknown constant.

A. $(c, -5)$

B. $(-c, -5 - c^2)$

C. $(c, c^2 - 5)$

D. none of the above.

12. Write the algebraic expression for the statement: The volume V of an enclosed ideal gas is directly proportional to the temperature T and inversely proportional to the pressure P.

A. $V = kT - P$ **B.** $V = \dfrac{kT}{kP}$ **C.** $V = k(T - P)$ **D.** $V = \dfrac{kT}{P}$

13. Let $f(x) = (x - a)^3(x + b)^2(x - c)^2$ where a, b, and c are distinct real numbers. Which of the following statements is not true?

A. f has a root at $x = -b$

B. $f(c) = f(a)$

C. The graph of f crosses the x-axis at $x = c$

D. $x = c$ is a zero of multiplicity two.

14. Which of the following is the solution to $\dfrac{1}{(x - 4)^2} \geq 0$?

A. All $x \neq 4$ **B.** All $x \neq -4$

C. $(4, \infty)$ **D.** $(-4, -\infty)$

15. Which of the following are solutions to the equation $x - \frac{4x}{x+2} = \frac{8}{x+2}$?

A. $x = -2$ or $x = 4$

B. only $x = 4$

C. only $x = -2$

D. There are no solutions.

For Problems 16–20, give the exact answer or, if necessary, approximate to two decimal places.

16. The cost of making a sweatshirt is \$13.00 per shirt. Fixed costs for the company are \$10,000 per week. If each sweatshirt sells for \$35.00, what is the minimum number of shirts that must be sold to make a profit each week?

16. ____________________

17. Let $g(x) = \frac{x}{x+1}$ and $f(x) = x^2 - 2$.

Evaluate $f(g(3))$.

17. ____________________

18. A baseball is thrown straight up with an initial velocity of 48 ft/sec from a height of 4 feet. What is the maximum height the ball will reach?

18. ____________________

19. The complex number $2 - i$ is a zero of $f(x) = 2x^3 - 7x^2 + 6x + 5$. Find the remaining zeros of $f(x)$ and write its linear factorization.

19. ____________________

20. How much pure acid must be added to 50 mL of a 25% acid solution to produce a mixture that is 60% acid?

20. ____________________

2 Contest Problems

1. If a is an integer such that $x - 5$ is a factor of $x^3 + ax^2 + 5x - 125$, then a is

A. 1

B. 25

C. 5

D. 0

E. -1

2. Suppose $x = 3 - 4i$ is a zero of the function $f(x) = x^4 - 6x^3 + 29x^2 - 24x + 100$. Which one of the following must be a factor of f?

A. $x^2 - 6x + 25$

B. $x^2 - 6x - 7$

C. $x^2 - 4$

D. $x^2 + 6x + 7$

E. $x^2 + 6x + 16$