INSTRUCTOR'S SOLUTIONS MANUAL

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PRECALCULUS

SIXTH EDITION

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TABLE OF CONTENTS for INSTRUCTOR SOLUTIONS

PRECALCULUS 6E

Chapter P
Prerequisites: Fundamental Concepts of Algebra

Section P.1

Check Point Exercises

1. $8+6(x-3)^2 = 8+6(13-3)^2$
$= 8+6(10)^2$
$= 8+6(100)$
$= 8+600$
$= 608$

2. **a.** Since 2014 is 14 years after 2000, substitute 14 for x.
$T = 4x^2 + 330x + 3310$
$= 4(14)^2 + 330(14) + 3310$
$= 8714$
The average cost of tuition and fees at public U.S. colleges for the school year ending in 2014 was $8714.

b. The formula underestimates the actual answer by $179.

3. The elements common to {3, 4, 5, 6, 7} and {3, 7, 8, 9} are 3 and 7.
$\{3,4,5,6,7\} \cap \{3,7,8,9\} = \{3,7\}$

4. The union is the set containing all the elements of either set.
$\{3,4,5,6,7\} \cup \{3,7,8,9\} = \{3,4,5,6,7,8,9\}$

5. $\left\{-9,\ -1.3,\ 0,\ 0.\bar{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

a. Natural numbers: $\sqrt{9}$ because $\sqrt{9} = 3$

b. Whole numbers: 0, $\sqrt{9}$

c. Integers: -9, 0, $\sqrt{9}$

d. Rational numbers: $-9,\ -1.3,\ 0,\ 0.\bar{3},\ \sqrt{9}$

e. Irrational numbers: $\frac{\pi}{2},\ \sqrt{10}$

f. Real numbers: $-9,\ -1.3,\ 0,\ 0.\bar{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}$

6. **a.** $\left|1-\sqrt{2}\right|$
Because $\sqrt{2} \approx 1.4$, the number inside the absolute value bars is negative. The absolute value of x when $x < 0$ is $-x$. Thus,
$\left|1-\sqrt{2}\right| = -\left(1-\sqrt{2}\right) = \sqrt{2}-1$

b. $|\pi - 3|$
Because $\pi \approx 3.14$, the number inside the absolute value bars is positive. The absolute value of a positive number is the number itself. Thus,
$|\pi - 3| = \pi - 3.$

c. $\frac{|x|}{x}$
Because $x > 0$, $|x| = x.$
Thus, $\frac{|x|}{x} = \frac{x}{x} = 1$

7. $|-4-(5)| = |-9| = 9$
The distance between -4 and 5 is 9.

8. $7(4x^2+3x)+2(5x^2+x)$
$= 7(4x^2+3x)+2(5x^2+x)$
$= 28x^2+21x+10x^2+2x$
$= 38x^2+23x$

9. $6+4[7-(x-2)]$
$= 6+4[7-x+2)]$
$= 6+4[9-x]$
$= 6+36-4x$
$= 42-4x$

Concept and Vocabulary Check P.1

1. expression

2. b to the nth power; base; exponent

3. formula; modeling; models

4. intersection; $A \cap B$

5. union; $A \cup B$

6. natural

7. whole

8. integers

9. rational

10. irrational

11. rational; irrational

12. absolute value; x, $-x$

13. $b+a$; ba

14. $a+(b+c)$; $(ab)c$

15. $ab+ac$

16. 0; inverse; 0; identity

17. inverse; 1; identity

18. simplified

19. a

Exercise Set P.1

1. $7+5(10)=7+50=57$

2. $8+6(5)=8+30=38$

3. $6(3)-8=18-8=10$

4. $8(3)-4=24-4=20$

5. $8^2+3(8)=64+24=88$

6. $6^2+5(6)=36+30=66$

7. $7^2-6(7)+3=49-42+3=7+3=10$

8. $8^2-7(8)+4=64-56+4=8+4=12$

9. $4+5(9-7)^3=4+5(2)^3$
$=4+5(8)=4+40=44$

10. $6+5(8-6)^3=6+5(2)^3$
$=6+5(8)$
$=6+40=46$

11. $8^2-3(8-2)=64-3(6)$
$=64-18=46$

12. $8^2-4(8-3)=64-4(5)=64-20=44$

13. $\frac{5(x+2)}{2x-14}=\frac{5(10+2)}{2(10)-14}$
$=\frac{5(12)}{6}$
$=5\cdot 2$
$=10$

14. $\frac{7(x-3)}{2x-16}=\frac{7(9-3)}{2(9)-16}=\frac{7(6)}{2}=7\cdot 3=21$

15. $\frac{2x+3y}{x+1}; x=-2, y=4$
$=\frac{2(-2)+3(4)}{-2+1}=\frac{-4+12}{-1}=\frac{8}{-1}=-8$

16. $\frac{2x+y}{xy-2x}$; $x=-2$ and $y=4$
$\frac{2(-2)+4}{(-2)(4)-2(-2)}=\frac{-4+4}{-8+4}=\frac{0}{4}=0$

17. $C=\frac{5}{9}(50-32)=\frac{5}{9}(18)=10$
50°F is equivalent to 10°C.

18. $C=\frac{5}{9}(F-32)=\frac{5}{9}(86-32)=\frac{5}{9}(54)=30$
86°F is equivalent to 30°C.

19. $h=4+60t-16t^2=4+60(2)-16(2)^2$
$=4+120-16(4)=4+120-64$
$=124-64=60$
Two seconds after it is kicked, the ball's height is 60 feet.

20. $h=4+60t-16t^2$
$=4+60(3)-16(3)^2$
$=4+180-16(9)$
$=4+180-144$
$=184-144=40$
Three seconds after it is kicked, the ball's height is 40 feet.

21. $\{1,2,3,4\}\cap\{2,4,5\}=\{2,4\}$

22. $\{1,3,7\}\cap\{2,3,8\}=\{3\}$

23. $\{s,e,t\}\cap\{t,e,s\}=\{s,e,t\}$

24. $\{r,e,a,l\}\cap\{l,e,a,r\}=\{r,e,a,l\}$

25. $\{1,3,5,7\}\cap\{2,4,6,8,10\}=\{\ \}$
The empty set is also denoted by $\varnothing$.

26. $\{1,3,5,7\}\cap\{-5,-3,-1\}=\{\ \}$ or $\varnothing$

27. $\{a,b,c,d\}\cap\varnothing=\varnothing$

28. $\{w,y,z\}\cap\varnothing=\varnothing$

29. $\{1,2,3,4\}\cup\{2,4,5\}=\{1,2,3,4,5\}$

30. $\{1,3,7,8\}\cup\{2,3,8\}=\{1,2,3,7,8\}$

31. $\{1,3,5,7\}\cup\{2,4,6,8,10\}$
$=\{1,2,3,4,5,6,7,8,10\}$

32. $\{0,1,3,5\}\cup\{2,4,6\}=\{0,1,2,3,4,5,6\}$

33. $\{a,e,i,o,u\}\cup\varnothing=\{a,e,i,o,u\}$

34. $\{e,m,p,t,y\}\cup\varnothing = \{e,m,p,t,y\}$

35. **a.** $\sqrt{100}$

b. $0, \sqrt{100}$

c. $-9, 0, \sqrt{100}$

d. $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$

e. $\sqrt{3}$

f. $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$

36. **a.** $\sqrt{49}$

b. $0, \sqrt{49}$

c. $-7, 0, \sqrt{49}$

d. $-7, -0.\overline{6}, 0, \sqrt{49}$

e. $\sqrt{50}$

f. $-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}$

37. **a.** $\sqrt{64}$

b. $0, \sqrt{64}$

c. $-11, 0, \sqrt{64}$

d. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$

e. $\sqrt{5}, \pi$

f. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$

38. **a.** $\sqrt{4}$

b. $0, \sqrt{4}$

c. $-5, 0, \sqrt{4}$

d. $-5, -0.\overline{3}, 0, \sqrt{4}$

e. $\sqrt{2}$

f. $-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}$

39. 0

40. Answers will vary. An example is $\frac{1}{2}$.

41. Answers will vary. An example is 2.

42. Answers will vary. An example is -2.

43. true; –13 is to the left of –2 on the number line.

44. false; –6 is to the left of 2 on the number line.

45. true; 4 is to the right of –7 on the number line.

46. true; –13 is to the left of –5 on the number line.

47. true; $-\pi=-\pi$

48. true; –3 is to the right of –13 on the number line.

49. true; 0 is to the right of –6 on the number line.

50. true; 0 is to the right of –13 on the number line.

51. $|300|=300$

52. $|-203|=203$

53. $|12-\pi| = 12-\pi$

54. $|7-\pi| = 7-\pi$

55. $|\sqrt{2}-5| = 5-\sqrt{2}$

56. $|\sqrt{5}-13| = 13-\sqrt{5}$

57. $\frac{-3}{|-3|} = \frac{-3}{3} = -1$

58. $\frac{-7}{|-7|} = \frac{-7}{7} = -1$

59. $||-3|-|-7|| = |3-7| = |-4| = 4$

60. $||-5|-|-13|| = |5-13| = |-8| = 8$

61. $|x+y| = |2+(-5)| = |-3| = 3$

62. $|x-y| = |2-(-5)| = |7| = 7$

63. $|x|+|y| = |2|+|-5| = 2+5 = 7$

64. $|x|-|y| = |2|-|-5| = 2-5 = -3$

65. $\frac{y}{|y|} = \frac{-5}{|-5|} = \frac{-5}{5} = -1$

66. $\frac{|x|}{x}+\frac{|y|}{y} = \frac{|2|}{2}+\frac{|-5|}{-5} = \frac{2}{2}+\frac{5}{-5} = 1+(-1) = 0$

67. The distance is $|2-17| = |-15| = 15$.

68. The distance is $|4-15| = |-11| = 11$.

69. The distance is $|-2-5| = |-7| = 7$.

70. The distance is $|-6-8| = |-14| = 14$.

71. The distance is $|-19-(-4)| = |-19+4| = |-15| = 15$.

72. The distance is $|-26-(-3)| = |-26+3| = |-23| = 23$.

73. The distance is $|-3.6-(-1.4)| = |-3.6+1.4| = |-2.2| = 2.2$.

74. The distance is $|-5.4-(-1.2)| = |-5.4+1.2| = |-4.2| = 4.2$.

75. 6 + (–4) = (–4) + 6;
commutative property of addition

76. $11\cdot(7+4) = 11\cdot 7 + 11\cdot 4$;
distributive property of multiplication over addition

77. 6 + (2 + 7) = (6 + 2) + 7;
associative property of addition

78. $6\cdot(2\cdot 3) = 6\cdot(3\cdot 2)$;
commutative property of multiplication

79. (2 + 3) + (4 + 5) = (4 + 5) + (2 + 3);
commutative property of addition

80. $7\cdot(11\cdot 8) = (11\cdot 8)\cdot 7$;
commutative property of multiplication

81. 2 (–8 + 6) = –16 + 12;
distributive property of multiplication over addition

82. $-8(3+11) = -24+(-88)$;
distributive property of multiplication over addition

83. $\frac{1}{x+3}(x+3) = 1; x \neq -3$,
inverse property of multiplication

84. $(x+4)+[-(x+4)] = 0$;
inverse property of addition

85.
$$\begin{aligned}5(3x+4)-4 &= 5\cdot 3x+5\cdot 4-4\\ &= 15x+20-4\\ &= 15x+16\end{aligned}$$

86.
$$\begin{aligned}2(5x+4)-3 &= 2\cdot 5x+2\cdot 4-3\\ &= 10x+8-3\\ &= 10x+5\end{aligned}$$

87.
$$\begin{aligned}5(3x-2)+12x &= 5\cdot 3x-5\cdot 2+12x\\ &= 15x-10+12x\\ &= 27x-10\end{aligned}$$

88.
$$\begin{aligned}2(5x-1)+14x &= 2\cdot 5x-2\cdot 1+14x\\ &= 10x-2+14x\\ &= 24x-2\end{aligned}$$

89.
$$\begin{aligned}&7(3y-5)+2(4y+3)\\ &= 7\cdot 3y-7\cdot 5+2\cdot 4y+2\cdot 3\\ &= 21y-35+8y+6\\ &= 29y-29\end{aligned}$$

90. $$\begin{aligned}&4(2y-6)+3(5y+10)\\&=4\cdot 2y-4\cdot 6+3\cdot 5y+3\cdot 10\\&=8y-24+15y+30\\&=23y+6\end{aligned}$$

91. $$\begin{aligned}5(3y-2)-(7y+2)&=15y-10-7y-2\\&=8y-12\end{aligned}$$

92. $$\begin{aligned}4(5y-3)-(6y+3)&=20y-12-6y-3\\&=14y-15\end{aligned}$$

93. $$\begin{aligned}7-4[3-(4y-5)]&=7-4[3-4y+5]\\&=7-4[8-4y]\\&=7-32+16y\\&=16y-25\end{aligned}$$

94. $$\begin{aligned}6-5[8-(2y-4)]&=6-5[8-2y+4]\\&=6-5[12-2y]\\&=6-60+10y\\&=10y-54\end{aligned}$$

95. $$\begin{aligned}&18x^2+4-\left[6\left(x^2-2\right)+5\right]\\&=18x^2+4-\left[6x^2-12+5\right]\\&=18x^2+4-\left[6x^2-7\right]\\&=18x^2+4-6x^2+7\\&=18x^2-6x^2+4+7\\&=(18-6)x^2+11=12x^2+11\end{aligned}$$

96. $$\begin{aligned}&14x^2+5-\left[7\left(x^2-2\right)+4\right]\\&=14x^2+5-\left[7x^2-14+4\right]\\&=14x^2+5-\left[7x^2-10\right]\\&=14x^2+5-7x^2+10\\&=14x^2-7x^2+5+10\\&=(14-7)x^2+15\\&=7x^2+15\end{aligned}$$

97. $-(-14x)=14x$

98. $-(-17y)=17y$

99. $-(2x-3y-6)=-2x+3y+6$

100. $-(5x-13y-1)=-5x+13y+1$

101. $\frac{1}{3}(3x)+[(4y)+(-4y)]=x+0=x$

102. $\frac{1}{2}(2y)+[(-7x)+7x]=y+0=y$

103. $$\begin{aligned}&|-6|\ \square\ |-3|\\&6\ \square\ 3\\&6>3\end{aligned}$$

Since $6>3$, $|-6|>|-3|$.

104. $$\begin{aligned}&|-20|\ \square\ |-50|\\&20\ \square\ 50\\&20<50\end{aligned}$$

Since $20<50$, $|-20|<|-50|$.

105. $$\begin{aligned}&\left|\frac{3}{5}\right|\ \square\ |-0.6|\\&|0.6|\ \square\ |-0.6|\\&0.6\ \square\ 0.6\\&0.6\ =\ 0.6\end{aligned}$$

Since $0.6=0.6$, $\left|\frac{3}{5}\right|=|-0.6|$.

106. $$\begin{aligned}&\left|\frac{5}{2}\right|\ \square\ |-2.5|\\&|2.5|\ \square\ |-2.5|\\&2.5\ \square\ 2.5\\&2.5\ =\ 2.5\end{aligned}$$

Since $2.5=2.5$, $\left|\frac{5}{2}\right|=|-2.5|$.

107. $$\begin{aligned}&\frac{30}{40}-\frac{3}{4}\ \square\ \frac{14}{15}\cdot\frac{15}{14}\\&\frac{30}{40}-\frac{30}{40}\ \square\ \frac{\cancel{14}}{\cancel{15}}\cdot\frac{\cancel{15}}{\cancel{14}}\\&0\ \square\ 1\\&0<1\end{aligned}$$

Since $0<1$, $\frac{30}{40}-\frac{3}{4}<\frac{14}{15}\cdot\frac{15}{14}$.

108. $$\begin{aligned}&\frac{17}{18}\cdot\frac{18}{17}\ \square\ \frac{50}{60}-\frac{5}{6}\\&\frac{\cancel{17}}{\cancel{18}}\cdot\frac{\cancel{18}}{\cancel{17}}\ \square\ \frac{50}{60}-\frac{50}{60}\\&1\ \square\ 0\\&1>0\end{aligned}$$

Since $1>0$, $\frac{17}{18}\cdot\frac{18}{17}>\frac{50}{60}-\frac{5}{6}$.

109. $$\begin{aligned}&\frac{8}{13}\div\frac{8}{13}\ \square\ |-1|\\&\frac{8}{13}\cdot\frac{13}{8}\ \square\ 1\\&1\ \square\ 1\\&1=1\end{aligned}$$

Since $1=1$, $\frac{8}{13}\div\frac{8}{13}=|-1|$.

110. $|-2| \square \frac{4}{17} \div \frac{4}{17}$
$2 \square \frac{4}{17} \cdot \frac{17}{4}$
$2 \square 1$
$2 > 1$

Since $2 > 1$, $|-2| > \frac{4}{17} \div \frac{4}{17}$.

111. $8^2 - 16 \div 2^2 \cdot 4 - 3 = 64 - 16 \div 4 \cdot 4 - 3 = 64 - 4 \cdot 4 - 3 = 64 - 16 - 3 = 48 - 3 = 45$

112. $10^2 - 100 \div 5^2 \cdot 2 - 3 = 100 - 100 \div 25 \cdot 2 - 3 = 100 - 4 \cdot 2 - 3 = 100 - 8 - 3 = 92 - 3 = 89$

113. $\frac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2} = \frac{5 \cdot 2 - 9}{[9 - (-2)]^2} = \frac{10 - 9}{[9 + 2]^2} = \frac{10 - 9}{11^2} = \frac{1}{121}$

114. $\frac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2} = \frac{5 + 12}{(12 - 6)^2} = \frac{17}{6^2} = \frac{17}{36}$

115. $8 - 3[-2(2-5) - 4(8-6)] = 8 - 3[-2(-3) - 4(2)] = 8 - 3[6 - 8] = 8 - 3[-2] = 8 + 6 = 14$

116. $8 - 3[-2(5-7) - 5(4-2)] = 8 - 3[-2(-2) - 5(2)] = 8 - 3[4 - 10] = 8 - 3[-6] = 8 + 18 = 26$

117. $\frac{2(-2) - 4(-3)}{5 - 8} = \frac{-4 + 12}{-3} = \frac{8}{-3} = -\frac{8}{3}$

118. $\frac{6(-4) - 5(-3)}{9 - 10} = \frac{-24 + 15}{-1} = \frac{-9}{-1} = 9$

119. $\frac{(5-6)^2 - 2|3-7|}{89 - 3 \cdot 5^2} = \frac{(-1)^2 - 2|-4|}{89 - 3 \cdot 25} = \frac{1 - 2(4)}{89 - 75} = \frac{1 - 8}{14} = \frac{-7}{14} = -\frac{1}{2}$

120. $\frac{12 \div 3 \cdot 5|2^2 + 3^2|}{7 + 3 - 6^2} = \frac{12 \div 3 \cdot 5|4 + 9|}{7 + 3 - 36} = \frac{4 \cdot 5|13|}{10 - 36} = \frac{20(13)}{-26} = \frac{260}{-26} = -10$

121. $x - (x + 4) = x - x - 4 = -4$

122. $x - (8 - x) = x - 8 + x = 2x - 8$

123. $6(-5x) = -30x$

124. $10(-4x) = -40x$

125. $5x - 2x = 3x$

126. $6x - (-2x) = 6x + 2x = 8x$

127. $8x - (3x + 6) = 8x - 3x - 6 = 5x - 6$

128. $8 - 3(x + 6) = 8 - 3x - 18 = -3x - 10$

129. **a.** $H = \frac{7}{10}(220 - a)$
$H = \frac{7}{10}(220 - 20) = \frac{7}{10}(200) = 140$

The lower limit of the heart rate for a 20-year-old with this exercise goal is 140 beats per minute.

b. $H = \frac{4}{5}(220 - a)$
$H = \frac{4}{5}(220 - 20)$
$= \frac{4}{5}(200)$
$= 160$

The upper limit of the heart rate for a 20-year-old with this exercise goal is 160 beats per minute.

130. a. $H = \frac{1}{2}(220 - a)$
$H = \frac{1}{2}(220 - 30)$
$= \frac{1}{2}(190)$
$= 95$

The lower limit of the heart rate for a 30-year-old with this exercise goal is 95 beats per minute.

b. $H = \frac{3}{5}(220 - a)$
$H = \frac{3}{5}(220 - 30)$
$= \frac{3}{5}(190)$
$= 114$

The upper limit of the heart rate for a 30-year-old with this exercise goal is 114 beats per minute.

131. a. $T = 21x^2 + 862x + 15,552$
$= 21(14)^2 + 862(14) + 15,552$
$= 31,736$

The formula estimates the cost to have been \$31,736 in 2014.

b. This overestimates the value in the graph by \$35.

c. $T = 21x^2 + 862x + 15,552$
$= 21(20)^2 + 862(20) + 15,552$
$= 41,192$

The formula projects the cost to be \$41,192 in 2020.

132. a. $T = 21x^2 + 862x + 15,552$
$= 21(12)^2 + 862(12) + 15,552$
$= 28,920$

The formula estimates the cost to have been \$28,920 in 2012.

b. This underestimates the value in the graph by \$136.

c. $T = 21x^2 + 862x + 15,552$
$= 21(22)^2 + 862(22) + 15,552$
$= 44,680$

The formula projects the cost to be \$44,680 in 2022.

133. a. $0.05x + 0.12(10,000 - x)$
$= 0.05x + 1200 - 0.12x$
$= 1200 - 0.07x$

b. $1200 - 0.07x = 1200 - 0.07(6000)$
$= \$780$

134. a. $0.06t + 0.5(50 - t) = 0.06t + 25 - 0.5t$
$= 25 - 0.44t$

b. $0.06(20) + 0.5(50 - 20)$
$= 1.2 + 0.5(30)$
$= 1.2 + 15$
$= 16.2$ miles

135. – 143. Answers will vary.

144. does not make sense; Explanations will vary. Sample explanation: Models do not always accurately predict future values.

145. does not make sense; Explanations will vary. Sample explanation: To use the model, substitute 0 for x.

146. makes sense

147. does not make sense; Explanations will vary. Sample explanation: The commutative property changes order and the associative property changes groupings.

148. false; Changes to make the statement true will vary. A sample change is: Some rational numbers are not integers.

149. false; Changes to make the statement true will vary. A sample change is: All whole numbers are integers.

150. true

151. false; Changes to make the statement true will vary. A sample change is: Some irrational numbers are negative.

152. false; Changes to make the statement true will vary. A sample change is: The term x has a coefficient of 1.

153. false; Changes to make the statement true will vary. A sample change is:
$5+3(x-4)=5+3x-12=3x-7.$

154. false; Changes to make the statement true will vary. A sample change is: $-x-x=-2x.$

155. true

156. $\sqrt{2}\approx 1.4$
$1.4<1.5$
$\sqrt{2}<1.5$

157. $-\pi>-3.5$

158. $-\frac{3.14}{2}=-1.57$
$-\frac{\pi}{2}\approx -1.571$
$-1.57>-1.571$
$-\frac{3.14}{2}>-\frac{\pi}{2}$

159. **a.** $b^4\cdot b^3=(b\cdot b\cdot b\cdot b)(b\cdot b\cdot b)=b^7$

b. $b^5\cdot b^5=(b\cdot b\cdot b\cdot b\cdot b)(b\cdot b\cdot b\cdot b\cdot b)=b^{10}$

c. add the exponents

160. **a.** $\frac{b^7}{b^3}=\frac{b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b}{b\cdot b\cdot b}=b^4$

b. $\frac{b^8}{b^2}=\frac{b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b\cdot b}{b\cdot b}=b^6$

c. subtract the exponents

161. $6.2\times 10^3=6.2\times 10\times 10\times 10=6200$
It moves the decimal point 3 places to the right.

Section P.2

Check Point Exercises

1. **a.** $\left(2x^3y^6\right)^4=(2)^4\left(x^3\right)^4\left(y^6\right)^4=16x^{12}y^{24}$

b. $\left(-6x^2y^5\right)\left(3xy^3\right)=(-6)\cdot 3\cdot x^2\cdot x\cdot y^5\cdot y^3$
$=-18x^3y^8$

c. $\frac{100x^{12}y^2}{20x^{16}y^{-4}}=\left(\frac{100}{20}\right)\left(\frac{x^{12}}{x^{16}}\right)\left(\frac{y^2}{y^{-4}}\right)$
$=5x^{12-16}y^{2-(-4)}$
$=5x^{-4}y^6$
$=\frac{5y^6}{x^4}$

d. $\left(\frac{5x}{y^4}\right)^{-2}=\frac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$=\frac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$=\frac{5^{-2}x^{-2}}{y^{-8}}$
$=\frac{y^8}{5^2x^2}$
$=\frac{y^8}{25x^2}$

2. **a.** $-2.6\times 10^9=-2,600,000,000$

b. $3.017\times 10^{-6}=0.000003017$

3. **a.** $5,210,000,000=5.21\times 10^9$

b. $-0.00000006893=-6.893\times 10^{-8}$

4. $410\times 10^7=\left(4.1\times 10^2\right)\times 10^7$
$=4.1\times\left(10^2\times 10^7\right)$
$=4.1\times 10^9$

5 **a.** $\left(7.1\times 10^5\right)\left(5\times 10^{-7}\right)$
$=7.1\cdot 5\times 10^5\cdot 10^{-7}$
$=35.5\times 10^{-2}$
$=\left(3.55\times 10^1\right)\times 10^{-2}$
$=3.55\times\left(10^1\times 10^{-2}\right)$
$=3.55\times 10^{-1}$

b. $\frac{1.2\times 10^6}{3\times 10^{-3}}=\frac{1.2}{3}\cdot\frac{10^6}{10^{-3}}$
$=0.4\times 10^{6-(-3)}$
$=0.4\times 10^9$
$=4\times 10^8$

6. $\frac{4.08\times10^{10}}{680,000}=\frac{4.08\times10^{10}}{6.8\times10^{5}}=\frac{4.08}{6.8}\cdot\frac{10^{10}}{10^{5}}$
$=0.6\times10^{5}$
$=60,000$

The average salary was $60,000 per U.S. police officer.

7. $S=(1.76\times10^{5})[(1.44\times10^{-2})-r^{2}]$
$=(1.76\times10^{5})[(1.44\times10^{-2})-0^{2}]$
$=2.5344\times10^{3}$
$=2534.4$

The speed of the blood at the central axis of the artery is 2534.4 centimeters per second.

Concept and Vocabulary Check P.2

1. b^{m+n}; add
2. b^{m-n}; subtract
3. 1
4. $\frac{1}{b^n}$
5. false
6. b^n
7. true
8. a number greater than or equal to 1 and less than 10; integer
9. true
10. false

Exercise Set P.2

1. $5^2\cdot2=(5\cdot5)\cdot2=25\cdot2=50$
2. $6^2\cdot2=(6\cdot6)\cdot2=36\cdot2=72$
3. $(-2)^6=(-2)(-2)(-2)(-2)(-2)(-2)=64$
4. $(-2)^4=(-2)(-2)(-2)(-2)=16$
5. $-2^6=-2\cdot2\cdot2\cdot2\cdot2\cdot2=-64$
6. $-2^4=-2\cdot2\cdot2\cdot2=-16$
7. $(-3)^0=1$
8. $(-9)^0=1$
9. $-3^0=-1$
10. $-9^0=-1$
11. $4^{-3}=\frac{1}{4^3}=\frac{1}{4\cdot4\cdot4}=\frac{1}{64}$
12. $2^{-6}=\frac{1}{2^6}=\frac{1}{2\cdot2\cdot2\cdot2\cdot2\cdot2}=\frac{1}{64}$
13. $2^2\cdot2^3=2^{2+3}=2^5=2\cdot2\cdot2\cdot2\cdot2=32$
14. $3^3\cdot3^2=3^{3+2}=3^5=3\cdot3\cdot3\cdot3\cdot3=243$
15. $(2^2)^3=2^{2\cdot3}=2^6=2\cdot2\cdot2\cdot2\cdot2\cdot2=64$
16. $(3^3)^2=3^{3\cdot2}=3^6=3\cdot3\cdot3\cdot3\cdot3\cdot3=729$
17. $\frac{2^8}{2^4}=2^{8-4}=2^4=2\cdot2\cdot2\cdot2=16$
18. $\frac{3^8}{3^4}=3^{8-4}=3^4=3\cdot3\cdot3\cdot3=81$
19. $3^{-3}\cdot3=3^{-3+1}=3^{-2}=\frac{1}{3^2}=\frac{1}{3\cdot3}=\frac{1}{9}$
20. $2^{-3}\cdot2=2^{-3+1}=2^{-2}=\frac{1}{2^2}=\frac{1}{2\cdot2}=\frac{1}{4}$
21. $\frac{2^3}{2^7}=2^{3-7}=2^{-4}=\frac{1}{2^4}=\frac{1}{2\cdot2\cdot2\cdot2}=\frac{1}{16}$
22. $\frac{3^4}{3^7}=3^{4-7}=3^{-3}=\frac{1}{3^3}=\frac{1}{3\cdot3\cdot3}=\frac{1}{27}$
23. $x^{-2}y=\frac{1}{x^2}\cdot y=\frac{y}{x^2}$
24. $xy^{-3}=x\cdot\frac{1}{y^3}=\frac{x}{y^3}$
25. $x^0y^5=1\cdot y^5=y^5$
26. $x^7\cdot y^0=x^7\cdot1=x^7$

27. $x^3 \cdot x^7 = x^{3+7} = x^{10}$

28. $x^{11} \cdot x^5 = x^{11+5} = x^{16}$

29. $x^{-5} \cdot x^{10} = x^{-5+10} = x^5$

30. $x^{-6} \cdot x^{12} = x^{-6+12} = x^6$

31. $(x^3)^7 = x^{3\cdot 7} = x^{21}$

32. $(x^{11})^5 = x^{11\cdot 5} = x^{55}$

33. $(x^{-5})^3 = x^{-5\cdot 3} = x^{-15} = \frac{1}{x^{15}}$

34. $(x^{-6})^4 = x^{-6\cdot 4} = x^{-24} = \frac{1}{x^{24}}$

35. $\frac{x^{14}}{x^7} = x^{14-7} = x^7$

36. $\frac{x^{30}}{x^{10}} = x^{30-10} = x^{20}$

37. $\frac{x^{14}}{x^{-7}} = x^{14-(-7)} = x^{14+7} = x^{21}$

38. $\frac{x^{30}}{x^{-10}} = x^{30-(-10)} = x^{30+10} = x^{40}$

39. $(8x^3)^2 = 8^2(x^3)^2 = 8^2 x^{3\cdot 2} = 64x^6$

40. $(6x^4)^2 = (6)^2(x^4)^2 = 6^2 x^{4\cdot 2} = 36x^8$

41. $\left(-\frac{4}{x}\right)^3 = \frac{(-4)^3}{x^3} = -\frac{64}{x^3}$

42. $\left(-\frac{6}{y}\right)^3 = \frac{(-6)^3}{y^3} = -\frac{216}{y^3}$

43.
$$\begin{aligned}(-3x^2y^5)^2 &= (-3)^2(x^2)^2\cdot(y^5)^2\\ &= 9x^{2\cdot 2}y^{5\cdot 2}\\ &= 9x^4y^{10}\end{aligned}$$

44.
$$\begin{aligned}(-3x^4y^6)^3 &= (-3)^3(x^4)^3(y^6)^3\\ &= -27x^{4\cdot 3}y^{6\cdot 3}\\ &= -27x^{12}y^{18}\end{aligned}$$

45. $(3x^4)(2x^7) = 3\cdot 2x^4\cdot x^7 = 6x^{4+7} = 6x^{11}$

46. $(11x^5)(9x^{12}) = 11\cdot 9x^5x^{12} = 99x^{5+12} = 99x^{17}$

47.
$$\begin{aligned}(-9x^3y)(-2x^6y^4) &= (-9)(-2)x^3x^6yy^4\\ &= 18x^{3+6}y^{1+4}\\ &= 18x^9y^5\end{aligned}$$

48.
$$\begin{aligned}(-5x^4y)(-6x^7y^{11}) &= (-5)(-6)x^4x^7yy^{11}\\ &= 30x^{4+7}y^{1+11}\\ &= 30x^{11}y^{12}\end{aligned}$$

49. $\frac{8x^{20}}{2x^4} = \left(\frac{8}{2}\right)\left(\frac{x^{20}}{x^4}\right) = 4x^{20-4} = 4x^{16}$

50. $\frac{20x^{24}}{10x^6} = \left(\frac{20}{10}\right)\left(\frac{x^{24}}{x^6}\right) = 2x^{24-6} = 2x^{18}$

51.
$$\begin{aligned}\frac{25a^{13}\cdot b^4}{-5a^2\cdot b^3} &= \left(\frac{25}{-5}\right)\left(\frac{a^{13}}{a^2}\right)\left(\frac{b^4}{b^3}\right)\\ &= -5a^{13-2}b^{4-3}\\ &= -5a^{11}b\end{aligned}$$

52.
$$\begin{aligned}\frac{35a^{14}b^6}{-7a^7b^3} &= \left(\frac{35}{-7}\right)\left(\frac{a^{14}}{a^7}\right)\left(\frac{b^6}{b^3}\right)\\ &= -5a^{14-7}b^{6-3}\\ &= -5a^7b^3\end{aligned}$$

53. $\frac{14b^7}{7b^{14}} = \left(\frac{14}{7}\right)\left(\frac{b^7}{b^{14}}\right) = 2\cdot b^{7-14} = 2b^{-7} = \frac{2}{b^7}$

54.
$$\begin{aligned}\frac{20b^{10}}{10b^{20}} &= \left(\frac{20}{10}\right)\left(\frac{b^{10}}{b^{20}}\right)\\ &= 2b^{10-20}\\ &= 2b^{-10}\\ &= \frac{2}{b^{10}}\end{aligned}$$

55.
$$\begin{aligned}(4x^3)^{-2} &= (4^{-2})(x^3)^{-2}\\ &= 4^{-2}x^{-6}\\ &= \frac{1}{4^2x^6}\\ &= \frac{1}{16x^6}\end{aligned}$$

56. $(10x^2)^{-3} = 10^{-3}x^{2\cdot(-3)} = 10^{-3}x^{-6} = \dfrac{1}{10^3x^6} = \dfrac{1}{1000x^6}$

57. $\dfrac{24x^3 \cdot y^5}{32x^7y^{-9}} = \dfrac{3}{4}x^{3-7}y^{5-(-9)} = \dfrac{3}{4}x^{-4}y^{14} = \dfrac{3y^{14}}{4x^4}$

58. $\dfrac{10x^4y^9}{30x^{12}y^{-3}} = \dfrac{1}{3}x^{4-12}y^{9-(-3)} = \dfrac{1}{3}x^{-8}y^{12} = \dfrac{y^{12}}{3x^8}$

59. $\left(\dfrac{5x^3}{y}\right)^{-2} = \dfrac{5^{-2}x^{-6}}{y^{-2}} = \dfrac{y^2}{25x^6}$

60. $\left(\dfrac{3x^4}{y}\right)^{-3} = \left(\dfrac{y}{3x^4}\right)^3 = \dfrac{y^3}{3^3x^{4\cdot 3}} = \dfrac{y^3}{27x^{12}}$

61. $\left(\dfrac{-15a^4b^2}{5a^{10}b^{-3}}\right)^3 = \left(\dfrac{-3b^{2-(-3)}}{a^{10-4}}\right)^3 = \left(\dfrac{-3b^5}{a^6}\right)^3 = \dfrac{-27b^{15}}{a^{18}}$

62. $\left(\dfrac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^3 = \left(\dfrac{-3b^{8-(-2)}}{a^{17-14}}\right)^3 = \left(\dfrac{-3b^{10}}{a^3}\right)^3 = \dfrac{-27b^{30}}{a^9}$

63. $\left(\dfrac{3a^{-5}b^2}{12a^3b^{-4}}\right)^0 = 1$

64. $\left(\dfrac{4a^{-5}b^3}{12a^3b^{-5}}\right)^0 = 1$

65. $3.8\times10^2 = 380$

66. $9.2\times10^2 = 920$

67. $6\times10^{-4} = 0.0006$

68. $7\times10^{-5} = 0.00007$

69. $-7.16\times10^6 = -7{,}160{,}000$

70. $-8.17\times10^6 = -8{,}170{,}000$

71. $7.9\times10^{-1} = 0.79$

72. $6.8\times10^{-1} = 0.68$

73. $-4.15\times10^{-3} = -0.00415$

74. $-3.14\times10^{-3} = -0.00314$

75. $-6.00001\times10^{10} = -60{,}000{,}100{,}000$

76. $-7.00001\times10^{10} = -70{,}000{,}100{,}000$

77. $32{,}000 = 3.2\times10^4$

78. $64{,}000 = 6.4\times10^4$

79. $638{,}000{,}000{,}000{,}000{,}000 = 6.38\times10^{17}$

80. $579{,}000{,}000{,}000{,}000{,}000 = 5.79\times10^{17}$

81. $-5716 = -5.716\times10^3$

82. $-3829 = -3.829\times10^3$

83. $0.0027 = 2.7\times10^{-3}$

84. $0.0083 = 8.3\times10^{-3}$

85. $-0.00000000504 = -5.04\times10^{-9}$

86. $-0.00000000405 = -4.05\times10^{-9}$

87. $\left(3\times10^4\right)\left(2.1\times10^3\right) = (3\times2.1)\left(10^4\times10^3\right)$
$= 6.3\times10^{4+3} = 6.3\times10^7$

88. $\left(2\times10^4\right)\left(4.1\times10^3\right) = 8.2\times10^7$

89. $\left(1.6\times10^{15}\right)\left(4\times10^{-11}\right) = (1.6\times4)\left(10^{15}\times10^{-11}\right)$
$= 6.4\times10^{15+(-11)}$
$= 6.4\times10^4$

90. $\left(1.4\times10^{15}\right)\left(3\times10^{-11}\right) = (1.4\times3)\left(10^{15}\times10^{-11}\right)$
$= 4.2\times10^{15+(-11)}$
$= 4.2\times10^4$

91. $\left(6.1\times10^{-8}\right)\left(2\times10^{-4}\right) = (6.1\times2)\left(10^{-8}\times10^{-4}\right)$
$= 12.2\times10^{-8+(-4)}$
$= 12.2\times10^{-12}$
$= 1.22\times10^{-11}$

92. $\left(5.1\times10^{-8}\right)\left(3\times10^{-4}\right) = 15.3\times10^{-12}$
$= 1.53\times10^{-11}$

93. $\left(4.3\times10^8\right)\left(6.2\times10^4\right)$
$= (4.3\times6.2)\left(10^8\times10^4\right)$
$= 26.66\times10^{8+4}$
$= 26.66\times10^{12}$
$= 2.666\times10^{13} \approx 2.67\times10^{13}$

94. $\left(8.2\times10^8\right)\left(4.6\times10^4\right)$
$= 37.72\times10^{8+4} = 37.72\times10^{12}$
$= 3.772\times10^{13} \approx 3.77\times10^{13}$

95. $\dfrac{8.4\times10^8}{4\times10^5} = \dfrac{8.4}{4}\times\dfrac{10^8}{10^5}$
$= 2.1\times10^{8-5} = 2.1\times10^3$

96. $\dfrac{6.9\times10^8}{3\times10^5} = 2.3\times10^{8-5} = 2.3\times10^3$

97. $\dfrac{3.6\times10^4}{9\times10^{-2}} = \dfrac{3.6}{9}\times\dfrac{10^4}{10^{-2}}$
$= 0.4\times10^{4-(-2)}$
$= 0.4\times10^6 = 4\times10^5$

98. $\dfrac{1.2\times10^4}{2\times10^{-2}} = 0.6\times10^{4-(-2)} = 0.6\times10^6$
$= \left(6\times10^{-1}\right)\times10^6 = 6\times10^5$

99. $\dfrac{4.8\times10^{-2}}{2.4\times10^6} = \dfrac{4.8}{2.4}\times\dfrac{10^{-2}}{10^6}$
$= 2\times10^{-2-6} = 2\times10^{-8}$

100. $\dfrac{7.5\times10^{-2}}{2.5\times10^6} = 3\times10^{-2-6} = 3\times10^{-8}$

101. $\dfrac{2.4\times10^{-2}}{4.8\times10^{-6}} = \dfrac{2.4}{4.8}\times\dfrac{10^{-2}}{10^{-6}}$
$= 0.5\times10^{-2-(-6)}$
$= 0.5\times10^4 = 5\times10^3$

102. $\dfrac{1.5\times10^{-2}}{5\times10^{-6}} = 0.5\times10^{-2-(-6)}$
$= 0.5\times10^4 = 5\times10^3$

103. $\dfrac{480{,}000{,}000{,}000}{0.00012} = \dfrac{4.8\times10^{11}}{1.2\times10^{-4}}$
$= \dfrac{4.8}{1.2}\times\dfrac{10^{11}}{10^{-4}}$
$= 4\times10^{11-(-4)}$
$= 4\times10^{15}$

104. $\dfrac{282{,}000{,}000{,}000}{0.00141} = \dfrac{2.82\times10^{11}}{1.41\times10^{-3}}$
$= 2\times10^{11-(-3)}$
$= 2\times10^{14}$

105. $\dfrac{0.00072\times0.003}{0.00024}$
$= \dfrac{\left(7.2\times10^{-4}\right)\left(3\times10^{-3}\right)}{2.4\times10^{-4}}$
$= \dfrac{7.2\times3}{2.4}\times\dfrac{10^{-4}\cdot10^{-3}}{10^{-4}} = 9\times10^{-3}$

106. $\dfrac{66000\times 0.001}{0.003\times 0.002}=\dfrac{\left(6.6\times 10^{4}\right)\left(1\times 10^{-3}\right)}{\left(3\times 10^{-3}\right)\left(2\times 10^{-3}\right)}$

$=\dfrac{6.6\times 10^{1}}{6\times 10^{-6}}=1.1\times 10^{1-(-6)}$

$=1.1\times 10^{7}$

107. $\dfrac{\left(x^{-2}y\right)^{-3}}{\left(x^{2}y^{-1}\right)^{3}}=\dfrac{x^{6}y^{-3}}{x^{6}y^{-3}}$

$=x^{6-6}y^{-3-(-3)}=x^{0}y^{0}=1$

108. $\dfrac{\left(xy^{-2}\right)^{-2}}{\left(x^{-2}y\right)^{-3}}=\dfrac{x^{-2}y^{4}}{x^{6}y^{-3}}$

$=x^{-2-6}y^{4-(-3)}=x^{-8}y^{7}=\dfrac{y^{7}}{x^{8}}$

109. $\left(2x^{-3}yz^{-6}\right)(2x)^{-5}=2x^{-3}yz^{-6}\cdot 2^{-5}x\cdot^{-5}$

$=2^{-4}x^{-8}yz^{-6}=\dfrac{y}{2^{4}x^{8}z^{6}}=\dfrac{y}{16x^{8}z^{6}}$

110. $\left(3x^{-4}yz^{-7}\right)(3x)^{-3}=3x^{-4}yz^{-7}\cdot 3^{-3}x\cdot^{-3}$

$=3^{-2}x^{-7}yz^{-7}=\dfrac{y}{3^{2}x^{7}z^{7}}=\dfrac{y}{9x^{7}z^{7}}$

111. $\left(\dfrac{x^{3}y^{4}z^{5}}{x^{-3}y^{-4}z^{-5}}\right)^{-2}=\left(x^{6}y^{8}z^{10}\right)^{-2}$

$=x^{-12}y^{-16}z^{-20}=\dfrac{1}{x^{12}y^{16}z^{20}}$

112. $\left(\dfrac{x^{4}y^{5}z^{6}}{x^{-4}y^{-5}z^{-6}}\right)^{-4}=\left(x^{8}y^{10}z^{12}\right)^{-4}$

$=x^{-32}y^{-40}z^{-48}=\dfrac{1}{x^{32}y^{40}z^{48}}$

113. $\dfrac{\left(2^{-1}x^{-2}y^{-1}\right)^{-2}\left(2x^{-4}y^{3}\right)^{-2}\left(16x^{-3}y^{3}\right)^{0}}{\left(2x^{-3}y^{-5}\right)^{2}}$

$=\dfrac{\left(2^{2}x^{2}y^{2}\right)\left(2^{-2}x^{8}y^{-6}\right)(1)}{\left(2^{2}x^{-6}y^{-10}\right)}$

$=\dfrac{x^{18}y^{6}}{4}$

114. $\dfrac{\left(2^{-1}x^{-3}y^{-1}\right)^{-2}\left(2x^{-6}y^{4}\right)^{-2}\left(9x^{3}y^{-3}\right)^{0}}{\left(2x^{-4}y^{-6}\right)^{2}}$

$=\dfrac{\left(2^{2}x^{6}y^{2}\right)\left(2^{-2}x^{12}y^{-8}\right)(1)}{\left(2^{2}x^{-8}y^{-12}\right)}$

$=\dfrac{x^{26}y^{6}}{4}$

115. **a.** 3.18×10^{12}

b. 3.20×10^{8}

c. $\dfrac{3.18\times 10^{12}}{3.20\times 10^{8}}=\dfrac{3.18}{3.20}\times\dfrac{10^{12}}{10^{8}}$

$\approx 0.9938\times 10^{4}$

≈ 9938

\$9938 per American

116. **a.** 3.02×10^{12}

b. 3.19×10^{8}

c. $\dfrac{3.02\times 10^{12}}{3.19\times 10^{8}}=\dfrac{3.02}{3.19}\times\dfrac{10^{12}}{10^{8}}$

$\approx 0.9467\times 10^{4}$

≈ 9467

\$9467 per American

117. **a.** 1.89×10^{13}

b. 6×10^{4}

c. $\dfrac{1.89\times 10^{13}}{6\times 10^{4}}=\dfrac{1.89}{6}\times\dfrac{10^{13}}{10^{4}}$

$=0.315\times 10^{9}$

$=3.15\times 10^{8}$

$=315,000,000$

315,00,000 Americans

118. **a.** 1.89×10^{13}

b. 2.54×10^{11}

c. $\dfrac{1.89\times 10^{13}}{2.54\times 10^{11}}=\dfrac{1.89}{2.54}\times\dfrac{10^{13}}{10^{11}}$

$\approx 0.74\times 10^{2}$

≈ 74

approximately 74 years

119. **a.** 1.09×10^{12}

b. 3.2×10^{7}

c. $\frac{1.09\times10^{12}}{3.2\times10^{7}}=\frac{1.09}{3.2}\times\frac{10^{12}}{10^{7}}$
$=0.340625\times10^{5}$
$=34,062.5$

34,062.5 years

120. – 128. Answers will vary.

129. does not make sense; Explanations will vary. Sample explanation: $36(x^3)^9=36x^{27}$ not $36x^{12}$.

130. makes sense

131. does not make sense; Explanations will vary. Sample explanation: 4.6×10^{12} represents over 4 trillion. The entire world population is measured in billions (10^9).

132. makes sense

133. false; Changes to make the statement true will vary. A sample change is: $4^{-2}>4^{-3}$.

134. true

135. false; Changes to make the statement true will vary. A sample change is: $(-2)^4\neq2^{-4}$ because $16\neq\frac{1}{16}$.

136. false; Changes to make the statement true will vary. A sample change is: $5^2\cdot5^{-2}=2^5\cdot2^{-5}$.

137. false; Changes to make the statement true will vary. A sample change is: $534.7\neq5347$.

138. false; Changes to make the statement true will vary. A sample change is:
$\frac{8\times10^{30}}{2\times10^{-5}}=4\times10^{30-(-5)}=4\times10^{35}$.

139. false; Changes to make the statement true will vary. A sample change is:
$(7\times10^5)+(2\times10^{-3})=700,000.002$.

140. true

141. The doctor has gathered:
$2^{-1}+2^{-2}=\frac{1}{2}+\frac{1}{2^2}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$

So, $1-\frac{3}{4}=\frac{1}{4}$ is remaining.

142. $b^A=MN, b^C=M, b^D=N$
$b^A=b^Cb^D$
$A=C+D$

143. $\frac{70\text{ bts}}{\text{min}}\cdot\frac{60\text{ min}}{\text{hr}}\cdot\frac{24\text{ hrs}}{\text{day}}\cdot\frac{365\text{ days}}{\text{yr}}\cdot80\text{ yrs}$
$=70\cdot60\cdot24\cdot365\cdot80$ beats
$=2943360000$ beats
$=2.94336\times10^9$ beats
$\approx2.94\times10^9$ beats

The heartbeats approximately 2.94×10^9 times over a lifetime of 80 years.

144. Answers will vary.

145. **a.** $\sqrt{16}\cdot\sqrt{4}=4\cdot2=8$

b. $\sqrt{16\cdot4}=\sqrt{64}=8$

c. $\sqrt{16}\cdot\sqrt{4}=\sqrt{16\cdot4}$

146. **a.** $\sqrt{300}\approx17.32$

b. $10\sqrt{3}\approx17.32$

c. $\sqrt{300}=10\sqrt{3}$

147. **a.** $21x+10x=31x$

b. $21\sqrt{2}+10\sqrt{2}=31\sqrt{2}$

Section P.3

Check Point Exercises

1. **a.** $\sqrt{81}=9$

b. $-\sqrt{9}=-3$

c. $\sqrt{\frac{1}{25}}=\frac{1}{5}$

d. $\sqrt{36+64}=\sqrt{100}=10$

e. $\sqrt{36}+\sqrt{64}=6+8=14$

2. **a.** $\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\sqrt{3}=5\sqrt{3}$

b.
$$\begin{aligned}\sqrt{5x}\cdot\sqrt{10x}&=\sqrt{5x\cdot 10x}\\&=\sqrt{50x^2}\\&=\sqrt{25\cdot 2x^2}\\&=\sqrt{25x^2}\cdot\sqrt{2}\\&=5x\sqrt{2}\end{aligned}$$

3. **a.** $\sqrt{\frac{25}{16}}=\frac{\sqrt{25}}{\sqrt{16}}=\frac{5}{4}$

b.
$$\begin{aligned}\frac{\sqrt{150x^3}}{\sqrt{2x}}&=\sqrt{\frac{150x^3}{2x}}\\&=\sqrt{75x^2}\\&=\sqrt{25x^2}\cdot\sqrt{3}\\&=5x\sqrt{3}\end{aligned}$$

4. **a.**
$$\begin{aligned}8\sqrt{13}+9\sqrt{13}&=(8+9)\sqrt{3}\\&=17\sqrt{13}\end{aligned}$$

b.
$$\begin{aligned}&\sqrt{17x}-20\sqrt{17x}\\&=1\sqrt{17x}-20\sqrt{17x}\\&=(1-20)\sqrt{17x}\\&=-19\sqrt{17x}\end{aligned}$$

5. **a.**
$$\begin{aligned}&5\sqrt{27}+\sqrt{12}\\&=5\sqrt{9\cdot 3}+\sqrt{4\cdot 3}\\&=5\cdot 3\sqrt{3}+2\sqrt{3}\\&=15\sqrt{3}+2\sqrt{3}\\&=(15+2)\sqrt{3}\\&=17\sqrt{3}\end{aligned}$$

b.
$$\begin{aligned}&6\sqrt{18x}-4\sqrt{8x}\\&=6\sqrt{9\cdot 2x}-4\sqrt{4\cdot 2x}\\&=6\cdot 3\sqrt{2x}-4\cdot 2\sqrt{2x}\\&=18\sqrt{2x}-8\sqrt{2x}\\&=(18-8)\sqrt{2x}\\&=10\sqrt{2x}\end{aligned}$$

6. **a.** If we multiply numerator and denominator by $\sqrt{3}$, the denominator becomes $\sqrt{3}\cdot\sqrt{3}=\sqrt{9}=3$. Therefore, multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{\sqrt{9}}=\frac{5\sqrt{3}}{3}$$

b. The *smallest* number that will produce a perfect square in the denominator of $\frac{6}{\sqrt{12}}$ is $\sqrt{3}$ because $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=6$. So multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{6}{\sqrt{12}}=\frac{6}{\sqrt{12}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{36}}=\frac{6\sqrt{3}}{6}=\sqrt{3}$$

7. Multiply by $\frac{4-\sqrt{5}}{4-\sqrt{5}}$.

$$\begin{aligned}\frac{8}{4+\sqrt{5}}&=\frac{8}{4+\sqrt{5}}\cdot\frac{4-\sqrt{5}}{4-\sqrt{5}}\\&=\frac{8(4-\sqrt{5})}{4^2-(\sqrt{5})^2}\\&=\frac{8(4-\sqrt{5})}{16-5}\\&=\frac{8(4-\sqrt{5})}{11}\text{ or }\frac{32-8\sqrt{5}}{11}\end{aligned}$$

8. **a.** $\sqrt[3]{40}=\sqrt[3]{8\cdot 5}=\sqrt[3]{8}\cdot\sqrt[3]{5}=2\sqrt[3]{5}$

b. $\sqrt[5]{8}\cdot\sqrt[5]{8}=\sqrt[5]{64}=\sqrt[5]{32}\cdot\sqrt[5]{2}=2\sqrt[5]{2}$

c. $\sqrt[3]{\frac{125}{27}}=\frac{\sqrt[3]{125}}{\sqrt[3]{27}}=\frac{5}{3}$

9. $3\sqrt[3]{81}-4\sqrt[3]{3}$
$=3\sqrt[3]{27\cdot 3}-4\sqrt[3]{3}$
$=3\cdot 3\sqrt[3]{3}-4\sqrt[3]{3}$
$=9\sqrt[3]{3}-4\sqrt[3]{3}$
$=(9-4)\sqrt[3]{3}$
$=5\sqrt[3]{3}$

10. a. $25^{\frac{1}{2}}=\sqrt{25}=5$

b. $8^{\frac{1}{3}}=\sqrt[3]{8}=2$

c. $-81^{\frac{1}{4}}=-\sqrt[4]{81}=-3$

d. $(-8)^{\frac{1}{3}}=\sqrt[3]{-8}=-2$

e. $27^{-\frac{1}{3}}=\dfrac{1}{27^{\frac{1}{3}}}=\dfrac{1}{\sqrt[3]{27}}=\dfrac{1}{3}$

11. a. $27^{\frac{4}{3}}=\left(\sqrt[3]{27}\right)^4=(3)^4=81$

b. $4^{\frac{3}{2}}=\left(\sqrt[2]{4}\right)^3=(2)^3=8$

c. $32^{-\frac{2}{5}}=\dfrac{1}{32^{\frac{2}{5}}}=\dfrac{1}{\left(\sqrt[5]{32}\right)^2}=\dfrac{1}{2^2}=\dfrac{1}{4}$

12. a. $\left(2x^{4/3}\right)\left(5x^{8/3}\right)$
$=2\cdot 5x^{4/3}\cdot x^{8/3}$
$=10x^{(4/3)+(8/3)}$
$=10x^{12/3}$
$=10x^4$

b. $\dfrac{20x^4}{5x^{3/2}}=\left(\dfrac{20}{5}\right)\left(\dfrac{x^4}{x^{3/2}}\right)$
$=4x^{4-(3/2)}$
$=4x^{(8/2)-(3/2)}$
$=4x^{5/2}$

13. $\sqrt[6]{x^3}=x^{3/6}=x^{1/2}=\sqrt{x}$

Concept and Vocabulary Check P.3

1. principal

2. 8^2

3. $|a|$

4. $\sqrt{a}\cdot\sqrt{b}$

5. $\dfrac{\sqrt{a}}{\sqrt{b}}$

6. $18\sqrt{3}$

7. 5; $6\sqrt{3}$

8. $7-\sqrt{3}$

9. $\sqrt{10}+\sqrt{2}$

10. index; radicand

11. $(-2)^5$

12. a; $|a|$

13. $\sqrt[n]{a}$

14. 2; 8

Exercise Set P.3

1. $\sqrt{36}=\sqrt{6^2}=6$

2. $\sqrt{25}=\sqrt{5^2}=5$

3. $-\sqrt{36}=-\sqrt{6^2}=-6$

4. $-\sqrt{25}=-\sqrt{5^2}=-5$

5. $\sqrt{-36}$, The square root of a negative number is not real.

6. $\sqrt{-25}$, The square root of a negative number is not real.

7. $\sqrt{25-16}=\sqrt{9}=3$

8. $\sqrt{144+25}=\sqrt{169}=13$

9. $\sqrt{25}-\sqrt{16}=5-4=1$

10. $\sqrt{144}+\sqrt{25}=12+5=17$

11. $\sqrt{(-13)^2}=\sqrt{169}=13$

12. $\sqrt{(-17)^2}=\sqrt{289}=17$

13. $\sqrt{50}=\sqrt{25\cdot 2}=\sqrt{25}\sqrt{2}=5\sqrt{2}$

14. $\sqrt{27}=\sqrt{9\cdot 3}=\sqrt{9}\sqrt{3}=3\sqrt{3}$

15.
$$\begin{aligned}\sqrt{45x^2}&=\sqrt{9x^2\cdot 5}\\&=\sqrt{9x^2}\sqrt{5}\\&=\sqrt{9}\sqrt{x^2}\sqrt{5}\\&=3|x|\sqrt{5}\end{aligned}$$

16.
$$\begin{aligned}\sqrt{125x^2}&=\sqrt{25x^2\cdot 5}\\&=\sqrt{25x^2}\sqrt{5}\\&=\sqrt{25}\sqrt{x^2}\sqrt{5}\\&=5|x|\sqrt{5}\end{aligned}$$

17.
$$\begin{aligned}\sqrt{2x}\cdot\sqrt{6x}&=\sqrt{2x\cdot 6x}\\&=\sqrt{12x^2}\\&=\sqrt{4x^2}\cdot\sqrt{3}\\&=2x\sqrt{3}\end{aligned}$$

18.
$$\begin{aligned}\sqrt{10x}\cdot\sqrt{8x}&=\sqrt{10x\cdot 8x}\\&=\sqrt{80x^2}\\&=\sqrt{16x^2}\cdot\sqrt{5}\\&=4x\sqrt{5}\end{aligned}$$

19. $\sqrt{x^3}=\sqrt{x^2}\cdot\sqrt{x}=x\sqrt{x}$

20. $\sqrt{y^3}=\sqrt{y^2}\cdot\sqrt{y}=y\sqrt{y}$

21.
$$\begin{aligned}\sqrt{2x^2}\cdot\sqrt{6x}&=\sqrt{2x^2\cdot 6x}\\&=\sqrt{12x^3}\\&=\sqrt{4x^2}\cdot\sqrt{3x}\\&=2x\sqrt{3x}\end{aligned}$$

22.
$$\begin{aligned}\sqrt{6x}\cdot\sqrt{3x^2}&=\sqrt{6x\cdot 3x^2}\\&=\sqrt{18x^3}\\&=\sqrt{9x^2}\cdot\sqrt{2x}\\&=3x\sqrt{2x}\end{aligned}$$

23. $\sqrt{\dfrac{1}{81}}=\dfrac{\sqrt{1}}{\sqrt{81}}=\dfrac{1}{9}$

24. $\sqrt{\dfrac{1}{49}}=\dfrac{\sqrt{1}}{\sqrt{49}}=\dfrac{1}{7}$

25. $\sqrt{\dfrac{49}{16}}=\dfrac{\sqrt{49}}{\sqrt{16}}=\dfrac{7}{4}$

26. $\sqrt{\dfrac{121}{9}}=\dfrac{\sqrt{121}}{\sqrt{9}}=\dfrac{11}{3}$

27. $\dfrac{\sqrt{48x^3}}{\sqrt{3x}}=\sqrt{\dfrac{48x^3}{3x}}=\sqrt{16x^2}=4x$

28. $\dfrac{\sqrt{72x^3}}{\sqrt{8x}}=\sqrt{\dfrac{72x^3}{8x}}=\sqrt{9x^2}=3x$

29.
$$\begin{aligned}\frac{\sqrt{150x^4}}{\sqrt{3x}}&=\sqrt{\frac{150x^4}{3x}}\\&=\sqrt{50x^3}\\&=\sqrt{25x^2}\cdot\sqrt{2x}\\&=5x\sqrt{2x}\end{aligned}$$

30.
$$\begin{aligned}\frac{\sqrt{24x^4}}{\sqrt{3x}}&=\sqrt{\frac{24x^4}{3x}}\\&=\sqrt{8x^3}\\&=\sqrt{4x^2}\cdot\sqrt{2x}\\&=2x\sqrt{2x}\end{aligned}$$

31.
$$\begin{aligned}\frac{\sqrt{200x^3}}{\sqrt{10x^{-1}}}&=\sqrt{\frac{200x^3}{10x^{-1}}}\\&=\sqrt{20x^{3-(-1)}}\\&=\sqrt{20x^4}\\&=\sqrt{4\cdot 5x^4}\\&=2x^2\sqrt{5}\end{aligned}$$

32.
$$\begin{aligned}\frac{\sqrt{500x^3}}{\sqrt{10x^{-1}}}&=\sqrt{\frac{500x^3}{10x^{-1}}}=\sqrt{50x^{3-(-1)}}\\&=\sqrt{50x^4}=\sqrt{25\cdot 2x^4}=5x^2\sqrt{2}\end{aligned}$$

33. $7\sqrt{3}+6\sqrt{3}=(7+6)\sqrt{3}=13\sqrt{3}$

34. $8\sqrt{5}+11\sqrt{5}=(8+11)\sqrt{5}=19\sqrt{5}$

35. $6\sqrt{17x} - 8\sqrt{17x} = (6-8)\sqrt{17x} = -2\sqrt{17x}$

36. $4\sqrt{13x} - 6\sqrt{13x} = (4-6)\sqrt{13x} = -2\sqrt{13x}$

37.
$$\begin{aligned}\sqrt{8} + 3\sqrt{2} &= \sqrt{4\cdot 2} + 3\sqrt{2}\\ &= 2\sqrt{2} + 3\sqrt{2}\\ &= (2+3)\sqrt{2}\\ &= 5\sqrt{2}\end{aligned}$$

38.
$$\begin{aligned}\sqrt{20} + 6\sqrt{5} &= \sqrt{4\cdot 5} + 6\sqrt{5}\\ &= 2\sqrt{5} + 6\sqrt{5}\\ &= (2+6)\sqrt{5}\\ &= 8\sqrt{5}\end{aligned}$$

39.
$$\begin{aligned}\sqrt{50x} - \sqrt{8x} &= \sqrt{25\cdot 2x} - \sqrt{4\cdot 2x}\\ &= 5\sqrt{2x} - 2\sqrt{2x}\\ &= (5-2)\sqrt{2x}\\ &= 3\sqrt{2x}\end{aligned}$$

40.
$$\begin{aligned}\sqrt{63x} - \sqrt{28x} &= \sqrt{9\cdot 7x} - \sqrt{4\cdot 7x}\\ &= 3\sqrt{7x} - 2\sqrt{7x}\\ &= (3-2)\sqrt{7x}\\ &= \sqrt{7x}\end{aligned}$$

41.
$$\begin{aligned}3\sqrt{18} + 5\sqrt{50} &= 3\sqrt{9\cdot 2} + 5\sqrt{25\cdot 2}\\ &= 3\cdot 3\sqrt{2} + 5\cdot 5\sqrt{2}\\ &= 9\sqrt{2} + 25\sqrt{2}\\ &= (9+25)\sqrt{2}\\ &= 34\sqrt{2}\end{aligned}$$

42.
$$\begin{aligned}4\sqrt{12} - 2\sqrt{75} &= 4\sqrt{4\cdot 3} - 2\sqrt{25\cdot 3}\\ &= 4\cdot 2\sqrt{3} - 2\cdot 5\sqrt{3}\\ &= 8\sqrt{3} - 10\sqrt{3}\\ &= (8-10)\sqrt{3}\\ &= -2\sqrt{3}\end{aligned}$$

43.
$$\begin{aligned}&3\sqrt{8} - \sqrt{32} + 3\sqrt{72} - \sqrt{75}\\ &= 3\sqrt{4\cdot 2} - \sqrt{16\cdot 2} + 3\sqrt{36\cdot 2} - \sqrt{25\cdot 3}\\ &= 3\cdot 2\sqrt{2} - 4\sqrt{2} + 3\cdot 6\sqrt{2} - 5\sqrt{3}\\ &= 6\sqrt{2} - 4\sqrt{2} + 18\sqrt{2} - 5\sqrt{3}\\ &= 20\sqrt{2} - 5\sqrt{3}\end{aligned}$$

44.
$$\begin{aligned}&3\sqrt{54} - 2\sqrt{24} - \sqrt{96} + 4\sqrt{63}\\ &= 3\sqrt{9\cdot 6} - 2\sqrt{4\cdot 6} - \sqrt{16\cdot 6} + 4\sqrt{9\cdot 7}\\ &= 3\cdot 3\sqrt{6} - 2\cdot 2\sqrt{6} - 4\sqrt{6} + 4\cdot 3\sqrt{7}\\ &= 9\sqrt{6} - 4\sqrt{6} - 4\sqrt{6} + 12\sqrt{7}\\ &= \sqrt{6} + 12\sqrt{7}\end{aligned}$$

45. $\dfrac{1}{\sqrt{7}} = \dfrac{1}{\sqrt{7}}\cdot\dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{\sqrt{7}}{7}$

46. $\dfrac{2}{\sqrt{10}} = \dfrac{2}{\sqrt{10}}\cdot\dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{2\sqrt{10}}{10} = \dfrac{\sqrt{10}}{5}$

47. $\dfrac{\sqrt{2}}{\sqrt{5}} = \dfrac{\sqrt{2}}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{10}}{5}$

48. $\dfrac{\sqrt{7}}{\sqrt{3}} = \dfrac{\sqrt{7}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{21}}{3}$

49.
$$\begin{aligned}\frac{13}{3+\sqrt{11}} &= \frac{13}{3+\sqrt{11}}\cdot\frac{3-\sqrt{11}}{3-\sqrt{11}}\\ &= \frac{13(3-\sqrt{11})}{3^2-(\sqrt{11})^2}\\ &= \frac{13(3-\sqrt{11})}{9-11}\\ &= \frac{13(3-\sqrt{11})}{-2}\end{aligned}$$

50.
$$\begin{aligned}\frac{3}{3+\sqrt{7}} &= \frac{3}{3+\sqrt{7}}\cdot\frac{3-\sqrt{7}}{3-\sqrt{7}}\\ &= \frac{3(3-\sqrt{7})}{3^2-(\sqrt{7})^2}\\ &= \frac{3(3-\sqrt{7})}{9-7}\\ &= \frac{3(3-\sqrt{7})}{2}\end{aligned}$$

51.
$$\begin{aligned}\frac{7}{\sqrt{5}-2} &= \frac{7}{\sqrt{5}-2}\cdot\frac{\sqrt{5}+2}{\sqrt{5}+2}\\ &= \frac{7(\sqrt{5}+2)}{(\sqrt{5})^2-2^2}\\ &= \frac{7(\sqrt{5}+2)}{5-4}\\ &= 7(\sqrt{5}+2)\end{aligned}$$

52.
$$\begin{aligned}\frac{5}{\sqrt{3}-1} &= \frac{5}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}\\ &= \frac{5(\sqrt{3}+1)}{(\sqrt{3})^2-1^2}\\ &= \frac{5(\sqrt{3}+1)}{3-1}\\ &= \frac{5(\sqrt{3}+1)}{2}\end{aligned}$$

53. $\frac{6}{\sqrt{5}+\sqrt{3}} = \frac{6}{\sqrt{5}+\sqrt{3}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$
$= \frac{6(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$
$= \frac{6(\sqrt{5}-\sqrt{3})}{5-3}$
$= \frac{6(\sqrt{5}-\sqrt{3})}{2}$
$= 3(\sqrt{5}-\sqrt{3})$

54. $\frac{11}{\sqrt{7}-\sqrt{3}} = \frac{11}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$
$= \frac{11(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2-(\sqrt{3})^2}$
$= \frac{11(\sqrt{7}+\sqrt{3})}{7-3}$
$= \frac{11(\sqrt{7}+\sqrt{3})}{4}$

55. $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

56. $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

57. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

58. $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

59. $\sqrt[4]{-16}$ is not a real number.

60. $\sqrt[4]{-81}$ is not a real number.

61. $\sqrt[4]{(-3)^4} = |-3| = 3$

62. $\sqrt[4]{(-2)^4} = |-2| = 2$

63. $\sqrt[5]{(-3)^5} = -3$

64. $\sqrt[5]{(-2)^5} = -2$

65. $\sqrt[5]{-\frac{1}{32}} = \sqrt[5]{-\frac{1}{2^5}} = -\frac{1}{2}$

66. $\sqrt[6]{\frac{1}{64}} = \frac{\sqrt[6]{1}}{\sqrt[6]{2^6}} = \frac{1}{2}$

67. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8}\,\sqrt[3]{4} = 2 \cdot \sqrt[3]{4}$

68. $\sqrt[3]{150}$ cannot be simplified further.

69. $\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = x \cdot \sqrt[3]{x}$

70. $\sqrt[3]{x^5} = \sqrt[3]{x^3 x^2} = x\sqrt[3]{x^2}$

71. $\sqrt[3]{9} \cdot \sqrt[3]{6} = \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} = 3\sqrt[3]{2}$

72. $\sqrt[3]{12} \cdot \sqrt[3]{4} = \sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = 2\sqrt[3]{6}$

73. $\frac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}} = \sqrt[5]{\frac{64x^6}{2x}} = \sqrt[5]{32x^5} = 2x$

74. $\frac{\sqrt[4]{162x^5}}{\sqrt[4]{2x}} = \sqrt[4]{\frac{162x^5}{2x}} = \sqrt[4]{81x^4} = 3x$

75. $4\sqrt[5]{2} + 3\sqrt[5]{2} = 7\sqrt[5]{2}$

76. $6\sqrt[5]{3} + 2\sqrt[5]{3} = 8\sqrt[5]{3}$

77. $5\sqrt[3]{16} + \sqrt[3]{54} = 5\sqrt[3]{8 \cdot 2} + \sqrt[3]{27 \cdot 2}$
$= 5 \cdot 2\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 10\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 13\sqrt[3]{2}$

78. $3\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{8 \cdot 3} + \sqrt[3]{27 \cdot 3}$
$= 3 \cdot 2\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 6\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 9\sqrt[3]{3}$

79. $\sqrt[3]{54xy^3} - y\sqrt[3]{128x}$
$= \sqrt[3]{27 \cdot 2xy^3} - y\sqrt[3]{64 \cdot 2x}$
$= 3y\sqrt[3]{2x} - 4y\sqrt[3]{2x}$
$= -y\sqrt[3]{2x}$

80. $\sqrt[3]{24xy^3} - y\sqrt[3]{81x}$
$= \sqrt[3]{8 \cdot 3xy^3} - y\sqrt[3]{27 \cdot 3x}$
$= 2y\sqrt[3]{3x} - 3y\sqrt[3]{3x}$
$= -y\sqrt[3]{3x}$

81. $\sqrt{2} + \sqrt[3]{8} = \sqrt{2} + 2$

82. $\sqrt{3} + \sqrt[3]{15}$ will not simplify.

83. $36^{1/2} = \sqrt{36} = 6$

84. $121^{1/2} = \sqrt{121} = 11$

85. $8^{1/3} = \sqrt[3]{8} = 2$

86. $27^{1/3} = \sqrt[3]{27} = 3$

87. $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

88. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4$

89. $32^{-4/5} = \dfrac{1}{32^{4/5}} = \dfrac{1}{2^4} = \dfrac{1}{16}$

90. $16^{-5/2} = \dfrac{1}{16^{5/2}} = \dfrac{1}{(\sqrt{16})^5} = \dfrac{1}{4^5} = \dfrac{1}{1024}$

91. $\left(7x^{1/3}\right)\left(2x^{1/4}\right) = 7 \cdot 2x^{1/3} \cdot x^{1/4}$
$= 14 \cdot x^{1/3+1/4}$
$= 14x^{7/12}$

92. $(3x^{2/3})(4x^{3/4}) = 3 \cdot 4x^{2/3} \cdot x^{3/4}$
$= 12 \cdot x^{2/3+3/4}$
$= 12x^{17/12}$

93. $\dfrac{20x^{1/2}}{5x^{1/4}} = \left(\dfrac{20}{5}\right)\left(\dfrac{x^{1/2}}{x^{1/4}}\right)$
$= 4 \cdot x^{1/2-1/4}$
$= 4x^{1/4}$

94. $\dfrac{72x^{3/4}}{9x^{1/3}} = \left(\dfrac{72}{9}\right)\left(\dfrac{x^{3/4}}{x^{1/3}}\right) = 8 \cdot x^{3/4-1/3} = 8x^{5/12}$

95. $\left(x^{2/3}\right)^3 = x^{2/3 \cdot 3} = x^2$

96. $(x^{4/5})^5 = x^{4/5 \cdot 5} = x^4$

97. $(25x^4y^6)^{1/2} = 25^{1/2}x^{4 \cdot 1/2}y^{6 \cdot 1/2} = 5x^2\left|y\right|^3$

98. $(125x^9y^6)^{1/3} = 125^{1/3}x^{9/3}y^{6/3} = 5x^3y^2$

99. $\dfrac{\left(3y^{\frac{1}{4}}\right)^3}{y^{\frac{1}{12}}} = \dfrac{27y^{\frac{3}{4}}}{y^{\frac{1}{12}}} = 27y^{\frac{3}{4}-\frac{1}{12}}$
$= 27y^{\frac{8}{12}} = 27y^{\frac{2}{3}}$

100. $\dfrac{\left(2y^{1/5}\right)^4}{y^{3/10}} = \dfrac{2^4\left(y^{1/5}\right)^4}{y^{3/10}}$
$= \dfrac{16y^{4/5}}{y^{3/10}} = 16y^{4/5-3/10} = 16y^{1/2}$

101. $\sqrt[4]{5^2} = 5^{2/4} = 5^{1/2} = \sqrt{5}$

102. $\sqrt[4]{7^2} = 7^{2/4} = 7^{1/2} = \sqrt{7}$

103. $\sqrt[3]{x^6} = x^{6/3} = x^2$

104. $\sqrt[4]{x^{12}} = x^{12/4} = |x|^3$

105. $\sqrt[6]{x^4} = \sqrt[6/2]{x^{4/2}} = \sqrt[3]{x^2}$

106. $\sqrt[9]{x^6} = \sqrt[9/3]{x^{6/3}} = \sqrt[3]{x^2}$

107. $\sqrt[9]{x^6y^3} = x^{\frac{6}{9}}y^{\frac{3}{9}} = x^{\frac{2}{3}}y^{\frac{1}{3}} = \sqrt[3]{x^2y}$

108. $\sqrt[12]{x^4y^8} = |x|^{\frac{4}{12}}|y|^{\frac{8}{12}} = |x|^{\frac{1}{3}}|y|^{\frac{2}{3}} = \sqrt[3]{|x|y^2}$

109. $\sqrt[3]{\sqrt[4]{16} + \sqrt{625}} = \sqrt[3]{2+25} = \sqrt[3]{27} = 3$

110. $\sqrt[3]{\sqrt{\sqrt{169}+\sqrt{9}} + \sqrt{\sqrt[3]{1000}+\sqrt[3]{216}}}$
$= \sqrt[3]{\sqrt{13+3} + \sqrt{10+6}}$
$= \sqrt[3]{\sqrt{16}+\sqrt{16}}$
$= \sqrt[3]{4+4} = \sqrt[3]{8}$
$= 2$

111. $\left(49x^{-2}y^4\right)^{-1/2}\left(xy^{1/2}\right)$
$= (49)^{-1/2}\left(x^{-2}\right)^{-1/2}\left(y^4\right)^{-1/2}\left(xy^{1/2}\right)$
$= \dfrac{1}{49^{1/2}}x^{(-2)(-1/2)}y^{(4)(-1/2)}\left(xy^{1/2}\right)$
$= \dfrac{1}{7}x^1y^{-2} \cdot xy^{1/2} = \dfrac{1}{7}x^{1+1}y^{-2+(1/2)}$
$= \dfrac{1}{7}x^2y^{-3/2} = \dfrac{x^2}{7y^{3/2}}$

112. $\left(8x^{-6}y^3\right)^{1/3}\left(x^{5/6}y^{-1/3}\right)^6$
$= 8^{1/3}x^{(-6)(1/3)}y^{(3)(1/3)}x^{(5/6)(6)}y^{(-1/3)(6)}$
$= 2x^{-2}y^1x^5y^{-2} = 2x^{-2+5}y^{1+(-2)}$
$= 2x^3y^{-1} = \dfrac{2x^3}{y}$

113. $\left(\dfrac{x^{-5/4}y^{1/3}}{x^{-3/4}}\right)^{-6} = \left(x^{(-5/4)-(-3/4)}y^{1/3}\right)^{-6}$
$= \left(x^{-2/4}y^{1/3}\right)^{-6} = x^{(-2/4)(-6)}y^{(1/3)(-6)}$
$= x^3y^{-2} = \dfrac{x^3}{y^2}$

114. $\left(\dfrac{x^{1/2}y^{-7/4}}{y^{-5/4}}\right)^{-4} = \left(x^{1/2}y^{(-7/4)-(-5/4)}\right)^{-4}$
$= \left(x^{1/2}y^{-2/4}\right)^{-4} = x^{(1/2)(-4)}y^{(-2/4)(-4)}$
$= x^{-2}y^2 = \dfrac{y^2}{x^2}$

115. The message is "Paige Fox is bad at math."

116. **a.** For 2030: $E = 5.8\sqrt{x} + 56.4$
$= 5.8\sqrt{10} + 56.4$

For 2060: $E = 5.8\sqrt{x} + 56.4$
$= 5.8\sqrt{40} + 56.4$
$= 5.8 \cdot 2\sqrt{10} + 56.4$
$= 11.6\sqrt{10} + 56.4$

Difference:
$(11.6\sqrt{10} + 56.4) - (5.8\sqrt{10} + 56.4)$
$= 11.6\sqrt{10} + 56.4 - 5.8\sqrt{10} - 56.4$
$= 11.6\sqrt{10} - 5.8\sqrt{10} + 56.4 - 56.4$
$= 5.8\sqrt{10}$

The difference is $5.8\sqrt{10}$.

b. $5.8\sqrt{10} \approx 18.3$
This underestimates the difference projected by the graph of $98.2 - 74.1 = 24.1$ by 5.8. This represents a difference of 5.8 million people.

117. $\dfrac{2}{\sqrt{5}-1} \cdot \dfrac{\sqrt{5}+1}{\sqrt{5}+1} = \dfrac{2(\sqrt{5}+1)}{5-1}$
$= \dfrac{2(\sqrt{5}+1)}{4}$
$= \dfrac{\sqrt{5}+1}{2}$
≈ 1.62

About 1.62 to 1.

118. $R_a = R_f\sqrt{1-\left(\dfrac{v}{c}\right)^2}$
$= R_f\sqrt{1-\left(\dfrac{0.9c}{c}\right)^2}$
$= R_f\sqrt{1-(0.9)^2}$
$= R_f\sqrt{0.19}$
$\approx 0.44R_f$

$R_a = 0.44R_f$
$44 = 0.44R_f$
$\dfrac{44}{0.44} = \dfrac{0.44R_f}{0.44}$
$100 = R_f$

If you are gone for 44 weeks, then 100 weeks will have passed for your friend.

119. Perimeter:
$P = 2l + 2w$
$= 2 \cdot \sqrt{125} + 2 \cdot 2\sqrt{20}$
$= 2 \cdot \sqrt{25 \cdot 5} + 4\sqrt{4 \cdot 5}$
$= 2 \cdot 5\sqrt{5} + 4 \cdot 2\sqrt{5}$
$= 10\sqrt{5} + 8\sqrt{5}$
$= 18\sqrt{5}$ feet

Area:
$A = lw$
$= \sqrt{125} \cdot 2\sqrt{20}$
$= 2\sqrt{125 \cdot 20}$
$= 2\sqrt{2500}$
$= 2 \cdot 50$
$= 100$ square feet

120. Perimeter:
$P = 2l + 2w$
$= 2 \cdot 4\sqrt{20} + 2 \cdot \sqrt{80}$
$= 8\sqrt{4 \cdot 5} + 2\sqrt{16 \cdot 5}$
$= 8 \cdot 2\sqrt{5} + 2 \cdot 4\sqrt{5}$
$= 16\sqrt{5} + 8\sqrt{5}$
$= 24\sqrt{5}$ feet

Area:
$A = lw$
$= 4\sqrt{20} \cdot \sqrt{80}$
$= 4\sqrt{20 \cdot 80}$
$= 4\sqrt{1600}$
$= 4 \cdot 40$
$= 160$ square feet

121. – 128. Answers will vary.

129. does not make sense; Explanations will vary. Sample explanation: The denominator is rationalized correctly.

130. makes sense

131. does not make sense; Explanations will vary. Sample explanation: $2\sqrt{20}+4\sqrt{75}$ simplifies to $4\sqrt{5}+20\sqrt{3}$ and thus the radical terms are not common.

132. does not make sense; Explanations will vary. Sample explanation: Finding the *n*th root first often gives smaller numbers on the middle step.

133. false; Changes to make the statement true will vary. A sample change is: $7^{1/2}\cdot 7^{1/2}=7^1=7$.

134. false; Changes to make the statement true will vary. A sample change is: $(8)^{-\frac{1}{3}}=\dfrac{1}{(8)^{\frac{1}{3}}}=\dfrac{1}{\sqrt[3]{8}}=\dfrac{1}{2}$.

135. false; Changes to make the statement true will vary. The cube root of –8 is the real number –2.

136. false; Changes to make the statement true will vary. A sample change is: $\dfrac{\sqrt{20}}{8}=\dfrac{\sqrt{5}}{4}$.

137. $\left(5+\sqrt{\boxed{3}}\right)\left(5-\sqrt{\boxed{3}}\right)=22$

$$25-\boxed{3}=22$$
$$\boxed{3}=3$$

138. $\sqrt{\boxed{25}x^{\boxed{14}}}=5x^7$

139. $\sqrt{13+\sqrt{2}+\dfrac{7}{3+\sqrt{2}}}$

$$\begin{aligned}&=\sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}}\cdot\frac{3-\sqrt{2}}{3-\sqrt{2}}}\\&=\sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{9-2}}\\&=\sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{7}}\\&=\sqrt{13+\sqrt{2}+3-\sqrt{2}}\\&=\sqrt{16}\\&=4\end{aligned}$$

140. a. $3^{\frac{1}{2}}\boxed{>}3^{\frac{1}{3}}$

Calculator Check: $1.7321>1.4422$

b. $\sqrt{7}+\sqrt{18}\boxed{>}\sqrt{7+18}$

Calculator Check: $6.8884>5$

141. a. $\frac{ab}{a^2+ab+b^2}+\left(\frac{ac-ad-bc+bd}{ac-ad+bc-bd}\div\frac{a^3-b^3}{a^3+b^3}\right)=\frac{ab}{a^2+ab+b^2}+\left(\frac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)}\cdot\frac{a^3+b^3}{a^3-b^3}\right)$

$=\frac{ab}{a^2+ab+b^2}+\left(\frac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}}\cdot\frac{\cancel{(a+b)}(a^2-ab+b^2)}{\cancel{(a-b)}(a^2+ab+b^2)}\right)=\frac{ab}{a^2+ab+b^2}+\frac{a^2-ab+b^2}{a^2+ab+b^2}$

$=\frac{ab+a^2-ab+b^2}{a^2+ab+b^2}=\frac{a^2+b^2}{a^2+ab+b^2}$

Her son is 8 years old.

b. Son's portion:

$$\frac{8^{-\frac{4}{3}}+2^{-2}}{16^{-\frac{3}{4}}+2^{-1}}=\frac{\frac{1}{\left(\sqrt[3]{8}\right)^4}+\frac{1}{2^2}}{\frac{1}{\left(\sqrt[4]{16}\right)^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{2^4}+\frac{1}{4}}{\frac{1}{2^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{16}+\frac{1}{4}}{\frac{1}{8}+\frac{1}{2}}$$

$$=\frac{\frac{5}{16}}{\frac{5}{8}}$$

$$=\frac{8}{16}$$

$$=\frac{1}{2}$$

Mom's portion:

$$\frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}$$

142. $(2x^3y^2)(5x^4y^7)=10x^7y^9$

143. $2x^4(8x^4+3x)=2x^4(8x^4)+2x^4(3x)$
$=16x^8+6x^5$

144. $2x(x^2+4x+5)+3(x^2+4x+5)$
$=2x^3+8x^2+10x+3x^2+12x+15$
$=2x^3+8x^2+3x^2+10x+12x+15$
$=2x^3+11x^2+22x+15$

Section P.4

Check Point Exercises

1. **a.** $(-17x^3+4x^2-11x-5)+(16x^3-3x^2+3x-15)$
$=(-17x^3+16x^3)+(4x^2-3x^2)+(-11x+3x)+(-5-15)$
$=-x^3+x^2-8x-20$

b. $(13x^2-9x^2-7x+1)-(-7x^3+2x^2-5x+9)$
$=(13x^3-9x^2-7x+1)+(7x^3-2x^2+5x-9)$
$=(13x^3+7x^3)+(-9x^2-2x^2)+(-7x+5x)+(1-9)$
$=20x^3-11x^2-2x-8$

2. $(5x-2)(3x^2-5x+4)$
$=5x(3x^2-5x+4)-2(3x^2-5x+4)$
$=5x\cdot 3x^2-5x\cdot 5x+5x\cdot 4-2\cdot 3x^2+2\cdot 5x-2\cdot 4$
$=15x^3-25x^2+20x-6x^2+10x-8$
$=15x^3-31x^2+30x-8$

3. $(7x-5)(4x-3)=7x\cdot 4x+7x(-3)+(-5)4x+(-5)(-3)$
$=28x^2-21x-20x+15$
$=28x^2-41x+15$

4. **a.** $(7x-6y)(3x-y)=(7x)(3x)+(7x)(-y)+(-6y)(3x)+(-6y)(-y)$
$=21x^2-7xy-18xy+6y^2$
$=21x^2-25xy+6y^2$

b. $(2x+4y)^2=(2x)^2+2(2x)(4y)+(4y)^2$
$=4x^2+16xy+16y^2$

5. **a.** $(3x+2+5y)(3x+2-5y)=(3x+2)^2-(5y)^2$
$=9x^2+12x+4-25y^2$
$=9x^2+12x-25y^2+4$

b. $(2x+y+3)^2=(2x+y)^2+2(2x+y)(3)+3^2$
$=4x^2+4xy+y^2+12x+6y+9$
$=4x^2+4xy+12x+y^2+6y+9$

Concept and Vocabulary Check P.4

1. whole

2. standard

3. monomial

4. binomial

5. trinomial

6. n

7. like;

8. distributive; $4x^3 - 8x^2 + 6$; $7x^3$

9. $5x$; 3; like

10. $3x^2$; $5x$; $21x$; 35

11. $A^2 - B^2$; minus

12. $A^2 + 2AB + B^2$; squared; product of the terms; squared

13. $A^2 - 2AB + B^2$; minus; product of the terms; plus

14. $n + m$

Exercise Set P.4

1. yes; $2x + 3x^2 - 5 = 3x^2 + 2x - 5$

2. no; The term $3x^{-1}$ does not have a whole number exponent.

3. no; The form of a polynomial involves addition and subtraction, not division.

4. yes; $x^2 - x^3 + x^4 - 5 = x^4 - x^3 + x^2 - 5$

5. $3x^2$ has degree 2
$-5x$ has degree 1
4 has degree 0
$3x^2 - 5x + 4$ has degree 2.

6. $-4x^3$ has degree 3
$7x^2$ has degree 2
-11 has degree 0
$-4x^3 + 7x^2 - 11$ has degree 3.

7. x^2 has degree 2
$-4x^3$ has degree 3
$9x$ has degree 1
$-12x^4$ has degree 4
63 has degree 0
$x^2 - 4x^3 + 9x - 12x^4 + 63$ has degree 4.

8. x^2 has degree 2
$-8x^3$ has degree 3
$15x^4$ has degree 4
91 has degree 0
$x^2 - 8x^3 + 15x^4 + 91$ has degree 4.

9. $$\begin{aligned}(-6x^3 + 5x^2 - 8x + 9) + (17x^3 + 2x^2 - 4x - 13) &= (-6x^3 + 17x^3) + (5x^2 + 2x^2) + (-8x - 4x) + (9 - 13) \\ &= 11x^3 + 7x^2 - 12x - 4\end{aligned}$$
The degree is 3.

10. $$\begin{aligned}(-7x^3 + 6x^2 - 11x + 13) + (19x^3 - 11x^2 + 7x - 17) &= (-7x^3 + 19x^3) + (6x^2 - 11x^2) + (-11x + 7x) + (13 - 17) \\ &= 12x^3 - 5x^2 - 4x - 4\end{aligned}$$
The degree is 3.

11. $$\begin{aligned}(17x^3 - 5x^2 + 4x - 3) - (5x^3 - 9x^2 - 8x + 11) &= (17x^3 - 5x^2 + 4x - 3) + (-5x^3 + 9x^2 + 8x - 11) \\ &= (17x^3 - 5x^3) + (-5x^2 + 9x^2) + (4x + 8x) + (-3 - 11) \\ &= 12x^3 + 4x^2 + 12x - 14\end{aligned}$$
The degree is 3.

12. $$\begin{aligned}(18x^4 - 2x^3 - 7x + 8) - (9x^4 - 6x^3 - 5x + 7) &= (18x^4 - 2x^3 - 7x + 8) + (-9x^4 + 6x^3 + 5x - 7) \\ &= (18x^4 - 9x^4) + (-2x^3 + 6x^3) + (-7x + 5x) + (8 - 7) \\ &= 9x^4 + 4x^3 - 2x + 1\end{aligned}$$
The degree is 4.

13. $$\begin{aligned}(5x^2 - 7x - 8) + (2x^2 - 3x + 7) - (x^2 - 4x - 3) &= (5x^2 - 7x - 8) + (2x^2 - 3x + 7) + (-x^2 + 4x + 3) \\ &= (5x^2 + 2x^2 - x^2) + (-7x - 3x + 4x) + (-8 + 7 + 3) \\ &= 6x^2 - 6x + 2\end{aligned}$$
The degree is 2.

14. $$\begin{aligned}(8x^2 + 7x - 5) - (3x^2 - 4x) - (-6x^3 - 5x^2 + 3) &= (8x^2 + 7x - 5) + (-3x^2 + 4x) + (6x^3 + 5x^2 - 3) \\ &= 6x^3 + (8x^2 - 3x^2 + 5x^2) + (7x + 4x) + (-5 - 3) \\ &= 6x^3 + 10x^2 + 11x - 8\end{aligned}$$
The degree is 3.

15. $$\begin{aligned}(x + 1)(x^2 - x + 1) &= x(x^2) - x \cdot x + x \cdot 1 + 1(x^2) - 1 \cdot x + 1 \cdot 1 \\ &= x^3 - x^2 + x + x^2 - x + 1 \\ &= x^3 + 1\end{aligned}$$

16. $$\begin{aligned}\left(x + 5\right)\left(x^2 - 5x + 25\right) &= x(x^2) - x(5x) + x(25) + 5(x^2) - 5(5x) + 5(25) \\ &= x^3 - 5x^2 + 25x + 5x^2 - 25x + 125) \\ &= x^3 + 125\end{aligned}$$

17. $$\begin{aligned}(2x - 3)(x^2 - 3x + 5) &= (2x)(x^2) + (2x)(-3x) + (2x)(5) + (-3)(x^2) + (-3)(-3x) + (-3)(5) \\ &= 2x^3 - 6x^2 + 10x - 3x^2 + 9x - 15 \\ &= 2x^3 - 9x^2 + 19x - 15\end{aligned}$$

18. $(2x-1)(x^2-4x+3)=(2x)(x^2)+(2x)(-4x)+(2x)(3)+(-1)(x^2)+(-1)(-4x)+(-1)(3)$
$=2x^3-8x^2+6x-x^2+4x-3$
$=2x^3-9x^2+10x-3$

19. $(x+7)(x+3)=x^2+3x+7x+21=x^2+10x+21$

20. $(x+8)(x+5)=x^2+5x+8x+40=x^2+13x+40$

21. $(x-5)(x+3)=x^2+3x-5x-15=x^2-2x-15$

22. $(x-1)(x+2)=x^2+2x-x-2=x^2+x-2$

23. $(3x+5)(2x+1)=(3x)(2x)+3x(1)+5(2x)+5=6x^2+3x+10x+5=6x^2+13x+5$

24. $(7x+4)(3x+1)=(7x)(3x)+7x(1)+4(3x)+4(1)=21x^2+7x+12x+4=21x^2+19x+4$

25. $(2x-3)(5x+3)=(2x)(5x)+(2x)(3)+(-3)(5x)+(-3)(3)=10x^2+6x-15x-9=10x^2-9x-9$

26. $(2x-5)(7x+2)=(2x)(7x)+(2x)(2)+(-5)(7x)+(-5)(2)=14x^2+4x-35x-10=14x^2-31x-10$

27. $(5x^2-4)(3x^2-7)=(5x^2)(3x^2)+(5x^2)(-7)+(-4)(3x^2)+(-4)(-7)=15x^4-35x^2-12x^2+28=15x^4-47x^2+28$

28. $(7x^2-2)(3x^2-5)=(7x^2)(3x^2)+(7x^2)(-5)+(-2)(3x^2)+(-2)(-5)=21x^4-35x^2-6x^2+10=21x^4-41x^2+10$

29. $\left(8x^3+3\right)\left(x^2-5\right)=\left(8x^3\right)\left(x^2\right)+\left(8x^3\right)\left(-5\right)+\left(3\right)\left(x^2\right)+\left(3\right)\left(-5\right)=8x^5-40x^3+3x^2-15$

30. $\left(7x^3+5\right)\left(x^2-2\right)=\left(7x^3\right)\left(x^2\right)+\left(7x^3\right)\left(-2\right)+\left(5\right)\left(x^2\right)+\left(5\right)\left(-2\right)=7x^5-14x^3+5x^2-10$

31. $(x+3)(x-3)=x^2-3^2=x^2-9$

32. $(x+5)(x-5)=x^2-5^2=x^2-25$

33. $(3x+2)(3x-2)=(3x)^2-2^2=9x^2-4$

34. $(2x+5)(2x-5)=(2x)^2-5^2=4x^2-25$

35. $(5-7x)(5+7x)=5^2-(7x)^2=25-49x^2$

36. $(4-3x)(4+3x)=4^2-(3x)^2=16-9x^2$

37. $(4x^2+5x)(4x^2-5x)=(4x^2)^2-(5x)^2=16x^4-25x^2$

38. $(3x^2+4x)(3x^2-4x)=(3x^2)^2-(4x)^2=9x^4-16x^2$

39. $\left(1-y^5\right)\left(1+y^5\right)=(1)^2-\left(y^5\right)^2=1-y^{10}$

40. $\left(2-y^5\right)\left(2+y^5\right)=(2)^2-\left(y^5\right)^2=4-y^{10}$

41. $(x+2)^2=x^2+2\cdot x\cdot 2+2^2=x^2+4x+4$

42. $(x+5)^2=x^2+2\cdot x\cdot 5+5^2=x^2+10x+25$

43. $(2x+3)^2=(2x)^2+2(2x)(3)+3^2=4x^2+12x+9$

44. $(3x+2)^2=(3x)^2+2(3x)(2)+2^2=9x^2+12x+4$

45. $(x-3)^2=x^2-2\cdot x\cdot 3+3^2=x^2-6x+9$

46. $(x-4)^2=x^2-2\cdot x\cdot 4+4^2=x^2-8x+16$

47. $(4x^2-1)^2=(4x^2)^2-2(4x^2)(1)+1^2=16x^4-8x^2+1$

48. $(5x^2-3)^2=(5x^2)^2-2(5x^2)(3)+3^2=25x^4-30x^2+9$

49. $(7-2x)^2=7^2-2(7)(2x)+(2x)^2=49-28x+4x^2=4x^2-28x+49$

50. $(9-5x)^2=9^2-2(9)(5x)+(5x)^2=81-90x+25x^2$ or $25x^2-90x+81$

51. $(x+1)^3=x^3+3\cdot x^2\cdot 1+3x\cdot 1^2+1^3=x^3+3x^2+3x+1$

52. $(x+2)^3=x^3+3\cdot x^2\cdot 2+3\cdot x\cdot 2^2+2^3=x^3+6x^2+12x+8$

53. $(2x+3)^3=(2x)^3+3\cdot(2x)^2\cdot 3+3(2x)\cdot 3^2+3^3=8x^3+36x^2+54x+27$

54. $(3x+4)^3=(3x)^3+3(3x)^2\cdot 4+3(3x)\cdot 4^2+4^3=27x^3+108x^2+144x+64$

55. $(x-3)^3=x^3-3\cdot x^3\cdot 3+3\cdot x\cdot 3^2-3^3=x^3-9x^2+27x-27$

56. $(x-1)^3=x^3-3x^2\cdot 1+3x\cdot 1^2-1^3=x^3-3x^2+3x-1$

57. $(3x-4)^3=(3x)^3-3(3x)^2\cdot 4+3(3x)\cdot 4^2-4^3=27x^3-108x^2+144x-64$

58. $(2x-3)^3=(2x)^3-3(2x)^2\cdot 3+3(2x)\cdot 3^2-3^3=8x^3-36x^2+54x-27$

59.
$$\begin{aligned}(x+5y)(7x+3y)&=x(7x)+x(3y)+(5y)(7x)+(5y)(3y)\\&=7x^2+3xy+35xy+15y^2\\&=7x^2+38xy+15y^2\end{aligned}$$

60.
$$\begin{aligned}(x+9y)(6x+7y)&=x(6x)+x(7y)+(9y)(6x)+(9y)(7y)\\&=6x^2+7xy+54xy+63y^2\\&=6x^2+61xy+63y^2\end{aligned}$$

61. $(x-3y)(2x+7y) = x(2x)+x(7y)+(-3y)(2x)+(-3y)(7y)$
$= 2x^2+7xy-6xy-21y^2$
$= 2x^2+xy-21y^2$

62. $(3x-y)(2x+5y) = (3x)(2x)+(3x)(5y)+(-y)(2x)+(-y)(5y)$
$= 6x^2+15xy-2xy-5y^2$
$= 6x^2+13xy-5y^2$

63. $(3xy-1)(5xy+2) = (3xy)(5xy)+(3xy)(2)+(-1)(5xy)+(-1)(2)$
$= 15x^2y^2+6xy-5xy-2$
$= 15x^2y^2+xy-2$

64. $(7x^2y+1)(2x^2y-3) = (7x^2y)(2x^2y)+(7x^2y)(-3)+(1)2x^2y+(1)(-3)$
$= 14x^4y^2-21x^2y+2x^2y-3$
$= 14x^4y^2-19x^2y-3$

65. $(7x+5y)^2 = (7x)^2+2(7x)(5y)+(5y)^2 = 49x^2+70xy+25y^2$

66. $(9x+7y)^2 = (9x)^2+2(9x)(7y)+(7y)^2 = 81x^2+126xy+49y^2$

67. $(x^2y^2-3)^2 = (x^2y^2)^2-2(x^2y^2)(3)+3^2 = x^4y^4-6x^2y^2+9$

68. $(x^2y^2-5)^2 = (x^2y^2)^2-2(x^2y^2)(5)+5^2 = x^4y^4-10x^2y^2+25$

69. $(x-y)(x^2+xy+y^2) = x(x^2)+x(xy)+x(y^2)+(-y)(x^2)+(-y)(xy)+(-y)(y^2)$
$= x^3+x^2y+xy^2-x^2y-xy^2-y^3$
$= x^3-y^3$

70. $(x+y)(x^2-xy+y^2) = x(x^2)+x(-xy)+x(y^2)+y(x^2)+y(-xy)+y(y^2)$
$= x^3-x^2y+xy^2+x^2y-xy^2+y^3$
$= x^3+y^3$

71. $(3x+5y)(3x-5y) = (3x)^2-(5y)^2 = 9x^2-25y^2$

72. $(7x+3y)(7x-3y) = (7x)^2-(3y)^2 = 49x^2-9y^2$

73. $(x+y+3)(x+y-3) = (x+y)^2-3^2 = x^2+2xy+y^2-9$

74. $(x+y+5)(x+y-5) = (x+y)^2-5^2 = x^2+2xy+y^2-25$

75. $(3x+7-5y)(3x+7+5y) = (3x+7)^2-(5y)^2 = 9x^2+42x+49-25y^2$

76. $(5x+7y-2)(5x+7y+2) = (5x+7y)^2-2^2 = 25x^2+70xy+49y^2-4$

77. $[5y-(2x+3)][5y+(2x+3)] = (5y)^2-(2x+3)^2 = 25y^2-(4x^2+12x+9) = 25y^2-4x^2-12x-9$

78. $[8y+(7-3x)][8y-(7-3x)] = (8y)^2-(7-3x)^2 = 64y^2-(49-42x+9x^2) = 64y^2-49+42x-9x^2$

79. $(x+y+1)^2 = (x+y)^2 + 2(x+y)+1 = x^2+2xy+y^2+2x+2y+1$

80. $(x+y+2)^2 = (x+y)^2 + 2(x+y)(2)+2^2 = x^2+2xy+y^2+4x+4y+4$

81. $(2x+y+1)^2 = (2x+y)^2 + 2(2x+y)+1 = 4x^2+4xy+y^2+4x+2y+1$

82. $(5x+1+6y)^2 = (5x+1)^2 + 2(5x+1)(6y)+(6y)^2 = 25x^2+10x+60xy+1+12y+36y^2$

83.
$$\begin{aligned}(3x+4y)^2-(3x-4y)^2 &= \left[(3x)^2+2(3x)(4y)+(4y)^2\right]-\left[(3x)^2-2(3x)(4y)+(4y)^2\right]\\ &= \left(9x^2+24xy+16y^2\right)-\left(9x^2-24xy+16y^2\right)\\ &= 9x^2+24xy+16y^2-9x^2+24xy-16y^2\\ &= 48xy\end{aligned}$$

84.
$$\begin{aligned}(5x+2y)^2-(5x-2y)^2 &= \left[(5x)^2+2(5x)(2y)+(2y)^2\right]-\left[(5x)^2-2(5x)(2y)+(2y)^2\right]\\ &= \left(25x^2+20xy+4y^2\right)-\left(25x^2-20xy+4y^2\right)\\ &= 25x^2+20xy+4y^2-25x^2+20xy-4y^2\\ &= 40xy\end{aligned}$$

85.
$$\begin{aligned}&(5x-7)(3x-2)-(4x-5)(6x-1)\\ &= \left[15x^2-10x-21x+14\right]-\left[24x^2-4x-30x+5\right]\\ &= \left(15x^2-31x+14\right)-\left(24x^2-34x+5\right)\\ &= 15x^2-31x+14-24x^2+34x-5\\ &= -9x^2+3x+9\end{aligned}$$

86.
$$\begin{aligned}&(3x+5)(2x-9)-(7x-2)(x-1)\\ &= \left(6x^2-27x+10x-45\right)-\left(7x^2-7x-2x+2\right)\\ &= \left(6x^2-17x-45\right)-\left(7x^2-9x+2\right)\\ &= 6x^2-17x-45-7x^2+9x-2\\ &= -x^2-8x-47\end{aligned}$$

87.
$$\begin{aligned}&(2x+5)(2x-5)\left(4x^2+25\right)\\ &= \left[(2x)^2-5^2\right]\left(4x^2+25\right)\\ &= \left(4x^2-25\right)\left(4x^2+25\right)\\ &= \left(4x^2\right)^2-(25)^2\\ &= 16x^4-625\end{aligned}$$

88.
$$\begin{aligned}&(3x+4)(3x-4)\left(9x^2+16\right)\\ &= \left[(3x)^2-4^2\right]\left(9x^2+16\right)\\ &= \left(9x^2-16\right)\left(9x^2+16\right)\\ &= \left(9x^2\right)^2-(16)^2\\ &= 81x^4-256\end{aligned}$$

89. $\dfrac{(2x-7)^5}{(2x-7)^3} = (2x-7)^{5-3}$

$= (2x-7)^2$
$= (2x)^2 - 2(2x)(7) + (7)^2$
$= 4x^2 - 28x + 49$

90. $\dfrac{(5x-3)^6}{(5x-3)^4} = (5x-3)^{6-4}$

$= (5x-3)^2$
$= (5x)^2 - 2(5x)(3) + (3)^2$
$= 25x^2 - 30x + 9$

91. **a.** $S = 0.2x^3 - 1.5x^2 + 3.4x + 25 + \left(0.1x^3 - 1.3x^2 + 3.3x + 5\right)$
$S = 0.2x^3 - 1.5x^2 + 3.4x + 25 + 0.1x^3 - 1.3x^2 + 3.3x + 5$
$S = 0.3x^3 - 2.8x^2 + 6.7x + 30$

b. $S = 0.3x^3 - 2.8x^2 + 6.7x + 30$
$S = 0.3(5)^3 - 2.8(5)^2 + 6.7(5) + 30$
$S = 31$
The model gives a score of 31 for the group in the 45-54 age range which is the same as the score displayed by the bar graph.

92. **a.** $S = -0.02x^3 + 0.4x^2 + 1.2x + 22 + \left(-0.01x^3 - 0.2x^2 + 1.1x + 2\right)$
$S = -0.02x^3 + 0.4x^2 + 1.2x + 22 - 0.01x^3 - 0.2x^2 + 1.1x + 2$
$S = -0.03x^3 + 0.2x^2 + 2.3x + 24$

b. $S = -0.03x^3 + 0.2x^2 + 2.3x + 24$
$S = -0.03(5)^3 + 0.2(5)^2 + 2.3(5) + 24$
$S = 36.75$
The model gives a score of 36.75 for the group of slightly conservative political identification group. This underestimates the score shown on the bar graph by 0.25.

93. $x(8-2x)(10-2x) = x\left(80 - 36x + 4x^2\right)$

$= 80x - 36x^2 + 4x^3$
$= 4x^3 - 36x^2 + 80x$

94. $x(8-2x)(5-2x) = x\left(40 - 26x + 4x^2\right)$

$= 40x - 26x^2 + 4x^3$
$= 4x^3 - 26x^2 + 40x$

95. $(x+9)(x+3) - (x+5)(x+1)$
$= x^2 + 12x + 27 - \left(x^2 + 6x + 5\right)$
$= x^2 + 12x + 27 - x^2 - 6x - 5$
$= 6x + 22$

96. $(x+4)(x+3)-(x+2)(x+1)$
$=x^2+7x+12-\left(x^2+3x+2\right)$
$=x^2+7x+12-x^2-3x-2$
$=4x+10$

97. – 102. Answers will vary.

103. makes sense

104. does not make sense; Explanations will vary. Sample explanation: FOIL is used to multiply two binomials.

105. makes sense

106. makes sense, although answers may vary

107. $(x+3)(x-1)+((x+3)-x)(x-(x-1))$
$=(x+3)(x-1)+3(x-x+1)$
$=x^2-x+3x-3+3$
$=x^2+2x$

108. $(2x-1)x(x+3)-x(x-2)x$
$=(2x^2+5x-3)(x+2)-x^2(x-2)$
$=2x^3+5x^2-3x-x^3+2x^2$
$=x^3+7x^2-3x$

109. $(x+5)(2x+1)(x+2)-3\cdot x(x+5)$
$=(2x^2+11x+5)(x+2)-3x^2-15x$
$=2x^3+15x^2+27x+10-3x^2-15x$
$=2x^3+12x^2+12x+10$

110. $(y^n+2)(y^n-2)-(y^n-3)^2$
$=y^{2n}-4-(y^{2n}-6y^n+9)$
$=y^{2n}-4-y^{2n}+6y^n-9$
$=6y^n-13$

111. $(x+3)(x+\boxed{4})=x^2+7x+12$

112. $(x-\boxed{2})(x-12)=x^2-14x+24$

113. $(4x+1)(2x-\boxed{3})=8x^2-10x-3$

Section P.5

Check Point Exercises

1. **a.** $10x^3 - 4x^2$
$= 2x^2(5x) - 2x^2(2)$
$= 2x^2(5x - 2)$

b. $2x(x-7) + 3(x-7)$
$= (x-7)(2x+3)$

2. $x^3 + 5x^2 - 2x - 10$
$= (x^3 + 5x^2) - (2x + 10)$
$= x^2(x+5) - 2(x+5)$
$= (x+5)(x^2 - 2)$

3. Find two numbers whose product is 40 and whose sum is 13. The required integers are 8 and 5. Thus,
$x^2 + 13x + 40 = (x+8)(x+5)$ or $(x+5)(x+8)$.

4. Find two numbers whose product is –14 and whose sum is –5. The required integers are –7 and 2. Thus,
$x^2 - 5x - 14 = (x-7)(x+2)$ or $(x+2)(x-7)$.

5. Find two First terms whose product is $6x^2$.
$6x^2 + 19x - 7 = (6x \quad)(x \quad)$
$6x^2 + 19x - 7 = (3x \quad)(2x \quad)$

Find two Last terms whose product is –7.
The possible factors are 1(–7) and –1(7).

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is 19*x*.

Possible Factors of $6x^2 + 19x - 7$	Sum of Outside and Inside Products (Should Equal 19*x*)
$(6x+1)(x-7)$	$-42x + x = -41x$
$(6x-7)(x+1)$	$6x - 7x = -x$
$(6x-1)(x+7)$	$42x - x = 41x$
$(6x+7)(x-1)$	$-6x + 7x = x$
$(3x+1)(2x-7)$	$-21x + 2x = -19x$
$(3x-7)(2x+1)$	$3x - 14x = -11x$
$(3x-1)(2x+7)$	$21x - 2x = 19x$
$(3x+7)(2x-1)$	$-3x + 14x = 11x$

Thus, $6x^2 + 19x - 7 = (3x-1)(2x+7)$ or $(2x+7)(3x-1)$.

6. Find two First terms whose product is $3x^2$.

$3x^2 - 13xy + 4y^2 = (3x \quad)(x \quad)$

Find two Last terms whose product is $4y^2$.

The possible factors are $(2y)(2y)$, $(-2y)(-2y)$, $(4y)(y)$, and $(-4y)(-y)$.

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is $-13xy$.

$3x^2 - 13xy + y^2 = (3x - y)(x - 4y)$ or $(x - 4y)(3x - y)$.

7. Express each term as the square of some monomial. Then use the formula for factoring $A^2 - B^2$.

a. $x^2 - 81 = x^2 - 9^2 = (x+9)(x-9)$

b. $36x^2 - 25 = (6x)^2 - 5^2 = (6x+5)(6x-5)$

8. Express $81x^4 - 16$ as the difference of two squares and use the formula for factoring $A^2 - B^2$.

$81x^4 - 16 = (9x^2)^2 - 4^2 = (9x^2 + 4)(9x^2 - 4)$

The factor $9x^2 - 4$ is the difference of two squares and can be factored. Express $9x^2 - 4$ as the difference of two squares and again use the formula for factoring $A^2 - B^2$.

$(9x^2 + 4)(9x^2 - 4) = (9x^2 + 4)\left[(3x)^2 - 2^2\right] = (9x^2 + 4)(3x + 2)(3x - 2)$

Thus, factored completely,

$81x^4 - 16 = (9x^2 + 4)(3x + 2)(3x - 2)$.

9. **a.** $x^2 + 14x + 49 = x^2 + 2 \cdot x \cdot 7 + 7^2 = (x+7)^2$

b. Since $16x^2 = (4x)^2$ and $49 = 7^2$, check to see if the middle term can be expressed as twice the product of $4x$ and 7. Since $2 \cdot 4x \cdot 7 = 56x$, $16x^2 - 56x + 49$ is a perfect square trinomial. Thus, $16x^2 - 56x + 49 = (4x)^2 - 2 \cdot 4x \cdot 7 + 7^2$

$= (4x - 7)^2$

10. **a.**

$$\begin{aligned} x^3 + 1 &= x^3 + 1^3 \\ &= (x+1)(x^2 - x \cdot 1 + 1^2) \\ &= (x+1)(x^2 - x + 1) \end{aligned}$$

b.

$$\begin{aligned} 125x^3 - 8 &= (5x)^3 - 2^3 \\ &= (5x - 2)\left[(5x)^2 + (5x)(2) + 2^2\right] \\ &= (5x-2)(25x^2 + 10x + 4) \end{aligned}$$

11. Factor out the greatest common factor.

$3x^3 - 30x^2 + 75x = 3x\left(x^2 - 10x + 25\right)$

Factor the perfect square trinomial.

$3x\left(x^2 - 10x + 25\right) = 3x(x-5)^2$

12. Reorder to write as a difference of squares.

$$\begin{aligned} &x^2-36a^2+20x+100 \\ &= x^2+20x+100-36a^2 \\ &= \left(x^2+20x+100\right)-36a^2 \\ &= (x+10)^2-36a^2 \\ &= (x+10+6a)(x+10-6a) \end{aligned}$$

13.

$$\begin{aligned} &x(x-1)^{-\frac{1}{2}}+(x-1)^{\frac{1}{2}} \\ &= (x-1)^{-\frac{1}{2}}\left[x+(x-1)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right] \\ &= (x-1)^{-\frac{1}{2}}\left[x+(x-1)\right] \\ &= (x-1)^{-\frac{1}{2}}(2x-1) \\ &= \frac{2x-1}{(x-1)^{\frac{1}{2}}} \end{aligned}$$

Concept and Vocabulary Check P.5

1. d

2. g

3. b

4. c

5. c

6. a

7. f

8. $(x+1)^{\frac{1}{2}}$

Exercise Set P.5

1. $18x+27=9\cdot 2x+9\cdot 3=9(2x+3)$

2. $16x-24=8(2x)+8(-3)=8(2x-3)$

3. $3x^2+6x=3x\cdot x+3x\cdot 2=3x(x+2)$

4. $4x^2-8x=4x(x)+4x(-2)=4x(x-2)$

5.

$$\begin{aligned} &9x^4-18x^3+27x^2 \\ &= 9x^2(x^2)+9x^2(-2x)+9x^2(3) \\ &= 9x^2(x^2-2x+3) \end{aligned}$$

6.

$$\begin{aligned} &6x^4-18x^3+12x^2 \\ &= 6x^2(x^2)+6x^2(-3x)+6x^2(2) \\ &= 6x^2\left(x^2-3x+2\right) \end{aligned}$$

7. $x(x+5)+3(x+5)=(x+5)(x+3)$

8. $x(2x+1)+4(2x+1)=(2x+1)(x+4)$

9. $x^2(x-3)+12(x-3)=(x-3)(x^2+12)$

10. $x^2(2x+5)+17(2x+5)=(2x+5)\left(x^2+17\right)$

11.

$$\begin{aligned} x^3-2x^2+5x-10 &= x^2(x-2)+5(x-2) \\ &= (x^2+5)(x-2) \end{aligned}$$

12.

$$\begin{aligned} x^3-3x^2+4x-12 &= x^2(x-3)+4(x-3) \\ &= (x-3)\left(x^2+4\right) \end{aligned}$$

13.

$$\begin{aligned} x^3-x^2+2x-2 &= x^2(x-1)+2(x-1) \\ &= (x-1)(x^2+2) \end{aligned}$$

14.

$$\begin{aligned} x^3+6x^2-2x-12 &= x^2(x+6)-2(x+6) \\ &= (x+6)\left(x^2-2\right) \end{aligned}$$

15.

$$\begin{aligned} 3x^3-2x^2-6x+4 &= x^2(3x-2)-2(3x-2) \\ &= (3x-2)(x^2-2) \end{aligned}$$

16.

$$\begin{aligned} x^3-x^2-5x+5 &= x^2(x-1)-5(x-1) \\ &= (x-1)\left(x^2-5\right) \end{aligned}$$

17. $x^2+5x+6=(x+2)(x+3)$

18. $x^2+8x+15=(x+3)(x+5)$

19. $x^2-2x-15=(x-5)(x+3)$

20. $x^2-4x-5=(x-5)(x+1)$

21. $x^2-8x+15=(x-5)(x-3)$

22. $x^2-14x+45=(x-5)(x-9)$

23. $3x^2-x-2=(3x+2)(x-1)$

24. $2x^2+5x-3=(2x-1)(x+3)$

25. $3x^2 - 25x - 28 = (3x - 28)(x + 1)$

26. $3x^2 - 2x - 5 = (3x - 5)(x + 1)$

27. $6x^2 - 11x + 4 = (2x - 1)(3x - 4)$

28. $6x^2 - 17x + 12 = (2x - 3)(3x - 4)$

29. $4x^2 + 16x + 15 = (2x + 3)(2x + 5)$

30. $8x^2 + 33x + 4 = (8x + 1)(x + 4)$

31. $9x^2 - 9x + 2 = (3x - 1)(3x - 2)$

32. $9x^2 + 5x - 4 = (9x - 4)(x + 1)$

33. $20x^2 + 27x - 8 = (5x + 8)(4x - 1)$

34. $15x^2 - 19x + 6 = (3x - 2)(5x - 3)$

35. $2x^2 + 3xy + y^2 = (2x + y)(x + y)$

36. $3x^2 + 4xy + y^2 = (3x + y)(x + y)$

37. $6x^2 - 5xy - 6y^2 = (3x + 2y)(2x - 3y)$

38. $6x^2 - 7xy - 5y^2 = (3x - 5y)(2x + y)$

39. $x^2 - 100 = x^2 - 10^2 = (x + 10)(x - 10)$

40. $x^2 - 144 = x^2 - 12^2 = (x + 12)(x - 12)$

41. $36x^2 - 49 = (6x)^2 - 7^2 = (6x + 7)(6x - 7)$

42. $64x^2 - 81 = (8x)^2 - 9^2 = (8x + 9)(8x - 9)$

43. $9x^2 - 25y^2 = (3x)^2 - (5y)^2$
$= (3x + 5y)(3x - 5y)$

44. $36x^2 - 49y^2 = (6x)^2 - (7y)^2$
$= (6x + 7y)(6x - 7y)$

45. $x^4 - 16 = (x^2)^2 - 4^2$
$= (x^2 + 4)(x^2 - 4)$
$= (x^2 + 4)(x + 2)(x - 2)$

46. $x^4 - 1 = (x^2)^2 - 1^2 = (x^2 + 1)(x^2 - 1)$
$= (x^2 + 1)(x + 1)(x - 1)$

47. $16x^4 - 81 = (4x^2)^2 - 9^2$
$= (4x^2 + 9)(4x^2 - 9)$
$= (4x^2 + 9)[(2x)^2 - 3^2]$
$= (4x^2 + 9)(2x + 3)(2x - 3)$

48. $81x^4 - 1 = (9x^2)^2 - 1^2$
$= (9x^2 + 1)(9x^2 - 1)$
$= (9x^2 + 1)[(3x)^2 - 1^2]$
$= (9x^2 + 1)(3x + 1)(3x - 1)$

49. $x^2 + 2x + 1 = x^2 + 2 \cdot x \cdot 1 + 1^2 = (x + 1)^2$

50. $x^2 + 4x + 4 = x^2 + 2 \cdot x \cdot 2 + 2^2 = (x + 2)^2$

51. $x^2 - 14x + 49 = x^2 - 2 \cdot x \cdot 7 + 7^2$
$= (x - 7)^2$

52. $x^2 - 10x + 25 = x^2 - 2 \cdot x \cdot 5 + 5^2 = (x - 5)^2$

53. $4x^2 + 4x + 1 = (2x)^2 + 2 \cdot 2x \cdot 1 + 1^2$
$= (2x + 1)^2$

54. $25x^2 + 10x + 1 = (5x)^2 + 2 \cdot 5x \cdot 1 + 1^2 = (5x + 1)^2$

55. $9x^2 - 6x + 1 = (3x)^2 - 2 \cdot 3x \cdot 1 + 1^2$
$= (3x - 1)^2$

56. $64x^2 - 16x + 1 = (8x)^2 - 2 \cdot 8x \cdot 1 + 1^2 = (8x - 1)^2$

57. $x^3 + 27 = x^3 + 3^3$
$= (x + 3)(x^2 - x \cdot 3 + 3^2)$
$= (x + 3)(x^2 - 3x + 9)$

58. $x^3 + 64 = x^3 + 4^3$
$= (x + 4)(x^2 - x \cdot 4 + 4^2)$
$= (x + 4)(x^2 - 4x + 16)$

59. $x^3 - 64 = x^3 - 4^3$
$= (x - 4)(x^2 + x \cdot 4 + 4^2)$
$= (x - 4)(x^2 + 4x + 16)$

60. $x^3 - 27 = x^3 - 3^3$
$= (x - 3)(x^2 + x \cdot 3 + 3^2)$
$= (x - 3)(x^2 + 3x + 9)$

61. $8x^3-1=(2x)^3-1^3$
$=(2x-1)[(2x)^2+(2x)(1)+1^2]$
$=(2x-1)(4x^2+2x+1)$

62. $27x^3-1=(3x)^3-1^3$
$=(3x-1)[(3x)^2+(3x)(1)+1^2]$
$=(3x-1)(9x^2+3x+1)$

63. $64x^3+27=(4x)^3+3^3$
$=(4x+3)[(4x)^2-(4x)(3)+3^2]$
$=(4x+3)(16x^2-12x+9)$

64. $8x^3+125=(2x)^3+5^3$
$=(2x+5)[(2x)^2-(2x)(5)+5^2]$
$=(2x+5)(4x^2-10x+25)$

65. $3x^3-3x=3x(x^2-1)=3x(x+1)(x-1)$

66. $5x^3-45x=5x(x^2-9)=5x(x+3)(x-3)$

67. $4x^2-4x-24=4(x^2-x-6)$
$=4(x+2)(x-3)$

68. $6x^2-18x-60=6(x^2-3x-10)$
$=6(x+2)(x-5)$

69. $2x^4-162=2(x^4-81)$
$=2[(x^2)^2-9^2]$
$=2(x^2+9)(x^2-9)$
$=2(x^2+9)(x^2-3^2)$
$=2(x^2+9)(x+3)(x-3)$

70. $7x^4-7=7(x^4-1)$
$=7[(x^2)^2-1^2]$
$=7(x^2+1)(x^2-1)$
$=7(x^2+1)(x+1)(x-1)$

71. $x^3+2x^2-9x-18=(x^3+2x^2)-(9x+18)$
$=x^2(x+2)-9(x+2)$
$=(x^2-9)(x+2)$
$=(x^2-3^2)(x+2)$
$=(x-3)(x+3)(x+2)$

72. $x^3+3x^2-25x-75=(x^3+3x^2)-(25x+75)$
$=x^2(x+3)-25(x+3)$
$=(x^2-25)(x+3)$
$=(x^2-5^2)(x+3)$
$=(x-5)(x+5)(x+3)$

73. $2x^2-2x-112=2(x^2-x-56)=2(x-8)(x+7)$

74. $6x^2-6x-12=6(x^2-x-2)$
$=6(x-2)(x+1)$

75. $x^3-4x=x(x^2-4)$
$=x(x^2-2^2)$
$=x(x-2)(x+2)$

76. $9x^3-9x=9x(x^2-1)=9x(x-1)(x+1)$

77. x^2+64 is prime.

78. x^2+36 is prime.

79. $x^3+2x^2-4x-8=(x^3+2x^2)+(-4x-8)$
$=x^2(x+2)-4(x+2)=(x^2-4)(x+2)=(x^2-2^2)(x+2)=(x-2)(x+2)(x+2)=(x-2)(x+2)^2$

80. x^3+2x^2-x-2
$=(x^3+2x^2)+(-x-2)=x^2(x+2)-1(x+2)=(x^2-1)(x+2)=(x^2-1^2)(x+2)=(x-1)(x+1)(x+2)$

81. y^5-81y
$=y(y^4-81)=y[(y^2)^2-9^2]=y(y^2+9)(y^2-9)=y(y^2+9)(y^2-3^2)=y(y^2+9)(y+3)(y-3)$

82. y^5-16y
$=y(y^4-16)=y[(y^2)^2-4^2]=y(y^2+4)(y^2-4)=y(y^2+4)(y^2-2^2)=y(y^2+4)(y+2)(y-2)$

83. $20y^4-45y^2=5y^2(4y^2-9)=5y^2[(2y)^2-3^2]=5y^2(2y+3)(2y-3)$

84. $48y^4-3y^2=3y^2(16y^2-1)=3y^2[(4y)^2-1^2]=3y^2(4y+1)(4y-1)$

85. $x^2-12x+36-49y^2=\left(x^2-12x+36\right)-49y^2=(x-6)^2-49y^2=(x-6+7y)(x-6-7y)$

86. $x^2-10x+25-36y^2=(x^2-10x+25)-36y^2=(x-5)^2-36y^2=(x-5+6y)(x-5-6y)$

87. $9b^2x-16y-16x+9b^2y$
$=\left(9b^2x+9b^2y\right)+(-16x-16y)=9b^2(x+y)-16(x+y)=(x+y)\left(9b^2-16\right)=(x+y)(3b+4)(3b-4)$

88. $16a^2x-25y-25x+16a^2y$
$=\left(16a^2x+16a^2y\right)+(-25y-25x)=16a^2(x+y)-25(x+y)=(x+y)\left(16a^2-25\right)=(x+y)(4a+5)(4a-5)$

89. $x^2y-16y+32-2x^2$
$=\left(x^2y-16y\right)+\left(-2x^2+32\right)=y\left(x^2-16\right)-2\left(x^2-16\right)=\left(x^2-16\right)(y-2)=(x+4)(x-4)(y-2)$

90. $12x^2y-27y-4x^2+9$
$=\left(12x^2y-27y\right)+\left(-4x^2+9\right)=3y\left(4x^2-9\right)-1\left(4x^2-9\right)=\left(4x^2-9\right)(3y-1)=(2x+3)(2x-3)(3y-1)$

91. $2x^3-8a^2x+24x^2+72x$
$=2x\left(x^2-4a^2+12x+36\right)=2x\left[\left(x^2+12x+36\right)-4a^2\right]=2x\left[(x+6)^2-4a^2\right]=2x(x+6-2a)(x+6+2a)$

92. $2x^3-98a^2x+28x^2+98x$
$=2x\left(x^2-49a^2+14x+49\right)=2x\left[\left(x^2+14x+49\right)-49a^2\right]=2x\left[(x+7)^2-49a^2\right]=2x(x+7-7a)(x+7+7a)$

93. $x^{\frac{3}{2}}-x^{\frac{1}{2}}=x^{\frac{1}{2}}\left(x^{\frac{3}{2}-\frac{1}{2}}\right)-1=x^{\frac{1}{2}}(x-1)$

94. $x^{\frac{3}{4}}-x^{\frac{1}{4}}=x^{\frac{1}{4}}\left(x^{\frac{3}{4}-\frac{1}{4}}-1\right)=x^{\frac{1}{4}}\left(x^{\frac{1}{2}}-1\right)$

95. $4x^{-\frac{2}{3}}+8x^{\frac{1}{3}}=4x^{-\frac{2}{3}}\left(1+2x^{\frac{1}{3}-\left(-\frac{2}{3}\right)}\right)=4x^{-\frac{2}{3}}(1+2x)=\dfrac{4(1+2x)}{x^{\frac{2}{3}}}$

96. $12x^{-\frac{3}{4}}+6x^{\frac{1}{4}}=6x^{-\frac{3}{4}}\left(2+x^{\frac{1}{4}-\left(-\frac{3}{4}\right)}\right)=6x^{-\frac{3}{4}}(2+x)=\dfrac{6(x+2)}{x^{\frac{3}{4}}}$

97. $(x+3)^{\frac{1}{2}}-(x+3)^{\frac{3}{2}}=(x+3)^{\frac{1}{2}}\left[1-(x+3)^{\frac{3}{2}-\frac{1}{2}}\right]=(x+3)^{\frac{1}{2}}[1-(x+3)]=(x+3)^{\frac{1}{2}}(-x-2)=-(x+3)^{\frac{1}{2}}(x+2)$

98. $\left(x^2+4\right)^{\frac{3}{2}}+\left(x^2+4\right)^{\frac{7}{2}}=\left(x^2+4\right)^{\frac{3}{2}}\left[1+\left(x^2+4\right)^{\frac{7}{2}-\frac{3}{2}}\right]=\left(x^2+4\right)^{\frac{3}{2}}\left[1+\left(x^2+4\right)^2\right]=\left(x^2+4\right)^{\frac{3}{2}}\left(x^4+8x^2+17\right)$

99. $(x+5)^{-\frac{1}{2}}-(x+5)^{-\frac{3}{2}}=(x+5)^{-\frac{3}{2}}\left[(x+5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)}-1\right]=(x+5)^{-\frac{3}{2}}[(x+5)-1]=(x+5)^{-\frac{3}{2}}(x+4)=\dfrac{x+4}{(x+5)^{\frac{3}{2}}}$

100. $\left(x^2+3\right)^{-\frac{2}{3}}+\left(x^2+3\right)^{-\frac{5}{3}}=\left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)^{-\frac{2}{3}-\left(-\frac{5}{3}\right)}+1\right]=\left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)+1\right]=\dfrac{x^2+4}{(x^2+3)^{5/3}}$

101. $(4x-1)^{\frac{1}{2}}-\frac{1}{3}(4x-1)^{\frac{3}{2}}$

$=(4x-1)^{\frac{1}{2}}\left[1-\frac{1}{3}(4x-1)^{\frac{3}{2}-\frac{1}{2}}\right]=(4x-1)^{\frac{1}{2}}\left[1-\frac{1}{3}(4x-1)\right]=(4x-1)^{\frac{1}{2}}\left[1-\frac{4}{3}x+\frac{1}{3}\right]$

$=(4x-1)^{\frac{1}{2}}\left(\frac{4}{3}-\frac{4}{3}x\right)=(4x-1)^{\frac{1}{2}}\frac{4}{3}(1-x)=\dfrac{-4(4x-1)^{\frac{1}{2}}(x-1)}{3}$

102. $-8(4x+3)^{-2}+10(5x+1)(4x+3)^{-1}=2(4x+3)^{-2}\left[-4+5(5x+1)(4x+3)\right]=\dfrac{2(100x^2+95x+11)}{(4x+3)^2}$

103. $10x^2(x+1)-7x(x+1)-6(x+1)=(x+1)\left(10x^2-7x-6\right)=(x+1)(5x-6)(2x+1)$

104. $12x^2(x-1)-4x(x-1)-5(x-1)=(x-1)\left(12x^2-4x-5\right)=(x-1)(6x-5)(2x+1)$

105. $6x^4+35x^2-6=\left(x^2+6\right)\left(6x^2-1\right)$

106. $7x^4+34x^2-5=\left(7x^2-1\right)\left(x^2+5\right)$

107. $y^7+y=y\left(y^6+1\right)=y\left[\left(y^2\right)^3+1^3\right]=y\left(y^2+1\right)\left(y^4-y^2+1\right)$

108. $(y+1)^3+1=(y+1)^3+1^3=\left[(y+1)+1\right]\left[(y+1)^2-(y+1)+1\right]=(y+2)\left[\left(y^2+2y+1\right)-y-1+1\right]$

$=(y+2)\left(y^2+2y+1-y-1+1\right)=(y+2)\left(y^2+y+1\right)$

109. $x^4-5x^2y^2+4y^4=\left(x^2-4y^2\right)\left(x^2-y^2\right)=(x+2y)(x-2y)(x+y)(x-y)$

110. $x^4-10x^2y^2+9y^4=\left(x^2-9y^2\right)\left(x^2-y^2\right)=(x+3y)(x-3y)(x+y)(x-y)$

111. $(x-y)^4-4(x-y)^2$

$=(x-y)^2\left((x-y)^2-4\right)=(x-y)^2((x-y)+2)((x-y)-2)=(x-y)^2(x-y+2)(x-y-2)$

112. $(x+y)^4-100(x+y)^2=(x+y)^2\left((x+y)^2-100\right)=(x+y)^2(x+y-10)(x+y+10)$

113. $2x^2-7xy^2+3y^4=\left(2x-y^2\right)\left(x-3y^2\right)$

114. $3x^2+5xy^2+2y^4=\left(3x+2y^2\right)\left(x+y^2\right)$

115. a. $(x-0.4x)-0.4(x-0.4x)=(x-0.4x)(1-0.4)=(0.6x)(0.6)=0.36x$

b. No, the computer is selling at 36% of its original price.

116. a. $(x-0.3x)-0.3(x-0.3x)=(x-0.3x)(1-0.3)=(0.7x)(0.7)=0.49x$

b. No, the computer is selling at 49% of its original price.

117. a. $(3x)^2-4\cdot 2^2=9x^2-16$

b. $9x^2-16=(3x+4)(3x-4)$

118. a. $(7x)^2-4\cdot 3^2=49x^2-36$

b. $49x^2-36=(7x+6)(7x-6)$

119. a. $x(x+y)-y(x+y)$

b. $x(x+y)-y(x+y)=(x+y)(x-y)$

120. a. $x^2+xy+xy+y^2=x^2+2xy+y^2$

b. $x^2+2xy+y^2=(x+y)^2$

121. $$\begin{aligned} V_{\text{shaded}} &= V_{\text{outside}}-V_{\text{inside}} \\ &= a\cdot a\cdot 4a-b\cdot b\cdot 4a \\ &= 4a^3-4ab^2 \\ &= 4a\left(a^2-b^2\right) \\ &= 4a(a+b)(a-b) \end{aligned}$$

122. $$\begin{aligned} V_{shaded} &= V_{outside}-V_{inside} \\ &= a\cdot a\cdot 3a-b\cdot b\cdot 3a \\ &= 3a^3-3ab^2 \\ &= 3a\left(a^2-b^2\right) \\ &= 3a(a+b)(a-b) \end{aligned}$$

123. – 129. Answers will vary.

130. makes sense

131. makes sense

132. does not make sense; Explanations will vary. Sample explanation: $4x^2-100=4(x^2-25)=4(x+5)(x-5)$

133. makes sense

134. false; Changes to make the statement true will vary. A sample change is:
$x^4-16=(x^2+4)(x^2-4)=(x^2+4)(x+2)(x-2)$

135. true

136. false; Changes to make the statement true will vary. A sample change is: The binomial $x^2 + 36$ is prime.

137. false; Changes to make the statement true will vary. A sample change is: $x^3 - 64 = (x-4)(x+4x+16)$

138. $x^{2n} + 6x^n + 8 = (x^n + 4)(x^n + 2)$

139. $-x^2 - 4x + 5 = -1(x^2 + 4x - 5) = -1(x+5)(x-1) = -(x+5)(x-1)$

140.
$$\begin{aligned} & x^4 - y^4 - 2x^3y + 2xy^3 \\ &= (x^4 - y^4) + (-2x^3y + 2xy^3) \\ &= (x^2 - y^2)(x^2 + y^2) - 2xy(x^2 - y^2) \\ &= (x^2 - y^2)(x^2 + y^2 - 2xy) \\ &= (x-y)(x+y)(x^2 - 2xy + y^2) \\ &= (x-y)(x+y)(x-y)^2 \\ &= (x-y)^3(x+y) \end{aligned}$$

141.
$$\begin{aligned} (x-5)^{-\frac{1}{2}}(x+5)^{-\frac{1}{2}} - (x+5)^{\frac{1}{2}}(x-5)^{-\frac{3}{2}} &= (x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)} - (x+5)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right] \\ &= (x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)-(x+5)\right] \\ &= (x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}(-10) = \frac{-10}{(x-5)^{3/2}(x+5)^{1/2}} \end{aligned}$$

142. $x^2 + bx + 15$, $b = 16$, -16, 8 or -8

143. $b = 0$, 3, 4, or $-c(c + 4)$, where $c > 0$ is an integer.

144. $\dfrac{x^2 + 6x + 5}{x^2 - 25} = \dfrac{(x+5)(x+1)}{(x+5)(x-5)} = \dfrac{x+1}{x-5}$

145.
$$\begin{aligned} \frac{5}{4} \cdot \frac{8}{15} &= \frac{5}{4} \cdot \frac{4 \cdot 2}{5 \cdot 3} \\ &= \frac{1}{1} \cdot \frac{2}{3} \\ &= \frac{2}{3} \end{aligned}$$

146.
$$\begin{aligned} \frac{1}{2} + \frac{2}{3} &= \frac{3}{6} + \frac{4}{6} \\ &= \frac{7}{6} \end{aligned}$$

Mid-Chapter P Check Point

1. $(3x+5)(4x-7)=(3x)(4x)+(3x)(-7)+(5)(4x)+(5)(-7)$
$$=12x^2-21x+20x-35$$
$$=12x^2-x-35$$

2. $(3x+5)-(4x-7)=3x+5-4x+7$
$$=3x-4x+5+7$$
$$=-x+12$$

3. $\sqrt{6}+9\sqrt{6}=10\sqrt{6}$

4. $3\sqrt{12}-\sqrt{27}=3\cdot 2\sqrt{3}-3\sqrt{3}=6\sqrt{3}-3\sqrt{3}=3\sqrt{3}$

5. $7x+3[9-(2x-6)]=7x+3[9-2x+6]=7x+3[15-2x]=7x+45-6x=x+45$

6. $(8x-3)^2=(8x)^2-2(8x)(3)+(3)^2=64x^2-48x+9$

7. $\left(x^{\frac{1}{3}}y^{-\frac{1}{2}}\right)^6=x^{\frac{1}{3}\cdot 6}y^{-\frac{1}{2}\cdot 6}=x^2y^{-3}=\dfrac{x^2}{y^3}$

8. $\left(\dfrac{2}{7}\right)^0-32^{-\frac{2}{5}}=1-\dfrac{1}{\left(\sqrt[5]{32}\right)^2}=1-\dfrac{1}{(2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

9. $(2x-5)-(x^2-3x+1)=2x-5-x^2+3x-1=-x^2+5x-6$

10. $(2x-5)(x^2-3x+1)=2x(x^2-3x+1)-5(x^2-3x+1)$
$$=2x(x^2-3x+1)-5(x^2-3x+1)$$
$$=2x^3-6x^2+2x-5x^2+15x-5$$
$$=2x^3-6x^2-5x^2+2x+15x-5$$
$$=2x^3-11x^2+17x-5$$

11. $x^3+x^3-x^3\cdot x^3=2x^3-x^6=-x^6+2x^3$

12. $(9a-10b)(2a+b)=(9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)$
$$=(9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)$$
$$=18a^2+9ab-20ab-10b^2$$
$$=18a^2-11ab-10b^2$$

13. $\{a,c,d,e\}\cup\{c,d,f,h\}=\{a,c,d,e,f,h\}$

14. $\{a,c,d,e\}\cap\{c,d,f,h\}=\{c,d\}$

15. $\left(3x^2y^3 - xy + 4y^2\right) - \left(-2x^2y^3 - 3xy + 5y^2\right) = 3x^2y^3 - xy + 4y^2 + 2x^2y^3 + 3xy - 5y^2$

$$= 3x^2y^3 - xy + 4y^2 + 2x^2y^3 + 3xy - 5y^2$$
$$= 3x^2y^3 + 2x^2y^3 - xy + 3xy + 4y^2 - 5y^2$$
$$= 5x^2y^3 + 2xy - y^2$$

16. $\dfrac{24x^2y^{13}}{-2x^5y^{-2}} = -12x^{2-5}y^{13-(-2)} = -12x^{-3}y^{15} = -\dfrac{12y^{15}}{x^3}$

17. $\left(\dfrac{1}{3}x^{-5}y^4\right)\left(18x^{-2}y^{-1}\right) = 6x^{-5-2}y^{4-1} = \dfrac{6y^3}{x^7}$

18. $\sqrt[12]{x^4} = x^{\frac{4}{12}} = x^{\frac{1}{3}} = \sqrt[3]{x}$

19. $\left[4y - (3x+2)\right]\left[4y + (3x+2)\right] = (4y)^2 - (3x+2)^2 = 16y^2 - (9x^2 + 12x + 4) = 16y^2 - 9x^2 - 12x - 4$

20. $(x - 2y - 1)^2 = x(x - 2y - 1) - 2y(x - 2y - 1) - (x - 2y - 1)$

$$= x^2 - 2xy - x - 2xy + 4y^2 + 2y - x + 2y + 1$$
$$= x^2 - 4xy + 4y^2 - 2x + 4y + 1$$

21. $\dfrac{24\times10^3}{2\times10^6} = \dfrac{24}{2}\cdot\dfrac{10^3}{10^6} = 12\times10^{-3} = \left(1.2\times10^1\right)\times10^{-3} = 1.2\times\left(10^1\times10^{-3}\right) = 1.2\times10^{-2}$

22. $\dfrac{\sqrt[3]{32}}{\sqrt[3]{2}} = \sqrt[3]{\dfrac{32}{2}} = \sqrt[3]{16} = \sqrt[3]{2^4} = 2\sqrt[3]{2}$

23. $(x^3 + 2)(x^3 - 2) = x^6 - 4$

24. $(x^2 + 2)^2 = (x^2)^2 + 2(x^2)(2) + (2)^2 = x^4 + 4x^2 + 4$

25. $\sqrt{50}\cdot\sqrt{6} = 5\sqrt{2}\cdot\sqrt{6} = 5\sqrt{2\cdot6} = 5\sqrt{12} = 5\cdot2\sqrt{3} = 10\sqrt{3}$

26. $\dfrac{11}{7-\sqrt{3}} = \dfrac{11}{7-\sqrt{3}}\cdot\dfrac{7+\sqrt{3}}{7+\sqrt{3}} = \dfrac{77+11\sqrt{3}}{49-3} = \dfrac{77+11\sqrt{3}}{46}$

27. $\dfrac{11}{\sqrt{3}} = \dfrac{11}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{11\sqrt{3}}{3}$

28. $7x^2 - 22x + 3 = (7x - 1)(x - 3)$

29. $x^2 - 2x + 4$ is prime.

30. $x^3 + 5x^2 + 3x + 15 = x^2(x+5) + 3(x+5)$

$$= x^2(x+5) + 3(x+5)$$
$$= (x^2 + 3)(x + 5)$$

31. $3x^2-4xy-7y^2=(3x-7y)(x+y)$

32. $64y-y^4=y\left(64-y^3\right)=y(4-y)\left(16+4y+y^2\right)$

33. $50x^3+20x^2+2x=2x\left(25x^2+10x+1\right)=2x(5x+1)^2$

34. $x^2-6x+9-49y^2=(x-3)^2-49y^2=\left[(x-3)+7y\right]\left[(x-3)-7y\right]=(x-3+7y)(x-3-7y)$

35. $x^{-\frac{3}{2}}-2x^{-\frac{1}{2}}+x^{\frac{1}{2}}=x^{-\frac{3}{2}}\left(1-2x+x^2\right)=\dfrac{(1-x)^2}{x^{\frac{3}{2}}}$

36. $\left(x^2+1\right)^{\frac{1}{2}}-10\left(x^2+1\right)^{-\frac{1}{2}}=\left(x^2+1\right)^{-\frac{1}{2}}\left[\left(x^2+1\right)-10\right]=\left(x^2+1\right)^{-\frac{1}{2}}\left(x^2-9\right)=\dfrac{(x+3)(x-3)}{\left(x^2+1\right)^{\frac{1}{2}}}$

37. $\left\{-11,\ -\dfrac{3}{7},\ 0,\ 0.45,\ \sqrt{25}\right\}$

38. Since $2-\sqrt{13}<0$ then $\left|2-\sqrt{13}\right|=\sqrt{13}-2$

39. Since $x<0$ then $|x|=-x$. Thus $x^2|x|=-x^2x=-x^3$

40. $4.6\cdot 3.0\times10^8=4.6\times10^8=13.8\times10^8=1.38\times10^9$
The U.S. produces 1.38×10^9 pounds of garbage per day.

41. $\dfrac{3\times10^{10}}{7.5\times10^9}=\dfrac{3}{7.5}\cdot\dfrac{10^{10}}{10^9}=0.4\times10=4$
A human brain has 4 times as many neurons as a gorilla brain.

42. **a.** Model 1:
$D=1188x+16{,}218$
$D=1188(1)+16{,}218$
$D=17{,}406$
Model 2:
$D=46x^2+541x+17{,}650$
$D=46(1)^2+541(1)+17{,}650$
$D=18{,}237$

Model 1 best describes the data in 2001.

b. $D=46x^2+541x+17{,}650$
$D=46(13)^2+541(13)+17{,}650$
$D=32{,}457$
Model 2 underestimates the average student-loan debt in 2013 by \$593.

Section P.6

Check Point Exercises

1. **a.** The denominator would equal zero if $x = -5$, so –5 must be excluded from the domain.

b. $x^2 - 36 = (x+6)(x-6)$

The denominator would equal zero if $x = -6$ or $x = 6$, so –6 and 6 both must be excluded from the domain.

c. $x^2 - 5x - 14 = (x+2)(x-7)$

The denominator would equal zero if $x = -2$ or $x = 7$, so –2 and 7 both must be excluded from the domain.

2. **a.**
$$\frac{x^3+3x^2}{x+3} = \frac{x^2(x+3)}{x+3} = \frac{x^2(x+3)}{x+3} = x^2,\ x \neq -3$$

Because the denominator is $x + 3$, $x \neq -3$

b.
$$\frac{x^2-1}{x^2+2x+1} = \frac{(x-1)(x+1)}{(x+1)(x+1)} = \frac{x-1}{x+1}, x \neq -1$$

Because the denominator is $(x+1)(x+1), x \neq -1$

3.
$$\frac{x+3}{x^2-4} \cdot \frac{x^2-x-6}{x^2+6x+9}$$
$$= \frac{x+3}{(x+2)(x-2)} \cdot \frac{(x-3)(x+2)}{(x+3)(x+3)}$$
$$= \frac{x+3}{(x+2)(x-2)} \cdot \frac{(x-3)(x+2)}{(x+3)(x+3)}$$
$$= \frac{x-3}{(x-2)(x+3)},\ x \neq -2,\ x \neq 2,\ x \neq -3$$

Because the denominator has factors of $x+2$, $x-2$, and $x+3$, $x \neq -2$, $x \neq 2$, and $x \neq -3$.

4.
$$\frac{x^2-2x+1}{x^3+x} \div \frac{x^2+x-2}{3x^2+3}$$
$$= \frac{x^2-2x+1}{x^3+x} \cdot \frac{3x^2+3}{x^2+x-2}$$
$$= \frac{(x-1)(x-1)}{x(x^2+1)} \cdot \frac{3(x^2+1)}{(x+2)(x-1)}$$
$$= \frac{3(x-1)}{x(x+2)},\ x \neq 1,\ x \neq 0,\ x \neq -2$$

5.
$$\frac{x}{x+1} - \frac{3x+2}{x+1} = \frac{x-3x-2}{x+1} = \frac{-2x-2}{x+1} = \frac{-2(x+1)}{x+1} = -2, x \neq -1$$

6. Factor each denominator completely.

$x+1 = 1(x+1)$
$x-1 = 1(x-1)$

List the factors of the first denominator.

$1,\ x+1$

Add any unlisted factors from the second denominator.

$1,\ x+1,\ x-1$

The least common denominator is the product of all factors in the final list.

$1(x+1)(x-1)$ or $(x+1)(x-1)$ is the least common denominator.

7. Factor each denominator completely.

$x^2 - 6x + 9 = (x-3)^2$
$x^2 - 9 = (x+3)(x-3)$

List the factors of the first denominator.

$x-3,\ x-3$

Add any unlisted factors from the second denominator.

$x-3,\ x-3,\ x+3$

The least common denominator is the product of all factors in the final list.

$(x-3)(x-3)(x+3)$ or $(x-3)^2(x+3)$ is the least common denominator.

8. Find the least common denominator.

$x-3 = 1(x-3)$
$x+3 = 1(x+3)$

The least common denominator is $(x-3)(x+3)$.
Write all rational expressions in terms of the least common denominator.

$$\frac{x}{x-3} + \frac{x-1}{x+3}$$
$$= \frac{x(x+3)}{(x-3)(x+3)} + \frac{(x-1)(x-3)}{(x-3)(x+3)}$$

Add numerators, putting this sum over the least common denominator.

$$= \frac{x(x+3)+(x-1)(x-3)}{(x-3)(x+3)}$$

$$= \frac{x^2+3x+\left(x^2-4x+3\right)}{(x-3)(x+3)}$$

$$= \frac{x^2+3x+x^2-4x+3}{(x-3)(x+3)}$$

$$= \frac{2x^2-x+3}{(x-3)(x+3)},\ x \neq 3, x \neq -3$$

9. Find the least common denominator.

$$x^2-10x+25=(x-5)^2$$
$$2x-10=2(x-5)$$

The least common denominator is $2(x-5)(x-5)$. Write all rational expressions in terms of the least common denominator.

$$\frac{x}{x^2-10x+25}-\frac{x-4}{2x-10}$$
$$=\frac{x}{(x-5)(x-5)}-\frac{x-4}{2(x-5)}$$
$$=\frac{2x}{2(x-5)(x-5)}-\frac{(x-4)(x-5)}{2(x-5)(x-5)}$$

Add numerators, putting this sum over the least common denominator.

$$=\frac{2x-(x-4)(x-5)}{2(x-5)(x-5)}$$
$$=\frac{2x-(x^2-5x-4x+20)}{2(x-5)(x-5)}$$
$$=\frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$=\frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$=\frac{-x^2+11x-20}{2(x-5)(x-5)}$$
$$=\frac{-x^2+11x-20}{2(x-5)^2},\ x \neq 5$$

10. $$\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}=\frac{\frac{2}{2x}-\frac{3x}{2x}}{\frac{4}{4x}+\frac{3x}{4x}},\ x \neq 0$$
$$=\frac{\frac{2-3x}{2x}}{\frac{4+3x}{4x}},\ x \neq \frac{-4}{3}$$
$$=\frac{2-3x}{2x} \div \frac{4+3x}{4x}$$
$$=\frac{2-3x}{2x} \cdot \frac{4x}{4+3x}$$
$$=\frac{2-3x}{4+3x} \cdot \frac{4}{2}$$
$$=\frac{2-3x}{4+3x} \cdot \frac{2}{1}$$
$$=\frac{2(2-3x)}{4+3x},\ x \neq 0,\ x \neq \frac{-4}{3}$$

11. Multiply each of the three terms, $\frac{1}{x+7}$, $\frac{1}{x}$, and 7 by the least common denominator of $x(x+7)$.

$$\frac{\frac{1}{x+7}-\frac{1}{x}}{7}=\frac{x(x+7)\left(\frac{1}{x+7}\right)-x(x+7)\left(\frac{1}{x}\right)}{7x(x+7)}$$
$$=\frac{x-(x+7)}{7x(x+7)}$$
$$=\frac{-7}{7x(x+7)}$$
$$=-\frac{1}{x(x+7)},\ x \neq 0, x \neq -7$$

12. $$\frac{\sqrt{x}+\frac{1}{\sqrt{x}}}{x}=\frac{\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\sqrt{x}}{x\sqrt{x}}$$
$$=\frac{x+1}{x^{3/2}} \text{ or } \frac{x+1}{\sqrt{x^3}}$$

13. $$\frac{\sqrt{x+3}-\sqrt{x}}{3}$$
$$=\frac{\sqrt{x+3}-\sqrt{x}}{3} \cdot \frac{\sqrt{x+3}+\sqrt{x}}{\sqrt{x+3}\cdot\sqrt{x}}$$
$$=\frac{\left(\sqrt{x+3}\right)^2-(\sqrt{x})^2}{3\left(\sqrt{x+3}+\sqrt{x}\right)}$$
$$=\frac{x+3-x}{3\left(\sqrt{x+3}+\sqrt{x}\right)}$$
$$=\frac{1}{\sqrt{x+3}+\sqrt{x}}$$

Concept and Vocabulary Check P.6

1. polynomials

2. domain; 0

3. factoring; common factors

4. $\frac{x^2}{15}$

5. $\frac{3}{5}$

6. $\frac{x^2-x+4}{3}$

7. $x+3$ and $x-2$; $x+3$ and $x+1$; $(x+3)(x-2)(x+1)$

8. $3x+4$

9. complex; complex

10. x; $x+3$; -3; $-\frac{1}{x(x+3)}$

11. $\sqrt{x}$

12. $\sqrt{x+7}+\sqrt{x}$

Exercise Set P.6

1. $\frac{7}{x-3}, x \neq 3$

2. $\frac{13}{x+9}, x \neq -9$

3. $\frac{x+5}{x^2-25} = \frac{x+5}{(x+5)(x-5)}, x \neq 5, -5$

4. $\frac{x+7}{x^2-49} = \frac{x+7}{(x+7)(x-7)}, x \neq 7, -7$

5. $\frac{x-1}{x^2+11x+10} = \frac{x-1}{(x+1)(x+10)}, x \neq -1, -10$

6. $\frac{x-3}{x^2+4x-45} = \frac{x-3}{(x+9)(x-5)}, x \neq -9, 5$

7. $\frac{3x-9}{x^2-6x+9} = \frac{3(x-3)}{(x-3)(x-3)}$
$= \frac{3}{x-3}, x \neq 3$

8. $\frac{4x-8}{x^2-4x+4} = \frac{4(x-2)}{(x-2)(x-2)} = \frac{4}{x-2}, \ x \neq 2$

9. $\frac{x^2-12x+36}{4x-24} = \frac{(x-6)(x-6)}{4(x-6)} = \frac{x-6}{4}.$
$x \neq 6$

10. $\frac{x^2-8x+16}{3x-12} = \frac{(x-4)(x-4)}{3(x-4)} = \frac{x-4}{3}, \ x \neq 4$

11. $\frac{y^2+7y-18}{y^2-3y+2} = \frac{(y+9)(y-2)}{(y-2)(y-1)} = \frac{y+9}{y-1},$
$y \neq 1, 2$

12. $\frac{y^2-4y-5}{y^2+5y+4} = \frac{(y-5)(y+1)}{(y+4)(y+1)} = \frac{y-5}{y+4}, y \neq -4, -1$

13. $\frac{x^2+12x+36}{x^2-36} = \frac{(x+6)^2}{(x+6)(x-6)} = \frac{x+6}{x-6},$
$x \neq 6, -6$

14. $\frac{x^2-14x+49}{x^2-49} = \frac{(x-7)^2}{(x-7)(x+7)}$
$= \frac{x-7}{x+7},$
$x \neq 7, -7$

15. $\frac{x-2}{3x+9} \cdot \frac{2x+6}{2x-4} = \frac{x-2}{3(x+3)} \cdot \frac{2(x+3)}{2(x-2)}$
$= \frac{2}{6} = \frac{1}{3}, x \neq 2, -3$

16. $\frac{6x+9}{3x-15} \cdot \frac{x-5}{4x+6} = \frac{3(2x+3)}{3(x-5)} \cdot \frac{x-5}{2(2x+3)}$
$= \frac{3}{6}$
$= \frac{1}{2},$
$x \neq 5, -\frac{3}{2}$

17. $\frac{x^2-9}{x^2}\cdot\frac{x^2-3x}{x^2+x-12}$
$=\frac{(x-3)(x+3)}{x^2}\cdot\frac{x(x-3)}{(x+4)(x-3)}$
$=\frac{(x-3)(x+3)}{x(x+4)}, x\neq 0,-4,3$

18. $\frac{x^2-4}{x^2-4x+4}\cdot\frac{2x-4}{x+2}=\frac{(x+2)(x-2)}{(x-2)^2}\cdot\frac{2(x-2)}{x+2}$
$=2,$
$x\neq 2,-2$

19. $\frac{x^2-5x+6}{x^2-2x-3}\cdot\frac{x^2-1}{x^2-4}$
$=\frac{(x-3)(x-2)}{(x-3)(x+1)}\cdot\frac{(x+1)(x-1)}{(x-2)(x+2)}$
$=\frac{x-1}{x+2},\ x\neq -2,-1,2,3$

20. $\frac{x^2+5x+6}{x^2+x-6}\cdot\frac{x^2-9}{x^2-x-6}$
$=\frac{(x+3)(x+2)}{(x+3)(x-2)}\cdot\frac{(x-3)(x+3)}{(x-3)(x+2)}=\frac{x+3}{x-2},$
$x\neq -3,-2,2,3$

21. $\frac{x^3-8}{x^2-4}\cdot\frac{x+2}{3x}=\frac{(x-2)\left(x^2+2x+4\right)}{(x-2)(x+2)}\cdot\frac{x+2}{3x}$
$=\frac{x^2+2x+4}{3x}, x\neq -2,0,2$

22. $\frac{x^2+6x+9}{x^3+27}\cdot\frac{1}{x+3}$
$=\frac{(x+3)(x+3)}{(x+3)\left(x^2-3x+9\right)}\cdot\frac{1}{x+3}=\frac{1}{x^2-3x+9},$
$x\neq -3$

23. $\frac{x+1}{3}\div\frac{3x+3}{7}=\frac{x+1}{3}\div\frac{3(x+1)}{7}$
$=\frac{x+1}{3}\cdot\frac{7}{3(x+1)}$
$=\frac{7}{9}, x\neq -1$

24. $\frac{x+5}{7}\div\frac{4x+20}{9}=\frac{x+5}{7}\div\frac{4(x+5)}{9}$
$=\frac{x+5}{7}\cdot\frac{9}{4(x+5)}$
$=\frac{9}{28},$
$x\neq -5$

25. $\frac{x^2-4}{x}\div\frac{x+2}{x-2}=\frac{(x-2)(x+2)}{x}\cdot\frac{x-2}{x+2}$
$=\frac{(x-2)^2}{x}; x\neq 0,-2,2$

26. $\frac{x^2-4}{x-2}\div\frac{x+2}{4x-8}=\frac{(x-2)(x+2)}{x-2}\div\frac{x+2}{4(x-2)}$
$=\frac{(x-2)(x+2)}{x-2}\cdot\frac{4(x-2)}{x+2}$
$=4(x-2),$
$x\neq 2,-2$

27. $\frac{4x^2+10}{x-3}\div\frac{6x^2+15}{x^2-9}$
$=\frac{2(2x^2+5)}{x-3}\div\frac{3(2x^2+5)}{(x-3)(x+3)}$
$=\frac{2(2x^2+5)}{x-3}\cdot\frac{(x-3)(x+3)}{3(2x^2+5)}$
$=\frac{2(x+3)}{3}, x\neq 3,-3$

28. $\frac{x^2+x}{x^2-4}\div\frac{x^2-1}{x^2+5x+6}$
$=\frac{x(x+1)}{(x-2)(x+2)}\div\frac{(x-1)(x+1)}{(x+2)(x+3)}$
$=\frac{x(x+1)}{(x-2)(x+2)}\cdot\frac{(x+2)(x+3)}{(x-1)(x+1)}$
$=\frac{x(x+3)}{(x-2)(x-1)},$
$x\neq 2,1,-1,-2,-3$

29. $\frac{x^2-25}{2x-2}\div\frac{x^2+10x+25}{x^2+4x-5}$
$=\frac{(x-5)(x+5)}{2(x-1)}\div\frac{(x+5)^2}{(x+5)(x-1)}$
$=\frac{(x-5)(x+5)}{2(x-1)}\cdot\frac{(x+5)(x-1)}{(x+5)^2}$
$=\frac{x-5}{2}, x\neq 1,-5$

30. $\frac{x^2-4}{x^2+3x-10} \div \frac{x^2+5x+6}{x^2+8x+15}$
$= \frac{(x+2)(x-2)}{(x+5)(x-2)} \div \frac{(x+2)(x+3)}{(x+3)(x+5)}$
$= \frac{(x+2)(x-2)}{(x+5)(x-2)} \cdot \frac{(x+3)(x+5)}{(x+2)(x+3)}$
$= 1$
$x \neq 2, -2, -3, -5$

31. $\frac{x^2+x-12}{x^2+x-30} \cdot \frac{x^2+5x+6}{x^2-2x-3} \div \frac{x+3}{x^2+7x+6}$
$= \frac{(x+4)(x-3)}{(x+6)(x-5)} \cdot \frac{(x+2)(x+3)}{(x+1)(x-3)} \cdot \frac{(x+6)(x+1)}{x+3}$
$= \frac{(x+4)(x+2)}{x-5}$
$x \neq -6, -3, -1, 3, 5$

32. $\frac{x^3-25x}{4x^2} \cdot \frac{2x^2-2}{x^2-6x+5} \div \frac{x^2+5x}{7x+7}$
$= \frac{x(x-5)(x+5)}{4x^2} \cdot \frac{2(x-1)(x+1)}{(x-1)(x-5)} \cdot \frac{7(x+1)}{x(x+5)}$
$= \frac{7(x+1)^2}{2x^2}$
$x \neq 0, 1, -1, 5, -5$

33. $\frac{4x+1}{6x+5} + \frac{8x+9}{6x+5} = \frac{4x+1+8x+9}{6x+5}$
$= \frac{12x+10}{6x+5}$
$= \frac{2(6x+5)}{6x+5} = 2, x \neq -\frac{5}{6}$

34. $\frac{3x+2}{3x+4} + \frac{3x+6}{3x+4} = \frac{3x+2+3x+6}{3x+4}$
$= \frac{6x+8}{3x+4}$
$= \frac{2(3x+4)}{3x+4}$
$= 2$
$x \neq -\frac{4}{3}$

35. $\frac{x^2-2x}{x^2+3x} + \frac{x^2+x}{x^2+3x} = \frac{x^2-2x+x^2+x}{x^2+3x}$
$= \frac{2x^2-x}{x^2+3x}$
$= \frac{x(2x-1)}{x(x+3)}$
$= \frac{2x-1}{x+3}, x \neq 0, -3$

36. $\frac{x^2-4x}{x^2-x-6} + \frac{4x-4}{x^2-x-6} = \frac{x^2-4x+4x-4}{x^2-x-6}$
$= \frac{x^2-4}{(x-3)(x+2)}$
$= \frac{(x-2)(x+2)}{(x-3)(x+2)}$
$= \frac{x-2}{x-3},$
$x \neq -2, 3$

37. $\frac{4x-10}{x-2} - \frac{x-4}{x-2} = \frac{4x-10-(x-4)}{x-2}$
$= \frac{4x-10-x+4}{x-2}$
$= \frac{3x-6}{x-2}$
$= \frac{3(x-2)}{x-2}$
$= 3, x \neq 2$

38. $\frac{2x+3}{3x-6} - \frac{3-x}{3x-6} = \frac{2x+3-(3-x)}{3x-6}$
$= \frac{2x+3-3+x}{3x-6}$
$= \frac{3x}{3(x-2)}$
$= \frac{x}{x-2},$
$x \neq 2$

39. $\frac{x^2+3x}{x^2+x-12} - \frac{x^2-12}{x^2+x-12}$
$= \frac{x^2+3x-(x^2-12)}{x^2+x-12}$
$= \frac{x^2+3x-x^2+12}{x^2+x-12}$
$= \frac{3x+12}{x^2+x-12}$
$= \frac{3(x+4)}{(x+4)(x-3)}$
$= \frac{3}{x-3}, x \neq 3, -4$

40. $\frac{x^2-4x}{x^2-x-6}-\frac{x-6}{x^2-x-6}$
$=\frac{x^2-4x-(x-6)}{x^2-x-6}$
$=\frac{x^2-4x-x+6}{x^2-x-6}$
$=\frac{x^2-5x+6}{x^2-x-6}$
$=\frac{(x-2)(x-3)}{(x-3)(x+2)}$
$=\frac{x-2}{x+2},\ x\neq -2,\ 3$

41. $\frac{3}{x+4}+\frac{6}{x+5}=\frac{3(x+5)+6(x+4)}{(x+4)(x+5)}$
$=\frac{3x+15+6x+24}{(x+4)(x+5)}$
$=\frac{9x+39}{(x+4)(x+5)},\ x\neq -4, -5$

42. $\frac{8}{x-2}+\frac{2}{x-3}=\frac{8(x-3)+2(x-2)}{(x-2)(x-3)}$
$=\frac{8x-24+2x-4}{(x-2)(x-3)}$
$=\frac{10x-28}{(x-2)(x-3)},$
$x\neq 2, 3$

43. $\frac{3}{x+1}-\frac{3}{x}=\frac{3x-3(x+1)}{x(x+1)}$
$=\frac{3x-3x-3}{x(x+1)}=-\frac{3}{x(x+1)},\ x\neq -1, 0$

44. $\frac{4}{x}-\frac{3}{x+3}=\frac{4(x+3)-3x}{x(x+3)}$
$=\frac{4x+12-3x}{x(x+3)}$
$=\frac{x+12}{x(x+3)}$
$x\neq -3, 0$

45. $\frac{2x}{x+2}+\frac{x+2}{x-2}=\frac{2x(x-2)+(x+2)(x+2)}{(x+2)(x-2)}$
$=\frac{2x^2-4x+x^2+4x+4}{(x+2)(x-2)}$
$=\frac{3x^2+4}{(x+2)(x-2)},\ x\neq -2, 2$

46. $\frac{3x}{x-3}-\frac{x+4}{x+2}=\frac{3x(x+2)-(x+4)(x-3)}{(x-3)(x+2)}$
$=\frac{3x^2+6x-(x^2+x-12)}{(x-3)(x+2)}$
$=\frac{2x^2+5x+12}{(x-3)(x+2)},$
$x\neq 3, -2$

47. $\frac{x+5}{x-5}+\frac{x-5}{x+5}$
$=\frac{(x+5)(x+5)+(x-5)(x-5)}{(x-5)(x+5)}$
$=\frac{x^2+10x+25+x^2-10x+25}{(x-5)(x+5)}$
$=\frac{2x^2+50}{(x-5)(x+5)},\ x\neq -5, 5$

48. $\frac{x+3}{x-3}+\frac{x-3}{x+3}=\frac{(x+3)(x+3)+(x-3)(x-3)}{(x-3)(x+3)}$
$=\frac{x^2+6x+9+x^2-6x+9}{(x-3)(x+3)}$
$=\frac{2x^2+18}{(x-3)(x+3)},$
$x\neq -3, 3$

49. $\frac{3}{2x+4}+\frac{2}{3x+6}=\frac{3}{2(x+2)}+\frac{2}{3(x+2)}$
$=\frac{9}{6(x+2)}+\frac{4}{6(x+2)}$
$=\frac{9+4}{6(x+2)}$
$=\frac{13}{6(x+2)}$
$x\neq -2$

50. $\frac{5}{2x+8}+\frac{7}{3x+12}=\frac{5}{2(x+4)}+\frac{7}{3(x+4)}$
$=\frac{15}{6(x+4)}+\frac{14}{6(x+4)}$
$=\frac{15+14}{6(x+4)}$
$=\frac{29}{6(x+4)}$
$x\neq -4$

51. $\frac{4}{x^2+6x+9}+\frac{4}{x+3}=\frac{4}{(x+3)^2}+\frac{4}{x+3}$
$=\frac{4+4(x+3)}{(x+3)^2}=\frac{4+4x+12}{(x+3)^2}=\frac{4x+16}{(x+3)^2}$,
$x\neq -3$

52. $\frac{3}{5x+2}+\frac{5x}{25x^2-4}=\frac{3}{5x+2}+\frac{5x}{(5x-2)(5x+2)}$
$=\frac{3(5x-2)+5x}{(5x-2)(5x+2)}$
$=\frac{15x-6+5x}{(5x-2)(5x+2)}$
$=\frac{20x-6}{(5x-2)(5x+2)}$,
$x\neq -\frac{2}{5},\frac{2}{5}$

53. $\frac{3x}{x^2+3x-10}-\frac{2x}{x^2+x-6}$
$=\frac{3x}{(x+5)(x-2)}-\frac{2x}{(x+3)(x-2)}$
$=\frac{3x(x+3)-2x(x+5)}{(x+5)(x-2)(x+3)}$
$=\frac{3x^2+9x-2x^2-10x}{(x+5)(x-2)(x+3)}$
$=\frac{x^2-x}{(x+5)(x-2)(x+3)}$, $x\neq -5, 2, -3$

54. $\frac{x}{x^2-2x-24}-\frac{x}{x^2-7x+6}$
$=\frac{x}{(x-6)(x+4)}-\frac{x}{(x-6)(x-1)}$
$=\frac{x(x-1)-x(x+4)}{(x-6)(x+4)(x-1)}$
$=\frac{x^2-x-x^2-4x}{(x-6)(x+4)(x-1)}$
$=-\frac{5x}{(x-6)(x-1)(x+4)}$,
$x\neq 6, 1, -4$

55. $\frac{x+3}{x^2-1}-\frac{x+2}{x-1}$
$=\frac{x+3}{(x+1)(x-1)}-\frac{x+2}{x-1}$
$=\frac{x+3}{(x+1)(x-1)}-\frac{(x+1)(x+2)}{(x+1)(x-1)}$
$=\frac{x+3}{(x+1)(x-1)}-\frac{x^2+3x+2}{(x+1)(x-1)}$
$=\frac{x+3-x^2-3x-2}{(x+1)(x-1)}$
$=\frac{-x^2-2x+1}{(x+1)(x-1)}$
$x\neq 1, -1$

56. $\frac{x+5}{x^2-4}-\frac{x+1}{x-2}$
$=\frac{x+5}{(x+2)(x-2)}-\frac{x+1}{x-2}$
$=\frac{x+5}{(x+2)(x-2)}-\frac{(x+2)(x+1)}{(x+2)(x-2)}$
$=\frac{x+5}{(x+2)(x-2)}-\frac{x^2+3x+2}{(x+2)(x-2)}$
$=\frac{x+5-x^2-3x-2}{(x+2)(x-2)}$
$=\frac{-x^2-2x+3}{(x+2)(x-2)}$
$x\neq 2, -2$

57. $\dfrac{4x^2+x-6}{x^2+3x+2}-\dfrac{3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-6}{(x+1)(x+2)}+\dfrac{-3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-5}{(x+1)(x+2)}+\dfrac{-3x(x+2)}{(x+1)(x+2)}+\dfrac{5(x+1)}{(x+1)(x+2)}$

$=\dfrac{4x^2+x-6-3x^2-6x+5x+5}{(x+1)(x+2)}$

$=\dfrac{x^2-1}{(x+1)(x+2)}$

$=\dfrac{(x-1)(x+1)}{(x+1)(x+2)}$

$=\dfrac{x-1}{x+2}; x\neq -2,-1$

58. $\dfrac{6x^2+17x-40}{x^2+x-20}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40}{(x+5)(x-4)}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40+3(x+5)-5x(x-4)}{(x+5)(x-4)}$

$=\dfrac{6x^2+17x-40+3x+15-5x^2+20x}{(x+5)(x-4)}$

$=\dfrac{x^2+40x-25}{(x+5)(x-4)}; x\neq -5,4$

59. $\dfrac{\frac{x}{3}-1}{x-3}=\dfrac{3\left[\frac{x}{3}-1\right]}{3[x-3]}=\dfrac{x-3}{3(x-3)}=\dfrac{1}{3},\ x\neq 3$

60. $\dfrac{\frac{x}{4}-1}{x-4}=\dfrac{4\left[\frac{x}{4}-1\right]}{4(x-4)}=\dfrac{x-4}{4(x-4)}=\dfrac{1}{4}, x\neq 4$

61. $\dfrac{1+\frac{1}{x}}{3-\frac{1}{x}}=\dfrac{x\left[1+\frac{1}{x}\right]}{x\left[3-\frac{1}{x}\right]}=\dfrac{x+1}{3x-1}, x\neq 0,\dfrac{1}{3}$

62. $\dfrac{8+\frac{1}{x}}{4-\frac{1}{x}}=\dfrac{x\left[8+\frac{1}{x}\right]}{x\left[4-\frac{1}{x}\right]}=\dfrac{8x+1}{4x-1},\ x\neq 0,\dfrac{1}{4}$

63. $\dfrac{\frac{1}{x}+\frac{1}{y}}{x+y}=\dfrac{xy\left[\frac{1}{x}+\frac{1}{y}\right]}{xy[x+y]}=\dfrac{y+x}{xy(x+y)}=\dfrac{1}{xy},$

$x\neq 0, y\neq 0, x\neq -y$

64. $\dfrac{1-\frac{1}{x}}{xy}=\dfrac{x\left[1-\frac{1}{x}\right]}{x(xy)}=\dfrac{x-1}{x^2y}, x\neq 0, y\neq 0$

65. $\dfrac{x-\frac{x}{x+3}}{x+2}=\dfrac{(x+3)\left[x-\frac{x}{x+3}\right]}{(x+3)(x+2)}=\dfrac{x(x+3)-x}{(x+3)(x+2)}$

$=\dfrac{x^2+3x-x}{(x+3)(x+2)}=\dfrac{x^2+2x}{(x+3)(x+2)}$

$=\dfrac{x(x+2)}{(x+3)(x+2)}=\dfrac{x}{x+3}, x\neq -2, -3$

66. $\dfrac{x-3}{x-\frac{3}{x-2}}=\dfrac{(x-2)[x-3]}{(x-2)\left[x-\frac{3}{x-2}\right]}=\dfrac{(x-2)(x-3)}{x(x-2)-3}$

$=\dfrac{(x-2)(x-3)}{x^2-2x-3}$

$=\dfrac{(x-2)(x-3)}{(x-3)(x+1)}=\dfrac{x-2}{x+1}, x\neq 2,3,-1$

67. $\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{x^2-4}}=\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{(x-2)(x+2)}}$

$=\dfrac{\left[\frac{3}{x-2}-\frac{4}{x+2}\right](x-2)(x+2)}{\left[\frac{7}{(x-2)(x+2)}\right](x-2)(x+2)}$

$=\dfrac{3(x+2)-4(x-2)}{7}$

$=\dfrac{3x+6-4x+8}{7}=\dfrac{-x+14}{7}$

$=-\dfrac{x-14}{7}\quad x\neq -2, 2$

68. $$\frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1}=\frac{\frac{x}{x-2}+1}{\frac{3}{(x-2)(x+2)}+1}$$
$$=\frac{\left[\frac{x}{x-2}+1\right](x-2)(x+2)}{\left[\frac{3}{(x-2)(x+2)}+1\right](x-2)(x+2)}$$
$$=\frac{x(x+2)+(x-2)(x+2)}{3+(x-2)(x+2)}$$
$$=\frac{x^2+2x+x^2-4}{3+x^2-4}=\frac{2x^2+2x-4}{x^2-1}$$
$$=\frac{2(x^2+x-2)}{(x-1)(x+1)}$$
$$=\frac{2(x+2)(x-1)}{(x-1)(x+1)}=\frac{2(x+2)}{x+1},$$
$x\neq 1,-1,\ 2,\ -2$

69. $$\frac{\frac{1}{x+1}}{\frac{1}{x^2-2x-3}+\frac{1}{x-3}}=\frac{\frac{1}{x+1}}{\frac{1}{(x+1)(x-3)}+\frac{1}{x-3}}$$
$$=\frac{\frac{(x+1)(x-3)}{x+1}}{\frac{(x+1)(x-3)}{(x+1)(x-3)}+\frac{(x+1)(x-3)}{x-3}}$$
$$=\frac{x-3}{1+x+1}$$
$$=\frac{x-3}{x+2}\quad x\neq -2,-1,3$$

70. $$\frac{\frac{6}{x^2+2x-15}-\frac{1}{x-3}}{\frac{1}{x+5}+1}=\frac{\frac{6}{(x+5)(x-3)}-\frac{1}{x-3}}{\frac{1}{x+5}+1}$$
$$=\frac{\frac{6(x+5)(x-3)}{(x+5)(x-3)}-\frac{(x+5)(x-3)}{x-3}}{\frac{(x+5)(x-3)}{x+5}+(x+5)(x-3)}$$
$$=\frac{6-(x+5)}{(x-3)+(x+5)(x-3)}$$
$$=\frac{6-x-5}{x-3+x^2+2x-15}$$
$$=\frac{1-x}{x^2+3x-18}$$
$$=\frac{1-x}{(x+6)(x-3)}\quad x\neq -6,-5,3$$

71. $$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2(x+h)^2}{(x+h)^2}-\frac{x^2(x+h)^2}{x^2}}{hx^2(x+h)^2}$$
$$=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$
$$=\frac{x^2-(x^2+2hx+h^2)}{hx^2(x+h)^2}$$
$$=\frac{x^2-x^2-2hx-h^2}{hx^2(x+h)^2}$$
$$=\frac{-2hx-h^2}{hx^2(x+h)^2}$$
$$=\frac{-h(2x+h)}{hx^2(x+h)^2}$$
$$=-\frac{(2x+h)}{x^2(x+h)^2}$$

72. $$\frac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h}=\frac{\frac{(x+h)(x+h+1)(x+1)}{x+h+1}-\frac{x(x+h+1)(x+1)}{x+1}}{h(x+h+1)(x+1)}$$
$$=\frac{(x+h)(x+1)-x(x+h+1)}{h(x+h+1)(x+1)}$$
$$=\frac{x^2+x+hx+h-x^2-hx-x}{h(x+h+1)(x+1)}$$
$$=\frac{h}{h(x+h+1)(x+1)}$$
$$=\frac{1}{(x+h+1)(x+1)}$$

73. $$\frac{\sqrt{x}-\frac{1}{3\sqrt{x}}}{\sqrt{x}}=\frac{\left(\sqrt{x}-\frac{1}{3\sqrt{x}}\right)(3\sqrt{x})}{\sqrt{x}(3\sqrt{x})}$$
$$=\frac{3x-1}{3x}$$
$$=1-\frac{1}{3x},\ x>0$$

74. $$\frac{\sqrt{x}-\frac{1}{4\sqrt{x}}}{\sqrt{x}}=\frac{\left(\sqrt{x}-\frac{1}{4\sqrt{x}}\right)(4\sqrt{x})}{\sqrt{x}(4\sqrt{x})}$$
$$=\frac{4x-1}{4x}$$
$$=1-\frac{1}{4x},\ x>0$$

75. $$\frac{\frac{x^2}{\sqrt{x^2+2}}-\sqrt{x^2+2}}{x^2}$$
$$=\frac{\left(\frac{x^2}{\sqrt{x^2+2}}-\sqrt{x^2+2}\right)\sqrt{x^2+2}}{x^2\sqrt{x^2+2}}$$
$$=\frac{x^2-(x^2+2)}{x^2\sqrt{x^2+2}}$$
$$=-\frac{2}{x^2\sqrt{x^2+2}}$$

76. $$\frac{\sqrt{5-x^2}+\frac{x^2}{\sqrt{5-x^2}}}{5-x^2}$$
$$=\frac{\left(\sqrt{5-x^2}+\frac{x^2}{\sqrt{5-x^2}}\right)\sqrt{5-x^2}}{\left(5-x^2\right)\sqrt{5-x^2}}$$
$$=\frac{5-x^2+x^2}{\left(5-x^2\right)\sqrt{5-x^2}}$$
$$=\frac{5}{\left(\sqrt{5-x^2}\right)^2\sqrt{5-x^2}}$$
$$=\frac{5}{\sqrt{\left(5-x^2\right)^3}}$$

77. $$\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}=\frac{\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)\sqrt{x+h}\sqrt{x}}{h\sqrt{x+h}\sqrt{x}}$$
$$=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x(x+h)}},\ h\neq 0$$

78. $$\frac{\frac{1}{\sqrt{x+3}}-\frac{1}{\sqrt{x}}}{3}=\frac{\left(\frac{1}{\sqrt{x+3}}-\frac{1}{\sqrt{x}}\right)\sqrt{x+3}\sqrt{x}}{3\sqrt{x+3}\sqrt{x}}$$
$$=\frac{\sqrt{x}-\sqrt{x+3}}{3\sqrt{x}\sqrt{x+3}}$$

79. $$\frac{\sqrt{x+5}-\sqrt{x}}{5}=\frac{\sqrt{x+5}-\sqrt{x}}{5}\cdot\frac{\sqrt{x+5}+\sqrt{x}}{\sqrt{x+5}+\sqrt{x}}$$
$$=\frac{(\sqrt{x+5})^2-(\sqrt{x})^2}{5(\sqrt{x+5}+\sqrt{x})}$$
$$=\frac{x+5-x}{5(\sqrt{x+5}+\sqrt{x})}$$
$$=\frac{1}{\sqrt{x+5}+\sqrt{x}}$$

80. $$\frac{\sqrt{x+7}-\sqrt{x}}{7}=\frac{\sqrt{x+7}-\sqrt{x}}{7}\cdot\frac{\sqrt{x+7}+\sqrt{x}}{\sqrt{x+7}+\sqrt{x}}$$
$$=\frac{(\sqrt{x+7})^2-(\sqrt{x})^2}{7(\sqrt{x+7}+\sqrt{x})}$$
$$=\frac{x+7-x}{7(\sqrt{x+7}+\sqrt{x})}$$
$$=\frac{1}{\sqrt{x+7}+\sqrt{x}}$$

81. $$\frac{\sqrt{x}+\sqrt{y}}{x^2-y^2}=\frac{\sqrt{x}+\sqrt{y}}{x^2-y^2}\cdot\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$
$$=\frac{(\sqrt{x})^2-(\sqrt{y})^2}{(x+y)(x-y)(\sqrt{x}-\sqrt{y})}$$
$$=\frac{x-y}{(x+y)(x-y)(\sqrt{x}-\sqrt{y})}$$
$$=\frac{1}{(x+y)(\sqrt{x}-\sqrt{y})}$$

82. $$\frac{\sqrt{x}-\sqrt{y}}{x^2-y^2}=\frac{\sqrt{x}-\sqrt{y}}{x^2-y^2}\cdot\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}$$
$$=\frac{(\sqrt{x})^2-(\sqrt{y})^2}{(x^2-y^2)(\sqrt{x}+\sqrt{y})}$$
$$=\frac{x-y}{(x+y)(x-y)(\sqrt{x}+\sqrt{y})}$$
$$=\frac{1}{(x+y)(\sqrt{x}+\sqrt{y})},\ x\neq y$$

83. $$\left(\frac{2x+3}{x+1}\cdot\frac{x^2+4x-5}{2x^2+x-3}\right)-\frac{2}{x+2}=\left(\frac{\cancel{(2x+3)}}{x+1}\cdot\frac{(x+5)\cancel{(x-1)}}{\cancel{(2x+3)}\cancel{(x-1)}}\right)-\frac{2}{x+2}=\frac{x+5}{x+1}-\frac{2}{x+2}$$
$$=\frac{(x+5)(x+2)}{(x+1)(x+2)}-\frac{2(x+1)}{(x+1)(x+2)}=\frac{(x+5)(x+2)-2(x+1)}{(x+1)(x+2)}=\frac{x^2+2x+5x+10-2x-2}{(x+1)(x+2)}=\frac{x^2+5x+8}{(x+1)(x+2)}$$

84. $$\frac{1}{x^2-2x-8}\cdot\left(\frac{1}{x-4}-\frac{1}{x+2}\right)=\frac{1}{(x-4)(x+2)}\div\left(\frac{(x+2)}{(x-4)(x+2)}-\frac{(x-4)}{(x-4)(x+2)}\right)$$
$$=\frac{1}{(x-4)(x+2)}\div\left(\frac{x+2-x+4}{(x-4)(x+2)}\right)=\frac{1}{(x-4)(x+2)}\div\left(\frac{6}{(x-4)(x+2)}\right)=\frac{1}{(x-4)(x+2)}\cdot\frac{(x-4)(x+2)}{6}=\frac{1}{6}$$

85. $$\left(2-\frac{6}{x+1}\right)\left(1+\frac{3}{x-2}\right)=\left(\frac{2(x+1)}{(x+1)}-\frac{6}{(x+1)}\right)\left(\frac{(x-2)}{(x-2)}+\frac{3}{(x-2)}\right)$$
$$=\left(\frac{2x+2-6}{x+1}\right)\left(\frac{x-2+3}{x-2}\right)=\left(\frac{2x-4}{x+1}\right)\left(\frac{x+1}{x-2}\right)=\frac{2\cancel{(x-2)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-2)}}=2$$

86. $$\left(4-\frac{3}{x+2}\right)\left(1+\frac{5}{x-1}\right)=\left(\frac{4(x+2)}{x+2}-\frac{3}{x+2}\right)\left(\frac{(x-1)}{x-1}+\frac{5}{x-1}\right)$$
$$=\left(\frac{4x+8-3}{x+2}\right)\left(\frac{x-1+5}{x-1}\right)=\frac{4x+5}{x+2}\cdot\frac{x+4}{x-1}=\frac{(4x+5)(x+4)}{(x+2)(x-1)}$$

87. $$\frac{y^{-1}-(y+5)^{-1}}{5}=\frac{\frac{1}{y}-\frac{1}{y+5}}{5}$$
LCD $= y(y+5)$
$$\frac{\frac{1}{y}-\frac{1}{y+5}}{5}=\frac{y(y+5)\left(\frac{1}{y}-\frac{1}{y+5}\right)}{y(y+5)(5)}=\frac{y+5-y}{5y(y+5)}=\frac{5}{5y(y+5)}=\frac{1}{y(y+5)}$$

88. $$\frac{y^{-1}-(y+2)^{-1}}{2}=\frac{\frac{1}{y}-\frac{1}{y+2}}{2}$$
LCD $= y(y+2)$
$$\frac{\frac{1}{y}-\frac{1}{y+2}}{2}=\frac{y(y+2)\left(\frac{1}{y}-\frac{1}{y+2}\right)}{y(y+2)(2)}=\frac{y+2-y}{2y(y+2)}=\frac{2}{2y(y+2)}=\frac{1}{y(y+2)}$$

89. $$\left(\frac{1}{a^3-b^3}\cdot\frac{ac+ad-bc-bd}{1}\right)-\frac{c-d}{a^2+ab+b^2}=\left(\frac{1}{(a-b)\left(a^2+ab+b^2\right)}\cdot\frac{a(c+d)-b(c+d)}{1}\right)-\frac{c-d}{a^2+ab+b^2}$$
$$=\left(\frac{1}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\cdot\frac{(c+d)\cancel{(a-b)}}{1}\right)-\frac{c-d}{a^2+ab+b^2}=\frac{c+d}{a^2+ab+b^2}-\frac{c-d}{a^2+bd+b^2}$$
$$=\frac{c+d-c+d}{a^2+ab+b^2}=\frac{2d}{a^2+ab+b^2}$$

90. $$\frac{ab}{a^2+ab+b^2}+\left(\frac{ac-ad-bc+bd}{ac-ad+bc-bd}\div\frac{a^3-b^3}{a^3+b^3}\right)=\frac{ab}{a^2+ab+b^2}+\left(\frac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)}\cdot\frac{a^3+b^3}{a^3-b^3}\right)$$
$$=\frac{ab}{a^2+ab+b^2}+\left(\frac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}}\cdot\frac{\cancel{(a+b)}\left(a^2-ab+b^2\right)}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\right)=\frac{ab}{a^2+ab+b^2}+\frac{a^2-ab+b^2}{a^2+ab+b^2}$$
$$=\frac{ab+a^2-ab+b^2}{a^2+ab+b^2}=\frac{a^2+b^2}{a^2+ab+b^2}$$

91. **a.** $\frac{130x}{100-x}$ is equal to

1. $\frac{130\cdot 40}{100-40}=\frac{130\cdot 40}{60}=86.67$,
when $x = 40$

2. $\frac{130\cdot 80}{100-80}=\frac{130\cdot 80}{20}=520$,
when $x = 80$

3. $\frac{130\cdot 90}{100-90}=\frac{130\cdot 90}{10}=1170$,
when $x = 90$

It costs $86,670,000 to inoculate 40% of the population against this strain of flu, and $520,000,000 to inoculate 80% of the population, and $1,170,000,000 to inoculate 90% of the population.

b. For $x = 100$, the function is not defined.

c. As x approaches 100, the value of the function increases rapidly. So it costs an astronomical amount of money to inoculate almost all of the people, and it is impossible to inoculate 100% of the population.

92. $\frac{2d}{\frac{d}{r_1}+\frac{d}{r_2}}$

LCD = r_1r_2

$$\frac{2d}{\frac{d}{r_1}+\frac{d}{r_2}}=\frac{r_1r_2(2d)}{r_1r_2\left(\frac{d}{r_1}+\frac{d}{r_2}\right)}$$
$$=\frac{2r_1r_2d}{r_2d+r_1d}$$
$$=\frac{2r_1r_2d}{d(r_2+r_1)}=\frac{2r_1r_2}{r_2+r_1}$$

If r_1 = 40 and r_2 = 30, the value of this expression will be

$$\frac{2\cdot 40\cdot 30}{30+40}=\frac{2400}{70}$$
$$=34\frac{2}{7}.$$

Your average speed will be $34\frac{2}{7}$ miles per hour.

93. **a.** Substitute 4 for x in the model.

$$W=-66x^2+526x+1030$$
$$W=-66(4)^2+526(4)+1030$$
$$W=2078$$

According to the model, women between the ages of 19 and 30 with this lifestyle need 2078 calories per day. This underestimates the actual value shown in the bar graph by 22 calories.

b. Substitute 4 for x in the model.

$$M=-120x^2+998x+590$$
$$M=-120(4)^2+998(4)+590$$
$$M=2662$$

According to the model, men between the ages of 19 and 30 with this lifestyle need 2662 calories per day. This underestimates the actual value shown in the bar graph by 38 calories.

c. $\frac{W}{M} = \frac{-66x^2 + 526x + 1030}{-120x^2 + 998x + 590}$
$= \frac{2(-33x^2 + 263x + 515)}{2(-60x^2 + 499x + 295)}$
$= \frac{-33x^2 + 263x + 515}{-60x^2 + 499x + 295}$

94. $R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$
$= \frac{R_1 R_2 R_3}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) R_1 R_2 R_3}$
$= \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$

$R(4, 8, 12) = \frac{4 \cdot 8 \cdot 12}{8 \cdot 12 + 4 \cdot 12 + 4 \cdot 8}$
$= \frac{384}{96 + 48 + 32}$
$= \frac{384}{176}$
$= \frac{24}{11}$

The parallel resistance is $\frac{24}{11}$ ohms.

95. $P = 2L + 2W$
$= 2\left(\frac{x}{x+3}\right) + 2\left(\frac{x}{x-4}\right)$
$= \frac{2x}{x+3} + \frac{2x}{x+4}$
$= \frac{2x(x+4)}{(x+3)(x+4)} + \frac{2x(x+3)}{(x+3)(x+4)}$
$= \frac{2x^2 + 8x + 2x^2 + 6x}{(x+3)(x+4)}$
$= \frac{4x^2 + 14x}{(x+3)(x+4)}$

96. $P = 2L + 2W$
$= 2 \quad \frac{x}{x+5} \quad + 2 \quad \frac{x}{x+6}$
$= \frac{2x}{x+5} + \frac{2x}{x+6}$
$= \frac{2x(x+6)}{(x+5)(x+6)} + \frac{2x(x+5)}{(x+5)(x+6)}$
$= \frac{2x^2 + 12x + 2x^2 + 10x}{(x+5)(x+6)}$
$= \frac{4x^2 + 22x}{(x+5)(x+6)}$

97. – 108. Answers will vary.

109. does not make sense; Explanations will vary. Sample explanation: $\frac{3x-3}{4x(x-1)} = \frac{3(1)-3}{4(1)(1-1)} = \frac{0}{0}$ which is undefined.

110. does not make sense; Explanations will vary. Sample explanation: The numerator and denominator of $\frac{7}{14+x}$ do not share a common factor.

111. does not make sense; Explanations will vary. Sample explanation: The first step is to invert the second fraction.

112. makes sense

113. false; Changes to make the statement true will vary. A sample change is: $\frac{x^2-25}{x-5} = \frac{(x+5)(x-5)}{x-5} = x+5$

114. true

115. true

116. false; Changes to make the statement true will vary. A sample change is: $6+\frac{1}{x} = \frac{6x}{x}+\frac{1}{x} = \frac{6x+1}{x}$

117.
$$\begin{aligned}&\frac{1}{x^n-1}-\frac{1}{x^n+1}-\frac{1}{x^{2n}-1}\\&=\frac{x^n+1}{x^{2n}-1}-\frac{x^n-1}{x^{2n}-1}-\frac{1}{x^{2n}-1}\\&=\frac{x^n+1-x^n+1-1}{x^{2n}-1}\\&=\frac{1}{x^{2n}-1}\end{aligned}$$

118.
$$\begin{aligned}\left(1-\frac{1}{x}\right)\left(1-\frac{1}{x+1}\right)\left(1-\frac{1}{x+2}\right)\left(1-\frac{1}{x+3}\right)&=\left(\frac{x}{x}-\frac{1}{x}\right)\left(\frac{x+1}{x+1}-\frac{1}{x+1}\right)\left(\frac{x+2}{x+2}-\frac{1}{x+2}\right)\left(\frac{x+3}{x+3}-\frac{1}{x+3}\right)\\&=\left(\frac{x-1}{x}\right)\left(\frac{(x+1)-1}{x+1}\right)\left(\frac{(x+2)-1}{x+2}\right)\left(\frac{(x+3)-1}{x+3}\right)\\&=\frac{x-1}{\cancel{x}}\cdot\frac{\cancel{x}}{\cancel{x+1}}\cdot\frac{\cancel{x+1}}{\cancel{x+2}}\cdot\frac{\cancel{x+2}}{x+3}=\frac{x-1}{x+3}\end{aligned}$$

119. $(x-y)^{-1}+(x-y)^{-2} = \frac{1}{(x-y)}+\frac{1}{(x-y)^2} = \frac{(x-y)}{(x-y)(x-y)}+\frac{1}{(x-y)^2} = \frac{x-y+1}{(x-y)^2}$

120. It cubes *x*.
$$\frac{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}}{\frac{1}{x^4}+\frac{1}{x^5}+\frac{1}{x^6}} = \frac{\frac{x^6}{x}+\frac{x^6}{x^2}+\frac{x^6}{x^3}}{\frac{x^6}{x^4}+\frac{x^6}{x^5}+\frac{x^6}{x^6}} = \frac{x^5+x^4+x^3}{x^2+x+1} = \frac{x^3\left(x^2+x+1\right)}{x^2+x+1} = x^3$$

121.
$$\begin{aligned}2(x-3)-17&=13-3(x+2)\\2(6-3)-17&=13-3(6+2)\\2(3)-17&=13-3(8)\\6-17&=13-24\\-11&=-11,\ \text{true}\end{aligned}$$

122.
$$\begin{aligned}12\left(\frac{x+2}{4}-\frac{x-1}{3}\right)&=12\left(\frac{x+2}{4}\right)-12\left(\frac{x-1}{3}\right)\\&=3(x+2)-4(x-1)\\&=3x+6-4x+4\\&=-x+10\end{aligned}$$

123.
$$\begin{aligned}\frac{-b-\sqrt{b^2-4ac}}{2a}&=\frac{-(9)-\sqrt{(9)^2-4(2)(-5)}}{2(2)}\\&=\frac{-9-\sqrt{81+40}}{4}\\&=\frac{-9-\sqrt{121}}{4}\\&=\frac{-9-11}{4}\\&=-5\end{aligned}$$

Section P.7

Check Point Exercises

1.
$$\begin{aligned}4(2x+1)-29&=3(2x-5)\\8x+4-29&=6x-15\\8x-25&=6x-15\\8x-25-6x&=6x-15-6x\\2x-25&=-15\\2x-25+25&=-15+25\\2x&=10\\\frac{2x}{2}&=\frac{10}{2}\\x&=5\end{aligned}$$

Check:
$$\begin{aligned}4(2x+1)-29&=3(2x-5)\\4[2(5)+1]-29&=3[2(5)-5]\\4[10+1]-29&=3[10-5]\\4[11]-29&=3[5]\\44-29&=15\\15&=15\ \text{true}\end{aligned}$$

The solution set is $\{5\}$.

2.
$$\begin{aligned}\frac{x-3}{4}&=\frac{5}{14}-\frac{x+5}{7}\\28\cdot\frac{x-3}{4}&=28\left(\frac{5}{14}-\frac{x+5}{7}\right)\\7(x-3)&=2(5)-4(x+5)\\7x-21&=10-4x-20\\7x-21&=-4x-10\\7x+4x&=-10+21\\11x&=11\\\frac{11x}{11}&=\frac{11}{11}\\x&=1\end{aligned}$$

Check:
$$\begin{aligned}\frac{x-3}{4}&=\frac{5}{14}-\frac{x+5}{7}\\\frac{1-3}{4}&=\frac{5}{14}-\frac{1+5}{7}\\\frac{-2}{4}&=\frac{5}{14}-\frac{6}{7}\\-\frac{1}{2}&=-\frac{1}{2}\end{aligned}$$

The solution set is $\{1\}$.

3.
$$\begin{aligned}\frac{6}{x+3}-\frac{5}{x-2}&=\frac{-20}{x^2+x-6}\\\frac{6}{x+3}-\frac{5}{x-2}&=\frac{-20}{(x+3)(x-2)}\\\frac{6(x+3)(x-2)}{x+3}-\frac{5(x+3)(x-2)}{x-2}&=\frac{-20(x+3)(x-2)}{(x+3)(x-2)}\\6(x-2)-5(x+3)&=-20\\6x-12-5x-15&=-20\\x-27&=-20\\x&=7\end{aligned}$$

The solution set is $\{7\}$.

4.
$$\begin{aligned}\frac{1}{x+2}&=\frac{4}{x^2-4}-\frac{1}{x-2}\\\frac{1}{x+2}&=\frac{4}{(x+2)(x-2)}-\frac{1}{x-2}\\\frac{1(x+2)(x-2)}{x+2}&=\frac{4(x+2)(x-2)}{(x+2)(x-2)}-\frac{1(x+2)(x-2)}{x-2}\\x-2&=4-(x+2)\\x-2&=4-x-2\\x-2&=2-x\\2x&=4\\x&=2\end{aligned}$$

2 must be rejected. The solution set is $\{\ \}$.

5. $$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1pqf}{p}+\frac{1pqf}{q}=\frac{1pqf}{f}$$
$$qf+pf=pq$$
$$qf-pq=-pf$$
$$q(f-p)=-pf$$
$$\frac{q(f-p)}{f-p}=\frac{-pf}{f-p}$$
$$q=\frac{pf}{p-f}$$

6. $$4|1-2x|-20=0$$
$$4|1-2x|=20$$
$$|1-2x|=5$$
$1-2x=5$ or $1-2x=-5$
$-2x=4$ $\quad$ $-2x=-6$
$x=-2$ $\quad$ $x=3$

The solution set is $\{-2, 3\}$.

7. a. $$3x^2-9x=0$$
$$3x(x-3)=0$$
$3x=0$ or $x-3=0$
$x=0$ $\quad$ $x=3$

The solution set is $\{0,3\}$.

b. $$2x^2+x=1$$
$$2x^2+x-1=0$$
$$(2x-1)(x+1)=0$$
$2x-1=0$ or $x+1=0$
$2x=1$ $\quad$ $x=-1$
$x=\frac{1}{2}$

The solution set is $\left\{\frac{1}{2},-1\right\}$.

8. a. $$3x^2-21=0$$
$$3x^2=21$$
$$\frac{3x^2}{3}=\frac{21}{3}$$
$$x^2=7$$
$$x=\pm\sqrt{7}$$

The solution set is $\left\{-\sqrt{7},\sqrt{7}\right\}$.

b. $$(x+5)^2=11$$
$$x+5=\pm\sqrt{11}$$
$$x=-5\pm\sqrt{11}$$

The solution set is $\left\{-5+\sqrt{11},-5-\sqrt{11}\right\}$.

9. $$x^2+4x-1=0$$
$$x^2+4x=1$$
$$x^2+4x+4=1+4$$
$$(x+2)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

The solution set is $\left\{-2\pm\sqrt{5}\right\}$.

10. $$2x^2+2x-1=0$$
$$a=2, b=2, c=-1$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}$$
$$=\frac{-2\pm\sqrt{4+8}}{4}$$
$$=\frac{-2\pm\sqrt{12}}{4}$$
$$=\frac{-2\pm2\sqrt{3}}{4}$$
$$=\frac{2(-1\pm\sqrt{3})}{4}$$
$$=\frac{-1\pm\sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1+\sqrt{3}}{2},\frac{-1-\sqrt{3}}{2}\right\}$.

11. $$3x^2-2x+5=0$$
$$a=3, b=-2, c=5$$
$$b^2-4ac=(-2)^2-4\cdot3\cdot5=4-60=-56$$

The discriminant is –56. The equation has no real solutions.

12. $$\sqrt{x+3}+3=x$$
$$\sqrt{x+3}=x-3$$
$$\left(\sqrt{x+3}\right)^2=(x-3)^2$$
$$x+3=x^2-6x+9$$
$$0=x^2-7x+6$$
$$0=(x-6)(x-1)$$
$x-6=0$ or $x-1=0$
$x=6$ $\quad$ $x=1$

1 does not check and must be rejected.
The solution set is $\{6\}$.

Concept and Vocabulary Check P.7

1. linear

2. equivalent

3. apply the distributive property

4. least common denominator; 12

5. 0

6. $x \neq 2$; $x \neq -1$

7. $5(x+3)+3(x+4)=12x+9$

8. isolated on one side

9. factoring

10. c; $-c$

11. $3x-1=7$; $3x-1=-7$

12. quadratic

13. $A=0$ or $B=0$

14. $\pm\sqrt{d}$

15. $\pm\sqrt{7}$

16. 9

17. $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

18. 2; 9; -5

19. 1; -4; -1

20. $2\pm\sqrt{2}$

21. b^2-4ac

22. no

23. two

24. the square root property

25. the quadratic formula

26. factoring and the zero-product principle

27. radical

28. extraneous

29. $2x+1$; $x^2-14x+49$

Exercise Set P.7

1.
$$\begin{aligned} 7x-5&=72\\ 7x&=77\\ x&=11 \end{aligned}$$
Check:
$$\begin{aligned} 7x-5&=72\\ 7(11)-5&=72\\ 77-5&=72\\ 72&=72 \end{aligned}$$
The solution set is $\{11\}$.

2.
$$\begin{aligned} 6x-3&=63\\ 6x&=66\\ x&=11 \end{aligned}$$
The solution set is $\{11\}$.

Check:
$$\begin{aligned} 6x-3&=63\\ 6(11)-3&=63\\ 66-3&=63\\ 63&=63 \end{aligned}$$

3.
$$\begin{aligned} 11x-(6x-5)&=40\\ 11x-6x+5&=40\\ 5x+5&=40\\ 5x&=35\\ x&=7 \end{aligned}$$
The solution set is $\{7\}$.

Check:
$$\begin{aligned} 11x-(6x-5)&=40\\ 11(7)-[6(7)-5]&=40\\ 77-(42-5)&=40\\ 77-(37)&=40\\ 40&=40 \end{aligned}$$

4. $5x - (2x - 10) = 35$
$5x - 2x + 10 = 35$
$3x + 10 = 35$
$3x = 25$
$x = \frac{25}{3}$

The solution set is $\left\{\frac{25}{3}\right\}$.

Check:

$$\begin{aligned} 5x-(2x-10)&=35\\ 5\left(\frac{25}{3}\right)-\left[2\left(\frac{25}{3}\right)-10\right]&=35\\ \frac{125}{3}-\left[\frac{50}{3}-10\right]&=35\\ \frac{125}{3}-\frac{20}{3}&=35\\ \frac{105}{3}&=35\\ 35&=35 \end{aligned}$$

5. $2x - 7 = 6 + x$
$x - 7 = 6$
$x = 13$

The solution set is $\{13\}$.

Check:

$$\begin{aligned} 2(13)-7&=6+13\\ 26-7&=19\\ 19&=19 \end{aligned}$$

6. $3x + 5 = 2x + 13$
$x + 5 = 13$
$x = 8$

The solution set is $\{8\}$.

Check:

$$\begin{aligned} 3x+5&=2x+13\\ 3(8)+5&=2(8)+13\\ 24+5&=16+13\\ 29&=29 \end{aligned}$$

7. $7x + 4 = x + 16$
$6x + 4 = 16$
$6x = 12$
$x = 2$

The solution set is $\{2\}$.

Check:

$$\begin{aligned} 7(2)+4&=2+16\\ 14+4&=18\\ 18&=18 \end{aligned}$$

8. $13x + 14 = 12x - 5$
$x + 14 = -5$
$x = -19$

The solution set is $\{-19\}$.

Check:

$$\begin{aligned} 13x+14&=12x-5\\ 13(-19)+14&=12(-19)-5\\ -247+14&=-228-5\\ -233&=-233 \end{aligned}$$

9. $3(x - 2) + 7 = 2(x + 5)$
$3x - 6 + 7 = 2x + 10$
$3x + 1 = 2x + 10$
$x + 1 = 10$
$x = 9$

The solution set is $\{9\}$.

Check:

$$\begin{aligned} 3(9-2)+7&=2(9+5)\\ 3(7)+7&=2(14)\\ 21+7&=28\\ 28&=28 \end{aligned}$$

10. $2(x - 1) + 3 = x - 3(x + 1)$
$2x - 2 + 3 = x - 3x - 3$
$2x + 1 = -2x - 3$
$4x + 1 = -3$
$4x = -4$
$x = -1$

The solution set is $\{-1\}$.

Check:

$$\begin{aligned} 2(x-1)+3&=x-3(x+1)\\ 2(-1-1)+3&=-1-3(-1+1)\\ 2(-2)+3&=-1-3(0)\\ -4+3&=-1+0\\ -1&=-1 \end{aligned}$$

11.

$$\begin{aligned} \frac{x+3}{6}&=\frac{3}{8}+\frac{x-5}{4}\\ 24\left[\frac{x+3}{6}\right.&=\left.\frac{3}{8}+\frac{x-5}{4}\right]\\ 4x+12&=9+6x-30\\ 4x-6x&=-21-12\\ -2x&=-33\\ x&=\frac{33}{2} \end{aligned}$$

The solution set is $\left\{\frac{33}{2}\right\}$.

12. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

$12\left[\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}\right]$

$3x+3=2+8-4x$

$3x+4x=10-3$

$7x=7$

$x=1$

The solution set is $\{1\}$.

13. $\frac{x}{4}=2+\frac{x-3}{3}$

$12\left[\frac{x}{4}=2+\frac{x-3}{3}\right]$

$3x = 24 + 4x - 12$

$3x - 4x = 12$

$-x = 12$

$x = -12$

The solution set is $\{-12\}$.

14. $5+\frac{x-2}{3}=\frac{x+3}{8}$

$24\left[5+\frac{x-2}{3}=\frac{x+3}{8}\right]$

$120+8x-16=3x+9$

$8x-3x=9-104$

$5x=-95$

$x=-19$

The solution set is $\{-19\}$.

15. $\frac{x+1}{3}=5-\frac{x+2}{7}$

$21\left[\frac{x+1}{3}=5-\frac{x+2}{7}\right]$

$7x + 7 = 105 - 3x - 6$

$7x + 3x = 99 - 7$

$10x = 92$

$x=\frac{92}{10}$

$x=\frac{46}{5}$

The solution set is $\left\{\frac{46}{5}\right\}$.

16. $\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}$

$30\left[\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}\right]$

$18x-15x+45=10x+20$

$3x-10x=20-45$

$-7x=-25$

$x=\frac{25}{7}$

The solution set is $\left\{\frac{25}{7}\right\}$.

17. **a.** $\frac{1}{x-1}+5=\frac{11}{x-1}\ (x\neq 1)$

b. $\frac{1}{x-1}+5=\frac{11}{x-1}$

$1+5(x-1)=11$

$1+5x-5=11$

$5x-4=11$

$5x=15$

$x=3$

The solution set is $\{3\}$.

18. **a.** $\frac{3}{x+4}-7=\frac{-4}{x+4}\ (x\neq -4)$

b. $\frac{3}{x+4}-7=\frac{-4}{x+4}$

$3-7(x+4)=-4$

$3-7x-28=-4$

$-7x=21$

$x=-3$

The solution set is $\{-3\}$.

19. **a.** $\frac{8x}{x+1}=4-\frac{8}{x+1}\ (x\neq -1)$

b. $\frac{8x}{x+1}=4-\frac{8}{x+1}$

$8x=4(x+1)-8$

$8x=4x+4-8$

$4x=-4$

$x=-1\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

20. **a.** $\frac{2}{x-2}=\frac{x}{x-2}-2\ (x\neq 2)$

b. $\frac{2}{x-2}=\frac{x}{x-2}-2$

$2=x-2(x-2)$

$2=x-2x+4$

$x=2\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

21. a. $\frac{3}{2x-2}+\frac{1}{2}=\frac{2}{x-1}\ (x\neq 1)$

b. $\frac{3}{2x-2}+\frac{1}{2}=\frac{2}{x-1}$

$\frac{3}{2(x-1)}+\frac{1}{2}=\frac{2}{x-1}$

$3+1(x-1)=4$

$3+x-1=4$

$x=2$

The solution set is $\{2\}$.

22. a. $\frac{3}{x+3}=\frac{5}{2x+6}+\frac{1}{x-2}\ (x\neq -3, x\neq 2)$

b. $\frac{3}{x+3}=\frac{5}{2(x+3)}+\frac{1}{x-2}$

$6(x-2)=5(x-2)+2(x+3)$

$6x-12=5x-10+2x+6$

$-x=8$

$x=-8$

The solution set is $\{-8\}$.

23. a. $\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}\ (x\neq 1, x\neq -1)$

b. $\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}$

$\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{(x+1)(x-1)}$

$2(x-1)-1(x+1)=2x$

$2x-2-x-1=2x$

$-x=3$

$x=-3$

The solution set is $\{-3\}$.

24. a. $\frac{4}{x+5}+\frac{2}{x-5}=\frac{32}{x^2-25}$; $x\neq 5,-5$

b. $\frac{4}{x+5}+\frac{2}{x-5}=\frac{32}{(x+5)(x-5)}$

$(x\neq 5, x\neq -5)$

$4(x-5)+2(x+5)=32$

$4x-20+2x+10=32$

$6x=42$

$x=7$

The solution set is $\{7\}$.

25. a. $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{(x-4)(x+2)}$; $(x\neq -2, 4)$

b. $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{x^2-2x-8}$

$\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{(x-4)(x+2)}$

$(x\neq 4, x\neq -2)$

$1(x+2)-5(x-4)=6$

$x+2-5x+20=6$

$-4x=-16$

$x=4$

The solution set is the empty set, $\varnothing$.

26. a. $\frac{1}{x-3}-\frac{2}{x+1}=\frac{8}{(x-3)(x+1)}$; $x\neq -1,3$

b. $\frac{1}{x-3}-\frac{2}{x+1}=\frac{8}{(x-3)(x+1)}$

$1(x+1)-2(x-3)=8$

$x+1-2x+6=8$

$-x+7=8$

$-x=1$

$x=-1$

The solution set is the empty set, $\varnothing$.

27. $I=Prt$

$P=\frac{I}{rt}$;

interest

28. $C=2\pi r$

$r=\frac{C}{2\pi}$;

circumference of a circle

29. $T=D+pm$

$T-D=pm$

$\frac{T-D}{m}=\frac{pm}{m}$

$\frac{T-D}{m}=p$

total of payment

30. $P=C+MC$

$P-C=MC$

$\frac{P-C}{C}=M$

markup based on cost

31. $A=\frac{1}{2}h(a+b)$

$2A=h(a+b)$

$\frac{2A}{h}=a+b$

$\frac{2A}{h}-b=a$

area of trapezoid

32. $A = \frac{1}{2}h(a+b)$

$2A = h(a+b)$

$\frac{2A}{h} = a+b$

$\frac{2A}{h} - a = b$

area of trapezoid

33. $S = P + Prt$

$S - P = Prt$

$\frac{S-P}{Pt} = r;$

interest

34. $S = P + Prt$

$S - P = Prt$

$\frac{S-P}{Pr} = t;$

interest

35. $B = \frac{F}{S-V}$

$B(S-V) = F$

$S - V = \frac{F}{B}$

$S = \frac{F}{B} + V$

36. $S = \frac{C}{1-r}$

$S(1-r) = C$

$1 - r = \frac{C}{S}$

$-r = \frac{C}{S} - 1$

$r = -\frac{C}{S} + 1$

markup based on selling price

37. $IR + Ir = E$

$I(R+r) = E$

$I = \frac{E}{R+r}$

electric current

38. $A = 2lw + 2lh + 2wh$

$A - 2lw = h(2l + 2w)$

$\frac{A-2lw}{2l+2w} = h$

surface area

39. $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$qf + pf = pq$

$f(q+p) = pq$

$f = \frac{pq}{p+q}$

thin lens equation

40. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$R_1R_2 = RR_2 + RR_1$

$R_1R_2 - RR_1 = RR_2$

$R_1(R_2 - R) = RR_2$

$R_1 = \frac{RR_2}{R_2 - R}$

resistance

41. $f = \frac{f_1f_2}{f_1+f_2}$

$f(f_1 + f_2) = f_1f_2$

$ff_1 + ff_2 = f_1f_2$

$ff_1 - f_1f_2 = -ff_2$

$f_1(f - f_2) = -ff_2$

$\frac{f_1(f-f_2)}{f-f_2} = \frac{-ff_2}{f-f_2}$

$f_1 = \frac{ff_2}{f_2 - f}$

focal length

42. $f = \frac{f_1f_2}{f_1+f_2}$

$f(f_1 + f_2) = f_1f_2$

$ff_1 + ff_2 = f_1f_2$

$ff_2 - f_1f_2 = -ff_1$

$f_2(f - f_1) = -ff_1$

$\frac{f_2(f-f_1)}{f-f_1} = \frac{-ff_1}{f-f_1}$

$f_2 = \frac{ff_1}{f_1 - f}$

focal length

43. $|x-2| = 7$

$x - 2 = 7 \quad x - 2 = -7$

$x = 9 \quad x = -5$

The solution set is $\{9, -5\}$.

44. $|x+1| = 5$

$x + 1 = 5 \quad x + 1 = -5$

$x = 4 \quad x = -6$

The solution set is $\{-6, 4\}$.

45. $|2x-1|=5$

$2x-1=5 \quad 2x-1=-5$
$2x=6 \quad 2x=-4$
$x=3 \quad x=-2$

The solution set is $\{3, -2\}$.

46. $|2x-3|=11$

$2x-3=11 \quad 2x-3=-11$
$2x=14 \quad 2x=-8$
$x=7 \quad x=-4$

The solution set is $\{-4, 7\}$.

47. $2|3x-2|=14$

$|3x-2|=7$
$3x-2=7 \quad 3x-2=-7$
$3x=9 \quad 3x=-5$
$x=3 \quad x=-5/3$

The solution set is $\{3, -5/3\}$

48. $3|2x-1|=21$

$|2x-1|=7$
$2x-1=7$ or $2x-1=-7$
$2x=8 \quad 2x=-6$
$x=4 \quad x=-3$

The solution set is $\{4, -3\}$

49. $2\left|4-\frac{5}{2}x\right|+6=18$

$$2\left|4-\frac{5}{2}x\right|=12$$
$$\left|4-\frac{5}{2}x\right|=6$$

$4-\frac{5}{2}x=6$ or $4-\frac{5}{2}x=-6$
$-\frac{5}{2}x=2 \quad -\frac{5}{2}x=-10$
$x=-\frac{4}{5} \quad x=4$

The solution set is $\left\{-\frac{4}{5}, 4\right\}$.

50. $4\left|1-\frac{3}{4}x\right|+7=10$

$$4\left|1-\frac{3}{4}x\right|=3$$
$$\left|1-\frac{3}{4}x\right|=\frac{3}{4}$$

$1-\frac{3}{4}x=\frac{3}{4}$ or $1-\frac{3}{4}x=-\frac{3}{4}$
$-\frac{3}{4}x=-\frac{1}{4} \quad -\frac{3}{4}x=-\frac{7}{4}$
$x=\frac{1}{3} \quad x=\frac{7}{3}$

The solution set is $\left\{\frac{1}{3}, \frac{7}{3}\right\}$.

51. $|x+1|+5=3$

$|x+1|=-2$
No solution
The solution set is $\{\ \}$.

52. $|x+1|+6=2$

$|x+1|=-4$ The solution set is $\{\ \}$.

53. $|2x-1|+3=3$

$|2x-1|=0$
$2x-1=0$
$2x=1$
$x=1/2$

The solution set is $\left\{\frac{1}{2}\right\}$.

54. $|3x-2|+4=4$

$|3x-2|=0$
$3x-2=0$
$3x=2$
$x=\frac{2}{3}$

The solution set is $\left\{\frac{2}{3}\right\}$.

55. $x^2-3x-10=0$

$(x-5)(x+2)=0$
$x-5=0$ or $x+2=0$
$x=5 \quad x=-2$

The solution set is $\{-2, 5\}$.

56. $x^2-13x+36=0$

$(x-4)(x-9)=0$
$x-4=0$ or $x-9=0$
$x=4 \quad x=9$

The solution set is $\{4, 9\}$.

57.
$$x^2 = 8x - 15$$
$$x^2 - 8x + 15 = 0$$
$$(x-3)(x-5) = 0$$
$x - 3 = 0$ or $x - 5 = 0$
$x = 3$ $\quad$ $x = 5$

The solution set is $\{3, 5\}$.

58.
$$x^2 = -11x - 10$$
$$x^2 + 11x + 10 = 0$$
$$(x+10)(x+1) = 0$$
$x + 10 = 0$ or $x + 1 = 0$
$x = -10$ $\quad$ $x = -1$

The solution set is $\{-10, -1\}$.

59.
$$5x^2 = 20x$$
$$5x^2 - 20x = 0$$
$$5x(x-4) = 0$$
$5x = 0$ or $x - 4 = 0$
$x = 0$ $\quad$ $x = 4$

The solution set is $\{0, 4\}$.

60.
$$3x^2 = 12x$$
$$3x^2 - 12x = 0$$
$$3x(x-4) = 0$$
$3x = 0$ or $x - 4 = 0$
$x = 0$ $\quad$ $x = 4$

The solution set is $\{0, 4\}$.

61.
$$3x^2 = 27$$
$$x^2 = 9$$
$$\sqrt{x^2} = \pm\sqrt{9}$$
$$x = \pm 3$$

The solution set is $\{\pm 3\}$.

62.
$$5x^2 = 45$$
$$x^2 = 9$$
$$\sqrt{x^2} = \pm\sqrt{9}$$
$$x = \pm 3$$

The solution set is $\{\pm 3\}$.

63.
$$5x^2 + 1 = 51$$
$$5x^2 = 50$$
$$x^2 = 10$$
$$\sqrt{x^2} = \pm\sqrt{10}$$
$$x = \pm\sqrt{10}$$

The solution set is $\{\pm\sqrt{10}\}$.

64.
$$3x^2 - 1 = 47$$
$$3x^2 = 48$$
$$x^2 = 16$$
$$\sqrt{x^2} = \pm\sqrt{16}$$
$$x = \pm 4$$

The solution set is $\{\pm 4\}$.

65.
$$3(x-4)^2 = 15$$
$$(x-4)^2 = 5$$
$$\sqrt{(x-4)^2} = \pm\sqrt{5}$$
$$x - 4 = \pm\sqrt{5}$$
$$x = 4 \pm \sqrt{5}$$

The solution set is $\{4 \pm \sqrt{5}\}$.

66.
$$3(x+4)^2 = 21$$
$$(x+4)^2 = 7$$
$$\sqrt{(x+4)^2} = \pm\sqrt{7}$$
$$x + 4 = \pm\sqrt{7}$$
$$x = -4 \pm \sqrt{7}$$

The solution set is $\{-4 \pm \sqrt{7}\}$.

67.
$$x^2 + 6x = 7$$
$$x^2 + 6x + 9 = 7 + 9$$
$$(x+3)^2 = 16$$
$$x + 3 = \pm 4$$
$$x = -3 \pm 4$$

The solution set is $\{-7, 1\}$.

68.
$$x^2 + 6x = -8$$
$$x^2 + 6x + 9 = -8 + 9$$
$$(x+3)^2 = 1$$
$$x + 3 = \pm 1$$
$$x = -3 \pm 1$$

The solution set is $\{-4, -2\}$.

69.
$$x^2 - 2x = 2$$
$$x^2 - 2x + 1 = 2 + 1$$
$$(x-1)^2 = 3$$
$$x - 1 = \pm\sqrt{3}$$
$$x = 1 \pm \sqrt{3}$$

The solution set is $\{1 + \sqrt{3}, 1 - \sqrt{3}\}$.

70.
$$x^2 + 4x = 12$$
$$x^2 + 4x + 4 = 12 + 4$$
$$(x+2)^2 = 16$$
$$x + 2 = \pm 4$$
$$x = -2 \pm 4$$

The solution set is $\{-6, 2\}$.

71. $x^2-6x-11=0$
$x^2-6x=11$
$x^2-6x+9=11+9$
$(x-3)^2=20$
$x-3=\pm\sqrt{20}$
$x=3\pm2\sqrt{5}$
The solution set is $\left\{3+2\sqrt{5}, 3-2\sqrt{5}\right\}$.

72. $x^2-2x-5=0$
$x^2-2x=5$
$x^2-2x+1=5+1$
$(x-1)^2=6$
$x-1=\pm\sqrt{6}$
$x=1\pm\sqrt{6}$
The solution set is $\left\{1+\sqrt{6}, 1-\sqrt{6}\right\}$.

73. $x^2+4x+1=0$
$x^2+4x=-1$
$x^2+4x+4=-1+4$
$(x+2)^2=3$
$x+2=\pm\sqrt{3}$
$x=-2\pm\sqrt{3}$
The solution set is $\left\{-2+\sqrt{3}, -2-\sqrt{3}\right\}$.

74. $x^2+6x-5=0$
$x^2+6x=5$
$x^2+6x+9=5+9$
$(x+3)^2=14$
$x+3=\pm\sqrt{14}$
$x=-3\pm\sqrt{14}$
The solution set is $\left\{-3+\sqrt{14}, -3-\sqrt{14}\right\}$.

75. $x^2+8x+15=0$
$$x=\frac{-8\pm\sqrt{8^2-4(1)(15)}}{2(1)}$$
$$x=\frac{-8\pm\sqrt{64-60}}{2}$$
$$x=\frac{-8\pm\sqrt{4}}{2}$$
$$x=\frac{-8\pm2}{2}$$
The solution set is $\{-5, -3\}$.

76. $x^2+8x+12=0$
$$x=\frac{-8\pm\sqrt{8^2-4(1)(12)}}{2(1)}$$
$$x=\frac{-8\pm\sqrt{64-48}}{2}$$
$$x=\frac{-8\pm\sqrt{16}}{2}$$
$$x=\frac{-8\pm4}{2}$$
The solution set is $\{-6, -2\}$.

77. $x^2+5x+3=0$
$$x=\frac{-5\pm\sqrt{5^2-4(1)(3)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{25-12}}{2}$$
$$x=\frac{-5\pm\sqrt{13}}{2}$$
The solution set is $\left\{\frac{-5+\sqrt{13}}{2}, \frac{-5-\sqrt{13}}{2}\right\}$.

78. $x^2+5x+2=0$
$$x=\frac{-5\pm\sqrt{5^2-4(1)(2)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{25-8}}{2}$$
$$x=\frac{-5\pm\sqrt{17}}{2}$$
The solution set is $\left\{\frac{-5+\sqrt{17}}{2}, \frac{-5-\sqrt{17}}{2}\right\}$.

79. $3x^2-3x-4=0$
$$x=\frac{3\pm\sqrt{(-3)^2-4(3)(-4)}}{2(3)}$$
$$x=\frac{3\pm\sqrt{9+48}}{6}$$
$$x=\frac{3\pm\sqrt{57}}{6}$$
The solution set is $\left\{\frac{3+\sqrt{57}}{6}, \frac{3-\sqrt{57}}{6}\right\}$

80. $5x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(5)(-2)}}{2(5)}$$
$$x = \frac{-1 \pm \sqrt{1+40}}{10}$$
$$x = \frac{-1 \pm \sqrt{41}}{10}$$

The solution set is $\left\{\frac{-1+\sqrt{41}}{10}, \frac{-1-\sqrt{41}}{10}\right\}$.

81. $4x^2 = 2x + 7$

$4x^2 - 2x - 7 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(4)(-7)}}{2(4)}$$
$$x = \frac{2 \pm \sqrt{4+112}}{8}$$
$$x = \frac{2 \pm \sqrt{116}}{8}$$
$$x = \frac{2 \pm 2\sqrt{29}}{8}$$
$$x = \frac{1 \pm \sqrt{29}}{4}$$

The solution set is $\left\{\frac{1+\sqrt{29}}{4}, \frac{1-\sqrt{29}}{4}\right\}$.

82. $3x^2 = 6x - 1$

$3x^2 - 6x + 1 = 0$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$$
$$x = \frac{6 \pm \sqrt{36-12}}{6}$$
$$x = \frac{6 \pm \sqrt{24}}{6}$$
$$x = \frac{6 \pm 2\sqrt{6}}{6}$$
$$x = \frac{3 \pm \sqrt{6}}{3}$$

The solution set is $\left\{\frac{3+\sqrt{6}}{3}, \frac{3-\sqrt{6}}{3}\right\}$.

83. $x^2 - 4x - 5 = 0$

$(-4)^2 - 4(1)(-5)$

$= 16 + 20$

$= 36$; 2 unequal real solutions

84. $4x^2 - 2x + 3 = 0$

$(-2)^2 - 4(4)(3)$

$= 4 - 48$

$= -44$; 2 complex imaginary solutions

85. $2x^2 - 11x + 3 = 0$

$(-11)^2 - 4(2)(3)$

$= 121 - 24$

$= 97$; 2 unequal real solutions

86. $2x^2 + 11x - 6 = 0$

$11^2 - 4(2)(-6)$

$= 121 + 48$

$= 169$; 2 unequal real solutions

87. $x^2 = 2x - 1$

$x^2 - 2x + 1 = 0$

$(-2)^2 - 4(1)(1)$

$= 4 - 4$

$= 0$; 1 real solution

88. $3x^2 = 2x - 1$

$3x^2 - 2x + 1 = 0$

$(-2)^2 - 4(3)(1)$

$= 4 - 12$

$= -8$; 2 complex imaginary solutions

89. $x^2 - 3x - 7 = 0$

$(-3)^2 - 4(1)(-7)$

$= 9 + 28$

$= 37$; 2 unequal real solutions

90. $3x^2 + 4x - 2 = 0$

$4^2 - 4(3)(-2)$

$= 16 + 24$

$= 40$; 2 unequal real solutions

91. $2x^2 - x = 1$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$2x + 1 = 0$ or $x - 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$ or $x = 1$

The solution set is $\left\{-\frac{1}{2}, 1\right\}$.

92. $3x^2 - 4x = 4$

$3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

$3x + 2$ or $x - 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$ or $x = -3$

The solution set is $\left\{-\frac{2}{3}, 2\right\}$.

93. $5x^2 + 2 = 11x$

$5x^2 - 11x + 2 = 0$

$(5x-1)(x-2) = 0$

$5x - 1 = 0$ or $x - 2 = 0$

$5x = 1$

$x = \frac{1}{5}$ or $x = 2$

The solution set is $\left\{\frac{1}{5}, 2\right\}$.

94. $5x^2 = 6 - 13x$

$5x^2 + 13x - 6 = 0$

$(5x-2)(x+3) = 0$

$5x - 2 = 0$ or $x + 3$

$5x = 2$

$x = \frac{2}{5}$ or $x = -3$

The solution set is $\left\{-3, \frac{2}{5}\right\}$.

95. $3x^2 = 60$

$x^2 = 20$

$x = \pm\sqrt{20}$

$x = \pm 2\sqrt{5}$

The solution set is $\left\{-2\sqrt{5}, 2\sqrt{5}\right\}$.

96. $2x^2 = 250$

$x^2 = 125$

$x = \pm\sqrt{125}$

$x = \pm 5\sqrt{5}$

The solution set is $\left\{-5\sqrt{5}, 5\sqrt{5}\right\}$.

97. $x^2 - 2x = 1$

$x^2 - 2x + 1 = 1 + 1$

$(x-1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

The solution set is $\left\{1+\sqrt{2}, 1-\sqrt{2}\right\}$.

98. $2x^2 + 3x = 1$

$2x^2 + 3x - 1 = 0$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$

$x = \frac{-3 \pm \sqrt{9+8}}{4}$

$x = \frac{-3 \pm \sqrt{17}}{4}$

The solution set is $\left\{\frac{-3+\sqrt{17}}{4}, \frac{-3-\sqrt{17}}{4}\right\}$.

99. $(2x+3)(x+4) = 1$

$2x^2 + 8x + 3x + 12 = 1$

$2x^2 + 11x + 11 = 0$

$x = \frac{-11 \pm \sqrt{11^2 - 4(2)(11)}}{2(2)}$

$x = \frac{-11 \pm \sqrt{121-88}}{4}$

$x = \frac{-11 \pm \sqrt{33}}{4}$

The solution set is $\left\{\frac{-11+\sqrt{33}}{4}, \frac{-11-\sqrt{33}}{4}\right\}$.

100. $(2x-5)(x+1) = 2$

$2x^2 + 2x - 5x - 5 = 2$

$2x^2 - 3x - 7 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$

$x = \frac{3 \pm \sqrt{9+56}}{4}$

$x = \frac{3 \pm \sqrt{65}}{4}$

The solution set is $\left\{\frac{3+\sqrt{65}}{4}, \frac{3-\sqrt{65}}{4}\right\}$.

101.
$$\begin{aligned}(3x-4)^2 &= 16\\ 3x-4 &= \pm\sqrt{16}\\ 3x-4 &= \pm 4\\ 3x &= 4\pm 4\\ 3x = 8 &\text{ or } 3x = 0\\ x = \frac{8}{3} &\text{ or } x = 0\end{aligned}$$

The solution set is $\left\{0, \frac{8}{3}\right\}$.

102.
$$\begin{aligned}(2x+7)^2 &= 25\\ 2x+7 &= \pm 5\\ 2x &= -7\pm 5\\ 2x = -12 \quad &\text{or} \quad 2x = -2\\ x = 6 \quad &\text{or} \quad x = -1\end{aligned}$$

The solution set is $\{-6, -1\}$.

103.
$$\begin{aligned}3x^2-12x+12 &= 0\\ x^2-4x+4 &= 0\\ (x-2)(x-2) &= 0\\ x-2 &= 0\\ x &= 2\end{aligned}$$

The solution set is $\{2\}$.

104.
$$\begin{aligned}9-6x+x^2 &= 0\\ x^2-6x+9 &= 0\\ (x-3)(x-3) &= 0\\ x-3 &= 0\\ x &= 3\end{aligned}$$

The solution set is $\{3\}$.

105.
$$\begin{aligned}4x^2-16 &= 0\\ 4x^2 &= 16\\ x^2 &= 4\\ x &= \pm 2\end{aligned}$$

The solution set is $\{-2, 2\}$.

106.
$$\begin{aligned}3x^2-27 &= 0\\ 3x^2 &= 27\\ x^2 &= 9\\ x &= \pm 3\end{aligned}$$

The solution set is $\{-3, 3\}$.

107.
$$\begin{aligned}x^2 &= 4x-2\\ x^2-4x+2 &= 0\\ x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\ x &= \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}\\ x &= \frac{4\pm\sqrt{8}}{2}\\ x &= 2\pm\sqrt{2}\end{aligned}$$

The solution set is $\left\{2\pm\sqrt{2}\right\}$.

108.
$$\begin{aligned}x^2 &= 6x-7\\ x^2-6x+7 &= 0\\ x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\ x &= \frac{-(-6)\pm\sqrt{(-6)^2-4(1)(7)}}{2(1)}\\ x &= \frac{6\pm\sqrt{8}}{2}\\ x &= 3\pm\sqrt{2}\end{aligned}$$

The solution set is $\left\{3\pm\sqrt{2}\right\}$.

109.
$$\begin{aligned}2x^2-7x &= 0\\ x(2x-7) &= 0\\ x = 0 &\text{ or } 2x-7 = 0\\ 2x &= 7\\ x = 0 &\text{ or } x = \frac{7}{2}\end{aligned}$$

The solution set is $\left\{0, \frac{7}{2}\right\}$.

110.
$$\begin{aligned}2x^2+5x &= 3\\ 2x^2+5x-3 &= 0\\ x &= \frac{-5\pm\sqrt{5^2-4(2)(-3)}}{2(2)}\\ x &= \frac{-5\pm\sqrt{25+24}}{4}\\ x &= \frac{-5\pm\sqrt{49}}{4}\\ x &= \frac{-5\pm 7}{4}\\ x &= -3, \frac{1}{2}\end{aligned}$$

The solution set is $\left\{-3, \frac{1}{2}\right\}$.

111. $\frac{1}{x}+\frac{1}{x+2}=\frac{1}{3}; x\neq 0,-2$

$$3x+6+3x=x^2+2x$$
$$0=x^2-4x-6$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-6)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{16+24}}{2}$$
$$x=\frac{4\pm\sqrt{40}}{2}$$
$$x=\frac{4\pm 2\sqrt{10}}{2}$$
$$x=2\pm\sqrt{10}$$

The solution set is $\{2+\sqrt{10},\ 2-\sqrt{10}\}$.

112. $\frac{1}{x}+\frac{1}{x+3}=\frac{1}{4}; x\neq 0,-3$

$$4x+12+4x=x^2+3x$$
$$0=x^2-5x-12$$
$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-12)}}{2(1)}$$
$$x=\frac{5\pm\sqrt{25+48}}{2}$$
$$x=\frac{5\pm\sqrt{73}}{2}$$

The solution set is $\left\{\frac{5+\sqrt{73}}{2},\frac{5-\sqrt{73}}{2}\right\}$.

113. $\frac{2x}{x-3}+\frac{6}{x+3}=\frac{-28}{x^2-9}; x\neq 3,-3$

$$2x(x+3)+6(x-3)=-28$$
$$2x^2+6x+6x-18=-28$$
$$2x^2+12x+10=0$$
$$x^2+6x+5=0$$
$$(x+1)(x+5)=0$$

The solution set is $\{-5,\ -1\}$.

114. $\frac{3}{x-3}+\frac{5}{x-4}=\frac{x^2-20}{x^2-7x+12}; x\neq 3,4$

$$3x-12+5x-15=x^2-20$$
$$0=x^2-8x+7$$
$$0=(x-7)(x-1)$$
$$x=7 \qquad x=1$$

The solution set is $\{1, 7\}$.

115. $\sqrt{3x+18}=x$

$$3x+18=x^2$$
$$x^2-3x-18=0$$
$$(x+3)(x-6)=0$$
$$x+3=0 \qquad x-6=0$$
$$x=-3 \qquad x=6$$
$$\sqrt{3(-3)+18}=-3 \qquad \sqrt{3(6)+18}=6$$
$$\sqrt{-9+18}=-3 \qquad \sqrt{18+18}=6$$
$$\sqrt{9}=-3 \text{ False} \qquad \sqrt{36}=6$$

The solution set is $\{6\}$.

116. $\sqrt{20-8x}=x$

$$20-8x=x^2$$
$$x^2+8x-20=0$$
$$(x+10)(x-2)=0$$
$$x+10=0 \qquad x-2=0$$
$$x=-10 \qquad x=2$$
$$\sqrt{20-8(-10)}=-10 \qquad \sqrt{20-8(2)}=2$$
$$\sqrt{20+80}=-10 \qquad \sqrt{20-16}=2$$
$$\sqrt{100}=-10 \qquad \sqrt{4}=2$$

False

The solution set is $\{2\}$.

117. $\sqrt{x+3}=x-3$

$$x+3=x^2-6x+9$$
$$x^2-7x+6=0$$
$$(x-1)(x-6)=0$$
$$x-1=0 \qquad x-6=0$$
$$x=1 \qquad x=6$$
$$\sqrt{1+3}=1-3 \qquad \sqrt{6+3}=6-3$$
$$\sqrt{4}=-2 \text{ False} \qquad \sqrt{9}=3$$

The solution set is $\{6\}$.

118. $\sqrt{x+10}=x-2$

$$x+10=(x-2)^2$$
$$x+10=x^2-4x+4$$
$$x^2-5x-6=0$$
$$(x+1)(x-6)=0$$
$$x+1=0 \qquad x-6=0$$
$$x=-1 \qquad x=6$$
$$\sqrt{-1+10}=-1-2 \qquad \sqrt{6+10}=6-2$$
$$\sqrt{9}=-3 \text{ False} \qquad \sqrt{16}=4$$

The solution set is $\{6\}$.

119.

$$\sqrt{2x+13} = x+7$$
$$2x+13 = (x+7)^2$$
$$2x+13 = x^2+14x+49$$
$$x^2+12x+36 = 0$$
$$(x+6)^2 = 0$$
$$x+6 = 0$$
$$x = -6$$
$$\sqrt{2(-6)+13} = -6+7$$
$$\sqrt{-12+13} = 1$$
$$\sqrt{1} = 1$$

The solution set is $\{-6\}$.

120.

$$\sqrt{6x+1} = x-1$$
$$6x+1 = (x-1)^2$$
$$6x+1 = x^2-2x+1$$
$$x^2-8x = 0$$
$$x(x-8) = 0$$
$$x-8 = 0 \qquad x = 0$$
$$x = 8$$
$$\sqrt{6(0)+1} = 0-1 \qquad \sqrt{6(8)+1} = 8-1$$
$$\sqrt{0+1} = -1 \qquad \sqrt{48+1} = 7$$
$$\sqrt{1} = -1 \text{ False} \qquad \sqrt{49} = 7$$

The solution set is $\{8\}$.

121.

$$x-\sqrt{2x+5} = 5$$
$$x-5 = \sqrt{2x+5}$$
$$(x-5)^2 = 2x+5$$
$$x^2-10x+25 = 2x+5$$
$$x^2-12x+20 = 0$$
$$(x-2)(x-10) = 0$$
$$x-2 = 0 \qquad x-10 = 0$$
$$x = 2 \qquad x = 10$$
$$2-\sqrt{2(2)+5} = 5 \qquad 10-\sqrt{2(10)+5} = 5$$
$$2-\sqrt{9} = 5 \qquad 10-\sqrt{25} = 5$$
$$2-3 = 5 \text{ False} \qquad 10-5 = 5$$

The solution set is $\{10\}$.

122.

$$x-\sqrt{x+11} = 1$$
$$x-1 = \sqrt{x+11}$$
$$(x-1)^2 = x+11$$
$$x^2-2x+1 = x+11$$
$$x^2-3x-10 = 0$$
$$(x+2)(x-5) = 0$$
$$x+2 = 0 \qquad x-5 = 0$$
$$x = -2 \qquad x = 5$$
$$-2-\sqrt{-2+11} = 1 \qquad 5-\sqrt{5+11} = 1$$
$$-2-\sqrt{9} = 1 \qquad 5-\sqrt{16} = 1$$
$$-2-3 = 1 \text{ False} \qquad 5-4 = 1$$

The solution set is $\{5\}$.

123. $\sqrt{2x+19}-8=x$

$\sqrt{2x+19}=x+8$

$\left(\sqrt{2x+19}\right)^2=(x+8)^2$

$2x+19=x^2+16x+64$

$0=x^2+14x+45$

$0=(x+9)(x+5)$

$x+9=0$ or $x+5=0$

$x=-9$ $\quad x=-5$

–9 does not check and must be rejected.

The solution set is $\{-5\}$.

124. $\sqrt{2x+15}-6=x$

$\sqrt{2x+15}=x+6$

$\left(\sqrt{2x+15}\right)^2=(x+6)^2$

$2x+15=x^2+12x+36$

$0=x^2+10x+21$

$0=(x+3)(x+7)$

$x+3=0$ or $x+7=0$

$x=-3$ $\quad x=-7$

–7 does not check and must be rejected.

The solution set is $\{-3\}$.

125.

$$25-[2+5y-3(y+2)]=-3(2y-5)-[5(y-1)-3y+3]$$
$$25-[2+5y-3y-6]=-6y+15-[5y-5-3y+3]$$
$$25-[2y-4]=-6y+15-[2y-2]$$
$$25-2y+4=-6y+15-2y+2$$
$$-2y+29=-8y+17$$
$$6y=-12$$
$$y=-2$$

The solution set is $\{-2\}$.

126.

$$45-[4-2y-4(y+7)]=-4(1+3y)-[4-3(y+2)-2(2y-5)]$$
$$45-[4-2y-4y-28]=-4-12y-[4-3y-6-4y+10]$$
$$45-[-6y-24]=-4-12y-[-7y+8]$$
$$45+6y+24=-4-12y+7y-8$$
$$6y+69=-5y-12$$
$$11y=-81$$
$$y=-\frac{81}{11}$$

The solution set is $\left\{-\frac{81}{11}\right\}$.

127. $7-7x=(3x+2)(x-1)$

$7-7x=3x^2-x-2$

$0=3x^2+6x-9$

$0=x^2+2x-3$

$0=(x+3)(x-1)$

$x+3=0$ or $x-1=0$

$x=-3$ $\quad x=1$

The solution set is $\{-3,1\}$.

128. $10x-1=(2x+1)^2$

$$10x-1=4x^2+4x+1$$
$$0=4x^2-6x+2$$
$$0=2x^2-3x+1$$
$$0=(2x-1)(x-1)$$

$2x-1=0$ or $x-1=0$

$x=\frac{1}{2}$ $\qquad x=1$

The solution set is $\left\{\frac{1}{2},1\right\}$.

129. $\left|x^2+2x-36\right|=12$

$x^2+2x-36=12$ $\qquad$ $x^2+2x-36=-12$

$x^2+2x-48=0$ or $x^2+2x-24=0$

$(x+8)(x-6)=0$ $\qquad$ $(x+6)(x-4)=0$

Setting each of the factors above equal to zero gives $x=-8,\quad x=6,\quad x=-6,\quad$ and $x=4$.

The solution set is $\{-8,-6,4,6\}$.

130. $\left|x^2+6x+1\right|=8$

$x^2+6x+1=8$ or $x^2+6x+1=-8$

$x^2+6x-7=0$ $\qquad$ $x^2+6x+9=0$

$(x+7)(x-1)=0$ $\qquad$ $(x+3)(x+3)=0$

Setting each of the factors above equal to zero gives $x=-7,\quad x=-3,\quad$ and $x=1$.

The solution set is $\{-7,-3,1\}$.

131.

$$\frac{1}{x^2-3x+2}=\frac{1}{x+2}+\frac{5}{x^2-4}$$
$$\frac{1}{(x-1)(x-2)}=\frac{1}{x+2}+\frac{5}{(x+2)(x-2)}$$

Multiply both sides of the equation by the least common denominator, $(x-1)(x-2)(x+2)$. This results in the following:

$$x+2=(x-1)(x-2)+5(x-1)$$
$$x+2=x^2-2x-x+2+5x-5$$
$$x+2=x^2+2x-3$$
$$0=x^2+x-5$$

Apply the quadratic formula: $a=1\quad b=1\quad c=-5$.

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-5)}}{2(1)}=\frac{-1\pm\sqrt{1-(-20)}}{2}=\frac{-1\pm\sqrt{21}}{2}$$

The solution set is $\left\{\frac{-1\pm\sqrt{21}}{2}\right\}$.

132. $\frac{x-1}{x-2}+\frac{x}{x-3}=\frac{1}{x^2-5x+6}$

$\frac{x-1}{x-2}+\frac{x}{x-3}=\frac{1}{(x-2)(x-3)}$

Multiply both sides of the equation by the least common denominator, $(x-2)(x-3)$. This results in the following:

$$(x-3)(x-1)+x(x-2)=1$$
$$x^2-x-3x+3+x^2-2x=1$$
$$2x^2-6x+3=1$$
$$2x^2-6x+2=0$$

Apply the quadratic formula: $a=2 \quad b=-6 \quad c=2$.

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(2)}}{2(2)}$$
$$=\frac{6\pm\sqrt{36-16}}{4}=\frac{6\pm\sqrt{20}}{4}$$
$$=\frac{6\pm\sqrt{4\cdot 5}}{4}=\frac{6\pm 2\sqrt{5}}{4}$$
$$=\frac{3\pm\sqrt{5}}{2}$$

The solution set is $\left\{\frac{3\pm\sqrt{5}}{2}\right\}$.

133. $\sqrt{x+8}-\sqrt{x-4}=2$

$$\sqrt{x+8}=\sqrt{x-4}+2$$
$$x+8=(\sqrt{x-4}+2)^2$$
$$x+8=x-4+4\sqrt{x-4}+4$$
$$x+8=x+4\sqrt{x-4}$$
$$8=4\sqrt{x-4}$$
$$2=\sqrt{x-4}$$
$$4=x-4$$
$$x=8$$

$$\sqrt{8+8}-\sqrt{8-4}=2$$
$$\sqrt{16}-\sqrt{4}=2$$
$$4-2=2$$

The solution set is $\{8\}$.

134. $\sqrt{x+5}-\sqrt{x-3}=2$

$$\sqrt{x+5}=\sqrt{x-3}+2$$
$$x+5=(\sqrt{x-3}+2)^2$$
$$x+5=x-3+4\sqrt{x-3}+4$$
$$x+5=x+1+4\sqrt{x-3}$$
$$5=1+4\sqrt{x-3}$$
$$4=4\sqrt{x-3}$$
$$1=\sqrt{x-3}$$
$$1=x-3$$
$$x=4$$

$$\sqrt{4+5}-\sqrt{4-3}=2$$
$$\sqrt{9}-\sqrt{1}=2$$
$$3-1=2$$

The solution set is $\{4\}$.

135. Values that make the denominator zero must be excluded**.**

$$2x^2+4x-9=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(4)\pm\sqrt{(4)^2-4(2)(-9)}}{2(2)}$$
$$x=\frac{-4\pm\sqrt{88}}{4}$$
$$x=\frac{-4\pm 2\sqrt{22}}{4}$$
$$x=\frac{-2\pm\sqrt{22}}{2}$$

136. Values that make the denominator zero must be excluded**.**

$$2x^2-8x+5=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(2)(5)}}{2(2)}$$
$$x=\frac{8\pm\sqrt{24}}{4}$$
$$x=\frac{8\pm 2\sqrt{6}}{4}$$
$$x=\frac{4\pm\sqrt{6}}{2}$$

137. a. $p=\frac{4x}{5}+25$

$$p=\frac{4(30)}{5}+25$$
$$p=24+25$$
$$p=49$$

According to the model, 49% of U.S. college freshman had an average grade of A in high school in 2010. This overestimates the value shown in the bar graph by 1%.

b. $p=\frac{4x}{5}+25$

$$57=\frac{4x}{5}+25$$
$$32=\frac{4x}{5}$$
$$160=4x$$
$$40=x$$

According to the model, 57% of U.S. college freshman will have an average grade of A in high school 40 years after 1980, or 2020.

138. a. $p = \frac{4x}{5} + 25$

$p = \frac{4(20)}{5} + 25$

$p = 16 + 25$

$p = 41$

According to the model, 41% of U.S. college freshman had an average grade of A in high school in 2000. This underestimates the value shown in the bar graph by 2%.

139. $C = \frac{x + 0.1(500)}{x + 500}$

$0.28 = \frac{x + 0.1(500)}{x + 500}$

$0.28(x + 500) = x + 0.1(500)$

$0.28x + 140 = x + 50$

$-0.72x = -90$

$\frac{-0.72x}{-0.72} = \frac{-90}{-0.72}$

$x = 125$

125 liters of pure peroxide must be added.

140. a. $C = \frac{x + 0.35(200)}{x + 200}$

b. $0.74 = \frac{x + 0.35(200)}{x + 200}$

$0.74(x + 200) = x + 0.35(200)$

$0.74x + 148 = x + 70$

$-0.26x = -78$

$\frac{-0.26x}{-0.26} = \frac{-78}{-0.26}$

$x = 300$

300 liters of pure acid must be added.

141. $f(x) = 0.013x^2 - 1.19x + 28.24$

$3 = 0.013x^2 - 1.19x + 28.24$

$0 = 0.013x^2 - 1.19x + 25.24$

Apply the quadratic formula:

$a = 0.013 \quad b = -1.19 \quad c = 25.24$

$$x = \frac{-(-1.19) \pm \sqrt{(-1.19)^2 - 4(0.013)(25.24)}}{2(0.013)}$$

$$= \frac{1.19 \pm \sqrt{1.4161 - 1.31248}}{0.026}$$

$$= \frac{1.19 \pm \sqrt{0.10362}}{0.026}$$

$$\approx \frac{1.19 \pm 0.32190}{0.026}$$

≈ 58.15 or 33.39

The solutions are approximately 33.39 and 58.15. Thus, 33 year olds and 58 year olds are expected to be in 3 fatal crashes per 100 million miles driven. The function models the actual data well.

142. $f(x) = 0.013x^2 - 1.19x + 28.24$

$10 = 0.013x^2 - 1.19x + 28.24$

$0 = 0.013x^2 - 1.19x + 18.24$

Apply the quadratic formula:

$a = 0.013 \quad b = -1.19 \quad c = 18.24$

$$x = \frac{-(-1.19) \pm \sqrt{(-1.19)^2 - 4(0.013)(18.24)}}{2(0.013)}$$

$$= \frac{1.19 \pm \sqrt{1.4161 - 0.94848}}{0.026}$$

$$= \frac{1.19 \pm \sqrt{0.46762}}{0.026} \approx \frac{1.19 \pm 0.68383}{0.026}$$

Evaluate the expression to obtain two solutions.

$x = \frac{1.19 + 0.68383}{0.026}$ or $x = \frac{1.19 - 0.68383}{0.026}$

$x = \frac{1.87383}{0.026}$ $\quad$ $x = \frac{0.50617}{0.026}$

$x \approx 72.1$ $\quad$ $x \approx 19$

Drivers of approximately age 19 and age 72 are expected to be involved in 10 fatal crashes per 100 million miles driven. The formula does not model the data very well. The formula overestimates the number of fatal accidents.

143. a. According to the line graph, about 47% ±1% of U.S. women participated in the labor force in 2010.

b. $p = 1.6\sqrt{t} + 38$

$p = 1.6\sqrt{40} + 38 \approx 48.1$

According to the formula, about 48.1% of U.S. women participated in the labor force in 2010.

c. $p = 1.6\sqrt{t} + 38$

$51 = 1.6\sqrt{t} + 38$

$13 = 1.6\sqrt{t}$

$\frac{13}{1.6} = \frac{1.6\sqrt{t}}{1.6}$

$\frac{13}{1.6} = \sqrt{t}$

$\left(\frac{13}{1.6}\right)^2 = \left(\sqrt{t}\right)^2$

$66 \approx t$

According to the formula, 51% of U.S. women will participate in the labor force 66 years after 1970, or 2036.

144. **a.** According to the line graph, about 53% ±1% of U.S. men participated in the labor force in 2010.

b. $p = -1.6\sqrt{t} + 62$
$p = -1.6\sqrt{40} + 62 \approx 51.9$
According to the formula, about 51.9% of U.S. men participated in the labor force in 2010.

c. $p = -1.6\sqrt{t} + 62$

$$49 = -1.6\sqrt{t} + 62$$
$$-13 = -1.6\sqrt{t}$$
$$\frac{-13}{-1.6} = \frac{-1.6\sqrt{t}}{-1.6}$$
$$\frac{-13}{-1.6} = \sqrt{t}$$
$$\left(\frac{-13}{-1.6}\right)^2 = \left(\sqrt{t}\right)^2$$
$$66 \approx t$$

According to the formula, 49% of U.S. men will participate in the labor force 66 years after 1970, or 2036.

145. – 158. Answers may vary.

159. does not make sense; Explanations will vary. Sample explanation: Substitute $n = 10$ into the equation to find *P*.

160. makes sense

161. does not make sense; Explanations will vary. Sample explanation: The factoring method would be quicker.

162. does not make sense; Explanations will vary. Sample explanation: You should substitute into the original equation.

163. false; Changes to make the statement true will vary.
A sample change is: $(2x-3)^2 = 25$

$$\sqrt{(2x-3)^2} = \pm\sqrt{25}$$
$$2x - 3 = \pm 5$$

164. false; Changes to make the statement true will vary. A sample change is: Some quadratics have one number in their solution sets.

165. true

166. false; Changes to make the statement true will vary. A sample change is: $ax^2 + c = 0$ can be solved using $b = 0$.

167.

$$\frac{7x+4}{b} + 13 = x$$
$$\frac{7(-6)+4}{b} + 13 = -6$$
$$\frac{-38}{b} = -19$$
$$-19b = -38$$
$$b = 2$$

168. $[x-(-3)][x-(5)] = 0$

$$(x+3)(x-5) = 0$$
$$x^2 - 2x - 15 = 0$$

169.

$$V = C - \frac{C-S}{L}N$$
$$VL = CL - (C-S)N$$
$$VL = CL - CN + SN$$
$$CN - CL = NS - LV$$
$$C(N-L) = NS - LV$$
$$\frac{C(N-L)}{N-L} = \frac{NS-LV}{N-L}$$
$$C = \frac{NS-LV}{N-L} \text{ or } \frac{LV-NS}{L-N}$$

170.

$$s = -16t^2 + v_0 t$$
$$0 = -16t^2 + v_0 t - s$$
$$a = -16,\ b = v_0,\ c = -s$$
$$t = \frac{-v_0 \pm \sqrt{(v_0)^2 - 4(-16)(-s)}}{2(-16)}$$
$$t = \frac{-v_0 \pm \sqrt{(v_0)^2 - 64s}}{-32}$$
$$t = \frac{v_0 \pm \sqrt{v_0^{\,2} - 64s}}{32}$$

171. $x + 150$

172. $20 + 0.05x$

173. $4x + 400$

Exercise Set P.8

Check Point Exercises

1. Let x = the average yearly salary, in thousands, of women with an associate's degree
Let x + 14 = the average yearly salary, in thousands, of women with a bachelor's degree
Let x + 26 = the average yearly salary, in thousands, of women with a master's degree

$$\begin{aligned} x+(x+14)+(x+26) &= 139 \\ x+x+14+x+26 &= 139 \\ 3x+40 &= 139 \\ 3x &= 99 \\ x &= 33 \end{aligned}$$

$x = 33$, associate's degree: \$33,000
$x + 14 = 47$, bachelor's degree: \$47,000
$x + 26 = 59$, master's degree: \$59,000

2. Let x = the number of years after 1969.

$$\begin{aligned} 85-0.9x &= 25 \\ -0.9x &= 25-85 \\ -0.9x &= -60 \\ x &= \frac{-60}{-0.9} \\ x &\approx 67 \end{aligned}$$

25% of freshmen will respond this way 67 years after 1969, or 2036.

3. Let x = the number of bridge crossings at which the costs of the two plans are the same.

$$\begin{aligned} \underbrace{5x}_{\text{No Pass}} &= \underbrace{40+3x}_{\text{Discount Pass}} \\ 5x-3x &= 40 \\ 2x &= 40 \\ x &= 20 \end{aligned}$$

The two plans cost the same for 20 bridge crossings.

4. Let x = the computer's price before the reduction.

$$\begin{aligned} x-0.30x &= 840 \\ 0.70x &= 840 \\ x &= \frac{840}{0.70} \\ x &= 1200 \end{aligned}$$

Before the reduction the computer's price was \$1200.

5. Let x = the amount invested at 9%.
Let 5000 – x = the amount invested at 11%.

$$\begin{aligned} 0.09x+0.11(5000-x) &= 487 \\ 0.09x+550-0.11x &= 487 \\ -0.02x+550 &= 487 \\ -0.02x &= -63 \\ x &= \frac{-63}{-0.02} \\ x &= 3150 \\ 5000-x &= 1850 \end{aligned}$$

\$3150 was invested at 9% and \$1850 was invested at 11%.

6. Let x = the width of the court.
Let x + 44 = the length of the court.

$$\begin{aligned} 2l+2w&=P \\ 2(x+44)+2x&=288 \\ 2x+88+2x&=288 \\ 4x+88&=288 \\ 4x&=200 \\ x&=\frac{200}{4} \\ x&=50 \\ x+44&=94 \end{aligned}$$

The dimensions of the court are 50 by 94.

7.
$$\begin{aligned} (16+2x)(12+2x)&=320 \\ 192+56x+4x^2&=320 \\ 4x^2+56x-128&=0 \\ x^2+14x-32&=0 \\ (x+16)(x-2)&=0 \end{aligned}$$

$x+16=0$ or $x-2=0$
$x=-16$ $\quad x=2$

–16 must be rejected.
The path must be 2 feet wide.

8.
$$\begin{aligned} a^2+b^2&=c^2 \\ a^2+(50)^2&=(130)^2 \\ a^2+2500&=16,900 \\ a^2&=14,400 \\ a&=\pm 120 \end{aligned}$$

–120 must be rejected.
The tower is 120 yards tall.

9.
$$\overbrace{\frac{5,000,000}{x}}^{\text{The original amount of money per person.}} - \overbrace{375,000}^{\text{reduction per winner}} = \overbrace{\frac{5,000,000}{x+3}}^{\text{The new amount of money per person.}}$$

$$\begin{aligned} x(x+3)\left(\frac{5,000,000}{x}-375,000\right)&=x(x+3)\frac{5,000,000}{x+3} \\ 5,000,000(x+3)-375,000x(x+3)&=5,000,000x \\ 5,000,000x+15,000,000-375,000x^2-1,125,000x&=5,000,000x \\ -375,000x^2-1,125,000x+15,000,000&=0 \\ x^2+3x-40&=0 \\ (x+8)(x-5)&=0 \end{aligned}$$

$x+8=0$ or $x-5=0$
$x=-8$ $\quad x=5$

–8 must be rejected. There were 5 people in the original group.

Concept and Vocabulary Check P.8

1. $x+658.6$

2. $31+2.4x$

3. $4+0.15x$

4. $x-0.15x$ or $0.85x$

5. $0.12x+0.09(30,000-x)$

6. $x+5$; $2(x+5)+2x$; $x(x+5)$

7. right; hypotenuse; legs

8. right; legs; the square of the length of the hypotenuse

9. $\dfrac{10{,}000}{x}$; $\dfrac{10{,}000}{x+2}$

Exercise Set P.8

1. Let $x =$ the number of years spent watching TV.
Let $x+19 =$ the number of years spent sleeping.

$$\begin{aligned} x+(x+19)&=37\\ x+x+19&=37\\ 2x+19&=37\\ 2x&=18\\ x&=9\\ x+19&=28 \end{aligned}$$

Americans will spend 9 years watching TV and 28 years sleeping.

2. Let $x =$ the number of years spent eating.
Let $x+24 =$ the number of years spent sleeping.

$$\begin{aligned} x+(x+24)&=32\\ x+x+24&=32\\ 2x+24&=32\\ 2x&=8\\ x&=4\\ x+24&=28 \end{aligned}$$

Americans will spend 4 years eating and 28 years sleeping.

3. Let $x =$ the average salary, in thousands, for an American whose final degree is a bachelor's.
Let $2x-70 =$ the average salary, in thousands, for an American whose final degree is a master's.

$$\begin{aligned} x+(2x-70)&=173\\ x+2x-70&=173\\ 3x-70&=173\\ 3x&=243\\ x&=81\\ 2x-70&=92 \end{aligned}$$

The average salary for an American whose final degree is a bachelor's is \$81 thousand and for an American whose final degree is a master's is \$92 thousand.

4. Let $x =$ the average salary, in thousands, for an American whose final degree is a bachelor's.
Let $2x-45 =$ the average salary, in thousands, for an American whose final degree is a doctorate.

$$\begin{aligned} x+(2x-45)&=198\\ x+2x-45&=198\\ 3x-45&=198\\ 3x&=243\\ x&=81\\ 2x-45&=117 \end{aligned}$$

The average salary for an American whose final degree is a bachelor's is \$81 thousand and for an American whose final degree is a doctorate is \$117 thousand.

5. Let $x =$ the number of years after 2014.

$$\begin{aligned} 37{,}600+1250x&=46{,}350\\ 1250x&=8750\\ \frac{1250x}{1250}&=\frac{8750}{1250}\\ x&=7 \end{aligned}$$

7 years after 2014, or in 2021, the average price of a new car will be \$46,350.

6. Let $x =$ the number of years after 2014.

$$\begin{aligned} 11.3+0.2x&=12.3\\ 0.2x&=1\\ \frac{0.2x}{0.2}&=\frac{1}{0.2}\\ x&=5 \end{aligned}$$

5 years after 2014, or in 2019, the average age of vehicles on U.S. roads will be 12.3 years.

7. **a.** $y = 24{,}000 - 3000x$

b.
$$\begin{aligned} y&=24{,}000-3000x\\ 9000&=24{,}000-3000x\\ 9000-24{,}000&=-3000x\\ -15{,}000&=-3000x\\ x&=\frac{-15{,}000}{-3000}\\ x&=5 \end{aligned}$$

The car's value will drop to \$9000 after 5 years.

8. **a.** $y = 45{,}000 - 5000x$

b.
$$\begin{aligned} y&=45{,}000-5000x\\ 10{,}000&=45{,}000-5000x\\ 10{,}000-45{,}000&=-5000x\\ -35{,}000&=-5000x\\ x&=\frac{-35{,}000}{-5000}\\ x&=7 \end{aligned}$$

The car's value will drop to \$10,000 after 7 years.

9. Let x = the number of months.
The cost for Club A: $25x+40$
The cost for Club B: $30x+15$

$$\begin{aligned}25x+40&=30x+15\\-5x+40&=15\\-5x&=-25\\x&=5\end{aligned}$$

The total cost for the clubs will be the same at 5 months. The cost will be
$25(5)+40=30(5)+15=\$165$

10. Let g = the number of video games rented

$$\begin{aligned}9g&=4g+50\\5g&=50\\g&=10\end{aligned}$$

The total amount spent at each store will be the same after 10 rentals.
$9g=9(10)=90$
The total amount spent will be \$90.

11. Let x = the number of uses.
Cost without discount pass: $1.25x$
Cost with discount pass: $15+0.75x$

$$\begin{aligned}1.25x&=15+0.75x\\0.50x&=15\\x&=30\end{aligned}$$

The bus must be used 30 times in a month for the costs to be equal.

12. Cost per crossing: \$5x
Cost with discount pass: \$30 + \$3.50x

$$\begin{aligned}5x&=30+3.50x\\1.50x&=30\\x&=20\end{aligned}$$

The bridge must be used 20 times in a month for the costs to be equal.

13. Let x = the number of years after 2010.
College A's enrollment: $13,300+1000x$

College B's enrollment: $26,800-500x$

$$\begin{aligned}13,300+1000x&=26,800-500x\\13,300+1500x&=26,800\\1500x&=13,500\\x&=9\end{aligned}$$

The two colleges will have the same enrollment 9 years after 2010, or 2019. That year the enrollment will be

$$\begin{aligned}&13,300+1000(9)\\&=26,800-500(9)\\&=22,300 \text{ students}\end{aligned}$$

14. Let x = the number of years after 2000

$$\begin{aligned}10,600,000-28,000x&=10,200,000-12,000x\\-16,000x&=-400,000\\x&=25\end{aligned}$$

The countries will have the same population 25 years after the year 2000, or the year 2025.

$$\begin{aligned}10,200,000-12,000x&=10,200,000-12,000(25)\\&=10,200,000-300,000\\&=9,900,000\end{aligned}$$

The population in the year 2025 will be 9,900,000.

15. Let x = the cost of the television set.

$$\begin{aligned}x-0.20x&=336\\0.80x&=336\\x&=420\end{aligned}$$

The television set's price is \$420.

16. Let x = the cost of the dictionary

$$\begin{aligned}x-0.30x&=30.80\\0.70x&=30.80\\x&=44\end{aligned}$$

The dictionary's price before the reduction was \$44.

17. Let x = the nightly cost

$$\begin{aligned}x+0.08x&=162\\1.08x&=162\\x&=150\end{aligned}$$

The nightly cost is \$150.

18. Let x = the nightly cost

$$\begin{aligned}x+0.05x&=252\\1.05x&=252\\x&=240\end{aligned}$$

The nightly cost is \$240.

19. Let c = the dealer's cost

$$\begin{aligned}584&=c+0.25c\\584&=1.25c\\467.20&=c\end{aligned}$$

The dealer's cost is \$467.20.

20. Let c = the dealer's cost

$$\begin{aligned}15&=c+0.25c\\15&=1.25c\\12&=c\end{aligned}$$

The dealer's cost is \$12.

21. Let x = the amount invested at 6%.
Let $7000 - x$ = the amount invested at 8%.

$$0.06x + 0.08(7000 - x) = 520$$
$$0.06x + 560 - 0.08x = 520$$
$$-0.02x + 560 = 520$$
$$-0.02x = -40$$
$$x = \frac{-40}{-0.02}$$
$$x = 2000$$
$$7000 - x = 5000$$

\$2000 was invested at 6% and \$5000 was invested at 8%.

22. Let x = the amount invested at 5%.
Let $11,000 - x$ = the amount invested at 8%.

$$0.05x + 0.08(11,000 - x) = 730$$
$$0.05x + 880 - 0.08x = 730$$
$$-0.03x + 880 = 730$$
$$-0.03x = -150$$
$$x = \frac{-150}{-0.03}$$
$$x = 5000$$
$$11,000 - x = 6000$$

\$5000 was invested at 5% and \$6000 was invested at 8%.

23. Let x = amount invested at 12%
$8000 - x$ = amount invested at 5% loss

$$.12x - .05(8000 - x) = 620$$
$$.12x - 400 + .05x = 620$$
$$.17x = 1020$$
$$x = 6000$$
$$8000\text{-}x = 2000$$

\$6000 at 12%, \$2000 at 5% loss

24. Let x = amount at 14%
$12000 - x$ = amount at 6%

$$.14x - 0.6(12000 - x) = 680$$
$$.14x - 720 + .06x = 680$$
$$.2x = 1400$$
$$x = 7000$$
$$12000 - 7000 = 5000$$

\$7000 at 14%, \$5000 at 6% loss

25. Let w = the width of the field
Let $2w$ = the length of the field

$$P = 2(\text{length}) + 2(\text{width})$$
$$300 = 2(2w) + 2(w)$$
$$300 = 4w + 2w$$
$$300 = 6w$$
$$50 = w$$

If $w = 50$, then $2w = 100$. Thus, the dimensions are 50 yards by 100 yards.

26. Let w = the width of the swimming pool,
Let $3w$ = the length of the swimming pool

$$P = 2(\text{length}) + 2(\text{width})$$
$$320 = 2(3w) + 2(w)$$
$$320 = 6w + 2w$$
$$320 = 8w$$
$$40 = w$$

If $w = 40$, $3w = 3(40) = 120$.
The dimensions are 40 feet by 120 feet.

27. Let w = the width of the field
Let $2w + 6$ = the length of the field

$$228 = 6w + 12$$
$$216 = 6w$$
$$36 = w$$

If $w = 36$, then $2w + 6 = 2(36) + 6 = 78$. Thus, the dimensions are 36 feet by 78 feet.

28. Let w = the width of the pool,
Let $2w - 6$ = the length of the pool

$$P = 2(\text{length}) + 2(\text{width})$$
$$126 = 2(2w - 6) + 2(w)$$
$$126 = 4w - 12 + 2w$$
$$126 = 6w - 12$$
$$138 = 6w$$
$$23 = w$$

Find the length.

$$2w - 6 = 2(23) - 6 = 46 - 6 = 40$$

The dimensions are 23 meters by 40 meters.

29. Let x = the width of the frame.
Total length: $16 + 2x$
Total width: $12 + 2x$

$$P = 2(\text{length}) + 2(\text{width})$$
$$72 = 2(16 + 2x) + 2(12 + 2x)$$
$$72 = 32 + 4x + 24 + 4x$$
$$72 = 8x + 56$$
$$16 = 8x$$
$$2 = x$$

The width of the frame is 2 inches.

30. Let w = the width of the path
Let $40 + 2w$ = the width of the pool and path
Let $60 + 2w$ = the length of the pool and path

$$2(40 + 2w) + 2(60 + 2w) = 248$$
$$80 + 4w + 120 + 4w = 248$$
$$200 + 8w = 248$$
$$8w = 48$$
$$w = 6$$

The width of the path is 6 feet.

31. Let w = the width
Let $w + 3$ = the length

$$\text{Area} = lw$$
$$54 = (w+3)w$$
$$54 = w^2 + 3w$$
$$0 = w^2 + 3w - 54$$
$$0 = (w+9)(w-6)$$

$w + 9 = 0$ $\quad$ $w - 6 = 0$
$w = -9$ $\quad$ $w = 6$

Disregard –9 because we can't have a negative length measurement. The width is 6 feet and the length is $6 + 3 = 9$ feet.

32. Let w = the width
Let $w + 3$ = the width

$$\text{Area} = lw$$
$$180 = (w+3)w$$
$$180 = w^2 + 3w$$
$$0 = w^2 + 3w - 180$$
$$0 = (w+15)(w-12)$$

$w + 15 = 0$ $\quad$ $w - 12 = 0$
~~$w = -15$~~ $\quad$ $w = 12$

The width is 12 yards and the length is 12 yards + 3 yards = 15 yards.

33. Let x = the length of the side of the original square
Let $x + 3$ = the length of the side of the new, larger square

$$(x+3)^2 = 64$$
$$x^2 + 6x + 9 = 64$$
$$x^2 + 6x - 55 = 0$$
$$(x+11)(x-5) = 0$$

Apply the zero product principle.

$x + 11 = 0$ $\quad$ $x - 5 = 0$
$x = -11$ $\quad$ $x = 5$

The solution set is $\{-11, 5\}$. Disregard –11 because we can't have a negative length measurement. This means that x, the length of the side of the original square, is 5 inches.

34. Let x = the side of the original square,
Let $x + 2$ = the side of the new, larger square

$$(x+2)^2 = 36$$
$$x^2 + 4x + 4 = 36$$
$$x^2 + 4x - 32 = 0$$
$$(x+8)(x-4) = 0$$

$x + 8 = 0$ $\quad$ $x - 4 = 0$
~~$x = -8$~~ $\quad$ $x = 4$

The length of the side of the original square, is 4 inches.

35. Let x = the width of the path

$$(20+2x)(10+2x) = 600$$
$$200 + 40x + 20x + 4x^2 = 600$$
$$200 + 60x + 4x^2 = 600$$
$$4x^2 + 60x + 200 = 600$$
$$4x^2 + 60x - 400 = 0$$
$$4(x^2 + 15x - 100) = 0$$
$$4(x+20)(x-5) = 0$$

Apply the zero product principle.

$4(x+20) = 0$ $\quad$ $x - 5 = 0$
$x + 20 = 0$ $\quad$ $x = 5$
$x = -20$

The solution set is $\{-20, 5\}$. Disregard –20 because we can't have a negative width measurement. The width of the path is 5 meters.

36. Let x = the width of the path

$$(12+2x)(15+2x) = 378$$
$$180 + 24x + 30x + 4x^2 = 378$$
$$4x^2 + 54x + 180 = 378$$
$$4x^2 + 54x - 198 = 0$$
$$2(2x^2 + 27x - 99) = 0$$
$$2(2x+33)(x-3) = 0$$

$2(2x+33) = 0$ $\quad$ $x - 3 = 0$
$2x + 33 = 0$ $\quad$ $x = 3$
$2x = -33$
~~$x = -\frac{33}{2}$~~

The width of the path is 3 meters.

37.
$$(20+2x)(30+2x) - (20)(30) = 336$$
$$600 + 100x + 4x^2 - 600 = 336$$
$$4x^2 + 100x - 336 = 0$$
$$x^2 + 25x - 84 = 0$$
$$(x-3)(x+28) = 0$$

$x - 3 = 0$ or $x + 28 = 0$
$x = 3$ $\quad$ $x = -28$

–28 must be rejected.
The width of the path is 3 feet

38.
$$(10+2x)(12+2x) - (10)(12) = 168$$
$$120 + 44x + 4x^2 - 120 = 168$$
$$4x^2 + 44x - 168 = 0$$
$$x^2 + 11x - 42 = 0$$
$$(x-3)(x+14) = 0$$

$x - 3 = 0$ or $x + 14 = 0$
$x = 3$ $\quad$ $x = -14$

–14 must be rejected.
The width of the path is 3 feet.

39. $a^2 + b^2 = c^2$
$a^2 + 15^2 = 20^2$
$a^2 + 225 = 400$
$a^2 = 175$
$a = \pm\sqrt{175}$
$a \approx \pm 13.2$
–13.2 must be rejected.
The ladder reaches 13.2 feet up the house.

40. $a^2 + b^2 = c^2$
$a^2 + 10^2 = 30^2$
$a^2 + 100 = 900$
$a^2 = 800$
$a = \pm\sqrt{800}$
$a \approx \pm 28.3$
–28.3 must be rejected.
The building is 28.3 feet tall.

41. $a^2 + b^2 = c^2$
$5^2 + x^2 = (x+1)^2$
$x^2 + 25 = x^2 + 2x + 1$
$25 = 2x + 1$
$24 = 2x$
$x = 12$
$x + 1 = 13$
The wire is 13 feet long.

42. $a^2 + b^2 = c^2$
$15^2 + x^2 = (x+4)^2$
$x^2 + 225 = x^2 + 8x + 16$
$225 = 8x + 16$
$209 = 8x$
$x = 26\frac{1}{8}$
$x + 4 = 30\frac{1}{8}$
The wire is $30\frac{1}{8}$ feet long.

43. Let x be the width.
$a^2 + b^2 = c^2$
$x^2 + (2x)^2 = 64^2$
$x^2 + 4x^2 = 4096$
$5x^2 = 4096$
$x^2 = \frac{4096}{5}$
$x = \pm\sqrt{\frac{4096}{5}}$
$x \approx 28.62$ feet
$2x \approx 57.24$ feet
The distance along the length and width is about 28.62 + 57.24, or about 85.9 feet. A person could save 85.9 – 64, or about 21.9 feet.

44. Let x be the width.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + (3x)^2 &= 92^2 \\ x^2 + 9x^2 &= 8464 \\ 10x^2 &= 8464 \\ x^2 &= 846.4 \\ x &= \pm\sqrt{846.4} \\ x &\approx 29.09 \text{ yd} \\ 3x &\approx 87.28 \text{ yd} \end{aligned}$$

The distance along the length and width is about 29.09 + 87.28, or about 116.4 yards. A person could save 116.4 – 92, or about 24.4 yards.

45.

$$\overbrace{\frac{20,000,000}{x}}^{\text{The original amount of money per person.}} - \overbrace{500,000}^{\text{reduction per winner}} = \overbrace{\frac{20,000,000}{x+2}}^{\text{The new amount of money per person.}}$$

$$\begin{aligned} x(x+2)\left(\frac{20,000,000}{x} - 500,000\right) &= x(x+2)\frac{20,000,000}{x+2} \\ 20,000,000(x+2) - 500,000x(x+2) &= 20,000,000x \\ 20,000,000x + 40,000,000 - 500,000x^2 - 1,000,000x &= 20,000,000x \\ 40,000,000 - 500,000x^2 - 1,000,000x &= 0 \\ x^2 + 2x - 80 &= 0 \\ (x+10)(x-8) &= 0 \end{aligned}$$

$x + 10 = 0$ or $x - 8 = 0$

$x = -10$ $\qquad x = 8$

–10 must be rejected. There were 8 people in the original group.

46.

$$\begin{aligned} \frac{480,000}{x} - 32,000 &= \frac{480,000}{x+4} \\ x(x+4)\left(\frac{480,000}{x} - 32,000\right) &= x(x+4)\frac{480,000}{x+4} \\ 480,000(x+4) - 32,000x(x+4) &= 480,000x \\ 480,000x + 1,920,000 - 32,000x^2 - 128,000x &= 480,000x \\ 1,920,000 - 32,000x^2 - 128,000x &= 0 \\ x^2 + 4x - 60 &= 0 \\ (x+10)(x-6) &= 0 \end{aligned}$$

$x + 10 = 0$ or $x - 6 = 0$

$x = -10$ $\qquad x = 6$

–10 must be rejected. There were 6 people in the original group.

47. Let x be the car's average velocity.

$$\overbrace{\frac{300}{x}}^{\text{car's time traveled}} = \overbrace{\frac{180}{x-20}}^{\text{bus's time traveled}}$$

$$\begin{aligned} 300(x-20) &= 180x \\ 300x - 6000 &= 180x \\ 120x &= 6000 \\ x &= 50 \\ x - 20 &= 30 \end{aligned}$$

The average velocity of the car is 50 miles per hour. The average velocity of the bus is 30 miles per hour.

48. Let x be the passenger train's average velocity.

$$\overbrace{\frac{240}{x}}^{\substack{\text{passenger train's}\\ \text{time traveled}}} = \overbrace{\frac{160}{x-20}}^{\substack{\text{freight train's}\\ \text{time traveled}}}$$

$$240(x-20)=160x$$
$$240x-4800=160x$$
$$80x=4800$$
$$x=60$$
$$x-20=40$$

The average velocity of the passenger train is 60 miles per hour. The average velocity of the freight train is 40 miles per hour.

49. Let x be the average velocity on the return trip.

$$\frac{5}{x+9}+\frac{5}{x}=\frac{7}{6}$$
$$6x(x+9)\left(\frac{5}{x+9}+\frac{5}{x}\right)=6x(x+9)\frac{7}{6}$$
$$30x+30(x+9)=7x(x+9)$$
$$30x+30x+270=7x^2+63x$$
$$0=7x^2+3x-270$$
$$0=(x-6)(7x+45)$$

$x-6=0$ or $7x+45=0$

$x=6$ $\quad x=-\frac{45}{7}$

$-\frac{45}{7}$ must be rejected. The average velocity on the return trip is 6 miles per hour.

50. Let x be the average velocity of the first engine.

$$\frac{140}{x}+\frac{200}{x+5}=9$$
$$\left(\frac{140}{x}+\frac{200}{x+5}\right)=9$$
$$x(x+5)\left(\frac{140}{x}+\frac{200}{x+5}\right)=9x(x+5)$$
$$140(x+5)+200x=9x(x+5)$$
$$140x+700+200x=9x^2+45x$$
$$0=9x^2-295x-700$$
$$0=(x-35)(9x-20)$$

$x-35=0$ or $9x+20=0$

$x=35$ $\quad x=-\frac{20}{9}$

$x+5=40$

$-\frac{20}{9}$ must be rejected. The average velocity of the first engine is 35 miles per hour. The average velocity of the second engine is 40 miles per hour.

51. Let x = number of hours

$35x$ = labor cost

$35x + 63 = 448$

$35x = 385$

$x = 11$

It took 11 hours.

52. Let x = number of hours

$63x$ = labor cost

$63x + 532 = 1603$

$63x = 1071$

$x = 17$

17 hours were required to repair the yacht.

53. Let x = inches over 5 feet

$100 + 5x = 135$

$5x = 35$

$x = 7$

A height of 5 feet 7 inches corresponds to 135 pounds.

54. Let g = the gross amount of the paycheck

$$\text{Yearly Salary} = 2(12)g+750$$
$$33150=24g+750$$
$$32400=24g$$
$$1350=g$$

The gross amount of each paycheck is \$1350.

55. Let x be the number of consecutive hits.

$$\frac{35+x}{140+x}=0.30$$
$$35+x=0.30(140+x)$$
$$35+x=42+0.30x$$
$$350+10x=420+3x$$
$$7x=70$$
$$x=10$$

You must get 10 consecutive hits to increase your batting average to 0.30.

56. Let x be the number of consecutive hits.

$$\frac{30+x}{120+x}=0.28$$
$$30+x=0.28(120+x)$$
$$30+x=33.6+0.28x$$
$$3000+100x=3360+28x$$
$$72x=360$$
$$x=5$$

You must get 5 consecutive hits to increase your batting average to 0.28.

57. – 60. Answers may vary.

61. does not make sense; Explanations will vary. Sample explanation: Though mathematical models can often provide excellent estimates about future attitudes, they cannot guaranty perfect precision.

62. makes sense

63. does not make sense; Explanations will vary. Sample explanation: The correct equation is $x-0.35x=780$.

64. makes sense

65. Let x be the length of one leg.

$$a^2+b^2=c^2$$
$$x^2+(x+1)^2=[12-x-(x+1)]^2$$
$$x^2+x^2+2x+1=[12-x-x-1]^2$$
$$2x^2+2x+1=(11-2x)^2$$
$$2x^2+2x+1=121-44x+4x^2$$
$$0=2x^2-46x+120$$
$$0=x^2-23x+60$$
$$0=(x-3)(x-20)$$
$$x-3=0 \quad \text{or} \quad x-20=0$$
$$x=3 \qquad x=20$$
$$x+1=4$$
$$12-(3+4)=5$$

20 must be rejected, as it is greater than the perimeter.
The lengths of the sides are 3, 4, and 5.

66. Let x = original price
$x - 0.4x = 0.6x$ = price after first reduction
$0.6x - 0.4(0.6x)$ = price after second reduction

$$0.6x-0.24x=72$$
$$0.36x=72$$
$$x=200$$

The original price was \$200.

67. Let x = woman's age
$3x$ = Coburn's age
$3x + 20 = 2(x + 20)$
$3x + 20 = 2x + 40$
$x + 20 = 40$
$x = 20$
Coburn is 60 years old the woman is 20 years old.

68. Let x = correct answers
$26 - x$ = incorrect answers

$$8x-5(26-x)=0$$
$$8x-130+5x=0$$
$$13x-130=0$$
$$13x=130$$
$$x=10$$

10 problems were solved correctly.

69. Let x = mother's amount
$2x$ = boy's amount
$\frac{x}{2}$ = girl's amount

$$x+2x+\frac{x}{2}=14,000$$
$$\frac{7}{2}x=14,000$$
$$x=\$4,000$$

The mother received \$4000, the boy received \$8000, and the girl received \$2000.

70. Let x = the number of plants originally stolen
After passing the first security guard, the thief has:

$$x-\left(\frac{1}{2}x+2\right)=x-\frac{1}{2}x-2=\frac{1}{2}x-2$$

After passing the second security guard, the thief has:

$$\frac{1}{2}x-2-\left(\frac{\frac{1}{2}x-2}{2}+2\right)=\frac{1}{4}x-3$$

After passing the third security guard, the thief has:

$$\frac{1}{4}x-3-\left(\frac{\frac{1}{4}x-3}{2}+2\right)=\frac{1}{8}x-\frac{7}{2}$$

Thus, $\frac{1}{8}x-\frac{7}{2}=1$

$$x-28=8$$
$$x=36$$

The thief stole 36 plants.

71. Answers may vary.

72.
$$3-2x\le 11$$
$$3-2(-1)\le 11$$
$$3+2\le 11$$
$$5\le 11, \text{ true}$$

-1 is a solution.

73.
$$-2x-4=x+5$$
$$-2x-x=5+4$$
$$-3x=9$$
$$x=\frac{9}{-3}$$
$$x=-3$$

The solution set is $\{-3\}$.

74.
$$\frac{x+3}{4}=\frac{x-2}{3}+\frac{1}{4}$$
$$12\left(\frac{x+3}{4}\right)=12\left(\frac{x-2}{3}+\frac{1}{4}\right)$$
$$3(x+3)=4(x-2)+3$$
$$3x+9=4x-8+3$$
$$3x+9=4x-5$$
$$3x-4x=-5-9$$
$$-x=-14$$
$$x=14$$

The solution set is $\{14\}$.

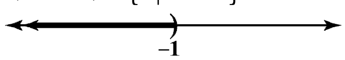
Section P.9

Check Point Exercises

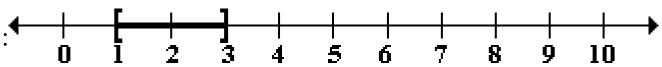
1. **a.** $[-2,5)=\{x|-2\le x<5\}$

b. $[1,3.5]=\{x|1\le x\le 3.5\}$

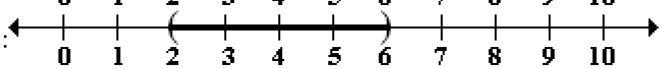
c. $(-\infty,-1)=\{x|x<-1\}$



2. **a.** Graph $[1,3]$:

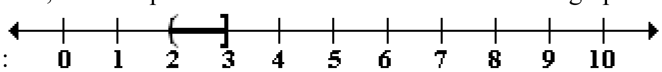


Graph $(2,6)$:



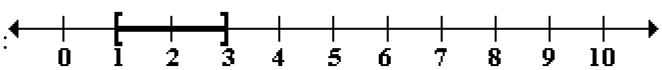
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[1,3]$ and $(2,6)$:

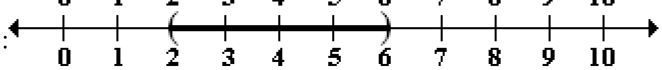


Thus, $[1,3]\cap(2,6)=(2,3]$.

b. Graph $[1,3]$:

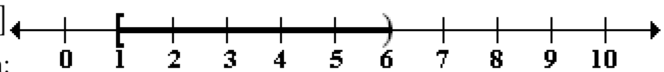


Graph $(2,6)$:



To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[1,3]$ or $(2,6)$ or both:



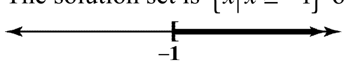
Thus, $[1,3]\cup(2,6)=[1,6)$.

3. $2-3x\le 5$

$-3x\le 3$

$x\ge -1$

The solution set is $\{x|x\ge -1\}$ or $[-1,\infty)$.



4. $3x+1>7x-15$

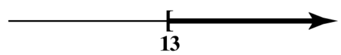
$-4x>-16$

$\frac{-4x}{-4}<\frac{-16}{-4}$

$x<4$

The solution set is $\{x|x<4\}$ or $(-\infty,4)$.

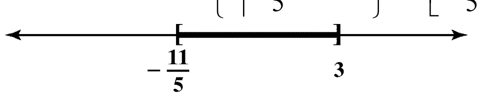
5. $$\frac{x-4}{2} \ge \frac{x-2}{3} + \frac{5}{6}$$
$$6\left(\frac{x-4}{2}\right) \ge 6\left(\frac{x-2}{3} + \frac{5}{6}\right)$$
$$3(x-4) \ge 2(x-2) + 5$$
$$3x - 12 \ge 2x - 4 + 5$$
$$3x - 12 \ge 2x + 1$$
$$3x - 2x \ge 1 + 12$$
$$x \ge 13$$
The solution set is $\{x | x \ge 13\}$ or $[13, \infty)$.



6. $$1 \le 2x + 3 < 11$$
$$-2 \le 2x < 8$$
$$-1 \le x < 4$$
The solution set is $\{x | -1 \le x < 4\}$ or $[-1, 4)$.

7. $$|x-2| < 5$$
$$-5 < x - 2 < 5$$
$$-3 < x < 7$$
The solution set is $\{x | -3 < x < 7\}$ or $(-3, 7)$.

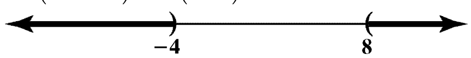
8. $$-3|5x-2| + 20 \ge -19$$
$$-3|5x-2| \ge -39$$
$$\frac{-3|5x-2|}{-3} \le \frac{-39}{-3}$$
$$|5x-2| \le 13$$
$$-13 \le 5x - 2 \le 13$$
$$-11 \le 5x \le 15$$
$$\frac{-11}{5} \le \frac{5x}{5} \le \frac{15}{5}$$
$$-\frac{11}{5} \le x \le 3$$
The solution set is $\left\{x \middle| -\frac{11}{5} \le x \le 3\right\}$ or $\left[-\frac{11}{5}, 3\right]$.



9. $18 < |6-3x|$

$$\begin{aligned} 6-3x &< -18 \\ -3x &< -24 \\ \frac{-3x}{-3} &> \frac{-24}{-3} \\ x &> 8 \end{aligned} \quad \text{or} \quad \begin{aligned} 6-3x &> 18 \\ -3x &> 12 \\ \frac{-3x}{-3} &< \frac{12}{-3} \\ x &< -4 \end{aligned}$$

The solution set is $\{x | x < -4 \text{ or } x > 8\}$ or $(-\infty,-4) \cup (8,\infty)$.



10. Let x = the number of miles driven in a week.

$260 < 80 + 0.25x$

$180 < 0.25x$

$720 < x$

Driving more than 720 miles in a week makes Basic the better deal.

Concept and Vocabulary Check P.9

1. 2; 5; 2; 5

2. greater than

3. less than or equal to

4. $(-\infty, 9)$; intersection

5. $(-\infty, 12)$; union

6. adding 4; dividing; -3; direction; $>$; $<$

7. middle

8. $-c$; c

9. $-c$; c

10. $-2 < x-7 < 2$

11. $x-7 < -2$ or $-7 > 2$

Exercise Set P.9

1. $1 < x \le 6$

2. $-2 < x \le 4$

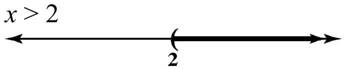
3. $-5 \le x < 2$

4. $-4 \le x < 3$

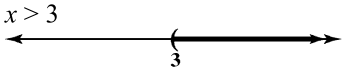
5. $-3 \le x \le 1$

6. $-2 \le x \le 5$

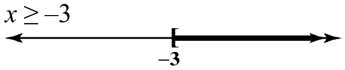
7. $x > 2$



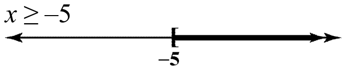
8. $x > 3$



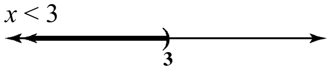
9. $x \ge -3$



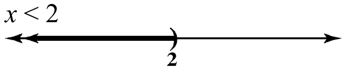
10. $x \ge -5$



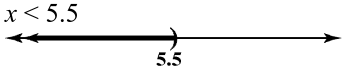
11. $x < 3$



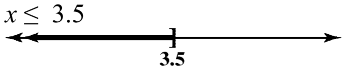
12. $x < 2$



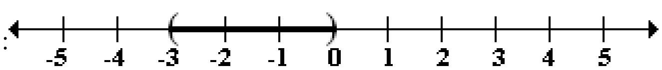
13. $x < 5.5$



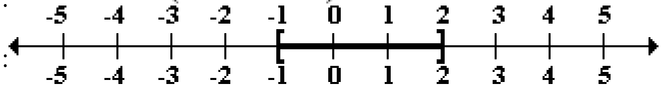
14. $x \le 3.5$



15. Graph $(-3,0)$:

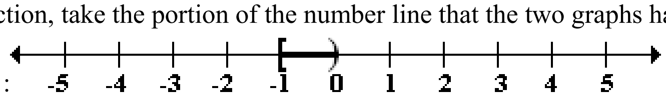


Graph $[-1,2]$:



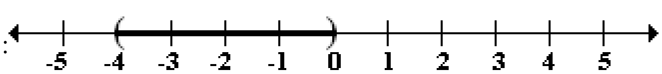
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-3,0)$ and $[-1,2]$:

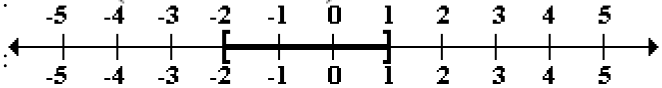


Thus, $(-3,0) \cap [-1,2] = [-1,0)$.

16. Graph $(-4,0)$:

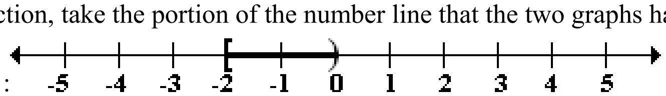


Graph $[-2,1]$:

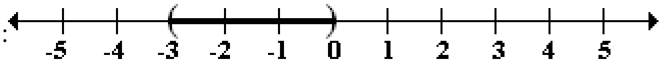


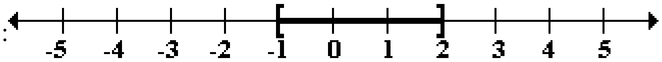
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-4,0)$ and $[-2,1]$:

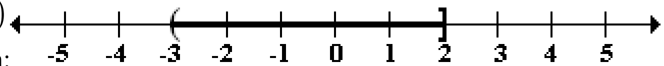


Thus, $(-4,0) \cap [-2,1] = [-2,0)$.

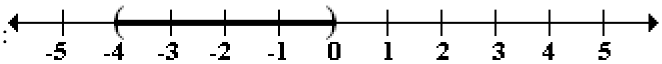
17. Graph $(-3,0)$: 

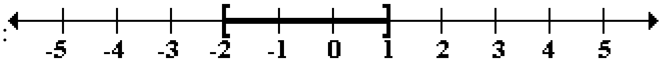
Graph $[-1,2]$: 

To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

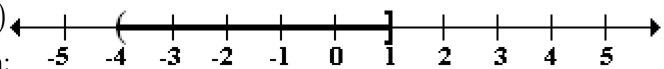
Numbers in either $(-3,0)$ or $[-1,2]$ or both: 

Thus, $(-3,0)\cup[-1,2]=(-3,2]$.

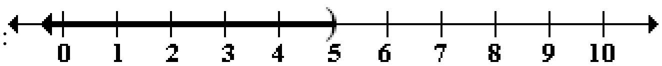
18. Graph $(-4,0)$: 

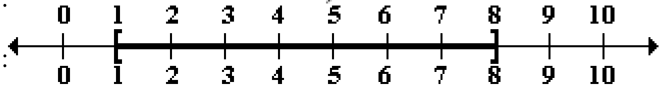
Graph $[-2,1]$: 

To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

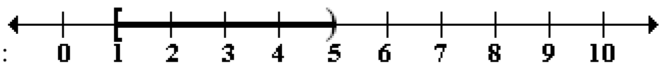
Numbers in either $(-4,0)$ or $[-2,1]$ or both: 

Thus, $(-4,0)\cup[-2,1]=(-4,1]$.

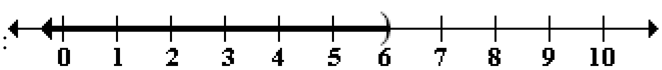
19. Graph $(-\infty,5)$: 

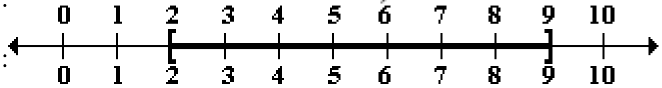
Graph $[1,8]$: 

To find the intersection, take the portion of the number line that the two graphs have in common.

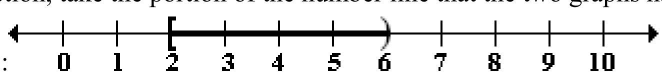
Numbers in both $(-\infty,5)$ and $[1,8]$: 

Thus, $(-\infty,5)\cap[1,8]=[1,5)$.

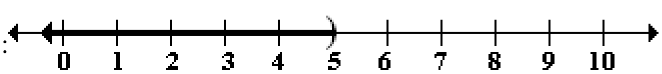
20. Graph $(-\infty,6)$: 

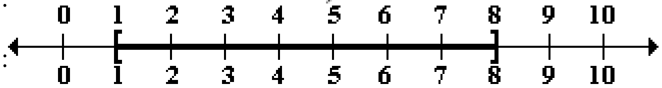
Graph $[2,9]$: 

To find the intersection, take the portion of the number line that the two graphs have in common.

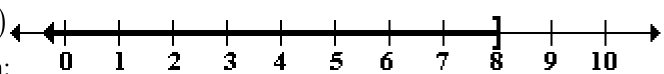
Numbers in both $(-\infty,6)$ and $[2,9]$: 

Thus, $(-\infty,6)\cap[2,9]=[2,6)$.

21. Graph $(-\infty,5)$: 

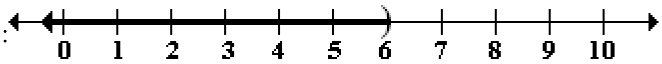
Graph $[1,8]$: 

To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

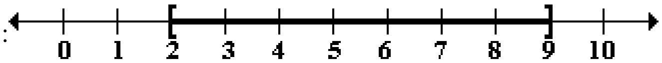
Numbers in either $(-\infty,5)$ or $[1,8]$ or both: 

Thus, $(-\infty,5)\cup[1,8]=(-\infty,8]$.

22. Graph $(-\infty, 6)$:

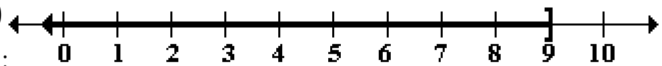


Graph $[2, 9]$:



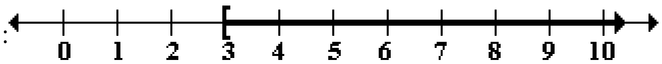
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-\infty, 6)$ or $[2, 9]$ or both:

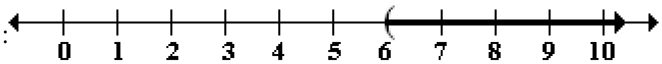


Thus, $(-\infty, 6) \cup [2, 9] = (-\infty, 9]$.

23. Graph $[3, \infty)$:

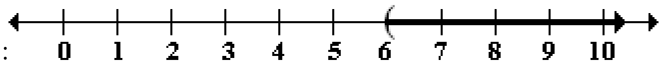


Graph $(6, \infty)$:



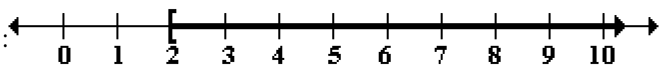
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[3, \infty)$ and $(6, \infty)$:

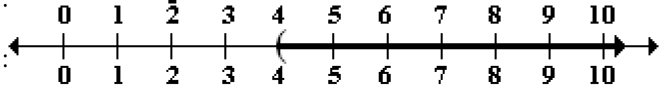


Thus, $[3, \infty) \cap (6, \infty) = (6, \infty)$.

24. Graph $[2, \infty)$:

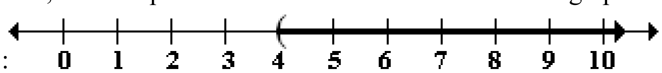


Graph $(4, \infty)$:



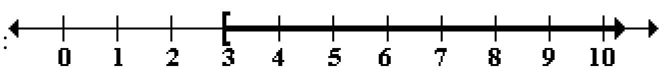
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[2, \infty)$ and $(4, \infty)$:

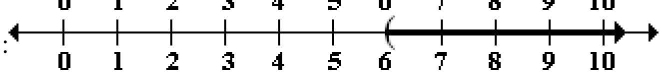


Thus, $[2, \infty) \cap (4, \infty) = (4, \infty)$.

25. Graph $[3, \infty)$:

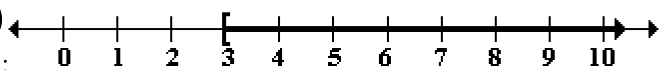


Graph $(6, \infty)$:



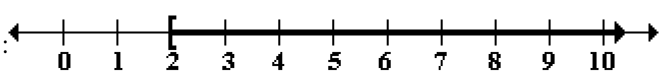
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[3, \infty)$ or $(6, \infty)$ or both:

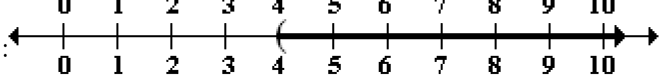


Thus, $[3, \infty) \cup (6, \infty) = [3, \infty)$.

26. Graph $[2, \infty)$:

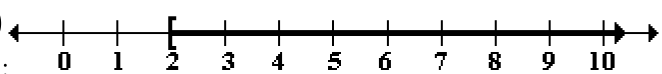


Graph $(4, \infty)$:



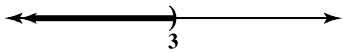
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[2, \infty)$ or $(4, \infty)$ or both:

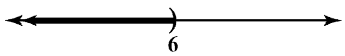


Thus, $[2, \infty) \cup (4, \infty) = [2, \infty)$.

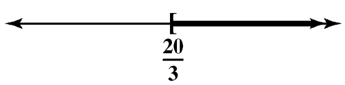
27. $5x + 11 < 26$
$5x < 15$
$x < 3$
The solution set is $\{x \mid x < 3\}$, or $(-\infty, 3)$.



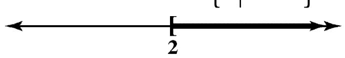
28. $2x + 5 < 17$
$2x < 12$
$x < 6$
The solution set is $\{x \mid x < 6\}$ or $(-\infty, 6)$.



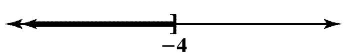
29. $3x - 7 \geq 13$
$3x \geq 20$
$x \geq \frac{20}{3}$
The solution set is $\left\{x \,\middle|\, x > \frac{20}{3}\right\}$, or $\left[\frac{20}{3}, \infty\right)$.



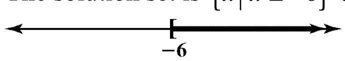
30. $8x - 2 \geq 14$
$8x \geq 16$
$x \geq 2$
The solution set is $\{x \mid x > 2\}$ or $[2, \infty)$.



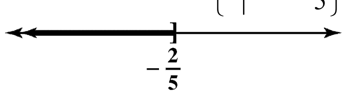
31. $-9x \geq 36$
$x \leq -4$
The solution set is $\{x \mid x \leq -4\}$, or $(-\infty, -4]$.



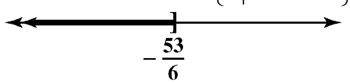
32. $-5x \leq 30$
$x \geq -6$
The solution set is $\{x \mid x \geq -6\}$ or $[-6, \infty)$.



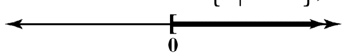
33. $8x - 11 \leq 3x - 13$
$8x - 3x \leq -13 + 11$
$5x \leq -2$
$x \leq -\frac{2}{5}$
The solution set is $\left\{x \,\middle|\, x \leq -\frac{2}{5}\right\}$, or $\left(-\infty, -\frac{2}{5}\right]$.



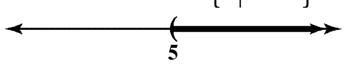
34. $18x + 45 \le 12x - 8$
$18x - 12x \le -8 - 45$
$6x \le -53$
$x \le -\frac{53}{6}$
The solution set is $\left\{x \,\middle|\, x \le -\frac{53}{6}\right\}$ or $\left(-\infty, -\frac{53}{6}\right]$.



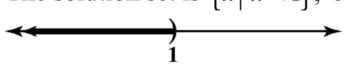
35. $4(x + 1) + 2 \ge 3x + 6$
$4x + 4 + 2 \ge 3x + 6$
$4x + 6 \ge 3x + 6$
$4x - 3x \ge 6 - 6$
$x \ge 0$
The solution set is $\{x \mid x > 0\}$, or $[0, \infty)$.



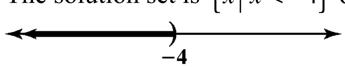
36. $8x + 3 > 3(2x + 1) + x + 5$
$8x + 3 > 6x + 3 + x + 5$
$8x + 3 > 7x + 8$
$8x - 7x > 8 - 3$
$x > 5$
The solution set is $\{x \mid x > 5\}$ or $(5, \infty)$.



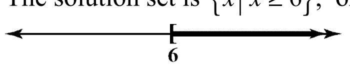
37. $2x - 11 < -3(x + 2)$
$2x - 11 < -3x - 6$
$5x < 5$
$x < 1$
The solution set is $\{x \mid x < 1\}$, or $(-\infty, 1)$.



38. $-4(x + 2) > 3x + 20$
$-4x - 8 > 3x + 20$
$-7x > 28$
$x < -4$
The solution set is $\{x \mid x < -4\}$ or $(-\infty, -4)$.



39. $1 - (x + 3) \ge 4 - 2x$
$1 - x - 3 \ge 4 - 2x$
$-x - 2 \ge 4 - 2x$
$x \ge 6$
The solution set is $\{x \mid x \ge 6\}$, or $[6, \infty)$.



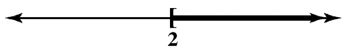
40. $5(3-x) \le 3x-1$

$15-5x \le 3x-1$

$-8x \le -16$

$x \ge 2$

The solution set is $\{x \mid x \ge 2\}$ or $[2, \infty)$.



41. $\frac{x}{4} - \frac{3}{2} \le \frac{x}{2} + 1$

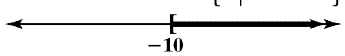
$\frac{4x}{4} - \frac{4 \cdot 3}{2} \le \frac{4 \cdot x}{2} + 4 \cdot 1$

$x - 6 \le 2x + 4$

$-x \le 10$

$x \ge -10$

The solution set is $\{x \mid x \ge -10\}$, or $[-10, \infty)$.



42. $\frac{3x}{10} + 1 \ge \frac{1}{5} - \frac{x}{10}$

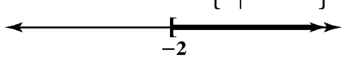
$10\left(\frac{3x}{10} + 1\right) \ge 10\left(\frac{1}{5} - \frac{x}{10}\right)$

$3x + 10 \ge 2 - x$

$4x \ge -8$

$x \ge -2$

The solution set is $\{x \mid x \ge -2\}$ or $[-2, \infty)$.

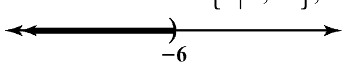


43. $1 - \frac{x}{2} > 4$

$-\frac{x}{2} > 3$

$x < -6$

The solution set is $\{x \mid x, -6\}$, or $(-\infty, -6)$.

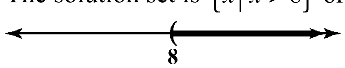


44. $7 - \frac{4}{5}x < \frac{3}{5}$

$-\frac{4}{5}x < -\frac{32}{5}$

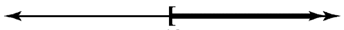
$x > 8$

The solution set is $\{x \mid x > 8\}$ or $(8, \infty)$.



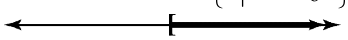
45.
$$\frac{x-4}{6} \geq \frac{x-2}{9} + \frac{5}{18}$$
$$3(x-4) \geq 2(x-2)+5$$
$$3x-12 \geq 2x-4+5$$
$$x \geq 13$$

The solution set is $\{x \mid x \geq 13\}$, or $[13, \infty)$.



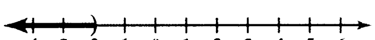
46.
$$\frac{4x-3}{6} + 2 \geq \frac{2x-1}{12}$$
$$2(4x-3)+24 \geq 2x-1$$
$$8x-6+24 \geq 2x-1$$
$$6x+18 \geq -1$$
$$6x \geq -19$$
$$x \geq -\frac{19}{6}$$

The solution set is $\left\{x \,\middle|\, x \geq \frac{-19}{6}\right\}$ or $\left[\frac{-19}{6}, \infty\right)$.



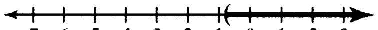
47.
$$3\left[3(x+5)+8x+7\right]+5\left[3(x-6)-2(3x-5)\right] < 2(4x+3)$$
$$3\left[3x+15+8x+7\right]+5\left[3x-18-6x+10\right] < 8x+6$$
$$3\left[11x+22\right]+5\left[-3x-8\right] < 8x+6$$
$$33x+66-15x-40 < 8x+6$$
$$18x+26 < 8x+6$$
$$10x < -20$$
$$x < -2$$

The solution set is $\{x \mid x < -2\}$ or $[-\infty, -2)$.



48.
$$5\left[3(2-3x)-2(5-x)\right]-6\left[5(x-2)-2(4x-3)\right] < 3x+19$$
$$5\left[6-9x-10+2x\right]-6\left[5x-10-8x+6\right] < 3x+19$$
$$5\left[-7x-4\right]-6\left[-3x-4\right] < 3x+19$$
$$-35x-20+18x+24 < 3x+19$$
$$-17x+4 < 3x+19$$
$$-20x < 15$$
$$\frac{-20x}{-20} > \frac{15}{-20}$$
$$x > -\frac{3}{4}$$

The solution set is $\left\{x \,\middle|\, x > -\frac{3}{4}\right\}$ or $\left[-\frac{3}{4}, \infty\right)$.



49. $6 < x + 3 < 8$
$6 - 3 < x + 3 - 3 < 8 - 3$
$3 < x < 5$
The solution set is $\{x \mid 3 < x < 5\}$, or (3, 5).

50. $7 < x + 5 < 11$
$7 - 5 < x + 5 - 5 < 11 - 5$
$2 < x < 6$
The solution set is $\{x \mid 2 < x < 6\}$ or (2, 6).

51. $-3 \le x - 2 < 1$
$-1 \le x < 3$
The solution set is $\{x \mid -1 \le x < 3\}$, or [–1, 3).

52. $-6 < x - 4 \le 1$
$-2 < x \le 5$
The solution set is $\{x \mid -2 < x \le 5\}$ or (–2, 5].

53. $-11 < 2x - 1 \le -5$
$-10 < 2x \le -4$
$-5 < x \le -2$
The solution set is $\{x \mid -5 < x \le -2\}$, or (–5, –2].

54. $3 \le 4x - 3 < 19$
$6 \le 4x < 22$
$\frac{6}{4} \le x < \frac{22}{4}$
$\frac{3}{2} \le x < \frac{11}{2}$
The solution set is $\left\{x \,\middle|\, \frac{3}{2} \le x < \frac{11}{2}\right\}$ or $\left[\frac{3}{2}, \frac{11}{2}\right)$.

55. $-3 \le \frac{2}{3}x - 5 < -1$
$2 \le \frac{2}{3}x < 4$
$3 \le x < 6$
The solution set is $\{x \mid 3 \le x < 6\}$, or [3, 6).

56. $-6 \le \frac{1}{2}x - 4 < -3$
$-2 \le \frac{1}{2}x < 1$
$-4 \le x < 2$
The solution set is $\{x \mid -4 \ge x < 2\}$ or $[-4, 2)$.

57. $|x| < 3$
$-3 < x < 3$
The solution set is $\{x \mid -3 < x < 3\}$, or (–3, 3).

58. $|x| < 5$
$-5 < x < 5$
The solution set is $\{x \mid -5 < x < 5\}$ or (–5, 5).

59. $|x - 1| \le 2$
$-2 \le x - 1 \le 2$
$-1 \le x \le 3$
The solution set is $\{x \mid -1 \le x \le 3\}$, or [–1, 3].

60. $|x + 3| \le 4$
$-4 \le x + 3 \le 4$
$-7 \le x \le 1$
The solution set is $\{x \mid -7 \le x \le 1\}$ or [–7, 1].

61. $|2x - 6| < 8$
$-8 < 2x - 6 < 8$
$-2 < 2x < 14$
$-1 < x < 7$
The solution set is $\{x \mid -1 < x < 7\}$, or (–1, 7).

62. $|3x + 5| < 17$
$-17 < 3x + 5 < 17$
$-22 < 3x < 12$
The solution set is $\left\{x \,\middle|\, -\frac{22}{3} < x < 4\right\}$ or $\left(-\frac{22}{3}, 4\right)$.

63. $|2(x - 1) + 4| \le 8$
$-8 \le 2(x - 1) + 4 \le 8$
$-8 \le 2x - 2 + 4 \le 8$
$-8 \le 2x + 2 \le 8$
$-10 \le 2x \le 6$
$-5 \le x \le 3$
The solution set is $\{x \mid -5 \le x \le 3\}$, or [–5, 3].

64. $|3(x - 1) + 2| \le 20$
$-20 \le 3(x - 1) + 2 \le 20$
$-20 \le 3x - 1 \le 20$
$-19 \le 3x \le 21$
$-\frac{19}{3} \le x \le 7$
The solution set is $\left\{x \,\middle|\, -\frac{19}{3} \le x \le 7\right\}$ or $\left[-\frac{19}{3}, 7\right]$.

65. $\left|\frac{2y + 6}{3}\right| < 2$
$-2 < \frac{2y + 6}{3} < 2$
$-6 < 2y + 6 < 6$
$-12 < 2y < 0$
$-6 < y < 0$
The solution set is $\{x \mid -6 < y < 0\}$, or (–6, 0).

66. $\left|\frac{3(x-1)}{4}\right| < 6$

$-6 < \frac{3(x-1)}{4} < 6$

$-24 < 3x - 3 < 24$

$-21 < 3x < 27$

$-7 < x < 9$

The solution set is $\{x \mid -7 < x < 9\}$ or (–7, 9).

67. $|x| > 3$

$x > 3$ or $x < -3$

The solution set is $\{x \mid x > 3 \text{ or } x < -3\}$, that is, $(-\infty, -3)$ or $(3, \infty)$.

68. $|x| > 5$

$x > 5$ or $x < -5$

The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$, that is, all x in $(-\infty, -5)$ or $(5, \infty)$.

69. $|x - 1| \geq 2$

$x - 1 \geq 2$ or $x - 1 \leq -2$

$x \geq 3$ $\quad$ $x \leq -1$

The solution set is $\{x \mid x \leq -1 \text{ or } x \geq 3\}$, that is, $(-\infty, -1]$ or $[3, \infty)$.

70. $|x + 3| \geq 4$

$x + 3 \geq 4$ or $x + 3 \leq -4$

$x \geq 1$ $\quad$ $x \leq -7$

The solution set is $\{x \mid x \leq -7 \text{ or } x \geq 1\}$ that is, $(-\infty, -7)$ or $(1, \infty)$.

71. $|3x - 8| > 7$

$3x - 8 > 7$ or $3x - 8 < -7$

$3x > 15$ $\quad$ $3x < 1$

$x > 5$ $\quad$ $x < \frac{1}{3}$

The solution set is $\left\{x \,\middle|\, x < \frac{1}{3} \text{ or } x > 5\right\}$, that is, $\left(-\infty, \frac{1}{3}\right)$ or $(5, \infty)$.

72. $|5x - 2| > 13$

$5x - 2 > 13$ or $5x - 2 < -13$

$5x > 15$ $\quad$ $5x < -11$

$x > 3$ $\quad$ $x < -\frac{11}{5}$

The solution set is $\left\{x \,\middle|\, x < \frac{-11}{5} \text{ or } x > 3\right\}$, that is, all x in $\left(-\infty, \frac{-11}{5}\right)$ or $(3, \infty)$

73. $\left|\frac{2x+2}{4}\right| \geq 2$

$\frac{2x+2}{4} \geq 2$ or $\frac{2x+2}{4} \leq -2$

$2x + 2 \geq 8$ $\quad$ $2x + 2 \leq -8$

$2x \geq 6$ $\quad$ $2x \leq -10$

$x \geq 3$ $\quad$ $x \leq -5$

The solution set is $\{x \mid x \leq -5 \text{ or } x \geq 3\}$, that is, $(-\infty, -5]$ or $[3, \infty)$.

74. $\left|\frac{3x-3}{9}\right| \geq 1$

$\frac{3x-3}{9} \geq 1$ or $\frac{3x-3}{9} \leq -1$

$3x - 3 \geq 9$ $\quad$ $3x - 3 \leq -9$

$3x \geq 12$ $\quad$ $3x \leq -6$

$x \geq 4$ $\quad$ $x \leq -2$

The solution set is $\{x \mid x \leq -2 \text{ or } x \geq 4\}$, or $(-\infty, -2]$ or $[4, \infty)$.

75. $\left|3 - \frac{2}{3}x\right| > 5$

$3 - \frac{2}{3}x > 5$ or $3 - \frac{2}{3}x < -5$

$-\frac{2}{3}x > 2$ $\quad$ $-\frac{2}{3}x < -8$

$x < -3$ $\quad$ $x > 12$

The solution set is $\{x \mid x < -3 \text{ or } x > 12\}$, that is, $(-\infty, -3)$ or $(12, \infty)$.

76. $\left|3-\frac{3}{4}x\right|>9$

$3-\frac{3}{4}x>9 \quad \text{or} \quad 3-\frac{3}{4}x<-9$

$-\frac{3}{4}x>6 \qquad -\frac{3}{4}x<-12$

$x<-8 \qquad x>16$

$\{x \mid x<-8 \text{ or } x>16\}$, that is all x in

$(-\infty,-8)$ or $(16,\infty)$.

77. $3|x-1|+2\geq 8$

$3|x-1|\geq 6$

$|x-1|\geq 2$

$x-1\geq 2 \quad \text{or} \quad x-1\leq -2$

$x\geq 3 \qquad x\leq -1$

The solution set is $\{x \mid x\leq 1 \text{ or } x\geq 3\}$, that is,

$(-\infty,-1]$ or $[3,\infty)$.

78. $5|2x+1|-3\geq 9$

$5|2x+1|\geq 12$

$|2x+1|\geq \frac{12}{5}$

$2x+1\geq \frac{12}{5} \qquad 2x+1\leq -\frac{12}{5}$

$2x\geq \frac{7}{5} \quad \text{or} \quad 2x\leq -\frac{17}{5}$

$x\geq \frac{7}{10} \qquad x\leq -\frac{17}{10}$

The solution set is $\left\{x \,\middle|\, x\leq -\frac{17}{10} \text{ or } x\geq \frac{7}{10}\right\}$.

79. $-2|x-4|\geq -4$

$\frac{-2|x-4|}{-2}\leq \frac{-4}{-2}$

$|x-4|\leq 2$

$-2\leq x-4\leq 2$

$2\leq x\leq 6$

The solution set is $\{x|2\leq x\leq 6\}$.

80. $-3|x+7|\geq -27$

$\frac{-3|x+7|}{-3}\leq \frac{-27}{-3}$

$|x+7|\leq 9$

$-9\leq x+7\leq 9$

$-16\leq x\leq 2$

The solution set is $\{x|-16\leq x\leq 2\}$.

81. $-4|1-x|<-16$

$\frac{-4|1-x|}{-4}>\frac{-16}{-4}$

$|1-x|>4$

$1-x>4 \qquad 1-x<-4$

$-x>3 \quad \text{or} \quad -x<-5$

$x<-3 \qquad x>5$

The solution set is $\{x \mid x<-3 \text{ or } x>5\}$.

82. $-2|5-x|<-6$

$-2|5-x|<-6$

$\frac{-2|5-x|}{-2}>\frac{-6}{-2}$

$|5-x|>3$

$5-x>3 \qquad 5-x<-3$

$-x>-2 \quad \text{or} \quad -x<-8$

$x<2 \qquad x>8$

The solution set is $\{x \mid x<2 \text{ or } x>8\}$.

83. $3\leq |2x-1|$

$2x-1\geq 3 \qquad 2x-1\leq -3$

$2x\geq 4 \quad \text{or} \quad 2x\leq -2$

$x\geq 2 \qquad x\leq -1$

The solution set is $\{x \mid x\leq -1 \text{ or } x\geq 2\}$.

84. $9\leq |4x+7|$

$4x+7\geq 9 \quad \text{or} \quad 4x+7\leq -9$

$4x\geq 2 \qquad 4x\leq -16$

$x\geq \frac{2}{4} \qquad x\leq -4$

$x\geq \frac{1}{2}$

The solution set is $\left\{x \,\middle|\, x\leq -4 \text{ or } x\geq \frac{1}{2}\right\}$.

85. $5 > |4-x|$ is equivalent to $|4-x| < 5$.

$-5 < 4-x < 5$

$-9 < -x < 1$

$\frac{-9}{-1} > \frac{-x}{-1} > \frac{1}{-1}$

$9 > x > -1$

$-1 < x < 9$

The solution set is $\{x|-1 < x < 9\}$.

86. $2 > |11-x|$ is equivalent to $|11-x| < 2$.

$-2 < 11-x < 2$

$-13 < -x < -9$

$\frac{-13}{-1} > \frac{-x}{-1} > \frac{-9}{-1}$

$13 > x > 9$

$9 < x < 13$

The solution set is $\{x|\ 9 < x < 13\}$.

87. $1 < |2-3x|$ is equivalent to $|2-3x| > 1$.

$2-3x > 1$ or $2-3x < -1$

$-3x > -1$ $\quad$ $-3x < -3$

$\frac{-3x}{-3} < \frac{-1}{-3}$ $\quad$ $\frac{-3x}{-3} > \frac{-3}{-3}$

$x < \frac{1}{3}$ $\quad$ $x > 1$

The solution set is $\left\{x \middle| x < \frac{1}{3} \text{ or } x > 1\right\}$.

88. $4 < |2-x|$ is equivalent to $|2-x| > 4$.

$2-x > 4$ or $2-x < -4$

$-x > 2$ $\quad$ $-x < -6$

$\frac{-x}{-1} < \frac{2}{-1}$ $\quad$ $\frac{-x}{-1} > \frac{-6}{-1}$

$x < -2$ $\quad$ $x > 6$

The solution set is $\{x|\ x < -2 \text{ or } x > 6\}$.

89. $12 < \left|-2x+\frac{6}{7}\right| + \frac{3}{7}$

$\frac{81}{7} < \left|-2x+\frac{6}{7}\right|$

$-2x+\frac{6}{7} > \frac{81}{7}$ or $-2x+\frac{6}{7} < -\frac{81}{7}$

$-2x > \frac{75}{7}$ $\quad$ $-2x < -\frac{87}{7}$

$x < -\frac{75}{14}$ $\quad$ $x > \frac{87}{14}$

The solution set is $\left\{x \middle| x < -\frac{75}{14} \text{ or } x > \frac{87}{14}\right\}$, that is, $\left(-\infty, -\frac{75}{14}\right)$ or $\left(\frac{87}{14}, \infty\right)$.

90. $1 < \left|x-\frac{11}{3}\right| + \frac{7}{3}$

$-\frac{4}{3} < \left|x-\frac{11}{3}\right|$

Since $\left|x-\frac{11}{3}\right| > -\frac{4}{3}$ is true for all x,

the solution set is $\{x|\ x \text{ is any real number}\}$ or $(-\infty, \infty)$.

91. $4 + \left|3-\frac{x}{3}\right| \ge 9$

$\left|3-\frac{x}{3}\right| \ge 5$

$3-\frac{x}{3} \ge 5$ or $3-\frac{x}{3} \le -5$

$-\frac{x}{3} \ge 2$ $\quad$ $-\frac{x}{3} \le -8$

$x \le -6$ $\quad$ $x \ge 24$

The solution set is $\{x|\ x \le -6 \text{ or } x \ge 24\}$, that is, $(-\infty, -6]$ or $[24, \infty)$.

92. $\left|2-\frac{x}{2}\right|-1\le 1$

$\left|2-\frac{x}{2}\right|\le 2$

$-2\le 2-\frac{x}{2}\le 2$

$-4\le -\frac{x}{2}\le 0$

$8\ge x\ge 0$

The solution set is $\{x\,|\,0\le x\le 8\}$ or $[0,8]$.

93. $y\ge 4$

$1-(x+3)+2x\ge 4$
$1-x-3+2x\ge 4$
$x-2\ge 4$
$x\ge 6$

The solution set is $[6,\infty)$.

94. $y\le 0$

$2x-11+3(x+2)\le 0$
$2x-11+3x+6\le 0$
$5x-5\le 0$
$5x\le 5$
$x\le 1$

The solution set is $(-\infty,1]$.

95. $y\le 4$

$7-\left|\frac{x}{2}+2\right|\le 4$

$-\left|\frac{x}{2}+2\right|\le -3$

$\left|\frac{x}{2}+2\right|\ge 3$

$\frac{x}{2}+2\ge 3$ or $\frac{x}{2}+2\le -3$
$x+4\ge 6$ $\quad$ $x+4\le -6$
$x\ge 2$ $\quad$ $x\le -10$

The solution set is $(-\infty,-10]\cup[2,\infty)$.

96. $y\ge 6$

$8-|5x+3|\ge 6$
$-|5x+3|\ge -2$
$-(-|5x+3|)\le -(-2)$
$|5x+3|\le 2$
$-2\le 5x+3\le 2$
$-5\le 5x\le -1$
$\frac{-5}{5}\le\frac{5x}{5}\le\frac{-1}{5}$
$-1\le x\le -\frac{1}{5}$

The solution set is $\left[-1,-\frac{1}{5}\right]$.

97. Let x be the number.
$|4-3x|\ge 5$ or $|3x-4|\ge 5$

$3x-4\ge 5$ or $3x-4\le -5$
$3x\ge 9$ $\quad$ $3x\le -1$
$x\ge 3$ $\quad$ $x\le -\frac{1}{3}$

The solution set is $\left\{x\,|\,x\le -\frac{1}{3}\text{ or }x\ge 3\right\}$ or

$\left(-\infty,-\frac{1}{3}\right]\cup[3,\infty)$.

98. Let x be the number.
$|5-4x|\le 13$ or $|4x-5|\le 13$

$-13\le 4x-5\le 13$
$-8\le 4x\le 18$
$-2\le x\le\frac{9}{2}$

The solution set is $\left\{x\,|-2\le x\le\frac{9}{2}\right\}$ or $\left[-2,-\frac{2}{9}\right]$.

99. $(0,4)$

100. $[0,5]$

101. passion ≤ intimacy or intimacy ≥ passion

102. commitment ≥ intimacy or
intimacy ≤ commitment

103. passion < commitment or
commitment > passion

104. commitment > passion or
passion < commitment

105. 9, after 3 years

106. after approximately $5\frac{1}{2}$ years

107. **a.** $I = \frac{1}{4}x + 26$

$\frac{1}{4}x + 26 > 33$

$\frac{1}{4}x > 7$

$x > 28$

More than 33% of U.S. households will have an interfaith marriage in years after 2016 (i.e. $1988 + 28$).

b. $N = \frac{1}{4}x + 6$

$\frac{1}{4}x + 6 > 14$

$\frac{1}{4}x > 8$

$x > 32$

More than 14% of U.S. households will have a person of faith married to someone with no religion in years after 2020 (i.e. $1988 + 32$).

c. More than 33% of U.S. households will have an interfaith marriage *and* more than 14% of U.S. households will have a person of faith married to someone with no religion in years after 2020.

d. More than 33% of U.S. households will have an interfaith marriage *or* more than 14% of U.S. households will have a person of faith married to someone with no religion in years after 2016.

108. **a.** $I = \frac{1}{4}x + 26$

$\frac{1}{4}x + 26 > 34$

$\frac{1}{4}x > 8$

$x > 32$

More than 34% of U.S. households will have an interfaith marriage in years after 2020 (i.e. $1988 + 32$).

b. $N = \frac{1}{4}x + 6$

$\frac{1}{4}x + 6 > 15$

$\frac{1}{4}x > 9$

$x > 36$

More than 15% of U.S. households will have a person of faith married to someone with no religion in years after 2024 (i.e. $1988 + 36$).

c. More than 34% of U.S. households will have an interfaith marriage *and* more than 15% of U.S. households will have a person of faith married to someone with no religion in years after 2024.

d. More than 34% of U.S. households will have an interfaith marriage *or* more than 15% of U.S. households will have a person of faith married to someone with no religion in years after 2020.

109. $15 \le \frac{5}{9}(F - 32) \le 35$

$\frac{9}{5}(15) \le \frac{9}{5}\left(\frac{5}{9}(F - 32)\right) \le \frac{9}{5}(35)$

$9(3) \le F - 32 \le 9(7)$

$27 \le F - 32 \le 63$

$59 \le F \le 95$

The range for Fahrenheit temperatures is $59°\text{F}$ to $95°\text{F}$, inclusive or $[59°\text{F}, 95°\text{F}]$.

110. $41 \le \frac{9}{5}C + 32 \le 50$

$41 - 32 \le \frac{9}{5}C + 32 - 32 \le 50 - 32$

$9 \le \frac{9}{5}C \le 18$

$\frac{5}{9}(9) \le \frac{5}{9}\left(\frac{9}{5}C\right) \le \frac{5}{9}(18)$

$5 \le C \le 10$

The range for Celsius temperatures is $5°\text{C}$ to $10°\text{C}$, inclusive or $[5°\text{C}, 10°\text{C}]$.

111. $\left|\frac{h - 50}{5}\right| \ge 1.645$

$\frac{h - 50}{5} \ge 1.645$ or $\frac{h - 50}{5} \le -1.645$

$h - 50 \ge 8.225$ $\quad$ $h - 50 \le -8.225$

$h \ge 58.225$ $\quad$ $h \le 41.775$

The number of outcomes would be 59 or more, or 41 or less.

112. $50 + 0.20x < 20 + 0.50x$

$30 < 0.3x$

$100 < x$

Basic Rental is a better deal when driving more than 100 miles per day.

113. $15 + 0.08x < 3 + .12x$

$12 < 0.04x$

$300 < x$

Plan A is a better deal when texting more than 300 times per month.

114. $1800 + 0.03x < 200 + 0.08x$
$1600 < 0.05x$
$32000 < x$
A home assessment of greater than $32,000 would make the first bill a better deal.

115. $2+0.08x<8+0.05x$
$0.03x<6$
$x<200$
The credit union is a better deal when writing less than 200 checks.

116. $2x>10,000+0.40x$
$1.6x>10,000$
$\frac{1.6x}{1.6}>\frac{10,000}{1.6}$
$x>6250$
More than 6250 tapes need to be sold a week to make a profit.

117. $3000+3x<5.5x$
$3000<2.5x$
$1200<x$
More then 1200 packets of stationary need to be sold each week to make a profit.

118. $265+65x\le 2800$
$65x\le 2535$
$x\le 39$
39 bags or fewer can be lifted safely.

119. $245+95x\le 3000$
$95x\le 2755$
$x\le 29$
29 bags or less can be lifted safely.

120. Let $x=$ the grade on the final exam.
$\frac{86+88+92+84+x+x}{6}\ge 90$
$86+88+92+84+x+x\ge 540$
$2x+350\ge 540$
$2x\ge 190$
$x\ge 95$
You must receive at least a 95% to earn an A.

121. a. $\frac{86+88+x}{3}\ge 90$
$\frac{174+x}{3}\ge 90$
$174+x\ge 270$
$x\ge 96$
You must get at least a 96.

b. $\frac{86+88+x}{3}<80$
$\frac{174+x}{3}<80$
$174+x<240$
$x<66$
This will happen if you get a grade less than 66.

122. Let x = the number of hours the mechanic works on the car.
$226\le 175+34x\le 294$
$51\le 34x\le 119$
$1.5\le x\le 3.5$
The man will be working on the job at least 1.5 and at most 3.5 hours.

123. Let x = the number of times the bridge is crossed per three month period
The cost with the 3-month pass is $C_3=7.50+0.50x$.
The cost with the 6-month pass is $C_6=30$.
Because we need to buy two 3-month passes per 6-month pass, we multiply the cost with the 3-month pass by 2.
$2(7.50+0.50x)<30$
$15+x<30$
$x<15$
We also must consider the cost without purchasing a pass. We need this cost to be less than the cost with a 3-month pass.
$3x>7.50+0.50x$
$2.50x>7.50$
$x>3$
The 3-month pass is the best deal when making more than 3 but less than 15 crossings per 3-month period.

124. – 131. Answers will vary.

132. makes sense

133. makes sense

134. makes sense

135. makes sense

136. true

137. false; Changes to make the statement true will vary.
A sample change is: $(-\infty,3)\cup(-\infty,-2)=(-\infty,3)$

138. false; Changes to make the statement true will vary.
A sample change is: $3x>6$ is equivalent to $x>2$.

139. true

140. Because $x > y$, $y - x$ represents a negative number. When both sides are multiplied by $(y - x)$ the inequality must be reversed.

141. a. $|x-4|<3$

b. $|x-4|\geq 3$

142. Answers will vary.

143. $y = 4 - x$

x	$y = 4 - x$
–3	$4-(-3)=7$
–2	$4-(-2)=6$
–1	$4-(-1)=5$
0	$4-(0)=4$
1	$4-(1)=3$
2	$4-(2)=2$
3	$4-(3)=1$

144. $y = 4 - x^2$

x	$y = 4 - x^2$
–3	$4-(-3)^2=-5$
–2	$4-(-2)^2=0$
–1	$4-(-1)^2=3$
0	$4-(0)^2=4$
1	$4-(1)^2=3$
2	$4-(2)^2=0$
3	$4-(3)^2=-5$

145. $y = |x+1|$

x	$y = \|x+1\|$
–4	$\|-4+1\|=3$
–3	$\|-3+1\|=2$
–2	$\|-2+1\|=1$
–1	$\|-1+1\|=0$
0	$\|0+1\|=1$
1	$\|1+1\|=2$
2	$\|2+1\|=3$

Chapter P Review Exercises

1.
$$\begin{aligned}3+6(x-2)^3 &= 3+6(4-2)^3\\ &= 3+6(2)^3\\ &= 3+6(8)\\ &= 3+48\\ &= 51\end{aligned}$$

2.
$$\begin{aligned}x^2-5(x-y) &= 6^2-5(6-2)\\ &= 36-5(4)\\ &= 36-20\\ &= 16\end{aligned}$$

3. $S = 0.015x^2 + x + 10$
$$\begin{aligned}S &= 0.015(60)^2+(60)+10\\ &= 0.015(3600)+60+10\\ &= 54+60+10\\ &= 124\end{aligned}$$

4. $A=\{a,b,c\}\quad B=\{a,c,d,e\}$

$\{a,b,c\}\cap\{a,c,d,e\}=\{a,c\}$

5. $A=\{a,b,c\}\quad B=\{a,c,d,e\}$

$\{a,b,c\}\cup\{a,c,d,e\}=\{a,b,c,d,e\}$

6. $A=\{a,b,c\}\quad C=\{a,d,f,g\}$

$\{a,b,c\}\cup\{a,d,f,g\}=\{a,b,c,d,f,g\}$

7. $A=\{a,b,c\}\quad C=\{a,d,f,g\}$

$\{a,d,f,g\}\cap\{a,b,c\}=\{a\}$

8. a. $\sqrt{81}$

b. $0, \sqrt{81}$

c. $-17, 0, \sqrt{81}$

d. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{81}$

e. $\sqrt{2}, \pi$

f. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}$

9. $|-103| = 103$

10. $|\sqrt{2}-1| = \sqrt{2}-1$

11. $\left|3-\sqrt{17}\right| = \sqrt{17}-3$ since $\sqrt{17}$ is greater than 3.

12. $|4-(-17)| = |4+17| = |21| = 21$

13. 3 + 17 = 17 + 3;
commutative property of addition.

14. $(6\cdot 3)\cdot 9 = 6\cdot(3\cdot 9)$;
associative property of multiplication.

15. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = \sqrt{15}+3$;
distributive property of multiplication over addition.

16. $(6\cdot 9)\cdot 2 = 2\cdot(6\cdot 9)$;
commutative property of multiplication.

17. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = (\sqrt{5}+\sqrt{3})\sqrt{3}$;
commutative property of multiplication.

18. $(3\cdot 7)+(4\cdot 7) = (4\cdot 7)+(3\cdot 7)$;
commutative property of addition.

19. $5(2x-3)+7x = 10x-15+7x = 17x-15$

20. $\frac{1}{5}(5x)+\left[(3y)+(-3y)\right]-(-x) = x+\left[0\right]+x = 2x$

21. $3(4y-5)-(7y+2) = 12y-15-7y-2 = 5y-17$

22.
$$\begin{aligned}8-2[3-(5x-1)] &= 8-2[3-5x+1]\\ &= 8-2[4-5x]\\ &= 8-8+10x\\ &= 10x\end{aligned}$$

23. $D = 0.005x^2+0.55x+34$
$D = 0.005(30)^2+0.55(30)+34$
$= 55$
The U.S. diversity index was 55% in 2010.
This is the same as the value displayed in the bar graph.

24. $(-3)^3(-2)^2 = (-27)\cdot(4) = -108$

25.
$$\begin{aligned}2^{-4}+4^{-1} &= \frac{1}{2^4}+\frac{1}{4}\\ &= \frac{1}{16}+\frac{1}{4}\\ &= \frac{1}{16}+\frac{4}{16}\\ &= \frac{5}{16}\end{aligned}$$

26. $5^{-3}\cdot 5 = 5^{-3}5^1 = 5^{-3+1} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

27. $\frac{3^3}{3^6} = 3^{3-6} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

28.
$$\begin{aligned}(-2x^4y^3)^3 &= (-2)^3(x^4)^3(y^3)^3\\ &= (-2)^3x^{4\cdot 3}y^{3\cdot 3}\\ &= -8x^{12}y^9\end{aligned}$$

29.
$$\begin{aligned}&(-5x^3y^2)(-2x^{-11}y^{-2})\\ &= (-5)(-2)x^3x^{-11}y^2y^{-2}\\ &= 10\cdot x^{3-11}y^{2-2}\\ &= 10x^{-8}y^0\\ &= \frac{10}{x^8}\end{aligned}$$

30.
$$\begin{aligned}(2x^3)^{-4} &= (2)^{-4}(x^3)^{-4}\\ &= 2^{-4}x^{-12}\\ &= \frac{1}{2^4x^{12}}\\ &= \frac{1}{16x^{12}}\end{aligned}$$

31.
$$\begin{aligned}\frac{7x^5y^6}{28x^{15}y^{-2}} &= \left(\frac{7}{28}\right)(x^{5-15})(y^{6-(-2)})\\ &= \frac{1}{4}x^{-10}y^8\\ &= \frac{y^8}{4x^{10}}\end{aligned}$$

32. $3.74\times 10^4 = 37,400$

33. $7.45\times 10^{-5} = 0.0000745$

34. $3,590,000 = 3.59\times 10^6$

35. $0.00725 = 7.25\times 10^{-3}$

36.
$$\begin{aligned}(3\times 10^3)(1.3\times 10^2) &= (3\times 1.3)\times(10^3\times 10^2)\\ &= 3.9\times 10^5\\ &= 390,000\end{aligned}$$

37.
$$\begin{aligned}\frac{6.9\times 10^3}{3\times 10^5} &= \left(\frac{6.9}{3}\right)\times 10^{3-5}\\ &= 2.3\times 10^{-2}\\ &= 0.023\end{aligned}$$

38. 1.35×10^{12}

39. $32,000,000 = 3.2\times10^7$

40. $\dfrac{1.35\times10^{12}}{3.2\times10^7} = \dfrac{1.35}{3.2}\cdot\dfrac{10^{12}}{10^7} \approx 0.42188\times10^5 = 42,188$

1.35×10^{12} seconds is approximately 42,188 years.

41. $\sqrt{300} = \sqrt{100\cdot3} = \sqrt{100}\cdot\sqrt{3} = 10\sqrt{3}$

42. $\sqrt{12x^2} = \sqrt{4x^2\cdot3} = \sqrt{4x^2}\cdot\sqrt{3} = 2|x|\sqrt{3}$

43. $\sqrt{10x}\cdot\sqrt{2x} = \sqrt{20x^2}$
$= \sqrt{4x^2}\cdot\sqrt{5}$
$= 2x\sqrt{5}$

44. $\sqrt{r^3} = \sqrt{r^2}\cdot\sqrt{r} = r\sqrt{r}$

45. $\sqrt{\dfrac{121}{4}} = \dfrac{\sqrt{121}}{\sqrt{4}} = \dfrac{11}{2}$

46. $\dfrac{\sqrt{96x^3}}{\sqrt{2x}} = \sqrt{\dfrac{96x^3}{2x}}$
$= \sqrt{48x^2}$
$= \sqrt{16x^2}\cdot\sqrt{3}$
$= 4x\sqrt{3}$

47. $7\sqrt{5} + 13\sqrt{5} = (7+13)\sqrt{5} = 20\sqrt{5}$

48. $2\sqrt{50} + 3\sqrt{8} = 2\sqrt{25\cdot2} + 3\sqrt{4\cdot2}$
$= 2\cdot5\sqrt{2} + 3\cdot2\sqrt{2}$
$= 10\sqrt{2} + 6\sqrt{2}$
$= 16\sqrt{2}$

49. $4\sqrt{72} - 2\sqrt{48} = 4\sqrt{36\cdot2} - 2\sqrt{16\cdot3}$
$= 4\cdot6\sqrt{2} - 2\cdot4\sqrt{3}$
$= 24\sqrt{2} - 8\sqrt{3}$

50. $\dfrac{30}{\sqrt{5}} = \dfrac{30}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{30\sqrt{5}}{5} = 6\sqrt{5}$

51. $\dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{2}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{6}}{3}$

52. $\dfrac{5}{6+\sqrt{3}} = \dfrac{5}{6+\sqrt{3}}\cdot\dfrac{6-\sqrt{3}}{6-\sqrt{3}}$
$= \dfrac{5(6-\sqrt{3})}{36-3}$
$= \dfrac{5(6-\sqrt{3})}{33}$

53. $\dfrac{14}{\sqrt{7}-\sqrt{5}} = \dfrac{14}{\sqrt{7}-\sqrt{5}}\cdot\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$
$= \dfrac{14(\sqrt{7}+\sqrt{5})}{7-5}$
$= \dfrac{14(\sqrt{7}+\sqrt{5})}{2}$
$= 7(\sqrt{7}+\sqrt{5})$

54. $\sqrt[3]{125} = 5$

55. $\sqrt[5]{-32} = -2$

56. $\sqrt[4]{-125}$ is not a real number.

57. $\sqrt[4]{(-5)^4} = \sqrt[4]{625} = \sqrt[4]{5^4} = 5$

58. $\sqrt[3]{81} = \sqrt[3]{27\cdot3} = \sqrt[3]{27}\cdot\sqrt[3]{3} = 3\sqrt[3]{3}$

59. $\sqrt[3]{y^5} = \sqrt[3]{y^3y^2} = y\sqrt[3]{y^2}$

60. $\sqrt[4]{8}\cdot\sqrt[4]{10} = \sqrt[4]{80} = \sqrt[4]{16\cdot5} = \sqrt[4]{16}\cdot\sqrt[4]{5} = 2\sqrt[4]{5}$

61. $4\sqrt[3]{16} + 5\sqrt[3]{2} = 4\sqrt[3]{8\cdot2} + 5\sqrt[3]{2}$
$= 4\cdot2\sqrt[3]{2} + 5\sqrt[3]{2}$
$= 8\sqrt[3]{2} + 5\sqrt[3]{2}$
$= 13\sqrt[3]{2}$

62. $\dfrac{\sqrt[4]{32x^5}}{\sqrt[4]{16x}} = \sqrt[4]{\dfrac{32x^5}{16x}} = \sqrt[4]{2x^4} = x\sqrt[4]{2}$

63. $16^{1/2} = \sqrt{16} = 4$

64. $25^{-1/2} = \dfrac{1}{25^{1/2}} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

65. $125^{1/3} = \sqrt[3]{125} = 5$

66. $27^{-1/3} = \dfrac{1}{27^{1/3}} = \dfrac{1}{\sqrt[3]{27}} = \dfrac{1}{3}$

67. $64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$

68. $27^{-4/3} = \dfrac{1}{27^{4/3}} = \dfrac{1}{(\sqrt[3]{27})^4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

69. $(5x^{2/3})(4x^{1/4}) = 5\cdot4x^{2/3+1/4} = 20x^{11/12}$

70. $\frac{15x^{3/4}}{5x^{1/2}} = \left(\frac{15}{5}\right)x^{3/4-1/2} = 3x^{1/4}$

71. $$\begin{aligned}(125\cdot x^6)^{2/3} &= (\sqrt[3]{125x^6})^2\\ &= (5x^2)^2\\ &= 25x^4\end{aligned}$$

72. $\sqrt[6]{y^3} = (y^3)^{1/6} = y^{3\cdot 1/6} = y^{1/2} = \sqrt{y}$

73. $$\begin{aligned}(-6x^3+7x^2-9x+3)+(14x^3+3x^2-11x-7) &= (-6x^3+14x^3)+(7x^2+3x^2)+(-9x-11x)+(3-7)\\ &= 8x^3+10x^2-20x-4\end{aligned}$$

The degree is 3.

74. $$\begin{aligned}(13x^4-8x^3+2x^2)-(5x^4-3x^3+2x^2-6) &= (13x^4-8x^3+2x^2)+(-5x^4+3x^3-2x^2+6)\\ &= (13x^4-5x^4)+(-8x^3+3x^3)+(2x^2-2x^2)+6\\ &= 8x^4-5x^3+6\end{aligned}$$

The degree is 4.

75. $$\begin{aligned}(3x-2)(4x^2+3x-5) &= (3x)(4x^2)+(3x)(3x)+(3x)(-5)+(-2)(4x^2)+(-2)(3x)+(-2)(-5)\\ &= 12x^3+9x^2-15x-8x^2-6x+10\\ &= 12x^3+x^2-21x+10\end{aligned}$$

76. $$\begin{aligned}(3x-5)(2x+1) &= (3x)(2x)+(3x)(1)+(-5)(2x)+(-5)(1)\\ &= 6x^2+3x-10x-5\\ &= 6x^2-7x-5\end{aligned}$$

77. $(4x+5)(4x-5) = (4x^2)-5^2 = 16x^2-25$

78. $(2x+5)^2 = (2x)^2+2(2x)\cdot 5+5^2 = 4x^2+20x+25$

79. $(3x-4)^2 = (3x)^2-2(3x)\cdot 4+(-4)^2 = 9x^2-24x+16$

80. $(2x+1)^3 = (2x)^3+3(2x)^2(1)+3(2x)(1)^2+1^3 = 8x^3+12x^2+6x+1$

81. $(5x-2)^3 = (5x)^3-3(5x)^2(2)+3(5x)(2)^2-2^3 = 125x^3-150x^2+60x-8$

82. $$\begin{aligned}(7x^2-8xy+y^2)+(-8x^2-9xy-4y^2) &= (7x^2-8x^2)+(-8xy-9xy)+(y^2-4y^2)\\ &= -x^2-17xy-3y^2\end{aligned}$$

The degree is 2.

83. $$\begin{aligned}&(13x^3y^2-5x^2y-9x^2)-(-11x^3y^2-6x^2y+3x^2-4)\\ &= (13x^3y^2-5x^2y-9x^2)+(11x^3y^2+6x^2y-3x^2+4)\\ &= (13x^3y^2+11x^3y^2)+(-5x^2y+6x^2y)+(-9x^2-3x^2)+4\\ &= 24x^3y^2+x^2y-12x^2+4\end{aligned}$$

The degree is 5.

84. $(x+7y)(3x-5y) = x(3x)+(x)(-5y)+(7y)(3x)+(7y)(-5y)$
$= 3x^2 - 5xy + 21xy - 35y^2$
$= 3x^2 + 16xy - 35y^2$

85. $(3x-5y)^2 = (3x)^2 - 2(3x)(5y) + (-5y)^2$
$= 9x^2 - 30xy + 25y^2$

86. $(3x^2+2y)^2 = (3x^2)^2 + 2(3x^2)(2y) + (2y)^2$
$= 9x^4 + 12x^2y + 4y^2$

87. $(7x+4y)(7x-4y) = (7x)^2 - (4y)^2$
$= 49x^2 - 16y^2$

88. $(a-b)(a^2+ab+b^2)$
$= a(a^2)+a(ab)+a(b^2)+(-b)(a^2)$
$+(-b)(ab)+(-b)(b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$

89. $15x^3 + 3x^2 = 3x^2 \cdot 5x + 3x^2 \cdot 1$
$= 3x^2(5x+1)$

90. $x^2 - 11x + 28 = (x-4)(x-7)$

91. $15x^2 - x - 2 = (3x+1)(5x-2)$

92. $64 - x^2 = 8^2 - x^2 = (8-x)(8+x)$

93. $x^2 + 16$ is prime.

94. $3x^4 - 9x^3 - 30x^2 = 3x^2(x^2 - 3x - 10)$
$= 3x^2(x-5)(x+2)$

95. $20x^7 - 36x^3 = 4x^3(5x^4 - 9)$

96. $x^3 - 3x^2 - 9x + 27 = x^2(x-3) - 9(x-3)$
$= (x^2-9)(x-3)$
$= (x+3)(x-3)(x-3)$
$= (x+3)(x-3)^2$

97. $16x^2 - 40x + 25 = (4x-5)(4x-5)$
$= (4x-5)^2$

98. $x^4 - 16 = (x^2)^2 - 4^2$
$= (x^2+4)(x^2-4)$
$= (x^2+4)(x+2)(x-2)$

99. $y^3 - 8 = y^3 - 2^3 = (y-2)(y^2+2y+4)$

100. $x^3 + 64 = x^3 + 4^3 = (x+4)(x^2 - 4x + 16)$

101. $3x^4 - 12x^2 = 3x^2(x^2-4)$
$= 3x^2(x-2)(x+2)$

102. $27x^3 - 125 = (3x)^3 - 5^3$
$= (3x-5)[(3x)^2 + (3x)(5) + 5^2]$
$= (3x-5)(9x^2 + 15x + 25)$

103. $x^5 - x = x(x^4 - 1)$
$= x(x^2-1)(x^2+1)$
$= x(x-1)(x+1)(x^2+1)$

104. $x^3 + 5x^2 - 2x - 10 = x^2(x+5) - 2(x+5)$
$= (x^2-2)(x+5)$

105. $x^2 + 18x + 81 - y^2 = \left(x^2 + 18x + 81\right) - y^2$
$= \left(x+9\right)^2 - y^2$
$= \left(x+9-y\right)\left(x+9+y\right)$

106. $16x^{-3/4} + 32x^{1/4} = 16x^{-3/4}\left(1 + 2x^{1/4-\left(-3/4\right)}\right)$
$= 16x^{-3/4}\left(1+2x\right)$
$= \dfrac{16\left(1+2x\right)}{x^{3/4}}$

107. $\left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}} - \left(x^2-4\right)^2\left(x^2+3\right)^{\frac{3}{2}}$
$= \left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x^2-4\right)\left(x^2+3\right)\right]$
$= \left(x-2\right)\left(x+2\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x-2\right)\left(x+2\right)\left(x^2+3\right)\right]$
$= (x-2)(x+2)(x^2+3)^{\frac{1}{2}}(-x^4 + x^2 + 13)$

108. $12x^{-\frac{1}{2}} + 6x^{-\frac{3}{2}} = 6x^{-\frac{3}{2}}\left(2x+1\right) = \dfrac{6(2x+1)}{x^{\frac{3}{2}}}$

109. $\dfrac{x^3 + 2x^2}{x+2} = \dfrac{x^2(x+2)}{x+2} = x^2$, $x \neq -2$

110. $\dfrac{x^2 + 3x - 18}{x^2 - 36} = \dfrac{(x+6)(x-3)}{(x+6)(x-6)} = \dfrac{x-3}{x-6}$,
$x \neq -6, 6$

111. $\dfrac{x^2 + 2x}{x^2 + 4x + 4} = \dfrac{x(x+2)}{(x+2)^2} = \dfrac{x}{x+2}$,
$x \neq -2$

112. $\dfrac{x^2 + 6x + 9}{x^2 - 4} \cdot \dfrac{x+3}{x-2} = \dfrac{(x+3)^2}{(x-2)(x+2)} \cdot \dfrac{x+3}{x-2}$
$= \dfrac{(x+3)^3}{(x-2)^2(x+2)}$,
$x \neq 2, -2$

113. $\dfrac{6x+2}{x^2-1} \div \dfrac{3x^2+x}{x-1}$
$= \dfrac{2(3x+1)}{(x-1)(x+1)} \div \dfrac{x(3x+1)}{x-1}$
$= \dfrac{2(3x+1)}{(x-1)(x+1)} \cdot \dfrac{x-1}{x(3x+1)}$
$= \dfrac{2}{x(x+1)}$,
$x \neq 0, 1, -1, -\dfrac{1}{3}$

114. $\dfrac{x^2 - 5x - 24}{x^2 - x - 12} \div \dfrac{x^2 - 10x + 16}{x^2 + x - 6}$
$= \dfrac{(x-8)(x+3)}{(x-4)(x+3)} \div \dfrac{(x-2)(x-8)}{(x+3)(x-2)}$
$= \dfrac{x-8}{x-4} \cdot \dfrac{x+3}{x-8}$
$= \dfrac{x+3}{x-4}$,
$x \neq -3, 4, 2, 8$

115. $\dfrac{2x-7}{x^2-9} - \dfrac{x-10}{x^2-9} = \dfrac{2x-7-(x-10)}{x^2-9}$
$= \dfrac{x+3}{(x+3)(x-3)}$
$= \dfrac{1}{x-3}$,
$x \neq 3, -3$

116. $\dfrac{3x}{x+2}+\dfrac{x}{x-2}=\dfrac{3x}{x+2}\cdot\dfrac{x-2}{x-2}+\dfrac{x}{x-2}\cdot\dfrac{x+2}{x+2}$
$=\dfrac{3x^2-6x+x^2+2x}{(x+2)(x-2)}$
$=\dfrac{4x^2-4x}{(x+2)(x-2)}$
$=\dfrac{4x(x-1)}{(x+2)(x-2)},$

$x\neq 2, -2$

117. $\dfrac{x}{x^2-9}+\dfrac{x-1}{x^2-5x+6}$
$=\dfrac{x}{(x-3)(x+3)}+\dfrac{x-1}{(x-2)(x-3)}$
$=\dfrac{x}{(x-3)(x+3)}\cdot\dfrac{x-2}{x-2}+\dfrac{x-1}{(x-2)(x-3)}\cdot\dfrac{x+3}{x+3}$
$=\dfrac{x(x-2)+(x-1)(x+3)}{(x-3)(x+3)(x-2)}$
$=\dfrac{x^2-2x+x^2+2x-3}{(x-3)(x+3)(x-2)}$
$=\dfrac{2x^2-3}{(x-3)(x+3)(x-2)}$

$x\neq 3, -3, 2$

118. $\dfrac{4x-1}{2x^2+5x-3}-\dfrac{x+3}{6x^2+x-2}$
$=\dfrac{4x-1}{(2x-1)(x+3)}-\dfrac{x+3}{(2x-1)(3x+2)}$
$=\dfrac{4x-1}{(2x-1)(x+3)}\cdot\dfrac{3x+2}{3x+2}$
$-\dfrac{x+3}{(2x-1)(3x+2)}\cdot\dfrac{x+3}{x+3}$
$=\dfrac{12x^2+8x-3x-2-x^2-6x-9}{(2x-1)(x+3)(3x+2)}$
$=\dfrac{11x^2-x-11}{(2x-1)(x+3)(3x+2)},$

$x\neq\dfrac{1}{2}, -3, -\dfrac{2}{3}$

119. $\dfrac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}=\dfrac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}\cdot\dfrac{6x}{6x}$
$=\dfrac{6-3x}{2x-x^2}$
$=\dfrac{-3(x-2)}{-x(x-2)}$
$=\dfrac{3}{x},$

$x\neq 0, 2$

120. $\dfrac{3+\frac{12}{x}}{1-\frac{16}{x^2}}=\dfrac{3+\frac{12}{x}}{1-\frac{16}{x^2}}\cdot\dfrac{x^2}{x^2}$
$=\dfrac{3x^2+12x}{x^2-16}$
$=\dfrac{3x(x+4)}{(x+4)(x-4)}$
$=\dfrac{3x}{x-4},$

$x\neq 0, 4, -4$

121. $\dfrac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}}=\dfrac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}}\cdot\dfrac{x+3}{x+3}$
$=\dfrac{3(x+3)-1}{3(x+3)+1}$
$=\dfrac{3x+9-1}{3x+9+1}$
$=\dfrac{3x+8}{3x+10},$

$x\neq -3, -\dfrac{10}{3}$

122 .
$$\frac{\sqrt{25-x^2}+\frac{x^2}{\sqrt{25-x^2}}}{25-x^2}$$
$$=\frac{\left(\sqrt{25-x^2}+\frac{x^2}{\sqrt{25-x^2}}\right)\sqrt{25-2x^2}}{(25-x^2)\sqrt{25-x^2}}$$
$$=\frac{25-x^2+x^2}{(25-x^2)\sqrt{25-x^2}}$$
$$=\frac{25}{\sqrt{(25-x^2)^3}}$$
$$=\frac{25}{\sqrt{(25-x^2)^3}}\cdot\frac{\sqrt{25-x^2}}{\sqrt{25-x^2}}$$
$$=\frac{25\sqrt{25-x^2}}{(25-x^2)}$$
$$=\frac{25\sqrt{25-x^2}}{(5-x)^2(5+x)^2}$$

123. $1 - 2(6 - x) = 3x + 2$
$1 - 12 + 2x = 3x + 2$
$-11 - x = 2$
$-x = 13$
$x = -13$
The solution set is $\{-13\}$.
This is a conditional equation.

124. $2(x-4)+3(x+5)=2x-2$
$2x-8+3x+15=2x-2$
$5x+7=2x-2$
$3x=-9$
$x=-3$
The solution set is $\{-3\}$.
This is a conditional equation.

125. $2x - 4(5x + 1) = 3x + 17$
$2x - 20x - 4 = 3x + 17$
$-18x - 4 = 3x + 17$
$-21x = 21$
$x = -1$
The solution set is $\{-1\}$.
This is a conditional equation.

126. $x \neq 1, x \neq -1$
$$\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}$$
$$\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{(x+1)(x-1)}$$
$x+1-(x-1)=2$
$x+1-x+1=2$
$2=2$
The solution set is all real numbers except 1 and –1.

127. $x \neq -2, x \neq 4$
$$\frac{4}{x+2}+\frac{2}{x-4}=\frac{30}{(x+2)(x-4)}$$
$4(x-4)+2(x+2)=30$
$4x-16+2x+4=30$
$6x-12=30$
$6x=42$
$x=7$
The solution set is $\{7\}$.

128. $-4|2x+1|+12=0$
$-4|2x+1|=-12$
$|2x+1|=3$

$2x+1=3$ or $2x+1=-3$
$2x=2$ $\quad$ $2x=-4$
$x=1$ $\quad$ $x=-2$

The solution set is $\{-2,1\}$.

129. $2x^2-11x+5=0$
$(2x - 1)(x - 5) = 0$
$2x - 1 = 0 \quad x - 5 = 0$
$x=\frac{1}{2}$ or $x = 5$

The solution set is $\left\{\frac{1}{2},5\right\}$.

130. $(3x+5)(x-3)=5$
$3x^2+5x-9x-15=5$
$3x^2-4x-20=0$
$$x=\frac{4\pm\sqrt{(-4)^2-4(3)(-20)}}{2(3)}$$
$$x=\frac{4\pm\sqrt{16+240}}{6}$$
$$x=\frac{4\pm\sqrt{256}}{6}$$
$$x=\frac{4\pm16}{6}$$
$$x=\frac{20}{6},\frac{-12}{6}$$
$$x=\frac{10}{3},-2$$

The solution set is $\left\{-2,\frac{10}{3}\right\}$.

131. $3x^2 - 7x + 1 = 0$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(3)(1)}}{2(3)}$$
$$x = \frac{7 \pm \sqrt{49-12}}{6}$$
$$x = \frac{7 \pm \sqrt{37}}{6}$$

The solution set is $\left\{\frac{7+\sqrt{37}}{6}, \frac{7-\sqrt{37}}{6}\right\}$.

132. $x^2 - 9 = 0$

$$x^2 = 9$$
$$x = \pm 3$$

The solution set is $\{-3, 3\}$.

133. $(x-3)^2 - 24 = 0$

$$(x-3)^2 = 24$$
$$\sqrt{(x-3)^2} = \pm\sqrt{24}$$
$$x - 3 = \pm 2\sqrt{6}$$
$$x = 3 \pm 2\sqrt{6}$$

134.

$$\frac{2x}{x^2+6x+8} = \frac{x}{x+4} - \frac{2}{x+2}$$
$$\frac{2x}{(x+4)(x+2)} = \frac{x}{x+4} - \frac{2}{x+2}$$
$$\frac{2x(x+4)(x+2)}{(x+4)(x+2)} = (x+4)(x+2)\left(\frac{x}{x+4} - \frac{2}{x+2}\right)$$
$$2x = x(x+2) - 2(x+4)$$
$$2x = x^2 + 2x - 2x - 8$$
$$0 = x^2 - 2x - 8$$
$$0 = (x+2)(x-4)$$

$x + 2 = 0$ or $x - 4 = 0$

$x = -2$ $\quad x = 4$

–2 must be rejected. The solution set is $\{4\}$.

135. $\sqrt{8-2x} - x = 0$

$$\sqrt{8-2x} = x$$
$$\left(\sqrt{8-2x}\right)^2 = x^2$$
$$8 - 2x = x^2$$
$$0 = x^2 + 2x - 8$$
$$0 = (x+4)(x-2)$$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ $\quad x = 2$

–4 must be rejected. The solution set is $\{2\}$.

136. $\sqrt{2x-3} + x = 3$

$$\sqrt{2x-3} = 3 - x$$
$$2x - 3 = 9 - 6x + x^2$$
$$x^2 - 8x + 12 = 0$$
$$x^2 - 8x = -12$$
$$x^2 - 8x + 16 = -12 + 16$$
$$(x-4)^2 = 4$$
$$x - 4 = \pm 2$$
$$x = 4 + 2$$
$$x = 6, 2$$

The solution set is $\{2\}$.

137. $vt + gt^2 = s$

$$gt^2 = s - vt$$
$$\frac{gt^2}{t^2} = \frac{s - vt}{t^2}$$
$$g = \frac{s - vt}{t^2}$$

138. $T = \frac{A-P}{\text{Pr}}$

$$Pr(T) = Pr\frac{A-P}{\text{Pr}}$$
$$PrT = A - P$$
$$PrT + P = A$$
$$P(rT + 1) = A$$
$$P = \frac{A}{1 + rT}$$

139. $x^2 = 2x - 19$

$$x^2 - 2x + 19 = 0$$
$$b^2 - 4ac = (-2)^2 - 4(1)(19) = -72$$

$-72 < 0,$ thus the equation has no real solutions.

140. $9x^2 - 30x + 25 = 0$

$$b^2 - 4ac = (-30)^2 - 4(9)(25) = 0$$

$b^2 - 4ac = 0,$ thus the equation has one repeated real solution.

141. Let $x =$ the number involving oversleeping.
Let $x + 10 =$ the number involving computer problems.
Let $x + 80 =$ the number involving illness.

$$x + (x+10) + (x+80) = 270$$
$$x + x + 10 + x + 80 = 270$$
$$3x + 90 = 270$$
$$3x = 180$$
$$x = 60$$
$$x + 10 = 70$$
$$x + 80 = 140$$

The number involving oversleeping, computer problems, and illness, respectively, is 60, 70, and 140.

142. Let x = the number of years after 1980.

$$2.69+0.17x=9.49$$
$$0.17x=6.8$$
$$x=40$$

The average price of a movie ticket will be \$9.49 40 years after 1980, or 2020.

143. Let x = the number of GB used.

Plan A: $C=52+18x$

Plan B: $C=32+22x$

Set the costs equal to each other.

$$52+18x=32+22x$$
$$52=32+4x$$
$$20=4x$$
$$5=x$$

The cost will be the same for 5 GB.

144. Let x = the original price of the phone

$$48=x-0.20x$$
$$48=0.80x$$
$$60=x$$

The original price is \$60.

145. Let x = the amount sold to earn \$800 in one week

$$800=300+0.05x$$
$$500=0.05x$$
$$10{,}000=x$$

Sales must be \$10,000 in one week to earn \$800.

146. Let x = the amount invested at 4%

Let y = the amount invested at 7%

$$x+y=9000$$
$$0.04x+0.07y=555$$

Multiply the first equation by –0.04 and add.

$$-0.04x-0.04y=-360$$
$$0.04x+0.07y=555$$
$$0.03y=195$$
$$y=6500$$

Back-substitute 6500 for y in one of the original equations to find x.

$$x+y=9000$$
$$x+6500=9000$$
$$x=2500$$

There was \$2500 invested at 4% and \$6500 invested at 7%.

147. Let $x=$ the amount invested at 2%

Let $8000-x=$ the amount invested at 5%.

$$0.05(8000-x)=0.02x+85$$
$$400-0.05x=0.02x+85$$
$$-0.05x-0.02x=85-400$$
$$-0.07x=-315$$
$$\frac{-0.07x}{-0.07}=\frac{-315}{-0.07}$$
$$x=4500$$
$$8000-x=3500$$

\$4500 was invested at 2% and \$3500 was invested at 5%.

148. Let w = the width of the playing field,

Let $3w-6$ = the length of the playing field

$$P=2(\text{length})+2(\text{width})$$
$$340=2(3w-6)+2w$$
$$340=6w-12+2w$$
$$340=8w-12$$
$$352=8w$$
$$44=w$$

The dimensions are 44 yards by 126 yards.

149. Check some points to determine that $y_1=14{,}100+1500x$ and $y_2=41{,}700-800x$.

Since $y_1=y_2=32{,}100$ when $x=12$, the two colleges will have the same enrollment in the year $2015+12=2027$. That year the enrollments will be 32,100 students.

150.

$$A=lw$$
$$15=l(2l-7)$$
$$15=2l^2-7l$$
$$0=2l^2-7l-15$$
$$0=(2l+3)(l-5)$$
$$l=5$$
$$2l-7=3$$

The length is 5 yards, the width is 3 yards.

151. Let x = height of building

$2x$ = shadow height

$$x^2+(2x)^2=300^2$$
$$x^2+4x^2=90{,}000$$
$$5x^2=90{,}000$$
$$x^2=18{,}000$$
$$x\approx\pm134.164$$

Discard negative height.

The building is approximately 134 meters high.

152.

$$(10+2x)(16+2x)=280$$
$$160+52x+4x^2=280$$
$$4x^2+52x-120=0$$
$$x^2+13x-30=0$$
$$(x+15)(x-2)=0$$

$x+15=0$ or $x-2=0$

$x=-15$ $\quad$ $x=2$

–15 must be rejected. The width of the frame is 2 inches.

153.

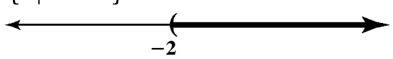
$$\frac{1500}{x}+100=\frac{1500}{x-4}$$
$$x(x-4)\left(\frac{1500}{x}+100\right)=x(x-4)\frac{1500}{x-4}$$
$$1500(x-4)+100x(x-4)=1500x$$
$$1500x-6000+100x^2-400x=1500x$$
$$15x-60+x^2-4x=15x$$
$$x^2-4x-60=0$$
$$(x+6)(x-10)=0$$

$x+6=0$ or $x-10=0$
$x=-6$ $\quad$ $x=10$

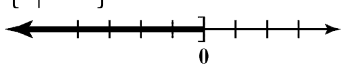
–6 must be rejected. There were originally 10 people.

154. $\{x|-3\le x<5\}$

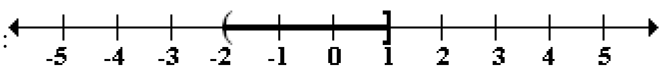
155. $\{x|x>-2\}$

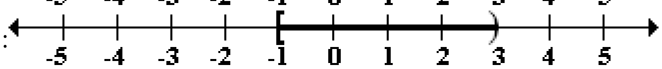


156. $\{x|x\le 0\}$

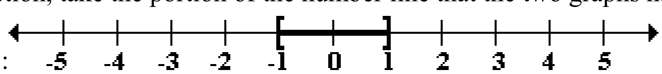


157.

Graph $(-2,1]$:


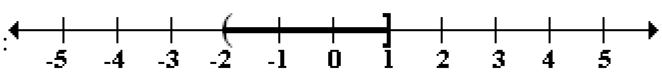
Graph $[-1,3)$:


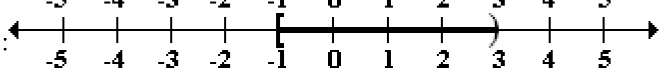
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-2,1]$ and $[-1,3)$:


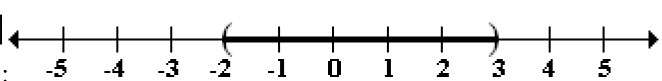
Thus, $(-2,1]\cap[-1,3)=[-1,1]$.

158.

Graph $(-2,1]$:


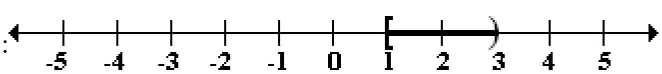
Graph $[-1,3)$:


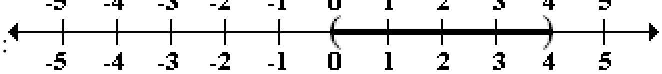
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-2,1]$ or $[-1,3)$ or both:


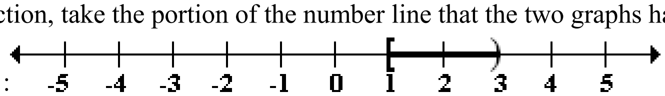
Thus, $(-2,1]\cup[-1,3)=(-2,3)$.

159.

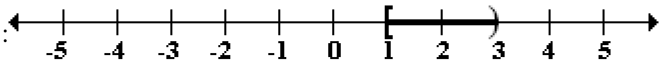
Graph $[1,3)$:


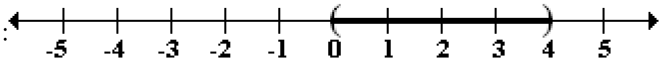
Graph $(0,4)$:


To find the intersection, take the portion of the number line that the two graphs have in common.

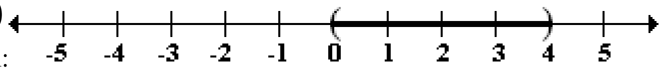
Numbers in both $[1,3)$ and $(0,4)$:


Thus, $[1,3)\cap(0,4)=[1,3)$.

160. Graph $[1,3)$: 

Graph $(0,4)$: 

To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

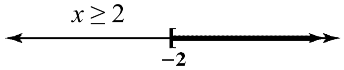
Numbers in either $[1,3)$ or $(0,4)$ or both: 

Thus, $[1,3) \cup (0,4) = (0,4)$.

161. $-6x + 3 \le 15$

$-6x \le 12$

$x \ge 2$

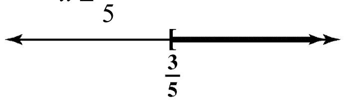


The solution set is $[-2, \infty)$.

162. $6x - 9 \ge -4x - 3$

$10x \ge 6$

$x \ge \frac{3}{5}$



The solution set is $\left[\frac{3}{5}, \infty\right)$.

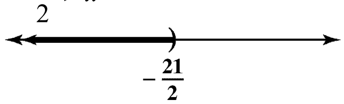
163. $\frac{x}{3} - \frac{3}{4} - 1 > \frac{x}{2}$

$12\left(\frac{x}{3} - \frac{3}{4} - 1\right) > 12\left(\frac{x}{2}\right)$

$4x - 9 - 12 > 6x$

$-21 > 2x$

$-\frac{21}{2} > x$



The solution set is $\left(-\infty, -\frac{21}{2}\right)$.

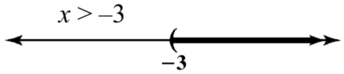
164. $6x + 5 > -2(x - 3) - 25$

$6x + 5 > -2x + 6 - 25$

$8x + 5 > -19$

$8x > -24$

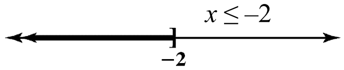
$x > -3$



The solution set is $(-3, \infty)$.

165. $3(2x-1)-2(x-4)\ge 7+2(3+4x)$

$$6x-3-2x+8\ge 7+6+8x$$
$$4x+5\ge 8x+13$$
$$-4x\ge 8$$
$$x\le -2$$



The solution set is $[-\infty,-2)$.

166. $7<2x+3\le 9$

$4<2x\le 6$

$2<x\le 3$

$(2, 3]$

The solution set is $[2,3)$.

167. $|2x+3|\le 15$

$-15\le 2x+3\le 15$

$-18\le 2x\le 12$

$-9\le x\le 6$

The solution set is $[-9,6]$.

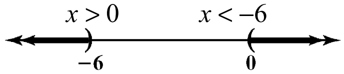
168. $\left|\dfrac{2x+6}{3}\right|>2$

$\dfrac{2x+6}{3}>2 \quad \dfrac{2x+6}{3}<-2$

$2x+6>6 \quad 2x+6<-6$

$2x>0 \quad 2x<-12$

$x>0 \quad x<-6$



The solution set is $(-\infty,-6)$ or $(0,\infty)$.

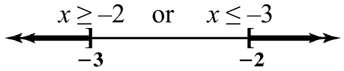
169. $|2x+5|-7\ge -6$

$|2x+5|\ge 1$

$2x+5\ge 1$ or $2x+5\le -1$

$2x\ge -4 \quad 2x\le -6$

$x\ge -2$ or $x\le -3$



The solution set is $(-\infty,-3]$ or $[-2,\infty)$.

170. $-4|x+2|+5\le -7$

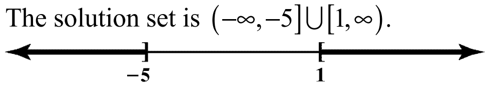
$-4|x+2|\le -12$

$|x+2|\ge 3$

$x+2\ge 3$ or $x+2\le -3$

$x\ge 1 \quad x\le -5$

The solution set is $(-\infty,-5]\cup[1,\infty)$.



171. $0.20x+24\le 40$

$0.20x\le 16$

$\dfrac{0.20x}{0.20}\le\dfrac{16}{0.20}$

$x\le 80$

A customer can drive no more than 80 miles.

172. $80\le\dfrac{95+79+91+86+x}{5}<90$

$400\le 95+79+91+86+x<450$

$400\le 351+x<450$

$49\le x<99$

A grade of at least 49% but less than 99% will result in a B.

Chapter P Test

1. $5(2x^2-6x)-(4x^2-3x)=10x^2-30x-4x^2+3x$
$=6x^2-27x$

2. $7+2[3(x+1)-2(3x-1)]$
$=7+2[3x+3-6x+2]$
$=7+2[-3x+5]$
$=7-6x+10$
$=-6x+17$

3. $\{1,2,5\}\cap\{5,a\}=\{5\}$

4. $\{1,2,5\}\cup\{5,a\}=\{1,2,5,a\}$

5. $\dfrac{30x^3y^4}{6x^9y^{-4}}=5x^{3-9}y^{4-(-4)}=5x^{-6}y^8=\dfrac{5y^8}{x^6}$

6. $\sqrt{6r}\cdot\sqrt{3r}=\sqrt{18r^2}=\sqrt{9r^2}\cdot\sqrt{2}=3r\sqrt{2}$

7. $4\sqrt{50}-3\sqrt{18}=4\sqrt{25\cdot 2}-3\sqrt{9\cdot 2}$
$=4\cdot 5\sqrt{2}-3\cdot 3\sqrt{2}$
$=20\sqrt{2}-9\sqrt{2}$
$=11\sqrt{2}$

8. $\dfrac{3}{5+\sqrt{2}}=\dfrac{3}{5+\sqrt{2}}\cdot\dfrac{5-\sqrt{2}}{5-\sqrt{2}}$
$=\dfrac{3(5-\sqrt{2})}{25-2}$
$=\dfrac{3(5-\sqrt{2})}{23}$

9. $\sqrt[3]{16x^4} = \sqrt[3]{8x^3 \cdot 2x}$
$= \sqrt[3]{8x^3} \cdot \sqrt[3]{2x}$
$= 2x\sqrt[3]{2x}$

10. $\dfrac{x^2+2x-3}{x^2-3x+2} = \dfrac{(x+3)(x-1)}{(x-2)(x-1)} = \dfrac{x+3}{x-2},$
$x \neq 2, 1$

11. $\dfrac{5\times10^{-6}}{20\times10^{-8}} = \dfrac{5}{20} \cdot \dfrac{10^{-6}}{10^{-8}} = 0.25\times10^2 = 2.5\times10^1$

12. $(2x-5)(x^2-4x+3)$
$= 2x^3 - 8x^2 + 6x - 5x^2 + 20x - 15$
$= 2x^3 - 13x^2 + 26x - 15$

13. $(5x+3y)^2 = (5x)^2 + 2(5x)(3y) + (3y)^2$
$= 25x^2 + 30xy + 9y^2$

14. $\dfrac{2x+8}{x-3} \div \dfrac{x^2+5x+4}{x^2-9}$
$= \dfrac{2(x+4)}{x-3} \div \dfrac{(x+1)(x+4)}{(x-3)(x+3)}$
$= \dfrac{2(x+4)}{x-3} \cdot \dfrac{(x-3)(x+3)}{(x+1)(x+4)}$
$= \dfrac{2(x+3)}{x+1},$
$x \neq 3, -1, -4, -3$

15. $\dfrac{x}{x+3} + \dfrac{5}{x-3}$
$= \dfrac{x}{x+3} \cdot \dfrac{x-3}{x-3} + \dfrac{5}{x-3} \cdot \dfrac{x+3}{x+3}$
$= \dfrac{x(x-3)+5(x+3)}{(x+3)(x-3)}$
$= \dfrac{x^2-3x+5x+15}{(x+3)(x-3)}$
$= \dfrac{x^2+2x+15}{(x+3)(x-3)}, x \neq 3, -3$

16. $\dfrac{2x+3}{x^2-7x+12} - \dfrac{2}{x-3}$
$= \dfrac{2x+3}{(x-3)(x-4)} - \dfrac{2}{x-3}$
$= \dfrac{2x+3}{(x-3)(x-4)} - \dfrac{2}{x-3} \cdot \dfrac{x-4}{x-4}$
$= \dfrac{2x+3-2(x-4)}{(x-3)(x-4)}$
$= \dfrac{2x+3-2(x-4)}{(x-3)(x-4)}$
$= \dfrac{2x+3-2x+8}{(x-3)(x-4)}$
$= \dfrac{11}{(x-3)(x-4)},$
$x \neq 3, 4$

17. $\dfrac{1-\frac{x}{x+2}}{1+\frac{1}{x}} = \dfrac{\left(1-\frac{x}{x+2}\right)(x+2)x}{\left(1+\frac{1}{x}\right)(x+2)x}$
$= \dfrac{x(x+2)-x^2}{x(x+2)+(x+2)}$
$= \dfrac{x^2+2x-x^2}{(x+1)(x+2)}$
$= \dfrac{2x}{x^2+3x+2}, x \neq 0$

18. $\dfrac{2x\sqrt{x^2+5} - \frac{2x^3}{\sqrt{x^2+5}}}{x^2+5}$
$= \dfrac{\left(2x\sqrt{x^2+5} - \frac{2x^3}{\sqrt{x^2+5}}\right)\sqrt{x^2+5}}{(x^2+5)\sqrt{x^2+5}}$
$= \dfrac{2x(x^2+5)-2x^3}{(x^2+5)\sqrt{x^2+5}}$
$= \dfrac{2x^3+10x-2x^3}{(x^2+5)\sqrt{x^2+5}}$
$= \dfrac{10x}{\sqrt{(x^2+5)^3}}$

19. $x^2 - 9x + 18 = (x-3)(x-6)$

20. $x^3 + 2x^2 + 3x + 6 = x^2(x+2) + 3(x+2)$
$= (x^2+3)(x+2)$

21. $25x^2 - 9 = (5x)^2 - 3^2 = (5x-3)(5x+3)$

22. $36x^2 - 84x + 49 = (6x)^2 - 2(6x)\cdot 7 + 7^2$
$= (6x-7)^2$

23. $y^3 - 125 = y^3 - 5^3 = (y-5)(y^2+5y+25)$

24. $(x^2+10x+25) - 9y^2$

$= (x+5)^2 - 9y^2$
$= (x+5-3y)(x+5+3y)$

25. $x(x+3)^{-\frac{3}{5}} + (x+3)^{\frac{2}{5}}$

$= (x+3)^{-\frac{3}{5}}\left[x+(x+3)\right]$

$= (x+3)^{-\frac{3}{5}}(2x+3) = \dfrac{2x+3}{(x+3)^{\frac{3}{5}}}$

26. $-7, -\dfrac{4}{5}, 0, 0.25, \sqrt{4}, \dfrac{22}{7}$ are rational numbers.

27. 3(2 + 5) = 3(5 + 2);
commutative property of addition

28. $6(7+4) = 6\cdot 7 + 6\cdot 4$
distributive property of multiplication over addition

29. $0.00076 = 7.6\times 10^{-4}$

30. $27^{-\frac{5}{3}} = \dfrac{1}{27^{\frac{5}{3}}} = \dfrac{1}{\left(\sqrt[3]{27}\right)^5} = \dfrac{1}{(3)^5} = \dfrac{1}{243}$

31. $2\left(6.6\times 10^9\right) = 13.2\times 10^9 = 1.32\times 10^{10}$

32. **a.** 2003 is 14 years after 1989.
$M = -0.28n + 47$
$M = -0.28(14) + 47$
$= 43.08$
In 2003, 43.08% of bachelor's degrees were awarded to men. This overestimates the actual percent shown by the bar graph by 0.08%.

b. $R = \dfrac{M}{W} = \dfrac{-0.28n+47}{0.28n+53}$

c. $R = \dfrac{-0.28n+47}{0.28n+53}$

$R = \dfrac{-0.28(25)+47}{0.28(25)+53}$

$= \dfrac{2}{3}$

Three women will receive bachelor's degrees for every two men. This describes the projections exactly.

33. $7(x-2) = 4(x+1) - 21$
$7x - 14 = 4x + 4 - 21$
$7x - 14 = 4x - 17$
$3x = -3$
$x = -1$
The solution set is $\{-1\}$.

34. $\dfrac{2x-3}{4} = \dfrac{x-4}{2} - \dfrac{x+1}{4}$
$2x - 3 = 2(x-4) - (x+1)$
$2x - 3 = 2x - 8 - x - 1$
$2x - 3 = x - 9$
$x = -6$
The solution set is $\{-6\}$.

35. $\dfrac{2}{x-3} - \dfrac{4}{x+3} = \dfrac{8}{(x-3)(x+3)}$

$2(x+3) - 4(x-3) = 8$
$2x + 6 - 4x + 12 = 8$
$-2x + 18 = 8$
$-2x = -10$
$x = 5$
The solution set is $\{5\}$.

36. $2x^2 - 3x - 2 = 0$
$(2x + 1)(x - 2) = 0$
$2x + 1 = 0$ or $x - 2 = 0$

$x = -\dfrac{1}{2}$ or $x = 2$

The solution set is $\left\{-\dfrac{1}{2}, 2\right\}$.

37. $(3x-1)^2 = 75$

$3x - 1 = \pm\sqrt{75}$
$3x = 1 \pm 5\sqrt{3}$
$x = \dfrac{1 \pm 5\sqrt{3}}{3}$

The solution set is $\left\{\dfrac{1-5\sqrt{3}}{3}, \dfrac{1+5\sqrt{3}}{3}\right\}$.

38. $x(x - 2) = 4$
$x^2 - 2x - 4 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2}$

$x = \dfrac{2 \pm 2\sqrt{5}}{2}$

$x = 1 \pm \sqrt{5}$

The solution set is $\left\{1-\sqrt{5}, 1+\sqrt{5}\right\}$.

39.
$$\sqrt{x-3}+5=x$$
$$\sqrt{x-3}=x-5$$
$$x-3=x^2-10x+25$$
$$x^2-11x+28=0$$
$$x=\frac{11\pm\sqrt{11^2-4(1)(28)}}{2(1)}$$
$$x=\frac{11\pm\sqrt{121-112}}{2}$$
$$x=\frac{11\pm\sqrt{9}}{2}$$
$$x=\frac{11\pm3}{2}$$
$x=7$ or $x=4$

4 does not check and must be rejected.
The solution set is $\{7\}$.

40.
$$\sqrt{8-2x}-x=0$$
$$\sqrt{8-2x}=x$$
$$\left(\sqrt{8-2x}\right)^2=(x)^2$$
$$8-2x=x^2$$
$$0=x^2+2x-8$$
$$0=(x+4)(x-2)$$
$x+4=0$ or $x-2=0$
$x=-4$ $\quad x=2$

–4 does not check and must be rejected.
The solution set is $\{2\}$.

41.
$$\left|\frac{2}{3}x-6\right|=2$$
$\frac{2}{3}x-6=2$ $\quad \frac{2}{3}x-6=-2$
$\frac{2}{3}x=8$ $\quad \frac{2}{3}x=4$
$x=12$ $\quad x=6$

The solution set is $\{6, 12\}$.

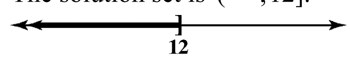
42.
$$-3|4x-7|+15=0$$
$$-3|4x-7|=-15$$
$$|4x-7|=5$$
$4x-7=5$ or $4x-7=-5$
$4x=12$ $\quad 4x=2$
$x=3$ $\quad x=\frac{1}{2}$

The solution set is $\left\{\frac{1}{2},3\right\}$

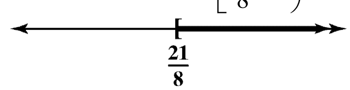
43.
$$\frac{2x}{x^2+6x+8}+\frac{2}{x+2}=\frac{x}{x+4}$$
$$\frac{2x}{(x+4)(x+2)}+\frac{2}{x+2}=\frac{x}{x+4}$$
$$\frac{2x(x+4)(x+2)}{(x+4)(x+2)}+\frac{2(x+4)(x+2)}{x+2}=\frac{x(x+4)(x+2)}{x+4}$$
$$2x+2(x+4)=x(x+2)$$
$$2x+2x+8=x^2+2x$$
$$2x+8=x^2$$
$$0=x^2-2x-8$$
$$0=(x-4)(x+2)$$
$x-4=0$ or $x+2=0$
$x=4$ $\quad x=-2$ (rejected)

The solution set is $\{4\}$.

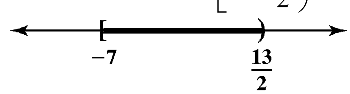
44.
$$3(x+4)\ge 5x-12$$
$$3x+12\ge 5x-12$$
$$-2x\ge -24$$
$$x\le 12$$
The solution set is $(-\infty, 12]$.



45.
$$\frac{x}{6}+\frac{1}{8}\le\frac{x}{2}-\frac{3}{4}$$
$$4x+3\le 12x-18$$
$$-8x\le -21$$
$$x\ge\frac{21}{8}$$
The solution set is $\left[\frac{21}{8},\infty\right)$.



46.
$$-3\le\frac{2x+5}{3}<6$$
$$-9\le 2x+5<18$$
$$-14\le 2x<13$$
$$-7\le x<\frac{13}{2}$$
The solution set is $\left[-7,\frac{13}{2}\right)$.



47. $|3x+2| \ge 3$

$3x+2 \ge 3$ or $3x+2 \le -3$

$3x \ge 1$ $\quad$ $3x \le -5$

$x \ge \frac{1}{3}$ $\quad$ $x \le -\frac{5}{3}$

The solution set is $\left(-\infty, -\frac{5}{3}\right] \cup \left[\frac{1}{3}, \infty\right)$.

48.
$$V = \frac{1}{3}lwh$$
$$3V = lwh$$
$$\frac{3V}{lw} = \frac{lwh}{lw}$$
$$\frac{3V}{lw} = h$$
$$h = \frac{3V}{lw}$$

49.
$$y - y_1 = m(x - x_1)$$
$$y - y_1 = mx - mx_1$$
$$-mx = y_1 - mx_1 - y$$
$$\frac{-mx}{-m} = \frac{y_1 - mx_1 - y}{-m}$$
$$x = \frac{y - y_1}{m} + x_1$$

50.
$$R = \frac{as}{a+s}$$
$$R(a+s) = as$$
$$Ra + Rs = as$$
$$Ra - as = -Rs$$
$$a(R-s) = -Rs$$
$$\frac{a(R-s)}{R-s} = \frac{-Rs}{R-s}$$
$$a = \frac{Rs}{s-R}$$

51.
$$43x + 575 = 1177$$
$$43x = 602$$
$$x = 14$$

The system's income will be \$1177 billion 14 years after 2004, or 2018.

52.
$$B = 0.07x^2 + 47.4x + 500$$
$$1177 = 0.07x^2 + 47.4x + 500$$
$$0 = 0.07x^2 + 47.4x - 677$$
$$0 = 0.07x^2 + 47.4x - 677$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(47.4) \pm \sqrt{(47.4)^2 - 4(0.07)(-677)}}{2(0.07)}$$
$$x \approx 14, \quad x \approx -691 \text{ (rejected)}$$

The system's income will be \$1177 billion 14 years after 2004, or 2018.

53. The formulas model the data quite well.

54. Let $x =$ the percentage of strikingly-attractive men.
Let $x + 57 =$ the percentage of average-looking men.
Let $x + 25 =$ the percentage of good-looking men.
$$(x) + (x+57) + (x+25) = 88$$
$$x + x + 57 + x + 25 = 88$$
$$3x + 82 = 88$$
$$3x = 6$$
$$x = 2$$
$$x + 57 = 59$$
$$x + 25 = 27$$

2% of men are strikingly-attractive.
59% of men are average-looking.
27% of men are good-looking.

55.
$$29700 + 150x = 5000 + 1100x$$
$$24700 = 950x$$
$$26 = x$$

In 26 years, the cost will be \$33,600.

56. Let x = amount invested at 8%
$10000 - x$ = amount invested at 10%
$$0.08x + 0.1(10000 - x) = 940$$
$$0.08x + 1000 - 0.1x = 940$$
$$-0.02x = -60$$
$$x = 3000$$
$$10000 - x = 7000$$

\$3000 at 8%, \$7000 at 10%

57.
$$l = 2w + 4$$
$$A = lw$$
$$48 = (2w+4)w$$
$$48 = 2w^2 + 4w$$
$$0 = 2w^2 + 4w - 48$$
$$0 = w^2 + 2w - 24$$
$$0 = (w+6)(w-4)$$

$w + 6 = 0$ $\quad$ $w - 4 = 0$

$w = -6$ $\quad$ $w = 4$

$2w + 4 = 2(4) + 4 = 12$

width is 4 feet, length is 12 feet

58.
$$24^2 + x^2 = 26^2$$
$$576 + x^2 = 676$$
$$x^2 = 100$$
$$x = \pm 10$$
The wire should be attached 10 feet up the pole.

59. Let x = the original selling price
$$20 = x - 0.60x$$
$$20 = 0.40x$$
$$50 = x$$
The original price is \$50.

60.
$$\frac{600,000}{x} - 6000 = \frac{600,000}{x+5}$$
$$x(x+5)\left(\frac{600,000}{x} - 6000\right) = x(x+5)\frac{600,000}{x+5}$$
$$600,000(x+5) - 6000x(x+5) = 600,000x$$
$$600,000x + 3,000,000 - 6000x^2 - 30,000x = 600,000x$$
$$-6000x^2 - 30,000x + 3,000,000 = 0$$
$$x^2 + 5x - 500 = 0$$
$$(x+25)(x-20) = 0$$

$x + 25 = 0$ or $x - 20 = 0$
$x = -25$ $\quad$ $x = 20$

–25 must be rejected. There were originally 20 people.

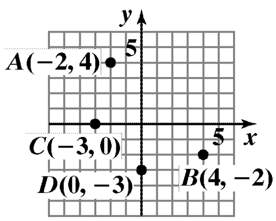
61. Let x = the number text messages.
The monthly cost using Plan A is $C_A = 25$.
The monthly cost using Plan B is $C_B = 13 + 0.06x$.
For Plan A to be better deal, it must cost less than Plan B.
$$C_A < C_B$$
$$25 < 13 + 0.06x$$
$$12 < 0.06x$$
$$200 < x$$
$$x > 200$$
Plan A is a better deal when more than 200 text messages per month are sent/received.

Chapter 1
Functions and Graphs

Section 1.1

Check Point Exercises

1. Plot points:



2. $x = -3, y = 7$

$x = -2, y = 6$

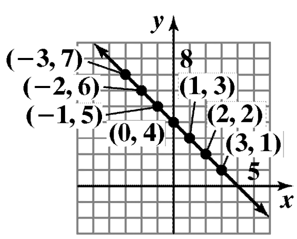
$x = -1, y = 5$

$x = 0, y = 4$

$x = 1, y = 3$

$x = 2, y = 2$

$x = 3, y = 1$



$y = 4 - x$

3. $x = -4, y = 3$

$x = -3, y = 2$

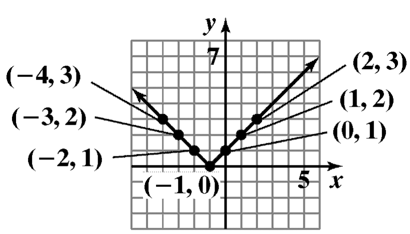
$x = -2, y = 1$

$x = -1, y = 0$

$x = 0, y = 1$

$x = 1, y = 2$

$x = 2, y = 3$



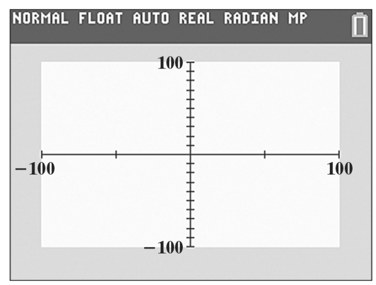
$y = |x + 1|$

4. The meaning of a $[-100, 100, 50]$ by $[-100, 100, 10]$ viewing rectangle is as follows:

$[\underbrace{-100}_{\text{minimum } x\text{-value}}, \underbrace{100}_{\text{maximum } x\text{-value}}, \underbrace{50}_{\text{distance between } x\text{-axis tick marks}}]$

by

$[\underbrace{-100}_{\text{minimum } y\text{-value}}, \underbrace{100}_{\text{maximum } y\text{-value}}, \underbrace{10}_{\text{distance between } y\text{-axis tick marks}}]$



5. **a.** The graph crosses the x-axis at (–3, 0).
Thus, the x-intercept is –3.
The graph crosses the y-axis at (0, 5).
Thus, the y-intercept is 5.

b. The graph does not cross the x-axis.
Thus, there is no x-intercept.
The graph crosses the y-axis at (0, 4).
Thus, the y-intercept is 4.

c. The graph crosses the x- and y-axes at the origin (0, 0).
Thus, the x-intercept is 0 and the y-intercept is 0.

d. The graph crosses the x-axis at (–1, 0) and (1, 0).
Thus, the x-intercepts are –1 and 1.
The graph crosses the y-axis at (0, 3).
Thus, the y-intercept is 3.

6. **a.** $d = 4n + 5$

$d = 4(15) + 5 = 65$

65% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage.

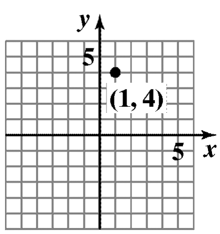
b. According to the line graph, 60% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage.

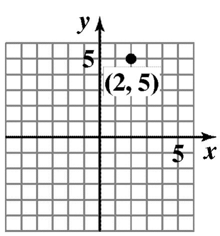
c. The mathematical model overestimates the actual percentage shown in the graph by 5%.

Concept and Vocabulary Check 1.1

1. x-axis
2. y-axis
3. origin
4. quadrants; four
5. x-coordinate; y-coordinate
6. solution; satisfies
7. x-intercept; zero
8. y-intercept; zero

Exercise Set 1.1

1.


2.


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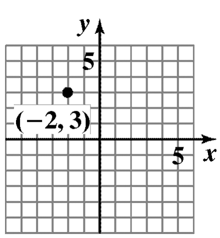
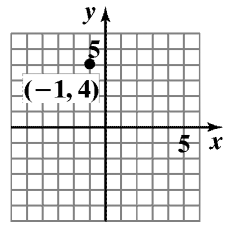


4.


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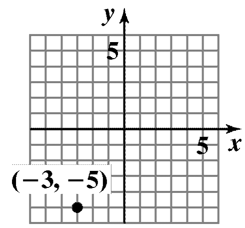
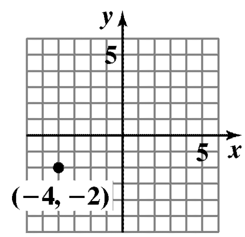
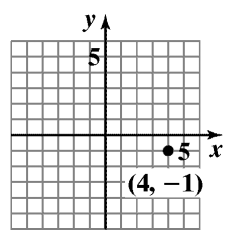
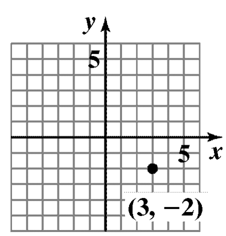
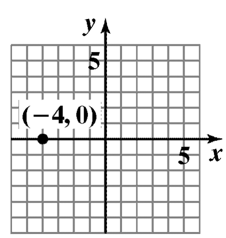


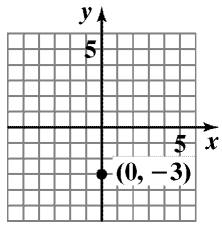
6.


7.


8.


9.


10.



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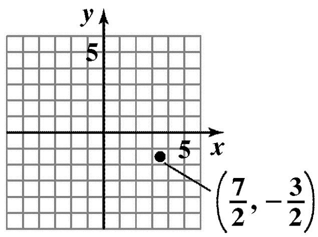
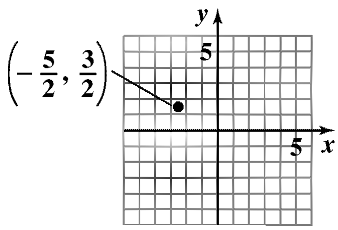
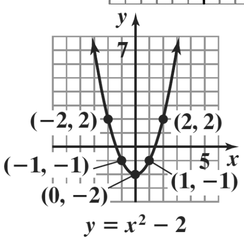


12.



13.



$x = -3, y = 7$

$x = -2, y = 2$

$x = -1, y = -1$

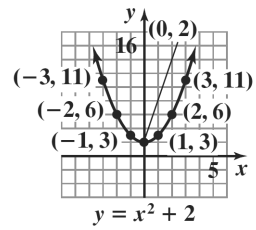
$x = 0, y = -2$

$x = 1, y = -1$

$x = 2, y = 2$

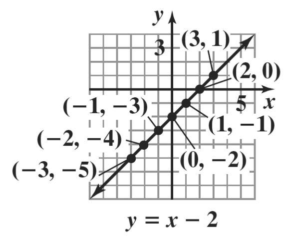
$x = 3, y = 7$

14.



$x = -3, y = 11$
$x = -2, y = 6$
$x = -1, y = 3$
$x = 0, y = 2$
$x = 1, y = 3$
$x = 2, y = 6$
$x = 3, y = 11$

15.



$x = -3, y = -5$

$x = -2, y = -4$

$x = -1, y = -3$

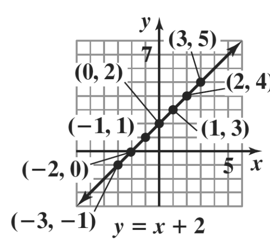
$x = 0, y = -2$

$x = 1, y = -1$

$x = 2, y = 0$

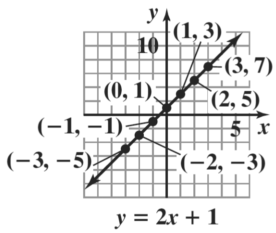
$x = 3, y = 1$

16.



$x = -3, y = -1$
$x = -2, y = 0$
$x = -1, y = 1$
$x = 0, y = 2$
$x = 1, y = 3$
$x = 2, y = 4$
$x = 3, y = 5$

17.



$x = -3, y = -5$

$x = -2, y = -3$

$x = -1, y = -1$

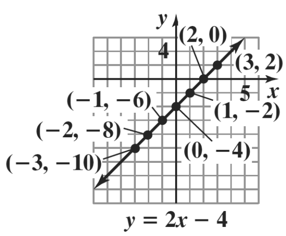
$x = 0, y = 1$

$x = 1, y = 3$

$x = 2, y = 5$

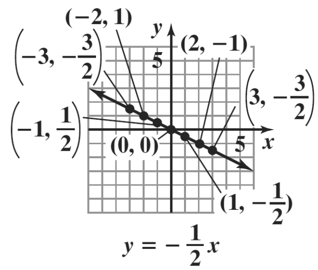
$x = 3, y = 7$

18.



$x = -3, y = -10$
$x = -2, y = -8$
$x = -1, y = -6$
$x = 0, y = -4$
$x = 1, y = -2$
$x = 2, y = 0$
$x = 3, y = 2$

19.



$x = -3, y = \dfrac{3}{2}$

$x = -2, y = 1$

$x = -1, y = \dfrac{1}{2}$

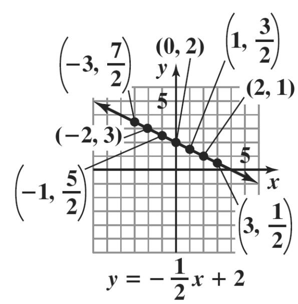
$x = 0, y = 0$

$x = 1, y = -\dfrac{1}{2}$

$x = 2, y = -1$

$x = 3, y = -\dfrac{3}{2}$

20.



$x = -3, y = \dfrac{7}{2}$

$x = -2, y = 3$

$x = -1, y = \dfrac{5}{2}$

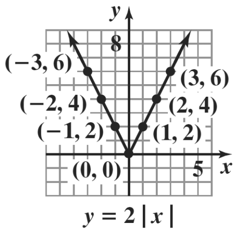
$x = 0, y = 2$

$x = 1, y = \dfrac{3}{2}$

$x = 2, y = 1$

$x = 3, y = \dfrac{1}{2}$

21.



$x=-3, y=6$

$x=-2, y=4$

$x=-1, y=2$

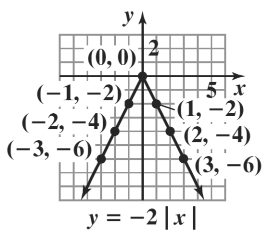
$x=0, y=0$

$x=1, y=2$

$x=2, y=4$

$x=3, y=6$

22.



$x=-3, y=-6$

$x=-2, y=-4$

$x=-1, y=-2$

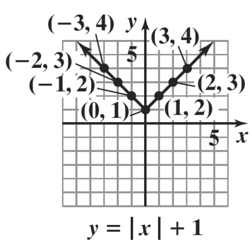
$x=0, y=0$

$x=1, y=-2$

$x=2, y=-4$

$x=3, y=-6$

23.



$x=-3, y=4$

$x=-2, y=3$

$x=-1, y=2$

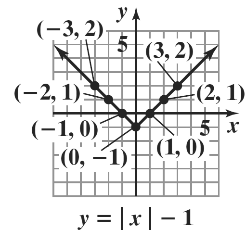
$x=0, y=1$

$x=1, y=2$

$x=2, y=3$

$x=3, y=4$

24.



$x=-3, y=2$

$x=-2, y=1$

$x=-1, y=0$

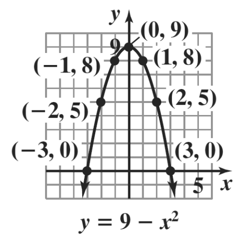
$x=0, y=-1$

$x=1, y=0$

$x=2, y=1$

$x=3, y=2$

25.



$x=-3, y=0$

$x=-2, y=5$

$x=-1, y=8$

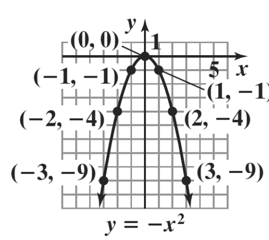
$x=0, y=9$

$x=1, y=8$

$x=2, y=5$

$x=3, y=0$

26.



$x=-3, y=-9$

$x=-2, y=-4$

$x=-1, y=-1$

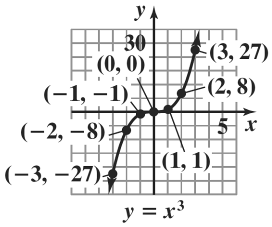
$x=0, y=0$

$x=1, y=-1$

$x=2, y=-4$

$x=3, y=-9$

27.



$x = -3, y = -27$

$x = -2, y = -8$

$x = -1, y = 1$

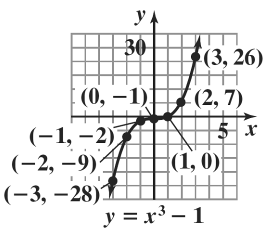
$x = 0, y = 0$

$x = 1, y = 1$

$x = 2, y = 8$

$x = 3, y = 27$

28.



$x = -3, y = -28$

$x = -2, y = -9$

$x = -1, y = -2$

$x = 0, y = -1$

$x = 1, y = 0$

$x = 2, y = 7$

$x = 3, y = 26$

29. (c) *x*-axis tick marks –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5; *y*-axis tick marks are the same.

30. (d) *x*-axis tick marks –10, –8, –6, –4, –2, 0, 2, 4, 6, 8, 10; *y*-axis tick marks –4, –2, 0, 2, 4

31. (b); *x*-axis tick marks –20, –10, 0, 10, 20, 30, 40, 50, 60, 70, 80; *y*-axis tick marks –30, –20, –10, 0, 10, 20, 30, 40, 50, 60, 70

32. (a) *x*-axis tick marks –40, –20, 0, 20, 40; *y*-axis tick marks –1000, –900, –800, –700, . . . , 700, 800, 900, 1000

33. The equation that corresponds to Y_2 in the table is (c), $y_2 = 2 - x$. We can tell because all of the points $(-3, 5)$, $(-2, 4)$, $(-1, 3)$, $(0, 2)$, $(1, 1)$, $(2, 0)$, and $(3, -1)$ are on the line $y = 2 - x$, but all are not on any of the others.

34. The equation that corresponds to Y_1 in the table is (b), $y_1 = x^2$. We can tell because all of the points $(-3, 9)$, $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$, and $(3, 9)$ are on the graph $y = x^2$, but all are not on any of the others.

35. No. It passes through the point $(0, 2)$.

36. Yes. It passes through the point $(0, 0)$.

37. $(2, 0)$

38. $(0, 2)$

39. The graphs of Y_1 and Y_2 intersect at the points $(-2, 4)$ and $(1, 1)$.

40. The values of Y_1 and Y_2 are the same when $x = -2$ and $x = 1$.

41. **a.** 2; The graph intersects the *x*-axis at (2, 0).

b. –4; The graph intersects the *y*-axis at (0,–4).

42. **a.** 1; The graph intersects the *x*-axis at (1, 0).

b. 2; The graph intersects the *y*-axis at (0, 2).

43. **a.** 1, –2; The graph intersects the *x*-axis at (1, 0) and (–2, 0).

b. 2; The graph intersects the *y*-axis at (0, 2).

44. **a.** 1, –1; The graph intersects the *x*-axis at (1, 0) and (–1, 0).

b. 1; The graph intersect the *y*-axis at (0, 1).

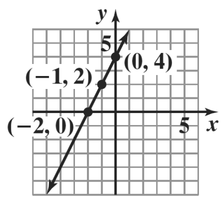
45. **a.** –1; The graph intersects the *x*-axis at (–1, 0).

b. none; The graph does not intersect the *y*-axis.

46. **a.** none; The graph does not intersect the *x*-axis.

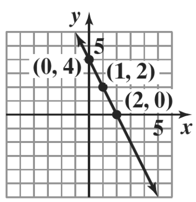
b. 2; The graph intersects the *y*-axis at (0, 2).

47.



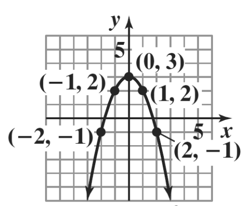
$y = 2x + 4$

48.



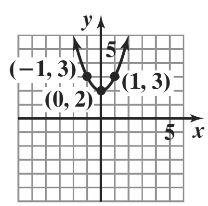
$y = 4 - 2x$

49.



$y = 3 - x^2$

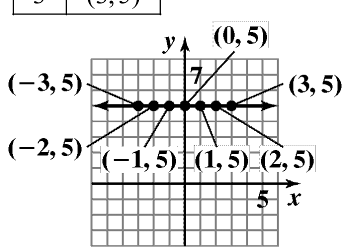
50.



$y = x^2 + 2$

51.

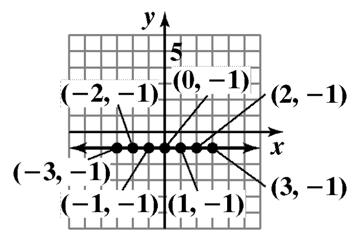
x	(x, y)
–3	(–3, 5)
–2	(–2, 5)
–1	(–1, 5)
0	(0, 5)
1	(1, 5)
2	(2, 5)
3	(3, 5)



$y = 5$

52.

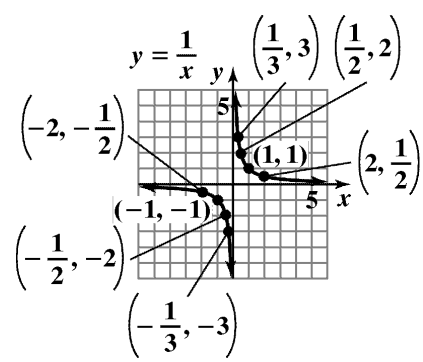
x	(x, y)
–3	(–3, –1)
–2	(–2, –1)
–1	(–1, –1)
0	(0, –1)
1	(1, –1)
2	(2, –1)
3	(3, –1)



$y = -1$

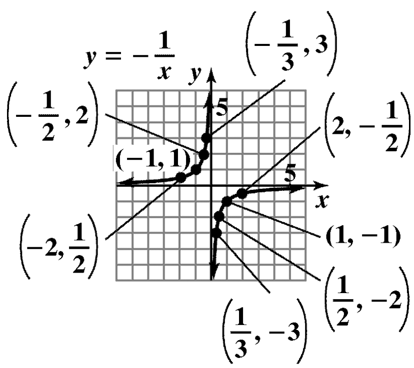
53.

x	(x, y)
–2	$\left(-2, -\frac{1}{2}\right)$
–1	$(-1, -1)$
$-\frac{1}{2}$	$\left(-\frac{1}{2}, -2\right)$
$-\frac{1}{3}$	$\left(-\frac{1}{3}, -3\right)$
$\frac{1}{3}$	$\left(\frac{1}{3}, 3\right)$
$\frac{1}{2}$	$\left(\frac{1}{2}, 2\right)$
1	$(1, 1)$
2	$\left(2, \frac{1}{2}\right)$



54.

x	(x, y)
-2	$\left(-2, \frac{1}{2}\right)$
-1	$(-1, 1)$
$-\frac{1}{2}$	$\left(-\frac{1}{2}, 2\right)$
$-\frac{1}{3}$	$\left(-\frac{1}{3}, 3\right)$
$\frac{1}{3}$	$\left(\frac{1}{3}, -3\right)$
$\frac{1}{2}$	$\left(\frac{1}{2}, -2\right)$
1	$(1, -1)$
2	$\left(2, -\frac{1}{2}\right)$



55. **a.** According to the line graph, about 44% of seniors used marijuana in 2010.

b. 2010 is 20 years after 1990.
$M = 0.1n + 43$
$M = 0.1(20) + 43 = 45$
According to formula, 45% of seniors used marijuana in 2010. It is greater than the estimate, although answers may vary.

c. According to the line graph, about 71% of seniors used alcohol in 2010.

d. 2010 is 20 years after 1990.
$A = -0.9n + 88$
$A = -0.9(20) + 88 = 70$
According to formula, 70% of seniors used alcohol in 2010. It is less than the estimate, although answers may vary.

e. The maximum for marijuana was reached in 2000.
According to the line graph, about 49% of seniors used marijuana in 1990.

56. **a.** According to the line graph, about 66% of seniors used alcohol in 2014.

b. 2014 is 24 years after 1990.
$A = -0.9n + 88$
$A = -0.9(24) + 88 = 66.4$
According to formula, 66.4% of seniors used alcohol in 2014. It is greater than the estimate, although answers may vary.

c. According to the line graph, about 44% of seniors used marijuana in 2014.

d. 2014 is 24 years after 1990.
$M = 0.1n + 43$
$M = 0.1(24) + 43 = 45.4$
According to formula, 45.4% of seniors used marijuana in 2014. It is greater than the estimate, although answers may vary.

e. The maximum for alcohol was reached in 1990. According to the line graph, about 90% of seniors used alcohol in 1990.

57. At age 8, women have the least number of awakenings, averaging about 1 awakening per night.

58. At age 65, men have the greatest number of awakenings, averaging about 8 awakenings per night.

59. The difference between the number of awakenings for 25-year-old men and women is about 1.9.

60. The difference between the number of awakenings for 18-year-old men and women is about 1.1.

61. – 66. Answers will vary.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: Most graphing utilities do not display numbers on the axes.

69. does not make sense; Explanations will vary. Sample explanation: These three points are not collinear.

70. does not make sense; Explanations will vary. Sample explanation: As the time of day goes up, the total calories burned will also go up.

71. false; Changes to make the statement true will vary. A sample change is: The product of the coordinates of a point in quadrant III is also positive.

72. false; Changes to make the statement true will vary. A sample change is: A point on the x-axis will have $y = 0$.

73. true

74. false; Changes to make the statement true will vary. A sample change is: $3(5) - 2(2) \neq -4$.

75. I, III

76. II, IV

77. IV

78. II

79. (a)

80. (d)

81. (b)

82. (c)

83. (b)

84. (a)

85. (c)

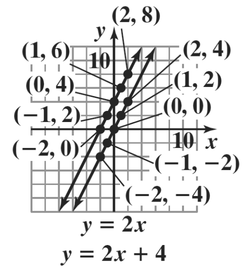
86. (b)

87. Set 1 has each x-coordinate paired with only one y-coordinate.

88.

x	$y = 2x$	(x, y)
–2	$y = 2(-2) = -4$	$(-2,-4)$
–1	$y = 2(-1)+4 = 2$	$(-1,-2)$
0	$y = 2(0) = 0$	$(0,0)$
1	$y = 2(1) = 2$	$(1,2)$
2	$y = 2(2) = 4$	$(2,4)$

x	$y = 2x+4$	(x, y)
–2	$y = 2(-2)+4 = 0$	$(-2,0)$
–1	$y = 2(-1)+4 = 2$	$(-1,2)$
0	$y = 2(0)+4 = 4$	$(0,4)$
1	$y = 2(1)+4 = 6$	$(1,6)$
2	$y = 2(2)+4 = 8$	$(2,8)$



89. **a.** When the x-coordinate is 2, the y-coordinate is 3.

b. When the y-coordinate is 4, the x-coordinates are –3 and 3.

c. The x-coordinates are all real numbers.

d. The y-coordinates are all real numbers greater than or equal to 1.

Section 1.2

Check Point Exercises

1. The domain is the set of all first components: {0, 10, 20, 30, 42}. The range is the set of all second components: {9.1, 6.7, 10.7, 13.2, 21.7}.

2. **a.** The relation is not a function since the two ordered pairs (5, 6) and (5, 8) have the same first component but different second components.

b. The relation is a function since no two ordered pairs have the same first component and different second components.

3. **a.** $2x + y = 6$
$y = 6 - 2x$
For each value of x, there is one and only one value for y, so the equation defines y as a function of x.

b. $x^2 + y^2 = 1$
$y^2 = 1 - x^2$
$y = \pm\sqrt{1 - x^2}$
Since there are values of x (all values between –1 and 1 exclusive) that give more than one value for y (for example, if $x = 0$, then $y = \pm\sqrt{1 - 0^2} = \pm 1$), the equation does not define y as a function of x.

4. **a.** $f(-5) = (-5)^2 - 2(-5) + 7$
$= 25 - (-10) + 7$
$= 42$

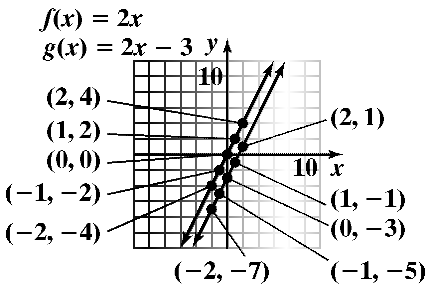
b. $f(x+4) = (x+4)^2 - 2(x+4) + 7$
$= x^2 + 8x + 16 - 2x - 8 + 7$
$= x^2 + 6x + 15$

c. $f(-x) = (-x)^2 - 2(-x) + 7$
$= x^2 - (-2x) + 7$
$= x^2 + 2x + 7$

5.

x	$f(x) = 2x$	(x, y)
-2	–4	$(-2, -4)$
-1	–2	$(-1, -2)$
0	0	$(0, 0)$
1	2	$(1, 2)$
2	4	$(2, 4)$

x	$g(x) = 2x - 3$	(x, y)
-2	$g(-2) = 2(-2) - 3 = -7$	$(-2, -7)$
-1	$g(-1) = 2(-1) - 3 = -5$	$(-1, -5)$
0	$g(0) = 2(0) - 3 = -3$	$(0, -3)$
1	$g(1) = 2(1) - 3 = -1$	$(1, -1)$
2	$g(2) = 2(2) - 3 = 1$	$(2, 1)$



The graph of g is the graph of f shifted down 3 units.

6. The graph (a) passes the vertical line test and is therefore is a function.
The graph (b) fails the vertical line test and is therefore not a function.
The graph (c) passes the vertical line test and is therefore is a function.
The graph (d) fails the vertical line test and is therefore not a function.

7. **a.** $f(5) = 400$

b. $x = 9$, $f(9) = 100$

c. The minimum T cell count in the asymptomatic stage is approximately 425.

8. **a.** domain: $\{x | -2 \le x \le 1\}$ or $[-2, 1]$.
range: $\{y | 0 \le y \le 3\}$ or $[0, 3]$.

b. domain: $\{x | -2 < x \le 1\}$ or $(-2, 1]$.
range: $\{y | -1 \le y < 2\}$ or $[-1, 2)$.

c. domain: $\{x | -3 \le x < 0\}$ or $[-3, 0)$.
range: $\{y | y = -3, -2, -1\}$.

Concept and Vocabulary Check 1.2

1. relation; domain; range

2. function

3. f; x

4. true

5. false

6. x; $x + 6$

7. ordered pairs

8. more than once; function

9. $[0, 3)$; domain

10. $[1, \infty)$; range

11. 0; 0; zeros

12. false

Exercise Set 1.2

1. The relation is a function since no two ordered pairs have the same first component and different second components. The domain is {1, 3, 5} and the range is {2, 4, 5}.

2. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {4, 6, 8} and the range is {5, 7, 8}.

3. The relation is not a function since the two ordered pairs (3, 4) and (3, 5) have the same first component but different second components (the same could be said for the ordered pairs (4, 4) and (4, 5)). The domain is {3, 4} and the range is {4, 5}.

4. The relation is not a function since the two ordered pairs (5, 6) and (5, 7) have the same first component but different second components (the same could be said for the ordered pairs (6, 6) and (6, 7)). The domain is {5, 6} and the range is {6, 7}.

5. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {3, 4, 5, 7} and the range is {–2, 1, 9}.

6. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {–2, –1, 5, 10} and the range is {1, 4, 6}.

7. The relation is a function since there are no same first components with different second components. The domain is {–3, –2, –1, 0} and the range is {–3, –2, –1, 0}.

8. The relation is a function since there are no ordered pairs that have the same first component but different second components. The domain is {–7, –5, –3, 0} and the range is {–7, –5, –3, 0}.

9. The relation is not a function since there are ordered pairs with the same first component and different second components. The domain is {1} and the range is {4, 5, 6}.

10. The relation is a function since there are no two ordered pairs that have the same first component and different second components. The domain is {4, 5, 6} and the range is {1}.

11. $x + y = 16$
$y = 16 - x$
Since only one value of y can be obtained for each value of x, y is a function of x.

12. $x + y = 25$
$y = 25 - x$
Since only one value of y can be obtained for each value of x, y is a function of x.

13. $x^2 + y = 16$
$y = 16 - x^2$
Since only one value of y can be obtained for each value of x, y is a function of x.

14. $x^2 + y = 25$
$y = 25 - x^2$
Since only one value of y can be obtained for each value of x, y is a function of x.

15. $x^2 + y^2 = 16$
$y^2 = 16 - x^2$
$y = \pm\sqrt{16 - x^2}$
If $x = 0$, $y = \pm 4$.
Since two values, $y = 4$ and $y = -4$, can be obtained for one value of x, y is not a function of x.

16. $x^2 + y^2 = 25$
$y^2 = 25 - x^2$
$y = \pm\sqrt{25 - x^2}$
If $x = 0$, $y = \pm 5$.
Since two values, $y = 5$ and $y = -5$, can be obtained for one value of x, y is not a function of x.

17. $x = y^2$
$y = \pm\sqrt{x}$
If $x = 1$, $y = \pm 1$.
Since two values, $y = 1$ and $y = -1$, can be obtained for $x = 1$, y is not a function of x.

18. $4x = y2$
$y = \pm\sqrt{4x} = \pm 2\sqrt{x}$
If $x = 1$, then $y = \pm 2$.
Since two values, $y = 2$ and $y = -2$, can be obtained for $x = 1$, y is not a function of x.

19. $y = \sqrt{x + 4}$
Since only one value of y can be obtained for each value of x, y is a function of x.

20. $y = -\sqrt{x+4}$
Since only one value of y can be obtained for each value of x, y is a function of x.

21. $x + y^3 = 8$
$y^3 = 8 - x$
$y = \sqrt[3]{8-x}$
Since only one value of y can be obtained for each value of x, y is a function of x.

22. $x + y^3 = 27$
$y^3 = 27 - x$
$y = \sqrt[3]{27-x}$
Since only one value of y can be obtained for each value of x, y is a function of x.

23. $xy + 2y = 1$
$y(x+2) = 1$
$y = \dfrac{1}{x+2}$
Since only one value of y can be obtained for each value of x, y is a function of x.

24. $xy - 5y = 1$
$y(x-5) = 1$
$y = \dfrac{1}{x-5}$
Since only one value of y can be obtained for each value of x, y is a function of x.

25. $|x| - y = 2$
$-y = -|x| + 2$
$y = |x| - 2$
Since only one value of y can be obtained for each value of x, y is a function of x.

26. $|x| - y = 5$
$-y = -|x| + 5$
$y = |x| - 5$
Since only one value of y can be obtained for each value of x, y is a function of x.

27. a. $f(6) = 4(6) + 5 = 29$

b. $f(x + 1) = 4(x + 1) + 5 = 4x + 9$

c. $f(-x) = 4(-x) + 5 = -4x + 5$

28. a. $f(4) = 3(4) + 7 = 19$

b. $f(x + 1) = 3(x + 1) + 7 = 3x + 10$

c. $f(-x) = 3(-x) + 7 = -3x + 7$

29. a. $g(-1) = (-1)^2 + 2(-1) + 3$
$= 1 - 2 + 3$
$= 2$

b. $g(x+5) = (x+5)^2 + 2(x+5) + 3$
$= x^2 + 10x + 25 + 2x + 10 + 3$
$= x^2 + 12x + 38$

c. $g(-x) = (-x)^2 + 2(-x) + 3$
$= x^2 - 2x + 3$

30. a. $g(-1) = (-1)^2 - 10(-1) - 3$
$= 1 + 10 - 3$
$= 8$

b. $g(x+2) = (x+2)^2 - 10(8+2) - 3$
$= x^2 + 4x + 4 - 10x - 20 - 3$
$= x^2 - 6x - 19$

c. $g(-x) = (-x)^2 - 10(-x) - 3$
$= x^2 + 10x - 3$

31. a. $h(2) = 2^4 - 2^2 + 1$
$= 16 - 4 + 1$
$= 13$

b. $h(-1) = (-1)^4 - (-1)^2 + 1$
$= 1 - 1 + 1$
$= 1$

c. $h(-x) = (-x)^4 - (-x)^2 + 1 = x^4 - x^2 + 1$

d. $h(3a) = (3a)^4 - (3a)^2 + 1$
$= 81a^4 - 9a^2 + 1$

32. a. $h(3) = 3^3 - 3 + 1 = 25$

b. $h(-2) = (-2)^3 - (-2) + 1$
$= -8 + 2 + 1$
$= -5$

c. $h(-x) = (-x)^3 - (-x) + 1 = -x^3 + x + 1$

d. $h(3a) = (3a)^3 - (3a) + 1$
$= 27a^3 - 3a + 1$

33. a. $f(-6) = \sqrt{-6+6} + 3 = \sqrt{0} + 3 = 3$

b. $f(10) = \sqrt{10+6} + 3$
$= \sqrt{16} + 3$
$= 4 + 3$
$= 7$

c. $f(x-6) = \sqrt{x-6+6} + 3 = \sqrt{x} + 3$

34. a. $f(16) = \sqrt{25-16} - 6 = \sqrt{9} - 6 = 3 - 6 = -3$

b. $f(-24) = \sqrt{25-(-24)} - 6$
$= \sqrt{49} - 6$
$= 7 - 6 = 1$

c. $f(25-2x) = \sqrt{25-(25-2x)} - 6$
$= \sqrt{2x} - 6$

35. a. $f(2) = \dfrac{4(2)^2 - 1}{2^2} = \dfrac{15}{4}$

b. $f(-2) = \dfrac{4(-2)^2 - 1}{(-2)^2} = \dfrac{15}{4}$

c. $f(-x) = \dfrac{4(-x)^2 - 1}{(-x)^2} = \dfrac{4x^2 - 1}{x^2}$

36. a. $f(2) = \dfrac{4(2)^3 + 1}{2^3} = \dfrac{33}{8}$

b. $f(-2) = \dfrac{4(-2)^3 + 1}{(-2)^3} = \dfrac{-31}{-8} = \dfrac{31}{8}$

c. $f(-x) = \dfrac{4(-x)^3 + 1}{(-x)^3} = \dfrac{-4x^3 + 1}{-x^3}$

or $\dfrac{4x^3 - 1}{x^3}$

37. a. $f(6) = \dfrac{6}{|6|} = 1$

b. $f(-6) = \dfrac{-6}{|-6|} = \dfrac{-6}{6} = -1$

c. $f(r^2) = \dfrac{r^2}{|r^2|} = \dfrac{r^2}{r^2} = 1$

38. a. $f(5) = \dfrac{|5+3|}{5+3} = \dfrac{|8|}{8} = 1$

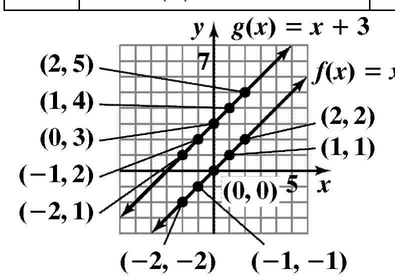
b. $f(-5) = \dfrac{|-5+3|}{-5+3} = \dfrac{|-2|}{-2} = \dfrac{2}{-2} = -1$

c. $f(-9-x) = \dfrac{|-9-x+3|}{-9-x+3}$
$= \dfrac{|-x-6|}{-x-6} = \begin{cases} 1, & \text{if } x < -6 \\ -1, & \text{if } x > -6 \end{cases}$

39.

x	$f(x) = x$	(x, y)
-2	$f(-2) = -2$	$(-2, -2)$
-1	$f(-1) = -1$	$(-1, -1)$
0	$f(0) = 0$	$(0, 0)$
1	$f(1) = 1$	$(1, 1)$
2	$f(2) = 2$	$(2, 2)$

x	$g(x) = x + 3$	(x, y)
-2	$g(-2) = -2 + 3 = 1$	$(-2, 1)$
-1	$g(-1) = -1 + 3 = 2$	$(-1, 2)$
0	$g(0) = 0 + 3 = 3$	$(0, 3)$
1	$g(1) = 1 + 3 = 4$	$(1, 4)$
2	$g(2) = 2 + 3 = 5$	$(2, 5)$

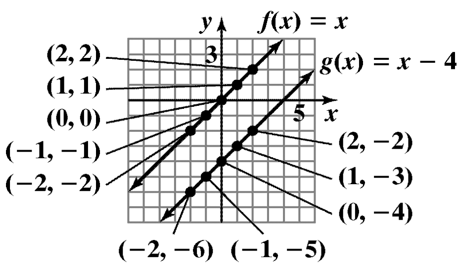


The graph of g is the graph of f shifted up 3 units.

40.

x	$f(x) = x$	(x, y)
–2	$f(-2) = -2$	$(-2, -2)$
–1	$f(-1) = -1$	$(-1, -1)$
0	$f(0) = 0$	$(0, 0)$
1	$f(1) = 1$	$(1, 1)$
2	$f(2) = 2$	$(2, 2)$

x	$g(x) = x - 4$	(x, y)
–2	$g(-2) = -2 - 4 = -6$	$(-2, -6)$
–1	$g(-1) = -1 - 4 = -5$	$(-1, -5)$
0	$g(0) = 0 - 4 = -4$	$(0, -4)$
1	$g(1) = 1 - 4 = -3$	$(1, -3)$
2	$g(2) = 2 - 4 = -2$	$(2, -2)$

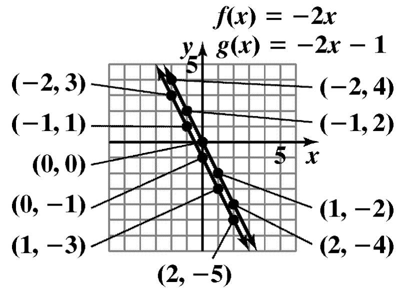


The graph of g is the graph of f shifted down 4 units.

41.

x	$f(x) = -2x$	(x, y)
–2	$f(-2) = -2(-2) = 4$	$(-2, 4)$
–1	$f(-1) = -2(-1) = 2$	$(-1, 2)$
0	$f(0) = -2(0) = 0$	$(0, 0)$
1	$f(1) = -2(1) = -2$	$(1, -2)$
2	$f(2) = -2(2) = -4$	$(2, -4)$

x	$g(x) = -2x - 1$	(x, y)
–2	$g(-2) = -2(-2) - 1 = 3$	$(-2, 3)$
–1	$g(-1) = -2(-1) - 1 = 1$	$(-1, 1)$
0	$g(0) = -2(0) - 1 = -1$	$(0, -1)$
1	$g(1) = -2(1) - 1 = -3$	$(1, -3)$
2	$g(2) = -2(2) - 1 = -5$	$(2, -5)$

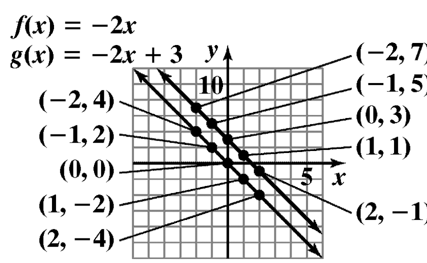


The graph of g is the graph of f shifted down 1 unit.

42.

x	$f(x) = -2x$	(x, y)
–2	$f(-2) = -2(-2) = 4$	$(-2, 4)$
–1	$f(-1) = -2(-1) = 2$	$(-1, 2)$
0	$f(0) = -2(0) = 0$	$(0, 0)$
1	$f(1) = -2(1) = -2$	$(1, -2)$
2	$f(2) = -2(2) = -4$	$(2, -4)$

x	$g(x) = -2x + 3$	(x, y)
–2	$g(-2) = -2(-2) + 3 = 7$	$(-2, 7)$
–1	$g(-1) = -2(-1) + 3 = 5$	$(-1, 5)$
0	$g(0) = -2(0) + 3 = 3$	$(0, 3)$
1	$g(1) = -2(1) + 3 = 1$	$(1, 1)$
2	$g(2) = -2(2) + 3 = -1$	$(2, -1)$

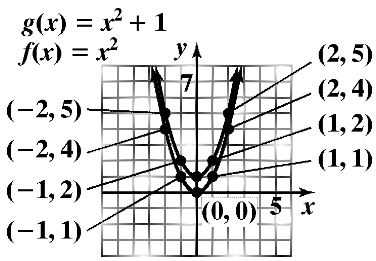


The graph of g is the graph of f shifted up 3 units.

43.

x	$f(x)=x^2$	(x,y)
-2	$f(-2)=(-2)^2=4$	$(-2,4)$
-1	$f(-1)=(-1)^2=1$	$(-1,1)$
0	$f(0)=(0)^2=0$	$(0,0)$
1	$f(1)=(1)^2=1$	$(1,1)$
2	$f(2)=(2)^2=4$	$(2,4)$

x	$g(x)=x^2+1$	(x,y)
-2	$g(-2)=(-2)^2+1=5$	$(-2,5)$
-1	$g(-1)=(-1)^2+1=2$	$(-1,2)$
0	$g(0)=(0)^2+1=1$	$(0,1)$
1	$g(1)=(1)^2+1=2$	$(1,2)$
2	$g(2)=(2)^2+1=5$	$(2,5)$

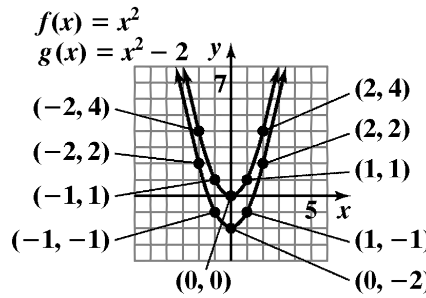


The graph of g is the graph of f shifted up 1 unit.

44.

x	$f(x)=x^2$	(x,y)
-2	$f(-2)=(-2)^2=4$	$(-2,4)$
-1	$f(-1)=(-1)^2=1$	$(-1,1)$
0	$f(0)=(0)^2=0$	$(0,0)$
1	$f(1)=(1)^2=1$	$(1,1)$
2	$f(2)=(2)^2=4$	$(2,4)$

x	$g(x)=x^2-2$	(x,y)
-2	$g(-2)=(-2)^2-2=2$	$(-2,2)$
-1	$g(-1)=(-1)^2-2=-1$	$(-1,-1)$
0	$g(0)=(0)^2-2=-2$	$(0,-2)$
1	$g(1)=(1)^2-2=-1$	$(1,-1)$
2	$g(2)=(2)^2-2=2$	$(2,2)$

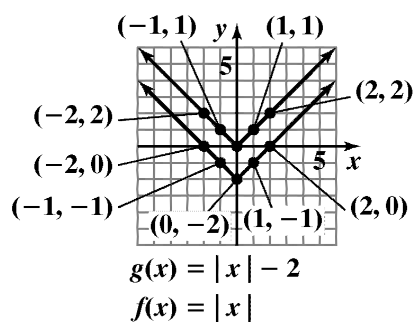


The graph of g is the graph of f shifted down 2 units.

45.

x	$f(x)=\lvert x\rvert$	(x,y)
-2	$f(-2)=\lvert -2\rvert=2$	$(-2,2)$
-1	$f(-1)=\lvert -1\rvert=1$	$(-1,1)$
0	$f(0)=\lvert 0\rvert=0$	$(0,0)$
1	$f(1)=\lvert 1\rvert=1$	$(1,1)$
2	$f(2)=\lvert 2\rvert=2$	$(2,2)$

x	$g(x)=\lvert x\rvert-2$	(x,y)
-2	$g(-2)=\lvert -2\rvert-2=0$	$(-2,0)$
-1	$g(-1)=\lvert -1\rvert-2=-1$	$(-1,-1)$
0	$g(0)=\lvert 0\rvert-2=-2$	$(0,-2)$
1	$g(1)=\lvert 1\rvert-2=-1$	$(1,-1)$
2	$g(2)=\lvert 2\rvert-2=0$	$(2,0)$

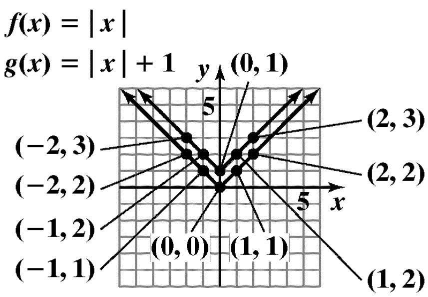


The graph of g is the graph of f shifted down 2 units.

46.

x	$f(x) = \lvert x \rvert$	(x, y)
-2	$f(-2) = \lvert -2 \rvert = 2$	$(-2, 2)$
-1	$f(-1) = \lvert -1 \rvert = 1$	$(-1, 1)$
0	$f(0) = \lvert 0 \rvert = 0$	$(0, 0)$
1	$f(1) = \lvert 1 \rvert = 1$	$(1, 1)$
2	$f(2) = \lvert 2 \rvert = 2$	$(2, 2)$

x	$g(x) = \lvert x \rvert + 1$	(x, y)
-2	$g(-2) = \lvert -2 \rvert + 1 = 3$	$(-2, 3)$
-1	$g(-1) = \lvert -1 \rvert + 1 = 2$	$(-1, 2)$
0	$g(0) = \lvert 0 \rvert + 1 = 1$	$(0, 1)$
1	$g(1) = \lvert 1 \rvert + 1 = 2$	$(1, 2)$
2	$g(2) = \lvert 2 \rvert + 1 = 3$	$(2, 3)$

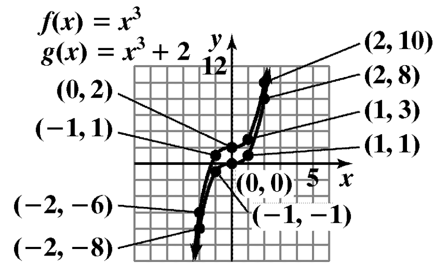


The graph of g is the graph of f shifted up 1 unit.

47.

x	$f(x) = x^3$	(x, y)
-2	$f(-2) = (-2)^3 = -8$	$(-2, -8)$
-1	$f(-1) = (-1)^3 = -1$	$(-1, -1)$
0	$f(0) = (0)^3 = 0$	$(0, 0)$
1	$f(1) = (1)^3 = 1$	$(1, 1)$
2	$f(2) = (2)^3 = 8$	$(2, 8)$

x	$g(x) = x^3 + 2$	(x, y)
-2	$g(-2) = (-2)^3 + 2 = -6$	$(-2, -6)$
-1	$g(-1) = (-1)^3 + 2 = 1$	$(-1, 1)$
0	$g(0) = (0)^3 + 2 = 2$	$(0, 2)$
1	$g(1) = (1)^3 + 2 = 3$	$(1, 3)$
2	$g(2) = (2)^3 + 2 = 10$	$(2, 10)$

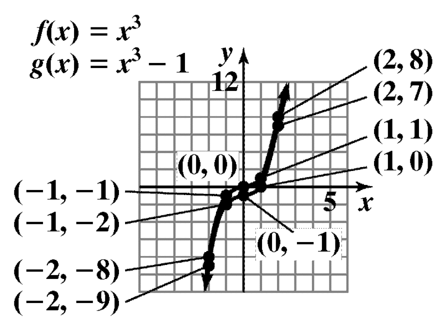


The graph of g is the graph of f shifted up 2 units.

48.

x	$f(x) = x^3$	(x, y)
-2	$f(-2) = (-2)^3 = -8$	$(-2, -8)$
-1	$f(-1) = (-1)^3 = -1$	$(-1, -1)$
0	$f(0) = (0)^3 = 0$	$(0, 0)$
1	$f(1) = (1)^3 = 1$	$(1, 1)$
2	$f(2) = (2)^3 = 8$	$(2, 8)$

x	$g(x) = x^3 - 1$	(x, y)
-2	$g(-2) = (-2)^3 - 1 = -9$	$(-2, -9)$
-1	$g(-1) = (-1)^3 - 1 = -2$	$(-1, -2)$
0	$g(0) = (0)^3 - 1 = -1$	$(0, -1)$
1	$g(1) = (1)^3 - 1 = 0$	$(1, 0)$
2	$g(2) = (2)^3 - 1 = 7$	$(2, 7)$

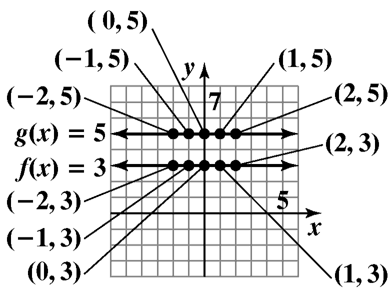


The graph of g is the graph of f shifted down 1 unit.

49.

x	$f(x)=3$	(x,y)
–2	$f(-2)=3$	$(-2,3)$
–1	$f(-1)=3$	$(-1,3)$
0	$f(0)=3$	$(0,3)$
1	$f(1)=3$	$(1,3)$
2	$f(2)=3$	$(2,3)$

x	$g(x)=5$	(x,y)
–2	$g(-2)=5$	$(-2,5)$
–1	$g(-1)=5$	$(-1,5)$
0	$g(0)=5$	$(0,5)$
1	$g(1)=5$	$(1,5)$
2	$g(2)=5$	$(2,5)$

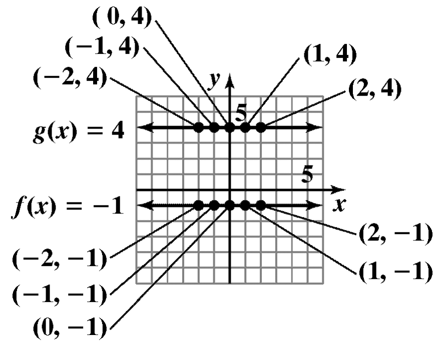


The graph of g is the graph of *f* shifted up 2 units.

50.

x	$f(x)=-1$	(x,y)
–2	$f(-2)=-1$	$(-2,-1)$
–1	$f(-1)=-1$	$(-1,-1)$
0	$f(0)=-1$	$(0,-1)$
1	$f(1)=-1$	$(1,-1)$
2	$f(2)=-1$	$(2,-1)$

x	$g(x)=4$	(x,y)
–2	$g(-2)=4$	$(-2,4)$
–1	$g(-1)=4$	$(-1,4)$
0	$g(0)=4$	$(0,4)$
1	$g(1)=4$	$(1,4)$
2	$g(2)=4$	$(2,4)$

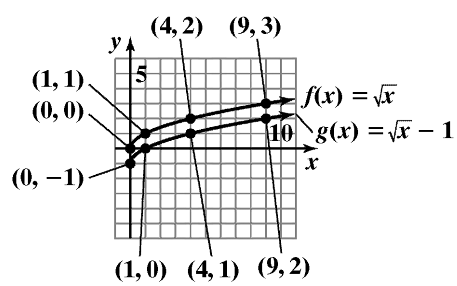


The graph of g is the graph of *f* shifted up 5 units.

51.

x	$f(x)=\sqrt{x}$	(x,y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x}-1$	(x,y)
0	$g(0)=\sqrt{0}-1=-1$	$(0,-1)$
1	$g(1)=\sqrt{1}-1=0$	$(1,0)$
4	$g(4)=\sqrt{4}-1=1$	$(4,1)$
9	$g(9)=\sqrt{9}-1=2$	$(9,2)$

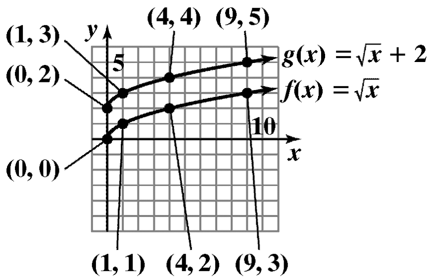


The graph of g is the graph of *f* shifted down 1 unit.

52.

x	$f(x)=\sqrt{x}$	(x,y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x}+2$	(x,y)
0	$g(0)=\sqrt{0}+2=2$	$(0,2)$
1	$g(1)=\sqrt{1}+2=3$	$(1,3)$
4	$g(4)=\sqrt{4}+2=4$	$(4,4)$
9	$g(9)=\sqrt{9}+2=5$	$(9,5)$

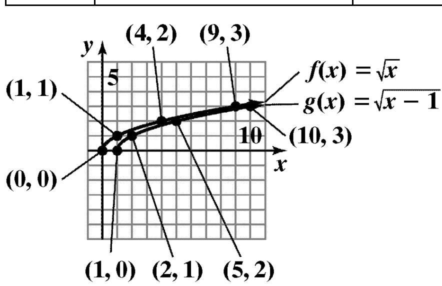


The graph of g is the graph of *f* shifted up 2 units.

53.

x	$f(x)=\sqrt{x}$	(x,y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x-1}$	(x,y)
1	$g(1)=\sqrt{1-1}=0$	$(1,0)$
2	$g(2)=\sqrt{2-1}=1$	$(2,1)$
5	$g(5)=\sqrt{5-1}=2$	$(5,2)$
10	$g(10)=\sqrt{10-1}=3$	$(10,3)$

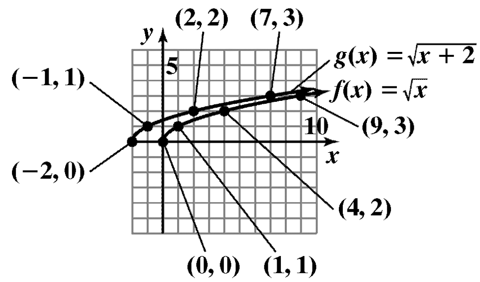


The graph of g is the graph of *f* shifted right 1 unit.

54.

x	$f(x)=\sqrt{x}$	(x,y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x+2}$	(x,y)
–2	$g(-2)=\sqrt{-2+2}=0$	$(-2,0)$
–1	$g(-1)=\sqrt{-1+2}=1$	$(-1,1)$
2	$g(2)=\sqrt{2+2}=2$	$(2,2)$
7	$g(7)=\sqrt{7+2}=3$	$(7,3)$



The graph of g is the graph of *f* shifted left 2 units.

55. function

56. function

57. function

58. not a function

59. not a function

60. not a function

61. function

62. not a function

63. function

64. function

65. $f(-2)=-4$

66. $f(2)=-4$

67. $f(4)=4$

68. $f(-4) = 4$

69. $f(-3) = 0$

70. $f(-1) = 0$

71. $g(-4) = 2$

72. $g(2) = -2$

73. $g(-10) = 2$

74. $g(10) = -2$

75. When $x = -2$, $g(x) = 1$.

76. When $x = 1$, $g(x) = -1$.

77. **a.** domain: $(-\infty, \infty)$

b. range: $[-4, \infty)$

c. x-intercepts: –3 and 1

d. y-intercept: –3

e. $f(-2) = -3$ and $f(2) = 5$

78. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, 4]$

c. x-intercepts: –3 and 1

d. y-intercept: 3

e. $f(-2) = 3$ and $f(2) = -5$

79. **a.** domain: $(-\infty, \infty)$

b. range: $[1, \infty)$

c. x-intercept: none

d. y-intercept: 1

e. $f(-1) = 2$ and $f(3) = 4$

80. **a.** domain: $(-\infty, \infty)$

b. range: $[0, \infty)$

c. x-intercept: –1

d. y-intercept: 1

e. $f(-4) = 3$ and $f(3) = 4$

81. **a.** domain: $[0, 5)$

b. range: $[-1, 5)$

c. x-intercept: 2

d. y-intercept: –1

e. $f(3) = 1$

82. **a.** domain: $(-6, 0]$

b. range: $[-3, 4)$

c. x-intercept: –3.75

d. y-intercept: –3

e. $f(-5) = 2$

83. **a.** domain: $[0, \infty)$

b. range: $[1, \infty)$

c. x-intercept: none

d. y-intercept: 1

e. $f(4) = 3$

84. **a.** domain: $[-1, \infty)$

b. range: $[0, \infty)$

c. x-intercept: –1

d. y-intercept: 1

e. $f(3) = 2$

85. **a.** domain: $[-2, 6]$

b. range: $[-2, 6]$

c. x-intercept: 4

d. y-intercept: 4

e. $f(-1) = 5$

86. **a.** domain: $[-3, 2]$

b. range: $[-5, 5]$

c. x-intercept: $-\frac{1}{2}$

d. y-intercept: 1

e. $f(-2) = -3$

87. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, -2]$

c. x-intercept: none

d. y-intercept: -2

e. $f(-4) = -5$ and $f(4) = -2$

88. **a.** domain: $(-\infty, \infty)$

b. range: $[0, \infty)$

c. x-intercept: $\{x \mid x \le 0\}$

d. y-intercept: 0

e. $f(-2) = 0$ and $f(2) = 4$

89. **a.** domain: $(-\infty, \infty)$

b. range: $(0, \infty)$

c. x-intercept: none

d. y-intercept: 1.5

e. $f(4) = 6$

90. **a.** domain: $(-\infty, 1) \cup (1, \infty)$

b. range: $(-\infty, 0) \cup (0, \infty)$

c. x-intercept: none

d. y-intercept: -1

e. $f(2) = 1$

91. **a.** domain: $\{-5, -2, 0, 1, 3\}$

b. range: $\{2\}$

c. x-intercept: none

d. y-intercept: 2

e. $f(-5) + f(3) = 2 + 2 = 4$

92. **a.** domain: $\{-5, -2, 0, 1, 4\}$

b. range: $\{-2\}$

c. x-intercept: none

d. y-intercept: -2

e. $f(-5) + f(4) = -2 + (-2) = -4$

93.
$$\begin{aligned} g(1) &= 3(1) - 5 = 3 - 5 = -2 \\ f(g(1)) &= f(-2) = (-2)^2 - (-2) + 4 \\ &= 4 + 2 + 4 = 10 \end{aligned}$$

94.
$$\begin{aligned} g(-1) &= 3(-1) - 5 = -3 - 5 = -8 \\ f(g(-1)) &= f(-8) = (-8)^2 - (-8) + 4 \\ &= 64 + 8 + 4 = 76 \end{aligned}$$

95.
$$\begin{aligned} &\sqrt{3-(-1)} - (-6)^2 + 6 \div (-6) \cdot 4 \\ &= \sqrt{3+1} - 36 + 6 \div (-6) \cdot 4 \\ &= \sqrt{4} - 36 + -1 \cdot 4 \\ &= 2 - 36 + -4 \\ &= -34 + -4 \\ &= -38 \end{aligned}$$

96.
$$\begin{aligned} &|-4-(-1)| - (-3)^2 + -3 \div 3 \cdot -6 \\ &= |-4+1| - 9 + -3 \div 3 \cdot -6 \\ &= |-3| - 9 + -1 \cdot -6 \\ &= 3 - 9 + 6 = -6 + 6 = 0 \end{aligned}$$

97.
$$\begin{aligned} &f(-x) - f(x) \\ &= (-x)^3 + (-x) - 5 - (x^3 + x - 5) \\ &= -x^3 - x - 5 - x^3 - x + 5 = -2x^3 - 2x \end{aligned}$$

98.
$$\begin{aligned} &f(-x) - f(x) \\ &= (-x)^2 - 3(-x) + 7 - (x^2 - 3x + 7) \\ &= x^2 + 3x + 7 - x^2 + 3x - 7 \\ &= 6x \end{aligned}$$

99. **a.** {(Iceland, 9.7), (Finland, 9.6), (New Zealand, 9.6), (Denmark, 9.5)}

b. Yes, the relation is a function because each country in the domain corresponds to exactly one corruption rating in the range.

c. {(9.7, Iceland), (9.6, Finland), (9.6, New Zealand), (9.5, Denmark)}

d. No, the relation is not a function because 9.6 in the domain corresponds to two countries in the range, Finland and New Zealand.

100. **a.** {(Bangladesh, 1.7), (Chad, 1.7), (Haiti, 1.8), (Myanmar, 1.8)}

b. Yes, the relation is a function because each country in the domain corresponds to exactly one corruption rating in the range.

c. {(1.7, Bangladesh), (1.7, Chad), (1.8, Haiti), (1.8, Myanmar)}

d. No, the relation is not a function because 1.7 in the domain corresponds to two countries in the range, Bangladesh and Chad.

101. **a.** $f(70) = 83$ which means the chance that a 60-year old will survive to age 70 is 83%.

b. $g(70) = 76$ which means the chance that a 60-year old will survive to age 70 is 76%.

c. Function *f* is the better model.

102. **a.** $f(90) = 25$ which means the chance that a 60-year old will survive to age 90 is 25%.

b. $g(90) = 10$ which means the chance that a 60-year old will survive to age 90 is 10%.

c. Function *f* is the better model.

103. **a.** $G(30) = -0.01(30)^2 + (30) + 60 = 81$
In 2010, the wage gap was 81%. This is represented as $(30, 81)$ on the graph.

b. $G(30)$ underestimates the actual data shown by the bar graph by 2%.

104. **a.** $G(10) = -0.01(10)^2 + (10) + 60 = 69$
In 1990, the wage gap was 69%. This is represented as $(10, 69)$ on the graph.

b. $G(10)$ underestimates the actual data shown by the bar graph by 2%.

105. $C(x) = 100{,}000 + 100x$
$C(90) = 100{,}000 + 100(90) = \$109{,}000$
It will cost $109,000 to produce 90 bicycles.

106. $V(x) = 22{,}500 - 3200x$
$V(3) = 22{,}500 - 3200(3) = \$12{,}900$
After 3 years, the car will be worth $12,900.

107. $T(x) = \dfrac{40}{x} + \dfrac{40}{x+30}$

$$T(30) = \frac{40}{30} + \frac{40}{30+30} = \frac{80}{60} + \frac{40}{60} = \frac{120}{60} = 2$$

If you travel 30 mph going and 60 mph returning, your total trip will take 2 hours.

108. $S(x) = 0.10x + 0.60(50 - x)$
$S(30) = 0.10(30) + 0.60(50 - 30) = 15$
When 30 mL of the 10% mixture is mixed with 20 mL of the 60% mixture, there will be 15 mL of sodium-iodine in the vaccine.

109. – 117. Answers will vary.

118. makes sense

119. does not make sense; Explanations will vary. Sample explanation: The parentheses used in function notation, such as $f(x)$, do not imply multiplication.

120. does not make sense; Explanations will vary. Sample explanation: The domain is the number of years worked for the company.

121. does not make sense; Explanations will vary. Sample explanation: This would not be a function because some elements in the domain would correspond to more than one age in the range.

122. false; Changes to make the statement true will vary. A sample change is: The domain is $[-4, 4]$.

123. false; Changes to make the statement true will vary. A sample change is: The range is $[-2, 2)$.

124. true

125. false; Changes to make the statement true will vary. A sample change is: $f(0) = 0.8$

126. $f(a+h) = 3(a+h) + 7 = 3a + 3h + 7$
$f(a) = 3a + 7$

$$\frac{f(a+h) - f(a)}{h} = \frac{(3a+3h+7) - (3a+7)}{h} = \frac{3a+3h+7-3a-7}{h} = \frac{3h}{h} = 3$$

127. Answers will vary.
An example is $\{(1,1),(2,1)\}$

128. It is given that $f(x+y)=f(x)+f(y)$ and $f(1)=3$.
To find $f(2)$, rewrite 2 as 1 + 1.

$$\begin{aligned}f(2)&=f(1+1)=f(1)+f(1)\\&=3+3=6\end{aligned}$$

Similarly:

$$\begin{aligned}f(3)&=f(2+1)=f(2)+f(1)\\&=6+3=9\end{aligned}$$

$$\begin{aligned}f(4)&=f(3+1)=f(3)+f(1)\\&=9+3=12\end{aligned}$$

While $f(x+y)=f(x)+f(y)$ is true for this function, it is not true for all functions. It is not true for $f(x)=x^2$, for example.

129.

$$\begin{aligned}-1+3(x-4)&=2x\\-1+3x-12&=2x\\3x-13&=2x\\-13&=-x\\13&=x\end{aligned}$$

The solution set is $\{13\}$.

130.

$$\begin{aligned}\frac{x-3}{5}-\frac{x-4}{2}&=5\\10\left(\frac{x-3}{5}\right)-10\left(\frac{x-4}{2}\right)&=10(5)\\2x-6-5x+20&=50\\-3x+14&=50\\-3x&=36\\x&=-12\end{aligned}$$

The solution set is $\{-12\}$.

131. Let x = the number of deaths by snakes, in thousands, in 2014
Let $x+661$ = the number of deaths by mosquitoes, in thousands, in 2014
Let $x+106$ = the number of deaths by snails, in thousands, in 2014

$$\begin{aligned}x+(x+661)+(x+106)&=1049\\x+x+661+x+106&=1049\\3x+767&=1049\\3x&=282\\x&=94\end{aligned}$$

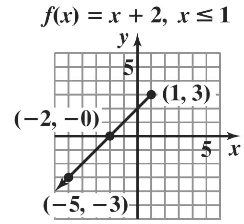
$x=94,$ thousand deaths by snakes
$x+661=755,$ thousand deaths by mosquitoes
$x+106=200,$ thousand deaths by snails

132.

$$\begin{aligned}C(t)&=20+0.40(t-60)\\C(100)&=20+0.40(100-60)\\&=20+0.40(40)\\&=20+16\\&=36\end{aligned}$$

For 100 calling minutes, the monthly cost is \$36.

133.



134.

$$\begin{aligned}&2(x+h)^2+3(x+h)+5-(2x^2+3x+5)\\&=2(x^2+2xh+h^2)+3x+3h+5-2x^2-3x-5\\&=2x^2+4xh+2h^2+3x+3h+5-2x^2-3x-5\\&=2x^2-2x^2+4xh+2h^2+3x-3x+3h+5-5\\&=4xh+2h^2+3h\end{aligned}$$

Section 1.3

Check Point Exercises

1. The function is increasing on the interval $(-\infty,-1)$, decreasing on the interval $(-1,1)$, and increasing on the interval $(1,\infty)$.

2. Test for symmetry with respect to the y-axis.

$$\begin{aligned}y&=x^2-1\\y&=(-x)^2-1\\y&=x^2-1\end{aligned}$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$\begin{aligned}y&=x^2-1\\-y&=x^2-1\\y&=-x^2+1\end{aligned}$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$\begin{aligned}y&=x^2-1\\-y&=(-x)^2-1\\-y&=x^2-1\\y&=-x^2+1\end{aligned}$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

3. Test for symmetry with respect to the y-axis.

$y^5 = x^3$
$y^5 = (-x)^3$
$y^5 = -x^3$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$y^5 = x^3$
$(-y)^5 = x^3$
$-y^5 = x^3$
$y^5 = -x^3$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$y^5 = x^3$
$(-y)^5 = (-x)^3$
$-y^5 = -x^3$
$y^5 = x^3$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

4. a. The graph passes the vertical line test and is therefore the graph of a function. The graph is symmetric with respect to the y-axis. Therefore, the graph is that of an even function.

b. The graph passes the vertical line test and is therefore the graph of a function. The graph is neither symmetric with respect to the y-axis nor the origin. Therefore, the graph is that of a function which is neither even nor odd.

c. The graph passes the vertical line test and is therefore the graph of a function. The graph is symmetric with respect to the origin. Therefore, the graph is that of an odd function.

5. a. $f(-x) = (-x)^2 + 6 = x^2 + 6 = f(x)$

The function is even. The graph is symmetric with respect to the y-axis.

b. $g(-x) = 7(-x)^3 - (-x) = -7x^3 + x = -f(x)$

The function is odd. The graph is symmetric with respect to the origin.

c. $h(-x) = (-x)^5 + 1 = -x^5 + 1$

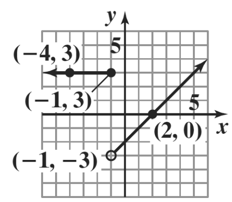
The function is neither even nor odd. The graph is neither symmetric to the y-axis nor the origin.

6. $C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t-60) & \text{if } t > 60 \end{cases}$

a. Since $0 \le 40 \le 60$, $C(40) = 20$
With 40 calling minutes, the cost is \$20.
This is represented by $(40, 20)$.

b. Since $80 > 60$,
$C(80) = 20 + 0.40(80 - 60) = 28$
With 80 calling minutes, the cost is \$28.
This is represented by $(80, 28)$.

7.



$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x - 2 & \text{if } x > -1 \end{cases}$

8. a.
$$f(x) = -2x^2 + x + 5$$
$$\begin{aligned} f(x+h) &= -2(x+h)^2 + (x+h) + 5 \\ &= -2(x^2 + 2xh + h^2) + x + h + 5 \\ &= -2x^2 - 4xh - 2h^2 + x + h + 5 \end{aligned}$$

b.
$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 5 - \left(-2x^2 + x + 5\right)}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 5 + 2x^2 - x - 5}{h}$$
$$= \frac{-4xh - 2h^2 + h}{h}$$
$$= \frac{h\left(-4x - 2h + 1\right)}{h}$$
$$= -4x - 2h + 1, \quad h \ne 0$$

Concept and Vocabulary Check 1.3

1. $< f(x_2)$; $> f(x_2)$; $= f(x_2)$
2. maximum; minimum
3. y-axis
4. x-axis
5. origin

6. $f(x)$; y-axis

7. $-f(x)$; origin

8. piecewise

9. less than or equal to x; 2; -3; 0

10. difference quotient; $x+h$; $f(x)$; h; h

11. false

12. false

Exercise Set 1.3

1. **a.** increasing: $(-1,\infty)$

b. decreasing: $(-\infty,-1)$

c. constant: none

2. **a.** increasing: $(-\infty, -1)$

b. decreasing: $(-1, \infty)$

c. constant: none

3. **a.** increasing: $(0,\infty)$

b. decreasing: none

c. constant: none

4. **a.** increasing: $(-1, \infty)$

b. decreasing: none

c. constant: none

5. **a.** increasing: none

b. decreasing: $(-2, 6)$

c. constant: none

6. **a.** increasing: $(-3, 2)$

b. decreasing: none

c. constant: none

7. **a.** increasing: $(-\infty,-1)$

b. decreasing: none

c. constant: $(-1,\infty)$

8. **a.** increasing: $(0, \infty)$

b. decreasing: none

c. constant: $(-\infty, 0)$

9. **a.** increasing: $(-\infty, 0)$ or $(1.5, 3)$

b. decreasing: $(0, 1.5)$ or $(3,\infty)$

c. constant: none

10. **a.** increasing: $(-5,-4)$ or $(-2,0)$ or $(2,4)$

b. decreasing: $(-4,-2)$ or $(0,2)$ or $(4,5)$

c. constant: none

11. **a.** increasing: $(-2, 4)$

b. decreasing: none

c. constant: $(-\infty,-2)$ or $(4,\infty)$

12. **a.** increasing: none

b. decreasing: $(-4, 2)$

c. constant: $(-\infty,-4)$ or $(2,\infty)$

13. **a.** $x = 0$, relative maximum = 4

b. $x = -3, 3$, relative minimum = 0

14. **a.** $x = 0$, relative maximum = 2

b. $x = -3, 3$, relative minimum = -1

15. **a.** $x = -2$, relative maximum = 21

b. $x = 1$, relative minimum = -6

16. **a.** $x = 1$, relative maximum = 30

b. $x = 4$, relative minimum = 3

17. Test for symmetry with respect to the y-axis.

$$y = x^2 + 6$$
$$y = (-x)^2 + 6$$
$$y = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = x^2 + 6$$
$$-y = x^2 + 6$$
$$y = -x^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = x^2 + 6$$
$$-y = (-x)^2 + 6$$
$$-y = x^2 + 6$$
$$y = -x^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

18. Test for symmetry with respect to the y-axis.

$$y = x^2 - 2$$
$$y = (-x)^2 - 2$$
$$y = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = x^2 - 2$$
$$-y = x^2 - 2$$
$$y = -x^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = x^2 - 2$$
$$-y = (-x)^2 - 2$$
$$-y = x^2 - 2$$
$$y = -x^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

19. Test for symmetry with respect to the y-axis.

$$x = y^2 + 6$$
$$-x = y^2 + 6$$
$$x = -y^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x = y^2 + 6$$
$$x = (-y)^2 + 6$$
$$x = y^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.
Test for symmetry with respect to the origin.

$$x = y^2 + 6$$
$$-x = (-y)^2 + 6$$
$$-x = y^2 + 6$$
$$x = -y^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

20. Test for symmetry with respect to the y-axis.

$$x = y^2 - 2$$
$$-x = y^2 - 2$$
$$x = -y^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x = y^2 - 2$$
$$x = (-y)^2 - 2$$
$$x = y^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x = y^2 - 2$$
$$-x = (-y)^2 - 2$$
$$-x = y^2 - 2$$
$$x = -y^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

21. Test for symmetry with respect to the y-axis.

$$y^2 = x^2 + 6$$
$$y^2 = (-x)^2 + 6$$
$$y^2 = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y^2 = x^2 + 6$$
$$(-y)^2 = x^2 + 6$$
$$y^2 = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y^2 = x^2 + 6$$
$$(-y)^2 = (-x)^2 + 6$$
$$y = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

22. Test for symmetry with respect to the y-axis.

$$y^2 = x^2 - 2$$
$$y^2 = (-x)^2 - 2$$
$$y^2 = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y^2 = x^2 - 2$$
$$(-y)^2 = x^2 - 2$$
$$y^2 = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y^2 = x^2 - 2$$
$$(-y)^2 = (-x)^2 - 2$$
$$y = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

23. Test for symmetry with respect to the y-axis.

$$y = 2x + 3$$
$$y = 2(-x) + 3$$
$$y = -2x + 3$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = 2x + 3$$
$$-y = 2x + 3$$
$$y = -2x - 3$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = 2x + 3$$
$$-y = 2(-x) + 3$$
$$y = 2x - 3$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

24. Test for symmetry with respect to the y-axis.

$$y = 2x + 5$$
$$y = 2(-x) + 5$$
$$y = -2x + 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = 2x + 5$$
$$-y = 2x + 5$$
$$y = -2x - 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = 2x + 5$$
$$-y = 2(-x) + 5$$
$$y = 2x - 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

25. Test for symmetry with respect to the *y*-axis.

$$x^2 - y^3 = 2$$
$$(-x)^2 - y^3 = 2$$
$$x^2 - y^3 = 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2 - y^3 = 2$$
$$x^2 - (-y)^3 = 2$$
$$x^2 + y^3 = 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2 - y^3 = 2$$
$$(-x)^2 - (-y)^3 = 2$$
$$x^2 + y^3 = 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

26. Test for symmetry with respect to the *y*-axis.

$$x^3 - y^2 = 5$$
$$(-x)^3 - y^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^3 - y^2 = 5$$
$$x^3 - (-y)^2 = 5$$
$$x^3 - y^2 = 5$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^3 - y^2 = 5$$
$$(-x)^3 - (-y)^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

27. Test for symmetry with respect to the *y*-axis.

$$x^2 + y^2 = 100$$
$$(-x)^2 + y^2 = 100$$
$$x^2 + y^2 = 100$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2 + y^2 = 100$$
$$x^2 + (-y)^2 = 100$$
$$x^2 + y^2 = 100$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2 + y^2 = 100$$
$$(-x)^2 + (-y)^2 = 100$$
$$x^2 + y^2 = 100$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

28. Test for symmetry with respect to the *y*-axis.

$$x^2 + y^2 = 49$$
$$(-x)^2 + y^2 = 49$$
$$x^2 + y^2 = 49$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2 + y^2 = 49$$
$$x^2 + (-y)^2 = 49$$
$$x^2 + y^2 = 49$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2 + y^2 = 49$$
$$(-x)^2 + (-y)^2 = 49$$
$$x^2 + y^2 = 49$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

29. Test for symmetry with respect to the y-axis.

$$x^2y^2 + 3xy = 1$$
$$(-x)^2 y^2 + 3(-x)y = 1$$
$$x^2y^2 - 3xy = 1$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x^2y^2 + 3xy = 1$$
$$x^2(-y)^2 + 3x(-y) = 1$$
$$x^2y^2 - 3xy = 1$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x^2y^2 + 3xy = 1$$
$$(-x)^2(-y)^2 + 3(-x)(-y) = 1$$
$$x^2y^2 + 3xy = 1$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

30. Test for symmetry with respect to the y-axis.

$$x^2y^2 + 5xy = 2$$
$$(-x)^2 y^2 + 5(-x)y = 2$$
$$x^2y^2 - 5xy = 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x^2y^2 + 5xy = 2$$
$$x^2(-y)^2 + 5x(-y) = 2$$
$$x^2y^2 - 5xy = 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x^2y^2 + 5xy = 2$$
$$(-x)^2(-y)^2 + 5(-x)(-y) = 2$$
$$x^2y^2 + 5xy = 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

31. Test for symmetry with respect to the y-axis.

$$y^4 = x^3 + 6$$
$$y^4 = (-x)^3 + 6$$
$$y^4 = -x^3 + 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y^4 = x^3 + 6$$
$$(-y)^4 = x^3 + 6$$
$$y^4 = x^3 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y^4 = x^3 + 6$$
$$(-y)^4 = (-x)^3 + 6$$
$$y^4 = -x^3 + 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

32. Test for symmetry with respect to the y-axis.

$$y^5 = x^4 + 2$$
$$y^5 = (-x)^4 + 2$$
$$y^5 = x^4 + 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y^5 = x^4 + 2$$
$$(-y)^5 = x^4 + 2$$
$$-y^5 = x^4 + 2$$
$$y^5 = -x^4 - 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y^4 = x^3 + 6$$
$$(-y)^4 = (-x)^3 + 6$$
$$y^4 = -x^3 + 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

33. The graph is symmetric with respect to the y-axis. The function is even.

34. The graph is symmetric with respect to the origin. The function is odd.

35. The graph is symmetric with respect to the origin. The function is odd.

36. The graph is not symmetric with respect to the y-axis or the origin. The function is neither even nor odd.

37. $f(x) = x^3 + x$
$f(-x) = (-x)^3 + (-x)$
$f(-x) = -x^3 - x = -(x^3 + x)$
$f(-x) = -f(x),$ odd function

38. $f(x) = x^3 - x$
$f(-x) = (-x)^3 - (-x)$
$f(-x) = -x^3 + x = -(x^3 - x)$
$f(-x) = -f(x),$ odd function

39. $g(x) = x^2 + x$
$g(-x) = (-x)^2 + (-x)$
$g(-x) = x^2 - x,$ neither

40. $g(x) = x^2 - x$
$g(-x) = (-x)^2 - (-x)$
$g(-x) = x^2 + x,$ neither

41. $h(x) = x^2 - x^4$
$h(-x) = (-x)^2 - (-x)^4$
$h(-x) = x^2 - x^4$
$h(-x) = h(x),$ even function

42. $h(x) = 2x^2 + x^4$
$h(-x) = 2(-x)^2 + (-x)^4$
$h(-x) = 2x^2 + x^4$
$h(-x) = h(x),$ even function

43. $f(x) = x^2 - x^4 + 1$
$f(-x) = (-x)^2 - (-x)^4 + 1$
$f(-x) = x^2 - x^4 + 1$
$f(-x) = f(x),$ even function

44. $f(x) = 2x^2 + x^4 + 1$
$f(-x) = 2(-x)^2 + (-x)^4 + 1$
$f(-x) = 2x^2 + x^4 + 1$
$f(-x) = f(x),$ even function

45. $f(x) = \frac{1}{5}x^6 - 3x^2$
$f(-x) = \frac{1}{5}(-x)^6 - 3(-x)^2$
$f(-x) = \frac{1}{5}x^6 - 3x^2$
$f(-x) = f(x)$, even function

46. $f(x) = 2x^3 - 6x^5$
$f(-x) = 2(-x)^3 - 6(-x)^5$
$f(-x) = -2x^3 + 6x^5$
$f(-x) = -(2x^3 - 6x^5)$
$f(-x) = -f(x),$ odd function

47. $f(x) = x\sqrt{1-x^2}$
$f(-x) = -x\sqrt{1-(-x)^2}$
$f(-x) = -x\sqrt{1-x^2}$
$= -\left(x\sqrt{1-x^2}\right)$
$f(-x) = -f(x)$, odd function

48. $f(x) = x^2\sqrt{1-x^2}$
$f(-x) = (-x)^2\sqrt{1-(-x)^2}$
$f(-x) = x^2\sqrt{1-x^2}$
$f(-x) = f(x)$, even function

49.
a. domain: $(-\infty, \infty)$

b. range: $[-4, \infty)$

c. x-intercepts: 1, 7

d. y-intercept: 4

e. $(4, \infty)$

f. $(0, 4)$

g. $(-\infty, 0)$

h. $x = 4$

i. $y = -4$

j. $f(-3) = 4$

k. $f(2) = -2$ and $f(6) = -2$

l. neither ; $f(-x) \neq x$, $f(-x) \neq -x$

50. **a.** domain: $(-\infty,\infty)$

b. range: $(-\infty,4]$

c. x-intercepts: –4, 4

d. y-intercept: 1

e. $(-\infty,-2)$ or $(0,3)$

f. $(-2,0)$ or $(3,\infty)$

g. $(-\infty,-4]$ or $[4,\infty)$

h. $x=-2$ and $x=3$

i. $f(-2)=4$ and $f(3)=2$

j. $f(-2)=4$

k. $x=-4$ and $x=4$

l. neither ; $f(-x)\neq x$, $f(-x)\neq -x$

51. **a.** domain: $(-\infty,3]$

b. range: $(-\infty,4]$

c. x-intercepts: –3, 3

d. $f(0)=3$

e. $(-\infty,1)$

f. $(1,3)$

g. $(-\infty,-3]$

h. $f(1)=4$

i. $x=1$

j. positive; $f(-1)=+2$

52. **a.** domain: $(-\infty,6]$

b. range: $(-\infty,1]$

c. zeros of f: –3, 3

d. $f(0)=1$

e. $(-\infty,-2)$

f. $(2,6)$

g. $(-2,2)$

h. $(-3,3)$

i. $x=-5$ and $x=5$

j. negative; $f(4)=-1$

k. neither

l. no; f(2) is not greater than the function values to the immediate left.

53. **a.** $f(-2) = 3(-2) + 5 = -1$

b. $f(0) = 4(0) + 7 = 7$

c. $f(3) = 4(3) + 7 = 19$

54. **a.** $f(-3) = 6(-3) - 1 = -19$

b. $f(0) = 7(0) + 3 = 3$

c. $f(4) = 7(4) + 3 = 31$

55. **a.** $g(0) = 0 + 3 = 3$

b. $g(-6) = -(-6 + 3) = -(-3) = 3$

c. $g(-3) = -3 + 3 = 0$

56. **a.** $g(0) = 0 + 5 = 5$

b. $g(-6) = -(-6 + 5) = -(-1) = 1$

c. $g(-5) = -5 + 5 = 0$

57. **a.** $h(5)=\dfrac{5^2-9}{5-3}=\dfrac{25-9}{2}=\dfrac{16}{2}=8$

b. $h(0)=\dfrac{0^2-9}{0-3}=\dfrac{-9}{-3}=3$

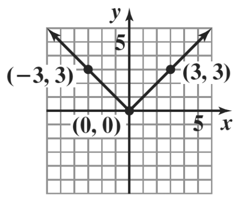
c. $h(3) = 6$

58. **a.** $h(7)=\dfrac{7^2-25}{7-5}=\dfrac{49-25}{2}=\dfrac{24}{2}=12$

b. $h(0)=\dfrac{0^2-25}{0-5}=\dfrac{-25}{-5}=5$

c. $h(5) = 10$

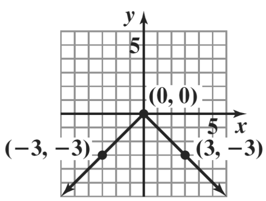
59. **a.**



$$f(x) = \begin{cases} -x \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$$

b. range: $[0, \infty)$

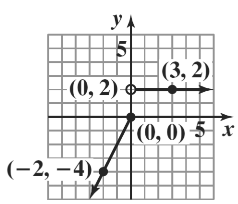
60. **a.**



$$f(x) = \begin{cases} x \text{ if } x < 0 \\ -x \text{ if } x \geq 0 \end{cases}$$

b. range: $(-\infty, 0]$

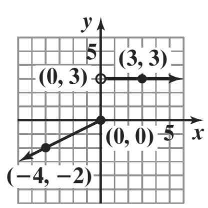
61. **a.**



$$f(x) = \begin{cases} 2x \text{ if } x \leq 0 \\ 2 \text{ if } x > 0 \end{cases}$$

b. range: $(-\infty, 0] \cup \{2\}$

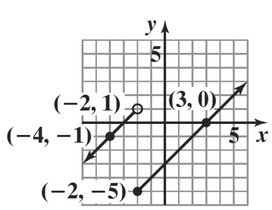
62. **a.**



$$f(x) = \begin{cases} \frac{1}{2}x \text{ if } x \leq 0 \\ 3 \text{ if } x > 0 \end{cases}$$

b. range: $(-\infty, 0] \cup \{3\}$

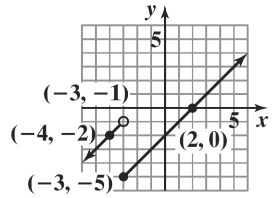
63. **a.**



$$f(x) = \begin{cases} x + 3 \text{ if } x < -2 \\ x - 3 \text{ if } x \geq -2 \end{cases}$$

b. range: $(-\infty, \infty)$

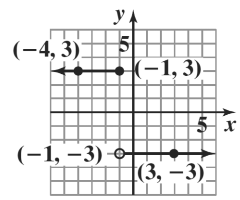
64. **a.**



$$f(x) = \begin{cases} x + 2 \text{ if } x < -3 \\ x - 2 \text{ if } x \geq -3 \end{cases}$$

b. range: $(-\infty, \infty)$

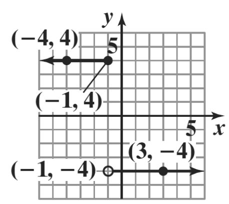
65. **a.**



$$f(x) = \begin{cases} 3 \text{ if } x \leq -1 \\ -3 \text{ if } x > -1 \end{cases}$$

b. range: $\{-3, 3\}$

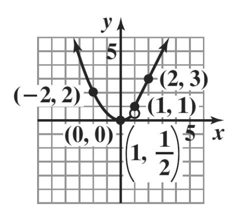
66. **a.**



$$f(x) = \begin{cases} 4 \text{ if } x \leq -1 \\ -4 \text{ if } x > -1 \end{cases}$$

b. range: $\{-4, 4\}$

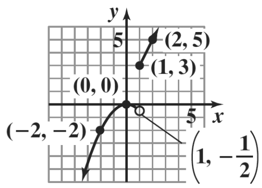
67. **a.**



$$f(x) = \begin{cases} \frac{1}{2}x^2 \text{ if } x < 1 \\ 2x - 1 \text{ if } x \geq 1 \end{cases}$$

b. range: $[0, \infty)$

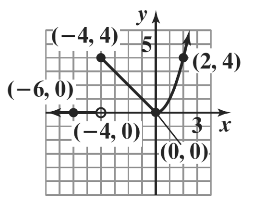
68. **a.**



$$f(x)=\begin{cases}-\frac{1}{2}x^2 & \text{if } x<1\\ 2x+1 & \text{if } x\ge 1\end{cases}$$

b. range: $(-\infty,0]\cup[3,\infty)$

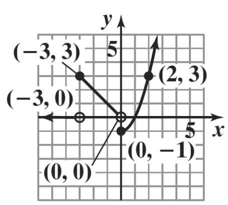
69. **a.**



$$f(x)=\begin{cases}0 & \text{if } x<-4\\ -x & \text{if } -4\le x<0\\ x^2 & \text{if } x\ge 0\end{cases}$$

b. range: $[0,\infty)$

70. **a.**



$$f(x)=\begin{cases}0 & \text{if } x<-3\\ -x & \text{if } -3\le x<0\\ x^2-1 & \text{if } x\ge 0\end{cases}$$

b. range: $[-1,\infty)$

71. $\frac{f(x+h)-f(x)}{h}$

$=\frac{4(x+h)-4x}{h}$

$=\frac{4x+4h-4x}{h}$

$=\frac{4h}{h}$

$=4$

72. $\frac{f(x+h)-f(x)}{h}$

$=\frac{7(x+h)-7x}{h}$

$=\frac{7x+7h-7x}{h}$

$=\frac{7h}{h}$

$=7$

73. $\frac{f(x+h)-f(x)}{h}$

$=\frac{3(x+h)+7-(3x+7)}{h}$

$=\frac{3x+3h+7-3x-7}{h}$

$=\frac{3h}{h}$

$=3$

74. $\frac{f(x+h)-f(x)}{h}$

$=\frac{6(x+h)+1-(6x+1)}{h}$

$=\frac{6x+6h+1-6x-1}{h}$

$=\frac{6h}{h}$

$=6$

75. $\frac{f(x+h)-f(x)}{h}$

$=\frac{(x+h)^2-x^2}{h}$

$=\frac{x^2+2xh+h^2-x^2}{h}$

$=\frac{2xh+h^2}{h}$

$=\frac{h(2x+h)}{h}$

$=2x+h$

76. $\frac{f(x+h)-f(x)}{h}$

$=\frac{2(x+h)^2-2x^2}{h}$

$=\frac{2(x^2+2xh+h^2)-2x^2}{h}$

$=\frac{2x^2+4xh+2h^2-2x^2}{h}$

$=\frac{4xh+2h^2}{h}$

$=\frac{h(4x+2h)}{h}$

$=4x+2h$

77. $\frac{f(x+h)-f(x)}{h}$

$=\frac{(x+h)^2-4(x+h)+3-(x^2-4x+3)}{h}$

$=\frac{x^2+2xh+h^2-4x-4h+3-x^2+4x-3}{h}$

$=\frac{2xh+h^2-4h}{h}$

$=\frac{h(2x+h-4)}{h}$

$=2x+h-4$

78. $\frac{f(x+h)-f(x)}{h}$

$=\frac{(x+h)^2-5(x+h)+8-(x^2-5x+8)}{h}$

$=\frac{x^2+2xh+h^2-5x-5h+8-x^2+5x-8}{h}$

$=\frac{2xh+h^2-5h}{h}$

$=\frac{h(2x+h-5)}{h}$

$=2x+h-5$

79. $\frac{f(x+h)-f(x)}{h}$

$=\frac{2(x+h)^2+(x+h)-1-(2x^2+x-1)}{h}$

$=\frac{2x^2+4xh+2h^2+x+h-1-2x^2-x+1}{h}$

$=\frac{4xh+2h^2+h}{h}$

$=\frac{h(4x+2h+1)}{h}$

$=4x+2h+1$

80. $\frac{f(x+h)-f(x)}{h}$

$=\frac{3(x+h)^2+(x+h)+5-(3x^2+x+5)}{h}$

$=\frac{3x^2+6xh+3h^2+x+h+5-3x^2-x-5}{h}$

$=\frac{6xh+3h^2+h}{h}$

$=\frac{h(6x+3h+1)}{h}$

$=6x+3h+1$

81. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-(x+h)^2+2(x+h)+4-(-x^2+2x+4)}{h}$

$=\frac{-x^2-2xh-h^2+2x+2h+4+x^2-2x-4}{h}$

$=\frac{-2xh-h^2+2h}{h}$

$=\frac{h(-2x-h+2)}{h}$

$=-2x-h+2$

82. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-(x+h)^2-3(x+h)+1-(-x^2-3x+1)}{h}$

$=\frac{-x^2-2xh-h^2-3x-3h+1+x^2+3x-1}{h}$

$=\frac{-2xh-h^2-3h}{h}$

$=\frac{h(-2x-h-3)}{h}$

$=-2x-h-3$

83. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-2(x+h)^2+5(x+h)+7-(-2x^2+5x+7)}{h}$

$=\frac{-2x^2-4xh-2h^2+5x+5h+7+2x^2-5x-7}{h}$

$=\frac{-4xh-2h^2+5h}{h}$

$=\frac{h(-4x-2h+5)}{h}$

$=-4x-2h+5$

84. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{-3(x+h)^2+2(x+h)-1-(-3x^2+2x-1)}{h}$

$= \dfrac{-3x^2-6xh-3h^2+2x+2h-1+3x^2-2x+1}{h}$

$= \dfrac{-6xh-3h^2+2h}{h}$

$= \dfrac{h(-6x-3h+2)}{h}$

$= -6x-3h+2$

85. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{-2(x+h)^2-(x+h)+3-(-2x^2-x+3)}{h}$

$= \dfrac{-2x^2-4xh-2h^2-x-h+3+2x^2+x-3}{h}$

$= \dfrac{-4xh-2h^2-h}{h}$

$= \dfrac{h(-4x-2h-1)}{h}$

$= -4x-2h-1$

86. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{-3(x+h)^2+(x+h)-1-(-3x^2+x-1)}{h}$

$= \dfrac{-3x^2-6xh-3h^2+x+h-1+3x^2-x+1}{h}$

$= \dfrac{-6xh-3h^2+h}{h}$

$= \dfrac{h(-6x-3h+1)}{h}$

$= -6x-3h+1$

87. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{6-6}{h} = \dfrac{0}{h} = 0$

88. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{7-7}{h} = \dfrac{0}{h} = 0$

89. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

$= \dfrac{\frac{x}{x(x+h)}+\frac{-(x+h)}{x(x+h)}}{h}$

$= \dfrac{\frac{x-x-h}{x(x+h)}}{h}$

$= \dfrac{\frac{-h}{x(x+h)}}{h}$

$= \dfrac{-h}{x(x+h)}\cdot\dfrac{1}{h}$

$= \dfrac{-1}{x(x+h)}$

90. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h}$

$= \dfrac{\frac{x}{2x(x+h)}-\frac{x+h}{2x(x+h)}}{h}$

$= \dfrac{\frac{-h}{2x(x+h)}}{h}$

$= \dfrac{-h}{2x(x+h)}\cdot\dfrac{1}{h}$

$= \dfrac{-1}{2x(x+h)}$

91. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \dfrac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \dfrac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$= \dfrac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$= \dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

92. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\sqrt{x+h-1}-\sqrt{x-1}}{h}$

$= \dfrac{\sqrt{x+h-1}-\sqrt{x-1}}{h} \cdot \dfrac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}$

$= \dfrac{x+h-1-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$= \dfrac{x+h-1-x+1}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$= \dfrac{h}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$= \dfrac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$

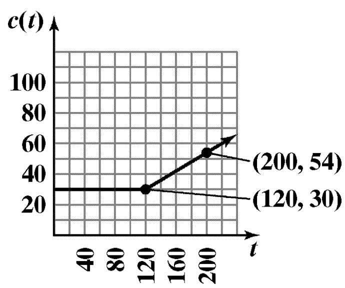
93. $\sqrt{f(-1.5)+f(-0.9)}-\left[f(\pi)\right]^2+f(-3)\div f(1)\cdot f(-\pi)$

$= \sqrt{1+0}-\left[-4\right]^2+2\div(-2)\cdot 3$
$= \sqrt{1}-16+(-1)\cdot 3$
$= 1-16-3$
$= -18$

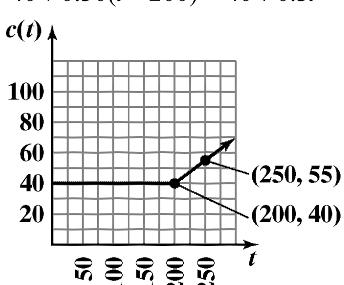
94. $\sqrt{f(-2.5)-f(1.9)}-\left[f(-\pi)\right]^2+f(-3)\div f(1)\cdot f(\pi)$

$\sqrt{f(-2.5)-f(1.9)}-\left[f(-\pi)\right]^2+f(-3)\div f(1)\cdot f(\pi)$
$= \sqrt{2-(-2)}-\left[3\right]^2+2\div(-2)\cdot(-4)$
$= \sqrt{4}-9+(-1)(-4)$
$= 2-9+4$
$= -3$

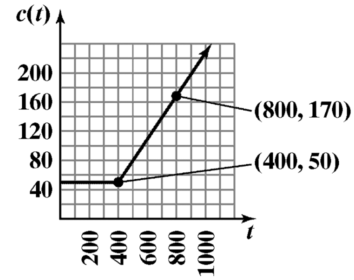
95. $30+0.30(t-120)=30+0.3t-36=0.3t-6$



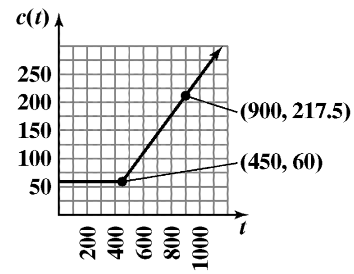
96. $40+0.30(t-200)=40+0.3t-60=0.3t-20$



97. $C(t)=\begin{cases} 50 & \text{if } 0 \le t \le 400 \\ 50+0.30(t-400) & \text{if } t > 400 \end{cases}$



98. $C(t)=\begin{cases} 60 & \text{if } 0 \le t \le 450 \\ 60+0.35(t-450) & \text{if } t > 450 \end{cases}$



99. increasing: (25, 55); decreasing: (55, 75)

100. increasing: (25, 65); decreasing: (65, 75)

101. The percent body fat in women reaches a maximum at age 55. This maximum is 38%.

102. The percent body fat in men reaches a maximum at age 65. This maximum is 26%.

103. domain: [25, 75]; range: [34, 38]

104. domain: [25, 75]; range: [23, 26]

105. This model describes percent body fat in men.

106. This model describes percent body fat in women.

107. $T(20{,}000)=850+0.15(20{,}000-8500)$
$= 2575$

A single taxpayer with taxable income of $20,000 owes $2575.

108. $T(50{,}000)=4750+0.25(50{,}000-34{,}500)$
$= 8625$

A single taxpayer with taxable income of $50,000 owes $8625.

109. $42{,}449+0.33(x-174{,}400)$

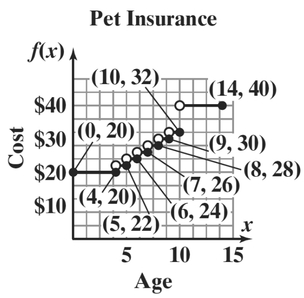
110. $110{,}016.50+0.35(x-(x-379{,}150)$

111. $f(3)=0.93$
The cost of mailing a first-class letter weighing 3 ounces is \$0.93.

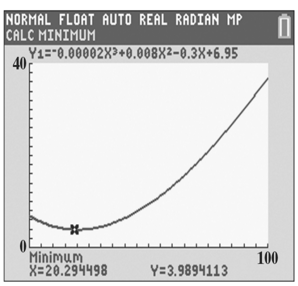
112. $f(3.5)=1.05$
The cost of mailing a first-class letter weighing 3.5 ounces is \$1.05.

113. The cost to mail a letter weighing 1.5 ounces is \$0.65.

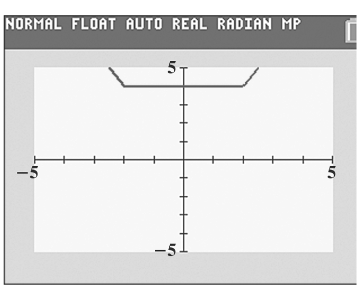
114. The cost to mail a letter weighing 1.8 ounces is \$0.65.

115.


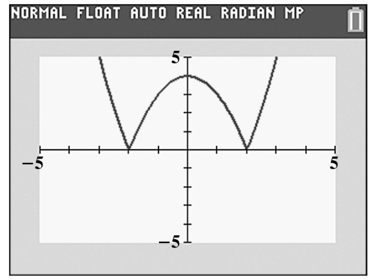
116. – 124. Answers will vary.

125.


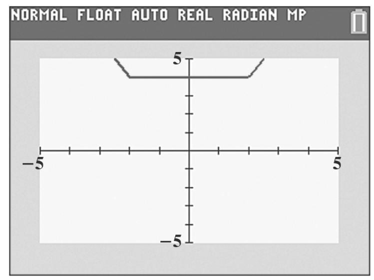
The number of doctor visits decreases during childhood and then increases as you get older. The minimum is (20.29, 3.99), which means that the minimum number of doctor visits, about 4, occurs at around age 20.

126.


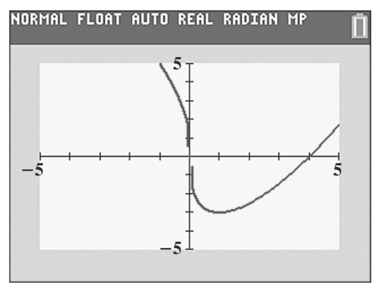
Increasing: $(-\infty, 1)$ or $(3, \infty)$
Decreasing: (1, 3)

127.


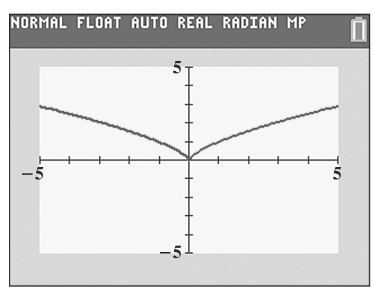
Increasing: (–2, 0) or (2, ∞)
Decreasing: (–∞, –2) or (0, 2)

128.


Increasing: $(2, \infty)$
Decreasing: $(-\infty, -2)$
Constant: (–2, 2)

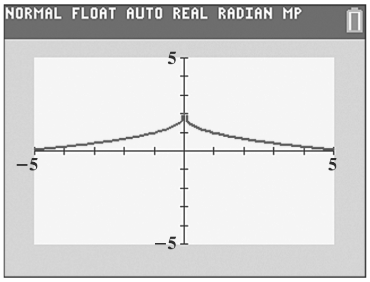
129.


Increasing: (1, ∞)
Decreasing: (–∞, 1)

130.


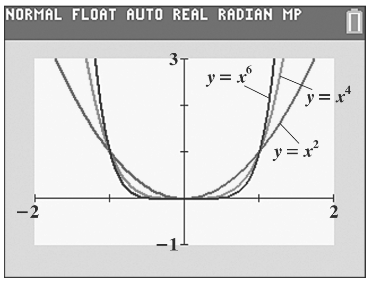
Increasing: $(0, \infty)$
Decreasing: $(-\infty, 0)$

131.

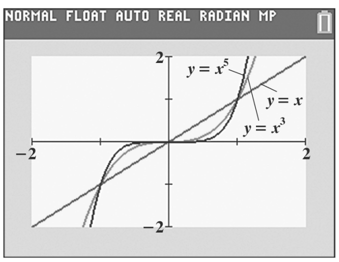


Increasing: (–∞, 0)
Decreasing: (0, ∞)

132. a.



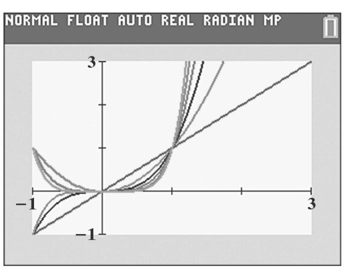
b.



c. Increasing: (0, ∞)
Decreasing: (–∞, 0)

d. $f(x) = x^n$ is increasing from (–∞, ∞) when *n* is odd.

e.



133. does not make sense; Explanations will vary. Sample explanation: It's possible the graph is not defined at *a*.

134. makes sense

135. makes sense

136. makes sense

137. answers will vary

138. answers will vary

139. a. *h* is even if both *f* and *g* are even or if both *f* and *g* are odd.
f and *g* are both even:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(x)} = h(x)$$

f and *g* are both odd:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{-g(x)} = \frac{f(x)}{g(x)} = h(x)$$

b. *h* is odd if *f* is odd and *g* is even or if *f* is even and *g* is odd.
f is odd and *g* is even:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{g(x)} = -\frac{f(x)}{g(x)} = -h(x)$$

f is even and *g* is odd:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)} = -h(x)$$

140. Let *x* = the amount invested at 5%.
Let 80,000 – *x* = the amount invested at 7%.

$$\begin{aligned} 0.05x + 0.07(80,000 - x) &= 5200 \\ 0.05x + 5600 - 0.07x &= 5200 \\ -0.02x + 5600 &= 5200 \\ -0.02x &= -400 \\ x &= 20,000 \\ 80,000 - x &= 60,000 \end{aligned}$$

$20,000 was invested at 5% and $60,000 was invested at 7%.

141.

$$\begin{aligned} C &= A + Ar \\ C &= A + Ar \\ C &= A(1+r) \\ \frac{C}{1+r} &= A \end{aligned}$$

142. $5x^2 - 6x - 8 = 0$

$a = 5, b = -6, c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-8)}}{2(5)}$$

$$x = \frac{6 \pm \sqrt{36+160}}{10}$$

$$x = \frac{6 \pm \sqrt{196}}{10}$$

$$x = \frac{6 \pm 14}{10}$$

$$x = \frac{6+14}{10} \text{ or } x = \frac{6-14}{10}$$

$$x = \frac{20}{10} \text{ or } x = \frac{-8}{10}$$

$$x = 2 \text{ or } x = -\frac{4}{5}$$

The solution set is $\left\{-\frac{4}{5}, 2\right\}$.

143. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{-2-(-3)} = \frac{3}{1} = 3$

144. When $y = 0$:

$$4x - 3y - 6 = 0$$
$$4x - 3(0) - 6 = 0$$
$$4x - 6 = 0$$
$$4x = 6$$
$$x = \frac{3}{2}$$

The point is $\left(\frac{3}{2}, 0\right)$.

When $x = 0$:

$$4x - 3y - 6 = 0$$
$$4(0) - 3y - 6 = 0$$
$$-3y - 6 = 0$$
$$-3y = 6$$
$$x = -2$$

The point is $(0, -2)$.

145. $3x + 2y - 4 = 0$

$$2y = -3x + 4$$
$$y = \frac{-3x+4}{2}$$

or

$$y = -\frac{3}{2}x + 2$$

Section 1.4

Check Point Exercises

1. **a.** $m = \frac{-2-4}{-4-(-3)} = \frac{-6}{-1} = 6$

b. $m = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$

2. Point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 6(x - 2)$$
$$y + 5 = 6(x - 2)$$

Slope-intercept form:

$$y + 5 = 6(x - 2)$$
$$y + 5 = 6x - 12$$
$$y = 6x - 17$$

3. $m = \frac{-6-(-1)}{-1-(-2)} = \frac{-5}{1} = -5$,

so the slope is –5.

Using the point (–2, –1), we get the following point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -5[x - (-2)]$$
$$y + 1 = -5(x + 2)$$

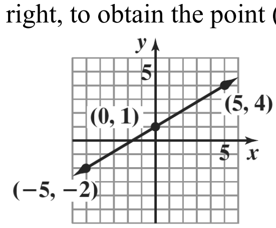
Using the point (–1, –6), we get the following point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - (-6) = -5[x - (-1)]$$
$$y + 6 = -5(x + 1)$$

Solve the equation for y:

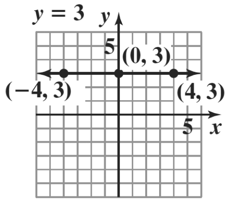
$$y + 1 = -5(x + 2)$$
$$y + 1 = -5x - 10$$
$$y = -5x - 11.$$

4. The slope m is $\frac{3}{5}$ and the y-intercept is 1, so one point on the line is (0, 1). We can find a second point on the line by using the slope $m = \frac{3}{5} = \frac{\text{Rise}}{\text{Run}}$: starting at the point (0, 1), move 3 units up and 5 units to the right, to obtain the point (5, 4).

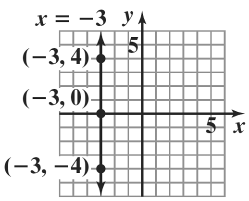


$f(x) = \frac{3}{5}x + 1$

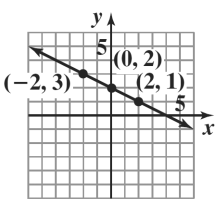
5. $y = 3$ is a horizontal line.



6. All ordered pairs that are solutions of $x = -3$ have a value of x that is always –3. Any value can be used for y.



7.
$$3x + 6y - 12 = 0$$
$$6y = -3x + 12$$
$$y = \frac{-3}{6}x + \frac{12}{6}$$
$$y = -\frac{1}{2}x + 2$$

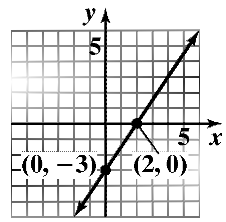


3x + 6y − 12 = 0

The slope is $-\frac{1}{2}$ and the y-intercept is 2.

8. Find the x-intercept:
$$3x - 2y - 6 = 0$$
$$3x - 2(0) - 6 = 0$$
$$3x - 6 = 0$$
$$3x = 6$$
$$x = 2$$

Find the y-intercept:
$$3x - 2y - 6 = 0$$
$$3(0) - 2y - 6 = 0$$
$$-2y - 6 = 0$$
$$-2y = 6$$
$$y = -3$$



3x − 2y = 6

9. First find the slope.
$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{57.64 - 57.04}{354 - 317} = \frac{0.6}{37} \approx 0.016$$
Use the point-slope form and then find slope-intercept form.
$$y - y_1 = m(x - x_1)$$
$$y - 57.04 = 0.016(x - 317)$$
$$y - 57.04 = 0.016x - 5.072$$
$$y = 0.016x + 51.968$$
$$f(x) = 0.016x + 52.0$$
Find the temperature at a concentration of 600 parts per million.
$$f(x) = 0.016x + 52.0$$
$$f(600) = 0.016(600) + 52.0$$
$$= 61.6$$
The temperature at a concentration of 600 parts per million would be 61.6°F.

Concept and Vocabulary Check 1.4

1. scatter plot; regression

2. $\frac{y_2 - y_1}{x_2 - x_1}$

3. positive

4. negative

5. zero

6. undefined

7. $y - y_1 = m(x - x_1)$

8. $y = mx + b$; slope; y-intercept

9. $(0, 3)$; 2; 5

10. horizontal

11. vertical

12. general

Exercise Set 1.4

1. $m = \frac{10-7}{8-4} = \frac{3}{4}$; rises

2. $m = \frac{4-1}{3-2} = \frac{3}{1} = 3$; rises

3. $m = \frac{2-1}{2-(-2)} = \frac{1}{4}$; rises

4. $m = \frac{4-3}{2-(-1)} = \frac{1}{3}$; rises

5. $m = \frac{2-(-2)}{3-4} = \frac{0}{-1} = 0$; horizontal

6. $m = \frac{-1-(-1)}{3-4} = \frac{0}{-1} = 0$; horizontal

7. $m = \frac{-1-4}{-1-(-2)} = \frac{-5}{1} = -5$; falls

8. $m = \frac{-2-(-4)}{4-6} = \frac{2}{-2} = -1$; falls

9. $m = \frac{-2-3}{5-5} = \frac{-5}{0}$ undefined; vertical

10. $m = \frac{5-(-4)}{3-3} = \frac{9}{0}$ undefined; vertical

11. $m = 2,\ x_1 = 3,\ y_1 = 5;$
point-slope form: $y - 5 = 2(x - 3)$;
slope-intercept form: $y - 5 = 2x - 6$
$y = 2x - 1$

12. point-slope form: $y - 3 = 4(x - 1)$;
$m = 4, x_1 = 1, y_1 = 3;$
slope-intercept form: $y = 4x - 1$

13. $m = 6,\ x_1 = -2,\ y_1 = 5;$
point-slope form: $y - 5 = 6(x + 2)$;
slope-intercept form: $y - 5 = 6x + 12$
$y = 6x + 17$

14. point-slope form: $y + 1 = 8(x - 4)$;
$m = 8, x_1 = 4, y_1 = -1;$
slope-intercept form: $y = 8x - 33$

15. $m = -3,\ x_1 = -2,\ y_1 = -3;$
point-slope form: $y + 3 = -3(x + 2)$;
slope-intercept form: $y + 3 = -3x - 6$
$y = -3x - 9$

16. point-slope form: $y + 2 = -5(x + 4)$;
$m = -5, x_1 = -4, y_1 = -2;$
slope-intercept form: $y = -5x - 22$

17. $m = -4,\ x_1 = -4,\ y_1 = 0;$
point-slope form: $y - 0 = -4(x + 4)$;
slope-intercept form: $y = -4(x + 4)$
$y = -4x - 16$

18. point-slope form: $y + 3 = -2(x - 0)$
$m = -2, x_1 = 0, y_1 = -3;$
slope-intercept form: $y = -2x - 3$

19. $m = -1,\ x_1 = \frac{-1}{2},\ y_1 = -2;$
point-slope form: $y + 2 = -1\left(x + \frac{1}{2}\right)$;
slope-intercept form: $y + 2 = -x - \frac{1}{2}$
$y = -x - \frac{5}{2}$

20. point-slope form: $y + \frac{1}{4} = -1(x + 4)$;
$m = -1, x_1 = -4, y_1 = -\frac{1}{4};$
slope-intercept form: $y = -x - \frac{17}{4}$

21. $m = \frac{1}{2},\ x_1 = 0,\ y_1 = 0;$
point-slope form: $y - 0 = \frac{1}{2}(x - 0)$;
slope-intercept form: $y = \frac{1}{2}x$

22. point-slope form: $y - 0 = \frac{1}{3}(x - 0)$;
$m = \frac{1}{3}, x_1 = 0, y_1 = 0;$
slope-intercept form: $y = \frac{1}{3}x$

23. $m=-\frac{2}{3},\ x_1=6,\ y_1=-2;$

point-slope form: $y+2=-\frac{2}{3}(x-6);$

slope-intercept form: $y+2=-\frac{2}{3}x+4$

$y=-\frac{2}{3}x+2$

24. point-slope form: $y+4=-\frac{3}{5}(x-10);$

$m=-\frac{3}{5}, x_1=10, y_1=-4;$

slope-intercept form: $y=-\frac{3}{5}x+2$

25. $m=\frac{10-2}{5-1}=\frac{8}{4}=2;$

point-slope form: $y-2=2(x-1)$ using $(x_1, y_1)=(1, 2)$, or $y-10=2(x-5)$ using $(x_1, y_1)=(5, 10);$

slope-intercept form: $y-2=2x-2$ or

$y-10=2x-10,$

$y=2x$

26. $m=\frac{15-5}{8-3}=\frac{10}{5}=2;$

point-slope form: $y-5=2(x-3)$ using $(x_1, y_1)=(3, 5)$, or $y-15=2(x-8)$ using $(x_1, y_1)=(8, 15);$

slope-intercept form: $y=2x-1$

27. $m=\frac{3-0}{0-(-3)}=\frac{3}{3}=1;$

point-slope form: $y-0=1(x+3)$ using $(x_1, y_1)=(-3, 0)$, or $y-3=1(x-0)$ using $(x_1, y_1)=(0, 3)$; slope-intercept form: $y=x+3$

28. $m=\frac{2-0}{0-(-2)}=\frac{2}{2}=1;$

point-slope form: $y-0=1(x+2)$ using $(x_1, y_1)=(-2, 0)$, or $y-2=1(x-0)$ using $(x_1, y_1)=(0, 2);$

slope-intercept form: $y=x+2$

29. $m=\frac{4-(-1)}{2-(-3)}=\frac{5}{5}=1;$

point-slope form: $y+1=1(x+3)$ using $(x_1, y_1)=(-3, -1)$, or $y-4=1(x-2)$ using $(x_1, y_1)=(2, 4)$; slope-intercept form:

$y+1=x+3$ or

$y-4=x-2$

$y=x+2$

30. $m=\frac{-1-(-4)}{1-(-2)}=\frac{3}{3}=1;$

point-slope form: $y+4=1(x+2)$ using $(x_1, y_1)=(-2, -4)$, or $y+1=1(x-1)$ using $(x_1, y_1)=(1, -1)$

slope-intercept form: $y=x-2$

31. $m=\frac{6-(-2)}{3-(-3)}=\frac{8}{6}=\frac{4}{3};$

point-slope form: $y+2=\frac{4}{3}(x+3)$ using

$(x_1, y_1)=(-3, -2)$, or $y-6=\frac{4}{3}(x-3)$ using

$(x_1, y_1)=(3, 6);$

slope-intercept form: $y+2=\frac{4}{3x}+4$ or

$y-6=\frac{4}{3}x-4,$

$y=\frac{4}{3}x+2$

32. $m=\frac{-2-6}{3-(-3)}=\frac{-8}{6}=-\frac{4}{3};$

point-slope form: $y-6=-\frac{4}{3}(x+3)$ using

$(x_1, y_1)=(-3, 6)$, or $y+2=-\frac{4}{3}(x-3)$ using

$(x_1, y_1)=(3, -2);$

slope-intercept form: $y=-\frac{4}{3}x+2$

33. $m=\frac{-1-(-1)}{4-(-3)}=\frac{0}{7}=0;$

point-slope form: $y+1=0(x+3)$ using $(x_1, y_1)=(-3, -1)$, or $y+1=0(x-4)$ using $(x_1, y_1)=(4, -1);$

slope-intercept form: $y+1=0,$ so

$y=-1$

34. $m = \dfrac{-5-(-5)}{6-(-2)} = \dfrac{0}{8} = 0$;

point-slope form: $y + 5 = 0(x + 2)$ using $(x_1, y_1) = (-2, -5)$, or $y + 5 = 0(x - 6)$ using $(x_1, y_1) = (6, -5)$;

slope-intercept form: $y + 5 = 0$, so $y = -5$

35. $m = \dfrac{0-4}{-2-2} = \dfrac{-4}{-4} = 1$;

point-slope form: $y - 4 = 1(x - 2)$ using $(x_1, y_1) = (2, 4)$, or $y - 0 = 1(x + 2)$ using $(x_1, y_1) = (-2, 0)$;

slope-intercept form: $y - 9 = x - 2$, or $y = x + 2$

36. $m = \dfrac{0-(-3)}{-1-1} = \dfrac{3}{-2} = -\dfrac{3}{2}$

point-slope form: $y + 3 = -\dfrac{3}{2}(x-1)$ using $(x_1, y_1) = (1, -3)$, or $y - 0 = -\dfrac{3}{2}(x+1)$ using $(x_1, y_1) = (-1, 0)$;

slope-intercept form: $y + 3 = -\dfrac{3}{2}x + \dfrac{3}{2}$, or $y = -\dfrac{3}{2}x - \dfrac{3}{2}$

37. $m = \dfrac{4-0}{0-\left(-\frac{1}{2}\right)} = \dfrac{4}{\frac{1}{2}} = 8$;

point-slope form: $y - 4 = 8(x - 0)$ using $(x_1, y_1) = (0, 4)$, or $y - 0 = 8\left(x + \frac{1}{2}\right)$ using $(x_1, y_1) = \left(-\frac{1}{2}, 0\right)$; or $y - 0 = 8\left(x + \frac{1}{2}\right)$

slope-intercept form: $y = 8x + 4$

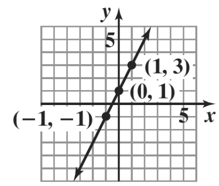
38. $m = \dfrac{-2-0}{0-4} = \dfrac{-2}{-4} = \dfrac{1}{2}$;

point-slope form: $y - 0 = \dfrac{1}{2}(x-4)$ using $(x_1, y_1) = (4, 0)$,

or $y + 2 = \dfrac{1}{2}(x-0)$ using $(x_1, y_1) = (0, -2)$;

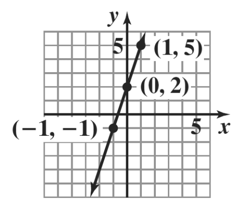
slope-intercept form: $y = \dfrac{1}{2}x - 2$

39. $m = 2$; $b = 1$



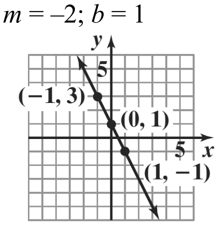
$y = 2x + 1$

40. $m = 3$; $b = 2$



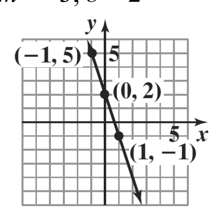
$y = 3x + 2$

41. $m = -2$; $b = 1$



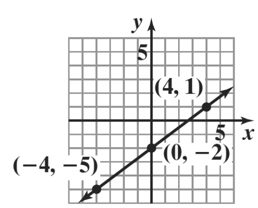
$f(x) = -2x + 1$

42. $m = -3$; $b = 2$



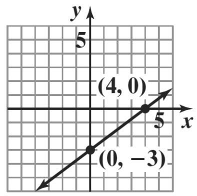
$f(x) = -3x + 2$

43. $m = \dfrac{3}{4}$; $b = -2$



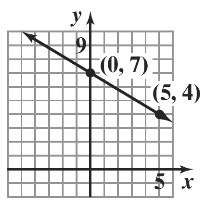
$f(x) = \dfrac{3}{4}x - 2$

44. $m = \frac{3}{4}$; $b = -3$



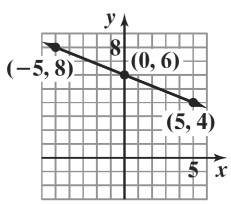
$f(x) = \frac{3}{4}x - 3$

45. $m = -\frac{3}{5}$; $b = 7$



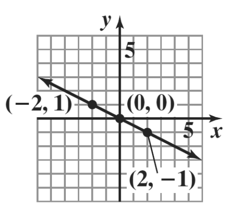
$y = -\frac{3}{5}x + 7$

46. $m = -\frac{2}{5}$; $b = 6$



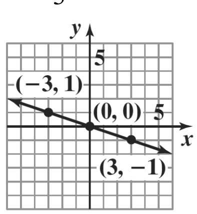
$y = -\frac{2}{5}x + 6$

47. $m = -\frac{1}{2}$; $b = 0$



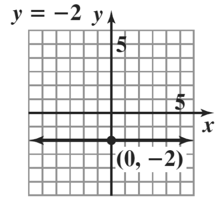
$g(x) = -\frac{1}{2}x$

48. $m = -\frac{1}{3}$; $b = 0$



$g(x) = -\frac{1}{3}x$

49.



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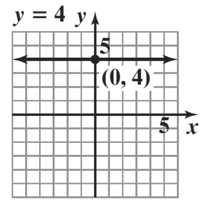

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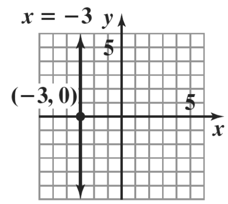

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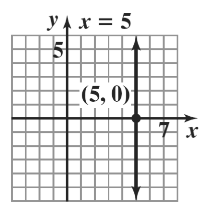

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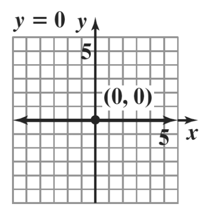

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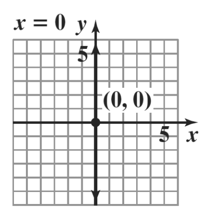

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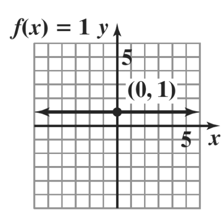

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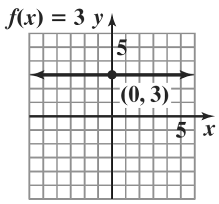
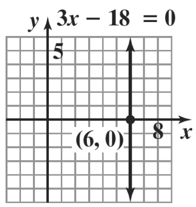


57. $3x-18=0$

$3x=18$

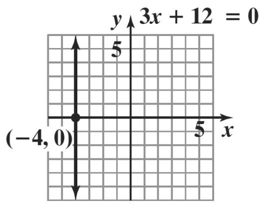
$x=6$



58. $3x+12=0$

$3x=-12$

$x=-4$



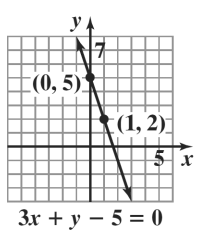
59. **a.** $3x+y-5=0$

$y-5=-3x$

$y=-3x+5$

b. $m=-3$; $b=5$

c.



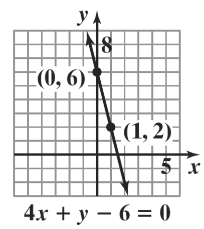
60. **a.** $4x+y-6=0$

$y-6=-4x$

$y=-4x+6$

b. $m=-4; b=6$

c.



61. **a.** $2x+3y-18=0$

$2x-18=-3y$

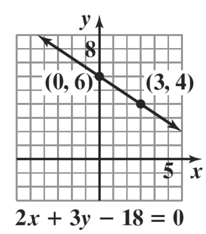
$-3y=2x-18$

$$y=\frac{2}{-3}x-\frac{18}{-3}$$

$$y=-\frac{2}{3}x+6$$

b. $m=-\frac{2}{3};\ b=6$

c.



62. **a.** $4x+6y+12=0$

$4x+12=-6y$

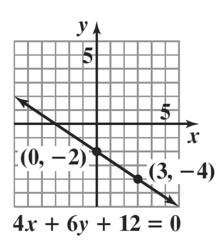
$-6y=4x+12$

$$y=\frac{4}{-6}x+\frac{12}{-6}$$

$$y=-\frac{2}{3}x-2$$

b. $m=-\frac{2}{3};\ b=-2$

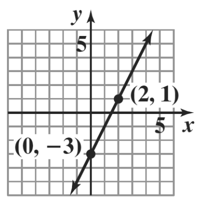
c.



63. a.
$$8x-4y-12=0$$
$$8x-12=4y$$
$$4y=8x-12$$
$$y=\frac{8}{4}x-\frac{12}{4}$$
$$y=2x-3$$

b. $m=2;\ b=-3$

c.

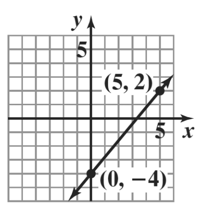


8x − 4y − 12 = 0

64. a.
$$6x-5y-20=0$$
$$6x-20=5y$$
$$5y=6x-20$$
$$y=\frac{6}{5}x-\frac{20}{5}$$
$$y=\frac{6}{5}x-4$$

b. $m=\frac{6}{5};b=-4$

c.

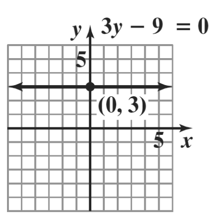


6x − 5y − 20 = 0

65. a.
$$3y-9=0$$
$$3y=9$$
$$y=3$$

b. $m=0;\ b=3$

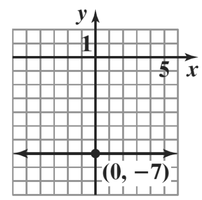
c.



66. a.
$$4y+28=0$$
$$4y=-28$$
$$y=-7$$

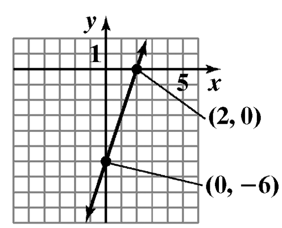
b. $m=0;\ b=-7$

c.



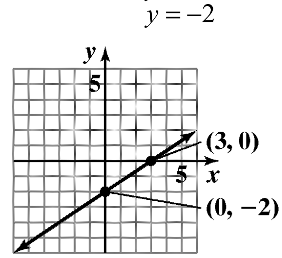
4y + 28 = 0

67. Find the x-intercept:
$$6x-2y-12=0$$
$$6x-2(0)-12=0$$
$$6x-12=0$$
$$6x=12$$
$$x=2$$
Find the y-intercept:
$$6x-2y-12=0$$
$$6(0)-2y-12=0$$
$$-2y-12=0$$
$$-2y=12$$
$$y=-6$$



6x − 2y − 12 = 0

68. Find the x-intercept:
$$6x-9y-18=0$$
$$6x-9(0)-18=0$$
$$6x-18=0$$
$$6x=18$$
$$x=3$$
Find the y-intercept:
$$6x-9y-18=0$$
$$6(0)-9y-18=0$$
$$-9y-18=0$$
$$-9y=18$$
$$y=-2$$



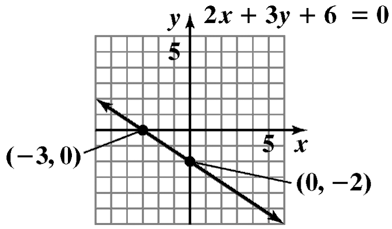
6x − 9y − 18 = 0

69. Find the x-intercept:

$$\begin{aligned} 2x+3y+6&=0\\ 2x+3(0)+6&=0\\ 2x+6&=0\\ 2x&=-6\\ x&=-3 \end{aligned}$$

Find the y-intercept:

$$\begin{aligned} 2x+3y+6&=0\\ 2(0)+3y+6&=0\\ 3y+6&=0\\ 3y&=-6\\ y&=-2 \end{aligned}$$

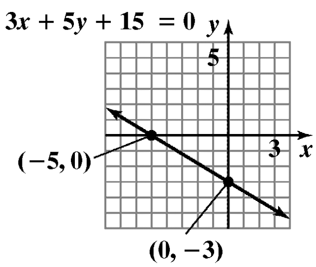


70. Find the x-intercept:

$$\begin{aligned} 3x+5y+15&=0\\ 3x+5(0)+15&=0\\ 3x+15&=0\\ 3x&=-15\\ x&=-5 \end{aligned}$$

Find the y-intercept:

$$\begin{aligned} 3x+5y+15&=0\\ 3(0)+5y+15&=0\\ 5y+15&=0\\ 5y&=-15\\ y&=-3 \end{aligned}$$

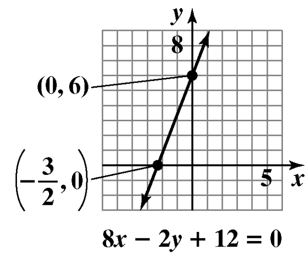


71. Find the x-intercept:

$$\begin{aligned} 8x-2y+12&=0\\ 8x-2(0)+12&=0\\ 8x+12&=0\\ 8x&=-12\\ \frac{8x}{8}&=\frac{-12}{8}\\ x&=\frac{-3}{2} \end{aligned}$$

Find the y-intercept:

$$\begin{aligned} 8x-2y+12&=0\\ 8(0)-2y+12&=0\\ -2y+12&=0\\ -2y&=-12\\ y&=-6 \end{aligned}$$

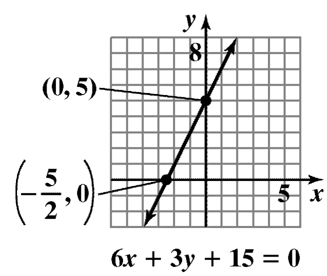


72. Find the x-intercept:

$$\begin{aligned} 6x-3y+15&=0\\ 6x-3(0)+15&=0\\ 6x+15&=0\\ 6x&=-15\\ \frac{6x}{6}&=\frac{-15}{6}\\ x&=-\frac{5}{2} \end{aligned}$$

Find the y-intercept:

$$\begin{aligned} 6x-3y+15&=0\\ 6(0)-3y+15&=0\\ -3y+15&=0\\ -3y&=-15\\ y&=5 \end{aligned}$$



73. $$m=\frac{0-a}{b-0}=\frac{-a}{b}=-\frac{a}{b}$$

Since a and b are both positive, $-\frac{a}{b}$ is negative. Therefore, the line falls.

74. $$m=\frac{-b-0}{0-(-a)}=\frac{-b}{a}=-\frac{b}{a}$$

Since a and b are both positive, $-\frac{b}{a}$ is negative. Therefore, the line falls.

75. $m = \frac{(b+c)-b}{a-a} = \frac{c}{0}$

The slope is undefined.
The line is vertical.

76. $m = \frac{(a+c)-c}{a-(a-b)} = \frac{a}{b}$

Since a and b are both positive, $\frac{a}{b}$ is positive.

Therefore, the line rises.

77. $Ax + By = C$
$By = -Ax + C$
$y = -\frac{A}{B}x + \frac{C}{B}$

The slope is $-\frac{A}{B}$ and the $y-$intercept is $\frac{C}{B}$.

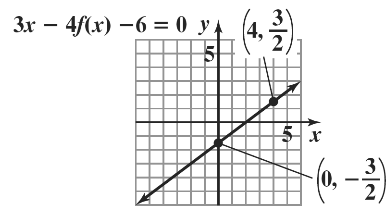
78. $Ax = By - C$
$Ax + C = By$
$\frac{A}{B}x + \frac{C}{B} = y$

The slope is $\frac{A}{B}$ and the $y-$intercept is $\frac{C}{B}$.

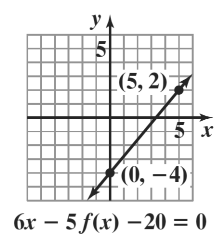
79.
$-3 = \frac{4-y}{1-3}$
$-3 = \frac{4-y}{-2}$
$6 = 4 - y$
$2 = -y$
$-2 = y$

80. $\frac{1}{3} = \frac{-4-y}{4-(-2)}$
$\frac{1}{3} = \frac{-4-y}{4+2}$
$\frac{1}{3} = \frac{-4-y}{6}$
$6 = 3(-4-y)$
$6 = -12 - 3y$
$18 = -3y$
$-6 = y$

81. $3x - 4f(x) = 6$
$-4f(x) = -3x + 6$
$f(x) = \frac{3}{4}x - \frac{3}{2}$



82. $6x - 5f(x) = 20$
$-5f(x) = -6x + 20$
$f(x) = \frac{6}{5}x - 4$



83. Using the slope-intercept form for the equation of a line:
$-1 = -2(3) + b$
$-1 = -6 + b$
$5 = b$

84. $-6 = -\frac{3}{2}(2) + b$
$-6 = -3 + b$
$-3 = b$

85. m_1, m_3, m_2, m_4

86. b_2, b_1, b_4, b_3

87. **a.** First, find the slope using $(20, 38.9)$ and $(30, 47.8)$.

$m = \frac{47.8 - 38.9}{30 - 20} = \frac{8.9}{10} = 0.89$

Then use the slope and one of the points to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$
$y - 47.8 = 0.89(x - 30)$
or
$y - 38.9 = 0.89(x - 20)$

b. $y-47.8=0.89(x-30)$
$y-47.8=0.89x-26.7$
$y=0.89x+21.1$
$f(x)=0.89x+21.1$

c. $f(40)=0.89(40)+21.1=56.7$

The linear function predicts the percentage of never married American females, ages 25 – 29, to be 56.7% in 2020.

88. **a.** First, find the slope using $(20,51.7)$ and $(30,62.6)$.

$$m=\frac{51.7-62.6}{20-30}=\frac{-10.9}{-10}=1.09$$

Then use the slope and one of the points to write the equation in point-slope form.

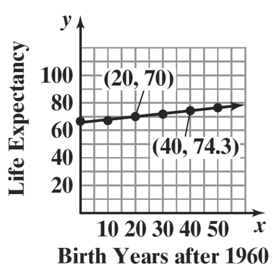
$y-y_1=m(x-x_1)$
$y-62.6=1.09(x-30)$
or
$y-51.7=1.09(x-20)$

b. $y-62.6=1.09(x-30)$
$y-62.6=1.09x-32.7$
$y=1.09x+29.9$
$f(x)=1.09x+29.9$

c. $f(35)=1.09(35)+29.9=68.05$

The linear function predicts the percentage of never married American males, ages 25 – 29, to be 68.05% in 2015.

89. **a.**



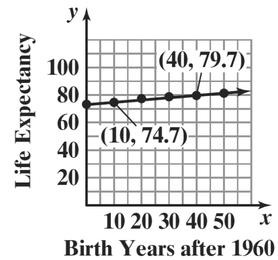
b. $m=\frac{\text{Change in } y}{\text{Change in } x}=\frac{74.3-70.0}{40-20}=0.215$

$y-y_1=m(x-x_1)$
$y-70.0=0.215(x-20)$
$y-70.0=0.215x-4.3$
$y=0.215x+65.7$
$E(x)=0.215x+65.7$

c. $E(x)=0.215x+65.7$
$E(60)=0.215(60)+65.7$
$=78.6$

The life expectancy of American men born in 2020 is expected to be 78.6.

90. **a.**



b. $m=\frac{\text{Change in } y}{\text{Change in } x}=\frac{79.7-74.7}{40-10}\approx 0.17$

$y-y_1=m(x-x_1)$
$y-74.7=0.17(x-10)$
$y-74.7=0.17x-1.7$
$y=0.17x+73$
$E(x)=0.17x+73$

c. $E(x)=0.17x+73$
$E(60)=0.17(60)+73$
$=83.2$

The life expectancy of American women born in 2020 is expected to be 83.2.

91. (10, 230) (60, 110) Points may vary.

$$m=\frac{110-230}{60-10}=-\frac{120}{50}=-2.4$$

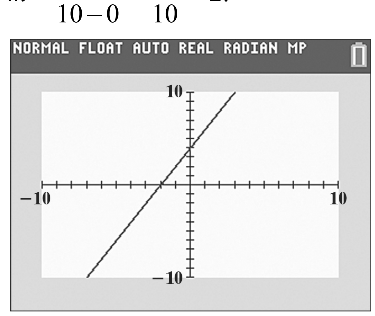
$y-230=-2.4(x-10)$
$y-230=-2.4x+24$
$y=-2.4x+254$

Answers will vary for predictions.

92. – 99. Answers will vary.

100. Two points are (0,4) and (10,24).

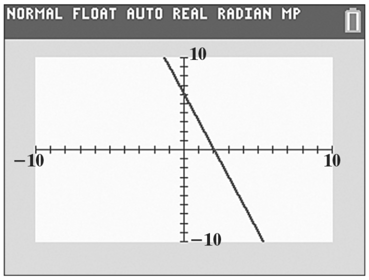
$$m=\frac{24-4}{10-0}=\frac{20}{10}=2.$$



101. Two points are (0, 6) and (10, –24).

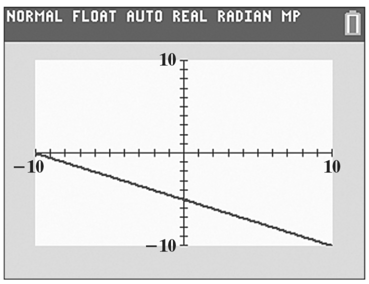
$m = \dfrac{-24-6}{10-0} = \dfrac{-30}{10} = -3.$

Check: $y = mx + b$: $y = -3x + 6$.



102. Two points are (0,–5) and (10,–10).

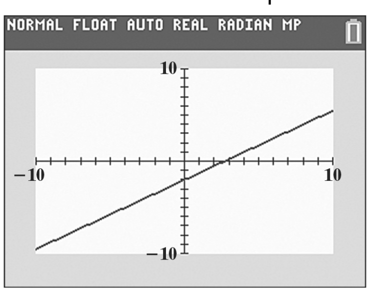
$m = \dfrac{-10-(-5)}{10-0} = \dfrac{-5}{10} = -\dfrac{1}{2}.$



103. Two points are (0, –2) and (10, 5.5).

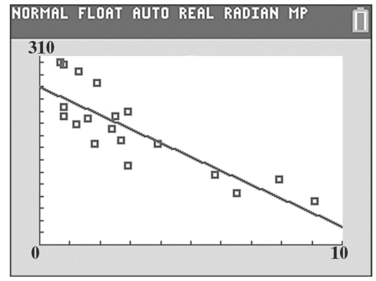
$m = \dfrac{5.5-(-2)}{10-0} = \dfrac{7.5}{10} = 0.75 \text{ or } \dfrac{3}{4}.$

Check: $y = mx + b$: $y = \dfrac{3}{4}x - 2$.



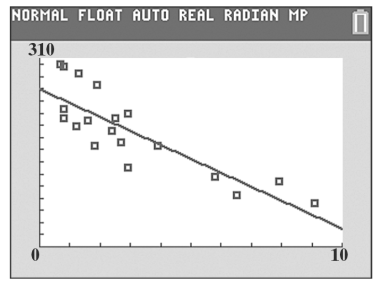
104. a. Enter data from table.

b.



c. $a = -22.96876741$
$b = 260.5633751$
$r = -0.8428126855$

d.



105. does not make sense; Explanations will vary. Sample explanation: Linear functions never change from increasing to decreasing.

106. does not make sense; Explanations will vary. Sample explanation: Since college cost are going up, this function has a positive slope.

107. does not make sense; Explanations will vary. Sample explanation: The slope of line's whose equations are in this form can be determined in several ways. One such way is to rewrite the equation in slope-intercept form.

108. makes sense

109. false; Changes to make the statement true will vary. A sample change is: It is possible for *m* to equal *b*.

110. false; Changes to make the statement true will vary. A sample change is: Slope-intercept form is $y = mx + b$. Vertical lines have equations of the form $x = a$. Equations of this form have undefined slope and cannot be written in slope-intercept form.

111. true

112. false; Changes to make the statement true will vary. A sample change is: The graph of $x = 7$ is a vertical line through the point (7, 0).

113. We are given that the $x-$intercept is -2 and the $y-$intercept is 4. We can use the points $(-2,0)$ and $(0,4)$ to find the slope.

$$m=\frac{4-0}{0-(-2)}=\frac{4}{0+2}=\frac{4}{2}=2$$

Using the slope and one of the intercepts, we can write the line in point-slope form.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-0&=2(x-(-2))\\ y&=2(x+2)\\ y&=2x+4\\ -2x+y&=4 \end{aligned}$$

Find the *x*– and *y*–coefficients for the equation of the line with right-hand-side equal to 12. Multiply both sides of $-2x+y=4$ by 3 to obtain 12 on the right-hand-side.

$$\begin{aligned} -2x+y&=4\\ 3(-2x+y)&=3(4)\\ -6x+3y&=12 \end{aligned}$$

Therefore, the coefficient of x is –6 and the coefficient of y is 3.

114. We are given that the $y-$intercept is -6 and the slope is $\frac{1}{2}$.

So the equation of the line is $y=\frac{1}{2}x-6$.

We can put this equation in the form $ax+by=c$ to find the missing coefficients.

$$\begin{aligned} y&=\frac{1}{2}x-6\\ y-\frac{1}{2}x&=-6\\ 2\left(y-\frac{1}{2}x\right)&=2(-6)\\ 2y-x&=-12\\ x-2y&=12 \end{aligned}$$

Therefore, the coefficient of x is 1 and the coefficient of y is –2.

115. Answers will vary.

116. Let (25, 40) and (125, 280) be ordered pairs (*M*, *E*) where *M* is degrees Madonna and *E* is degrees Elvis. Then

$m=\frac{280-40}{125-25}=\frac{240}{100}=2.4$. Using $(x_1,y_1)=(25,40)$, point-slope form tells us that

$E - 40 = 2.4\ (M - 25)$ or

$E = 2.4\ M - 20$.

117. Answers will vary.

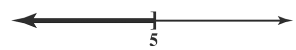
118. Let x = the number of years after 1994.

$$\begin{aligned} 714-17x&=289\\ -17x&=-425\\ x&=25 \end{aligned}$$

Violent crime incidents will decrease to 289 per 100,000 people 25 years after 1994, or 2019.

119.

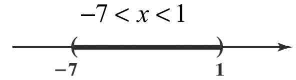
$$\begin{aligned} \frac{x+3}{4}&\geq\frac{x-2}{3}+1\\ 12\left(\frac{x+3}{4}\right)&\geq 12\left(\frac{x-2}{3}+1\right)\\ 3(x+3)&\geq 4(x-2)+12\\ 3x+9&\geq 4x-8+12\\ 3x+9&\geq 4x+4\\ 5&\geq x\\ x&\leq 5 \end{aligned}$$



The solution set is $\{x|x\leq 5\}$ or $(-\infty,5]$.

120. $3|2x+6|-9<15$

$$\begin{aligned} 3|2x+6|&<24\\ \frac{3|2x+6|}{3}&<\frac{24}{3}\\ |2x+6|&<8\\ -8<2x&+6<8\\ -14<&2x<2\\ -7<&x<1 \end{aligned}$$



The solution set is $\{x|-7<x<1\}$ or $(-7,1)$.

121. Since the slope is the same as the slope of $y=2x+1$, then $m=2$.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-1&=2(x-(-3))\\ y-1&=2(x+3)\\ y-1&=2x+6\\ y&=2x+7 \end{aligned}$$

122. Since the slope is the negative reciprocal of $-\frac{1}{4}$, then $m = 4$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= 4(x - 3) \\ y + 5 &= 4x - 12 \\ -4x + y + 17 &= 0 \\ 4x - y - 17 &= 0 \end{aligned}$$

123. $$\begin{aligned} \frac{f(x_2) - f(x_1)}{x_2 - x_1} &= \frac{f(4) - f(1)}{4 - 1} \\ &= \frac{4^2 - 1^2}{4 - 1} \\ &= \frac{15}{3} \\ &= 5 \end{aligned}$$

Section 1.5

Check Point Exercises

1. The slope of the line $y = 3x + 1$ is 3.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= 3(x - (-2)) \\ y - 5 &= 3(x + 2) \quad \text{point-slope} \\ y - 5 &= 3x + 6 \\ y &= 3x + 11 \quad \text{slope-intercept} \end{aligned}$$

2. **a.** Write the equation in slope-intercept form:

$$\begin{aligned} x + 3y - 12 &= 0 \\ 3y &= -x + 12 \\ y &= -\frac{1}{3}x + 4 \end{aligned}$$

The slope of this line is $-\frac{1}{3}$ thus the slope of any line perpendicular to this line is 3.

b. Use $m = 3$ and the point (–2, –6) to write the equation.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-6) &= 3(x - (-2)) \\ y + 6 &= 3(x + 2) \\ y + 6 &= 3x + 6 \\ -3x + y &= 0 \\ 3x - y &= 0 \quad \text{general form} \end{aligned}$$

3. $$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{15 - 11.2}{2013 - 2000} = \frac{3.8}{13} \approx 0.29$$

The slope indicates that the number of U.S. men living alone increased at a rate of 0.29 million each year.
The rate of change is 0.29 million men per year.

4. **a.** $$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{1^3 - 0^3}{1 - 0} = 1$$

b. $$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{2^3 - 1^3}{2 - 1} = \frac{8 - 1}{1} = 7$$

c. $$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0^3 - (-2)^3}{0 - (-2)} = \frac{8}{2} = 4$$

5. $$\begin{aligned} \frac{f(x_2) - f(x_1)}{x_2 - x_1} &= \frac{f(3) - f(1)}{3 - 1} \\ &= \frac{0.05 - 0.03}{3 - 1} \\ &= 0.01 \end{aligned}$$

The average rate of change in the drug's concentration between 1 hour and 3 hours is 0.01 mg per 100 mL per hour.

6. **a.** $s(1) = 4(1)^2 = 4$

$s(2) = 4(2)^2 = 16$

$\frac{\Delta s}{\Delta t} = \frac{16 - 4}{2 - 1} = 12$ feet per second

b. $s(1) = 4(1)^2 = 4$

$s(1.5) = 4(1.5)^2 = 9$

$\frac{\Delta s}{\Delta t} = \frac{9 - 4}{1.5 - 1} = 10$ feet per second

c. $s(1) = 4(1)^2 = 4$

$s(1.01) = 4(1.01)^2 = 4.0804$

$\frac{\Delta s}{\Delta t} = \frac{4.0804 - 4}{1.01 - 1} = 8.04$ feet per second

Concept and Vocabulary Check 1.5

1. the same

2. -1

3. $-\frac{1}{3}$; 3

4. -2; $\frac{1}{2}$

5. y; x

6. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

7. $\dfrac{s(6)-s(3)}{6-3}$

Exercise Set 1.5

1. Since L is parallel to $y = 2x$, we know it will have slope $m = 2$. We are given that it passes through $(4, 2)$. We use the slope and point to write the equation in point-slope form.
$y - y_1 = m(x - x_1)$
$y - 2 = 2(x - 4)$
Solve for y to obtain slope-intercept form.
$y - 2 = 2(x - 4)$
$y - 2 = 2x - 8$
$y = 2x - 6$
In function notation, the equation of the line is $f(x) = 2x - 6.$

2. L will have slope $m = -2$. Using the point and the slope, we have $y - 4 = -2(x - 3)$. Solve for y to obtain slope-intercept form.
$y - 4 = -2x + 6$
$y = -2x + 10$
$f(x) = -2x + 10$

3. Since L is perpendicular to $y = 2x$, we know it will have slope $m = -\frac{1}{2}$. We are given that it passes through $(2, 4)$. We use the slope and point to write the equation in point-slope form.
$y - y_1 = m(x - x_1)$
$y - 4 = -\frac{1}{2}(x - 2)$
Solve for y to obtain slope-intercept form.
$y - 4 = -\frac{1}{2}(x - 2)$
$y - 4 = -\frac{1}{2}x + 1$
$y = -\frac{1}{2}x + 5$
In function notation, the equation of the line is $f(x) = -\frac{1}{2}x + 5.$

4. L will have slope $m = \frac{1}{2}$. The line passes through $(-1, 2)$. Use the slope and point to write the equation in point-slope form.
$y - 2 = \frac{1}{2}(x - (-1))$
$y - 2 = \frac{1}{2}(x + 1)$
Solve for y to obtain slope-intercept form.
$y - 2 = \frac{1}{2}x + \frac{1}{2}$
$y = \frac{1}{2}x + \frac{1}{2} + 2$
$y = \frac{1}{2}x + \frac{5}{2}$
$f(x) = \frac{1}{2}x + \frac{5}{2}$

5. $m = -4$ since the line is parallel to $y = -4x + 3$; $x_1 = -8$, $y_1 = -10$;
point-slope form: $y + 10 = -4(x + 8)$
slope-intercept form: $y + 10 = -4x - 32$
$y = -4x - 42$

6. $m = -5$ since the line is parallel to $y = -5x + 4$;
$x_1 = -2, y_1 = -7$;
point-slope form: $y + 7 = -5(x + 2)$
slope-intercept form: $y + 7 = -5x - 10$
$y = -5x - 17$

7. $m = -5$ since the line is perpendicular to $y = \frac{1}{5}x + 6$; $x_1 = 2$, $y_1 = -3$;
point-slope form: $y + 3 = -5(x - 2)$
slope-intercept form: $y + 3 = -5x + 10$
$y = -5x + 7$

8. $m = -3$ since the line is perpendicular to $y = \frac{1}{3}x + 7$;
$x_1 = -4, y_1 = 2$;
point-slope form: $y - 2 = -3(x + 4)$
slope-intercept form: $y - 2 = -3x - 12$
$y = -3x - 10$

9. $2x-3y-7=0$

$-3y=-2x+7$

$y=\frac{2}{3}x-\frac{7}{3}$

The slope of the given line is $\frac{2}{3}$, so $m=\frac{2}{3}$ since the lines are parallel.

point-slope form: $y-2=\frac{2}{3}(x+2)$

general form: $2x-3y+10=0$

10. $3x-2y-=0$

$-2y=-3x+5$

$y=\frac{3}{2}x-\frac{5}{2}$

The slope of the given line is $\frac{3}{2}$, so $m=\frac{3}{2}$ since the lines are parallel.

point-slope form: $y-3=\frac{3}{2}(x+1)$

general form: $3x-2y+9=0$

11. $x-2y-3=0$

$-2y=-x+3$

$y=\frac{1}{2}x-\frac{3}{2}$

The slope of the given line is $\frac{1}{2}$, so $m = -2$ since the lines are perpendicular.

point-slope form: $y+7=-2(x-4)$

general form: $2x+y-1=0$

12. $x+7y-12=0$

$7y=-x+12$

$y=\frac{-1}{7}x+\frac{12}{7}$

The slope of the given line is $-\frac{1}{7}$, so $m = 7$ since the lines are perpendicular.

point-slope form: $y + 9 = 7(x - 5)$

general form: $7x-y-44=0$

13. $\frac{15-0}{5-0}=\frac{15}{5}=3$

14. $\frac{24-0}{4-0}=\frac{24}{4}=6$

15. $\frac{5^2+2\cdot 5-(3^2+2\cdot 3)}{5-3}=\frac{25+10-(9+6)}{2}$

$=\frac{20}{2}$

$=10$

16. $\frac{6^2-2(6)-(3^2-2\cdot 3)}{6-3}=\frac{36-12-(9-6)}{3}=\frac{21}{3}=7$

17. $\frac{\sqrt{9}-\sqrt{4}}{9-4}=\frac{3-2}{5}=\frac{1}{5}$

18. $\frac{\sqrt{16}-\sqrt{9}}{16-9}=\frac{4-3}{7}=\frac{1}{7}$

19. **a.** $s(3)=10(3)^2=90$

$s(4)=10(4)^2=160$

$\frac{\Delta s}{\Delta t}=\frac{160-90}{4-3}=70$ feet per second

b. $s(3)=10(3)^2=90$

$s(3.5)=10(3.5)^2=122.5$

$\frac{\Delta s}{\Delta t}=\frac{122.5-90}{3.5-3}=65$ feet per second

c. $s(3)=10(3)^2=90$

$s(3.01)=10(3.01)^2=90.601$

$\frac{\Delta s}{\Delta t}=\frac{90.601-90}{3.01-3}=60.1$ feet per second

d. $s(3)=10(3)^2=90$

$s(3.001)=10(3.001)^2=90.06$

$\frac{\Delta s}{\Delta t}=\frac{90.06-90}{3.001-3}=60.01$ feet per second

20. **a.** $s(3)=12(3)^2=108$

$s(4)=12(4)^2=192$

$\frac{\Delta s}{\Delta t}=\frac{108-192}{4-3}=84$ feet per second

b. $s(3)=12(3)^2=108$

$s(3.5)=12(3.5)^2=147$

$\frac{\Delta s}{\Delta t}=\frac{147-108}{3.5-2}=78$ feet per second

c. $s(3) = 12(3)^2 = 108$

$s(3.01) = 12(3.01)^2 = 108.7212$

$\frac{\Delta s}{\Delta t} = \frac{108.7212 - 108}{3.01 - 3} = 72.12$ feet per second

d. $s(3) = 12(3)^2 = 108$

$s(3.001) = 12(3.001)^2 = 108.07201$

$\frac{\Delta s}{\Delta t} = \frac{108.07201 - 108}{3.001 - 3} = 72.01$ feet per second

21. Since the line is perpendicular to $x = 6$ which is a vertical line, we know the graph of f is a horizontal line with 0 slope. The graph of f passes through $(-1, 5)$, so the equation of f is $f(x) = 5$.

22. Since the line is perpendicular to $x = -4$ which is a vertical line, we know the graph of f is a horizontal line with 0 slope. The graph of f passes through $(-2, 6)$, so the equation of f is $f(x) = 6$.

23. First we need to find the equation of the line with $x-$intercept of 2 and $y-$intercept of -4. This line will pass through $(2, 0)$ and $(0, -4)$. We use these points to find the slope.

$m = \frac{-4 - 0}{0 - 2} = \frac{-4}{-2} = 2$

Since the graph of f is perpendicular to this line, it will have slope $m = -\frac{1}{2}$.

Use the point $(-6, 4)$ and the slope $-\frac{1}{2}$ to find the equation of the line.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 4 &= -\frac{1}{2}(x - (-6)) \\
y - 4 &= -\frac{1}{2}(x + 6) \\
y - 4 &= -\frac{1}{2}x - 3 \\
y &= -\frac{1}{2}x + 1 \\
f(x) &= -\frac{1}{2}x + 1
\end{aligned}$$

24. First we need to find the equation of the line with $x-$intercept of 3 and $y-$intercept of -9. This line will pass through $(3, 0)$ and $(0, -9)$. We use these points to find the slope.

$m = \frac{-9 - 0}{0 - 3} = \frac{-9}{-3} = 3$

Since the graph of f is perpendicular to this line, it will have slope $m = -\frac{1}{3}$.

Use the point $(-5, 6)$ and the slope $-\frac{1}{3}$ to find the equation of the line.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 6 &= -\frac{1}{3}(x - (-5)) \\
y - 6 &= -\frac{1}{3}(x + 5) \\
y - 6 &= -\frac{1}{3}x - \frac{5}{3} \\
y &= -\frac{1}{3}x + \frac{13}{3} \\
f(x) &= -\frac{1}{3}x + \frac{13}{3}
\end{aligned}$$

25. First put the equation $3x - 2y - 4 = 0$ in slope-intercept form.

$$\begin{aligned}
3x - 2y - 4 &= 0 \\
-2y &= -3x + 4 \\
y &= \frac{3}{2}x - 2
\end{aligned}$$

The equation of f will have slope $-\frac{2}{3}$ since it is perpendicular to the line above and the same $y-$intercept -2.

So the equation of f is $f(x) = -\frac{2}{3}x - 2$.

26. First put the equation $4x - y - 6 = 0$ in slope-intercept form.

$$\begin{aligned}
4x - y - 6 &= 0 \\
-y &= -4x + 6 \\
y &= 4x - 6
\end{aligned}$$

The equation of f will have slope $-\frac{1}{4}$ since it is perpendicular to the line above and the same $y-$intercept -6.

So the equation of f is $f(x) = -\frac{1}{4}x - 6$.

27. $p(x) = -0.25x + 22$

28. $p(x) = 0.22x + 3$

29. $m = \frac{1163 - 617}{1998 - 1994} = \frac{546}{4} \approx 137$

There was an average increase of approximately 137 discharges per year.

30. $m = \frac{623 - 1273}{2006 - 2001} = \frac{-650}{5} \approx -130$

There was an average decrease of approximately 130 discharges per year.

31. **a.** $f(x) = 1.1x^3 - 35x^2 + 264x + 557$

$f(0) = 1.1(0)^3 - 35(0)^2 + 264(0) + 557 = 557$

$f(4) = 1.1(4)^3 - 35(4)^2 + 264(4) + 557 = 1123.4$

$m = \frac{1123.4 - 557}{4 - 0} \approx 142$

b. This overestimates by 5 discharges per year.

32. **a.** $f(x) = 1.1x^3 - 35x^2 + 264x + 557$

$f(0) = 1.1(7)^3 - 35(7)^2 + 264(7) + 557 = 1067.3$

$f(12) = 1.1(12)^3 - 35(12)^2 + 264(12) + 557 = 585.8$

$m = \frac{585.8 - 1067.3}{12 - 7} \approx -96$

b. This underestimates the decrease by 34 discharges per year.

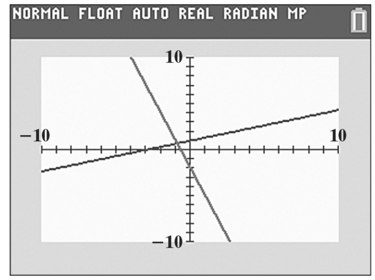
33. – 38. Answers will vary.

39. $y = \frac{1}{3}x + 1$

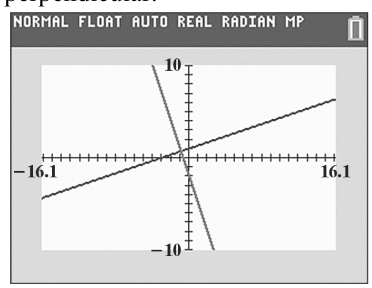
$y = -3x - 2$

a. The lines are perpendicular because their slopes are negative reciprocals of each other. This is verified because product of their slopes is –1.

b. The lines do not appear to be perpendicular.



c. The lines appear to be perpendicular. The calculator screen is rectangular and does not have the same width and height. This causes the scale of the *x*–axis to differ from the scale on the *y*–axis despite using the same scale in the window settings. In part (b), this causes the lines not to appear perpendicular when indeed they are. The zoom square feature compensates for this and in part (c), the lines appear to be perpendicular.



40. does not make sense; Explanations will vary. Sample explanation: Perpendicular lines have slopes with opposite signs.

41. makes sense

42. does not make sense; Explanations will vary. Sample explanation: Slopes can be used for segments of the graph.

43. makes sense

44. Write $Ax + By + C = 0$ in slope-intercept form.

$$Ax + By + C = 0$$
$$By = -Ax - C$$
$$\frac{By}{B} = \frac{-Ax}{B} - \frac{C}{B}$$
$$y = -\frac{A}{B}x - \frac{C}{B}$$

The slope of the given line is $-\frac{A}{B}$.

The slope of any line perpendicular to $Ax + By + C = 0$ is $\frac{B}{A}$.

45. The slope of the line containing $(1,-3)$ and $(-2,4)$ has slope $m=\dfrac{4-(-3)}{-2-1}=\dfrac{4+3}{-3}=\dfrac{7}{-3}=-\dfrac{7}{3}$

Solve $Ax+y-2=0$ for y to obtain slope-intercept form.

$$Ax+y-2=0$$
$$y=-Ax+2$$

So the slope of this line is $-A$.

This line is perpendicular to the line above so its slope is $\dfrac{3}{7}$. Therefore, $-A=\dfrac{3}{7}$ so $A=-\dfrac{3}{7}$.

46.
$$24+3(x+2)=5(x-12)$$
$$24+3x+6=5x-60$$
$$3x+30=5x-60$$
$$90=2x$$
$$45=x$$

The solution set is $\{45\}$.

47. Let x = the television's price before the reduction.

$$x-0.30x=980$$
$$0.70x=980$$
$$x=\frac{980}{0.70}$$
$$x=1400$$

Before the reduction the television's price was $1400.

48.
$$\sqrt{x+7}+5=x$$
$$\sqrt{x+7}=x-5$$
$$x+7=x^2-10x+25$$
$$0=x^2-11x+18$$
$$0=(x-9)(x-2)$$

$x-9=0$ $x-2=0$

$x=9$ $x=2$

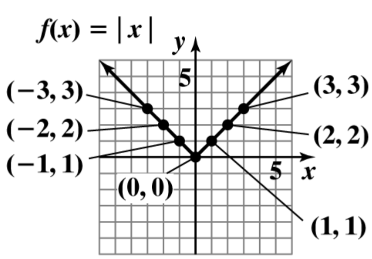
$\sqrt{9+7}+5=9$ $\sqrt{2+7}+5=2$

$\sqrt{16}+5=9$ $\sqrt{9}+5=2$

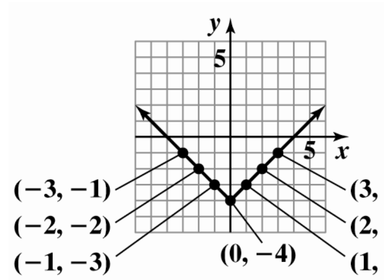
$9=9$ $8=2$ False

The solution set is $\{9\}$.

49. **a.**

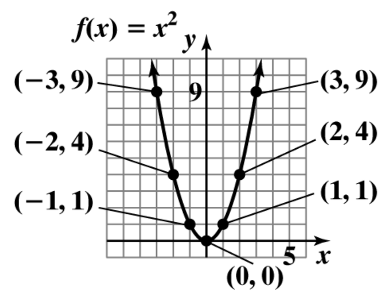


b.

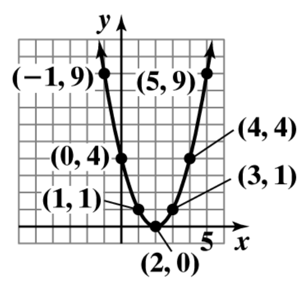


c. The graph in part (b) is the graph in part (a) shifted down 4 units.

50. **a.**

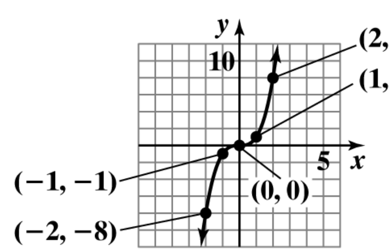


b.

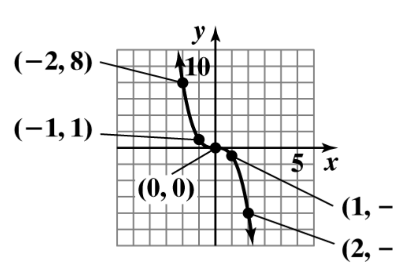


c. The graph in part (b) is the graph in part (a) shifted to the right 2 units.

51. **a.**



b.



c. The graph in part (b) is the graph in part (a) reflected across the *y*-axis.

Mid-Chapter 1 Check Point

1. The relation is not a function.
The domain is $\{1, 2\}$.
The range is $\{-6, 4, 6\}$.

2. The relation is a function.
The domain is $\{0, 2, 3\}$.
The range is $\{1, 4\}$.

3. The relation is a function.
The domain is $\{x \mid -2 \le x < 2\}$.
The range is $\{y \mid 0 \le y \le 3\}$.

4. The relation is not a function.
The domain is $\{x \mid -3 < x \le 4\}$.
The range is $\{y \mid -1 \le y \le 2\}$.

5. The relation is not a function.
The domain is $\{-2, -1, 0, 1, 2\}$.
The range is $\{-2, -1, 1, 3\}$.

6. The relation is a function.
The domain is $\{x \mid x \le 1\}$.
The range is $\{y \mid y \ge -1\}$.

7. $x^2 + y = 5$

$y = -x^2 + 5$

For each value of x, there is one and only one value for y, so the equation defines y as a function of x.

8. $x + y^2 = 5$

$y^2 = 5 - x$

$y = \pm\sqrt{5 - x}$

Since there are values of x that give more than one value for y (for example, if $x = 4$, then $y = \pm\sqrt{5-4} = \pm 1$), the equation does not define y as a function of x.

9. No vertical line intersects the graph in more than one point. Each value of x corresponds to exactly one value of y.

10. Domain: $(-\infty, \infty)$

11. Range: $(-\infty, 4]$

12. x-intercepts: –6 and 2

13. y-intercept: 3

14. increasing: $(-\infty, -2)$

15. decreasing: $(-2, \infty)$

16. $x = -2$

17. $f(-2) = 4$

18. $f(-4) = 3$

19. $f(-7) = -2$ and $f(3) = -2$

20. $f(-6) = 0$ and $f(2) = 0$

21. $(-6, 2)$

22. $f(100)$ is negative.

23. neither; $f(-x) \ne x$ and $f(-x) \ne -x$

24. $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(-4)}{4 - (-4)} = \frac{-5 - 3}{4 + 4} = -1$

25. Test for symmetry with respect to the y-axis.

$x = y^2 + 1$

$-x = y^2 + 1$

$x = -y^2 - 1$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$x = y^2 + 1$

$x = (-y)^2 + 1$

$x = y^2 + 1$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$x = y^2 + 1$

$-x = (-y)^2 + 1$

$-x = y^2 + 1$

$x = -y^2 - 1$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

26. Test for symmetry with respect to the y-axis.

$y = x^3 - 1$
$y = (-x)^3 - 1$
$y = -x^3 - 1$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$y = x^3 - 1$
$-y = x^3 - 1$
$y = -x^3 + 1$

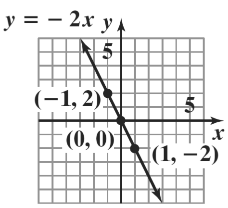
The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

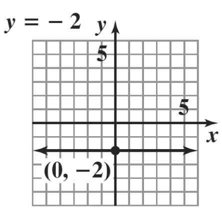
$y = x^3 - 1$
$-y = (-x)^3 - 1$
$-y = -x^3 - 1$
$y = x^3 + 1$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

27.



28.



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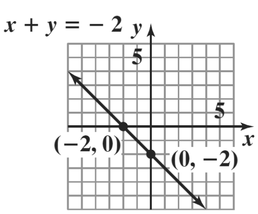
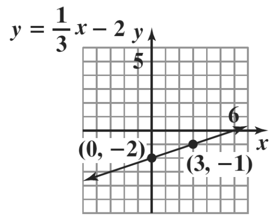


30.



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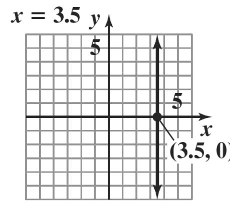
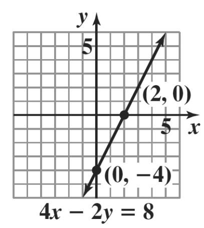


32.



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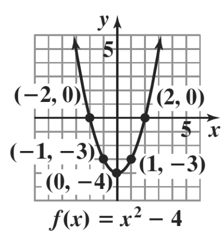
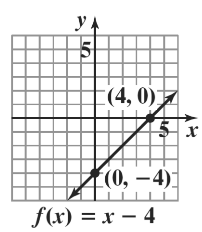
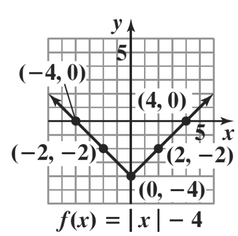


34.

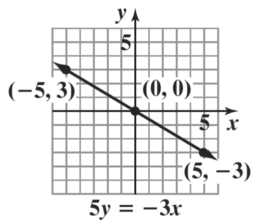


35.



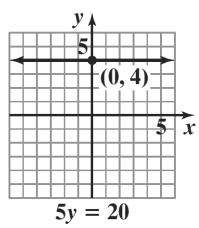
36. $5y = -3x$

$y = -\frac{3}{5}x$

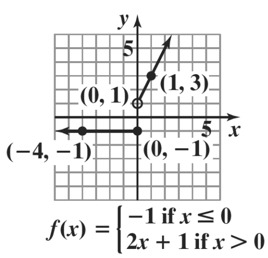


37. $5y = 20$

$y = 4$



38.



39. **a.** $f(-x) = -2(-x)^2 - x - 5$

$= -2x^2 - x - 5$

neither; $f(-x) \neq x$ and $f(-x) \neq -x$

b. $\frac{f(x+h) - f(x)}{h}$

$= \frac{-2(x+h)^2 + (x+h) - 5 - (-2x^2 + x - 5)}{h}$

$= \frac{-2x^2 - 4xh - 2h^2 + x + h - 5 + 2x^2 - x + 5}{h}$

$= \frac{-4xh - 2h^2 + h}{h}$

$= \frac{h(-4x - 2h + 1)}{h}$

$= -4x - 2h + 1$

40. $C(x) = \begin{cases} 30 & \text{if} \quad 0 \leq t \leq 200 \\ 30 + 0.40(t - 200) & \text{if} \quad t > 200 \end{cases}$

a. $C(150) = 30$

b. $C(250) = 30 + 0.40(250 - 200) = 50$

41. $y - y_1 = m(x - x_1)$

$y - 3 = -2(x - (-4))$

$y - 3 = -2(x + 4)$

$y - 3 = -2x - 8$

$y = -2x - 5$

$f(x) = -2x - 5$

42. $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{1 - (-5)}{2 - (-1)} = \frac{6}{3} = 2$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$y = 2x - 3$

$f(x) = 2x - 3$

43. $3x - y - 5 = 0$

$-y = -3x + 5$

$y = 3x - 5$

The slope of the given line is 3, and the lines are parallel, so $m = 3$.

$y - y_1 = m(x - x_1)$

$y - (-4) = 3(x - 3)$

$y + 4 = 3x - 9$

$y = 3x - 13$

$f(x) = 3x - 13$

44. $2x - 5y - 10 = 0$

$-5y = -2x + 10$

$\frac{-5y}{-5} = \frac{-2x}{-5} + \frac{10}{-5}$

$y = \frac{2}{5}x - 2$

The slope of the given line is $\frac{2}{5}$, and the lines are perpendicular, so $m = -\frac{5}{2}$.

$y - y_1 = m(x - x_1)$

$y - (-3) = -\frac{5}{2}(x - (-4))$

$y + 3 = -\frac{5}{2}x - 10$

$y = -\frac{5}{2}x - 13$

$f(x) = -\frac{5}{2}x - 13$

45. $m_1 = \frac{\text{Change in } y}{\text{Change in } x} = \frac{0 - (-4)}{7 - 2} = \frac{4}{5}$

$m_2 = \frac{\text{Change in } y}{\text{Change in } x} = \frac{6 - 2}{1 - (-4)} = \frac{4}{5}$

The slope of the lines are equal thus the lines are parallel.

46. **a.** $m = \dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{42-26}{180-80} = \dfrac{16}{100} = 0.16$

b. For each minute of brisk walking, the percentage of patients with depression in remission increased by 0.16%. The rate of change is 0.16% per minute of brisk walking.

47. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1} = \dfrac{f(2)-f(-1)}{2-(-1)}$

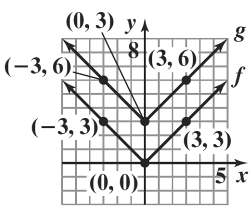
$= \dfrac{\left(3(2)^2-2\right)-\left(3(-1)^2-(-1)\right)}{2+1}$

$= 2$

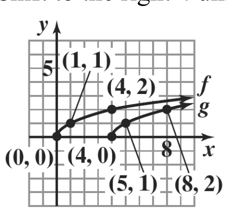
Section 1.6

Check Point Exercises

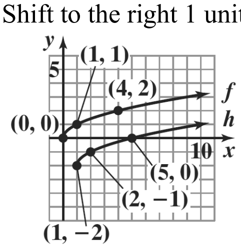
1. Shift up vertically 3 units.



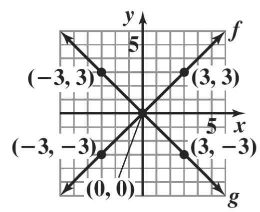
2. Shift to the right 4 units.



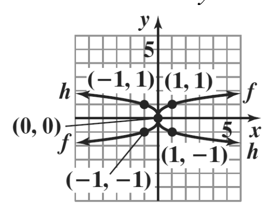
3. Shift to the right 1 unit and down 2 units.



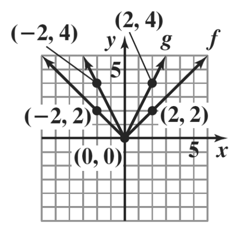
4. Reflect about the x-axis.



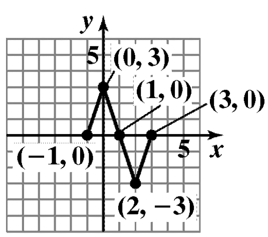
5. Reflect about the y-axis.



6. Vertically stretch the graph of $f(x) = |x|$.

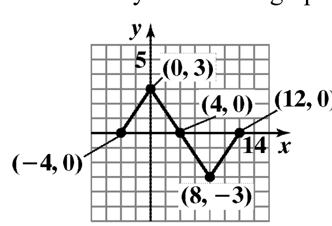


7. **a.** Horizontally shrink the graph of $y = f(x)$.



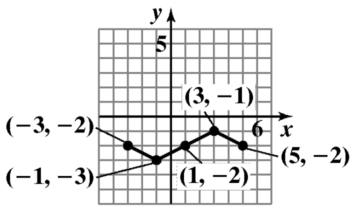
$g(x) = f(2x)$

b. Horizontally stretch the graph of $y = f(x)$.



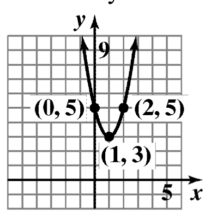
$h(x) = f\left(\dfrac{1}{2}x\right)$

8. The graph of $y = f(x)$ is shifted 1 unit left, shrunk by a factor of $\frac{1}{3}$, reflected about the x-axis, then shifted down 2 units.



$y = -\frac{1}{3}f(x+1) - 2$

9. The graph of $f(x) = x^2$ is shifted 1 unit right, stretched by a factor of 2, then shifted up 3 units.



$g(x) = 2(x-1)^2 + 3$

Concept and Vocabulary Check 1.6

1. vertical; down

2. horizontal; to the right

3. x-axis

4. y-axis

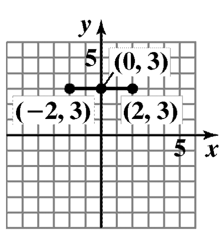
5. vertical; y

6. horizontal; x

7. false

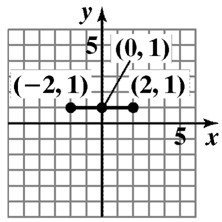
Exercise Set 1.6

1.



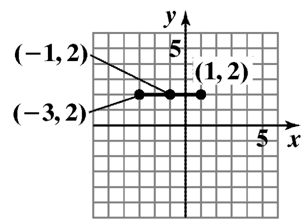
$g(x) = f(x) + 1$

2.



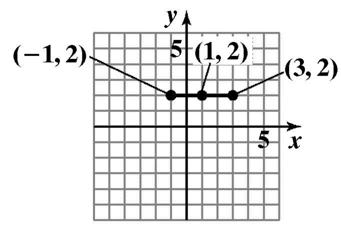
$g(x) = f(x) - 1$

3.



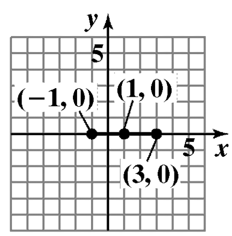
$g(x) = f(x+1)$

4.



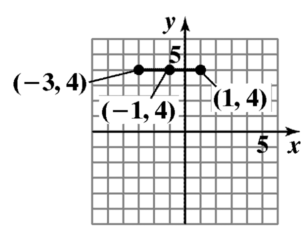
$g(x) = f(x-1)$

5.



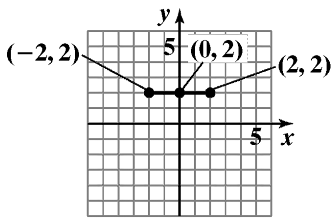
$g(x) = f(x-1) - 2$

6.



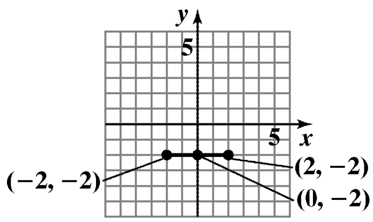
$g(x) = f(x+1) + 2$

7.



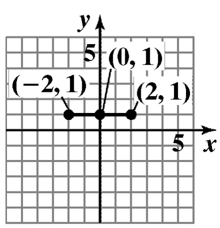
$g(x) = -f(x)$

8.



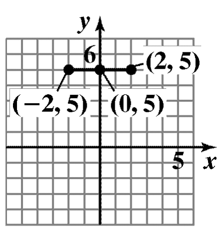
$g(x) = -f(x)$

9.



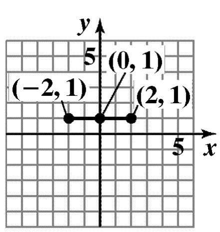
$g(x) = -f(x) + 3$

10.



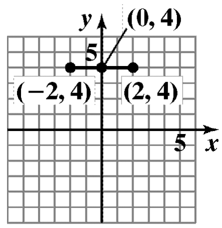
$g(x) = f(-x) + 3$

11.



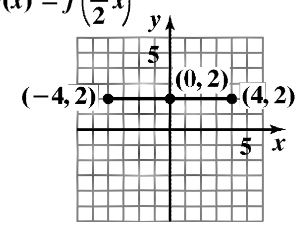
$g(x) = \frac{1}{2}f(x)$

12.

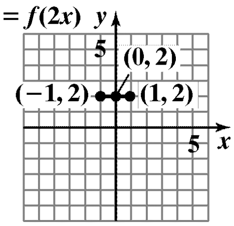


$g(x) = 2f(x)$

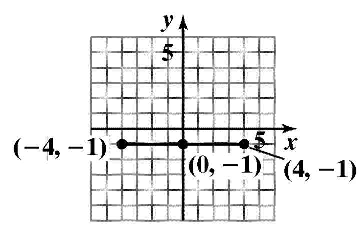
13. $g(x) = f\left(\frac{1}{2}x\right)$



14. $g(x) = f(2x)$

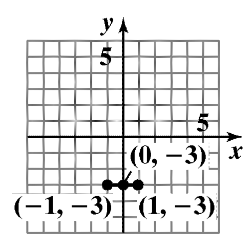


15.



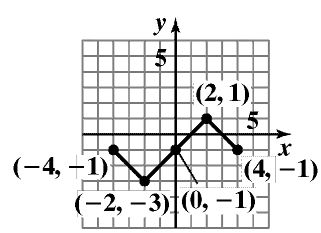
$g(x) = -f\left(\frac{1}{2}x\right) + 1$

16.



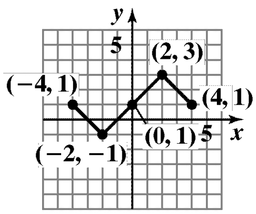
$g(x) = -f(2x) - 1$

17.



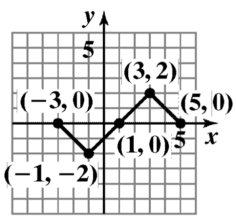
$g(x) = f(x) - 1$

18.



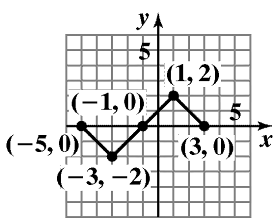
$g(x) = f(x) + 1$

19.



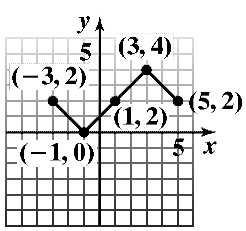
$g(x) = f(x - 1)$

20.



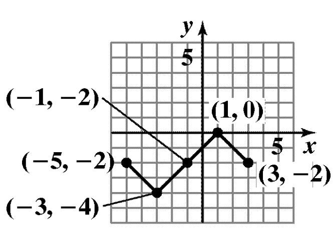
$g(x) = f(x + 1)$

21.



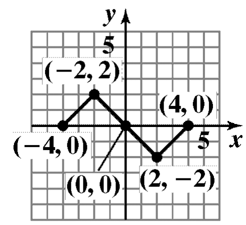
$g(x) = f(x - 1) + 2$

22.



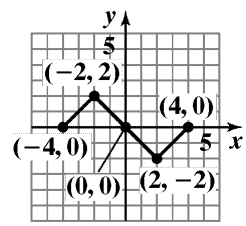
$g(x) = f(x + 1) - 2$

23.



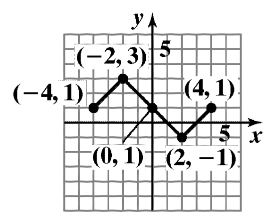
$g(x) = -f(x)$

24.



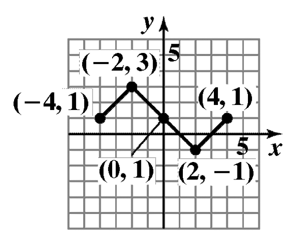
$g(x) = f(-x)$

25.



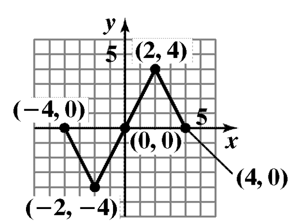
$g(x) = f(-x) + 1$

26.



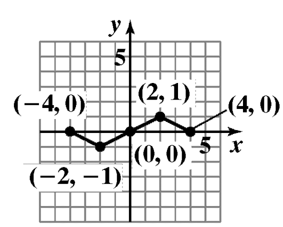
$g(x) = -f(x) + 1$

27.



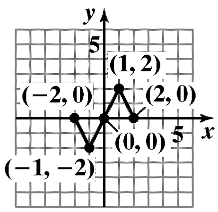
$g(x) = 2f(x)$

28.



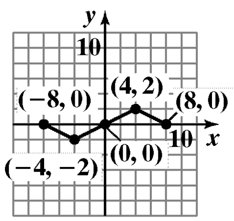
$g(x) = \frac{1}{2}f(x)$

29.



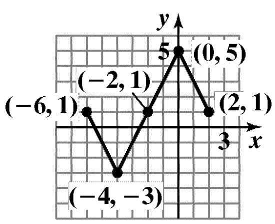
$g(x) = f(2x)$

30.



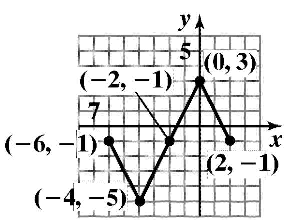
$g(x) = f\left(\frac{1}{2}x\right)$

31.



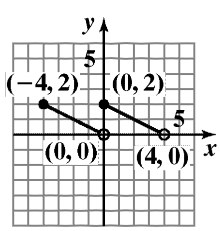
$g(x) = 2f(x + 2) + 1$

32.



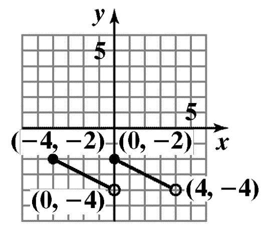
$g(x) = 2f(x + 2) - 1$

33.



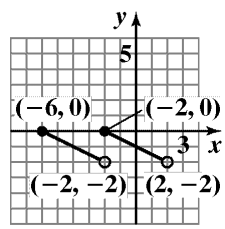
$g(x) = f(x) + 2$

34.



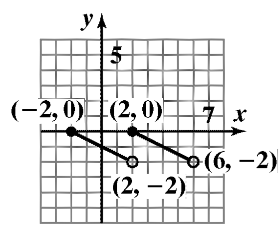
$g(x) = f(x) - 2$

35.



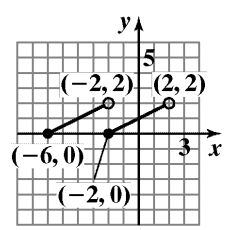
$g(x) = f(x + 2)$

36.



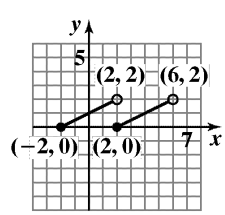
$g(x) = f(x - 2)$

37.



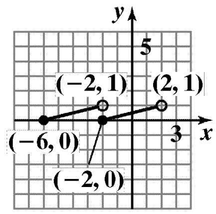
$g(x) = -f(x + 2)$

38.



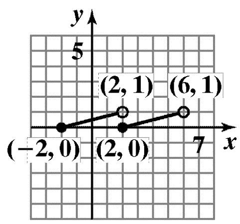
$g(x) = -f(x - 2)$

39.



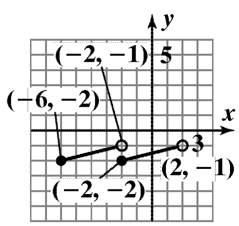
$g(x) = -\frac{1}{2}f(x + 2)$

40.



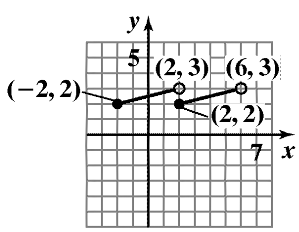
$g(x) = -\frac{1}{2}f(x - 2)$

41.



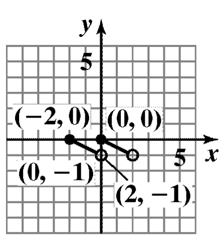
$g(x) = -\frac{1}{2}f(x + 2) - 2$

42.



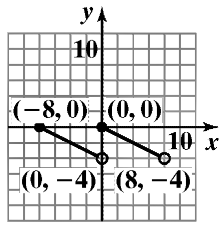
$g(x) = -\frac{1}{2}f(x - 2) + 2$

43.



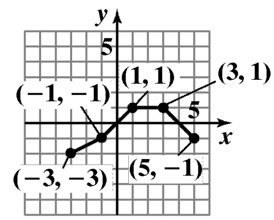
$g(x) = \frac{1}{2}f(2x)$

44.



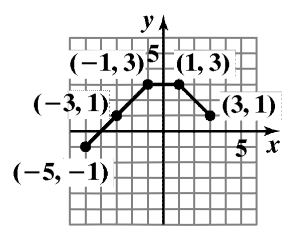
$g(x) = 2f\left(\frac{1}{2}x\right)$

45.



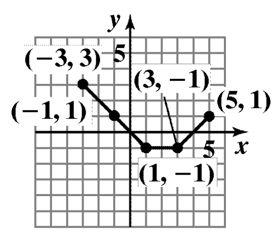
$g(x) = f(x - 1) - 1$

46.



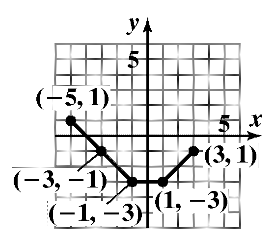
$g(x) = f(x + 1) + 1$

47.



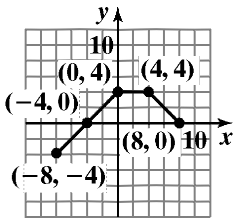
$g(x) = -f(x - 1) + 1$

48.



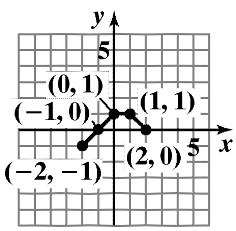
$g(x) = -f(x + 1) - 1$

49.



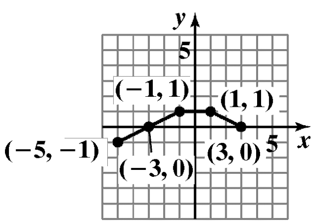
$g(x) = 2f\left(\frac{1}{2}x\right)$

50.



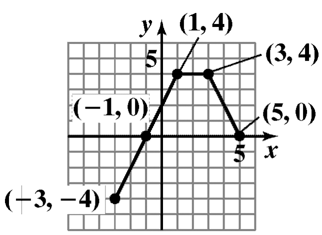
$g(x) = \frac{1}{2}f(2x)$

51.



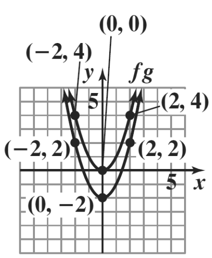
$g(x) = \frac{1}{2}f(x + 1)$

52.



$g(x) = 2f(x - 1)$

53.



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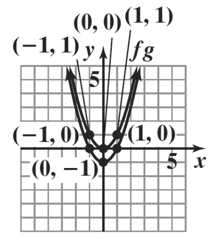

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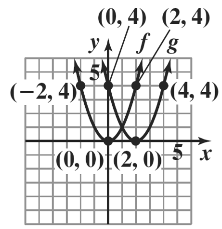

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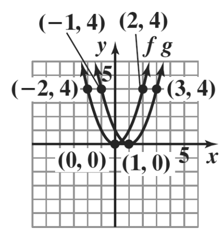

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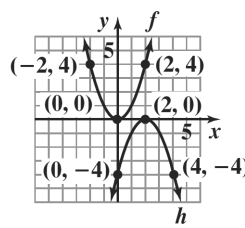

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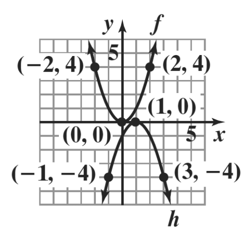

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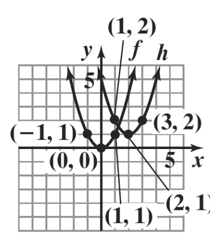

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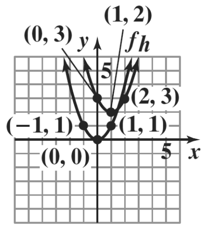

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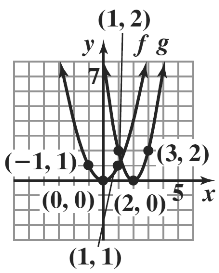

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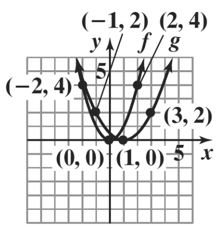

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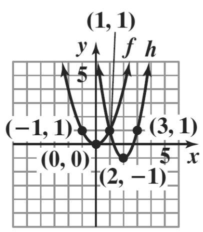

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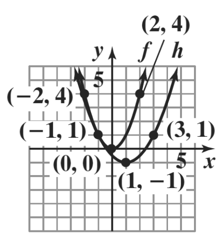

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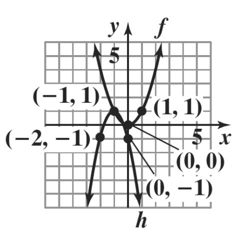

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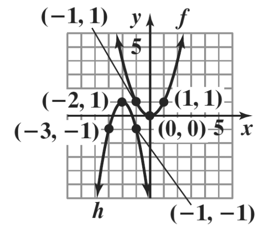

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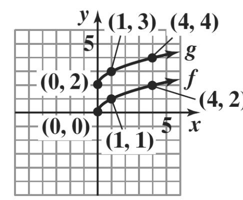

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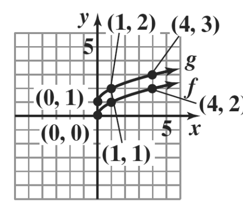

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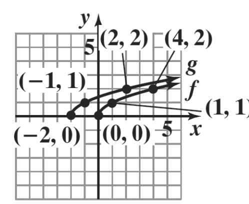

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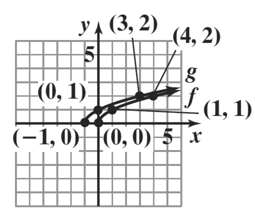

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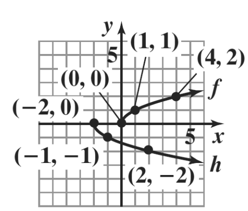

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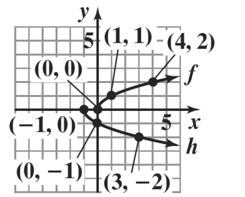

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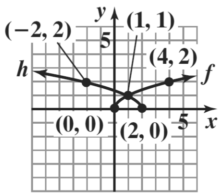

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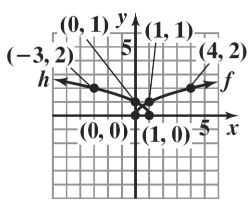

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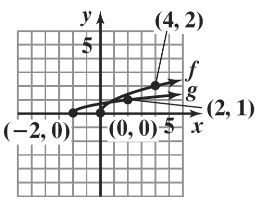

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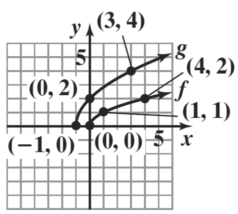

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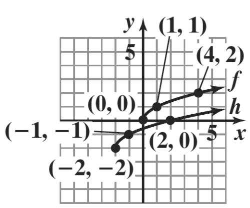

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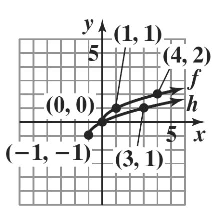

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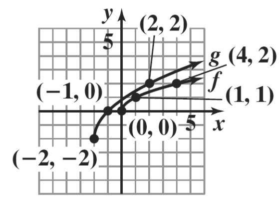

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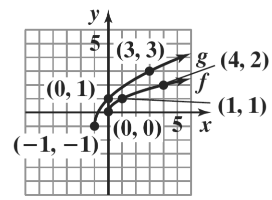

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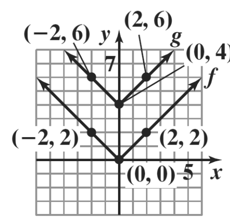

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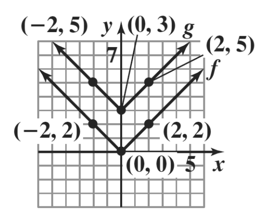

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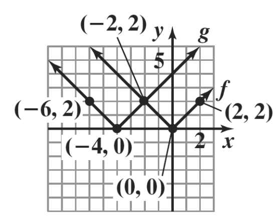

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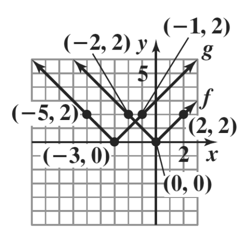

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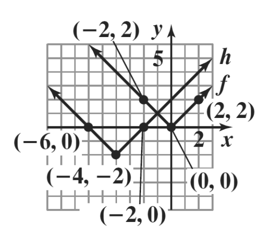

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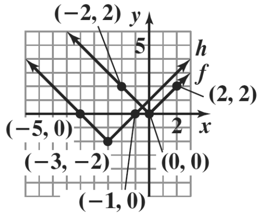

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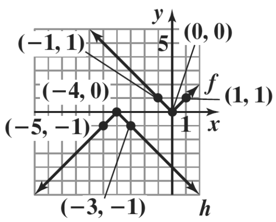

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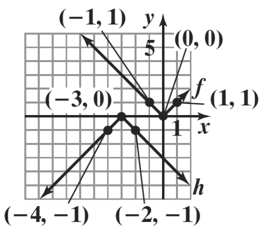

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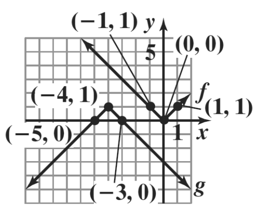

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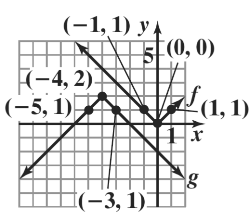

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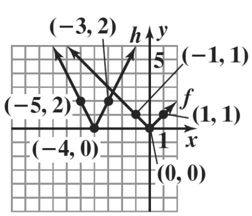

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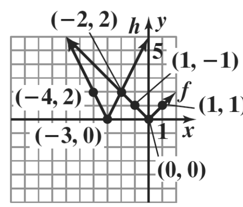

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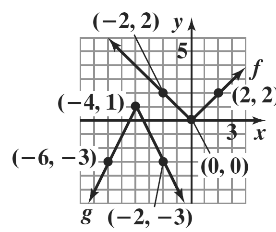

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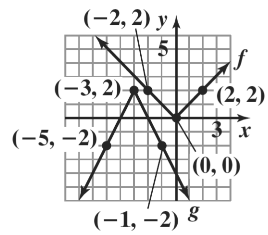

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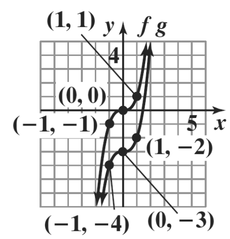

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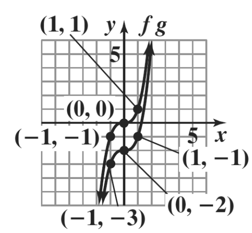

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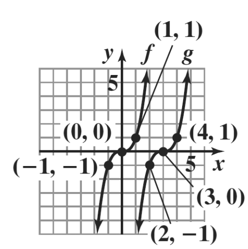

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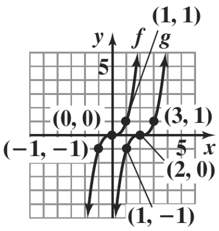

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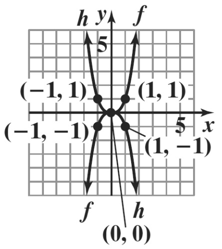

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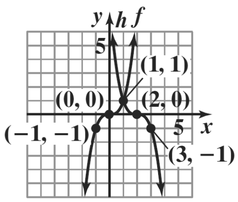

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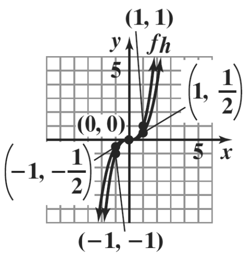

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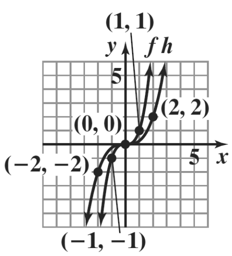

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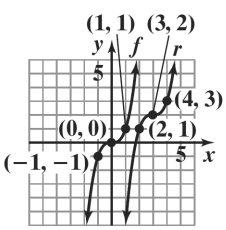

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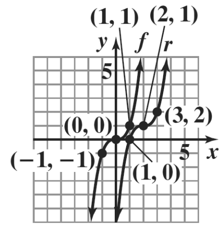

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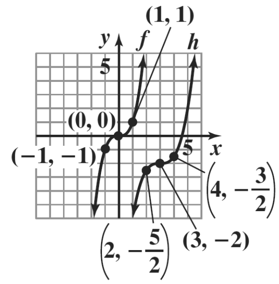

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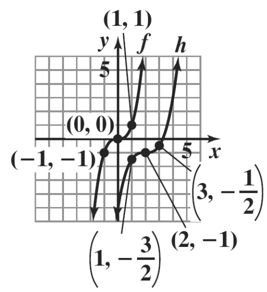

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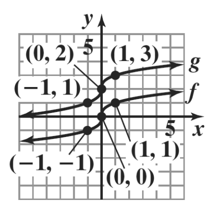

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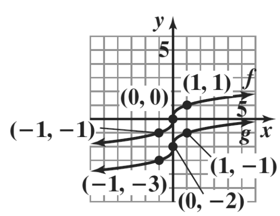

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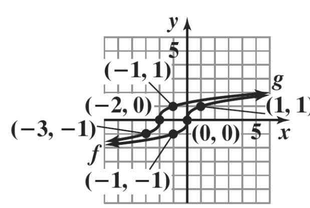

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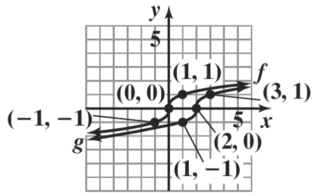

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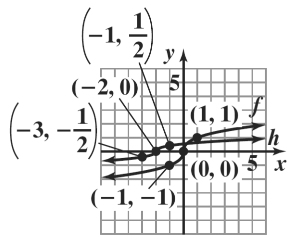

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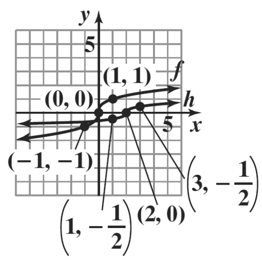

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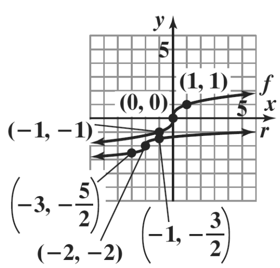

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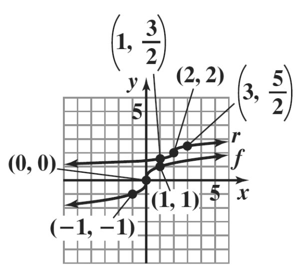

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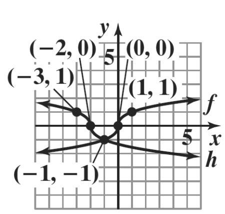

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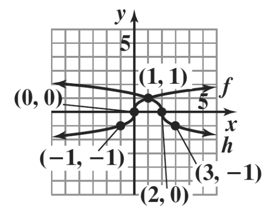

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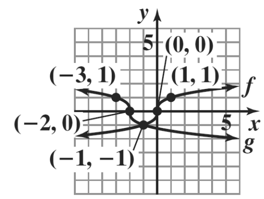

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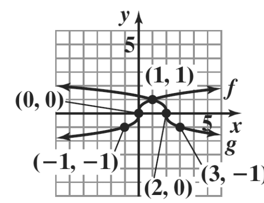

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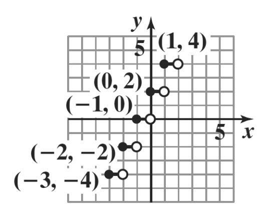
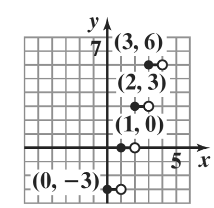


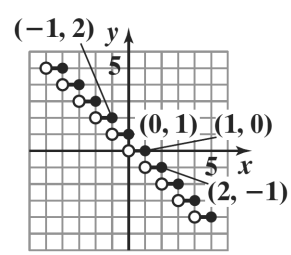
$g(x) = 2 \text{ int } (x + 1)$

120.



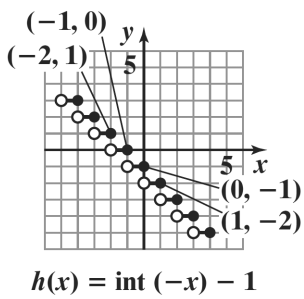
$g(x) = 3 \text{ int } (x - 1)$

121.



$h(x) = \text{int } (-x) + 1$

122.



123. $y=\sqrt{x-2}$

124. $y=-x^3+2$

125. $y=(x+1)^2-4$

126. $y=\sqrt{x-2}+1$

127. a. First, vertically stretch the graph of $f(x)=\sqrt{x}$ by the factor 2.9; then shift the result up 20.1 units.

b. $f(x)=2.9\sqrt{x}+20.1$

$f(48)=2.9\sqrt{48}+20.1\approx 40.2$

The model describes the actual data very well.

c. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(10)-f(0)}{10-0}$

$=\frac{(2.9\sqrt{10}+20.1)-(2.9\sqrt{0}+20.1)}{10-0}$

$=\frac{29.27-20.1}{10}$

≈ 0.9

0.9 inches per month

d. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(60)-f(50)}{60-50}$

$=\frac{(2.9\sqrt{60}+20.1)-(2.9\sqrt{50}+20.1)}{60-50}$

$=\frac{42.5633-40.6061}{10}$

≈ 0.2

This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

128. a. First, vertically stretch the graph of $f(x)=\sqrt{x}$ by the factor 3.1; then shift the result up 19 units.

b. $f(x)=3.1\sqrt{x}+19$

$f(48)=3.1\sqrt{48}+19\approx 40.5$

The model describes the actual data very well.

c. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(10)-f(0)}{10-0}$

$=\frac{(3.1\sqrt{10}+19)-(3.1\sqrt{0}+19)}{10-0}$

$=\frac{28.8031-19}{10}$

≈ 1.0

1.0 inches per month

d. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(60)-f(50)}{60-50}$

$=\frac{(3.1\sqrt{60}+19)-(3.1\sqrt{50}+19)}{60-50}$

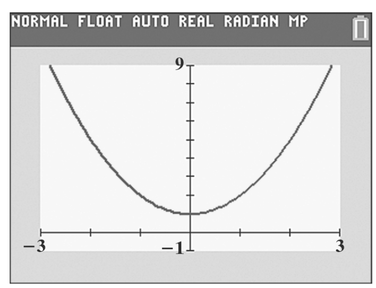
$=\frac{43.0125-40.9203}{10}$

≈ 0.2

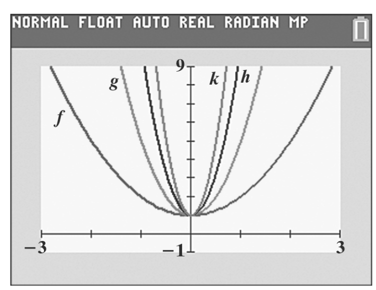
This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

129. – 134. Answers will vary.

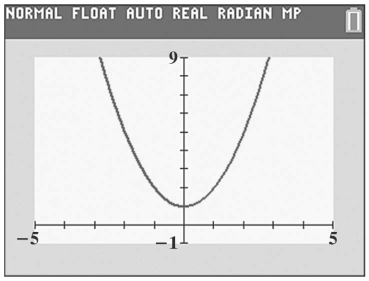
135. a.



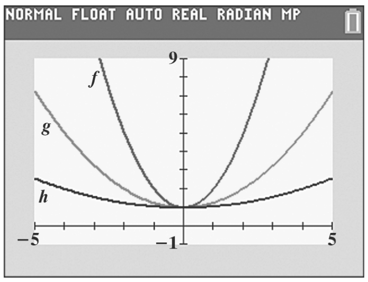
b.



136. a.



b.



137. makes sense

138. makes sense

139. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t+1)$.

140. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t-1)$.

141. false; Changes to make the statement true will vary. A sample change is: The graph of *g* is a translation of *f* three units to the left and three units upward.

142. false; Changes to make the statement true will vary. A sample change is: The graph of *f* is a reflection of the graph of $y=\sqrt{x}$ in the *x*-axis, while the graph of *g* is a reflection of the graph of $y=\sqrt{x}$ in the *y*-axis.

143. false; Changes to make the statement true will vary. A sample change is: The stretch will be 5 units and the downward shift will be 10 units.

144. true

145. $g(x) = -(x+4)^2$

146. $g(x) = -|x-5|+1$

147. $g(x) = -\sqrt{x-2}+2$

148. $g(x) = -\frac{1}{4}\sqrt{16-x^2}-1$

149. $(-a, b)$

150. $(a, 2b)$

151. $(a+3, b)$

152. $(a, b-3)$

153. Let x = the width of the rectangle.
Let $x + 13$ = the length of the rectangle.

$$\begin{aligned}2l+2w&=P\\2(x+13)+2x&=82\\2x+26+2x&=82\\4x+26&=82\\4x&=56\\x&=\frac{56}{4}\\x&=14\\x+13&=27\end{aligned}$$

The dimensions of the rectangle are 14 yards by 27 yards.

154.

$$\begin{aligned}\sqrt{x+10}-4&=x\\\sqrt{x+10}&=x+4\\\left(\sqrt{x+10}\right)^2&=(x+4)^2\\x+10&=x^2+8x+16\\0&=x^2+7x+6\\0&=(x+6)(x+1)\end{aligned}$$

$x+6=0$ or $x+1=0$
$x=-6$ $\quad x=-1$

–6 does not check and must be rejected.
The solution set is $\{-1\}$.

155.

$$f(x)=x^2+3x+2$$

$$\begin{aligned}f(x+h)&=(x+h)^2+3(x+h)+2\\&=x^2+2xh+h^2+3x+3h+2\end{aligned}$$

$$\begin{aligned}&\frac{f(x+h)-f(x)}{h}\\&=\frac{x^2+2xh+h^2+3x+3h+2-\left(x^2+3x+2\right)}{h}\\&=\frac{x^2+2xh+h^2+3x+3h+2-x^2-3x-2}{h}\\&=\frac{2xh+h^2+3h}{h}\\&=\frac{h(2x+h+3)}{h}\\&=2x+h+3,\quad h\neq 0\end{aligned}$$

156.

$$\begin{aligned}(2x-1)(x^2+x-2)&=2x(x^2+x-2)-1(x^2+x-2)\\&=2x^3+2x^2-4x-x^2-x+2\\&=2x^3+2x^2-x^2-4x-x+2\\&=2x^3+x^2-5x+2\end{aligned}$$

157. $(f(x))^2 - 2f(x) + 6 = (3x-4)^2 - 2(3x-4) + 6$
$= 9x^2 - 24x + 16 - 6x + 8 + 6$
$= 9x^2 - 24x - 6x + 16 + 8 + 6$
$= 9x^2 - 30x + 30$

158. $\dfrac{2}{\frac{3}{x} - 1} = \dfrac{2x}{\frac{3x}{x} - x} = \dfrac{2x}{3-x}$

Section 1.7

Check Point Exercises

1. **a.** The function $f(x) = x^2 + 3x - 17$ contains neither division nor an even root. The domain of *f* is the set of all real numbers or $(-\infty, \infty)$.

b. The denominator equals zero when $x = 7$ or $x = -7$. These values must be excluded from the domain.
domain of $g = (-\infty, -7) \cup (-7, 7) \cup (7, \infty)$.

c. Since $h(x) = \sqrt{9x - 27}$ contains an even root; the quantity under the radical must be greater than or equal to 0.
$9x - 27 \geq 0$
$9x \geq 27$
$x \geq 3$
Thus, the domain of *h* is $\{x | x \geq 3\}$, or the interval $[3, \infty)$.

d. Since the denominator of $j(x)$ contains an even root; the quantity under the radical must be greater than or equal to 0. But that quantity must also not be 0 (because we cannot have division by 0). Thus, $24 - 3x$ must be strictly greater than 0.
$24 - 3x > 0$
$-3x > -24$
$x < 8$
Thus, the domain of *j* is $\{x | x < 8\}$, or the interval $(-\infty, 8)$.

2. **a.** $(f+g)(x) = f(x) + g(x)$
$= x - 5 + (x^2 - 1)$
$= x - 5 + x^2 - 1$
$= -x^2 + x - 6$
domain: $(-\infty, \infty)$

b. $(f-g)(x) = f(x) - g(x)$
$= x - 5 - (x^2 - 1)$
$= x - 5 - x^2 + 1$
$= -x^2 + x - 4$
domain: $(-\infty, \infty)$

c. $(fg)(x) = (x-5)(x^2 - 1)$
$= x(x^2 - 1) - 5(x^2 - 1)$
$= x^3 - x - 5x^2 + 5$
$= x^3 - 5x^2 - x + 5$
domain: $(-\infty, \infty)$

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$
$= \dfrac{x-5}{x^2 - 1}, \; x \neq \pm 1$
domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

3. **a.** $(f+g)(x) = f(x) + g(x)$
$= \sqrt{x-3} + \sqrt{x+1}$

b. domain of *f*: $x - 3 \geq 0$
$x \geq 3$
$[3, \infty)$
domain of g: $x + 1 \geq 0$
$x \geq -1$
$[-1, \infty)$
The domain of $f + g$ is the set of all real numbers that are common to the domain of *f* and the domain of *g*. Thus, the domain of $f + g$ is $[3, \infty)$.

4. **a.** $(B+D)(x)$
$= B(x) + D(x)$
$= (-2.6x^2 + 49x + 3994) + (-0.6x^2 + 7x + 2412)$
$= -2.6x^2 + 49x + 3994 - 0.6x^2 + 7x + 2412$
$= -3.2x^2 + 56x + 6406$

b. $(B+D)(x) = -3.2x^2 + 56x + 6406$
$(B+D)(5) = -3.2(3)^2 + 56(3) + 6406$
$= 6545.2$
The number of births and deaths in the U.S. in 2003 was 6545.2 thousand.

c. $(B+D)(x)$ overestimates the actual number of births and deaths in 2003 by 7.2 thousand.

5. a. $(f \circ g)(x) = f(g(x))$
$= 5\left(2x^2 - x - 1\right) + 6$
$= 10x^2 - 5x - 5 + 6$
$= 10x^2 - 5x + 1$

b. $(g \circ f)(x) = g(f(x))$
$= 2(5x+6)^2 - (5x+6) - 1$
$= 2(25x^2 + 60x + 36) - 5x - 6 - 1$
$= 50x^2 + 120x + 72 - 5x - 6 - 1$
$= 50x^2 + 115x + 65$

c. $(f \circ g)(x) = 10x^2 - 5x + 1$
$(f \circ g)(-1) = 10(-1)^2 - 5(-1) + 1$
$= 10 + 5 + 1$
$= 16$

6. a. $(f \circ g)(x) = \dfrac{4}{\frac{1}{x} + 2} = \dfrac{4x}{1 + 2x}$

b. domain: $\left\{x \middle| x \neq 0, x \neq -\frac{1}{2}\right\}$
or $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 0\right) \cup (0, \infty)$

7. $h(x) = f \circ g$ where $f(x) = \sqrt{x}$; $g(x) = x^2 + 5$

Concept and Vocabulary Check 1.7

1. zero

2. negative

3. $f(x) + g(x)$

4. $f(x) - g(x)$

5. $f(x) \cdot g(x)$

6. $\dfrac{f(x)}{g(x)}$; $g(x)$

7. $(-\infty, \infty)$

8. $(2, \infty)$

9. $(0, 3)$; $(3, \infty)$

10. composition; $f(g(x))$

11. f; $g(x)$

12. composition; $g(f(x))$

13. g; $f(x)$

14. false

15. false

16. 2

Exercise Set 1.7

1. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

2. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

3. The denominator equals zero when $x = 4$. This value must be excluded from the domain.
domain: $(-\infty, 4) \cup (4, \infty)$.

4. The denominator equals zero when $x = -5$. This value must be excluded from the domain.
domain: $(-\infty, -5) \cup (-5, \infty)$.

5. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

6. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

7. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

8. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

9. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty, -7) \cup (-7, 9) \cup (9, \infty)$

10. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty,-8)\cup(-8,10)\cup(10,\infty)$

11. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

12. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

13. Exclude x for $x=0$.

Exclude x for $\frac{3}{x}-1=0$.

$$\frac{3}{x}-1=0$$
$$x\left(\frac{3}{x}-1\right)=x(0)$$
$$3-x=0$$
$$-x=-3$$
$$x=3$$

domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$

14. Exclude x for $x=0$.

Exclude x for $\frac{4}{x}-1=0$.

$$\frac{4}{x}-1=0$$
$$x\left(\frac{4}{x}-1\right)=x(0)$$
$$4-x=0$$
$$-x=-4$$
$$x=4$$

domain: $(-\infty,0)\cup(0,4)\cup(4,\infty)$

15. Exclude x for $x-1=0$.
$$x-1=0$$
$$x=1$$

Exclude x for $\frac{4}{x-1}-2=0$.

$$\frac{4}{x-1}-2=0$$
$$(x-1)\left(\frac{4}{x-1}-2\right)=(x-1)(0)$$
$$4-2(x-1)=0$$
$$4-2x+2=0$$
$$-2x+6=0$$
$$-2x=-6$$
$$x=3$$

domain: $(-\infty,1)\cup(1,3)\cup(3,\infty)$

16. Exclude x for $x-2=0$.
$$x-2=0$$
$$x=2$$

Exclude x for $\frac{4}{x-2}-3=0$.

$$\frac{4}{x-2}-3=0$$
$$(x-2)\left(\frac{4}{x-2}-3\right)=(x-2)(0)$$
$$4-3(x-2)=0$$
$$4-3x+6=0$$
$$-3x+10=0$$
$$-3x=-10$$
$$x=\frac{10}{3}$$

domain: $(-\infty,2)\cup\left(2,\frac{10}{3}\right)\cup\left(\frac{10}{3},\infty\right)$

17. The expression under the radical must not be negative.
$$x-3\ge 0$$
$$x\ge 3$$
domain: $[3,\infty)$

18. The expression under the radical must not be negative.
$$x+2\ge 0$$
$$x\ge -2$$
domain: $[-2,\infty)$

19. The expression under the radical must be positive.
$$x-3>0$$
$$x>3$$
domain: $(3,\infty)$

20. The expression under the radical must be positive.
$$x+2>0$$
$$x>-2$$
domain: $(-2,\infty)$

21. The expression under the radical must not be negative.
$$5x+35\ge 0$$
$$5x\ge -35$$
$$x\ge -7$$
domain: $[-7,\infty)$

22. The expression under the radical must not be negative.
$$7x-70\ge 0$$
$$7x\ge 70$$
$$x\ge 10$$
domain: $[10,\infty)$

23. The expression under the radical must not be negative.

$$24-2x \ge 0$$
$$-2x \ge -24$$
$$\frac{-2x}{-2} \le \frac{-24}{-2}$$
$$x \le 12$$

domain: $(-\infty,12]$

24. The expression under the radical must not be negative.

$$84-6x \ge 0$$
$$-6x \ge -84$$
$$\frac{-6x}{-6} \le \frac{-84}{-6}$$
$$x \le 14$$

domain: $(-\infty,14]$

25. The expressions under the radicals must not be negative.

$x-2 \ge 0$, $x \ge 2$ and $x+3 \ge 0$, $x \ge -3$

To make both inequalities true, $x \ge 2$.

domain: $[2,\infty)$

26. The expressions under the radicals must not be negative.

$x-3 \ge 0$, $x \ge 3$ and $x+4 \ge 0$, $x \ge -4$

To make both inequalities true, $x \ge 3$.

domain: $[3,\infty)$

27. The expression under the radical must not be negative.

$$x-2 \ge 0$$
$$x \ge 2$$

The denominator equals zero when $x = 5$.

domain: $[2,5)\cup(5,\infty)$.

28. The expression under the radical must not be negative.

$$x-3 \ge 0$$
$$x \ge 3$$

The denominator equals zero when $x = 6$.

domain: $[3,6)\cup(6,\infty)$.

29. Find the values that make the denominator equal zero and must be excluded from the domain.

$$x^3-5x^2-4x+20$$
$$=x^2(x-5)-4(x-5)$$
$$=(x-5)(x^2-4)$$
$$=(x-5)(x+2)(x-2)$$

–2, 2, and 5 must be excluded.

domain: $(-\infty,-2)\cup(-2,2)\cup(2,5)\cup(5,\infty)$

30. Find the values that make the denominator equal zero and must be excluded from the domain.

$$x^3-2x^2-9x+18$$
$$=x^2(x-2)-9(x-2)$$
$$=(x-2)(x^2-9)$$
$$=(x-2)(x+3)(x-3)$$

–3, 2, and 3 must be excluded.

domain: $(-\infty,-3)\cup(-3,2)\cup(2,3)\cup(3,\infty)$

31. $(f+g)(x) = 3x+2$

domain: $(-\infty,\infty)$

$$(f-g)(x) = f(x)-g(x)$$
$$= (2x+3)-(x-1)$$
$$= x+4$$

domain: $(-\infty,\infty)$

$$(fg)(x) = f(x)\cdot g(x)$$
$$= (2x+3)\cdot(x-1)$$
$$= 2x^2+x-3$$

domain: $(-\infty,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+3}{x-1}$$

domain: $(-\infty,1)\cup(1,\infty)$

32. $(f+g)(x) = 4x-2$

domain: $(-\infty, \infty)$

$(f-g)(x) = (3x-4)-(x+2) = 2x-6$

domain: $(-\infty, \infty)$

$(fg)(x) = (3x-4)(x+2) = 3x^2+2x-8$

domain: $(-\infty, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{3x-4}{x+2}$$

domain: $(-\infty,-2)\cup(-2,\infty)$

33. $(f+g)(x) = 3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(f-g)(x) = -3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-5)(3x^2) = 3x^3 - 15x^2$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{x-5}{3x^2}$
domain: $(-\infty, 0) \cup (0, \infty)$

34. $(f+g)(x) = 5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(f-g)(x) = -5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-6)(5x^2) = 5x^3 - 30x^2$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{x-6}{5x^2}$
domain: $(-\infty, 0) \cup (0, \infty)$

35. $(f+g)(x) = 2x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 2x^2 - 2x - 4$
domain: $(-\infty, \infty)$
$$(fg)(x) = (2x^2 - x - 3)(x+1) = 2x^3 + x^2 - 4x - 3$$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{2x^2 - x - 3}{x+1} = \frac{(2x-3)(x+1)}{(x+1)} = 2x - 3$$
domain: $(-\infty, -1) \cup (-1, \infty)$

36. $(f+g)(x) = 6x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 6x^2 - 2x$
domain: $(-\infty, \infty)$
$(fg)(x) = (6x^2 - x - 1)(x-1) = 6x^3 - 7x^2 + 1$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{6x^2 - x - 1}{x-1}$
domain: $(-\infty, 1) \cup (1, \infty)$

37. $$(f+g)(x) = (3 - x^2) + (x^2 + 2x - 15) = 2x - 12$$
domain: $(-\infty, \infty)$
$$(f-g)(x) = (3 - x^2) - (x^2 + 2x - 15) = -2x^2 - 2x + 18$$
domain: $(-\infty, \infty)$
$$(fg)(x) = (3 - x^2)(x^2 + 2x - 15) = -x^4 - 2x^3 + 18x^2 + 6x - 45$$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{3 - x^2}{x^2 + 2x - 15}$
domain: $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$

38. $$(f+g)(x) = (5 - x^2) + (x^2 + 4x - 12) = 4x - 7$$
domain: $(-\infty, \infty)$
$$(f-g)(x) = (5 - x^2) - (x^2 + 4x - 12) = -2x^2 - 4x + 17$$
domain: $(-\infty, \infty)$
$$(fg)(x) = (5 - x^2)(x^2 + 4x - 12) = -x^4 - 4x^3 + 17x^2 + 20x - 60$$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{5 - x^2}{x^2 + 4x - 12}$
domain: $(-\infty, -6) \cup (-6, 2) \cup (2, \infty)$

39. $(f+g)(x) = \sqrt{x} + x - 4$
domain: $[0, \infty)$
$(f-g)(x) = \sqrt{x} - x + 4$
domain: $[0, \infty)$
$(fg)(x) = \sqrt{x}(x-4)$
domain: $[0, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x-4}$
domain: $[0, 4) \cup (4, \infty)$

40. $(f+g)(x)=\sqrt{x}+x-5$
domain: $[0,\infty)$
$(f-g)(x)=\sqrt{x}-x+5$
domain: $[0,\infty)$
$(fg)(x)=\sqrt{x}(x-5)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x}}{x-5}$
domain: $[0,5)\cup(5,\infty)$

41. $(f+g)(x)=2+\frac{1}{x}+\frac{1}{x}=2+\frac{2}{x}=\frac{2x+2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x)=2+\frac{1}{x}-\frac{1}{x}=2$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x)=\left(2+\frac{1}{x}\right)\cdot\frac{1}{x}=\frac{2}{x}+\frac{1}{x^2}=\frac{2x+1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{2+\frac{1}{x}}{\frac{1}{x}}=\left(2+\frac{1}{x}\right)\cdot x=2x+1$
domain: $(-\infty,0)\cup(0,\infty)$

42. $(f+g)(x)=6-\frac{1}{x}+\frac{1}{x}=6$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x)=6-\frac{1}{x}-\frac{1}{x}=6-\frac{2}{x}=\frac{6x-2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x)=\left(6-\frac{1}{x}\right)\cdot\frac{1}{x}=\frac{6}{x}-\frac{1}{x^2}=\frac{6x-1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{6-\frac{1}{x}}{\frac{1}{x}}=\left(6-\frac{1}{x}\right)\cdot x=6x-1$
domain: $(-\infty,0)\cup(0,\infty)$

43. $(f+g)(x)=f(x)+g(x)$
$$=\frac{5x+1}{x^2-9}+\frac{4x-2}{x^2-9}$$
$$=\frac{9x-1}{x^2-9}$$
domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$
$(f-g)(x)=f(x)-g(x)$
$$=\frac{5x+1}{x^2-9}-\frac{4x-2}{x^2-9}$$
$$=\frac{x+3}{x^2-9}$$
$$=\frac{1}{x-3}$$
domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$
$(fg)(x)=f(x)\cdot g(x)$
$$=\frac{5x+1}{x^2-9}\cdot\frac{4x-2}{x^2-9}$$
$$=\frac{(5x+1)(4x-2)}{\left(x^2-9\right)^2}$$
domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$
$$\left(\frac{f}{g}\right)(x)=\frac{\frac{5x+1}{x^2-9}}{\frac{4x-2}{x^2-9}}$$
$$=\frac{5x+1}{x^2-9}\cdot\frac{x^2-9}{4x-2}$$
$$=\frac{5x+1}{4x-2}$$
The domain must exclude –3, 3, and any values that make $4x-2=0$.
$4x-2=0$
$4x=2$
$x=\frac{1}{2}$
domain: $(-\infty,-3)\cup\left(-3,\frac{1}{2}\right)\cup\left(\frac{1}{2},3\right)\cup(3,\infty)$

44. $(f+g)(x) = f(x)+g(x)$

$$= \frac{3x+1}{x^2-25} + \frac{2x-4}{x^2-25}$$

$$= \frac{5x-3}{x^2-25}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(f-g)(x) = f(x)-g(x)$

$$= \frac{3x+1}{x^2-25} - \frac{2x-4}{x^2-25}$$

$$= \frac{x+5}{x^2-25}$$

$$= \frac{1}{x-5}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{3x+1}{x^2-25} \cdot \frac{2x-4}{x^2-25}$$

$$= \frac{(3x+1)(2x-4)}{\left(x^2-25\right)^2}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{3x+1}{x^2-25}}{\frac{2x-4}{x^2-25}}$$

$$= \frac{3x+1}{x^2-25} \cdot \frac{x^2-25}{2x-4}$$

$$= \frac{3x+1}{2x-4}$$

The domain must exclude –5, 5, and any values that make $2x-4=0$.

$2x-4=0$
$2x=4$
$x=2$

domain: $(-\infty,-5)\cup(-5,2)\cup(2,5)\cup(5,\infty)$

45. $(f+g)(x) = f(x)+g(x)$

$$= \frac{8x}{x-2} + \frac{6}{x+3}$$

$$= \frac{8x(x+3)}{(x-2)(x+3)} + \frac{6(x-2)}{(x-2)(x+3)}$$

$$= \frac{8x^2+24x}{(x-2)(x+3)} + \frac{6x-12}{(x-2)(x+3)}$$

$$= \frac{8x^2+30x-12}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$(f+g)(x) = f(x)-g(x)$

$$= \frac{8x}{x-2} - \frac{6}{x+3}$$

$$= \frac{8x(x+3)}{(x-2)(x+3)} - \frac{6(x-2)}{(x-2)(x+3)}$$

$$= \frac{8x^2+24x}{(x-2)(x+3)} - \frac{6x-12}{(x-2)(x+3)}$$

$$= \frac{8x^2+18x+12}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{8x}{x-2} \cdot \frac{6}{x+3}$$

$$= \frac{48x}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{8x}{x-2}}{\frac{6}{x+3}}$$

$$= \frac{8x}{x-2} \cdot \frac{x+3}{6}$$

$$= \frac{4x(x+3)}{3(x-2)}$$

The domain must exclude –3, 2, and any values that make $3(x-2)=0$.

$3(x-2)=0$
$3x-6=0$
$3x=6$
$x=2$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

46. $(f+g)(x) = f(x)+g(x)$

$$= \frac{9x}{x-4}+\frac{7}{x+8}$$
$$= \frac{9x(x+8)}{(x-4)(x+8)}+\frac{7(x-4)}{(x-4)(x+8)}$$
$$= \frac{9x^2+72x}{(x-4)(x+8)}+\frac{7x-28}{(x-4)(x+8)}$$
$$= \frac{9x^2+79x-28}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$(f+g)(x) = f(x)-g(x)$

$$= \frac{9x}{x-4}-\frac{7}{x+8}$$
$$= \frac{9x(x+8)}{(x-4)(x+8)}-\frac{7(x-4)}{(x-4)(x+8)}$$
$$= \frac{9x^2+72x}{(x-4)(x+8)}-\frac{7x-28}{(x-4)(x+8)}$$
$$= \frac{9x^2+65x+28}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{9x}{x-4}\cdot\frac{7}{x+8}$$
$$= \frac{63x}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{9x}{x-4}}{\frac{7}{x+8}}$$
$$= \frac{9x}{x-4}\cdot\frac{x+8}{7}$$
$$= \frac{9x(x+8)}{7(x-4)}$$

The domain must exclude –8, 4, and any values that make $7(x-4)=0$.

$7(x-4)=0$
$7x-28=0$
$7x=28$
$x=4$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

47. $(f+g)(x) = \sqrt{x+4}+\sqrt{x-1}$

domain: $[1,\infty)$

$(f-g)(x) = \sqrt{x+4}-\sqrt{x-1}$

domain: $[1,\infty)$

$(fg)(x) = \sqrt{x+4}\cdot\sqrt{x-1} = \sqrt{x^2+3x-4}$

domain: $[1,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$$

domain: $(1,\infty)$

48. $(f+g)(x) = \sqrt{x+6}+\sqrt{x-3}$

domain: $[3,\infty)$

$(f-g)(x) = \sqrt{x+6}-\sqrt{x-3}$

domain: $[3,\infty)$

$(fg)(x) = \sqrt{x+6}\cdot\sqrt{x-3} = \sqrt{x^2+3x-18}$

domain: $[3,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+6}}{\sqrt{x-3}}$$

domain: $(3,\infty)$

49. $(f+g)(x) = \sqrt{x-2}+\sqrt{2-x}$

domain: $\{2\}$

$(f-g)(x) = \sqrt{x-2}-\sqrt{2-x}$

domain: $\{2\}$

$(fg)(x) = \sqrt{x-2}\cdot\sqrt{2-x} = \sqrt{-x^2+4x-4}$

domain: $\{2\}$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-2}}{\sqrt{2-x}}$$

domain: $\varnothing$

50. $(f+g)(x) = \sqrt{x-5}+\sqrt{5-x}$

domain: $\{5\}$

$(f-g)(x) = \sqrt{x-5}-\sqrt{5-x}$

domain: $\{5\}$

$(fg)(x) = \sqrt{x-5}\cdot\sqrt{5-x} = \sqrt{-x^2+10x-25}$

domain: $\{5\}$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-5}}{\sqrt{5-x}}$$

domain: $\varnothing$

51. $f(x) = 2x$; $g(x) = x + 7$

a. $(f \circ g)(x) = 2(x+7) = 2x+14$

b. $(g \circ f)(x) = 2x+7$

c. $(f \circ g)(2) = 2(2)+14 = 18$

52. $f(x) = 3x$; $g(x) = x - 5$

a. $(f \circ g)(x) = 3(x-5) = 3x-15$

b. $(g \circ f)(x) = 3x-5$

c. $(f \circ g)(2) = 3(2)-15 = -9$

53. $f(x) = x + 4$; $g(x) = 2x + 1$

a. $(f \circ g)(x) = (2x+1)+4 = 2x+5$

b. $(g \circ f)(x) = 2(x+4)+1 = 2x+9$

c. $(f \circ g)(2) = 2(2)+5 = 9$

54. $f(x) = 5x + 2$; $g(x) = 3x - 4$

a. $(f \circ g)(x) = 5(3x-4)+2 = 15x-18$

b. $(g \circ f)(x) = 3(5x+2)-4 = 15x+2$

c. $(f \circ g)(2) = 15(2)-18 = 12$

55. $f(x) = 4x - 3$; $g(x) = 5x^2 - 2$

a. $(f \circ g)(x) = 4(5x^2-2)-3$
$= 20x^2 - 11$

b. $(g \circ f)(x) = 5(4x-3)^2 - 2$
$= 5(16x^2 - 24x + 9) - 2$
$= 80x^2 - 120x + 43$

c. $(f \circ g)(2) = 20(2)^2 - 11 = 69$

56. $f(x) = 7x+1$; $g(x) = 2x^2 - 9$

a. $(f \circ g)(x) = 7(2x^2-9)+1 = 14x^2 - 62$

b. $(g \circ f)(x) = 2(7x+1)^2 - 9$
$= 2(49x^2 + 14x + 1) - 9$
$= 98x^2 + 28x - 7$

c. $(f \circ g)(2) = 14(2)^2 - 62 = -6$

57. $f(x) = x^2 + 2$; $g(x) = x^2 - 2$

a. $(f \circ g)(x) = (x^2-2)^2 + 2$
$= x^4 - 4x^2 + 4 + 2$
$= x^4 - 4x^2 + 6$

b. $(g \circ f)(x) = (x^2+2)^2 - 2$
$= x^4 + 4x^2 + 4 - 2$
$= x^4 + 4x^2 + 2$

c. $(f \circ g)(2) = 2^4 - 4(2)^2 + 6 = 6$

58. $f(x) = x^2 + 1$; $g(x) = x^2 - 3$

a. $(f \circ g)(x) = (x^2-3)^2 + 1$
$= x^4 - 6x^2 + 9 + 1$
$= x^4 - 6x^2 + 10$

b. $(g \circ f)(x) = (x^2+1)^2 - 3$
$= x^4 + 2x^2 + 1 - 3$
$= x^4 + 2x^2 - 2$

c. $(f \circ g)(2) = 2^4 - 6(2)^2 + 10 = 2$

59. $f(x) = 4 - x$; $g(x) = 2x^2 + x + 5$

a. $(f \circ g)(x) = 4 - \left(2x^2 + x + 5\right)$
$= 4 - 2x^2 - x - 5$
$= -2x^2 - x - 1$

b. $(g \circ f)(x) = 2\left(4-x\right)^2 + \left(4-x\right) + 5$
$= 2(16 - 8x + x^2) + 4 - x + 5$
$= 32 - 16x + 2x^2 + 4 - x + 5$
$= 2x^2 - 17x + 41$

c. $(f \circ g)(2) = -2(2)^2 - 2 - 1 = -11$

60. $f(x)=5x-2;\quad g(x)=-x^2+4x-1$

a. $(f\circ g)(x)=5\left(-x^2+4x-1\right)-2$
$=-5x^2+20x-5-2$
$=-5x^2+20x-7$

b. $(g\circ f)(x)=-\left(5x-2\right)^2+4\left(5x-2\right)-1$
$=-(25x^2-20x+4)+20x-8-1$
$=-25x^2+20x-4+20x-8-1$
$=-25x^2+40x-13$

c. $(f\circ g)(2)=-5(2)^2+20(2)-7=13$

61. $f(x)=\sqrt{x};\ g(x)=x-1$

a. $(f\circ g)(x)=\sqrt{x-1}$

b. $(g\circ f)(x)=\sqrt{x}-1$

c. $(f\circ g)(2)=\sqrt{2-1}=\sqrt{1}=1$

62. $f(x)=\sqrt{x}\,;g(x)=x+2$

a. $(f\circ g)(x)=\sqrt{x+2}$

b. $(g\circ f)(x)=\sqrt{x}+2$

c. $(f\circ g)(2)=\sqrt{2+2}=\sqrt{4}=2$

63. $f(x)=2x-3;\ g(x)=\dfrac{x+3}{2}$

a. $(f\circ g)(x)=2\left(\dfrac{x+3}{2}\right)-3$
$=x+3-3$
$=x$

b. $(g\circ f)(x)=\dfrac{(2x-3)+3}{2}=\dfrac{2x}{2}=x$

c. $(f\circ g)(2)=2$

64. $f(x)=6x-3;g(x)=\dfrac{x+3}{6}$

a. $(f\circ g)(x)=6\left(\dfrac{x+3}{6}\right)-3=x+3-3=x$

b. $(g\circ f)(x)=\dfrac{6x-3+3}{6}=\dfrac{6x}{6}=x$

c. $(f\circ g)(2)=2$

65. $f(x)=\dfrac{1}{x};\quad g(x)=\dfrac{1}{x}$

a. $(f\circ g)(x)=\dfrac{1}{\frac{1}{x}}=x$

b. $(g\circ f)(x)=\dfrac{1}{\frac{1}{x}}=x$

c. $(f\circ g)(2)=2$

66. $f(x)=\dfrac{2}{x};\quad g(x)=\dfrac{2}{x}$

a. $(f\circ g)(x)=\dfrac{2}{\frac{2}{x}}=x$

b. $(g\circ f)(x)=\dfrac{2}{\frac{2}{x}}=x$

c. $(f\circ g)(2)=2$

67. a. $(f\circ g)(x)=f\left(\dfrac{1}{x}\right)=\dfrac{2}{\frac{1}{x}+3},x\neq 0$
$=\dfrac{2(x)}{\left(\frac{1}{x}+3\right)(x)}$
$=\dfrac{2x}{1+3x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\dfrac{1}{3}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\dfrac{1}{3}\right)\cup\left(-\dfrac{1}{3},0\right)\cup\left(0,\infty\right).$

68. a. $f\circ g(x)=f\left(\dfrac{1}{x}\right)=\dfrac{5}{\frac{1}{x}+4}=\dfrac{5x}{1+4x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\dfrac{1}{4}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\dfrac{1}{4}\right)\cup\left(-\dfrac{1}{4},0\right)\cup\left(0,\infty\right).$

69. a. $(f \circ g)(x) = f\left(\frac{4}{x}\right) = \frac{\frac{4}{x}}{\frac{4}{x}+1}$

$= \frac{\left(\frac{4}{x}\right)(x)}{\left(\frac{4}{x}+1\right)(x)}$

$= \frac{4}{4+x}, x \neq -4$

b. We must exclude 0 because it is excluded from g.
We must exclude -4 because it causes the denominator of $f \circ g$ to be 0.
domain: $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

70. a. $f \circ g(x) = f\left(\frac{6}{x}\right) = \frac{\frac{6}{x}}{\frac{6}{x}+5} = \frac{6}{6+5x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\frac{6}{5}$ because it causes the denominator of $f \circ g$ to be 0.
domain: $\left(-\infty,-\frac{6}{5}\right)\cup\left(-\frac{6}{5},0\right)\cup(0,\infty)$.

71. a. $f \circ g(x) = f(x-2) = \sqrt{x-2}$

b. The expression under the radical in $f \circ g$ must not be negative.
$x-2 \geq 0$
$x \geq 2$
domain: $[2,\infty)$.

72. a. $f \circ g(x) = f(x-3) = \sqrt{x-3}$

b. The expression under the radical in $f \circ g$ must not be negative.
$x-3 \geq 0$
$x \geq 3$
domain: $[3,\infty)$.

73. a. $(f \circ g)(x) = f(\sqrt{1-x})$

$= \left(\sqrt{1-x}\right)^2 + 4$
$= 1-x+4$
$= 5-x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.
$1-x \geq 0$
$-x \geq -1$
$x \leq 1$
domain: (• •, 1].

74. a. $(f \circ g)(x) = f(\sqrt{2-x})$

$= \left(\sqrt{2-x}\right)^2 + 1$
$= 2-x+1$
$= 3-x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.
$2-x \geq 0$
$-x \geq -2$
$x \leq 2$
domain: (• •, 2].

75. $f(x) = x^4 \quad g(x) = 3x-1$

76. $f(x) = x^3; g(x) = 2x-5$

77. $f(x) = \sqrt[3]{x} \quad g(x) = x^2 - 9$

78. $f(x) = \sqrt{x}; g(x) = 5x^2 + 3$

79. $f(x) = |x| \quad g(x) = 2x - 5$

80. $f(x) = |x|;\ g(x) = 3x - 4$

81. $f(x) = \frac{1}{x} \quad g(x) = 2x-3$

82. $f(x) = \frac{1}{x}; g(x) = 4x+5$

83. $(f+g)(-3) = f(-3)+g(-3) = 4+1 = 5$

84. $(g-f)(-2) = g(-2)-f(-2) = 2-3 = -1$

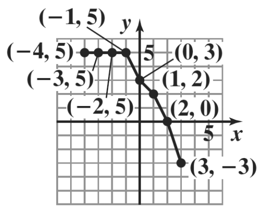
85. $(fg)(2) = f(2)g(2) = (-1)(1) = -1$

86. $\left(\frac{g}{f}\right)(3) = \frac{g(3)}{f(3)} = \frac{0}{-3} = 0$

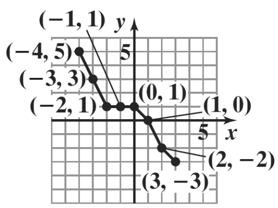
87. The domain of $f+g$ is $[-4,3]$.

88. The domain of $\frac{f}{g}$ is $(-4,3)$.

89. The graph of $f+g$



90. The graph of $f-g$



91. $(f\circ g)(-1)=f(g(-1))=f(-3)=1$

92. $(f\circ g)(1)=f(g(1))=f(-5)=3$

93. $(g\circ f)(0)=g(f(0))=g(2)=-6$

94. $(g\circ f)(-1)=g(f(-1))=g(1)=-5$

95.
$$\begin{aligned}(f\circ g)(x)&=7\\ 2\left(x^2-3x+8\right)-5&=7\\ 2x^2-6x+16-5&=7\\ 2x^2-6x+11&=7\\ 2x^2-6x+4&=0\\ x^2-3x+2&=0\\ (x-1)(x-2)&=0\end{aligned}$$
$x-1=0$ or $x-2=0$
$x=1$ $\quad x=2$

96.
$$\begin{aligned}(f\circ g)(x)&=-5\\ 1-2\left(3x^2+x-1\right)&=-5\\ 1-6x^2-2x+2&=-5\\ -6x^2-2x+3&=-5\\ -6x^2-2x+8&=0\\ 3x^2+x-4&=0\\ (3x+4)(x-1)&=0\end{aligned}$$
$3x+4=0$ or $x-1=0$
$3x=-4$ $\quad x=1$
$x=-\frac{4}{3}$

97. **a.**
$$\begin{aligned}(M+F)(x)&=M(x)+F(x)\\&=(1.48x+115.1)+(1.44x+120.9)\\&=2.92x+236\end{aligned}$$

b.
$$\begin{aligned}(M+F)(x)&=2.92x+236\\(M+F)(25)&=2.92(25)+236\\&=309\end{aligned}$$
The total U.S. population in 2010 was 309 million.

c. It is the same.

98. **a.**
$$\begin{aligned}(F-M)(x)&=F(x)-M(x)\\&=(1.44x+120.9)-(1.48x+115.1)\\&=-0.04x+5.8\end{aligned}$$

b.
$$\begin{aligned}(F-M)(x)&=-0.04x+5.8\\(F-M)(25)&=-0.04(25)+5.8\\&=4.8\end{aligned}$$
In 2010 there were 4.8 million more women than men.

c. The result in part (b) underestimates the actual difference by 0.2 million.

99.
$$\begin{aligned}&(R-C)(20,000)\\&=65(20,000)-(600,000+45(20,000))\\&=-200,000\end{aligned}$$
The company lost \$200,000 since costs exceeded revenues.

$(R-C)(30,000)$
$= 65(30,000) - (600,000 + 45(30,000))$
$= 0$
The company broke even.

$(R-C)(40,000)$
$= 65(40,000) - (600,000 + 45(40,000))$
$= 200,000$
The company gained \$200,000 since revenues exceeded costs.

100. **a.** The slope for f is -0.44 This is the decrease in profits for the first store for each year after 2012.

b. The slope of g is 0.51 This is the increase in profits for the second store for each year after 2012.

c. $f+g = -.044x + 13.62 + 0.51x + 11.14$
$= 0.07x + 24.76$
The slope for $f+g$ is 0.07 This is the profit for the two stores combined for each year after 2012.

101. a. *f* gives the price of the computer after a $400 discount. *g* gives the price of the computer after a 25% discount.

b. $(f \circ g)(x) = 0.75x - 400$

This models the price of a computer after first a 25% discount and then a $400 discount.

c. $(g \circ f)(x) = 0.75(x - 400)$

This models the price of a computer after first a $400 discount and then a 25% discount.

d. The function $f \circ g$ models the greater discount, since the 25% discount is taken on the regular price first.

102. a. *f* gives the cost of a pair of jeans for which a $5 rebate is offered.
g gives the cost of a pair of jeans that has been discounted 40%.

b. $(f \circ g)(x) = 0.6x - 5$

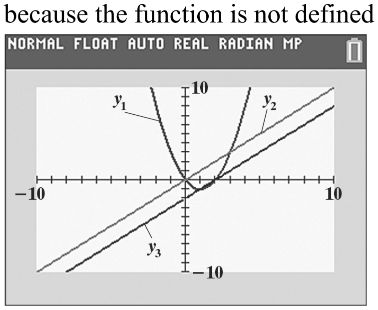
The cost of a pair of jeans is 60% of the regular price minus a $5 rebate.

c. $(g \circ f)(x) = 0.6(x - 5)$
$= 0.6x - 3$
The cost of a pair of jeans is 60% of the regular price minus a $3 rebate.

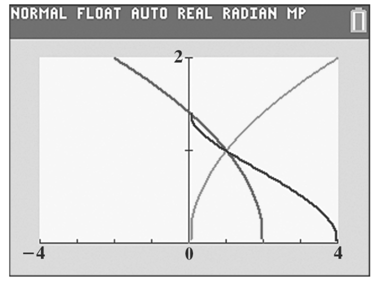
d. $f \circ g$ because of a $5 rebate.

103. – 107. Answers will vary.

108. When your trace reaches $x = 0$, the y value disappears because the function is not defined at $x = 0$.



109.



$(f \circ g)(x) = \sqrt{2 - \sqrt{x}}$

The domain of g is $[0, \infty)$.

The expression under the radical in $f \circ g$ must not be negative.

$$\begin{aligned} 2 - \sqrt{x} &\ge 0 \\ -\sqrt{x} &\ge -2 \\ \sqrt{x} &\le 2 \\ x &\le 4 \end{aligned}$$

domain: $[0, 4]$

110. makes sense

111. makes sense

112. does not make sense; Explanations will vary. Sample explanation: It is common that $f \circ g$ and $g \circ f$ are not the same.

113. does not make sense; Explanations will vary. Sample explanation: The diagram illustrates $g(f(x)) = x^2 + 4$.

114. false; Changes to make the statement true will vary.

A sample change is: $(f \circ g)(x) = f\left(\sqrt{x^2 - 4}\right)$

$$\begin{aligned} &= \left(\sqrt{x^2 - 4}\right)^2 - 4 \\ &= x^2 - 4 - 4 \\ &= x^2 - 8 \end{aligned}$$

115. false; Changes to make the statement true will vary.
A sample change is:

$f(x) = 2x; g(x) = 3x$

$(f \circ g)(x) = f(g(x)) = f(3x) = 2(3x) = 6x$

$(g \circ f)(x) = g(f(x)) = g(f(x)) = 3(2x) = 6x$

116. false; Changes to make the statement true will vary.
A sample change is:

$(f \circ g)(4) = f(g(4)) = f(7) = 5$

117. true

118. $(f \circ g)(x) = (f \circ g)(-x)$

$f(g(x)) = f(g(-x))$ since g is even
$f(g(x)) = f(g(x))$ so $f \circ g$ is even

119. Answers will vary.

120. $\frac{x-1}{5} - \frac{x+3}{2} = 1 - \frac{x}{4}$

$$20\left(\frac{x-1}{5} - \frac{x+3}{2}\right) = 20\left(1 - \frac{x}{4}\right)$$
$$4(x-1) - 10(x+3) = 20 - 5x$$
$$4x - 4 - 10x - 30 = 20 - 5x$$
$$-6x - 34 = 20 - 5x$$
$$-6x + 5x = 20 + 34$$
$$-1x = 54$$
$$x = -54$$

The solution set is {-54}.

121. Let x = the number of bridge crossings at which the costs of the two plans are the same.

$$\underbrace{6x}_{\text{No Pass}} = \underbrace{30 + 4x}_{\text{Discount Pass}}$$
$$6x - 4x = 30$$
$$2x = 30$$
$$x = 15$$

The two plans cost the same for 15 bridge crossings.
The monthly cost is $\$6(15) = \$90$.

122. $Ax + By = Cy + D$

$$By - Cy = D - Ax$$
$$y(B - C) = D - Ax$$
$$y = \frac{D - Ax}{B - C}$$

123. $\{(4,-2),(1,-1),(1,1),(4,2)\}$

The element 1 in the domain corresponds to two elements in the range.
Thus, the relation is not a function.

124. $x = \frac{5}{y} + 4$

$$y(x) = y\left(\frac{5}{y} + 4\right)$$
$$xy = 5 + 4y$$
$$xy - 4y = 5$$
$$y(x-4) = 5$$
$$y = \frac{5}{x-4}$$

125. $x = y^2 - 1$

$$x + 1 = y^2$$
$$\sqrt{x+1} = \sqrt{y^2}$$
$$\sqrt{x+1} = y$$
$$y = \sqrt{x+1}$$

Section 1.8

Check Point Exercises

1. $f(g(x)) = 4\left(\frac{x+7}{4}\right) - 7$

$$= x + 7 - 7$$
$$= x$$
$$g(f(x)) = \frac{(4x-7)+7}{4}$$
$$= \frac{4x - 7 + 7}{4}$$
$$= \frac{4x}{4}$$
$$= x$$
$$f(g(x)) = g(f(x)) = x$$

2. $f(x) = 2x + 7$

Replace $f(x)$ with y:

$y = 2x + 7$

Interchange x and y:

$x = 2y + 7$

Solve for y:

$$x = 2y + 7$$
$$x - 7 = 2y$$
$$\frac{x-7}{2} = y$$

Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \frac{x-7}{2}$$

3. $f(x) = 4x^3 - 1$

Replace $f(x)$ with y:

$y = 4x^3 - 1$

Interchange x and y:

$x = 4y^3 - 1$

Solve for y:

$$x = 4y^3 - 1$$
$$x + 1 = 4y^3$$
$$\frac{x+1}{4} = y^3$$
$$\sqrt[3]{\frac{x+1}{4}} = y$$

Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$$

Alternative form for answer:

$$f(x)^{-1} = \sqrt[3]{\frac{x+1}{4}} = \frac{\sqrt[3]{x+1}}{\sqrt[3]{4}}$$
$$= \frac{\sqrt[3]{x+1}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2x+2}}{\sqrt[3]{8}}$$
$$= \frac{\sqrt[3]{2x+2}}{2}$$

4. $f(x) = \dfrac{x+1}{x-5}, x \neq 5$

Replace $f(x)$ with y:

$$y = \frac{x+1}{x-5}$$

Interchange x and y:

$$x = \frac{y+1}{y-5}$$

Solve for y:

$$x = \frac{y+1}{y-5}$$
$$x(y-5) = y+1$$
$$xy - 5x = y+1$$
$$xy - y = 5x+1$$
$$y(x-1) = 5x+1$$
$$y = \frac{5x+1}{x-1}$$

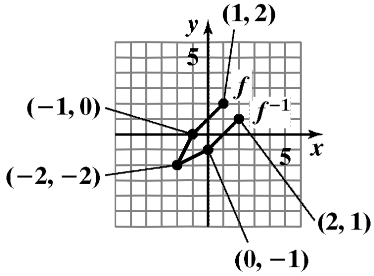
Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \frac{5x+1}{x-1}$$

5. The graphs of (b) and (c) pass the horizontal line test and thus have an inverse.

6. Find points of f^{-1}.

$f(x)$	$f^{-1}(x)$
$(-2,-2)$	$(-2,-2)$
$(-1,0)$	$(0,-1)$
$(1,2)$	$(2,1)$



7. $f(x) = x^2 + 1$

Replace $f(x)$ with y:

$$y = x^2 + 1$$

Interchange x and y:

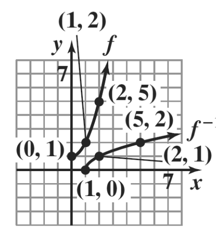
$$x = y^2 + 1$$

Solve for y:

$$x = y^2 + 1$$
$$x - 1 = y^2$$
$$\sqrt{x-1} = y$$

Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \sqrt{x-1}$$



Concept and Vocabulary Check 1.8

1. inverse

2. x; x

3. horizontal; one-to-one

4. $y = x$

Exercise Set 1.8

1. $f(x) = 4x; g(x) = \dfrac{x}{4}$

$$f(g(x)) = 4\left(\frac{x}{4}\right) = x$$
$$g(f(x)) = \frac{4x}{4} = x$$

f and *g* are inverses.

2. $f(x) = 6x; g(x) = \dfrac{x}{6}$

$$f\left(g(x)\right) = 6\left(\frac{x}{6}\right) = x$$
$$g\left(f(x)\right) = \frac{6x}{6} = x$$

f and *g* are inverses.

3. $f(x) = 3x + 8;\ g(x) = \frac{x-8}{3}$

$$f(g(x)) = 3\left(\frac{x-8}{3}\right) + 8 = x - 8 + 8 = x$$
$$g(f(x)) = \frac{(3x+8)-8}{3} = \frac{3x}{3} = x$$

f and *g* are inverses.

4. $f(x) = 4x + 9; g(x) = \frac{x-9}{4}$

$$f\left(g(x)\right) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$
$$g\left(f(x)\right) = \frac{(4x+9)-9}{4} = \frac{4x}{4} = x$$

f and *g* are inverses.

5. $f(x) = 5x - 9;\ g(x) = \frac{x+5}{9}$

$$f(g(x)) = 5\left(\frac{x+5}{9}\right) - 9$$
$$= \frac{5x+25}{9} - 9$$
$$= \frac{5x-56}{9}$$
$$g(f(x)) = \frac{5x-9+5}{9} = \frac{5x-4}{9}$$

f and *g* are not inverses.

6. $f(x) = 3x - 7; g(x) = \frac{x+3}{7}$

$$f\left(g(x)\right) = 3\left(\frac{x+3}{7}\right) - 7 = \frac{3x+9}{7} - 7 = \frac{3x-40}{7}$$
$$g\left(f(x)\right) = \frac{3x-7+3}{7} = \frac{3x-4}{7}$$

f and *g* are not inverses.

7. $f(x) = \frac{3}{x-4}; g(x) = \frac{3}{x} + 4$

$$f(g(x)) = \frac{3}{\frac{3}{x} + 4 - 4} = \frac{3}{\frac{3}{x}} = x$$
$$g(f(x)) = \frac{3}{\frac{3}{x-4}} + 4$$
$$= 3 \cdot \left(\frac{x-4}{3}\right) + 4$$
$$= x - 4 + 4$$
$$= x$$

f and *g* are inverses.

8. $f(x) = \frac{2}{x-5}; g(x) = \frac{2}{x} + 5$

$$f(g(x)) = \frac{2}{\left(\frac{2}{x} + 5\right) - 5} = \frac{2x}{2} = x$$
$$g\left(f(x)\right) = \frac{2}{\frac{2}{x-5}} + 5 = 2\left(\frac{x-5}{2}\right) + 5 = x - 5 + 5 = x$$

f and *g* are inverses.

9. $f(x) = -x; g(x) = -x$

$$f(g(x)) = -(-x) = x$$
$$g(f(x)) = -(-x) = x$$

f and *g* are inverses.

10. $f(x) = \sqrt[3]{x-4}; g(x) = x^3 + 4$

$$f\left(g(x)\right) = \sqrt[3]{x^3 + 4 - 4} = \sqrt[3]{x^3} = x$$
$$g\left(f(x)\right) = \left(\sqrt[3]{x-4}\right)^3 + 4 = x - 4 + 4 = x$$

f and *g* are inverses.

11. a.

$$f(x) = x + 3$$
$$y = x + 3$$
$$x = y + 3$$
$$y = x - 3$$
$$f^{-1}(x) = x - 3$$

b.

$$f(f^{-1}(x)) = x - 3 + 3 = x$$
$$f^{-1}(f(x)) = x + 3 - 3 = x$$

12. a.

$$f(x) = x + 5$$
$$y = x + 5$$
$$x = y + 5$$
$$y = x - 5$$
$$f^{-1}(x) = x - 5$$

b.

$$f\left(f^{-1}(x)\right) = x - 5 + 5 = x$$
$$f^{-1}\left(f(x)\right) = x + 5 - 5 = x$$

13. a.

$$f(x) = 2x$$
$$y = 2x$$
$$x = 2y$$
$$y = \frac{x}{2}$$
$$f^{-1}(x) = \frac{x}{2}$$

b.

$$f(f^{-1}(x)) = 2\left(\frac{x}{2}\right) = x$$
$$f^{-1}(f(x)) = \frac{2x}{2} = x$$

14. **a.**

$$\begin{aligned} f(x) &= 4x \\ y &= 4x \\ x &= 4y \\ y &= \frac{x}{4} \\ f^{-1}(x) &= \frac{x}{4} \end{aligned}$$

b.

$$f\left(f^{-1}(x)\right) = 4\left(\frac{x}{4}\right) = x$$

$$f^{-1}\left(f(x)\right) = \frac{4x}{4} = x$$

15. **a.**

$$\begin{aligned} f(x) &= 2x+3 \\ y &= 2x+3 \\ x &= 2y+3 \\ x-3 &= 2y \\ y &= \frac{x-3}{2} \\ f^{-1}(x) &= \frac{x-3}{2} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= 2\left(\frac{x-3}{2}\right)+3 \\ &= x-3+3 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \frac{2x+3-3}{2} = \frac{2x}{2} = x$$

16. **a.**

$$\begin{aligned} f(x) &= 3x-1 \\ y &= 3x-1 \\ x &= 3y-1 \\ x+1 &= 3y \\ y &= \frac{x+1}{3} \\ f^{-1}(x) &= \frac{x+1}{3} \end{aligned}$$

b.

$$f\left(f^{-1}(x)\right) = 3\left(\frac{x+1}{3}\right) - 1 = x+1-1 = x$$

$$f^{-1}\left(f(x)\right) = \frac{3x-1+1}{3} = \frac{3x}{3} = x$$

17. **a.**

$$\begin{aligned} f(x) &= x^3+2 \\ y &= x^3+2 \\ x &= y^3+2 \\ x-2 &= y^3 \\ y &= \sqrt[3]{x-2} \\ f^{-1}(x) &= \sqrt[3]{x-2} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= \left(\sqrt[3]{x-2}\right)^3 + 2 \\ &= x-2+2 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3+2-2} = \sqrt[3]{x^3} = x$$

18. **a.**

$$\begin{aligned} f(x) &= x^3-1 \\ y &= x^3-1 \\ x &= y^3-1 \\ x+1 &= y^3 \\ y &= \sqrt[3]{x+1} \\ f^{-1}(x) &= \sqrt[3]{x+1} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= \left(\sqrt[3]{x+1}\right)^3 - 1 \\ &= x+1-1 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3-1+1} = \sqrt[3]{x^3} = x$$

19. **a.**

$$\begin{aligned} f(x) &= (x+2)^3 \\ y &= (x+2)^3 \\ x &= (y+2)^3 \\ \sqrt[3]{x} &= y+2 \\ y &= \sqrt[3]{x}-2 \\ f^{-1}(x) &= \sqrt[3]{x}-2 \end{aligned}$$

b.

$$f(f^{-1}(x)) = \left(\sqrt[3]{x}-2+2\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$

$$\begin{aligned} f^{-1}(f(x)) &= \sqrt[3]{(x+2)^3} - 2 \\ &= x+2-2 \\ &= x \end{aligned}$$

20. **a.**

$$\begin{aligned} f(x) &= (x-1)^3 \\ y &= (x-1)^3 \\ x &= (y-1)^3 \\ \sqrt[3]{x} &= y-1 \\ y &= \sqrt[3]{x}+1 \end{aligned}$$

b.

$$f\left(f^{-1}(x)\right) = \left(\sqrt[3]{x}+1-1\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$

$$f^{-1}\left(f(x)\right) = \sqrt[3]{(x-1^3} + 1 = x-1+1 = x$$

21. a.
$$f(x)=\frac{1}{x}$$
$$y=\frac{1}{x}$$
$$x=\frac{1}{y}$$
$$xy=1$$
$$y=\frac{1}{x}$$
$$f^{-1}(x)=\frac{1}{x}$$

b.
$$f(f^{-1}(x))=\frac{1}{\frac{1}{x}}=x$$
$$f^{-1}(f(x))=\frac{1}{\frac{1}{x}}=x$$

22. a.
$$f(x)=\frac{2}{x}$$
$$y=\frac{2}{x}$$
$$x=\frac{2}{y}$$
$$xy=2$$
$$y=\frac{2}{x}$$
$$f^{-1}(x)=\frac{2}{x}$$

b.
$$f(f^{-1}(x))=\frac{2}{\frac{2}{x}}=2\cdot\frac{x}{2}=x$$
$$f^{-1}(f(x))=\frac{2}{\frac{2}{x}}=2\cdot\frac{x}{2}=x$$

23. a.
$$f(x)=\sqrt{x}$$
$$y=\sqrt{x}$$
$$x=\sqrt{y}$$
$$y=x^2$$
$$f^{-1}(x)=x^2,\ x\ge 0$$

b.
$$f(f^{-1}(x))=\sqrt{x^2}=|x|=x \text{ for } x\ge 0.$$
$$f^{-1}(f(x))=(\sqrt{x})^2=x$$

24. a.
$$f(x)=\sqrt[3]{x}$$
$$y=\sqrt[3]{x}$$
$$x=\sqrt[3]{y}$$
$$y=x^3$$
$$f^{-1}(x)=x^3$$

b.
$$f\left(f^{-1}(x)\right)=\sqrt[3]{x^3}=x$$
$$f^{-1}\left(f(x)\right)=\left(\sqrt[3]{x}\right)^3=x$$

25. a.
$$f(x)=\frac{x+4}{x-2}$$
$$y=\frac{x+4}{x-2}$$
$$x=\frac{y+4}{y-2}$$
$$xy-2x=y+4$$
$$xy-y=2x+4$$
$$y(x-1)=2x+4$$
$$y=\frac{2x+4}{x-1}$$
$$f^{-1}(x)=\frac{2x+4}{x-1},\ x\ne 1$$

b.
$$f\left(f^{-1}(x)\right)=\frac{\frac{2x+4}{x-1}+4}{\frac{2x+4}{x-1}-2}$$
$$=\frac{2x+4+4(x-1)}{2x+4-2(x-1)}$$
$$=\frac{6x}{6}$$
$$=x$$
$$f^{-1}\left(f(x)\right)=\frac{2\left(\frac{x+4}{x-2}\right)+4}{\frac{x+4}{x-2}-1}$$
$$=\frac{2x+8+4(x-2)}{x+4-(x-2)}$$
$$=\frac{6x}{6}$$
$$=x$$

26. **a.** $f(x)=\dfrac{x+5}{x-6}$

$$y=\frac{x+5}{x-6}$$
$$x=\frac{y+5}{y-6}$$
$$xy-6x=y+5$$
$$xy-y=6x+5$$
$$y(x-1)=6x+5$$
$$y=\frac{6x+5}{x-1}$$
$$f^{-1}(x)=\frac{6x+5}{x-1}, x\neq 1$$

b. $$f\left(f^{-1}(x)\right)=\frac{\frac{6x+5}{x-1}+5}{\frac{6x+5}{x-1}-6}$$
$$=\frac{6x+5+5(x-1)}{6x+5-6(x-1)}$$
$$=\frac{11x}{11}$$
$$=x$$

$$f^{-1}\left(f(x)\right)=\frac{6\left(\frac{x+5}{x-6}\right)+5}{\frac{x+5}{x-6}-1}$$
$$=\frac{6x+30+5(x-6)}{x+5-(x-6)}$$
$$=\frac{11x}{11}$$
$$=x$$

27. **a.** $f(x)=\dfrac{2x+1}{x-3}$

$$y=\frac{2x+1}{x-3}$$
$$x=\frac{2y+1}{y-3}$$
$$x(y-3)=2y+1$$
$$xy-3x=2y+1$$
$$xy-2y=3x+1$$
$$y(x-2)=3x+1$$
$$y=\frac{3x+1}{x-2}$$
$$f^{-1}(x)=\frac{3x+1}{x-2}$$

b. $$f(f^{-1}(x))=\frac{2\left(\frac{3x+1}{x-2}\right)+1}{\frac{3x+1}{x-2}-3}$$
$$=\frac{2(3x+1)+x-2}{3x+1-3(x-2)}=\frac{6x+2+x-2}{3x+1-3x+6}$$
$$=\frac{7x}{7}=x$$

$$f^{-1}(f(x))=\frac{3\left(\frac{2x+1}{x-3}\right)+1}{\frac{2x+1}{x-3}-2}$$
$$=\frac{3(2x+1)+x-3}{2x+1-2(x-3)}$$
$$=\frac{6x+3+x-3}{2x+1-2x+6}=\frac{7x}{7}=x$$

28. **a.** $f(x)=\dfrac{2x-3}{x+1}$

$$y=\frac{2x-3}{x+1}$$
$$x=\frac{2y-3}{y+1}$$
$$xy+x=2y-3$$
$$y(x-2)=-x-3$$
$$y=\frac{-x-3}{x-2}$$
$$f^{-1}(x)=\frac{-x-3}{x-2},\quad x\neq 2$$

b. $$f\left(f^{-1}(x)\right)=\frac{2\left(\frac{-x-3}{x-2}\right)-3}{\frac{-x-3}{x-2}+1}$$
$$=\frac{-2x-6-3x+6}{-x-3+x-2}=\frac{-5x}{-5}=x$$
$$f^{-1}(f(x))=\frac{-\left(\frac{2x-3}{x+1}\right)-3}{\frac{2x-3}{x+1}-2}$$
$$=\frac{-2x+3-3x-3}{2x-3-2x-2}=\frac{-5x}{-5}=x$$

29. The function fails the horizontal line test, so it does not have an inverse function.

30. The function passes the horizontal line test, so it does have an inverse function.

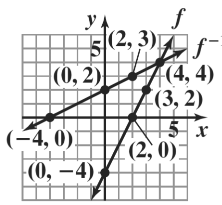
31. The function fails the horizontal line test, so it does not have an inverse function.

32. The function fails the horizontal line test, so it does not have an inverse function.

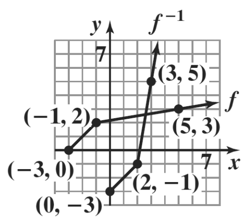
33. The function passes the horizontal line test, so it does have an inverse function.

34. The function passes the horizontal line test, so it does have an inverse function.

35.



36.



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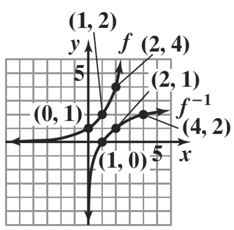

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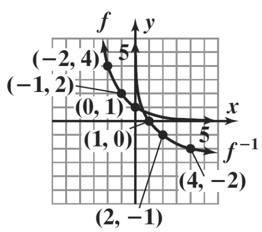
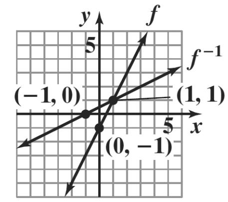


39. **a.**

$$f(x) = 2x - 1$$
$$y = 2x - 1$$
$$x = 2y - 1$$
$$x + 1 = 2y$$
$$\frac{x+1}{2} = y$$
$$f^{-1}(x) = \frac{x+1}{2}$$

b.



c. domain of f: $(-\infty, \infty)$

range of f: $(-\infty, \infty)$

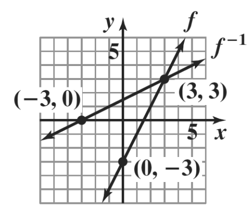
domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

40. **a.**

$$f(x) = 2x - 3$$
$$y = 2x - 3$$
$$x = 2y - 3$$
$$x + 3 = 2y$$
$$\frac{x+3}{2} = y$$
$$f^{-1}(x) = \frac{x+3}{2}$$

b.



c. domain of f: $(-\infty, \infty)$

range of f: $(-\infty, \infty)$

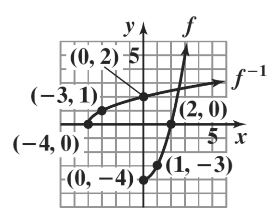
domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

41. **a.**

$$f(x) = x^2 - 4$$
$$y = x^2 - 4$$
$$x = y^2 - 4$$
$$x + 4 = y^2$$
$$\sqrt{x+4} = y$$
$$f^{-1}(x) = \sqrt{x+4}$$

b.



c. domain of f: $[0,\infty)$

range of f: $[-4,\infty)$

domain of f^{-1}: $[-4,\infty)$

range of f^{-1}: $[0,\infty)$

42. **a.** $f(x)=x^2-1$

$y=x^2-1$

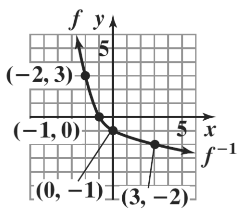
$x=y^2-1$

$x+1=y^2$

$-\sqrt{x+1}=y$

$f^{-1}(x)=-\sqrt{x+1}$

b.



c. domain of f: $(-\infty,0]$

range of f: $[-1,\infty)$

domain of f^{-1}: $[-1,\infty)$

range of f^{-1}: $(-\infty,0]$

43. **a.** $f(x)=(x-1)^2$

$y=(x-1)^2$

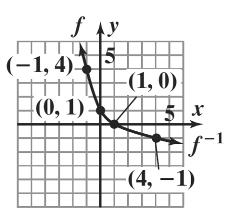
$x=(y-1)^2$

$-\sqrt{x}=y-1$

$-\sqrt{x}+1=y$

$f^{-1}(x)=1-\sqrt{x}$

b.



c. domain of f: $(-\infty,1]$

range of f: $[0,\infty)$

domain of f^{-1}: $[0,\infty)$

range of f^{-1}: $(-\infty,1]$

44. **a.** $f(x)=(x-1)^2$

$y=(x-1)^2$

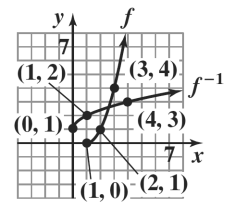
$x=(y-1)^2$

$\sqrt{x}=y-1$

$\sqrt{x}+1=y$

$f^{-1}(x)=1+\sqrt{x}$

b.



c. domain of f: $[1,\infty)$

range of f: $[0,\infty)$

domain of f^{-1}: $[0,\infty)$

range of f^{-1}: $[1,\infty)$

45. **a.** $f(x)=x^3-1$

$y=x^3-1$

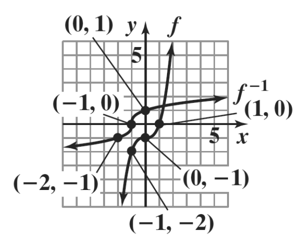
$x=y^3-1$

$x+1=y^3$

$\sqrt[3]{x+1}=y$

$f^{-1}(x)=\sqrt[3]{x+1}$

b.



c. domain of f: $(-\infty,\infty)$

range of f: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

46. **a.** $f(x)=x^3+1$

$y=x^3+1$

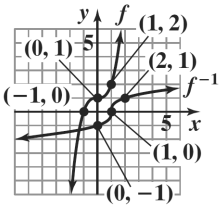
$x=y^3+1$

$x-1=y^3$

$\sqrt[3]{x-1}=y$

$f^{-1}(x)=\sqrt[3]{x-1}$

b.



c. domain of f: $(-\infty,\infty)$

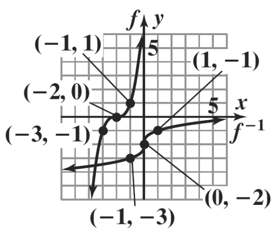
range of f: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

47. a.
$$f(x)=(x+2)^3$$
$$y=(x+2)^3$$
$$x=(y+2)^3$$
$$\sqrt[3]{x}=y+2$$
$$\sqrt[3]{x}-2=y$$
$$f^{-1}(x)=\sqrt[3]{x}-2$$

b.



c. domain of f: $(-\infty,\infty)$

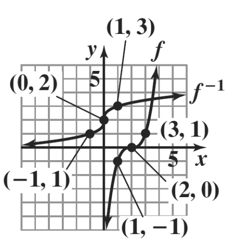
range of f: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

48. a.
$$f(x)=(x-2)^3$$
$$y=(x-2)^3$$
$$x=(y-2)^3$$
$$\sqrt[3]{x}=y-2$$
$$\sqrt[3]{x}+2=y$$
$$f^{-1}(x)=\sqrt[3]{x}+2$$

b.



c. domain of f: $(-\infty,\infty)$

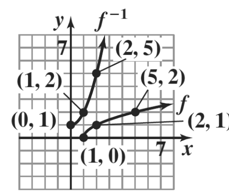
range of f: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

49. a.
$$f(x)=\sqrt{x-1}$$
$$y=\sqrt{x-1}$$
$$x=\sqrt{y-1}$$
$$x^2=y-1$$
$$x^2+1=y$$
$$f^{-1}(x)=x^2+1$$

b.



c. domain of f: $[1,\infty)$

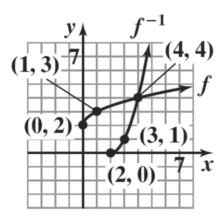
range of f: $[0,\infty)$

domain of f^{-1}: $[0,\infty)$

range of f^{-1}: $[1,\infty)$

50. a.
$$f(x)=\sqrt{x}+2$$
$$y=\sqrt{x}+2$$
$$x=\sqrt{y}+2$$
$$x-2=\sqrt{y}$$
$$(x-2)^2=y$$
$$f^{-1}(x)=(x-2)^2$$

b.



c. domain of f: $[0,\infty)$

range of f: $[2,\infty)$

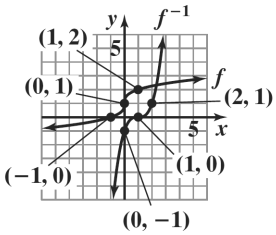
domain of f^{-1}: $[2,\infty)$

range of f^{-1}: $[0,\infty)$

51. **a.** $f(x) = \sqrt[3]{x} + 1$

$$\begin{aligned} y &= \sqrt[3]{x} + 1 \\ x &= \sqrt[3]{y} + 1 \\ x - 1 &= \sqrt[3]{y} \\ (x-1)^3 &= y \\ f^{-1}(x) &= (x-1)^3 \end{aligned}$$

b.



c. domain of f: $(-\infty, \infty)$

range of f: $(-\infty, \infty)$

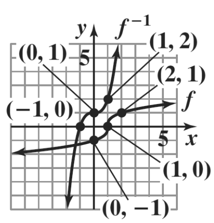
domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

52. **a.** $f(x) = \sqrt[3]{x-1}$

$$\begin{aligned} y &= \sqrt[3]{x-1} \\ x &= \sqrt[3]{y-1} \\ x^3 &= y - 1 \\ x^3 + 1 &= y \\ f^{-1}(x) &= x^3 + 1 \end{aligned}$$

b.



c. domain of f: $(-\infty, \infty)$

range of f: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

53. $f(g(1)) = f(1) = 5$

54. $f(g(4)) = f(2) = -1$

55. $(g \circ f)(-1) = g(f(-1)) = g(1) = 1$

56. $(g \circ f)(0) = g(f(0)) = g(4) = 2$

57. $f^{-1}(g(10)) = f^{-1}(-1) = 2$, since $f(2) = -1$.

58. $f^{-1}(g(1)) = f^{-1}(1) = -1$, since $f(-1) = 1$.

59.
$$\begin{aligned} (f \circ g)(0) &= f(g(0)) \\ &= f(4 \cdot 0 - 1) \\ &= f(-1) = 2(-1) - 5 = -7 \end{aligned}$$

60.
$$\begin{aligned} (g \circ f)(0) &= g(f(0)) \\ &= g(2 \cdot 0 - 5) \\ &= g(-5) = 4(-5) - 1 = -21 \end{aligned}$$

61. Let $f^{-1}(1) = x$. Then

$$\begin{aligned} f(x) &= 1 \\ 2x - 5 &= 1 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Thus, $f^{-1}(1) = 3$

62. Let $g^{-1}(7) = x$. Then

$$\begin{aligned} g(x) &= 7 \\ 4x - 1 &= 7 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

Thus, $g^{-1}(7) = 2$

63.
$$\begin{aligned} g(f[h(1)]) &= g\left(f\left[1^2 + 1 + 2\right]\right) \\ &= g(f(4)) \\ &= g(2 \cdot 4 - 5) \\ &= g(3) \\ &= 4 \cdot 3 - 1 = 11 \end{aligned}$$

64.
$$\begin{aligned} f(g[h(1)]) &= f\left(g\left[1^2 + 1 + 2\right]\right) \\ &= f(g(4)) \\ &= f(4 \cdot 4 - 1) \\ &= f(15) \\ &= 2 \cdot 15 - 5 = 25 \end{aligned}$$

65. **a.** {(Zambia, 4.2), (Colombia, 4.5), (Poland, 3.3), (Italy, 3.3), (United States, 2.5)}

b. {(4.2, Zambia), (4.5 , Colombia), (3.3 , Poland), (3.3, Italy), (2.5, United States)}

f is not a one-to-one function because the inverse of *f* is not a function.

66. **a.** {(Zambia,- 7.3), (Colombia, - 4.5), (Poland, - 2.8), (Italy, - 2.8), (United States, - 1.9)}

b. { (- 7.3, Zambia), (- 4.5, Colombia), (- 2.8, Po land), (- 2.8, Italy), (- 1.9, United States)}

g is not a one-to-one function because the inverse of g is not a function.

67. **a.** It passes the horizontal line test and is one-to-one.

b. $f^{-1}(0.25) = 15$ If there are 15 people in the room, the probability that 2 of them have the same birthday is 0.25.
$f^{-1}(0.5) = 21$ If there are 21 people in the room, the probability that 2 of them have the same birthday is 0.5.
$f^{-1}(0.7) = 30$ If there are 30 people in the room, the probability that 2 of them have the same birthday is 0.7.

68. **a.** This function fails the horizontal line test. Thus, this function does not have an inverse.

b. The average happiness level is 3 at 12 noon and at 7 p.m. These values can be represented as $(12,3)$ and $(19,3)$.

c. The graph does not represent a one-to-one function. $(12,3)$ and $(19,3)$ are an example of two x-values that correspond to the same y-value.

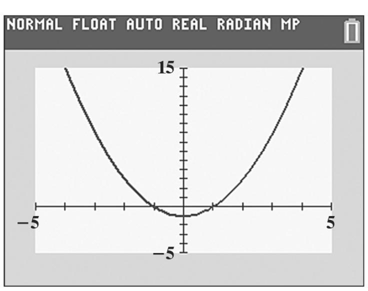
69.
$$\begin{aligned} f(g(x)) &= \frac{9}{5}\left[\frac{5}{9}(x-32)\right]+32 \\ &= x-32+32 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= \frac{5}{9}\left[\left(\frac{9}{5}x+32\right)-32\right] \\ &= x+32-32 \\ &= x \end{aligned}$$

f and g are inverses.

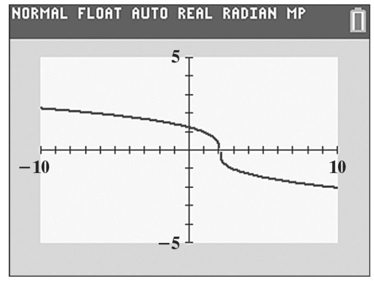
70. – 75. Answers will vary.

76.



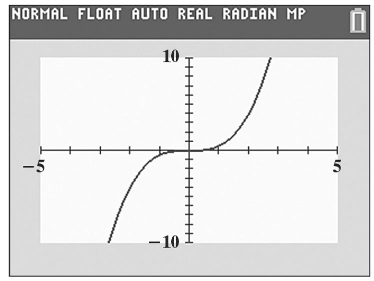
not one-to-one

77.



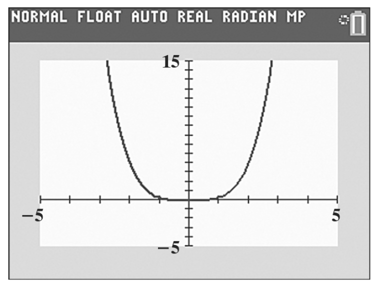
one-to-one

78.



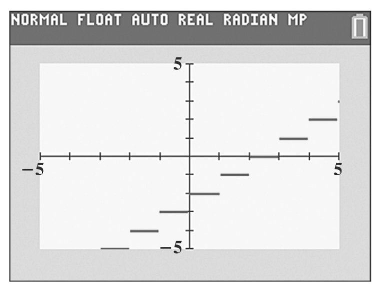
one-to-one

79.



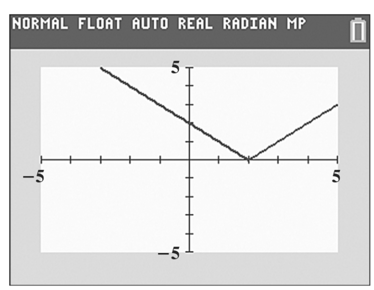
not one-to-one

80.



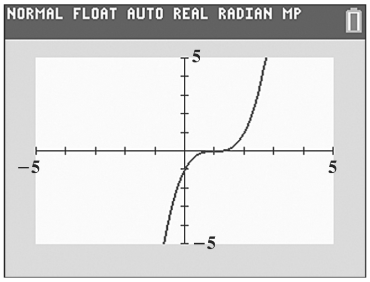
not one-to-one

81.



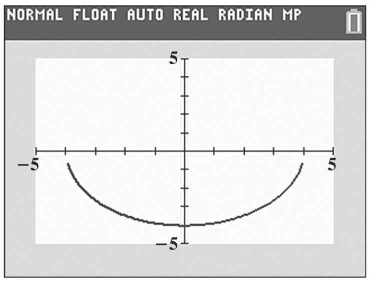
not one-to-one

82.



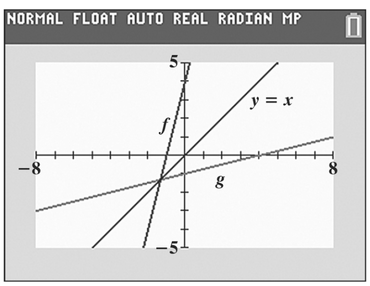
one-to-one

83.



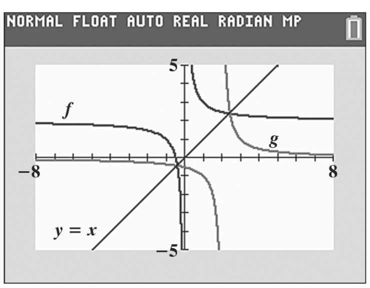
not one-to-one

84.



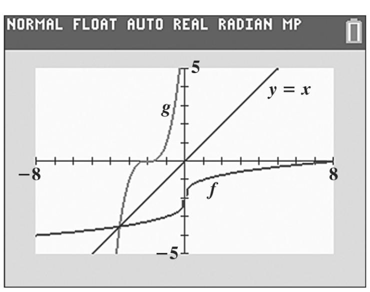
f and g are inverses

85.



f and g are inverses

86.



f and g are inverses

87. makes sense

88. makes sense

89. makes sense

90. does not make sense; Explanations will vary. Sample explanation: The vertical line test is used to determine if a relation is a function, but does not tell us if a function is one-to-one.

91. false; Changes to make the statement true will vary. A sample change is: The inverse is $\{(4,1), (7,2)\}$.

92. false; Changes to make the statement true will vary. A sample change is: $f(x) = 5$ is a horizontal line, so it does not pass the horizontal line test.

93. false; Changes to make the statement true will vary. A sample change is: $f^{-1}(x) = \frac{x}{3}$.

94. true

95. $(f \circ g)(x) = 3(x+5) = 3x+15.$

$$y = 3x+15$$
$$x = 3y+15$$
$$y = \frac{x-15}{3}$$
$$(f \circ g)^{-1}(x) = \frac{x-15}{3}$$

$$g(x) = x+5$$
$$y = x+5$$
$$x = y+5$$
$$y = x-5$$
$$g^{-1}(x) = x-5$$

$$f(x) = 3x$$
$$y = 3x$$
$$x = 3y$$
$$y = \frac{x}{3}$$
$$f^{-1}(x) = \frac{x}{3}$$

$$\left(g^{-1} \circ f^{-1}\right)(x) = \frac{x}{3} - 5 = \frac{x-15}{3}$$

96.
$$f(x)=\frac{3x-2}{5x-3}$$
$$y=\frac{3x-2}{5x-3}$$
$$x=\frac{3y-2}{5y-3}$$
$$x(5y-3)=3y-2$$
$$5xy-3x=3y-2$$
$$5xy-3y=3x-2$$
$$y(5x-3)=3x-2$$
$$y=\frac{3x-2}{5x-3}$$
$$f^{-1}(x)=\frac{3x-2}{5x-3}$$

Note: An alternative approach is to show that $(f\circ f)(x)=x$.

97. No, there will be 2 times when the spacecraft is at the same height, when it is going up and when it is coming down.

98.
$$8+f^{-1}(x-1)=10$$
$$f^{-1}(x-1)=2$$
$$f(2)=x-1$$
$$6=x-1$$
$$7=x$$
$$x=7$$

99. Answers will vary.

100.
$$2x^2-5x+1=0$$
$$x^2-\frac{5}{2}x+\frac{1}{2}=0$$
$$x^2-\frac{5}{2}x=-\frac{1}{2}$$
$$x^2-\frac{5}{2}x+\frac{25}{16}=-\frac{1}{2}+\frac{25}{16}$$
$$\left(x-\frac{5}{4}\right)^2=\frac{17}{16}$$
$$x-\frac{5}{4}=\pm\sqrt{\frac{17}{16}}$$
$$x-\frac{5}{4}=\pm\frac{\sqrt{17}}{4}$$
$$x=\frac{5}{4}\pm\frac{\sqrt{17}}{4}$$
$$x=\frac{5\pm\sqrt{17}}{4}$$

The solution set is $\left\{\frac{5\pm\sqrt{17}}{4}\right\}$.

101.
$$28^2+15.7^2=x^2$$
$$784+246.49=x^2$$
$$1030.49=x^2$$
$$32\approx x$$

The size of the screen is about 32 inches.

102.
$$3|2x-1|\ge 21$$
$$|2x-1|\ge 7$$

$2x-1\le -7$ or $2x-1\ge 7$

$2x\le -6$ $\qquad$ $2x\ge 8$

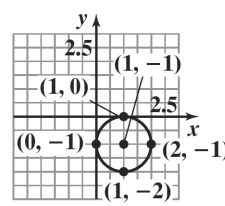
$\frac{2x}{2}\le\frac{-6}{2}$ $\qquad$ $\frac{2x}{2}\ge\frac{8}{2}$

$x\le -3$ $\qquad$ $x\ge 4$

The solution set is $\{x\,|\,x\le -3 \text{ or } x\ge 4\}$ or $(-\infty,-3]\cup[4,\infty)$.

103.
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(1-7)^2+(-1-2)^2}$$
$$=\sqrt{(-6)^2+(-3)^2}$$
$$=\sqrt{36+9}$$
$$=\sqrt{45}$$
$$=3\sqrt{5}$$

104.



105.
$$y^2-6y-4=0$$
$$y^2-6y=4$$
$$y^2-6y+9=4+9$$
$$(y-3)^2=13$$
$$y-3=\pm\sqrt{13}$$
$$y=3\pm\sqrt{13}$$

Solution set: $\{3\pm\sqrt{13}\}$

Section 1.9

Check Point Exercises

1. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\begin{aligned}d&=\sqrt{(2-(-1))^2+(3-(-3))^2}\\&=\sqrt{3^2+6^2}\\&=\sqrt{9+36}\\&=\sqrt{45}\\&=3\sqrt{5}\\&\approx 6.71\end{aligned}$$

2. $\left(\dfrac{1+7}{2},\dfrac{2+(-3)}{2}\right)=\left(\dfrac{8}{2},\dfrac{-1}{2}\right)=\left(4,-\dfrac{1}{2}\right)$

3. $h=0,\ k=0,\ r=4;$

$$\begin{aligned}(x-0)^2+(y-0)^2&=4^2\\x^2+y^2&=16\end{aligned}$$

4. $h=0,\ k=-6,\ r=10;$

$$\begin{aligned}(x-0)^2+[y-(-6)]^2&=10^2\\(x-0)^2+(y+6)^2&=100\\x^2+(y+6)^2&=100\end{aligned}$$

5. **a.** $(x+3)^2+(y-1)^2=4$

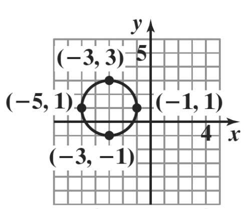
$[x-(-3)]^2+(y-1)^2=2^2$

So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$, we have $h=-3,\ k=1,\ r=2.$

center: $(h,\ k)=(-3,\ 1)$

radius: $r=2$

b.



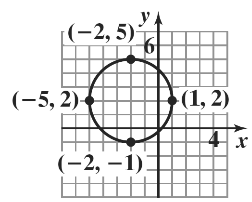
$(x+3)^2+(y-1)^2=4$

c. domain: $[-5,-1]$

range: $[-1,3]$

6.
$$\begin{aligned}x^2+y^2+4x-4y-1&=0\\x^2+y^2+4x-4y-1&=0\\(x^2+4x\quad)+(y^2-4y\quad)&=0\\(x^2+4x+4)+(y^2+4y+4)&=1+4+4\\(x+2)^2+(y-2)^2&=9\\[x-(-x)]^2+(y-2)^2&=3^2\end{aligned}$$

So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$, we have $h=-2,\ k=2,\ r=3$.



$x^2+y^2+4x-4y-1=0$

Concept and Vocabulary Check 1.9

1. $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

2. $\dfrac{x_1+x_2}{2};\ \dfrac{y_1+y_2}{2}$

3. circle; center; radius

4. $(x-h)^2+(y-k)^2=r^2$

5. general

6. 4; 16

Exercise Set 1.9

1.
$$\begin{aligned}d&=\sqrt{(14-2)^2+(8-3)^2}\\&=\sqrt{12^2+5^2}\\&=\sqrt{144+25}\\&=\sqrt{169}\\&=13\end{aligned}$$

2.
$$\begin{aligned}d&=\sqrt{(8-5)^2+(5-1)^2}\\&=\sqrt{3^2+4^2}\\&=\sqrt{9+16}\\&=\sqrt{25}\\&=5\end{aligned}$$

3. $d = \sqrt{(-6-4)^2 + (3-(-1))^2}$
$= \sqrt{(-10)^2 + (4)^2}$
$= \sqrt{100+16}$
$= \sqrt{116}$
$= 2\sqrt{29}$
≈ 10.77

4. $d = \sqrt{(-1-2)^2 + (5-(-3))^2}$
$= \sqrt{(-3)^2 + (8)^2}$
$= \sqrt{9+64}$
$= \sqrt{73}$
≈ 8.54

5. $d = \sqrt{(-3-0)^2 + (4-0)^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
$= 5$

6. $d = \sqrt{(3-0)^2 + (-4-0)^2}$
$= \sqrt{3^2 + (-4)^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
$= 5$

7. $d = \sqrt{[3-(-2)]^2 + [-4-(-6)]^2}$
$= \sqrt{5^2 + 2^2}$
$= \sqrt{25+4}$
$= \sqrt{29}$
≈ 5.39

8. $d = \sqrt{[2-(-4)]^2 + [-3-(-1)]^2}$
$= \sqrt{6^2 + (-2)^2}$
$= \sqrt{36+4}$
$= \sqrt{40}$
$= 2\sqrt{10}$
≈ 6.32

9. $d = \sqrt{(4-0)^2 + [1-(-3)]^2}$
$= \sqrt{4^2 + 4^2}$
$= \sqrt{16+16}$
$= \sqrt{32}$
$= 4\sqrt{2}$
≈ 5.66

10. $d = \sqrt{(4-0)^2 + [3-(-2)]^2}$
$= \sqrt{4^2 + [3+2]^2}$
$= \sqrt{16+5^2}$
$= \sqrt{16+25}$
$= \sqrt{41}$
≈ 6.40

11. $d = \sqrt{(-.5-3.5)^2 + (6.2-8.2)^2}$
$= \sqrt{(-4)^2 + (-2)^2}$
$= \sqrt{16+4}$
$= \sqrt{20}$
$= 2\sqrt{5}$
≈ 4.47

12. $d = \sqrt{(1.6-2.6)^2 + (-5.7-1.3)^2}$
$= \sqrt{(-1)^2 + (-7)^2}$
$= \sqrt{1+49}$
$= \sqrt{50}$
$= 5\sqrt{2}$
≈ 7.07

13. $d = \sqrt{(\sqrt{5}-0)^2 + [0-(-\sqrt{3})]^2}$
$= \sqrt{(\sqrt{5})^2 + (\sqrt{3})^2}$
$= \sqrt{5+3}$
$= \sqrt{8}$
$= 2\sqrt{2}$
≈ 2.83

14. $d = \sqrt{\left(\sqrt{7}-0\right)^2 + \left[0-\left(-\sqrt{2}\right)\right]^2}$
$= \sqrt{\left(\sqrt{7}\right)^2 + \left[-\sqrt{2}\right]^2}$
$= \sqrt{7+2}$
$= \sqrt{9}$
$= 3$

15. $d = \sqrt{(-\sqrt{3}-3\sqrt{3})^2 + (4\sqrt{5}-\sqrt{5})^2}$
$= \sqrt{(-4\sqrt{3})^2 + (3\sqrt{5})^2}$
$= \sqrt{16(3)+9(5)}$
$= \sqrt{48+45}$
$= \sqrt{93}$
≈ 9.64

16. $d=\sqrt{\left(-\sqrt{3}-2\sqrt{3}\right)^2+\left(5\sqrt{6}-\sqrt{6}\right)^2}$
$=\sqrt{\left(-3\sqrt{3}\right)^2+\left(4\sqrt{6}\right)^2}$
$=\sqrt{9\cdot 3+16\cdot 6}$
$=\sqrt{27+96}$
$=\sqrt{123}$
≈ 11.09

17. $d=\sqrt{\left(\frac{1}{3}-\frac{7}{3}\right)^2+\left(\frac{6}{5}-\frac{1}{5}\right)^2}$
$=\sqrt{(-2)^2+1^2}$
$=\sqrt{4+1}$
$=\sqrt{5}$
≈ 2.24

18. $d=\sqrt{\left[\frac{3}{4}-\left(-\frac{1}{4}\right)\right]^2+\left[\frac{6}{7}-\left(-\frac{1}{7}\right)\right]^2}$
$=\sqrt{\left(\frac{3}{4}+\frac{1}{4}\right)^2+\left[\frac{6}{7}+\frac{1}{7}\right]^2}$
$=\sqrt{1^2+1^2}$
$=\sqrt{2}$
≈ 1.41

19. $\left(\frac{6+2}{2},\frac{8+4}{2}\right)=\left(\frac{8}{2},\frac{12}{2}\right)=(4,6)$

20. $\left(\frac{10+2}{2},\frac{4+6}{2}\right)=\left(\frac{12}{2},\frac{10}{2}\right)=(6,5)$

21. $\left(\frac{-2+(-6)}{2},\frac{-8+(-2)}{2}\right)$
$=\left(\frac{-8}{2},\frac{-10}{2}\right)=(-4,-5)$

22. $\left(\frac{-4+(-1)}{2},\frac{-7+(-3)}{2}\right)=\left(\frac{-5}{2},\frac{-10}{2}\right)$
$=\left(\frac{-5}{2},-5\right)$

23. $\left(\frac{-3+6}{2},\frac{-4+(-8)}{2}\right)$
$=\left(\frac{3}{2},\frac{-12}{2}\right)=\left(\frac{3}{2},-6\right)$

24. $\left(\frac{-2+(-8)}{2}\frac{-1+6}{2}\right)=\left(\frac{-10}{2},\frac{5}{2}\right)=\left(-5,\frac{5}{2}\right)$

25. $\left(\frac{\frac{-7}{2}+\left(-\frac{5}{2}\right)}{2},\frac{\frac{3}{2}+\left(-\frac{11}{2}\right)}{2}\right)$
$=\left(\frac{\frac{-12}{2}}{2},\frac{\frac{-8}{2}}{2}\right)=\left(-\frac{6}{2},\frac{-4}{2}\right)=(-3,-2)$

26. $\left(\frac{-\frac{2}{5}+\left(-\frac{2}{5}\right)}{2},\frac{\frac{7}{15}+\left(-\frac{4}{15}\right)}{2}\right)=\left(\frac{-\frac{4}{5}}{2},\frac{\frac{3}{15}}{2}\right)$
$=\left(-\frac{4}{5}\cdot\frac{1}{2},\frac{3}{15}\cdot\frac{1}{2}\right)=\left(-\frac{2}{5},\frac{1}{10}\right)$

27. $\left(\frac{8+(-6)}{2},\frac{3\sqrt{5}+7\sqrt{5}}{2}\right)$
$=\left(\frac{2}{2},\frac{10\sqrt{5}}{2}\right)=\left(1,5\sqrt{5}\right)$

28. $\left(\frac{7\sqrt{3}+3\sqrt{3}}{2},\frac{-6+(-2)}{2}\right)=\left(\frac{10\sqrt{3}}{2},\frac{-8}{2}\right)$
$=\left(5\sqrt{3},-4\right)$

29. $\left(\frac{\sqrt{18}+\sqrt{2}}{2},\frac{-4+4}{2}\right)$
$=\left(\frac{3\sqrt{2}+\sqrt{2}}{2},\frac{0}{2}\right)=\left(\frac{4\sqrt{2}}{2},0\right)=(2\sqrt{2},0)$

30. $\left(\frac{\sqrt{50}+\sqrt{2}}{2},\frac{-6+6}{2}\right)=\left(\frac{5\sqrt{2}+\sqrt{2}}{2},\frac{0}{2}\right)$
$=\left(\frac{6\sqrt{2}}{2},0\right)=\left(3\sqrt{2},0\right)$

31. $(x-0)^2+(y-0)^2=7^2$
$x^2+y^2=49$

32. $(x-0)^2+(y-0)^2=8^2$
$x^2+y^2=64$

33. $(x-3)^2+(y-2)^2=5^2$
$(x-3)^2+(y-2)^2=25$

34. $(x-2)^2+[y-(-1)]^2=4^2$

$(x-2)^2+(y+1)^2=16$

35. $[x-(-1)]^2+(y-4)^2=2^2$

$(x+1)^2+(y-4)^2=4$

36. $[x-(-3)]^2+(y-5)^2=3^2$

$(x+3)^2+(y-5)^2=9$

37. $[x-(-3)]^2+[y-(-1)]^2=\left(\sqrt{3}\right)^2$

$(x+3)^2+(y+1)^2=3$

38. $[x-(-5)]^2+[y-(-3)]^2=\left(\sqrt{5}\right)^2$

$(x+5)^2+(y+3)^2=5$

39. $[x-(-4)]^2+(y-0)^2=10^2$

$(x+4)^2+(y-0)^2=100$

40. $[x-(-2)]^2+(y-0)^2=6^2$

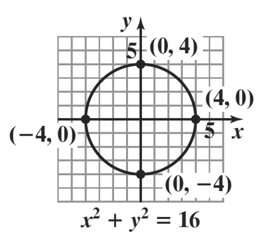
$(x+2)^2+y^2=36$

41. $x^2+y^2=16$

$(x-0)^2+(y-0)^2=y^2$

$h=0,\ k=0,\ r=4;$

center = (0, 0); radius = 4



domain: $[-4,4]$

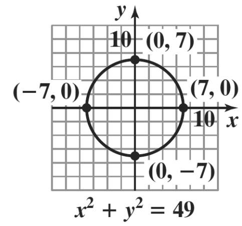
range: $[-4,4]$

42. $x^2+y^2=49$

$(x-0)^2+(y-0)^2=7^2$

$h=0, k=0, r=7;$

center = (0, 0); radius = 7



domain: $[-7,7]$

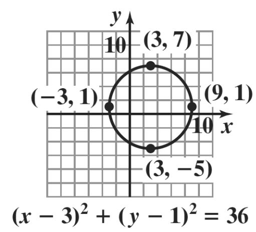
range: $[-7,7]$

43. $(x-3)^2+(y-1)^2=36$

$(x-3)^2+(y-1)^2=6^2$

$h=3,\ k=1,\ r=6;$

center = (3, 1); radius = 6



domain: $[-3,9]$

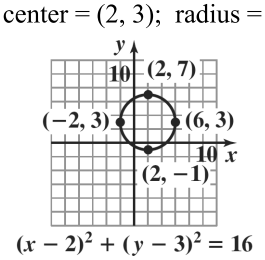
range: $[-5,7]$

44. $(x-2)^2+(y-3)^2=16$

$(x-2)^2+(y-3)^2=4^2$

$h=2, k=3, r=4;$

center = (2, 3); radius = 4



domain: $[-2,6]$

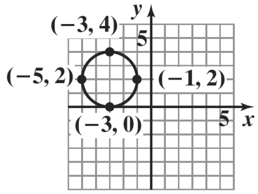
range: $[-1,7]$

45. $(x+3)^2+(y-2)^2=4$

$[x-(-3)]^2+(y-2)^2=2^2$

$h=-3,\ k=2,\ r=2$

center = (–3, 2); radius = 2



$(x+3)^2+(y-2)^2=4$

domain: $[-5,-1]$

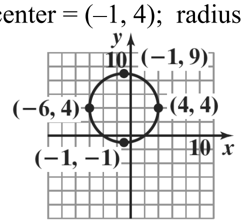
range: $[0,4]$

46. $(x+1)^2+(y-4)^2=25$

$[x-(-1)]^2+(y-4)^2=5^2$

$h=-1, k=4, r=5;$

center = (–1, 4); radius = 5



$(x+1)^2+(y-4)^2=25$

domain: $[-6,4]$

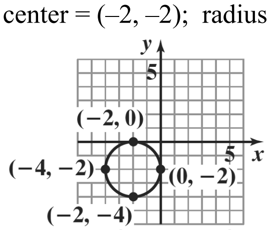
range: $[-1,9]$

47. $(x+2)^2+(y+2)^2=4$

$[x-(-2)]^2+[y-(-2)]^2=2^2$

$h=-2,\ k=-2,\ r=2$

center = (–2, –2); radius = 2



$(x+2)^2+(y+2)^2=4$

domain: $[-4,0]$

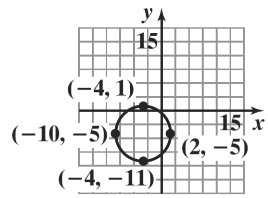
range: $[-4,0]$

48. $(x+4)^2+(y+5)^2=36$

$[x-(-4)]^2+[y-(-5)]^2=6^2$

$h=-4, k=-5, r=6;$

center = (–4, –5); radius = 6



$(x+4)^2+(y+5)^2=36$

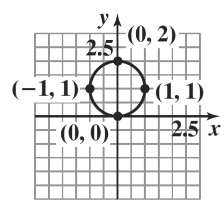
domain: $[-10,2]$

range: $[-11,1]$

49. $x^2+(y-1)^2=1$

$h=0, k=1, r=1;$

center = (0, 1); radius = 1



$x^2+(y-1)^2=1$

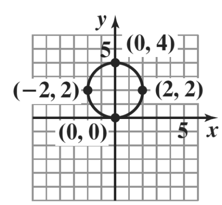
domain: $[-1,1]$

range: $[0,2]$

50. $x^2+(y-2)^2=4$

$h=0, k=2, r=2;$

center = (0,2); radius = 2



$x^2+(y-2)^2=4$

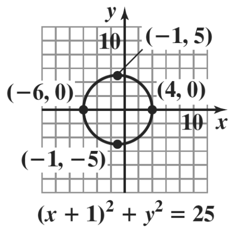
domain: $[-2,2]$

range: $[0,4]$

51. $(x+1)^2 + y^2 = 25$

$h = -1, k = 0, r = 5;$

center = (–1,0); radius = 5



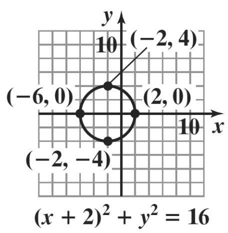
domain: $[-6, 4]$

range: $[-5, 5]$

52. $(x+2)^2 + y^2 = 16$

$h = -2, k = 0, r = 4;$

center = (–2,0); radius = 4



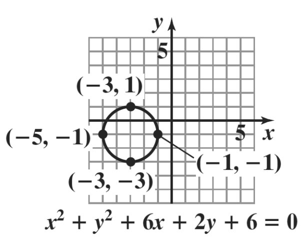
domain: $[-6, 2]$

range: $[-4, 4]$

53.

$$x^2 + y^2 + 6x + 2y + 6 = 0$$
$$\left(x^2 + 6x\right) + \left(y^2 + 2y\right) = -6$$
$$\left(x^2 + 6x + 9\right) + \left(y^2 + 2y + 1\right) = 9 + 1 - 6$$
$$(x+3)^2 + (y+1)^2 = 4$$
$$[x - (-3)]^2 + [9 - (-1)]^2 = 2^2$$

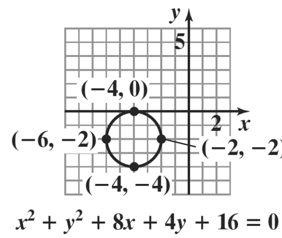
center = (–3, –1); radius = 2



54. $x^2 + y^2 + 8x + 4y + 16 = 0$

$$\left(x^2 + 8x\right) + \left(y^2 + 4y\right) = -16$$
$$\left(x^2 + 8x + 16\right) + \left(y^2 + 4y + 4\right) = 20 - 16$$
$$(x+4)^2 + (y+2)^2 = 4$$
$$[x - (-4)]^2 + [y - (-2)]^2 = 2^2$$

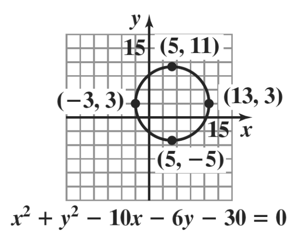
center = (–4, –2); radius = 2



55.

$$x^2 + y^2 - 10x - 6y - 30 = 0$$
$$\left(x^2 - 10x\right) + \left(y^2 - 6y\right) = 30$$
$$\left(x^2 - 10x + 25\right) + \left(y^2 - 6y + 9\right) = 25 + 9 + 30$$
$$(x-5)^2 + (y-3)^2 = 64$$
$$(x-5)^2 + (y-3)^2 = 8^2$$

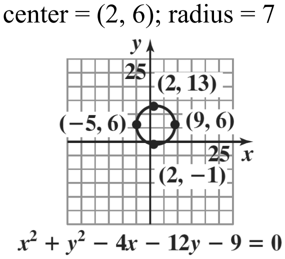
center = (5, 3); radius = 8



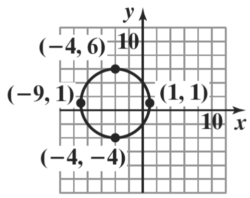
56. $x^2 + y^2 - 4x - 12y - 9 = 0$

$$\left(x^2 - 4x\right) + \left(y^2 - 12y\right) = 9$$
$$\left(x^2 - 4x + 4\right) + \left(y^2 - 12y + 36\right) = 4 + 36 + 9$$
$$(x-2)^2 + (y-6)^2 = 49$$
$$(x-2)^2 + (y-6)^2 = 7^2$$

center = (2, 6); radius = 7

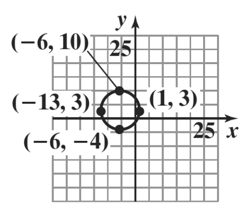


57.
$$x^2+y^2+8x-2y-8=0$$
$$\left(x^2+8x\right)+\left(y^2-2y\right)=8$$
$$\left(x^2+8x+16\right)+\left(y^2-2y+1\right)=16+1+8$$
$$(x+4)^2+(y-1)^2=25$$
$$[x-(-4)]^2+(y-1)^2=5^2$$
center = (–4, 1); radius = 5



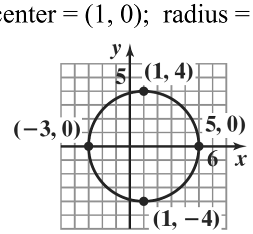
$x^2 + y^2 + 8x - 2y - 8 = 0$

58.
$$x^2+y^2+12x-6y-4=0$$
$$\left(x^2+12x\right)+\left(y^2-6y\right)=4$$
$$\left(x^2+12x+36\right)+\left(y^2-6y+9\right)=36+9+4$$
$$[x-(-6)]^2+(y-3)^2=7^2$$
center = (–6, 3); radius = 7



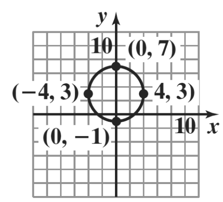
$x^2 + y^2 + 12x - 6y - 4 = 0$

59.
$$x^2-2x+y^2-15=0$$
$$\left(x^2-2x\right)+y^2=15$$
$$\left(x^2-2x+1\right)+(y-0)^2=1+0+15$$
$$(x-1)^2+(y-0)^2=16$$
$$(x-1)^2+(y-0)^2=4^2$$
center = (1, 0); radius = 4



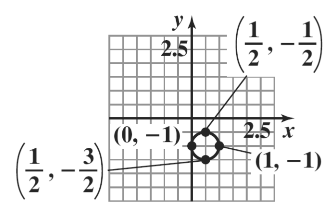
$x^2 - 2x + y^2 - 15 = 0$

60.
$$x^2+y^2-6y-7=0$$
$$x^2+\left(y^2-6y\right)=7$$
$$(x-0)^2=\left(y^2-6y+9\right)=0+9+7$$
$$(x-0)^2+(y-3)^2=16$$
$$(x-0)^2+(y-3)^2=4^2$$
center = (0, 3); radius = 4



$x^2 + y^2 - 6y - 7 = 0$

61.
$$x^2+y^2-x+2y+1=0$$
$$x^2-x\quad+y^2+2y\quad=-1$$
$$x^2-x+\frac{1}{4}+y^2+2y+1=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\frac{1}{4}$$
center = $\left(\frac{1}{2},-1\right)$; radius = $\frac{1}{2}$



$x^2 + y^2 - x + 2y + 1 = 0$

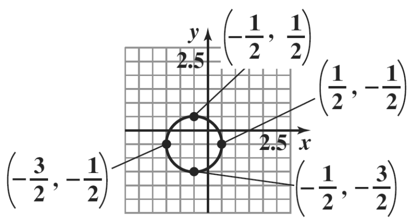
62. $x^2+y^2+x+y-\frac{1}{2}=0$

$x^2+x\quad+y^2+y\quad=\frac{1}{2}$

$x^2+x+\frac{1}{4}+y^2+y+\frac{1}{4}=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$

$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1$

center = $\left(-\frac{1}{2},-\frac{1}{2}\right)$; radius = 1



$x^2+y^2+x+y-\frac{1}{2}=0$

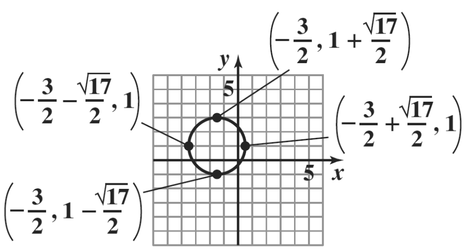
63. $x^2+y^2+3x-2y-1=0$

$x^2+3x\quad+y^2-2y\quad=1$

$x^2+3x+\frac{9}{4}+y^2-2y+1=1+\frac{9}{4}+1$

$\left(x+\frac{3}{2}\right)^2+(y-1)^2=\frac{17}{4}$

center = $\left(-\frac{3}{2},1\right)$; radius = $\frac{\sqrt{17}}{2}$



$x^2+y^2+3x-2y-1=0$

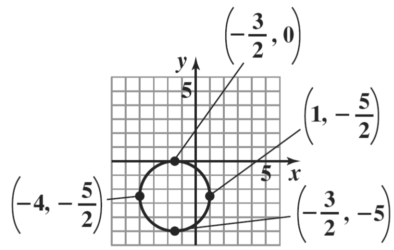
64. $x^2+y^2+3x+5y+\frac{9}{4}=0$

$x^2+3x\quad+y^2+5y\quad=-\frac{9}{4}$

$x^2+3x+\frac{9}{4}+y^2+5y+\frac{25}{4}=-\frac{9}{4}+\frac{9}{4}+\frac{25}{4}$

$\left(x+\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{25}{4}$

center = $\left(-\frac{3}{2},-\frac{5}{2}\right)$; radius = $\frac{5}{2}$



$x^2+y^2+3x+5y+\frac{9}{4}=0$

65. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$=\left(\frac{3+7}{2},\frac{9+11}{2}\right)=\left(\frac{10}{2},\frac{20}{2}\right)$

$=(5,10)$

The center is $(5,10)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,9)$, we get:

$d=\sqrt{(5-3)^2+(10-9)^2}$

$=\sqrt{2^2+1^2}=\sqrt{4+1}$

$=\sqrt{5}$

The radius is $\sqrt{5}$ units.

c. $(x-5)^2+(y-10)^2=\left(\sqrt{5}\right)^2$

$(x-5)^2+(y-10)^2=5$

66. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{3+5}{2}, \frac{6+4}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right)$$
$$= (4, 5)$$

The center is $(4, 5)$.

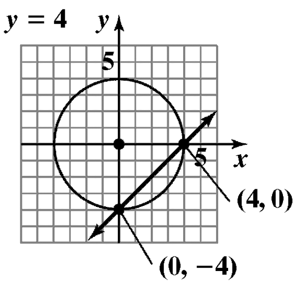
b. The radius is the distance from the center to one of the points on the circle. Using the point $(3, 6)$, we get:

$$d = \sqrt{(4-3)^2 + (5-6)^2}$$
$$= \sqrt{1^2 + (-1)^2} = \sqrt{1+1}$$
$$= \sqrt{2}$$

The radius is $\sqrt{2}$ units.

c.
$$(x-4)^2 + (y-5)^2 = \left(\sqrt{2}\right)^2$$
$$(x-4)^2 + (y-5)^2 = 2$$

67. $x^2 + y^2 = 16$
$x - y = 4$



Intersection points: $(0, -4)$ and $(4, 0)$

Check $(0, -4)$:

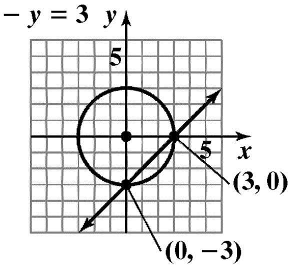
$0^2 + (-4)^2 = 16$ $0 - (-4) = 4$
$16 = 16$ true $4 = 4$ true

Check $(4, 0)$:

$4^2 + 0^2 = 16$ $4 - 0 = 4$
$16 = 16$ true $4 = 4$ true

The solution set is $\{(0, -4), (4, 0)\}$.

68. $x^2 + y^2 = 9$
$x - y = 3$



Intersection points: $(0, -3)$ and $(3, 0)$

Check $(0, -3)$:

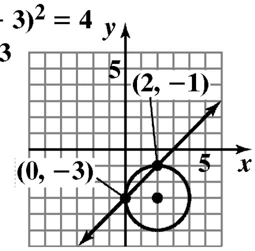
$0^2 + (-3)^2 = 9$ $0 - (-3) = 3$
$9 = 9$ true $3 = 3$ true

Check $(3, 0)$:

$3^2 + 0^2 = 9$ $3 - 0 = 3$
$9 = 9$ true $3 = 3$ true

The solution set is $\{(0, -3), (3, 0)\}$.

69. $(x-2)^2 + (y+3)^2 = 4$
$y = x - 3$



Intersection points: $(0, -3)$ and $(2, -1)$

Check $(0, -3)$:

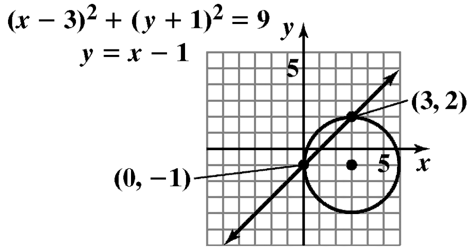
$(0-2)^2 + (-3+3)^2 = 9$ $-3 = 0 - 3$
$(-2)^2 + 0^2 = 4$ $-3 = -3$ true
$4 = 4$
true

Check $(2, -1)$:

$(2-2)^2 + (-1+3)^2 = 4$ $-1 = 2 - 3$
$0^2 + 2^2 = 4$ $-1 = -1$ true
$4 = 4$
true

The solution set is $\{(0, -3), (2, -1)\}$.

70.



Intersection points: $(0,-1)$ and $(3,2)$

Check $(0,-1)$:

$(0-3)^2+(-1+1)^2=9$ $\quad$ $-1=0-1$

$(-3)^2+0^2=9$ $\quad$ $-1=-1$ true

$9=9$

true

Check $(3,2)$:

$(3-3)^2+(2+1)^2=9$ $\quad$ $2=3-1$

$0^2+3^2=9$ $\quad$ $2=2$ true

$9=9$

true

The solution set is $\{(0,-1),(3,2)\}$.

71. $d=\sqrt{(8495-4422)^2+(8720-1241)^2}\cdot\sqrt{0.1}$

$d=\sqrt{72,524,770}\cdot\sqrt{0.1}$

$d\approx 2693$

The distance between Boston and San Francisco is about 2693 miles.

72. $d=\sqrt{(8936-8448)^2+(3542-2625)^2}\cdot\sqrt{0.1}$

$d=\sqrt{1,079,033}\cdot\sqrt{0.1}$

$d\approx 328$

The distance between New Orleans and Houston is about 328 miles.

73. If we place L.A. at the origin, then we want the equation of a circle with center at $(-2.4,-2.7)$ and radius 30.

$(x-(-2.4))^2+(y-(-2.7))^2=30^2$

$(x+2.4)^2+(y+2.7)^2=900$

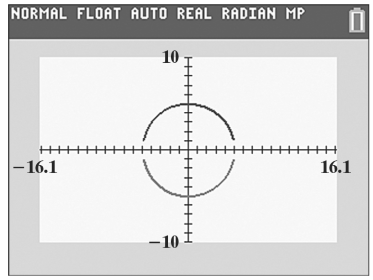
74. $C(0, 68+14)=(0, 82)$

$(x-0)^2+(y-82)^2=68^2$

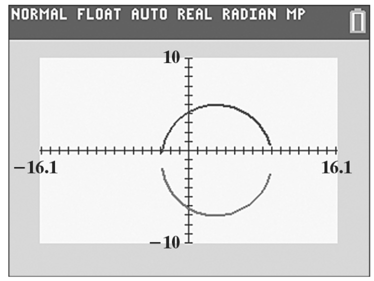
$x^2+(y-82)^2=4624$

75. – 82. Answers will vary.

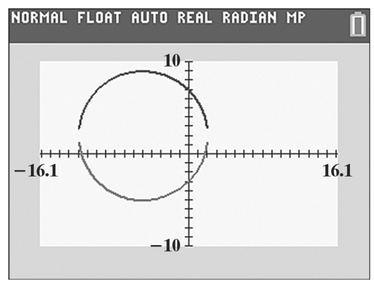
83.



84.



85.



86. makes sense

87. makes sense

88. does not make sense; Explanations will vary. Sample explanation: Since $r^2=-4$ this is not the equation of a circle.

89. makes sense

90. false; Changes to make the statement true will vary. A sample change is: The equation would be $x^2+y^2=256$.

91. false; Changes to make the statement true will vary. A sample change is: The center is at (3, –5).

92. false; Changes to make the statement true will vary. A sample change is: This is not an equation for a circle.

93. false; Changes to make the statement true will vary. A sample change is: Since $r^2 = -36$ this is not the equation of a circle.

94. The distance for A to B:

$$\begin{aligned}\overline{AB} &= \sqrt{(3-1)^2 + [3+d-(1+d)]^2}\\ &= \sqrt{2^2+2^2}\\ &= \sqrt{4+4}\\ &= \sqrt{8}\\ &= 2\sqrt{2}\end{aligned}$$

The distance from B to C:

$$\begin{aligned}\overline{BC} &= \sqrt{(6-3)^2 + [3+d-(6+d)]^2}\\ &= \sqrt{3^2+(-3)^2}\\ &= \sqrt{9+9}\\ &= \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

The distance for A to C:

$$\begin{aligned}\overline{AC} &= \sqrt{(6-1)^2 + [6+d-(1+d)]^2}\\ &= \sqrt{5^2+5^2}\\ &= \sqrt{25+25}\\ &= \sqrt{50}\\ &= 5\sqrt{2}\end{aligned}$$

$$\begin{aligned}\overline{AB}+\overline{BC} &= \overline{AC}\\ 2\sqrt{2}+3\sqrt{2} &= 5\sqrt{2}\\ 5\sqrt{2} &= 5\sqrt{2}\end{aligned}$$

95. a. d_1 is distance from (x_1, x_2) to midpoint

$$d_1 = \sqrt{\left(\frac{x_1+x_2}{2}-x_1\right)^2 + \left(\frac{y_1+y_2}{2}-y_1\right)^2}$$
$$d_1 = \sqrt{\left(\frac{x_1+x_2-2x_1}{2}\right)^2 + \left(\frac{y_1+y_2-2y_1}{2}\right)^2}$$
$$d_1 = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$
$$d_1 = \sqrt{\frac{x_2-2x_1x_2+x_1^2}{4} + \frac{y_2^2-2y_2y_1+y_1^2}{4}}$$
$$d_1 = \sqrt{\frac{1}{4}\left(x_2-2x_1x_2+x_1+y_2^2-2y_2y_1+y_1^2\right)}$$
$$d_1 = \frac{1}{2}\sqrt{x_2-2x_1x_2+x_1+y_2^2-2y_2y_1+y_1^2}$$

d_2 is distance from midpoint to (x_2, y_2)

$$d_2 = \sqrt{\left(\frac{x_1+x_2}{2}-x_2\right)^2 + \left(\frac{y_1+y_2}{2}-y_2\right)^2}$$
$$d_2 = \sqrt{\left(\frac{x_1+x_2-2x_2}{2}\right)^2 + \left(\frac{y_1+y_2-2y_2}{2}\right)^2}$$
$$d_2 = \sqrt{\left(\frac{x_1-x_2}{2}\right)^2 + \left(\frac{y_1-y_2}{2}\right)^2}$$
$$d_2 = \sqrt{\frac{x_1^2-2x_1x_2+x_2^2}{4} + \frac{y_1^2-2y_2y_1+y_2^2}{4}}$$
$$d_2 = \sqrt{\frac{1}{4}\left(x_1^2-2x_1x_2+x_2^2+y_1^2-2y_2y_1+y_2^2\right)}$$
$$d_2 = \frac{1}{2}\sqrt{x_1^2-2x_1x_2+x_2^2+y_1^2-2y_2y_1+y_2^2}$$
$$d_1 = d_2$$

b. d_3 is the distance from (x_1, y_1) to $(x_2 y_2)$

$$d_3 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d_3 = \sqrt{x_2^2-2x_1x_2+x_1^2+y_2^2-2y_2y_1+y_1^2}$$

$d_1 + d_2 = d_3$ because $\frac{1}{2}\sqrt{a}+\frac{1}{2}\sqrt{a}=\sqrt{a}$

96. Both circles have center (2, –3). The smaller circle has radius 5 and the larger circle has radius 6. The smaller circle is inside of the larger circle. The area between them is given by

$$\begin{aligned}\pi(6)^2-\pi(5)^2 &= 36\pi-25\pi\\ &= 11\pi\\ &\approx 34.56 \text{ square units.}\end{aligned}$$

97. The circle is centered at (0,0). The slope of the radius with endpoints (0,0) and (3,–4) is $m = -\frac{-4-0}{3-0} = -\frac{4}{3}$. The line perpendicular to the radius has slope $\frac{3}{4}$. The tangent line has slope $\frac{3}{4}$ and passes through (3,–4), so its equation is: $y+4=\frac{3}{4}(x-3)$.

98. Test for symmetry with respect to the y-axis.

$$x^2-y^3=2$$
$$(-x)^2-y^3=2$$
$$x^2-y^3=2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$x^2 - y^3 = 2$
$x^2 - (-y)^3 = 2$
$x^2 + y^3 = 2$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$x^2 - y^3 = 2$
$(-x)^2 - (-y)^3 = 2$
$x^2 + y^3 = 2$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

99. **a.** The relation is not a function since the two ordered pairs (1, 6) and (1, 8) have the same first component but different second components.

b. The relation is a function since no two ordered pairs have the same first component and different second components.

100.
$$\frac{2}{x+3} - \frac{4}{x+5} = \frac{6}{x^2+8x+15}$$
$$\frac{2}{x+3} - \frac{4}{x+5} = \frac{6}{(x+3)(x+5)}$$
$$2(x+5) - 4(x+3) = 6$$
$$2x + 10 - 4x - 12 = 6$$
$$-2x - 2 = 6$$
$$-2x = 8$$
$$x = -4$$

The solution set is $\{-4\}$.

101. $x - 200$

102. **a.** $p = 2l + 2w = 2(40) + 2(30) = 140$
$A = lw = (40)(30) = 1200$
The perimeter is 140 yd; the area is 1200 sq yd

b. $p = 2l + 2w = 2(50) + 2(20) = 140$
$A = lw = (50)(20) = 1000$
The perimeter is 140 yd; the area is 1000 sq yd

103. $\pi r^2 h = 22$
$$h = \frac{22}{\pi r^2}$$
$$2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\frac{22}{\pi r^2}\right)$$
$$= 2\pi r^2 + \frac{44}{r}$$

Section 1.10

Check Point Exercises

1. **a.** $f(x) = 15 + 0.08x$

b. $g(x) = 3 + 0.12x$

c. $15 + 0.08x = 3 + 0.12x$
$12 = 0.04x$
$300 = x$
The plans cost the same for 300 text messages.

2. **a.** $N(x) = 8000 - 100(x - 100)$
$= 8000 - 100x + 10000$
$= 18{,}000 - 100x$

b. $R(x) = (18{,}000 - 100x)x$
$= -100x^2 + 18{,}000x$

3. $V(x) = (15 - 2x)(8 - 2x)x$
$= (120 - 46x + 4x^2)x$
$= 4x^3 - 46x^2 + 120x$

Since x represents the inches to be cut off, $x > 0$. The smallest side is 8, so must cut less than 4 off each side. The domain of V is $\{x | 0 < x < 4\}$ or, in interval notation, $(0, 4)$.

4. $2l + 2w = 200$
$2l = 200 - 2w$
$l = 100 - w$

Let x = width, then length = $100 - x$
$A(x) = x(100 - x)$
$= 100x - x^2$ square feet

5. $V = \pi r^2 h$

$$1000 = \pi r^2 h$$
$$\frac{1000}{\pi r^2} = h$$

$$\begin{aligned} A(r) &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right) \\ &= 2\pi r^2 + \frac{2000}{r} \end{aligned}$$

6. $$\begin{aligned} d &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} \end{aligned}$$

$$\begin{aligned} y &= x^3 \\ d &= \sqrt{x^2 + \left(x^3\right)^2} \\ &= \sqrt{x^2 + x^6} \end{aligned}$$

Concept and Vocabulary Check 1.10

1. $4 + 0.15x$

2. $x - 300$; $50(x - 300)$; 5000; $50(x - 300)$

3. $10 - 2x$; $10 - 2x$; x; $10 - 2x$; $10 - 2x$; x

4. xy; $2x - 2y$; $90 - x$; $x(90 - x)$

5. $\sqrt{x^2 + y^2}$; $\sqrt{x^2 + x^6}$

Exercise Set 1.10

1. **a.** $f(x) = 200 + 0.15x$

b. $$\begin{aligned} 320 &= 200 + 0.15x \\ 120 &= 01.5x \\ 800 &= x \end{aligned}$$

800 miles

2. **a.** $f(x) = 180 + 0.25x$

b. $$\begin{aligned} 395 &= 180 + 0.25x \\ 215 &= 0.25x \\ 860 &= x \end{aligned}$$

You drove 860 miles for $395.

3. **a.** $M(x) = 239.4 - 0.3x$

b. $$\begin{aligned} 180 &= 239.4 - 0.3x \\ 0.3x &= 59.4 \\ x &= 198 \end{aligned}$$

198 years after 1954, in 2152, someone will run a 3 minute mile.

4. **a.** $P(x) = 28 + 0.6x$

b. $$\begin{aligned} 40 &= 28 + 06x \\ 12 &= 0.6x \\ 20 &= x \end{aligned}$$

20 years after 1990, in 2010, 40% of babies born were out of wedlock.

5. **a.** $f(x) = 1.25x$

b. $g(x) = 21 + 0.5x$

c. $$\begin{aligned} 1.25x &= 21 + 0.5x \\ 0.75x &= 21 \\ x &= 28 \end{aligned}$$
$$f(28) = 1.25(28) = 35$$
$$g(28) = 21 + 0.5(28) = 35$$

If a person crosses the bridge 28 times the cost will be $35 for both options

6. **a.** $f(x) = 2.5x$

b. $g(x) = 21 + x$

c. $$\begin{aligned} 2.5x &= 21 + x \\ 1.5x &= 21 \\ x &= 14 \end{aligned}$$

$$f(14) = 2.5(14) = 35$$
$$g(14) = 21 + 14 = 35$$

To cross the bridge 14 times costs the same, $35, for either method.

7. **a.** $f(x) = 100 + 0.8x$

b. $g(x) = 40 + 0.9x$

c. $$\begin{aligned} 100 + 0.8x &= 40 + 0.9x \\ 60 &= 0.1x \\ 600 &= x \end{aligned}$$

For $600 worth of merchandise, your cost is $580 for both plans

8. **a.** $f(x) = 300 + 0.7x$

b. $g(x) = 40 + 0.9x$

c. $$\begin{aligned} 300 + 0.7x &= 40 + 0.9x \\ 260 &= 0.2x \\ 1300 &= x \end{aligned}$$

$$f(1300) = 300 + 0.7(1300) = 1210$$
$$g(1300) = 40 + 0.9(1300) = 1210$$

You would have to purchase \$1300 in merchandise at a total cost of \$1210.

9. **a.** $$\begin{aligned} N(x) &= 30{,}000 - 500(x - 20) \\ &= 30{,}000 - 500x + 10000 \\ &= 40{,}000 - 500x \end{aligned}$$

b. $$\begin{aligned} R(x) &= (40{,}000 - 500x)x \\ &= -500x^2 + 40{,}000x \end{aligned}$$

10. **a.** $$\begin{aligned} N(x) &= 20{,}000 - 400(x - 15) \\ &= 20{,}000 - 400x + 6000 \\ &= 26{,}000 - 400x \end{aligned}$$

b. $$\begin{aligned} R(x) &= (26{,}000 - 400x)x \\ &= -400x^2 + 26{,}000x \end{aligned}$$

11. **a.** $$\begin{aligned} N(x) &= 9000 + 50(150 - x) \\ &= 9000 - 50x + 7500 \\ &= 16500 - 50x \end{aligned}$$

b. $$\begin{aligned} R(x) &= (16500 - 50x)x \\ &= -50x^2 + 16500x \end{aligned}$$

12. **a.** $$\begin{aligned} N(x) &= 7{,}000 + 60(90 - x) \\ &= 7000 - 60x + 5400 \\ &= 12400 - 60x \end{aligned}$$

b. $$\begin{aligned} R(x) &= (12400 - 60x)x \\ &= -60x^2 + 12400x \end{aligned}$$

13. **a.** $$\begin{aligned} Y(x) &= 320 - 4(x - 50) \\ &= 320 - 4x + 200 \\ &= 520 - 4x \end{aligned}$$

b. $$\begin{aligned} T(x) &= (520 - 4x)x \\ &= -4x^2 + 520x \end{aligned}$$

14. **a.** $$\begin{aligned} Y(x) &= 270 - 3(x - 30) \\ &= 270 - 3x + 90 \\ &= 360 - 3x \end{aligned}$$

b. $$\begin{aligned} T(x) &= (360 - 3x)x \\ &= -3x^2 + 360x \end{aligned}$$

15. **a.** $$\begin{aligned} V(x) &= (24 - 2x)(24 - 2x)x \\ &= (576 - 96x + 4x^2)x \\ &= 4x^3 - 96x^2 + 576x \end{aligned}$$

b. $V(2) = 4(2)^3 - 96(2)^2 + 576(2) = 800$ If 2-inch squares are cut off each corner, the volume will be 800 square inches.

$V(3) = 4(3)^3 - 96(3)^2 + 576(3) = 972$ If 3-inch squares are cut off each corner, the volume will be 972 square inches.

$V(4) = 4(4)^3 - 96(4)^2 + 576(4) = 1024$ If 4-inch squares are cut off each corner, the volume will be 1024 square inches.

$V(5) = 4(5)^3 - 96(5)^2 + 576(5) = 980$ If 5-inch squares are cut off each corner, the volume will be 980 square inches.

$V(6) = 4(6)^3 - 96(6)^2 + 576(6) = 864$ If 6-inch squares are cut off each corner, the volume will be 864 square inches.

c. If x is the inches to be cut off, $x > 0$. Since each side is 24, you must cut less than 12 inches off each end.
$0 < x < 12$

16. **a.** $$\begin{aligned} V(x) &= (30 - 2x)(30 - 2x)x \\ &= (900 - 120x + 4x^2)x \\ &= 4x^3 - 120x^2 + 900x \end{aligned}$$

b. $V(3) = 4(3^3) - 120(3^2) + 900(3) = 1728$
If 3 inches are cut from each side, the volume will be 1728 square inches.

$V(4) = 4(4^3) - 120(4^2) + 900(4) = 1936$
If 4 inches are cut from each side, the volume will be 1936 square inches.

$V(5) = 4(5^3) - 120(5^2) + 900(5) = 2000$
If 5 inches are cut from each side, the volume will be 2000 square inches.

$V(6) = 4(6^3) - 120(6^2) + 900(6) = 1944$
If 6 inches are cut from each side, the volume will be 1944 square inches.

$V(7) = 4(7^3) - 120(7^2) + 900(7) = 1792$
If 7 inches are cut from each side, the volume will be 1792 square inches.

c. Since x is the number of inches to be cut from each side, $x > 0$. Since each side is 30 inches, you must cut less than 15 inches from each side.
$0 < x < 15$ or (0, 15)

17. $A(x) = x(20-2x)$
$= -2x^2 + 20x$

18.
$$A(x) = \left(\frac{x}{4}\right)^2 + \left(\frac{8-x}{4}\right)^2 = \frac{x^2}{16} + \frac{64-16x+x^2}{16} = \frac{2x^2-16x+64}{16} = \frac{x^2-8x+32}{8}$$

19. $P(x) = x(66-x)$
$= -x^2 + 66x$

20. $P(x) = x(50-x)$
$= -x^2 + 50x$

21. $A(x) = x(400-x)$
$= -x^2 + 400x$

22. $A(x) = x(300-x)$
$= -x^2 + 300x$

23. $2w + l = 800$
$l = 800 - 2w$
Let $x = w$
$A(x) = x(800-2x)$
$= -2x^2 + 800x$

24. $2w + l = 600$
$l = 600 - 2l$
let $x =$ width, $600 - 2x =$ length

$A(x) = (600-2x)x$
$= -2x^2 + 600x$

25. $2x + 3y = 1000$
$3y = 1000 - 2x$
$$y = \frac{1000-2x}{3}$$
$$A(x) = x\left(\frac{1000-2x}{3}\right) = \frac{x(1000-2x)}{3}$$

26. $2x + 4y = 1200$
$4y = 1200 - 2x$
$$y = \frac{1200-2x}{4}$$
$$A(x) = x\left(\frac{1200-2x}{4}\right) = \frac{x(1200-2x)}{4} = \frac{2x(600-x)}{4} = \frac{x(600-x)}{2}$$

27. $2x =$ distance around 2 straight sides
$\pi 2r =$ distance around 2 curved sides

$2x + 2\pi r = 440$
$2x = 440 - 2\pi r$
$x = 220 - \pi r$

$A(r) = (220 - \pi r)2r + \pi r^2$
$= 440r - 2\pi r^2 + \pi r^2$
$= 440r - \pi r^2$

28. $2x =$ distance around the 2 straight sides
$2\pi r =$ distance around the 2 curved sides
$2x + 2\pi r = 880$
$2x = 880 - 2\pi r$
$x = 440 - \pi r$

$A(x) = r(440 - \pi r) + \pi r^2$
$= 440r - \pi r^2 + \pi r^2$
$= 440r$

29. $xy = 4000$
$$y = \frac{4000}{x}$$
$$C(x) = \left[2x + 2\left(\frac{4000}{x}\right)\right]175 + 125x = 350x + \frac{1,400,000}{x} + 125x = 475x + \frac{1,400,000}{x}$$

30. $125 = lw$
$\frac{125}{l} = w$; let $x = l$
$$C(x) = 20\left(2\left(\frac{125}{x}\right) + x\right) + 9x = \frac{5000}{x} + 20x + 9x = \frac{5000}{x} + 29x$$

31.
$$10 = x^2 y$$
$$\frac{10}{x^2} = y$$

$$A(x) = x^2 + 4\left(x \cdot \frac{10}{x^2}\right) = x^2 + \frac{40}{x}$$

32.
$$400 = x^2 y$$
$$\frac{400}{x^2} = y$$

$$A = x^2 + 5\left(\frac{400}{x^2}\right)x = x^2 + \frac{2000}{x}$$

33.
$$300 = y + 4x$$
$$300 - 4x = y^2$$

$$A(x) = x^2(300 - 4x) = -4x^3 + 300x^2$$

34.
$$108 = y + 4x$$
$$108 - 4x = y$$

$$A = x^2(108 - 4x) = -4x^3 + 108x^2$$

35.
$$\begin{aligned} d &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} \\ &= \sqrt{x^2 + \left(x^2 - 4\right)^2} \\ &= \sqrt{x^2 + x^4 - 8x^2 + 16} \\ &= \sqrt{x^4 - 7x^2 + 16} \end{aligned}$$

36.
$$\begin{aligned} d &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} \\ &= \sqrt{x^2 + \left(x^2 - 8\right)^2} \\ &= \sqrt{x^2 + x^4 - 16x^2 + 64} \\ &= \sqrt{x^4 - 15x^2 + 64} \end{aligned}$$

37.
$$\begin{aligned} d &= \sqrt{(x-1)^2 + y^2} \\ &= \sqrt{x^2 - 2x + 1 + \left(\sqrt{x}\right)^2} \\ &= \sqrt{x^2 - 2x + 1 + x} \\ &= \sqrt{x^2 - x + 1} \end{aligned}$$

38.
$$\begin{aligned} d &= \sqrt{(x-2)^2 + y^2} \\ &= \sqrt{x^2 - 4x + 4 + \left(\sqrt{x}\right)^2} \\ &= \sqrt{x^2 - 3x + 4} \end{aligned}$$

39. **a.**
$$A(x) = 2xy = 2x\sqrt{4 - x^2}$$

b.
$$P(x) = 2(2x) + 2y = 4x + 2\sqrt{4 - x^2}$$

40. **a.**
$$A(x) = 2xy = 2x\sqrt{9 - x^2}$$

b.
$$P(x) = 2(2x) + 2y = 4x + 2\sqrt{9 - x^2}$$

41. 6-foot pole
$$c^2 = 6^2 + x^2$$
$$x = \sqrt{36 + x^2}$$
8-foot pole
$$c^2 = 8^2 + (10 - x)^2$$
$$c = \sqrt{64 + 100 - 20x + x^2}$$
$$c = \sqrt{x^2 - 20x + 164}$$
total length
$$f(x) = \sqrt{36 + x^2} + \sqrt{x^2 - 20x + 164}$$

42. Road from Town A:
$$c^2 = 6^2 + x^2$$
$$c = \sqrt{36 + x^2}$$

Road from Town B:
$$c^2 = 3^2 + (12 - x)^2$$
$$c = \sqrt{9 + 144 - 24x + x^2}$$
$$c = \sqrt{x^2 - 24x + 153}$$

$$f(x) = \sqrt{36 + x^2} + \sqrt{x^2 - 24x + 153}$$

43.
$$A(x) = \tfrac{1}{2}x(x-5) + \tfrac{1}{2}x(x+3) + (x+2)\left[(x-5) + (x+3)\right]$$
$$A(x) = \tfrac{1}{2}x^2 - \tfrac{5}{2}x + \tfrac{1}{2}x^2 + \tfrac{3}{2}x + (x+2)\left[2x - 2\right]$$
$$A(x) = x^2 - x + 2x^2 + 2x - 4$$
$$A(x) = 3x^2 + x - 4$$

44. $A(x)=\frac{1}{2}x(2x)+\frac{1}{2}(6x-4x)(x+2)$
$\quad +(4x)(x+2)+2x(8)$
$A(x)=x^2+x(x+2)+4x^2+8x+16x$
$A(x)=x^2+x^2+2x+4x^2+8x+16x$
$A(x)=6x^2+26$

45. $V(x)=(x+5)(2x+1)(x+2)-(x+5)(3)(x)$
$V(x)=(x+5)(2x^2+5x+2)-3x(x+5)$
$V(x)=2x^3+15x^2+27x+10-3x^2-15x$
$V(x)=2x^3+12x^2+12x+10$

46. $V(x)=(x)(2x-1)(x+3)$
$\quad -(x)(x)\left[(2x-1)-(x+1)\right]$
$V(x)=(x)(2x^2+5x-3)-x^2(x-2)$
$V(x)=2x^3+5x^2-3x-x^3+2x^2$
$V(x)=x^3+7x^2-3x$

47. – 58. Answers may vary.

59. does not make sense; Explanations will vary. Sample explanation: This model is not reasonable, as it suggests a per minute charge of $30.

60. does not make sense; Explanations will vary. Sample explanation: The decrease in passengers is modeled by $60(x-300)$.

61. does not make sense; Explanations will vary. Sample explanation: The area of a rectangle is not solely determined by its perimeter. For example: A 4 by 6 rectangle and a 3 by 7 rectangle both have perimeters of 20 units, yet their areas are different from each other.

62. makes sense

63. Distance and time rowed:
$d^2=2^2+x^2$
$d=\sqrt{4+x^2}$
$rt=d$
$2t=\sqrt{4+x^2}$
$t=\dfrac{\sqrt{4+x^2}}{2}$
Distance and time walked:
$d=6-x$
$rt=d$
$5t=6-x$
$t=\dfrac{6-x}{5}$
Total time:
$$T(x)=\frac{\sqrt{4+x^2}}{2}+\frac{6-x}{5}$$

64. $A(x)=(20+2x)(10+2x)-10(20)$
$=4x^2+60x+200-200$
$=4x^2+60x$

65.
$P=2h+2r+\frac{1}{2}(\pi 2r)$
$12=2h+2r+\pi r$
$12-2r-\pi r=2h$
$\dfrac{12-2r-\pi r}{2}=h$

$A=\left(\dfrac{12-2r-\pi r}{2}\right)2r+\frac{1}{2}\left(\pi r^2\right)$
$=12r-2r^2-\pi r^2+\frac{1}{2}\pi r^2$
$=12r-2r^2-\frac{1}{2}\pi r^2$

66.
$r=\frac{1}{2}h$

$V(h)=\frac{1}{3}\pi r^2h$
$=\frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$
$=\frac{1}{3}\pi\frac{1}{4}h^2h$
$=\frac{\pi}{12}h^3$

67.
$\dfrac{2x+1}{9}-\dfrac{x+4}{6}=1$
$18\left(\dfrac{2x+1}{9}-\dfrac{x+4}{6}\right)=(1)18$
$2(2x+1)-3(x+4)=18$
$4x+2-3x-12=18$
$x-10=18$
$x=28$
The solution set is $\{28\}$.

68.
$2x^2+x=6$
$2x^2+x-6=0$
$(2x-3)(x+2)=0$
$2x-3=0$ or $x+2=0$
$x=3/2 \qquad x=-2$
The solution set is $\left\{\frac{3}{2},-2\right\}$.

69. $\sqrt{3x+7}+1=x$

$\sqrt{3x+7}=x-1$

$3x+7=x^2-2x+1$

$x^2-5x-6=0$

$(x+1)(x-6)=0$

$x+1=0 \quad x-6=0$

$x=-1 \quad\quad x=6$

$\sqrt{-3+7}+1=-1 \qquad \sqrt{18+7}+1=6$

$\sqrt{4}+1=-1 \qquad \sqrt{25}+1=6$

$3=-1$ False $\qquad 6=6$

The solution set is $\{6\}$.

70. $(7-3x)(-2-5x)=-14-35x+6x+15x^2$

$=-14-29x+15x^2$

or

$=15x^2-29x-14$

71. $\sqrt{18}-\sqrt{8}=\sqrt{9\cdot 2}-\sqrt{4\cdot 2}$

$=3\sqrt{2}-2\sqrt{2}$

$=\sqrt{2}$

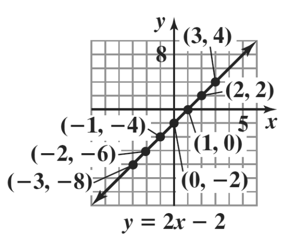
72. $\dfrac{7+4\sqrt{2}}{2-5\sqrt{2}}\cdot\dfrac{2+5\sqrt{2}}{2+5\sqrt{2}}=\dfrac{14+35\sqrt{2}+8\sqrt{2}+40}{4+10\sqrt{2}-10\sqrt{2}-50}$

$=\dfrac{54+43\sqrt{2}}{-46}$

$=-\dfrac{54+43\sqrt{2}}{46}$

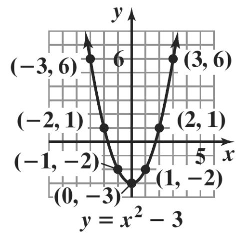
Chapter 1 Review Exercises

1.



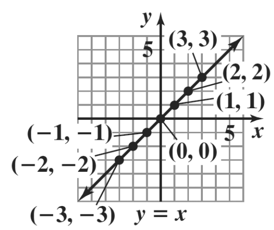
$x=-3, y=-8$
$x=-2, y=-6$
$x=-1, y=-4$
$x=0, y=-2$
$x=1, y=0$
$x=2, y=2$
$x=3, y=4$

2.



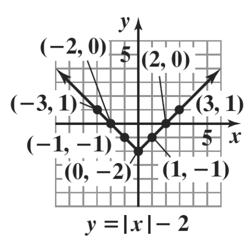
$x=-3, y=6$
$x=-2, y=1$
$x=-1, y=-2$
$x=0, y=-3$
$x=1, y=-2$
$x=2, y=1$
$x=3, y=6$

3.



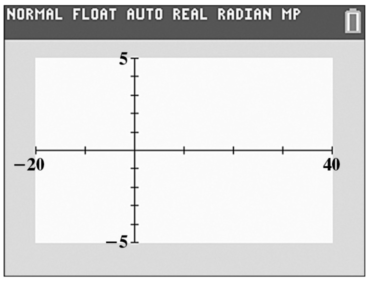
$x=-3, y=-3$
$x=-2, y=-2$
$x=-1, y=-1$
$x=0, y=0$
$x=1, y=1$
$x=2, y=2$
$x=3, y=3$

4.



$x=-3, y=1$
$x=-2, y=0$
$x=-1, y=-1$
$x=0, y=-2$
$x=1, y=-1$
$x=2, y=0$
$x=3, y=1$

5. A portion of Cartesian coordinate plane with minimum x-value equal to –20, maximum x-value equal to 40, x-scale equal to 10 and with minimum y-value equal to –5, maximum y-value equal to 5, and y-scale equal to 1.



6. x-intercept: –2; The graph intersects the x-axis at $(-2,\ 0)$.

y-intercept: 2; The graph intersects the y-axis at $(0,\ 2)$.

7. x-intercepts: 2, –2; The graph intersects the x-axis at $(-2,\ 0)$ and $(2,\ 0)$.

y-intercept: –4; The graph intercepts the y-axis at $(0,\ -4)$.

8. x-intercept: 5; The graph intersects the x-axis at $(5,\ 0)$.

y-intercept: None; The graph does not intersect the y-axis.

9. The coordinates are (20, 8). This means that 8% of college students anticipated a starting salary of $20 thousand.

10. The starting salary that was anticipated by the greatest percentage of college students was $30 thousand. 22% of students anticipated this salary.

11. The starting salary that was anticipated by the least percentage of college students was $70 thousand. 2% of students anticipated this salary.

12. Starting salaries of $25 thousand and $30 thousand were anticipated by more than 20% of college students

13. 14% of students anticipated a starting salary of $40 thousand.

14. $p = -0.01s^2 + 0.8s + 3.7$
$p = -0.01(40)^2 + 0.8(40) + 3.7$
$p = 19.7$

This is greater than the estimate of the previous question.

15. function
domain: {2, 3, 5}
range: {7}

16. function
domain: {1, 2, 13}
range: {10, 500, π}

17. not a function
domain: {12, 14}
range: {13, 15, 19}

18. $2x + y = 8$
$y = -2x + 8$

Since only one value of y can be obtained for each value of x, y is a function of x.

19. $3x^2 + y = 14$
$y = -3x^2 + 14$

Since only one value of y can be obtained for each value of x, y is a function of x.

20. $2x + y^2 = 6$
$y^2 = -2x + 6$
$y = \pm\sqrt{-2x + 6}$

Since more than one value of y can be obtained from some values of x, y is not a function of x.

21. $f(x) = 5 - 7x$

a. $f(4) = 5 - 7(4) = -23$

b. $f(x+3) = 5 - 7(x+3)$
$= 5 - 7x - 21$
$= -7x - 16$

c. $f(-x) = 5 - 7(-x) = 5 + 7x$

22. $g(x) = 3x^2 - 5x + 2$

a. $g(0) = 3(0)^2 - 5(0) + 2 = 2$

b. $g(-2) = 3(-2)^2 - 5(-2) + 2$
$= 12 + 10 + 2$
$= 24$

c. $g(x-1)=3(x-1)^2-5(x-1)+2$
$=3(x^2-2x+1)-5x+5+2$
$=3x^2-11x+10$

d. $g(-x)=3(-x)^2-5(-x)+2$
$=3x^2+5x+2$

23. a. $g(13)=\sqrt{13-4}=\sqrt{9}=3$

b. $g(0) = 4 – 0 = 4$

c. $g(–3) = 4 – (–3) = 7$

24. a. $f(-2)=\frac{(-2)^2-1}{-2-1}=\frac{3}{-3}=-1$

b. $f(1) = 12$

c. $f(2)=\frac{2^2-1}{2-1}=\frac{3}{1}=3$

25. The vertical line test shows that this is not the graph of a function.

26. The vertical line test shows that this is the graph of a function.

27. The vertical line test shows that this is the graph of a function.

28. The vertical line test shows that this is not the graph of a function.

29. The vertical line test shows that this is not the graph of a function.

30. The vertical line test shows that this is the graph of a function.

31. a. domain: [–3, 5)

b. range: [–5, 0]

c. x-intercept: –3

d. y-intercept: –2

e. increasing: $(-2, 0)$ or $(3, 5)$
decreasing: $(-3, -2)$ or $(0, 3)$

f. $f(–2) = –3$ and $f(3) = –5$

32. a. domain: $(-\infty, \infty)$

b. range: $(-\infty, 3]$

c. x-intercepts: –2 and 3

d. y-intercept: 3

e. increasing: (–∞, 0)
decreasing: $(0, \infty)$

f. $f(–2) = 0$ and $f(6) = –3$

33. a. domain: $(-\infty, \infty)$

b. range: [–2, 2]

c. x-intercept: 0

d. y-intercept: 0

e. increasing: (–2, 2)
constant: $(-\infty, -2)$ or $(2, \infty)$

f. $f(–9) = –2$ and $f(14) = 2$

34. a. 0, relative maximum −2

b. −2, 3, relative minimum −3, –5

35. a. 0, relative maximum 3

b. none

36. Test for symmetry with respect to the y-axis.
$y=x^2+8$
$y=(-x)^2+8$
$y=x^2+8$
The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.
$y=x^2+8$
$-y=x^2+8$
$y=-x^2-8$
The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = x^2 + 8$$
$$-y = (-x)^2 + 8$$
$$-y = x^2 + 8$$
$$y = -x^2 - 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

37. Test for symmetry with respect to the y-axis.

$$x^2 + y^2 = 17$$
$$(-x)^2 + y^2 = 17$$
$$x^2 + y^2 = 17$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x^2 + y^2 = 17$$
$$x^2 + (-y)^2 = 17$$
$$x^2 + y^2 = 17$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x^2 + y^2 = 17$$
$$(-x)^2 + (-y)^2 = 17$$
$$x^2 + y^2 = 17$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

38. Test for symmetry with respect to the y-axis.

$$x^3 - y^2 = 5$$
$$(-x)^3 - y^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x^3 - y^2 = 5$$
$$x^3 - (-y)^2 = 5$$
$$x^3 - y^2 = 5$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x^3 - y^2 = 5$$
$$(-x)^3 - (-y)^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

39. The graph is symmetric with respect to the origin. The function is odd.

40. The graph is not symmetric with respect to the y-axis or the origin. The function is neither even nor odd.

41. The graph is symmetric with respect to the y-axis. The function is even.

42.
$$f(x) = x^3 - 5x$$
$$f(-x) = (-x)^3 - 5(-x)$$
$$= -x^3 + 5x$$
$$= -f(x)$$

The function is odd. The function is symmetric with respect to the origin.

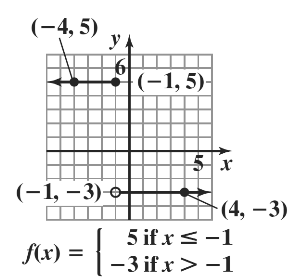
43.
$$f(x) = x^4 - 2x^2 + 1$$
$$f(-x) = (-x)^4 - 2(-x)^2 + 1$$
$$= x^4 - 2x^2 + 1$$
$$= f(x)$$

The function is even. The function is symmetric with respect to the y-axis.

44.
$$f(x) = 2x\sqrt{1-x^2}$$
$$f(-x) = 2(-x)\sqrt{1-(-x)^2}$$
$$= -2x\sqrt{1-x^2}$$
$$= -f(x)$$

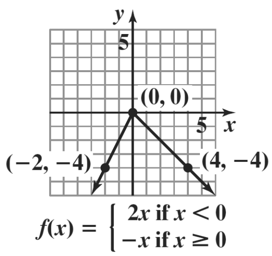
The function is odd. The function is symmetric with respect to the origin.

45. **a.**



b. range: {–3, 5}

46. **a.**



b. range: $\{y \mid y \le 0\}$

47. $\dfrac{8(x+h)-11-(8x-11)}{h}$

$= \dfrac{8x+8h-11-8x+11}{h}$

$= \dfrac{8h}{8}$

$= 8$

48. $\dfrac{-2(x+h)^2+(x+h)+10-\left(-2x^2+x+10\right)}{h}$

$= \dfrac{-2\left(x^2+2xh+h^2\right)+x+h+10+2x^2-x-10}{h}$

$= \dfrac{-2x^2-4xh-2h^2+x+h+10+2x^2-x-10}{h}$

$= \dfrac{-4xh-2h^2+h}{h}$

$= \dfrac{h\left(-4x-2h+1\right)}{h}$

$-4x-2h+1$

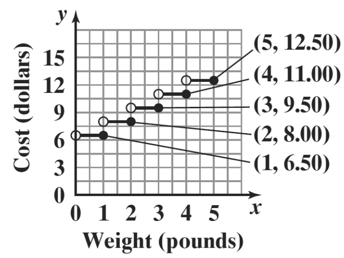
49. **a.** Yes, the eagle's height is a function of time since the graph passes the vertical line test.

b. Decreasing: (3, 12)
The eagle descended.

c. Constant: (0, 3) or (12, 17)
The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds.

d. Increasing: (17, 30)
The eagle was ascending.

50.



51. $m = \dfrac{1-2}{5-3} = \dfrac{-1}{2} = -\dfrac{1}{2}$; falls

52. $m = \dfrac{-4-(-2)}{-3-(-1)} = \dfrac{-2}{-2} = 1$; rises

53. $m = \dfrac{\frac{1}{4}-\frac{1}{4}}{6-(-3)} = \dfrac{0}{9} = 0$; horizontal

54. $m = \dfrac{10-5}{-2-(-2)} = \dfrac{5}{0}$ undefined; vertical

55. point-slope form: $y - 2 = -6(x + 3)$
slope-intercept form: $y = -6x - 16$

56. $m = \dfrac{2-6}{-1-1} = \dfrac{-4}{-2} = 2$
point-slope form: $y - 6 = 2(x - 1)$
or $y - 2 = 2(x + 1)$
slope-intercept form: $y = 2x + 4$

57. $3x + y - 9 = 0$
$y = -3x + 9$
$m = -3$
point-slope form:
$y + 7 = -3(x - 4)$
slope-intercept form:
$y = -3x + 12 - 7$
$y = -3x + 5$

58. perpendicular to $y = \dfrac{1}{3}x + 4$
$m = -3$
point-slope form:
$y - 6 = -3(x + 3)$
slope-intercept form:
$y = -3x - 9 + 6$
$y = -3x - 3$

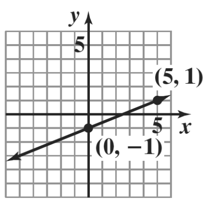
59. Write $6x - y - 4 = 0$ in slope intercept form.

$$\begin{aligned} 6x - y - 4 &= 0 \\ -y &= -6x + 4 \\ y &= 6x - 4 \end{aligned}$$

The slope of the perpendicular line is 6, thus the slope of the desired line is $m = -\frac{1}{6}$.

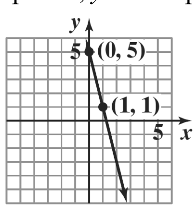
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= -\tfrac{1}{6}(x - (-12)) \\ y + 1 &= -\tfrac{1}{6}(x + 12) \\ y + 1 &= -\tfrac{1}{6}x - 2 \\ 6y + 6 &= -x - 12 \\ x + 6y + 18 &= 0 \end{aligned}$$

60. slope: $\frac{2}{5}$; y-intercept: –1



$y = \frac{2}{5}x - 1$

61. slope: –4; y-intercept: 5

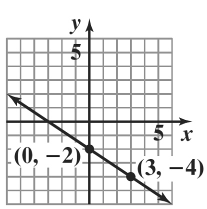


$f(x) = -4x + 5$

62.

$$\begin{aligned} 2x + 3y + 6 &= 0 \\ 3y &= -2x - 6 \\ y &= -\frac{2}{3}x - 2 \end{aligned}$$

slope: $-\frac{2}{3}$; y-intercept: –2

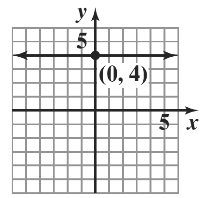


$2x + 3y + 6 = 0$

63.

$$\begin{aligned} 2y - 8 &= 0 \\ 2y &= 8 \\ y &= 4 \end{aligned}$$

slope: 0; y-intercept: 4



$2y - 8 = 0$

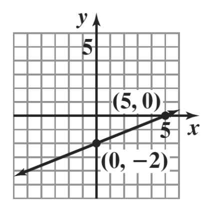
64. $2x - 5y - 10 = 0$

Find x-intercept:

$$\begin{aligned} 2x - 5(0) - 10 &= 0 \\ 2x - 10 &= 0 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

Find y-intercept:

$$\begin{aligned} 2(0) - 5y - 10 &= 0 \\ -5y - 10 &= 0 \\ -5y &= 10 \\ y &= -2 \end{aligned}$$

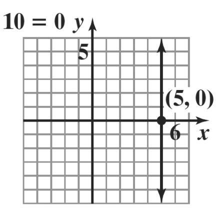


$2x - 5y - 10 = 0$

65.

$$\begin{aligned} 2x - 10 &= 0 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

$2x - 10 = 0$



66. a. First, find the slope using the points $(2, 28.2)$ and $(4, 28.6)$.

$$m = \frac{28.6 - 28.2}{4 - 2} = \frac{0.4}{2} = 0.2$$

Then use the slope and one of the points to write the equation in point-slope form.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 28.2 &= 0.2(x - 2) \end{aligned}$$

or

$$y - 28.6 = 0.2(x - 4)$$

b. Solve for y to obtain slope-intercept form.

$$\begin{aligned} y-28.2 &= 0.2(x-2) \\ y-28.2 &= 0.2x-0.4 \\ y &= 0.2x+27.8 \\ f(x) &= 0.2x+27.8 \end{aligned}$$

c.
$$\begin{aligned} f(x) &= 0.2x+27.8 \\ f(7) &= 0.2(12)+27.8 \\ &= 30.2 \end{aligned}$$

The linear function predicts men's average age of first marriage will be 30.2 years in 2020.

67. **a.** $m=\dfrac{27-21}{2010-1980}=\dfrac{6}{30}=0.2$

b. For the period shown, the number of the percentage of liberal college freshman increased each year by approximately 0.2. The rate of change was 0.2% per year.

68. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{[9^2-4(9)]-[4^2-4\cdot 5]}{9-5}=10$

69. **a.** $S(0)=-16(0)^2+64(0)+80=80$
$S(2)=-16(2)^2+64(2)+80=144$

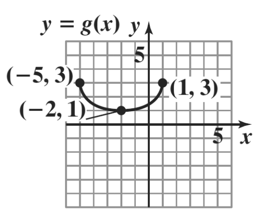
$\dfrac{144-80}{2-0}=32$

b. $S(4)=-16(4)^2+64(4)+80=80$

$\dfrac{80-144}{4-2}=-32$

c. The ball is traveling up until 2 seconds, then it starts to come down.

70.



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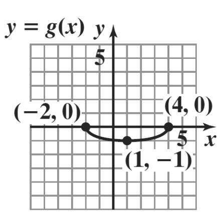

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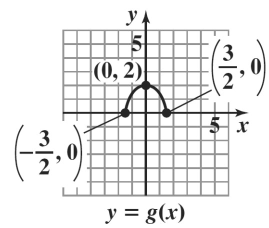

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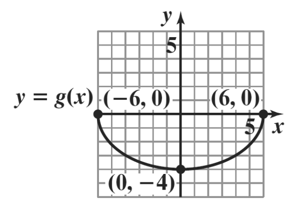

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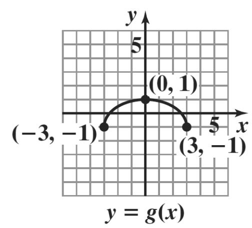

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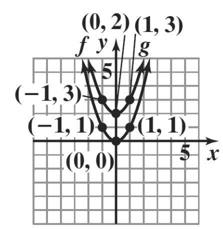

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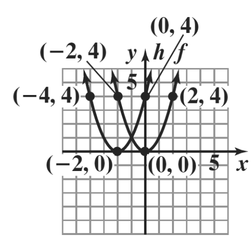

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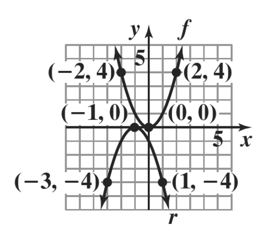

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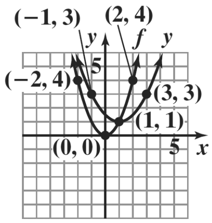
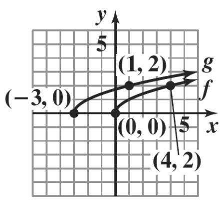
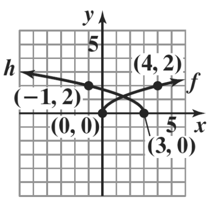


79.



80.



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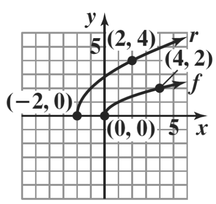

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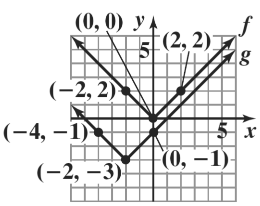

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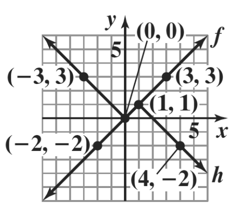

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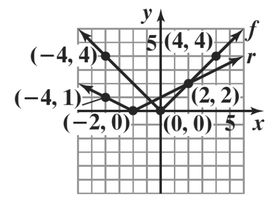

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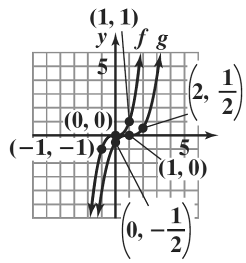

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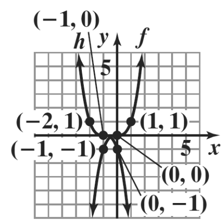

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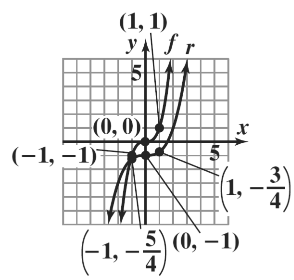

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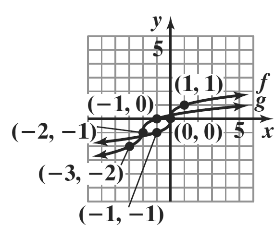

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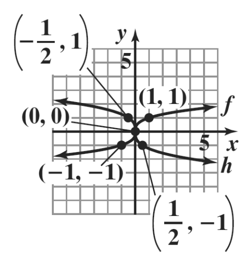
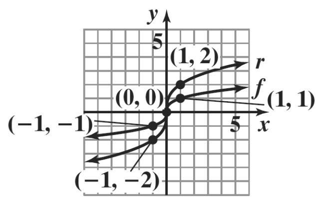


90. 

91. domain: $(-\infty, \infty)$

92. The denominator is zero when $x = 7$. The domain is $(-\infty, 7) \cup (7, \infty)$.

93. The expressions under each radical must not be negative.
$8 - 2x \ge 0$
$-2x \ge -8$
$x \le 4$
domain: $(-\infty, 4]$.

94. The denominator is zero when $x = -7$ or $x = 3$.
domain: $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$

95. The expressions under each radical must not be negative. The denominator is zero when $x = 5$.
$x - 2 \ge 0$
$x \ge 2$
domain: $[2, 5) \cup (5, \infty)$

96. The expressions under each radical must not be negative.
$x - 1 \ge 0$ and $x + 5 \ge 0$
$x \ge 1$ $\quad$ $x \ge -5$
domain: $[1, \infty)$

97. $f(x) = 3x - 1$; $g(x) = x - 5$
$(f + g)(x) = 4x - 6$
domain: $(-\infty, \infty)$
$(f - g)(x) = (3x - 1) - (x - 5) = 2x + 4$
domain: $(-\infty, \infty)$
$(fg)(x) = (3x - 1)(x - 5) = 3x^2 - 16x + 5$
domain: $(-\infty, \infty)$
$\left(\dfrac{f}{g}\right)(x) = \dfrac{3x - 1}{x - 5}$
domain: $(-\infty, 5) \cup (5, \infty)$

98. $f(x) = x^2 + x + 1$; $g(x) = x^2 - 1$
$(f + g)(x) = 2x^2 + x$
domain: $(-\infty, \infty)$
$(f - g)(x) = (x^2 + x + 1) - (x^2 - 1) = x + 2$
domain: $(-\infty, \infty)$
$(fg)(x) = (x^2 + x + 1)(x^2 - 1)$
$= x^4 + x^3 - x - 1$
$\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + x + 1}{x^2 - 1}$
domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

99. $f(x) = \sqrt{x + 7}$; $g(x) = \sqrt{x - 2}$
$(f + g)(x) = \sqrt{x + 7} + \sqrt{x - 2}$
domain: $[2, \infty)$
$(f - g)(x) = \sqrt{x + 7} - \sqrt{x - 2}$
domain: $[2, \infty)$
$(fg)(x) = \sqrt{x + 7} \cdot \sqrt{x - 2}$
$= \sqrt{x^2 + 5x - 14}$
domain: $[2, \infty)$
$\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x + 7}}{\sqrt{x - 2}}$
domain: $(2, \infty)$

100. $f(x) = x^2 + 3$; $g(x) = 4x - 1$

a. $(f \circ g)(x) = (4x - 1)^2 + 3$
$= 16x^2 - 8x + 4$

b. $(g \circ f)(x) = 4(x^2 + 3) - 1$
$= 4x^2 + 11$

c. $(f \circ g)(3) = 16(3)^2 - 8(3) + 4 = 124$

101. $f(x) = \sqrt{x}$; $g(x) = x + 1$

a. $(f \circ g)(x) = \sqrt{x + 1}$

b. $(g \circ f)(x) = \sqrt{x} + 1$

c. $(f \circ g)(3) = \sqrt{3 + 1} = \sqrt{4} = 2$

102. a.
$$(f\circ g)(x)=f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}+1}{\frac{1}{x}-2}=\frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-2\right)x}=\frac{1+x}{1-2x}$$

b. $x\neq 0$ $\qquad 1-2x\neq 0$, $\quad x\neq\frac{1}{2}$

$$(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$$

103. a. $(f\circ g)(x)=f(x+3)=\sqrt{x+3-1}=\sqrt{x+2}$

b. $x+2\geq 0$, $\quad x\geq -2$ $\qquad [-2,\infty)$

104. $f(x)=x^4 \qquad g(x)=x^2+2x-1$

105. $f(x)=\sqrt[3]{x} \qquad g(x)=7x+4$

106. $f(x)=\frac{3}{5}x+\frac{1}{2};\ g(x)=\frac{5}{3}x-2$

$$f(g(x))=\frac{3}{5}\left(\frac{5}{3}x-2\right)+\frac{1}{2}=x-\frac{6}{5}+\frac{1}{2}=x-\frac{7}{10}$$

$$g(f(x))=\frac{5}{3}\left(\frac{3}{5}x+\frac{1}{2}\right)-2=x+\frac{5}{6}-2=x-\frac{7}{6}$$

f and *g* are not inverses of each other.

107. $f(x)=2-5x;\ g(x)=\frac{2-x}{5}$

$$f(g(x))=2-5\left(\frac{2-x}{5}\right)=2-(2-x)=x$$

$$g(f(x))=\frac{2-(2-5x)}{5}=\frac{5x}{5}=x$$

f and *g* are inverses of each other.

108. a.
$$f(x)=4x-3$$
$$y=4x-3$$
$$x=4y-3$$
$$y=\frac{x+3}{4}$$
$$f^{-1}(x)=\frac{x+3}{4}$$

b.
$$f(f^{-1}(x))=4\left(\frac{x+3}{4}\right)-3=x+3-3=x$$
$$f^{-1}(f(x))=\frac{(4x-3)+3}{4}=\frac{4x}{4}=x$$

109. a.
$$f(x)=8x^3+1$$
$$y=8x^3+1$$
$$x=8y^3+1$$
$$x-1=8y^3$$
$$\frac{x-1}{8}=y^3$$
$$\sqrt[3]{\frac{x-1}{8}}=y$$
$$\frac{\sqrt[3]{x-1}}{2}=y$$
$$f^{-1}(x)=\frac{\sqrt[3]{x-1}}{2}$$

b.
$$f\left(f^{-1}(x)\right)=8\left(\frac{\sqrt[3]{x-1}}{2}\right)^3+1=8\left(\frac{x-1}{8}\right)+1=x-1+1=x$$
$$f^{-1}\left(f(x)\right)=\frac{\sqrt[3]{\left(8x^3+1\right)-1}}{2}=\frac{\sqrt[3]{8x^3}}{2}=\frac{2x}{2}=x$$

110. a.
$$f(x)=\frac{x-7}{x+2}$$
$$y=\frac{x-7}{x+2}$$
$$x=\frac{y-7}{y+2}$$
$$xy+2x=y-7$$
$$xy-y=-2x-7$$
$$y(x-1)=-2x-7$$
$$y=\frac{-2x-7}{x-1}$$
$$f^{-1}(x)=\frac{-2x-7}{x-1}, x\neq 1$$

b.
$$f\left(f^{-1}(x)\right)=\frac{\frac{-2x-7}{x-1}-7}{\frac{-2x-7}{x-1}+2}$$
$$=\frac{-2x-7-7(x-1)}{-2x-7+2(x-1)}$$
$$=\frac{-9x}{-9}$$
$$=x$$

$$f^{-1}(f(x))=\frac{-2\left(\frac{x-7}{x+2}\right)-7}{\frac{x-7}{x+2}-1}$$
$$=\frac{-2x+14-7(x+2)}{x-7-(x+2)}$$
$$=\frac{-9x}{-9}$$
$$=x$$

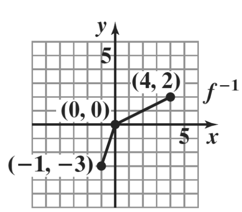
111. The inverse function exists.

112. The inverse function does not exist since it does not pass the horizontal line test.

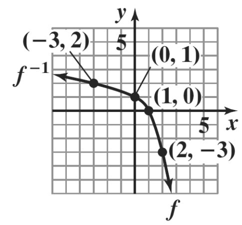
113. The inverse function exists.

114. The inverse function does not exist since it does not pass the horizontal line test.

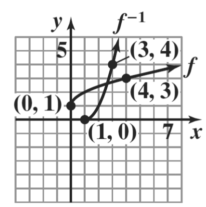
115.



116. $f(x)=1-x^2$
$$y=1-x^2$$
$$x=1-y^2$$
$$y^2=1-x$$
$$y=\sqrt{1-x}$$
$$f^{-1}(x)=\sqrt{1-x}$$



117. $f(x)=\sqrt{x}+1$
$$y=\sqrt{x}+1$$
$$x=\sqrt{y}+1$$
$$x-1=\sqrt{y}$$
$$(x-1)^2=y$$
$$f^{-1}(x)=(x-1)^2,\quad x\geq 1$$



118. $d=\sqrt{[3-(-2)]^2+[9-(-3)]^2}$
$$=\sqrt{5^2+12^2}$$
$$=\sqrt{25+144}$$
$$=\sqrt{169}$$
$$=13$$

119. $d=\sqrt{[-2-(-4)]^2+(5-3)^2}$
$$=\sqrt{2^2+2^2}$$
$$=\sqrt{4+4}$$
$$=\sqrt{8}$$
$$=2\sqrt{2}$$
$$\approx 2.83$$

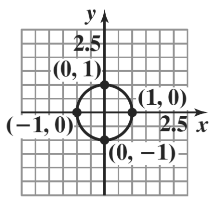
120. $\left(\frac{2+(-12)}{2},\frac{6+4}{2}\right)=\left(\frac{-10}{2},\frac{10}{2}\right)=(-5,5)$

121. $\left(\frac{4+(-15)}{2},\frac{-6+2}{2}\right)=\left(\frac{-11}{2},\frac{-4}{2}\right)=\left(\frac{-11}{2},-2\right)$

122. $x^2+y^2=3^2$
$$x^2+y^2=9$$

123. $(x-(-2))^2+(y-4)^2=6^2$
$(x+2)^2+(y-4)^2=36$

124. center: (0, 0); radius: 1

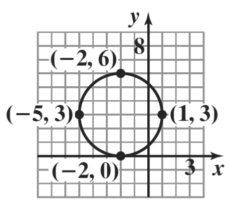


$x^2+y^2=1$

domain: $[-1,1]$

range: $[-1,1]$

125. center: (–2, 3); radius: 3

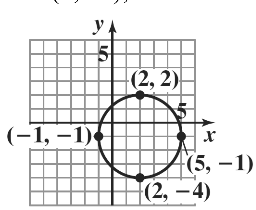


$(x+2)^2+(y-3)^2=9$

domain: $[-5,1]$

range: $[0,6]$

126.
$$x^2+y^2-4x+2y-4=0$$
$$x^2-4x\quad+y^2+2y\quad=4$$
$$x^2-4x+4+y^2+2y+1=4+4+1$$
$$(x-2)^2+(y+1)^2=9$$
center: (2, –1); radius: 3



$x^2+y^2-4x+2y-4=0$

domain: $[-1,5]$

range: $[-4,2]$

127. a. $W(x)=567+15x$

b.
$$882=567+15x$$
$$315=15x$$
$$21=x$$
21 years after 2000, or 2021, the average weekly sales will be $882.

128. a. $f(x)=15+0.05x$

b. $g(x)=5+0.07x$

c.
$$15+0.05x=5+0.07x$$
$$10=0.02x$$
$$500=x$$
For 500 minutes, the two plans cost the same.

129. a.
$$N(x)=400-2(x-120)$$
$$=400-2x+240$$
$$=640-2x$$

b.
$$R(x)=x(640-2x)$$
$$=-2x^2+640x$$

130. a.
$$w=16-2x \qquad l=24-2x$$
$$V(x)=(16-2x)(24-2x)x$$

b. $0<x<8$

131.
$$2l+3w=400$$
$$2l=400-3w$$
$$l=\frac{400-3w}{2}$$
Let x = width
$$A(x)=x\left(\frac{400-3w}{2}\right)$$
$$=\frac{x(400-3w)}{2}$$

132.
$$V=lwh$$
$$8=x\cdot x\cdot h$$
$$\frac{8}{x^2}=h$$
$$A(x)=2x\cdot x+4hx$$
$$=2x^2+4\left(\frac{8}{x^2}\right)x$$
$$=2x^2+\frac{32}{x}$$

Chapter 1 Test

1. (b), (c), and (d) are not functions.

2. a. $f(4)-f(-3)=3-(-2)=5$

b. domain: (–5, 6]

c. range: [–4, 5]

d. increasing: (–1, 2)

e. decreasing: $(-5, -1)$ or $(2, 6)$

f. 2, $f(2) = 5$

g. (–1, –4)

h. *x*-intercepts: –4, 1, and 5.

i. *y*-intercept: –3

3. **a.** –2, 2

b. –1, 1

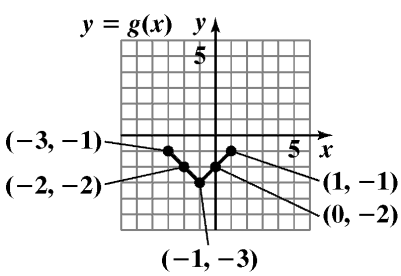
c. 0

d. even; $f(-x) = f(x)$

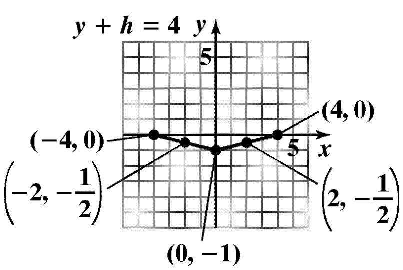
e. no; f fails the horizontal line test

f. $f(0)$ is a relative minimum.

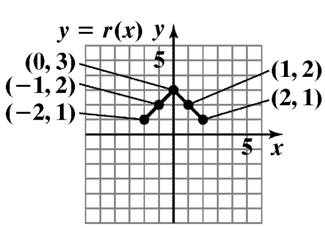
g.



h.

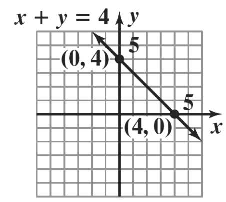


i.



j. $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{-1-0}{1-(-2)} = -\frac{1}{3}$

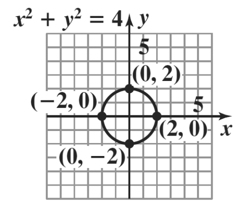
4.



domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

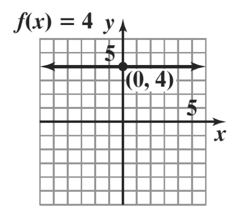
5.



domain: $[-2, 2]$

range: $[-2, 2]$

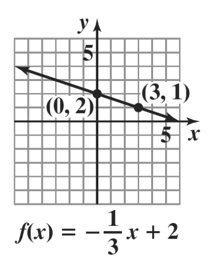
6.



domain: $(-\infty, \infty)$

range: $\{4\}$

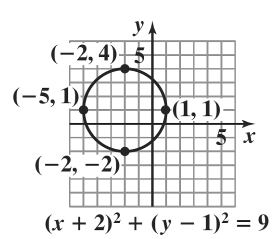
7.



domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

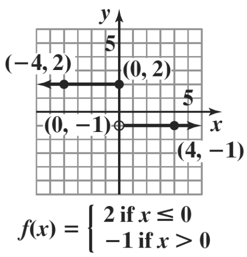
8.



domain: $[-5,1]$

range: $[-2,4]$

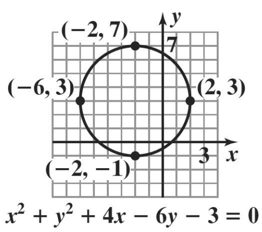
9.



domain: $(-\infty,\infty)$

range: $\{-1,2\}$

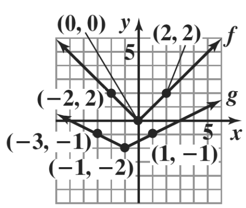
10.



domain: $[-6,2]$

range: $[-1,7]$

11.



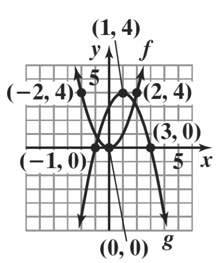
domain of f: $(-\infty,\infty)$

range of f: $[0,\infty)$

domain of g: $(-\infty,\infty)$

range of g: $[-2,\infty)$

12.



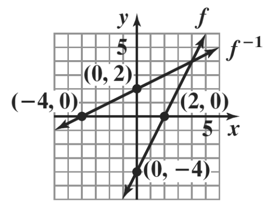
domain of f: $(-\infty,\infty)$

range of f: $[0,\infty)$

domain of g: $(-\infty,\infty)$

range of g: $(-\infty,4]$

13.



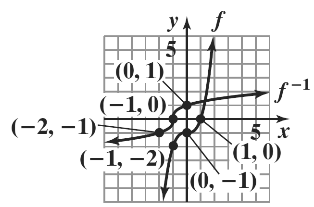
domain of f: $(-\infty,\infty)$

range of f: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

14.



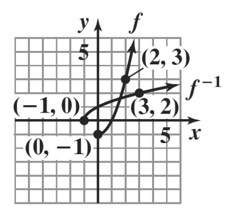
domain of f: $(-\infty,\infty)$

range of f: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

15.



domain of f: $[0,\infty)$

range of f: $[-1,\infty)$

domain of f^{-1}: $[-1,\infty)$

range of f^{-1}: $[0,\infty)$

16.

$$f(x) = x^2 - x - 4$$

$$\begin{aligned} f(x-1) &= (x-1)^2 - (x-1) - 4 \\ &= x^2 - 2x + 1 - x + 1 - 4 \\ &= x^2 - 3x - 2 \end{aligned}$$

17. $\frac{f(x+h)-f(x)}{h}$

$= \frac{(x+h)^2-(x+h)-4-\left(x^2-x-4\right)}{h}$

$= \frac{x^2+2xh+h^2-x-h-4-x^2+x+4}{h}$

$= \frac{2xh+h^2-h}{h}$

$= \frac{h(2x+h-1)}{h}$

$= 2x+h-1$

18. $(g-f)(x) = 2x-6-\left(x^2-x-4\right)$

$= 2x-6-x^2+x+4$

$= -x^2+3x-2$

19. $\left(\frac{f}{g}\right)(x) = \frac{x^2-x-4}{2x-6}$

domain: $(-\infty,3)\cup(3,\infty)$

20. $(f\circ g)(x) = f\left(g(x)\right)$

$= (2x-6)^2-(2x-6)-4$

$= 4x^2-24x+36-2x+6-4$

$= 4x^2-26x+38$

21. $(g\circ f)(x) = g\left(f(x)\right)$

$= 2\left(x^2-x-4\right)-6$

$= 2x^2-2x-8-6$

$= 2x^2-2x-14$

22. $g\left(f(-1)\right) = 2\left((-1)^2-(-1)-4\right)-6$

$= 2(1+1-4)-6$

$= 2(-2)-6$

$= -4-6$

$= -10$

23. $f(x) = x^2-x-4$

$f(-x) = (-x)^2-(-x)-4$

$= x^2+x-4$

f is neither even nor odd.

24. Test for symmetry with respect to the y-axis.

$x^2+y^3=7$

$(-x)^2+y^3=7$

$x^2+y^3=7$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$x^2+y^3=7$

$x^2+(-y)^3=7$

$x^2-y^3=7$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$x^2+y^3=7$

$(-x)^2+(-y)^3=7$

$x^2-y^3=7$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

25. $m = \frac{-8-1}{-1-2} = \frac{-9}{-3} = 3$

point-slope form: $y - 1 = 3(x - 2)$

or $y + 8 = 3(x + 1)$

slope-intercept form: $y = 3x - 5$

26. $y = -\frac{1}{4}x+5$ so $m = 4$

point-slope form: $y - 6 = 4(x + 4)$

slope-intercept form: $y = 4x + 22$

27. Write $4x+2y-5=0$ in slope intercept form.

$4x+2y-5=0$

$2y=-4x+5$

$y=-2x+\frac{5}{2}$

The slope of the parallel line is –2, thus the slope of the desired line is $m=-2$.

$y-y_1=m(x-x_1)$

$y-(-10)=-2\left(x-(-7)\right)$

$y+10=-2(x+7)$

$y+10=-2x-14$

$2x+y+24=0$

28. **a.** Find slope: $m = \frac{25.8-24.6}{20-10} = \frac{1.2}{10} = 0.12$

point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 24.6 = 0.12(x-10)$$

b. slope-intercept form:

$$y - 24.6 = 0.12(x-10)$$
$$y - 24.6 = 0.12x - 1.2$$
$$y = 0.12x + 23.4$$
$$f(x) = 0.12x + 23.4$$

c. $f(x) = 0.12x + 23.4$

$$= 0.12(40) + 23.4$$
$$= 28.2$$

According to the model, 28.2% of U.S. households will be one-person households in 2020.

29. $\frac{3(10)^2 - 5 - [3(6)^2 - 5]}{10-6}$

$$= \frac{205-103}{4}$$
$$= \frac{192}{4}$$
$$= 48$$

30. $g(-1) = 3 - (-1) = 4$

$g(7) = \sqrt{7-3} = \sqrt{4} = 2$

31. The denominator is zero when $x = 1$ or $x = -5$.

domain: $(-\infty,-5)\cup(-5,1)\cup(1,\infty)$

32. The expressions under each radical must not be negative.

$x + 5 \ge 0$ and $x - 1 \ge 0$

$x \ge -5$ $\quad x \ge 1$

domain: $[1,\infty)$

33. $(f \circ g)(x) = \frac{7}{\frac{2}{x} - 4} = \frac{7x}{2-4x}$

$x \ne 0,\quad 2 - 4x \ne 0$

$x \ne \frac{1}{2}$

domain: $(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

34. $f(x) = x^7 \qquad g(x) = 2x + 3$

35. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-2)^2 + (2-(-2))^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+5}{2}, \frac{-2+2}{2}\right)$$
$$= \left(\frac{7}{2}, 0\right)$$

The length is 5 and the midpoint is $\left(\frac{7}{2},0\right)$ or $(3.5, 0)$.

36. **a.** $T(x) = 41.78 - 0.19x$

b.

$$35.7 = 41.78 - 0.19x$$
$$-6.08 = -0.19x$$
$$32 = x$$

32 years after 1980, in 2012, the winning time will be 35.7 seconds.

37. **a.** $Y(x) = 50 - 1.5(x-30)$

$$= 50 - 1.5x + 45$$
$$= 95 - 1.5x$$

b. $T(x) = x(95 - 1.5x)$

$$= -1.5x^2 + 95x$$

38. $2l + 2w = 600$

$$2l = 600 - 2w$$
$$l = 300 - w$$

Let $x = w$

$$A(x) = x(300-x)$$
$$= -x^2 + 300x$$

39. $V = lwh$

$$8000 = x \cdot x \cdot h$$
$$\frac{8000}{x^2} = h$$

$$A(x) = 2x^2 + 4x\left(\frac{8000}{x^2}\right)$$
$$= 2x^2 + \frac{32,000}{x}$$

Chapter 2
Polynomial and Rational Functions

Section 2.1

Check Point Exercises

1. **a.** $(5-2i)+(3+3i)$
$=5-2i+3+3i$
$=(5+3)+(-2+3)i$
$=8+i$

b. $(2+6i)-(12-i)$
$=2+6i-12+i$
$=(2-12)+(6+1)i$
$=-10+7i$

2. **a.** $7i(2-9i)=7i(2)-7i(9i)$
$=14i-63i^2$
$=14i-63(-1)$
$=63+14i$

b. $(5+4i)(6-7i)=30-35i+24i-28i^2$
$=30-35i+24i-28(-1)$
$=30+28-35i+24i$
$=58-11i$

3. $\frac{5+4i}{4-i}=\frac{5+4i}{4-i}\cdot\frac{4+i}{4+i}$
$=\frac{20+5i+16i+4i^2}{16+4i-4i-i^2}$
$=\frac{20+21i-4}{16+1}$
$=\frac{16+21i}{17}$
$=\frac{16}{17}+\frac{21}{17}i$

4. **a.** $\sqrt{-27}+\sqrt{-48}=i\sqrt{27}+i\sqrt{48}$
$=i\sqrt{9\cdot 3}+i\sqrt{16\cdot 3}$
$=3i\sqrt{3}+4i\sqrt{3}$
$=7i\sqrt{3}$

b. $(-2+\sqrt{-3})^2=(-2+i\sqrt{3})^2$
$=(-2)^2+2(-2)(i\sqrt{3})+(i\sqrt{3})^2$
$=4-4i\sqrt{3}+3i^2$
$=4-4i\sqrt{3}+3(-1)$
$=1-4i\sqrt{3}$

c. $\frac{-14+\sqrt{-12}}{2}=\frac{-14+i\sqrt{12}}{2}$
$=\frac{-14+2i\sqrt{3}}{2}$
$=\frac{-14}{2}+\frac{2i\sqrt{3}}{2}$
$=-7+i\sqrt{3}$

5. $x^2-2x+2=0$
$a=1, b=-2, c=2$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$
$x=\frac{2\pm\sqrt{4-8}}{2}$
$x=\frac{2\pm\sqrt{-4}}{2}$
$x=\frac{2\pm 2i}{2}$
$x=1\pm i$
The solution set is $\{1+i, 1-i\}$.

Concept and Vocabulary Check 2.1

1. $\sqrt{-1}$; -1

2. complex; imaginary; real

3. $-6i$

4. $14i$

5. 18 ; $-15i$; $12i$; $-10i^2$; 10

6. $2+9i$

7. $2+5i$

8. i; i; $2i\sqrt{5}$

9. $-1\pm i\frac{\sqrt{6}}{2}$

Exercise Set 2.1

1. $(7 + 2i) + (1 - 4i) = 7 + 2i + 1 - 4i$
$= 7 + 1 + 2i - 4i$
$= 8 - 2i$

2. $(-2 + 6i) + (4 - i)$
$= -2 + 6i + 4 - i$
$= -2 + 4 + 6i - i$
$= 2 + 5i$

3. $(3 + 2i) - (5 - 7i) = 3 - 5 + 2i + 7i$
$= 3 + 2i - 5 + 7i$
$= -2 + 9i$

4. $(-7 + 5i) - (-9 - 11i) = -7 + 5i + 9 + 11i$
$= -7 + 9 + 5i + 11i$
$= 2 + 16i$

5. $6 - (-5 + 4i) - (-13 - i) = 6 + 5 - 4i + 13 + i$
$= 24 - 3i$

6. $7 - (-9 + 2i) - (-17 - i) = 7 + 9 - 2i + 17 + i$
$= 33 - i$

7. $8i - (14 - 9i) = 8i - 14 + 9i$
$= -14 + 8i + 9i$
$= -14 + 17i$

8. $15i - (12 - 11i) = 15i - 12 + 11i$
$= -12 + 15i + 11i$
$= -12 + 26i$

9. $-3i(7i - 5) = -21i^2 + 15i$
$= -21(-1) + 15i$
$= 21 + 15i$

10. $-8i\,(2i - 7) = -16i^2 + 56i = -16(-1) + 56i$
$= 9 - 25i^2 = 9 + 25 = 34 = 16 + 56i$

11. $(-5 + 4i)(3 + i) = -15 - 5i + 12i + 4i^2$
$= -15 + 7i - 4$
$= -19 + 7i$

12. $(-4 - 8i)(3 + i) = -12 - 4i - 24i - 8i^2$
$= -12 - 28i + 8$
$= -4 - 28i$

13. $(7 - 5i)(-2 - 3i) = -14 - 21i + 10i + 15i^2$
$= -14 - 15 - 11i$
$= -29 - 11i$

14. $(8 - 4i)(-3 + 9i) = -24 + 72i + 12i - 36i^2$
$= -24 + 36 + 84i$
$= 12 + 84i$

15. $(3 + 5i)(3 - 5i) = 9 - 15i + 15i - 25i^2$
$= 9 + 25$
$= 34$

16. $(2 + 7i)(2 - 7i) = 4 - 49i^2 = 4 + 49 = 53$

17. $(-5 + i)(-5 - i) = 25 + 5i - 5i - i^2$
$= 25 + 1$
$= 26$

18. $(-7 + i)(-7 - i) = 49 + 7i - 7i - i^2$
$= 49 + 1$
$= 50$

19. $(2 + 3i)^2 = 4 + 12i + 9i^2$
$= 4 + 12i - 9$
$= -5 + 12i$

20. $(5 - 2i)^2 = 25 - 20i + 4i^2$
$= 25 - 20i - 4$
$= 21 - 20i$

21.
$$\frac{2}{3-i} = \frac{2}{3-i} \cdot \frac{3+i}{3+i} = \frac{2(3+i)}{9+1} = \frac{2(3+i)}{10} = \frac{3+i}{5} = \frac{3}{5} + \frac{1}{5}i$$

22.
$$\frac{3}{4+i} = \frac{3}{4+i} \cdot \frac{4-i}{4-i} = \frac{3(4-i)}{16-i^2} = \frac{3(4-i)}{17} = \frac{12}{17} - \frac{3}{17}i$$

23. $\frac{2i}{1+i}=\frac{2i}{1+i}\cdot\frac{1-i}{1-i}=\frac{2i-2i^2}{1+1}=\frac{2+2i}{2}=1+i$

24. $\frac{5i}{2-i}=\frac{5i}{2-i}\cdot\frac{2+i}{2+i}$
$=\frac{10i+5i^2}{4+1}$
$=\frac{-5+10i}{5}$
$=-1+2i$

25. $\frac{8i}{4-3i}=\frac{8i}{4-3i}\cdot\frac{4+3i}{4+3i}$
$=\frac{32i+24i^2}{16+9}$
$=\frac{-24+32i}{25}$
$=-\frac{24}{25}+\frac{32}{25}i$

26. $\frac{-6i}{3+2i}=\frac{-6i}{3+2i}\cdot\frac{3-2i}{3-2i}=\frac{-18i+12i^2}{9+4}$
$=\frac{-12-18i}{13}=-\frac{12}{13}-\frac{18}{13}i$

27. $\frac{2+3i}{2+i}=\frac{2+3i}{2+i}\cdot\frac{2-i}{2-i}$
$=\frac{4+4i-3i^2}{4+1}$
$=\frac{7+4i}{5}$
$=\frac{7}{5}+\frac{4}{5}i$

28. $\frac{3-4i}{4+3i}=\frac{3-4i}{4+3i}\cdot\frac{4-3i}{4-3i}$
$=\frac{12-25i+12i^2}{16+9}$
$=\frac{-25i}{25}$
$=-i$

29. $\sqrt{-64}-\sqrt{-25}=i\sqrt{64}-i\sqrt{25}$
$=8i-5i=3i$

30. $\sqrt{-81}-\sqrt{-144}=i\sqrt{81}-i\sqrt{144}=9i-12i$
$=-3i$

31. $5\sqrt{-16}+3\sqrt{-81}=5(4i)+3(9i)$
$=20i+27i=47i$

32. $5\sqrt{-8}+3\sqrt{-18}$
$=5i\sqrt{8}+3i\sqrt{18}=5i\sqrt{4\cdot 2}+3i\sqrt{9\cdot 2}$
$=10i\sqrt{2}+9i\sqrt{2}$
$=19i\sqrt{2}$

33. $\left(-2+\sqrt{-4}\right)^2=\left(-2+2i\right)^2$
$=4-8i+4i^2$
$=4-8i-4$
$=-8i$

34. $\left(-5-\sqrt{-9}\right)^2=(-5-i\sqrt{9})^2=\left(-5-3i\right)^2$
$=25+30i+9i^2$
$=25+30i-9$
$=16+30i$

35. $\left(-3-\sqrt{-7}\right)^2=\left(-3-i\sqrt{7}\right)^2$
$=9+6i\sqrt{7}+i^2\left(7\right)$
$=9-7+6i\sqrt{7}$
$=2+6i\sqrt{7}$

36. $\left(-2+\sqrt{-11}\right)^2=\left(-2+i\sqrt{11}\right)^2$
$=4-4i\sqrt{11}+i^2\left(11\right)$
$=4-11-4i\sqrt{11}$
$=-7-4i\sqrt{11}$

37. $\frac{-8+\sqrt{-32}}{24}=\frac{-8+i\sqrt{32}}{24}$
$=\frac{-8+i\sqrt{16\cdot 2}}{24}$
$=\frac{-8+4i\sqrt{2}}{24}$
$=-\frac{1}{3}+\frac{\sqrt{2}}{6}i$

38. $\frac{-12+\sqrt{-28}}{32}=\frac{-12+i\sqrt{28}}{32}=\frac{-12+i\sqrt{4\cdot 7}}{32}$
$=\frac{-12+2i\sqrt{7}}{32}=-\frac{3}{8}+\frac{\sqrt{7}}{16}i$

39. $\dfrac{-6-\sqrt{-12}}{48}=\dfrac{-6-i\sqrt{12}}{48}$
$=\dfrac{-6-i\sqrt{4\cdot 3}}{48}$
$=\dfrac{-6-2i\sqrt{3}}{48}$
$=-\dfrac{1}{8}-\dfrac{\sqrt{3}}{24}i$

40. $\dfrac{-15-\sqrt{-18}}{33}=\dfrac{-15-i\sqrt{18}}{33}=\dfrac{-15-i\sqrt{9\cdot 2}}{33}$
$=\dfrac{-15-3i\sqrt{2}}{33}=-\dfrac{5}{11}-\dfrac{\sqrt{2}}{11}i$

41. $\sqrt{-8}\left(\sqrt{-3}-\sqrt{5}\right)=i\sqrt{8}(i\sqrt{3}-\sqrt{5})$
$=2i\sqrt{2}\left(i\sqrt{3}-\sqrt{5}\right)$
$=-2\sqrt{6}-2i\sqrt{10}$

42. $\sqrt{-12}\left(\sqrt{-4}-\sqrt{2}\right)=i\sqrt{12}(i\sqrt{4}-\sqrt{2})$
$=2i\sqrt{3}\left(2i-\sqrt{2}\right)$
$=4i^2\sqrt{3}-2i\sqrt{6}$
$=-4\sqrt{3}-2i\sqrt{6}$

43. $\left(3\sqrt{-5}\right)\left(-4\sqrt{-12}\right)=\left(3i\sqrt{5}\right)\left(-8i\sqrt{3}\right)$
$=-24i^2\sqrt{15}$
$=24\sqrt{15}$

44. $\left(3\sqrt{-7}\right)\left(2\sqrt{-8}\right)$
$=(3i\sqrt{7})(2i\sqrt{8})=(3i\sqrt{7})(2i\sqrt{4\cdot 2})$
$=\left(3i\sqrt{7}\right)\left(4i\sqrt{2}\right)=12i^2\sqrt{14}=-12\sqrt{14}$

45. $x^2-6x+10=0$
$x=\dfrac{6\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}$
$x=\dfrac{6\pm\sqrt{36-40}}{2}$
$x=\dfrac{6\pm\sqrt{-4}}{2}$
$x=\dfrac{6\pm 2i}{2}$
$x=3\pm i$
The solution set is $\{3+i,3-i\}$.

46. $x^2-2x+17=0$
$x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(17)}}{2(1)}$
$x=\dfrac{2\pm\sqrt{4-68}}{2}$
$x=\dfrac{2\pm\sqrt{-64}}{2}$
$x=\dfrac{2\pm 8i}{2}$
$x=1\pm 4i$
The solution set is $\{1+4i,1-4i\}$.

47. $4x^2+8x+13=0$
$x=\dfrac{-8\pm\sqrt{8^2-4(4)(13)}}{2(4)}$
$=\dfrac{-8\pm\sqrt{64-208}}{8}$
$=\dfrac{-8\pm\sqrt{-144}}{8}$
$=\dfrac{-8\pm 12i}{8}$
$=\dfrac{4(-2\pm 3i)}{8}$
$=\dfrac{-2\pm 3i}{2}$
$=-1\pm\dfrac{3}{2}i$

The solution set is $\left\{-1+\dfrac{3}{2}i,-1-\dfrac{3}{2}i\right\}$.

48. $2x^2 + 2x + 3 = 0$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{4-24}}{4}$$
$$= \frac{-2 \pm \sqrt{-20}}{4}$$
$$= \frac{-2 \pm 2i\sqrt{5}}{4}$$
$$= \frac{2(-1 \pm i\sqrt{5})}{4}$$
$$= \frac{-1 \pm i\sqrt{5}}{2}$$
$$= -\frac{1}{2} \pm \frac{\sqrt{5}}{2}i$$

The solution set is $\left\{-\frac{1}{2} + \frac{\sqrt{5}}{2}i, -\frac{1}{2} - \frac{\sqrt{5}}{2}i\right\}$.

49. $3x^2 - 8x + 7 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(7)}}{2(3)}$$
$$= \frac{8 \pm \sqrt{64-84}}{6}$$
$$= \frac{8 \pm \sqrt{-20}}{6}$$
$$= \frac{8 \pm 2i\sqrt{5}}{6}$$
$$= \frac{2(4 \pm i\sqrt{5})}{6}$$
$$= \frac{4 \pm i\sqrt{5}}{3}$$
$$= \frac{4}{3} \pm \frac{\sqrt{5}}{3}i$$

The solution set is $\left\{\frac{4}{3} + \frac{\sqrt{5}}{3}i, \frac{4}{3} - \frac{\sqrt{5}}{3}i\right\}$.

50. $3x^2 - 4x + 6 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(6)}}{2(3)}$$
$$= \frac{4 \pm \sqrt{16-72}}{6}$$
$$= \frac{4 \pm \sqrt{-56}}{6}$$
$$= \frac{4 \pm 2i\sqrt{14}}{6}$$
$$= \frac{2(2 \pm i\sqrt{14})}{6}$$
$$= \frac{2 \pm i\sqrt{14}}{3}$$
$$= \frac{2}{3} \pm \frac{\sqrt{14}}{3}i$$

The solution set is $\left\{\frac{2}{3} + \frac{\sqrt{14}}{3}i, \frac{2}{3} - \frac{\sqrt{14}}{3}i\right\}$.

51. $(2-3i)(1-i)-(3-i)(3+i)$

$$= \left(2 - 2i - 3i + 3i^2\right) - \left(3^2 - i^2\right)$$
$$= 2 - 5i + 3i^2 - 9 + i^2$$
$$= -7 - 5i + 4i^2$$
$$= -7 - 5i + 4(-1)$$
$$= -11 - 5i$$

52. $(8+9i)(2-i)-(1-i)(1+i)$

$$= \left(16 - 8i + 18i - 9i^2\right) - \left(1^2 - i^2\right)$$
$$= 16 + 10i - 9i^2 - 1 + i^2$$
$$= 15 + 10i - 8i^2$$
$$= 15 + 10i - 8(-1)$$
$$= 23 + 10i$$

53. $(2+i)^2 - (3-i)^2$

$$= \left(4 + 4i + i^2\right) - \left(9 - 6i + i^2\right)$$
$$= 4 + 4i + i^2 - 9 + 6i - i^2$$
$$= -5 + 10i$$

54. $(4-i)^2-(1+2i)^2$
$=(16-8i+i^2)-(1+4i+4i^2)$
$=16-8i+i^2-1-4i-4i^2$
$=15-12i-3i^2$
$=15-12i-3(-1)$
$=18-12i$

55. $5\sqrt{-16}+3\sqrt{-81}$
$=5\sqrt{16}\sqrt{-1}+3\sqrt{81}\sqrt{-1}$
$=5\cdot 4i+3\cdot 9i$
$=20i+27i$
$=47i$ or $0+47i$

56. $5\sqrt{-8}+3\sqrt{-18}$
$=5\sqrt{4}\sqrt{2}\sqrt{-1}+3\sqrt{9}\sqrt{2}\sqrt{-1}$
$=5\cdot 2\sqrt{2}\,i+3\cdot 3\sqrt{2}\,i$
$=10i\sqrt{2}+9i\sqrt{2}$
$=(10+9)i\sqrt{2}$
$=19i\sqrt{2}$ or $0+19i\sqrt{2}$

57. $f(x)=x^2-2x+2$
$f(1+i)=(1+i)^2-2(1+i)+2$
$=1+2i+i^2-2-2i+2$
$=1+i^2$
$=1-1$
$=0$

58. $f(x)=x^2-2x+5$
$f(1-2i)=(1-2i)^2-2(1-2i)+5$
$=1-4i+4i^2-2+4i+5$
$=4+4i^2$
$=4-4$
$=0$

59. $f(x)=\dfrac{x^2+19}{2-x}$
$f(3i)=\dfrac{(3i)^2+19}{2-3i}$
$=\dfrac{9i^2+19}{2-3i}$
$=\dfrac{-9+19}{2-3i}$
$=\dfrac{10}{2-3i}$
$=\dfrac{10}{2-3i}\cdot\dfrac{2+3i}{2+3i}$
$=\dfrac{20+30i}{4-9i^2}$
$=\dfrac{20+30i}{4+9}$
$=\dfrac{20+30i}{13}$
$=\dfrac{20}{13}+\dfrac{30}{13}i$

60. $f(x)=\dfrac{x^2+11}{3-x}$
$f(4i)=\dfrac{(4i)^2+11}{3-4i}=\dfrac{16i^2+11}{3-4i}$
$=\dfrac{-16+11}{3-4i}$
$=\dfrac{-5}{3-4i}$
$=\dfrac{-5}{3-4i}\cdot\dfrac{3+4i}{3+4i}$
$=\dfrac{-15-20i}{9-16i^2}$
$=\dfrac{-15-20i}{9+16}$
$=\dfrac{-15-20i}{25}$
$=\dfrac{-15}{25}-\dfrac{20}{25}i$
$=-\dfrac{3}{5}-\dfrac{4}{5}i$

61. $E = IR = (4-5i)(3+7i)$
$= 12 + 28i - 15i - 35i^2$
$= 12 + 13i - 35(-1)$
$= 12 + 35 + 13i = 47 + 13i$
The voltage of the circuit is
$(47 + 13i)$ volts.

62. $E = IR = (2-3i)(3+5i)$
$= 6 + 10i - 9i - 15i^2 = 6 + i - 15(-1)$
$= 6 + i + 15 = 21 + i$
The voltage of the circuit is $(21+i)$ volts.

63. Sum:
$(5 + i\sqrt{15}) + (5 - i\sqrt{15})$
$= 5 + i\sqrt{15} + 5 - i\sqrt{15}$
$= 5 + 5$
$= 10$
Product:
$(5 + i\sqrt{15})(5 - i\sqrt{15})$
$= 25 - 5i\sqrt{15} + 5i\sqrt{15} - 15i^2$
$= 25 + 15$
$= 40$

64. – 72. Answers will vary.

73. makes sense

74. does not make sense; Explanations will vary. Sample explanation: Imaginary numbers are not undefined.

75. does not make sense; Explanations will vary. Sample explanation: $i = \sqrt{-1}$; It is not a variable in this context.

76. makes sense

77. false; Changes to make the statement true will vary. A sample change is: All irrational numbers are complex numbers.

78. false; Changes to make the statement true will vary. A sample change is: $(3 + 7i)(3 - 7i) = 9 + 49 = 58$ which is a real number.

79. false; Changes to make the statement true will vary. A sample change is:
$$\frac{7+3i}{5+3i} = \frac{7+3i}{5+3i} \cdot \frac{5-3i}{5-3i} = \frac{44-6i}{34} = \frac{22}{17} - \frac{3}{17}i$$

80. true

81.
$$\frac{4}{(2+i)(3-i)} = \frac{4}{6-2i+3i-i^2}$$
$$= \frac{4}{6+i+1}$$
$$= \frac{4}{7+i}$$
$$= \frac{4}{7+i} \cdot \frac{7-i}{7-i}$$
$$= \frac{28-4i}{49-i^2}$$
$$= \frac{28-4i}{49+1}$$
$$= \frac{28-4i}{50}$$
$$= \frac{28}{50} - \frac{4}{50}i$$
$$= \frac{14}{25} - \frac{2}{25}i$$

82.
$$\frac{1+i}{1+2i} + \frac{1-i}{1-2i}$$
$$= \frac{(1+i)(1-2i)}{(1+2i)(1-2i)} + \frac{(1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$= \frac{(1+i)(1-2i) + (1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$= \frac{1-2i+i-2i^2+1+2i-i-2i^2}{1-4i^2}$$
$$= \frac{1-2i+i+2+1+2i-i+2}{1+4}$$
$$= \frac{6}{5}$$
$$= \frac{6}{5} + 0i$$

83. $\dfrac{8}{1+\frac{2}{i}} = \dfrac{8}{\frac{i}{i}+\frac{2}{i}}$

$= \dfrac{8}{\frac{2+i}{i}}$

$= \dfrac{8i}{2+i}$

$= \dfrac{8i}{2+i} \cdot \dfrac{2-i}{2-i}$

$= \dfrac{16i-8i^2}{4-i^2}$

$= \dfrac{16i+8}{4+1}$

$= \dfrac{8+16i}{5}$

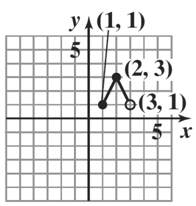
$= \dfrac{8}{5}+\dfrac{16}{5}i$

84. domain: $[0,2)$

range: $[0,2]$

85. $f(x)=1$ at $\frac{1}{2}$ and $\frac{3}{2}$.

86.



$g(x) = f(x-1)+1$

87. $0=-2(x-3)^2+8$

$2(x-3)^2=8$

$(x-3)^2=4$

$x-3=\pm\sqrt{4}$

$x=3\pm 2$

$x=1, 5$

88. $-x^2-2x+1=0$

$x^2+2x-1=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

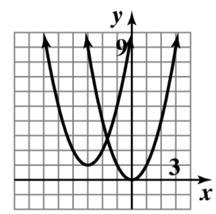
$=\dfrac{2\pm\sqrt{8}}{2}$

$=\dfrac{2\pm 2\sqrt{2}}{2}$

$=1\pm\sqrt{2}$

The solution set is $\{1\pm\sqrt{2}\}$.

89. The graph of *g* is the graph of *f* shifted 1 unit up and 3 units to the left.



$f(x)=x^2$
$g(x)=(x+3)^2+1$

Section 2.2

Check Point Exercises

1. $f(x) = -(x-1)^2 + 4$

$$f(x) = \overset{a=-1}{-}\left(x - \overset{h=1}{1}\right)^2 + \overset{k=4}{4}$$

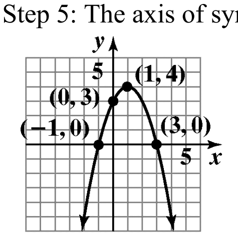
Step 1: The parabola opens down because $a < 0$.
Step 2: find the vertex: (1, 4)
Step 3: find the x-intercepts:

$$0 = -(x-1)^2 + 4$$
$$(x-1)^2 = 4$$
$$x - 1 = \pm 2$$
$$x = 1 \pm 2$$

$x = 3$ or $x = -1$
Step 4: find the y-intercept:

$$f(0) = -(0-1)^2 + 4 = 3$$

Step 5: The axis of symmetry is $x = 1$.



$f(x) = -(x-1)^2 + 4$

2. $f(x) = (x-2)^2 + 1$

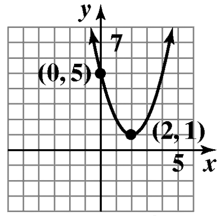
Step 1: The parabola opens up because $a > 0$.
Step 2: find the vertex: (2, 1)
Step 3: find the x-intercepts:

$$0 = (x-2)^2 + 1$$
$$(x-2)^2 = -1$$
$$x - 2 = \sqrt{-1}$$
$$x = 2 \pm i$$

The equation has no real roots, thus the parabola has no x-intercepts.
Step 4: find the y-intercept:

$$f(0) = (0-2)^2 + 1 = 5$$

Step 5: The axis of symmetry is $x = 2$.



$f(x) = (x-2)^2 + 1$

3. $f(x) = -x^2 + 4x + 1$

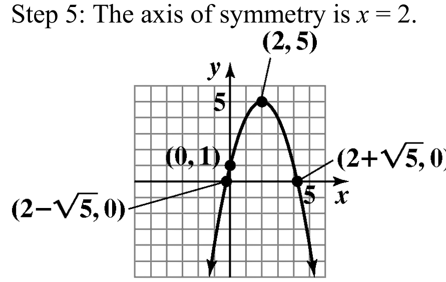
Step 1: The parabola opens down because $a < 0$.
Step 2: find the vertex:

$$x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$
$$f(2) = -2^2 + 4(2) + 1 = 5$$

The vertex is (2, 5).
Step 3: find the x-intercepts:

$$0 = -x^2 + 4x + 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(1)}}{2(-1)}$$
$$x = \frac{-4 \pm \sqrt{20}}{-2}$$
$$x = 2 \pm \sqrt{5}$$

The x-intercepts are $x \approx -0.2$ and $x \approx -4.2$.
Step 4: find the y-intercept: $f(0) = -0^2 + 4(0) + 1 = 1$
Step 5: The axis of symmetry is $x = 2$.



$f(x) = -x^2 + 4x + 1$

4. $f(x) = 4x^2 - 16x + 1000$

a. $a = 4$. The parabola opens upward and has a minimum value.

b. $x = \dfrac{-b}{2a} = \dfrac{16}{8} = 2$

$f(2) = 4(2)^2 - 16(2) + 1000 = 984$
The minimum point is 984 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[984, \infty)$

5. $f(x) = -0.005x^2 + 2x + 5$

a. The information needed is found at the vertex.
x-coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-2}{2(-0.005)} = 200$$

y-coordinate of vertex

$$y = -0.005(200)^2 + 2(200) + 5 = 205$$

The vertex is (200,205).
The maximum height of the arrow is 205 feet.
This occurs 200 feet from its release.

b. The arrow will hit the ground when the height reaches 0.

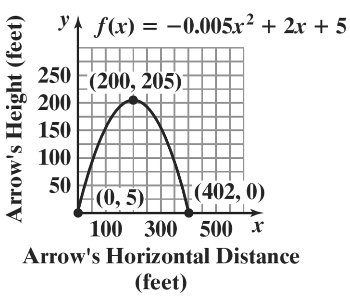
$$f(x) = -0.005x^2 + 2x + 5$$
$$0 = -0.005x^2 + 2x + 5$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(-0.005)(5)}}{2(-0.005)}$$
$$x \approx -2 \text{ or } x \approx 402$$

The arrow travels 402 feet before hitting the ground.

c. The starting point occurs when $x = 0$. Find the corresponding y-coordinate.

$$f(x) = -0.005(0)^2 + 2(0) + 5 = 5$$

Plot $(0,5)$, $(402,0)$, and $(200,205)$, and connect them with a smooth curve.



6. Let x = one of the numbers;
$x - 8$ = the other number.
The product is $f(x) = x(x-8) = x^2 - 8x$
The x-coordinate of the minimum is

$$x = -\frac{b}{2a} = -\frac{-8}{2(1)} = -\frac{-8}{2} = 4.$$

$$f(4) = (4)^2 - 8(4) = 16 - 32 = -16$$

The vertex is $(4,-16)$.
The minimum product is -16. This occurs when the two numbers are 4 and $4 - 8 = -4$.

7. Maximize the area of a rectangle constructed with 120 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 2y = 120$$
$$2y = 120 - 2x$$
$$y = \frac{120 - 2x}{2} = 60 - x$$

We need to maximize $A = xy = x(60 - x)$. Rewrite A as a function of x.

$$A(x) = x(60 - x) = -x^2 + 60x$$

Since $a = -1$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{60}{2(-1)} = -\frac{60}{-2} = 30.$$

When the length x is 30, the width y is
$y = 60 - x = 60 - 30 = 30.$
The dimensions of the rectangular region with maximum area are 30 feet by 30 feet. This gives an area of $30 \cdot 30 = 900$ square feet.

Concept and Vocabulary Check 2.2

1. standard; parabola; (h,k); >0; <0

2. $-\frac{b}{2a}$; $f\left(-\frac{b}{2a}\right)$; $-\frac{b}{2a}$; $f\left(-\frac{b}{2a}\right)$

3. true

4. false

5. true

6. $x - 8$; $x^2 - 8x$

7. $40 - x$; $-x^2 + 40x$

Exercise Set 2.2

1. vertex: (1, 1)
$h(x) = (x-1)^2 + 1$

2. vertex: (–1, 1)
$g(x) = (x+1)^2 + 1$

3. vertex: (1, –1)
$j(x) = (x-1)^2 - 1$

4. vertex: (–1, –1)
$f(x) = (x+1)^2 - 1$

5. The graph is $f(x) = x^2$ translated down one.
$h(x) = x^2 - 1$

6. The point (–1, 0) is on the graph and $f(-1) = 0$. $f(x) = x^2 + 2x + 1$

7. The point (1, 0) is on the graph and $g(1) = 0$. $g(x) = x^2 - 2x + 1$

8. The graph is $f(x) = -x^2$ translated down one.
$j(x) = -x^2 - 1$

9. $f(x) = 2(x-3)^2 + 1$
$h = 3, k = 1$
The vertex is at (3, 1).

10. $f(x) = -3(x-2)^2 + 12$
$h = 2, k = 12$
The vertex is at (2, 12).

11. $f(x) = -2(x+1)^2 + 5$
$h = -1, k = 5$
The vertex is at (–1, 5).

12. $f(x) = -2(x+4)^2 - 8$
$h = -4, k = -8$
The vertex is at (–4, –8).

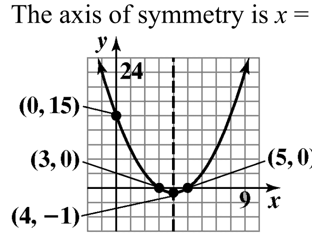
13. $f(x) = 2x^2 - 8x + 3$
$x = \frac{-b}{2a} = \frac{8}{4} = 2$
$f(2) = 2(2)^2 - 8(2) + 3$
$= 8 - 16 + 3 = -5$
The vertex is at (2, –5).

14. $f(x) = 3x^2 - 12x + 1$
$x = \frac{-b}{2a} = \frac{12}{6} = 2$
$f(2) = 3(2)^2 - 12(2) + 1$
$= 12 - 24 + 1 = -11$
The vertex is at (2, –11).

15. $f(x) = -x^2 - 2x + 8$
$x = \frac{-b}{2a} = \frac{2}{-2} = -1$
$f(-1) = -(-1)^2 - 2(-1) + 8$
$= -1 + 2 + 8 = 9$
The vertex is at (–1, 9).

16. $f(x) = -2x^2 + 8x - 1$
$x = \frac{-b}{2a} = \frac{-8}{-4} = 2$
$f(2) = -2(2)^2 + 8(2) - 1$
$= -8 + 16 - 1 = 7$
The vertex is at (2, 7).

17. $f(x) = (x-4)^2 - 1$
vertex: (4, –1)
x-intercepts:
$0 = (x-4)^2 - 1$
$1 = (x-4)^2$
$\pm 1 = x - 4$
$x = 3$ or $x = 5$
y-intercept:
$f(0) = (0-4)^2 - 1 = 15$
The axis of symmetry is $x = 4$.



$f(x) = (x-4)^2 - 1$

domain: $(-\infty, \infty)$
range: $[-1, \infty)$

18. $f(x)=(x-1)^2-2$

vertex: (1, –2)

x-intercepts:

$0=(x-1)^2-2$

$(x-1)^2=2$

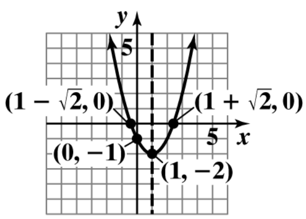
$x-1=\pm\sqrt{2}$

$x=1\pm\sqrt{2}$

y-intercept:

$f(0)=(0-1)^2-2=-1$

The axis of symmetry is $x = 1$.



$f(x) = (x-1)^2-2$

domain: $(-\infty,\infty)$

range: $[-2,\infty)$

19. $f(x)=(x-1)^2+2$

vertex: (1, 2)

x-intercepts:

$0=(x-1)^2+2$

$(x-1)^2=-2$

$x-1=\pm\sqrt{-2}$

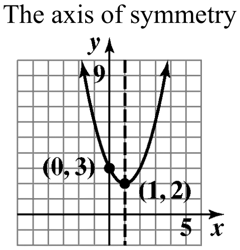
$x=1\pm i\sqrt{2}$

No x-intercepts.

y-intercept:

$f(0)=(0-1)^2+2=3$

The axis of symmetry is $x = 1$.



$f(x) = (x-1)^2+2$

domain: $(-\infty,\infty)$

range: $[2,\infty)$

20. $f(x)=(x-3)^2+2$

vertex: (3, 2)

x-intercepts:

$0=(x-3)^2+2$

$(x-3)^2=-2$

$x-3=\pm i\sqrt{2}$

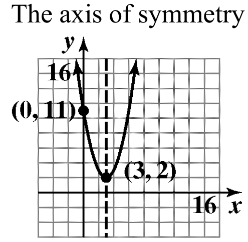
$x=3\pm i\sqrt{2}$

No x-intercepts.

y-intercept:

$f(0)=(0-3)^2+2=11$

The axis of symmetry is $x = 3$.



$f(x) = (x-3)^2+2$

domain: $(-\infty,\infty)$

range: $[2,\infty)$

21. $y-1=(x-3)^2$

$y=(x-3)^2+1$

vertex: (3, 1)

x-intercepts:

$0=(x-3)^2+1$

$(x-3)^2=-1$

$x-3=\pm i$

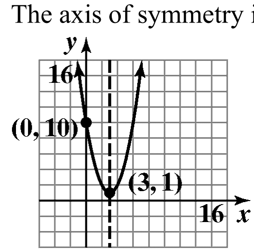
$x=3\pm i$

No x-intercepts.

y-intercept: 10

$y=(0-3)^2+1=10$

The axis of symmetry is $x = 3$.



$y-1=(x-3)^2$

domain: $(-\infty,\infty)$

range: $[1,\infty)$

22. $y-3=(x-1)^2$

$y=(x-1)^2+3$

vertex: (1, 3)

x-intercepts:

$0=(x-1)^2+3$

$(x-1)^2=-3$

$x-1=\pm i\sqrt{3}$

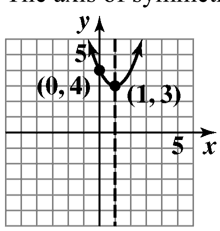
$x=1\pm i\sqrt{3}$

No x-intercepts

y-intercept:

$y=(0-1)^2+3=4$

The axis of symmetry is $x = 1$.



domain: $(-\infty,\infty)$

range: $[3,\infty)$

23. $f(x)=2(x+2)^2-1$

vertex: (–2, –1)

x-intercepts:

$0=2(x+2)^2-1$

$2(x+2)^2=1$

$(x+2)^2=\frac{1}{2}$

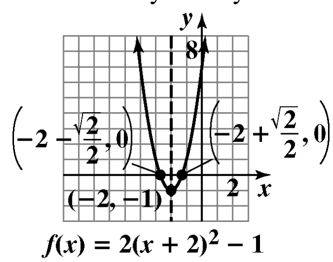
$x+2=\pm\frac{1}{\sqrt{2}}$

$x=-2\pm\frac{1}{\sqrt{2}}=-2\pm\frac{\sqrt{2}}{2}$

y-intercept:

$f(0)=2(0+2)^2-1=7$

The axis of symmetry is $x = -2$.



domain: $(-\infty,\infty)$

range: $[-1,\infty)$

24. $f(x)=\frac{5}{4}-\left(x-\frac{1}{2}\right)^2$

$f(x)=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

vertex: $\left(\frac{1}{2},\frac{5}{4}\right)$

x-intercepts:

$0=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

$\left(x-\frac{1}{2}\right)^2=\frac{5}{4}$

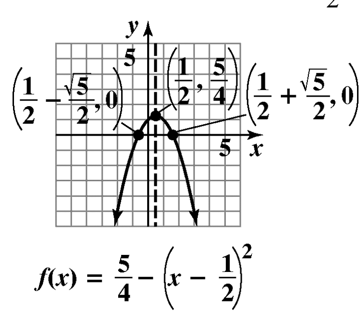
$x-\frac{1}{2}=\pm\frac{\sqrt{5}}{2}$

$x=\frac{1\pm\sqrt{5}}{2}$

y-intercept:

$f(0)=-\left(0-\frac{1}{2}\right)^2+\frac{5}{4}=1$

The axis of symmetry is $x=\frac{1}{2}$.



domain: $(-\infty,\infty)$

range: $\left(-\infty,\frac{5}{4}\right]$

25. $f(x) = 4 - (x-1)^2$

$f(x) = -(x-1)^2 + 4$

vertex: (1, 4)

x-intercepts:

$0 = -(x-1)^2 + 4$

$(x-1)^2 = 4$

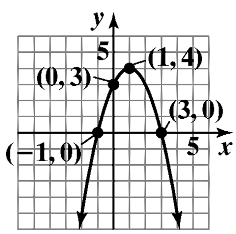
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept:

$f(x) = -(0-1)^2 + 4 = 3$

The axis of symmetry is $x = 1$.



$f(x) = 4 - (x - 1)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 4]$

26. $f(x) = 1 - (x-3)^2$

$f(x) = -(x-3)^2 + 1$

vertex: (3, 1)

x-intercepts:

$0 = -(x-3)^2 + 1$

$(x-3)^2 = 1$

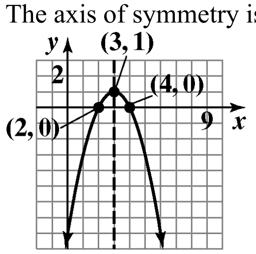
$x - 3 = \pm 1$

$x = 2$ or $x = 4$

y-intercept:

$f(0) = -(0-3)^2 + 1 = -8$

The axis of symmetry is $x = 3$.



$f(x) = 1 - (x - 3)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 1]$

27. $f(x) = x^2 - 2x - 3$

$f(x) = (x^2 - 2x + 1) - 3 - 1$

$f(x) = (x-1)^2 - 4$

vertex: (1, –4)

x-intercepts:

$0 = (x-1)^2 - 4$

$(x-1)^2 = 4$

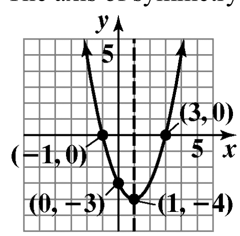
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept: –3

$f(0) = 0^2 - 2(0) - 3 = -3$

The axis of symmetry is $x = 1$.



$f(x) = x^2 + 3x - 10$

domain: $(-\infty, \infty)$

range: $[-4, \infty)$

28. $f(x) = x^2 - 2x - 15$

$f(x) = (x^2 - 2x + 1) - 15 - 1$

$f(x) = (x-1)^2 - 16$

vertex: (1, –16)

x-intercepts:

$0 = (x-1)^2 - 16$

$(x-1)^2 = 16$

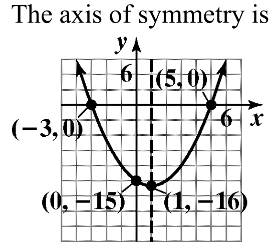
$x - 1 = \pm 4$

$x = -3$ or $x = 5$

y-intercept:

$f(0) = 0^2 - 2(0) - 15 = -15$

The axis of symmetry is $x = 1$.



$f(x) = x^2 - 2x - 15$

domain: $(-\infty, \infty)$

range: $[-16, \infty)$

29. $f(x)=x^2+3x-10$

$f(x)=\left(x^2+3x+\frac{9}{4}\right)-10-\frac{9}{4}$

$f(x)=\left(x+\frac{3}{2}\right)^2-\frac{49}{4}$

vertex: $\left(-\frac{3}{2},-\frac{49}{4}\right)$

x-intercepts:

$0=\left(x+\frac{3}{2}\right)^2-\frac{49}{4}$

$\left(x+\frac{3}{2}\right)^2=\frac{49}{4}$

$x+\frac{3}{2}=\pm\frac{7}{2}$

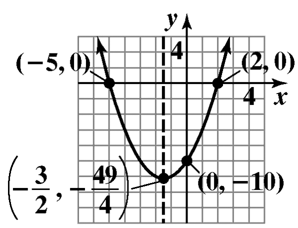
$x=-\frac{3}{2}\pm\frac{7}{2}$

$x = 2$ or $x = –5$

y-intercept:

$f(x)=0^2+3(0)-10=-10$

The axis of symmetry is $x=-\frac{3}{2}$.



$f(x) = x^2 + 3x - 10$

domain: $(-\infty,\infty)$

range: $\left[-\frac{49}{4},\infty\right)$

30. $f(x)=2x^2-7x-4$

$f(x)=2\left(x^2-\frac{7}{2}x+\frac{49}{16}\right)-4-\frac{49}{8}$

$f(x)=2\left(x-\frac{7}{4}\right)^2-\frac{81}{8}$

vertex: $\left(\frac{7}{4},-\frac{81}{8}\right)$

x-intercepts:

$0=2\left(x-\frac{7}{4}\right)^2-\frac{81}{8}$

$2\left(x-\frac{7}{4}\right)^2=\frac{81}{8}$

$\left(x-\frac{7}{4}\right)^2=\frac{81}{16}$

$x-\frac{7}{4}=\pm\frac{9}{4}$

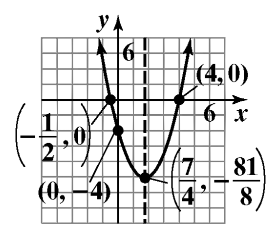
$x=\frac{7}{4}\pm\frac{9}{4}$

$x=-\frac{1}{2}$ or $x=4$

y-intercept:

$f(0)=2(0)^2-7(0)-4=-4$

The axis of symmetry is $x=\frac{7}{4}$.



$f(x) = 2x^2 - 7x - 4$

domain: $(-\infty,\infty)$

range: $\left[-\frac{81}{8},\infty\right)$

31. $f(x)=2x-x^2+3$

$f(x)=-x^2+2x+3$

$f(x)=-\left(x^2-2x+1\right)+3+1$

$f(x)=-(x-1)^2+4$

vertex: (1, 4)

x-intercepts:

$0=-(x-1)^2+4$

$(x-1)^2=4$

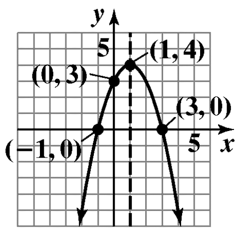
$x – 1 = ±2$

$x = –1$ or $x = 3$

y-intercept:

$f(0)=2(0)-(0)^2+3=3$

The axis of symmetry is $x = 1$.



$f(x) = 2x - x^2 + 3$

domain: $(-\infty, \infty)$

range: $(-\infty, 4]$

32. $f(x) = 5 - 4x - x^2$

$f(x) = -x^2 - 4x + 5$

$f(x) = -\left(x^2 + 4x + 4\right) + 5 + 4$

$f(x) = -(x+2)^2 + 9$

vertex: (–2, 9)

x-intercepts:

$0 = -(x+2)^2 + 9$

$(x+2)^2 = 9$

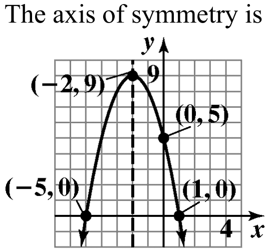
$x + 2 = \pm 3$

$x = -5, 1$

y-intercept:

$f(0) = 5 - 4(0) - (0)^2 = 5$

The axis of symmetry is $x = -2$.



$f(x) = 5 - 4x - x^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 9]$

33. $f(x) = x^2 + 6x + 3$

$f(x) = (x^2 + 6x + 9) + 3 - 9$

$f(x) = (x+3)^2 - 6$

vertex: $(-3, -6)$

x-intercepts:

$0 = (x+3)^2 - 6$

$(x+3)^2 = 6$

$x + 3 = \pm\sqrt{6}$

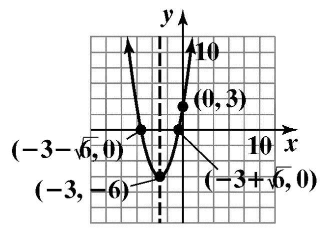
$x = -3 \pm \sqrt{6}$

y-intercept:

$f(0) = (0)^2 + 6(0) + 3$

$f(0) = 3$

The axis of symmetry is $x = -3$.



$f(x) = x^2 + 6x + 3$

domain: $(-\infty, \infty)$

range: $[-6, \infty)$

34. $f(x) = x^2 + 4x - 1$

$f(x) = (x^2 + 4x + 4) - 1 - 4$

$f(x) = (x+2)^2 - 5$

vertex: $(-2, -5)$

x-intercepts:

$0 = (x+2)^2 - 5$

$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

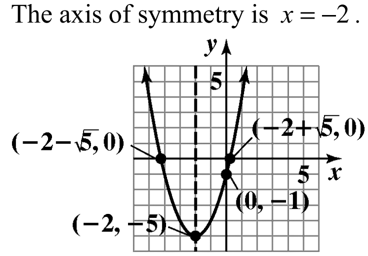
$x = -2 \pm \sqrt{5}$

y-intercept:

$f(0) = (0)^2 + 4(0) - 1$

$f(0) = -1$

The axis of symmetry is $x = -2$.



$f(x) = x^2 + 4x - 1$

domain: $(-\infty, \infty)$

range: $[-5, \infty)$

35. $f(x) = 2x^2 + 4x - 3$
$f(x) = 2(x^2 + 2x \quad) - 3$
$f(x) = 2(x^2 + 2x + 1) - 3 - 2$
$f(x) = 2(x+1)^2 - 5$

vertex: $(-1, -5)$

x-intercepts:

$0 = 2(x+1)^2 - 5$

$2(x+1)^2 = 5$

$(x+1)^2 = \frac{5}{2}$

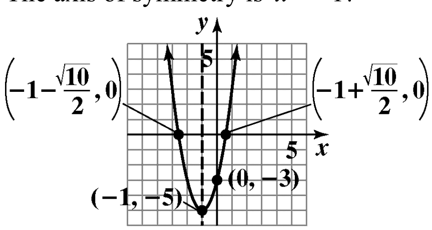
$x + 1 = \pm\sqrt{\frac{5}{2}}$

$x = -1 \pm \frac{\sqrt{10}}{2}$

y-intercept:

$f(0) = 2(0)^2 + 4(0) - 3$
$f(0) = -3$

The axis of symmetry is $x = -1$.



$f(x) = 2x^2 + 4x - 3$

domain: $(-\infty, \infty)$

range: $[-5, \infty)$

36. $f(x) = 3x^2 - 2x - 4$

$f(x) = 3\left(x^2 - \frac{2}{3}x \quad \right) - 4$

$f(x) = 3\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) - 4 - \frac{1}{3}$

$f(x) = 3\left(x - \frac{1}{3}\right)^2 - \frac{13}{3}$

vertex: $\left(\frac{1}{3}, -\frac{13}{3}\right)$

x-intercepts:

$0 = 3\left(x - \frac{1}{3}\right)^2 - \frac{13}{3}$

$3\left(x - \frac{1}{3}\right)^2 = \frac{13}{3}$

$\left(x - \frac{1}{3}\right)^2 = \frac{13}{9}$

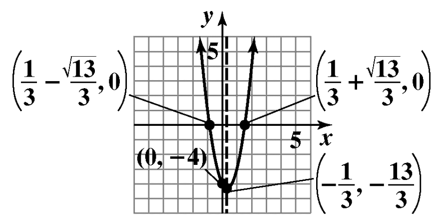
$x - \frac{1}{3} = \pm\sqrt{\frac{13}{9}}$

$x = \frac{1}{3} \pm \frac{\sqrt{13}}{3}$

y-intercept:

$f(0) = 3(0)^2 - 2(0) - 4$
$f(0) = -4$

The axis of symmetry is $x = \frac{1}{3}$.



$f(x) = 3x^2 - 2x - 4$

domain: $(-\infty, \infty)$

range: $\left[-\frac{13}{3}, \infty\right)$

37. $f(x) = 2x - x^2 - 2$
$f(x) = -x^2 + 2x - 2$
$f(x) = -\left(x^2 - 2x + 1\right) - 2 + 1$
$f(x) = -(x-1)^2 - 1$

vertex: (1, –1)
x-intercepts:

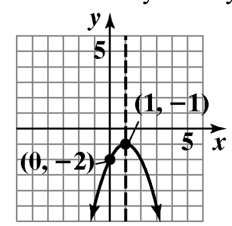
$0 = -(x-1)^2 - 1$

$(x-1)^2 = -1$

$x - 1 = \pm i$
$x = 1 \pm i$
No x-intercepts.
y-intercept:

$f(0) = 2(0) - (0)^2 - 2 = -2$

The axis of symmetry is $x = 1$.



$f(x) = 2x - x^2 - 2$

domain: $(-\infty, \infty)$

range: $(-\infty, -1]$

38. $f(x) = 6 - 4x + x^2$

$f(x) = x^2 - 4x + 6$

$f(x) = \left(x^2 - 4x + 4\right) + 6 - 4$

$f(x) = (x-2)^2 + 2$

vertex: (2, 2)

x-intercepts:

$0 = (x-2)^2 + 2$

$(x-2)^2 = -2$

$x - 2 = \pm i\sqrt{2}$

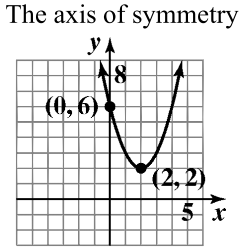
$x = 2 \pm i\sqrt{2}$

No x-intercepts

y-intercept:

$f(0) = 6 - 4(0) + (0)^2 = 6$

The axis of symmetry is $x = 2$.



$f(x) = 6 - 4x + x^2$

domain: $(-\infty, \infty)$

range: $[2, \infty)$

39. $f(x) = 3x^2 - 12x - 1$

a. $a = 3$. The parabola opens upward and has a minimum value.

b. $x = \dfrac{-b}{2a} = \dfrac{12}{6} = 2$

$f(2) = 3(2)^2 - 12(2) - 1 = 12 - 24 - 1 = -13$

The minimum is –13 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-13, \infty)$

40. $f(x) = 2x^2 - 8x - 3$

a. $a = 2$. The parabola opens upward and has a minimum value.

b. $x = \dfrac{-b}{2a} = \dfrac{8}{4} = 2$

$f(2) = 2(2)^2 - 8(2) - 3$
$= 8 - 16 - 3 = -11$

The minimum is –11 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-11, \infty)$

41. $f(x) = -4x^2 + 8x - 3$

a. $a = -4$. The parabola opens downward and has a maximum value.

b. $x = \dfrac{-b}{2a} = \dfrac{-8}{-8} = 1$

$f(1) = -4(1)^2 + 8(1) - 3$
$= -4 + 8 - 3 = 1$

The maximum is 1 at $x = 1$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 1]$

42. $f(x) = -2x^2 - 12x + 3$

a. $a = -2$. The parabola opens downward and has a maximum value.

b. $x = \dfrac{-b}{2a} = \dfrac{12}{-4} = -3$

$f(-3) = -2(-3)^2 - 12(-3) + 3$
$= -18 + 36 + 3 = 21$

The maximum is 21 at $x = -3$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 21]$

43. $f(x) = 5x^2 - 5x$

a. $a = 5$. The parabola opens upward and has a minimum value.

b. $x = \dfrac{-b}{2a} = \dfrac{5}{10} = \dfrac{1}{2}$

$f\left(\dfrac{1}{2}\right) = 5\left(\dfrac{1}{2}\right)^2 - 5\left(\dfrac{1}{2}\right)$

$= \dfrac{5}{4} - \dfrac{5}{2} = \dfrac{5}{4} - \dfrac{10}{4} = \dfrac{-5}{4}$

The minimum is $\dfrac{-5}{4}$ at $x = \dfrac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\dfrac{-5}{4}, \infty\right)$

44. $f(x) = 6x^2 - 6x$

a. $a = 6$. The parabola opens upward and has minimum value.

b. $x = \frac{-b}{2a} = \frac{6}{12} = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right)$$
$$= \frac{6}{4} - 3 = \frac{3}{2} - \frac{6}{2} = \frac{-3}{2}$$

The minimum is $\frac{-3}{2}$ at $x = \frac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\frac{-3}{2}, \infty\right)$

45. Since the parabola opens up, the vertex $(-1, -2)$ is a minimum point.
domain: $(-\infty, \infty)$. range: $[-2, \infty)$

46. Since the parabola opens down, the vertex $(-3, -4)$ is a maximum point.
domain: $(-\infty, \infty)$. range: $(-\infty, -4]$

47. Since the parabola has a maximum, it opens down from the vertex $(10, -6)$.
domain: $(-\infty, \infty)$. range: $(-\infty, -6]$

48. Since the parabola has a minimum, it opens up from the vertex $(-6, 18)$.
domain: $(-\infty, \infty)$. range: $[18, \infty)$

49. $(h, k) = (5, 3)$

$$f(x) = 2(x-h)^2 + k = 2(x-5)^2 + 3$$

50. $(h, k) = (7, 4)$

$$f(x) = 2(x-h)^2 + k = 2(x-7)^2 + 4$$

51. $(h, k) = (-10, -5)$

$$f(x) = 2(x-h)^2 + k$$
$$= 2[x-(-10)]^2 + (-5)$$
$$= 2(x+10)^2 - 5$$

52. $(h, k) = (-8, -6)$

$$f(x) = 2(x-h)^2 + k$$
$$= 2[x-(-8)]^2 + (-6)$$
$$= 2(x+8)^2 - 6$$

53. Since the vertex is a maximum, the parabola opens down and $a = -3$.
$(h, k) = (-2, 4)$

$$f(x) = -3(x-h)^2 + k$$
$$= -3[x-(-2)]^2 + 4$$
$$= -3(x+2)^2 + 4$$

54. Since the vertex is a maximum, the parabola opens down and $a = -3$.
$(h, k) = (5, -7)$

$$f(x) = -3(x-h)^2 + k$$
$$= -3(x-5)^2 + (-7)$$
$$= -3(x-5)^2 - 7$$

55. Since the vertex is a minimum, the parabola opens up and $a = 3$.
$(h, k) = (11, 0)$

$$f(x) = 3(x-h)^2 + k$$
$$= 3(x-11)^2 + 0$$
$$= 3(x-11)^2$$

56. Since the vertex is a minimum, the parabola opens up and $a = 3$.
$(h, k) = (9, 0)$

$$f(x) = 3(x-h)^2 + k$$
$$= 3(x-9)^2 + 0$$
$$= 3(x-9)^2$$

57. **a.** $y = -0.01x^2 + 0.7x + 6.1$
$a = -0.01,\ b = 0.7,\ c = 6.1$

x-coordinate of vertex

$$= \frac{-b}{2a} = \frac{-0.7}{2(-0.01)} = 35$$

y-coordinate of vertex

$$y = -0.01x^2 + 0.7x + 6.1$$
$$y = -0.01(35)^2 + 0.7(35) + 6.1 = 18.35$$

The maximum height of the shot is about 18.35 feet. This occurs 35 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$y = -0.01x^2 + 0.7x + 6.1$
$0 = -0.01x^2 + 0.7x + 6.1$
$a = -0.01, \ b = 0.7, \ c = 6.1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0.7 \pm \sqrt{0.7^2 - 4(-0.01)(6.1)}}{2(-0.01)}$$

$x \approx 77.8$ or $x \approx -7.8$

The maximum horizontal distance is 77.8 feet.

c. The initial height can be found at $x = 0$.

$y = -0.01x^2 + 0.7x + 6.1$
$y = -0.01(0)^2 + 0.7(0) + 6.1 = 6.1$

The shot was released at a height of 6.1 feet.

58. a. $y = -0.04x^2 + 2.1x + 6.1$
$a = -0.04, \ b = 2.1, \ c = 6.1$

x-coordinate of vertex

$$= \frac{-b}{2a} = \frac{-2.1}{2(-0.04)} = 26.25$$

y-coordinate of vertex

$y = -0.04x^2 + 2.1x + 6.1$
$y = -0.04(26.25)^2 + 2.1(26.25) + 6.1 \approx 33.7$

The maximum height of the shot is about 33.7 feet. This occurs 26.25 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$y = -0.04x^2 + 2.1x + 6.1$
$0 = -0.04x^2 + 2.1x + 6.1$
$a = -0.04, \ b = 2.1, \ c = 6.1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2.1 \pm \sqrt{2.1^2 - 4(-0.04)(6.1)}}{2(-0.04)}$$

$x \approx 55.3$ or $x \approx -2.8$

The maximum horizontal distance is 55.3 feet.

c. The initial height can be found at $x = 0$.

$y = -0.04x^2 + 2.1x + 6.1$
$y = -0.04(0)^2 + 2.1(0) + 6.1 = 6.1$

The shot was released at a height of 6.1 feet.

59. $y = -0.8x^2 + 2.4x + 6$

a. The information needed is found at the vertex.
x-coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-2.4}{2(-0.8)} = 1.5$$

y-coordinate of vertex

$y = -0.8(1.5)^2 + 2.4(1.5) + 6 = 7.8$

The vertex is (1.5, 7.8).
The maximum height of the ball is 7.8 feet.
This occurs 1.5 feet from its release.

b. The ball will hit the ground when the height reaches 0.

$y = -0.8x^2 + 2.4x + 6$
$0 = -0.8x^2 + 2.4x + 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2.4 \pm \sqrt{2.4^2 - 4(-0.8)(6)}}{2(-0.8)}$$

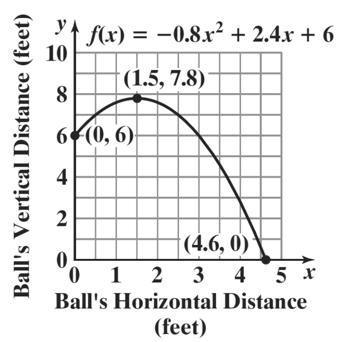
$x \approx -1.6$ or $x \approx 4.6$

The ball travels 4.6 feet before hitting the ground.

c. The starting point occurs when $x = 0$. Find the corresponding y-coordinate.

$y = -0.8(0)^2 + 2.4(0) + 6 = 6$

Plot $(0, 6)$, $(1.5, 7.8)$, and $(4.7, 0)$, and connect them with a smooth curve.



60. $y = -0.8x^2 + 3.2x + 6$

a. The information needed is found at the vertex.
x-coordinate of vertex

$$x = \frac{-b}{2a} = \frac{-3.2}{2(-0.8)} = 2$$

y-coordinate of vertex

$y = -0.8(2)^2 + 3.2(2) + 6 = 9.2$

The vertex is (2, 9.2).
The maximum height of the ball is 9.2 feet.
This occurs 2 feet from its release.

b. The ball will hit the ground when the height reaches 0.

$y = -0.8x^2 + 3.2x + 6$
$0 = -0.8x^2 + 3.2x + 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3.2 \pm \sqrt{3.2^2 - 4(-0.8)(6)}}{2(-0.8)}$$

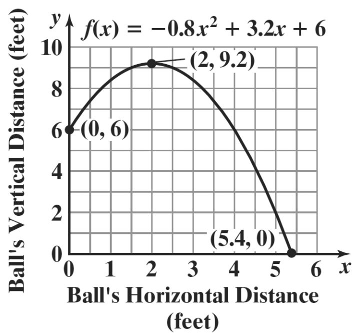
$x \approx -1.4$ or $x \approx 5.4$

The ball travels 5.4 feet before hitting the ground.

c. The starting point occurs when $x = 0$. Find the corresponding y-coordinate.

$y = -0.8(0)^2 + 3.2(0) + 6 = 6$

Plot $(0,6)$, $(2,9.2)$, and $(5.4,0)$, and connect them with a smooth curve.



61. Let x = one of the numbers;
$16 - x$ = the other number.

The product is $f(x) = x(16-x) = 16x - x^2 = -x^2 + 16x$

The x-coordinate of the maximum is

$$x = -\frac{b}{2a} = -\frac{16}{2(-1)} = -\frac{16}{-2} = 8.$$

$f(8) = -8^2 + 16(8) = -64 + 128 = 64$

The vertex is (8, 64). The maximum product is 64. This occurs when the two numbers are 8 and $16 - 8 = 8$.

62. Let x = one of the numbers
Let $20 - x$ = the other number

$P(x) = x(20-x) = 20x - x^2 = -x^2 + 20x$

$$x = -\frac{b}{2a} = -\frac{20}{2(-1)} = -\frac{20}{-2} = 10$$

The other number is $20 - x = 20 - 10 = 10$.

The numbers which maximize the product are 10 and 10. The maximum product is $10 \cdot 10 = 100$.

63. Let x = one of the numbers;
$x - 16$ = the other number.

The product is $f(x) = x(x-16) = x^2 - 16x$

The x-coordinate of the minimum is

$$x = -\frac{b}{2a} = -\frac{-16}{2(1)} = -\frac{-16}{2} = 8.$$

$f(8) = (8)^2 - 16(8)$
$= 64 - 128 = -64$

The vertex is $(8,-64)$. The minimum product is -64.

This occurs when the two numbers are 8 and $8 - 16 = -8$.

64. Let x = the larger number. Then $x - 24$ is the smaller number. The product of these two numbers is given by

$P(x) = x(x-24) = x^2 - 24x$

The product is minimized when

$$x = -\frac{b}{2a} = -\frac{(-24)}{2(1)} = 12$$

Since $12 - (-12) = 24$, the two numbers whose difference is 24 and whose product is minimized are 12 and -12.

The minimum product is $P(12) = 12(12-24) = -144$.

65. Maximize the area of a rectangle constructed along a river with 600 feet of fencing.
Let x = the width of the rectangle;
$600 - 2x$ = the length of the rectangle
We need to maximize.

$A(x) = x(600-2x)$
$= 600x - 2x^2 = -2x^2 + 600x$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{600}{2(-2)} = -\frac{600}{-4} = 150.$$

When the width is $x = 150$ feet, the length is $600 - 2(150) = 600 - 300 = 300$ feet.

The dimensions of the rectangular plot with maximum area are 150 feet by 300 feet. This gives an area of $150 \cdot 300 = 45,000$ square feet.

66. From the diagram, we have that x is the width of the rectangular plot and $200-2x$ is the length. Thus, the area of the plot is given by

$A = l \cdot w = (200-2x)(x) = -2x^2 + 200x$

Since the graph of this equation is a parabola that opens down, the area is maximized at the vertex.

$$x = -\frac{b}{2a} = -\frac{200}{2(-2)} = 50$$

$$A = -2(50)^2 + 200(50) = -5000 + 10{,}000 = 5000$$

The maximum area is 5000 square feet when the length is 100 feet and the width is 50 feet.

67. Maximize the area of a rectangle constructed with 50 yards of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 2y = 50$$
$$2y = 50 - 2x$$
$$y = \frac{50-2x}{2} = 25 - x$$

We need to maximize $A = xy = x(25-x)$. Rewrite A as a function of x.

$$A(x) = x(25-x) = -x^2 + 25x$$

Since $a = -1$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{25}{2(-1)} = -\frac{25}{-2} = 12.5.$$

When the length x is 12.5, the width y is
$y = 25 - x = 25 - 12.5 = 12.5.$

The dimensions of the rectangular region with maximum area are 12.5 yards by 12.5 yards. This gives an area of $12.5 \cdot 12.5 = 156.25$ square yards.

68. Let x = the length of the rectangle
Let y = the width of the rectangle

$$2x + 2y = 80$$
$$2y = 80 - 2x$$
$$y = \frac{80-2x}{2}$$
$$y = 40 - x$$

$$A(x) = x(40-x) = -x^2 + 40x$$

$$x = -\frac{b}{2a} = -\frac{40}{2(-1)} = -\frac{40}{-2} = 20.$$

When the length x is 20, the width y is
$y = 40 - x = 40 - 20 = 20.$

The dimensions of the rectangular region with maximum area are 20 yards by 20 yards. This gives an area of $20 \cdot 20 = 400$ square yards.

69. Maximize the area of the playground with 600 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 3y = 600$$
$$3y = 600 - 2x$$
$$y = \frac{600-2x}{3}$$
$$y = 200 - \frac{2}{3}x$$

We need to maximize $A = xy = x\left(200 - \frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x) = x\left(200 - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + 200x$$

Since $a = -\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{200}{2\left(-\frac{2}{3}\right)} = -\frac{200}{-\frac{4}{3}} = 150.$$

When the length x is 150, the width y is

$$y = 200 - \frac{2}{3}x = 200 - \frac{2}{3}(150) = 100.$$

The dimensions of the rectangular playground with maximum area are 150 feet by 100 feet. This gives an area of $150 \cdot 100 = 15{,}000$ square feet.

70. Maximize the area of the playground with 400 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 3y = 400$$
$$3y = 400 - 2x$$
$$y = \frac{400-2x}{3}$$
$$y = \frac{400}{3} - \frac{2}{3}x$$

We need to maximize $A = xy = x\left(\frac{400}{3} - \frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x) = x\left(\frac{400}{3} - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + \frac{400}{3}x$$

Since $a = -\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{\frac{400}{3}}{2\left(-\frac{2}{3}\right)} = -\frac{\frac{400}{3}}{-\frac{4}{3}} = 100.$$

When the length x is 100, the width y is

$$y = \frac{400}{3} - \frac{2}{3}x = \frac{400}{3} - \frac{2}{3}(100) = \frac{200}{3} = 66\frac{2}{3}.$$

The dimensions of the rectangular playground with maximum area are 100 feet by $66\frac{2}{3}$ feet. This gives an area of $100 \cdot 66\frac{2}{3} = 6666\frac{2}{3}$ square feet.

71. Maximize the cross-sectional area of the gutter:

$$A(x) = x(20-2x)$$
$$= 20x - 2x^2 = -2x^2 + 20x.$$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{20}{2(-2)} = -\frac{20}{-4} = 5.$$

When the height x is 5, the width is

$$20 - 2x = 20 - 2(5) = 20 - 10 = 10.$$

$$A(5) = -2(5)^2 + 20(5)$$
$$= -2(25) + 100 = -50 + 100 = 50$$

The maximum cross-sectional area is 50 square inches. This occurs when the gutter is 5 inches deep and 10 inches wide.

72.

$$A(x) = x(12-2x) = 12x - 2x^2$$
$$= -2x^2 + 12x$$

$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = -\frac{12}{-4} = 3$$

When the height x is 3, the width is

$$12 - 2x = 12 - 2(3) = 12 - 6 = 6.$$

$$A(3) = -2(3)^2 + 12(3) = -2(9) + 36$$
$$= -18 + 36 = 18$$

The maximum cross-sectional area is 18 square inches. This occurs when the gutter is 3 inches deep and 6 inches wide.

73. x = increase

$$A = (50 + x)(8000 - 100x)$$
$$= 400,000 + 3000x - 100x^2$$

$$x = \frac{-b}{2a} = \frac{-3000}{2(-100)} = 15$$

The maximum price is 50 + 15 = \$65.
The maximum revenue = 65(800 – 100·15) = \$422,500.

74. Maximize $A = (30 + x)(200 - 5x)$
$= 6000 + 50x - 5x^2$

$$x = \frac{-(50)}{2(-5)} = 5$$

Maximum rental = 30 + 5 = \$35
Maximum revenue = 35(200 – 5·5) = \$6125

75. x = increase

$$A = (20 + x)(60 - 2x)$$
$$= 1200 + 20x - 2x^2$$

$$x = \frac{-b}{2a} = \frac{-20}{2(-2)} = 5$$

The maximum number of trees is 20 + 5 = 25 trees.
The maximum yield is 60 – 2·5=50 pounds per tree, 50 x 25 = 1250 pounds.

76. Maximize $A = (30 + x)(50 - x)$
$= 1500 + 20x - x^2$

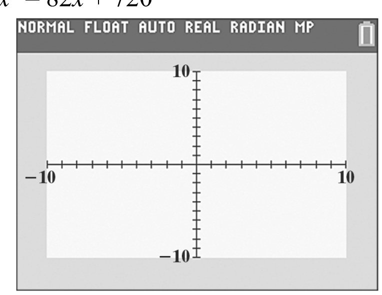
$$x = \frac{-20}{2(-1)} = 10$$

Maximum number of trees = 30 + 10 = 40 trees
Maximum yield = (30 + 10)(50 – 10) = 1600 pounds

77. – 83. Answers will vary.

84. $y = 2x^2 - 82x + 720$

a.



You can only see a little of the parabola.

b. $a = 2$; $b = -82$

$$x = -\frac{b}{2a} = -\frac{-82}{4} = 20.5$$

$$\begin{aligned} y &= 2(20.5)^2 - 82(20.5) + 720 \\ &= 840.5 - 1681 + 720 \\ &= -120.5 \end{aligned}$$

vertex: (20.5, –120.5)

c. Ymax = 750

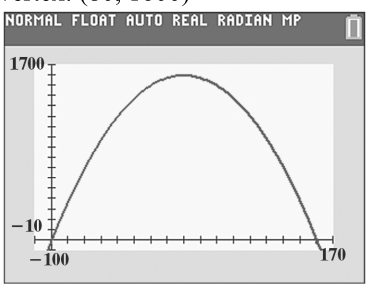
d. You can choose Xmin and Xmax so the x-value of the vertex is in the center of the graph. Choose Ymin to include the y-value of the vertex.

85. $y = -0.25x^2 + 40x$

$$x = \frac{-b}{2a} = \frac{-40}{-0.5} = 80$$

$$\begin{aligned} y &= -0.25(80)^2 + 40(80) \\ &= 1600 \end{aligned}$$

vertex: (80, 1600)

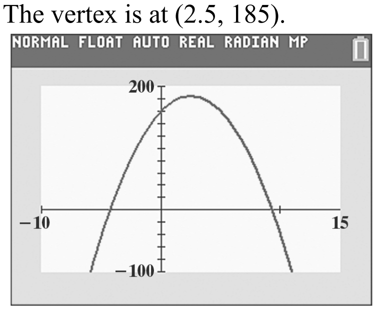


86. $y = -4x^2 + 20x + 160$

$$x = \frac{-b}{2a} = \frac{-20}{-8} = 2.5$$

$$\begin{aligned} y &= -4(2.5)^2 + 20(2.5) + 160 \\ &= -2.5 + 50 + 160 = 185 \end{aligned}$$

The vertex is at (2.5, 185).

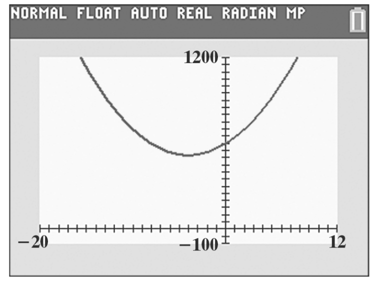


87. $y = 5x^2 + 40x + 600$

$$x = \frac{-b}{2a} = \frac{-40}{10} = -4$$

$$\begin{aligned} y &= 5(-4)^2 + 40(-4) + 600 \\ &= 80 - 160 + 600 = 520 \end{aligned}$$

vertex: (–4, 520)

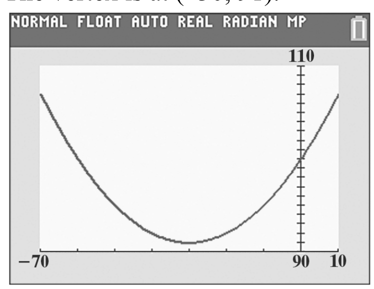


88. $y = 0.01x^2 + 0.6x + 100$

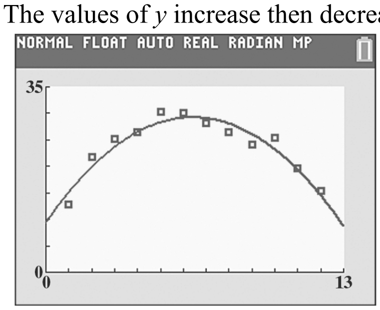
$$x = \frac{-b}{2a} = \frac{-0.6}{0.02} = -30$$

$$\begin{aligned} y &= 0.01(-30)^2 + 0.6(-30) + 100 \\ &= 9 - 18 + 100 = 91 \end{aligned}$$

The vertex is at (–30, 91).



89. a. The values of y increase then decrease.



b. $y = -0.48x^2 + 6.17x + 9.57$

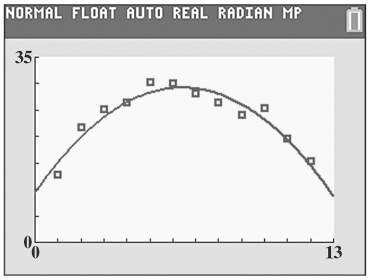
c. $x = \dfrac{-(6.17)}{2(-0.48)} \approx 6$

$y = -0.48(6)^2 + 6.17(6) + 9.57 \approx 29.3$

According to the model in part (b), *American Idol* had the greatest number of viewers, 29.3 million, in Season 6.

d. The greatest number of viewers actually occurred in Season 5, not Season 6, and the model underestimates the greatest number by 1.1 million.

e. Scatter plot and quadratic function of best fit:



90. does not make sense; Explanations will vary. Sample explanation: Some parabolas have the *y*-axis as the axis of symmetry.

91. makes sense

92. does not make sense; Explanations will vary. Sample explanation: If it is thrown vertically, its path will be a line segment.

93. does not make sense; Explanations will vary. Sample explanation: The football's path is better described by a quadratic model.

94. true

95. false; Changes to make the statement true will vary. A sample change is: The vertex is $(5,-1)$.

96. false; Changes to make the statement true will vary. A sample change is: The graph has no *x*–intercepts. To find *x*–intercepts, set $y = 0$ and solve for *x*.

$$0 = -2(x+4)^2 - 8$$
$$2(x+4)^2 = -8$$
$$(x+4)^2 = -4$$

Because the solutions to the equation are imaginary, we know that there are no *x*–intercepts.

97. false; Changes to make the statement true will vary. A sample change is: The *x*-coordinate of the maximum is $-\frac{b}{2a} = -\frac{1}{2(-1)} = -\frac{1}{-2} = \frac{1}{2}$ and the *y*–coordinate of the vertex of the parabola is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = \frac{5}{4}.$$

The maximum *y*–value is $\frac{5}{4}$.

98. $f(x) = 3(x+2)^2 - 5$; $(-1, -2)$
axis: $x = -2$
$(-1, -2)$ is one unit right of $(-2, -2)$. One unit left of $(-2, -2)$ is $(-3, -2)$.
point: $(-3, -2)$

99. Vertex $(3, 2)$ Axis: $x = 3$
second point $(0, 11)$

100. We start with the form $f(x) = a(x-h)^2 + k$.
Since we know the vertex is $(h,k) = (-3,-4)$, we have $f(x) = a(x+3)^2 - 4$. We also know that the graph passes through the point $(1,4)$, which allows us to solve for *a*.

$$4 = a(1+3)^2 - 4$$
$$8 = a(4)^2$$
$$8 = 16a$$
$$\frac{1}{2} = a$$

Therefore, the function is $f(x) = \frac{1}{2}(x+3)^2 - 4$.

101. We know $(h,k) = (-3,-4)$, so the equation is of the form

$$f(x) = a(x-h)^2 + k$$
$$= a[x-(-3)]^2 + (-1)$$
$$= a(x+3)^2 - 1$$

We use the point $(-2,-3)$ on the graph to determine the value of *a*:

$$f(x) = a(x+3)^2 - 1$$
$$-3 = a(-2+3)^2 - 1$$
$$-3 = a(1)^2 - 1$$
$$-3 = a - 1$$
$$-2 = a$$

Thus, the equation of the parabola is $f(x) = -2(x+3)^2 - 1$.

102. $2x + y - 2 = 0$
$y = 2 - 2x$

$d = \sqrt{x^2 + (2-2x)^2}$
$d = \sqrt{x^2 + 4 - 8x + 4x^2}$
$d = \sqrt{5x^2 - 8x + 4}$

Minimize $5x^2 - 8x + 4$

$x = \dfrac{-(-8)}{2(5)} = \dfrac{4}{5}$

$y = 2 - 2\left(\dfrac{4}{5}\right) = \dfrac{2}{5}$

$\left(\dfrac{4}{5}, \dfrac{2}{5}\right)$

103. $f(x) = (80 + x)(300 - 3x) - 10(300 - 3x)$
$= 24000 + 60x - 3x^2 - 3000 + 30x$
$= -3x^2 + 90x + 21000$

$x = \dfrac{-b}{2a} = \dfrac{-90}{2(-3)} = \dfrac{3}{2} = 15$

The maximum charge is 80 + 15 = \$95.00. the maximum profit is $-3(15)^2 + 9(15) + 21000 =$ \$21,675.

104. $440 = 2x + \pi y$
$440 - 2x = \pi y$
$\dfrac{440 - 2x}{\pi} = y$

Maximize $A = x\left(\dfrac{440 - 2x}{\pi}\right) = -\dfrac{2}{\pi}x^2 + \dfrac{440}{\pi}x$

$x = \dfrac{-\frac{440}{\pi}}{2\left(-\frac{2}{\pi}\right)} = \dfrac{\frac{-440}{\pi}}{-\frac{4}{\pi}} = \dfrac{440}{4} = 110$

$\dfrac{440 - 2(110)}{\pi} = \dfrac{220}{\pi}$

The dimensions are 110 yards by $\dfrac{220}{\pi}$ yards.

105. Answers will vary.

106. $3x + y^2 = 10$
$y^2 = 10 - 3x$
$y = \pm\sqrt{10 - 3x}$

Since there are values of x that give more than one value for y (for example, if $x = 0$, then $y = \pm\sqrt{10 - 0} = \pm\sqrt{10}$), the equation does not define y as a function of x.

107. a. domain: $\{x \mid -\infty < x < \infty\}$ or $(-\infty, \infty)$.

b. range: $\{y \mid -3 \le y < \infty\}$ or $[-3, \infty)$.

c. The x-intercepts are –2 and 4.

d. The y-intercept is –2.

e. $f(-4) = 2$

108. $f(x) = 4x^2 - 2x + 7$
$f(x+h) = 4(x+h)^2 - 2(x+h) + 7$
$= 4(x^2 + 2xh + h^2) - 2x - 2h + 7$
$= 4x^2 + 8xh + 4h^2 - 2x - 2h + 7$

$\dfrac{f(x+h) - f(x)}{h}$
$= \dfrac{4x^2 + 8xh + 4h^2 - 2x - 2h + 7 - \left(4x^2 - 2x + 7\right)}{h}$
$= \dfrac{4x^2 + 8xh + 4h^2 - 2x - 2h + 7 - 4x^2 + 2x - 7}{h}$
$= \dfrac{8xh + 4h^2 - 2h}{h}$
$= \dfrac{h\left(8x + 4h - 2\right)}{h}$
$= 8x + 4h - 2, \quad h \ne 0$

109. $x^3 + 3x^2 - x - 3 = x^2(x+3) - 1(x+3)$
$= (x+3)(x^2 - 1)$
$= (x+3)(x+1)(x-1)$

110. $f(x) = x^3 - 2x - 5$
$f(2) = (2)^3 - 2(2) - 5 = -1$
$f(3) = (3)^3 - 2(3) - 5 = 16$

The graph passes through (2, –1), which is below the x-axis, and (3, 16), which is above the x-axis. Since the graph of f is continuous, it must cross the x-axis somewhere between 2 and 3 to get from one of these points to the other.

111. $f(x) = x^4 - 2x^2 + 1$
$f(-x) = (-x)^4 - 2(-x)^2 + 1$
$= x^4 - 2x^2 + 1$

Since $f(-x) = f(x)$, the function is even.

Thus, the graph is symmetric with respect to the y-axis.

Section 2.3

Check Point Exercises

1. Since n is even and $a_n > 0$, the graph rises to the left and to the right.

2. It is not necessary to multiply out the polynomial to determine its degree. We can find the degree of the polynomial by adding the degrees of each of its factors. $f(x) = 2\underbrace{x^3}_{\text{degree 3}}\underbrace{(x-1)}_{\text{degree 1}}\underbrace{(x+5)}_{\text{degree 1}}$ has degree $3+1+1=5$.

$f(x) = 2x^3(x-1)(x+5)$ is of odd degree with a positive leading coefficient. Thus, the graph falls to the left and rises to the right.

3. Since n is odd and the leading coefficient is negative, the function falls to the right. Since the ratio cannot be negative, the model won't be appropriate.

4. The graph does not show the function's end behavior. Since $a_n > 0$ and n is odd, the graph should fall to the left but doesn't appear to do so.

5. $f(x) = x^3 + 2x^2 - 4x - 8$

$0 = x^2(x+2) - 4(x+2)$

$0 = (x+2)(x^2-4)$

$0 = (x+2)^2(x-2)$

$x = -2$ or $x = 2$

The zeros are –2 and 2.

6. $f(x) = x^4 - 4x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2-4) = 0$

$x^2(x+2)(x-2) = 0$

$x = 0$ or $x = -2$ or $x = 2$

The zeros are –2, 0, and 2.

7. $f(x) = -4\left(x+\frac{1}{2}\right)^2(x-5)^3$

$-4\left(x+\frac{1}{2}\right)^2(x-5)^3 = 0$

$x = -\frac{1}{2}$ or $x = 5$

The zeros are $-\frac{1}{2}$, with multiplicity 2, and 5, with multiplicity 3.

Because the multiplicity of $-\frac{1}{2}$ is even, the graph touches the x-axis and turns around at this zero. Because the multiplicity of 5 is odd, the graph crosses the x-axis at this zero.

8. $f(x) = 3x^3 - 10x + 9$

$f(-3) = 3(-3)^3 - 10(-3) + 9 = -42$

$f(-2) = 3(-2)^3 - 10(-2) + 9 = 5$

The sign change shows there is a zero between –3 and –2.

9. $f(x) = x^3 - 3x^2$

Since $a_n > 0$ and n is odd, the graph falls to the left and rises to the right.

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0$ or $x = 3$

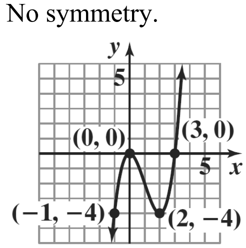
The x-intercepts are 0 and 3.

$f(0) = 0^3 - 3(0)^2 = 0$

The y-intercept is 0.

$f(-x) = (-x)^3 - 3(-x)^2 = -x^3 - 3x^2$

No symmetry.



$f(x) = x^3 - 3x^2$

10. $f(x) = 2(x+2)^2(x-3)$

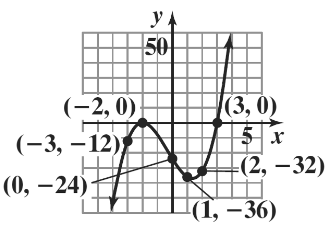
The leading term is $2 \cdot x^2 \cdot x$, or $2x^3$.
Since $a_n > 0$ and n is odd, the graph falls to the left and rises to the right.

$2(x+2)^2(x-3) = 0$
$x = -2$ or $x = 3$
The x-intercepts are –2 and 3.

$f(0) = 2(0+2)^2(0-3) = -12$
The y-intercept is –12.

$$f(-x) = 2((-x)+2)^2((-x)-3)$$
$$= 2(-x+2)^2(-x-3)$$

No symmetry.



$f(x) = 2(x+2)^2(x-3)$

Concept and Vocabulary Check 2.3

1. 5; –2
2. false
3. end; leading
4. falls; rises
5. rises; falls
6. rises; rises
7. falls; falls
8. true
9. true
10. x-intercept
11. turns around; crosses
12. 0; Intermediate Value
13. $n-1$

Exercise Set 2.3

1. polynomial function;
degree: 3
2. polynomial function;
degree: 4
3. polynomial function;
degree: 5
4. polynomial function;
degree: 7
5. not a polynomial function
6. not a polynomial function
7. not a polynomial function
8. not a polynomial function
9. not a polynomial function
10. polynomial function;
degree: 2
11. polynomial function
12. Not a polynomial function because graph is not smooth.
13. Not a polynomial function because graph is not continuous.
14. polynomial function
15. (b)
16. (c)
17. (a)
18. (d)
19. $f(x) = 5x^3 + 7x^2 - x + 9$
Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.
20. $f(x) = 11x^3 - 6x^2 + x + 3$
Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.
21. $f(x) = 5x^4 + 7x^2 - x + 9$
Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

22. $f(x) = 11x^4 - 6x^2 + x + 3$
Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

23. $f(x) = -5x^4 + 7x^2 - x + 9$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

24. $f(x) = -11x^4 - 6x^2 + x + 3$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

25. $f(x) = 2(x-5)(x+4)^2$
$x = 5$ has multiplicity 1;
The graph crosses the x-axis.
$x = -4$ has multiplicity 2;
The graph touches the x-axis and turns around.

26. $f(x) = 3(x+5)(x+2)^2$
$x = -5$ has multiplicity 1;
The graph crosses the x-axis.
$x = -2$ has multiplicity 2;
The graph touches the x-axis and turns around.

27. $f(x) = 4(x-3)(x+6)^3$
x = 3 has multiplicity 1;
The graph crosses the x-axis.
$x = -6$ has multiplicity 3;
The graph crosses the x-axis.

28. $f(x) = -3\left(x + \frac{1}{2}\right)(x-4)^3$

$x = -\frac{1}{2}$ has multiplicity 1;
The graph crosses the x-axis.
$x = 4$ has multiplicity 3;
The graph crosses the x-axis.

29. $f(x) = x^3 - 2x^2 + x$
$= x\left(x^2 - 2x + 1\right)$
$= x(x-1)^2$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = 1$ has multiplicity 2;
The graph touches the x-axis and turns around.

30. $f(x) = x^3 + 4x^2 + 4x$
$= x\left(x^2 + 4x + 4\right)$
$= x(x+2)^2$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = -2$ has multiplicity 2;
The graph touches the x-axis and turns around.

31. $f(x) = x^3 + 7x^2 - 4x - 28$
$= x^2(x+7) - 4(x+7)$
$= \left(x^2 - 4\right)(x+7)$
$= (x-2)(x+2)(x+7)$
$x = 2$, $x = -2$ and $x = -7$ have multiplicity 1;
The graph crosses the x-axis.

32. $f(x) = x^3 + 5x^2 - 9x - 45$
$= x^2(x+5) - 9(x+5)$
$= \left(x^2 - 9\right)(x+5)$
$= (x-3)(x+3)(x+5)$
$x = 3$, $x = -3$ and $x = -5$ have multiplicity 1;
The graph crosses the x-axis.

33. $f(x) = x^3 - x - 1$
$f(1) = -1$
$f(2) = 5$
The sign change shows there is a zero between the given values.

34. $f(x) = x^3 - 4x^2 + 2$
$f(0) = 2$
$f(1) = -1$
The sign change shows there is a zero between the given values.

35. $f(x) = 2x^4 - 4x^2 + 1$
$f(-1) = -1$
$f(0) = 1$
The sign change shows there is a zero between the given values.

36. $f(x) = x^4 + 6x^3 - 18x^2$
$f(2) = -8$
$f(3) = 81$
The sign change shows there is a zero between the given values.

37. $f(x) = x^3 + x^2 - 2x + 1$

$f(-3) = -11$
$f(-2) = 1$
The sign change shows there is a zero between the given values.

38. $f(x) = x^5 - x^3 - 1$

$f(1) = -1$
$f(2) = 23$
The sign change shows there is a zero between the given values.

39. $f(x) = 3x^3 - 10x + 9$

$f(-3) = -42$
$f(-2) = 5$
The sign change shows there is a zero between the given values.

40. $f(x) = 3x^3 - 8x^2 + x + 2$

$f(2) = -4$
$f(3) = 14$
The sign change shows there is a zero between the given values.

41. $f(x) = x^3 + 2x^2 - x - 2$

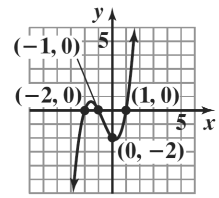
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b.
$$x^3 + 2x^2 - x - 2 = 0$$
$$x^2(x+2) - (x+2) = 0$$
$$(x+2)(x^2 - 1) = 0$$
$$(x+2)(x-1)(x+1) = 0$$
$$x = -2, x = 1, x = -1$$
The zeros at –2, –1, and 1 have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c.
$$f(0) = (0)^3 + 2(0)^2 - 0 - 2$$
$$= -2$$
The y-intercept is –2.

d.
$$f(-x) = (-x) + 2(-x)^2 - (-x) - 2$$
$$= -x^3 + 2x^2 + x - 2$$
$$-f(x) = -x^3 - 2x^2 + x + 2$$
The graph has neither origin symmetry nor y-axis symmetry.

e. The graph has 2 turning points and $2 \le 3 - 1$.



$f(x) = x^3 + 2x^2 - x - 2$

42. $f(x) = x^3 + x^2 - 4x - 4$

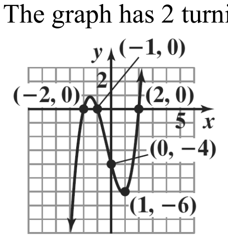
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b.
$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x+1)(x^2 - 4) = 0$$
$$(x+1)(x-2)(x+2) = 0$$
$x = -1$, or $x = 2$, or $x = -2$
The zeros at –2, –1 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The x-intercepts are –2, –1, and 2.

c. $f(0) = 0^3 + (0)^2 - 4(0) - 4 = -4$
The y-intercept is –4.

d.
$$f(-x) = -x^3 + x^2 + 4x - 4$$
$$-f(x) = -x^3 - x^2 + 4x + 4$$
neither symmetry

e. The graph has 2 turning points and $2 \le 3 - 1$.



43. $f(x) = x^4 - 9x^2$

a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

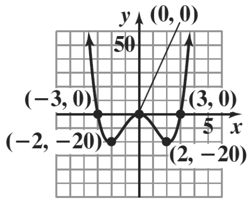
b.
$$x^4 - 9x^2 = 0$$
$$x^2(x^2 - 9) = 0$$
$$x^2(x-3)(x+3) = 0$$
$$x = 0, x = 3, x = -3$$

The zeros at –3 and 3 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - 9x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 9x^2$

44. $f(x) = x^4 - x^2$

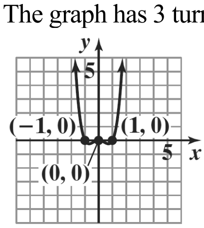
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - x^2 = 0$
$x^2(x^2 - 1) = 0$
$x^2(x-1)(x+1) = 0$
$x = 0, x = 1, x = -1$
f touches but does not cross the x-axis at 0.

c. $f(0) = (0)^4 - (0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - x^2$

45. $f(x) = -x^4 + 16x^2$

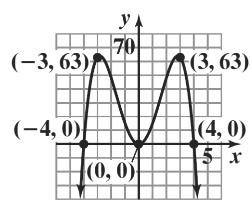
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-x^4 + 16x^2 = 0$
$x^2(-x^2 + 16) = 0$
$x^2(4-x)(4+x) = 0$
$x = 0, x = 4, x = -4$
The zeros at –4 and 4 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = -x^4 + 16x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^4 + 16x^2$

46. $f(x) = -x^4 + 4x^2$

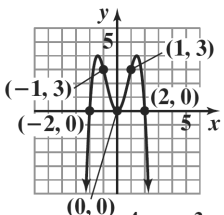
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-x^4 + 4x^2 = 0$
$x^2(4 - x^2) = 0$
$x^2(2-x)(2+x) = 0$
$x = 0, x = 2, x = -2$
The x-intercepts are –2,0, and 2. Since f has a double root at 0, it touches but does not cross the x-axis at 0.

c. $f(0) = -(0)^4 + 4(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = -x^4 + 4x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4-1$.



47. $f(x) = x^4 - 2x^3 + x^2$

a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - 2x^3 + x^2 = 0$

$x^2(x^2 - 2x + 1) = 0$

$x^2(x-1)(x-1) = 0$

$x = 0, x = 1$

The zeros at 1 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 0 and 1.

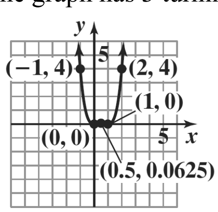
c. $f(0) = (0)^4 - 2(0)^3 + (0)^2 = 0$

The y-intercept is 0.

d. $f(-x) = x^4 + 2x^3 + x^2$

The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4-1$.



48. $f(x) = x^4 - 6x^3 + 9x^2$

a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - 6x^3 + 9x^2 = 0$

$x^2(x^2 - 6x + 9) = 0$

$x^2(x-3)^2 = 0$

$x = 0, x = 3$

The zeros at 3 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 3 and 0.

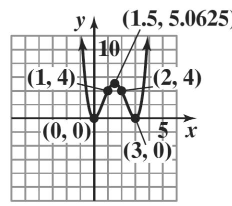
c. $f(0) = (0)^4 - 6(0)^3 + 9(0)^2 = 0$

The y-intercept is 0.

d. $f(-x) = x^4 + 6x^3 + 9x^2$

The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4-1$.



49. $f(x) = -2x^4 + 4x^3$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4 + 4x^3 = 0$

$x^3(-2x + 4) = 0$

$x = 0, x = 2$

The zeros at 0 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

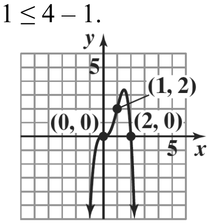
c. $f(0) = -2(0)^4 + 4(0)^3 = 0$

The y-intercept is 0.

d. $f(-x) = -2x^4 - 4x^3$

The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4-1$.



50. $f(x) = -2x^4 + 2x^3$

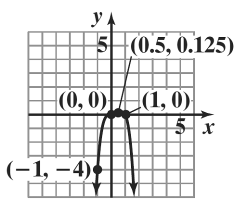
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4 + 2x^3 = 0$
$x^3(-2x + 2) = 0$
$x = 0, x = 1$
The zeros at 0 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. The y-intercept is 0.

d. $f(-x) = -2x^4 - 2x^3$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 4 - 1$.



$f(x) = -2x^4 + 2x^3$

51. $f(x) = 6x^3 - 9x - x^5$

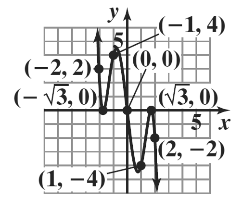
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5 + 6x^3 - 9x = 0$
$-x(x^4 - 6x^2 + 9) = 0$
$-x(x^2 - 3)(x^2 - 3) = 0$
$x = 0, x = \pm\sqrt{3}$
The root at 0 has odd multiplicity so $f(x)$ crosses the x-axis at (0, 0). The zeros at $-\sqrt{3}$ and $\sqrt{3}$ have even multiplicity so $f(x)$ touches the x-axis at $\sqrt{3}$ and $-\sqrt{3}$.

c. $f(0) = -(0)^5 + 6(0)^3 - 9(0) = 0$
The y-intercept is 0.

d. $f(-x) = x^5 - 6x^3 + 9x$
$f(-x) = -f(x)$
The graph has origin symmetry.

e. The graph has 4 turning points and $4 \le 5 - 1$.



$f(x) = 6x^3 - 9x - x^5$

52. $f(x) = 6x - x^3 - x^5$

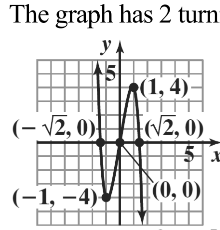
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5 - x^3 + 6x = 0$
$-x(x^4 + x^2 - 6) = 0$
$-x(x^2 + 3)(x^2 - 2) = 0$
$x = 0, x = \pm\sqrt{2}$
The zeros at $-\sqrt{2}$, 0, and $\sqrt{2}$ have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -(0)^5 - (0)^3 + 6(0) = 0$
The y-intercept is 0.

d. $f(-x) = x^5 + x^3 - 6x$
$f(-x) = -f(x)$
The graph has origin symmetry.

e. The graph has 2 turning points and $2 \le 5 - 1$.



$f(x) = 6x - x^3 - x^5$

53. $f(x) = 3x^2 - x^3$

a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^3 + 3x^2 = 0$
$-x^2(x - 3) = 0$
$x = 0, x = 3$
The zero at 3 has odd multiplicity so $f(x)$ crosses the x-axis at that point. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

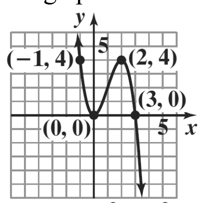
c. $f(0) = -(0)^3 + 3(0)^2 = 0$

The y-intercept is 0.

d. $f(-x) = x^3 + 3x^2$

The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 3 - 1$.



$f(x) = 3x^2 - x^3$

54. $f(x) = \frac{1}{2} - \frac{1}{2}x^4$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-\frac{1}{2}x^4 + \frac{1}{2} = 0$

$-\frac{1}{2}\left(x^4 - 1\right) = 0$

$-\frac{1}{2}\left(x^2 + 1\right)\left(x^2 - 1\right) = 0$

$-\frac{1}{2}\left(x^2 + 1\right)(x - 1)(x + 1) = 0$

$x = \pm 1$

The zeros at –1 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -\frac{1}{2}(0)^4 + \frac{1}{2} = \frac{1}{2}$

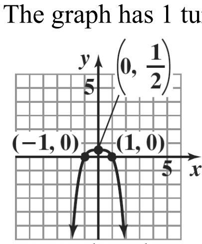
The y-intercept is $\frac{1}{2}$.

d. $f(-x) = \frac{1}{2} - \frac{1}{2}x^4$

$f(-x) = f(x)$

The graph has y-axis symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = \frac{1}{2} - \frac{1}{2}x^4$

55. $f(x) = -3(x-1)^2\left(x^2 - 4\right)$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-3(x-1)^2\left(x^2 - 4\right) = 0$

$x = 1, x = –2, x = 2$

The zeros at –2 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 1 has even multiplicity, so $f(x)$ touches the x-axis at (1, 0).

c. $f(0) = -3(0-1)^2(0^2 - 4)^3$

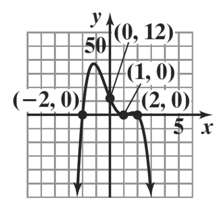
$= -3(1)(-4) = 12$

The y-intercept is 12.

d. $f(-x) = -3(-x-1)^2\left(x^2 - 4\right)$

The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -3(x-1)^2(x^2 - 4)$

56. $f(x) = -2(x-4)^2\left(x^2 - 25\right)$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2(x-4)^2\left(x^2 - 25\right) = 0$

$x = 4, x = –5, x = 5$

The zeros at –5 and 5 have odd multiplicity so $f(x)$ crosses the x-axis at these points. The root at 4 has even multiplicity so $f(x)$ touches the x-axis at (4, 0).

c. $f(0) = -2(0-4)^2(0^2 - 25)$

$= -2(16)(-25)$

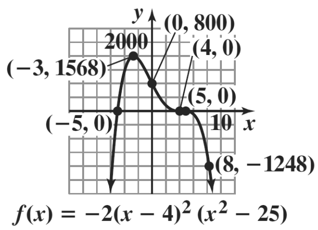
$= 800$

The y-intercept is 800.

d. $f(-x) = -2(-x-4)^2\left(x^2 - 2\right)$

The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4-1$.



57. $f(x) = x^2(x-1)^3(x+2)$

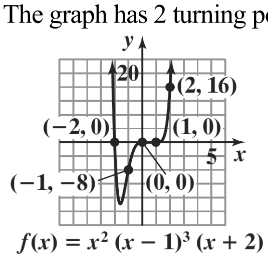
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = 1, x = -2$
The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = 0^2(0-1)^3(0+2) = 0$
The y-intercept is 0.

d. $f(-x) = x^2(-x-1)^3(-x+2)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6-1$.



58. $f(x) = x^3(x+2)^2(x+1)$

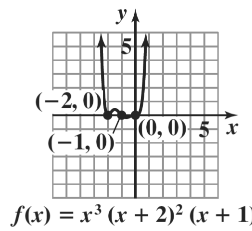
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = -2, x = -1$
The roots at 0 and –1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –2 has even multiplicity so $f(x)$ touches the axis at (–2, 0).

c. $f(0) = 0^3(0+2)^2(0+1) = 0$
The y-intercept is 0.

d. $f(-x) = -x^3(-x+2)^2(-x+1)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 6-1$.



59. $f(x) = -x^2(x-1)(x+3)$

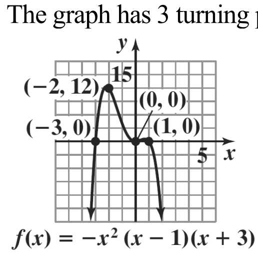
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -3$
The zeros at 1 and –3 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = -0^2(0-1)(0+3) = 0$
The y-intercept is 0.

d. $f(-x) = -x^2(-x-1)(-x+3)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4-1$.



60. $f(x) = -x^2(x+2)(x-2)$

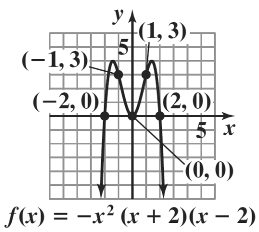
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 2, x = -2$
The zeros at 2 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = -0^2(0+2)(0-2) = 0$
The y-intercept is 0.

d. $f(-x) = -x^2(-x+2)(-x-2)$
$f(-x) = -x^2(-1)(x-2)(-1)(x+2)$
$f(-x) = -x^2(x+2)(x-2)$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



61. $f(x) = -2x^3(x-1)^2(x+5)$

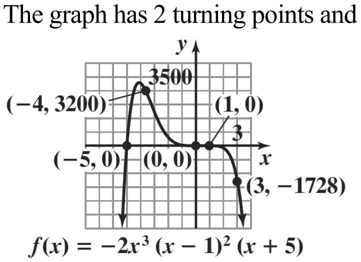
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -5$
The roots at 0 and –5 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 1 has even multiplicity so $f(x)$ touches the axis at (1, 0).

c. $f(0) = -2(0)^3(0-1)^2(0+5) = 0$
The y-intercept is 0.

d. $f(-x) = 2x^3(-x-1)^2(-x+5)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



62. $f(x) = -3x^3(x-1)^2(x+3)$

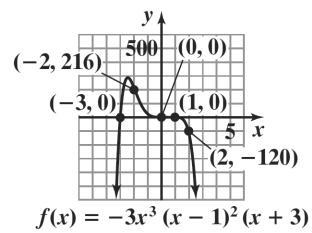
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0, x = 1, x = -3$
The roots at 0 and –3 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 1 has even multiplicity so $f(x)$ touches the axis at (1, 0).

c. $f(0) = -3(0)^3(0-1)^2(0+3) = 0$
The y-intercept is 0.

d. $f(-x) = 3x^3(-x-1)^2(-x+3)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



63. $f(x) = (x-2)^2(x+4)(x-1)$

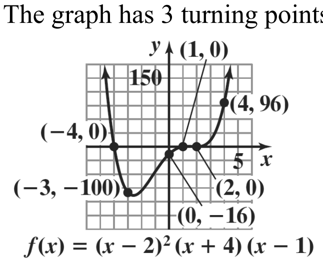
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and rises the right.

b. $x = 2, x = -4, x = 1$
The zeros at –4 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 2 has even multiplicity so $f(x)$ touches the axis at (2, 0).

c. $f(0) = (0-2)^2(0+4)(0-1) = -16$
The y-intercept is –16.

d. $f(-x) = (-x-2)^2(-x+4)(-x-1)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



64. $f(x) = (x+3)(x+1)^3(x+4)$

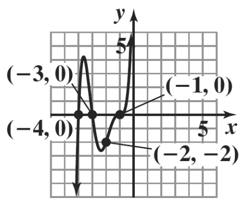
a. Since $a_n > 0$ and n is odd, $f(x)$ falls to the left and rises to the right.

b. $x = -3, x = -1, x = -4$
The zeros at all have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c. $f(0) = (0+3)(0+1)^3(0+4) = 12$
The y-intercept is 12.

d. $f(-x) = (-x+3)(-x+1)^3(-x+4)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points



$f(x) = (x+3)(x+1)^3(x+4)$

65. a. The x-intercepts of the graph are -2, 1, and 4, so they are the zeros. Since the graph actually crosses the x-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2, 1, and 4 are the zeros, $x+2$, $x-1$, and $x-4$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+2)(x-1)(x-4)$.

c. $f(0) = (0+2)(0-1)(0-4) = 8$

66. a. The x-intercepts of the graph are -3, 2, and 5, so they are the zeros. Since the graph actually crosses the x-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -3, 2, and 5 are the zeros, $x+3$, $x-2$, and $x-5$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+3)(x-2)(x-5)$.

c. $f(0) = (0+3)(0-2)(0-5) = 30$

67. a. The x-intercepts of the graph are -1 and 3, so they are the zeros. Since the graph crosses the x-axis at -1, it has odd multiplicity. Since the graph touches the x-axis and turns around at 3, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 3 are the zeros, $x+1$ and $x-3$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+1)(x-3)^2$.

c. $f(0) = (0+1)(0-3)^2 = 9$

68. a. The x-intercepts of the graph are -2 and 1, so they are the zeros. Since the graph crosses the x-axis at -2, it has odd multiplicity. Since the graph touches the x-axis and turns around at 1, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2 and 1 are the zeros, $x+2$ and $x-1$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+2)(x-1)^2$.

c. $f(0) = (0+2)(0-1)^2 = 2$

69. a. The x-intercepts of the graph are -3 and 2, so they are the zeros. Since the graph touches the x-axis and turns around at both -3 and 2, both have even multiplicity.

b. Since the graph has three turning points, the function must be at least of degree 4. Since -3 and 2 are the zeros, $x+3$ and $x-2$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x) = -(x+3)^2(x-2)^2$.

c. $f(0) = -(0+3)^2(0-2)^2 = -36$

70. **a.** The x-intercepts of the graph are -1 and 4, so they are the zeros. Since the graph touches the x-axis and turns around at both -1 and 4, both have even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 4 are the zeros, $x+1$ and $x-4$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x)=-(x+1)^2(x-4)^2$.

c. $f(0)=-(0+1)^2(0-4)^2=-16$

71. **a.** The x-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the x-axis at -1 and 1, they both have odd multiplicity. Since the graph touches the x-axis and turns around at -2, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is
$f(x)=(x+2)^2(x+1)(x-1)^3$.

c. $f(0)=(0+2)^2(0+1)(0-1)^3=-4$

72. **a.** The x-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the x-axis at -2 and 1, they both have odd multiplicity. Since the graph touches the x-axis and turns around at -1, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is
$f(x)=(x+2)(x+1)^2(x-1)^3$.

c. $f(0)=(0+2)(0+1)^2(0-1)^3=-2$

73. **a.**
$$f(x)=0.76x^3-30x^2-882x+37{,}807$$
$$f(40)=0.76(40)^3-30(40)^2-882(40)+37807$$
$$=3167$$
The world tiger population in 2010 (40 years after 1970) was about 3167.
This is represented by the point $(40, 3167)$.

b. This underestimates the actual data shown in the bar graph by 33.

c. The leading coefficient is positive, thus the graph rises to the right.
No, if conservation efforts fail, the model will not be useful. The model indicates an increasing world tiger population that will actually decrease without conservation efforts.

74. **a.**
$$f(x)=0.76x^3-30x^2-882x+37{,}807$$
$$f(10)=0.76(10)^3-30(10)^2-882(10)+37{,}807$$
$$=26{,}747$$
The world tiger population in 1980 (10 years after 1970) was about 26,747.
This is represented by the point (10, 26,747).

b. This underestimates the actual data shown in the bar graph by 1253.

c. The leading coefficient is positive, thus the graph rises to the right.
Yes, if conservation efforts succeed, the model will be useful. The model indicates an increasing world tiger population that might actually increase with conservation efforts.

b. Since the degree of g is odd and the leading coefficient is negative, the graph rises to the right. Based on the end behavior, the function will be a useful model over an extended period of time.

75. **a.** The woman's heart rate was increasing from 1 through 4 minutes and from 8 through 10 minutes.

b. The woman's heart rate was decreasing from 4 through 8 minutes and from 10 through 12 minutes.

c. There were 3 turning points during the 12 minutes.

d. Since there were 3 turning points, a polynomial of degree 4 would provide the best fit.

e. The leading coefficient should be negative. The graph falls to the left and to the right.

f. The woman's heart rate reached a maximum of about 116 ± 1 beats per minute. This occurred after 10 minutes.

g. The woman's heart rate reached a minimum of about 64 ± 1 beats per minute. This occurred after 8 minutes.

76. **a.** The average price per gallon in January was increasing from 2005 to 2006, 2007 to 2008, and 2009 to 2011.

b. The average price per gallon in January was decreasing from 2006 to 2007, and 2008 to 2009.

c. 4 turning points are shown in the graph.

d. Since there are 4 turning points, the degree of the polynomial function of best fit would be 5

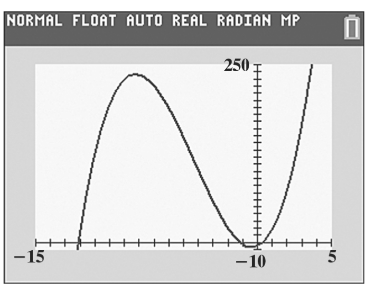
e. The leading coefficient would be positive because the graph falls to the left and rises to the right.

f. The maximum average January price per gallon was about \$3.15. This occurred in 2011.

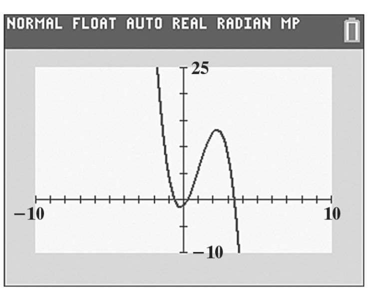
g. The minimum average January price per gallon was about \$1.85. This occurred in 2009.

77. – 93. Answers will vary.

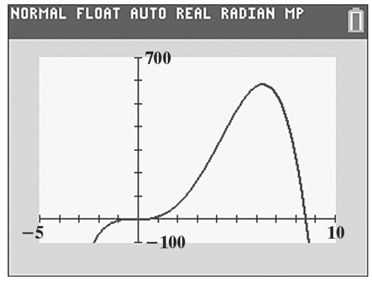
94.



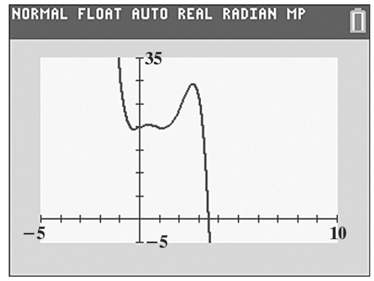
95.



96.



97.



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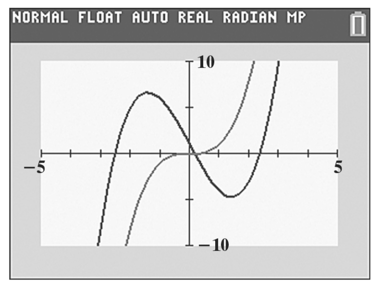

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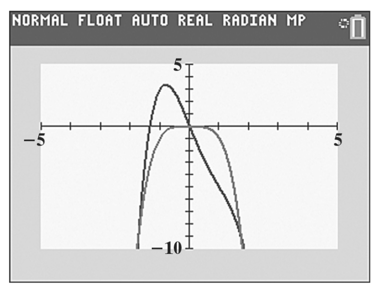


100. makes sense

101. does not make sense; Explanations will vary. Sample explanation: Since $(x+2)$ is raised to an odd power, the graph crosses the x-axis at –2.

102. does not make sense; Explanations will vary. Sample explanation: A forth degree function has at most 3 turning points.

103. makes sense

104. false; Changes to make the statement true will vary. A sample change is: $f(x)$ falls to the left and rises to the right.

105. false; Changes to make the statement true will vary. A sample change is: Such a function falls to the right and will eventually have negative values.

106. true

107. false; Changes to make the statement true will vary. A sample change is: A function with origin symmetry either falls to the left and rises to the right, or rises to the left and falls to the right.

108. $f(x) = x^3 + x^2 - 12x$

109. $f(x) = x^3 - 2x^2$

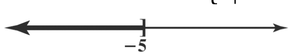
110. Let x = the number of years after 1995.

$$315 - 13x = 29$$
$$-13x = -286$$
$$x = \frac{-286}{-13}$$
$$x = 22$$

Juries will render 29 death sentences 22 years after 1995, or 2017.

111.
$$\frac{2x-3}{4} \ge \frac{3x}{4} + \frac{1}{2}$$
$$4\left(\frac{2x-3}{4}\right) \ge 4\left(\frac{3x}{4} + \frac{1}{2}\right)$$
$$2x - 3 \ge 3x + 2$$
$$-5 \ge x$$
$$x \le -5$$

The solution set is $\{x | x \le -5\}$ or $(-\infty, -5]$.



112. $m = \frac{3-(-5)}{-10-(-2)} = \frac{8}{-8} = -1$,

so the slope is –1.

Using the point (–10, 3), we get the following point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -1[x - (-10)]$$
$$y - 3 = -(x + 10)$$

Solve the equation for y to find the slope-intercept form:

$$y - 3 = -(x + 10)$$
$$y - 3 = -x - 10$$
$$y = -x - 7.$$

113. $\frac{737}{21} = 35 + \frac{2}{21}$ or $35\frac{2}{21}$.

114. $6x^3 - x^2 - 5x + 4$

115. $2x^3 - x^2 - 11x + 6 = (x-3)(2x^2 + 3x - 2)$
$= (x-3)(2x-1)(x+2)$

Section 2.4

Check Point Exercises

1.
$$\begin{array}{r} x+5 \\ x+9\overline{)x^2+14x+45} \\ \underline{x^2+9x} \\ 5x+45 \\ \underline{5x+45} \\ 0 \end{array}$$

The answer is $x + 5$.

2.
$$\begin{array}{r} 2x^2+3x-2 \\ x-3\overline{)2x^3-3x^2-11x+7} \\ \underline{2x^3-6x^2} \\ 3x^2-11x \\ \underline{3x^2-9x} \\ -2x+7 \\ \underline{-2x+6} \\ 1 \end{array}$$

The answer is $2x^2 + 3x - 2 + \frac{1}{x-3}$.

3.
$$\begin{array}{r} 2x^2+7x+14 \\ x^2-2x\overline{)2x^4+3x^3+0x^2-7x-10} \\ \underline{2x^4-4x^3} \\ 7x^3+0x^2 \\ \underline{7x^3-14x^2} \\ 14x^2-7x \\ \underline{14x^2-28x} \\ 21x-10 \end{array}$$

The answer is $2x^2 + 7x + 14 + \frac{21x-10}{x^2-2x}$.

4.

–2	1	0	–7	–6
		–2	4	6
	1	–2	–3	0

The answer is $x^2 - 2x - 3$.

5.

–4	3	4	–5	3
		–12	32	–108
	3	–8	27	–105

$f(-4) = -105$

6.

–1	15	14	–3	–2
		–15	1	2
	15	–1	–2	0

$$15x^2 - x - 2 = 0$$
$$(3x+1)(5x-2) = 0$$
$$x = -\frac{1}{3} \quad \text{or} \quad x = \frac{2}{5}$$

The solution set is $\left\{-1, -\frac{1}{3}, \frac{2}{5}\right\}$.

Concept and Vocabulary Check 2.4

1. $2x^3 + 0x^2 + 6x - 4$
2. $6x^3$; $3x$; $2x^2$; $7x^2$
3. $2x^2$; $5x-2$; $10x^3 - 4x^2$; $10x^3 + 6x^2$
4. $6x^2 - 10x$; $6x^2 + 8x$; $18x$; -4; $18x - 4$
5. 9; $3x-5$; 9; $3x - 5 + \frac{9}{2x+1}$
6. divisor; quotient; remainder; dividend
7. 4; 1; 5; -7; 1
8. -5; 4; 0; -8; -2
9. true
10. $f(c)$
11. $x - c$

Exercise Set 2.4

1.

$$\begin{array}{r} x+3 \\ x+5\overline{)x^2+8x+15} \\ \underline{x^2+5x} \\ 3x+15 \\ \underline{3x+15} \\ 0 \end{array}$$

The answer is $x+3$.

2.

$$\begin{array}{r} x+5 \\ x-2\overline{)x^2+3x-10} \\ \underline{x^2-2x} \\ 5x-10 \\ \underline{5x-10} \\ 0 \end{array}$$

The answer is $x+5$.

3.

$$\begin{array}{r} x^2+3x+1 \\ x+2\overline{)x^3+5x^2+7x+2} \\ \underline{x^3+2x^2} \\ 3x^2+7x \\ \underline{3x^2+6x} \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$

The answer is x^2+3x+1.

4.

$$\begin{array}{r} x^2+x-2 \\ x-3\overline{)x^3-2x^2-5x+6} \\ \underline{x^3-3x^2} \\ x^2-5x \\ \underline{x^2-3x} \\ -2x+6 \\ \underline{-2x+6} \\ 0 \end{array}$$

The answer is x^2+x-2.

5.

$$\begin{array}{r} 2x^2+3x+5 \\ 3x-1\overline{)6x^3+7x^2+12x-5} \\ \underline{6x^3-2x^2} \\ 9x^2+12x \\ \underline{9x^2-3x} \\ 15x-5 \\ \underline{15x-5} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

6.

$$\begin{array}{r} 2x^2+3x+5 \\ 3x+4\overline{)6x^3+17x^2+27x+20} \\ \underline{6x^3+8x^2} \\ 9x^2+27x \\ \underline{9x^2+12x} \\ 15x+20 \\ \underline{15x+20} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

7.

$$
\begin{array}{r}
4x+3+\dfrac{2}{3x-2}\\
3x-2\overline{)12x^2+x-4}\\
\underline{12x^2-8x}\\
9x-4\\
\underline{9x-6}\\
2
\end{array}
$$

The answer is $4x+3+\dfrac{2}{3x-2}$.

8.

$$
\begin{array}{r}
2x-3+\dfrac{3}{2x-1}\\
2x-1\overline{)4x^2-8x+6}\\
\underline{4x^2-2x}\\
-6x+6\\
\underline{-6x+6}\\
3
\end{array}
$$

The answer is $2x-3+\dfrac{3}{2x-1}$.

9.

$$
\begin{array}{r}
2x^2+x+6-\dfrac{38}{x+3}\\
x+3\overline{)2x^3+7x^2+9x-20}\\
\underline{2x^3+6x^2}\\
x^2+9x\\
\underline{x^2+3x}\\
6x-20\\
\underline{6x+18}\\
-38
\end{array}
$$

The answer is $2x^2+x+6-\dfrac{38}{x+3}$.

10.

$$
\begin{array}{r}
3x+7+\dfrac{26}{x-3}\\
x-3\overline{)3x^2-2x+5}\\
\underline{3x^2-9x}\\
7x+5\\
\underline{7x-21}\\
26
\end{array}
$$

The answer is $3x+7+\dfrac{26}{x-3}$.

11.

$$
\begin{array}{r}
4x^3+16x^2+60x+246+\dfrac{984}{x-4}\\
x-4\overline{)4x^4-4x^2+6x}\\
\underline{4x^4-16x^3}\\
16x^3-4x^2\\
\underline{16x^3-64x^2}\\
60x^2+6x\\
\underline{60x^2-240x}\\
246x\\
\underline{246x-984}\\
984
\end{array}
$$

The answer is

$$4x^3+16x^2+60x+246+\frac{984}{x-4}.$$

12.

$$
\begin{array}{r}
x^3+3x^2+9x+27\\
x-3\overline{)x^4\qquad\qquad\qquad-81}\\
\underline{x^4-3x^3}\\
3x^3\\
\underline{3x^3-9x^2}\\
9x^2\\
\underline{9x^2-27x}\\
27x-81\\
\underline{27x-81}\\
0
\end{array}
$$

The answer is $x^3+3x^2+9x+27$.

13.

$$
\begin{array}{r}
2x+5\\
3x^2-x-3\overline{)6x^3+13x^2-11x-15}\\
\underline{6x^3-2x^2-6x}\\
15x^2-5x-15\\
\underline{15x^2-5x-15}\\
0
\end{array}
$$

The answer is $2x+5$.

14.

$$
\begin{array}{r}
x^2+x-3\\
x^2+x-2\overline{)x^4+2x^3-4x^2-5x-6}\\
\underline{x^4+x^3-2x^2}\\
x^3-2x^2-5x\\
\underline{x^3+x^2-2x}\\
-3x^2-3x-6\\
\underline{-3x^2-3x+6}\\
-12
\end{array}
$$

The answer is $x^2+x-3-\dfrac{12}{x^2+x-2}$.

15.

$$
\begin{array}{r|l}
 & 6x^2+3x-1 \\
\hline
3x^2+1 & 18x^4+9x^3+3x^2 \\
 & \underline{18x^4+6x^2} \\
 & 9x^3-3x^2 \\
 & \underline{9x^3+3x} \\
 & -3x^2-3x \\
 & \underline{-3x^2-1} \\
 & -3x+1
\end{array}
$$

The answer is $6x^2+3x-1-\dfrac{3x-1}{3x^2+1}$.

16.

$$
\begin{array}{r|l}
 & x^2-4x+1 \\
\hline
2x^3+1 & 2x^5-8x^4+2x^3+x^2 \\
 & \underline{2x^5+x^2} \\
 & -8x^4+2x^3 \\
 & \underline{-8x^4-4x} \\
 & 2x^3+4x \\
 & \underline{2x^3+1} \\
 & 4x-1
\end{array}
$$

The answer is $x^2-4x+1+\dfrac{4x-1}{2x^3+1}$.

17. $(2x^2+x-10)\div(x-2)$

2	2	1	–10
		4	10
	2	5	0

The answer is $2x+5$.

18. $(x^2+x-2)\div(x-1)$

1	1	1	–2
		1	2
	1	2	0

The answer is $x + 2$.

19. $(3x^2+7x-20)\div(x+5)$

–5	3	7	–20
		–15	40
	3	–8	20

The answer is $3x-8+\dfrac{20}{x+5}$.

20. $(5x^2-12x-8)\div(x+3)$

–3	5	12	–8
		–15	81
	5	–27	73

The answer is $5x-27+\dfrac{73}{x+3}$.

21. $(4x^3-3x^2+3x-1)\div(x-1)$

1	4	–3	3	–1
		4	1	4
	4	1	4	3

The answer is $4x^2+x+4+\dfrac{3}{x-1}$.

22. $(5x^3-6x^2+3x+11)\div(x-2)$

2	5	–6	3	11
		10	8	22
	5	4	11	33

The answer is $5x^2+4x+11+\dfrac{33}{x-2}$.

23. $(6x^5-2x^3+4x^2-3x+1)\div(x-2)$

2	6	0	–2	4	–3	1
		12	24	44	96	186
	6	12	22	48	93	187

The answer is

$6x^4+12x^3+22x^2+48x+93+\dfrac{187}{x-2}$.

24. $(x^5+4x^4-3x^2+2x+3)\div(x-3)$

3	1	4	0	–3	2	3
		3	21	63	180	546
	1	7	21	60	182	549

The answer is

$x^4+7x^3+21x^2+60x+182+\dfrac{549}{x-3}$.

25. $\left(x^2-5x-5x^3+x^4\right)\div(5+x)\Rightarrow$
$\left(x^4-5x^3+x^2-5x\right)\div(x+5)$

–5	1	–5	1	–5	0
		–5	50	–255	1300
	1	–10	51	–260	1300

The answer is
$x^3-10x^2+51x-260+\dfrac{1300}{x+5}$.

26. $\left(x^2-6x-6x^3+x^4\right)\div(6+x)\Rightarrow$
$\left(x^4-6x^3+x^2-6x\right)\div(x+6)$

–6	1	–6	1	–6	0
		–6	72	–438	2664
	1	–12	73	–444	2664

The answer is $x^3-12x^2+73x-444+\dfrac{2664}{x+6}$.

27. $\dfrac{x^5+x^3-2}{x-1}$

1	1	0	1	0	0	–2
		1	1	2	2	2
	1	1	2	2	2	0

The answer is $x^4+x^3+2x^2+2x+2$.

28. $\dfrac{x^7+x^5-10x^3+12}{x+2}$

–2	1	0	1	0	–10	0	0	12
		–2	4	–10	20	–20	40	–80
	1	–2	5	–10	10	–20	40	–68

The answer is $x^6-2x^5+5x^4-10x^3+10x^2$
$-20x+40-\dfrac{68}{x+2}$.

29. $\dfrac{x^4-256}{x-4}$

4	1	0	0	0	–256
		4	16	64	256
	1	4	16	64	0

The answer is $x^3+4x^2+16x+64$.

30. $\dfrac{x^7-128}{x-2}$

2	1	0	0	0	0	0	0	–128
		2	4	8	16	32	64	128
	1	2	4	8	16	32	64	0

The answer is
$x^6+2x^5+4x^4+8x^3+16x^2+32x+64$.

31. $\dfrac{2x^5-3x^4+x^3-x^2+2x-1}{x+2}$

–2	2	–3	1	–1	2	–1
		–4	14	–30	62	–128
	2	–7	15	–31	64	–129

The answer is
$2x^4-7x^3+15x^2-31x+64-\dfrac{129}{x+2}$.

32. $\dfrac{x^5-2x^4-x^3+3x^2-x+1}{x-2}$

2	1	–2	–1	3	–1	1
		2	0	–2	2	2
	1	0	–1	1	1	3

The answer is $x^4-x^2+x+1+\dfrac{3}{x-2}$.

33. $f(x)=2x^3-11x^2+7x-5$

4	2	–11	7	–5
		8	–12	–20
	2	–3	–5	–25

$f(4)=-25$

34.

3	1	–7	5	–6
		3	–12	–21
	1	–4	–7	–27

$f(3)=-27$

35. $f(x)=3x^3-7x^2-2x+5$

–3	3	–7	–2	5
		–9	48	–138
	3	–16	46	–133

$f(-3)=-133$

36.

−2	4	5	−6	−4
		−8	6	0
	4	−3	0	−4

$f(-2) = -4$

37. $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

3	1	5	5	−5	−6
		3	24	87	246
	1	8	29	82	240

$f(3) = 240$

38.

2	1	−5	5	5	−6
		2	−6	−2	6
	1	−3	−1	3	0

$f(2) = 0$

39. $f(x) = 2x^4 - 5x^3 - x^2 + 3x + 2$

$-\frac{1}{2}$	2	−5	−1	3	2
		−1	3	−1	−1
	2	−6	2	2	1

$f\left(-\frac{1}{2}\right) = 1$

40.

$-\frac{2}{3}$	6	10	5	1	1
		−4	−4	$-\frac{2}{3}$	$-\frac{2}{9}$
	6	6	1	$\frac{1}{3}$	$\frac{7}{9}$

$f\left(-\frac{2}{3}\right) = \frac{7}{9}$

41. Dividend: $x^3 - 4x^2 + x + 6$
Divisor: $x + 1$

−1	1	−4	1	6
		−1	5	−6
	1	−5	6	0

The quotient is $x^2 - 5x + 6$.
$(x+1)(x^2 - 5x + 6) = 0$
$(x + 1)(x - 2)(x - 3) = 0$
$x = -1, x = 2, x = 3$
The solution set is $\{-1, 2, 3\}$.

42. Dividend: $x^3 - 2x^2 - x + 2$
Divisor: $x + 1$

−1	1	−2	−1	2
		−1	3	−2
	1	−3	2	0

The quotient is $x^2 - 3x + 2$.
$(x+1)(x^2 - 3x + 2) = 0$
$(x + 1)(x - 2)(x - 1) = 0$
$x = -1, x = 2, x = 1$
The solution set is $\{-1, 2, 1\}$.

43. $2x^3 - 5x^2 + x + 2 = 0$

2	2	−5	1	2
		4	−2	−2
	2	−1	−1	0

$(x-2)(2x^2 - x - 1) = 0$
$(x - 2)(2x + 1)(x - 1) = 0$
$x = 2,\ x = -\frac{1}{2},\ x = 1$
The solution set is $\left\{-\frac{1}{2}, 1, 2\right\}$.

44. $2x^3 - 3x^2 - 11x + 6 = 0$

−2	2	−3	−11	6
		−4	14	−6
	2	−7	3	0

$(x + 2)(2x^2 - 7x + 3) = 0$
$(x + 2)(2x - 1)(x - 3) = 0$
$x = -2,\ x = \frac{1}{2},\ x = 3$
The solution set is $\left\{-2, \frac{1}{2}, 3\right\}$.

45. $12x^3 + 16x^2 - 5x - 3 = 0$

$-\frac{3}{2}$	12	16	–5	–3
		–18	3	3
	12	–2	–2	0

$\left(x+\frac{3}{2}\right)(12x^2 - 2x - 2) = 0$

$\left(x+\frac{3}{2}\right)2\left(6x^2 - x - 1\right) = 0$

$\left(x+\frac{3}{2}\right)2(3x+1)(2x-1) = 0$

$x = -\frac{3}{2},\ x = -\frac{1}{3},\ x = \frac{1}{2}$

The solution set is $\left\{-\frac{3}{2}, -\frac{1}{3}, \frac{1}{2}\right\}$.

46. $3x^3 + 7x^2 - 22x - 8 = 0$

$-\frac{1}{3}$	3	7	–22	–8
		–1	–2	8
	3	6	–24	0

$\left(x+\frac{1}{3}\right)3x^2 + 6x - 24 = 0$

$\left(x+\frac{1}{3}\right)3(x+4)(x-2) = 0$

$x = -4, x = 2, x = -\frac{1}{3}$

The solution set is $\left\{-4, -\frac{1}{3}, 2\right\}$.

47. The graph indicates that 2 is a solution to the equation.

2	1	2	–5	–6
		2	8	6
	1	4	3	0

The remainder is 0, so 2 is a solution.

$x^3 + 2x^2 - 5x - 6 = 0$

$(x-2)\left(x^2 + 4x + 3\right) = 0$

$(x-2)(x+3)(x+1) = 0$

The solutions are 2, -3, and -1, or $\{-3, -1, 2\}$.

48. The graph indicates that -3 is a solution to the equation.

–3	2	1	–13	6
		–6	15	–6
	2	–5	2	0

The remainder is 0, so -3 is a solution.

$2x^3 + x^2 - 13x + 6 = 0$

$(x+3)\left(2x^2 - 5x + 2\right) = 0$

$(x+3)(2x-1)(x-2) = 0$

The solutions are -3, $\frac{1}{2}$, and 2, or $\left\{-3, \frac{1}{2}, 2\right\}$.

49. The table indicates that 1 is a solution to the equation.

1	6	–11	6	–1
		6	–5	1
	6	–5	1	0

The remainder is 0, so 1 is a solution.

$6x^3 - 11x^2 + 6x - 1 = 0$

$(x-1)\left(6x^2 - 5x + 1\right) = 0$

$(x-1)(3x-1)(2x-1) = 0$

The solutions are 1, $\frac{1}{3}$, and $\frac{1}{2}$, or $\left\{\frac{1}{3}, \frac{1}{2}, 1\right\}$.

50. The table indicates that 1 is a solution to the equation.

1	2	11	–7	–6
		2	13	–6
	2	13	6	0

The remainder is 0, so 1 is a solution.

$2x^3 + 11x^2 - 7x - 6 = 0$

$(x-1)\left(2x^2 + 13x + 6\right) = 0$

$(x-1)(2x+1)(x+6) = 0$

The solutions are 1, $-\frac{1}{2}$, and -6, or

$\left\{-6, -\frac{1}{2}, 1\right\}$.

51. a. $14x^3-17x^2-16x-177=0$

3	14	−17	−16	−177
		42	75	177
	14	25	59	0

The remainder is 0 so 3 is a solution.

$14x^3-17x^2-16x-177$
$=(x-3)(14x^2+25x+59)$

b. $f(x)=14x^3-17x^2-16x+34$

We need to find x when $f(x)=211$.

$$f(x)=14x^3-17x^2-16x+34$$
$$211=14x^3-17x^2-16x+34$$
$$0=14x^3-17x^2-16x-177$$

This is the equation obtained in part **a.** One solution is 3. It can be used to find other solutions (if they exist).

$$14x^3-17x^2-16x-177=0$$
$$(x-3)(14x^2+25x+59)=0$$

The polynomial $14x^2+25x+59$ cannot be factored, so the only solution is $x=3$. The female moth's abdominal width is 3 millimeters.

52. a.

2	2	14	0	−72
		4	36	72
	2	18	36	0

$2h^3+14h^2-72=(h-2)(2h^2+18h+36)$

b.
$$V=lwh$$
$$72=(h+7)(2h)(h)$$
$$72=2h^3+14h^2$$
$$0=2h^3+14h^2-72$$
$$0=(h-2)(2h^2+18h+36)$$
$$0=(h-2)(2(h^2+9h+18))$$
$$0=(h-2)(2(h+6)(h+3))$$
$$0=2(h-2)(h+6)(h+3)$$

$2(h-2)=0$ $\quad h+6=0 \quad h+3=0$
$h-2=0 \quad$ ~~$h=-6$~~ $\quad$ ~~$h=-3$~~
$h=2$

The height is 2 inches, the width is $2\cdot 2=4$ inches and the length is $2+7=9$ inches. The dimensions are 2 inches by 4 inches by 9 inches.

53. $A=l\cdot w$ so

$$l=\frac{A}{w}=\frac{0.5x^3-0.3x^2+0.22x+0.06}{x+0.2}$$

−0.2	0.5	−0.3	0.22	0.06
		−0.1	0.08	−0.06
	0.5	−0.4	0.3	0

Therefore, the length of the rectangle is $0.5x^2-0.4x+0.3$ units.

54. $A=l\cdot w$ so,

$$l=\frac{A}{w}=\frac{8x^3-6x^2-5x+3}{x+\frac{3}{4}}$$

$-\frac{3}{4}$	8	−6	−5	3
		−6	9	−3
	8	−12	4	0

Therefore, the length of the rectangle is $8x^2-12x+4$ units.

55. a.
$$f(30)=\frac{80(30)-8000}{30-110}=70$$

(30, 70) At a 30% tax rate, the government tax revenue will be \$70 ten billion.

b.

110	80	-8000
		8800
	80	800

$$f(x)=80+\frac{800}{x-110}$$
$$f(30)=80+\frac{800}{80-110}=70$$

(30, 70) same answer as in **a.**

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

56. a.
$$f(40)=\frac{80(40)-8000}{40-110}=68.57$$

(40, 68.57) At a 40% tax rate, the government's revenue is \$68.57 ten billion.

b.

110	80	-8000
		8800
	80	800

$$f(x) = 80 + \frac{800}{x-110}$$
$$f(40) = 80 + \frac{800}{40-110} = 68.57$$

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

57. – 65. Answers will vary.

66. does not make sense; Explanations will vary. Sample explanation: The division must account for the zero coefficients on the x^4, x^3, x^2 and x terms.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: The remainder theorem provides an alternative method for evaluating a function at a given value.

69. does not make sense; Explanations will vary. Sample explanation: The zeros of f are the same as the solutions of $f(x) = 0$.

70. false; Changes to make the statement true will vary. A sample change is: The degree of the quotient is 3, since $\frac{x^6}{x^3} = x^3$.

71. true

72. true

73. false; Changes to make the statement true will vary. A sample change is: The divisor is a factor of the divided only if the remainder is the whole number 0.

74.
$$\begin{array}{r} 5x^2 + 2x - 4 \\ 4x+3\overline{\smash{\big)}\,20x^3 + 23x^2 - 10x + k} \\ \underline{20x^3 + 15x^2} \qquad\qquad\quad \\ 8x^2 - 10 \qquad\quad \\ \underline{8x^2 + 6x} \qquad\quad \\ -16x + k \\ \underline{-16x - 12} \end{array}$$

To get a remainder of zero, k must equal –12.
$k = -12$

75.
$$f(x) = d(x)\cdot q(x) + r(x)$$
$$2x^2 - 7x + 9 = d(x)(2x-3) + 3$$
$$2x^2 - 7x + 6 = d(x)(2x-3)$$
$$\frac{2x^2 - 7x + 6}{2x-3} = d(x)$$

$$\begin{array}{r} x - 2 \\ 2x-3\overline{\smash{\big)}\,2x^2 - 7x + 6} \\ \underline{2x^2 - 3x} \qquad \\ -4x + 6 \\ \underline{-4x + 6} \end{array}$$

The polynomial is $x - 2$.

76.
$$\begin{array}{r} x^{2n} - x^n + 1 \\ x^n + 1\overline{\smash{\big)}\,x^{3n} \qquad\qquad + 1} \\ \underline{x^{3n} + x^{2n}} \qquad\qquad \\ -x^{2n} \qquad\qquad \\ \underline{-x^{2n} - x^n} \qquad \\ x^n + 1 \\ \underline{x^n + 1} \\ 0 \end{array}$$

77. $2x - 4 = 2(x-2)$
Use synthetic division to divide by $x – 2$. Then divide the quotient by 2.

78. $x^4 - 4x^3 - 9x^2 + 16x + 20 = 0$

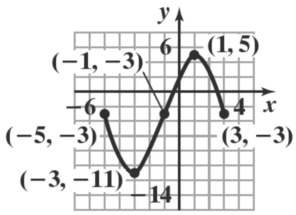
5	1	–4	–9	16	20
		5	5	–20	–20
	1	1	–4	–4	0

The remainder is zero and 5 is a solution to the equation.
$$x^4 - 4x^3 - 9x^2 + 16x + 20 = (x-5)\left(x^3 + x^2 - 4x - 4\right)$$
To solve the equation, we set it equal to zero and factor.
$$(x-5)\left(x^3 + x^2 - 4x - 4\right) = 0$$
$$(x-5)\left(x^2(x+1) - 4(x+1)\right) = 0$$
$$(x-5)(x+1)\left(x^2 - 4\right) = 0$$
$$(x-5)(x+1)(x+2)(x-2) = 0$$
Apply the zero product principle.
$x - 5 = 0$, $x = 5$ $\qquad$ $x + 1 = 0$, $x = -1$

$x + 2 = 0$, $x = -2$ $\qquad$ $x - 2 = 0$, $x = 2$

The solutions are –2, –1, 2 and 5 and the solution set is $\{-2, -1, 2, 5\}$.

79. The graph of $y = f(x)$ is shifted 1 unit left, stretched by a factor of 2, reflected about the x-axis, then shifted down 3 units.



80. **a.** $(f \circ g)(x) = f(g(x))$
$= 2(2x^2 - x + 5) - 3$
$= 4x^2 - 2x + 10 - 3$
$= 4x^2 - 2x + 7$

b. $(g \circ f)(x) = g(f(x))$
$= 2(2x-3)^2 - (2x-3) + 5$
$= 2(4x^2 - 12x + 9) - 2x + 3 + 5$
$= 8x^2 - 24x + 18 - 2x + 3 + 5$
$= 8x^2 - 26x + 26$

c. $(g \circ f)(x) = 8x^2 - 26x + 26$
$(g \circ f)(1) = 8(1)^2 - 26(1) + 26$
$= 8 - 26 + 26$
$= 8$

81. $f(x) = \dfrac{x-10}{x+10}$

Replace $f(x)$ with y:

$y = \dfrac{x-10}{x+10}$

Interchange x and y:

$x = \dfrac{y-10}{y+10}$

Solve for y:

$$x = \frac{y-10}{y+10}$$
$$x(y+10) = y - 10$$
$$xy + 10x = y - 10$$
$$xy - y = -10x - 10$$
$$y(x-1) = -10x - 10$$
$$y = \frac{-10x-10}{x-1}$$

Replace y with $f^{-1}(x)$:

$f^{-1}(x) = \dfrac{-10x-10}{x-1}$

82. $x^2 + 4x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{20}}{2}$$
$$x = \frac{-4 \pm 2\sqrt{5}}{2}$$
$$x = -2 \pm \sqrt{5}$$

The solution set is $\{-2 \pm \sqrt{5}\}$.

83. $x^2 + 4x + 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(6)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{-8}}{2}$$
$$x = \frac{-4 \pm 2i\sqrt{2}}{2}$$
$$x = -2 \pm i\sqrt{2}$$

The solution set is $\{-2 \pm i\sqrt{2}\}$.

84. $f(x) = a_n(x^4 - 3x^2 - 4)$

$$f(3) = -150$$
$$a_n\left((3)^4 - 3(3)^2 - 4\right) = -150$$
$$a_n(81 - 27 - 4) = -150$$
$$a_n(50) = -150$$
$$a_n = -3$$

Section 2.5

Check Point Exercises

1. $p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$
are the possible rational zeros.

2. $p: \pm 1, \pm 3$
$q: \pm 1, \pm 2, \pm 4$
$\frac{p}{q}: \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}$
are the possible rational zeros.

3. $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$ are possible rational zeros

1	1	8	11	–20
		1	9	20
	1	9	20	0

1 is a zero.
$x^2 + 9x + 20 = 0$
$(x+4)(x+5) = 0$
$x = -4$ or $x = -5$
The zeros are $-5, -4,$ and 1.

4. $\pm 1, \pm 2$ are possible rational zeros

2	1	1	-5	–2
		2	6	2
	1	3	1	0

2 is a zero.
$x^2 + 3x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{5}}{2}$$

The zeros are $2, \frac{-3-\sqrt{5}}{2},$ and $\frac{-3+\sqrt{5}}{2}$. .

5. $\pm 1, \pm 13$ are possible rational zeros.

1	1	–6	22	–30	13
		1	–5	17	–13
	1	–5	17	–13	0

1 is a zero.

1	1	5	17	–13
		1	–4	13
	1	–4	13	0

1 is a double root.
$x^2 - 4x + 13 = 0$

$$x = \frac{4 \pm \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2} = 2 + 3i$$

The solution set is $\{1, 2-3i, 2+3i\}$.

6. $(x+3)(x-i)(x+i) = (x+3)(x^2+1)$
$f(x) = a_n(x+3)(x^2+1)$
$f(1) = a_n(1+3)(1^2+1) = 8a_n = 8$
$a_n = 1$
$f(x) = (x+3)(x^2+1)$
$f(x) = x^3 + 3x^2 + x + 3$

7. $f(x) = x^4 - 14x^3 + 71x^2 - 154x + 120$
$f(-x) = x^4 + 14x^3 + 71x^2 + 154x + 120$
Since $f(x)$ has 4 changes of sign, there are 4, 2, or 0 positive real zeros.
Since $f(-x)$ has no changes of sign, there are no negative real zeros.

Concept and Vocabulary Check 2.5

1. a_0; a_n

2. true

3. false

4. n

5. $a - bi$

6. -6; $(x+6)(2x^2 - x - 1) = 0$

7. n; 1

8. false

9. true

10. true

Exercise Set 2.5

1. $f(x) = x^3 + x^2 - 4x - 4$
$p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

2. $f(x) = x^3 + 3x^2 - 6x - 8$
$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8$

3. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$
$p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 3$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$

4. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$
$p: \pm 1, \pm 3, \pm 5, \pm 15$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

5. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$
$p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 2, \pm 4$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}$

6. $f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$
$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1, \pm 3$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

7. $f(x) = x^5 - x^4 - 7x^3 + 7x^2 - 12x - 12$
$p: \pm 1, \pm 2, \pm 3 \pm 4 \pm 6 \pm 12$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3 \pm 4 \pm 6 \pm 12$

8. $f(x) = 4x^5 - 8x^4 - x + 2$
$p: \pm 1, \pm 2$
$q: \pm 1, \pm 2, \pm 4$
$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$

9. $f(x) = x^3 + x^2 - 4x - 4$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.

2	1	1	–4	–4
		2	6	4
	1	3	2	0

2 is a zero.
2, –2, –1 are rational zeros.

c. $x^3 + x^2 - 4x - 4 = 0$
$(x-2)(x^2 + 3x + 2) = 0$
$(x-2)(x+2)(x+1) = 0$

$x - 2 = 0 \quad x + 2 = 0 \quad x + 1 = 0$
$x = 2, \; x = -2, \; x = -1$
The solution set is $\{2, -2, -1\}$.

10. a. $f(x) = x^3 - 2x - 11x + 12$
$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.

4	1	–2	–11	12
		4	8	–12
	1	2	–3	0

4 is a zero.
4, –3, 1 are rational zeros.

c. $x^3 - 2x^2 - 11x + 12 = 0$
$(x-4)\left(x^2 + 2x - 3\right) = 0$
$(x-4)(x+3)(x-1) = 0$
$x = 4, \; x = -3, \; x = 1$
The solution set is $\{4, -3, 1\}$.

11. $f(x) = 2x^3 - 3x^2 - 11x + 6$

a. $p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

b.

3	2	–3	–11	6
		6	9	–6
	2	3	–2	0

3 is a zero.

$3, \frac{1}{2}, -2$ are rational zeros.

c. $2x^3 - 3x^2 - 11x + 6 = 0$

$(x-3)(2x^2+3x-2) = 0$

$(x-3)(2x-1)(x+2) = 0$

$x = 3,\ x = \frac{1}{2},\ x = -2$

The solution set is $\left\{3, \frac{1}{2}, -2\right\}$.

12. a. $f(x) = 2x^3 - 5x^2 + x + 2$

$p: \pm 1, \pm 2$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}$

b.

2	2	–5	1	2
		4	–2	–2
	2	–1	–1	0

2 is a zero.

$2, -\frac{1}{2}, 1$ are rational zeros.

c. $2x^3 - 5x^2 + x + 2 = 0$

$(x-2)\left(2x^2 - x - 1\right) = 0$

$(x-2)\ (2x+1)(x-1) = 0$

$x = 2,\ x = -\frac{1}{2},\ x = 1$

The solution set is $\left\{2, -\frac{1}{2}, 1\right\}$.

13. a. $f(x) = x^3 + 4x^2 - 3x - 6$

$p: \pm 1, \pm 2, \pm 3, \pm 6$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

b.

–1	1	4	–3	–6
		–1	–3	6
	1	3	–6	0

–1 is a rational zero.

c. $x^2 + 3x - 6 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-6)}}{2(1)}$

$= \frac{-3 \pm \sqrt{33}}{2}$

The solution set is $\left\{-1, \frac{-3+\sqrt{33}}{2}, \frac{-3-\sqrt{33}}{2}\right\}$.

14. a. $f(x) = 2x^3 + x^2 - 3x + 1$

$p: \pm 1$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm\frac{1}{2}$

b.

$\frac{1}{2}$	2	1	–3	1
		1	1	–1
	2	2	–2	0

$\frac{1}{2}$ is a rational zero.

c. $2x^2 + 2x - 2 = 0$

$x^2 + x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$

$= \frac{-1 \pm \sqrt{5}}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}\right\}$.

15. a. $f(x) = 2x^3 + 6x^2 + 5x + 2$

$p: \pm 1, \pm 2$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}$

b.

–2	2	6	5	2
		–4	–4	–2
	2	2	1	0

–2 is a rational zero.

c. $2x^2+2x+1=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-2\pm\sqrt{2^2-4(2)(1)}}{2(2)}$

$=\dfrac{-2\pm\sqrt{-4}}{4}$

$=\dfrac{-2\pm 2i}{4}$

$=\dfrac{-1\pm i}{2}$

The solution set is $\left\{-2,\dfrac{-1+i}{2},\dfrac{-1-i}{2}\right\}$.

16. a. $f(x)=x^3-4x^2+8x-5$

p: ±1, ±5

q: ±1

$\dfrac{p}{q}$: ±1, ±5

b.

1	1	–4	8	–5
		1	–3	5
	1	–3	5	0

1 is a rational zero.

c. $x^2-3x+5=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(1)(5)}}{2(1)}$

$=\dfrac{3\pm\sqrt{-11}}{2}$

$=\dfrac{3\pm i\sqrt{11}}{2}$

The solution set is $\left\{1,\dfrac{3+i\sqrt{11}}{2},\dfrac{3-i\sqrt{11}}{2}\right\}$.

17. $x^3-2x^2-11x+12=0$

a. p: ±1, ±2, ±3, ±4, ±6, ±12

q: ±1

$\dfrac{p}{q}$: ±1, ±2, ±3, ±4, ±6, ±12

b.

4	1	–2	–11	12
		4	8	–12
	1	2	–3	0

4 is a root.

–3, 1, 4 are rational roots.

c. $x^3-2x^2-11x+12$

$(x-4)(x^2+2x-3)=0$

$(x-4)(x+3)(x-1)=0$

$x-4=0 \quad x+3=0 \quad x-1=0$

$x=4 \quad x=-3 \quad x=1$

The solution set is {–3, 1, 4}.

18. a. $x^3-2x^2-7x-4=0$

p: ±1, ±2, ±4

q: ±1

$\dfrac{p}{q}$: ±1, ±2, ±4

b.

4	1	–2	–7	–4
		4	8	4
	1	2	1	0

4 is a root.

–1, 4 are rational roots.

c. $x^3+2x^2-7x-4=0$

$(x-4)\left(x^2+2x+1\right)=0$

$(x-4) \quad (x+1)^2$

$x=4, \quad x=-1$

The solution set is $\{4,-1\}$.

19. $x^3-10x-12=0$

a. p: ±1, ±2, ±3, ±4, ±6, ±12

q: ±1

$\dfrac{p}{q}$: ±1, ±2, ±3, ±4, ±6, ±12

b.

–2	1	0	–10	–12
		–2	4	12
	1	–2	–6	0

–2 is a rational root.

c. $x^3-10x-12=0$

$(x+2)(x^2-2x-6)=0$

$x=\dfrac{2\pm\sqrt{4+24}}{2}=\dfrac{2\pm\sqrt{28}}{2}$

$=\dfrac{2\pm 2\sqrt{7}}{2}=1\pm\sqrt{7}$

The solution set is $\left\{-2, 1+\sqrt{7}, 1-\sqrt{7}\right\}$.

20. **a.** $x^3-5x^2+17x-13=0$
p: ±1, ±13
q: ±1
$\frac{p}{q}: \pm 1, \pm 13$

b.

1	1	–5	17	–13
		1	–4	13
	1	–4	13	0

1 is a rational root.

c. $x^3-5x^2+17x-13=0$
$(x-1)\left(x^2-4x+13\right)=0$
$x=\frac{4+\sqrt{16-52}}{2}=\frac{4\pm\sqrt{-36}}{2}$
$=\frac{4\pm 6i}{2}=2\pm 3i$
The solution set is $\{1,\ 2+3i,\ 2,-3i\}$.

21. $6x^3+25x^2-24x+5=0$

a. $p: \pm 1, \pm 5$
$q: \pm 1, \pm 2, \pm 3, \pm 6$
$\frac{p}{q}: \pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

b.

–5	6	25	–24	5
		–30	25	–5
	6	–5	1	0

–5 is a root.
$-5, \frac{1}{2}, \frac{1}{3}$ are rational roots.

c. $6x^3+25x^2-24x+5=0$
$(x+5)(6x^2-5x+1)=0$
$(x+5)(2x-1)(3x-1)=0$
$x+5=0 \quad 2x-1=0 \quad 3x-1=0$
$x=-5, \quad x=\frac{1}{2}, \quad x=\frac{1}{3}$
The solution set is $\left\{-5, \frac{1}{2}, \frac{1}{3}\right\}$.

22. **a.** $2x^3-5x^2-6x+4=0$
p: ±1, ±2, ±4
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm \frac{1}{2}$

b.

$\frac{1}{2}$	2	–5	–6	4
		1	–2	–4
	2	–4	–8	0

$\frac{1}{2}$ is a rational root.

c. $2x^3-5x^2-6x+4=0$
$(x-\frac{1}{2})\left(2x^2-4x-8\right)=0$
$2\left(x-\frac{1}{2}\right)\left(x^2-2x-4\right)=0$
$x=\frac{2\pm 2\sqrt{5}}{2}=1\pm\sqrt{5}$
The solution set is $\left\{\frac{1}{2}, 1+\sqrt{5}, 1-\sqrt{5}\right\}$.

23. $x^4-2x^3-5x^2+8x+4=0$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.

2	1	–2	–5	8	4
		2	0	–10	–4
	1	0	–5	–2	0

2 is a root.
$-2, 2$ are rational roots.

c. $x^4-2x^3-5x^2+8x+4=0$
$(x-2)(x^3-5x-2)=0$

–2	1	0	–5	–2
		–2	4	2
	1	–2	–1	0

–2 is a zero of $x^3-5x-2=0$.

$(x-2)(x+2)\left(x^2-2x-1\right)=0$
$x=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm 2\sqrt{2}}{2}$
$=1\pm\sqrt{2}$
The solution set is
$\left\{-2, 2, 1+\sqrt{2}, 1-\sqrt{2}\right\}$.

24. **a.** $x^4 - 2x^2 - 16x - 15 = 0$

$p: \pm 1, \pm 3, \pm 5, \pm 15$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 3 \pm 5 \pm 15$

b.

3	1	0	–2	–16	–15
		3	9	21	15
	1	3	7	5	0

3 is a root.

$-1, 3$ are rational roots.

c. $x^4 - 2x^2 - 16x - 15 = 0$

$(x-3)\left(x^3 + 3x^2 + 7x + 5\right) = 0$

–1	1	3	7	5
		–1	–2	–5
	1	2	5	0

-1 is a root of $x^3 + 3x^2 + 7x + 5$

$(x-3)\ (x+1)\ \left(x^2 + 2x + 5\right)$

$x = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$

$= \frac{-2 \pm 4i}{2} = -1 \pm 2i$

The solution set is $\{3, -1, -1+2i, -1-2i\}$.

25. $(x-1)\ (x+5i)\ (x-5i)$

$= (x-1)\ \left(x^2 + 25\right)$

$= x^3 + 25x - x^2 - 25$

$= x^3 - x^2 + 25x - 25$

$f(x) = a_n\left(x^3 - x^2 + 25x - 25\right)$

$f(-1) = a_n(-1 - 1 - 25 - 25)$

$-104 = a_n(-52)$

$a_n = 2$

$f(x) = 2\left(x^3 - x^2 + 25x - 25\right)$

$f(x) = 2x^3 - 2x^2 + 50x - 50$

26. $(x-4)\ (x+2i)\ (x-2i)$

$= (x-4)\ \left(x^2 + 4\right)$

$= x^3 - 4x^2 + 4x - 16$

$f(x) = a_n\left(x^3 - 4x^2 + 4x - 16\right)$

$f(-1) = a_n(-1 - 4 - 4 - 16)$

$-50 = a_n(-25)$

$a_n = 2$

$f(x) = 2\left(x^3 - 4x^2 + 4x - 16\right)$

$f(x) = 2x^3 - 8x^2 + 8x - 32$

27. $(x+5)(x-4-3i)(x-4+3i)$

$= (x+5)\left(x^2 - 4x + 3ix - 4x + 16 - 12i\right.$

$\left.-3ix + 12i - 9i^2\right)$

$= (x+5)\left(x^2 - 8x + 25\right)$

$= \left(x^3 - 8x^2 + 25x + 5x^2 - 40x + 125\right)$

$= x^3 - 3x^2 - 15x + 125$

$f(x) = a_n(x^3 - 3x^2 - 15x + 125)$

$f(2) = a_n\left(2^3 - 3(2)^2 - 15(2) + 125\right)$

$91 = a_n(91)$

$a_n = 1$

$f(x) = 1\left(x^3 - 3x^2 - 15x + 125\right)$

$f(x) = x^3 - 3x^2 - 15x + 125$

28. $(x-6)\ (x+5+2i)\ (x+5-2i)$

$= (x-6)\ \left(x^2 + 5x - 2ix + 5x + 25 - 10i + 2ix + 10i - 4i^2\right)$

$= (x-6)\ \left(x^2 + 10x + 29\right)$

$= x^3 + 10x^2 + 29x - 6x^2 - 60x - 174$

$= x^3 + 4x^2 - 31x - 174$

$f(x) = a_n\left(x^3 + 4x^2 - 31x - 174\right)$

$f(2) = a_n(8 + 16 - 62 - 174)$

$-636 = a_n(-212)$

$a_n = 3$

$f(x) = 3\left(x^3 + 4x^2 - 31x - 174\right)$

$f(x) = 3x^3 + 12x^2 - 93x - 522$

29. $(x-i)(x+i)(x-3i)(x+3i)$

$=\left(x^2-i^2\right)\left(x^2-9i^2\right)$

$=\left(x^2+1\right)\left(x^2+9\right)$

$=x^4+10x^2+9$

$f(x)=a_n(x^4+10x^2+9)$

$f(-1)=a_n((-1)^4+10(-1)^2+9)$

$20=a_n(20)$

$a_n=1$

$f(x)=x^4+10x^2+9$

30. $(x+2)\left(x+\frac{1}{2}\right)(x-i)(x+i)$

$=\left(x^2+\frac{5}{2}x+1\right)\left(x^2+1\right)$

$=x^4+x^2+\frac{5}{2}x^3+\frac{5}{2}x+x^2+1$

$=x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1$

$f(x)=a_n\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(1)=a_n\left[(1)^4+\frac{5}{2}(1)^3+2(1)^2+\frac{5}{2}(1)+1\right]$

$18=a_n(9)$

$a_n=2$

$f(x)=2\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(x)=2x^4+5x^3+4x^2+5x+2$

31. $(x+2)\ (x-5)\ (x-3+2i)\ (x-3-2i)$

$=\left(x^2-3x-10\right)\left(x^2-3x-2ix-3x+9+6i+2ix-6i-4i^2\right)$

$=\left(x^2-3x-10\right)\left(x^2-6x+13\right)$

$=x^4-6x+13x^2-3x^3+18x^2-39x-10x^2+60x-130$

$=x^4-9x^3+21x^2+21x-130$

$f(x)=a_n\left(x^4-9x^3+21x^2+21x-130\right)$

$f(1)=a_n(1-9+21+21-130)$

$-96=a_n(-96)$

$a_n=1$

$f(x)=x^4-9x^3+21x^2+21x-130$

32. $(x+4)\ (3x-1)\ (x-2+3i)\ (x-2-3i)$

$$=\left(3x^2+11x-4\right)\left(x^2-2x-3ix-2x+4+6i+3ix-6i-9i^2\right)$$
$$=\left(3x^2+11x-4\right)\left(x^2-4x+13\right)$$
$$=3x^4-12x^3+39x^2+11x^3-44x^2+143x-4x^2+16x-52$$
$$=3x^4-x^3-9x^2+159x-52$$
$$f(x)=a_n\left(3x^4-x^3-9x^2+159x-52\right)$$
$$f(1)=a_n(3-1-9+159-52)$$
$$100=a_n(100)$$
$$a_n=1$$
$$f(x)=3x^4-x^3-9x^2+159x-52$$

33. $f(x)=x^3+2x^2+5x+4$

Since $f(x)$ has no sign variations,
no positive real roots exist.

$f(-x)=-x^3+2x^2-5x+4$

Since $f(-x)$ has 3 sign variations,
3 or 1 negative real roots exist.

34. $f(x)=x^3+7x^2+x+7$

Since $f(x)$ has no sign variations no positive real roots exist.

$f(-x)=-x^3+7x^2-x+7$

Since $f(-x)$ has 3 sign variations, 3 or 1 negative real roots exist.

35. $f(x)=\ 5x^3-3x^2+3x-1$

Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.

$f(-x)=-5x^3-3x^2-3x-1$

Since $f(-x)$ has no sign variations, no negative real roots exist.

36. $f(x)=-2x^3+x^2-x+7$

Since $f(x)$ has 3 sign variations,
3 or 1 positive real roots exist.

$f(-x)=2x^3+x^2+x+7$

Since $f(-x)$ has no sign variations,
no negative real roots exist.

37. $f(x)=2x^4-5x^3-x^2-6x+4$

Since $f(x)$ has 2 sign variations, 2 or 0 positive real roots exist.

$f(-x)=2x^4+5x^3-x^2+6x+4$

Since $f(-x)$ has 2 sign variations, 2 or 0 negative real roots exist.

38. $f(x)=4x^4-x^3+5x^2-2x-6$

Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.

$f(-x)=4x^4+x^3+5x^2+2x-6$

Since $f(x)$ has 1 sign variations, 1 negative real roots exist.

39. $f(x) = x^3 - 4x^2 - 7x + 10$

$p: \pm 1, \pm 2, \pm 5, \pm 10$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 5, \pm 10$

Since $f(x)$ has 2 sign variations, 0 or 2 positive real zeros exist.

$f(-x) = -x^3 - 4x^2 + 7x + 10$

Since $f(-x)$ has 1 sign variation, exactly one negative real zeros exists.

–2	1	–4	–7	10
		–2	12	–10
	1	–6	5	0

–2 is a zero.

$f(x) = (x+2)\left(x^2 - 6x + 5\right)$
$= (x+2)(x-5)(x-1)$

$x = -2, x = 5, x = 1$

The solution set is $\{-2, 5, 1\}$.

40. $f(x) = x^3 + 12x^2 + 2x + 10$

$p: \pm 1, \pm 2, \pm 5, \pm 10$

$q: \pm 1,$

$\frac{p}{q}: \pm 1, \pm 2 \pm 5 \pm 10$

Since $f(x)$ has no sign variations, no positive zeros exist.

$f(-x) = -x^3 + 12x^2 - 21x + 10$

Since $f(-x)$ has 3 sign variations, 3 or 1 negative zeros exist.

–1	1	12	21	10
		–1	–11	–10
	1	11	10	0

–1 is a zero.

$f(x) = (x+1)\left(x^2 + 11x + 10\right)$
$= (x+1)(x+10)(x+1)$
$x = -1, x = -10$

The solution set is $\{-1, -10\}$.

41. $2x^3 - x^2 - 9x - 4 = 0$

$p: \pm 1, \pm 2, \pm 4$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4 \pm \frac{1}{2}$

1 positive real root exists.

$f(-x) = -2x^3 - x^2 + 9x - 4$ 2 or no negative real roots exist.

$-\frac{1}{2}$	2	–1	–9	–4
		–1	1	4
	2	–2	–8	0

$-\frac{1}{2}$ is a root.

$\left(x + \frac{1}{2}\right)\left(2x^2 - 2x - 8\right) = 0$

$2\left(x + \frac{1}{2}\right)\left(x^2 - x - 4\right) = 0$

$x = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$

The solution set is $\left\{-\frac{1}{2}, \frac{1+\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2}\right\}$.

42. $3x^3 - 8x^2 - 8x + 8 = 0$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1, \pm 3$

$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

Since $f(x)$ has 2 sign variations, 2 or no positive real roots exist.

$f(-x) = -3x^3 - 8x^2 + 8x + 8$

Since $f(-x)$ has 1 sign changes, exactly 1 negative real zero exists.

$\frac{2}{3}$	3	–8	–8	8
		2	–4	–8
	3	–6	–12	0

$\frac{2}{3}$ is a zero.

$f(x) = \left(x - \frac{2}{3}\right)\left(3x^2 - 6x - 12\right)$

$x = \frac{6 \pm \sqrt{36+144}}{6} = \frac{6 \pm 6\sqrt{5}}{6} = 1 \pm \sqrt{5}$

The solution set is $\left\{\frac{2}{3}, 1+\sqrt{5}, 1-\sqrt{5}\right\}$.

43. $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$
$p: \pm 1, \ \pm 2, \ \pm 4, \ \pm 8$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \ \pm 2, \ \pm 4, \ \pm 8$
Since $f(x)$ has 2 sign changes, 0 or 2 positive roots exist.
$f(-x) = (-x)^4 - 2(-x)^3 + (-x)^2 - 12x + 8$
$= x^4 + 2x^3 + x^2 - 12x + 8$
Since $f(-x)$ has 2 sign changes, 0 or 2 negative roots exist.

–1	1	–2	1	12	8
		–1	4	–4	–8
	1	–3	4	8	0

–1	1	–3	4	8
		–1	4	–8
	1	–4	8	0

$0 = x^2 - 4x + 8$
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$
$x = \frac{4 \pm \sqrt{16 - 32}}{2}$
$x = \frac{4 \pm \sqrt{-16}}{2}$
$x = \frac{4 \pm 4i}{2}$
$x = 2 \pm 2i$
The solution set is $\{-1, -1, 2 + 2i, 2 - 2i\}$.

44. $f(x) = x^4 - 4x^3 - x^2 + 14x + 10$
p: ±1, ±2, ±5, ±10
q: ±1
$\frac{p}{q}$: ±1, ±2, ±5, ±10

–1	1	-4	-1	14	10
		–1	5	-4	-10
	1	-5	4	10	0

–1	1	-5	4	10
		–1	6	–10
	1	-6	10	0

$f(x) = (x-1)(x-1)(x^2 - 6x + 10)$
$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$ $\quad x = 1$
$x = \frac{6 \pm \sqrt{36 - 40}}{2}$
$x = \frac{6 \pm \sqrt{-4}}{2}$
$x = \frac{6 \pm 2i}{2}$
$x = 3 \pm i$

The solution set is $\{-1, 3 - i, 3 + i\}$

45. $x^4 - 3x^3 - 20x^2 - 24x - 8 = 0$
$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4 \pm 8$
1 positive real root exists.
3 or 1 negative real roots exist.

–1	1	–3	–20	–24	–8
		–1	4	16	8
	1	–4	–16	–8	0

$(x+1)\left(x^3 - 4x^2 - 16x - 8\right) = 0$

–2	1	–4	–16	–8
		–2	12	8
	1	–6	–4	0

$(x+1)(x+2)\left(x^2 - 6x - 4\right) = 0$
$x = \frac{6 \pm \sqrt{36 + 16}}{2} = \frac{6 \pm \sqrt{52}}{2}$
$= \frac{6 \pm 2\sqrt{13}}{2} = \frac{3 \pm \sqrt{13}}{2}$
The solution set is
$\left\{-1, -2, 3 \pm \sqrt{13}, 3 - \sqrt{13}\right\}$.

46. $x^4 - x^3 + 2x^2 - 4x - 8 = 0$

p: ±1, ±2, ±4, ±8

q: ±1

$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8$

1 negative real root exists.

–1	1	–1	2	–4	–8
		–1	2	–4	8
	1	–2	4	–8	0

$(x+1)\left(x^3 - 2x^2 + 4x - 8\right)$

2	1	–2	4	–8
		2	0	8
	1	0	4	0

$(x+1) \quad (x-2) \quad \left(x^2+4\right)$

$x+1=0 \quad x-2=0 \quad x^2+4=0$

$x=-1 \quad x=2 \quad x^2=-4$

$x = \pm 2i$

The solution set is $\{-1, 2, 2i, -2i\}$.

47. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

$p: \pm 1, \pm 2, \pm 3, \pm 6$

$q: \pm 1, \pm 3$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$

2 or no positive real zeros exists.

$f(-x) = 3x^4 + 11x^3 - x^2 - 19x + 6$

2 or no negative real zeros exist.

–1	3	–11	–1	19	6
		–3	14	–13	–6
	3	–14	13	6	0

$f(x) = (x+1)\left(3x^3 - 14x^2 + 13x + 6\right)$

2	3	–14	13	6
		6	–16	–6
	3	–8	–3	0

$f(x) = (x+1)(x-2)\left(3x^2 - 8x - 3\right)$

$= (x+1)(x-2)(3x+1)(x-3)$

$x=-1,\ x=2\ x=-\frac{1}{3},\ x=3$

The solution set is $\left\{-1, 2, -\frac{1}{3}, 3\right\}$.

48. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

p: ±1, ±3, ±5, ±15

q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

2 or no positive real zeros exist.

$f(-x) = 2x^4 - 3x^3 - 11x^2 + 9x + 15$

2 or no negative real zeros exist.

1	2	3	–11	–9	15
		2	5	–6	–15
	2	5	–6	–15	0

$f(x) = (x-1)\left(2x^3 + 5x^2 - 6x - 15\right)$

$-\frac{5}{2}$	2	5	–6	–15
		–5	0	15
	2	0	–6	0

$f(x) = (x-1)\left(x+\frac{5}{2}\right)\left(2x^2-6\right)$

$= 2(x-1)\left(x+\frac{5}{2}\right)\left(x^2-3\right)$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x=1,\ x=-\frac{5}{2},\ x=\sqrt{3},\ x=-\sqrt{3}$

The solution set is $\left\{1, -\frac{5}{2}, \sqrt{3}, -\sqrt{3}\right\}$.

49. $4x^4 - x^3 + 5x^2 - 2x - 6 = 0$
$p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 2, \pm 4$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$
3 or 1 positive real roots exists.
1 negative real root exists.

1	4	–1	5	–2	–6
		4	3	8	6
	4	3	8	6	0

$(x-1)(4x^3 + 3x^2 + 8x + 6) = 0$

$4x^3 + 3x^2 + 8x + 6 = 0$ has no positive real roots.

$-\frac{3}{4}$	4	3	8	6
		–3	0	–6
	4	0	8	0

$(x-1)\left(x+\frac{3}{4}\right)\left(4x^2+8\right) = 0$
$4(x-1)\left(x+\frac{3}{4}\right)\left(x^2+2\right) = 0$

$x^2 + 2 = 0$
$x^2 = -2$
$x = \pm i\sqrt{2}$
The solution set is $\left\{1, -\frac{3}{4}, i\sqrt{2}, -i\sqrt{2}\right\}$.

50. $3x^4 - 11x^3 - 3x^2 - 6x + 8 = 0$
p: ±1, ±2, ±4, ±8
q: ±1, ±3
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$
2 or no positive real roots exist.
$f(-x) = 3x^4 + 11x^3 - 3x^2 + 6x + 8$ 2 or no negative real roots exist.

4	3	–11	–3	–6	8
		12	4	4	–8
	3	1	1	–2	0

$(x-4)\left(3x^3 + x^2 + x - 2\right) = 0$

Another positive real root must exist.

$\frac{2}{3}$	3	1	1	–2
		2	2	2
	3	3	3	0

$(x-4)\left(x-\frac{2}{3}\right)\left(3x^2+3x+3\right) = 0$
$3(x-4)\left(x-\frac{2}{3}\right)\left(x^2+x+1\right) = 0$
$x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$
The solution set is $\left\{4, \frac{2}{3}, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$.

51. $2x^5 + 7x^4 - 18x^2 - 8x + 8 = 0$
$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{2}$
2 or no positive real roots exists.
3 or 1 negative real root exist.

–2	2	7	0	–18	–8	8
		–4	–6	12	12	–8
	2	3	–6	–6	4	0

$(x+2)(2x^4 + 3x^3 - 6x^2 - 6x + 4) = 0$

$4x^3 + 3x^2 + 8x + 6 = 0$ has no positive real roots.

–2	2	3	–6	–6	4
		–4	2	8	–4
	2	–1	–4	2	0

$(x+2)^2(2x^3 - x^2 - 4x + 2)$

$\frac{1}{2}$	2	–1	–4	2
		1	0	2
	2	0	–4	0

$(x+2)^2\left(x-\frac{1}{2}\right)\left(2x^2-4\right) = 0$
$2(x+2)^2\left(x-\frac{1}{2}\right)\left(x^2-2\right) = 0$

$x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$
The solution set is $\left\{-2, \frac{1}{2}, \sqrt{2}, -\sqrt{2}\right\}$.

52. $4x^5+12x^4-41x^3-99x^2+10x+24=0$

$p: \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$q: \pm1, \pm2, \pm4$

$\frac{p}{q}: \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

2 or no positive real roots exist.

$f(-x)=-4x^5+12x^4+41x^3-99x^2-10x+24$

3 or 1 negative real roots exist.

3	4	12	–41	–99	10	24
		12	72	93	–18	–24
	4	24	31	–6	–8	0

$(x-3)\left(4x^4+24x^3+31x^2-6x-8\right)=0$

–2	4	24	31	–6	–8
		–8	–32	2	8
	4	16	–1	–4	0

$(x-3)(x+2)\left(4x^3+16x^2-x-4\right)=0$

–4	4	16	–1	4
		–16	0	4
	4	0	–1	0

$(x-3)(x+2)(x+4)\left(4x^2-1\right)=0$

$4x^2-1=0$
$4x^2=1$
$x^2=\frac{1}{4}$
$x=\pm\frac{1}{2}$

The solution set is $\left\{3, -2, -4, \frac{1}{2}, -\frac{1}{2}\right\}$.

53. $f(x)=-x^3+x^2+16x-16$

a. From the graph provided, we can see that -4 is an x-intercept and is thus a zero of the function. We verify this below:

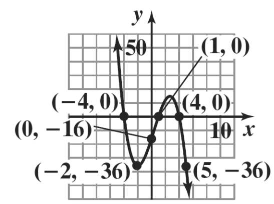
–4	–1	1	16	–16
		4	–20	16
	–1	5	–4	0

Thus,
$$-x^3+x^2+16x-16=0$$
$$(x+4)\left(-x^2+5x-4\right)=0$$
$$-(x+4)\left(x^2-5x+4\right)=0$$
$$-(x+4)(x-1)(x-4)=0$$

$x+4=0$ or $x-1=0$ or $x-4=0$
$x=-4$ $\quad x=1$ $\quad x=4$

The zeros are -4, 1, and 4.

b.



$f(x) = -x^3 + x^2 + 16x - 16$

54. $f(x)=-x^3+3x^2-4$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

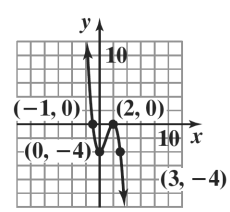
–1	–1	3	0	–4
		1	–4	4
	–1	4	–4	0

Thus,
$$-x^3+3x^2-4=0$$
$$(x+1)\left(-x^2+4x-4\right)=0$$
$$-(x+1)\left(x^2-4x+4\right)=0$$
$$-(x+1)(x-2)^2=0$$

$x+1=0$ or $(x-2)^2=0$
$x=-1$ $\quad x-2=0$
$x=2$

The zeros are -1 and 2.

b.



$f(x) = -x^3 + 3x^2 - 4$

55. $f(x) = 4x^3 - 8x^2 - 3x + 9$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrr} -1 & 4 & -8 & -3 & 9 \\ & & -4 & 12 & -9 \\ \hline & 4 & -12 & 9 & 0 \end{array}$$

Thus,
$$4x^3 - 8x^2 - 3x + 9 = 0$$
$$(x+1)(4x^2 - 12x + 9) = 0$$
$$(x+1)(2x-3)^2 = 0$$

$x + 1 = 0$ or $(2x-3)^2 = 0$

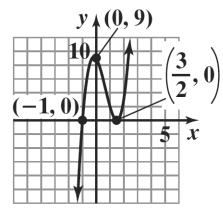
$x = -1$ $\quad 2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

The zeros are -1 and $\frac{3}{2}$.

b.



$f(x) = 4x^3 - 8x^2 - 3x + 9$

56. $f(x) = 3x^3 + 2x^2 + 2x - 1$

a. From the graph provided, we can see that $\frac{1}{3}$ is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 2 & 2 & -1 \\ & & 1 & 1 & 1 \\ \hline & 3 & 3 & 3 & 0 \end{array}$$

Thus,
$$3x^3 + 2x^2 + 2x - 1 = 0$$
$$\left(x - \frac{1}{3}\right)(3x^2 + 3x + 3) = 0$$
$$3\left(x - \frac{1}{3}\right)(x^2 + x + 1) = 0$$

Note that $x^2 + x + 1$ will not factor, so we use the quadratic formula:

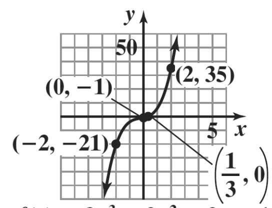
$x - \frac{1}{3} = 0$ or $x^2 + x + 1 = 0$

$x = \frac{1}{3}$ $\quad a = 1 \quad b = 1 \quad c = 1$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{-3}}{2}$$
$$= \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

The zeros are $\frac{1}{3}$ and $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

b.



$f(x) = 3x^3 + 2x^2 + 2x - 1$

57. $f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$

a. From the graph provided, we can see that $\frac{1}{2}$ is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -3 & -7 & -8 & 6 \\ & & 1 & -1 & -4 & -6 \\ \hline & 2 & -2 & -8 & -12 & 0 \end{array}$$

Thus,
$$2x^4 - 3x^3 - 7x^2 - 8x + 6 = 0$$
$$\left(x - \frac{1}{2}\right)(2x^3 - 2x^2 - 8x - 12) = 0$$
$$2\left(x - \frac{1}{2}\right)(x^3 - x^2 - 4x - 6) = 0$$

To factor $x^3 - x^2 - 4x - 6$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term -6:

$\pm 1, \pm 2, \pm 3, \pm 6$

Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -6}{\text{Factors of } 1} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$$
$$= \pm 1, \pm 2, \pm 3, \pm 6$$

We test values from above until we find a zero. One possibility is shown next:

Test 3:

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -4 & -6 \\ & & 3 & 6 & 6 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

The remainder is 0, so 3 is a zero of *f*.

$$2x^4-3x^3-7x^2-8x+6=0$$
$$\left(x-\frac{1}{2}\right)\left(2x^3-2x^2-8x-12\right)=0$$
$$2\left(x-\frac{1}{2}\right)\left(x^3-x^2-4x-6\right)=0$$
$$2\left(x-\frac{1}{2}\right)(x-3)\left(x^2+2x+2\right)=0$$

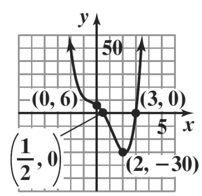
Note that x^2+x+1 will not factor, so we use the quadratic formula:

$a=1 \quad b=2 \quad c=2$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(2)}}{2(1)}$$
$$=\frac{-2\pm\sqrt{-4}}{2}=\frac{-2\pm 2i}{2}=-1\pm i$$

The zeros are $\frac{1}{2}$, 3, and $-1\pm i$.

b.



$f(x)=2x^4-3x^3-7x^2-8x+6$

58. $f(x)=2x^4+2x^3-22x^2-18x+36$

a. From the graph provided, we can see that 1 and 3 are *x*-intercepts and are thus zeros of the function. We verify this below:

1	2	2	−22	−18	36
		2	4	−18	−36
	2	4	−18	−36	0

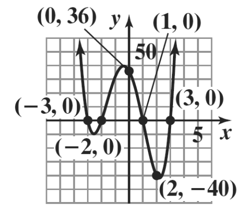
Thus, $2x^4+2x^3-22x^2-18x+36$
$$=(x-1)\left(2x^3+4x^2-18x-36\right)$$

3	2	4	−18	−36
		6	30	36
	2	10	12	0

Thus, $2x^4+2x^3-22x^2-18x+36=0$
$$(x-1)(x-3)\left(2x^2+10x+12\right)=0$$
$$2(x-1)(x-3)\left(x^2+5x+6\right)=0$$
$$2(x-1)(x-3)(x+3)(x+2)=0$$
$$x=1,\ x=3,\ x=-3,\ x=-2$$

The zeros are -3, -2, 1, and 3.

b.



$f(x)=2x^4+2x^3-22x^2-18x+36$

59. $f(x)=3x^5+2x^4-15x^3-10x^2+12x+8$

a. From the graph provided, we can see that 1 and 2 are *x*-intercepts and are thus zeros of the function. We verify this below:

1	3	2	−15	−10	12	8
		3	5	−10	−20	−8
	3	5	−10	−20	−8	0

Thus, $3x^5+2x^4-15x^3-10x^2+12x+8$
$$=(x-1)\left(3x^4+5x^3-10x^2-20x-8\right)$$

2	3	5	−10	−20	−8
		6	22	24	8
	3	11	12	4	0

Thus, $3x^5+2x^4-15x^3-10x^2+12x+8$
$$=(x-1)\left(3x^4+5x^3-10x^2-20x-8\right)$$
$$=(x-1)(x-2)\left(3x^3+11x^2+12x+4\right)$$

To factor $3x^3+11x^2+12x+4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1,\ \pm2,\ \pm4$

Factors of the leading coefficient 3: $\pm1,\ \pm3$

The possible rational zeros are:

$$\frac{\text{Factors of 4}}{\text{Factors of 3}}=\frac{\pm1,\ \pm2,\ \pm4}{\pm1,\ \pm3}$$
$$=\pm1,\ \pm2,\ \pm4,\ \pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm\frac{4}{3}$$

We test values from above until we find a zero. One possibility is shown next:

Test -1:

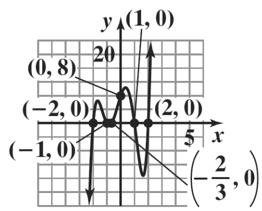
−1	3	11	12	4
		−3	−8	−4
	3	8	4	0

The remainder is 0, so -1 is a zero of *f*. We can now finish the factoring:

$$3x^5+2x^4-15x^3-10x^2+12x+8=0$$
$$(x-1)\left(3x^4+5x^3-10x^2-20x-8\right)=0$$
$$(x-1)(x-2)\left(3x^3+11x^2+12x+4\right)=0$$
$$(x-1)(x-2)(x+1)\left(3x^2+8x+4\right)=0$$
$$(x-1)(x-2)(x+1)(3x+2)(x+2)=0$$
$$x=1,\ x=2,\ x=-1,\ x=-\frac{2}{3},\ x=-2$$

The zeros are -2, -1, $-\frac{2}{3}$, 1 and 2.

b.



$f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

60. $f(x)=-5x^4+4x^3-19x^2+16x+4$

a. From the graph provided, we can see that 1 is an x-intercept and is thus a zero of the function. We verify this below:

1	−5	4	−19	16	4
		−5	−1	−20	−4
	−5	−1	−20	−4	0

Thus, $-5x^4+4x^3-19x^2+16x+4=0$
$$(x-1)\left(-5x^3-x^2-20x-4\right)=0$$
$$-(x-1)\left(5x^3+x^2+20x+4\right)=0$$

To factor $5x^3+x^2+20x+4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1,\ \pm2,\ \pm4$

Factors of the leading coefficient 5: $\pm1,\ \pm5$

The possible rational zeros are:

$$\frac{\text{Factors of 4}}{\text{Factors of 5}}=\frac{\pm1,\ \pm2,\ \pm4}{\pm1,\ \pm5}$$
$$=\pm1,\ \pm2,\ \pm4,\ \pm\frac{1}{5},\ \pm\frac{2}{5},\ \pm\frac{4}{5}$$

We test values from above until we find a zero. One possibility is shown next:

Test $-\frac{1}{5}$:

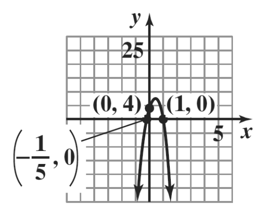
$-\frac{1}{5}$	5	1	20	4
		−1	0	−4
	5	0	20	0

The remainder is 0, so $-\frac{1}{5}$ is a zero of *f*.

$$-5x^4+4x^3-19x^2+16x+4=0$$
$$(x-1)\left(-5x^3-x^2-20x-4\right)=0$$
$$-(x-1)\left(5x^3+x^2+20x+4\right)=0$$
$$-(x-1)\left(x+\frac{1}{5}\right)\left(5x^2+20\right)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)\left(x^2+4\right)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)(x+2i)(x-2i)=0$$
$$x=1,\ x=-\frac{1}{5},\ x=-2i,\ x=2i$$

The zeros are $-\frac{1}{5}$, 1, and $\pm2i$.

b.



$f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$

61. $V(x)=x(x+10)(30-2x)$
$$2000=x(x+10)(30-2x)$$
$$2000=-2x^3+10x^2+300x$$
$$2x^3-10x^2-300x+2000=0$$
$$x^3-5x^2-150x+1000=0$$

Find the roots.

10	1	−5	−150	1000
		10	50	−1000
	1	5	−100	0

Use the remaining quadratic to find the other 2 roots.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(-100)}}{2(1)}$$
$$x\approx-12.8,\ 7.8$$

Since the depth must be positive, reject the negative value.

The depth can be 10 inches or 7.8 inches to obtain a volume of 2000 cubic inches.

62. $V(x) = x(x+10)(30-2x)$

$$1500 = x(x+10)(30-2x)$$
$$1500 = -2x^3 + 10x^2 + 300x$$
$$2x^3 - 10x^2 - 300x + 1500 = 0$$
$$x^3 - 5x^2 - 150x + 750 = 0$$

Find the roots.

5	1	−5	−150	750
		5	0	−750
	1	0	−150	0

Use the remaining quadratic to find the other 2 roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-150)}}{2(1)}$$
$$x \approx -12.2,\ 12.2$$

Since the depth must be positive, reject the negative value.
The depth can be 5 inches or 12.2 inches to obtain a volume of 1500 cubic inches.

63. **a.** The answers correspond to the points $(7.8, 2000)$ and $(10, 2000)$.

b. The range is $(0, 15)$.

64. **a.** The answers correspond to the points $(5, 1500)$ and $(12.2, 1500)$.

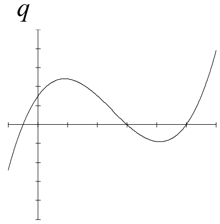
b. The range is $(0, 15)$.

65. – 71. Answers will vary.

72. $2x^3 - 15x^2 + 22x + 15 = 0$

p: ±1, ±3, ±5, ±15
q: ±1, ±2

$$\frac{p}{q}: \pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$$



From the graph we see that the solutions are $-\frac{1}{2}$, 3 and 5.

73. $6x^3 - 19x^2 + 16x - 4 = 0$

p: ±1, ±2, ±4
q: ±1, ±2, ±3, ±6

$$\frac{p}{q}: \pm1, \pm2, \pm4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{6}$$

From the graph, we see that the solutions are $\frac{1}{2}, \frac{2}{3}$ and 2.

74. $2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$

p: ±1, ±2, ±3, ±6, ±9, ±18
q: ±1, ±2

$$\frac{p}{q}: \pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$$

From the graph we see the solutions are $-3,\ -\frac{3}{2},\ -1,\ 2.$

75. $4x^4 + 4x^3 + 7x^2 - x - 2 = 0$

p: ±1, ±2
q: ±1, ±2, ±4

$$\frac{p}{q}: \pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$$

From the graph, we see that the solutions are $-\frac{1}{2}$ and $\frac{1}{2}$.

76. $f(x) = 3x^4 + 5x^2 + 2$

Since $f(x)$ has no sign variations, it has no positive real roots.

$f(-x) = 3x^4 + 5x^2 + 2$

Since $f(-x)$ has no sign variations, no negative roots exist.
The polynomial's graph doesn't intersect the x-axis.
From the graph, we see that there are no real solutions.

77. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 8$

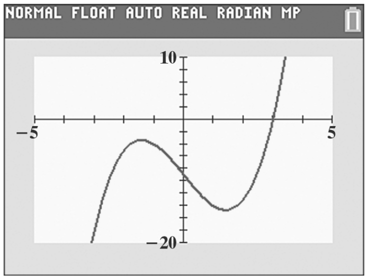
$f(x)$ has 5 sign variations, so either 5, 3, or 1 positive real roots exist.

$f(-x) = -x^5 - x^4 - x^3 - x^2 - x - 8$

$f(-x)$ has no sign variations, so no negative real roots exist.

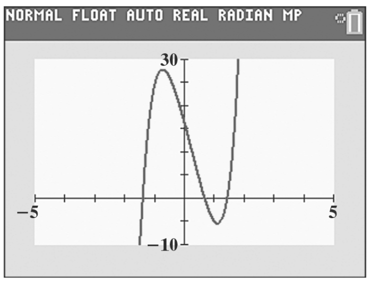
78. Odd functions must have at least one real zero. Even functions do not.

79. $f(x) = x^3 - 6x - 9$



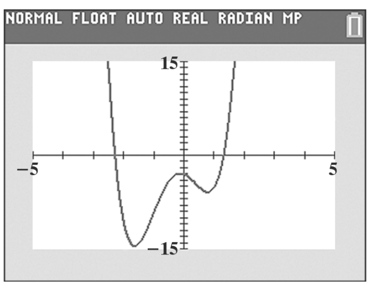
1 real zero
2 nonreal complex zeros

80. $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$

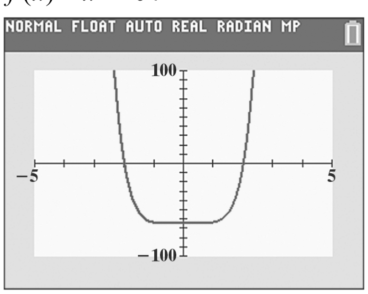


3 real zeros
2 nonreal complex zeros

81. $f(x) = 3x^4 + 4x^3 - 7x^2 - 2x - 3$



82. $f(x) = x^6 - 64$



2 real zeros
4 nonreal complex zeros

83. makes sense

84. does not make sense; Explanations will vary. Sample explanation: The quadratic formula is can be applied only of equations of degree 2.

85. makes sense

86. makes sense

87. false; Changes to make the statement true will vary. A sample change is: The equation has 0 sign variations, so no positive roots exist.

88. false; Changes to make the statement true will vary. A sample change is: Descartes' Rule gives the maximum possible number of real roots.

89. true

90. false; Changes to make the statement true will vary. A sample change is: Polynonials of degree *n* have at most *n* distinct solutions.

91. $(2x+1)(x+5)(x+2) - 3x(x+5) = 208$

$(2x^2 + 11x + 5)(x+2) - 3x^2 - 15x = 208$

$2x^3 + 4x^2 + 11x^2 + 22x + 5x$

$+10 - 3x^2 - 15x = 208$

$2x^3 + 15x^2 + 27x - 3x^2 - 15x - 198 = 0$

$2x^3 + 12x^2 + 12x - 198 = 0$

$2(x^3 + 6x^2 + 6x - 99) = 0$

3	1	6	6	–99
		3	27	99
	1	9	33	0

$x^2 + 9x + 33 = 0$

$b^2 - 4ac = -51$

$x = 3$ in.

92. Answers will vary

93. Because the polynomial has two obvious changes of direction; the smallest degree is 3.

94. Because the polynomial has no obvious changes of direction but the graph is obviously not linear, the smallest degree is 3.

95. Because the polynomial has two obvious changes of direction and two roots have multiplicity 2, the smallest degree is 5.

96. Two roots appear twice, the smallest degree is 5.

97. Answers will vary.

98. **a.** $(f \circ g)(x) = f(g(x))$
$= 4 - (x+5)^2$
$= 4 - x^2 - 10x - 25$
$= -x^2 - 10x - 21$

b. $\frac{f(x+h) - f(x)}{h}$
$= \frac{4 - x^2 - 2xh - h^2 - (4 - x^2)}{h}$
$= \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h}$
$= \frac{-2xh - h^2}{h}$
$= \frac{h(-2x - h)}{h}$
$= -2x - h, \quad h \neq 0$

99. Write the equation in slope-intercept form:
$x + 5y - 7 = 0$
$5y = -x + 7$
$y = -\frac{1}{5}x + \frac{7}{5}$

The slope of this line is $-\frac{1}{5}$ thus the slope of any line perpendicular to this line is 5.

Use $m = 5$ and the point (–5, 3) to write the equation.
$y - y_1 = m(x - x_1)$
$y - 3 = 5(x - (-5))$
$y - 3 = 5(x + 5)$
$y - 3 = 5x + 25$
$-5x + y - 28 = 0$
$5x - y + 28 = 0$ general form

100. $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\sqrt{9} - \sqrt{4}}{9 - 4} = \frac{3 - 2}{5} = \frac{1}{5}$

101. The function is undefined at $x = 1$ and $x = 2$.

102. The equation of the vertical asymptote is $x = 1$.

103. The equation of the horizontal asymptote is $y = 0$.

Mid-Chapter 2 Check Point

1. $(6 - 2i) - (7 - i) = 6 - 2i - 7 + i = -1 - i$

2. $3i(2 + i) = 6i + 3i^2 = -3 + 6i$

3. $(1 + i)(4 - 3i) = 4 - 3i + 4i - 3i^2$
$= 4 + i + 3 = 7 + i$

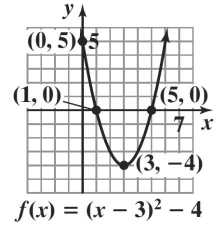
4. $\frac{1+i}{1-i} = \frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1 + i + i + i^2}{1 - i^2}$
$= \frac{1 + 2i - 1}{1 + 1}$
$= \frac{2i}{2}$
$= i$

5. $\sqrt{-75} - \sqrt{-12} = 5i\sqrt{3} - 2i\sqrt{3} = 3i\sqrt{3}$

6. $(2 - \sqrt{-3})^2 = (2 - i\sqrt{3})^2$
$= 4 - 4i\sqrt{3} + 3i^2$
$= 4 - 4i\sqrt{3} - 3$
$= 1 - 4i\sqrt{3}$

7. $x(2x - 3) = -4$
$2x^2 - 3x = -4$
$2x^2 - 3x + 4 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$
$x = \frac{3 \pm \sqrt{-23}}{4}$
$x = \frac{3}{4} \pm \frac{\sqrt{23}}{4}i$

8. $f(x) = (x - 3)^2 - 4$
The parabola opens up because $a > 0$.
The vertex is (3, –4).
x-intercepts:
$0 = (x - 3)^2 - 4$
$(x - 3)^2 = 4$
$x - 3 = \pm\sqrt{4}$
$x = 3 \pm 2$
The equation has x-intercepts at $x = 1$ and $x = 5$.
y-intercept:
$f(0) = (0 - 3)^2 - 4 = 5$
domain: $(-\infty, \infty)$ range: $[-4, \infty)$



$f(x) = (x - 3)^2 - 4$

9. $f(x) = 5 - (x+2)^2$

The parabola opens down because $a < 0$.
The vertex is (–2, 5).
x-intercepts:

$0 = 5 - (x+2)^2$

$(x+2)^2 = 5$

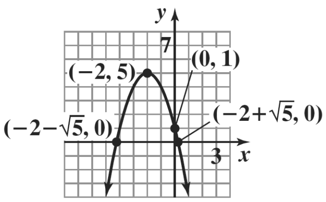
$x+2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

y-intercept:

$f(0) = 5 - (0+2)^2 = 1$

domain: $(-\infty, \infty)$ range: $(-\infty, 5]$



10. $f(x) = -x^2 - 4x + 5$

The parabola opens down because $a < 0$.

vertex: $x = -\frac{b}{2a} = -\frac{-4}{2(-1)} = -2$

$f(-2) = -(-2)^2 - 4(-2) + 5 = 9$

The vertex is (–2, 9).
x-intercepts:

$0 = -x^2 - 4x + 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-1)(5)}}{2(-1)}$

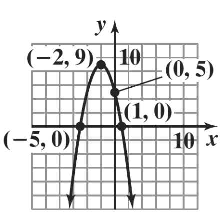
$x = \frac{4 \pm \sqrt{36}}{-2}$

$x = -2 \pm 3$

The x-intercepts are $x = 1$ and $x = -5$.
y-intercept:

$f(0) = -0^2 - 4(0) + 5 = 5$

domain: $(-\infty, \infty)$ range: $(-\infty, 9]$



11. $f(x) = 3x^2 - 6x + 1$

The parabola opens up because $a > 0$.

vertex: $x = -\frac{b}{2a} = -\frac{-6}{2(3)} = 1$

$f(1) = 3(1)^2 - 6(1) + 1 = -2$

The vertex is (1, –2).
x-intercepts:

$0 = 3x^2 - 6x + 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$

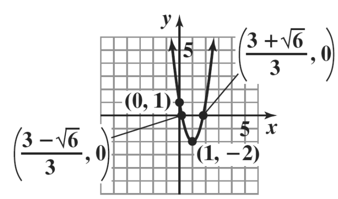
$x = \frac{6 \pm \sqrt{24}}{6}$

$x = \frac{3 \pm \sqrt{6}}{3}$

y-intercept:

$f(0) = 3(0)^2 - 6(0) + 1 = 1$

domain: $(-\infty, \infty)$ range: $[-2, \infty)$



12. $f(x) = (x-2)^2(x+1)^3$

$(x-2)^2(x+1)^3 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^3 = 0$

$x - 2 = 0$ $\quad$ $x + 1 = 0$

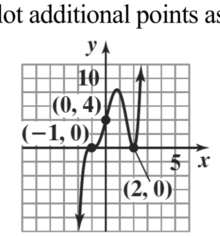
$x = 2$ $\quad$ $x = -1$

The zeros are -1 and 2.

The graph of f crosses the x-axis at -1, since the zero has multiplicity 3. The graph touches the x-axis and turns around at 2 since the zero has multiplicity 2.

Since f is an odd-degree polynomial, degree 5, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



13. $f(x) = -(x-2)^2(x+1)^2$

$-(x-2)^2(x+1)^2 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^2 = 0$

$x-2 = 0$ $\quad$ $x+1 = 0$

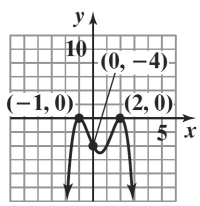
$x = 2$ $\quad$ $x = -1$

The zeros are -1 and 2.

The graph touches the x-axis and turns around both at -1 and 2 since both zeros have multiplicity 2.

Since f is an even-degree polynomial, degree 4, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x-2)^2(x+1)^2$

14. $f(x) = x^3 - x^2 - 4x + 4$

$x^3 - x^2 - 4x + 4 = 0$

$x^2(x-1) - 4(x-1) = 0$

$(x^2-4)(x-1) = 0$

$(x+2)(x-2)(x-1) = 0$

Apply the zero-product principle:

$x+2 = 0$ or $x-2 = 0$ or $x-1 = 0$

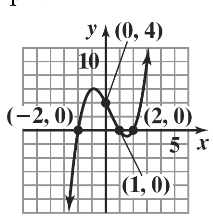
$x = -2$ $\quad$ $x = 2$ $\quad$ $x = 1$

The zeros are -2, 1, and 2.

The graph of f crosses the x-axis at all three zeros, -2, 1, and 2, since all have multiplicity 1.

Since f is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^3 - x^2 - 4x + 4$

15. $f(x) = x^4 - 5x^2 + 4$

$x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2-1) = 0$

$(x+2)(x-2)(x+1)(x-1) = 0$

Apply the zero-product principle,

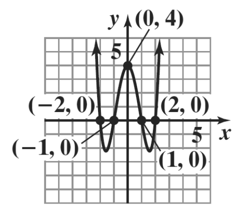
$x = -2,\ x = 2,\ x = -1,\ x = 1$

The zeros are -2, -1, 1, and 2.

The graph crosses the x-axis at all four zeros, -2, -1, 1, and 2., since all have multiplicity 1.

Since f is an even-degree polynomial, degree 4, and since the leading coefficient, 1, is positive, the graph rises to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^4 - 5x^2 + 4$

16. $f(x) = -(x+1)^6$

$-(x+1)^6 = 0$

$(x+1)^6 = 0$

$x+1 = 0$

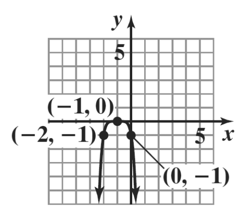
$x = -1$

The zero is are -1.

The graph touches the x-axis and turns around at -1 since the zero has multiplicity 6.

Since f is an even-degree polynomial, degree 6, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x+1)^6$

17. $f(x) = -6x^3 + 7x^2 - 1$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -1: ± 1
List all factors of the leading coefficient -6:
$\pm 1, \pm 2, \pm 3, \pm 6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } -6} = \frac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$$
$$= \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$$

We test values from the above list until we find a zero. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & -6 & 7 & 0 & -1 \\ & & -6 & 1 & 1 \\ \hline & -6 & 1 & 1 & 0 \end{array}$$

The remainder is 0, so 1 is a zero. Thus,

$$-6x^3 + 7x^2 - 1 = 0$$
$$(x-1)\left(-6x^2 + x + 1\right) = 0$$
$$-(x-1)\left(6x^2 - x - 1\right) = 0$$
$$-(x-1)(3x+1)(2x-1) = 0$$

Apply the zero-product property:

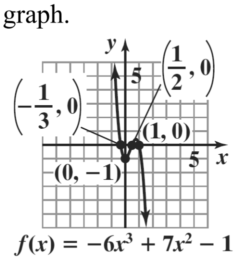
$x = 1, \quad x = -\frac{1}{3}, \quad x = \frac{1}{2}$

The zeros are $-\frac{1}{3}$, $\frac{1}{2}$, and 1.

The graph of *f* crosses the *x*-axis at all three zeros, $-\frac{1}{3}$, $\frac{1}{2}$, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -6, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



18. $f(x) = 2x^3 - 2x$

$$2x^3 - 2x = 0$$
$$2x\left(x^2 - 1\right) = 0$$
$$2x(x+1)(x-1) = 0$$

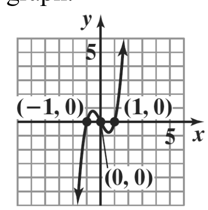
Apply the zero-product principle:

$x = 0, \qquad x = -1, \qquad x = 1$

The zeros are -1, 0, and 1.
The graph of *f* crosses the *x*-axis at all three zeros, -1, 0, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 2, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



19. $f(x) = x^3 - 2x^2 + 26x$

$$x^3 - 2x^2 + 26x = 0$$
$$x\left(x^2 - 2x + 26\right) = 0$$

Note that $x^2 - 2x + 26$ does not factor, so we use the quadratic formula:

$x = 0$ or $x^2 - 2x + 26 = 0$
$a = 1, \quad b = -2, \quad c = 26$

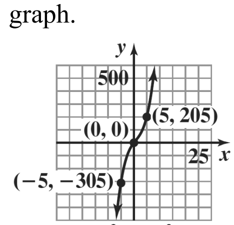
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(26)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{-100}}{2} = \frac{2 \pm 10i}{2} = 1 \pm 5i$$

The zeros are 0 and $1 \pm 5i$.

The graph of *f* crosses the *x*-axis at 0 (the only real zero), since it has multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



20. $f(x) = -x^3 + 5x^2 - 5x - 3$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -3: $\pm 1, \pm 3$
List all factors of the leading coefficient -1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -3}{\text{Factors of } -1} = \frac{\pm 1, \ \pm 3}{\pm 1} = \pm 1, \ \pm 3$$

We test values from the previous list until we find a zero. One is shown next:

Test 3:

$$\begin{array}{r|rrrr} 3 & -1 & 5 & -5 & -3 \\ & & -3 & 6 & 3 \\ \hline & -1 & 2 & 1 & 0 \end{array}$$

The remainder is 0, so 3 is a zero. Thus,

$$\begin{aligned} -x^3 + 5x^2 - 5x - 3 &= 0 \\ (x-3)\left(-x^2 + 2x + 1\right) &= 0 \\ -(x-3)\left(x^2 - 2x - 1\right) &= 0 \end{aligned}$$

Note that $x^2 - 2x - 1$ does not factor, so we use the quadratic formula:

$x - 3 = 0$ or $x^2 - 2x - 1 = 0$
$x = 3$ $\quad a = 1, \ b = -2, \ c = -1$

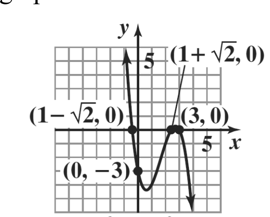
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

The zeros are 3 and $1 \pm \sqrt{2}$.

The graph of *f* crosses the *x*-axis at all three zeros, 3 and $1 \pm \sqrt{2}$, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -1, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -x^3 + 5x^2 - 5x - 3$

21. $x^3 - 3x + 2 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 2: $\pm 1, \pm 2$
Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } 2}{\text{Factors of } 1} = \frac{\pm 1, \ \pm 2}{\pm 1} = \pm 1, \ \pm 2$$

We test values from above until we find a root. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

The remainder is 0, so 1 is a root of the equation. Thus,

$$\begin{aligned} x^3 - 3x + 2 &= 0 \\ (x-1)\left(x^2 + x - 2\right) &= 0 \\ (x-1)(x+2)(x-1) &= 0 \\ (x-1)^2 (x+2) &= 0 \end{aligned}$$

Apply the zero-product property:

$(x-1)^2 = 0$ or $x + 2 = 0$
$x - 1 = 0$ $\quad x = -2$
$x = 1$

The solutions are -2 and 1, and the solution set is $\{-2, 1\}$.

22. $6x^3 - 11x^2 + 6x - 1 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -1: ± 1
Factors of the leading coefficient 6:
$\pm 1, \ \pm 2, \ \pm 3, \ \pm 6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } 6} = \frac{\pm 1}{\pm 1, \ \pm 2, \ \pm 3, \ \pm 6}$$
$$= \pm 1, \ \pm \frac{1}{2}, \ \pm \frac{1}{3}, \ \pm \frac{1}{6}$$

We test values from above until we find a root. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & 6 & -11 & 6 & -1 \\ & & 6 & -5 & 1 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

The remainder is 0, so 1 is a root of the equation. Thus,

$$6x^3 - 11x^2 + 6x - 1 = 0$$
$$(x-1)\left(6x^2 - 5x + 1\right) = 0$$
$$(x-1)(3x-1)(2x-1) = 0$$

Apply the zero-product property:

$x - 1 = 0$ or $3x - 1 = 0$ or $2x - 1 = 0$

$x = 1$ $\quad x = \frac{1}{3}$ $\quad x = \frac{1}{2}$

The solutions are $\frac{1}{3}$, $\frac{1}{2}$ and 1, and the solution set is $\left\{\frac{1}{3}, \frac{1}{2}, 1\right\}$.

23. $(2x+1)(3x-2)^3(2x-7) = 0$

Apply the zero-product property:

$2x + 1 = 0$ or $(3x-2)^3 = 0$ or $2x - 7 = 0$ T

$x = -\frac{1}{2}$ $\quad 3x - 2 = 0$ $\quad x = \frac{7}{2}$

$x = \frac{2}{3}$

he solutions are $-\frac{1}{2}$, $\frac{2}{3}$ and $\frac{7}{2}$, and the solution set is $\left\{-\frac{1}{2}, \frac{2}{3}, \frac{7}{2}\right\}$.

24. $2x^3 + 5x^2 - 200x - 500 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -500:
$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 25, \pm 50, \pm 100, \pm 125, \pm 250, \pm 500$

Factors of the leading coefficient 2: $\pm 1, \pm 2$

The possible rational zeros are:

$$\frac{\text{Factors of } 500}{\text{Factors of } 2} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 25, \pm 50, \pm 100, \pm 125, \pm 250, \pm 500, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{25}{2}, \pm\frac{125}{2}$$

We test values from above until we find a root. One is shown next:

Test 10:

10	2	5	−200	−500
		20	250	500
	2	25	50	0

The remainder is 0, so 10 is a root of the equation. Thus,

$$2x^3 + 5x^2 - 200x - 500 = 0$$
$$(x-10)\left(2x^2 + 25x + 50\right) = 0$$
$$(x-10)(2x+5)(x+10) = 0$$

Apply the zero-product property:

$x - 10 = 0$ or $2x + 5 = 0$ or $x + 10 = 0$

$x = 10$ $\quad x = -\frac{5}{2}$ $\quad x = -10$

The solutions are -10, $-\frac{5}{2}$, and 10, and the solution set is $\left\{-10, -\frac{5}{2}, 10\right\}$.

25.
$$x^4 - x^3 - 11x^2 = x + 12$$
$$x^4 - x^3 - 11x^2 - x - 12 = 0$$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -12:
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -12}{\text{Factors of } 1} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

We test values from this list we find a root. One possibility is shown next:

Test -3:

−3	1	−1	−11	−1	−12
		−3	12	−3	12
	1	−4	1	−4	0

The remainder is 0, so -3 is a root of the equation. Using the Factor Theorem, we know that $x - 1$ is a factor. Thus,

$$x^4 - x^3 - 11x^2 - x - 12 = 0$$
$$(x+3)\left(x^3 - 4x^2 + x - 4\right) = 0$$
$$(x+3)\left[x^2(x-4) + 1(x-4)\right] = 0$$
$$(x+3)(x-4)\left(x^2 + 1\right) = 0$$

As this point we know that -3 and 4 are roots of the equation. Note that $x^2 + 1$ does not factor, so we use the square-root principle:

$$x^2 + 1 = 0$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

The roots are -3, 4, and $\pm i$, and the solution set is $\{-3, 4, \pm i\}$.

26. $2x^4 + x^3 - 17x^2 - 4x + 6 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 6: $\pm 1, \ \pm 2, \ \pm 3, \ \pm 6$
Factors of the leading coefficient 4: $\pm 1, \ \pm 2$

The possible rational roots are:

$$\frac{\text{Factors of 6}}{\text{Factors of 2}} = \frac{\pm 1, \ \pm 2, \ \pm 3, \ \pm 6}{\pm 1, \ \pm 2}$$
$$= \pm 1, \ \pm 2, \ \pm 3, \ \pm 6, \ \pm\frac{1}{2}, \ \pm\frac{3}{2}$$

We test values from above until we find a root. One possibility is shown next:

Test -3:

-3	2	1	-17	-4	6
		-6	15	6	-6
	2	-5	-2	2	0

The remainder is 0, so -3 is a root. Using the Factor Theorem, we know that $x+3$ is a factor of the polynomial. Thus,

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)\left(2x^3 - 5x^2 - 2x + 2\right) = 0$$

To solve the equation above, we need to factor $2x^3 - 5x^2 - 2x + 2$. We continue testing potential roots:

Test $\frac{1}{2}$:

$\frac{1}{2}$	2	-5	-2	2
		1	-2	-2
	2	-4	-4	0

The remainder is 0, so $\frac{1}{2}$ is a zero and $x - \frac{1}{2}$ is a factor.

Summarizing our findings so far, we have

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)\left(2x^3 - 5x^2 - 2x + 2\right) = 0$$
$$(x+3)\left(x - \frac{1}{2}\right)\left(2x^2 - 4x - 4\right) = 0$$
$$2(x+3)\left(x - \frac{1}{2}\right)\left(x^2 - 2x - 2\right) = 0$$

At this point, we know that -3 and $\frac{1}{2}$ are roots of the equation. Note that $x^2 - 2x - 2$ does not factor, so we use the quadratic formula:

$$x^2 - 2x - 2 = 0$$
$$a = 1, \ \ b = -2, \ \ c = -2$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

The solutions are -3, $\frac{1}{2}$, and $1 \pm \sqrt{3}$, and the solution set is $\left\{-3, \ \frac{1}{2}, \ 1 \pm \sqrt{3}\right\}$.

27. $P(x) = -x^2 + 150x - 4425$

Since $a = -1$ is negative, we know the function opens down and has a maximum at

$$x = -\frac{b}{2a} = -\frac{150}{2(-1)} = -\frac{150}{-2} = 75.$$

$$P(75) = -75^2 + 150(75) - 4425$$
$$= -5625 + 11,250 - 4425 = 1200$$

The company will maximize its profit by manufacturing and selling 75 cabinets per day. The maximum daily profit is \$1200.

28. Let x = one of the numbers;
$-18 - x$ = the other number

The product is $f(x) = x(-18 - x) = -x^2 - 18x$

The x-coordinate of the maximum is

$$x = -\frac{b}{2a} = -\frac{-18}{2(-1)} = -\frac{-18}{-2} = -9.$$

$$f(-9) = -9\left[-18 - (-9)\right]$$
$$= -9(-18 + 9) = -9(-9) = 81$$

The vertex is $(-9, 81)$. The maximum product is 81. This occurs when the two numbers are -9 and $-18 - (-9) = -9$.

29. Let x = height of triangle;
$40 - 2x$ = base of triangle

$$A = \frac{1}{2}bh = \frac{1}{2}x(40 - 2x)$$

$$A(x) = 20x - x^2$$

The height at which the triangle will have maximum area is $x = -\frac{b}{2a} = -\frac{20}{2(-1)} = 10.$

$$A(10) = 20(10) - (10)^2 = 100$$

The maximum area is 100 squares inches.

30.

$$\begin{array}{r} 2x^2 - x - 3 \\ 3x^2 - 1 \overline{)6x^4 - 3x^3 - 11x^2 + 2x + 4} \\ \underline{6x^4 \quad - 2x^2} \\ -3x^3 - 9x^2 + 2x \\ \underline{-3x^3 \quad + x} \\ -9x^2 + x + 4 \\ \underline{-9x^2 \quad + 3} \\ x + 1 \end{array}$$

$$2x^2 - x - 3 + \frac{x+1}{3x^2-1}$$

31. $\left(2x^4 - 13x^3 + 17x^2 + 18x - 24\right) \div (x-4)$

4	2	–13	17	18	–24
		8	-20	–12	24
	2	–5	–3	6	0

The quotient is $2x^3 - 5x^2 - 3x + 6$.

32. $(x-1)(x-i)(x+i) = (x-1)(x^2+1)$

$f(x) = a_n(x-1)(x^2+1)$
$f(-1) = a_n(-1-1)\left((-1)^2+1\right) = -4a_n = 8$
$a_n = -2$
$f(x) = -2(x-1)(x^2+1)$ or $-2x^3 + 2x^2 - 2x + 2$

33. $(x-2)(x-2)(x-3i)(x+3i)$

$= (x-2)(x-2)(x^2+9)$

$f(x) = a_n(x-2)(x-2)(x^2+9)$
$f(0) = a_n(0-2)(0-2)(0^2+9)$
$36 = 36a_n$
$a_n = 1$
$f(x) = 1(x-2)(x-2)(x^2+9)$
$f(x) = x^4 - 4x^3 + 13x^2 - 36x + 36$

34. $f(x) = x^3 - x - 5$

$f(1) = 1^3 - 1 - 5 = -5$

$f(2) = 2^3 - 2 - 5 = 1$

Yes, the function must have a real zero between 1 and 2 because $f(1)$ and $f(2)$ have opposite signs.

Section 2.6

Check Point Exercises

1. Because division by 0 is undefined, we must exclude from the domain of each function values of x that cause the polynomial function in the denominator to be 0.

a. $x - 5 = 0$
$x = 5$

$\{x | x \neq 5\}$ or $(-\infty, 5) \cup (5, \infty)$.

b. $x^2 - 25 = 0$

$x^2 = 25$
$x = \pm 5$
$\{x \mid x \neq 5, x \neq -5\}$ or $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$.

c. The denominator cannot equal zero.
All real numbers or $(-\infty, \infty)$..

2. a. $x^2 - 1 = 0$
$x^2 = 1$
$x = 1, x = -1$

b. $g(x) = \dfrac{x-1}{x^2-1} = \dfrac{x-1}{(x-1)(x+1)} = \dfrac{1}{x+1}$

$x = -1$

c. The denominator cannot equal zero.
No vertical asymptotes.

3. a. The degree of the numerator, 2, is equal to the degree of the denominator, 2. Thus, the leading coefficients of the numerator and denominator, 9 and 3, are used to obtain the equation of the horizontal asymptote.

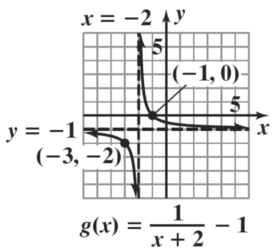
$y = \dfrac{9}{3} = 3$

$y = 3$ is a horizontal asymptote.

b. The degree of the numerator, 1, is less than the degree of the denominator, 2. Thus, the graph has the x-axis as a horizontal asymptote
$y = 0$ is a horizontal asymptote.

c. The degree of the numerator, 3, is greater than the degree of the denominator, 2. Thus, the graph has no horizontal asymptote.

4. Begin with the graph of $f(x)=\frac{1}{x}$.



Shift the graph 2 units to the left by subtracting 2 from each x-coordinate. Shift the graph 1 unit down by subtracting 1 from each y-coordinate.

5. $f(x)=\frac{3x-3}{x-2}$

$f(-x)=\frac{3(-x)-3}{-x-2}=\frac{-3x-3}{-x-2}=\frac{3x+3}{x+2}$

no symmetry

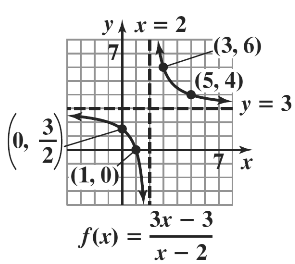
$f(0)=\frac{3(0)-3}{0-2}=\frac{3}{2}$

The y-intercept is $\frac{3}{2}$.

$3x-3=0$
$3x=3$
$x=1$

The x-intercept is 1.
Vertical asymptote:
$x-2=0$
$x=2$
Horizontal asymptote:

$y=\frac{3}{1}=3$



6. $f(x)=\frac{2x^2}{x^2-9}$

$f(-x)=\frac{2(-x)^2}{(-x)^2-9}=\frac{2x^2}{x^2-9}=f(x)$

The y-axis symmetry.

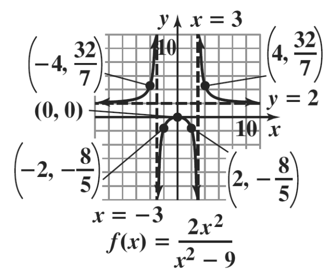
$f(0)=\frac{2(0)^2}{0^2-9}=0$

The y-intercept is 0.

$2x^2=0$
$x=0$

The x-intercept is 0.
vertical asymptotes:
$x^2-9=0$
$x=3, x=-3$
horizontal asymptote:

$y=\frac{2}{1}=2$



7. $f(x)=\frac{x^4}{x^2+2}$

$f(-x)=\frac{(-x)^4}{(-x)^2+2}=\frac{x^4}{x^2+2}=f(x)$

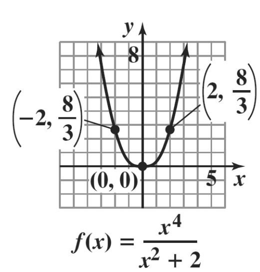
y-axis symmetry

$f(0)=\frac{0^4}{0^2+2}=0$

The y-intercept is 0.

$x^4=0$
$x=0$

The x-intercept is 0.
vertical asymptotes:
$x^2+2=0$
$x^2=-2$
no vertical asymptotes
horizontal asymptote:
Since $n>m$, there is no horizontal asymptote.



8.

2	2	–5	7
		4	–2
	2	–1	5

the equation of the slant asymptote is $y = 2x - 1$.

9. **a.** $C(x) = 500,000 + 400x$

b. $\bar{C}(x) = \dfrac{500,000 + 400x}{x}$

c. $\bar{C}(1000) = \dfrac{500,000 + 400(1000)}{1000} = 900$

$\bar{C}(10,000) = \dfrac{500,000 + 400(10,000)}{10,000} = 450$

$\bar{C}(100,000) = \dfrac{500,000 + 400(100,000)}{100,000} = 405$

The average cost per wheelchair of producing 1000, 10,000, and 100,000 wheelchairs is \$900, \$450, and \$405, respectively.

d. $y = \dfrac{400}{1} = 400$

The cost per wheelchair approaches \$400 as more wheelchairs are produced.

10. $x - 10$ = the average velocity on the return trip. The function that expresses the total time required to complete the round trip is $T(x) = \dfrac{20}{x} + \dfrac{20}{x-10}$.

Concept and Vocabulary Check 2.6

1. polynomial

2. false

3. true

4. vertical asymptote; $x = -5$

5. horizontal asymptote; $y = 0$; $y = \dfrac{1}{3}$

6. true

7. left; down

8. one more than

9. $y = 3x + 5$

10. $x - 20$; $\dfrac{30}{x-20}$

Exercise Set 2.6

1. $f(x) = \dfrac{5x}{x-4}$

$\{x \mid x \neq 4\}$

2. $f(x) = \dfrac{7x}{x-8}$

$\{x \mid x \neq 8\}$

3. $g(x) = \dfrac{3x^2}{(x-5)(x+4)}$

$\{x \mid x \neq 5, x \neq -4\}$

4. $g(x) = \dfrac{2x^2}{(x-2)(x+6)}$

$\{x \mid x \neq 2, x \neq -6\}$

5. $h(x) = \dfrac{x+7}{x^2-49}$

$x^2 - 49 = (x-7)(x+7)$
$\{x \mid x \neq 7, x \neq -7\}$

6. $h(x) = \dfrac{x+8}{x^2-64}$

$x^2 - 64 = (x-8)(x+8)$
$\{x \mid x \neq 8, x \neq -8\}$

7. $f(x) = \dfrac{x+7}{x^2+49}$

all real numbers

8. $f(x) = \dfrac{x+8}{x^2+64}$

all real numbers

9. $-\infty$

10. $+\infty$

11. $-\infty$

12. $+\infty$

13. 0

14. 0

15. $+\infty$

16. $-\infty$

17. $-\infty$

18. $+\infty$

19. 1

20. 1

21. $f(x) = \frac{x}{x+4}$

$x+4=0$
$x=-4$
vertical asymptote: $x=-4$
There are no holes.

22. $f(x) = \frac{x}{x-3}$

$x-3=0$
$x=3$
vertical asymptote: $x=3$
There are no holes.

23. $g(x) = \frac{x+3}{x(x+4)}$

$x(x+4)=0$
$x=0, x=-4$
vertical asymptotes: $x=0,\ x=-4$
There are no holes.

24. $g(x) = \frac{x+3}{x(x-3)}$

$x(x-3)=0$
$x=0, x=3$
vertical asymptotes: $x=0, x=3$
There are no holes.

25. $h(x) = \frac{x}{x(x+4)} = \frac{1}{x+4}$

$x+4=0$
$x=-4$
vertical asymptote: $x=-4$
There is a hole at $x=0$.

26. $h(x) = \frac{x}{x(x-3)} = \frac{1}{x-3}$

$x-3=0$
$x=3$
vertical asymptote: $x=3$
There is a hole at $x=0$.

27. $r(x) = \frac{x}{x^2+4}$

x^2+4 has no real zeros
There are no vertical asymptotes.
There are no holes.

28. $r(x) = \frac{x}{x^2+3}$

x^2+3 has no real zeros
There is no vertical asymptotes.
There are no holes.

29. $f(x) = \frac{x^2-9}{x-3}$

$= \frac{(x+3)(x-3)}{x-3}$

$= x+3$

There are no vertical asymptotes.
There is a hole at $x=3$.

30. $f(x) = \frac{x^2-25}{x-5}$

$= \frac{(x+5)(x-5)}{x-5}$

$= x+5$

There are no vertical asymptotes.
There is a hole at $x=5$.

31. $g(x) = \frac{x-3}{x^2-9}$

$= \frac{x-3}{(x+3)(x-3)}$

$= \frac{1}{x+3}$

vertical asymptote: $x=-3$
There is a hole at $x=3$.

32. $g(x) = \frac{x-5}{x^2-25}$

$= \frac{x-5}{(x+5)(x-5)}$

$= \frac{1}{x+5}$

vertical asymptote: $x=-5$
There is a hole at $x=5$.

33. $h(x) = \dfrac{x+7}{x^2+4x-21}$
$= \dfrac{x+7}{(x+7)(x-3)}$
$= \dfrac{1}{x-3}$
vertical asymptote: $x = 3$
There is a hole at $x = -7$.

34. $h(x) = \dfrac{x+6}{x^2+2x-24}$
$= \dfrac{x+6}{(x+6)(x-4)}$
$= \dfrac{1}{x-4}$
vertical asymptote: $x = 4$
There is a hole at $x = -6$.

35. $r(x) = \dfrac{x^2+4x-21}{x+7}$
$= \dfrac{(x+7)(x-3)}{x+7}$
$= x-3$
There are no vertical asymptotes.
There is a hole at $x = -7$.

36. $r(x) = \dfrac{x^2+2x-24}{x+6}$
$= \dfrac{(x+6)(x-4)}{x+6}$
$= x-4$
There are no vertical asymptotes.
There is a hole at $x = -6$.

37. $f(x) = \dfrac{12x}{3x^2+1}$
$n < m$
horizontal asymptote: $y = 0$

38. $f(x) = \dfrac{15x}{3x^2+1}$
$n < m$
horizontal asymptote: $y = 0$

39. $g(x) = \dfrac{12x^2}{3x^2+1}$
$n = m$,
horizontal asymptote: $y = \dfrac{12}{3} = 4$

40. $g(x) = \dfrac{15x^2}{3x^2+1}$
$n = m$
horizontal asymptote: $y = \dfrac{15}{3} = 5$

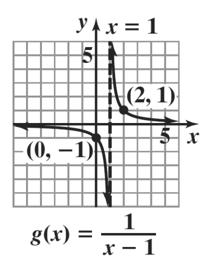
41. $h(x) = \dfrac{12x^3}{3x^2+1}$
$n > m$
no horizontal asymptote

42. $h(x) = \dfrac{15x^3}{3x^2+1}$
$n > m$
no horizontal asymptote

43. $f(x) = \dfrac{-2x+1}{3x+5}$
$n = m$
horizontal asymptote: $y = -\dfrac{2}{3}$

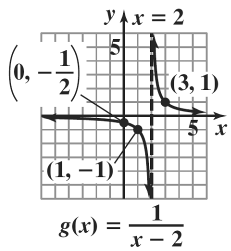
44. $f(x) = \dfrac{-3x+7}{5x-2}$
$n = m$ s
horizontal asymptote: $y = -\dfrac{3}{5}$

45. $g(x) = \dfrac{1}{x-1}$
Shift the graph of $f(x) = \dfrac{1}{x}$ 1 unit to the right.



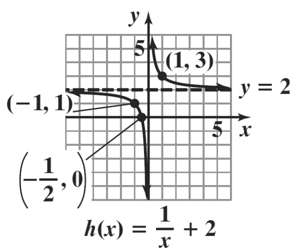
46. $g(x)=\dfrac{1}{x-2}$

Shift the graph of $f(x)=\dfrac{1}{x}$ 2 units to the right.



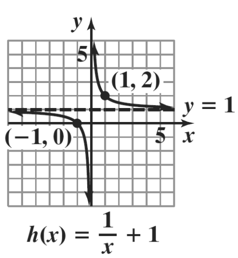
47. $h(x)=\dfrac{1}{x}+2$

Shift the graph of $f(x)=\dfrac{1}{x}$ 2 units up.



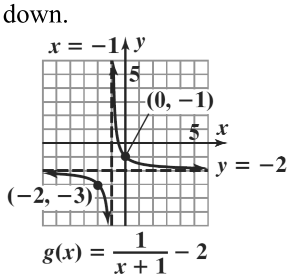
48. $h(x)=\dfrac{1}{x}+1$

Shift the graph of $f(x)=\dfrac{1}{x}$ 1 unit up.



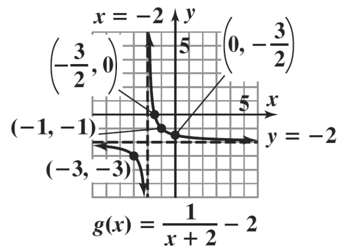
49. $g(x)=\dfrac{1}{x+1}-2$

Shift the graph of $f(x)=\dfrac{1}{x}$ 1 unit left and 2 units down.



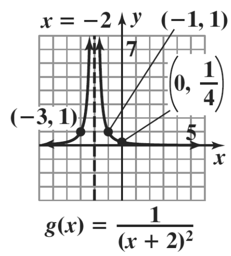
50. $g(x)=\dfrac{1}{x+2}-2$

Shift the graph of $f(x)=\dfrac{1}{x}$ 2 units left and 2 units down.



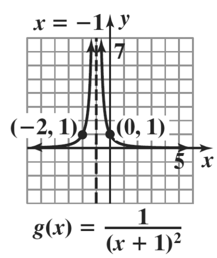
51. $g(x)=\dfrac{1}{(x+2)^2}$

Shift the graph of $f(x)=\dfrac{1}{x^2}$ 2 units left.



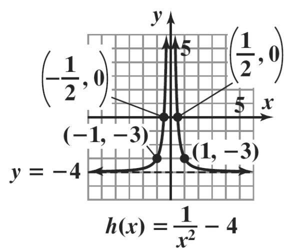
52. $g(x)=\dfrac{1}{(x+1)^2}$

Shift the graph of $f(x)=\dfrac{1}{x^2}$ 1 unit left.



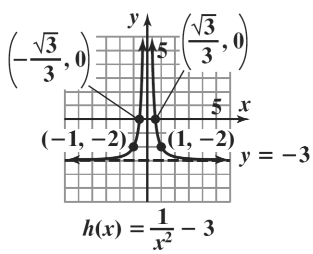
53. $h(x) = \frac{1}{x^2} - 4$

Shift the graph of $f(x) = \frac{1}{x^2}$ 4 units down.



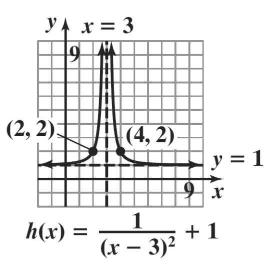
54. $h(x) = \frac{1}{x^2} - 3$

Shift the graph of $f(x) = \frac{1}{x^2}$ 3 units down.



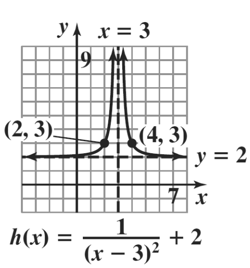
55. $h(x) = \frac{1}{(x-3)^2} + 1$

Shift the graph of $f(x) = \frac{1}{x^2}$ 3 units right and 1 unit up.



56. $h(x) = \frac{1}{(x-3)^2} + 2$

Shift the graph of $f(x) = \frac{1}{x^2}$ 3 units right and 2 units up.



57. $f(x) = \frac{4x}{x-2}$

$f(-x) = \frac{4(-x)}{(-x)-2} = \frac{4x}{x+2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \frac{4(0)}{0-2} = 0$

x-intercept: $4x = 0$

$x = 0$

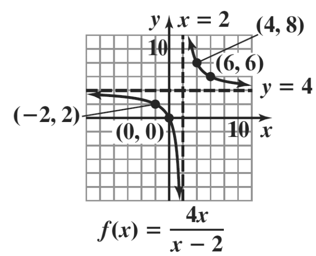
vertical asymptote:

$x - 2 = 0$

$x = 2$

horizontal asymptote:

$n = m$, so $y = \frac{4}{1} = 4$



58. $f(x) = \frac{3x}{x-1}$

$f(-x) = \frac{3(-x)}{(-x)-1} = \frac{3x}{x+1}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \frac{3(0)}{0-1} = 0$

x-intercept: $3x = 0$

$x = 0$

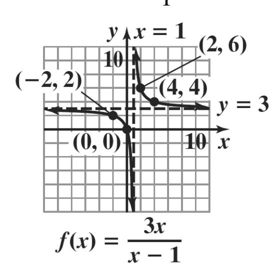
vertical asymptote:

$x - 1 = 0$

$x = 1$

horizontal asymptote:

$n = m$, so $y = \frac{3}{1} = 3$



59. $f(x)=\dfrac{2x}{x^2-4}$

$f(-x)=\dfrac{2(-x)}{(-x)^2-4}=-\dfrac{2x}{x^2-4}=-f(x)$

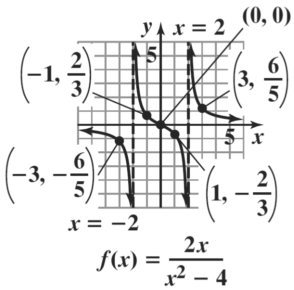
Origin symmetry

y-intercept: $\dfrac{2(0)}{0^2-4}=\dfrac{0}{-4}=0$

x-intercept:
$2x=0$
$x=0$

vertical asymptotes:
$x^2-4=0$
$x=\pm2$

horizontal asymptote:
$n<m$ so $y=0$



60. $f(x)=\dfrac{4x}{x^2-1}$

$f(-x)=\dfrac{4(-x)}{(-x)^2-1}=-\dfrac{4x}{x^2-1}=-f(x)$

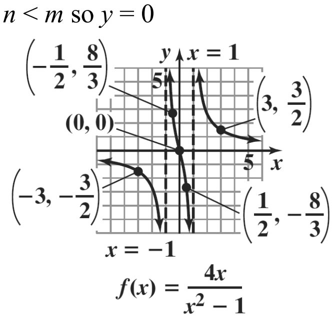
Origin symmetry

y-intercept: $\dfrac{4(0)}{0^2-1}=0$

x-intercept: $4x=0$
$x=0$

vertical asymptotes:
$x^2-1=0$
$(x-1)(x+1)=0$
$x=\pm1$

horizontal asymptote:
$n<m$ so $y=0$



61. $f(x)=\dfrac{2x^2}{x^2-1}$

$f(-x)=\dfrac{2(-x)^2}{(-x)^2-1}=\dfrac{2x^2}{x^2-1}=f(x)$

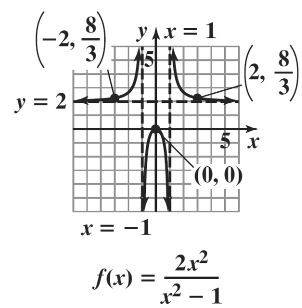
y-axis symmetry

y-intercept: $y=\dfrac{2(0)^2}{0^2-1}=\dfrac{0}{1}=0$

x-intercept:
$2x^2=0$
$x=0$

vertical asymptote:
$x^2-1=0$
$x^2=1$
$x=\pm1$

horizontal asymptote:
$n=m$, so $y=\dfrac{2}{1}=2$



62. $f(x)=\dfrac{4x^2}{x^2-9}$

$f(-x)=\dfrac{4(-x)^2}{(-x)^2-9}=\dfrac{4x^2}{x^2-9}=f(x)$

y-axis symmetry

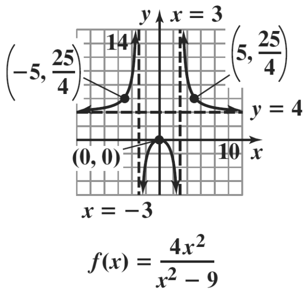
y-intercept: $y=\dfrac{4(0)^2}{0^2-9}=0$

x-intercept:
$4x^2=0$
$x=0$

vertical asymptotes:
$x^2-9=0$
$(x-3)(x+3)=0$
$x=\pm3$

horizontal asymptote:

$n = m$, so $y = \dfrac{4}{1} = 4$



63. $f(x) = \dfrac{-x}{x+1}$

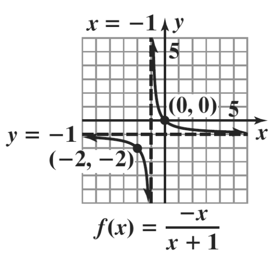
$f(-x) = \dfrac{-(-x)}{(-x)+1} = \dfrac{x}{-x+1}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{-(0)}{0+1} = \dfrac{0}{1} = 0$

x-intercept:
$-x = 0$
$x = 0$
vertical asymptote:
$x + 1 = 0$
$x = -1$
horizontal asymptote:

$n = m$, so $y = \dfrac{-1}{1} = -1$



64. $f(x) = \dfrac{-3x}{x+2}$

$f(-x) = \dfrac{-3(-x)}{(-x)+2} = \dfrac{3x}{-x+2}$

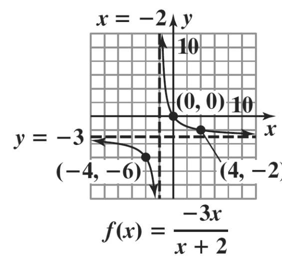
$f-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry
y-intercept:

$y = \dfrac{-3(0)}{0+2} = 0$

x-intercept:
$-3x = 0$
$x = 0$

vertical asymptote:
$x + 2 = 0$
$x = -2$
horizontal asymptote:

$n = m$, so $y = \dfrac{-3}{1} = -3$



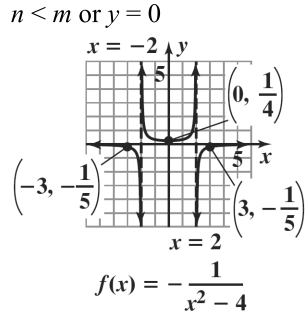
65. $f(x) = -\dfrac{1}{x^2-4}$

$f(-x) = -\dfrac{1}{(-x)^2-4} = -\dfrac{1}{x^2-4} = f(x)$

y-axis symmetry

y-intercept: $y = -\dfrac{1}{0^2-4} = \dfrac{1}{4}$

x-intercept: $-1 \neq 0$
no x-intercept
vertical asymptotes:
$x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$
horizontal asymptote:
$n < m$ or $y = 0$



66. $f(x) = -\dfrac{2}{x^2 - 1}$

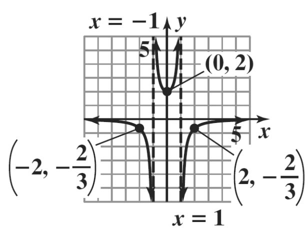
$f(-x) = -\dfrac{2}{(-x)^2 - 1} = -\dfrac{2}{x^2 - 1} = f(x)$

y-axis symmetry
y-intercept:

$y = -\dfrac{2}{0^2 - 1} = -\dfrac{2}{-1} = 2$

x-intercept: $-2 = 0$
no x-intercept
vertical asymptotes:

$x^2 - 1 = 0$
$(x - 1)(x + 1)$
$x = \pm 1$
horizontal asymptote:
$n < m$, so $y = 0$



$f(x) = -\dfrac{2}{x^2 - 1}$

67. $f(x) = \dfrac{2}{x^2 + x - 2}$

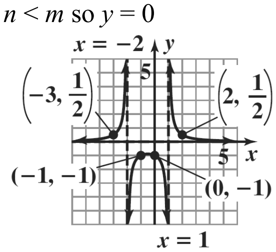
$f(-x) = -\dfrac{2}{(-x)^2 - x - 2} = \dfrac{2}{x^2 - x - 2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{2}{0^2 + 0 - 2} = \dfrac{2}{-2} = -1$

x-intercept: none
vertical asymptotes:

$x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$
$x = -2, x = 1$
horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{2}{x^2 + x - 2}$

68. $f(x) = \dfrac{-2}{x^2 - x - 2}$

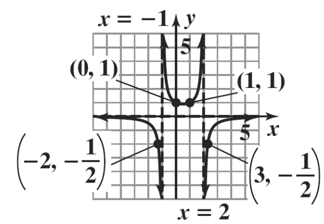
$f(-x) = \dfrac{-2}{(-x)^2 - (-x) - 2} = \dfrac{-2}{x^2 + x - 2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{-2}{0^2 - 0 - 2} = 1$

x-intercept: none
vertical asymptotes:

$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x = 2, x = -1$
horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{-2}{x^2 - x - 2}$

69. $f(x) = \dfrac{2x^2}{x^2 + 4}$

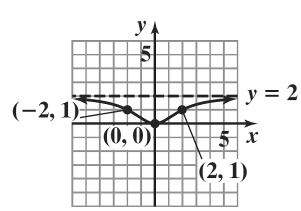
$f(-x) = \dfrac{2(-x)^2}{(-x)^2 + 4} = \dfrac{2x^2}{x^2 + 4} = f(x)$

y axis symmetry

y-intercept: $y = \dfrac{2(0)^2}{0^2 + 4} = 0$

x-intercept: $2x^2 = 0$
$x = 0$
vertical asymptote: none
horizontal asymptote:

$n = m$, so $y = \dfrac{2}{1} = 2$



$f(x) = \dfrac{2x^2}{x^2 + 4}$

70. $f(x)=\dfrac{4x^2}{x^2+1}$

$f(-x)=\dfrac{4(-x)^2}{(-x)^2+1}=\dfrac{4x^2}{x^2+1}=f(x)$

y axis symmetry

y-intercept: $y=\dfrac{4(0)^2}{0^2+1}=0$

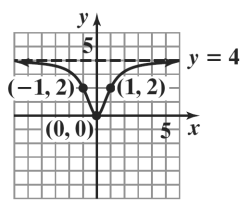
x-intercept: $4x^2=0$

$x=0$

vertical asymptote: none

horizontal asymptote:

$n=m$, so $y=\dfrac{4}{1}=4$



$f(x)=\dfrac{4x^2}{x^2+1}$

71. $f(x)=\dfrac{x+2}{x^2+x-6}$

$f(-x)=\dfrac{-x+2}{(-x)^2-(-x)-6}=\dfrac{-x+2}{x^2+x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0+2}{0^2+0-6}=-\dfrac{2}{6}=-\dfrac{1}{3}$

x-intercept:

$x+2=0$

$x=-2$

vertical asymptotes:

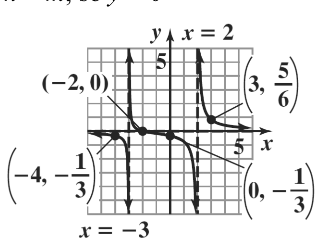
$x^2+x-6=0$

$(x+3)(x-2)$

$x=-3, x=2$

horizontal asymptote:

$n<m$, so $y=0$



$f(x)=\dfrac{x+2}{x^2+x-6}$

72. $f(x)=\dfrac{x-4}{x^2-x-6}$

$f(-x)=\dfrac{-x-4}{(-x)^2-(-x)-6}=-\dfrac{x+4}{x^2+x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0-4}{0^2-0-6}=\dfrac{2}{3}$

x-intercept:

$x-4=0, x=4$

vertical asymptotes:

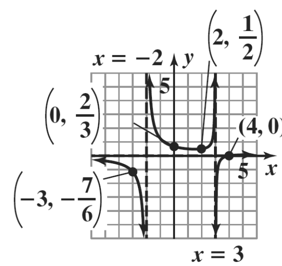
$x^2-x-6=0$

$(x-3)(x+2)$

$x=3, x=-2$

horizontal asymptote:

$n<m$, so $y=0$



$f(x)=\dfrac{x-4}{x^2-x-6}$

73. $f(x)=\dfrac{x-2}{x^2-4}$

$f(-x)=\dfrac{-x-2}{(-x)^2-4}=\dfrac{-x-2}{x^2-4}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0-2}{0^2-4}=\dfrac{-2}{-4}=\dfrac{1}{2}$

x-intercept:

$x-2=0, x=2$

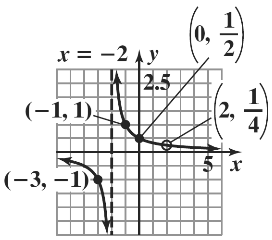
vertical asymptotes:

$$f(x)=\frac{x-2}{x^2-4}=\frac{x-2}{(x-2)(x+2)}=\frac{1}{x+2}$$

$x=-2$ is a vertical asymptote.

Furthermore, the value 2 causes the original denominator to be zero, but the reduced form of the function's equation does not cause the denominator to be zero. Thus, there is a hole at $x=2$.

horizontal asymptote:
$n < m$, so $y = 0$



$$f(x) = \frac{x-2}{x^2-4}$$

74. $f(x) = \dfrac{x-3}{x^2-9}$

$$f(-x) = \frac{-x-3}{(-x)^2-9} = \frac{-x-3}{x^2-9}$$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

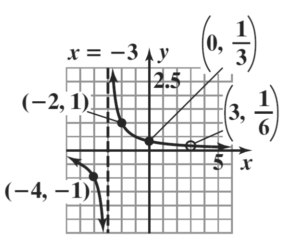
y-intercept: $y = \dfrac{0-3}{0^2-9} = \dfrac{-3}{-9} = \dfrac{1}{3}$

x-intercept:
$x - 3 = 0, x = 3$

vertical asymptotes:

$$f(x) = \frac{x-3}{x^2-9} = \frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3}$$

$x = -3$ is a vertical asymptote.
Furthermore, the value 2 causes the original denominator to be zero, but the reduced form of the function's equation does not cause the denominator to be zero. Thus, there is a hole at $x = 3$.
horizontal asymptote:
$n < m$, so $y = 0$



$$f(x) = \frac{x-3}{x^2-9}$$

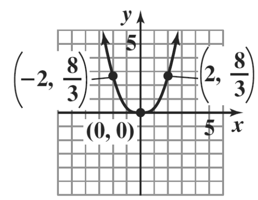
75. $f(x) = \dfrac{x^4}{x^2+2}$

$$f(-x) = \frac{(-x)^4}{(-x)^2+2} = \frac{x^4}{x^2+2} = f(x)$$

y-axis symmetry

y-intercept: $y = \dfrac{0^4}{0^2+2} = 0$

x-intercept: $x^4 = 0$
$x = 0$
vertical asymptote: none
horizontal asymptote:
$n > m$, so none



$$f(x) = \frac{x^4}{x^2+2}$$

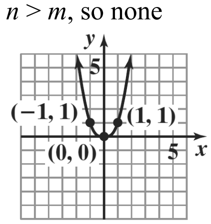
76. $f(x) = \dfrac{2x^4}{x^2+1}$

$$f(-x) = \frac{2(-x)^4}{(-x)^2+1} = \frac{2x^4}{x^2+1} = f(x)$$

y-axis symmetry

y-intercept: $y = \dfrac{2\left(0^4\right)}{0^2+2} = 0$

x-intercept: $2x^4 = 0$
$x = 0$
vertical asymptote: none
horizontal asymptote:
$n > m$, so none



$$f(x) = \frac{2x^4}{x^2+1}$$

77. $f(x)=\dfrac{x^2+x-12}{x^2-4}$

$f(-x)=\dfrac{(-x)^2-x-12}{(-x)^2-4}=\dfrac{x^2-x-12}{x^2-4}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^2+0-12}{0^2-4}=3$

x-intercept: $x^2+x-12=0$

$(x-3)(x+4)=0$

$x=3, x=-4$

vertical asymptotes:

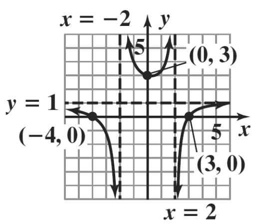
$x^2-4=0$

$(x-2)(x+2)=0$

$x=2, x=-2$

horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$f(x)=\dfrac{x^2+x-12}{x^2-4}$

78. $f(x)=\dfrac{x^2}{x^2+x-6}$

$f(-x)=\dfrac{(-x)^2}{(-x)^2-x-6}=\dfrac{x^2}{x^2-x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^2}{0^2+0-6}=0$

x-intercept: $x^2=0, x=0$

vertical asymptotes:

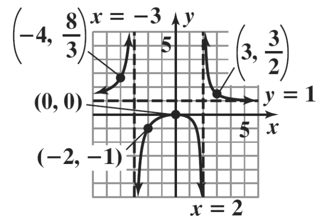
$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3, x=2$

horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$f(x)=\dfrac{x^2}{x^2+x-6}$

79. $f(x)=\dfrac{3x^2+x-4}{2x^2-5x}$

$f(-x)=\dfrac{3(-x)^2-x-4}{2(-x)^2+5x}=\dfrac{3x^2-x-4}{2x^2+5x}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{3(0)^2+0-4}{2(0)^2-5(0)}=\dfrac{-4}{0}$

no y-intercept

x-intercepts:

$3x^2+x-4=0$

$(3x+4)(x-1)=0$

$3x+4=0 \quad x-1=0$

$3x=-4$

$x=-\dfrac{4}{3}, x=1$

vertical asymptotes:

$2x^2-5x=0$

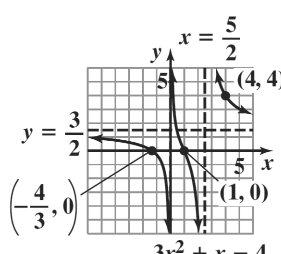
$x(2x-5)=0$

$x=0, 2x=5$

$x=\dfrac{5}{2}$

horizontal asymptote:

$n=m$, so $y=\dfrac{3}{2}$



$f(x)=\dfrac{3x^2+x-4}{2x^2-5x}$

80. $f(x)=\dfrac{x^2-4x+3}{(x+1)^2}$

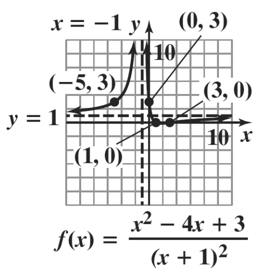
$f(-x)=\dfrac{(-x)^2-4(-x)+3}{(-x+1)^2}=\dfrac{x^2+4x+3}{(-x+1)^2}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$
no symmetry

y-intercept: $y=\dfrac{0^2-4(0)+3}{(0+1)^2}=\dfrac{3}{1}=3$

x-intercept:
$x^2-4x+3=0$
$(x-3)(x-1)=0$
$x=3$ and $x=1$
vertical asymptote:
$(x+1)^2=0$
$x=-1$

horizontal asymptote:
$n=m$, so $y=\dfrac{1}{1}=1$



81. **a.** Slant asymptote:
$f(x)=x-\dfrac{1}{x}$
$y=x$

b. $f(x)=\dfrac{x^2-1}{x}$

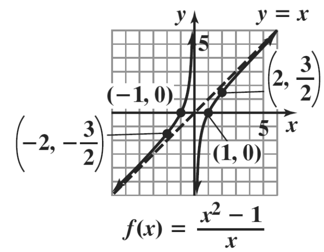
$f(-x)=\dfrac{(-x)^2-1}{(-x)}=\dfrac{x^2-1}{-x}=-f(x)$

Origin symmetry

y-intercept: $y=\dfrac{0^2-1}{0}=\dfrac{-1}{0}$

no y-intercept
x-intercepts: $x^2-1=0$
$x=\pm 1$
vertical asymptote: $x=0$

horizontal asymptote:
$n<m$, so none exist.



82. $f(x)=\dfrac{x^2-4}{x}$

a. slant asymptote:
$f(x)=x-\dfrac{4}{x}$
$y=x$

b. $f(x)=\dfrac{x^2-4}{x}$

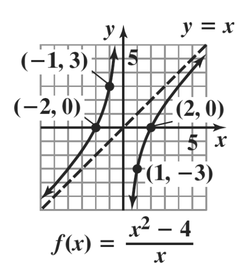
$f(-x)=\dfrac{(-x)^2-4}{-x}=\dfrac{x^2-4}{-x}=-f(x)$

origin symmetry

y-intercept: $y=\dfrac{0^2-4}{0}=-\dfrac{4}{0}$

no y-intercept
x-intercept:
$x^2-4=0$
$x=\pm 2$
vertical asymptote: $x=0$

horizontal asymptote:
$n>m$, so none exist.



83. **a.** Slant asymptote:

$$f(x) = x + \frac{1}{x}$$

$y = x$

b. $$f(x) = \frac{x^2+1}{x}$$

$$f(-x) = \frac{(-x)^2+1}{-x} = \frac{x^2+1}{-x} = -f(x)$$

Origin symmetry

y-intercept: $y = \frac{0^2+1}{0} = \frac{1}{0}$

no y-intercept

x-intercept:

$x^2+1=0$

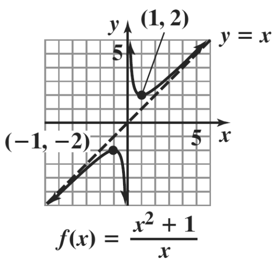
$x^2=-1$

no x-intercept

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



84. $$f(x) = \frac{x^2+4}{x}$$

a. slant asymptote:

$$g(x) = x + \frac{4}{x}$$

$y = x$

b. $$f(x) = \frac{x^2+4}{x}$$

$$f(-x) = \frac{(-x)^2+4}{-x} = \frac{x^2+4}{-x} = -f(x)$$

origin symmetry

y-intercept: $y = \frac{0^2+4}{0} = \frac{4}{0}$

no y-intercept

$x^2+4=0$

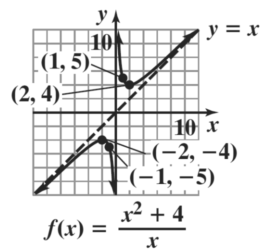
$x^2=-4$

no x-intercept

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



85. **a.** Slant asymptote:

$$f(x) = x + 4 + \frac{6}{x-3}$$

$y = x + 4$

b. $$f(x) = \frac{x^2+x-6}{x-3}$$

$$f(-x) = \frac{(-x)^2+(-x)-6}{-x-3} = \frac{x^2-x-6}{-x-3}$$

$f(-x) \neq g(x)$, $g(-x) \neq -g(x)$

No symmetry

y-intercept: $y = \frac{0^2+0-6}{0-3} = \frac{-6}{-3} = 2$

x-intercept:

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ and $x = 2$

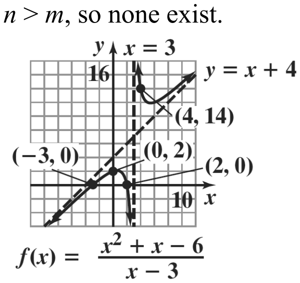
vertical asymptote:

$x - 3 = 0$

$x = 3$

horizontal asymptote:

$n > m$, so none exist.



86. $$f(x) = \frac{x^2-x+1}{x-1}$$

a. slant asymptote:

$$g(x) = x + \frac{1}{x-1}$$

$y = x$

b. $f(x) = \dfrac{x^2 - x - 1}{x - 1}$

$f(-x) = \dfrac{(-x)^2 - (-x) + 1}{-x - 1} = \dfrac{x^2 + x + 1}{-x - 1}$

no symmetry

$f(-x) \neq f(x), f(-x) \neq -g(x)$

y-intercept: $y = \dfrac{0^2 - 0 + 1}{0 - 1} = \dfrac{1}{-1} = -1$

x-intercept:

$x^2 - x + 1 = 0$

no x-intercept

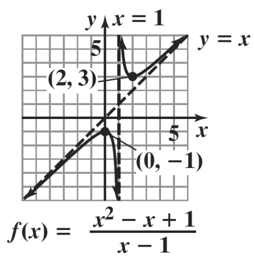
vertical asymptote:

$x - 1 = 0$

$x = 1$

horizontal asymptote:

$n > m$, so none



87. $f(x) = \dfrac{x^3 + 1}{x^2 + 2x}$

a. slant asymptote:

$$\begin{array}{r} x - 2 \\ x^2 + 2x \overline{\big) x^3 \qquad + 1} \\ \underline{x^3 + 2x^2} \\ -2x^2 \\ \underline{-2x^2 + 4x} \\ -4x + 1 \end{array}$$

$y = x - 2$

b. $f(-x) = \dfrac{(-x)^3 + 1}{(-x)^2 + 2(-x)} = \dfrac{-x^3 + 1}{x^2 - 2x}$

$f(-x) \neq f(x), \ f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^3 + 1}{0^2 + 2(0)} = \dfrac{1}{0}$

no y-intercept

x-intercept: $x^3 + 1 = 0$

$x^3 = -1$

$x = -1$

vertical asymptotes:

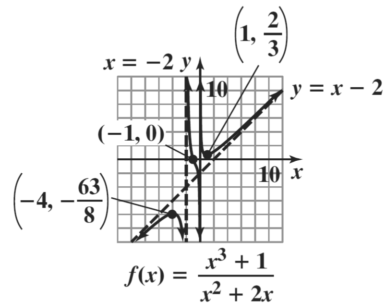
$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 0, x = -2$

horizontal asymptote:

$n > m$, so none



88. $f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

a. slant asymptote:

$$\begin{array}{r} x + \dfrac{9x - 1}{x^2 - 9} \\ x^2 - 9 \overline{\big) x^3 \qquad - 1} \\ \underline{x^3 - 9x} \\ 9x - 1 \end{array}$$

$y = x$

b. $f(-x) = \dfrac{(-x)^3 - 1}{(-x)^2 - 9} = \dfrac{-x^3 - 1}{x^2 - 9}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^3 - 1}{0^2 - 9} = \dfrac{1}{9}$

x-intercept: $x^3 - 1 = 0$

$x^3 = 1$

$x = 1$

vertical asymptotes:

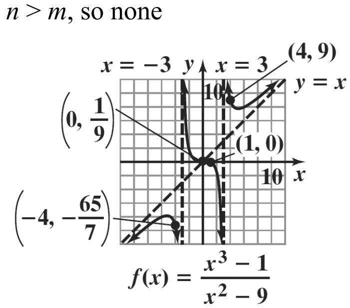
$x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x = 3, x = -3$

horizontal asymptote:

$n > m$, so none

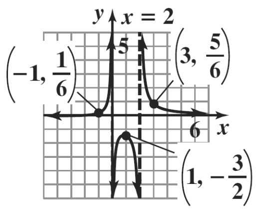


89. $\dfrac{5x^2}{x^2-4}\cdot\dfrac{x^2+4x+4}{10x^3}$

$=\dfrac{\cancel{5}\,\cancel{x^2}}{\cancel{(x+2)}(x-2)}\cdot\dfrac{(x+2)^{\cancel{2}}}{\underset{2}{\cancel{10}}\,x^{\cancel{3}1}}$

$=\dfrac{x+2}{2x(x-2)}$

So, $f(x)=\dfrac{x+2}{2x(x-2)}$



$f(x) = \dfrac{x+2}{2x(x-2)}$

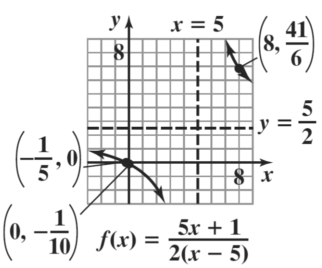
90. $\dfrac{x-5}{10x-2}\div\dfrac{x^2-10x+25}{25x^2-1}$

$=\dfrac{x-5}{10x-2}\cdot\dfrac{25x^2-1}{x^2-10x+25}$

$=\dfrac{\cancel{x-5}}{2\cancel{(5x-1)}}\cdot\dfrac{(5x+1)\cancel{(5x-1)}}{(x-5)^{\cancel{2}}}$

$=\dfrac{5x+1}{2(x-5)}$

So, $f(x)=\dfrac{5x+1}{2(x-5)}$



$f(x) = \dfrac{5x+1}{2(x-5)}$

91. $\dfrac{x}{2x+6}-\dfrac{9}{x^2-9}$

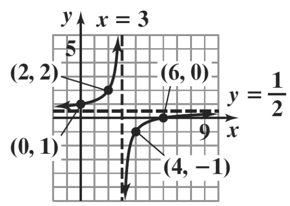
$\dfrac{x}{2x+6}-\dfrac{9}{x^2-9}=\dfrac{x}{2(x+3)}-\dfrac{9}{(x+3)(x-3)}$

$=\dfrac{x(x-3)-9(2)}{2(x+3)(x-3)}$

$=\dfrac{x^2-3x-18}{2(x+3)(x-3)}$

$=\dfrac{(x-6)\cancel{(x+3)}}{2\cancel{(x+3)}(x-3)}=\dfrac{x-6}{2(x-3)}$

So, $f(x)=\dfrac{x-6}{2(x-3)}$



$f(x) = \dfrac{x-6}{2(x-3)}$

92. $\dfrac{2}{x^2+3x+2}-\dfrac{4}{x^2+4x+3}$

$=\dfrac{2}{(x+2)(x+1)}-\dfrac{4}{(x+3)(x+1)}$

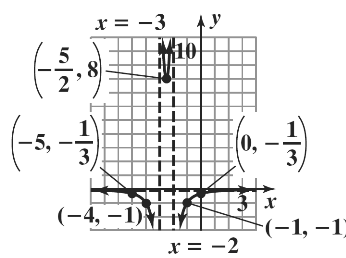
$=\dfrac{2(x+3)-4(x+2)}{(x+2)(x+1)(x+3)}$

$=\dfrac{2x+6-4x-8}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2x-2}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2\cancel{(x+1)}}{(x+2)\cancel{(x+1)}(x+3)}=\dfrac{-2}{(x+2)(x+3)}$

So, $f(x)=\dfrac{-2}{(x+2)(x+3)}$



$f(x) = \dfrac{-2}{(x+2)(x+3)}$

93. $\dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}}=\dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}}\cdot\dfrac{(x+2)(x-2)}{(x+2)(x-2)}$

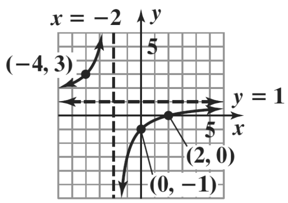
$=\dfrac{(x+2)(x-2)-3(x-2)}{(x+2)(x-2)+(x+2)}$

$=\dfrac{x^2-4-3x+6}{x^2-4+x+2}$

$=\dfrac{x^2-3x+2}{x^2+x-2}$

$=\dfrac{(x-2)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}=\dfrac{x-2}{x+2}$

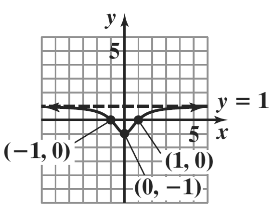
So, $f(x)=\dfrac{x-2}{x+2}$



$f(x)=\dfrac{x-2}{x+2}$

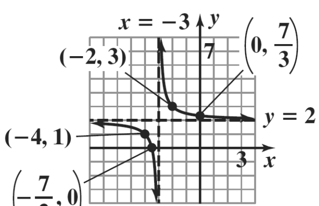
94. $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}}\cdot\dfrac{x}{x}=\dfrac{x^2-1}{x^2+1}=\dfrac{(x-1)(x+1)}{x^2+1}$

So, $f(x)=\dfrac{(x-1)(x+1)}{x^2+1}$



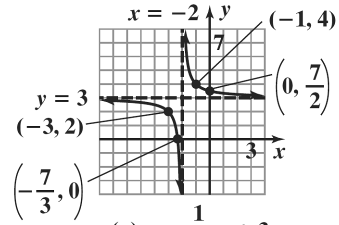
$f(x)=\dfrac{(x-1)(x+1)}{x^2+1}$

95. $g(x)=\dfrac{2x+7}{x+3}=\dfrac{1}{x+3}+2$



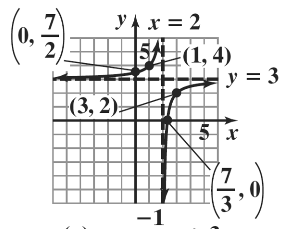
$g(x)=\dfrac{1}{x+3}+2$

96. $g(x)=\dfrac{3x+7}{x+2}=\dfrac{1}{x+2}+3$



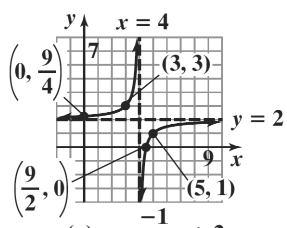
$g(x)=\dfrac{1}{x+2}+3$

97. $g(x)=\dfrac{3x-7}{x-2}=\dfrac{-1}{x-2}+3$



$g(x)=\dfrac{-1}{x-2}+3$

98. $g(x)=\dfrac{2x-9}{x-4}=\dfrac{-1}{x-4}+2$



$g(x)=\dfrac{-1}{x-4}+2$

99. **a.** $C(x)=100x+100{,}000$

b. $\bar{C}(x)=\dfrac{100x+100{,}000}{x}$

c. $\bar{C}(500)=\dfrac{100(500)+100{,}000}{500}=\$300$

When 500 bicycles are manufactured, it costs $300 to manufacture each.

$\bar{C}(1000)=\dfrac{100(1000)+100{,}000}{1000}=\$200$

When 1000 bicycles are manufactured, it costs $200 to manufacture each.

$\bar{C}(2000)=\dfrac{100(2000)+100{,}000}{2000}=\$150$

When 2000 bicycles are manufactured, it costs $150 to manufacture each.

$\overline{C}(4000) = \frac{100(4000)+100,000}{4000} = \$125$

When 4000 bicycles are manufactured, it costs \$125 to manufacture each.
The average cost decreases as the number of bicycles manufactured increases.

d. $n = m$, so $y = \frac{100}{1} = 100$.

As greater numbers of bicycles are manufactured, the average cost approaches \$100.

100. a. $C(x) = 30x + 300,000$

b. $\overline{C} = \frac{300,000+30x}{x}$

c. $\overline{C}(1000) = \frac{300000+30(1000)}{1000} = 330$

When 1000 shoes are manufactured, it costs \$330 to manufacture each.

$\overline{C}(10000) = \frac{300000+30(10000)}{10000} = 60$

When 10,000 shoes are manufactured, it costs \$60 to manufacture each.

$\overline{C}(100,00) = \frac{300,000+30(100,000)}{100,000} = 33$

When 100,000 shoes are manufactured, it costs \$33 to manufacture each.
The average cost decreases as the number of shoes manufactured increases.

d. $n = m$, so $y = \frac{30}{1} = 30$.

As greater numbers of shoes are manufactured, the average cost approaches \$30.

101. a. From the graph the pH level of the human mouth 42 minutes after a person eats food containing sugar will be about 6.0.

b. From the graph, the pH level is lowest after about 6 minutes.

$f(6) = \frac{6.5(6)^2 - 20.4(6) + 234}{6^2 + 36}$
$= 4.8$

The pH level after 6 minutes (i.e. the lowest pH level) is 4.8.

c. From the graph, the pH level appears to approach 6.5 as time goes by. Therefore, the normal pH level must be 6.5.

d. $y = 6.5$

Over time, the pH level rises back to the normal level.

e. During the first hour, the pH level drops quickly below normal, and then slowly begins to approach the normal level.

102. a. From the graph, the drug's concentration after three hours appears to be about 1.5 milligrams per liter.

$$C(3) = \frac{5(3)}{3^2+1} = \frac{15}{10} = 1.5$$

This verifies that the drug's concentration after 3 hours will be 1.5 milligrams per liter.

b. The degree of the numerator, 1, is less than the degree of the denominator, 2, so the the horizontal asymptote is $y = 0$.

Over time, the drug's concentration will approach 0 milligrams per liter.

103. $P(10) = \frac{100(10-1)}{10} = 90 \quad (10, 90)$

For a disease that smokers are 10 times more likely to contact than non-smokers, 90% of the deaths are smoking related.

104. $P(9) = \frac{100(9-1)}{9} = 89 \quad (9, 89)$

For a disease that smokers are 9 times more likely to have than non-smokers, 89% of the deaths are smoking related.

105. $y = 100$ As incidence of the diseases increases, the percent of death approaches, but never gets to be, 100%.

106. No, the percentage approaches 100%, but never reaches 100%.

107. a. $f(x) = \frac{p(x)}{q(x)} = \frac{1.75x^2 - 15.9x + 160}{2.1x^2 - 3.5x + 296}$

b. According to the graph, $\frac{2504.0}{3720.7} \approx 0.67$ or 67% of federal expenditures were spent on human resources in 2010.

c. According to the function,

$f(x) = \frac{1.75(40)^2 - 15.9(40) + 160}{2.1(40)^2 - 3.5(40) + 296} \approx 0.66$ or

66% of federal expenditures were spent on human resources in 2010.

d. The degree of the numerator, 2, is equal to the degree of the denominator, 2. The leading coefficients of the numerator and denominator are 1.75 and 2.1, respectively. The equation of the horizontal asymptote is $y = \frac{1.75}{2.1}$ which is about 83%
Thus, about 83% of federal expenditures will be spent on human resources over time.

108. $x - 10$ = the average velocity on the return trip.
The function that expresses the total time required to complete the round trip is
$T(x) = \frac{600}{x} + \frac{600}{x-10}$.

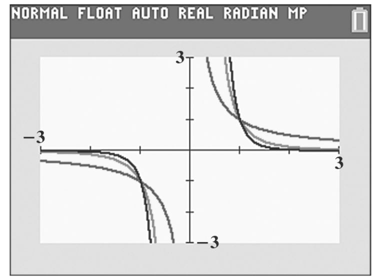
109. $T(x) = \frac{90}{9x} + \frac{5}{x} = \frac{10}{x} + \frac{5}{x}$
The function that expresses the total time for driving and hiking is $T(x) = \frac{10}{x} + \frac{5}{x}$.

110. $A = xy = 2500$
$y = \frac{2500}{x}$
$P = 2x + 2y = 2x + 2\left(\frac{2500}{x}\right) = 2x + \frac{5000}{x}$
The perimeter of the floor, P, as a function of the width, x is $P(x) = 2x + \frac{5000}{x}$.

111. $A = lw$
$xy = 50$
$l = y + 2 = \frac{50}{x} + 2$
$w = x + 1$
$A = \left(\frac{50}{x} + 2\right)(x+1)$
$= 50 + \frac{50}{x} + 2x + 2$
$= 2x + \frac{50}{x} + 52$
The total area of the page is
$A(x) = 2x + \frac{50}{x} + 52.$

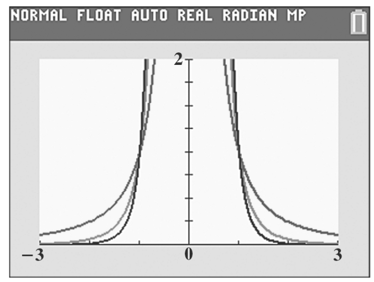
112. – 121. Answers will vary.

122.



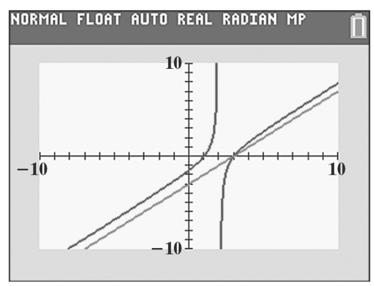
The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

123.



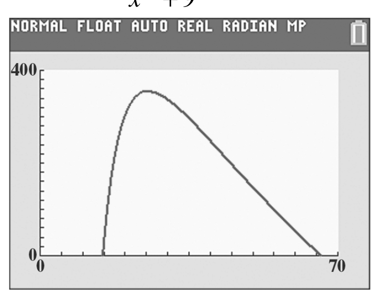
The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

124.



$g(x)$ is the graph of a line where $f(x)$ is the graph of a rational function with a slant asymptote.
In $g(x)$, $x - 2$ is a factor of $x^2 - 5x + 6$.

125. a. $f(x) = \frac{27725(x-14)}{x^2+9} - 5x$



b. The graph increases from late teens until about the age of 25, and then the number of arrests decreases.

c. At age 25 the highest number arrests occurs. There are about 356 arrests for every 100,000 drivers.

126. does not make sense; Explanations will vary. Sample explanation: A rational function can have at most one horizontal asymptote.

127. does not make sense; Explanations will vary. Sample explanation: The function has one vertical asymptote, $x = 2$.

128. makes sense

129. does not make sense; Explanations will vary. Sample explanation: As production level increases, the average cost for a company to produce each unit of its product decreases.

130. false; Changes to make the statement true will vary. A sample change is: The graph of a rational function may have both a vertical asymptote and a horizontal asymptote.

131. true

132. true

133. true

134. – 137. Answers will vary.

138. Let x = the number of miles driven in a week.

$$20 + 0.10x < 30 + 0.05x$$
$$0.05x < 10$$
$$x < 200$$

Driving less than 200 miles in a week makes Basic the better deal.

139. The graph (a) passes the vertical line test and is therefore is a <u>function</u>.
The graph (b) fails the vertical line test and is therefore <u>not a function</u>.
The graph (c) fails the vertical line test and is therefore <u>not a function</u>.
The graph (d) passes the vertical line test and is therefore is a <u>function</u>.

140. The graphs of (b) and (d) pass the horizontal line test and thus have an inverse.

141.

$$2x^2 + x = 15$$
$$2x^2 + x - 15 = 0$$
$$(2x-5)(x+3) = 0$$
$$2x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = \frac{5}{2} \qquad\qquad x = -3$$

The solution set is $\left\{-3, \frac{5}{2}\right\}$.

142.

$$x^3 + x^2 = 4x + 4$$
$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x+1)(x^2 - 4) = 0$$
$$(x+1)(x+2)(x-2) = 0$$

The solution set is $\{-2, -1, 2\}$.

143.

$$\frac{x+1}{x+3} - 2 = \frac{x+1}{x+3} - \frac{2(x+3)}{x+3}$$
$$= \frac{x+1}{x+3} - \frac{2x+6}{x+3}$$
$$= \frac{x+1-2x-6}{x+3}$$
$$= \frac{-x-5}{x+3} \text{ or } -\frac{x+5}{x+3}$$

Section 2.7

Check Point Exercises

1.

$$x^2 - x > 20$$
$$x^2 - x - 20 > 0$$
$$(x+4)(x-5) > 0$$

Solve the related quadratic equation.

$(x+4)(x-5) = 0$

Apply the zero product principle.

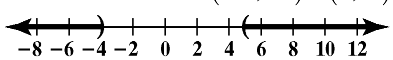
$x+4=0$ or $x-5=0$

$x=-4$ $\quad$ $x=5$

The boundary points are –2 and 4.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-4)$	–5	$(-5)^2-(-5)>20$ $30>20$, true	$(-\infty,-4)$ belongs to the solution set.
$(-4,5)$	0	$(0)^2-(0)>20$ $0>20$, false	$(-4,5)$ does not belong to the solution set.
$(5,\infty)$	10	$(10)^2-(10)>20$ $90>20$, true	$(5,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-4)\cup(5,\infty)$ or $\{x \mid x<-4 \text{ or } x>5\}$.



2.

$$2x^2 \le -6x-1$$
$$2x^2+6x+1 \le 0$$

Solve the related quadratic equation to find the boundary points.

$2x^2+6x+1=0$

$a=2 \quad b=6 \quad c=1$

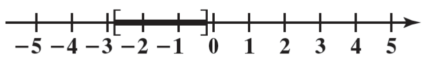
$$x=\frac{-(6)\pm\sqrt{(6)^2-4(2)(1)}}{2(2)}$$
$$=\frac{-6\pm\sqrt{36-8}}{4}$$
$$=\frac{-6\pm\sqrt{28}}{4}$$
$$=\frac{-6\pm 2\sqrt{7}}{4}$$
$$=\frac{-3\pm\sqrt{7}}{2}$$

$x=\frac{-3+\sqrt{7}}{2}$ or $x=\frac{-3-\sqrt{7}}{2}$

$x\approx -0.2 \qquad x\approx -2.8$

Interval	Test Value	Test	Conclusion
$\left(-\infty, \frac{-3-\sqrt{7}}{2}\right)$	-10	$2(-10)^2 \le -6(-10)-1$ $200 \le 59$, false	$\left(-\infty, \frac{-3-\sqrt{7}}{2}\right)$ is not part of the solution set
$\left(\frac{-3-\sqrt{7}}{2}, \frac{-3+\sqrt{7}}{2}\right)$	-1	$2(-1)^2 \le -6(-1)-1$ $2 \le 5$, true	$\left(\frac{-3-\sqrt{7}}{2}, \frac{-3+\sqrt{7}}{2}\right)$ is part of the solution set
$\left(\frac{-3+\sqrt{7}}{2}, \infty\right)$	0	$2(0)^2 \le -6(0)-1$ $0 \le -1$, false	$\left(\frac{-3+\sqrt{7}}{2}, \infty\right)$ is not part of the solution set

The solution set is $\left(\frac{-3-\sqrt{7}}{2}, \frac{-3+\sqrt{7}}{2}\right)$.



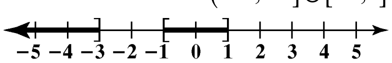
3.

$$x^3 + 3x^2 \le x + 3$$
$$x^3 + 3x^2 - x - 3 \le 0$$
$$(x+1)(x-1)(x+3) \le 0$$
$$(x+1)(x-1)(x+3) = 0$$

$x+1=0$ or $x-1=0$ or $x+3=0$
$x=-1$ $\quad x=1$ $\quad x=-3$

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$(-4)^3+3(-4)^2 \le (-4)+3$ $-16 \le -1$ true	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1]$	–2	$(-2)^3+3(-2)^2 \le (-2)+3$ $4 \le 1$ false	$(-3,-1]$ does not belong to the solution set.
$[-1,1]$	0	$(0)^3+3(0)^2 \le (0)+3$ $0 \le 3$ true	$[-1,1]$ belongs to the solution set.
$[1,\infty)$	2	$(6+3)(6-5) > 0$ true	$[1,\infty)$ does not belong to the solution set.

The solution set is $(-\infty,-3] \cup [-1,1]$ or $\{x \mid x \le -3 \text{ or } -1 \le x \le 1\}$.



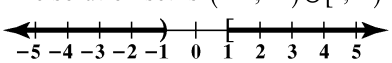
4.

$$\frac{2x}{x+1} \geq 1$$
$$\frac{2x}{x+1} - 1 \geq 0$$
$$\frac{x-1}{x+1} \geq 0$$

$x - 1 = 0$ or $x + 1 = 0$
$x = 1$ $\quad x = -1$

Test Interval	Test Number	Test	Conclusion
$(-\infty, -1)$	-2	$\frac{2(-2)}{-2+1} \geq 1$ $4 \geq 1$, true	$(-\infty, -1)$ belongs to the solution set.
$(-1, 1]$	0	$\frac{2(0)}{0+1} \geq 1$ $0 \geq 1$, false	$(-1, 1]$ does not belong to the solution set.
$[1, \infty)$	2	$\frac{2(2)}{2+1} \geq 1$ $\frac{4}{3} \geq 1$, true	$[1, \infty)$ belongs to the solution set.

The solution set is $(-\infty, -1) \cup [1, \infty)$ or $\{x \mid x < -1 \text{ or } x \geq 1\}$.



5.

$$-16t^2 + 80t > 64$$
$$-16t^2 + 80t - 64 > 0$$
$$-16(t-1)(t-4) > 0$$

$t - 1 = 0$ or $t - 4 = 0$
$t = 1$ $\quad t = 4$

Test Interval	Test Number	Test	Conclusion
$(-\infty, 1)$	0	$-16(0)^2 + 80(0) > 64$ $0 > 64$, false	$(-\infty, 1)$ does not belong to the solution set.
$(1, 4)$	2	$-16(2)^2 + 80(2) > 64$ $96 > 64$, true	$(1, 4)$ belongs to the solution set.
$(4, \infty)$	5	$-16(5)^2 + 80(5) > 64$ $0 > 64$, false	$(4, \infty)$ does not belong to the solution set.

The object will be more than 64 feet above the ground between 1 and 4 seconds.

Concept and Vocabulary Check 2.7

1. $x^2 + 8x + 15 = 0$; boundary

2. $(-\infty, -5)$; $(-5, -3)$; $(-3, \infty)$

3. true

4. true

5. $[-\infty, -2) \cup [1, \infty)$

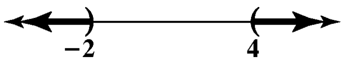
Exercise Set 2.7

1. $(x-4)(x+2) > 0$
$x = 4$ or $x = -2$

T	F	T

$-2 \qquad 4$

Test –3: $(-3-4)(-3+2) > 0$
$7 > 0$ True
Test 0: $(0-4)(0+2) > 0$
$-8 > 0$ False
Test 5: $(5-4)(5+2) > 0$
$7 > 0$ True
$(-\infty, -2)$ or $(4, \infty)$

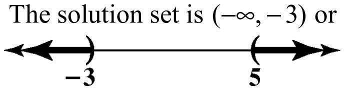


2. $(x+3)(x-5) > 0$
$x = -3$ or $x = 5$

T	F	T

$-3 \qquad 5$

Test –4: $(-4+3)(-4-5) > 0$
$9 > 0$ True
Test 0: $(0+3)(0-5) > 0$
$-15 > 0$ False
Test 6: $(6+3)(6-5) > 0$
$18 > 0$ True
The solution set is $(-\infty, -3)$ or $(5, \infty)$.

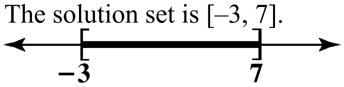


3. $(x-7)(x+3) \le 0$
$x = 7$ or $x = -3$

F	T	F

$-3 \qquad 7$

Test –4: $(-4-7)(-4+3) \le 0$
$11 \le 0$ False
Test 0: $(0-7)(0+3) \le 0$
$-21 \le 0$ True
Test 8: $(8-7)(8+3) \le 0$
$11 \le 0$ False
The solution set is $[-3, 7]$.

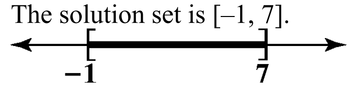


4. $(x+1)(x-7) \le 0$
$x = -1$ or $x = 7$

F	T	F

$-1 \qquad 7$

Test –2: $(-2+1)(-2-7) \le 0$
$9 \le 0$ False
Test 0: $(0+1)(0-7) \le 0$
$-7 \le 0$ True
Test 8: $(8+1)(8-7) \le 0$
$9 \le 0$ False
The solution set is $[-1, 7]$.

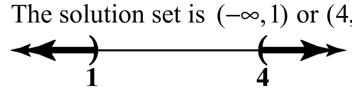


5. $x^2 - 5x + 4 > 0$
$(x-4)(x-1) > 0$
$x = 4$ or $x = 1$

T	F	T

$1 \qquad 4$

Test 0: $0^2 - 5(0) + 4 > 0$
$4 > 0$ True
Test 2: $2^2 - 5(2) + 4 > 0$
$-2 > 0$ False
Test 5: $5^2 - 5(5) + 4 > 0$
$4 > 0$ True
The solution set is $(-\infty, 1)$ or $(4, \infty)$.

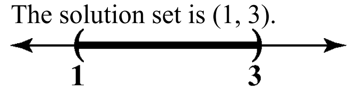


6. $x^2 - 4x + 3 < 0$
$(x-1)(x-3) < 0$
$x = 1$ or $x = 3$

F	T	F

$1 \qquad 3$

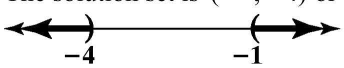
Test 0: $0^2 - 4(0) + 3 < 0$
$3 < 0$ False
Test 2: $2^2 - 4(2) + 3 < 0$
$-1 < 0$ True
Test 4: $4^2 - 4(4) + 3 < 0$
$3 < 0$ False
The solution set is $(1, 3)$.



7. $x^2 + 5x + 4 > 0$
$(x+1)(x+4) > 0$
$x = -1$ or $x = -4$

T	F	T
	−4	−1

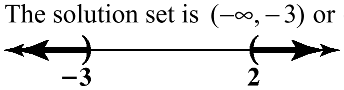
Test −5: $(-5)^2 + 5(-5) + 4 > 0$
$4 > 0$ True
Test −3: $(-3)^2 + 5(-3) + 4 > 0$
$-2 > 0$ False
Test 0: $0^2 + 5(0) + 4 > 0$
$4 > 0$ True
The solution set is $(-\infty, -4)$ or $(-1, \infty)$.



8. $x^2 + x - 6 > 0$
$(x+3)(x-2) > 0$
$x = -3$ or $x = 2$

T	F	T
	−3	2

Test −4: $(-4)^2 - 4 - 6 > 0$
$6 > 0$ True
Test 0: $(0)^2 + 0 - 6 > 0$
$-6 > 0$ False
Test 3: $3^2 + 3 - 6 > 0$
$6 > 0$ True
The solution set is $(-\infty, -3)$ or $(2, \infty)$.



9. $x^2 - 6x + 9 < 0$
$(x-3)(x-3) < 0$
$x = 3$

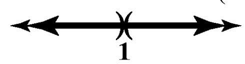
F	F
3	

Test 0: $0^2 - 6(0) + 9 < 0$
$9 < 0$ False
Test 4: $4^2 - 6(4) + 9 < 0$
$1 < 0$ False
The solution set is the empty set, $\varnothing$.

10. $x^2 - 2x + 1 > 0$
$(x-1)(x-1) > 0$
$x = 1$

T	T
1	

Test 0: $0^2 - 2(0) + 1 > 0$
$1 > 0$ True
Test 2: $2^2 - 2(2) + 1 > 0$
$1 > 0$ True
The solution set is $(-\infty, 1)$ or $(1, \infty)$.

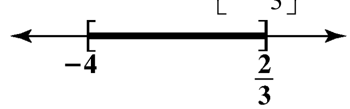


11. $3x^2 + 10x - 8 \le 0$
$(3x-2)(x+4) \le 0$

$x = \frac{2}{3}$ or $x = -4$

F	T	F
	−4	$\frac{2}{3}$

Test −5: $3(-5)^2 + 10(-5) - 8 \le 0$
$17 \le 0$ False
Test 0: $3(0)^2 + 10(0) - 8 \le 0$
$8 \le 0$ True
Test 1: $3(1)^2 + 10(1) - 8 \le 0$
$5 \le 0$ False
The solution set is $\left[-4, \frac{2}{3}\right]$.



12. $9x^2 + 3x - 2 \ge 0$

$(3x-1)(3x+2) \ge 0$

$3x = 1 \quad 3x = -2$

$x = \frac{1}{3} \quad x = \frac{-2}{3}$

T	F	T

$\frac{-2}{3}$ $\quad$ $\frac{1}{3}$

Test –1: $9(-1)^2 + 3(-1) - 2 \ge 0$

$4 \ge 0$ True

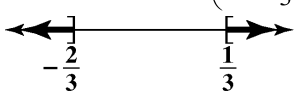
Test 0: $9(0)^2 + 3(0) - 2 \ge 0$

$-2 \ge 0$ False

Test 1: $9(1)^2 + 3(1) - 2 \le 0$

$10 \ge 0$ True

The solution set is $\left(-\infty, \frac{-2}{3}\right]$ or $\left[\frac{1}{3}, \infty\right)$.



13. $2x^2 + x < 15$

$2x^2 + x - 15 < 0$

$(2x-5)(x+3) < 0$

$2x - 5 = 0$ or x+3=0

$2x = 5$

$x = \frac{5}{2}$ or $x = -3$

F	T	F

-3 $\quad$ $\frac{5}{2}$

Test –4: $2(-4)^2 + (-4) < 15$

$28 < 15$ False

Test 0: $2(0)^2 + 0 < 15$

$0 < 15$ True

Test 3: $2(3)^2 + 3 < 15$

$21 < 15$ False

The solution set is $\left(-3, \frac{5}{2}\right)$.

14. $6x^2 + x > 1$

$6x^2 + x - 1 > 0$

$(2x+1)(3x-1) > 0$

$2x + 1 = 0$ or $3x - 1 = 0$

$2x = -1 \quad 3x = 1$

$x = -\frac{1}{2} \quad x = \frac{1}{3}$

T	F	T

$-\frac{1}{2}$ $\quad$ $\frac{1}{3}$

Test –1: $6(-1)^2 + (-1) > 1$

$5 > 1$ True

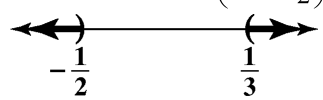
Test 0: $6(0)^2 + 0 > 1$

$0 > 1$ False

Test 1: $6(1)^2 + 1 > 1$

$7 > 1$ True

The solution set is $\left(-\infty, -\frac{1}{2}\right)$ or $\left(\frac{1}{3}, \infty\right)$.



15. $4x^2 + 7x < -3$

$4x^2 + 7x + 3 < 0$

$(4x+3)(x+1) < 0$

$4x + 3 = 0$ or x+1=0

4x-3=0

$x = -\frac{3}{4}$ or $x = -1$

F	T	F

-1 $\quad$ $-\frac{3}{4}$

Test –2: $4(-2)^2 + 7(-2) < -3$

$2 < -3$ False

Test $-\frac{7}{8}$: $4\left(-\frac{7}{8}\right)^2 + 7\left(-\frac{7}{8}\right) < -3$

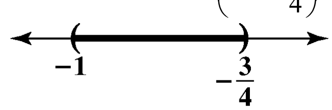
$\frac{49}{16} - \frac{49}{8} < -3$

$-\frac{49}{16} < -3$ True

Test 0: $4(0)^2 + 7(0) < -3$

$0 < -3$ False

The solution set is $\left(-1, -\frac{3}{4}\right)$.



16. $3x^2 + 16x < -5$

$3x^2 + 16x + 5 < 0$

$(3x+1)(x+5) < 0$

$3x + 1 = 0$ or $x + 5 = 0$

$3x = -1$

$x = -\frac{1}{3}$ $\quad x = -5$

F	T	F
	-5	$-\frac{1}{3}$

Test –6: $3(-6)^2 + 16(-6) < -5$

$12 < -5$ False

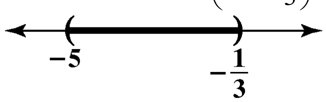
Test –2: $3(-2)^2 + 16(-2) < -5$

$-20 < -5$ True

Test 0: $3(0)^2 + 16(0) < -5$

$0 < -5$ False

The solution set is $\left(-5, -\frac{1}{3}\right)$.



17. $5x \le 2 - 3x^2$

$3x^2 + 5x - 2 \le 0$

$(3x-1)(x+2) \le 0$

$3x - 1 = 0$ or $x + 2 = 0$

$3x = 1$

$3x - 1 = 0$ or $x + 2 = 0$

$3x = 1$

$x = \frac{1}{3}$ or $x = -2$

F	T	F
	-2	$\frac{1}{3}$

Test –3: $5(-3) \le 2 - 3(-3)^2$

$-15 \le -25$ False

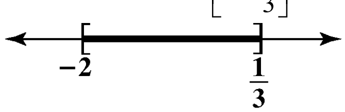
Test 0: $5(0) \le 2 - 3(0)^2$

$0 \le 2$ True

Test 1: $5(1) \le 2 - 3(1)^2$

$5 \le -1$ False

The solution set is $\left[-2, \frac{1}{3}\right]$.



18. $4x^2 + 1 \ge 4x$

$4x^2 - 4x + 1 \ge 0$

$(2x-1)(2x-1) \ge 0$

$2x - 1 = 0$

$x = \frac{1}{2}$

T	T
	$\frac{1}{2}$

Test 0: $4(0)^2 + 1 \ge 4(0)$

$1 \ge 0$ True

Test 1: $4(1)^2 + 1 \ge 4(1)$

$5 \ge 4$ True

The solution set is $(-\infty, \infty)$.

19. $x^2 - 4x \ge 0$

$x(x-4) \ge 0$

$x = 0$ or $x - 4 = 0$

$x = 4$

T	F	T
	0	4

Test –1: $(-1)^2 - 4(-1) \ge 0$

$5 \ge 0$ True

Test 1: $(1)^2 - 4(1) \ge 0$

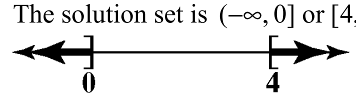
$-3 \ge 0$ False

$0 \le 2$ True

Test 5: $5^2 - 4(5) \ge 0$

$5 \ge 0$ True

The solution set is $(-\infty, 0]$ or $[4, \infty)$.

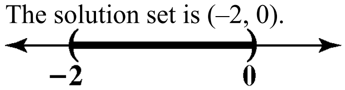


20. $x^2 + 2x < 0$
$x(x+2) < 0$
$x = 0$ or $x = -2$

F	T	F

$-2 \quad 0$

Test –3: $(-3)^2 + 2(-3) < 0$
$3 < 0$ False
Test –1: $(-1)^2 + 2(-1) < 0$
$-1 < 0$ True
Test 1: $(1)^2 + 2(1) < 0$
$3 < 0$ False
The solution set is $(-2, 0)$.



21. $2x^2 + 3x > 0$
$x(2x+3) > 0$
$x = 0$ or $x = -\frac{3}{2}$

T	F	T

$-\frac{3}{2} \quad 0$

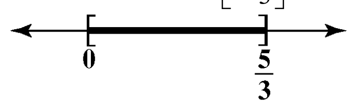
Test –2: $2(-2)^2 + 3(-2) > 0$
$2 > 0$ True
Test –1: $2(-1)^2 + 3(-1) > 0$
$-1 > 0$ False
Test 1: $2(1)^2 + 3(1) > 0$
$5 > 0$ True
The solution set is $\left(-\infty, -\frac{3}{2}\right)$ or $(0, \infty)$.

22. $3x^2 - 5x \le 0$
$x(3x-5) \le 0$
$x = 0$ or $x = \frac{5}{3}$

F	T	F

$0 \quad \frac{5}{3}$

Test –1: $3(-1)^2 - 5(-1) \le 0$
$8 \le 0$ False
Test 1: $3(1)^2 - 5(1) \le 0$
$-2 \le 0$ True
Test 2: $3(2)^2 - 5(2) \le 0$
$2 \le 0$ False
The solution set is $\left[0, \frac{5}{3}\right]$.

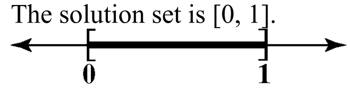


23. $-x^2 + x \ge 0$
$x^2 - x \le 0$
$x(x-1) \le 0$
$x = 0$ or $x = 1$

F	T	F

$0 \quad 1$

Test –1: $-(-1)^2 + (-1) \ge 0$
$-2 \ge 0$ False
Test $\frac{1}{2}$: $-\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) \ge 0$
$\frac{1}{4} \ge 0$ True
Test 2: $-(2)^2 + 2 \ge 0$
$-2 \ge 0$ False
The solution set is $[0, 1]$.



24. $-x^2 + 2x \geq 0$

$x(-x+2) \geq 0$

$x = 0$ or $x = 2$

F	T	F
	0	2

Test –1: $-(-1)^2 + 2(-1) \geq 0$

$-3 \geq 0$ False

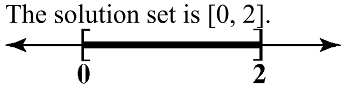
Test 1: $-(1)^2 + 2(1) \geq 0$

$1 \geq 0$ True

Test 3: $-(3)^2 + 2(3) \geq 0$

$-3 \geq 0$ False

The solution set is [0, 2].



25. $x^2 \leq 4x - 2$

$x^2 - 4x + 2 \leq 0$

Solve $x^2 - 4x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

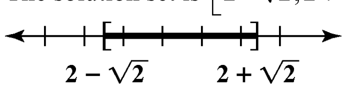
$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= 2 \pm \sqrt{2}$$

$x \approx 0.59$ or $x \approx 3.41$

F	T	F
	0.59	3.41

The solution set is $\left[2-\sqrt{2}, 2+\sqrt{2}\right]$ or $[0.59, 3.41]$.



26. $x^2 \leq 2x + 2$

$x^2 - 2x - 2 \leq 0$

Solve $x^2 - 2x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

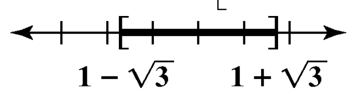
$$= \frac{2 \pm \sqrt{12}}{2}$$

$$= 1 \pm \sqrt{3}$$

$x \approx -0.73$ or $x \approx 2.73$

F	T	F
	–0.73	2.73

The solution set is $\left[1-\sqrt{3}, 1+\sqrt{3}\right]$ or $[-0.73, 2.73]$.



27. $x^2 - 6x + 9 < 0$

Solve $x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$(x-3)^2 = 0$

$x = 3$

F	F
	3

The solution set is the empty set, $\varnothing$.

28. $4x^2 - 4x + 1 \geq 0$

Solve $4x^2 - 4x + 1 = 0$

$(2x-1)(2x-1) = 0$

$(2x-1)^2 = 0$

$$x = \frac{1}{2}$$

T	T
	$\frac{1}{2}$

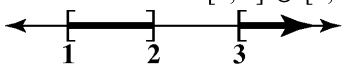
The solution set is $(-\infty, \infty)$.

29. $(x-1)(x-2)(x-3) \geq 0$

Boundary points: 1, 2, and 3
Test one value in each interval.

F	T	F	T
	1	2	3

The solution set is [1, 2] ∪ [3, ∞).

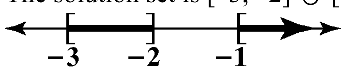


30. $(x+1)(x+2)(x+3) \geq 0$

Boundary points: –1, –2, and –3
Test one value in each interval.

F	T	F	T
	–3	–2	–1

The solution set is [–3, –2] ∪ [–1, ∞).

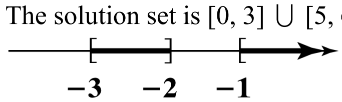


31. $x(3-x)(x-5) \leq 0$

Boundary points: 0, 3, and 5
Test one value in each interval.

F	T	F	T
	0	3	5

The solution set is [0, 3] ∪ [5, ∞).

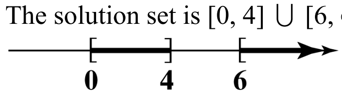


32. $x(4-x)(x-6) \leq 0$

Boundary points: 0, 3, and 5
Test one value in each interval.

F	T	F	T
	0	4	6

The solution set is [0, 4] ∪ [6, ∞).



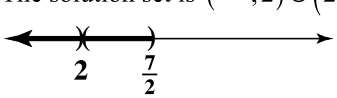
33. $(2-x)^2\left(x-\frac{7}{2}\right) < 0$

Boundary points: 2, and $\frac{7}{2}$

Test one value in each interval.

T	T	F
	2	$\frac{7}{2}$

The solution set is $\left(-\infty, 2\right) \cup \left(2, \frac{7}{2}\right)$.



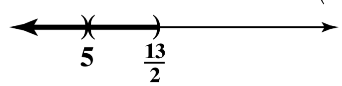
34. $(5-x)^2\left(x-\frac{13}{2}\right) < 0$

Boundary points: 5, and $\frac{13}{2}$

Test one value in each interval.

T	T	F
	5	$\frac{13}{2}$

The solution set is $\left(-\infty, 5\right) \cup \left(5, \frac{13}{2}\right)$.



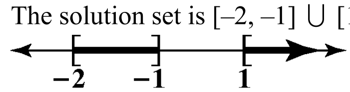
35. $x^3 + 2x^2 - x - 2 \geq 0$

$x^2(x+2) - 1(x+2) \geq 0$
$(x+2)(x^2-1) \geq 0$
$(x+2)(x-1)(x+1) \geq 0$

Boundary points: –2, –1, and 2
Test one value in each interval.

F	T	F	T
	–2	–1	2

The solution set is [–2, –1] ∪ [1, ∞).



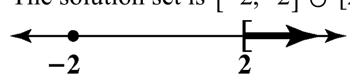
36. $x^3 + 2x^2 - 4x - 8 \geq 0$

$x^2(x+2) - 4(x+1) \geq 0$
$(x+2)(x^2-4) \geq 0$
$(x+2)(x+2)(x-2) \geq 0$

Boundary points: –2, and 2
Test one value in each interval.

F	F	T
	–2	2

The solution set is $[-2,-2]$ ∪ [2, ∞).



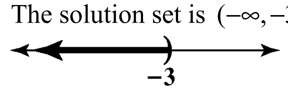
37. $x^3 + 2x^2 - x - 2 \geq 0$

$x^2(x-3) - 9(x-3) \geq 0$
$(x-3)(x^2-9) \geq 0$
$(x-3)(x+3)(x-3) \geq 0$

Boundary points: –3 and 3
Test one value in each interval.

T	F	F
	–3	3

The solution set is $(-\infty, -3]$.



38. $x^3+7x^2-x-7<0$

$x^2(x+7)-(x+7)<0$

$(x+7)(x^2-1)<0$

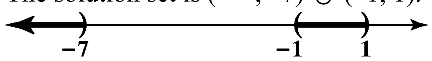
$(x+7)(x+1)(x-1)<0$

Boundary points: –7, –1 and 1

Test one value in each interval.

T	F	T	F
	–7	–1	1

The solution set is $(-\infty, -7) \cup (-1, 1)$.



39. $x^3+x^2+4x+4>0$

$x^2(x+1)+4(x+1)\ge 0$

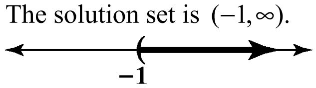
$(x+1)(x^2+4)\ge 0$

Boundary point: –1

Test one value in each interval.

F	T
	–1

The solution set is $(-1,\infty)$.



40. $x^3-x^2+9x-9>0$

$x^2(x-1)+9(x-1)\ge 0$

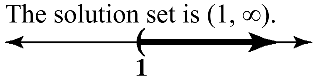
$(x-1)(x^2+9)\ge 0$

Boundary point: 1.

Test one value in each interval.

F	T
	1

The solution set is $(1, \infty)$.



41. $x^3-9x^2\ge 0$

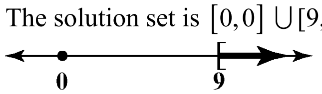
$x^2(x-9)\ge 0$

Boundary points: 0 and 9

Test one value in each interval.

F	F	T
	0	9

The solution set is $[0,0]\cup[9, \infty)$.



42. $x^3-4x^2\le 0$

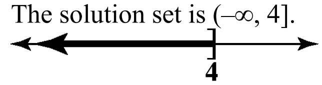
$x^2(x-4)\le 0$

Boundary points: 0 and 4.

Test one value in each interval.

T	T	F
	0	4

The solution set is $(-\infty, 4]$.



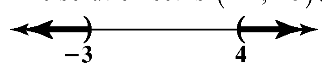
43. $\frac{x-4}{x+3}>0$

$x-4=0 \quad x+3=0$

$x=4 \quad x=-3$

T	F	T
	–3	4

The solution set is $(-\infty,-3)\cup(4,\infty)$.

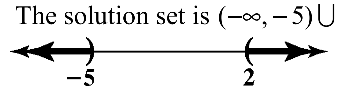


44. $\frac{x+5}{x-2}>0$

$x=-5$ or $x=2$

T	F	T
	–5	2

The solution set is $(-\infty,-5)\cup(2,\infty)$.

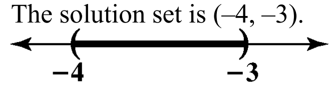


45. $\frac{x+3}{x+4}<0$

$x=-3$ or $x=-4$

F	T	F
	–4	–3

The solution set is $(-4, -3)$.

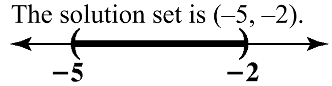


46. $\frac{x+5}{x+2}<0$

$x=-5$ or $x=-2$

F	T	F
	–5	–2

The solution set is $(-5, -2)$.

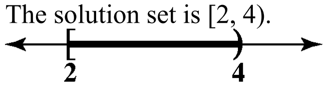


47. $\frac{-x+2}{x-4} \geq 0$

$x = 2$ or $x = 4$

F	T	F
	2	4

The solution set is [2, 4).

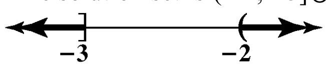


48. $\frac{-x-3}{x+2} \leq 0$

$x = -3$ or $x = -2$

T	F	T
	–3	–2

The solution set is $(-\infty, -3] \cup (-2, \infty)$.



49. $\frac{4-2x}{3x+4} \leq 0$

$x = 2$ or $x = -\frac{4}{3}$

T	F	T
	$-\frac{4}{3}$	2

The solution set is $\left(-\infty, \frac{-4}{3}\right) \cup [2, \infty)$.

50. $\frac{3x+5}{6-2x} \geq 0$

$x = -\frac{5}{3}$ or $x = 3$

F	T	F
	$-\frac{5}{3}$	3

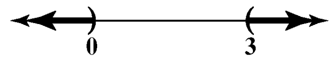
The solution set is $\left[-\frac{5}{3}, 3\right)$.

51. $\frac{x}{x-3} > 0$

$x = 0$ or $x = 3$

T	F	T
	0	3

The solution set is $(-\infty, 0) \cup (3, \infty)$.

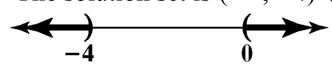


52. $\frac{x+4}{x} > 0$

$x = -4$ or $x = 0$

T	F	T
	–4	0

The solution set is $(-\infty, -4) \cup (0, \infty)$.



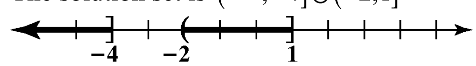
53. $\frac{(x+4)(x-1)}{x+2} \leq 0$

$x = -4$ or $x = -2$ or $x = 1$.

T	F	T	F
	–4	–2	1

Values of $x = -4$ or $x = 1$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -4] \cup (-2, 1]$



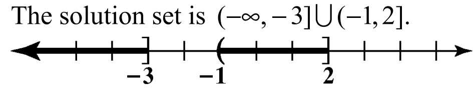
54. $\frac{(x+3)(x-2)}{x+1} \leq 0$

$x = -3$ or $x = -1$ or $x = 2$.

T	F	T	F
	–3	–1	2

Values of $x = -3$ or $x = 2$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -3] \cup (-1, 2]$.



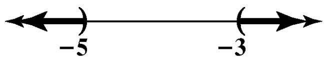
55.

$$\frac{x+1}{x+3}<2$$
$$\frac{x+1}{x+3}-2<0$$
$$\frac{x+1-2(x+3)}{x+3}<0$$
$$\frac{x+1-2x-6}{x+3}<0$$
$$\frac{-x-5}{x+3}<0$$

$x=$ or $x=-3$

T	F	T
	–5	–3

The solution set is $(-\infty,-5)\cup(-3,\infty)$.



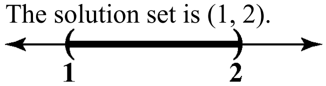
56.

$$\frac{x}{x-1}>2$$
$$\frac{x}{x-1}-2>0$$
$$\frac{x}{x-1}-\frac{2(x-1)}{x-1}>0$$
$$\frac{x-2x+2}{x-1}>0$$
$$\frac{-x+2}{x-1}>0$$

$x=2$ or $x=1$

F	T	F
	1	2

The solution set is (1, 2).

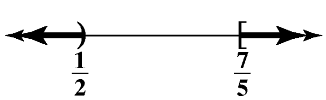


57.

$$\frac{x+4}{2x-1}\le 3$$
$$\frac{x+4}{2x-1}-3\le 0$$
$$\frac{x+4-3(2x-1)}{2x-1}\le 0$$
$$\frac{x+4-6x+3}{2x-1}\le 0$$
$$\frac{-5x+7}{2x-1}\le 0$$

T	F	T
	$\frac{1}{2}$	$\frac{7}{5}$

$x=\frac{7}{5}$ or $x=\frac{1}{2}$



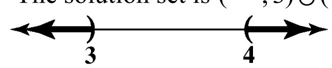
58.

$$\frac{1}{x-3}<1$$
$$\frac{1}{x-3}-1<0$$
$$\frac{1}{x-3}-\frac{x-3}{x-3}<0$$
$$\frac{1-x+3}{x-3}<0$$
$$\frac{-x+4}{x-3}<0$$

$x=4$ or $x=3$

T	F	T
	3	4

The solution set is $(-\infty,3)\cup(4,\infty)$.



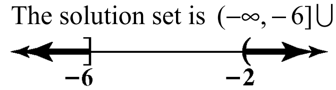
59.

$$\frac{x-2}{x+2}\le 2$$
$$\frac{x-2}{x+2}-2\le 0$$
$$\frac{x-2-2(x+2)}{x+2}\le 0$$
$$\frac{x-2-2x-4}{x+2}\le 0$$
$$\frac{-x-6}{x+2}\le 0$$

$x=-6$ or $x=-2$

T	F	T
	-6	-2

The solution set is $(-\infty,-6]\cup(-2,\infty)$.



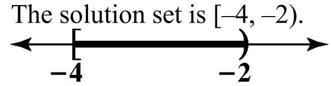
60.

$$\frac{x}{x+2}\ge 2$$
$$\frac{x}{x+2}-2\ge 0$$
$$\frac{x}{x+2}-\frac{2(x+2)}{x+2}\ge 0$$
$$\frac{x-2x-4}{x+2}\ge 0$$
$$\frac{-x-4}{x+2}\ge 0$$

$x=-4$ or $x=-2$

F	T	F
	–4	–2

The solution set is [–4, –2).



61. $f(x)=\sqrt{2x^2-5x+2}$

The domain of this function requires that $2x^2-5x+2\ge 0$

Solve $2x^2-5x+2=0$

$(x-2)(2x-1)=0$

$x=\frac{1}{2}$ or $x=2$

T	F	T
	$\frac{1}{2}$	2

The domain is $\left(-\infty,\frac{1}{2}\right]\cup[2,\infty)$.

62. $f(x)=\dfrac{1}{\sqrt{4x^2-9x+2}}$

The domain of this function requires that $4x^2-9x+2>0$

Solve $4x^2-9x+2=0$

$(x-2)(4x-1)=0$

$x=\frac{1}{4}$ or $x=2$

T	F	T
	$\frac{1}{4}$	2

The domain is $\left(-\infty,\frac{1}{4}\right)\cup(2,\infty)$.

63. $f(x)=\sqrt{\dfrac{2x}{x+1}-1}$

The domain of this function requires that $\dfrac{2x}{x+1}-1\ge 0$ or $\dfrac{x-1}{x+1}\ge 0$

$x=-1$ or $x=1$

T	F	T
	-1	1

The value $x=1$ results in 0 and, thus, it must be included in the domain.

The domain is $(-\infty,-1)\cup[1,\infty)$.

64. $f(x) = \sqrt{\dfrac{x}{2x-1} - 1}$

The domain of this function requires that $\dfrac{x}{2x-1} - 1 \geq 0$ or $\dfrac{-x+1}{2x-1} \geq 0$

$x = \dfrac{1}{2}$ or $x = 1$

F	T	F
	$\frac{1}{2}$	1

The value $x = 1$ results in 0 and, thus, it must be included in the domain.
The domain is $(-\infty, -1) \cup [1, \infty)$.

65. $\left|x^2 + 2x - 36\right| > 12$

Express the inequality without the absolute value symbol:

$x^2 + 2x - 36 < -12$ or $x^2 + 2x - 36 > 12$
$x^2 + 2x - 24 < 0$ $\quad$ $x^2 + 2x - 48 > 0$

Solve the related quadratic equations.

$x^2 + 2x - 24 = 0$ or $x^2 + 2x - 48 = 0$
$(x+6)(x-4) = 0$ $\quad$ $(x+8)(x-6) = 0$

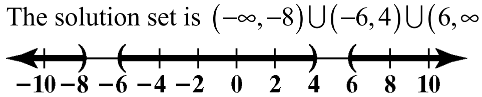
Apply the zero product principle.

$x + 6 = 0$ or $x - 4 = 0$ or $x + 8 = 0$ or $x - 6 = 0$
$x = -6$ $\quad$ $x = 4$ $\quad$ $x = -8$ $\quad$ $x = 6$

The boundary points are -8, -6, 4 and 6 .

Test Interval	Test Number	Test	Conclusion
$(-\infty, -8)$	-9	$\left\|(-9)^2 + 2(-9) - 36\right\| > 12$ $27 > 12$, True	$(-\infty, -8)$ belongs to the solution set.
$(-8, -6)$	-7	$\left\|(-7)^2 + 2(-7) - 36\right\| > 12$ $1 > 12$, False	$(-8, -6)$ does not belong to the solution set.
$(-6, 4)$	0	$\left\|0^2 + 2(0) - 36\right\| > 12$ $36 > 12$, True	$(-6, 4)$ belongs to the solution set.
$(4, 6)$	5	$\left\|5^2 + 2(5) - 36\right\| > 12$ $1 > 12$, False	$(4, 6)$ does not belong to the solution set.
$(6, \infty)$	7	$\left\|7^2 + 2(7) - 36\right\| > 12$ $27 > 12$, True	$(6, \infty)$ belongs to the solution set.

The solution set is $(-\infty, -8) \cup (-6, 4) \cup (6, \infty)$ or $\{x \mid x < -8 \text{ or } -6 < x < 4 \text{ or } x > 6\}$.



66. $\left|x^2+6x+1\right|>8$

Express the inequality without the absolute value symbol:

$x^2+6x+1<-8 \quad \text{or} \quad x^2+6x+1>8$
$x^2+6x+9<0 \qquad\quad x^2+6x-7>0$

Solve the related quadratic equations.

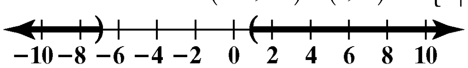
$x^2+6x+9=0 \quad \text{or} \quad x^2+6x-7=0$
$(x+3)^2=0 \qquad\quad (x+7)(x-1)=0$

$x+3=\pm\sqrt{0} \quad \text{or} \quad x+7=0 \quad \text{or} \quad x-1=0$
$x+3=0 \qquad\qquad x=-7 \qquad\qquad x=1$
$x=-3$

The boundary points are $-7, -3,$ and 1.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-7)$	-8	$\left\|(-8)^2+6(-8)+1\right\|>8$ $17\geq 8$, True	$(-\infty,-7)$ belongs to the solution set.
$(-7,-3)$	-5	$\left\|(-5)^2+6(-5)+1\right\|>8$ $4\geq 8$, False	$(-7,-3)$ does not belong to the solution set.
$(-3,1)$	0	$\left\|0^2+6(0)+1\right\|>8$ $1\geq 8$, False	$(-3,1)$ does not belong to the solution set.
$(1,\infty)$	2	$\left\|2^2+6(2)+1\right\|>8$ $17\geq 8$, True	$(1,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-7)\cup(1,\infty)$ or $\{x \mid x<-7 \text{ or } x>1\}$.



67. $\dfrac{3}{x+3}>\dfrac{3}{x-2}$

Express the inequality so that one side is zero.

$$\frac{3}{x+3}-\frac{3}{x-2}>0$$
$$\frac{3(x-2)}{(x+3)(x-2)}-\frac{3(x+3)}{(x+3)(x-2)}>0$$
$$\frac{3x-6-3x-9}{(x+3)(x-2)}<0$$
$$\frac{-15}{(x+3)(x-2)}<0$$

Find the values of x that make the denominator zero.

$x+3=0 \qquad x-2=0$
$x=-3 \qquad\quad x=2$

The boundary points are -3 and 2.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\frac{3}{-4+3}>\frac{3}{-4-2}$ $-3>\frac{1}{2}$, False	$(-\infty,-3)$ does not belong to the solution set.
$(-3,2)$	0	$\frac{3}{0+3}>\frac{3}{0-2}$ $1>-\frac{3}{2}$, True	$(-3,2)$ belongs to the solution set.
$(2,\infty)$	3	$\frac{3}{3+3}>\frac{3}{3-2}$ $\frac{1}{2}>3$, False	$(2,\infty)$ does not belong to the solution set.

The solution set is $(-3,2)$ or $\{x \mid -3<x<2\}$.

68. $\frac{1}{x+1}>\frac{2}{x-1}$

Express the inequality so that one side is zero.

$$\frac{1}{x+1}-\frac{2}{x-1}>0$$

$$\frac{x-1}{(x+1)(x-1)}-\frac{2(x+1)}{(x+1)(x-1)}>0$$

$$\frac{x-1-2x-2}{(x+1)(x-1)}<0$$

$$\frac{-x-3}{(x+1)(x-1)}<0$$

Find the values of *x* that make the numerator and denominator zero.

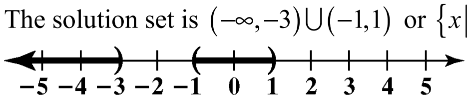
$-x-3=0$ $\quad$ $x+1=0$ $\quad$ $x-1=0$

$-3=x$ $\quad$ $x=-1$ $\quad$ $x=1$

The boundary points are -3, -1, and 1.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\frac{1}{-4+1}>\frac{2}{-3-1}$ $-\frac{1}{3}>-\frac{1}{2}$, True	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1)$	-2	$\frac{1}{-2+1}>\frac{2}{-2-1}$ $-1>-\frac{2}{3}$, False	$(-3,-1)$ does not belong to the solution set.
$(-1,1)$	0	$\frac{1}{0+1}>\frac{2}{0-1}$ $1>-2$, True	$(-3,1)$ belongs to the solution set.
$(1,\infty)$	2	$\frac{1}{2+1}>\frac{2}{2-1}$ $\frac{1}{3}>1$, False	$(1,\infty)$ does not belong to the solution set.

The solution set is $(-\infty,-3)\cup(-1,1)$ or $\{x \mid x<-3 \text{ or } -1<x<1\}$.



69. $\dfrac{x^2-x-2}{x^2-4x+3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-x-2=0 \qquad x^2-4x+3=0$

$(x-2)(x+1)=0 \qquad (x-3)(x-1)=0$

Apply the zero product principle.

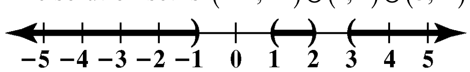
$x-2=0$ or $x+1=0 \qquad x-3=0$ or $x-1=0$

$x=2 \qquad x=-1 \qquad x=3 \qquad x=1$

The boundary points are -1, 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2	$\dfrac{(-2)^2-(-2)-2}{(-2)^2-4(-2)+3}>0$ $\dfrac{4}{15}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\dfrac{0^2-0-2}{0^2-4(0)+3}>0$ $-\dfrac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\dfrac{1.5^2-1.5-2}{1.5^2-4(1.5)+3}>0$ $\dfrac{5}{3}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\dfrac{2.5^2-2.5-2}{2.5^2-4(2.5)+3}>0$ $-\dfrac{7}{3}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\dfrac{4^2-4-2}{4^2-4(4)+3}>0$ $\dfrac{10}{3}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x \mid x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.



70. $\dfrac{x^2-3x+2}{x^2-2x-3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-3x+2=0 \qquad x^2-2x-3=0$

$(x-2)(x-1)=0 \qquad (x-3)(x+1)=0$

Apply the zero product principle.

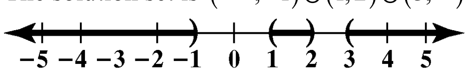
$x-2=0$ or $x-1=0 \qquad x-3=0$ or $x+1=0$

$x=2 \qquad x=1 \qquad x=3 \qquad x=-1$

The boundary points are -1, 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2 $\dfrac{x^2-3x+2}{x^2-2x-3}>0$	$\dfrac{(-2)^2-3(-2)+2}{(-2)^2-2(-2)-3}>0$ $\dfrac{12}{5}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\dfrac{0^2-3(0)+2}{0^2-2(0)-3}>0$ $-\dfrac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\dfrac{1.5^2-3(1.5)+2}{1.5^2-2(1.5)-3}>0$ $\dfrac{1}{15}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\dfrac{2.5^2-3(2.5)+2}{2.5^2-2(2.5)-3}>0$ $-\dfrac{3}{7}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\dfrac{4^2-3(4)+2}{4^2-2(4)-3}>0$ $\dfrac{6}{5}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x \mid x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.



71.

$$2x^3+11x^2 \ge 7x+6$$
$$2x^3+11x^2-7x-6 \ge 0$$

The graph of $f(x)=2x^3+11x^2-7x-6$ appears to cross the *x*-axis at -6, $-\frac{1}{2}$, and 1. We verify this numerically by substituting these values into the function:

$$f(-6)=2(-6)^3+11(-6)^2-7(-6)-6=2(-216)+11(36)-(-42)-6=-432+396+42-6=0$$

$$f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+11\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)-6=2\left(-\frac{1}{8}\right)+11\left(\frac{1}{4}\right)-\left(-\frac{7}{2}\right)-6=-\frac{1}{4}+\frac{11}{4}+\frac{7}{2}-6=0$$

$$f(1)=2(1)^3+11(1)^2-7(1)-6=2(1)+11(1)-7-6=2+11-7-6=0$$

Thus, the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x)\ge 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. Now, the curve is above the *x*-axis when $-6<x<-\frac{1}{2}$ and when $x>1$. Thus, the solution set is $\left\{x \middle| -6\le x\le -\frac{1}{2} \text{ or } x\ge 1\right\}$ or $\left[-6,-\frac{1}{2}\right]\cup[1,\infty)$.

72.

$$2x^3+11x^2 < 7x+6$$
$$2x^3+11x^2-7x-6<0$$

In Problem 63, we verified that the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x)<0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x<-6$ and when $-\frac{1}{2}<x<1$. Thus, the solution set is $\left\{x \middle| x<-6 \text{ or } -\frac{1}{2}<x<1\right\}$ or $(-\infty,-6)\cup\left(-\frac{1}{2},1\right)$.

73.

$$\frac{1}{4(x+2)} \le -\frac{3}{4(x-2)}$$
$$\frac{1}{4(x+2)}+\frac{3}{4(x-2)} \le 0$$

Simplify the left side of the inequality:

$$\frac{x-2}{4(x+2)}+\frac{3(x+2)}{4(x-2)}=\frac{x-2+3x+6}{4(x+2)(x-2)}=\frac{4x+4}{4(x+2)(x-2)}=\frac{4(x+1)}{4(x+2)(x-2)}=\frac{x+1}{x^2-4}.$$

The graph of $f(x)=\frac{x+1}{x^2-4}$ crosses the *x*-axis at -1, and has vertical asymptotes at $x=-2$ and $x=2$. Thus, the boundaries are -2, -1, and 1. We need to find the intervals on which $f(x)\le 0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x<-2$ and when $-1<x<2$. Thus, the solution set is $\{x \mid x<-2 \text{ or } -1\le x<2\}$ or $(-\infty,-2)\cup[-1,2)$.

74.

$$\frac{1}{4(x+2)} > -\frac{3}{4(x-2)}$$

$$\frac{1}{4(x+2)} + \frac{3}{4(x-2)} > 0$$

$$\frac{x+1}{(x+2)(x-2)} > 0$$

The boundaries are -2, -1, and 2. We need to find the intervals on which $f(x) > 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. The curve is above the *x*-axis when $-2 < x < -1$ and when $x > 2$. Thus, the solution set is $\{x | -2 < x < -1 \text{ or } x > 2\}$ or $(-2,-1) \cup (2,\infty)$.

75. $s(t) = -16t^2 + 8t + 87$

The diver's height will exceed that of the cliff when $s(t) > 87$

$$-16t^2 + 8t + 87 > 87$$
$$-16t^2 + 8t > 0$$
$$-8t(2t-1) > 0$$

The boundaries are 0 and $\frac{1}{2}$. Testing each interval shows that the diver will be higher than the cliff for the first half second after beginning the jump. The interval is $\left(0, \frac{1}{2}\right)$.

76. $s(t) = -16t^2 + 48t + 160$

The ball's height will exceed that of the rooftop when $s(t) > 160$

$$-16t^2 + 48t + 160 > 160$$
$$-16t^2 + 48t > 0$$
$$-16t(t-3) > 0$$

The boundaries are 0 and 3. Testing each interval shows that the ball will be higher than the rooftop for the first three seconds after the throw. The interval is $(0,3)$.

77. $f(x) = 0.0875x^2 - 0.4x + 66.6$

$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(35) = 0.0875(35)^2 - 0.4(35) + 66.6 \approx 160$ feet

$g(35) = 0.0875(35)^2 + 1.9(35) + 11.6 \approx 185$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well. The function values and the data are identical.

d. $0.0875x^2 - 0.4x + 66.6 > 540$
$0.0875x^2 - 0.4x + 473.4 > 0$
Solve the related quadratic equation.
$0.0875x^2 - 0.4x + 473.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.4) \pm \sqrt{(-0.4)^2 - 4(0.0875)(473.4)}}{2(0.0875)}$$

$x \approx -71$ or 76

Since the function's domain is $x \geq 30$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30, 76)$	50	$0.0875(50)^2 - 0.4(50) + 66.6 > 540$ $265.35 > 540$, False	$(30, 76)$ does not belong to the solution set.
$(76, \infty)$	100	$0.0875(100)^2 - 0.4(100) + 66.6 > 540$ $901.6 > 540$, True	$(76, \infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 540 feet for speeds exceeding 76 miles per hour. This is represented on graph (b) to the right of point (76, 540).

78. $f(x) = 0.0875x^2 - 0.4x + 66.6$
$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(55) = 0.0875(55)^2 - 0.4(55) + 66.6 \approx 309$ feet
$g(55) = 0.0875(55)^2 + 1.9(55) + 11.6 \approx 381$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well.

d. $0.0875x^2 + 1.9x + 11.6 > 540$
$0.0875x^2 + 1.9x + 528.4 > 0$
Solve the related quadratic equation.
$0.0875x^2 + 1.9x + 528.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1.9) \pm \sqrt{(1.9)^2 - 4(0.0875)(528.4)}}{2(0.0875)}$$

$x \approx -89$ or 68

Since the function's domain is $x \geq 30$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30, 68)$	50	$0.0875(50)^2 + 1.9(50) + 11.6 > 540$ $325.35 > 540$, False	$(30, 68)$ does not belong to the solution set.
$(68, \infty)$	100	$0.0875(100)^2 + 1.9(100) + 11.6 > 540$ $1076.6 > 540$, True	$(68, \infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 540 feet for speeds exceeding 68 miles per hour. This is represented on graph (a) to the right of point (68, 540).

79. Let x = the length of the rectangle.
Since Perimeter $= 2(\text{length}) + 2(\text{width})$, we know

$$50 = 2x + 2(\text{width})$$
$$50 - 2x = 2(\text{width})$$
$$\text{width} = \frac{50-2x}{2} = 25 - x$$

Now, $A = (\text{length})(\text{width})$, so we have that

$$A(x) \le 114$$
$$x(25-x) \le 114$$
$$25x - x^2 \le 114$$

Solve the related equation

$$25x - x^2 = 114$$
$$0 = x^2 - 25x + 114$$
$$0 = (x-19)(x-6)$$

Apply the zero product principle:

$x - 19 = 0$ or $x - 6 = 0$
$x = 19$ $\quad$ $x = 6$

The boundary points are 6 and 19.

Test Interval	Test Number	Test	Conclusion
$(-\infty, 6)$	0	$25(0) - 0^2 \le 114$ $0 \le 114$, True	$(-\infty, 6)$ belongs to the solution set.
$(6, 19)$	10	$25(10) - 10^2 \le 114$ $150 \le 114$, False	$(6, 19)$ does not belong to the solution set.
$(19, \infty)$	20	$25(20) - 20^2 \le 114$ $100 \le 114$, True	$(19, \infty)$ belongs to the solution set.

If the length is 6 feet, then the width is 19 feet. If the length is less than 6 feet, then the width is greater than 19 feet. Thus, if the area of the rectangle is not to exceed 114 square feet, the length of the shorter side must be 6 feet or less.

80. $2l + 2w = P$
$2l + 2w = 180$
$2l = 180 - 2w$
$l = 90 - w$

We want to restrict the area to 800 square feet. That is,

$$A \le 800$$
$$l \cdot w \le 800$$
$$(90 - w)w \le 800$$
$$90w - w^2 \le 800$$
$$-w^2 + 90w - 800 \le 0$$
$$w^2 - 90w + 800 \ge 0$$

$$w^2 - 90w + 800 = 0$$
$$(w - 80)(w - 10) = 0$$

$w - 80 = 0$ or $w - 10 = 0$
$w = 80$ $\quad w = 10$

Assuming the width is the shorter side, we ignore the larger solution.

Test Interval	Test Number	Test	Conclusion
$(0,10)$	5	$90(5) - (5)^2 \le 800$ true	$(0,10)$ is part of the solution set
$(10,45)$	20	$90(20) - (20)^2 \le 800$ false	$(10,45)$ is not part of the solution set

The solution set is $\{w \mid 0 < w \le 10\}$ or $(0,10]$.

The length of the shorter side cannot exceed 10 feet.

81. – 85. Answers will vary.

86. The solution set is $(-\infty, -5) \cup (2, \infty)$.

87. The solution set is $\left\{x \middle| -3 \le x \le \frac{1}{2}\right\}$ or $\left[-3, \frac{1}{2}\right]$.

88. The solution set is $(-2, -1)$ or $(2, \infty)$.

89. The solution set is $(1, 4]$.

90. Graph $y_1 = \frac{x+2}{x-3}$ and $y_2 = 2$

y_1 less than or equal to y_2 for $x < 3$ or $x \ge 8$.

The solution set is $(-\infty, 3) \cup [8, \infty)$

91. Graph $y_1 = \frac{1}{x+1}$ and $y_2 = \frac{2}{x+4}$

y_1 less than or equal to y_2 for $-4 < x < -1$ or $x \ge 2$.

The solution set is $(-4, -1) \cup [2, \infty)$

92. **a.** $f(x) = 0.1125x^2 - 0.1x + 55.9$

b. $0.1125x^2 - 0.1x + 55.9 > 455$

$0.1125x^2 - 0.1x + 399.1 > 0$

Solve the related quadratic equation.

$0.1125x^2 - 0.1x + 399.1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.1) \pm \sqrt{(-0.1)^2 - 4(0.1125)(399.1)}}{2(0.1125)}$$

$x \approx -59$ or 60

Since the function's domain must be $x \geq 0$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(0, 60)$	50	$0.1125(50)^2 - 0.1(50) + 55.9 > 455$ $332.15 > 455$, False	$(0, 60)$ does not belong to the solution set.
$(60, \infty)$	100	$0.1125(100)^2 - 0.1(100) + 55.9 > 455$ $1170.9 > 455$, True	$(60, \infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 455 feet for speeds exceeding 60 miles per hour.

93. **a.** $f(x) = 0.1375x^2 + 0.7x + 37.8$

b. $0.1375x^2 + 0.7x + 37.8 > 446$

$0.1375x^2 + 0.7x + 408.2 > 0$

Solve the related quadratic equation.

$0.1375x^2 + 0.7x + 408.2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.7) \pm \sqrt{(0.7)^2 - 4(0.1375)(408.2)}}{2(0.1375)}$$

$x \approx -57$ or 52

Since the function's domain must be $x \geq 0$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(0, 52)$	10	$0.1375(10)^2 + 0.7(10) + 37.8 > 446$ $58.55 > 446$, False	$(0, 52)$ does not belong to the solution set.
$(52, \infty)$	100	$0.1375(100)^2 + 0.7(100) + 37.8 > 446$ $1482.8 > 446$, True	$(52, \infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 446 feet for speeds exceeding 52 miles per hour.

94. makes sense

95. does not make sense; Explanations will vary. Sample explanation: Polynomials are defined for all values.

96. makes sense

97. does not make sense; Explanations will vary. Sample explanation: To solve this inequality you must first subtract 2 from both sides.

98. false; Changes to make the statement true will vary. A sample change is: The solution set is $\{x | x < -5 \text{ or } x > 5\}$ or $(-\infty, -5) \cup (5, \infty)$.

99. false; Changes to make the statement true will vary. A sample change is: The inequality cannot be solved by multiplying both sides by $x + 3$. We do not know if $x + 3$ is positive or negative. Thus, we would not know whether or not to reverse the order of the inequality.

100. false; Changes to make the statement true will vary. A sample change is: The inequalities have different solution sets. The value, 1, is included in the domain of the first inequality, but not included in the domain of the second inequality.

101. true

102. One possible solution: $x^2 - 2x - 15 \le 0$

103. One possible solution: $\dfrac{x-3}{x+4} \ge 0$

104. Because any non-zero number squared is positive, the solution is all real numbers except 2.

105. Because any number squared other than zero is positive, the solution includes only 2.

106. Because any number squared is positive, the solution is the empty set, $\varnothing$.

107. Because any number squared other than zero is positive, and the reciprocal of zero is undefined, the solution is all real numbers except 2.

108. **a.** The solution set is all real numbers.

b. The solution set is the empty set, $\varnothing$.

c. $4x^2 - 8x + 7 > 0$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(7)}}{2(4)}$$
$$x = \frac{8 \pm \sqrt{64 - 112}}{8}$$
$$x = \frac{8 \pm \sqrt{-48}}{8} \Rightarrow \text{imaginary}$$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 > 0$

$7 > 0$ True

The inequality is true for all numbers.

$4x^2 - 8x + 7 < 0$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 = 7 < 0$ False

The solution set is the empty set.

109. $\sqrt{27 - 3x^2} \ge 0$

$27 - 3x^2 \ge 0$

$9 - x^2 \ge 0$

$(3 - x)(3 + x) \ge 0$

$3 - x = 0 \qquad 3 + x = 0$

$x = 3$ or $\qquad x = -3$

–3	T	3

Test –4: $\sqrt{27 - 3(-4)^2} \ge 0$

$\sqrt{27 - 48} \ge 0$

$\sqrt{-21} \ge 0$

no graph- imaginary

Test 0: $\sqrt{27 - 3(0)^2} \ge 0$

$\sqrt{27} \ge 0$ True

Test 4: $\sqrt{27 - 3(4)^2} \ge 0$

$\sqrt{27 - 48} \ge 0$

$\sqrt{-21} \ge 0$

no graph -imaginary

The solution set is [–3, 3].

110. The slope of the line $y = -\frac{1}{4}x + \frac{1}{3}$ is $-\frac{1}{4}$. Thus the slope of the line perpendicular to this line is 4.

$y - y_1 = m(x - x_1)$

$y - 5 = 4(x - (-2))$

$y - 5 = 4(x + 2)$ point-slope

$y - 5 = 4x + 8$

$y = 4x + 13$ slope-intercept

$4x - y + 13 = 0$ general

111. Since $h(x) = \sqrt{36 - 2x}$ contains an even root; the quantity under the radical must be greater than or equal to 0.

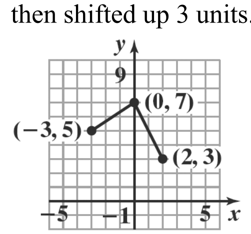
$36 - 2x \ge 0$

$-2x \ge -36$

$x \le 18$

Thus, the domain of h is $\{x | x \le 18\}$, or the interval $(-\infty, 18]$.

112. The graph of $y = f(x)$ is reflected about the y-axis, then shifted up 3 units.



113. **a.**
$$y = kx^2$$
$$64 = k \cdot 2^2$$
$$64 = 4k$$
$$16 = k$$

b.
$$y = kx^2$$
$$y = 16x^2$$

c.
$$y = kx^2$$
$$y = 16x^2$$
$$y = 16 \cdot 5^2$$
$$y = 400$$

114. **a.**
$$y = \frac{k}{x}$$
$$12 = \frac{k}{8}$$
$$96 = k$$

b.
$$y = \frac{k}{x}$$
$$y = \frac{96}{x}$$

c.
$$y = \frac{96}{x}$$
$$y = \frac{96}{3}$$
$$y = 32$$

115.
$$S = \frac{kA}{P}$$
$$12,000 = \frac{k \cdot 60,000}{40}$$
$$\frac{12,000 \cdot 40}{60,000} = k$$
$$8 = k$$

Section 2.8

Check Point Exercises

1. y varies directly as x is expressed as $y = kx.$
The volume of water, W, varies directly as the time, t can be expressed as $W = kt$.
Use the given values to find k.
$$W = kt$$
$$30 = k(5)$$
$$6 = k$$
Substitute the value of k into the equation.
$$W = kt$$
$$W = 6t$$
Use the equation to find W when $t = 11$.
$$W = 6t$$
$$= 6(11)$$
$$= 66$$
A shower lasting 11 minutes will use 66 gallons of water.

2. y varies directly as the cube of x is expressed as $y = kx^3$.
The weight, w, varies directly as the cube of the length, l can be expressed as $w = kl^2$.
Use the given values to find k.
$$w = kl^3$$
$$2025 = k(15)^3$$
$$0.6 = k$$
Substitute the value of k into the equation.
$$w = kl^3$$
$$w = 0.6l^3$$
Use the equation to find w when $l = 25$.
$$w = 0.6l^3$$
$$= 0.6(25)^3$$
$$= 9375$$
The 25-foot long shark was 9375 pounds.

3. y varies inversely as x is expressed as $y = \frac{k}{x}$.
The length, L, varies inversely as the frequency, f can be expressed as $L = \frac{k}{f}$.
Use the given values to find k.
$$L = \frac{k}{f}$$
$$8 = \frac{k}{640}$$
$$5120 = k$$
Substitute the value of k into the equation.
$$L = \frac{k}{f}$$
$$L = \frac{5120}{f}$$
Use the equation to find f when $L = 10$.
$$L = \frac{5120}{f}$$
$$10 = \frac{5120}{f}$$
$$10f = 5120$$
$$f = 512$$
A 10 inch violin string will have a frequency of 512 cycles per second.

4. let M represent the number of minutes
let Q represent the number of problems
let P represent the number of people
M varies directly as Q and inversely as P is expressed as $M = \frac{kQ}{P}$.

Use the given values to find k.

$$M = \frac{kQ}{P}$$
$$32 = \frac{k(16)}{4}$$
$$8 = k$$

Substitute the value of k into the equation.

$$M = \frac{kQ}{P}$$
$$M = \frac{8Q}{P}$$

Use the equation to find M when $P = 8$ and $Q = 24$.

$$M = \frac{8Q}{P}$$
$$M = \frac{8(24)}{8}$$
$$M = 24$$

It will take 24 minutes for 8 people to solve 24 problems.

5. V varies jointly with h and r^2 and can be modeled as $V = khr^2$.

Use the given values to find k.

$$V = khr^2$$
$$120\pi = k(10)(6)^2$$
$$\frac{\pi}{3} = k$$

Therefore, the volume equation is $V = \frac{1}{3}hr^2$.

$$V = \frac{\pi}{3}(2)(12)^2 = 96\pi \text{ cubic feet}$$

Concept and Vocabulary Check 2.8

1. $y - kx$; constant of variation

2. $y = kx^n$

3. $y = \frac{k}{x}$

4. $y = \frac{kx}{z}$

5. $y = kxz$

6. directly; inversely

7. jointly; inversely

Exercise Set 2.8

1. Use the given values to find k.

$$y = kx$$
$$65 = k \cdot 5$$
$$\frac{65}{5} = \frac{k \cdot 5}{5}$$
$$13 = k$$

The equation becomes $y = 13x$.
When $x = 12$, $y = 13x = 13 \cdot 12 = 156$.

2.

$$y = kx$$
$$45 = k \cdot 5$$
$$9 = k$$

$$y = 9x = 9 \cdot 13 = 117$$

3. Since y varies inversely with x, we have $y = \frac{k}{x}$.

Use the given values to find k.

$$y = \frac{k}{x}$$
$$12 = \frac{k}{5}$$
$$5 \cdot 12 = 5 \cdot \frac{k}{5}$$
$$60 = k$$

The equation becomes $y = \frac{60}{x}$.

When $x = 2$, $y = \frac{60}{2} = 30$.

4.

$$y = \frac{k}{x}$$
$$6 = \frac{k}{3}$$
$$18 = k$$

$$y = \frac{18}{9} = 2$$

5. Since y varies inversely as x and inversely as the square of z, we have $y=\frac{kx}{z^2}$.

Use the given values to find k.

$$y=\frac{kx}{z^2}$$
$$20=\frac{k(50)}{5^2}$$
$$20=\frac{k(50)}{25}$$
$$20=2k$$
$$10=k$$

The equation becomes $y=\frac{10x}{z^2}$.

When $x = 3$ and $z = 6$,
$$y=\frac{10x}{z^2}=\frac{10(3)}{6^2}=\frac{10(3)}{36}=\frac{30}{36}=\frac{5}{6}.$$

6.
$$a=\frac{kb}{c^2}$$
$$7=\frac{k(9)}{(6)^2}$$
$$7=\frac{k(9)}{36}$$
$$7=\frac{k}{4}$$
$$28=k$$

$$a=\frac{28(4)}{(8)^2}=\frac{28(4)}{64}=\frac{7}{4}$$

7. Since y varies jointly as x and z, we have $y=kxz$.

Use the given values to find k.

$$y=kxz$$
$$25=k(2)(5)$$
$$25=k(10)$$
$$\frac{25}{10}=\frac{k(10)}{10}$$
$$\frac{5}{2}=k$$

The equation becomes $y=\frac{5}{2}xz$.

When $x = 8$ and $z = 12$, $y=\frac{5}{2}(8)(12)=240$.

8.
$$C=kAT$$
$$175=k(2100)(4)$$
$$175=k(8400)$$
$$\frac{1}{48}=k$$

$$C=\frac{1}{48}(2400)(6)=\frac{14400}{48}=300$$

9. Since y varies jointly as a and b and inversely as the square root of c, we have $y=\frac{kab}{\sqrt{c}}$.

Use the given values to find k.

$$y=\frac{kab}{\sqrt{c}}$$
$$12=\frac{k(3)(2)}{\sqrt{25}}$$
$$12=\frac{k(6)}{5}$$
$$12(5)=\frac{k(6)}{5}(5)$$
$$60=6k$$
$$\frac{60}{6}=\frac{6k}{6}$$
$$10=k$$

The equation becomes $y=\frac{10ab}{\sqrt{c}}$.

When $a = 5$, $b = 3$, $c = 9$,
$$y=\frac{10ab}{\sqrt{c}}=\frac{10(5)(3)}{\sqrt{9}}=\frac{150}{3}=50.$$

10.
$$y=\frac{kmn^2}{p}$$
$$15=\frac{k(2)(1)^2}{6}$$
$$15=\frac{2k}{6}$$
$$15(6)=\frac{2k}{6}(6)$$
$$90=2k$$
$$k=45$$

$$y=\frac{45mn^2}{p}=\frac{45(3)(4)^2}{10}=\frac{2160}{10}=216$$

11. $x = kyz$;

Solving for y:

$$x = kyz$$
$$\frac{x}{kz} = \frac{kyz}{yz}.$$
$$y = \frac{x}{kz}$$

12. $x = kyz^2$;

Solving for y:

$$x = kyz^2$$
$$\frac{x}{kz^2} = \frac{kyz^2}{kz^2}$$
$$y = \frac{x}{kz^2}$$

13. $x = \frac{kz^3}{y}$;

Solving for y

$$x = \frac{kz^3}{y}$$
$$xy = y \cdot \frac{kz^3}{y}$$
$$xy = kz^3$$
$$\frac{xy}{x} = \frac{kz^3}{x}$$
$$y = \frac{kz^3}{x}$$

14.

$$x = \frac{k\sqrt[3]{z}}{y}$$
$$yx = y \cdot \frac{k\sqrt[3]{z}}{y}$$
$$yx = k\sqrt[3]{z}$$
$$\frac{yx}{x} = \frac{k\sqrt[3]{z}}{x}$$
$$y = \frac{k\sqrt[3]{z}}{x}$$

15. $x = \frac{kyz}{\sqrt{w}}$;

Solving for y:

$$x = \frac{kyz}{\sqrt{w}}$$
$$x\left(\sqrt{w}\right) = \left(\sqrt{w}\right)\frac{kyz}{\sqrt{w}}$$
$$x\sqrt{w} = kyz$$
$$\frac{x\sqrt{w}}{kz} = \frac{kyz}{kz}$$
$$y = \frac{x\sqrt{w}}{kz}$$

16.

$$x = \frac{kyz}{w^2}$$
$$\left(\frac{w^2}{kz}\right)x = \frac{w^2}{kz}\frac{kyz}{w^2}$$
$$y = \frac{xw^2}{kz}$$

17. $x = kz(y + w)$;

Solving for y:

$$x = kz(y + w)$$
$$x = kzy + kzw$$
$$x - kzw = kzy$$
$$\frac{x - kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x - kzw}{kz}$$

18.

$$x = kz(y - w)$$
$$x = kzy - kzw$$
$$x + kzw = kzy$$
$$\frac{x + kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x + kzw}{kz}$$

19. $x = \dfrac{kz}{y-w}$;

Solving for *y:*

$$\begin{aligned}
x &= \frac{kz}{y-w} \\
(y-w)x &= (y-w)\frac{kz}{y-w} \\
xy - wx &= kz \\
xy &= kz + wx \\
\frac{xy}{x} &= \frac{kz+wx}{x} \\
y &= \frac{xw+kz}{x}
\end{aligned}$$

20.

$$\begin{aligned}
x &= \frac{kz}{y+w} \\
(y+w)x &= (y+w)\frac{kz}{y+w} \\
yx + xw &= kz \\
yx &= kz - xw \\
\frac{yx}{x} &= \frac{kz-xw}{x} \\
y &= \frac{kz-xw}{x}
\end{aligned}$$

21. Since *T* varies directly as *B*, we have $T = kB$. Use the given values to find *k*.

$$\begin{aligned}
T &= kB \\
3.6 &= k(4) \\
\frac{3.6}{4} &= \frac{k(4)}{4} \\
0.9 &= k
\end{aligned}$$

The equation becomes $T = 0.9B$.

When $B = 6$, $T = 0.9(6) = 5.4$.

The tail length is 5.4 feet.

22.

$$\begin{aligned}
M &= kE \\
60 &= k(360) \\
\frac{60}{360} &= \frac{k(360)}{360} \\
\frac{1}{6} &= k
\end{aligned}$$

$$M = \frac{1}{6}(186) = 31$$

A person who weighs 186 pounds on Earth will weigh 31 pounds on the moon.

23. Since *B* varies directly as *D*, we have $B = kD$. Use the given values to find *k*.

$$\begin{aligned}
B &= kD \\
8.4 &= k(12) \\
\frac{8.4}{12} &= \frac{k(12)}{12} \\
k &= \frac{8.4}{12} = 0.7
\end{aligned}$$

The equation becomes $B = 0.7D$.

When $B = 56$,

$$\begin{aligned}
56 &= 0.7D \\
\frac{56}{0.7} &= \frac{0.7D}{0.7} \\
D &= \frac{56}{0.7} = 80
\end{aligned}$$

It was dropped from 80 inches.

24.

$$\begin{aligned}
d &= kf \\
9 &= k(12) \\
\frac{9}{12} &= \frac{k(12)}{12} \\
0.75 &= k \\
d &= 0.75f \\
15 &= 0.75f \\
\frac{15}{0.75} &= \frac{0.75f}{0.75} \\
20 &= f
\end{aligned}$$

A force of 20 pounds is needed.

25. Since a man's weight varies directly as the cube of his height, we have $w = kh^3$. Use the given values to find *k*.

$$\begin{aligned}
w &= kh^3 \\
170 &= k(70)^3 \\
170 &= k(343,000) \\
\frac{170}{343,000} &= \frac{k(343,000)}{343,000} \\
0.000496 &= k
\end{aligned}$$

The equation becomes $w = 0.000496h^3$.

When $h = 107$,

$$\begin{aligned}
w &= 0.000496(107)^3 \\
&= 0.000496(1,225,043) \approx 607.
\end{aligned}$$

Robert Wadlow's weight was approximately 607 pounds.

26.
$$h = kd^2$$
$$50 = k \cdot 10^2$$
$$0.5 = k$$

$$h = 0.5d^2$$

a.
$$h = 0.5d^2$$
$$h = 0.5(30)^2$$
$$h = 450$$
A water pipe with a 30 centimeter diameter can serve 450 houses.

b.
$$h = 0.5d^2$$
$$1250 = 0.5d^2$$
$$d^2 = 625$$
$$d = \sqrt{625}$$
$$d = 25$$
A water pipe with a 25 centimeter diameter can serve 1250 houses.

27. Since the banking angle varies inversely as the turning radius, we have $B = \frac{k}{r}$.

Use the given values to find *k*.
$$B = \frac{k}{r}$$
$$28 = \frac{k}{4}$$
$$28(4) = 28\left(\frac{k}{4}\right)$$
$$112 = k$$
The equation becomes $B = \frac{112}{r}$.

When $r = 3.5$, $B = \frac{112}{r} = \frac{112}{3.5} = 32$.

The banking angle is 32° when the turning radius is 3.5 feet.

28.
$$t = \frac{k}{d}$$
$$4.4 = \frac{k}{1000}$$
$$(1000)4.4 = (1000)\frac{k}{1000}$$
$$4400 = k$$

$$t = \frac{4400}{d} = \frac{4400}{5000} = 0.88$$
The water temperature is 0.88° Celsius at a depth of 5000 meters.

29. Since intensity varies inversely as the square of the distance, we have pressure, we have $I = \frac{k}{d}$.

Use the given values to find *k*.
$$I = \frac{k}{d^2}.$$
$$62.5 = \frac{k}{3^2}$$
$$62.5 = \frac{k}{9}$$
$$9(62.5) = 9\left(\frac{k}{9}\right)$$
$$562.5 = k$$
The equation becomes $I = \frac{562.5}{d^2}$.

When $d = 2.5$, $I = \frac{562.5}{2.5^2} = \frac{562.5}{6.25} = 90$

The intensity is 90 milliroentgens per hour.

30.
$$i = \frac{k}{d^2}$$
$$3.75 = \frac{k}{40^2}$$
$$3.75 = \frac{k}{1600}$$
$$(1600)3.75 = (1600)\frac{k}{1600}$$
$$6000 = k$$

$$i = \frac{6000}{d^2} = \frac{6000}{50^2} = \frac{6000}{2500} = 2.4$$
The illumination is 2.4 foot-candles at a distance of 50 feet.

31. Since index varies directly as weight and inversely as the square of one's height, we have $I = \frac{kw}{h^2}$.

Use the given values to find *k*.

$$I = \frac{kw}{h^2}$$
$$35.15 = \frac{k(180)}{60^2}$$
$$35.15 = \frac{k(180)}{3600}$$
$$(3600)35.15 = \frac{k(180)}{3600}$$
$$126540 = k(180)$$
$$k = \frac{126540}{180} = 703$$

The equation becomes $I = \frac{703w}{h^2}$.

When $w = 170$ and $h = 70$,

$I = \frac{703(170)}{(70)^2} \approx 24.4.$

This person has a BMI of 24.4 and is not overweight.

32.

$$i = \frac{km}{c}$$
$$125 = \frac{k(25)}{20}$$
$$20(125) = (20)\frac{k(25)}{20}$$
$$2500 = 25k$$
$$\frac{2500}{25} = \frac{25k}{25}$$
$$100 = k$$

$$i = \frac{100m}{c}$$
$$80 = \frac{100(40)}{c}$$
$$80 = \frac{4000}{c}$$
$$80c = c \cdot \frac{4000}{c}$$
$$80c = 4000$$
$$\frac{80c}{80} = \frac{4000}{80}$$
$$c = 50$$

The chronological age is 50.

33. Since heat loss varies jointly as the area and temperature difference, we have $L = kAD$. Use the given values to find *k*.

$$L = kAD$$
$$1200 = k(3 \cdot 6)(20)$$
$$1200 = 360k$$
$$\frac{1200}{360} = \frac{360k}{360}$$
$$k = \frac{10}{3}$$

The equation becomes $L = \frac{10}{3}AD$

When $A = 6 \cdot 9 = 54$, $D = 10$,

$L = \frac{10}{3}(9 \cdot 6)(10) = 1800$.

The heat loss is 1800 Btu .

34.

$$e = kmv^2$$
$$36 = k(8)(3)^2$$
$$36 = k(8)(9)$$
$$36 = 72k$$
$$\frac{36}{72} = \frac{72k}{72}$$
$$k = 0.5$$

$e = 0.5mv^2 = 0.5(4)(6)^2 = 0.5(4)(36) = 72$

A mass of 4 grams and velocity of 6 centimeters per second has a kinetic energy of 72 ergs.

35. Since intensity varies inversely as the square of the distance from the sound source, we have $I = \frac{k}{d^2}$. If you move to a seat twice as far, then $d = 2d$. So we have $I = \frac{k}{(2d)^2} = \frac{k}{4d^2} = \frac{1}{4} \cdot \frac{k}{d^2}$. The intensity will be multiplied by a factor of $\frac{1}{4}$. So the sound intensity is $\frac{1}{4}$ of what it was originally.

36.

$$t = \frac{k}{a}$$
$$t = \frac{k}{3a} = \frac{1}{3} \cdot \frac{k}{a}$$

A year will seem to be $\frac{1}{3}$ of a year.

37. **a.** Since the average number of phone calls varies jointly as the product of the populations and inversely as the square of the distance, we have $C = \frac{kP_1P_2}{d^2}$.

b. Use the given values to find *k*.

$$C = \frac{kP_1P_2}{d^2}$$
$$326,000 = \frac{k(777,000)(3,695,000)}{(420)^2}$$
$$326,000 = \frac{k\left(2.87\times10^{12}\right)}{176,400}$$
$$326,000 = 16269841.27k$$
$$0.02 \approx k$$

The equation becomes $C = \frac{0.02P_1P_2}{d^2}$.

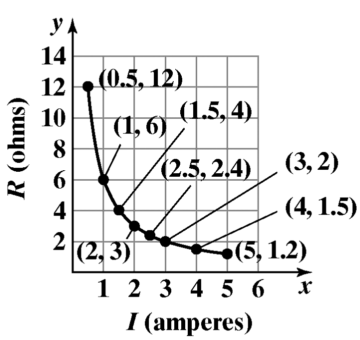
c. $$C = \frac{0.02(650,000)(490,000)}{(400)^2} \approx 39,813$$

There are approximately 39,813 daily phone calls.

38.
$$f = kas^2$$
$$150 = k(4\cdot 5)(30)^2$$
$$150 = k(20)(900)$$
$$150 = 18000k$$
$$\frac{150}{18000} = \frac{18000k}{150}$$
$$\frac{1}{120} = k$$
$$f = \frac{1}{120}as^2 = \frac{1}{120}(3\cdot 4)(60)^2$$
$$= \frac{1}{120}(12)(3600)$$
$$= 360$$

Yes, the wind will exert a force of 360 pounds on the window.

39. **a.**



b. Current varies inversely as resistance. Answers will vary.

c. Since the current varies inversely as resistance we have $R = \frac{k}{I}$. Using one of the given ordered pairs to find k.

$$12 = \frac{k}{0.5}$$
$$12(0.5) = \frac{k}{0.5}(0.5)$$
$$k = 6$$

The equation becomes $R = \frac{6}{I}$.

40. – 48. Answers will vary.

49. does not make sense; Explanations will vary. Sample explanation: For an inverse variation, the independent variable can not be zero.

50. does not make sense; Explanations will vary. Sample explanation: A direct variation with a positive constant of variation will have both variables increase simultaneously.

51. makes sense

52. makes sense

53. Pressure, *P*, varies directly as the square of wind velocity, *v*, can be modeled as $P = kv^2$.

If $v = x$ then $P = k(x)^2 = kx^2$

If $v = 2x$ then $P = k(2x)^2 = 4kx^2$

If the wind speed doubles the pressure is 4 times more destructive.

54. Illumination, *I*, varies inversely as the square of the distance, *d*, can be modeled as $I = \frac{k}{d^2}$.

If $d = 15$ then $I = \frac{k}{15^2} = \frac{k}{225}$

If $d = 30$ then $I = \frac{k}{30^2} = \frac{k}{900}$

Note that $\frac{225}{900} = \frac{1}{4}$

If the distance doubles the illumination will be $\frac{1}{4}$ as intense.

55. The Heat, *H*, varies directly as the square of the voltage, *v*, and inversely as the resistance, *r*.

$$H = \frac{kv^2}{r}$$

If the voltage remains constant, to triple the heat the resistant must be reduced by a multiple of 3.

56. Illumination, I, varies inversely as the square of the distance, d, can be modeled as $I = \frac{k}{d^2}$.

If $I = x$ then $x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{k}{x}}$.

If $I = \frac{1}{50}x$ then $\frac{1}{50}x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{50k}{x}} = \sqrt{50}\sqrt{\frac{k}{x}}$.

Since $\sqrt{50} \approx 7$, the Hubble telescope is able to see about 7 times farther than a ground-based telescope.

57. Answers will vary.

58. Let x = the amount invested at 7%.
Let 20,000 – x = the amount invested at 9%.

$$0.07x + 0.09(20,000 - x) = 1550$$
$$0.07x + 1800 - 0.09x = 1550$$
$$-0.02x + 1800 = 1550$$
$$-0.02x = -250$$
$$x = 12,500$$
$$20,000 - x = 7500$$

$12,500 was invested at 7% and $7500 was invested at 9%.

59.
$$\sqrt{x+7} - 1 = x$$
$$\sqrt{x+7} = x + 1$$
$$x + 7 = x^2 + 2x + 1$$
$$x^2 + x - 6 = 0$$
$$(x-2)(x+3) = 0$$

$x - 2 = 0 \quad x + 3 = 0$
$x = 2 \quad x = -3$

The check indicates that 2 is a solution.
The solution set is $\{2\}$.

60. $f(x) = x^3 + 2$

Replace $f(x)$ with y:

$y = x^3 + 2$

Interchange x and y:

$x = y^3 + 2$

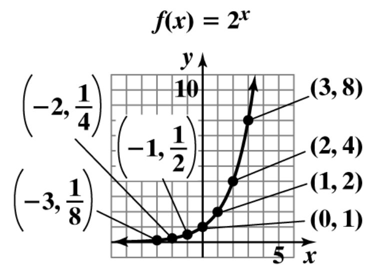
Solve for y:

$$x = y^3 + 2$$
$$x - 2 = y^3$$
$$\sqrt[3]{x-2} = y$$

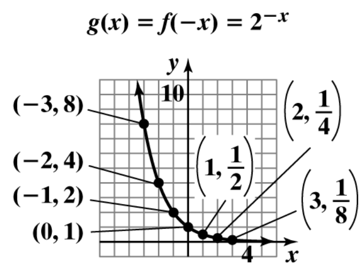
Replace y with $f^{-1}(x)$:

$f^{-1}(x) = \sqrt[3]{x-2}$

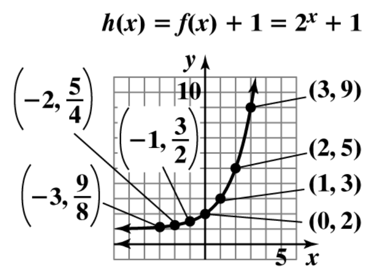
61.



62.



63.



Chapter 2 Review Exercises

1. $(8 - 3i) - (17 - 7i) = 8 - 3i - 17 + 7i$
$= -9 + 4i$

2. $4i(3i - 2) = (4i)(3i) + (4i)(-2)$
$= 12i^2 - 8i$
$= -12 - 8i$

3. $(7 - i)(2 + 3i)$
$= 7 \cdot 2 + 7(3i) + (-i)(2) + (-i)(3i)$
$= 14 + 21i - 2i + 3$
$= 17 + 19i$

4. $(3 - 4i)^2 = 3^2 + 2 \cdot 3(-4i) + (-4i)^2$
$= 9 - 24i - 16$
$= -7 - 24i$

5. $(7 + 8i)(7 - 8i) = 7^2 + 8^2 = 49 + 64 = 113$

6. $\frac{6}{5+i} = \frac{6}{5+i}\cdot\frac{5-i}{5-i}$
$= \frac{30-6i}{25+1}$
$= \frac{30-6i}{26}$
$= \frac{15-3i}{13}$
$= \frac{15}{13} - \frac{3}{13}i$

7. $\frac{3+4i}{4-2i} = \frac{3+4i}{4-2i}\cdot\frac{4+2i}{4+2i}$
$= \frac{12+6i+16i+8i^2}{16-4i^2}$
$= \frac{12+22i-8}{16+4}$
$= \frac{4+22i}{20}$
$= \frac{1}{5} + \frac{11}{10}i$

8. $\sqrt{-32} - \sqrt{-18} = i\sqrt{32} - i\sqrt{18}$
$= i\sqrt{16\cdot 2} - i\sqrt{9\cdot 2}$
$= 4i\sqrt{2} - 3i\sqrt{2}$
$= (4i-3i)\sqrt{2}$
$= i\sqrt{2}$

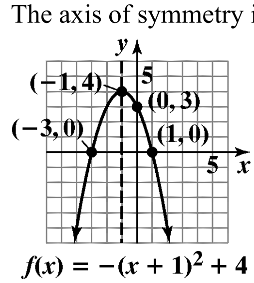
9. $(-2+\sqrt{-100})^2 = (-2+i\sqrt{100})^2$
$= (-2+10i)^2$
$= 4-40i+(10i)^2$
$= 4-40i-100$
$= -96-40i$

10. $\frac{4+\sqrt{-8}}{2} = \frac{4+i\sqrt{8}}{2} = \frac{4+2i\sqrt{2}}{2} = 2+i\sqrt{2}$

11. $x^2 - 2x + 4 = 0$
$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$
$x = \frac{2\pm\sqrt{4-16}}{2}$
$x = \frac{2\pm\sqrt{-12}}{2}$
$x = \frac{2\pm 2i\sqrt{3}}{2}$
$x = 1\pm i\sqrt{3}$
The solution set is $\{1-i\sqrt{3},\ 1+i\sqrt{3}\}$.

12. $2x^2 - 6x + 5 = 0$
$x = \frac{-(-6)\pm\sqrt{(-6)^2-4(2)(5)}}{2(2)}$
$x = \frac{6\pm\sqrt{36-40}}{4}$
$x = \frac{6\pm\sqrt{-4}}{4}$
$x = \frac{6\pm 2i}{4}$
$x = \frac{6}{4} \pm \frac{2i}{4}$
$= \frac{3}{2} \pm \frac{1}{2}i$
The solution set is $\left\{\frac{3}{2}-\frac{1}{2}i,\ \frac{3}{2}+\frac{1}{2}i\right\}$.

13. $f(x) = -(x+1)^2 + 4$
vertex: (–1, 4)
x-intercepts:
$0 = -(x+1)^2 + 4$
$(x+1)^2 = 4$
$x+1 = \pm 2$
$x = -1\pm 2$
$x = -3$ or $x = 1$
y-intercept:
$f(0) = -(0+1)^2 + 4 = 3$
The axis of symmetry is $x = -1$.



domain: $(-\infty,\infty)$ range: $(-\infty,4]$

14. $f(x) = (x+4)^2 - 2$

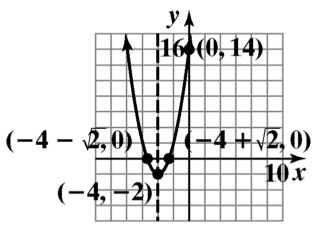
vertex: (–4, –2)

x-intercepts:

$$0 = (x+4)^2 - 2$$
$$(x+4)^2 = 2$$
$$x+4 = \pm\sqrt{2}$$
$$x = -4 \pm \sqrt{2}$$

y-intercept:

$$f(0) = (0+4)^2 - 2 = 14 \ = -1$$



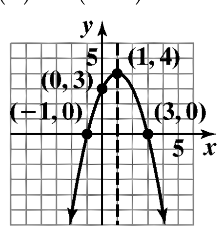
***f*(*x*) = (*x* + 4)² − 2**

The axis of symmetry is $x = -4$.

domain: $(-\infty, \infty)$ range: $[-2, \infty)$

15. $f(x) = -x^2 + 2x + 3$

$$= -\left(x^2 - 2x + 1\right) + 3 + 1$$
$$f(x) = -(x-1)^2 + 4$$

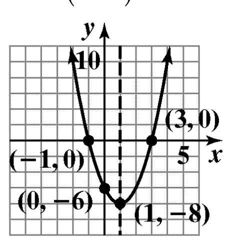


***f*(*x*) = − *x*² + 2*x* + 3**

domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

16. $f(x) = 2x^2 - 4x - 6$

$$f(x) = 2\left(x^2 - 2x + 1\right) - 6 - 2$$
$$2(x-1)^2 - 8$$



***f*(*x*) = 2*x*² − 4*x* − 6**

axis of symmetry: $x = 1$

domain: $(-\infty, \infty)$ range: $[-8, \infty)$

17. $f(x) = -x^2 + 14x - 106$

a. Since $a < 0$ the parabola opens down with the maximum value occurring at

$$x = -\frac{b}{2a} = -\frac{14}{2(-1)} = 7.$$

The maximum value is $f(7)$.

$$f(7) = -(7)^2 + 14(7) - 106 = -57$$

b. domain: $(-\infty, \infty)$ range: $(-\infty, -57]$

18. $f(x) = 2x^2 + 12x + 703$

a. Since $a > 0$ the parabola opens up with the minimum value occurring at

$$x = -\frac{b}{2a} = -\frac{12}{2(2)} = -3.$$

The minimum value is $f(-3)$.

$$f(-3) = 2(-3)^2 + 12(-3) + 703 = 685$$

b. domain: $(-\infty, \infty)$ range: $[685, \infty)$

19. **a.** The maximum height will occur at the vertex.

$$f(x) = -0.025x^2 + x + 6$$
$$x = -\frac{b}{2a} = -\frac{1}{2(-0.025)} = 20$$
$$f(20) = -0.025(20)^2 + (20) + 6 = 16$$

The maximum height of 16 feet occurs when the ball is 20 yards downfield.

b. $f(x) = -0.025x^2 + x + 6$

$$f(0) = -0.025(0)^2 + (0) + 6 = 6$$

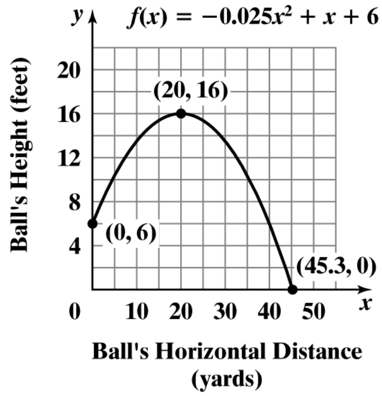
The ball was tossed at a height of 6 feet.

c. The ball is at a height of 0 when it hits the ground.

$$f(x) = -0.025x^2 + x + 6$$
$$0 = -0.025x^2 + x + 6$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(-0.025)(6)}}{2(-0.025)}$$
$$x \approx 45.3,\ -5.3 \text{(reject)}$$

The ball will hit the ground 45.3 yards downfield.

d. The football's path:



20. Maximize the area using $A = lw$.

$A(x) = x(1000 - 2x)$

$A(x) = -2x^2 + 1000x$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{1000}{2(-2)} = -\frac{1000}{-4} = 250.$$

The maximum area is achieved when the width is 250 yards. The maximum area is

$$\begin{aligned} A(250) &= 250(1000 - 2(250)) \\ &= 250(1000 - 500) \\ &= 250(500) = 125,000. \end{aligned}$$

The area is maximized at 125,000 square yards when the width is 250 yards and the length is $1000 - 2 \cdot 250 = 500$ yards.

21. Let x = one of the numbers
Let $14 + x$ = the other number

We need to minimize the function

$$\begin{aligned} P(x) &= x(14 + x) \\ &= 14x + x^2 \\ &= x^2 + 14x. \end{aligned}$$

The minimum is at

$$x = -\frac{b}{2a} = -\frac{14}{2(1)} = -\frac{14}{2} = -7.$$

The other number is $14 + x = 14 + (-7) = 7$.

The numbers which minimize the product are 7 and -7. The minimum product is $-7 \cdot 7 = -49$.

22. $3x + 4y = 1000$

$4y = 1000 - 3x$

$$y = \frac{1000 - 3x}{4}$$

$$\begin{aligned} A &= x\left(\frac{1000 - 3x}{4}\right) \\ &= -\frac{3}{4}x^2 + 250x \end{aligned}$$

$$x = \frac{-b}{2a} = \frac{-250}{2\left(-\frac{3}{4}\right)} = 125$$

$$y = \frac{1000 - 3(125)}{4} = 166\tfrac{2}{3}$$

125 feet by $166\frac{2}{3}$ feet will maximize the area.

23. $y = (35 + x)(150 - 4x)$

$y = 5250 + 10x - 4x^2$

$$x = \frac{-b}{2a} = \frac{-10}{2(-4)} = \frac{5}{4} = 1.25 \text{ or } 1 \text{ tree}$$

The maximum number of trees should be 35 + 1 = 36 trees.

$y = 36(150 - 4x) = 36(150 - 4 \cdot 1) = 5256$

The maximum yield will be 5256 pounds.

24. $f(x) = -x^3 + 12x^2 - x$

The graph rises to the left and falls to the right and goes through the origin, so graph (c) is the best match.

25. $g(x) = x^6 - 6x^4 + 9x^2$

The graph rises to the left and rises to the right, so graph (b) is the best match.

26. $h(x) = x^5 - 5x^3 + 4x$

The graph falls to the left and rises to the right and crosses the y-axis at zero, so graph (a) is the best match.

27. $f(x) = -x^4 + 1$

$f(x)$ falls to the left and to the right so graph (d) is the best match.

28. **a.** Since n is odd and $a_n > 0$, the graph rises to the right.

b. No, the model will not be useful. The model indicates increasing deforestation despite a declining rate in which the forest is being cut down.

c. The graph of function g falls to the right.

d. No, the model will not be useful. The model indicates the amount of forest cleared, in square kilometers, will eventually be negative, which is not possible.

29. In the polynomial, $f(x) = -x^4 + 21x^2 + 100$, the leading coefficient is –1 and the degree is 4. Applying the Leading Coefficient Test, we know that even-degree polynomials with negative leading coefficient will fall to the left and to the right. Since the graph falls to the right, we know that the elk population will die out over time.

30. $f(x) = -2(x-1)(x+2)^2(x+5)^3$
$x = 1$, multiplicity 1, the graph crosses the x-axis
$x = -2$, multiplicity 2, the graph touches the x-axis
$x = -5$, multiplicity 5, the graph crosses the x-axis

31. $f(x) = x^3 - 5x^2 - 25x + 125$
$= x^2(x-5) - 25(x-5)$
$= (x^2 - 25)(x-5)$
$= (x+5)(x-5)^2$
$x = -5$, multiplicity 1, the graph crosses the x-axis
$x = 5$, multiplicity 2, the graph touches the x-axis

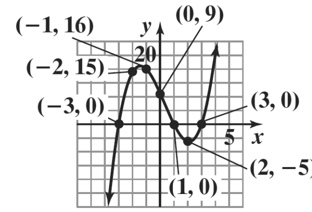
32. $f(x) = x^3 - 2x - 1$
$f(1) = (1)^3 - 2(1) - 1 = -2$
$f(2) = (2)^3 - 2(2) - 1 = 3$
The sign change shows there is a zero between the given values.

33. $f(x) = x^3 - x^2 - 9x + 9$

a. Since n is odd and $a_n > 0$, the graph falls to the left and rises to the right.

b. $f(-x) = (-x)^3 - (-x)^2 - 9(-x) + 9$
$= -x^3 - x^2 + 9x + 9$
$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

c. $f(x) = (x-3)(x+3)(x-1)$
zeros: 3, −3, 1



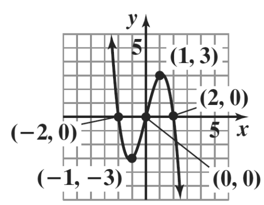
$f(x) = x^3 - x^2 - 9x + 9$

34. $f(x) = 4x - x^3$

a. Since n is odd and $a_n < 0$, the graph rises to the left and falls to the right.

b. $f(-x) = -4x + x^3$
$f(-x) = -f(x)$
origin symmetry

c. $f(x) = x(x^2 - 4) = x(x-2)(x+2)$
zeros: $x = 0, 2, -2$



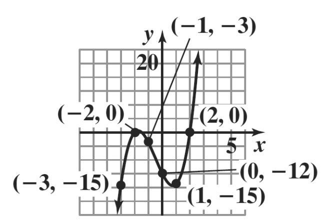
$f(x) = 4x - x^3$

35. $f(x) = 2x^3 + 3x^2 - 8x - 12$

a. Since h is odd and $a_n > 0$, the graph falls to the left and rises to the right.

b. $f(-x) = -2x^3 + 3x^2 + 8x - 12$
$f(-x) \neq f(x),\ f(-x) = -f(x)$
no symmetry

c. $f(x) = (x-2)(x+2)(2x+3)$
zeros: $x = 2, -2, -\frac{3}{2}$



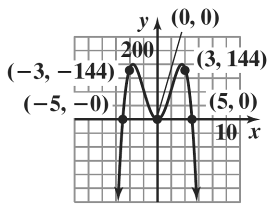
$f(x) = 2x^3 + 3x^2 - 8x - 12$

36. $g(x) = -x^4 + 25x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 25(-x)^2$
$= -x^4 + 25x^2 = f(x)$
y-axis symmetry

c. $-x^4 + 25x^2 = 0$
$-x^2\left(x^2 - 25\right) = 0$
$-x^2(x-5)(x+5) = 0$
zeros: $x = –5, 0, 5$



$f(x) = -x^4 + 25x^2$

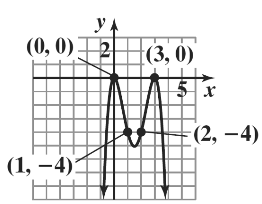
37. $f(x) = -x^4 + 6x^3 - 9x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 6(-x)^3 - 9(-x)$
$= -x^4 - 6x^3 - 9x^2 \; f(-x) \neq f(x)$
$f(-x) \neq -f(x)$
no symmetry

c. $= -x^2\left(x^2 - 6x + 9\right) = 0$

$-x^2(x-3)(x-3) = 0$
zeros: $x = 0, 3$



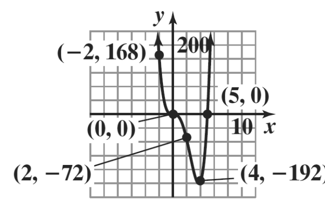
$f(x) = -x^4 + 6x^3 - 9x^2$

38. $f(x) = 3x^4 - 15x^3$

a. The graph rises to the left and to the right.

b. $f(-x) = 3(-x)^4 - 15(-x)^2 = 3x^4 + 15x^3$
$f(-x) \neq f(x),\; f(-x) \neq -f(x)$
no symmetry

c $3x^4 - 15x^3 = 0$
$3x^3(x-5) = 0$
zeros: $x = 0, 5$



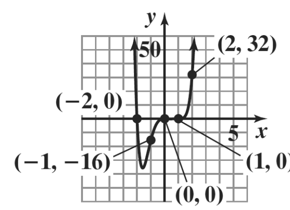
$f(x) = 3x^4 - 15x^3$

39. $f(x) = 2x^2(x-1)^3(x+2)$

Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.
$x = 0, x = 1, x = –2$
The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0)

$f(0) = 2(0)^2(0-1)^3(0+2) = 0$

The y-intercept is 0.



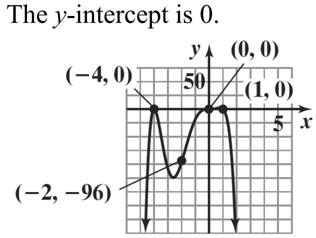
$f(x) = 2x^2(x-1)^3(x+2)$

40. $f(x) = -x^3(x+4)^2(x-1)$

Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.
$x = 0, x = –4, x = 1$
The roots at 0 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –4 has even multiplicity so $f(x)$ touches the axis at (–4, 0)

$f(0) = -(0)^3(0+4)^2(0-1) = 0$

The y-intercept is 0.



$f(x) = -x^3(x+4)^2(x-1)$

41.

$$\begin{array}{r}
4x^2-7x+5 \\
x+1\overline{)4x^3-3x^2-2x+1} \\
\underline{4x^3+4x^2} \\
-7x^2-2x \\
\underline{-7x^2-7x} \\
5x+1 \\
\underline{5x+5} \\
-4
\end{array}$$

Quotient: $4x^2-7x+5-\dfrac{4}{x+1}$

42.

$$\begin{array}{r}
2x^2-4x+1 \\
5x-3\overline{)10x^3-26x^2+17x-13} \\
\underline{10x^3+6x^2} \\
-20x^2+17x \\
\underline{-20x^2+12x} \\
5x-13 \\
\underline{5x-\ \ 3} \\
-10
\end{array}$$

Quotient: $2x^2-4x+1-\dfrac{10}{5x-3}$

43.

$$\begin{array}{r}
2x^2+3x-1 \\
2x^2+1\overline{)4x^4+6x^3+3x-1} \\
\underline{4x^2+2x^2} \\
6x^3-2x^2+3x \\
\underline{6x^2+3x} \\
-2x^2-1 \\
\underline{-2x^2-1} \\
0
\end{array}$$

44. $(3x^4+11x^3-20x^3+7x+35)\div(x+5)$

–5	3	11	–20	7	35
		–15	20	0	–35
	3	–4	0	7	0

Quotient: $3x^3-4x^2+7$

45. $(3x^4-2x^2-10x)\div(x-2)$

2	3	0	–2	–10	0
		6	12	20	20
	3	6	10	10	20

Quotient: $3x^3+6x^2+10x+10+\dfrac{20}{x-2}$

46. $f(x)=2x^3-7x^2+9x-3$

–13	2	–7	9	–3
		–26	429	–5694
	2	–33	438	–5697

Quotient: $f(-13)=-5697$

47. $f(x)=2x^3+x^2-13x+6$

2	2	1	–13	6
		4	10	–6
	2	5	–3	0

$f(x)=(x-2)(2x^2+5x-3)$
$\quad=(x-2)(2x-1)(x+3)$

Zeros: $x=2,\dfrac{1}{2},-3$

48. $x^3-17x+4=0$

4	1	0	–17	4
		4	16	–4
	1	4	–1	0

$(x-4)\left(x^2+4x-1\right)=0$

$x=\dfrac{-4\pm\sqrt{16+4}}{2}=\dfrac{-4\pm2\sqrt{5}}{2}=-2\pm\sqrt{5}$

The solution set is $\left\{4,-2+\sqrt{5},-2-\sqrt{5}\right\}$.

49. $f(x)=x^4-6x^3+14x^2-14x+5$

$p:\pm1,\pm5$
$q:\pm1$
$\dfrac{p}{q}:\pm1,\pm5$

50. $f(x)=3x^5-2x^4-15x^3+10x^2+12x-8$

$p:\pm1,\pm2,\pm4,\pm8$
$q:\pm1,\pm3$
$\dfrac{p}{q}:\pm1,\pm2,\pm4,\pm8,\pm\dfrac{8}{3},\pm\dfrac{4}{3},\pm\dfrac{2}{3},\pm\dfrac{1}{3}$

51. $f\left(x\right)=3x^4-2x^3-8x+5$

$f(x)$ has 2 sign variations, so $f(x)=0$ has 2 or 0 positive solutions.

$f\left(-x\right)=3x^4+2x^3+x+5$

$f(-x)$ has no sign variations, so $f(x)=0$ has no negative solutions.

52. $f(x)=2x^5-3x^3-5x^2+3x-1$

$f(x)$ has 3 sign variations, so $f(x)=0$ has 3 or 1 positive real roots.

$f(-x)=-2x^5+3x^3-5x^2-3x-1$

$f(-x)$ has 2 sign variations, so $f(x)=0$ has 2 or 0 negative solutions.

53. $f(x)=f(-x)=2x^4+6x^2+8$

No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

54. $f(x)=x^3+3x^2-4$

a. $p:\pm1,\pm2,\pm4$
$q:\pm1$
$\frac{p}{q}:\pm1,\pm2,\pm4$

b. 1 sign variation $\Rightarrow$ 1 positive real zero

$f(-x)=-x^3+3x^2-4$

2 sign variations $\Rightarrow$ 2 or no negative real zeros

c.

1	1	3	0	–4
		1	4	–4
	1	4	4	0

1 is a zero.
1, –2 are rational zeros.

d. $(x-1)(x^2+4x+4)=0$
$(x-1)(x+2)^2=0$
$x=1$ or $x=-2$
The solution set is $\{1,-2\}$.

55. $f(x)=6x^3+x^2-4x+1$

a. p: ±1
q: ±1, ±2, ±3, ±6
$\frac{p}{q}:\pm1,\pm\frac{1}{2},\pm\frac{1}{3},\pm\frac{1}{6}$

b. $f(x)=6x^3+x^2-4x+1$

2 sign variations; 2 or 0 positive real zeros.

$f(-x)=-6x^3+x^2+4x+1$

1 sign variation; 1 negative real zero.

c.

–1	6	1	–4	1
		–6	5	–1
	6	–5	1	0

–1 is a zero.

$-1,\frac{1}{3},\frac{1}{2}$ are rational zeros.

d.
$$6x^3+x^2-4x+1=0$$
$$(x+1)(6x^2-5x+1)=0$$
$$(x+1)(3x-1)(2x-1)=0$$
$$x=-1 \text{ or } x=\frac{1}{3} \text{ or } x=\frac{1}{2}$$

The solution set is $\left\{-1,\frac{1}{3},\frac{1}{2}\right\}$.

56. $f(x)=8x^3-36x^2+46x-15$

a. p: ±1, ±3, ±5, ±15
q: ±1, ±2, ±4, ±8

$\frac{p}{q}:\pm1,\pm3,\pm5,\pm15,\pm\frac{1}{2},\pm\frac{1}{4},\pm\frac{1}{8},$
$\pm\frac{3}{2},\pm\frac{3}{4},\pm\frac{3}{8},\pm\frac{5}{2},\pm\frac{5}{4},$
$\pm\frac{5}{8},\pm\frac{15}{2},\pm\frac{15}{4},\pm\frac{15}{8}$

b. $f(x)=8x^3-36x^2+46x-15$

3 sign variations; 3 or 1 positive real solutions.

$f(-x)=-8x^3-36x^2-46x-15$

0 sign variations; no negative real solutions.

c.

$\frac{1}{2}$	8	–36	46	–15
		4	–16	15
	8	–32	30	0

$\frac{1}{2}$ is a zero.

$\frac{1}{2},\frac{3}{2},\frac{5}{2}$ are rational zeros.

d.

$$8x^3 - 36x^2 + 46x - 15 = 0$$
$$\left(x - \frac{1}{2}\right)(8x^2 - 32x + 30) = 0$$
$$2\left(x - \frac{1}{2}\right)(4x - 16x + 15) = 0$$
$$2\left(x - \frac{1}{2}\right)(2x - 5)(2x - 3) = 0$$

$x = \frac{1}{2}$ or $x = \frac{5}{2}$ or $x = \frac{3}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$.

57. $2x^3 + 9x^2 - 7x + 1 = 0$

a. p: ±1
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm \frac{1}{2}$

b. $f(x) = 2x^3 + 9x^2 - 7x + 1$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = -2x^3 + 9x^2 + 7x + 1$
1 sign variation; 1 negative real zero.

c.

$\frac{1}{2}$	2	9	–7	1
		1	5	–1
	2	10	–2	0

$\frac{1}{2}$ is a rational zero.

d.

$$2x^3 + 9x^2 - 7x + 1 = 0$$
$$\left(x - \frac{1}{2}\right)(2x^2 + 10x - 2) = 0$$
$$2\left(x - \frac{1}{2}\right)(x^2 + 5x - 1) = 0$$

Solving $x^2 + 5x - 1 = 0$ using the quadratic formula gives $x = \frac{-5 \pm \sqrt{29}}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{-5 + \sqrt{29}}{2}, \frac{-5 - \sqrt{29}}{2}\right\}$.

58. $x^4 - x^3 - 7x^2 + x + 6 = 0$

a. $p = \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

b. $f(x) = x^4 - x^3 - 7x^2 + x + 6$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = x^4 + x^3 - 7x^2 - x + 6$
2 sign variations; 2 or 0 negative real zeros.

c.

1	1	–1	–7	1	6
		1	0	–7	–6
	1	0	–7	–6	0

–1	1	0	–7	–6
		–1	1	6
	1	–1	–6	0

$-2, -1, 1, 3$ are rational zeros.

d.

$$x^4 - x^3 - 7x^2 + x + 6 = 0$$
$$(x-1)(x+1)(x^2 - x + 6) = 0$$
$$(x-1)(x+1)(x-3)(x+2) = 0$$

The solution set is $\{-2, -1, 1, 3\}$.

59. $4x^4 + 7x^2 - 2 = 0$

a. p: ±1, ±2
q: ±1, ±2, ±4
$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$

b. $f(x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 positive real zero.
$f(-x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 negative real zero.

c.

$\frac{1}{2}$	4	0	7	0	–2
		2	1	4	2
	4	2	8	4	0

$-\frac{1}{2}$	4	2	8	4
		–2	0	–4
	4	0	8	0

$-\frac{1}{2}, \frac{1}{2}$ are rational zeros.

d. $4x^4+7x^2-2=0$

$\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(4x^2+8)=0$

$4\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(x^2+2)=0$

Solving $x^2+2=0$ using the quadratic formula gives $x=\pm 2i$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}, 2i, -2i\right\}$.

60. $f(x)=2x^4+x^3-9x^2-4x+4$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$

b. $f(x)=2x^4+x^3-9x^2-4x+4$
2 sign variations; 2 or 0 positive real zeros.
$f(-x)=2x^4-x^3-9x^2+4x+4$
2 sign variations; 2 or 0 negative real zeros.

c.

2	2	1	–9	–4	4
		4	10	2	–4
	2	5	1	–2	0

–1	2	5	1	–2
		–2	–3	2
	2	3	–2	0

$-2, -1, \frac{1}{2}, 2$ are rational zeros.

d. $2x^2+3x-2=0$
$(2x-1)(x+2)=0$

$x=-2$ or $x=\frac{1}{2}$

The solution set is $\left\{-2, -1, \frac{1}{2}, 2\right\}$.

61. $f(x)=a_n(x-2)(x-2+3i)(x-2-3i)$
$f(x)=a_n(x-2)(x^2-4x+13)$
$f(1)=a_n(1-2)\left[1^2-4(1)+13\right]$
$-10=-10a_n$
$a_n=1$

$f(x)=1(x-2)(x^2-4x+13)$
$f(x)=x^3-4x^2+13x-2x^2+8x-26$
$f(x)=x^3-6x^2+21x-26$

62. $f(x)=a_n(x-i)(x+i)(x+3)^2$
$f(x)=a_n(x^2+1)(x^2+6x+9)$
$f(-1)=a_n\left[(-1)^2+1\right]\left[(-1)^2+6(-1)+9\right]$
$16=8a_n$
$a_n=2$

$f(x)=2(x^2+1)(x^2+6x+9)$
$f(x)=2(x^4+6x^3+9x^2+x^2+6x+9)$
$f(x)=2x^4+12x^3+20x^2+12x+18$

63. $f(x)=2x^4+3x^3+3x-2$
$p: \pm 1, \pm 2$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}$

–2	2	3	0	3	–2
		–4	2	–4	2
	2	–1	2	–1	0

$2x^4+3x^3+3x-2=0$
$(x+2)(2x^3-x^2+2x-1)=0$
$(x+2)[x^2(2x-1)+(2x-1)]=0$
$(x+2)(2x-1)(x^2+1)=0$

$x=-2,\ x=\frac{1}{2}$ or $x=\pm i$

The zeros are $-2, \frac{1}{2}, \pm i$.
$f(x)=(x-i)(x+i)(x+2)(2x-1)$

64. $g(x) = x^4 - 6x^3 + x^2 + 24x + 16$

p: ±1, ±2, ±4, ±8, ±16

q: ±1

$\frac{p}{q}$: ±1, ±2, ±4, ±8, ±16

–1	1	–6	1	24	16
		–1	7	–8	–16
	1	–7	8	16	0

$x^4 - 6x^3 + x^2 + 24x + 16 = 0$

$(x+1)(x^3 - 7x^2 + 8x + 16) = 0$

–1	1	–7	8	16
		–1	8	–16
	1	–8	16	0

$(x+1)^2(x^2 - 8x + 16) = 0$

$(x+1)^2(x-4)^2 = 0$

$x = -1$ or $x = 4$

$g(x) = (x+1)^2(x-4)^2$

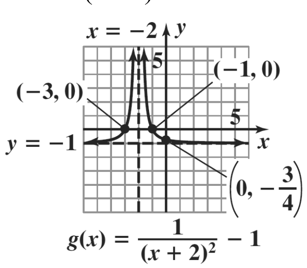
65. 4 real zeros, one with multiplicity two

66. 3 real zeros; 2 nonreal complex zeros

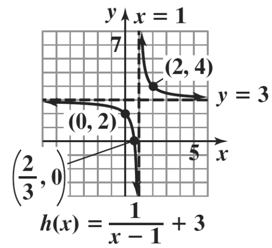
67. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros

68. 1 real zero; 4 nonreal complex zeros

69. $g(x) = \frac{1}{(x+2)^2} - 1$



70. $h(x) = \frac{1}{x-1} + 3$



71. $f(x) = \frac{2x}{x^2 - 9}$

Symmetry: $f(-x) = -\frac{2x}{x^2 - 9} = -f(x)$

origin symmetry

x-intercept:

$0 = \frac{2x}{x^2 - 9}$

$2x = 0$

$x = 0$

y-intercept: $y = \frac{2(0)}{0^2 - 9} = 0$

Vertical asymptote:

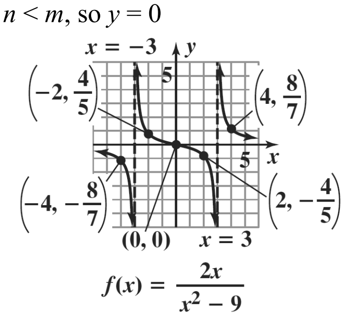
$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = 3$ and $x = -3$

Horizontal asymptote:

$n < m$, so $y = 0$



72. $g(x) = \frac{2x-4}{x+3}$

Symmetry: $g(-x) = \frac{-2x-4}{x+3}$

$g(-x) \neq g(x)$, $g(-x) \neq -g(x)$

No symmetry

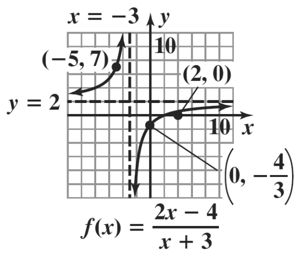
x-intercept:

$2x - 4 = 0$

$x = 2$

y-intercept: $y = \frac{2(0)-4}{(0)+3} = -\frac{4}{3}$

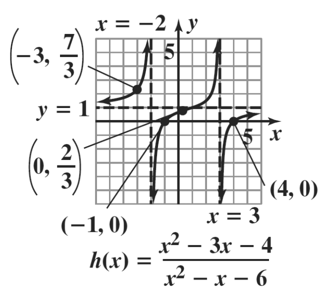
Vertical asymptote:
$x + 3 = 0$
$x = -3$
Horizontal asymptote:
$n = m$, so $y = \frac{2}{1} = 2$



73. $h(x) = \dfrac{x^2 - 3x - 4}{x^2 - x - 6}$

Symmetry: $h(-x) = \dfrac{x^2 + 3x - 4}{x^2 + x - 6}$
$h(-x) \neq h(x)$, $h(-x) \neq -h(x)$
No symmetry
x-intercepts:
$x^2 - 3x - 4 = 0$
$(x - 4)(x + 1)$
$x = 4 \quad x = -1$
y-intercept: $y = \dfrac{0^2 - 3(0) - 4}{0^2 - 0 - 6} = \dfrac{2}{3}$
Vertical asymptotes:
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3, -2$

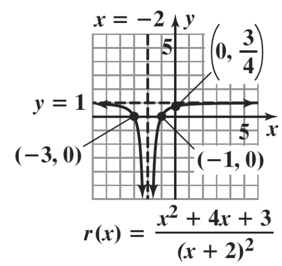
Horizontal asymptote:
$n = m$, so $y = \frac{1}{1} = 1$



74. $r(x) = \dfrac{x^2 + 4x + 3}{(x+2)^2}$

Symmetry: $r(-x) = \dfrac{x^2 - 4x + 3}{(-x+2)^2}$
$r(-x) \neq r(x)$, $r(-x) \neq -r(x)$
No symmetry
x-intercepts:
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3, -1$
y-intercept: $y = \dfrac{0^2 + 4(0) + 3}{(0+2)^2} = \dfrac{3}{4}$
Vertical asymptote:
$x + 2 = 0$
$x = -2$

Horizontal asymptote:
$n = m$, so $y = \frac{1}{1} = 1$



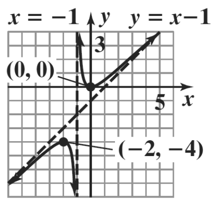
75. $y = \dfrac{x^2}{x+1}$

Symmetry: $f(-x) = \dfrac{x^2}{-x+1}$
$f(-x) \neq f(x)$, $f(-x) \neq -f(x)$
No symmetry
x-intercept:
$x^2 = 0$
$x = 0$
y-intercept: $y = \dfrac{0^2}{0+1} = 0$
Vertical asymptote:
$x + 1 = 0$
$x = -1$
$n > m$, no horizontal asymptote.

Slant asymptote:

$y = x - 1 + \frac{1}{x+1}$

$y = x - 1$



$y = \frac{x^2}{x+1}$

76. $y = \frac{x^2 + 2x - 3}{x - 3}$

Symmetry: $f(-x) = \frac{x^2 - 2x - 3}{-x - 3}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercepts:

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3, 1$

y-intercept: $y = \frac{0^2 + 2(0) - 3}{0 - 3} = \frac{-3}{-3} = 1$

Vertical asymptote:

$x - 3 = 0$

$x = 3$

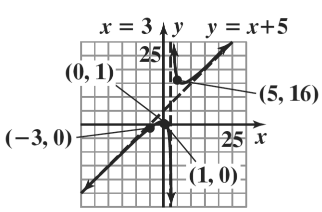
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$y = x + 5 + \frac{12}{x - 3}$

$y = x + 5$



$y = \frac{x^2 + 2x - 3}{x - 3}$

77. $f(x) = \frac{-2x^3}{x^2 + 1}$

Symmetry: $f(-x) = \frac{2}{x^2 + 1} = -f(x)$

Origin symmetry

x-intercept:

$-2x^3 = 0$

$x = 0$

y-intercept: $y = \frac{-2(0)^3}{0^2 + 1} = \frac{0}{1} = 0$

Vertical asymptote:

$x^2 + 1 = 0$

$x^2 = -1$

No vertical asymptote.

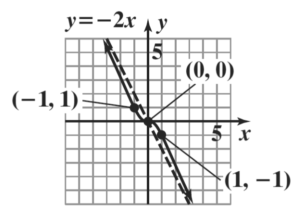
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$f(x) = -2x + \frac{2x}{x^2 + 1}$

$y = -2x$



$f(x) = \frac{-2x^3}{x^2 + 1}$

78. $g(x) = \frac{4x^2 - 16x + 16}{2x - 3}$

Symmetry: $g(-x) = \frac{4x^2 + 16x + 16}{-2x - 3}$

$g(-x) \neq g(x), g(-x) \neq -g(x)$

No symmetry

x-intercept:

$4x^2 - 16x + 16 = 0$

$4(x - 2)^2 = 0$

$x = 2$

y-intercept:

$y = \frac{4(0)^2 - 16(0) + 16}{2(0) - 3} = -\frac{16}{3}$

Vertical asymptote:

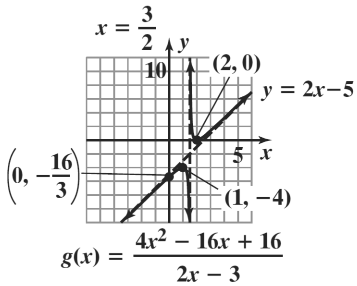
$2x - 3 = 0$

$x = \frac{3}{2}$

Horizontal asymptote:
$n > m$, so no horizontal asymptote.
Slant asymptote:

$$g(x) = 2x - 5 + \frac{1}{2x-3}$$

$y = 2x - 5$



79. **a.** $C(x) = 50{,}000 + 25x$

b. $\overline{C}(x) = \frac{25x + 50{,}000}{x}$

c. $\overline{C}(50) = \frac{25(50) + 50{,}000}{50} = 1025$

When 50 calculators are manufactured, it costs \$1025 to manufacture each.

$\overline{C}(100) = \frac{25(100) + 50{,}000}{100} = 525$

When 100 calculators are manufactured, it costs \$525 to manufacture each.

$\overline{C}(1000) = \frac{25(1000) + 50{,}000}{1000} = 75$

When 1,000 calculators are manufactured, it costs \$75 to manufacture each.

$\overline{C}(100{,}000) = \frac{25(100{,}000) + 50{,}000}{100{,}000} = 25.5$ Wh
en 100,000 calculators are manufactured, it costs \$25.50 to manufacture each.

d. $n = m$, so $y = \frac{25}{1} = 25$ is the horizontal asymptote. Minimum costs will approach \$25.

80. $f(x) = \frac{150x + 120}{0.05x + 1}$

$n = m$, so $y = \frac{150}{0.05} = 3000$

The number of fish available in the pond approaches 3000.

81. $P(x) = \frac{72{,}900}{100x^2 + 729}$

$n < m$ so $y = 0$

As the number of years of education increases the percentage rate of unemployment approaches zero.

82. **a.**
$$\begin{aligned} P(x) &= M(x) + F(x) \\ &= 1.48x + 115.1 + 1.44x + 120.9 \\ &= 2.92x + 236 \end{aligned}$$

b. $R(x) = \frac{M(x)}{P(x)} = \frac{1.48x + 115.1}{2.92x + 236}$

c. $y = \frac{1.48}{2.92} \approx 0.51$

Over time, the percentage of men in the U.S. population will approach 51%.

83. $T(x) = \frac{4}{x+3} + \frac{2}{x}$

84.
$$1000 = lw$$
$$\frac{1000}{w} = l$$
$$P = 2x + 2\left(\frac{1000}{x}\right)$$
$$P = 2x + \frac{2000}{x}$$

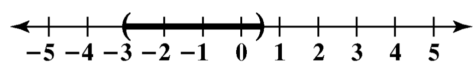
85. $2x^2 + 5x - 3 < 0$

Solve the related quadratic equation.

$$2x^2 + 5x - 3 = 0$$
$$(2x-1)(x+3) = 0$$

The boundary points are -3 and $\frac{1}{2}$.

Testing each interval gives a solution set of $\left(-3, \frac{1}{2}\right)$



86. $2x^2 + 9x + 4 \geq 0$

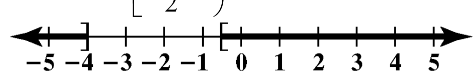
Solve the related quadratic equation.

$$2x^2 + 9x + 4 = 0$$
$$(2x+1)(x+4) = 0$$

The boundary points are -4 and $-\frac{1}{2}$.

Testing each interval gives a solution set of

$(-\infty, -4] \cup \left[-\frac{1}{2}, \infty\right)$

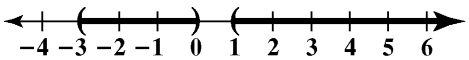


87. $x^3 + 2x^2 > 3x$

Solve the related equation.

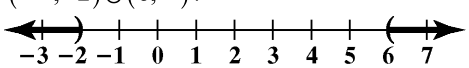
$$x^3 + 2x^2 = 3x$$
$$x^3 + 2x^2 - 3x = 0$$
$$x\left(x^2 + 2x - 3\right) = 0$$
$$x(x+3)(x-1) = 0$$

The boundary points are -3, 0, and 1.
Testing each interval gives a solution set of
$(-3,0)\cup(1,\infty)$



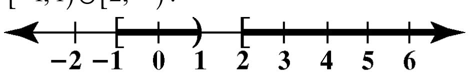
88. $\dfrac{x-6}{x+2} > 0$

Find the values of x that make the numerator and denominator zero.
The boundary points are –2 and 6.
Testing each interval gives a solution set of
$(-\infty,-2)\cup(6,\infty)$.



89. $\dfrac{(x+1)(x-2)}{x-1} \geq 0$

Find the values of x that make the numerator and denominator zero.
The boundary points are –1, 1 and 2. We exclude 1 from the solution set, since this would make the denominator zero.
Testing each interval gives a solution set of
$[-1,1)\cup[2,\infty)$.



90. $\dfrac{x+3}{x-4} \leq 5$

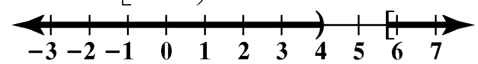
Express the inequality so that one side is zero.

$$\frac{x+3}{x-4} - 5 \leq 0$$
$$\frac{x+3}{x-4} - \frac{5(x-4)}{x-4} \leq 0$$
$$\frac{-4x+23}{x-4} \leq 0$$

Find the values of x that make the numerator and denominator zero.

The boundary points are 4 and $\dfrac{23}{4}$. We exclude 4 from the solution set, since this would make the denominator zero.

Testing each interval gives a solution set of
$(-\infty,4)\cup\left[\dfrac{23}{4},\infty\right)$.



91. **a.** $g(x) = 0.125x^2 + 2.3x + 27$

$g(35) = 0.125(35)^2 + 2.3(35) + 27 \approx 261$

The stopping distance on wet pavement for a motorcycle traveling 35 miles per hour is about 261 feet. This overestimates the distance shown in the graph by 1 foot.

b. $f(x) = 0.125x^2 - 0.8x + 99$

$$0.125x^2 - 0.8x + 99 > 267$$
$$0.125x^2 - 0.8x - 168 > 0$$

Solve the related quadratic equation.

$$0.125x^2 - 0.8x - 168 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.125)(-168)}}{2(0.125)}$$
$$x = -33.6,\ 40$$

Testing each interval gives a solution set of
$(-\infty,-33.6)\cup(40,\infty)$.

Thus, speeds exceeding 40 miles per hour on dry pavement will require over 267 feet of stopping distance.

92. $s = -16t^2 + v_0t + s_0$

$$32 < -16t^2 + 48t + 0$$
$$0 < -16t^2 + 48t - 32$$
$$0 < -16\left(t^2 - 3t + 2\right)$$
$$0 < -16(t-2)(t-1)$$

F	T	F
	1	2

The projectile's height exceeds 32 feet during the time period from 1 to 2 seconds.

93.

$$w = ks$$
$$28 = k \cdot 250$$
$$0.112 = k$$

Thus, $w = 0.112s$.

$w = 0.112(1200) = 134.4$

1200 cubic centimeters of melting snow will produce 134.4 cubic centimeters of water.

94. $d = kt^2$
$144 = k(3)^2$
$k = 16$
$d = 16t^2$
$d = 16(10)^2 = 1{,}600$ ft

95. $p = \frac{k}{w}$
$660 = \frac{k}{1.6}$
$1056 = k$
Thus, $p = \frac{1056}{w}$.
$p = \frac{1056}{2.4} = 440$
The pitch is 440 vibrations per second.

96. $l = \frac{k}{d^2}$
$28 = \frac{k}{8^2}$
$k = 1792$
$l = \frac{1792}{d^2}$
$l = \frac{1792}{4^2} = 112$ decibels

97. $t = \frac{kc}{w}$
$10 = \frac{k \cdot 30}{6}$
$10 = 5h$
$h = 2$
$t = \frac{2c}{w}$
$t = \frac{2(40)}{5} = 16$ hours

98. $V = khB$
$175 = k \cdot 15 \cdot 35$
$k = \frac{1}{3}$
$V = \frac{1}{3}hB$
$V = \frac{1}{3} \cdot 20 \cdot 120 = 800 \text{ ft}^3$

99. **a.** Use $L = \frac{k}{R}$ to find *k*.
$L = \frac{k}{R}$
$30 = \frac{k}{63}$
$63 \cdot 30 = 63 \cdot \frac{k}{63}$
$1890 = k$
Thus, $L = \frac{1890}{R}$.

b. This is an approximate model.

c. $L = \frac{1890}{R}$
$L = \frac{1890}{27} = 70$
The average life span of an elephant is 70 years.

Chapter 2 Test

1. $(6-7i)(2+5i) = 12 + 30i - 14i - 35i^2$
$= 12 + 16i + 35$
$= 47 + 16i$

2. $\frac{5}{2-i} = \frac{5}{2-i} \cdot \frac{2+i}{2+i}$
$= \frac{5(2+i)}{4+1}$
$= \frac{5(2+i)}{5}$
$= 2 + i$

3. $2\sqrt{-49} + 3\sqrt{-64} = 2(7i) + 3(8i)$
$= 14i + 24i$
$= 38i$

4. $x^2 = 4x - 8$
$x^2 - 4x + 8 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$
$x = \frac{4 \pm \sqrt{-16}}{2}$
$x = \frac{4 \pm 4i}{2}$
$x = 2 \pm 2i$

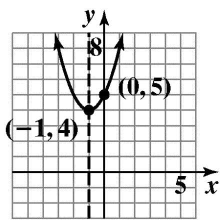
5. $f(x) = (x+1)^2 + 4$

vertex: (–1, 4)
axis of symmetry: $x = -1$
x-intercepts:

$$(x+1)^2 + 4 = 0$$
$$x^2 + 2x + 5 = 0$$
$$x = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

no x-intercepts
y-intercept:

$$f(0) = (0+1)^2 + 4 = 5$$



$f(x) = (x+1)^2 + 4$
domain: $(-\infty, \infty)$; range: $[4, \infty)$

6. $f(x) = x^2 - 2x - 3$

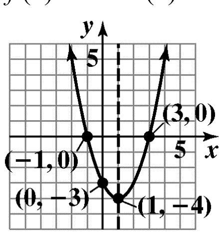
$$x = \frac{-b}{2a} = \frac{2}{2} = 1$$
$$f(1) = 1^2 - 2(1) - 3 = -4$$

vertex: (1, –4)
axis of symmetry $x = 1$
x-intercepts:

$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3 \text{ or } x = -1$$

y-intercept:

$$f(0) = 0^2 - 2(0) - 3 = -3$$



$f(x) = x^2 - 2x - 3$
domain: $(-\infty, \infty)$; range: $[-4, \infty)$

7. $f(x) = -2x^2 + 12x - 16$

Since the coefficient of x^2 is negative, the graph of $f(x)$ opens down and $f(x)$ has a maximum point.

$$x = \frac{-12}{2(-2)} = 3$$

$$f(3) = -2(3)^2 + 12(3) - 16$$
$$= -18 + 36 - 16$$
$$= 2$$

Maximum point: (3, 2)
domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

8. $f(x) = -x^2 + 46x - 360$

$$x = -\frac{b}{2a} = \frac{-46}{-2} = 23$$

23 computers will maximize profit.

$$f(23) = -(23)^2 + 46(23) - 360 = 169$$

Maximum daily profit = \$16,900.

9. Let x = one of the numbers;
$14 - x$ = the other number.
The product is $f(x) = x(14-x)$

$$f(x) = x(14-x) = -x^2 + 14x$$

The x-coordinate of the maximum is

$$x = -\frac{b}{2a} = -\frac{14}{2(-1)} = -\frac{14}{-2} = 7.$$

$$f(7) = -7^2 + 14(7) = 49$$

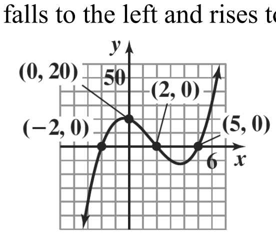
The vertex is (7, 49). The maximum product is 49. This occurs when the two numbers are 7 and $14 - 7 = 7$.

10. **a.** $f(x) = x^3 - 5x^2 - 4x + 20$

$$x^3 - 5x^2 - 4x + 20 = 0$$
$$x^2(x-5) - 4(x-5) = 0$$
$$(x-5)(x-2)(x+2) = 0$$

$x = 5, 2, -2$
The solution set is $\{5, 2, -2\}$.

b. The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



$f(x) = x^3 - 5x^2 - 4x + 20$

11. $f(x) = x^5 - x$

Since the degree of the polynomial is odd and the leading coefficient is positive, the graph of *f* should fall to the left and rise to the right. The *x*-intercepts should be –1 and 1.

12. a. The integral root is 2.

b.

2	6	–19	16	–4
		12	–14	4
	6	–7	2	0

$6x^2 - 7x + 2 = 0$

$(3x - 2)(2x - 1) = 0$

$x = \frac{2}{3}$ or $x = \frac{1}{2}$

The other two roots are $\frac{1}{2}$ and $\frac{2}{3}$.

13. $2x^3 + 11x^2 - 7x - 6 = 0$

p: ±1, ±2, ±3, ±6

q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

14. $f(x) = 3x^5 - 2x^4 - 2x^2 + x - 1$

f(*x*) has 3 sign variations.

$f(-x) = -3x^5 - 2x^4 - 2x^2 - x - 1$

f(–*x*) has no sign variations.

There are 3 or 1 positive real solutions and no negative real solutions.

15. $x^3 + 9x^2 + 16x - 6 = 0$

Since the leading coefficient is 1, the possible rational zeros are the factors of 6

$p = \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

–3	1	9	16	–6
		–3	–18	6
	1	6	–2	0

Thus $x = 3$ is a root.

Solve the quotient $x^2 + 6x - 2 = 0$ using the quadratic formula to find the remaining roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{44}}{2}$$

$$= -3 \pm \sqrt{11}$$

The zeros are -3, $-3 + \sqrt{11}$, and $-3 - \sqrt{11}$.

16. $f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15$

a. Possible rational zeros are:

$p: \pm 1, \pm 3, \pm 5, \pm 15$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

b. Verify that -1 and $\frac{3}{2}$ are zeros as it appears in the graph:

–1	2	–1	–13	5	15
		–2	3	10	–15
	2	–3	–10	15	0

$\frac{3}{2}$	2	–3	–10	15
		3	0	–15
	2	0	–10	0

Thus, -1 and $\frac{3}{2}$ are zeros, and the polynomial factors as follows:

$$2x^4 - x^3 - 13x^2 + 5x + 15 = 0$$

$$(x+1)(2x^3 - 3x^2 - 10x + 15) = 0$$

$$(x+1)\left(x - \frac{3}{2}\right)(2x^2 - 10) = 0$$

Find the remaining zeros by solving:

$2x^2 - 10 = 0$

$2x^2 = 10$

$x^2 = 5$

$x = \pm\sqrt{5}$

The zeros are -1, $\frac{3}{2}$, and $\pm\sqrt{5}$.

17. *f*(*x*) has zeros at –2 and 1. The zero at –2 has multiplicity of 2.

$x^3 + 3x^2 - 4 = (x - 1)(x + 2)^2$

18. $f(x) = a_0(x+1)(x-1)(x+i)(x-i)$
$= a_0(x^2-1)(x^2+1)$
$= a_0(x^4-1)$

Since $f(3)=160$, then

$$a_0(3^4-1)=160$$
$$a_0(80)=160$$
$$a_0=\frac{160}{80}$$
$$a_0=2$$

$f(x)=2(x^4-1)=2x^4-2$

19. $f(x)=-3x^3-4x^2+x+2$

The graph shows a root at $x=-1$.
Use synthetic division to verify this root.

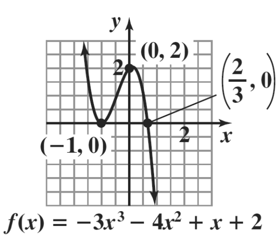
−1	−3	−4	1	2
		3	1	4
	−3	−1	2	0

Factor the quotient to find the remaining zeros.

$$-3x^2-x+2=0$$
$$-(3x-2)(x+1)=0$$

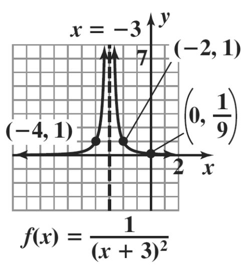
The zeros (x-intercepts) are –1 and $\frac{2}{3}$.

The y-intercept is $f(0)=2$



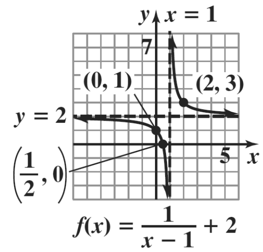
20. $f(x)=\frac{1}{(x+3)^2}$

domain: $\{x \mid x \neq -3\}$ or $(-\infty,-3)\cup(-3,\infty)$



21. $f(x)=\frac{1}{x-1}+2$

domain: $\{x \mid x \neq 1\}$ or $(-\infty,1)\cup(1,\infty)$



22. $f(x)=\frac{x}{x^2-16}$

domain: $\{x \mid x \neq 4, x \neq -4\}$

Symmetry: $f(-x)=\frac{-x}{x^2-16}=-f(x)$

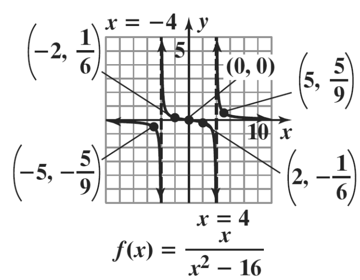
y-axis symmetry
x-intercept: $x = 0$

y-intercept: $y=\frac{0}{0^2-16}=0$

Vertical asymptotes:

$x^2-16=0$
$(x-4)(x+4)=0$
$x=4, -4$

Horizontal asymptote:
$n < m$, so $y = 0$ is the horizontal asymptote.



23. $f(x)=\frac{x^2-9}{x-2}$

domain: $\{x \mid x \neq 2\}$

Symmetry: $f(-x)=\frac{x^2-9}{-x-2}$

$f(-x) \neq f(x)$, $f(-x) \neq -f(x)$
No symmetry

x-intercepts:

$x^2-9=0$
$(x-3)(x+3)=0$
$x=3, -3$

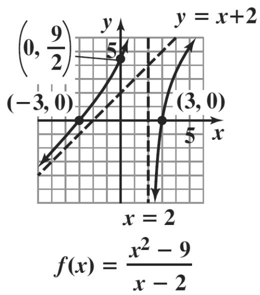
y-intercept: $y = \frac{0^2 - 9}{0 - 2} = \frac{9}{2}$

Vertical asymptote:
$x - 2 = 0$
$x = 2$

Horizontal asymptote:
$n > m$, so no horizontal asymptote exists.

Slant asymptote: $f(x) = x + 2 - \frac{5}{x - 2}$

$y = x + 2$



24. $f(x) = \frac{x+1}{x^2 + 2x - 3}$

$x^2 + 2x - 3 = (x+3)(x-1)$

domain: $\{x \mid x \neq -3, x \neq 1\}$

Symmetry: $f(-x) = \frac{-x+1}{x^2 - 2x - 3}$

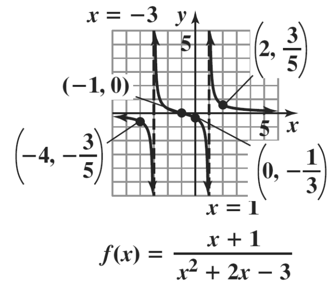
$f(-x) \neq f(x), f(-x) \neq -f(x)$
No symmetry

x-intercept:
$x + 1 = 0$
$x = -1$

y-intercept: $y = \frac{0+1}{0^2 + 2(0) - 3} = -\frac{1}{3}$

Vertical asymptotes:
$x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
x –3, 1

Horizontal asymptote:
$n < m$, so $y = 0$ is the horizontal asymptote.



25. $f(x) = \frac{4x^2}{x^2 + 3}$

domain: all real numbers

Symmetry: $f(-x) = \frac{4x^2}{x^2 + 3} = f(x)$

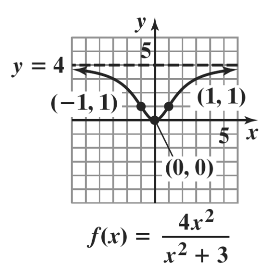
y-axis symmetry

x-intercept:
$4x^2 = 0$
$x = 0$

y-intercept: $y = \frac{4(0)^2}{0^2 + 3} = 0$

Vertical asymptote:
$x^2 + 3 = 0$
$x^2 = -3$
No vertical asymptote.

Horizontal asymptote:
$n = m$, so $y = \frac{4}{1} = 4$ is the horizontal asymptote.



26. **a.** $\overline{C}(x) = \frac{300{,}000 + 10x}{x}$

b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $x = \frac{10}{1} = 10$.
This represents the fact that as the number of satellite radio players produced increases, the production cost approaches \$10 per radio.

27. $x^2 < x + 12$

$x^2 - x - 12 < 0$

$(x+3)(x-4) < 0$

Boundary values: –3 and 4

Solution set: $(-3, 4)$

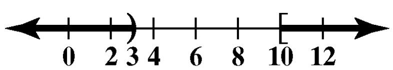
28. $\dfrac{2x+1}{x-3} \le 3$

$\dfrac{2x+1}{x-3} - 3 \le 0$

$\dfrac{10-x}{x-3} \le 0$

Boundary values: 3 and 10

Solution set: $(-\infty, 3) \cup [10, \infty)$



29. $i = \dfrac{k}{d^2}$

$20 = \dfrac{k}{15^2}$

$4500 = k$

$i = \dfrac{4500}{d^2} = \dfrac{4500}{10^2} = 45$ foot-candles

Cumulative Review Exercises (Chapters P–2)

1. domain: $(-2, 2)$ range: $[0, \infty)$

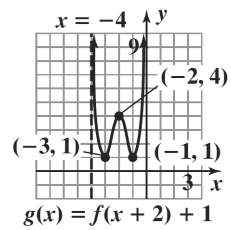
2. The zero at –1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.
The zero at 1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.

3. There is a relative maximum at the point (0, 3).

4. $(f \circ f)(-1) = f\left(f(-1)\right) = f(0) = 3$

5. $f(x) \to \infty$ as $\underline{x \to -2^+}$ or as $\underline{x \to 2^-}$

6.



7. $|2x - 1| = 3$

$2x - 1 = 3$

$2x = 4$

$x = 2$

$2x - 1 = -3$

$2x = -2$

$x = -1$

The solution set is $\{-1, 2\}$.

8. $3x^2 - 5x + 1 = 0$

$x = \dfrac{5 \pm \sqrt{25-12}}{6} = \dfrac{5 \pm \sqrt{13}}{6}$

The solution set is $\left\{\dfrac{5+\sqrt{13}}{6}, \dfrac{5-\sqrt{13}}{6}\right\}$.

9. $9 + \dfrac{3}{x} = \dfrac{2}{x^2}$

$9x^2 + 3x = 2$

$9x^2 + 3x - 2 = 0$

$(3x-1)(3x+2) = 0$

$3x - 1 = 0 \qquad 3x + 2 = 0$

$x = \dfrac{1}{3} \quad$ or $\quad x = -\dfrac{2}{3}$

The solution set is $\left\{-\dfrac{2}{3}, \dfrac{1}{3}\right\}$.

10. $x^3 + 2x^2 - 5x - 6 = 0$

p: ±1, ±2, ±3, ±6

q: ±1

$\dfrac{p}{q}$: ±1, ±2, ±3, ±6

–3	1	2	–5	–6
		–3	3	6
	1	–1	–2	0

$x^3 + 2x^2 - 5x - 6 = 0$

$(x+3)(x^2 - x - 2) = 0$

$(x+3)(x+1)(x-2) = 0$

$x = -3$ or $x = -1$ or $x = 2$

The solution set is $\{-3, -1, 2\}$.

11. $|2x-5|>3$

$$2x-5>3$$
$$2x>8$$
$$x>4$$
$$2x-5<-3$$
$$2x<2$$
$$x<1$$

$(-\infty, 1)$ or $(4, \infty)$

12.

$$3x^2>2x+5$$
$$3x^2-2x-5>0$$
$$3x^2-2x-5=0$$
$$(3x-5)(x+1)=0$$
$$x=\frac{5}{3} \text{ or } x=-1$$

Test intervals are $(-\infty, -1)$, $\left(-1, \frac{5}{3}\right)$, $\left(\frac{5}{3}, \infty\right)$.

Testing points, the solution is $(-\infty, -1)$ or $\left(\frac{5}{3}, \infty\right)$.

13. $f(x)=x^3-4x^2-x+4$

x-intercepts:

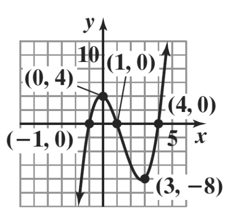
$$x^3-4x^2-x+4=0$$
$$x^2(x-4)-1(x-4)=0$$
$$(x-4)(x^2-1)=0$$
$$(x-4)(x+1)(x-1)=0$$

$x=-1, 1, 4$

x-intercepts:

$$f(0)=0^3-4(0)^2-0+4=4$$

The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



$f(x) = x^3 - 4x^2 - x + 4$

14. $f(x)=x^2+2x-8$

$$x=\frac{-b}{2a}=\frac{-2}{2}=-1$$
$$f(-1)=(-1)^2+2(-1)-8$$
$$=1-2-8=-9$$

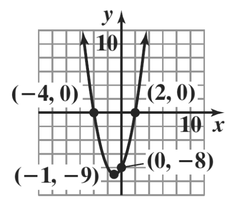
vertex: $(-1, -9)$

x-intercepts:

$$x^2+2x-8=0$$
$$(x+4)(x-2)=0$$

$x=-4$ or $x=2$

y-intercept: $f(0)=-8$



$f(x) = x^2 + 2x - 8$

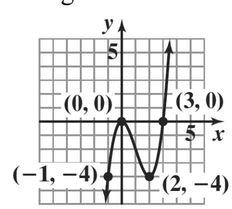
15. $f(x)=x^2(x-3)$

zeros: $x=0$ (multiplicity 2) and $x=3$

y-intercept: $y=0$

$$f(x)=x^3-3x^2$$

$n=3, a_n=0$ so the graph falls to the left and rises to the right.



$f(x) = x^2(x - 3)$

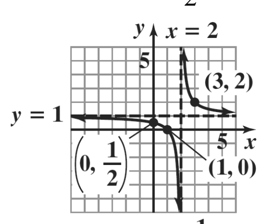
16. $f(x)=\dfrac{x-1}{x-2}$

vertical asymptote: $x=2$

horizontal asymptote: $y=1$

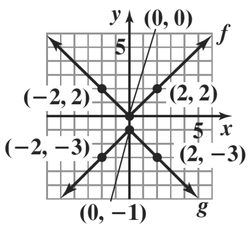
x-intercept: $x=1$

y-intercept: $y=\dfrac{1}{2}$

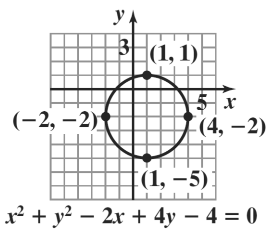


$f(x) = \dfrac{x-1}{x-2}$

17.



18.



19. $(f\circ g)(x) = f\left(g(x)\right)$

$$= 2(4x-1)^2 - (4x-1) - 1$$
$$= 32x^2 - 20x + 2$$

20. $\dfrac{f(x+h)-f(x)}{h}$

$$= \frac{\left[2(x+h)^2 - (x+h) - 1\right] - \left[2x^2 - x - 1\right]}{h}$$
$$= \frac{2x^2 + 4hx - x + 2h^2 - h - 1 - 2x^2 + x + 1}{h}$$
$$= \frac{4hx + 2h^2 - h}{h}$$
$$= 4x + 2h - 1$$

