Appendix A

Section 1 Radicals and Rational Exponents

Section 1 Exercises

For Exercises 1–6, recall that there are two real nth roots if n is even, and only one if n is odd.

1. $\sqrt{81} = 9$ or -9, since $81 = (\pm 9)^2$

2. $\sqrt[4]{81} = 3$ or -3, since $81 = (\pm 3)^4$

3. $\sqrt[3]{64} = 4$, since $64 = 4^3$

4. $\sqrt[5]{243} = 3$, since $243 = 3^5$

5. $\sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$ or $-\frac{4}{3}$, since $\frac{16}{9} = \left(\pm\frac{4}{3}\right)^2$

6. $\sqrt[3]{-\frac{27}{8}} = -\frac{\sqrt[3]{27}}{\sqrt[3]{8}} = -\frac{3}{2}$

7. $\sqrt{144} = 12$ since $12 \cdot 12 = 144$

8. No real answer — no real number multiplied by itself gives -16

9. $\sqrt[3]{216} = -6$ since $(-6)^3 = -216$

10. $\sqrt[3]{216} = 6$, since $6^3 = 216$

11. $\sqrt[3]{-\frac{64}{27}} = -\frac{4}{3}$, since $\left(-\frac{4}{3}\right)^3 = -\frac{64}{27}$

12. $\sqrt{\frac{64}{25}} = \frac{8}{5}$, since $8^2 = 64$ and $5^2 = 25$

13. 4

14. 5

15. $\frac{5}{2}$ or 2.5

16. $\frac{7}{2}$ or 3.5

17. 729

18. 32

19. $\frac{1}{4}$ or 0.25

20. $\frac{1}{81}$ or 0.012345679

21. -2

22. $-\frac{4}{5}$ or -0.8

23. $\sqrt{1.69} = 1.3$, since $1.3^2 = 1.69$

24. $\sqrt{19.4481} = 4.41$, since $4.41 = 19.4481$

25. $\sqrt[4]{19.4481} = 2.1$, since $2.1^4 = 19.4481$

26. $\sqrt[3]{3.375} = 1.5$, since $1.5^3 = 3.375$

27. $\sqrt{288} = \sqrt{12^2 \cdot 2} = \sqrt{12^2} \cdot \sqrt{2} = 12\sqrt{2}$

28. $\sqrt[3]{500} = \sqrt[3]{5^3 \cdot 4} = \sqrt[3]{5^3} \cdot \sqrt[3]{4} = 5\sqrt[3]{4}$

29. $\sqrt[3]{-250} = \sqrt[3]{(-5)^3 \cdot 2} = -5\sqrt[3]{2}$

30. $\sqrt[4]{192} = \sqrt[4]{2^4} \cdot \sqrt[4]{12} = 2\sqrt[4]{12}$

31. $\sqrt{2x^3y^4} = \sqrt{(xy^2)^2 \cdot 2x}$
$= \sqrt{(xy^2)^2} \cdot \sqrt{2x} = |x|y^2\sqrt{2x}$

32. $\sqrt[3]{-27x^3y^6} = \sqrt[3]{(-3xy^2)^3} = -3xy^2$

33. $\sqrt[4]{3x^8y^6} = \sqrt[4]{(x^2y)^4 \cdot 3y^2}$
$= \sqrt[4]{(x^2y)^4} \cdot \sqrt[4]{3y^2} = |x^2y|\sqrt[4]{3y^2} = x^2|y|\sqrt[4]{3y^2}$

34. $\sqrt[3]{8x^6y^4} = \sqrt[3]{(2x^2y)^3 \cdot y}$
$= \sqrt[3]{(2x^2y)^3} \cdot \sqrt[3]{y} = 2x^2y\sqrt[3]{y}$

35. $\sqrt[5]{96x^{10}} = \sqrt[5]{(2x^2)^5 \cdot 3} = \sqrt[5]{(2x^2)^5} \cdot \sqrt[5]{3} = 2x^2\sqrt[5]{3}$

36. $\sqrt{108x^4y^9} = \sqrt{(6x^2y^4)^2 \cdot 3y}$
$= \sqrt{(6x^2y^4)^2} \cdot \sqrt{3y} = 6x^2y^4\sqrt{3y}$

37. $\frac{4}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{4\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{4\sqrt[3]{4}}{2} = 2\sqrt[3]{4}$

38. $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{25}} = \frac{\sqrt{5}}{5}$

39. $\frac{1}{\sqrt[5]{x^2}} \cdot \frac{\sqrt[5]{x^3}}{\sqrt[5]{x^3}} = \frac{\sqrt[5]{x^3}}{\sqrt[5]{x^5}} = \frac{\sqrt[5]{x^3}}{x}$

40. $\frac{2}{\sqrt[4]{y}} \cdot \frac{\sqrt[4]{y^3}}{\sqrt[4]{y^3}} = \frac{2\sqrt[4]{y^3}}{\sqrt[4]{y^4}} = \frac{2\sqrt[4]{y^3}}{y}$

41. $\sqrt[3]{\frac{x^2}{y}} = \frac{\sqrt[3]{x^2}}{\sqrt[3]{y}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \frac{\sqrt[3]{x^2y^2}}{\sqrt[3]{y^3}} = \frac{\sqrt[3]{x^2y^2}}{y}$

42. $\sqrt[5]{\frac{a^3}{b^2}} = \frac{\sqrt[5]{a^3}}{\sqrt[5]{b^2}} \cdot \frac{\sqrt[5]{b^3}}{\sqrt[5]{b^3}} = \frac{\sqrt[5]{a^3b^3}}{\sqrt[5]{b^5}} = \frac{\sqrt[5]{a^3b^3}}{b}$

43. $[(a + 2b)^2]^{1/3} = (a + 2b)^{2/3}$

44. $(x^2y^3)^{1/5} = (x^2)^{1/5}(y^3)^{1/5} = x^{2/5}y^{3/5}$

45. $2x(x^2y)^{1/3} = 2x(x^2)^{1/3}y^{1/3} = 2x^{3/3}x^{2/3}y^{1/3} = 2x^{5/3}y^{1/3}$

46. $xy(xy^3)^{1/4} = xyx^{1/4}(y^3)^{1/4} = x^{4/4}y^{4/4}x^{1/4}y^{3/4} = x^{5/4}y^{7/4}$

47. $a^{3/4}b^{1/4} = \sqrt[4]{a^3} \cdot \sqrt[4]{b} = \sqrt[4]{a^3b}$

48. $x^{2/3}y^{1/3} = \sqrt[3]{x^2} \cdot \sqrt[3]{y} = \sqrt[3]{x^2y}$

49. $x^{-5/3} = \sqrt[3]{x^{-5}} = \frac{1}{\sqrt[3]{x^5}}$

50. $(xy)^{-3/4} = \sqrt[4]{x^{-3}y^{-3}} = \frac{1}{\sqrt[4]{x^3y^3}}$

51. $\sqrt{\sqrt{2x}} = [(2x)^{1/2}]^{1/2} = (2x)^{1/4} = \sqrt[4]{2x}$

52. $\sqrt{\sqrt[3]{x^2}} = [(3x)^{1/3}]^{1/2} = (3x^2)^{1/6} = \sqrt[6]{3x^2}$

53. $\sqrt[4]{\sqrt{xy}} = [(xy)^{1/2}]^{1/4} = (xy)^{1/8} = \sqrt[8]{xy}$

54. $\sqrt[3]{\sqrt{ab}} = [(ab)^{1/2}]^{1/3} = (ab)^{1/6} = \sqrt[6]{ab}$

55. $\frac{\sqrt[5]{a^2}}{\sqrt[3]{a}} = \frac{a^{2/5}}{a^{1/3}} = a^{2/5 - 1/3} = a^{1/15} = \sqrt[15]{a}$

56. $\sqrt{a}\sqrt[3]{a^2} = a^{1/2}a^{2/3} = a^{1/2 + 2/3} = a^{7/6} = \sqrt[6]{a^7} = a\sqrt[6]{a}$

57. $a^{3/5}a^{1/3}a^{-3/2} = a^{3/5 + 1/3 - 3/2} = a^{-17/30} = \frac{1}{a^{17/30}}$

58. $\sqrt{x^2y^4} = \sqrt{(xy^2)^2} = |xy^2| = |x|y^2$

59. $(a^{5/3}b^{3/4})(3a^{1/3}b^{5/4}) = 3 \cdot a^{5/3}a^{1/3} \cdot b^{3/4}b^{5/4}$
$= 3 \cdot a^{6/3} \cdot b^{8/4} = 3a^2b^2 \ (b \geq 0)$

60. $\left(\frac{x^{1/2}}{y^{2/3}}\right)^6 = \frac{(x^{1/2})^6}{(y^{2/3})^6} = \frac{x^{6/2}}{y^{12/3}} = \frac{x^3}{y^4} \ (x \geq 0)$

61. $\left(\frac{-8x^6}{y^{-3}}\right)^{2/3} = (-8x^6y^3)^{2/3} = (-8)^{2/3}(x^6)^{2/3}(y^3)^{2/3}$
$= [(-8)^2]^{1/3}\,x^{12/3}y^{6/3} = 64^{1/3}x^4y^2 = 4x^4y^2$

62. $\frac{(p^2q^4)^{1/2}}{(27q^3p^6)^{1/3}} = \frac{\sqrt{p^2q^4}}{\sqrt[3]{27q^3p^6}} = \frac{\sqrt{(pq^2)^2}}{\sqrt[3]{(3qp^2)^3}} = \frac{|pq^2|}{3qp^2} = \frac{|p|q^2}{3qp^2}$
$= \frac{q}{3|p|}$

63. $\frac{(x^9y^6)^{-1/3}}{(x^6y^2)^{-1/2}} = \frac{(x^6y^2)^{1/2}}{(x^9y^6)^{1/3}} = \frac{\sqrt{x^6y^2}}{\sqrt[3]{x^9y^6}} = \frac{|x^3y|}{x^3y^2} = \frac{|x^3| \cdot |y|}{x^3y^2}$
$= \frac{1}{|y|} \cdot \frac{|x|}{x} = \frac{|x|}{x|y|}$

64. $\left(\frac{2x^{1/2}}{y^{2/3}}\right)\left(\frac{3x^{-2/3}}{y^{1/2}}\right) = \frac{6x^{1/2-2/3}}{y^{2/3+1/2}} = \frac{6x^{-1/6}}{y^{7/6}} = \frac{6}{x^{1/6}y^{7/6}}$

65. $\sqrt{9x^{-6}y^4} = |3x^{-3}y^2| = 3y^2|x^{-3}| = \frac{3y^2}{|x^3|}$

66. $\sqrt{16y^8z^{-2}} = |4y^4z^{-1}| = 4y^4|z^{-1}| = \frac{4y^4}{|z|}$

67. $\sqrt[4]{\frac{3x^8y^2}{8x^2}}\sqrt[4]{\frac{2 \cdot 3x^8y^2}{2 \cdot 8x^2}} = \frac{\sqrt[4]{6x^6y^2}}{2} = \frac{\sqrt[4]{6x^4x^2y^2}}{2}$
$= \frac{|x|\sqrt[4]{6x^2y^2}}{2}$

68. $\sqrt[5]{\frac{4x^6y}{9x^3}} = \sqrt[5]{\frac{27 \cdot 4x^6y}{27 \cdot 9x^3}} = \sqrt[5]{\frac{108x^6y}{3^5x^3}} = \frac{\sqrt[5]{108x^3y}}{3}$

69. $\sqrt[3]{\frac{4x^2}{y^2}} \cdot \sqrt[3]{\frac{2x^2}{y}} = \sqrt[3]{\frac{(4x^2)(2x^2)}{(y^2)(y)}} = \sqrt[3]{\frac{8x^4}{y^3}} = \frac{2\sqrt[3]{x^4}}{y}$
$= \frac{2x\sqrt[3]{x}}{y}$

70. $\sqrt[5]{9ab^6} \cdot \sqrt[5]{27a^2b^{-1}} = \sqrt[5]{(9ab^6)(27a^2b^{-1})} = \sqrt[5]{243a^3b^5}$
$= 3b\sqrt[5]{a^3}$

71. $3\sqrt{4^2 \cdot 3} - 2\sqrt{6^2 \cdot 3} = 3 \cdot 4\sqrt{3} - 2 \cdot 6\sqrt{3}$
$= 12\sqrt{3} - 12\sqrt{3} = 0$

72. $2\sqrt{5^2 \cdot 7} - 4\sqrt{2^2 \cdot 7} = 2 \cdot 5\sqrt{7} - 4 \cdot 2\sqrt{7}$
$= 10\sqrt{7} - 8\sqrt{7} = 2\sqrt{7}$

73. $\sqrt{x^2 \cdot x} - \sqrt{(2y)^2 \cdot x} = |x|\sqrt{x} - 2|y| \cdot \sqrt{x}$
$= (|x| - 2|y|)\sqrt{x} = (x - 2|y|)\sqrt{x}$ (since the square root is undefined when $x < 0$)

74. $\sqrt{(3x)^2 \cdot 2y} + \sqrt{y^2 \cdot 2y} = 3|x|\sqrt{2y} + |y| \cdot \sqrt{2y}$
$= (3|x| + |y|)\sqrt{2y} = (3|x| + y)\sqrt{2y}$ (since the square root is undefined when $y < 0$)

For #75–82, evaluate each side using a calculator or paper and pencil.

75. $\sqrt{2+6} < \sqrt{2} + \sqrt{6}$ $(2.828\ldots < 3.863\ldots)$

76. $\sqrt{4} + \sqrt{9} > \sqrt{4+9}$ $(5 > 3.605\ldots)$

77. $(3^{-2})^{-1/2} = 3$

78. $(2^{-3})^{1/3} < 2 \left(\frac{1}{2} < 2\right)$

79. $\sqrt[4]{(-2)^4} > -2 \ (2 > -2)$

80. $\sqrt[3]{(-2)^3} = -2$

81. $2^{2/3} < 3^{3/4}$ $(1.587\ldots < 2.279\ldots)$

82. $4^{-2/3} < 3^{-3/4}$ $(0.396\ldots < 0.438\ldots)$

83. $t = 1.1\sqrt{10} \approx 3.48$ sec

84. $t = 0.45\sqrt{200} = 4.5\sqrt{2} \approx 6.36$ sec

85. If n is even, then there are two real nth roots of a (when $a > 0$): $\sqrt[n]{a}$ and $-\sqrt[n]{a}$.

■ Section 2 Polynomials and Factoring

Section 2 Exercises

1. $3x^2 + 2x - 1$; degree 2

2. $-2x^3 + x^2 - 2x + 1$; degree 3

3. $-x^7 + 1$; degree 7

4. $-x^4 + x^2 + x - 3$; degree 4

5. No — cannot have a negative exponent like x^{-1}

6. No — cannot have a variable in the denominator

7. Yes

8. Yes

9. $(x^2 - 3x + 7) + (3x^2 + 5x - 3) = (x^2 + 3x^2)$
$+ (-3x + 5x) + (7 - 3) = 4x^2 + 2x + 4$

10. $(-3x^2 - 5) + (-x^2 - 7x - 12) = (-3x^2 - x^2)$
$-7x + (-5 - 12) = -4x^2 - 7x - 17$

11. $(4x^3 - x^2 + 3x) + (-x^3 - 12x + 3)$
$= (4x^3 - x^3) - x^2 + (3x - 12x) + 3$
$= 3x^3 - x^2 - 9x + 3$

12. $(-y^2 - 2y + 3) + (5y^2 + 3y + 4)$
$= (-y^2 + 5y^2) + (-2y + 3y) + (3 + 4)$
$= 4y^2 + y + 7$

13. $2x(x^2) - 2x(x) + 2x(3) = 2x^3 - 2x^2 + 6x$

14. $y^2(2y^2) + y^2(3y) - y^2(4) = 2y^4 + 3y^3 - 4y^2$

15. $(-3u)(4u) + (-3u)(-1) = -12u^2 + 3u$

16. $(-4v)(2) + (-4v)(-3v^3) = -8v + 12v^4 = 12v^4 - 8v$

17. $2(5x) - x(5x) - 3x^2(5x) = 10x - 5x^2 - 15x^3$
$= -15x^3 - 5x^2 + 10x$

18. $1(2x) - x^2(2x) + x^4(2x) = 2x - 2x^3 + 2x^5$
$= 2x^5 - 2x^3 + 2x$

19. $x(x + 5) - 2(x + 5) = (x)(x) + (x)(5) - (2)(x)$
$- (2)(5) = x^2 + 5x - 2x - 10 = x^2 + 3x - 10$

20. $2x(4x + 1) + 3(4x + 1) = (2x)(4x) + (2x)(1)$
$+ (3)(4x) + (3)(1) = 8x^2 + 2x + 12x + 3$
$= 8x^2 + 14x + 3$

21. $3x(x + 2) - 5(x + 2) = (3x)(x) + (3x)(2)$
$- (5)(x) - (5)(2) = 3x^2 + 6x - 5x - 10$
$= 3x^2 + x - 10$

22. $(2x)^2 - (3)^2 = 4x^2 - 9$

23. $(3x)^2 - (y)^2 = 9x^2 - y^2$

24. $(3)^2 - 2(3)(5x) + (5x)^2 = 9 - 30x + 25x^2$
$= 25x^2 - 30x + 9$

25. $(3x)^2 + 2(3x)(4y) + (4y)^2 = 9x^2 + 24xy + 16y^2$

26. $(x)^3 - 3(x)^2(1) + 3(x)(1)^2 - (1)^3$
$= x^3 - 3x^2 + 3x - 1$

27. $(2u)^3 - 3(2u)^2(v) + 3(2u)(v)^2 - (v)^3$
$= 8u^3 - 3v(4u^2) + 6uv^2 - v^3$
$= 8u^3 - 12u^2v + 6uv^2 - v^3$

28. $(u)^3 + 3(u)^2(3v) + 3(u)(3v)^2 + (3v)^3 = u^3 + 9u^2v$
$+ 3u(9v^2) + 27v^3 = u^3 + 9u^2v + 27uv^2 + 27v^3$

29. $(2x^3)^2 - (3y)^2 = 4x^6 - 9y^2$

30. $(5x^3)^2 - 2(5x^3)(1) + (1)^2 = 25x^6 - 10x^3 + 1$

31. $x^2(x + 4) - 2x(x + 4) + 3(x + 4)$
$= (x^2)(x) + (x^2)(4) - (2x)(x) - (2x)(4)$
$+ (3)(x) + (3)(4)$
$= x^3 + 4x^2 - 2x^2 - 8x + 3x + 12$
$= x^3 + 2x^2 - 5x + 12$

32. $x^2(x - 3) + 3x(x - 3) - 2(x - 3)$
$= (x^2)(x) + (x^2)(-3) + (3x)(x) + (3x)(-3)$
$- (2)(x) - (2)(-3)$
$= x^3 - 3x^2 + 3x^2 - 9x - 2x + 6 = x^3 - 11x + 6$

33.
$$\begin{array}{r} x^2 + x - 3 \\ \underline{x^2 + x + 1} \\ x^4 + x^3 - 3x^2 \qquad\qquad \\ x^3 + x^2 - 3x \qquad \\ \underline{x^2 + x - 3} \\ x^4 + 2x^3 - x^2 - 2x - 3 \end{array}$$

34.
$$\begin{array}{r} 2x^2 - 3x + 1 \\ \underline{x^2 - x + 2} \\ 2x^4 - 3x^3 + x^2 \qquad\qquad \\ - 2x^3 + 3x^2 - x \qquad \\ \underline{4x^2 - 6x + 2} \\ 2x^4 - 5x^3 + 8x^2 - 6x + 2 \end{array}$$

35. $(x)^2 - (\sqrt{2})^2 = x^2 - 2$

36. $(x^{1/2})^2 - (y^{1/2})^2 = x - y$, $x \geq 0$ and $y \geq 0$

37. $(\sqrt{u})^2 - (\sqrt{v})^2 = u - v$, $u \geq 0$ and $v \geq 0$

38. $(x^2)^2 - (\sqrt{3})^2 = x^4 - 3$

39. $x(x^2 + 2x + 4) - 2(x^2 + 2x + 4) = (x)(x^2)$
$+ (x)(2x) + (x)(4) - (2)(x^2) - (2)(2x) - (2)(4)$
$= x^3 + 2x^2 + 4x - 2x^2 - 4x - 8 = x^3 - 8$

40. $x(x^2 - x + 1) + 1(x^2 - x + 1) = (x)(x^2) +$
$(x)(-x) + (x)(1) + (1)(x^2) + (1)(-x) + (1)(1)$
$= x^3 - x^2 + x + x^2 - x + 1 = x^3 + 1$

41. $5(x - 3)$

42. $5x(x^2 - 4)$

43. $yz(z^2 - 3z + 2)$

44. $(x + 3)(2x - 5)$

45. $z^2 - 7^2 = (z + 7)(z - 7)$

46. $(3y)^2 - 4^2 = (3y + 4)(3y - 4)$

47. $8^2 - (5y)^2 = (8 + 5y)(8 - 5y)$

48. $4^2 - (x + 2)^2 = [4 + (x + 2)]$
$[(4 - (x + 2)] = (6 + x)(2 - x)$

49. $y^2 + 2(y)(4) + 4^2 = (y + 4)^2$

50. $(6y)^2 + 2(6y)(1) + 1^2 = (6y + 1)^2$

51. $(2z)^2 - 2(2z)(1) + 1^2 = (2z - 1)^2$

52. $(3z)^2 - 2(3z)(4) + 4^2 = (3z - 4)^2$

53. $y^3 - 2^3 = (y - 2)[y^2 + (y)(2) + 2^2]$
$= (y - 2)(y^2 + 2y + 4)$

54. $z^3 + 4^3 = (z + 4)[z^2 - (z)(4) + 4^2]$
$= (z + 4)(z^2 - 4z + 16)$

55. $(3y)^3 - 2^3 = (3y - 2)[(3y)^2 + (3y)(2) + 2^2]$
$= (3y - 2)(9y^2 + 6y + 4)$

56. $(4z)^3 + 3^3 = (4z + 3)[(4z)^2 - (4z)(3) + 3^2]$
$= (4z + 3)(16z^2 - 12z + 9)$

57. $1^3 - x^3 = (1 - x)[1^2 + (1)(x) + x^2]$
$= (1 - x)(1 + x + x^2)$

58. $3^3 - y^3 = (3 - y)[3^2 + (3)(y) + y^2]$
$= (3 - y)(9 + 3y + y^2)$

59. $(x + 2)(x + 7)$

60. $(y - 5)(y - 6)$

61. $(z - 8)(z + 3)$

62. $(2t + 1)(3t + 1)$

63. $(2u - 5)(7u + 1)$

64. $(2v + 3)(5v + 4)$

65. $(3x + 5)(4x - 3)$

66. $(x - y)(2x - y)$

67. $(2x + 5y)(3x - 2y)$

68. $(3x + 7y)(5x - 2y)$

69. $(x^3 - 4x^2) + (5x - 20) = x^2(x - 4) + 5(x - 4)$
$= (x - 4)(x^2 + 5)$

70. $(2x^3 - 3x^2) + (2x - 3) = x^2(2x - 3) + 1(2x - 3)$
$= (2x - 3)(x^2 + 1)$

71. $(x^6 - 3x^4) + (x^2 - 3) = x^4(x^2 - 3) + 1(x^2 - 3)$
$= (x^2 - 3)(x^4 + 1)$

72. $(x^6 + 2x^4) + (x^2 + 2) = x^4(x^2 + 2) + 1(x^2 + 2)$
$= (x^2 + 2)(x^4 + 1)$

73. $(2ac + 6ad) - (bc + 3bd) = 2a(c + 3d) - b(c + 3d)$
$= (c + 3d)(2a - b)$

74. $(3uw + 12uz) - (2vw + 8vz) = 3u(w + 4z)$
$-2v(w + 4z) = (w + 4z)(3u - 2v)$

75. $x(x^2 + 1)$

76. $y(4y^2 - 20y + 25) = y[(2y)^2 - 2(2y)(5) + 5^2]$
$= y(2y - 5)^2$

77. $2y(9y^2 + 24y + 16) = 2y[(3y)^2 + 2(3y)(4)$
$+ 4^2] = 2y(3y + 4)^2$

78. $2x(x^2 - 8x + 7) = 2x(x - 1)(x - 7)$

79. $y(16 - y^2) = y(4^2 - y^2) = y(4 + y)(4 - y)$

80. $3x(x^3 + 8) = 3x(x^3 + 2^3)$
$= 3x(x + 2)[x^2 - (x)(2) + 2^2]$
$= 3x(x + 2)(x^2 - 2x + 4)$

81. $y(5 + 3y - 2y^2) = y(1 + y)(5 - 2y)$

82. $z(1 - 8z^3) = z[1^3 - (2z)^3]$
$= z(1 - 2z)[1^2 + (1)(2z) + (2z)^2]$
$= z(1 - 2z)(1 + 2z + 4z^2)$

83. $2[(5x + 1)^2 - 9] = 2[(5x + 1)^2 - 3^2]$
$= 2[(5x + 1) + 3][(5x + 1) - 3]$
$= 2(5x + 4)(5x - 2)$

84. $5[(2x - 3)^2 - 4] = 5[(2x - 3)^2 - 2^2]$
$= 5[(2x - 3) + 2][(2x - 3) - 2]$
$= 5(2x - 1)(2x - 5)$

85. $2(6x^2 + 11x - 10) = 2(2x + 5)(3x - 2)$

86. $(x + 5y)(3x - 2y)$

87. $(2ac + 4ad) - (2bd + bc) = 2a(c + 2d) - b(2d + c)$
$= (c + 2d)(2a - b) = (2a - b)(c + 2d)$

88. $(6ac + 4bc) - (2bd + 3ad)$
$= 2c(3a + 2b) - d(2b + 3a) = (3a + 2b)(2c - d)$

89. $(x^3 - 3x^2) - (4x - 12) = x^2(x - 3) - 4(x - 3)$
$= (x - 3)(x^2 - 4) = (x - 3)(x + 2)(x - 2)$

90. $x(x^3 - 4x^2 - x + 4)$
$= x(x - 1)(x^2 - 3x - 4)$
$= x(x - 1)(x + 1)(x - 4)$

91. $(2ac + bc) - (2ad + bd)$
$= c(2a + b) - d(2a + b) = (c - d)(2a + b)$
Neither of the groupings $(2ac - bd)$ and $(-2ad + bc)$ has a common factor to remove.

Section 3 Fractional Expressions

Section 3 Exercises

1. $\frac{5}{9} + \frac{10}{9} = \frac{5 + 10}{9} = \frac{15}{9} = \frac{5}{3}$

2. $\frac{17}{32} - \frac{9}{32} = \frac{17 - 9}{32} = \frac{8}{32} = \frac{1}{4}$

3. $\frac{20}{21} \cdot \frac{9}{22} = \frac{20 \cdot 9}{21 \cdot 22} = \frac{180}{462} = \frac{30}{77}$

4. $\frac{33}{25} \cdot \frac{20}{77} = \frac{33 \cdot 20}{25 \cdot 77} = \frac{660}{1925} = \frac{12}{35}$

5. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4} = \frac{2 \cdot 5}{3 \cdot 4} = \frac{10}{12} = \frac{5}{6}$

6. $\frac{9}{4} \div \frac{15}{10} = \frac{9}{4} \div \frac{3}{2} = \frac{9}{4} \cdot \frac{2}{3} = \frac{9 \cdot 2}{4 \cdot 3} = \frac{18}{12} = \frac{3}{2}$

7. The LCD of the denominators is $2 \cdot 7 \cdot 3 \cdot 5 = 210$:
$\frac{1}{14} + \frac{4}{15} - \frac{5}{21} = \frac{15}{210} + \frac{56}{210} - \frac{50}{210}$
$= \frac{15 + 56 - 50}{210} = \frac{21}{210} = \frac{1}{10}$

8. The LCD of the denominators is $2 \cdot 3 \cdot 5 \cdot 7 = 210$:
$\frac{1}{6} + \frac{6}{35} - \frac{4}{15} = \frac{35}{210} + \frac{36}{210} - \frac{56}{210}$
$= \frac{35 + 36 - 56}{210} = \frac{15}{210} = \frac{1}{14}$

9. No values are restricted, so the domain is all real numbers.

10. No values are restricted, so the domain is all real numbers.

11. The value under the radical must be nonnegative, so $x - 4 \ge 0$: $x \ge 4$ or $[4, \infty)$.

12. The value under the radical must be positive, so $x + 3 > 0$: $x > -3$ or $(-3, \infty)$.

13. The denominator cannot be 0, so $x^2 + 3x \ne 0$ or $x(x + 3) \ne 0$. Then $x \ne 0$ and $x + 3 \ne 0$: $x \ne 0$ and $x \ne -3$.

14. The denominator cannot be 0, so $x^2 - 4 \ne 0$ or $(x + 2)(x - 2) \ne 0$. Then $x + 2 \ne 0$ and $x - 2 \ne 0$: $x \ne -2$ and $x \ne 2$.

15. The denominator cannot be 0, so $x - 1 \ne 0$, or $x \ne 1$. Then $x \ne 2$ and $x \ne 1$.

16. The denominator cannot be 0, so $x - 2 \ne 0$, or $x \ne 2$. Then $x \ne 2$ and $x \ne 0$.

17. $x^{-1} = \frac{1}{x}$ and the denominator cannot be 0, so $x \ne 0$.

18. $x(x + 1)^{-2} = \frac{x}{(x + 1)^2}$ and the denominator cannot be 0, so $(x + 1)^2 \ne 0$ or $x + 1 \ne 0$: $x \ne -1$.

19. The denominator is $12x^3 = (3x)(4x^2)$, so the new numerator is $2(4x^2) = 8x^2$.

20. The numerator is $15y = (5)(3y)$, so the new denominator is $(2y)(3y) = 6y^2$.

21. The numerator is $x^2 - 4x = (x - 4)(x)$, so the new denominator is $(x)(x) = x^2$.

22. The denominator is $x^2 - 4 = (x - 2)(x + 2)$, so the new numerator is $x(x - 2) = x^2 - 2x$.

23. The denominator is $x^2 + 2x - 8 = (x + 4)(x - 2)$, so the new numerator is $(x + 3)(x + 4) = x^2 + 7x + 12$.

24. The numerator is $x^2 - x - 12 = (x - 4)(x + 3)$, so the new denominator is $(x + 5)(x + 3) = x^2 + 8x + 15$.

25. The numerator is $x^2 - 3x = x(x - 3)$, so the new denominator is $x(x^2 + 2x)$ or $x^3 + 2x^2$.

26. The denominator is $x^2 - 9 = (x + 3)(x - 3)$, so the new numerator is
$(x + 3)(x^2 + x - 6) = x(x^2 + x - 6)$
$+ 3(x^2 + x - 6) = x^3 + x^2 - 6x + 3x^2$
$+ 3x - 18 = x^3 + 4x^2 - 3x - 18$.

27. $(x - 2)(x + 7)$ cancels out during simplification; the restriction indicates that the values 2 and -7 were not valid in the original expression.

28. $(x + 1)(x - 2)$ cancels out during simplification; the restriction indicates that the values -1 and 2 were not valid in the original expression.

29. No factors were removed from the expression; we can see by inspection that $\frac{2}{3}$ and 5 are not valid.

30. x cancels out during simplification; the restriction indicates that 0 was not valid in the original expression.

31. $(x - 3)$ ends up in the numerator of the simplified expression; the restriction reminds us that it began in the denominator so that 3 is not allowed.

32. When $a = b$ in the original, we get division by 0; this is not apparent in the simplified expression because we canceled a factor of $b - a$.

33. $\frac{3x(6x^2)}{3x(5)} = \frac{6x^2}{5}, x \ne 0$

34. $\frac{3y^2(25)}{3y^2(3y^2)} = \frac{25}{3y^2}$

35. $\frac{x(x^2)}{x(x - 2)} = \frac{x^2}{x - 2}, x \ne 0$

36. $\frac{2y(y + 3)}{4(y + 3)} = \frac{y}{2}, y \ne -3$

37. $\frac{z(z - 3)}{(3 - z)(3 + z)} = -\frac{z}{z + 3}, z \ne 3$

38. $\frac{(x + 3)^2}{(x + 3)(x - 4)} = \frac{x + 3}{x - 4}, x \ne -3$

39. $\dfrac{(y+5)(y-6)}{(y+3)(y-6)} = \dfrac{y+5}{y+3}, y \neq 6$

40. $\dfrac{y(y^2+4y-21)}{(y+7)(y-7)} = \dfrac{y(y+7)(y-3)}{(y+7)(y-7)}$
$= \dfrac{y(y-3)}{y-7}, y \neq -7$

41. $\dfrac{(2z)^3 - 1^3}{(z+3)(2z-1)} = \dfrac{(2z-1)[(2z)^2+(2z)(1)+1^2]}{(z+3)(2z-1)}$
$= \dfrac{4z^2+2z+1}{z+3}, z \neq \dfrac{1}{2}$

42. $\dfrac{2z(z^2+3z+9)}{z^3-3^3} = \dfrac{2z(z^2+3z+9)}{(z-3)[z^2+(z)(3)+3^2]}$
$= \dfrac{2z(z^2+3z+9)}{(z-3)(z^2+3z+9)} = \dfrac{2z}{z-3}$

43. $\dfrac{(x^3+2x^2)-(3x+6)}{x^2(x+2)} = \dfrac{x^2(x+2)-3(x+2)}{x^2(x+2)}$
$= \dfrac{(x+2)(x^2-3)}{x^2(x+2)} = \dfrac{x^2-3}{x^2}, x \neq -2$

44. $\dfrac{y(y+3)}{(y^3+3y^2)-(5y+15)} = \dfrac{y(y+3)}{y^2(y+3)-5(y+3)}$
$= \dfrac{y(y+3)}{(y+3)(y^2-5)} = \dfrac{y}{y^2-5}, y \neq -3$

45. $\dfrac{1}{x-1} \cdot \dfrac{(x+1)(x-1)}{3} = \dfrac{x+1}{3}, x \neq 1$

46. $\dfrac{x+3}{7} \cdot \dfrac{14}{2(x+3)} = 1, x \neq -3$

47. $\dfrac{x+3}{x-1} \cdot \dfrac{-(x-1)}{(x+3)(x-3)} = -\dfrac{1}{x-3}, x \neq 1$ and $x \neq -3$

48. $\dfrac{3x(6x-1)}{3xy} \cdot \dfrac{12y^2}{6x-1} = 12y, x \neq 0, y \neq 0$ and $x \neq \dfrac{1}{6}$

49. $\dfrac{(x-1)(x^2+x+1)}{2x^2} \cdot \dfrac{4x}{x^2+x+1} = \dfrac{2(x-1)}{x}$

50. $\dfrac{y(y^2+2y+4)}{y^2(y+2)} \cdot \dfrac{(y+2)(y-2)}{(y-2)(y^2+2y+4)} = \dfrac{1}{y},$
$y \neq -2$ and $y \neq 2$

51. $\dfrac{(y+5)(2y-1)}{(y+5)(y-5)} \cdot \dfrac{y-5}{y(2y-1)} = \dfrac{1}{y}, y \neq 5, y \neq -5,$ and
$y \neq \dfrac{1}{2}$

52. $\dfrac{(y+4)^2}{(3y+2)(y-1)} \cdot \dfrac{y(3y+2)}{y+4} = \dfrac{y(y+4)}{y-1}, y \neq -4$ and
$y \neq -\dfrac{2}{3}$

53. $\dfrac{1}{2x} \cdot \dfrac{4}{1} = \dfrac{2}{x}$

54. $\dfrac{4x}{y} \cdot \dfrac{x}{8y} = \dfrac{x^2}{2y^2}, x \neq 0$

55. $\dfrac{x(x-3)}{14y} \cdot \dfrac{3y^2}{2xy} = \dfrac{3(x-3)}{28}, x \neq 0$ and $y \neq 0$

56. $\dfrac{7(x-y)}{14(x-y)} = \dfrac{3}{8}, x \neq y$ and $y \neq 0$

57. $\dfrac{2x^2y}{(x-3)^2} \cdot \dfrac{x-3}{8xy} = \dfrac{x}{4(x-3)}, x \neq 0$ and $y \neq 0$

58. $\dfrac{(x+y)(x-y)}{2xy} \cdot \dfrac{4x^2y}{(y+x)(y-x)} = -2x,$
$x \neq 0, y \neq 0, x \neq y,$ and $x \neq -y$

59. $\dfrac{2x+1-3}{x+5} = \dfrac{2x-2}{x+5}$

60. $\dfrac{3+x+1}{x-2} = \dfrac{x+4}{x-2}$

61. $\dfrac{3}{x(x+3)} - \dfrac{1}{x} - \dfrac{6}{(x+3)(x-3)}$
$= \dfrac{3(x-3)}{x(x+3)(x-3)} - \dfrac{1(x+3)(x-3)}{x(x+3)(x-3)}$
$- \dfrac{6x}{x(x+3)(x-3)}$
$= \dfrac{(3x-9)-(x^2-9)-(6x)}{x(x+3)(x-3)}$
$= \dfrac{-x^2-3x}{x(x+3)(x-3)} = -\dfrac{x(x+3)}{x(x+3)(x-3)}$
$= -\dfrac{1}{x-3} = \dfrac{1}{3-x}, x \neq 0$ and $x \neq -3$

62. $\dfrac{5}{(x+3)(x-2)} - \dfrac{2}{x-2} + \dfrac{4}{(x+2)(x-2)}$
$= \dfrac{5(x+2)}{(x+2)(x+3)(x-2)} - \dfrac{2(x+2)(x+3)}{(x+2)(x+3)(x-2)}$
$+ \dfrac{4(x+3)}{(x+2)(x+3)(x-2)}$
$= \dfrac{(5x+10)-(2x^2+10x+12)+(4x+12)}{(x+2)(x+3)(x-2)}$
$= \dfrac{-2x^2-x+10}{(x+2)(x+3)(x-2)}$
$= -\dfrac{(2x+5)(x-2)}{(x+2)(x+3)(x-2)}$
$= -\dfrac{2x+5}{(x+2)(x+3)} = -\dfrac{2x+5}{x^2+5x+6}, x \neq 2$

63. $\dfrac{\dfrac{x^3-y^3}{x^2y^2}}{\dfrac{x^2-y^2}{x^2y^2}} = \dfrac{x^3-y^3}{x^2y^2} \cdot \dfrac{x^2y^2}{x^2-y^2}$
$= \dfrac{(x-y)(x^2+xy+y^2)}{(x-y)(x+y)} = \dfrac{x^2+xy+y^2}{x+y},$
$x \neq y, x \neq 0,$ and $y \neq 0$

64. $\dfrac{\dfrac{y+x}{xy}}{\dfrac{y^2-x^2}{x^2y^2}} = \dfrac{y+x}{xy}\cdot\dfrac{x^2y^2}{y^2-x^2}$

$= \dfrac{xy(y+x)}{(y-x)(x+y)} = \dfrac{xy}{y-x},$

$x \neq -y$, $x \neq 0$, and $y \neq 0$

65. $\dfrac{\dfrac{2x(x-4)+13x-3}{x-4}}{\dfrac{2x(x-4)+x+3}{x-4}} = \dfrac{2x^2+5x-3}{x-4}\cdot\dfrac{x-4}{2x^2-7x+3}$

$= \dfrac{(2x-1)(x+3)}{(2x-1)(x-3)} = \dfrac{x+3}{x-3}$, $x \neq 4$ and $x \neq \dfrac{1}{2}$

66. $\dfrac{\dfrac{2(x+5)-13}{x+5}}{\dfrac{2(x-3)+3}{x-3}} = \dfrac{2x-3}{x+5}\cdot\dfrac{x-3}{2x-3} = \dfrac{x-3}{x+5},$

$x \neq 3$, and $x \neq \dfrac{3}{2}$

67. $\dfrac{\dfrac{x^2-(x+h)^2}{x^2(x+h)^2}}{h} = \dfrac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}\cdot\dfrac{1}{h}$

$= \dfrac{-2xh-h^2}{hx^2(x+h)^2} = \dfrac{-h(2x+h)}{hx^2(x+h)^2}$

$= -\dfrac{2x+h}{x^2(x+h)^2}$, $h \neq 0$

68. $\dfrac{\dfrac{(x+h)(x+2)-x(x+h+2)}{(x+h+2)(x+2)}}{h}$

$= \dfrac{x^2+2x+hx+2h-x^2-hx-2x}{(x+h+2)(x+2)}\cdot\dfrac{1}{h}$

$= \dfrac{2h}{h(x+h+2)(x+2)}$

$= \dfrac{2}{(x+h+2)(x+2)}$, $h \neq 0$

69. $\dfrac{\dfrac{b^2-a^2}{ab}}{\dfrac{b-a}{ab}} = \dfrac{(b+a)(b-a)}{ab}\cdot\dfrac{ab}{b-a} = b+a$

$= a+b$, $a \neq 0$, $b \neq 0$, and $a \neq b$

70. $\dfrac{\dfrac{b+a}{ab}}{\dfrac{b^2-a^2}{ab}} = \dfrac{b+a}{ab}\cdot\dfrac{ab}{(b+a)(b-a)} = \dfrac{1}{b-a}$

$a \neq 0$, $b \neq 0$, and $a \neq -b$

71. $\left(\dfrac{x+y}{xy}\right)\left(\dfrac{1}{x+y}\right) = \dfrac{1}{xy}$, $x \neq -y$

72. $\dfrac{x-y}{x+y}$, $x \neq y$

73. $\dfrac{1}{x}+\dfrac{1}{y} = \dfrac{y}{xy}+\dfrac{x}{xy} = \dfrac{x+y}{xy}$

74. $\dfrac{1}{x^{-1}+y^{-1}} = \dfrac{1}{\dfrac{1}{x}+\dfrac{1}{y}} = \dfrac{1}{\dfrac{y+x}{xy}} = \dfrac{xy}{y+x},$

$x \neq 0$, and $y \neq 0$

Appendix B

■ Section 1 Logic: An Introduction

Section 1 Exercises

1. (a) $2 + 4 = 8$ is a false statement.

(b) "Shut the window" is an instruction, which is neither true nor false. It is not a statement.

(c) "Los Angeles is a state" is a false statement.

(d) "He is in town" is neither true nor false when "he" is unspecified. It is not a statement.

(e) "What time is it?" is a question, which is neither true nor false. Is it not a statement.

(f) $5x = 15$ is neither true nor false when x is unspecified. It is not a statement.

(g) $3 \cdot 2 = 6$ is a true statement.

(h) $2x^2 > x$ is neither true nor false when x is unspecified. It is not a statement.

(i) "This statement is false" is true if it is false, and false if it is true. So it fails to be either true or false but not both. It is not a statement.

(j) "Stay put!" is a command, which is neither true nor false. It is not a statement.

2. (a) The equation is true when $x = 3$: There exists a natural number x such that $x + 8 = 11$.

(b) Apply the Identity Property of Addition: For all natural numbers x, $x + 0 = x$.

(c) The equation is true when $x = 2$ or -2: There exists a natural number x such that $x^2 = 4$. (Read "a natural number" as "at least one natural number.")

(d) Subtracting x from both sides produces $1 = 2$, which is impossible. There is no natural number x such that $x + 1 = x + 2$.

3. In each case, negate the corresponding quantified statement from Exercise 2.

(a) There is no natural number x such that $x + 8 = 11$.

(b) It is not true that for all natural numbers x, $x + 0 = x$. That is: There exists a natural number x such that $x + 0 \neq x$.

(c) There is no natural number x such that $x^2 = 4$.

(d) There exists a natural number x such that $x + 1 = x + 2$.

4. (a)

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

(b)

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

(c) Yes, p and $\sim(\sim p)$ are equivalent, because they always have the same truth value.

(d) No, $p \vee \sim p$ and $p \wedge \sim p$ are not equivalent, because they sometimes have different truth values. Indeed, they *always* have different truth values: $p \vee \sim p$ is always true and $p \wedge \sim p$ is always false.

5. (a) The book does not have 500 pages.

(b) Six is not less than eight.

(c) $3 \cdot 5 \neq 15$

(d) It is not true that some people have blond hair. In other words: No people have blond hair.

(e) Not all dogs have four legs. In other words: Some dogs do not have four legs.

(f) It is not true that some cats do not have nine lives. In other words: All cats have nine lives.

(g) Not all squares are rectangles. In other words: Some squares are not rectangles.

(h) All rectangles are squares.

(i) It is not true that for all natural numbers x, $x + 3 = 3 + x$. In other words: There exists a natural number x such that $x + 3 \neq 3 + x$.

(j) There does not exist a natural number x such that $3 \cdot (x + 2) = 12$. In other words: For all natural numbers x, $3 \cdot (x + 2) \neq 12$.

(k) Not every counting number is divisible by itself and 1. In other words: Some natural counting number is not divisible by itself and 1.

(l) All natural numbers are divisible by 2.

(m) It is not true that for all natural numbers x, $5x + 4x = 9x$. In other words: For some natural number x, $5x + 4x \neq 9x$.

6. Use q: "This course is easy" and r: "Lazy students do not study."

(a) The conjunction of q and r is $q \wedge r$.

(b) The disjunction that joins r to the negation of q is $r \vee \sim q$.

(c) The negation of the statement in part (a) is $\sim(q \wedge r)$.

(d) The negation of q is $\sim q$.

7. Use the truth tables for the connectives.

(a) $p \wedge q$ is false because p is false (and a true conjunction requires both sides true).

(b) $p \vee q$ is true because q is true (and a true disjunction requires only one side true).

(c) $\sim p$ is true because p is false.

(d) $\sim q$ is false because q is true.

(e) $\sim(\sim p)$ is false because $\sim p$ is true [part **(c)**].

(f) $\sim p \vee q$ is true because $\sim p$ and q are both true. (Either one would suffice.)

(g) $p \wedge \sim q$ is false because p and $\sim q$ are both false. (Either one false would suffice to make the conjunction false.)

(h) $\sim(p \vee q)$ is false because $p \vee q$ is true [part **(b)**].

(i) $\sim(\sim p \wedge q)$ is false because $\sim p$ and q both true makes $\sim p \wedge q$ true.

(j) $\sim q \wedge \sim p$ is false because $\sim q$ is false [part **(d)**].

8. Use the truth tables for the connectives.

(a) $p \wedge q$ is false because p and q are both false. (Either one false would suffice to make the conjunction false.)

(b) $p \vee q$ is false because p and q are both false (and a true disjunction requires at least one side true).

(c) $\sim p$ is true because p is false.

(d) $\sim q$ is true because q is false.

(e) $\sim(\sim p)$ is false because $\sim p$ is true [part **(c)**].

(f) $\sim p \vee q$ is true because $\sim p$ is true.

(g) $p \wedge \sim q$ is false because p is false.

(h) $\sim(p \vee q)$ is true because $p \vee q$ is false [part **(b)**].

(i) $\sim(\sim p \wedge q)$ is true because q false makes $\sim p \wedge q$ false.

(j) $\sim q \wedge \sim p$ is true because $\sim q$ and $\sim p$ are both true.

9. (a) r, $\vee$, and s are analogous to R, $\cup$, and S, respectively, so $r \vee s$ corresponds to $R \cup S$.

(b) q, $\wedge$, and $\sim q$ are analogous to Q, $\cap$, and $\overline{Q}$, respectively, so $q \wedge \sim q$ corresponds to $Q \cap \overline{Q}$.

(c) r, $\vee$, and q are analogous to R, $\cup$, and Q, respectively, so $\sim(r \vee q)$ corresponds to $\overline{R \cup Q}$.

(d) p, $\wedge$, r, $\vee$, and s are analogous to P, $\cap$, R, $\cup$, and S, respectively, so $p \wedge (r \vee s)$ corresponds to $P \cap (R \cup S)$.

10. (a)

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	T
F	T	T	F	T	F	T
F	F	F	T	T	T	T
			*			*

The statements $\sim(p \vee q)$ and $\sim p \vee \sim q$ are not equivalent, because their truth values can differ.

(b)

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T
			*			*

The statements $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are equivalent, because their truth values are always the same.

(c)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	T	T	T	T
			*			*

The statements $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not equivalent, because their truth values can differ.

(d)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T
			*			*

The statements $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are equivalent, because their truth values are always the same.

11. (a) The statements $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are equivalent, and the statements $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are equivalent.

(b) The corresponding DeMorgan's Laws for sets are $\overline{P \cup Q} = \overline{P} \cap \overline{Q}$ and $\overline{P \cap Q} = \overline{P} \cup \overline{Q}$. The analogy comes from letting p mean "x is a member of P" and letting q mean "x is a member of Q." Then, for the first law, $\sim(p \vee q)$ means "x is a member of $\overline{P \cup Q}$," which is equivalent to "x is a member of $\overline{P} \cap \overline{Q}$," which translates into $\sim p \wedge \sim q$. Similar reasoning holds for the second law.

12.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Note that the column for $\sim q$ is not really necessary.

13. Restate using DeMorgan's Laws.

(a) Today is not Wednesday or the month is not June.

(b) I did not eat breakfast yesterday, or I did not watch television yesterday.

(c) It is not true that both it is raining and it is July.

Section 2 Conditionals and Biconditionals

Section 2 Exercises

1. Use p: "It is raining" and q: "The grass is wet."

(a) The conditional "If p, then q" is $p \rightarrow q$.

(b) The conditional "If not-p, then q" is $\sim p \rightarrow q$.

(c) The conditional "If p, then not-q" is $p \rightarrow \sim q$.

(d) The conditional "q if p" is $p \rightarrow q$.

(e) The conditional "Not-q implies not-p" is $\sim q \rightarrow \sim p$.

(f) The biconditional "q if, and only if, p" is $q \leftrightarrow p$.

2. (a)

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

(b)

p	q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(c)

p	$\sim p$	$\sim(\sim p)$	$p \leftrightarrow \sim(\sim p)$
T	F	T	T
F	T	F	T

(d)

p	q	$p \to q$	$\sim(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

3. If the implication is $p \to q$, then the converse is $q \to p$, the inverse is $\sim p \to \sim q$, and the contrapositive is $\sim q \to \sim p$.

(a) Converse: If you are good in sports, then you eat Meaties; Inverse: If you do not eat Meaties, then you are not good in sports; Contrapositive: If you are not good in sports, then you do not eat Meaties.

(b) Converse: If you do not like mathematics, then you do not like this book; Inverse: If you like this book, then you like mathematics; Contrapositive: If you like mathematics, then you like this book.

(c) Converse: If you have cavities, then you do not use Ultra Brush toothpaste; Inverse: If you use Ultra Brush toothpaste, then you do not have cavities; Contrapositive: If you do not have cavities, then you use Ultra Brush toothpaste.

(d) Converse: If your grades are high, then you are good at logic; Inverse: If you are not good at logic, then your grades are not high; Contrapositive: If your grades are not high, then you are not good at logic.

4. No, an implication and its converse cannot both be false. For $p \to q$ to be false, p must be true and q must be false. But then $q \to p$ is true.

5. Use the truth tables for the connectives.

(a) $\sim p \to \sim q$ is true because $\sim p$ is false and $\sim q$ is true.

(Either one would suffice to make the implication true.)

(b) $\sim(p \to q)$ is true because p true and q false makes $p \to q$ false.

(c) $(p \vee q) \to (p \wedge q)$ is false because p true and q false makes $p \vee q$ true and $p \wedge q$ false.

(d) $p \to \sim p$ is false because p is true and $\sim p$ is false.

(e) $(p \vee \sim p) \to p$ is true because p is true. ($p \vee \sim p$ is always true.)

(f) $(p \vee q) \leftrightarrow (p \wedge q)$ is false because $(p \vee q) \to (p \wedge q)$ is false [part (c)].

6. Use the truth tables for the connectives.

(a) $\sim p \to \sim q$ is true because $\sim q$ is true.

(b) $\sim(p \to q)$ is false because p false makes $p \to q$ true.

(c) $(p \vee q) \to (p \wedge q)$ is true because p and q both false makes $p \vee q$ false.

(d) $p \to \sim p$ is true because p is false and $\sim p$ is true. (Either one would suffice to make the implication true.)

(e) $(p \vee \sim p) \to p$ is false because p is false. ($p \vee \sim p$ is always true.)

(f) $(p \vee q) \leftrightarrow (p \wedge q)$ is true because p and q both false makes $p \vee q$ and $p \wedge q$ both false.

7. No. If it does not rain and Iris goes to the movies, then the first statement is true, but the second statement is false.

8. (a) This is the inverse, which is not logically equivalent to the original implication.

(b) This is the contrapositive, which *is* equivalent to the original implication.

(c) This is the converse, which is not logically equivalent to the original implication.

9. The contrapositive is logically equivalent: "If a number is not a multiple of 4, it is not a multiple of 8."

10. (a)

p	q	r	$p \to q$	$p \wedge r$	$(p \wedge r) \to q$	$(p \to q) \to [(p \wedge r) \to q]$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

$(p \to q) \to [(p \wedge r) \to q]$ is always true.

(b)

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$[(p \to q) \wedge p] \to q$ is always true.

(c)

p	q	$\sim p$	$\sim q$	$p \to q$	$(p \to q) \wedge \sim q$	$[(p \to q) \wedge \sim q] \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

$[(p \to q) \wedge \sim q] \to \sim p$ is always true.

(d)

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is always true.

11. (a) For $r \to s$ to be true when s is false, r must be false also. By the same reasoning, q must be false, and so must p.

(b) p must be false, because if p were true, $p \wedge q$ would be true, and then $(p \wedge q) \to r$ could not be true while r was false.

(c) Not only can q be true, but it has to be true, since q true and p false makes $p \to q$ true and is the only way for $q \to p$ to be false.

12. (a) Use p: "Mary's little lamb follows her to school," q: "The lamb's appearance at school will break the rules," and r: "Mary will be sent home." Then the statement translates as $p \to (q \wedge r)$.

(b) Use p: "Jack is nimble," q: "Jack is quick," and r: "Jack will make it over the candlestick." Then the statement translates as $\sim(p \wedge q) \to \sim r$.

(c) Use p: "The apple hit Newton on the head" and q: "The laws of gravity were discovered." Then the statement translates as $\sim p \to \sim q$.

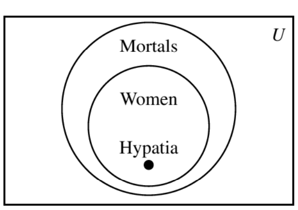
13. (a) All college students are poor.
Helen is a college student.
Helen is poor.
(A Venn diagram will confirm that this is valid.)

(b) Some freshmen like mathematics.
All people who like mathematics are intelligent.
Some freshmen are intelligent.
(A Venn diagram will confirm that this is valid.)

(c) If I study for the final, then I will pass the final.
If I pass the final, then I will pass the course.
If I pass the course, then I will look for a teaching job.
If I study for the final, then I will look for a teaching job.
(This involves two successive applications of the chain rule.)

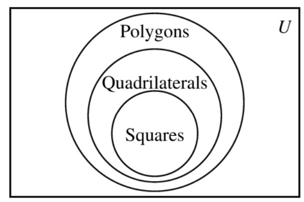
(d) Every equilateral triangle is isosceles.
There exist triangles that are equilateral.
There exist triangles that are isosceles.
(A Venn diagram will confirm that this is valid.)

14. (a) Draw a Venn diagram satisfying the hypotheses, and the conclusion is automatically satisfied.



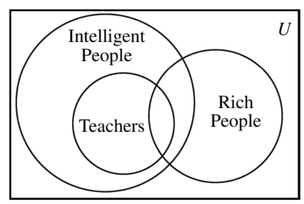
The argument is valid.

(b) Draw a Venn diagram satisfying the hypotheses, and the conclusion is automatically satisfied.



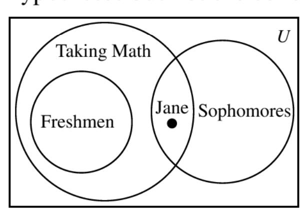
The argument is valid.

(c) Draw a Venn diagram satisfying the hypotheses, and the conclusion is automatically satisfied.



The argument is valid.

(d) It is possible to draw a Venn diagram that satisfies the hypotheses but not the conclusion.



The argument is invalid.

15. (a) If a figure is a square, then it is a rectangle.

(b) If a number is an integer, then it is a rational number.

(c) If a figure has exactly three sides, then it may be a triangle.

(d) If it rains, it is cloudy.