CHAPTER 2

Section 2.1 Solutions

1. b Vertex $(-2,-5)$ and opens up

2. d Vertex (1,3) and opens up

3. a Vertex $(-3,2)$ and opens down

4. c Vertex (2,3) and opens down

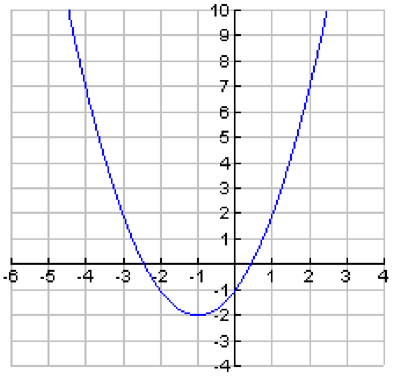
5. Since the coefficient of x^2 is positive, the parabola opens up, so it must be *b* or *d*. Since the coefficient of *x* is positive, the *x*-coordinate of the vertex is negative. So, the graph is **b.**

6. As in #5, the parabola opens up, so it must be *b* or *d*. Since the coefficient of *x* is negative, the *x*-coordinate of the vertex is positive. So, the graph is **d**.

7. Since the coefficient of x^2 is negative, the parabola opens down, so it must be *a* or *c*. In comparison to #8, this one will grow more slowly in the negative *y*-direction. So, the graph is **c**.

8. Since the coefficient of x^2 is -2, the parabola opens down and twice as quickly as the graph of $y=-x^2$. So, the graph is **a**.

9.



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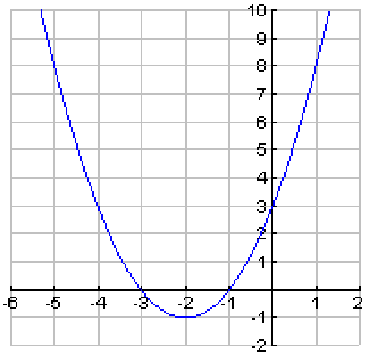

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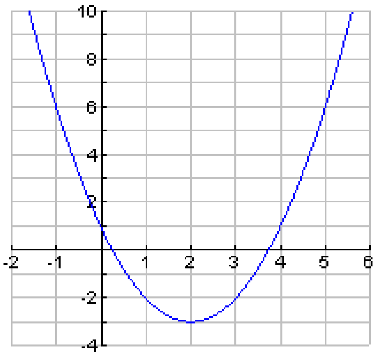

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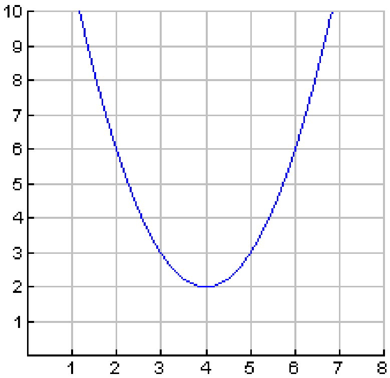

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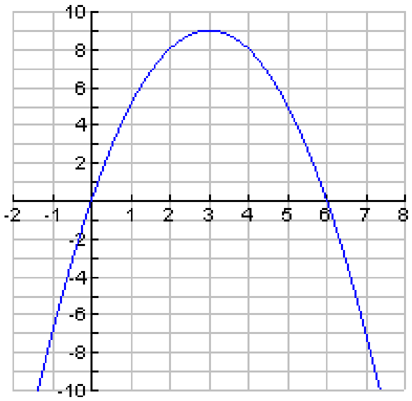

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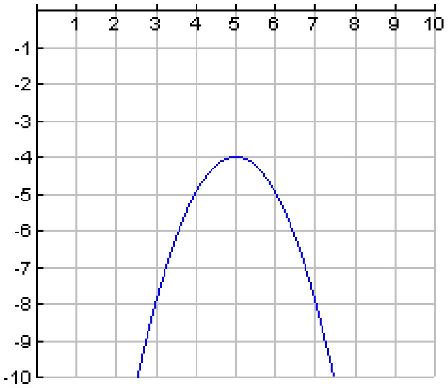

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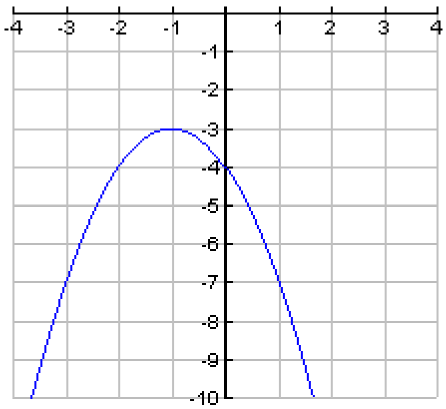

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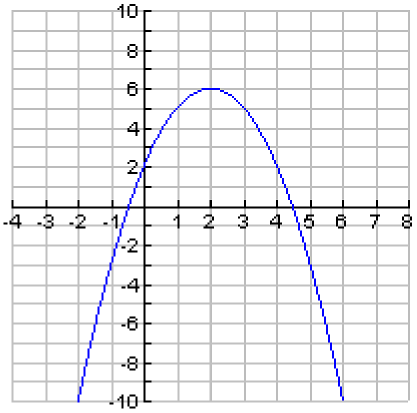

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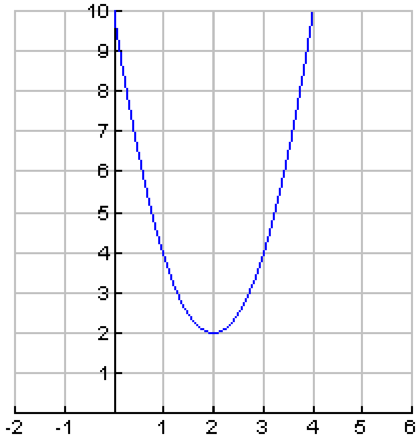

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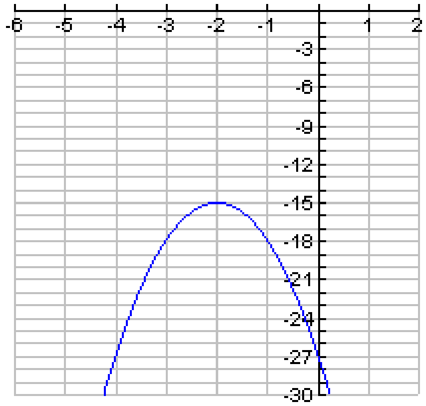

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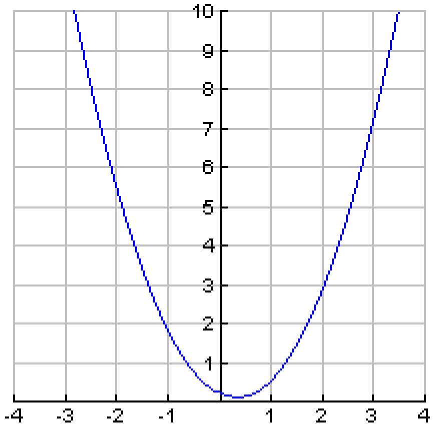

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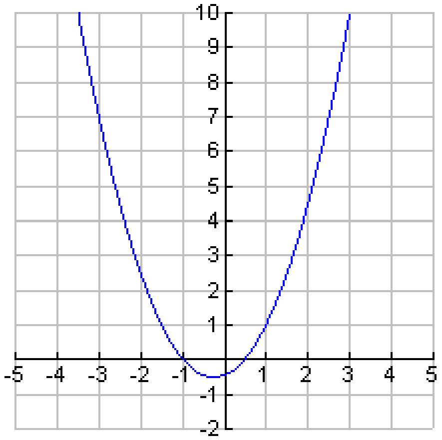


21.

$$\begin{aligned} f(x) &= \left(x^2+6x+9\right)-3-9 \\ &= (x+3)^2-12 \end{aligned}$$

22.

$$\begin{aligned} f(x) &= \left(x^2+8x+16\right)+2-16 \\ &= (x+4)^2-14 \end{aligned}$$

23.

$$\begin{aligned} f(x) &= -\left(x^2+10x \quad\right)+3 \\ &= -\left(x^2+10x+25\right)+3+25 \\ &= -(x+5)^2+28 \end{aligned}$$

24.

$$\begin{aligned} f(x) &= -\left(x^2+12x \quad\right)+6 \\ &= -\left(x^2+12x+36\right)+6+36 \\ &= -(x+6)^2+42 \end{aligned}$$

25.

$$\begin{aligned} f(x) &= 2\left(x^2+4x \quad\right)-2 \\ &= 2\left(x^2+4x+4\right)-2-8 \\ &= 2(x+2)^2-10 \end{aligned}$$

26.

$$\begin{aligned} f(x) &= 3\left(x^2-3x \quad\right)+11 \\ &= 3\left(x^2-3x+\tfrac{9}{4}\right)+11-\tfrac{27}{4} \\ &= 3\left(x-\tfrac{3}{2}\right)^2+\tfrac{17}{4} \end{aligned}$$

27.

$$\begin{aligned} f(x) &= -4\left(x^2-4x \quad\right)-7 \\ &= -4\left(x^2-4x+4\right)-7+16 \\ &= -4(x-2)^2+9 \end{aligned}$$

28.

$$\begin{aligned} f(x) &= -5\left(x^2-20x \quad\right)-36 \\ &= -5\left(x^2-20x+100\right)-36+500 \\ &= -5(x-10)^2+464 \end{aligned}$$

29.

$$\begin{aligned} f(x) &= \left(x^2+10x+25\right)-25 \\ &= (x+5)^2-25 \end{aligned}$$

30.

$$\begin{aligned} f(x) &= -4\left(x^2-3x\quad\right)-2 \\ &= -4\left(x^2-3x+\tfrac{9}{4}\right)-2+9 \\ &= -4\left(x-\tfrac{3}{2}\right)^2+7 \end{aligned}$$

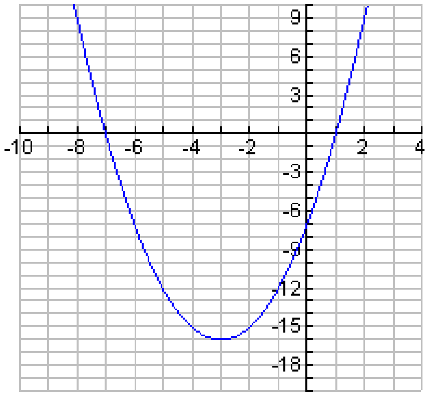
31.

$$\begin{aligned} f(x) &= \tfrac{1}{2}\left(x^2-8x\quad\right)+3 \\ &= \tfrac{1}{2}\left(x^2-8x+16\right)+3-8 \\ &= \tfrac{1}{2}(x-4)^2-5 \end{aligned}$$

32.

$$\begin{aligned} f(x) &= -\tfrac{1}{3}\left(x^2-18x\quad\right)+4 \\ &= -\tfrac{1}{3}\left(x^2-18x+81\right)+4+27 \\ &= -\tfrac{1}{3}(x-9)^2+31 \end{aligned}$$

33.



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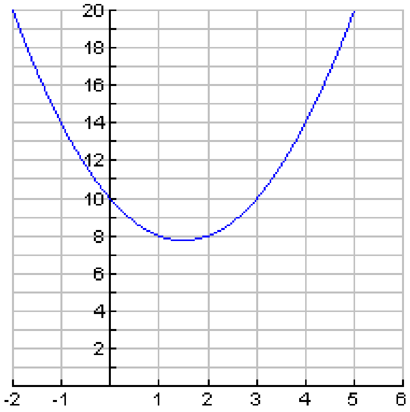

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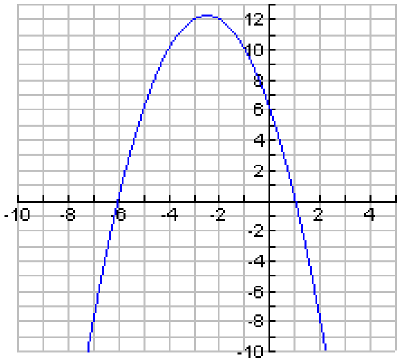

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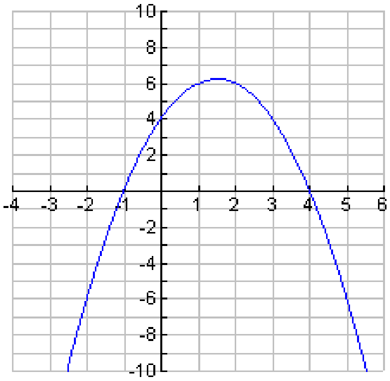

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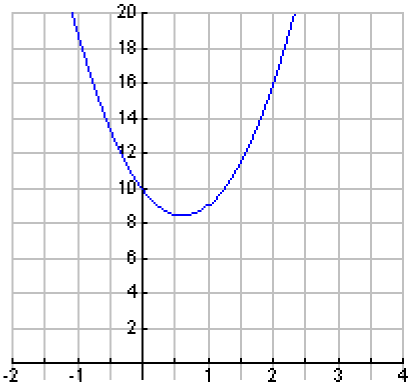

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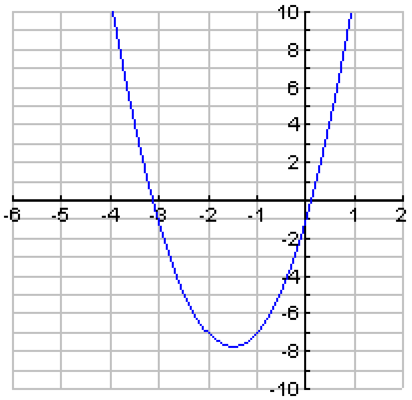

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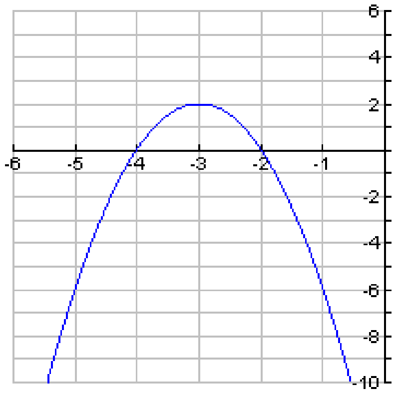

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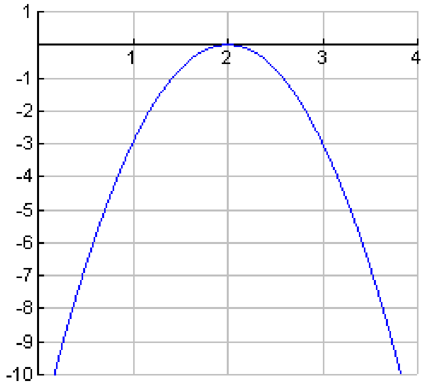


41.

$$\left(-\tfrac{b}{2a},c-\tfrac{b^2}{4a}\right)=\left(-\tfrac{-2}{2(33)},15-\tfrac{(-2)^2}{4(33)}\right)$$
$$=\left(\tfrac{1}{33},\tfrac{494}{33}\right)$$

42.

$$\left(-\tfrac{b}{2a},c-\tfrac{b^2}{4a}\right)=\left(-\tfrac{4}{2(17)},-3-\tfrac{(4)^2}{4(17)}\right)$$
$$=\left(-\tfrac{2}{17},-\tfrac{55}{17}\right)$$

43.

$$\left(-\tfrac{b}{2a},c-\tfrac{b^2}{4a}\right)=\left(-\tfrac{-7}{2(\frac{1}{2})},5-\tfrac{(-7)^2}{4(\frac{1}{2})}\right)$$
$$=\left(7,-\tfrac{39}{2}\right)$$

44.

$$\left(-\tfrac{b}{2a},c-\tfrac{b^2}{4a}\right)=\left(-\tfrac{\frac{2}{5}}{2(-\frac{1}{3})},4-\tfrac{(\frac{2}{5})^2}{4(-\frac{1}{3})}\right)$$
$$=\left(\tfrac{3}{5},\tfrac{103}{25}\right)$$

45.

$$\left(-\tfrac{b}{2a},c-\tfrac{b^2}{4a}\right)=\left(-\tfrac{-2.6}{2(0.06)},3.52-\tfrac{(-2.6)^2}{4(0.06)}\right)$$
$$=\left(21.67,-24.65\right)$$

46.

$$\left(-\tfrac{b}{2a},c-\tfrac{b^2}{4a}\right)=\left(-\tfrac{0.8}{2(-3.2)},-0.14-\tfrac{(0.8)^2}{4(-3.2)}\right)$$
$$=\left(0.125,-0.09\right)$$

47. Since the vertex is $(-1,4)$, the function has the form $y = a(x+1)^2 + 4$. To find *a*, use the fact that the point (0,2) is on the graph:

$$2 = a(0+1)^2 + 4$$
$$-2 = a$$

So, the function is $y = -2(x+1)^2 + 4$.

48. Since the vertex is $(2,-3)$, the function has the form $y = a(x-2)^2 - 3$. To find *a*, use the fact that the point (0,1) is on the graph:

$$1 = a(0-2)^2 - 3$$
$$1 = 4a - 3$$
$$1 = a$$

So, the function is $y = (x-2)^2 - 3$.

49. Since the vertex is $(2,5)$, the function has the form $y = a(x-2)^2 + 5$. To find *a*, use the fact that the point (3,0) is on the graph:

$$0 = a(3-2)^2 + 5$$
$$-5 = a$$

So, the function is $y = -5(x-2)^2 + 5$.

50. Since the vertex is $(1,3)$, the function has the form $y = a(x-1)^2 + 3$. To find *a*, use the fact that the point $(-2,0)$ is on the graph:

$$0 = a(-2-1)^2 + 3$$
$$0 = 9a + 3$$
$$-\tfrac{1}{3} = a$$

So, the function is $y = -\tfrac{1}{3}(x-1)^2 + 3$.

51. Since the vertex is $(-1,-3)$, the function has the form $y = a(x+1)^2 - 3$. To find *a*, use the fact that the point $(-4,2)$ is on the graph:

$$2 = a(-4+1)^2 - 3$$
$$2 = 9a - 3$$
$$\tfrac{5}{9} = a$$

So, the function is $y = \tfrac{5}{9}(x+1)^2 - 3$.

52. Since the vertex is $(0,-2)$, the function has the form $y = a(x-0)^2 - 2$. To find *a*, use the fact that the point $(3,10)$ is on the graph:

$$10 = a(3-0)^2 - 2$$
$$10 = 9a - 2$$
$$\tfrac{12}{9} = \tfrac{4}{3} = a$$

So, the function is $y = \tfrac{4}{3}(x-0)^2 - 2$.

53. Since the vertex is $(-2,-4)$, the function has the form $y = a(x+2)^2 - 4$. To find *a*, use the fact that the point $(-1,6)$ is on the graph:

$$6 = a(-1+2)^2 - 4$$
$$6 = a - 4$$
$$10 = a$$

So, the function is $y = 10(x+2)^2 - 4$.

54. Since the vertex is $(5,4)$, the function has the form $y = a(x-5)^2 + 4$. To find *a*, use the fact that the point $(2,-5)$ is on the graph:

$$-5 = a(2-5)^2 + 4$$
$$-5 = 9a + 4$$
$$-1 = a$$

So, the function is $y = -(x-5)^2 + 4$.

55. Since the vertex is $(\frac{1}{2},\frac{-3}{4})$, the function has the form $y = a(x-\frac{1}{2})^2 - \frac{3}{4}$. To find *a*, use the fact that the point $(\frac{3}{4},0)$ is on the graph:

$$\begin{aligned} 0 &= a(\tfrac{3}{4}-\tfrac{1}{2})^2 - \tfrac{3}{4} \\ 0 &= \tfrac{1}{16}a - \tfrac{3}{4} \\ 12 &= a \end{aligned}$$

So, the function is $y = 12(x-\frac{1}{2})^2 - \frac{3}{4}$.

56. Since the vertex is $(-\frac{5}{6},\frac{2}{3})$, the function has the form $y = a(x+\frac{5}{6})^2 + \frac{2}{3}$. To find *a*, use the fact that the point $(0,0)$ is on the graph:

$$\begin{aligned} 0 &= a(0+\tfrac{5}{6})^2 + \tfrac{2}{3} \\ 0 &= \tfrac{25}{36}a + \tfrac{2}{3} \\ -\tfrac{24}{25} &= a \end{aligned}$$

So, the function is $y = -\frac{24}{25}(x+\frac{5}{6})^2 + \frac{2}{3}$.

57. Completing the square will enable you to identify the vertex of the parabola, which is precisely where the maximum occurs.

$$\begin{aligned} P(x) &= -0.0001\left(x^2 - 700{,}000x\right) + 12{,}500 \\ &= -0.0001\left(x^2 - 700{,}000x + 350{,}000^2\right) + 12{,}500 + 12{,}250{,}000 \\ &= -0.0001\left(x - 350{,}000\right)^2 + 12{,}262{,}500 \end{aligned}$$

a. Maximum profit occurs when 350,000 units are sold.

b. The maximum profit is *P*(350,000) = \$12,262,500.

58. Completing the square will enable you to identify the vertex of the parabola, which is precisely where the minimum occurs.

$$\begin{aligned} P(x) &= 0.5x^2 - 20x + 1{,}600 \\ &= 0.5\left(x^2 - 40x\right) + 1{,}600 \\ &= 0.5\left(x^2 - 40x + 400\right) + 1{,}600 - 200 \\ &= 0.5(x-20)^2 + 1{,}400 \end{aligned}$$

a. Minimum profit occurs when $x = 20$, which corresponds to when 20,000 units are sold.

b. The minimum profit is *P*(20) = 1,400 hundred dollars, which corresponds to \$140,000.

59. Complete the square to identify the vertex. Since the coefficient of t^2 is negative, *W*(*t*) will be increasing to the left of the vertex and decreasing to the right of it.

$$\begin{aligned} W(t) &= -\tfrac{2}{3}\left(t^2 - \tfrac{39}{10}t\right) + \tfrac{433}{5} \\ &= -\tfrac{2}{3}\left(t^2 - \tfrac{39}{10}t + \left(\tfrac{39}{20}\right)^2\right) + \tfrac{433}{5} + \tfrac{2}{3}\left(\tfrac{39}{20}\right)^2 \\ &= -\tfrac{2}{3}\left(t - \tfrac{39}{20}\right)^2 + \tfrac{17{,}827}{200} \end{aligned}$$

So, gaining weight through most of January 2010 and then losing weight during the second to eighteenth months, namely Feb 2010 to June 2011.

60. The maximum weight of the *y*-coordinate of the vertex and is $\frac{17{,}827}{200} \approx 89\,\text{kg}$.

61. a. The maximum occurs at the vertex, which is $(-5,40)$. So, the maximum height is $\boxed{\text{120 feet}}$.

b. If the height of the ball is assumed to be zero when the ball is kicked, and is zero when it lands, then we need to simply compute the *x*-intercepts of *h* and determine the distance between them. To this end, solve

$$0 = -\tfrac{8}{125}(x+5)^2 + 40 \;\Rightarrow\; \underbrace{40\left(\tfrac{125}{8}\right)}_{=625} = (x+5)^2 \;\Rightarrow\; \pm 25 = x+5 \;\Rightarrow\; x = -30, 20$$

So, the distance the ball covers is $\boxed{\text{50 yards}}$.

62. a. The maximum occurs at the vertex, which is $(30,50)$. So, the maximum height is $\boxed{\text{150 feet}}$ = 50 yards.

b. If the height of the ball is assumed to be zero when the ball is kicked, and is zero when it lands, then we need to simply compute the *x*-intercepts of *h* and determine the distance between them. To this end, solve

$$0 = -\tfrac{5}{40}(x-30)^2 + 50 \;\Rightarrow\; \underbrace{50\left(\tfrac{40}{5}\right)}_{=400} = (x-30)^2 \;\Rightarrow\; \pm 20 = x-30 \;\Rightarrow\; x = 10, 50$$

So, the distance the ball covers is $\boxed{\text{40 yards}}$.

63. Let *x* = length and *y* = width.

The total amount of fence is given by: $4x + 3y = 10{,}000$ so that $y = \dfrac{10{,}000 - 4x}{3}$ **(1)**.

The combined area of the two identical pens is $2xy$. Substituting **(1)** in for *y*, we see that the area is described by the function:

$$A(x) = 2x\left(\frac{10{,}000 - 4x}{3}\right) = -\tfrac{8}{3}x^2 + \tfrac{20{,}000}{3}x$$

Since this parabola opens downward (since the coefficient of x^2 is negative), the maximum occurs at the *x*-coordinate of the vertex, namely

$$x = -\tfrac{b}{2a} = \frac{-\frac{20{,}000}{3}}{2\left(-\frac{8}{3}\right)} = 1250\,.$$

The corresponding width of the pen (from **(1)**) is $y = \dfrac{10{,}000 - 4(1250)}{3} \approx 1666.67$

So, each of the two pens would have area $\boxed{\cong 2{,}083{,}333 \text{ sq. ft.}}$

64. Let *x* = length of one of the four pastures, *y* = width of one of the four pastures.

The total amount of fence is given by: $8x+5y=30,000$ so that $y=\dfrac{30,000-8x}{5}$ **(1)**.

The combined area of the four identical pastures is $4xy$. Substituting **(1)** in for *y*, we see that the area is described by the function:

$$A(x)=4x\left(\frac{30,000-8x}{5}\right)=-\tfrac{32}{5}x^2+24,000x$$

Since this parabola opens downward (since the coefficient of x^2 is negative), the maximum occurs at the *x*-coordinate of the vertex, namely

$$x=-\tfrac{b}{2a}=\frac{-24,000}{2\left(-\frac{32}{5}\right)}=1875$$

The corresponding width of the pen (from **(1)**) is $y=\dfrac{30,000-8(1875)}{5}=3000$

So, each of the four pastures would have area $\boxed{5,625,000 \text{ sq. ft.}}$

65. a. Completing the square on *h* yields

$$\begin{aligned} h(t)&=-16\left(t^2-2t\right)+100 \\ &=-16\left(t^2-2t+1\right)+100+16 \\ &=-16(t-1)^2+116 \end{aligned}$$

So, it takes 1 second to reach maximum height of 116 ft.

b. Solve $h(t)=0$.

$$\begin{aligned} -16(t-1)^2+116&=0 \\ (t-1)^2&=\tfrac{116}{16} \\ t-1&=\pm\sqrt{\tfrac{116}{16}} \\ t&=1\pm\sqrt{\tfrac{116}{16}} \end{aligned}$$

Since time must be positive, we conclude that the rock hits the water after about $t=1+\sqrt{\tfrac{116}{16}}\cong 3.69$ seconds (assuming the time started at $t=0$).

c. The rock is above the cliff between 0 and 2 seconds (0<t<2).

66. Solve $-16t^2+1200t=0$:

$$-16t^2+1200t=0$$
$$t(-16t+1200)=0$$
$$t=0,75$$

So, the person has 75 seconds to get out of the way of the bullet.

67.

$$A(x)=-0.0003\left(x^2-31,000x\right)-46,075$$
$$=-0.0003\left(x-15,500\right)^2-46,075+72,075$$
$$=-0.0003(x-15,500)^2+26,000$$

Also, we need the *x*-intercepts to determine the horizontal distance. Observe

$$-0.0003(x-15,500)^2+26,000=20,000$$
$$(x-15,500)^2=\frac{6,000}{0.0003}\approx 20,000,000$$
$$x-15,500\approx \pm 4472.14$$
$$x\approx 15,500\pm 4472.14$$
$$=11,027.86 \text{ and } 19,972.14$$

So, the maximum altitude is 26,000 ft. over a horizontal distance of 8,944 ft.

68. a. Since the vertex is $(50,30)$, the function has the form $y=a(x-50)^2+30$. To find *a*, use the fact that the point $(70,25)$ is on the graph:

$$25=a(75-50)^2+30$$
$$-5=400a \text{ so that } -0.0125=a$$

So, the function is $y(x)=-0.0125(x-50)^2+30$.

b. Since $y(90)=-0.0125(90-50)^2+30=10$, you would expect 10 mpg.

69. First, completing the square yields

$$P(x)=(100-x)x-1000-20x$$
$$=-x^2+80x-1000=-\left(x^2-80x\right)-1000=-\left(x^2-80x+1600\right)-1000+1600$$
$$=-(x-40)^2+600$$

Now, solve $-(x-40)^2+600=0$:

$$-(x-40)^2+600=0$$
$$(x-40)^2=600$$
$$x-40=\pm\sqrt{600} \text{ so that } x=40\pm\sqrt{600}\approx 15.5,\ 64.5$$

So, $\boxed{\text{15 to 16 units to break even, or 64 or 65 units to break even.}}$

70. Using #81, we see that the maximum profit occurs at the *y*-coordinate of the vertex, namely $600.

71. a. We are given that the vertex is $(h,k) = (225, 400)$, that (50, 93.75) is on the graph, and that the graph opens down ($a < 0$) since the peak occurs at the vertex. We need to find a such that the equation governing the situation is

$$y = a(t-225)^2 + 400. \quad \textbf{(1)}$$

To do this, we use the fact that (50, 93.75) satisfies **(1)**:

$$93.75 = a(50-225)^2 + 400 \Rightarrow a = -0.01$$

Thus, the equation is $\boxed{y = -0.01(t-225)^2 + 400}$.

b. We must find the value(s) of t for which $0 = -0.01(t-225)^2 + 400$. To this end,

$$\begin{aligned} 0 = -0.01(t-225)^2 + 400 \;&\Rightarrow\; (t-225)^2 = 40{,}000 \\ &\Rightarrow\; t - 225 = \pm\sqrt{40{,}000} = \pm 200 \\ &\Rightarrow\; t = 225 \pm 200 = 25{,}425 \end{aligned}$$

So, it takes $\boxed{\text{425 minutes}}$ for the drug to be eliminated from the bloodstream.

72. a. We know that the points (70, 20) and (50, 25) lie on this line. Hence, the slope is $m = \frac{25-20}{50-70} = -\frac{1}{4}$. Using point-slope form, the price function is:

$$p - 20 = -\tfrac{1}{4}(x-70) \;\Rightarrow\; \boxed{p(x) = -\tfrac{1}{4}x + \tfrac{75}{2}}$$

b. $R(x) = xp(x) = -\frac{1}{4}x^2 + \frac{75}{2}x$

c. The maximum revenue occurs at the vertex of $R(x)$. Completing the square yields

$$\begin{aligned} -\tfrac{1}{4}x^2 + \tfrac{75}{2}x &= -\tfrac{1}{4}\left(x^2 - 150x\right) \\ &= -\tfrac{1}{4}\left(x^2 - 150x + 5{,}625\right) + \tfrac{5{,}625}{4} \\ &= -\tfrac{1}{4}\left(x - 75\right)^2 + \tfrac{5{,}625}{4} \end{aligned}$$

The vertex is $\left(75, \frac{5{,}625}{4}\right)$. So, he needs to wash $\boxed{\text{75 cars}}$ in order to maximize revenue.

d. To maximize revenue, he should charge $p(x) = -\frac{1}{4}(75) + \frac{75}{2} = \frac{75}{4} = \boxed{\$18.75}$.

73. Step 2 is wrong: Vertex is $(-3,-1)$

Step 4 is wrong: The x-intercepts are $(-2,0)$, $(-4,0)$. So, should graph $y = (x+3)^2 - 1$.

74. Step 3 is wrong:

$f(9) = a(9-2)^2 - 3 = 0$

$49a - 3 = 0$

$a = \frac{3}{49}$

So, $f(x) = \frac{3}{49}(x-2)^2 - 3$.

75. True. $f(x) = a(x-h)^2 + k$, so that $f(0) = ah^2 + k$.

76. False. Consider $f(x) = -1 - x^2$.

77. False. The graph would not pass the vertical line test in such case, and hence wouldn't define a function.

78. True. Consider $f(x) = x^2 - 1$.

79. Completing the square yields

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \tfrac{b}{a}x\right) + c \\ &= a\left(x^2 + \tfrac{b}{a}x + \left(\tfrac{b}{2a}\right)^2\right) + c - a\left(\tfrac{b}{2a}\right)^2 \\ &= a\left(x + \tfrac{b}{2a}\right)^2 + c - \tfrac{b^2}{4a} = a\left(x + \tfrac{b}{2a}\right)^2 + \tfrac{4ac-b^2}{4a} \end{aligned}$$

So, the vertex is $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$. Observe that $f\left(-\frac{b}{2a}\right) = a\underbrace{\left(-\frac{b}{2a} + \frac{b}{2a}\right)^2}_{=0} + c - \frac{b^2}{4a} = c - \frac{b^2}{4a}$.

80. Given that $f(x) = a(x-h)^2 + k$, we have:

<u>*y*-intercept</u>: $a(0-h)^2 + k = ah^2 + k$, so that the *y*-intercept is $\left(0, ah^2 + k\right)$.

<u>*x*-intercepts</u>: Solve $a(x-h)^2 + k = 0$.

$$\begin{aligned} a(x-h)^2 + k &= 0 \\ (x-h)^2 &= -\tfrac{k}{a} \\ x - h &= \pm\sqrt{-\tfrac{k}{a}} \\ x &= h \pm \sqrt{-\tfrac{k}{a}} \end{aligned}$$

So, the *x*-intercepts are $\left(h + \sqrt{-\frac{k}{a}}, 0\right), \left(h - \sqrt{-\frac{k}{a}}, 0\right)$.

81. a. Let *x* = width of rectangular pasture, *y* = length of rectangular pasture.
Then, the total amount of fence is described by $2x + 2y = 1000$ (so that $y = 500 - x$ **(1)**).

The area of the pasture is xy. Substituting in **(1)** yields $x(500 - x) = -x^2 + 500x$.

The maximum area occurs at the *y*-coordinate of the vertex (since the coefficient of x^2 is negative); in this case this value is $c - \frac{b^2}{4a} = 0 - \frac{500^2}{4(-1)} = 62{,}500$.

So the maximum area is 62,500 sq. ft.

b. Let *x* = radius of circular pasture.
Then, the total amount of fence is described by $2\pi x = 1000$ (so that $x = \frac{1000}{2\pi} = \frac{500}{\pi}$ **(2)**).

The area of the pasture is πx^2, which in this case must be (by **(2)**) $\pi\left(\frac{500}{\pi}\right)^2 = \frac{500^2}{\pi}$.

So, the area of the pasture must be approximately 79,577 sq. ft.

82. Let x = number of increases in room rate. Then, the monthly income for the hotel is

$$I(x) = (90+5x)(600-10x) = -50x^2 + 2100x + 54{,}000$$

Completing the square then yields

$$\begin{aligned} -50x^2 + 2100x + 54{,}000 &= -50\left(x^2 - 42x\right) + 54{,}000 \\ &= -50\left(x^2 - 42x + 441\right) + 54{,}000 + 22{,}050 \\ &= -50(x-21)^2 + 76{,}050 \end{aligned}$$

Since the coefficient of x^2 is negative, the maximum income must be \$76,050, which occurs when there are 21 increases in room rate (i.e., at the vertex). Hence, the room rate that yields the maximum profit is $90 + 21(5) = \$195$.

83. Observe that

$$\begin{aligned} &\frac{1}{x+11} + \frac{1}{x+4} = \frac{25}{144} \\ &\frac{x+4+x+11}{(x+11)(x+4)} = \frac{25}{144} \\ &144(2x+15) = 25(x+11)(x+4) \\ &288x + 2160 = 25x^2 + 375x + 1100 \\ &25x^2 + 87x - 1060 = 0 \\ &x = \frac{-87 \pm \sqrt{87^2 + 4(25)(1060)}}{2(25)} = 5,\ \cancel{-8.48} \end{aligned}$$

Since speed cannot be negative, we conclude that $x = 5$.

84. Let W= original width and L = original length

After reduction by 25%, the new width and length are:

.25W = new width
.25L = new length = W

(0.25W)(0.25L) = 26
(0.25W)(W)= 36
0.25 W^2 = 36
W^2 = 144
W = 12

Since W = 0.25L then 0.25L = 12 or ¼ L = 12 or L =48.

85.

$$4(x-0)^2+9(y^2-4y)=0$$
$$4(x-0)^2+9(y^2-4y+4)=36$$
$$4(x-0)^2+9(y-2)^2=36$$
$$\frac{(x-0)^2}{9}+\frac{(y-2)^2}{4}=1 \quad \text{ellipse}$$

86.

$$x^2-2x-4y^2+16y=19$$
$$(x^2-2x+1)-4(y^2-4y+4)=19+1-16$$
$$(x-1)^2-4(y-2)^2=4$$
$$\frac{(x-1)^2}{4}-(y-2)^2=1 \quad \text{hyperbola}$$

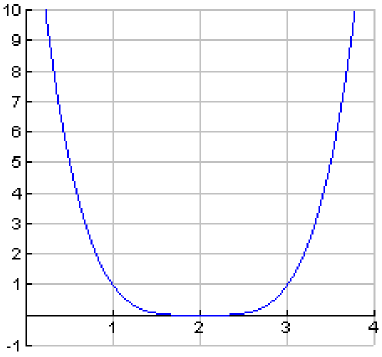
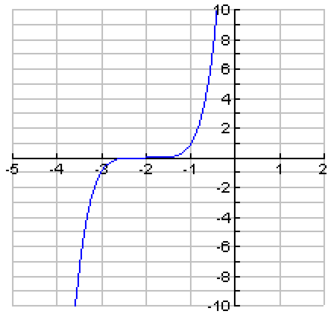
87.

$$x^2+6x=20y-5$$
$$x^2+6x+9=20y-5+9$$
$$(x+3)^2=20y+4$$
$$(x+3)^2=20(y+\tfrac{1}{5}) \quad \text{parabola}$$

88.

$$x^2-6x+4y^2+40y=-105$$
$$(x^2-6x+9)+4(y^2+10y+25)=-105+9+100$$
$$(x-3)^2+4(y+5)^2=4$$
$$\frac{(x-3)^2}{4}+(y+5)^2=1 \quad \text{ellipse}$$

Section 2.2 Solutions ---

1. Polynomial with degree 5	**2.** Polynomial with degree 6
3. Polynomial with degree 7	**4.** Polynomial with degree 9
5. Not a polynomial (due to the term $x^{\frac{1}{2}}$)	**6.** Not a polynomial (due to the term $x^{\frac{1}{2}}$)
7. Not a polynomial (due to the term $x^{\frac{1}{3}}$)	**8.** Not a polynomial (due to the term $\frac{2}{3x}$)
9. Not a polynomial (due to the terms $\frac{1}{x},\frac{1}{x^2}$)	**10.** Polynomial with degree 2
11. h linear function	**12. g** Parabola that opens down
13. b Parabola that opens up	**14. f** Note that $-2x^3+4x^2-6x=-2x\left(x^2-2x+3\right)$. Since x^2-2x+3 is a parabola opening up with vertex (1,2), it has no real roots. So, this polynomial has only 1 *x*-intercept at which it crosses.
15. e $x^3-x^2=x^2(x-1)$ So, there are two *x*-intercepts: the graph is tangent at 0 and crosses at 1.	**16. d** Note that $2x^4-18x^2=2x^2\left(x^2-9\right)=2x^2(x-3)(x+3)$ There are three *x*-intercepts (0, 3, -3) and it crosses at each of them.
17. c $-x^4+5x^3=-x^3(x-5)$ So, there are two *x*-intercepts (0, 5) and the graph crosses at each of them.	**18. a** $x^5-5x^3+4x=x\left(x^4-5x^2+4\right)$ $=x\left(x^2-4\right)\left(x^2-1\right)$ $=x(x-1)(x+1)(x-2)(x+2)$ So, there are five *x*-intercepts (0, 1, 2, -1, -2) and the graph crosses at each of them.
19. 	**20.** 

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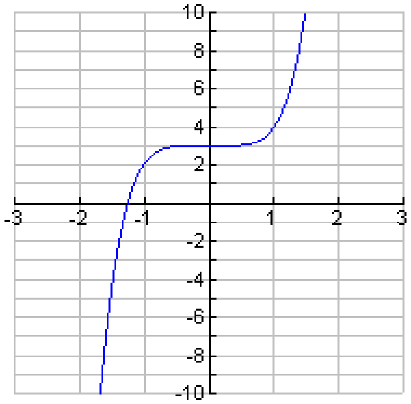

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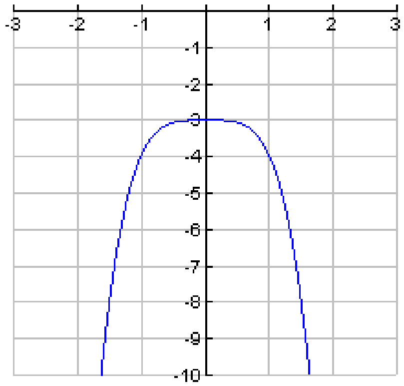

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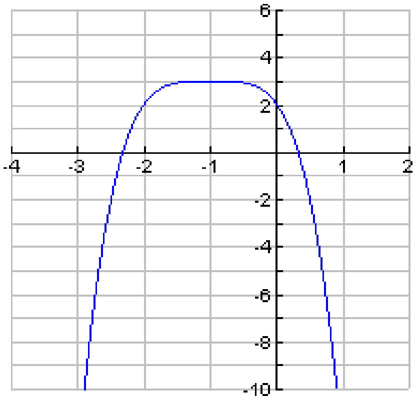

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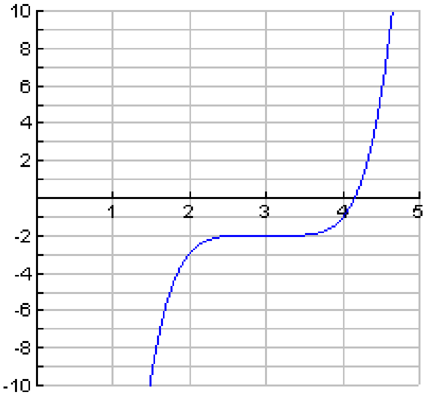


25. 3 (multiplicity 1)
–4 (multiplicity 3)

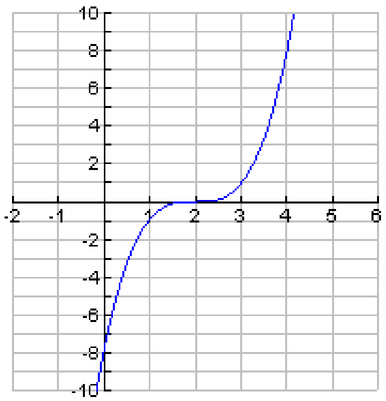
26. –2 (multiplicity 3)
1 (multiplicity 2)

27. 0 (multiplicity 2)
7 (multiplicity 2)
–4 (multiplicity 1)

28. 0 (multiplicity 3)
–1 (multiplicity 4)
6 (multiplicity 1)

29. 0 (multiplicity 2)
1 (multiplicity 2)
Note: $x^2 + 4 = 0$ has no real solutions.

30. 0 (multiplicity 2)
–1 (multiplicity 1)
1 (multiplicity 1)

31. $8x^3+6x^2-27x=x\left(8x^2+6x-27\right)$ $=x(2x-3)(4x+9)$ So, the zeros are: 0 (multiplicity 1) $\frac{3}{2}$ (multiplicity 1) $-\frac{9}{4}$ (multiplicity 1)	**32.** $2x^4+5x^3-3x^2=x^2\left(2x^2+5x-3\right)$ $=x^2(2x-1)(x+3)$ So, the zeros are: 0 (multiplicity 2) $\frac{1}{2}$ (multiplicity 1) -3 (multiplicity 1)
33. $P(x)=x(x+3)(x-1)(x-2)$	**34.** $P(x)=x(x+2)(x-2)$
35. $P(x)=x(x+5)(x+3)(x-2)(x-6)$	**36.** $P(x)=x(x-1)(x-3)(x-5)(x-10)$
37. $P(x)=(2x+1)(3x-2)(4x-3)$	**38.** $P(x)=x(4x+3)(3x+1)(2x-1)$
39. $P(x)=\left(x-(1-\sqrt{2})\right)\left(x-(1+\sqrt{2})\right)$ $=x^2-2x-1$	**40.** $P(x)=\left(x-(1-\sqrt{3})\right)\left(x-(1+\sqrt{3})\right)$ $=x^2-2x-2$
41. $P(x)=x^2(x+2)^3$	**42.** $P(x)=(x+4)^2(x-5)^3$
43. $P(x)=(x+3)^2(x-7)^5$	**44.** $P(x)=x(x-10)^3$
45. $P(x)=x^2(x+1)(x+\sqrt{3})^2(x-\sqrt{3})^2$	**46.** $P(x)=x(x-1)^2(x+\sqrt{5})^2(x-\sqrt{5})^2$
47. $f(x)=(x-2)^3$ **a.** <u>Zeros</u>: 2 (multiplicity 3) **b.** Crosses at 2 **c.** <u>y-intercept</u>: $f(0)=-8$, so $(0,-8)$ **d.** <u>End behavior</u>: Behaves like $y=x^3$. Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.	**e.** 

48. $f(x) = -(x+3)^3$

a. Zeros: -3 (multiplicity 3)

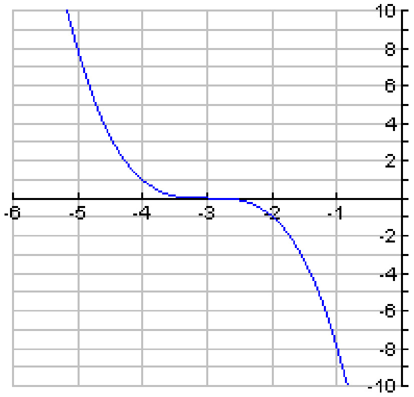
b. Crosses at -3

c. y-intercept: $f(0) = -27$, so $(0, -27)$

d. End behavior: Behaves like $y = -x^3$.

Odd degree and leading coefficient negative, so graph falls without bound to the right and rises to the left.

e.



49. $f(x) = x^3 - 9x = x(x-3)(x+3)$

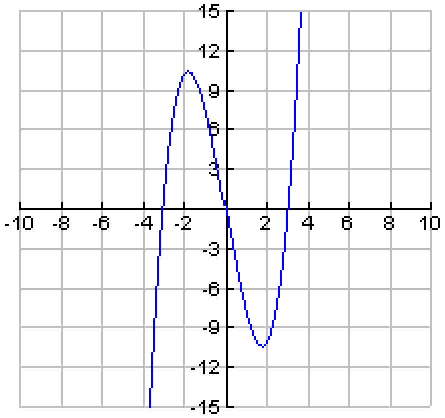
a. Zeros: 0, 3, -3 (multiplicity 1)

b. Crosses at each zero

c. y-intercept: $f(0) = 0$, so $(0,0)$

d. End behavior: Behaves like $y = x^3$.

Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.



50. $f(x) = -x^3 + 4x^2 = -x^2(x-4)$

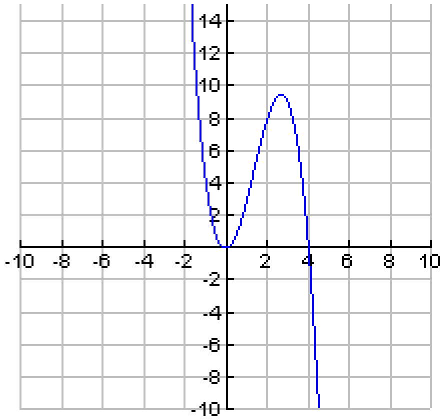
a. Zeros: 0 (multiplicity 2),
4 (multiplicity 1)

b. Crosses at 4, touches at 0

c. y-intercept: $f(0) = 0$, so $(0,0)$

d. End behavior: Behaves like $y = -x^3$.

Odd degree and leading coefficient negative, so graph falls without bound to the right and rises to the left.



51. $f(x)=-x^3+x^2+2x=-x(x-2)(x+1)$

a. Zeros: 0, 2, -1 (multiplicity 1)

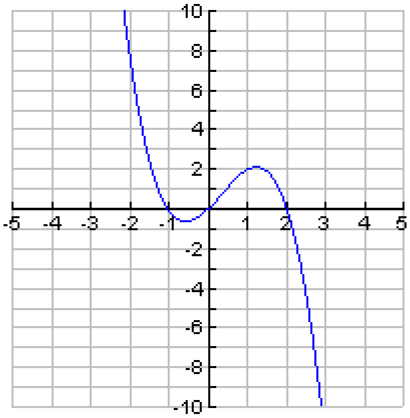
b. Crosses at each zero

c. y-intercept: $f(0)=0$, so $(0,0)$

d. End behavior: Behaves like

$y=-x^3$.

Odd degree and leading coefficient negative, so graph falls without bound to the right and rises to the left.



52. $f(x)=x^3-6x^2+9x=x(x-3)^2$

a. Zeros: 0 (multiplicity 1)
3 (multiplicity 2)

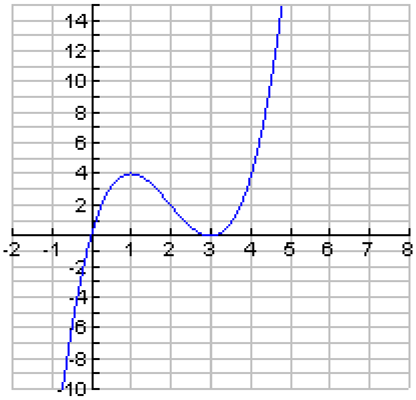
b. Crosses at 0, touches at 3

c. y-intercept: $f(0)=0$, so $(0,0)$

d. End behavior: Behaves like

$y=x^3$.

Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.



53. $f(x)=-x^4-3x^3=-x^3(x+3)$

a. Zeros: 0 (multiplicity 3)
-3 (multiplicity 1)

b. Crosses at both 0 and -3

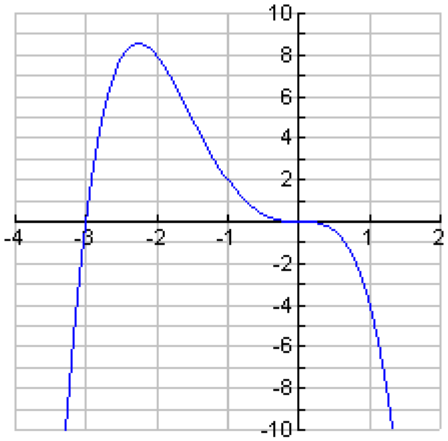
c. y-intercept: $f(0)=0$, so $(0,0)$

d. End behavior: Behaves like

$y=-x^4$.

Even degree and leading coefficient negative, so graph falls without bound to left and right.

e.



54.

$f(x) = x^5 - x^3 = x^3(x^2 - 1) = x^3(x-1)(x+1)$

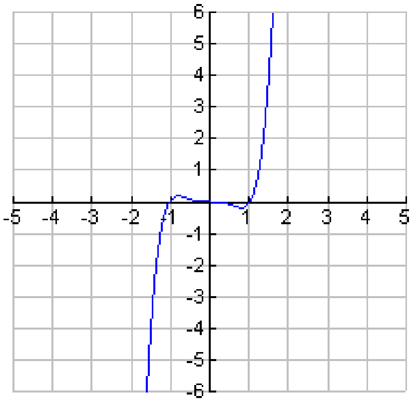
a. Zeros: 0 (multiplicity 3)
1 (multiplicity 1), -1 (multiplicity 1)

b. Crosses at each of 0, 1, and -1

c. y-intercept: $f(0) = 0$, so $(0,0)$

d. End behavior: Behaves like $y = x^5$.
Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.

e.



55.

$$f(x) = 12x^6 - 36x^5 - 48x^4$$
$$= 12x^4\left(x^2 - 3x - 4\right)$$
$$= 12x^4(x-4)(x+1)$$

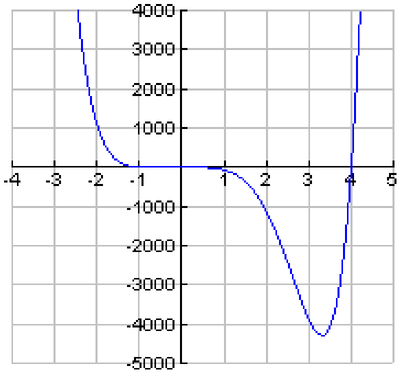
a. Zeros: 0 (multiplicity 4),
4 (multiplicity 1), -1 (multiplicity 1)

b. Touches at 0 and crosses at 4 and -1.

c. y-intercept: $f(0) = 0$, so $(0,0)$

d. End behavior: Behaves like $y = x^6$.
Even degree and leading coefficient positive, so graph rises without bound to the left and right.

e.



56.

$$f(x) = 7x^5 - 14x^4 - 21x^3$$
$$= 7x^3\left(x^2 - 2x - 3\right)$$
$$= 7x^3(x-3)(x+1)$$

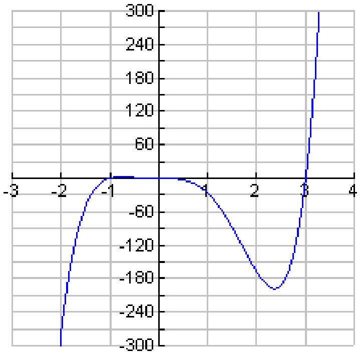
a. Zeros: 0 (multiplicity 3),
3 (multiplicity 1), -1 (multiplicity 1)

b. Crosses at each of 0, 3, and -1

c. y-intercept: $f(0) = 0$, so $(0,0)$

d. End behavior: Behaves like $y = x^5$. Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.

e.



57.

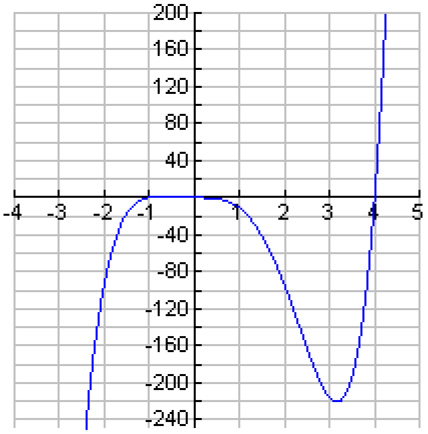
$$f(x) = 2x^5 - 6x^4 - 8x^3$$
$$= 2x^3(x-4)(x+1)$$

a. Zeros: 0 (multiplicity 3), 4(multiplicity 1), -1 (multiplicity 1)

b. Crosses at each zero

c. y-intercept: $f(0) = 0$, so $(0, 0)$

d. End behavior: Behaves like $y = x^5$. Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.



58.

$$f(x) = -5x^4 + 10x^3 - 5x^2 = -5x^2(x-1)^2$$

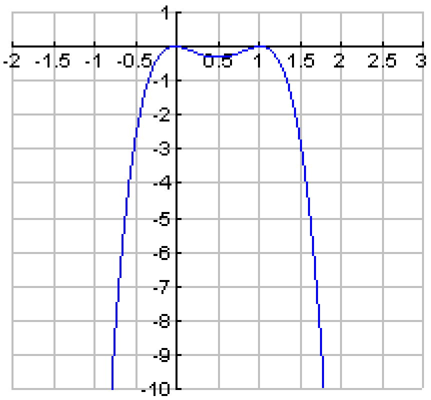
a. Zeros: 0 (multiplicity 2)
1 (multiplicity 2)

b. Touches at each zero

c. y-intercept: $f(0) = 0$, so $(0, 0)$

d. End behavior: Behaves like $y = -x^4$.

Even degree and leading coefficient negative, so graph falls without bound to left and right.



59.

$$f(x) = x^3 - x^2 - 4x + 4$$
$$= \left(x^3 - x^2\right) - 4(x-1)$$
$$= x^2(x-1) - 4(x-1)$$
$$= (x-2)(x+2)(x-1)$$

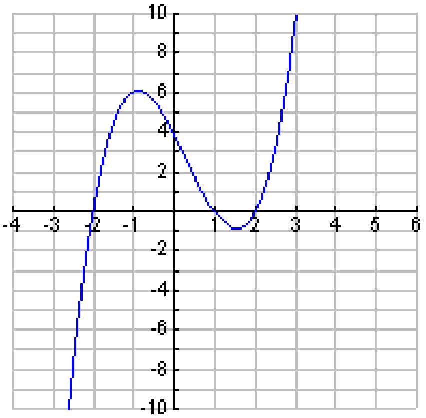
a. Zeros: 1, 2, -2 (multiplicity 1)

b. Crosses at each zero

c. y-intercept: $f(0) = 4$, so $(0, 4)$

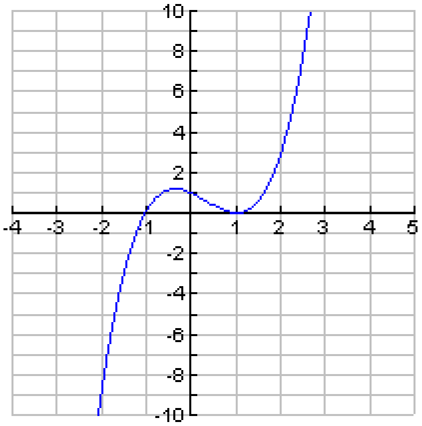
d. End behavior: Behaves like $y = x^3$.

Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.



60.

$$\begin{aligned} f(x) &= x^3 - x^2 - x + 1 \\ &= \left(x^3 - x^2\right) - (x-1) \\ &= x^2(x-1) - (x-1) \\ &= (x^2-1)(x-1) \\ &= (x-1)^2(x+1) \end{aligned}$$



a. Zeros: -1 (multiplicity 1), 1 (multiplicity 2)

b. Crosses at -1, touches at 1

c. y-intercept: $f(0) = 1$, so $(0,1)$

d. End behavior: Behaves like $y = x^3$.

Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.

---

61. $f(x) = -(x+2)^2(x-1)^2$

a. Zeros: -2 (multiplicity 2)
1 (multiplicity 2)

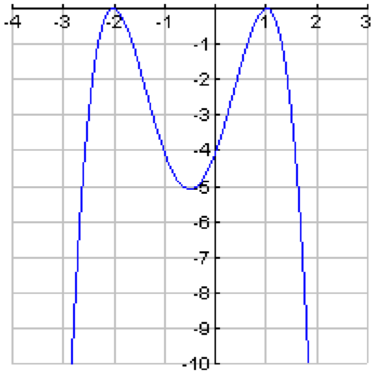
b. Touches at both -2 and 1

c. y-intercept: $f(0) = -4$, so $(0,-4)$

d. End behavior: Behaves like $y = -x^4$.

Even degree and leading coefficient negative, so graph falls without bound to left and right.

e.



62. $f(x) = (x-2)^3(x+1)^3$

a. Zeros: -1 (multiplicity 3)
2 (multiplicity 3)

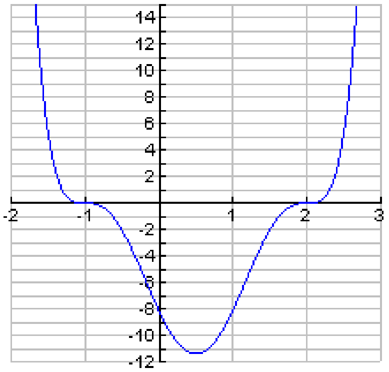
b. Crosses at both -1 and 2

c. y-intercept: $f(0) = -8$, so $(0, -8)$

d. End behavior: Behaves like $y = x^6$.

Even degree and leading coefficient positive, so graph rises without bound to the left and right.

e.



63. $f(x) = x^2(x-2)^3(x+3)^2$

a. Zeros: 0 (multiplicity 2)
2 (multiplicity 3)
-3 (multiplicity 2)

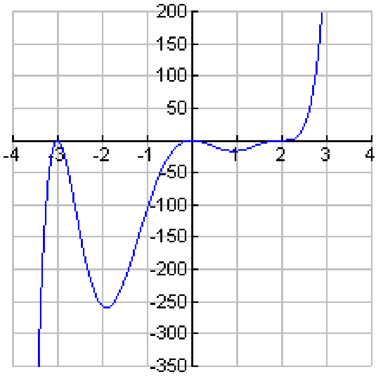
b. Touches at both 0 and -3, and crosses at 2.

c. y-intercept: $f(0) = 0$, so $(0, 0)$

d. End behavior: Behaves like $y = x^7$.

Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.

e.



64. $f(x) = -x^3(x-4)^2(x+2)^2$

a. Zeros: 0 (multiplicity 3)
4 (multiplicity 2)
-2 (multiplicity 2)

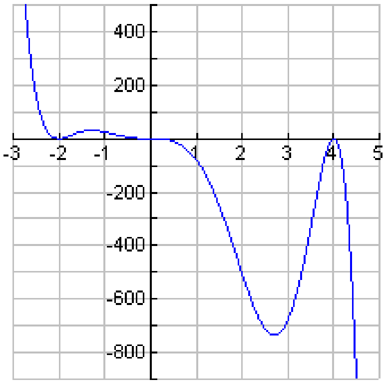
b. Touches at 4 and -2, and crosses at 0.

c. y-intercept: $f(0) = 0$, so $(0, 0)$

d. Long-term behavior: Behaves like $y = -x^7$.

Odd degree and leading coefficient negative, so graph falls without bound to the right and rises to the left.

e.



65. a. zeros: -3 (multiplicity 1)
-1 (multiplicity 2)
2 (multiplicity 1)
b. degree of polynomial: even
c. sign of leading coefficient: negative
d. y-intercept: (0,6)
e. $f(x) = -(x+1)^2(x-2)(x+3)$.

66. a. zeros: -2 (multiplicity 1)
2 (multiplicity 2)
0 (multiplicity 1)
b. degree of polynomial: even
c. sign of leading coefficient: positive
d. y-intercept: (0,0)
e. $f(x) = x(x+2)(x-2)^2$.

67. a. zeros: 0 (multiplicity 2)
-2 (multiplicity 2)
$\frac{3}{2}$ (multiplicity 1)
b. degree of polynomial: odd
c. sign of leading coefficient: positive
d. y-intercept: (0,0)
e. $f(x) = x^2(2x-3)(x+2)^2$.

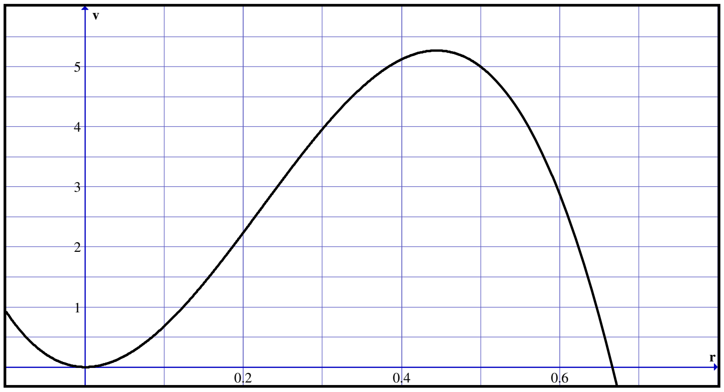
68. a. zeros: -3 (multiplicity 1)
0 (multiplicity 1)
$-\frac{3}{2}$ (multiplicity 1)
1 (multiplicity 2)
b. degree of polynomial: odd
c. sign of leading coefficient: negative
d. y-intercept: (0,0)
e. $f(x) = -x(2x+3)(x+3)(x-1)^2$.

69. a. Revenue for the company is increasing when advertising costs are less than \$400,000. Revenue for the company is decreasing when advertising costs are between \$400,000 and \$600,000.

b. The zeros of the revenue function occur when \$0 and \$600,000 are spent on advertising. When either \$0 or \$600,000 is spent on advertising the company's revenue is \$0.

70. The company's maximum revenue is \$32,000,000 when \$400,000 is spent on advertising.

71. The velocity of air in the trachea is increasing when the radius of the trachea is between 0 and 0.45 cm and decreasing when the radius of the trachea is between 0 and 0.65 cm.



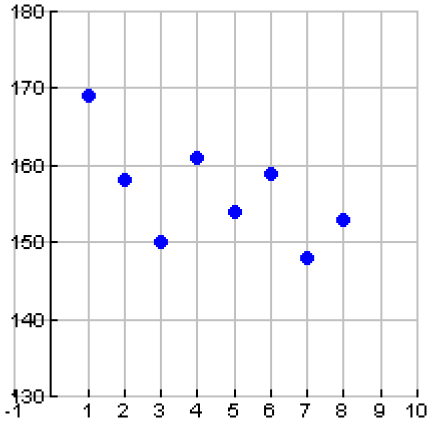
72. r = 0.45 cm ; v = 5.265 meters per second.

73. From the data, there is a turning point at 2. Since it is a <u>third</u> degree polynomial, one would expect the price to go down.

74. From the data, we know that there is a turning point at 3. Since a <u>third</u> degree polynomial is used to model the stock, we expect there to be a turning point at 4, so that the stock should go up in the fifth period.

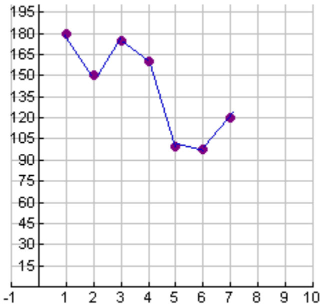
75. From the data, the turning points occur between months 3 & 4, 4 & 5, 5 & 6, 6 & 7, and 7 & 8. So, since there are at least 5 turning points, the degree of the polynomial must be at least 6.

<u>Note</u>: There could be more turning points if there was more oscillation during these month-long periods, but the data doesn't reveal such behavior.



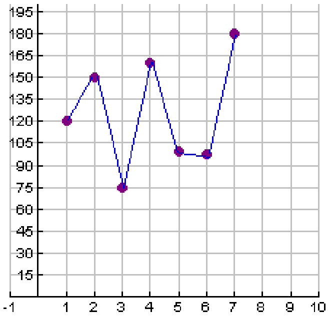
76. From the data and scatterplot in #75, the leading coefficient should be negative.

77. Let Monday correspond to $x = 1$, Tuesday $x = 2$, etc… We have the following data:



Note that a continuous curve passing through these points would have 3 turning points. Hence, its minimum degree is 4.

78. Let Monday correspond to $x = 1$, Tuesday $x = 2$, etc… We have the following data:



Note that a continuous curve passing through these points would have 2 turning points. Hence, its minimum degree is 3.

79. If h is a zero of a polynomial, then $(x-h)$ is a factor of it. So, in this case the function would be:

$$f(x) = (x+2)(x+1)(x-3)(x-4)$$

80. It should be similar to $y = x^4$, which rises to the left and right without bound.

81. False. A polynomial has general form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 .$$

So, $f(0) = a_0$.

82. True. Consider $f(x) = x^2 + 1$

83. True.

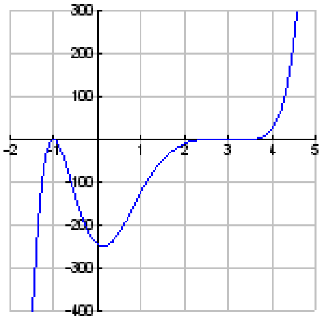
84. False. Consider $f(x) = x^2 + 1$. Its range is $[1, \infty)$.

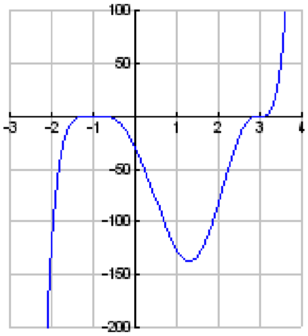
85. A polynomial of degree *n* can have at most *n* zeros.

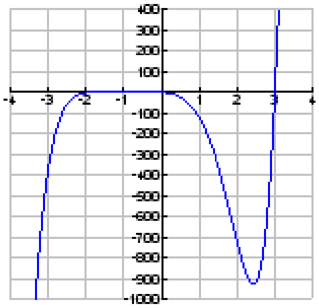
86. An *n*th degree polynomial can have at most (*n* – 1) turning points, and this would occur if it had *n* distinct real zeros, each with multiplicity 1.

87. It touches at -1, so the multiplicity of this zero must be 2, 4, or 6.
It crosses at 3, so the multiplicity of this zero must be 1, 3, or 5.
Thus, the following polynomials would work:

$$f(x) = (x+1)^2(x-3)^5, \quad g(x) = (x+1)^4(x-3)^3, \quad h(x) = (x+1)^6(x-3)$$



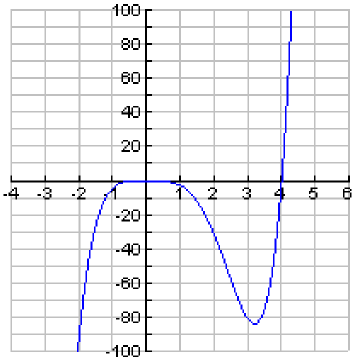


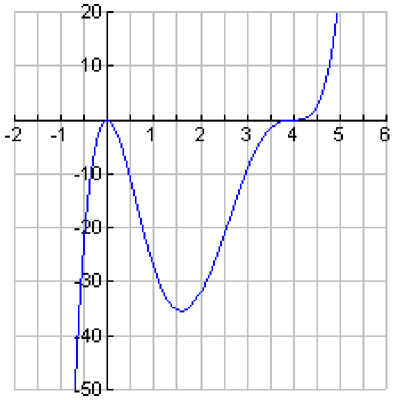


88. A 5th degree polynomial satisfying these properties is of one of the following two forms. You can multiply each by a nonzero real constant to obtain others.

$P_1(x) = (x-0)^4(x-4)$

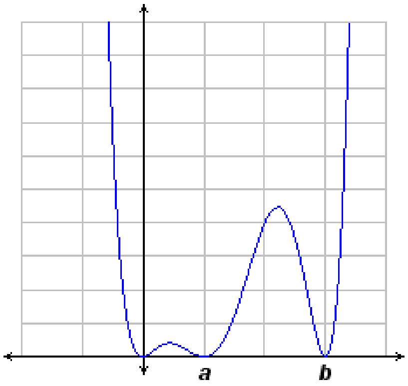
$P_2(x) = (x-0)^2(x-4)^3$



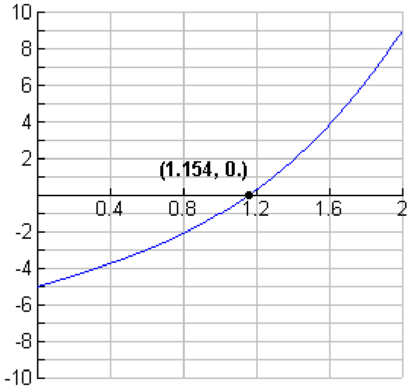


89. Observe that $x^3 + (b-a)x^2 - abx = x\left(x^2 + (b-a)x - ab\right) = x(x-a)(x+b)$. So, the zeros are 0, *a*, and $-b$.

90. The function $f(x) = x^2(x-a)^2(x-b)^2$ has zeros of 0, a, and b, all of which touch the x-axis. The graph is as follows:

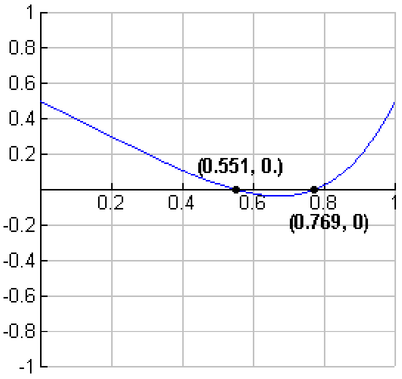


91. The graph is:



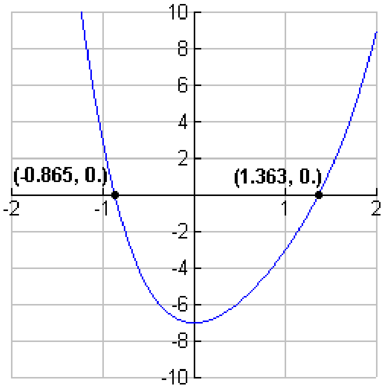
There is one root of *f* in this interval, namely approximately $x = 1.154$.

92. The graph is:



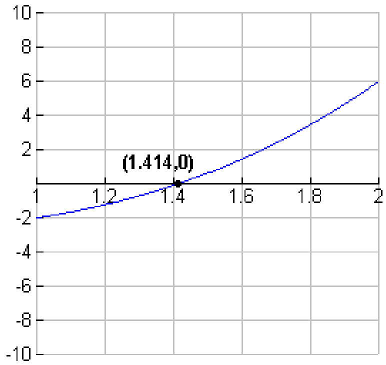
There are two roots of *f* in this interval, namely approximately $x = 0.551$ and $x = 0.769$.

93. The graph is :



There are two roots of *f* in this interval, namely approximately x = -0.865 and x = 1.363.

94. The graph is:



There is one root of *f* in this interval, namely approximately x = 1.414.

Section 2.3 Solutions --

1.

$$\begin{array}{r} 3x-3 \\ x-2\overline{\smash{)}\,3x^2-9x-5} \\ \underline{-(3x^2-6x)} \\ -3x-5 \\ \underline{-(-3x+6)} \\ -11 \end{array}$$

So, $\boxed{Q(x)=3x-3,\ \ r(x)=-11}$.

2.

$$\begin{array}{r} x+5 \\ x-1\overline{\smash{)}\,x^2+4x-3} \\ \underline{-(x^2-x)} \\ 5x-3 \\ \underline{-(5x-5)} \\ 2 \end{array}$$

So, $\boxed{Q(x)=x+5,\ \ r(x)=2}$.

3.

$$\begin{array}{r} 3x-28 \\ x+5\overline{\smash{)}\,3x^2-13x-10} \\ \underline{-(3x^2+15x)} \\ -28x-10 \\ \underline{-(-28x-140)} \\ 130 \end{array}$$

So, $\boxed{Q(x)=3x-28,\ \ r(x)=130}$.

4.

$$\begin{array}{r} 3x+2 \\ x-5\overline{\smash{)}\,3x^2-13x-10} \\ \underline{-(3x^2-15x)} \\ 2x-10 \\ \underline{-(2x-10)} \\ 0 \end{array}$$

So, $\boxed{Q(x)=3x+2,\ \ r(x)=0}$.

5.

$$\begin{array}{r} x-4 \\ x+4\overline{\big)\; x^2+0x-4} \\ \underline{-(x^2+4x)} \\ -4x-4 \\ \underline{-(-4x-16)} \\ 12 \end{array}$$

So, $\boxed{Q(x)=x-4,\quad r(x)=12}$.

6.

$$\begin{array}{r} x+2 \\ x-2\overline{\big)\; x^2+0x-9} \\ \underline{-(x^2-2x)} \\ 2x-9 \\ \underline{-(2x-4)} \\ -5 \end{array}$$

So, $\boxed{Q(x)=x+2,\quad r(x)=-5}$.

7.

$$\begin{array}{r} 3x+5 \\ 3x-5\overline{\big)\; 9x^2+0x-25} \\ \underline{-(9x^2-15x)} \\ 15x-25 \\ \underline{-(15x-25)} \\ 0 \end{array}$$

So, $\boxed{Q(x)=3x+5,\quad r(x)=0}$.

8.

$$\begin{array}{r} 5x-5 \\ x+1\overline{\big)\; 5x^2+0x-3} \\ \underline{-(5x^2+5x)} \\ -5x-3 \\ \underline{-(-5x-5)} \\ 2 \end{array}$$

So, $\boxed{Q(x)=5x-5,\quad r(x)=2}$.

9.

$$\begin{array}{r} 2x-3 \\ 2x+3\overline{\big)\; 4x^2+0x-9} \\ \underline{-(4x^2+6x)} \\ -6x-9 \\ \underline{-(-6x-9)} \\ 0 \end{array}$$

So, $\boxed{Q(x)=2x-3,\quad r(x)=0}$.

10.

$$\begin{array}{r} 4x^2-6x+9 \\ 2x+3\overline{\big)\; 8x^3+0x^2+0x+27} \\ \underline{-(8x^3+12x^2)} \\ -12x^2+0x \\ \underline{-(-12x^2-18x)} \\ 18x+27 \\ \underline{-(18x+27)} \\ 0 \end{array}$$

So, $\boxed{Q(x)=4x^2-6x+9,\quad r(x)=0}$.

11.

$$
\begin{array}{r}
4x^2+4x+1 \\
3x+2\,\overline{)\,12x^3+20x^2+11x+2} \\
\underline{-(12x^3+8x^2)} \\
12x^2+11x \\
\underline{-(12x^2+8x)} \\
3x+2 \\
\underline{-(3x+2)} \\
0
\end{array}
$$

So, $\boxed{Q(x)=4x^2+4x+1,\ \ r(x)=0}$.

12.

$$
\begin{array}{r}
6x^2+7x+2 \\
2x+1\,\overline{)\,12x^3+20x^2+11x+2} \\
\underline{-(12x^3+6x^2)} \\
14x^2+11x \\
\underline{-(14x^2+7x)} \\
4x+2 \\
\underline{-(4x+2)} \\
0
\end{array}
$$

So, $\boxed{Q(x)=6x^2+7x+2,\ \ r(x)=0}$.

13.

$$
\begin{array}{r}
2x^2-x-\frac{1}{2} \\
2x+1\,\overline{)\,4x^3+0x^2-2x+7} \\
\underline{-(4x^3+2x^2)} \\
-2x^2-2x \\
\underline{-(-2x^2-x)} \\
-x+7 \\
\underline{-(-x-\frac{1}{2})} \\
\frac{15}{2}
\end{array}
$$

So, $\boxed{Q(x)=2x^2-x-\frac{1}{2},\ \ r(x)=\frac{15}{2}}$.

14.

$$
\begin{array}{r}
-2x^3-\frac{4}{3}x^2-\frac{2}{9}x-\frac{4}{27} \\
-3x+2\,\overline{)\,6x^4+0x^3-2x^2+0x+5} \\
\underline{-(6x^4-4x^3)} \\
4x^3-2x^2 \\
\underline{-(4x^3-\frac{8}{3}x^2)} \\
\frac{2}{3}x^2+0x \\
\underline{-(\frac{2}{3}x^2-\frac{4}{9}x)} \\
\frac{4}{9}x+5 \\
\underline{-(\frac{4}{9}x-\frac{8}{27})} \\
\frac{143}{27}
\end{array}
$$

So,

$\boxed{Q(x)=-2x^3-\frac{4}{3}x^2-\frac{2}{9}x-\frac{4}{27},\ \ r(x)=\frac{143}{27}}$.

15.

$$
\begin{array}{r}
4x^2-10x-6 \\
x-\frac{1}{2}\,\overline{)\,4x^3-12x^2-x+3} \\
\underline{-(4x^3-2x^2)} \\
-10x^2-x \\
\underline{-(-10x^2+5x)} \\
-6x+3 \\
\underline{-(-6x+3)} \\
0
\end{array}
$$

So, $\boxed{Q(x)=4x^2-10x-6,\ \ r(x)=0}$.

16.

$$
\begin{array}{r}
12x^2+12x+3 \\
x+\frac{1}{3}\,\overline{)\,12x^3+16x^2+7x+1} \\
\underline{-(12x^3+4x^2)} \\
12x^2+7x \\
\underline{-(12x^2+4x)} \\
3x+1 \\
\underline{-(3x+1)} \\
0
\end{array}
$$

So, $\boxed{Q(x)=12x^2+12x+3,\ \ r(x)=0}$.

17.

$$\begin{array}{r}
-2x^2-3x-9 \\
x^3-3x^2+0x+1\overline{\big)\,-2x^5+3x^4+0x^3-2x^2+0x+0} \\
\underline{-(-2x^5+6x^4+0x^3-2x^2)} \\
-3x^4+0x^3+0x^2+0x \\
\underline{-(-3x^4+9x^3+0x^2-3x)} \\
-9x^3+0x^2+3x+0 \\
\underline{-(-9x^3+27x^2+0x-9)} \\
-27x^2+3x+9
\end{array}$$

So, $\boxed{Q(x)=-2x^2-3x-9,\quad r(x)=-27x^2+3x+9}$.

18.

$$\begin{array}{r}
-3x^2+0x+\frac{7}{3} \\
3x^4+0x^3+0x^2-2x+1\overline{\big)\,-9x^6+0x^5+7x^4-2x^3+0x^2+0x+5} \\
\underline{-(-9x^6+0x^5+0x^4+6x^3-3x^2)} \\
7x^4-8x^3+3x^2+0x+5 \\
\underline{-(7x^4+0x^3+0x^2-\frac{14}{3}x+\frac{7}{3})} \\
-8x^3+3x^2+\frac{14}{3}x+\frac{8}{3}
\end{array}$$

So, $\boxed{Q(x)=-3x^2+\frac{7}{3},\quad r(x)=-8x^3+3x^2+\frac{14}{3}x+\frac{8}{3}}$.

19.

$$\begin{array}{r}
x^2+0x+1 \\
x^2+0x-1\overline{\big)\,x^4+0x^3+0x^2+0x-1} \\
\underline{-(x^4+0x^3-x^2)} \\
x^2+0x-1 \\
\underline{-(x^2+0x-1)} \\
0
\end{array}$$

So, $\boxed{Q(x)=x^2+1,\quad r(x)=0}$.

20.

$$\begin{array}{r}
x^2+0x-3 \\
x^2+0x+3\overline{\big)\,x^4+0x^3+0x^2+0x-9} \\
\underline{-(x^4+0x^3+3x^2)} \\
-3x^2+0x-9 \\
\underline{-(-3x^2+0x-9)} \\
0
\end{array}$$

So, $\boxed{Q(x)=x^2-3,\quad r(x)=0}$.

21.

$$\begin{array}{r} x^2+x+\frac{1}{6} \\ 6x^2+x-2\overline{\big)\,6x^4+7x^3+0x^2-22x+40} \\ \underline{-(6x^4+x^3-2x^2)} \\ 6x^3+2x^2-22x \\ \underline{-(6x^3+x^2-2x)} \\ x^2-20x+40 \\ \underline{-(x^2+\frac{1}{6}x-\frac{1}{3})} \\ -\frac{121}{6}x+\frac{121}{3} \end{array}$$

So, $\boxed{Q(x)=x^2+x+\frac{1}{6},\ \ r(x)=-\frac{121}{6}x+\frac{121}{3}}$.

22.

$$\begin{array}{r} x^2+0x-1 \\ 4x^2+0x-9\overline{\big)\,4x^4+0x^3-13x^2+0x+9} \\ \underline{-(4x^4+0x^3-9x^2)} \\ -4x^2+0x+9 \\ \underline{-(-4x^2+0x+9)} \\ 0 \end{array}$$

So, $\boxed{Q(x)=x^2-1,\ \ r(x)=0}$.

23.

$$\begin{array}{r|rrr} -2 & 3 & 7 & 2 \\ & & -6 & -2 \\ \hline & 3 & 1 & 0 \end{array}$$

So, $\boxed{Q(x)=3x+1,\ \ r(x)=0}$.

24.

$$\begin{array}{r|rrr} -5 & 2 & 7 & -15 \\ & & -10 & 15 \\ \hline & 2 & -3 & 0 \end{array}$$

So, $\boxed{Q(x)=2x-3,\ \ r(x)=0}$.

25.

$$\begin{array}{r|rrr} -1 & 7 & -3 & 5 \\ & & -7 & 10 \\ \hline & 7 & -10 & 15 \end{array}$$

So, $\boxed{Q(x)=7x-10,\ \ r(x)=15}$.

26.

$$\begin{array}{r|rrr} 2 & 4 & 1 & 1 \\ & & 8 & 18 \\ \hline & 4 & 9 & 19 \end{array}$$

So, $\boxed{Q(x)=4x+9,\ \ r(x)=19}$.

27.

$$\begin{array}{r|rrrrr} -2 & -1 & -2 & 3 & 4 & -4 \\ & & 2 & 0 & -6 & 4 \\ \hline & -1 & 0 & 3 & -2 & 0 \end{array}$$

So, $\boxed{Q(x)=-x^3+3x-2,\ \ r(x)=0}$.

28.

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

So, $\boxed{Q(x)=x^2+4x+4,\ \ r(x)=0}$.

29.

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & 0 & 0 & 1 \\ & & -1 & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & -1 & 2 \end{array}$$

So, $\boxed{Q(x)=x^3-x^2+x-1,\ \ r(x)=2}$.

30.

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 0 & 0 & 9 \\ & & -3 & 9 & -27 & 81 \\ \hline & 1 & -3 & 9 & -27 & 90 \end{array}$$

So, $\boxed{Q(x)=x^3-3x^2+9x-27,\ \ r(x)=90}$.

31.

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 0 & 0 & -16 \\ & & -2 & 4 & -8 & 16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

So, $\boxed{Q(x)=x^3-2x^2+4x-8,\ \ r(x)=0}$.

32.

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 0 & 0 & -81 \\ & & 3 & 9 & 27 & 81 \\ \hline & 1 & 3 & 9 & 27 & 0 \end{array}$$

So, $\boxed{Q(x)=x^3+3x^2+9x+27,\ \ r(x)=0}$.

33.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -5 & -1 & 1 \\ & & -1 & 3 & -1 \\ \hline & 2 & -6 & 2 & 0 \end{array}$$

So, $\boxed{Q(x)=2x^2-6x+2,\ \ r(x)=0}$.

34.

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -8 & 0 & 1 \\ & & -1 & 3 & -1 \\ \hline & 3 & -9 & 3 & 0 \end{array}$$

So, $\boxed{Q(x)=3x^2-9x+3,\ \ r(x)=0}$.

35.

$$\begin{array}{r|rrrrr} \frac{2}{3} & 2 & -3 & 7 & 0 & -4 \\ & & \frac{4}{3} & -\frac{10}{9} & \frac{106}{27} & \frac{212}{81} \\ \hline & 2 & -\frac{5}{3} & \frac{53}{9} & \frac{106}{27} & -\frac{112}{81} \end{array}$$

So,

$\boxed{Q(x)=2x^3-\frac{5}{3}x^2+\frac{53}{9}x+\frac{106}{27},\ \ r(x)=-\frac{112}{81}}$.

36.

$$\begin{array}{r|rrrrr} \frac{3}{4} & 3 & 1 & 0 & 2 & -3 \\ & & \frac{9}{4} & \frac{39}{16} & \frac{117}{64} & \frac{735}{256} \\ \hline & 3 & \frac{13}{4} & \frac{39}{16} & \frac{245}{64} & -\frac{33}{256} \end{array}$$

So,

$\boxed{Q(x)=3x^3+\frac{13}{4}x^2+\frac{39}{16}x+\frac{245}{64},\ \ r(x)=-\frac{33}{256}}$

37.

$$\begin{array}{r|rrrrr} -1.5 & 2 & 9 & -9 & -81 & -81 \\ & & -3 & -9 & 27 & 81 \\ \hline & 2 & 6 & -18 & -54 & 0 \end{array}$$

So, $\boxed{Q(x)=2x^3+6x^2-18x-54,\ \ r(x)=0}$.

38.

$$\begin{array}{r|rrrr} -0.8 & 5 & -1 & 6 & 8 \\ & & -4 & 4 & -8 \\ \hline & 5 & -5 & 10 & 0 \end{array}$$

So, $\boxed{Q(x)=5x^2-5x+10,\ \ r(x)=0}$.

39.

$$\begin{array}{r|rrrrrrrr} 1 & 1 & 0 & 0 & -8 & 0 & 3 & 0 & 1 \\ & & 1 & 1 & 1 & -7 & -7 & -4 & -4 \\ \hline & 1 & 1 & 1 & -7 & -7 & -4 & -4 & -3 \end{array}$$

So,

$\boxed{\begin{array}{l} Q(x)=x^6+x^5+x^4-7x^3-7x^2-4x-4, \\ r(x)=-3 \end{array}}$.

40.

$$\begin{array}{r|rrrrrrr} -1 & 1 & 4 & 0 & -2 & 0 & 0 & 7 \\ & & -1 & -3 & 3 & -1 & 1 & -1 \\ \hline & 1 & 3 & -3 & 1 & -1 & 1 & 6 \end{array}$$

So,

$\boxed{\begin{array}{l} Q(x)=x^5+3x^4-3x^3+x^2-x+1, \\ r(x)=6 \end{array}}$.

41.

$$\begin{array}{r|rrrrrrr} \sqrt{5} & 1 & 0 & -49 & 0 & -25 & 0 & 1225 \\ & & \sqrt{5} & 5 & -44\sqrt{5} & -220 & -245\sqrt{5} & -1225 \\ \hline & 1 & \sqrt{5} & -44 & -44\sqrt{5} & -245 & -245\sqrt{5} & 0 \end{array}$$

So, $\boxed{Q(x)=x^5+\sqrt{5}x^4-44x^3-44\sqrt{5}x^2-245x-245\sqrt{5}, \quad r(x)=0}$.

42.

$$\begin{array}{r|rrrrrrr} \sqrt{3} & 1 & 0 & -4 & 0 & -9 & 0 & 36 \\ & & \sqrt{3} & 3 & -\sqrt{3} & -3 & -12\sqrt{3} & -36 \\ \hline & 1 & \sqrt{3} & -1 & -\sqrt{3} & -12 & -12\sqrt{3} & 0 \end{array}$$

So, $\boxed{Q(x)=x^5+\sqrt{3}x^4-x^3-\sqrt{3}x^2-12x-12\sqrt{3}, \quad r(x)=0}$.

43.

$$\begin{array}{r} 2x-7 \\ 3x-1\overline{)\,6x^2-23x+7} \\ \underline{-(6x^2-2x)} \\ -21x+7 \\ \underline{-(-21x+7)} \\ 0 \end{array}$$

So, $\boxed{Q(x)=2x-7, \quad r(x)=0}$.

44.

$$\begin{array}{r} 3x+2 \\ 2x-1\overline{)\,6x^2+x-2} \\ \underline{-(6x^2-3x)} \\ 4x-2 \\ \underline{-(4x-2)} \\ 0 \end{array}$$

So, $\boxed{Q(x)=3x+2 \quad r(x)=0}$.

45.

$$\begin{array}{r|rrrr} \underline{1} & 1 & -1 & -9 & 9 \\ & & 1 & 0 & -9 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

So, $\boxed{Q(x)=x^2-9, \quad r(x)=0}$.

46.

$$\begin{array}{r|rrrr} \underline{-2} & 1 & 2 & -6 & -12 \\ & & -2 & 0 & 12 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

So, $\boxed{Q(x)=x^2-6, \quad r(x)=0}$.

47.

$$\begin{array}{r}
x+6 \\
x^2+0x-1\overline{\big)\; x^3+6x^2-2x-5} \\
\underline{-(x^3+0x-x)} \\
6x^2-x-5 \\
\underline{-(6x^2+0x-6)} \\
-x+1
\end{array}$$

So, $\boxed{Q(x)=x+6,\quad r(x)=-x+1}$.

48.

$$\begin{array}{r}
3x^2-3x+5 \\
x^3+x^2-x+1\overline{\big)3x^5+0x^4-x^3+2x^2+0x-1} \\
\underline{-(3x^5+3x^4-3x^3+3x^2)} \\
-3x^4+2x^3-x^2+0x \\
\underline{-(-3x^4-3x^3+3x^2-3x)} \\
5x^3-4x^2+3x-1 \\
\underline{-(5x^3+5x^2-5x+5)} \\
-9x^2+8x-6
\end{array}$$

So,

$\boxed{Q(x)=3x^2-3x+5,\quad r(x)=-9x^2+8x-6}$

49.

$$\begin{array}{r}
x^4-2x^3-4x+7 \\
x^2+0x+1\overline{\big)x^6-2x^5+x^4-6x^3+7x^2-4x+7} \\
\underline{-(x^6+0x^5+x^4)} \\
-2x^5+0x^4-6x^3 \\
\underline{-(-2x^5+0x^4-2x^3)} \\
-4x^3+7x^2-4x \\
\underline{-(-4x^3+0x^2-4x)} \\
7x^2+0x+7 \\
\underline{-(7x^2+0x+7)} \\
0
\end{array}$$

So, $\boxed{Q(x)=x^4-2x^3-4x+7,\quad r(x)=0}$

50.

$$\begin{array}{r}
x^4-x^3+x-1 \\
x^2+x+1\overline{\big)x^6+0x^5+0x^4+0x^3+0x^2+0x-1} \\
\underline{-(x^6+x^5+x^4)} \\
-x^5-x^4+0x^3 \\
\underline{-(-x^5-x^4-x^3)} \\
x^3+0x^2+0x \\
\underline{-(x^3+x^2+x)} \\
-x^2-x-1 \\
\underline{-(-x^2-x-1)} \\
0
\end{array}$$

So, $\boxed{Q(x)=x^4-x^3+x-1,\quad r(x)=0}$

51.

$$\begin{array}{r|rrrrrr}
2 & 1 & 0 & 4 & 2 & 0 & -1 \\
 & & 2 & 4 & 16 & 36 & 72 \\ \hline
 & 1 & 2 & 8 & 18 & 36 & 71
\end{array}$$

So,

$\boxed{\begin{array}{l} Q(x)=x^4+2x^3+8x^2+18x+36, \\ r(x)=71 \end{array}}$.

52.

$$\begin{array}{r|rrrrr}
-5 & 1 & 0 & -1 & 3 & -10 \\
 & & -5 & 25 & -120 & 585 \\ \hline
 & 1 & -5 & 24 & -117 & 575
\end{array}$$

So,

$\boxed{Q(x)=x^3-5x^2+24x-117,\ r(x)=575}$.

53.

$$\begin{array}{r} x^2+0x+1 \\ x^2+0x-1\overline{\smash{)}\,x^4+0x^3+0x^2+0x-25} \\ \underline{-(x^4+0x^3-x^2)} \\ x^2+0x-25 \\ \underline{-(x^2+0x-1)} \\ -24 \end{array}$$

So, $\boxed{Q(x)=x^2+1,\quad r(x)=-24}$.

54.

$$\begin{array}{r} x \\ x^2+0x-2\overline{\smash{)}\,x^3+0x^2+0x-8} \\ \underline{-(x^3+0x^2-2x)} \\ 2x-8 \end{array}$$

So, $\boxed{Q(x)=x,\quad r(x)=2x-8}$.

55.

$$\begin{array}{r|rrrrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

So,

$\boxed{\begin{array}{l} Q(x)=x^6+x^5+x^4+x^3+x^2+x+1, \\ r(x)=0 \end{array}}$.

56.

$$\begin{array}{r|rrrrrrr} 3 & 1 & 0 & 0 & 0 & 0 & 0 & -27 \\ & & 3 & 9 & 27 & 81 & 243 & 729 \\ \hline & 1 & 3 & 9 & 27 & 81 & 243 & 702 \end{array}$$

So,

$\boxed{\begin{array}{l} Q(x)=x^5+3x^4+9x^3+27x^2+81x+243, \\ r(x)=702 \end{array}}$.

57. Area = length × width. So, solving for width, we see that width = Area ÷ length. So, we have:

$$\begin{array}{r} 3x^2+2x+1 \\ 2x^2+0x-1\overline{\smash{)}\,6x^4+4x^3-x^2-2x-1} \\ \underline{-(6x^4+0x^3-3x^2)} \\ 4x^3+2x^2-2x \\ \underline{-(4x^3+0x^2-2x)} \\ 2x^2+0x-1 \\ \underline{-(2x^2+0x-1)} \\ 0 \end{array}$$

Thus, the width (in terms of x) is $\boxed{3x^2+2x+1\text{ feet}}$.

58. Volume = (Area of base) × height. So, solving for height, we see that height = Volume ÷ (Area of base). So, we have:

$$\begin{array}{r} 3x+1 \\ 6x^4+4x^3-x^2-2x-1\overline{\big)\,18x^5+18x^4+x^3-7x^2-5x-1} \\ \underline{-(18x^5+12x^4-3x^3-6x^2-3x)} \\ 6x^4+4x^3-x^2-2x-1 \\ \underline{-(6x^4+4x^3-x^2-2x-1)} \\ 0 \end{array}$$

So, the height (in terms of x) is $\boxed{3x+1 \text{ feet}}$.

59. Distance = Rate × Time. So, solving for Time, we have: Time = Distance ÷ Rate. So, we calculate $\left(x^3+60x^2+x+60\right)\div(x+60)$ using synthetic division:

$$\begin{array}{r|rrrr} \underline{-60} & 1 & 60 & 1 & 60 \\ & & -60 & 0 & -60 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

So, the time is $\boxed{x^2+1 \text{ hours}}$.

60. Distance = Rate × Time. So, solving for Rate, we have: Rate = Distance ÷ Time. So, we calculate $\left(-x^2-5x+50\right)\div(5-x)$ using long division:

$$\begin{array}{r} x+10 \\ -x+5\overline{\big)\,-x^2-5x+50} \\ \underline{-(-x^2+5x)} \\ -10x+50 \\ \underline{-(-10x+50)} \\ 0 \end{array}$$

So, the rate is $\boxed{x+10 \text{ yards per second}}$.

61. Should have subtracted each term in the long division rather than adding them.

62. The zero of the divisor is used in synthetic division. So, 2 should replace -2 as the divisor.

63. Forgot the "0" placeholder.

64. Cannot use synthetic division with a quadratic divisor. Use long division instead.

65. True.

66. False. For instance,

$$\left(x^3-x^2+x-1\right)\div\left(x-1\right)=x^2+1.$$

67. False. Only use when the divisor has degree 1.

68. True.

69. False. For example, $\frac{x+2}{x+1} \neq 1$.

70. True.

71.

$$\begin{array}{r|rrrr}
\underline{-b} & 1 & 2b-a & b^2-2ab & -ab^2 \\
 & & -b & -b^2+ab & ab^2 \\
\hline
 & 1 & b-a & -ab & 0
\end{array}$$

Since the remainder is 0 upon using synthetic division, YES, $(x+b)$ is a factor of $x^3+(2b-a)x^2+\left(b^2-2ab\right)x-ab^2$.

72.

$$\begin{array}{r|rrrrr}
\underline{-b} & 1 & 0 & b^2-a^2 & 0 & -a^2b^2 \\
 & & -b & b^2 & -2b^3+a^2b & 2b^4-a^2b^2 \\
\hline
 & 1 & -b & 2b^2-a^2 & -2b^3+a^2b & 2b^4-2a^2b^2
\end{array}$$

Since the remainder is not 0 upon using synthetic division, NO, $(x+b)$ is not a factor of the given polynomial, <u>unless</u> $b=0$, in which case the above simplifies to saying $x-0$ is a factor of $x^4-a^2x^2$.

73.

$$\begin{array}{r}
x^{2n}+2x^n+1 \\
x^n-1\overline{\big)\,x^{3n}+x^{2n}-x^n-1} \\
\underline{-(x^{3n}-x^{2n})} \\
2x^{2n}-x^n \\
\underline{-(2x^{2n}-2x^n)} \\
x^n-1 \\
\underline{-(x^n-1)} \\
0
\end{array}$$

So, $\boxed{Q(x)=x^{2n}+2x^n+1,\quad r(x)=0}$

74. First, we rewrite the polynomial in a more familiar form using the substitution $y=x^n$. Doing so yields

$x^{3n}+5x^{2n}+8x^n+4=y^3+5y^2+8y+4$.

Now, apply synthetic division:

$$\begin{array}{r|rrrr}
\underline{-1} & 1 & 5 & 8 & 4 \\
 & & -1 & -4 & -4 \\
\hline
 & 1 & 4 & 4 & 0
\end{array}$$

Thus,

$$\begin{aligned} y^3+5y^2+8y+4 &= \left(y+1\right)\left(y^2+4y+4\right) \\ &= \left(y+1\right)\left(y+2\right)^2. \end{aligned}$$

Going back to the original polynomial, this says:

$x^{3n}+5x^{2n}+8x^n+4=\left(x^n+1\right)\left(x^n+2\right)^2$.

75. Using synthetic division gives us:

$$\begin{array}{r|rrr}
\underline{-2} & 2 & -1 & 0 \\
 & & -4 & 10 \\
\hline
 & 2 & -5 & 10
\end{array}$$

So, $\frac{2x^2-x}{x+2}=(2x-5)+\frac{10}{x+2}$.

76. Using synthetic division gives us:

$$\begin{array}{r|rrrr}
\underline{3} & 5 & 2 & -3 & 0 \\
 & & 15 & 51 & 144 \\
\hline
 & 5 & 17 & 48 & 144
\end{array}$$

So,

$$\frac{5x^3+2x^2-3x}{x-3}=\left(5x^2+17x+48\right)+\frac{144}{x-3}$$

77.

$$\begin{array}{r}
2x^2-2x+3 \\
x^2+x+1\overline{\big)\,2x^4+0x^3+3x^2+0x+6} \\
\underline{-(2x^4+2x^3+2x^2)} \\
-2x^3+x^2+0x \\
\underline{-\left(-2x^3-2x^2-2x\right)} \\
3x^2+2x+6 \\
\underline{-\left(3x^2+3x+3\right)} \\
-x+3
\end{array}$$

So, $\dfrac{2x^4+3x^2+6}{x^2+x+1}=\left(2x^2-2x+3\right)-\dfrac{x-3}{x^2+x+1}$.

78.

$$\begin{array}{r}
3x^3-3x^2-14x+30 \\
x^2+x+5\overline{\big)\,3x^5+0x^4-2x^3+x^2+x-6} \\
\underline{-(3x^5+3x^4+15x^3)} \\
-3x^4-17x^3+x^2 \\
\underline{-(-3x^4-3x^3+15x^2)} \\
-14x^3+16x^2+x \\
\underline{-(-14x^3-14x^2-70x)} \\
30x^2+71x-6 \\
\underline{-(30x^2+30x+150)} \\
41x-156
\end{array}$$

So, $\dfrac{3x^5-2x^3+x^2+x-6}{x^2+x+5}=\left(3x^3-3x^2-14x+30\right)+\dfrac{41x-156}{x^2+x+5}$

Section 2.4 Solutions ---

1.

$$\begin{array}{r|rrrr} \underline{1} & 1 & 0 & -13 & 12 \\ & & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-1)(x^2+x-12) \\ &= (x-1)(x+4)(x-3) \end{aligned}.$$

The zeros are 1, 3 and -4.

2.

$$\begin{array}{r|rrrr} \underline{3} & 1 & 3 & -10 & -24 \\ & & 3 & 18 & 24 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-3)(x^2+6x+8) \\ &= (x-3)(x+4)(x+2) \end{aligned}.$$

The zeros are 3, -2 and -4.

3.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 1 & -13 & 6 \\ & & 1 & 1 & -6 \\ \hline & 2 & 2 & -12 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-\tfrac{1}{2})(2x^2+2x-12) \\ &= 2(x-\tfrac{1}{2})(x^2+x-6) \\ &= 2(x-\tfrac{1}{2})(x+3)(x-2) \\ &= (2x-1)(x+3)(x-2) \end{aligned}.$$

The zeros are $\frac{1}{2}, -3$ and 2.

4.

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -14 & 7 & 4 \\ & & -1 & 5 & -4 \\ \hline & 3 & -15 & 12 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x+\tfrac{1}{3})(3x^2-15x+12) \\ &= 3(x+\tfrac{1}{3})(x^2-5x+4) \\ &= 3(x+\tfrac{1}{3})(x-4)(x-1) \\ &= (3x+1)(x-4)(x-1) \end{aligned}.$$

The zeros are 1, 4 and $-\frac{1}{3}$.

5. Since -3 and 5 are both zeros, we know that $(x+3)$ and $(x-5)$ are factors of $P(x)$ and hence, must divide $P(x)$ evenly. (<u>Note</u>: $(x+3)(x-5) = x^2-2x-15$.)

$$\begin{array}{r} x^2+4 \\ x^2-2x-15\overline{\big)\,x^4-2x^3-11x^2-8x-60} \\ \underline{-(x^4-2x^3-15x^2)} \\ 4x^2-8x-60 \\ \underline{-(4x^2-8x-60)} \\ 0 \end{array}$$

So, $P(x) = (x^2+4)(x^2-2x-15) = \boxed{(x^2+4)(x-5)(x+3)}$. The real zeros are 5 and -3.

6. Since -1 and 2 are both zeros, we know that $(x+1)$ and $(x-2)$ are factors of $P(x)$ and hence, must divide $P(x)$ evenly. (<u>Note</u>: $(x+1)(x-2)=x^2-x-2$.)

$$\begin{array}{r} x^2+9 \\ x^2-x-2\overline{\smash{\big)}\,x^4-x^3+7x^2-9x-18} \\ \underline{-(x^4-x^3-2x^2)} \\ 9x^2-9x-18 \\ \underline{-(9x^2-9x-18)} \\ 0 \end{array}$$

So, $P(x)=(x^2+9)(x^2-x-2)=(x^2+9)(x-2)(x+1)$. The real zeros are 2 and -1 .

7. Since -3 and 1 are both zeros, we know that $(x+3)$ and $(x-1)$ are factors of $P(x)$ and hence, must divide $P(x)$ evenly. (<u>Note</u>: $(x+3)(x-1)=x^2+2x-3$.)

$$\begin{array}{r} x^2-2x+2 \\ x^2+2x-3\overline{\smash{\big)}\,x^4+0x^3-5x^2+10x-6} \\ \underline{-(x^4+2x^3-3x^2)} \\ -2x^3-2x^2+10x \\ \underline{-(-2x^3-4x^2+6x)} \\ 2x^2+4x-6 \\ \underline{-(2x^2+4x-6)} \\ 0 \end{array}$$

So, $P(x)=(x^2+2x-3)(x^2-2x+2)=(x-1)(x+3)(x^2-2x+2)$. The real zeros are 1 and -3.

8. Since -2 and 4 are both zeros, we know that $(x+2)$ and $(x-4)$ are factors of $P(x)$ and hence, must divide $P(x)$ evenly. (<u>Note</u>: $(x+2)(x-4)=x^2-2x-8$.)

$$\begin{array}{r} x^2-2x+5 \\ x^2-2x-8\overline{\smash{\big)}\,x^4-4x^3+x^2+6x-40} \\ \underline{-(x^4-2x^3-8x^2)} \\ -2x^3+9x^2+6x \\ \underline{-(-2x^3+4x^2+16x)} \\ 5x^2-10x-40 \\ \underline{-(5x^2-10x-40)} \\ 0 \end{array}$$

So, $P(x)=(x^2-2x+5)(x^2-2x-8)=(x^2-2x+5)(x-4)(x+2)$. The real zeros are 4 and -2.

9.

$$\begin{array}{r|rrrrr} -2 & 1 & 6 & 13 & 12 & 4 \\ & & -2 & -8 & -10 & -4 \\ \hline -2 & 1 & 4 & 5 & 2 & 0 \\ & & -2 & -4 & -2 & \\ \hline & 1 & 2 & 1 & 0 & \end{array}$$

So,

$P(x) = (x+2)^2(x^2+2x+1) = (x+2)^2(x+1)^2$

The zeros are -2 and -1, both with multiplicity 2.

10.

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -2 & -12 & 9 \\ & & 1 & 5 & 3 & -9 \\ \hline 1 & 1 & 5 & 3 & -9 & 0 \\ & & 1 & 6 & 9 & \\ \hline & 1 & 6 & 9 & 0 & \end{array}$$

So,

$P(x) = (x-1)^2(x^2+6x+9) = (x-1)^2(x+3)^2$

The zeros are -3 and 1, both with multiplicity 2.

11.

Factors of 4: $\pm 1, \pm 2, \pm 4$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

12.

Factors of 4: $\pm 1, \pm 2, \pm 4$

Factors of -1: ± 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

13.

Factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Factors of 1: ± 1

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

14.

Factors of 9: $\pm 1, \pm 3, \pm 9$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 3, \pm 9$

15.

Factors of 8: $\pm 1, \pm 2, \pm 4, \pm 8$

Factors of 2: $\pm 1, \pm 2$

Possible rational zeros:

$\pm \frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

16.

Factors of -10: $\pm 1, \pm 2, \pm 5, \pm 10$

Factors of 3: $\pm 1, \pm 3$

Possible rational zeros:

$\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}$

17.

Factors of -20:

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

Factors of 5: $\pm 1, \pm 5$

Possible rational zeros:

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{4}{5}$

18.

Factors of -21: $\pm 1, \pm 3, \pm 7, \pm 21$

Factors of 4: $\pm 1, \pm 2, \pm 4$

Possible rational zeros:

$\pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2},$

$\pm \frac{7}{2}, \pm \frac{21}{2}, \pm \frac{3}{4}, \pm \frac{7}{4}, \pm \frac{21}{4}$

19.

Factors of 8: $\pm 1, \pm 2, \pm 4, \pm 8$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

Testing: $P(1) = P(-1) = P(2) = P(-4) = 0$

20.
Factors of 3: $\pm 1, \pm 3$
Factors of 1: ± 1
Possible rational zeros: $\pm 1, \pm 3$
Testing: $P(1) = P(-1) = P(-3) = 0$

21.
Factors of -3: $\pm 1, \pm 3$
Factors of 2: $\pm 1, \pm 2$
Possible rational zeros: $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$
Testing: $P(1) = P(3) = P\left(\frac{1}{2}\right) = 0$

22.
Factors of -8: $\pm 1, \pm 2, \pm 4, \pm 8$
Factors of 3: $\pm 1, \pm 3$
Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$
Testing: $P(-2) = P(4) = P\left(-\frac{1}{3}\right) = 0$

23.
Number of sign variations for $P(x)$: 1
$P(-x) = P(x)$, so
Number of sign variations for $P(-x)$: 1
Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
1	1

24.
Number of sign variations for $P(x)$: 0
$P(-x) = P(x)$, so
Number of sign variations for $P(-x)$: 0
Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
0	0

25.

Number of sign variations for $P(x)$: 1

$P(-x) = (-x)^5 - 1 = -x^5 - 1$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 5, there are 5 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
1	0

26.

Number of sign variations for $P(x)$: 0

$P(-x) = -x^5 + 1$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 5, there are 5 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
0	1

27.

Number of sign variations for $P(x)$: 2

$P(-x) = -x^5 + 3x^3 + x + 2$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 5, there are 5 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
2	1
0	1

28.

Number of sign variations for $P(x)$: 1

$P(-x) = P(x)$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
1	1

29.

Number of sign variations for $P(x)$: 1

$P(-x)=-9x^7-2x^5+x^3+x$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 7, there are 7 zeros. But, 0 is also a zero. So, we classify the remaining real zeros to the right:

Positive Real Zeros	**Negative Real Zeros**
1	1

30.

Number of sign variations for $P(x)$: 3

$P(-x)=-16x^7-3x^4-2x-1$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 7, there are 7 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
3	0
1	0

31.

Number of sign variations for $P(x)$: 2

$P(-x)=P(x)$, so

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 6, there are 6 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
2	2
0	2
2	0
0	0

32.

Number of sign variations for $P(x)$: 1

$P(-x)=-7x^6-5x^4-x^2-2x+1$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 6, there are 6 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
1	1

33.

Number of sign variations for $P(x)$: 4

$P(-x)=-3x^4-2x^3-4x^2-x-11$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified to the right:

Positive Real Zeros	**Negative Real Zeros**
4	0
2	0
0	0

34.

Number of sign variations for $P(x)$: 2

$P(-x)=2x^4+3x^3+7x^2-3x+2$, so

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which classified to the right:

Positive Real Zeros	**Negative Real Zeros**
2	2
2	0
0	2
0	0

35.

a. Number of sign variations for $P(x)$: 0

$P(-x)=-x^3+6x^2-11x+6$, so

Number of sign variations for $P(-x)$: 3

Since $P(x)$ is degree 3, there are zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
0	3
0	1

b. Factors of 6: $\pm1, \pm2, \pm3, \pm6$

Factors of 1: ±1

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

c. Note that $P(-1)=P(-2)=P(-3)=0$. So, the rational zeros are $-1,-2,-3$. These are the only zeros since *P* has degree 3.

d. $P(x)=(x+1)(x+2)(x+3)$

36.

a. Number of sign variations for $P(x)$: 3

$P(-x)=-x^3-6x^2-11x-6$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
3	0
1	0

b. Factors of -6: $\pm1, \pm2, \pm3, \pm6$

Factors of 1: ±1

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

c. Note that $P(1)=P(2)=P(3)=0$. So, the rational zeros are $1,2,3$. These are the only zeros since *P* has degree 3.

d. $P(x)=(x-1)(x-2)(x-3)$

37.

a. Number of sign variations for $P(x)$: 2

$P(-x) = -x^3 - 7x^2 + x + 7$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
2	1
0	1

b. Factors of 7: $\pm 1, \pm 7$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 7$

c. Note that $P(-1) = P(1) = P(7) = 0$. So, the rational zeros are $-1, 1, 7$. These are the only zeros since *P* has degree 3.

d. $P(x) = (x+1)(x-1)(x-7)$

38.

a. Number of sign variations for $P(x)$: 2

$P(-x) = -x^3 - 5x^2 + 4x + 20$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
2	1
0	1

b. Factors of 20:

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

Factors of 1: ± 1

Possible rational zeros:

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

c. Note that $P(-2) = P(2) = P(5) = 0$. So, the rational zeros are $-2, 2, 5$. These are the only zeros since *P* has degree 3.

d. $P(x) = (x+2)(x-2)(x-5)$

39.

a. Number of sign variations for $P(x)$: 1

$P(-x) = x^4 - 6x^3 + 3x^2 + 10x$, so

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 4, there are 4 zeros, one of which is 0. We classify the remaining real zeros below:

Positive Real Zeros	**Negative Real Zeros**
1	2
1	0

b. $P(x) = x\left(x^3 + 6x^2 + 3x - 10\right)$ We list the possible nonzero rational zeros below:

Factors of -10: $\pm 1, \pm 2, \pm 5, \pm 10$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 2, \pm 5, \pm 10$

c. Note that

$P(0) = P(1) = P(-2) = P(-5) = 0$.

So, the rational zeros are $0, 1, -2, -5$. These are the only zeros since *P* has degree 4.

d. $P(x) = x(x-1)(x+2)(x+5)$

40.

a. Number of sign variations for $P(x)$: 2

$P(-x)=x^4+x^3-14x^2-24x$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 4, there are 4 zeros, one of which is 0. We classify the remaining real zeros below:

Positive Real Zeros	**Negative Real Zeros**
2	1
0	1

b. $P(x)=x\left(x^3-x^2-14x+24\right)$ We list the possible nonzero rational zeros below:

Factors of 24:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$
Factors of 1: ±1
Possible rational zeros:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

c. Note that

$$P(0)=P(-4)=P(2)=P(3)=0.$$

So, the rational zeros are $0, -4, 2, 3$. These are the only zeros since *P* has degree 4.

d. $P(x)=x(x+4)(x-2)(x-3)$

41.

a. Number of sign variations for $P(x)$: 4

$P(-x)=x^4+7x^3+27x^2+47x+26$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
4	0
2	0
0	0

b. Factors of 26: $\pm1, \pm2, \pm13, \pm26$
Factors of 1: ±1
Possible rational zeros:
$\pm1, \pm2, \pm13, \pm26$

c. Note that $P(1)=P(2)=0$. After testing the others, it is found that the only rational zeros are $1, 2$. So, we at least know that $(x-1)(x-2)=x^2-3x+2$ divides $P(x)$ evenly. To find the remaining zeros, we long divide:

$$\begin{array}{r}
x^2-4x+13 \\
x^2-3x+2\overline{\big)\,x^4-7x^3+27x^2-47x+26} \\
\underline{-(x^4-3x^3+2x^2)} \\
-4x^3+25x^2-47x \\
\underline{-(-4x^3+12x^2-8x)} \\
13x^2-39x+26 \\
\underline{-(13x^2-39x+26)} \\
0
\end{array}$$

Since $x^2-4x+13$ is irreducible, the real zeros are 1 and 2.

d. $P(x)=(x-1)(x-2)\left(x^2-4x+13\right)$

42.

a. Number of sign variations for $P(x)$: 3

$P(-x)=x^4+5x^3+5x^2-25x-26$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
3	1
1	1

b. Factors of -26: $\pm 1, \pm 2, \pm 13, \pm 26$

Factors of 1: ± 1

Possible rational zeros:

$\pm 1, \pm 2, \pm 13, \pm 26$

c. Note that $P(1)=P(-2)=0$. After testing the others, it is found that the only rational zeros are $1, -2$. So, we at least know that $(x-1)(x+2)=x^2+x-2$ divides $P(x)$ evenly. To find the remaining zeros, we long divide:

$$\begin{array}{r} x^2-6x+13 \\ x^2+x-2\overline{)\,x^4-5x^3+5x^2+25x-26} \\ \underline{-(x^4+x^3-2x^2)} \\ -6x^3+7x^2+25x \\ \underline{-(-6x^3-6x^2+12x)} \\ 13x^2+13x-26 \\ \underline{-(13x^2+13x-26)} \\ 0 \end{array}$$

Since $x^2-6x+13$ is irreducible, the real zeros are 1 and -2.

d. $P(x)=(x-1)(x+2)\left(x^2-6x+13\right)$

43.

a. Number of sign variations for $P(x)$: 2

$P(-x)=-10x^3-7x^2+4x+1$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
2	1
0	1

b. Factors of 1: ± 1

Factors of 10: $\pm 1, \pm 2, \pm 5, \pm 10$

Possible rational zeros: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}$

c. Note that $P(1)=P(-\frac{1}{2})=P(\frac{1}{5})=0$. So, the rational zeros are $1, -\frac{1}{2}, \frac{1}{5}$. These are the only zeros since *P* has degree 3.

d. $P(x)=(x-1)(2x+1)(5x-1)$

44.

a. Number of sign variations for $P(x)$: 3

$P(-x) = -12x^3 - 13x^2 - 2x - 1$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
3	0
1	0

b. Factors of -1: ±1

Factors of 12:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Possible rational zeros:

$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$

c. After testing, we conclude that the only rational zero is 1. To determine the other zeros, we use synthetic division:

$$\begin{array}{r|rrrr} \underline{1} & 12 & -13 & 2 & -1 \\ & & 12 & -1 & 1 \\ \hline & 12 & -1 & 1 & 0 \end{array}$$

Since $12x^2 - x + 1$ is irreducible, there are no other rational zeros.

d. $P(x) = (x-1)\left(12x^2 - x + 1\right)$

45.

a. Number of sign variations for $P(x)$: 1

$P(-x) = -6x^3 + 17x^2 - x - 10$, so

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
1	2
1	0

b. Factors of -10: $\pm 1, \pm 2, \pm 5, \pm 10$

Factors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

Possible rational zeros:

$\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{2}{3},$

$\pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \pm\frac{10}{3}$

c. Note that $P(-1) = P(-\frac{5}{2}) = P(\frac{2}{3}) = 0$.

So, the rational zeros are $-1, -\frac{5}{2}, \frac{2}{3}$. These are the only zeros since *P* has degree 3.

d.
$$\begin{aligned} P(x) &= (x+1)(2x+5)(3x-2) \\ &= 6(x+1)\left(x+\tfrac{5}{2}\right)\left(x-\tfrac{2}{3}\right) \end{aligned}$$

46.

a. Number of sign variations for $P(x)$: 1

$P(-x) = -6x^3 + x^2 + 5x - 2$, so

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
1	2
1	0

b. Factors of -2: $\pm 1, \pm 2$

Factors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

Possible rational zeros:

$\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{2}{3}$

c. Note that $P(1) = P(-\frac{1}{2}) = P(-\frac{2}{3}) = 0$.

So, the rational zeros are $1, -\frac{1}{2}, -\frac{2}{3}$. These are the only zeros since *P* has degree 3.

d.
$$\begin{aligned} P(x) &= (x-1)(3x+2)(2x+1) \\ &= 6(x-1)\left(x+\tfrac{2}{3}\right)\left(x+\tfrac{1}{2}\right) \end{aligned}$$

47.

a. Number of sign variations for $P(x)$: 4

$P(-x) = x^4 + 2x^3 + 5x^2 + 8x + 4$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
4	0
2	0
0	0

b. Factors of 4: $\pm 1, \pm 2, \pm 4$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

c. Note that $P(1) = 0$. After testing the others, it is found that the only rational zeros is 1. Hence, by **a**, 1 must have multiplicity 2 or 4. So, we know that at least $(x-1)^2 = x^2 - 2x + 1$ divides $P(x)$ evenly. To find the remaining zeros, we long divide:

$$\begin{array}{r|l} & x^2 + 4 \\ \hline x^2 - 2x + 1 & x^4 - 2x^3 + 5x^2 - 8x + 4 \\ & \underline{-(x^4 - 2x^3 + x^2)} \\ & 4x^2 - 8x + 4 \\ & \underline{-(4x^2 - 8x + 4)} \\ & 0 \end{array}$$

Since $x^2 + 4$ is irreducible, the only real zero is 1 (multiplicity 2).

d. $P(x) = (x-1)^2(x^2 + 4)$

48.

a. Number of sign variations for $P(x)$: 0

$P(-x) = x^4 - 2x^3 + 10x^2 - 18x + 9$, so

Number of sign variations for $P(-x)$: 4

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
0	4
0	2
0	0

b. Factors of 9: $\pm 1, \pm 3, \pm 9$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 3, \pm 9$

c. Note that $P(-1) = 0$. After testing the others, it is found that the only rational zero is -1. Hence, by **a**, -1 must have at least multiplicity 2. So, we know that $(x+1)^2 = x^2 + 2x + 1$ divides $P(x)$ evenly. To find the remaining zeros, we long divide:

$$\begin{array}{r|l} & x^2 + 9 \\ \hline x^2 + 2x + 1 & x^4 + 2x^3 + 10x^2 + 18x + 9 \\ & \underline{-(x^4 + 2x^3 + x^2)} \\ & 9x^2 + 18x + 9 \\ & \underline{-(9x^2 + 18x + 9)} \\ & 0 \end{array}$$

Since $x^2 + 9$ is irreducible, the only real zero is -1 (multiplicity 2).

d. $P(x) = (x+1)^2(x^2 + 9)$

49.

a. Number of sign variations for $P(x)$: 1

$P(-x) = P(x)$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 6, there are 6 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
1	1

b. Factors of -36:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18, \pm36$

Factors of 1: ±1

Possible rational zeros:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18, \pm36$

c. Note that $P(-1) = P(1) = 0$. From **a**, there can be no other rational zeros for *P*.

We know that $(x+1)(x-1) = x^2 - 1$ divides $P(x)$ evenly. To find the remaining zeros, we long divide:

$$\begin{array}{r} x^4+13x^2+36 \\ x^2-1\overline{)\,x^6+0x^5+12x^4+0x^3+23x^2+0x-36} \\ \underline{-(x^6+0x^5-x^4)}\qquad\qquad\qquad\qquad \\ 13x^4+0x^3+23x^2\qquad\qquad \\ \underline{-(13x^4+0x^3-13x^2)}\qquad\qquad \\ 36x^2+0x-36 \\ \underline{-(36x^2+0x-36)} \\ 0 \end{array}$$

Observe that

$$x^4+13x^2+36 = (x^2+9)(x^2+4),$$

both of which are irreducible. So, the real zeros are: -1 and 1

d. $P(x) = (x+1)(x-1)(x^2+9)(x^2+4)$

50.

a. Number of sign variations for $P(x)$: 2

$P(-x) = x^4 + x^3 - 16x^2 + 16$, so

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified as:

Positive Real Zeros	**Negative Real Zeros**
2	2
2	0
0	2
0	0

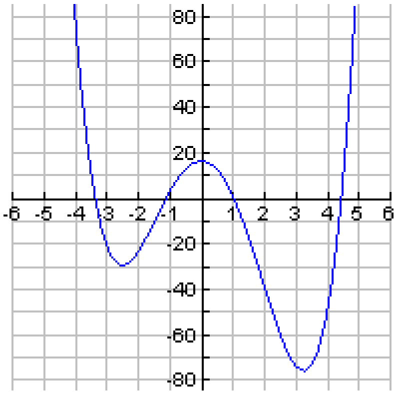
b. Factors of 16: $\pm1, \pm2, \pm4, \pm8, \pm16$

Factors of 1: ±1

Possible rational zeros:

$\pm1, \pm2, \pm4, \pm8, \pm16$

c. Note that $P(1) = 0$. After testing, we conclude that the only rational zero is 1. The best we can do is estimate the remaining zeros graphically:



From the graph, we see that the other real zeros are approximately

$-3.35026, -1.07838,$ and 4.42864.

d. An approximate factorization of $P(x)$ is:

$P(x) = (x-1)(x+3.35026)(x+1.07838)(x-4.42864)$

51.

a. Number of sign variations for $P(x)$: 4

$$P(-x) = 4x^4 + 20x^3 + 37x^2 + 24x + 5,$$

so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified as:

Positive Real Zeros	Negative Real Zeros
4	0
2	0
0	0

b. Factors of 5: ±1, ±5

Factors of 4: ±1, ±2, ±4

Possible rational zeros:

$$\pm 1, \pm 5, \pm\tfrac{1}{2}, \pm\tfrac{1}{4}, \pm\tfrac{5}{2}, \pm\tfrac{5}{4}$$

c. Note that $P(\tfrac{1}{2}) = 0$. After testing, we conclude that there the only rational zero is $\tfrac{1}{2}$, which has multiplicity 2 or 4. So, we know that at least $(x - \tfrac{1}{2})^2$ divides $P(x)$ evenly:

$$\begin{array}{r|rrrrr} \tfrac{1}{2} & 4 & -20 & 37 & -24 & 5 \\ & & 2 & -9 & 14 & -5 \\ \hline \tfrac{1}{2} & 4 & -18 & 28 & -10 & 0 \\ & & 2 & -8 & 10 & \\ \hline & 4 & -16 & 20 & 0 & \end{array}$$

d. So,

$$\begin{aligned} P(x) &= (x - \tfrac{1}{2})^2 (4x^2 - 16x + 20) \\ &= 4(x - \tfrac{1}{2})^2 (x^2 - 4x + 5) \\ &= (2x - 1)^2 (x^2 - 4x + 5) \end{aligned}$$

52.

a. Number of sign variations for $P(x)$: 2

$$P(-x) = 4x^4 + 8x^3 + 7x^2 - 30x + 50, \text{ so}$$

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 4, there are 4 zeros, the real ones of which are classified as:

Positive Real Zeros	Negative Real Zeros
2	2
0	2
0	0
2	0

b. Factors of 50:

$$\pm 1, \pm 2, \pm 5, \pm 10, \pm 25, \pm 50$$

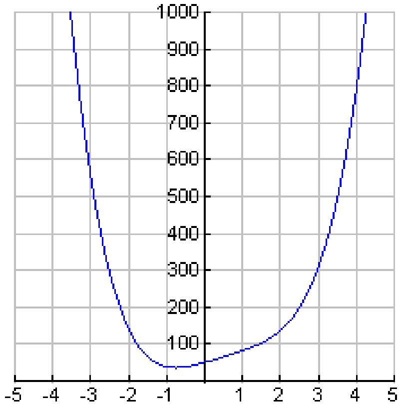
Factors of 4: ±1, ±2, ±4

Possible rational zeros:

$$\pm 1, \pm 2, \pm 5, \pm 10, \pm 25, \pm 50,$$

$$\pm\tfrac{1}{2}, \pm\tfrac{5}{2}, \pm\tfrac{25}{2}, \pm\tfrac{1}{4}, \pm\tfrac{5}{4}, \pm\tfrac{25}{4}$$

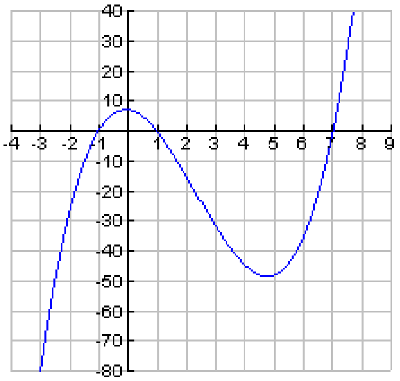
c. After testing, we conclude that there are no rational zeros. The best we can do is graph the polynomial to locate any real zeros:



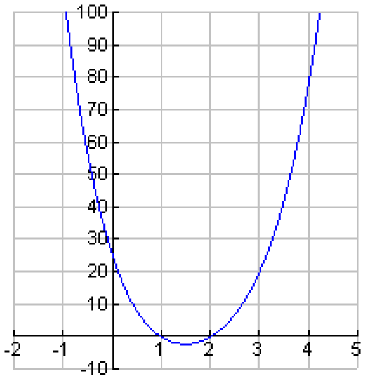
As seen from the graph, there are no real zeros.

d. An accurate factorization of $P(x)$ is not possible since we don't have values for the zeros.

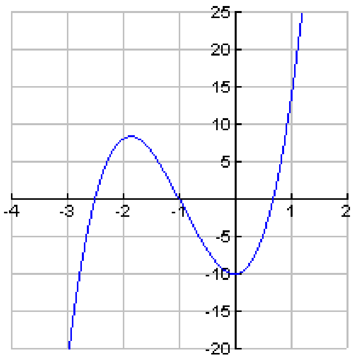
53.



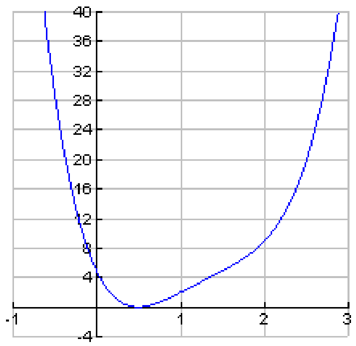
54.



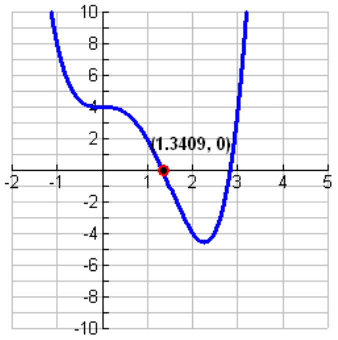
55.



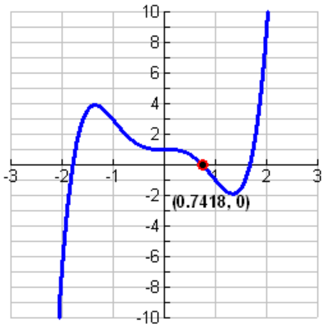
56.



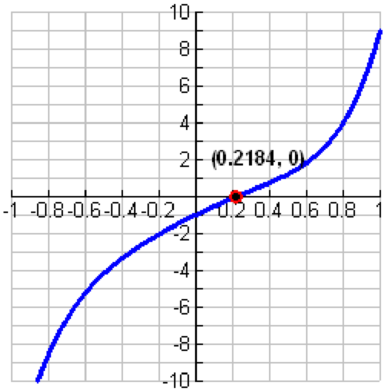
57. Observe that $f(1)=2,\ f(2)=-4$. So, by Intermediate Value Theorem, there must exist a zero in the interval (1,2). Graphically, we approximate this zero to be approximately $\boxed{1.34}$, as seen below:



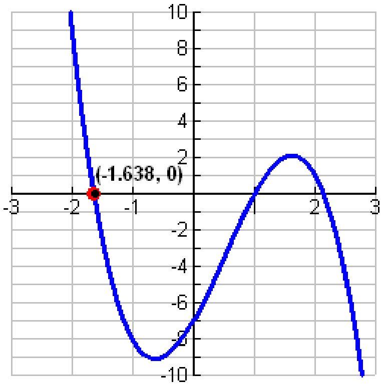
58. Observe that $f(0)=1,\ f(1)=-1$. So, by Intermediate Value Theorem, there must exist a zero in the interval (0,1). Graphically, we approximate this zero to be approximately $\boxed{0.74}$, as seen below:



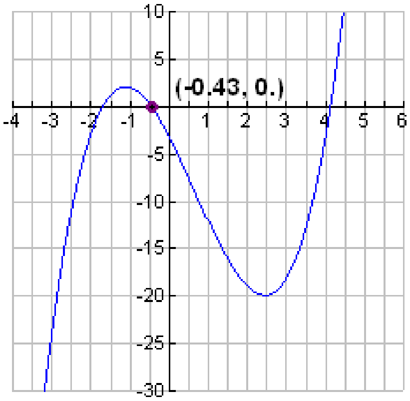
59. Observe that $f(0)=-1,\ f(1)=9$. So, by Intermediate Value Theorem, there must exist a zero in the interval (0,1). Graphically, we approximate this zero to be approximately $\boxed{0.22}$, as seen below:



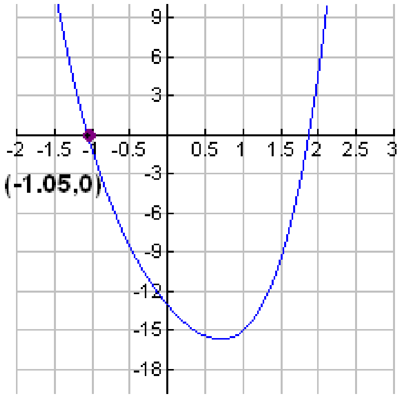
60. Observe that $f(-2)=9,\ f(-1)=-8$. So, by Intermediate Value Theorem, there must exist a zero in the interval $(-2,-1)$. Graphically, we approximate this zero to be approximately $\boxed{-1.64}$, as seen below:



61. Observe that $f(-1)=2,\ f(0)=-3$. So, by Intermediate Value Theorem, there must exist a zero in the interval (-1, 0). Graphically, we approximate this zero to be approximately $\boxed{-0.43}$, as seen below:



62. Observe that $f(-2)=33,\ f(-1)=-1$. So, by Intermediate Value Theorem, there must exist a zero in the interval (-2,-1). Graphically, we approximate this zero to be approximately -1.05, as seen below:



63. Let x = width. Then, the other leg has length x +2 and the diagonal has length x+4 (since it must be the longest side). By the Pythagorean theorem,

$$\begin{aligned} x^2+(x+2)^2 &= (x+4)^2 \\ x^2+x^2+4x+4 &= x^2+8x+16 \\ x^2-4x-12 &= 0 \\ (x-6)(x+2) &= 0 \\ x &= 6, \cancel{-2} \end{aligned}$$

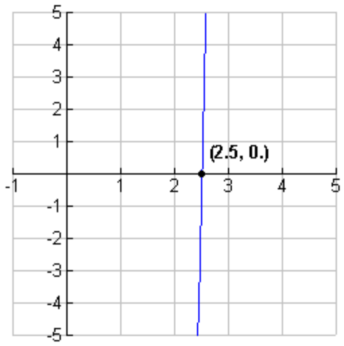
So, the width is 6 in. and length is 8 in.

64. Let x = width of the base.
Then, the length is x + 3.5 and the height is x + 4.0. The volume of the box is then given by

$$x\left(x+3.5\right)\left(x+4\right)=97.5$$

$$x^3+7.5x^2+14x-97.5=0$$

The graph of the left-side on [2,3] is given by



So, x = 2.5. Thus, the dimensions of the box are 2.5 in x 6 in. x 6.5 in.

65. We must solve $x^3+21x^2-1480x-1500=0$. We attempt to factor the left-side using synthetic division:

$$\begin{array}{r|rrrr} 30 & 1 & 21 & -1480 & -1500 \\ & & 30 & 1530 & 1500 \\ \hline & 1 & 51 & 50 & 0 \end{array}$$

Hence,

$$x^3+21x^2-1480x-1500=(x-30)\left(x^2+51x+50\right)=(x-30)(x+50)(x+1)=0.$$

As such, $x=30, \cancel{-50}, \cancel{-1}$, so that there are 30 cows.

66. We must solve $2x^4 - 7x^3 + 3x^2 + 8x - 4 = 0$. We attempt to factor the left-side using synthetic division:

$$\begin{array}{r|rrrrr}
\underline{2} & 2 & -7 & 3 & 8 & -4 \\
 & & 4 & -6 & -6 & 4 \\ \hline
\underline{2} & 2 & -3 & -3 & 2 & 0 \\
 & & 4 & 2 & -2 & \\ \hline
 & 2 & 1 & -1 & 0 &
\end{array}$$

Hence,

$$2x^4 - 7x^3 + 3x^2 + 8x - 4 = (x-2)^2\left(2x^2 + x - 1\right) = (x-2)^2(2x-1)(x+1) = 0\,.$$

As such, $x = 2,\ \cancel{\tfrac{1}{2}},\ \cancel{-1}$. So, there are 2 loaves.

67.

$$\begin{aligned}
P(x) &= xp(x) - C(x) \\
&= x(28 - 0.0002x) - (20x + 1{,}500) \\
&= 28x - 0.0002x^2 - 20x - 1{,}500 \\
&= -0.0002x^2 + 8x - 1{,}500
\end{aligned}$$

By Descartes Rule of Signs, there are either 0 or 2 positive real zeros.

68. Solve $P(x) = 0$.

$$-0.0002x^2 + 8x - 1{,}500 = 0$$

$$x^2 - 40{,}000x + 7{,}500{,}000 = 0 \text{ (cleared the fractions)}$$

$$\begin{aligned}
x &= \frac{40{,}000 \pm \sqrt{40{,}000^2 - 4(7{,}500{,}000)}}{2(1)} \\
&\approx 188 \text{ or } 39{,}812
\end{aligned}$$

The break even points are 188 and 39, 812 units. When fewer than 188 units or more than 39,812 units are produced and sold, profit is negative- money is lost. When the number of units being produced and sold is between 188 and 39,812 a profit is being made on the product.

69. Solve $C(t) = 0$.

$$\begin{aligned}
15.4 - 0.05t^2 &= 0 \\
1{,}540 - 5t^2 &= 0 \\
5(308 - t^2) &= 0
\end{aligned}$$

$$t = \pm\sqrt{308} \approx 17.55 \text{ hours}$$

So, it takes about 18 hours to eliminate the drug from the bloodstream.

70. Solve $C(t) = 0$.

$$\begin{aligned} 60-0.75t^2 &= 0 \\ 6{,}000-75t^2 &= 0 \\ 75(80-t^2) &= 0 \\ t = \pm\sqrt{80} &\approx 9 \text{ hours} \end{aligned}$$

So, it takes about 9 hours to eliminate the drug from the bloodstream.

71. It is true that one can get 5 negative zeros here, but there may be just 1 or 3.

Positive Real Zeros	**Negative Real Zeros**
0	5
0	3
0	1

72. Use 2, not -2.

73. True

74. False. Consider $f(x) = x^2 + 1$. There are no real zeros.

75. False. For instance, $f(x) = \left(x^2+1\right)\left(x^2+2\right)$ cannot be factored over the reals.

76. False. This is one possibility, but if there are 2 or more such sign changes, then there could be fewer.

77. False. For instance, $f(x) = (x-1)^2$ has only one x-intercept.

78. False. For instance, $f(x) = (x-1)^2(x-3)^2(x-4)^2$ has three x-intercepts, but degree 6.

79.

$$\begin{array}{r|rrrr} \underline{a} & 1 & -(a+b+c) & (ab+ac+bc) & -abc \\ & & a & -ab-ac & abc \\ \hline & 1 & -b-c & bc & 0 \end{array}$$

So, $P(x) = (x-a)(x^2-(b+c)x+bc) = (x-a)(x-b)(x-c)$. So, the other zeros are b, c.

80.

$$\begin{array}{r|rrrr} \underline{a} & 1 & -a+b-c & -(ab+bc-ac) & abc \\ & & a & ab-ac & -abc \\ \hline & 1 & b-c & -bc & 0 \end{array}$$

So, $P(x) = (x-a)(x^2+(b-c)x-bc) = (x-a)(x+b)(x-c)$. So, the other zeros are –b, c.

81. First, note that

$$\begin{array}{r|rrrrr} \underline{b} & 1 & -(a+b) & (ab-c^2) & (a+b)c^2 & -abc^2 \\ & & b & -ab & -bc^2 & abc^2 \\ \hline & 1 & -a & -c^2 & ac^2 & 0 \end{array}$$

At this point, the possible rational zeros are a, c, and $-c$. Continuing the synthetic division yields

$$\begin{array}{r|rrrr} \underline{a} & 1 & -a & -c^2 & ac^2 \\ & & a & 0 & -ac^2 \\ \hline & 1 & 0 & -c^2 & \end{array}$$

Thus, we see that

$$P(x) = (x-b)(x-a)\left(x^2-c^2\right) = (x-b)(x-a)(x-c)(x+c).$$

So, the other three zeros are a, c, and $-c$.

82. The possible rational zeros are $\pm a, \pm b$. Using synthetic division yields

$$\begin{array}{r|rrrrr} \underline{a} & 1 & 2(b-a) & a^2-4ab+b^2 & 2ab(a-b) & a^2b^2 \\ & & a & 2ab-a^2 & ab^2-2a^2b & -a^2b^2 \\ \hline \underline{a} & 1 & 2b-a & b^2-2ab & -ab^2 & 0 \\ & & a & 2ab & ab^2 & \\ \hline & 1 & 2b & b^2 & 0 & \end{array}$$

Thus, we see that

$$P(x) = (x-a)^2\left(x^2+2bx+b^2\right) = (x-a)^2(x+b)^2.$$

So, the other zeros are a, $-b$ (multiplicity 2).

83. The possible rational zeros of $f(x) = x^3-4x^2-7x+10$ are $\pm 1, \pm 2, \pm 5, \pm 10$. Using synthetic division yields:

$$\begin{array}{r|rrrr} \underline{1} & 1 & -4 & -7 & 10 \\ & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

So, $f(x) = (x-1)(x^2-3x-10) = (x-1)(x-5)(x+2)$ and the zeros are 1, 5, -2.

The graph of f is above the x-axis on the set: $(-2,1)\cup(5,\infty)$.

84. Factors of 8: $\pm 1, \pm 2, \pm 4, \pm 8$ Factors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

Using synthetic division yields:

$$\begin{array}{r|rrrr} \underline{-1} & 6 & -13 & -11 & 8 \\ & & -6 & 19 & -8 \\ \hline & 6 & -19 & 8 & 0 \end{array}$$

So,

$$f(x) = (x+1)\left(6x^2 - 19x + 8\right) = (x+1)(3x-8)(2x-1),$$

and the zeros are $-1, \frac{8}{3}, \frac{1}{2}$.

The graph of *f* is above the *x*-axis on the set: $\left(-1, \frac{1}{2}\right) \cup \left(\frac{8}{3}, \infty\right)$.

85. Factors of -6: $\pm 1, \pm 2, \pm 3, \pm 6$ Factors of -2: $\pm 1, \pm 2$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

Using synthetic division yields:

$$\begin{array}{r|rrrrr} \underline{3} & -2 & 5 & 7 & -10 & -6 \\ & & -6 & -3 & 12 & 6 \\ \hline \underline{-\frac{1}{2}} & -2 & -1 & 4 & 2 & 0 \\ & & 1 & 0 & -2 & \\ \hline & -2 & 0 & 4 & & \end{array}$$

So,

$$f(x) = (x-3)(x+\tfrac{1}{2})(-2x^2+4) = -2(x-3)(x+\tfrac{1}{2})(x^2-2) = -2(x-3)(x+\tfrac{1}{2})(x-\sqrt{2})(x+\sqrt{2}),$$

and the zeros are $-\frac{1}{2}, 3, \pm\sqrt{2}$.

The graph of *f* is above the *x*-axis on the set: $\left(-\sqrt{2}, -\frac{1}{2}\right) \cup \left(\sqrt{2}, 3\right)$.

86. Factors of -8: $\pm 1, \pm 2, \pm 4, \pm 8$ Factors of -3: $\pm 1, \pm 3$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

Using synthetic division yields:

$$\begin{array}{r|rrrrr} \underline{4} & -3 & 14 & -11 & 14 & -8 \\ & & -12 & 8 & -12 & 8 \\ \hline \underline{\frac{2}{3}} & -3 & 2 & -3 & 2 & 0 \\ & & -2 & 0 & -2 & \\ \hline & -3 & 0 & -3 & & \end{array}$$

So, $f(x) = (x-4)(x-\frac{2}{3})(-3x^2-3) = -3(x-4)(x-\frac{2}{3})(x^2+1)$ and the real zeros are 4, $\frac{2}{3}$.

The graph of *f* is above the *x*-axis on the set: $\left(\frac{2}{3}, 4\right)$.

Section 2.5 Solutions ---

1. $P(x)=(x+2i)(x-2i)$. Zeros are $\pm 2i$.	**2.** $P(x)=(x+3i)(x-3i)$. Zeros are $\pm 3i$
3. Note that the zeros are $x^2-2x+2=0 \Rightarrow$ $x=\frac{2\pm\sqrt{4-4(2)}}{2}=\frac{2\pm 2i}{2}=1\pm i$ So, $P(x)=(x-(1-i))(x-(1+i))$.	**4.** Note that the zeros are $x^2-4x+5=0 \Rightarrow$ $x=\frac{4\pm\sqrt{16-4(5)}}{2}=\frac{4\pm 2i}{2}=2\pm i$ So, $P(x)=(x-(2-i))(x-(2+i))$.
5. Observe that $P(x)=\left(x^2-4\right)\left(x^2+4\right)$ $=(x-2)(x+2)(x-2i)(x+2i)$ So, the zeros are $\pm 2,\ \pm 2i$.	**6.** Observe that $P(x)=\left(x^2-9\right)\left(x^2+9\right)$ $=(x-3)(x+3)(x-3i)(x+3i)$ So, the zeros are $\pm 3,\ \pm 3i$.
7. Observe that $P(x)=\left(x^2-5\right)\left(x^2+5\right)$ $=\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)\left(x-i\sqrt{5}\right)\left(x+i\sqrt{5}\right)$ So, the zeros are $\pm\sqrt{5},\ \pm i\sqrt{5}$.	**8.** Observe that $P(x)=\left(x^2-3\right)\left(x^2+3\right)$ $=\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)\left(x-i\sqrt{3}\right)\left(x+i\sqrt{3}\right)$ So, the zeros are $\pm\sqrt{3},\ \pm i\sqrt{3}$.
9. If *i* is a zero, then so is its conjugate –*i*. Since $P(x)$ has degree 3, this is the only missing zero.	**10.** If -*i* is a zero, then so is its conjugate *i*. Since $P(x)$ has degree 3, this is the only missing zero.
11. Since $2i$ and $3-i$ are zeros, so are their conjugates $-2i$ and $3+i$, respectively. Since $P(x)$ has degree 4, these are the only missing zeros.	**12.** Since $3i$ and $2+i$ are zeros, so are their conjugates $-3i$ and $2-i$, respectively. Since $P(x)$ has degree 4, these are the only missing zeros.
13. Since $1-3i$ and $2+5i$ are zeros, so are their conjugates $1+3i$ and $2-5i$, respectively. Since $P(x)$ has degree 6 and 0 is a zero with multiplicity 2, these are the only missing zeros.	**14.** Since $1-5i$ and $2+3i$ are zeros, so are their conjugates $1+5i$ and $2-3i$, respectively. Since $P(x)$ has degree 6 and -2 is a zero with multiplicity 2, these are the only missing zeros.
15. Since $-i$ and $1-i$ are zeros, so are their conjugates i and $1+i$, respectively. Since $1-i$ has multiplicity 2, so does its conjugate. Since $P(x)$ has degree 6, these are the only missing zeros.	**16.** Since $2i$ and $1+i$ are zeros, so are their conjugates $-2i$ and $1-i$, respectively. Since $1+i$ has multiplicity 2, so does its conjugate. Since $P(x)$ has degree 6, these are the only missing zeros.

17. Let $P(x)$ be the desired polynomial. Since 0 is a zero of P, x is a factor of P. Also, since $1\pm 2i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-(1-2i))(x-(1+2i))=x^2-2x+5$ So, $P(x)$ is given by $x\left(x^2-2x+5\right)=x^3-2x^2+5x$.	**18.** Let $P(x)$ be the desired polynomial. Since 0 is a zero of P, x is a factor of P. Also, since $2\pm i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-(2-i))(x-(2+i))=x^2-4x+5$ So, $P(x)$ is given by $x\left(x^2-4x+5\right)=x^3-4x^2+5x$.
19. Let $P(x)$ be the desired polynomial. Since 1 is a zero of P, $x-1$ is a factor of P. Also, since $1\pm 5i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-(1-5i))(x-(1+5i))=x^2-2x+26$ So, $P(x)$ is given by $(x-1)\left(x^2-2x+26\right)=x^3-3x^2+28x-26$.	**20.** Let $P(x)$ be the desired polynomial. Since 2 is a zero of P, $x-2$ is a factor of P. Also, since $4\pm i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-(4-i))(x-(4+i))=x^2-8x+17$ So, $P(x)$ is given by $(x-2)\left(x^2-8x+17\right)=x^3-10x^2+33x-34$.
21. Let $P(x)$ be the desired polynomial. Since $1\pm i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-(1-i))(x-(1+i))=x^2-2x+2$ Also, since $\pm 3i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-3i)(x+3i)=x^2+9$ So, $P(x)$ is given by $\begin{array}{c}\left(x^2+9\right)\left(x^2-2x+2\right)=\\x^4-2x^3+11x^2-18x+18\end{array}.$	**22.** Let $P(x)$ be the desired polynomial. Since $\pm i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-i)(x+i)=x^2+1$ Also, since $1\pm 2i$ is a conjugate pair, the following must divide into $P(x)$ evenly: $(x-(1-2i))(x-(1+2i))=x^2-2x+5$ So, $P(x)$ is given by $\begin{array}{c}\left(x^2+1\right)\left(x^2-2x+5\right)=\\x^4-2x^3+6x^2-2x+5\end{array}.$
23. Since -2i is a zero of $P(x)$, so is its conjugate 2i. As such, $(x-2i)(x+2i)=$ x^2+4 divides $P(x)$ evenly. Indeed, observe that	**24.** Since 3i is a zero of $P(x)$, so is its conjugate -3i. As such, $(x-3i)(x+3i)=$ x^2+9 divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-2x-15 \\ x^2+0x+4\overline{\big)\,x^4-2x^3-11x^2-8x-60} \\ \underline{-(x^4+0x^3+4x^2)} \\ -2x^3-15x^2-8x \\ \underline{-(-2x^3+0x^2-8x)} \\ -15x^2-60 \\ \underline{-(-15x^2-60)} \\ 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-2i)(x+2i)\left(x^2-2x-15\right) \\ &= (x-2i)(x+2i)(x-5)(x+3) \end{aligned}$$

So, the zeros are $\pm 2i,\,-3,\,5$.

$$\begin{array}{r} x^2-x-2 \\ x^2+0x+9\overline{\big)\,x^4-x^3+7x^2-9x-18} \\ \underline{-(x^4+0x^3+9x^2)} \\ -x^3-2x^2-9x \\ \underline{-(-x^3+0x^2-9x)} \\ -2x^2-18 \\ \underline{-(-2x^2-18)} \\ 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-3i)(x+3i)\left(x^2-x-2\right) \\ &= (x-3i)(x+3i)(x-2)(x+1) \end{aligned}$$

So, the zeros are $\pm 3i,\,-1,\,2$.

25. Since *i* is a zero of $P(x)$, so is its conjugate -*i*. As such, $(x-i)(x+i) = x^2+1$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-4x+3 \\ x^2+0x+1\overline{\big)\,x^4-4x^3+4x^2-4x+3} \\ \underline{-(x^4+0x^3+x^2)} \\ -4x^3+3x^2-4x \\ \underline{-(-4x^3+0x^2-4x)} \\ 3x^2+3 \\ \underline{-(3x^2+3)} \\ 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-i)(x+i)\left(x^2-4x+3\right) \\ &= (x-i)(x+i)(x-3)(x-1) \end{aligned}$$

So, the zeros are $\pm i,\,1,\,3$.

26. Since -2*i* is a zero of $P(x)$, so is its conjugate 2*i*. As such, $(x-2i)(x+2i) = x^2+4$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-x-2 \\ x^2+0x+4\overline{\big)\,x^4-x^3+2x^2-4x-8} \\ \underline{-(x^4+0x^3+4x^2)} \\ -x^3-2x^2-4x \\ \underline{-(-x^3+0x^2-4x)} \\ -2x^2-8 \\ \underline{-(-2x^2-8)} \\ 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-2i)(x+2i)\left(x^2-x-2\right) \\ &= (x-2i)(x+2i)(x-2)(x+1) \end{aligned}$$

So, the zeros are $\pm 2i,\,-1,\,2$.

27. Since -3*i* is a zero of $P(x)$, so is its conjugate 3*i*. As such, $(x-3i)(x+3i) = x^2+9$ divides $P(x)$ evenly. Indeed, observe that

28. Since 5*i* is a zero of $P(x)$, so is its conjugate -5*i*. As such, $(x-5i)(x+5i) = x^2+25$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-2x+1 \\ x^2+0x+9\overline{\smash{)}x^4-2x^3+10x^2-18x+9} \\ \underline{-(x^4+0x^3+9x^2)} \\ -2x^3+x^2-18x \\ \underline{-(-2x^3+0x^2-18x)} \\ x^2+9 \\ \underline{-(x^2+9)} \\ 0 \end{array}$$

So,

$$P(x)=(x-3i)(x+3i)\left(x^2-2x+1\right)$$
$$=(x-3i)(x+3i)(x-1)^2$$

So, the zeros are $\pm 3i$ and 1 (multiplicity 2).

$$\begin{array}{r} x^2-3x-4 \\ x^2+0x+25\overline{\smash{)}x^4-3x^3+21x^2-75x-100} \\ \underline{-(x^4+0x^3+25x^2)} \\ -3x^3-4x^2-75x \\ \underline{-(-3x^3+0x^2-75x)} \\ -4x^2-100 \\ \underline{-(-4x^2-100)} \\ 0 \end{array}$$

So,

$$P(x)=(x-5i)(x+5i)\left(x^2-3x-4\right)$$
$$=(x-5i)(x+5i)(x-4)(x+1)$$

So, the zeros are $\pm 5i, -1, 4$.

29. Since 1+ *i* is a zero of $P(x)$, so is its conjugate 1 – *i*. As such, $(x-(1+i))(x-(1-i))=\ x^2-2x+2$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2+2x-7 \\ x^2-2x+2\overline{\smash{)}x^4+0x^3-9x^2+18x-14} \\ \underline{-(x^4-2x^3+2x^2)} \\ 2x^3-11x^2+18x \\ \underline{-(2x^3-4x^2+4x)} \\ -7x^2+14x-14 \\ \underline{-(-7x^2+14x-14)} \\ 0 \end{array}$$

Now, we find the roots of x^2+2x-7:

$$x=\frac{-2\pm\sqrt{4-4(-7)}}{2}=-1\pm 2\sqrt{2}$$

So,

$$P(x)=(x-(1+i))(x-(1-i))\cdot$$
$$(x-(-1-2\sqrt{2}))(x-(-1+2\sqrt{2}))$$

So, the zeros are $1\pm i, -1\pm 2\sqrt{2}$.

30. Since 1 - 2*i* is a zero of $P(x)$, so is its conjugate 1 + 2*i*. As such, $(x-(1+2i))(x-(1-2i))=x^2-2x+5$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-2x-8 \\ x^2-2x+5\overline{\smash{)}x^4-4x^3+x^2+6x-40} \\ \underline{-(x^4-2x^3+5x^2)} \\ -2x^3-4x^2+6x \\ \underline{-(-2x^3+4x^2-10x)} \\ -8x^2+16x-40 \\ \underline{-(-8x^2+16x-40)} \\ 0 \end{array}$$

So,

$$P(x)=(x-(1+2i))(x-(1-2i))(x-4)(x+2)$$

So, the zeros are $1\pm 2i, -2, 4$.

31. Since 3 - *i* is a zero of $P(x)$, so is its conjugate 3 + *i*. As such, $(x-(3+i))(x-(3-i)) = x^2-6x+10$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-4 \\ x^2-6x+10\overline{\big)\, x^4-6x^3+6x^2+24x-40} \\ \underline{-(x^4-6x^3+10x^2)} \\ -4x^2+24x-40 \\ \underline{-(-4x^2+24x-40)} \\ 0 \end{array}$$

So,

$P(x)=(x-(3+i))(x-(3-i))(x-2)(x+2)$

So, the zeros are $3\pm i, \pm 2$.

32. Since 2+ *i* is a zero of $P(x)$, so is its conjugate 2- *i*. As such, $(x-(2+i))(x-(2-i)) = x^2-4x+5$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-1 \\ x^2-4x+5\overline{\big)\, x^4-4x^3+4x^2+4x-5} \\ \underline{-(x^4-4x^3+5x^2)} \\ -x^2+4x-5 \\ \underline{-(-x^2+4x-5)} \\ 0 \end{array}$$

So,

$P(x)=(x-(2+i))(x-(2-i))(x-1)(x+1)$

So, the zeros are $2\pm i, \pm 1$.

33. Since 2-*i* is a zero of $P(x)$, so is its conjugate 2+*i*. As such, $(x-(2+i))(x-(2-i)) = x^2-4x+5$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-5x+4 \\ x^2-4x+5\overline{\big)\, x^4-9x^3+29x^2-41x+20} \\ \underline{-(x^4-4x^3+5x^2)} \\ -5x^3+24x^2-41x \\ \underline{-(-5x^3+20x^2-25x)} \\ 4x^2-16x+20 \\ \underline{-(4x^2-16x+20)} \\ 0 \end{array}$$

So,

$P(x)=(x-(2+i))(x-(2-i))(x-1)(x-4)$

So, the zeros are $2\pm i, 1, 4$.

34. Since 3+*i* is a zero of $P(x)$, so is its conjugate 3-*i*. As such, $(x-(3+i))(x-(3-i)) = x^2-6x+10$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-x-2 \\ x^2-6x+10\overline{\big)\, x^4-7x^3+14x^2+2x-20} \\ \underline{-(x^4-6x^3+10x^2)} \\ -x^3+4x^2+2x \\ \underline{-(-x^3+6x^2-10x)} \\ -2x^2+12x-20 \\ \underline{-(-2x^2+12x-20)} \\ 0 \end{array}$$

So,

$P(x)=(x-(3+i))(x-(3-i))(x-2)(x+1)$

So, the zeros are $3\pm i, -1, 2$.

35.

$$\begin{aligned} x^3-x^2+9x-9 &= \left(x^3-x^2\right)+9\left(x-1\right) \\ &= x^2(x-1)+9\left(x-1\right) \\ &= \left(x^2+9\right)(x-1) \\ &= (x+3i)(x-3i)(x-1) \end{aligned}$$

36.

$$\begin{aligned} x^3-2x^2+4x-8 &= \left(x^3-2x^2\right)+4\left(x-2\right) \\ &= x^2(x-2)+4\left(x-2\right) \\ &= \left(x^2+4\right)(x-2) \\ &= (x+2i)(x-2i)(x-2) \end{aligned}$$

37.

$$\begin{aligned} x^3-5x^2+x-5 &= \left(x^3-5x^2\right)+\left(x-5\right) \\ &= x^2(x-5)+\left(x-5\right) \\ &= \left(x^2+1\right)(x-5) \\ &= (x+i)(x-i)(x-5) \end{aligned}$$

38.

$$\begin{aligned} x^3-7x^2+x-7 &= \left(x^3-7x^2\right)+\left(x-7\right) \\ &= x^2(x-7)+\left(x-7\right) \\ &= \left(x^2+1\right)(x-7) \\ &= (x+i)(x-i)(x-7) \end{aligned}$$

39.

$$\begin{aligned} x^3+x^2+4x+4 &= \left(x^3+x^2\right)+4\left(x+1\right) \\ &= x^2(x+1)+4\left(x+1\right) \\ &= \left(x^2+4\right)(x+1) \\ &= (x+2i)(x-2i)(x+1) \end{aligned}$$

40. Consider $P(x)=x^3+x^2-2$.
By the Rational Zero Theorem, the only possible rational roots are $\pm1, \pm2$. Note that

$$\begin{array}{r|rrrr} 1 & 1 & 1 & 0 & -2 \\ & & 1 & 2 & 2 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

So, $P(x)=(x-1)\left(x^2+2x+2\right)$. Now, we find the roots of x^2+2x+2:

$$x=\frac{-2\pm\sqrt{4-4(2)}}{2}=-1\pm i$$

So, $P(x)=(x-1)(x-(-1+i))(x-(-1-i))$.

41. Consider $P(x)=x^3-x^2-18$.
By the Rational Zero Theorem, the only possible rational roots are
$\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$. Note that

42. Consider $P(x)=x^4-2x^3-2x^2-2x-3$.
By the Rational Zero Theorem, the only possible rational roots are $\pm1, \pm3$. Note that

$$\begin{array}{r|rrrr} 3 & 1 & -1 & 0 & -18\\ & & 3 & 6 & 18\\ \hline & 1 & 2 & 6 & 0 \end{array}$$

So, $P(x)=(x-3)\left(x^2+2x+6\right)$. Now, we find the roots of x^2+2x+6:

$$x=\frac{-2\pm\sqrt{4-4(6)}}{2}=-1\pm i\sqrt{5}$$

So,

$P(x)=(x-3)(x-(-1+i\sqrt{5}))(x-(-1-i\sqrt{5}))$.

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & -2 & -2 & -3\\ & & -1 & 3 & -1 & 3\\ \hline & 1 & -3 & 1 & -3 & 0 \end{array}$$

So,

$$\begin{aligned} P(x)&=(x+1)\left(x^3-3x^2+x-3\right)\\ &=(x+1)\left[x^2(x-3)+(x-3)\right].\\ &=(x+1)\left(x^2+1\right)(x-3)\\ &=(x+1)(x-3)(x+i)(x-i) \end{aligned}$$

43. Consider

$P(x)=x^4-2x^3-11x^2-8x-60$.

By the Rational Zero Theorem, the only possible rational roots are

$\pm1, \pm2, \pm3, \pm4, \pm5, \pm6,$

$\pm10, \pm12, \pm15, \pm20, \pm30, \pm60.$

Note that

$$\begin{array}{r|rrrrr} -3 & 1 & -2 & -11 & -8 & -60\\ & & -3 & 15 & -12 & 60\\ \hline & 1 & -5 & 4 & -20 & 0 \end{array}$$

So,

$$\begin{aligned} P(x)&=(x+3)\left(x^3-5x^2+4x-20\right)\\ &=(x+3)\left[x^2(x-5)+4(x-5)\right].\\ &=(x+3)\left(x^2+4\right)(x-5)\\ &=(x+3)(x-5)(x+2i)(x-2i) \end{aligned}$$

44. Consider $P(x)=x^4-x^3+7x^2-9x-18$.

By the Rational Zero Theorem, the only possible rational roots are

$\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$.

Note that

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 7 & -9 & -18\\ & & -1 & 2 & -9 & 18\\ \hline & 1 & -2 & 9 & -18 & 0 \end{array}$$

So,

$$\begin{aligned} P(x)&=(x+1)\left(x^3-2x^2+9x-18\right)\\ &=(x+1)\left[x^2(x-2)+9(x-2)\right].\\ &=(x+1)\left(x^2+9\right)(x-2)\\ &=(x+1)(x-2)(x+3i)(x-3i) \end{aligned}$$

45. Consider

$P(x)=x^4-4x^3-x^2-16x-20$.

By the Rational Zero Theorem, the only possible rational roots are

$\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

Note that

46. Consider

$P(x)=x^4-3x^3+11x^2-27x+18$.

By the Rational Zero Theorem, the only possible rational roots are

$\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

Note that

$$\begin{array}{r|rrrrr} \underline{-1} & 1 & -4 & -1 & -16 & -20 \\ & & -1 & 5 & -4 & 20 \\ \hline & 1 & -5 & 4 & -20 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x+1)\left(x^3-5x^2+4x-20\right) \\ &= (x+1)\left[x^2(x-5)+4(x-5)\right]. \\ &= (x+1)\left(x^2+4\right)(x-5) \\ &= (x+1)(x-5)(x+2i)(x-2i) \end{aligned}$$

$$\begin{array}{r|rrrrr} \underline{1} & 1 & -3 & 11 & -27 & 18 \\ & & 1 & -2 & 9 & -18 \\ \hline & 1 & -2 & 9 & -18 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-1)\left(x^3-2x^2+9x-18\right) \\ &= (x-1)\left[x^2(x-2)+9(x-2)\right]. \\ &= (x-1)\left(x^2+9\right)(x-2) \\ &= (x-1)(x-2)(x+3i)(x-3i) \end{aligned}$$

47. Consider

$P(x) = x^4-7x^3+27x^2-47x+26$.

By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1, \pm 2, \pm 13, \pm 26$$

Note that

$$\begin{array}{r|rrrrr} \underline{1} & 1 & -7 & 27 & -47 & 26 \\ & & 1 & -6 & 21 & -26 \\ \hline \underline{2} & 1 & -6 & 21 & -26 & 0 \\ & & 2 & -8 & 26 & \\ \hline & 1 & -4 & 13 & 0 & \end{array}$$

So, $P(x) = (x-1)(x-2)\left(x^2-4x+13\right)$.

Next, we find the roots of $x^2-4x+13$:

$$x = \frac{4 \pm \sqrt{16-4(13)}}{2} = 2 \pm 3i$$

So,

$P(x) = (x-1)(x-2)(x-(2-3i))(x-(2+3i))$

48. Consider

$P(x) = x^4-5x^3+5x^2+25x-26$.

By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1, \pm 2, \pm 13, \pm 26$$

Note that

$$\begin{array}{r|rrrrr} \underline{1} & 1 & -5 & 5 & 25 & -26 \\ & & 1 & -4 & 1 & 26 \\ \hline \underline{-2} & 1 & -4 & 1 & 26 & 0 \\ & & -2 & 12 & -26 & \\ \hline & 1 & -6 & 13 & 0 & \end{array}$$

So, $P(x) = (x-1)(x+2)\left(x^2-6x+13\right)$.

Next, we find the roots of $x^2-6x+13$:

$$x = \frac{6 \pm \sqrt{36-4(13)}}{2} = 3 \pm 2i$$

So,

$P(x) = (x-1)(x+2)(x-(3-2i))(x-(3+2i))$

49. Consider

$P(x) = -x^4-3x^3+x^2+13x+10$.

By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1, \pm 2, \pm 5, \pm 10$$

Note that

50. Consider

$P(x) = -x^4-x^3+12x^2+26x+24$.

By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$

Note that

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & -1 & -13 & -10 \\ & & -1 & -2 & 3 & 10 \\ \hline 2 & 1 & 2 & -3 & -10 & 0 \\ & & 2 & 8 & 10 & \\ \hline & 1 & 4 & 5 & 0 & \end{array}$$

So, $P(x) = -(x+1)(x-2)\left(x^2+4x+5\right)$.

Next, we find the roots of x^2+4x+5:

$$x = \frac{-4 \pm \sqrt{16-4(5)}}{2} = -2 \pm i$$

So,
$P(x) = -(x+1)(x-2)(x-(-2-i))(x-(-2+i))$

$$\begin{array}{r|rrrrr} -3 & 1 & 1 & -12 & -26 & -24 \\ & & -3 & 6 & 18 & 24 \\ \hline 4 & 1 & -2 & -6 & -8 & 0 \\ & & 4 & 8 & 8 & \\ \hline & 1 & 2 & 2 & 0 & \end{array}$$

So, $P(x) = -(x+3)(x-4)\left(x^2+2x+2\right)$.

Next, we find the roots of x^2+2x+2:

$$x = \frac{-2 \pm \sqrt{4-4(2)}}{2} = -1 \pm i$$

So,
$P(x) = -(x+3)(x-4)(x-(-1-i))(x-(-1+i))$

51. Consider
$P(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$.
By the Rational Zero Theorem, the only possible rational roots are $\pm 1, \pm 2, \pm 4$.
Note that

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & 5 & -8 & 4 \\ & & 1 & -1 & 4 & -4 \\ \hline & 1 & -1 & 4 & -4 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-1)\left(x^3 - x^2 + 4x - 4\right) \\ &= (x-1)\left[x^2(x-1) + 4(x-1)\right] \\ &= (x-1)\left(x^2+4\right)(x-1) \\ &= (x-1)^2(x+2i)(x-2i) \end{aligned}.$$

52. Consider
$P(x) = x^4 + 2x^3 + 10x^2 + 18x + 9$.
By the Rational Zero Theorem, the only possible rational roots are $\pm 1, \pm 3, \pm 9$. Note that

$$\begin{array}{r|rrrrr} -1 & 1 & 2 & 10 & 18 & 9 \\ & & -1 & -1 & -9 & -9 \\ \hline & 1 & 1 & 9 & 9 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x+1)\left(x^3 + x^2 + 9x + 9\right) \\ &= (x+1)\left[x^2(x+1) + 9(x+1)\right] \\ &= (x+1)\left(x^2+9\right)(x+1) \\ &= (x+1)^2(x+3i)(x-3i) \end{aligned}.$$

53. Consider
$P(x) = x^6 + 12x^4 + 23x^2 - 36$.
By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1, \pm 2, \pm 4, \pm 6, \pm 9, \pm 18, \pm 36$$

Note that

54. Consider
$P(x) = x^6 - 2x^5 + 9x^4 - 16x^3 + 24x^2 - 32x + 16$.
By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$$

Note that

$$\begin{array}{r|rrrrrrr} \underline{1} & 1 & 0 & 12 & 0 & 23 & 0 & -36 \\ & & 1 & 1 & 13 & 13 & 36 & 36 \\ \hline \underline{-1} & 1 & 1 & 13 & 13 & 36 & 36 & 0 \\ & & -1 & 0 & -13 & 0 & -36 & \\ \hline & 1 & 0 & 13 & 0 & 36 & 0 & \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-1)(x+1)\left(x^4+13x^2+36\right) \\ &= (x-1)(x+1)\left(x^2+4\right)\left(x^2+9\right) \\ &= (x-1)(x+1)(x-2i)(x+2i)(x-3i)(x+3i) \end{aligned}$$

$$\begin{array}{r|rrrrrrr} \underline{1} & 1 & -2 & 9 & -16 & 24 & -32 & 16 \\ & & 1 & -1 & 8 & -8 & 16 & -16 \\ \hline \underline{1} & 1 & -1 & 8 & -8 & 16 & -16 & 0 \\ & & 1 & 0 & 8 & 0 & 16 & \\ \hline & 1 & 0 & 8 & 0 & 16 & 0 & \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-1)^2\left(x^4+8x^2+16\right) \\ &= (x-1)^2\left(x^2+4\right)^2 \\ &= (x-1)^2(x-2i)^2(x+2i)^2 \end{aligned}$$

55. Consider $P(x)=4x^4-20x^3+37x^2-24x+5$.
By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1,\ \pm 5,\ \pm\tfrac{1}{2},\ \pm\tfrac{5}{2},\ \pm\tfrac{1}{4},\ \pm\tfrac{5}{4}$$

Note that

$$\begin{array}{r|rrrrr} \underline{\tfrac{1}{2}} & 4 & -20 & 37 & -24 & 5 \\ & & 2 & -9 & 14 & -5 \\ \hline \underline{\tfrac{1}{2}} & 4 & -18 & 28 & -10 & 0 \\ & & 2 & -8 & 10 & \\ \hline & 4 & -16 & 20 & 0 & \end{array}$$

So, $P(x)=4\left(x-\tfrac{1}{2}\right)^2\left(x^2-4x+5\right)$.

Next, we find the roots of x^2-4x+5:

$$x=\frac{4\pm\sqrt{16-4(5)}}{2}=2\pm i$$

So,

$$\begin{aligned} P(x) &= 4\left(x-\tfrac{1}{2}\right)^2(x-(2-i))(x-(2+i)) \\ &= (2x-1)^2(x-(2-i))(x-(2+i)) \end{aligned}$$

56. Consider $P(x)=4x^4-44x^3+145x^2-114x+26$.
By the Rational Zero Theorem, the only possible rational roots are

$$\pm 1,\ \pm 2,\ \pm\tfrac{1}{2},\ \pm\tfrac{13}{2},\ \pm\tfrac{13}{4},\ \pm 13,\ \pm 26$$

Note that

$$\begin{array}{r|rrrrr} \underline{\tfrac{1}{2}} & 4 & -44 & 145 & -114 & 26 \\ & & 2 & -21 & 62 & -26 \\ \hline \underline{\tfrac{1}{2}} & 4 & -42 & 124 & -52 & 0 \\ & & 2 & -20 & 52 & \\ \hline & 4 & -40 & 104 & 0 & \end{array}$$

So,

$$\begin{aligned} P(x) &= \left(x-\tfrac{1}{2}\right)^2\left(4x^2-40x+104\right) \\ &= 4\left(x-\tfrac{1}{2}\right)^2\left(x^2-10x+26\right). \end{aligned}$$

Next, we find the roots of $x^2-10x+26$:

$$x=\frac{10\pm\sqrt{100-4(26)}}{2}=5\pm i$$

So, $P(x)=4\left(x-\tfrac{1}{2}\right)^2(x-(5-i))(x-(5+i))$

57. Consider $P(x)=3x^5-2x^4+9x^3-6x^2-12x+8$.
By the Rational Zero Theorem, the only possible rational roots are

58. Consider $P(x)=2x^5-5x^4+4x^3-26x^2+50x-25$.
By the Rational Zero Theorem, the only possible rational roots are

$\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

Note that

$$\begin{array}{r|rrrrrr}
-1 & 3 & -2 & 9 & -6 & -12 & 8 \\
 & & -3 & 5 & -14 & 20 & -8 \\ \hline
1 & 3 & -5 & 14 & -20 & 8 & 0 \\
 & & 3 & -2 & 12 & -8 & \\ \hline
\frac{2}{3} & 3 & -2 & 12 & -8 & 0 & \\
 & & 2 & 0 & 8 & & \\ \hline
 & 3 & 0 & 12 & 0 & &
\end{array}$$

So,

$$\begin{aligned} P(x) &= (x-1)(x+1)(x-\tfrac{2}{3})(3x^2+12) \\ &= (x-1)(x+1)(3x-2)(x-2i)(x+2i) \end{aligned}$$

$\pm1, \pm5, \pm25, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{25}{2}$

Note that

$$\begin{array}{r|rrrrrr}
1 & 2 & -5 & 4 & -26 & 50 & -25 \\
 & & 2 & -3 & 1 & -25 & 25 \\ \hline
1 & 2 & -3 & 1 & -25 & 25 & 0 \\
 & & 2 & -1 & 0 & -25 & \\ \hline
\frac{5}{2} & 2 & -1 & 0 & -25 & 0 & \\
 & & 5 & 10 & 25 & & \\ \hline
 & 2 & 4 & 10 & 0 & &
\end{array}$$

So, $P(x) = (x-1)^2(x-\frac{5}{2})(2x^2+4x+10)$

$= (x-1)^2(2x-5)(x^2+2x+5)$

Next, we find the roots of x^2+2x+5:

$$x = \frac{-2 \pm \sqrt{4-4(5)}}{2} = -1 \pm 2i$$

So,

$P(x) = (x-1)^2(2x-5)(x+1-2i)(x+1+2i)$.

59. Yes. In such case, $P(x)$ never touches the *x*-axis (since crossing it would require $P(x)$ to have a real root) and is always above it since the leading coefficient is positive, indicating that the end behavior should resemble that of $y = x^{2n}$, for some positive integer *n*. So, profit is always positive and increasing.

60. No. In such case, $P(x)$ never touches the *x*-axis (since crossing it would require $P(x)$ to have a real root) and is always below it since the leading coefficient is negative. So, never have a positive profit.

61. No. In such case, it crosses the *x*-axis and looks like $y = -x^3$. So, profit is decreasing.

62. Yes. In such case, it crosses the *x*-axis and looks like $y = x^3$. So, profit is increasing.

63. Since the profit function is a third-degree polynomial we know that the function has three zeros and at most two turning points. Looking at the graph we can see there is one real zero where $t \le 0$. There are no real zeros when $t > 0$, therefore the other two zeros must be complex conjugates. Therefore, the company always has a profit greater than approximately 5.1 million dollars and, in fact, the profit will increase towards infinity as *t* increases.

64. Since the profit function is a fourth-degree polynomial with a negative leading we know the function has four zeros and at most three turning points. The end behavior is towards negative infinity because of the negative leading coefficient of an even degree polynomial and there will be two real zeros; one where $t \le 0$ and one where $t \ge 6$. The remaining two zeros are a complex conjugate pair. Therefore, the company will have profits of greater than approximately 5.1 million dollars during the first six months. Sometime later ($t \geq 6$) the company's profit will be zero. Then the company will start losing money and, in fact, the profit will decrease towards negative infinity as time increases.

65. Since the concentration function is a third degree polynomial, we know the function has three zeros and at most two turning points. Looking at the graph we can see there will be one real zero at some time $t \geq 8$. The remaining zeros are a pair of complex conjugates. Therefore, the concentration of the drug in the blood stream will decrease to zero as the hours go by. Note that the concentration will not approach negative infinity since concentration is a non-negative quantity.

66. Since the concentration function is a fourth degree polynomial, we know the function has four zeros. The negative leading coefficient indicates negative end behavior (opening down). Since the function opens down there is a real zero for $t \leq 0$ and there will be a real zero for $t \geq 8$. Note that the concentration will not approach negative infinity since concentration is a non-negative quantity.

67. Step 2 is an error. In general, the additive inverse of a real root need not be a root. This is being confused with the fact that complex roots occur in conjugate pairs.

68. Possible rational roots include $\pm\frac{1}{2}$ from the Rational Zero theorem.

69. False. For example, consider $P(x) = (x-1)(x+3)$. Note that 1 is a zero of *P*, but -1 is not.

70. False. Complex zeros do not correspond to *x*-intercepts.

71. True. It has *n* complex zeros.

72. True.

73. No. Complex zeros occur in conjugate pairs. So, the collection of complex solutions contribute an even number of zeros, thereby requiring there to be at least one real zero.

74. Yes. For example, $P(x) = x^2 + 4$ has zeros $\pm 2i$, both of which are imaginary.

75. Since *bi* is a zero of multiplicity 3, its conjugate -*bi* is also a zero of multiplicity 3. Hence,

$$\begin{aligned} P(x) &= (x-bi)^3(x+bi)^3 \\ &= \left[(x-bi)(x+bi)\right]^3 \\ &= \left(x^2+b^2\right)^3 \\ &= x^6 + 3b^2x^4 + 3b^4x^2 + b^6 \end{aligned}$$

76. Since $a+bi$ is a zero of multiplicity 2, its conjugate $a-bi$ is also a zero of multiplicity 2. Hence,

$$\begin{aligned} P(x) &= \left(x-(a+bi)\right)^2\left(x-(a-bi)\right)^2 \\ &= \left[\left(x-(a+bi)\right)\left(x-(a-bi)\right)\right]^2 \\ &= x^2 - 2ax + (a^2+b^2) \end{aligned}$$

77. Since *ai* is a zero with multiplicity 2 and *bi* is a zero, it follows that their conjugates –*ai* and –*bi* satisfy the same conditions, so that $\left(x^2+a^2\right)^2$ and $\left(x^2+b^2\right)$ are factors. These must divide *P*(*x*) evenly. Hence, a 6th degree polynomial satisfying these conditions is:

$$P(x)=\left(x^2+a^2\right)^2\left(x^2+b^2\right)=\left(x^4+2a^2x^2+a^4\right)\left(x^2+b^2\right)=x^6+\left(2a^2+b^2\right)x^4+\underbrace{\left(a^4+2a^2b^2\right)}_{=a^2\left(a^2+2b^2\right)}x^2+b^2a^4$$

78. Since *ai* and *bi* are both zeros, it follows that their conjugates –*ai* and –*bi* are also. So, $\left(x^2+a^2\right)$ and $\left(x^2+b^2\right)$ are factors. These must divide *P*(*x*) evenly. Hence, a polynomial with minimal degree satisfying these conditions is

$$P(x)=\left(x^2+a^2\right)\left(x^2+b^2\right)=x^4+\left(a^2+b^2\right)x^2+a^2b^2.$$

79. The possible rational zeros are ± 1. Using synthetic division yields

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 1 & 1 \\ & & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

a.) Factoring over the complex numbers yields $f(x)=(x+1)\left(x^2+1\right)=(x+1)(x-i)(x+i)$.

b.) Factoring over the real numbers yields $f(x)=(x+1)\left(x^2+1\right)$.

80. The possible rational zeros are $\pm 1,\ \pm 2,\ \pm 13, \pm 26$. Using synthetic division yields

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 21 & -26 \\ & & 2 & -8 & 26 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

a.) Observe that $x^2-4x+13=0 \Rightarrow x=\frac{4\pm\sqrt{16-4(13)}}{2}=2\pm 3i$. So, factoring over the complex numbers yields $f(x)=(x-2)(x-2-3i)(x-2+3i)$.

b.) Factoring over the real numbers yields $f(x)=(x-2)\left(x^2-4x+13\right)$.

81. a.) Factoring over the complex numbers yields

$$f(x)=\left(x^2+4\right)\left(x^2+1\right)=(x+2i)(x-2i)(x+i)(x-i).$$

b.) Factoring over the real numbers yields $f(x)=\left(x^2+4\right)\left(x^2+1\right)$.

82. The possible rational zeros are $\pm 1,\ \pm 2,\ \pm 3, \pm 6, \pm 9, \pm 18$. Using synthetic division yields

$$\begin{array}{r|rrrrr} 3 & 1 & -2 & -7 & 18 & -18 \\ & & 3 & 3 & -12 & 18 \\ \hline -3 & 1 & 1 & -4 & 6 & \\ & & -3 & 6 & -6 & \\ \hline & 1 & -2 & 2 & & \end{array}$$

a.) Observe that $x^2-2x+2=0 \Rightarrow x=\frac{2\pm\sqrt{4-4(2)}}{2}=1\pm i$. So, factoring over the complex numbers yields $f(x)=(x-3)(x+3)(x-1-i)(x-1+i)$.

b.) Factoring over the real numbers yields $f(x)=(x-3)(x+3)(x^2-2x+2)$.

Section 2.6 Solutions --

1. Note that $x^2+x-12=(x+4)(x-3)$. Domain: $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$	**2.** Note that $x^2+2x-3=(x+3)(x-1)$. Domain: $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$
3 Note that $x\neq\pm2$. So, Domain: **.** $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$	**4.** Note that $x\neq\pm7$. So, Domain: $(-\infty,-7)\cup(-7,7)\cup(7,\infty)$
5. Note that x^2+16 is never zero. Domain: $(-\infty,\infty)$	**6.** Note that x^2+9 is never zero. Domain: $(-\infty,\infty)$
7. Note that $2\left(x^2-x-6\right)=2(x-3)(x+2)$. Domain: $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$	**8.** Note that $x^2-x-6=(x-3)(x+2)$**.** Domain: $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$
9. Vertical Asymptote: $x+2=0$, so $x=-2$ is the VA. Horizontal Asymptote: Since the degree of the numerator is less than degree of the denominator, $y=0$ is the HA.	**10.** Vertical Asymptote: $5-x=0$, so $x=5$ is the VA. Horizontal Asymptote: Since the degree of the numerator is less than degree of the denominator, $y=0$ is the HA.
11. Vertical Asymptote: $x+5=0$, so $x=-5$ is the VA. Horizontal Asymptote: Since the degree of the numerator is greater than degree of the denominator, there is no HA.	**12.** Vertical Asymptote: $2x-7=0$, so $x=\frac{7}{2}$ is the VA. Horizontal Asymptote: Since the degree of the numerator is greater than degree of the denominator, there is no HA.
13. Vertical Asymptote: $6x^2+5x-4=(2x-1)(3x+4)=0$, so $x=\frac{1}{2}$, $x=-\frac{4}{3}$ are the VAs. Horizontal Asymptote: Since the degree of the numerator is greater than degree of the denominator, there is no HA.	**14.** Vertical Asymptote: $3x^2-5x-2=(3x+1)(x-2)=0$, so $x=2$, $x=-\frac{1}{3}$ are the VAs. Horizontal Asymptote: Since the degree of the numerator equals the degree of the denominator, $y=\frac{6}{3}=2$ is the HA.

15. Vertical Asymptote:

$x^2+\frac{1}{9}$ is never 0, so there is no VA.

Horizontal Asymptote:

Since the degree of the numerator equals the degree of the denominator, $y=\frac{1}{3}$ is the HA.

16. Vertical Asymptote:

$(2x-1)=0$, so

$x=\frac{1}{2}$ is the VA.

Horizontal Asymptote:

Since the degree of the numerator is greater than degree of the denominator, there is no HA.

17. To find the slant asymptote, we use synthetic division:

$$\begin{array}{r|rrr} -4 & 1 & 10 & 25 \\ & & -4 & -24 \\ \hline & 1 & 6 & 1 \end{array}$$

So, the slant asymptote is $y=x+6$.

18. To find the slant asymptote, we use synthetic division:

$$\begin{array}{r|rrr} 3 & 1 & 9 & 20 \\ & & 3 & 36 \\ \hline & 1 & 12 & 56 \end{array}$$

So, the slant asymptote is $y=x+12$.

19. To find the slant asymptote, we use synthetic division:

$$\begin{array}{r|rrr} 5 & 2 & 14 & 7 \\ & & 10 & 120 \\ \hline & 2 & 24 & 127 \end{array}$$

So, the slant asymptote is $y=2x+24$.

20. To find the slant asymptote, we use long division:

$$\begin{array}{r l} & \quad 3x+7 \\ x^2-x-30 \big) & \overline{3x^3+4x^2-6x+1} \\ & \underline{-(3x^3-3x^2-90x)} \\ & \qquad 7x^2+84x+1 \\ & \qquad \underline{-(7x^2-7x-210)} \\ & \qquad\qquad 91x+211 \end{array}$$

So, the slant asymptote is $y=3x+7$.

21. To find the slant asymptote, we use long division:

$$\begin{array}{r l} & \qquad\qquad 4x+\frac{11}{2} \\ 2x^3-x^2+3x-1 \big) & \overline{8x^4+7x^3+0x^2+2x-5} \\ & \underline{-(8x^4-4x^3+12x^2-4x)} \\ & \qquad 11x^3-12x^2+6x-5 \\ & \qquad \underline{-(11x^3-\frac{11}{2}x^2+\frac{33}{2}x-\frac{11}{2})} \\ & \qquad\qquad -\frac{13}{2}x^2-\frac{21}{2}x+\frac{1}{2} \end{array}$$

So, the slant asymptote is $y=4x+\frac{11}{2}$.

22. To find the slant asymptote, we use long division:

$$\begin{array}{r} 2x \\ x^5+0x^4+0x^3+0x^2+0x-1\overline{\big)\,2x^6+0x^5+0x^4+0x^3+0x^2+0x+1} \\ \underline{-(2x^6+0x^5+0x^4+0x^3+0x^2-2x)} \\ 2x+1 \end{array}$$

So, the slant asymptote is $y = 2x$.

23. b Vertical Asymptote: $x = 4$
Horizontal Asymptote: $y = 0$

24. d Vertical Asymptote: $x = 4$
Horizontal Asymptote: $y = 3$

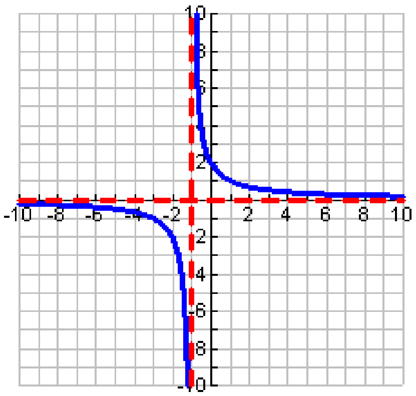
25. a Vertical Asymptotes: $x = 2,\ x = -2$
Horizontal Asymptote: $y = 3$

26. f Vertical Asymptote: None
Horizontal Asymptote: $y = -3$
Graph never goes above the *x*-axis

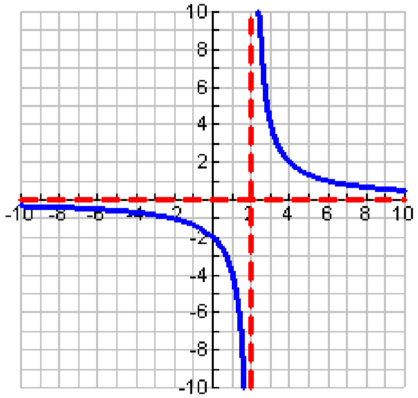
27. e Vertical Asymptotes: $x = 2,\ x = -2$
Horizontal Asymptote: $y = -3$

28. c Vertical Asymptote: $x = -4$
Horizontal Asymptote: None

29. Vertical Asymptote: $x = -1$
Horizontal Asymptote: $y = 0$



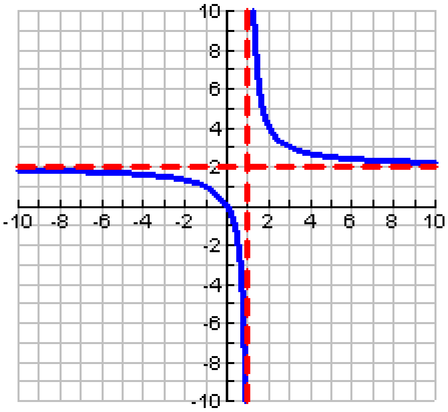
30. Vertical Asymptote: $x = 2$
Horizontal Asymptote: $y = 0$



31. Vertical Asymptote: $x = 1$

Horizontal Asymptote: $y = 2$

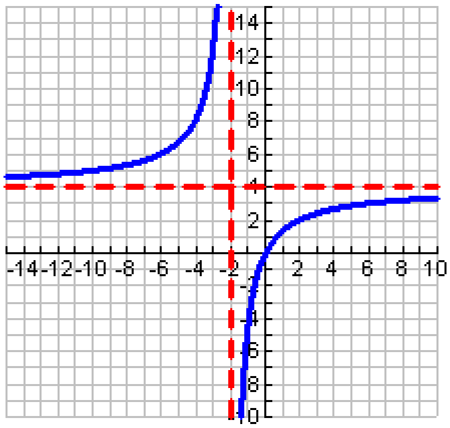
Intercept: (0,0)



32. Vertical Asymptote: $x = -2$

Horizontal Asymptote: $y = 4$

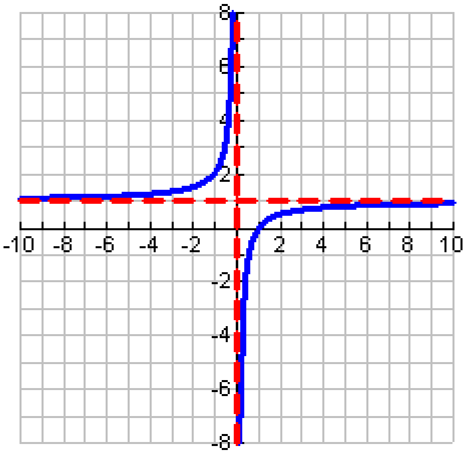
Intercept: (0,0)



33. Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 1$

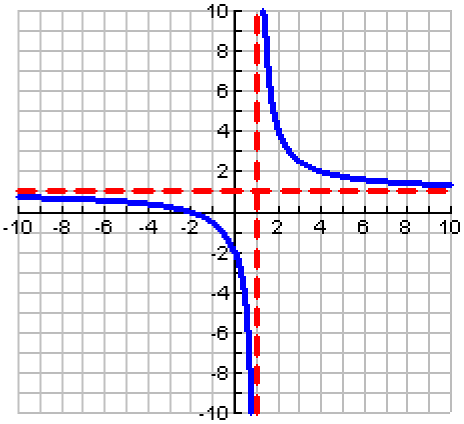
Intercept: (1,0)



34. Vertical Asymptote: $x = 1$

Horizontal Asymptote: $y = 1$

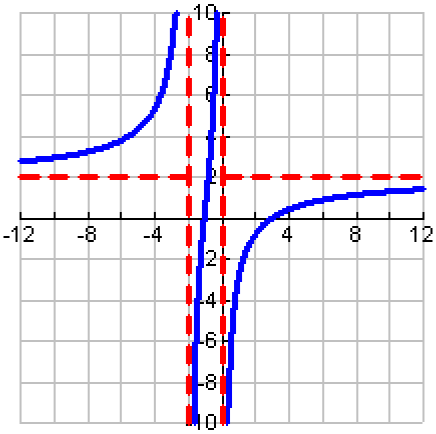
Intercepts: $(-2, 0)$, $(0, -2)$



35. Vertical Asymptotes: $x=0,\ x=-2$

Horizontal Asymptote: $y=2$

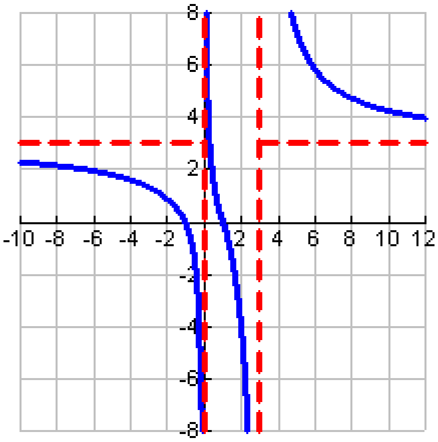
Intercepts: $(3,0),\ (-1,0)$



36. Vertical Asymptotes: $x=0,\ x=3$

Horizontal Asymptote: $y=3$

Intercepts: $(1,0),\ (-1,0)$

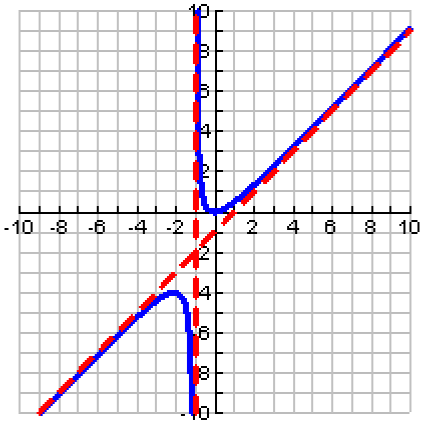


37. Vertical Asymptote: $x=-1$

Intercept: $(0,0)$

Slant Asymptote: $y=x-1$

$$\begin{array}{r} x-1 \\ x+1\overline{)\,x^2+0x+0} \\ \underline{-(x^2+x)} \\ -x+0 \\ \underline{-(-x-1)} \\ 1 \end{array}$$

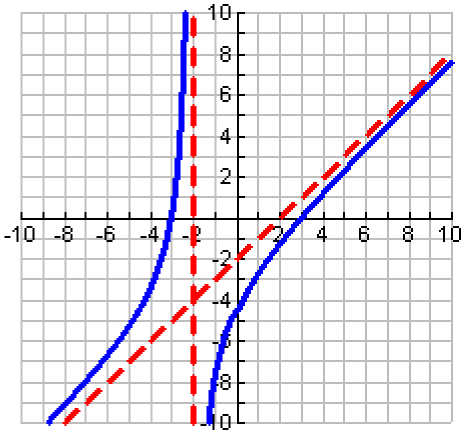


38. Vertical Asymptote: $x=-2$

Intercepts: $(3,0)$, $(-3,0)$

Slant Asymptote: $y=x-2$

$$\begin{array}{r} x-2 \\ x+2\overline{)\,x^2+0x-9} \\ \underline{-(x^2+2x)} \\ -2x-9 \\ \underline{-(-2x-4)} \\ -5 \end{array}$$

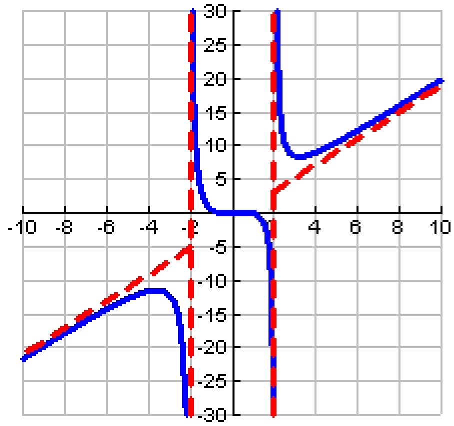


39. Vertical Asymptotes: $x=-2,\ x=2$

Intercepts: $(0,0)$, $(-\frac{1}{2},0)$, $(1,0)$

Slant Asymptote: $y=2x-1$

$$\begin{array}{r} 2x-1 \\ x^2+0x-4\overline{)\,2x^3-x^2-x+0} \\ \underline{-(2x^3+0x^2-8x)} \\ -x^2+7x+0 \\ \underline{-(-x^2+0x+4)} \\ 7x-4 \end{array}$$

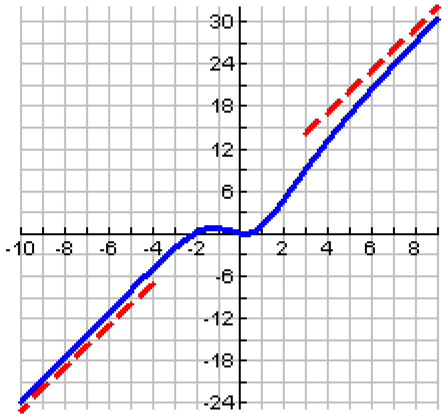


40. Vertical Asymptote: None

Intercepts: $(0,0)$, $(\frac{1}{3},0)$, $(-2,0)$

Slant Asymptote: $y=3x+5$

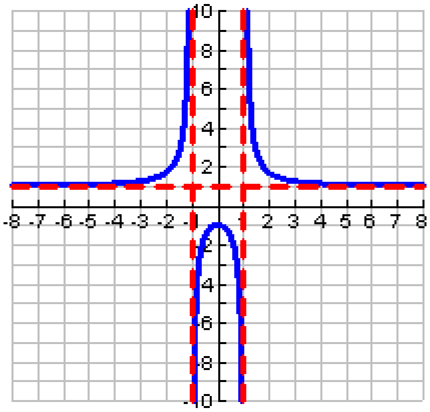
$$\begin{array}{r} 3x+5 \\ x^2+0x+4\overline{)\,3x^3+5x^2-2x+0} \\ \underline{-(3x^3+0x^2+12x)} \\ 5x^2-14x+0 \\ \underline{-(5x^2+0x+20)} \\ -14x-20 \end{array}$$



41. Vertical Asymptote: $x = 1,\ x = -1$

Horizontal Asymptote: $y = 1$

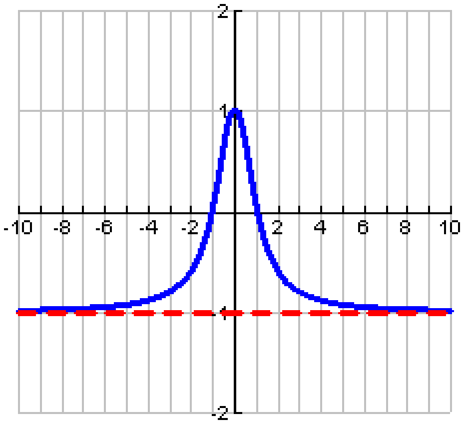
Intercept: $(0, -1)$



42. Vertical Asymptote: None

Horizontal Asymptote: $y = -1$

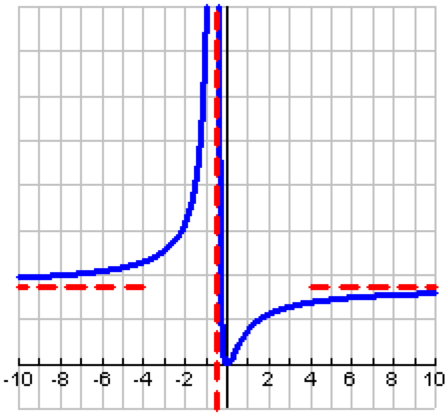
Intercepts: $(1,0)$, $(-1,0)$, $(0,1)$



43. Vertical Asymptote: $x = -\frac{1}{2}$

Horizontal Asymptote: $y = \frac{7}{4}$

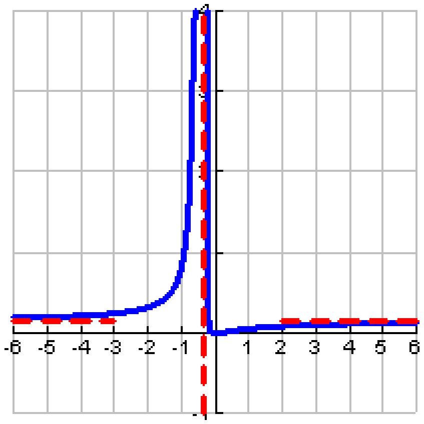
Intercept: $(0,0)$



44. Vertical Asymptote: $x = -\frac{1}{3}$

Horizontal Asymptote: $y = \frac{4}{27}$

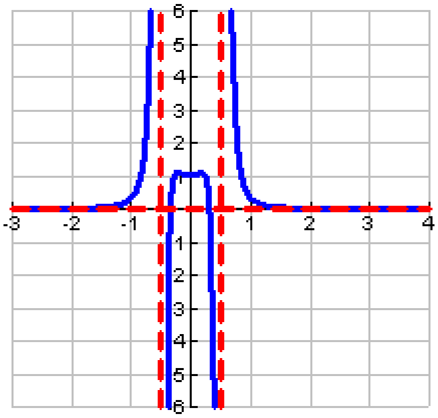
Intercept: (0,0)



45. Vertical Asymptotes: $x = -\frac{1}{2}$, $x = \frac{1}{2}$

Horizontal Asymptote: $y = 0$

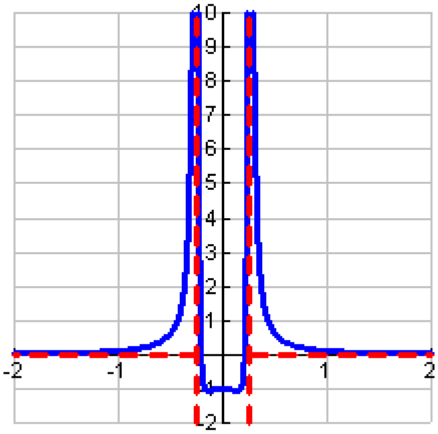
Intercepts: $(0,1)$, $(\frac{1}{3},0)$, $(-\frac{1}{3},0)$



46. Vertical Asymptotes: $x = -\frac{1}{4}$, $x = \frac{1}{4}$

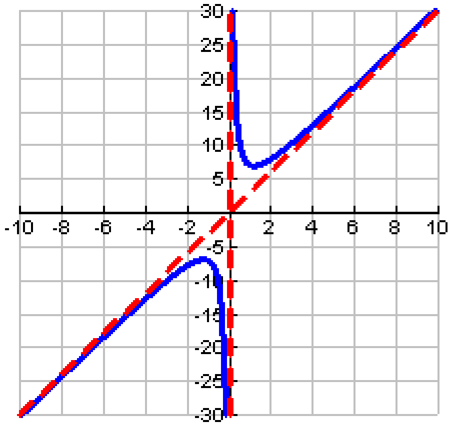
Horizontal Asymptote: $y = 0$

Intercepts: $(0,-1)$, $(\frac{1}{5},0)$, $(-\frac{1}{5},0)$



47. Vertical Asymptotes: $x = 0$

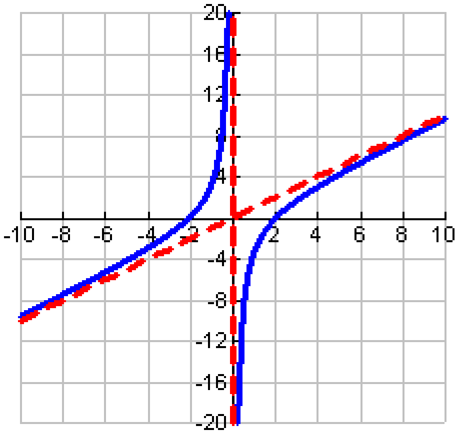
Slant Asymptote: $y = 3x$



48. Vertical Asymptotes: $x = 0$

Slant Asymptote: $y = x$

Intercepts: $(2,0)$, $(-2,0)$



49. Observe that

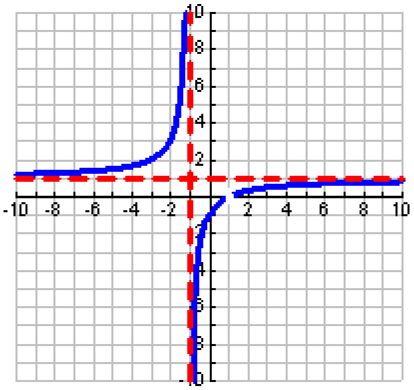
$$f(x) = \frac{(x-1)^2}{(x-1)(x+1)} = \frac{x-1}{x+1}.$$

Open hole: $(1,0)$

Vertical Asymptote: $x = -1$

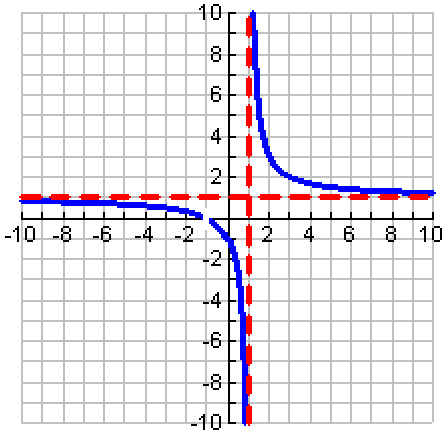
Horizontal Asymptote: $y = 1$

Intercepts: $(0,-1)$ and $(1,0)$



50. Observe that

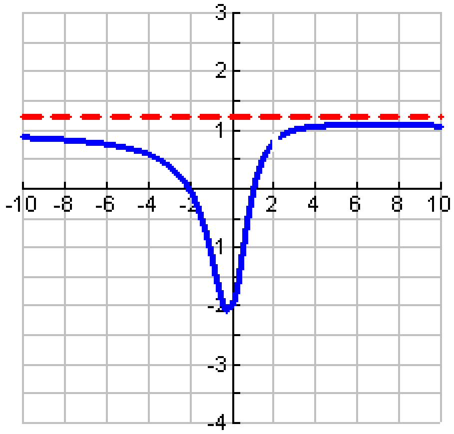
$$f(x)=\frac{(x+1)^2}{(x-1)(x+1)}=\frac{x+1}{x-1}.$$



Open hole: (-1,0)
Vertical Asymptote: $x=1$
Horizontal Asymptote: $y=1$
Intercepts: $(0,-1)$ and $(-1,0)$

51. Observe that

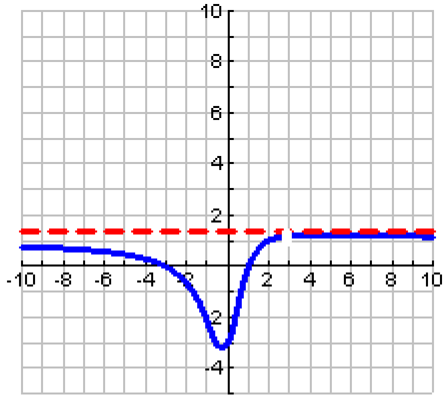
$$f(x)=\frac{(x-1)(x-2)(x+2)}{(x-2)(x^2+1)}=\frac{(x-1)(x+2)}{x^2+1}.$$



Open hole: (2,4/5)
Vertical Asymptote: None
Horizontal Asymptote: $y=1$
Intercepts: $(0,-2)$, $(1,0)$, and $(-2,0)$

52. Observe that

$$f(x)=\frac{(x-1)(x-3)(x+3)}{(x-3)(x^2+1)}=\frac{(x-1)(x+3)}{x^2+1}.$$

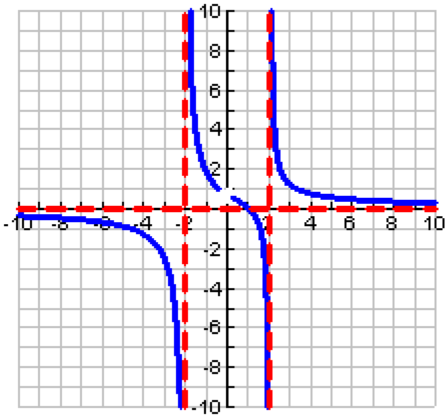


Open hole: (3,6/5)
Vertical Asymptote: None
Horizontal Asymptote: $y=1$
Intercepts: $(0,-3)$, $(1,0)$, and $(-3,0)$

53. Observe that

$$f(x)=\frac{3x(x-1)}{x(x-2)(x+2)}=\frac{3x-3}{(x-2)(x+2)}.$$

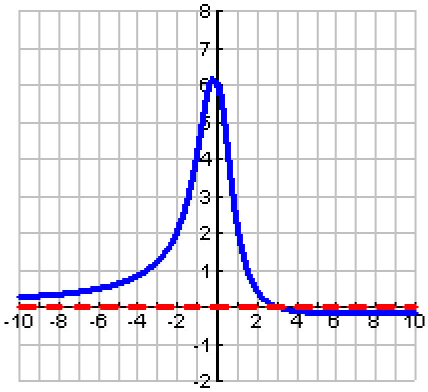
Open hole: (0,3/4)
Vertical Asymptote: $x=2,\ x=-2$
Horizontal Asymptote: $y=0$
Intercepts: $(1,0)$



54. Observe that

$$f(x)=\frac{-2x(x-3)}{x\left(x^2+1\right)}=\frac{-2(x-3)}{x^2+1}.$$

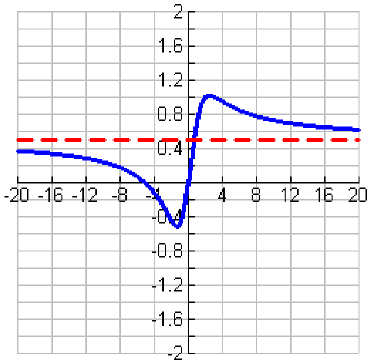
Open hole: (0,6)
Vertical Asymptote: None
Horizontal Asymptote: $y=0$
Intercepts: $(3,0)$



55. Observe that

$$f(x)=\frac{x^2(x+5)}{2x\left(x^2+3\right)}=\frac{x^2+5x}{2\left(x^2+3\right)},\ x\neq 0.$$

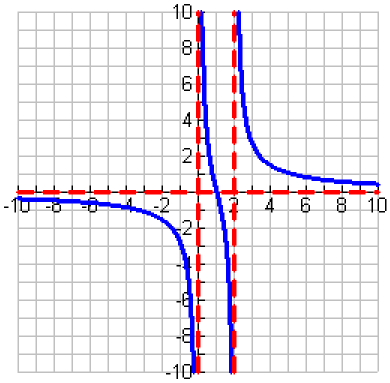
Open hole: (0,0)
Vertical Asymptote: None
Horizontal Asymptote: $y=\frac{1}{2}$
Intercepts: $(-5,0)$



56. Observe that

$$f(x)=\frac{4x(x-1)(x+2)}{x^2(x-2)(x+2)}=\frac{4(x-1)}{x(x-2)},\quad x\neq 0,-2$$

.



Open hole: (-2, $-\frac{3}{2}$)

Vertical Asymptote: x=0, x=2

Horizontal Asymptote: $y=0$

Intercepts: (1,0)

57. a. x-intercept: (2,0)

y-intercept: (0,0.5)

b. horizontal asymptote: $y=0$

vertical asymptotes: $x=-1,\ x=4$

c. $f(x)=\dfrac{x-2}{(x+1)(x-4)}$

58. a. x-intercepts: (-0.5,0), (3,0)

y-intercept: (0,0.5)

b. horizontal asymptote: $y=2$

vertical asymptotes: $x=-3,\ x=2$

c. $f(x)=\dfrac{(2x+1)(x-3)}{(x+3)(x-2)}$

59. a. x-intercept: (0,0)

y-intercept: (0,0)

b. horizontal asymptote: $y=-3$

vertical asymptotes: $x=-4,\ x=4$

c. $f(x)=\dfrac{-3x^2}{(x+4)(x-4)}$

60. a. x-intercept: (0,0), (2,0)

y-intercept: (0,0)

b. horizontal asymptote: None

vertical asymptote: $x=-1$

slant asymptote: $y=x-3$

c. $f(x)=\dfrac{x(x-2)}{x+1}$

61. a.) $n(4)=4500$

b.) Observe that

$$\tfrac{9500t-2000}{4+t}=5500$$

$$9500t-2000=5500(4+t)$$

$$4000t=24{,}000$$

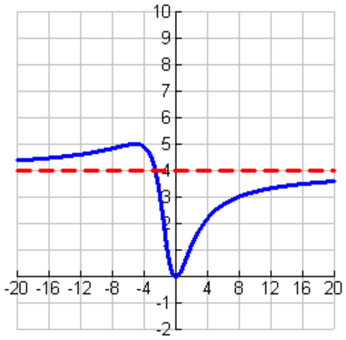
$$t=6 \text{ months}$$

c.) $n(t)$ gets closer to 9500 as t gets larger. So, the number of infected people stabilizes around 9500.

62. a.) $r(8)=3.01$, so about 3%.

b.) $r(20)=3.60$, so about 3.6%

c.) The graph of $r(x)=\dfrac{4x^2}{x^2+2x+5}$ is:



Observe that the graph increases towards 4% as x gets large.

63. a. $C(1)=\frac{2(1)}{1^2+100}=\frac{2}{101}\cong 0.0198$

b. Since 1 hour = 60 minutes, we have $C(60)=\frac{2(60)}{60^2+100}\cong 0.0324$.

c. Since 5 hours = 300 minutes, we have $C(300)=\frac{2(300)}{300^2+100}\cong 0\ .0067$.

d. The horizontal asymptote is $y=0$. So, after several days, C is approximately 0.

64. a. $C\left(\frac{1}{2}\right)\cong 0.0124$

b. $C(1)\cong 0.0243$

c. $C(4)\cong 0.0714$

d. Even though the values temporarily get larger, eventually they decrease toward 0. So, the horizontal asymptote is $y=0$. So, after several days, C is approximately 0.

65. a. $N(0)=52$ wpm

b. $N(12)\cong 107$ wpm

c. Since 3 years = 36 months, $N(36)\cong 120$wpm

d. The horizontal asymptote is $y=130$. So, expect to type approximately 130 wpm as time goes on.

66. $N(3)\cong 78$, $N(16)\cong 267$

The horizontal asymptote is $y=600$. So, expect to remember at most 600 names.

67. Solve for x:

$$\frac{-x^3+10x^2}{x}=16$$

$$-x^2+10x-16=0$$

$$(x-8)(x-2)=0 \;\Rightarrow\; x=2,8$$

So, must sell either 2,000 or 8,000 units to get this average profict. This yields an average value of \$16 per unit.

68. Solve for x:

$$\frac{-x^3+10x^2}{x}=25$$

$$-x^2+10x-25=0$$

$$(x-5)^2=0 \;\Rightarrow\; x=5$$

So, must sell 5,000 units to get this average profict. This yields an average value of \$25 per unit.

69. $C(15)=\dfrac{22(14)}{15^2+1}+24\approx 25.4$ mcg/mL.

The times t for which this concentration is achieved are found by solving $C(t)=25.4$, as follows:

$$\frac{22(t-1)}{t^2+1}+24=25.4$$

$$22(t-1)-1.4\left(t^2+1\right)=0$$

$$1.4t^2-22t+23.4=0$$

$$t=\frac{22\pm\sqrt{22^2-4(1.4)(23.4)}}{2(1.4)}\approx 1,\,15$$

So, there are two times, 1 hours and 15 hours, after taking the medication at which the concentration of the drug in the bloodstream is approximately 25.4 mcg/mL. The first time, approximately 1 hour, occurs as the concentration of the drug is increasing to a level high enough that the body will be able to maintain a concentration of approximately 25 mcg/mL throughout the day. The second time, approximately 15 hours, occurs many hours later in the day as the concentration of the medication in the bloodstream drops.

70. The times *t* for which this concentration is achieved are found by solving $C(t)=25$, as follows:

$$\frac{22(t-1)}{t^2+1}+24=25$$

$$22(t-1)-\left(t^2+1\right)=0$$

$$t^2-22t+23=0$$

$$t=\frac{22\pm\sqrt{22^2-4(1)(23)}}{2(1)}\approx \cancel{1.10},\ 21$$

The concentration of the drug in the bloodstream is 25 mcg/mL approximately 21 hours after taking the medication. After 24 hours the concentration of the medication in the bloodstream has dropped to 24.9 mcg/mL. As the drug becomes inert during the 25th hour this concentration will drop to 0 mcg/mL. Thus it is important to take the next dose 24 hours after the previous does so that as the previous dose becomes inert the new dose has time to build up the concentration of the drug in the bloodstream. At the end of the 25th hour the previous dose will no longer be in the patient's system but the new dose will provide a concentration of 24 mcg/mL.

71. $f(x)=\dfrac{x-1}{x^2-1}=\dfrac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}=\dfrac{1}{x+1}$

with a hole at $x=1$. So, $x=1$ is not a vertical asymptote.

72. "Degree of numerator = Degree of denominator – 1" is not the criterion for the existence of an oblique asymptote. In this case, there is a horizontal asymptote, namely $y=0$, but no oblique asymptote.

73. True. The only way to have a slant asymptote is for the degree of the numerator to be greater than the degree of the denominator (by 1). In such case, there is no horizontal asymptote.

74. False. Consider $f(x)=\dfrac{1}{(x-2)(x+2)}$.

The vertical asymptotes are $x=-2,\ x=2$.

75. False. This would require the denominator to equal 0, causing the function to be undefined.

76. True. Intersections with neither of these types of asymptotes creates a division by 0.

77. Vertical Asymptotes: $x=c,\ x=-d$

Horizontal Asymptote: $y=1$

78. There are no vertical asymptotes since $x^2+a^2\neq 0$. The horizontal asymptote is $y=3$. (Note: The actual values of *a* and *b* do not impact this result.)

79. Two such possibilities are:

$y=\dfrac{4x^2}{(x+3)(x-1)}$ and $y=\dfrac{4x^5}{(x+3)^3(x-1)^2}$

80. $f(x)=\dfrac{x-3}{x^2+1}$ is such a function.

81. There are many different answers. Here is one approach.

Since there is an oblique asymptote, the degree of the numerator equals 1 + degree of the denominator. We are also given that $f(0)=1,\ f(-1)=0$.

The denominator cannot be linear since it would then have a vertical asymptote. So, it must be at least quadratic. Note that $x^2+a^2 \neq 0$, for any $a \neq 0$.

Guided by these observations, we assume the general form of *f* is: $f(x)=\frac{x^3+K}{x^2+a^2}$.

(Note that *y*=*x* is, in fact, an oblique asymptote for *f*.)

We must find values of *K* and *a* using the two points on the curve – this leads to the following system:

$$\begin{cases} f(0)=\frac{K}{a^2}=1 \Rightarrow K=a^2 \\ f(-1)=\frac{-1+K}{1+a^2}=0 \Rightarrow K=1 \end{cases}$$

From this system, we see that *K* = 1 and *a* = ± 1.

Thus, one function that works is $f(x)=\frac{x^3+1}{x^2+1}$.

82. There are many different answers. Here is one approach.

Since $x=-3, x=1$ are vertical asymptotes, the denominator has factors $(x+3)(x-1)$.

Further, since *y*=3*x* is an oblique asymptote for *f*, we know it has form:

$$f(x)=\frac{3x^3+K}{a(x+3)(x-1)}$$

We must determine values of *K* and *a* that satisfy $f(0)=2, f(2)=0$. This leads to the system:

$$\begin{cases} f(0)=\frac{K}{-3a}=2 \Rightarrow K=-6a \\ f(2)=\frac{24+k}{5a}=0 \Rightarrow K=-24 \end{cases}$$

From this system, we see that *K* = -24 and *a* = 4.

Thus, one function that works is $f(x)=\dfrac{3x^3-24}{4(x+3)(x-1)}$.

83. We must factor the denominator. Observe that the possible rational zeros are: $\pm 1, \pm 2, \pm 5, \pm 10$. Using synthetic division yields

$$\begin{array}{r|rrrr} \underline{-2} & 1 & -2 & -13 & -10 \\ & & -2 & 8 & 10 \\ \hline \underline{-1} & 1 & -4 & -5 & \\ & & -1 & 5 & \\ \hline & 1 & -5 & 0 & \end{array}$$

So, $x^3-2x^2-13x-10=(x+2)(x+1)(x-5)$. None of these factors cancels with one in the numerator. So, the vertical asymptotes are $x=-2, x=-1, x=5$. So, the integral of *f* exists on [0,3].

84. We must factor the denominator. Observe that the possible rational zeros are: $\pm 1, \pm 2, \pm 5, \pm 10, \pm 25, \pm 50$. Using synthetic division yields

$$\begin{array}{r|rrrr} 5 & 1 & 2 & -25 & -50 \\ & & 5 & 35 & 50 \\ \hline -5 & 1 & 7 & 10 & \\ & & -5 & -10 & \\ \hline & 1 & 2 & 0 & \end{array}$$

So, $x^3 + 2x^2 - 25x - 50 = (x-5)(x+5)(x-2)$. None of these factors cancels with one in the numerator. So, the vertical asymptotes are $x = -5, x = 2, x = 5$. So, the integral of *f* might not exist on [-3,2].

85. Note that the denominator factors as $6x^2 - x - 2 = (3x-2)(2x+1)$. Neither of these factors cancels with one in the numerator. So, the vertical asymptotes are $x = \frac{2}{3}, x = -\frac{1}{2}$. So, the integral of *f* might not exist on [-2,0].

86. Observe that $f(x) = \dfrac{2x(3-x)}{x\left(x^2+1\right)} = \dfrac{2(3-x)}{x^2+1}$, which has no vertical asymptotes. Hence, the integral of *f* exists on [-1,1].

Chapter 2 Review Solutions --

1. b Opens down, vertex is $(-6,3)$

2. c Opens up, vertex is (4,2)

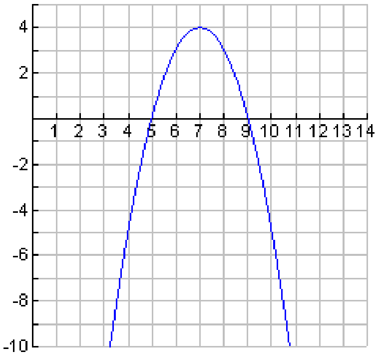
3. a Opens up, vertex is $\left(-\frac{1}{2}, -\frac{25}{4}\right)$.

$$\begin{aligned} x^2 + x - 6 &= (x^2 + x + \tfrac{1}{4}) - 6 - \tfrac{1}{4} \\ &= (x + \tfrac{1}{2})^2 - \tfrac{25}{4} \end{aligned}$$

4. d Opens down, vertex is $\left(-\frac{5}{3}, \frac{49}{3}\right)$

$$\begin{aligned} -3x^2 - 10x + 8 &= -3(x^2 + \tfrac{10}{3}x \quad) + 8 \\ &= -3(x^2 + \tfrac{10}{3}x + \tfrac{25}{9} \quad) + 8 + \tfrac{25}{3} \\ &= -3(x + \tfrac{5}{3})^2 + \tfrac{49}{3} \end{aligned}$$

5.



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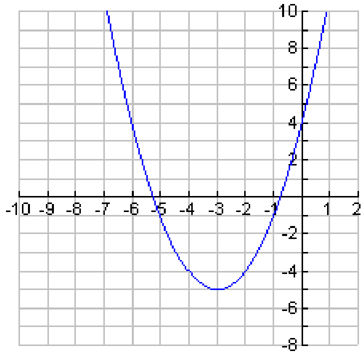

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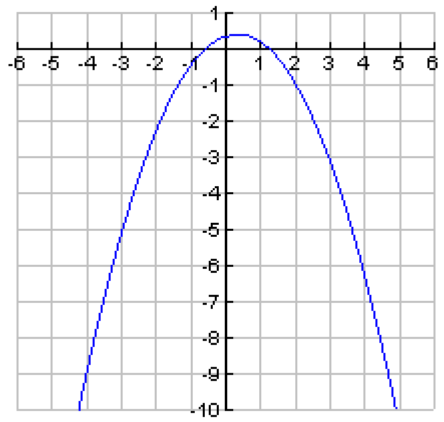

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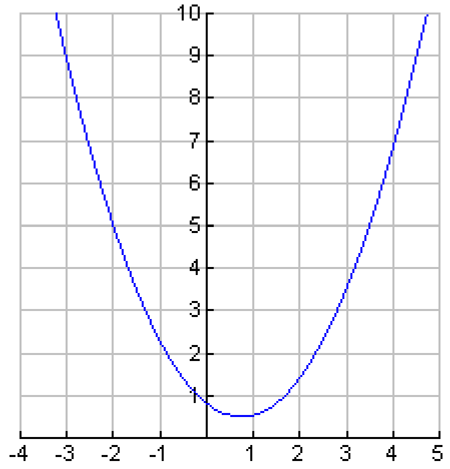


9.

$$x^2 - 3x - 10 = (x^2 - 3x + \tfrac{9}{4}) - 10 - \tfrac{9}{4}$$
$$= (x - \tfrac{3}{2})^2 - \tfrac{49}{4}$$

10.

$$x^2 - 2x - 24 = (x^2 - 2x + 1) - 24 - 1$$
$$= (x-1)^2 - 25$$

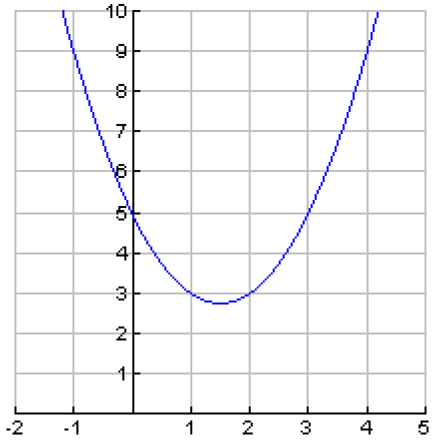
11.

$$4x^2 + 8x - 7 = 4(x^2 + 2x \quad) - 7$$
$$= 4(x^2 + 2x + 1) - 7 - 4$$
$$= 4(x+1)^2 - 11$$

12.

$$-\tfrac{1}{4}x^2 + 2x - 4 = -\tfrac{1}{4}(x^2 - 8x \quad) - 4$$
$$= -\tfrac{1}{4}(x^2 - 8x + 16) - 4 + 4$$
$$= -\tfrac{1}{4}(x-4)^2$$

13.



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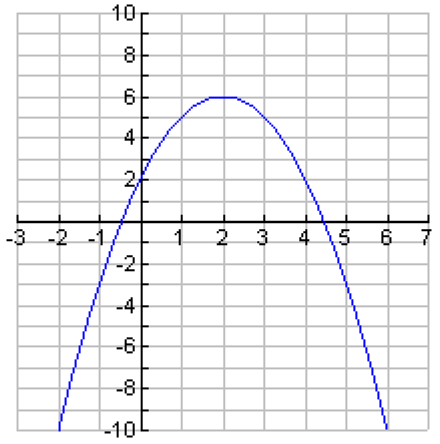

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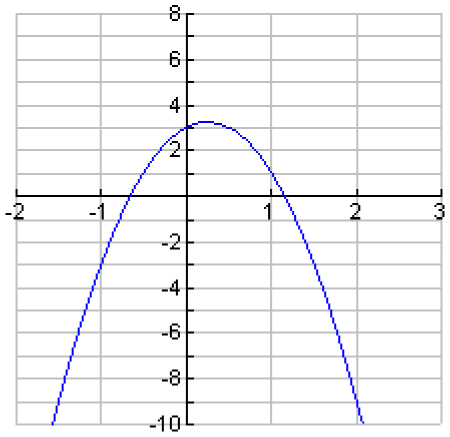

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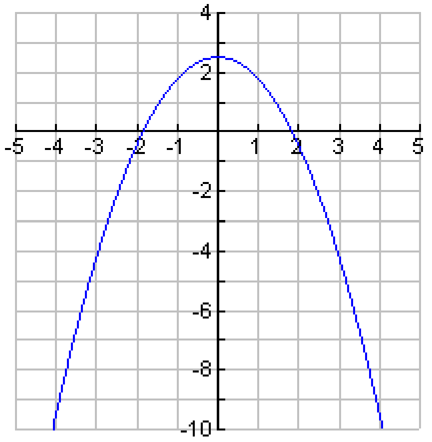


17. Vertex is

$$\left(-\frac{b}{2a}, c-\frac{b^2}{4a}\right)=\left(-\frac{-5}{2(13)}, 12-\frac{(-5)^2}{4(13)}\right)$$

$$=\left(\frac{5}{26}, \frac{599}{52}\right)$$

18. Vertex is

$$\left(-\frac{b}{2a}, c-\frac{b^2}{4a}\right)=\left(-\frac{-4}{2(\frac{2}{5})}, 3-\frac{(-4)^2}{4(\frac{2}{5})}\right)$$

$$=\left(5,-7\right)$$

19. Vertex is

$$\left(-\frac{b}{2a}, c-\frac{b^2}{4a}\right)=\left(-\frac{-0.12}{2(-0.45)}, 3.6-\frac{(-0.12)^2}{4(-0.45)}\right)$$

$$=\left(-\frac{2}{15}, \frac{451}{125}\right)$$

20. Vertex is

$$\left(-\frac{b}{2a}, c-\frac{b^2}{4a}\right)=\left(-\frac{\frac{2}{5}}{2(-\frac{3}{4})}, 4-\frac{(\frac{2}{5})^2}{4(-\frac{3}{4})}\right)$$

$$=\left(\frac{4}{15}, \frac{304}{75}\right)$$

21. Since the vertex is $(-2,3)$, the function has the form $y=a(x+2)^2+3$. To find *a*, use the fact that the point $(1,4)$ is on the graph:

$$4=a(1+2)^2+3$$

$$4=9a+3$$

$$\tfrac{1}{9}=a$$

So, the function is $y=\frac{1}{9}(x+2)^2+3$.

22. Since the vertex is $(4,7)$, the function has the form $y=a(x-4)^2+7$. To find *a*, use the fact that the point $(-3,1)$ is on the graph:

$$1=a(-3-4)^2+7$$

$$1=49a+7$$

$$-\tfrac{6}{49}=a$$

So, the function is $y=-\frac{6}{49}(x-4)^2+7$.

23. Since the vertex is $(2.7, 3.4)$, the function has the form $y = a(x-2.7)^2 + 3.4$. To find *a*, use the fact that the point (3.2, 4.8) is on the graph:

$$4.8 = a(3.2-2.7)^2 + 3.4$$
$$4.8 = 0.25a + 3.4$$
$$5.6 = a$$

So, the function is $y = 5.6(x-2.7)^2 + 3.4$.

24. Since the vertex is $(-\frac{5}{2}, \frac{7}{4})$, the function has the form $y = a(x+\frac{5}{2})^2 + \frac{7}{4}$. To find *a*, use the fact that the point $(\frac{1}{2}, \frac{3}{5})$ is on the graph:

$$\tfrac{3}{5} = a(\tfrac{1}{2}+\tfrac{5}{2})^2 + \tfrac{7}{4}$$
$$\tfrac{3}{5} = 9a + \tfrac{7}{4}$$
$$-\tfrac{23}{180} = a$$

So, the function is $y = -\frac{23}{180}(x+\frac{5}{2})^2 + \frac{7}{4}$.

25.

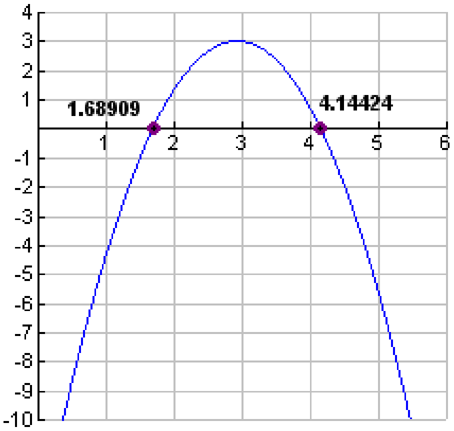
a.

$$P(x) = R(x) - C(x)$$
$$= \left(-2x^2 + 12x - 12\right) - \left(\tfrac{1}{3}x + 2\right)$$
$$= -2x^2 + \tfrac{35}{3}x - 14$$

b. Solve $P(x) = 0$.

$$-2x^2 + \tfrac{35}{3}x - 14 = 0$$
$$-2\left(x^2 - \tfrac{35}{6}x \quad\right) - 14 = 0$$
$$-2\left(x^2 - \tfrac{35}{6}x + \tfrac{1225}{144} \quad\right) - 14 + \tfrac{1225}{72} = 0$$
$$-2(x - \tfrac{35}{12})^2 + \tfrac{217}{72} = 0$$
$$(x - \tfrac{35}{12})^2 = \tfrac{217}{144}$$
$$x = \tfrac{35}{12} \pm \sqrt{\tfrac{217}{144}}$$
$$= \tfrac{35 \pm \sqrt{217}}{12}$$
$$\cong 4.1442433,\ 1.68909$$

c.



d. The range is approximately $\left(1.6891, 4.144\right)$, which corresponds to where the graph is above the *x*-axis.

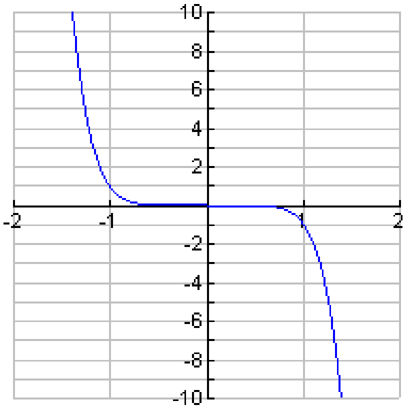
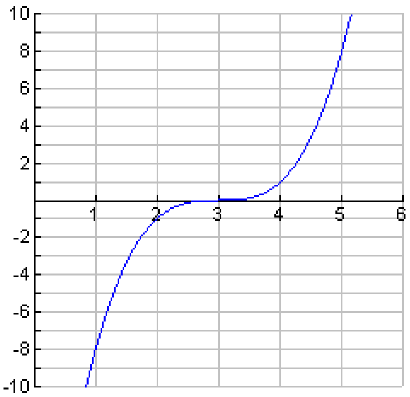
26. Area is

$$A(x) = (2x-4)(x+7) = 2x^2 + 10x - 28$$

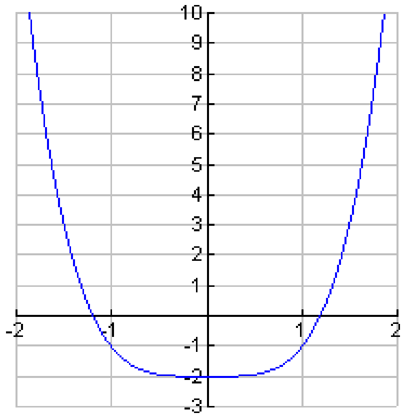
27. Area is

$$A(x) = \tfrac{1}{2}(x+2)(4-x) = -\tfrac{1}{2}(x^2 - 2x \quad) + 4$$
$$= -\tfrac{1}{2}(x^2 - 2x + 1) + 4 + \tfrac{1}{2}$$
$$= -\tfrac{1}{2}(x-1)^2 + \tfrac{9}{2}$$

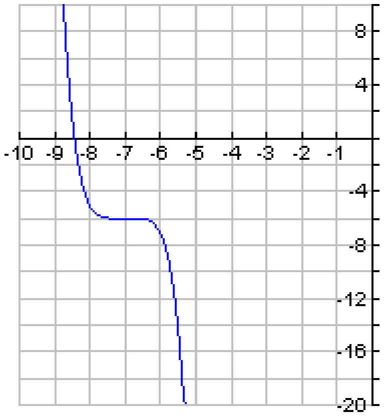
Note that $A(x)$ has a maximum at $x = 1$ (since its graph is a parabola that opens down). The corresponding dimensions are both base and height are 3 units.

28. a. $h(t) = -12(t^2 - \frac{20}{3}t)$ $= -12(t^2 - \frac{20}{3}t + \frac{100}{9}) + 12\left(\frac{100}{9}\right)$ $= -12(t - \frac{10}{3})^2 + \frac{400}{3}$ So, the maximum height is approximately 133.33 units.	**b.** Solve $h(t) = 0$: $t(-12t + 80) = 0$ $t = 0,\ \frac{80}{12} \cong 6.67$ So, after approximately 6.7 seconds, it will hit the ground.
29. Polynomial with degree 6	**30.** Polynomial with degree 5
31. Not a polynomial (due to the term $x^{1/4}$)	**32.** Polynomial with degree 3
33. d linear	**34. b** Parabola opens down
35. a 4th degree polynomial, looks like $y = x^4$ for x very large.	**36. c** 7th degree polynomial, looks like $y = x^7$ for x very large.
37. Reflect the graph of x^7 over the x-axis  	**38.** Shift the graph of x^3 to the right 3 units. 

39. Shift the graph of x^4 down 2 units.



40. Shift the graph of x^5 left 7 units, then reflect over the *x*-axis, and then move down 6 units and.



41. 6 (multiplicity 5)
–4 (multiplicity 2)

42. 0 (multiplicity 1)
2 (multiplicity 3)
–5 (multiplicity 1)

43.

$$x^5 - 13x^3 + 36x = x(x^2 - 9)(x^2 - 4)$$
$$= x(x+3)(x-3)(x+2)(x-2)$$

So, the zeros are $0, -2, 2, 3, -3$, all with multiplicity 1.

44.

$$4.2x^4 - 2.6x^2 \cong 4.2x^2\left(x^2 - 0.619047\right)$$
$$= 4.2x(x - 0.786795)(x + 0.786795)$$

So, the zeros are approximately:
0 (multiplicity 2)
0.786795, –0.786795 (multiplicity 1)

45. $x(x+3)(x-4)$

46. $(x-2)(x-4)(x-6)(x+8)$

47. $x(x+\frac{2}{5})(x-\frac{3}{4}) = x(5x+2)(4x-3)$

48. $(x-(2-\sqrt{5}))(x-(2+\sqrt{5}))$

49.

$$(x+2)^2(x-3)^2 =$$
$$x^4 - 2x^3 - 11x^2 + 12x + 36$$

50. $(x-3)^2 x^3 (x+1)^2$

51. $f(x)=x^2-5x-14=(x-7)(x+2)$

a. Zeros: -2, 7 (both multiplicity 1)

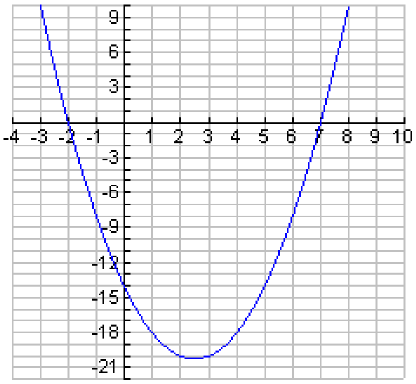
b. Crosses at both -2, 7

c. *y*-intercept: $f(0)=-14$, so $(0,-14)$

d. Long-term behavior: Behaves like $y=x^2$.

Even degree and leading coefficient positive, so graph rises without bound to the left and right.

e.



52. $f(x)=-(x-5)^5$

a. Zeros: 5 (multiplicity 5)

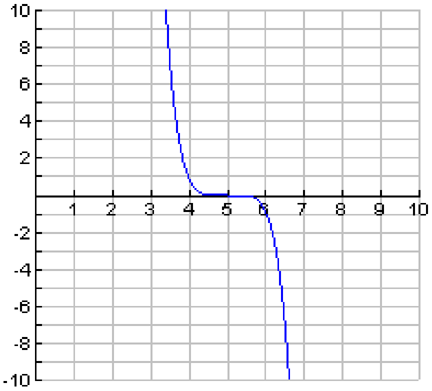
b. Crosses at 5

c. *y*-intercept: $f(0)=-\left(-5\right)^5=3125$, so $(0,3125)$

d. Long-term behavior: Behaves like $y=-x^5$.

Odd degree and leading coefficient negative, so graph falls without bound to the right and rises to the left.

e.



53. $f(x) = 6x^7 + 3x^5 - x^2 + x - 4$

a. Zeros: We first try to apply the Rational Root Test:

Factors of -4: $\pm 1, \pm 2, \pm 4$

Factors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

Possible rational zeros:

$\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$

Unfortunately, it can be shown that none of these are zeros of *f*. To get a feel for the possible number of irrational and complex root, we apply Descartes' Rule of Signs:

Number of sign variations for $f(x)$: 3

$f(-x) = -6x^7 - 3x^5 - x^2 - x - 4$, so

Number of sign variations for $f(-x)$: 0

Since $f(x)$ is degree 7, there are 7 zeros, classified as:

Positive Real Zeros	**Negative Real Zeros**	**Imaginary Zeros**
3	0	4
1	0	6

Need to actually graph the polynomial to determine the approximate zeros. From **e.**, we see there is a zero at approximately (0.8748,0) with multiplicity 1.

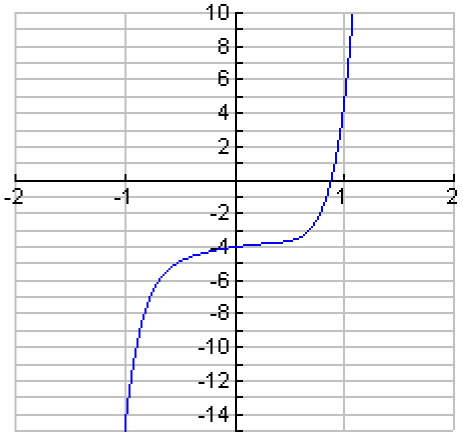
b. From the analysis in part **a**, we know the graph crosses at its only real zero.

c. y-intercept: $f(0) = -4$, so $(0, -4)$

d. Long-term behavior: Behaves like $y = x^7$.

Odd degree and leading coefficient positive, so graph falls without bound to the left and rises to the right.

e.



54. $f(x) = -x^4(3x+6)^3(x-7)^3$

a. Zeros: 0 (multiplicity 4)

-2 (multiplicity 3)

7 (multiplicity 3)

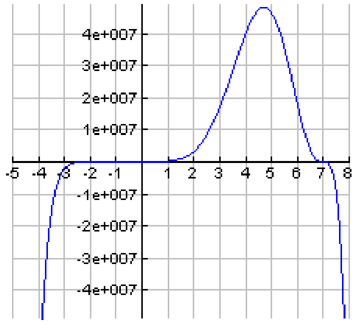
b. Touches at 0, and crosses at both -2, 7

c. y-intercept: $f(0) = 0$, so $(0,0)$

d. Long-term behavior: Behaves like $y = -x^{10}$.

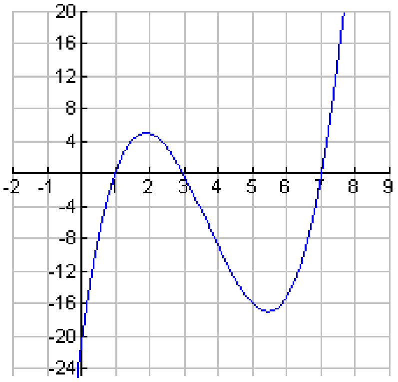
Even degree and leading coefficient negative, so graph falls without bound to the right and left.

e.



55. $f(x) = (x-1)(x-3)(x-7)$

a.

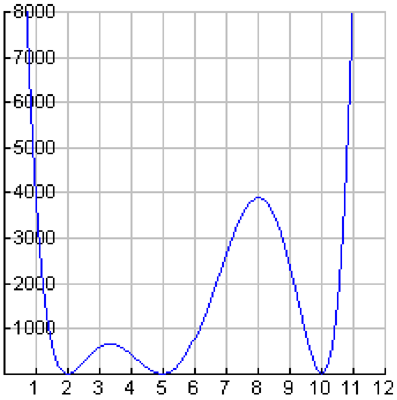


b. Zeros: 1, 3, 7 (all with multiplicity 1)

c. Want intervals where the graph of *f* is above the *x*-axis. From **a.**, this occurs on the intervals $(1,3)$ and $(7,\infty)$.

So, between 1 and 3 hours, and more than 7 hours would be financially beneficial.

56. Consider the graph below. The peak seasons occur during the summer and in Nov. and Dec.



57.

$$\begin{array}{r} x+4 \\ x-2\overline{)\,x^2+2x-6} \\ \underline{-(x^2-2x)} \\ 4x-6 \\ \underline{-(4x-8)} \\ 2 \end{array}$$

So, $\boxed{Q(x) = x+4, \quad r(x) = 2}$.

58.

$$\begin{array}{r} x-1 \\ 2x-3\overline{)\;2x^2-5x-1} \\ \underline{-(2x^2-3x)} \\ -2x-1 \\ \underline{-(-2x+3)} \\ -4 \end{array}$$

So, $\boxed{Q(x)=x-1,\ \ r(x)=-4}$.

59.

$$\begin{array}{r} 2x^3-4x^2-2x-\tfrac{7}{2} \\ 2x-4\overline{)\;4x^4-16x^3+12x^2+x-9} \\ \underline{-(4x^4-8x^3)} \\ -8x^3+12x^2 \\ \underline{-(-8x^3+16x^2)} \\ -4x^2+x \\ \underline{-(-4x^2+8x)} \\ -7x-9 \\ \underline{-(-7x+14)} \\ -23 \end{array}$$

So,

$\boxed{Q(x)=2x^3-4x^2-2x-\tfrac{7}{2},\ \ r(x)=-23}$.

60.

$$\begin{array}{r} -2x^2+2x-2 \\ 2x^2+x-4\overline{)\;-4x^4+2x^3+6x^2-x+2} \\ \underline{-(-4x^4-2x^3+8x^2)} \\ 4x^3-2x^2-x \\ \underline{-(4x^3+2x^2-8x)} \\ -4x^2+7x+2 \\ \underline{-(-4x^2-2x+8)} \\ 9x-6 \end{array}$$

So, $\boxed{Q(x)=-2x^2+2x-2,\ \ r(x)=9x-6}$.

61.

$$\begin{array}{r|rrrrr} \underline{-2} & 1 & 4 & 5 & -2 & -8 \\ & & -2 & -4 & -2 & 8 \\ \hline & 1 & 2 & 1 & -4 & 0 \end{array}$$

So, $Q(x)=x^3+2x^2+x-4,\ \ r(x)=0$.

62.

$$\begin{array}{r|rrrr} \underline{-2} & 1 & 0 & -10 & 3 \\ & & -2 & 4 & 12 \\ \hline & 1 & -2 & -6 & 15 \end{array}$$

So, $Q(x)=x^2-2x-6,\ \ r(x)=15$.

63.

$$\begin{array}{r|rrrrrrr} -8 & 1 & 0 & 0 & 0 & 0 & 0 & -64 \\ & & -8 & 64 & -512 & 4096 & -32,768 & 262,144 \\ \hline & 1 & -8 & 64 & -512 & 4096 & -32,768 & 262,080 \end{array}$$

So, $Q(x) = x^5 - 8x^4 + 64x^3 - 512x^2 + 4096x - 32,768, \quad r(x) = 262,080$.

64.

$$\begin{array}{r|rrrrrr} \frac{3}{4} & 2 & 4 & -2 & 0 & 7 & 5 \\ & & \frac{3}{2} & \frac{33}{8} & \frac{51}{32} & \frac{153}{128} & \frac{3147}{512} \\ \hline & 2 & \frac{11}{2} & \frac{17}{8} & \frac{51}{32} & \frac{1049}{128} & \frac{5707}{512} \end{array}$$

So, $Q(x) = 2x^4 + \frac{11}{2}x^3 + \frac{17}{8}x^2 + \frac{51}{32}x + \frac{1049}{128}, \quad r(x) = \frac{5707}{512}$.

65.

$$\begin{array}{r} x+3 \\ 5x^2-7x+3\overline{)\;5x^3+8x^2-22x+1} \\ \underline{-(5x^3-7x^2+3x)} \\ 15x^2-25x+1 \\ \underline{-(15x^2-21x+9)} \\ -4x-8 \end{array}$$

So, $Q(x) = x + 3, \quad r(x) = -4x - 8$.

66.

$$\begin{array}{r|rrrrr} 3 & 1 & 2 & -5 & 4 & 2 \\ & & 3 & 15 & 30 & 102 \\ \hline & 1 & 5 & 10 & 34 & 104 \end{array}$$

So, $Q(x) = x^3 + 5x^2 + 10x + 34, \; r(x) = 104$.

67.

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 2 & -8 \\ & & -1 & 5 & -7 \\ \hline & 1 & -5 & 7 & -15 \end{array}$$

So, $Q(x) = x^2 - 5x + 7, \; r(x) = -15$.

68.

$$\begin{array}{r} x-5 \\ x^2+0x+4\overline{)\;x^3-5x^2+4x-20} \\ \underline{-(x^3+0x^2+4x)} \\ -5x^2+0x-20 \\ \underline{-(-5x^2+0x-20)} \\ 0 \end{array}$$

So, $Q(x) = x - 5, \quad r(x) = 0$.

69. Area = length × width. So, solving for length, we see that length = Area ÷ width. So, in this case,

$$\text{length} = \frac{6x^4 - 8x^3 - 10x^2 + 12x - 16}{2x-4} = \frac{3x^4 - 4x^3 - 5x^2 + 6x - 8}{x-2}.$$

We compute this quotient using synthetic division:

$$\begin{array}{r|rrrrr} 2 & 3 & -4 & -5 & 6 & -8 \\ & & 6 & 4 & -2 & 8 \\ \hline & 3 & 2 & -1 & 4 & 0 \end{array}$$

Thus, the length (in terms of x) is $\boxed{3x^3 + 2x^2 - x + 4 \text{ feet}}$.

70. Let x = width (= length) of corner square.
Then, the dimensions of the box formed are:

$$\text{width} = 10 - 2x$$
$$\text{length} = 15 - 2x$$
$$\text{height} = x$$

Thus, the volume of the box is given by

$$V(x) = x(10-2x)(15-2x).$$

71.

$$\begin{array}{r|rrrrrr} -2 & 6 & 1 & 0 & -7 & 1 & -1 \\ & & -12 & 22 & -44 & 102 & -206 \\ \hline & 6 & -11 & 22 & -51 & 103 & -207 \end{array}$$

So, $f(-2) = -207$.

72.

$$\begin{array}{r|rrrrrr} 1 & 6 & 1 & 0 & -7 & 1 & -1 \\ & & 6 & 7 & 7 & 0 & 1 \\ \hline & 6 & 7 & 7 & 0 & 1 & 0 \end{array}$$

So, $f(1) = 0$.

73.

$$\begin{array}{r|rrrr} 1 & 1 & 2 & 0 & -3 \\ & & 1 & 3 & 3 \\ \hline & 1 & 3 & 3 & 0 \end{array}$$

So, $g(1) = 0$.

74.

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 0 & -3 \\ & & -1 & -1 & 1 \\ \hline & 1 & 1 & -1 & -2 \end{array}$$

So, $g(-1) = -2$.

75.

$P(-3) = (-3)^3 - 5(-3)^2 + 4(-3) + 2 = -82$.

So, it is not a zero.

76. $P(-2) = P(2) = 0$

Yes, they are zeros.

77. $P(1) = 2(1)^4 - 2(1) = 0$

Yes, it is a zero.

78. $P(4) = (4)^4 - 2(4)^3 - 8(4) = 96$

No, it is not a zero.

79. $P(x) = x\left(x^3 - 6x^2 + 32\right)$

Observe that since -2 is a zero, synthetic division yields:

$$\begin{array}{r|rrrr} \underline{-2} & 1 & -6 & 0 & 32 \\ & & -2 & 16 & -32 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= x(x+2)(x^2 - 8x + 16) \\ &= x(x+2)(x-4)^2 \end{aligned}.$$

80. Observe that since 3 is a zero, synthetic division yields:

$$\begin{array}{r|rrrr} \underline{3} & 1 & -7 & 0 & 36 \\ & & 3 & -12 & -36 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-3)(x^2 - 4x - 12) \\ &= (x-3)(x-6)(x+2) \end{aligned}.$$

81. $P(x) = x^2\left(x^3 - x^2 - 8x + 12\right)$

We need to factor $x^3 - x^2 - 8x + 12$. To do so, we begin by applying the Rational Root Test:

Factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Factors of 1: ± 1

Possible rational zeros:

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

Observe that both -3 (multiplicity 1) and 2 (multiplicity 2) are zeros. So,

$$P(x) = x^2(x+3)(x-2)^2.$$

82. P(x) =

$$\begin{aligned} x^4 - 32x^2 - 144 &= (x^2 - 36)(x^2 + 4) \\ &= (x-6)(x+6)(x-2i)(x+2i) \end{aligned}$$

83.

Number of sign variations for $P(x)$: 1

$P(-x) = x^4 - 3x^3 - 16$, so

Number of sign variations for $P(-x)$: 1

Positive Real Zeros	**Negative Real Zeros**
1	1

84.

Number of sign variations for $P(x)$: 1

$$P(-x)=-x^5-6x^3+4x-2\text{, so}$$

Number of sign variations for $P(-x)$: 2

Positive Real Zeros	**Negative Real Zeros**
1	2
1	0

85.

Number of sign variations for $P(x)$: 5

$$P(-x)=-x^9+2x^7+x^4+3x^3-2x-1\text{, so}$$

Number of sign variations for $P(-x)$: 2

Positive Real Zeros	**Negative Real Zeros**
5	2
5	0
3	2
3	0
1	2
1	0

86.

Number of sign variations for $P(x)$: 3

$$P(-x)=-2x^5+4x^3+2x^2-7\text{, so}$$

Number of sign variations for $P(-x)$: 2

Positive Real Zeros	**Negative Real Zeros**
3	2
1	2
3	0
1	0

87.

Factors of 6: $\pm1, \pm2, \pm3, \pm6$

Factors of 1: ±1

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

88.

Factors of -8: $\pm1, \pm2, \pm4, \pm8$

Factors of 1: ±1

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8$

89.

Factors of 64:

$\pm1, \pm2, \pm4, \pm8, \pm16, \pm32, \pm64$

Factors of 2: $\pm1, \pm2$

Possible rational zeros:

$\pm1, \pm2, \pm4, \pm8, \pm16, \pm32, \pm64, \pm\frac{1}{2}$

90.

Factors of 2: $\pm1, \pm2$

Factors of -4: $\pm1, \pm2, \pm4$

Possible rational zeros: $\pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$

91.

Factors of 1: ± 1

Factors of 2: $\pm 1, \pm 2$

Possible rational zeros: $\pm 1, \pm\frac{1}{2}$

The only rational zero is $\frac{1}{2}$.

92.

Factors of 3: $\pm 1, \pm 3$

Factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Possible rational zeros:

$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}$

The rational zeros are $-\frac{3}{2}, \frac{1}{3}, \frac{1}{2}$.

93.

Factors of -16: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Factors of 1: ± 1

Possible rational zeros:

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

The rational zeros are 1, 2, 4, and -2.

94.

Factors of -2: $\pm 1, \pm 2$

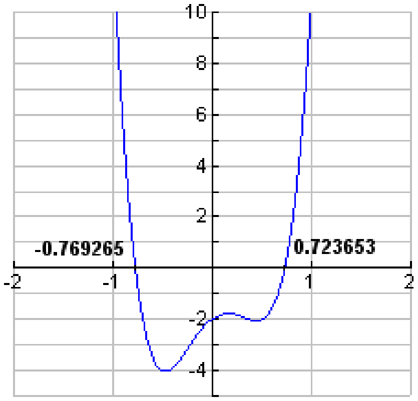
Factors of 24:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

Possible rational zeros:

$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{8}, \pm\frac{1}{12}, \pm\frac{1}{24}, \pm 2, \pm\frac{2}{3}$

There are no rational zeros. Indeed, see the graph to the right:



95.

a. Number of sign variations for $P(x)$: 1

$P(-x) = -x^3 - 3x - 5$, so

Number of sign variations for $P(-x)$: 0

Since $P(x)$ is degree 3, there are 3 zeros, the real ones of which are classified as:

Positive Real Zeros	Negative Real Zeros
1	0

b. Factors of -5: $\pm 1, \pm 5$

Factors of 1: ± 1

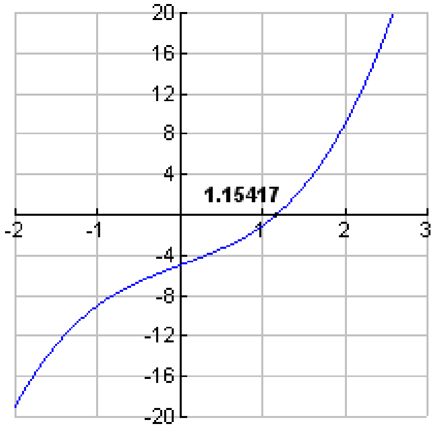
Possible rational zeros: $\pm 1, \pm 5$

c. -1 is a lower bound, 5 is an upper bound

d. There are no rational zeros.

e. Not possible to accurately factor since we do not have the zeros.

f.



96.

a. Number of sign variations for $P(x)$: 1

$P(-x) = -x^3 + 3x^2 + 6x - 8$, so

Number of sign variations for $P(-x)$: 2

Positive Real Zeros	Negative Real Zeros
1	0
1	2

b. Factors of -8: $\pm 1, \pm 2, \pm 4, \pm 8$

Factors of 1: ± 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

c. -8 is a lower bound, 4 is upper bound (check by synthetic division)

d. Observe that

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -6 & -8 \\ & & 2 & 10 & 8 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

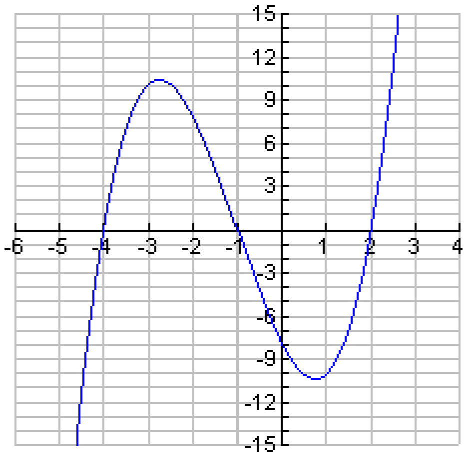
So,

$P(x) = (x-2)(x^2 + 5x + 4) = (x-2)(x+4)(x$

Hence, the zeros are -4, 1, and 2.

e. $P(x) = (x-2)(x+4)(x+1)$

f.



97.

a. Number of sign variations for $P(x)$: 3

$P(-x) = -x^3 - 9x^2 - 20x - 12$, so

Number of sign variations for $P(-x)$: 0

Positive Real Zeros	**Negative Real Zeros**
3	0
1	0

b. Factors of -12:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Factors of 1: ± 1

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

c. -1 is a lower bound, 12 is upper bound (check by synthetic division)

d. Observe that

$$\begin{array}{r|rrrr} \underline{1} & 1 & -9 & 20 & -12 \\ & & 1 & -8 & 12 \\ \hline & 1 & -8 & 12 & 0 \end{array}$$

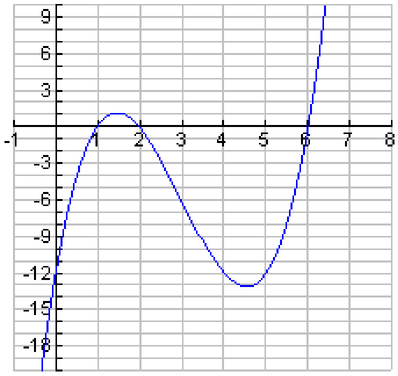
So,

$P(x) = (x-1)(x^2 - 8x + 12) = (x-1)(x-6)(x$

Hence, the zeros are 1, 2, and 6.

e. $P(x) = (x-1)(x-6)(x-2)$

f.



98.

a. Number of sign variations for $P(x)$: 2

$P(-x)=x^4+x^3-7x^2-x+6$, so

Number of sign variations for $P(-x)$: 2

Positive Real Zeros	**Negative Real Zeros**
2	2
2	0
0	2
0	0

b. Factors of 6: $\pm1, \pm2, \pm3, \pm6$

Factors of 1: ±1

Possible rational zeros:

$\pm1, \pm2, \pm3, \pm6$

c. -3 is a lower bound, 6 is upper bound (check by synthetic division)

d. Observe that

$$\begin{array}{r|rrrrr}
1 & 1 & -1 & -7 & 1 & 6 \\
 & & 1 & 0 & -7 & -6 \\
\hline
-2 & 1 & 0 & -7 & -6 & 0 \\
 & & -2 & 4 & 6 & \\
\hline
 & 1 & -2 & -3 & 0 &
\end{array}$$

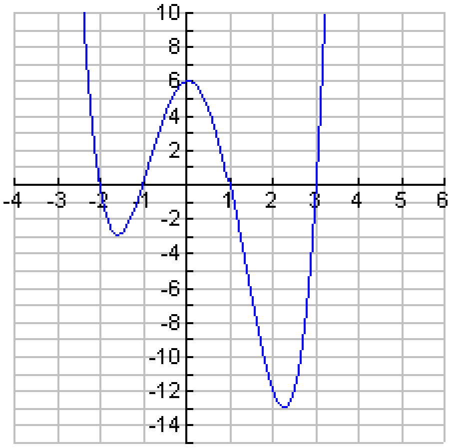
So,

$$P(x)=(x-1)(x+2)(x^2-2x-3)$$
$$=(x-1)(x+2)(x-3)(x+1)$$

Hence, the zeros are -2, -1, 1, and 3.

e. $P(x)=(x-1)(x+2)(x-3)(x+1)$

f.



99.

a. Number of sign variations for $P(x)$: 2

$P(-x) = x^4 + 5x^3 - 10x^2 - 20x + 24$, so

Number of sign variations for $P(-x)$: 2

Positive Real Zeros	Negative Real Zeros
0	0
0	2
2	2
2	0

b. Factors of 24:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

Factors of 1: ± 1

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

c. -3 is a lower bound, 8 is an upper bound (check by synthetic division)

d. Observe that

$$\begin{array}{r|rrrrr}
2 & 1 & -5 & -10 & 20 & 24 \\
 & & 2 & -6 & -32 & -24 \\
\hline
-1 & 1 & -3 & -16 & -12 & 0 \\
 & & -1 & 4 & 12 & \\
\hline
 & 1 & -4 & -12 & 0 &
\end{array}$$

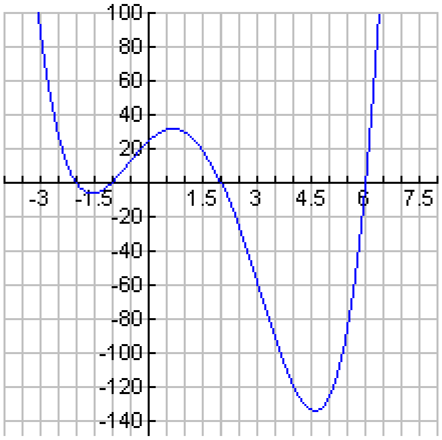
So,

$$\begin{aligned} P(x) &= (x-2)(x+1)(x^2 - 4x + 12) \\ &= (x-2)(x+1)(x+2)(x-6) \end{aligned}$$

Hence, the zeros are -2, -1, 2, and 6.

e. $P(x) = (x-2)(x+1)(x+2)(x-6)$

f.



100.

a. Number of sign variations for $P(x)$: 2

$P(-x) = -x^5 + 3x^3 - 6x^2 - 8x$, so

Number of sign variations for $P(-x)$: 2

Since $P(x)$ is degree 5, there are 5 zeros.

Also note that 0 is a zero of P

Positive Real Zeros	**Negative Real Zeros**
2	2
2	0
0	2
0	0

b. $P(x) = x\left(x^4 - 3x^2 - 6x + 8\right)$

Factors of 8: ±1, ±2, ±4, ±8

Factors of 1: ±1

Possible rational zeros: ±1, ±2, ±4, ±8

c. -2 is a lower bound.

d. The rational zeros are 0, 1, 2.

e. From **d.**, we know that $x(x-1)(x-2)$ divides $P(x)$ evenly. We use synthetic division using 1 and 2 to determine the quotient $P(x) \div x(x-1)(x-2)$:

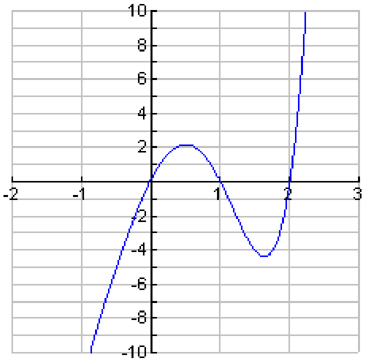
$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -3 & -6 & 8 \\ & & 2 & 4 & 2 & -8 \\ \hline 1 & 1 & 2 & 1 & -4 & 0 \\ & & 1 & 3 & 4 & \\ \hline & 1 & 3 & 4 & 0 & \end{array}$$

So, $P(x) = x(x-1)(x-2)(x^2 + 3x + 4)$

Now, solve $x^2 + 3x + 4 = 0$ using the quadratic formula: $x = \frac{-3 \pm \sqrt{9-16}}{2} = \frac{-3 \pm i\sqrt{7}}{2}$ So,

$P(x) = x(x-1)(x-2)(x - (\frac{-3+i\sqrt{7}}{2}))(x - (\frac{-3-i\sqrt{7}}{2}))$

f.



101. P(x) = $x^2 + 25 = (x - 5i)(x + 5i)$

102. P(x) $= x^2 + 16 = (x - 4i)(x + 4i)$

103. Note that the zeros are

$$x^2 - 2x + 5 = 0 \Rightarrow$$

$$x = \frac{2 \pm \sqrt{4 - 4(5)}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

So, $P(x) = (x - (1 - 2i))(x - (1 + 2i))$.

104. Note that the zeros are

$$x^2 + 4x + 5 = 0 \Rightarrow$$

$$x = \frac{-4 \pm \sqrt{16 - 4(5)}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

So, $P(x) = (x - (-2 - i))(x - (-2 + i))$.

105. Since $-2i$ and $3 + i$ are zeros, so are their conjugates $2i$ and $3 - i$, respectively. Since $P(x)$ has degree 4, these are the only missing zeros.

106. Since $3i$ and $2 - i$ are zeros, so are their conjugates $-3i$ and $2 + i$, respectively. Since $P(x)$ has degree 4, these are the only missing zeros.

107. Since *i* is a zero, then so is its conjugate –*i*. Also, since 2 – *i* is a zero of multiplicity 2, then its conjugate 2 + *i* is also a zero of multiplicity 2. These are all of the zeros.

108. Since 2*i* is a zero, then so is its conjugate –2*i*. Also, since 1 – *i* is a zero of multiplicity 2, then its conjugate 1 + *i* is also a zero of multiplicity 2. These are all of the zeros.

109. Since *i* is a zero of $P(x)$, so is its conjugate -*i*. As such, $(x-i)(x+i)=x^2+1$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-3x-4 \\ x^2+0x+1\overline{\big)x^4-3x^3-3x^2-3x-4} \\ \underline{-(x^4+0x^3+x^2)} \\ -3x^3-4x^2-3x \\ \underline{-(-3x^3+0x^2-3x)} \\ -4x^2+0x-4 \\ \underline{-(-4x^2+0x-4)} \\ 0 \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-i)(x+i)\left(x^2-3x-4\right) \\ &= (x-i)(x+i)(x-4)(x+1) \end{aligned}$$

110. Since 2-*i* is a zero of $P(x)$, so is its conjugate 2+*i*. As such, $(x-(2+i))(x-(2-i))=x^2-4x+5$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-4 \\ x^2-4x+5\overline{\big)x^4-4x^3+x^2+16x-20} \\ \underline{-(x^4-4x^3+5x^2)} \\ -4x^2+16x-20 \\ \underline{-(-4x^2+16x-20)} \\ 0 \end{array}$$

So,

$P(x)=(x-(2+i))(x-(2-i))(x-2)(x+2)$.

111. Since -3*i* is a zero of $P(x)$, so is its conjugate 3*i*. As such, $(x-3i)(x+3i)=x^2+9$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2-2x+2 \\ x^2+0x+9\overline{\big)x^4-2x^3+11x^2-18x+18} \\ \underline{-(x^4+0x^3+9x^2)} \\ -2x^3+2x^2-18x \\ \underline{-(-2x^3+0x^2-18x)} \\ 2x^2+0x+18 \\ \underline{-(2x^2+0x+18)} \\ 0 \end{array}$$

Next, we find the roots of x^2-2x+2:

$$x=\frac{2\pm\sqrt{4-4(2)}}{2}=1\pm i$$

So,

$P(x)=(x-3i)(x+3i)(x-(1+i))(x-(1-i))$.

112. Since 1+*i* is a zero of $P(x)$, so is its conjugate 1-*i*. As such, $(x-(1+i))(x-(1-i))=x^2-2x+2$ divides $P(x)$ evenly. Indeed, observe that

$$\begin{array}{r} x^2+2x-3 \\ x^2-2x+2\overline{\big)x^4+0x^3-5x^2+10x-6} \\ \underline{-(x^4-2x^3+2x^2)} \\ 2x^3-7x^2+10x \\ \underline{-(2x^3-4x^2+4x)} \\ -3x^2+6x-6 \\ \underline{-(-3x^2+6x-6)} \\ 0 \end{array}$$

So,

$P(x)=(x-(1+i))(x-(1-i))(x-1)(x+3)$.

113.

$$P(x) = x^4 - 81 = \left(x^2 - 9\right)\left(x^2 + 9\right)$$
$$= (x-3)(x+3)(x-3i)(x+3i)$$

114.

$$P(x) = x^3 - 6x^2 + 12x = x\left(x^2 - 6x + 12\right)$$

We need to find the roots of $x^2 - 6x + 12$:

$$x = \frac{6 \pm \sqrt{36 - 4(12)}}{2} = 3 \pm i\sqrt{3}$$

So, $P(x) = x(x - (3 + i\sqrt{3}))(x - (3 - i\sqrt{3}))$.

115. $x^3 - x^2 + 4x - 4 = x^2(x-1) + 4(x-1) = \left(x^2 + 4\right)(x-1) = (x-2i)(x+2i)(x-1)$

116. $P(x) = x^4 - 5x^3 + 12x^2 - 2x - 20$

Factors of -20: ±1, ±2, ±4, ±5, ±10, ±20

Factors of 1: ±1

Possible rational zeros:

±1, ±2, ±4, ±5, ±10, ±20

$$\begin{array}{r|rrrrr}
\underline{-1} & 1 & -5 & 12 & -2 & -20 \\
 & & -1 & 6 & -18 & 20 \\
\hline
\underline{2} & 1 & -6 & 18 & -20 & 0 \\
 & & 2 & -8 & 20 & \\
\hline
 & 1 & -4 & 10 & 0 &
\end{array}$$

So, $P(x) = (x+1)(x-2)\left(x^2 - 4x + 10\right)$.

Next, we find the roots of $x^2 - 4x + 10$:

$$x = \frac{4 \pm \sqrt{16 - 4(10)}}{2} = 2 \pm i\sqrt{6}$$

So,

$P(x) = (x+1)(x-2)(x - (2 + i\sqrt{6}))(x - (2 - i\sqrt{6}))$.

117. Vertical Asymptote: $x = -2$

Horizontal Asymptote:

Since the degree of the numerator equals the degree of the denominator, $y = -1$ is the HA.

118. Vertical Asymptote: $x = 1$

Horizontal Asymptote:

Since the degree of the numerator is less than degree of the denominator, $y = 0$ is the HA.

119. Vertical Asymptote: $x = -1$
Slant Asymptote: $y = 4x - 4$
To find the slant asymptote, we use long division:

$$\begin{array}{r}
4x-4 \\
x+1\overline{)\;4x^2+0x+0} \\
\underline{-(4x^2+4x)} \\
-4x+0 \\
\underline{-(-4x-4)} \\
4
\end{array}$$

No horizontal asymptote

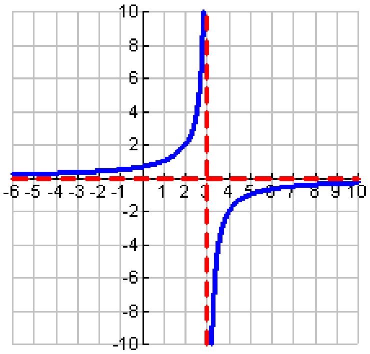
120. Vertical Asymptote: None since $x^2 + 9 \neq 0$
Horizontal Asymptote:
Since the degree of the numerator equals the degree of the denominator, $y = 3$ is the HA.

121. No vertical asymptotes,
Horizontal asymptote: $y = 2$

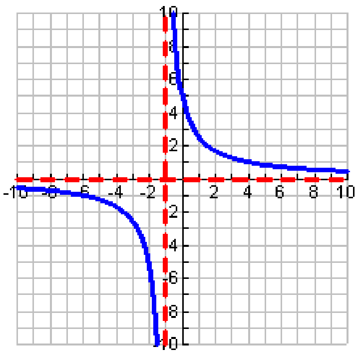
122. vertical asymptotes: $x = -5$
Horizontal asymptote: None
Slant asymptote: $y = -2x + 13$, as seen by the result of this synthetic division:

$$\begin{array}{r|rrr}
-5 & -2 & 3 & 5 \\
 & & 10 & -65 \\
\hline
 & -2 & 13 & -60
\end{array}$$

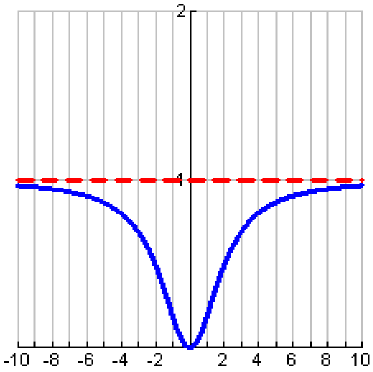
123.



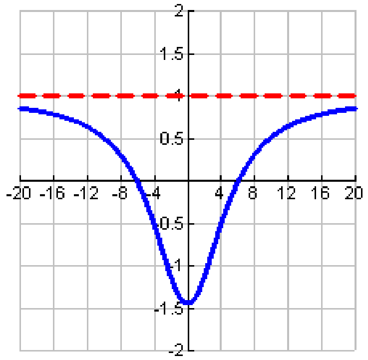
124.



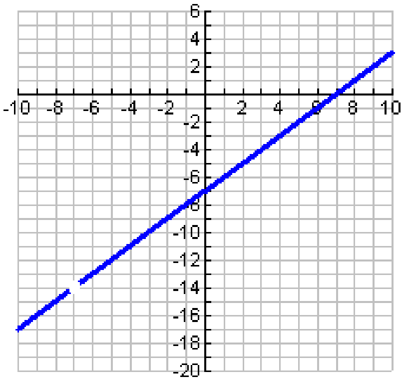
125.



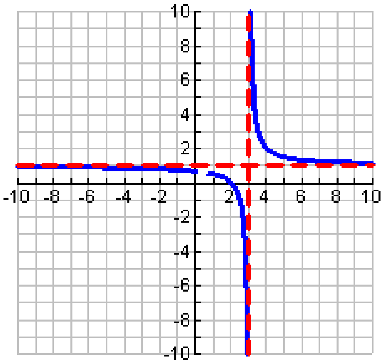
126.



127. Note the hole at $x = -7$.

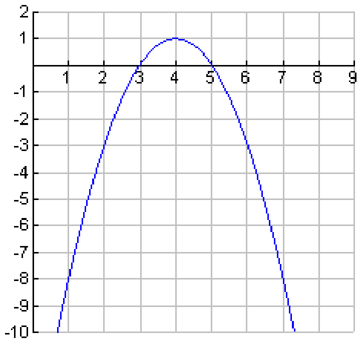


128. Note the hole at $x = -\frac{1}{2}$.



Chapter 2 Practice Test Solutions ---

1.



2.

$$
\begin{aligned}
y &= -\left(x^2 - 4x\right) - 1 \\
&= -(x^2 - 4x + 4) - 1 + 4 \\
&= -(x-2)^2 + 3
\end{aligned}
$$

3.

$$
\begin{aligned}
y &= -\tfrac{1}{2}\left(x^2 - 6x\right) - 4 \\
&= -\tfrac{1}{2}(x^2 - 6x + 9) - 4 + \tfrac{9}{2} \\
&= -\tfrac{1}{2}(x-3)^2 + \tfrac{1}{2}
\end{aligned}
$$

So, the vertex is $(3, \tfrac{1}{2})$.

4. Since the vertex is $(-3,-1)$, the function has the form $y = a(x+3)^2 - 1$. To find *a*, use the fact that the point $(-4,1)$ is on the graph:

$$
\begin{aligned}
1 &= a(-4+3)^2 - 1 \\
1 &= a - 1 \\
2 &= a
\end{aligned}
$$

So, the function is $y = 2(x+3)^2 - 1$.

5. $f(x) = x(x-2)^3(x-1)^2$

6. $f(x)=x\left(x^3+6x^2-7\right)$

a. Zeros: Certainly, 0 is a zero. To find the remaining three, we first try to apply the Rational Root Test:

Factors of -7: $\pm1, \pm7$

Factors of 1: ±1

Possible rational zeros: $\pm1, \pm7$

Observe that 1 is a zero. So, using synthetic division, we compute the quotient $\left(x^3+6x^2-7\right)\div(x-1)$:

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 0 & -7 \\ & & 1 & 7 & 7 \\ \hline & 1 & 7 & 7 & 0 \end{array}$$

So, $f(x)=x(x-1)(x^2+7x+7)$. Now, we solve $x^2+7x+7=0$ using the quadratic formula: $x=\frac{-7\pm\sqrt{49-4(7)}}{2}=\frac{-7\pm\sqrt{21}}{2}$

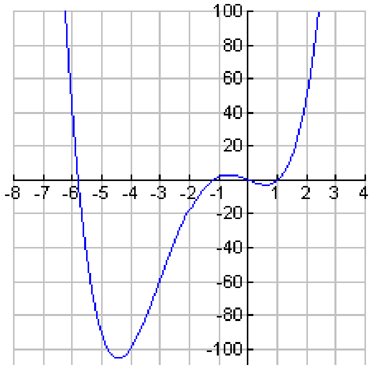
So, there are four distinct x-intercepts.

b. Crosses at all four zeros.

c. y-intercept: $f(0)=0$, so $(0,0)$

d. Long-term behavior: Behaves like $y=x^4$. Even degree and leading coefficient positive, so graph rises without bound to the left and right.

e.



7.

$$\begin{array}{r}
-2x^2-2x-\frac{11}{2} \\
2x^2-3x+1\overline{\big)\,-4x^4+2x^3-7x^2+5x-2} \\
\underline{-(-4x^4+6x^3-2x^2)} \\
-4x^3-5x^2+5x \\
\underline{-(-4x^3+6x^2-2x)} \\
-11x^2+7x-2 \\
\underline{-(-11x^2+\frac{33}{2}x-\frac{11}{2})} \\
-\frac{19}{2}x+\frac{7}{2}
\end{array}$$

So, $\boxed{Q(x)=-2x^2-2x-\frac{11}{2},\quad r(x)=-\frac{19}{2}x+\frac{7}{2}}$.

8.

$$\begin{array}{r|rrrrrr} -2 & 17 & 0 & -4 & 0 & 2 & -10 \\ & & -34 & 68 & -128 & 256 & -516 \\ \hline & 17 & -34 & 64 & -128 & 258 & -526 \end{array}$$

So,

$Q(x) = 17x^4 - 34x^3 + 64x^2 - 128x + 258$,

$r(x) = -526$

9.

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -13 & -1 & 12 \\ & & 3 & 12 & -3 & -12 \\ \hline & 1 & 4 & -1 & -4 & 0 \end{array}$$

Yes, $x-3$ is a factor of $x^4 + x^3 - 13x^2 - x + 12$.

10. $P(-1) = -1 - 2 + 5 - 7 + 3 + 2 = 0$.

So, yes it is a zero.

11.

$$\begin{array}{r|rrrr} 7 & 1 & -6 & -9 & 14 \\ & & 7 & 7 & -14 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

So,

$P(x) = (x-7)(x^2 + x - 2) = (x-7)(x+2)(x-1)$

12. Since $3i$ is a zero, its conjugate $-3i$ must also be a zero. So, we know that $(x-3i)(x+3i) = x^2 + 9$ divides $P(x)$ evenly. This gives us the following, after long division:

$$\begin{array}{r} x^2 - 3x + 10 \\ x^2 + 9 \overline{\smash{\big)}\, x^4 - 3x^3 + 19x^2 - 27x + 90} \\ \underline{-(x^4 + 0x^3 + 9x^2)} \\ -3x^3 + 10x^2 - 27x \\ \underline{-(-3x^3 + 0x^2 - 27x)} \\ 10x^2 + 0x + 90 \\ \underline{-(10x^2 + 0x + 90)} \\ 0 \end{array}$$

So, $P(x) = (x-3i)(x+3i)(x^2 - 3x + 10) = (x-3i)(x+3i)(x-5)(x+2)$.

The zeros are -3*i*, 3*i*, -2, and 5.

13. Yes, a complex zero cannot be an *x*-intercept.

14. Number of sign variations for $P(x)$: 4

$P(-x) = -3x^5 + 2x^4 + 3x^3 + 2x^2 + x + 1$, so

Number of sign variations for $P(-x)$: 1

Since $P(x)$ is degree 5, there are 5 zeros that we classify as:

Positive Real Zeros	**Negative Real Zeros**	**Imaginary Zeros**
4	1	0
2	1	2
0	1	4

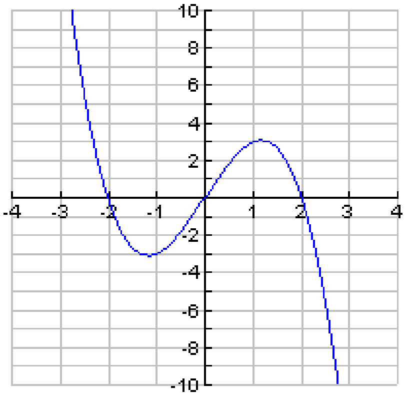
15.

Factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ Factors of 3: $\pm 1, \pm 3$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

16. $P(x) = -x^3 + 4x = -x(x-2)(x+2)$

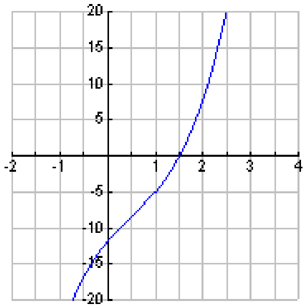
Zeros are $0, \pm 2$.



17.

$$\begin{aligned} P(x) &= 2x^3 - 3x^2 + 8x - 12 \\ &= x^2(2x-3) + 4(2x-3) \\ &= (x^2+4)(2x-3) \\ &= (x+2i)(x-2i)(2x-3) \end{aligned}$$

The only real zero is $\frac{3}{2}$, the other two are complex, namely $\pm 2i$.



18. Factors of 9: $\pm1, \pm3, \pm9$

Factors of 1: ±1

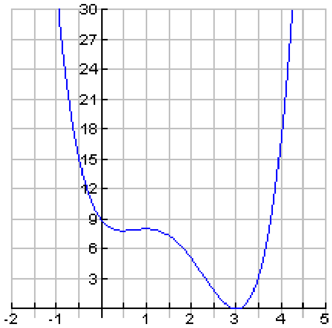
Possible rational zeros: $\pm1, \pm3, \pm9$

Observe that

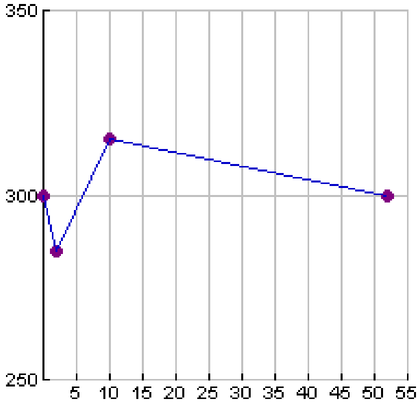
$$\begin{array}{r|rrrrr} 3 & 1 & -6 & 10 & -6 & 9 \\ & & 3 & -9 & 3 & -9 \\ \hline 3 & 1 & -3 & 1 & -3 & 0 \\ & & 3 & 0 & 3 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

So, $P(x) = (x-3)^2(x^2+1)$.

So, the only real zero is 3 (multiplicity 2).



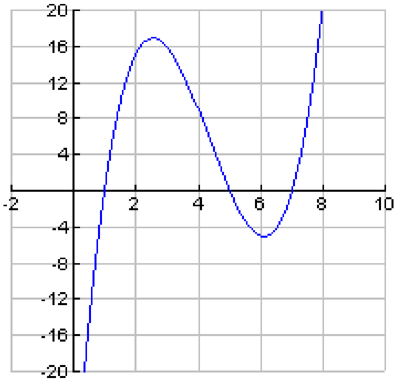
19. The polynomial can have degree 3 since there are 2 turning points. See the graph below:



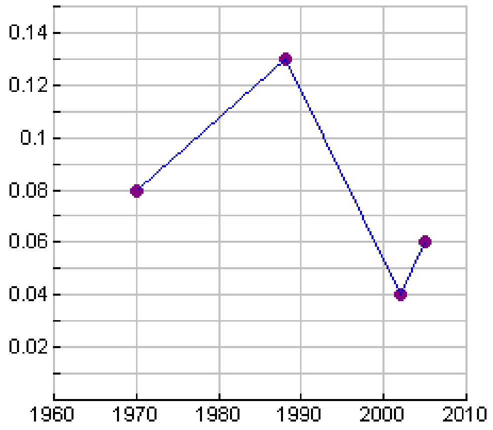
20. We need to solve

$$x^3 - 13x^2 + 47x - 35 = 0.$$

From the graph to the right, we observe that the zeros are 1, 5, and 7. So, you will break even at any of these values.

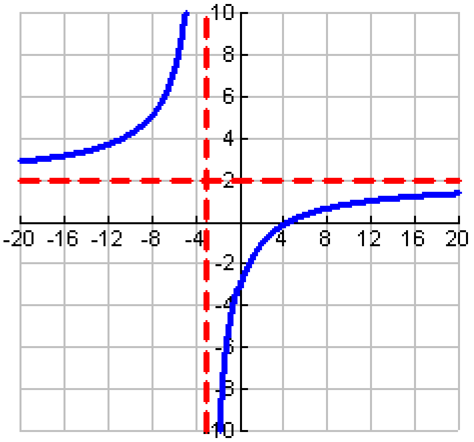


21. Since there are 2 turning points (seen on the graph to the right), the polynomial must be at least degree 3.



22. x-intercept: $(\frac{9}{2},0)$

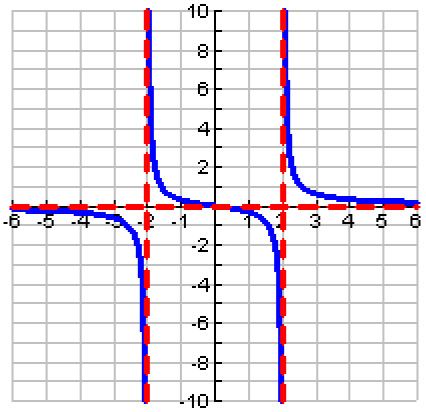
y-intercept: (0,-3)
Vertical Asymptote: $x = -3$
Horizontal Asymptote: $y = 2$
Slant Asymptote: None



23. Observe that

$$g(x)=\frac{x}{x^2-4}=\frac{x}{(x-2)(x+2)}.$$

x-intercept: (0,0)
y-intercept: (0,0)
Vertical Asymptote: $x=\pm 2$
Horizontal Asymptote: $y=0$
Slant Asymptote: None

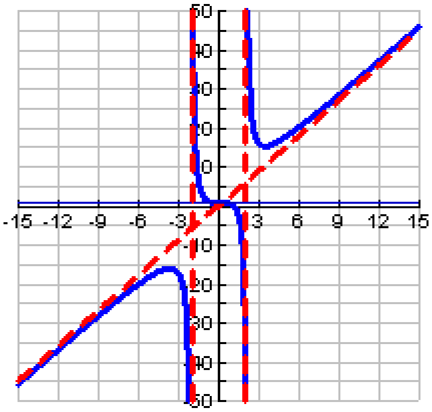


24. Observe that

$$h(x)=\frac{3x^3-3}{x^2-4}=\frac{3(x^3-1)}{(x-2)(x+2)}.$$

x-intercept: (1,0)
y-intercept: $(0,\frac{3}{4})$
Vertical Asymptote: $x=\pm 2$
Horizontal Asymptote: None
Slant Asymptote: $y=3x$, as seen by the following long division:

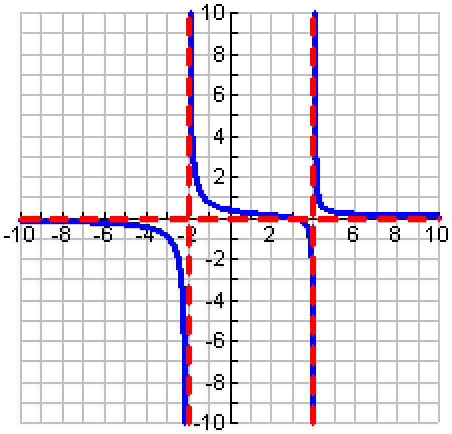
$$\begin{array}{r} 3x \\ x^2+0x-4\overline{\smash{)}3x^3+0x^2+0x-3} \end{array}$$



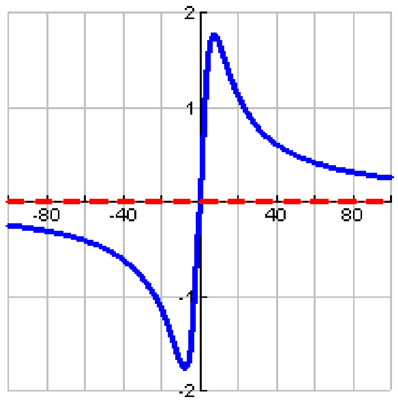
25. Observe that

$$F(x)=\frac{x-3}{x^2-2x-8}=\frac{x-3}{(x-4)(x+2)}.$$

x-intercept: (3,0)
y-intercept: $(0,\frac{3}{8})$
Vertical Asymptote: $x=-2,\ x=4$
Horizontal Asymptote: $y=0$
Slant Asymptote: None



26.



Chapter 2 Cumulative Review ---

1.

$$f(2)=8-\tfrac{1}{\sqrt{4}}=\tfrac{15}{2}$$
$$f(-1)=-4-\tfrac{1}{\sqrt{1}}=-5$$
$$f(1+h)=4(1+h)-\tfrac{1}{\sqrt{1+h+2}}=4+4h-\tfrac{1}{\sqrt{h+3}}$$
$$f(-x)=-4x-\tfrac{1}{\sqrt{-x+2}}=-4x-\tfrac{1}{\sqrt{2-x}}$$

2.

$$f(1)=0^4-\sqrt{5}=-\sqrt{5}$$
$$f(3)=2^4-\sqrt{9}=13$$
$$f(x+h)=(x+h-1)^4-\sqrt{2x+2h+3}$$

3. Note that $f(x)=\dfrac{3x-5}{-(x+2)(x-1)}$.

$$f(-3)=\tfrac{-14}{-(-1)(-4)}=\tfrac{7}{2}$$
$$f(0)=\tfrac{-5}{-(2)(-1)}=-\tfrac{5}{2}$$
$$f(4)=\tfrac{7}{-(6)(3)}=-\tfrac{7}{18}$$

$f(1)$ is undefined

4.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h}&=\frac{\left[4\left(x+h\right)^3-3\left(x+h\right)^2+5\right]-\left[4x^3-3x^2+5\right]}{h}\\&=\frac{4\left[x^3+3x^2h+3xh^2+h^3\right]-3x^2-6xh-3h^2+5-4x^3+3x^2+5}{h}\\&=\frac{h\left(12x^2+12xh+4h^2-6x-3h\right)}{h}=\boxed{12x^2+12xh+4h^2-6x-3h}\end{aligned}$$

5.

$$\frac{f(x+h)-f(x)}{h}=\frac{\left[\sqrt{x+h}-\frac{1}{(x+h)^2}\right]-\left[\sqrt{x}-\frac{1}{x^2}\right]}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}-\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}-\frac{x^2-(x+h)^2}{hx^2(x+h)^2}=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}-\frac{x^2-x^2-2xh-h^2}{hx^2(x+h)^2}$$

$$=\frac{1}{\sqrt{x+h}+\sqrt{x}}+\frac{2x+h}{x^2(x+h)^2}$$

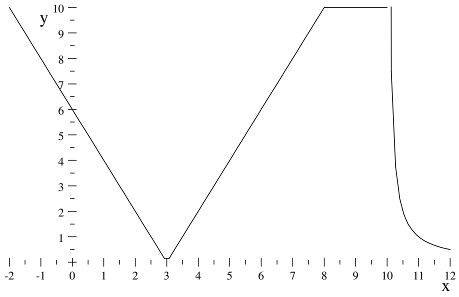
6.

$$f(-5)=0,\quad f(0)=0,$$
$$f(3)=3(3)+3^2=18$$
$$f(4)=3(4)+4^2=28$$
$$f(5)=\left|2(5)-5^3\right|=115$$

7.

Domain	$(-\infty,10)\cup(10,\infty)$
Range	$[0,\infty)$
Increasing	$(3,8)$
Decreasing	$(-\infty,3)\cup(10,\infty)$
Constant	$(8,10)$

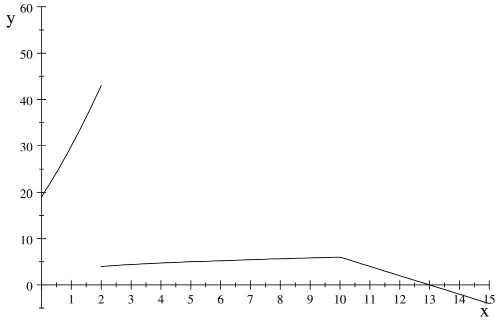
The graph of *f* is:



8.

Domain	$(-\infty,14]$
Range	$[-6,\infty)$
Increasing	$(-5,10)$
Decreasing	$(-\infty,-5)\cup(10,14)$
Constant	None

The graph of *f* is:



9.

$$\frac{y(9)-y(5)}{9-5}=\frac{\frac{2(9)}{9^2+3}-\frac{2(5)}{5^2+3}}{4}=\frac{\frac{18}{84}-\frac{10}{28}}{4}=\boxed{-\frac{1}{28}}$$

10. Need $6x-7\geq 0$, so that $x\geq \frac{7}{6}$. So, the domain is $\left[\frac{7}{6},\infty\right)$.

11. Observe that

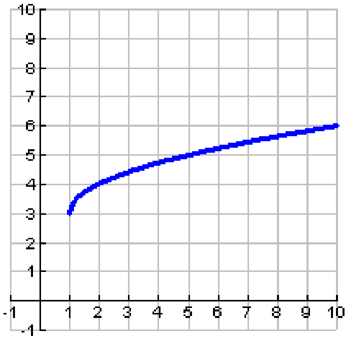
$$g(-x)=\sqrt{-x+10}\neq g(x)$$

$$-g(-x)=-\sqrt{-x+10}\neq g(x)$$

So, neither.

12. Shift the graph of $y=x^2$ left 1 unit, then reflect over the *x*-axis, and then move up 2 units.

13. Translate the graph of $y=\sqrt{x}$ to the right 1 unit and then up 3 units.



14.

$$f(g(x))=\left(\sqrt{x+2}\right)^2-3=x+2-3=x-1$$

Domain: $[-2,\infty)$

15.

$f(-1)=7-2(-1)^2=5$

$g(f(-1))=2(5)-10=0$

16. To find the inverse, switch the *x* and *y* and solve for *y*:

$$x=(y-4)^2+2 \quad\Rightarrow\quad x-2=(y-4)^2 \quad\Rightarrow\quad y=4\pm\sqrt{x-2}$$

Since we are assuming $y\geq 4$, use the positive root above. So, $f^{-1}(x)=4+\sqrt{x-2},\ x\geq 2$.

17. Since the vertex is (-2,3), the equation so far is $f(x)=a(x+2)^2+3$. Use (-1,4) to find *a*:

$$4=a(-1+2)^2+3=a+3 \;\Rightarrow\; a=1$$

So, $f(x)=(x+2)^2+3$.

18. $f(x)=-3.7x^3(x+4)$

So, the zeros are 0 (multiplicity 3) and -4 (multiplicity 1)

19. Observe that

$$\begin{array}{r} 4x^2+4x+1 \\ -5x+3\overline{)\,-20x^3-8x^2+7x-5} \\ \underline{-(-20x^3+12x^2)} \\ -20x^2+7x \\ \underline{-(-20x^2+12x)} \\ -5x-5 \\ \underline{-(-5x+3)} \\ -8 \end{array}$$

So, $Q(x)=4x^2+4x+1,\ r(x)=-8$

20. Observe that

$$\begin{array}{r|rrrr} 3 & 2 & 3 & -11 & 6 \\ & & 6 & 27 & 48 \\ \hline & 2 & 9 & 16 & 54 \end{array}$$

So, $Q(x)=2x^2+9x+16,\ r(x)=54$.

21. $P(x)=12x^3+29x^2+7x-6$

Factors of -6: $\pm1, \pm2, \pm3, \pm6$

Factors of 12: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Possible rational zeros:

$$\pm1, \pm2, \pm3, \pm6, \pm\tfrac{1}{2}, \pm\tfrac{1}{3},$$
$$\pm\tfrac{1}{4}, \pm\tfrac{1}{6}, \pm\tfrac{1}{12}, \pm\tfrac{2}{3}, \pm\tfrac{3}{2}, \pm\tfrac{3}{4}, \pm\tfrac{1}{4}$$

Note that

$$\begin{array}{r|rrrr} -2 & 12 & 29 & 7 & -6 \\ & & -24 & -10 & 6 \\ \hline & 12 & 5 & -3 & 0 \end{array}$$

So, $P(x)=(x+2)(12x^2+5x-3)$
$\qquad\quad =(x+2)(4x+3)(3x-1)$

So, the zeros are $-2, -\tfrac{3}{4}, \tfrac{1}{3}$.

22. Since 5 is a zero, we know that $x-5$ divides $P(x)$ evenly. So,

$$\begin{array}{r|rrrr} 5 & 2 & -3 & -32 & -15 \\ & & 10 & 35 & 15 \\ \hline & 2 & 7 & 3 & 0 \end{array}$$

So,

$$\begin{aligned} P(x)&=(x-5)(2x^2+7x+3) \\ &=(x-5)(2x+1)(x+3) \end{aligned}$$

So, the zeros are $-3, -\tfrac{1}{2}, 5$.

23. Possible rational zeros are:

$$\pm 1, \pm 2, \pm 4, \pm 8$$

Using synthetic division yields

$$\begin{array}{r|rrrr} \underline{-1} & 1 & -5 & 2 & 8 \\ & & -1 & 6 & -8 \\ \hline \end{array}$$

$$\begin{array}{r|rrr} \underline{2} & 1 & -6 & 8 \\ & & 2 & -8 \\ \hline & 1 & -4 & \end{array}$$

So, $P(x) = (x+1)(x-2)(x-4)$.

24. Possible rational zeros are:

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

Using synthetic division yields

$$\begin{array}{r|rrrrrr} \underline{1} & 1 & 7 & 15 & 5 & -16 & -12 \\ & & 1 & 8 & 23 & 28 & 12 \\ \hline \end{array}$$

$$\begin{array}{r|rrrrr} \underline{-1} & 1 & 8 & 23 & 28 & 12 \\ & & -1 & -7 & -16 & -12 \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} \underline{-3} & 1 & 7 & 16 & 12 \\ & & -3 & -12 & -12 \\ \hline & 1 & 4 & 4 & \end{array}$$

So,

$$\begin{aligned} P(x) &= (x-1)(x+1)(x+3)\left(x^2+4x+4\right) \\ &= (x-1)(x+1)(x+3)(x+2)^2 \end{aligned}$$

25. Vertical asymptotes: $x = \pm 2$ Horizontal asymptote: $y = 0$

26. Observe that

$$f(x) = \frac{x\left(2x^2-x-1\right)}{(x-1)(x+1)} = \frac{x(2x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$$

Open hole: $(1, \frac{3}{2})$

x-intercepts: (0,0), $(-\frac{1}{2}, 0)$

y-intercept: (0,0)

Vertical Asymptote: $x = -1$

Horizontal Asymptote: None

Slant Asymptote: $y = 2x - 1$

$$\begin{array}{r|rrr} \underline{-1} & 2 & 1 & 0 \\ & & -2 & 1 \\ \hline & 2 & -1 & 1 \end{array}$$