Mini-Lecture 1.1
Functions

Learning Objectives:

1. Determine Whether a Relation Represents a Function
2. Find the Value of a Function
3. Find the Difference Quotient of a Function
4. Find the Domain of a Function Defined by an Equation
5. Form the Sum, Difference, Product, and Quotient of Two Functions

Examples:

1. Determine whether the equation defines y as function of x.

   a) $y = x^2 - 2x$ b) $y^2 = 3x - 4$ c) $5x + 7y = 10$ d) $y = \frac{2}{x-3}$

2. For $f(x) = -x^2 + 2x - 3$ find

   a) $f(0)$ b) $f(-1)$ c) $f(3)$ d) $f(a+1)$ e) $f(x+h)$.

3. Find the domain of each function.

   a) $f(x) = 2x + 3$ b) $f(x) = \frac{2}{x^2}$ c) $f(x) = \frac{5}{\sqrt{x+4}}$ d) $f(x) = \frac{2x}{x^2+1}$

4. For $f(x) = 2x - 3$ and $g(x) = 2x^2$ find

   a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(f \cdot g)(2)$ d) $\left(\frac{f}{g}\right)(3)$

5. Find and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$ for $f(x) = -2x^2 - x + 3$.

Teaching Notes:

- Many students do not relate to f in $f(x)$ as the name of the function. To make the point, give two functions names like *Fred(x)* and *Ginger(x)*.
- When determining if an equation is a function, first substitute several values for x. This reinforces the concepts of "one input" leads to "one output". Be sure to work at least one that is not a function. Then, work the examples in the book that solve for y.
- Emphasize "Finding the Domain of a Function Defined by an Equation" in the book.
- Remind students to review interval notation before discussing the domain.
- Many students have trouble simplifying the difference quotient. Remind them that $(x+h)^2 \neq x^2+h^2$. Also, finding $f(x+h)$ first and then substituting it and $f(x)$ into the difference quotient seems to keep students from getting so overwhelmed by the algebra.
- Emphasize that y is "what" (the value of the function) and x is "where" (the location of the value). This will be helpful later in discussing relative maxima and minima.

Answers:

1. a) yes b) no c) yes d) yes

2. a) -3 b) -6 c) -6 d) $-a^2-2$ e) $-x^2-2xh-h^2+2x+2h-3$

3. a) $(-\infty,\infty)$ b) $(-\infty,0)\cup(0,\infty)$ c) $(-4,\infty)$ d) $(-\infty,\infty)$

4. a) $2x^2+2x-3$ b) $-2x^2+2x-3$ c) 8 d) $\frac{1}{6}$

5. $-4x-2h-1$

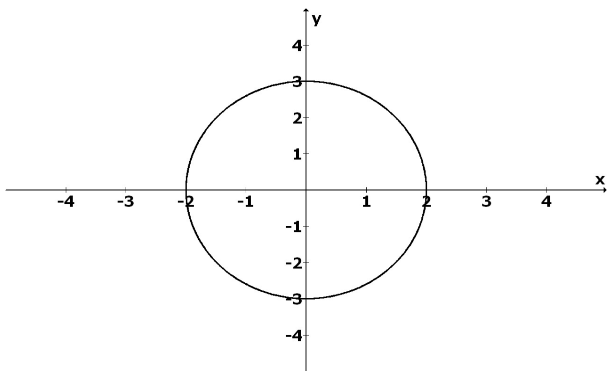
Mini-Lecture 1.2
The Graph of a Function

Learning Objectives:

1. Identify the Graph of a Function
2. Obtain Information from or about the Graph of a Function (p. 58)

Examples:

a)



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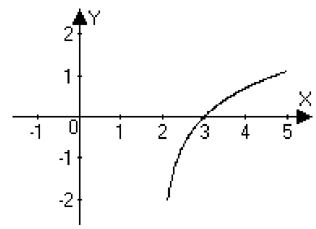


2. For $f(x) = \dfrac{2x}{x-2}$ answer the following questions.

   a) Is the point $(3, 6)$ on the graph of f ?

   b) For $x = -2$ what is $f(x)$? What point is on the graph of f ?

   c) If $f(x) = 3$, what is x ?

   d) What is the domain of f ?

   e) List any zeros of *f*. What points are on the graph of f ?

   f) List the y-intercept, if there is one, of the graph of f .

Teaching Notes:

- Be sure to draw a relation that is not a function. Have the points along a vertical line through the function labeled so that students can see the shared first coordinates.
- Some students fail to see the connection between $f(x)$ and y. When they are asked to find the value of x on the graph given $f(x)$, remind them that $f(x)$ is the y-coordinate.
- Remind students when they are finding the domain of a function from its graph that they should be looking to the left and right (or horizontally) on the graph. Even if they know the definition of the domain, sometimes they fail to visualize it properly on the graph of the function. Students have the same trouble with the range.
- Remind students that $f(x) > 0$ means the y-coordinates are positive and are located above the x-axis.

Answers:

1) a) No b) Function, Domain= $(2,\infty)$, Range $=(-\infty,\infty)$, x-intercept $=3$,
No symmetry

2) a) yes b) 1; $(-2,1)$ c) $x=6$ (d) $(-\infty,2)\cup(2,\infty)$ e) zero $=0$; $(0,0)$

f) y-intercept $=0$

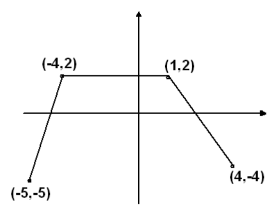
Mini-Lecture 1.3
Properties of Functions

Learning Objectives:

1. Determine Even and Odd Functions from a Graph
2. Identify Even and Odd Functions from the Equation
3. Use a Graph to Determine Where a Function is Increasing, Decreasing, or Constant
4. Use a Graph to Locate Local Maxima and Local Minima
5. Use a Graph to Locate the Absolute Maximum and the Absolute Minimum
6. Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function is Increasing or Decreasing
7. Find the Average Rate of Change of a Function

Examples:

1. For the graph below
   (a) State the intervals where the function is increasing, decreasing, or constant.
   (b) State the domain and range.
   (c) State whether the graph is odd, even, or neither.
   (d) Locate the maxima and minima.



2. Determine algebraically whether the function $f(x) = x^3 - 2x + 1$ is odd, even, or neither.

3. Find the average rate of change of $f(x) = -x^3 + 3x^2$ from $x = -1$ to $x = 4$.

Teaching Notes:

- Reinforce that the values of the relative maxima and minima are y-values.
- When finding even and odd functions, remind students that a negative number raised to an even power is positive and a negative number raised to an odd power is negative.
- When determining where a function is increasing or decreasing by looking at its graph, emphasize that you should observe what is happening to the y-values as you move from the left to the right.
- Show how to use MAXIMUM and MINIMUM on a graphing utility to approximate local maxima and minima. Using TRACE can also help students visualize the increasing or decreasing nature of the function.

Answers:

1. a) Increasing on $(-5,-4)$, Decreasing on $(1,4)$, Constant on $(-4,1)$

   b) Domain $=[-5,4]$, Range $=[-5,2]$

   c) Not odd or even

   d) Local Maximum $=2$, Local Minimum $=-5$

2) Neither

3) -4

Mini-Lecture 1.4
Library of Functions; Piecewise-defined Functions

Learning Objectives:

1. Graph the Functions Listed in the Library of Functions
2. Graph Piecewise-defined Functions

Examples:

1. Sketch the graph of each function.

a) $f(x)=\begin{cases} 2x-1 & \text{if } x>2 \\ 2-x & \text{if } x\le 2 \end{cases}$

b) $f(x)=\begin{cases} \dfrac{1}{x} & \text{if } x<0 \\ \sqrt{x} & \text{if } x\ge 0 \end{cases}$

c) $f(x)=\begin{cases} x^3 & \text{if } x<1 \\ |x| & \text{if } x\ge 1 \end{cases}$

d) $f(x)=\begin{cases} x^2 & x<0 \\ 1 & x=0 \\ \sqrt[3]{x} & x>0 \end{cases}$

2. Find $h(-5)$, if

$$h(x)=\begin{cases} x+4 & \text{if} \quad x\ge -4 \\ -(x+4) & \text{if} \quad x<-4 \end{cases}$$

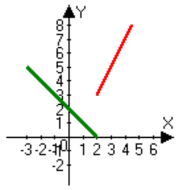
3. Find the domain and the range of the function, $f(x)=\begin{cases} |x| & \text{if } -3\le x\le 1 \\ \sqrt[3]{x} & \text{if } x>1 \end{cases}$.

Teaching Notes:

- It is a good idea to have the students memorize the graphs of the functions listed in the library of functions in this section, along with 3 points on each one. This will help them when they apply the transformations in the next section.
- Students often have trouble graphing piecewise-defined functions. If you can show the different pieces in different colors, this can help them visualize the way the function is divided.
- Many students have trouble evaluating piecewise-defined functions. Emphasize locating the desired piece of the function first based on the given value of x. Then, have the student literally cover up the other piece before evaluating.
- Remind students of the importance of correctly using an open point or a closed point on the graph of a piecewise-defined function.

Answers:

1.) a)



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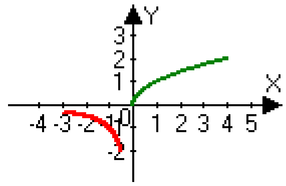

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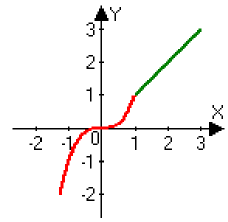

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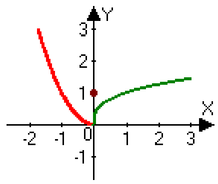


2. $h(-5) = 1$

3. domain-$[-3, \infty)$, range-$[0, \infty)$

Mini-Lecture 1.5
Graphing Techniques: Transformations

Learning Objectives:

1. Graph Functions Using Vertical and Horizontal Shifts
2. Graph Functions Using Compressions and Stretches
3. Graph Functions Using Reflections about the x-Axis and the y-Axis

Examples:

1. Sketch the graph of each function.

a) $f(x)=x^2-2$ b) $f(x)=x^3+3$ c) $f(x)=\sqrt{x+5}$ d) $f(x)=|x-2|$

e) $f(x)=2x^2$ f) $f(x)=\frac{1}{2}x^3$ g) $f(x)=-\dfrac{1}{x}$ h) $f(x)=\sqrt{3-x}$

i) $f(x)=(x-1)^2+2$ j) $f(x)=-\sqrt{2-x}+1$

2. Find the equation of the function that is finally graphed after the following transformations are applied to the graph of $y=x^2$.

   a. Shift right 3 units.
   b. Reflect about the x-axis.
   c. Shift down 2 units.

3. Suppose that the x-intercepts of the graph of $y=f(x)$ are 2 and -4. What are the x-intercepts of $2f(x)$.

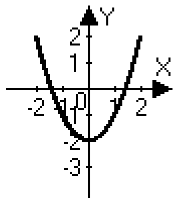
Teaching Notes:

- Before shifting graphs of equations, practice shifting points first.
- One helpful type of exercise is to give the students the standard function and a word description of the transformed function, then have them find the equation of the transformed function.
- Emphasize the "Summary of Graphing Techniques" in the book.
- Use a graphing utility calculator to show transformations. Begin with the standard function and show one transformation at a time on the screen.
- Before graphing review the difference between $cf(x)$ and $f(cx)$.

Answers:

1.

(a)



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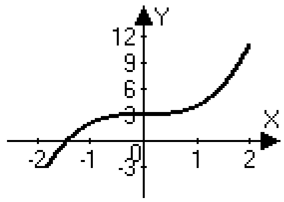

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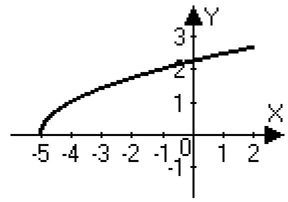

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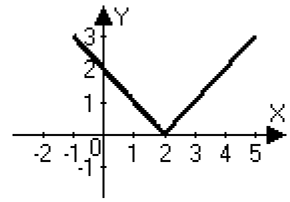

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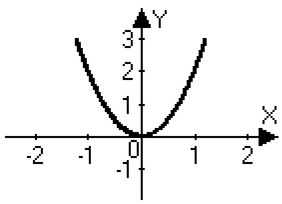

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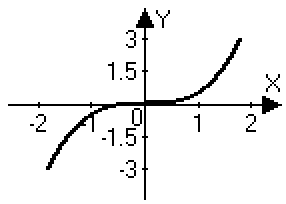

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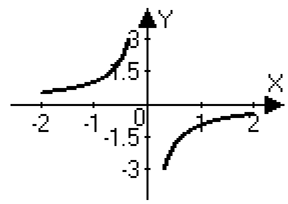

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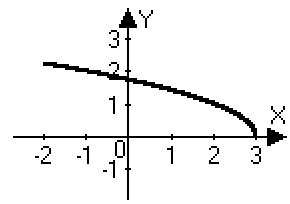
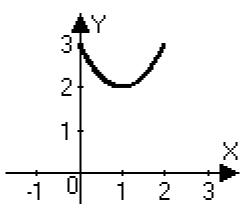


(i)



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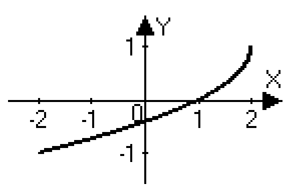

2. $f(x) = -(x-3)^2 - 2$

3. 2 and -4

Mini-Lecture 1.6
Mathematical Model: Building Functions

Learning Objectives:

1. Build and Analyze Functions

Examples:

1. A rectangle is inscribed in a circle of radius 3. Let $P=(x,y)$ be a point in quadrant I that is a vertex of the rectangle and is on the circle. Express the area A of the rectangle as a function of x and the perimeter P of the rectangle as a function of x.

2. Let $P=(x,y)$ be a point on the graph of $y=x^3$. Express the distance d from P to the point $(2,0)$ as a function of x. What is d if $x=-1$?

3. A right triangle has one vertex on the graph of $y=16-x^2$, $x>0$, at (x,y), another at the origin, and the third on the positive x-axis at $(x,0)$. Express the area A of the triangle as a function of x.

4. An open box with a square base is to be made from a square piece of cardboard 16 inches on a side by cutting out a square from each corner and turning up the sides. Express the volume V of the box as a function of the length x of the side of the square cut from each corner. Find the volume if a 2-inch square is cut out.

Teaching Notes:

- Have students review basic geometric formulas for perimeter, area, and volume.
- Encourage students to draw graphs or figures before attempting to work the problems in this section.
- Emphasize proper function notation when giving answers.

Answers:

1. $A(x)=4x\sqrt{9-x^2}$, $P(x)=4x+4\sqrt{9-x^2}$

2. $d(x)=\sqrt{x^6+x^2-4x+4}$, $d=\sqrt{10}$

3. $A(x)=8x-\frac{1}{2}x^3$

4. $V(x)=4x(8-x)^2$, $V(2)=288\ in^2$

Mini-Lecture 1.7
Building Mathematical Models using Variation

Learning Objectives:

1. Construct a Model using Direct Variation
2. Construct a Model Using Indirect Variation
3. Construct a Model Using Joint or Combined Variation

Examples:

1. Write a general formula to describe each variation:

   a) A varies directly as r; $A = 5\pi$ when $r = 2$.
   b) y varies inversely as the cube of x; $y = 5$ when $x = 2$.
   c) X varies jointly as y and z^2 and inversely as w; $X = 45$ when $y = 2$, $z = 5$, and $w = 3$

2. Write a general formula to describe the variation.

   x varies directly with y and inversely with the fourth power of z.

3. The time t that it takes to get to your job varies inversely with your average speed s. Suppose that it takes you 30 minutes to get to work when your average speed is 40 miles per hour. a) Express the driving time to work in terms of the average speed. b) Suppose that your average speed to work is 50 miles per hour. How long will it take you to get to work?

4. The volume of a sphere varies directly with the cube of the radius. If the volume of a sphere with a radius of 2 inches is $\frac{32}{3}\pi$ cubic inches, what would be the volume of a sphere with a radius of 6 inches?

5. The force that it requires to stretch a spring varies directly with the distance that it is stretched. If a 10 pound force can stretch a spring 8 inches, how much force would it take to stretch the spring 12 inches?

Teaching Notes:

- Remind students to always find the constant of variation first, if enough information is given in the problem.
- Emphasize the graphs in the figures that accompany the examples on direct and inverse variation in the book. These graphs will help the student visualize the type of variation.
- Tell students to read each problem carefully. A common mistake is to read *square root* as *squared.*

- Emphasize the difference between joint variation and combined variation. Some students will get these confused.

Answers:

1) a) $A = \frac{5\pi}{2} r$ b) $y = \frac{40}{x^3}$ c) $X = \frac{2.7yz^2}{w}$

2. $x = \frac{ky}{z^4}$

3. a) $t = \frac{20}{s}$ b) 24 minutes

4. $288\pi\ in^3$

5. 15 *lbs*