CHAPTER

2

Kinematics in One Dimension

Of all the intellectual hurdles which the human mind has confronted and has overcome in the last fifteen hundred years, the one which seems to me to have been the most amazing in character and the most stupendous in the scope of its consequences is the one relating to the problem of motion.

Herbert Butterfield—*The Origins of Modern Science*

Recommended class days: 3 minimum, 4 preferred

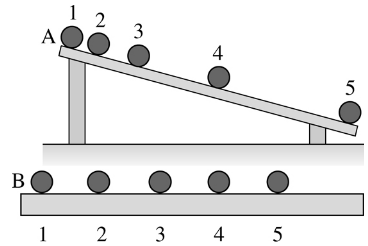
Background Information

Chapter 2 is a large and difficult chapter. Although to physicists the chapter says nothing more than $v = dx/dt$ and $a = dv/dt$, these are symbolic expressions for difficult, abstract concepts. Student ideas about force and motion are largely non-Newtonian, and they cannot begin to grasp Newton's laws without first coming to a better conceptual understanding of motion.

As Butterfield notes in the above quote, the "problem of motion" was an immense intellectual hurdle. Galileo was perhaps the first to understand what it means to *quantify* observations about nature and to apply mathematical analysis to those observations. He was also the first to recognize the need to separate the *how* of motion—kinematics—from the *why* of motion—dynamics. These are very difficult ideas, and we should not be surprised that kinematics is also an immense intellectual hurdle for students.

Student difficulties with kinematics have been well researched (Trowbridge and McDermott, 1980 and 1981; Rosenquist and McDermott, 1987; McDermott et al., 1987, Thornton and Sokoloff, 1990). Arons (1990) gives an excellent summary and makes many useful suggestions for teaching kinematics. Student difficulties can be placed in several categories.

Difficulties with concepts: Students have a rather undifferentiated view of motion, without clear distinctions between position, velocity, and acceleration. Chapter 1 will have given them a start at making these distinctions, but they'll need additional practice.



In one study, illustrated in the figure above, students were shown two balls on tracks. Ball A is released from rest and rolls down an incline while ball B rolls horizontally at constant speed. Ball B overtakes ball A near the beginning, as the motion diagram shows, but later ball A overtakes ball B. Students were asked to identify the time or times (if any) at which the two balls have the same speed. Prior to instruction, roughly half the students in a calculus-based physics class identify frames 2 and 4, when the balls have equal *positions*, as being times when they have equal speeds.

Similarly (see references for details), students often identify situations in which two objects have the same velocity as indicating that the objects have the same acceleration. Confusion of velocity and acceleration is particularly pronounced at a turning point, where a majority of students think that the acceleration is zero. McDermott and her co-workers found that roughly 80% of students beginning calculus-based physics make errors when asked to identify or compare accelerations, and that the error rate was still roughly 60% after conventional instruction. Thornton and Sokoloff (1990) report very similar pre-instruction and post-instruction error rates for students' abilities to interpret graphs of velocity and acceleration versus time.

In addition:

- Students have a very difficult time with the idea *of instantaneous* quantities.
- Students are often confused by the significance of positive and negative signs, especially for velocity and acceleration. Many students interpret positive and negative accelerations as *always* meaning that the object is speeding up or slowing down. This seems to be an especially difficult idea to change.

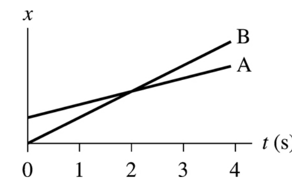
Difficulties with graphs: Nearly all students can graph a set of data or can read a value from a graph. Their difficulties are with *interpreting* information presented graphically. In particular:

- Many students don't know the meaning of "Graph *a*-versus-*b*." They graph the first quantity on the horizontal axis, ending up with the two axes reversed.
- Many students think that the slope of a straight-line graph is found from y/x (using any point on the graph) rather than $\Delta y/\Delta x$.
- Students don't recognize that a slope has *units* or how to determine those units.
- Many students don't understand the idea of the "slope at a point" on a curvilinear graph. They cannot readily compare the slopes at different points.
- Very few students are familiar with the idea of "area under a curve." Even students who have already started calculus, and who "know" that an integral can be understood as an area, have little or no idea how to use this information if presented with an actual curve.
- Many students interpret "slope of a curve" or "area under a curve" literally, as the graph is drawn, rather than with reference to the scales and units along the axes. To them, a line drawn at 45° *always* has a slope of 1 (no units), and they may answer an area-under-the-curve question with "about three square inches."
- Students don't recognize that an "area under the curve" has *units* or how the units of an "area" can be something other than area units. We tell them, "Distance traveled is the area under the *v*-versus-*t* curve." But distance is a length? How can a length equal an area?

A recitation hour spent interpreting and using graphs is an hour well spent for all students.

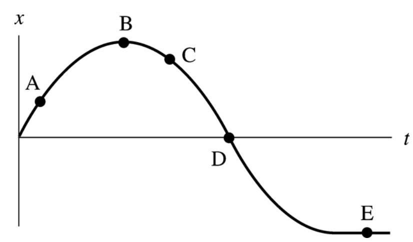
Difficulties relating graphs to motion: Nearly all students have a very difficult time relating the *physical* ideas of motion to a *graphical representation* of motion. If students observe a motion—a ball rolling down an incline, for example—and are then asked to draw an x-versus-t graph, many will draw a *picture* of the motion as they saw it. Confusion between graphs and pictures underlies many of the difficulties of relating graphs to motion.

Part of the difficulty is that we measure position along a *horizontal* axis (for horizontal motion), but then we graph the position on a *vertical* axis. This choice is never explained, as it seems obvious to physicists, but it's a confusing issue for students who aren't sure what a function is or how graphs are interpreted.

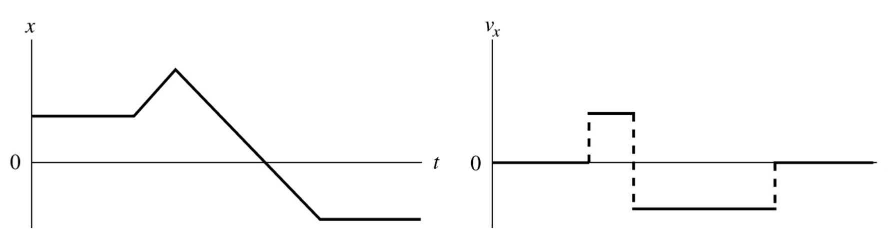


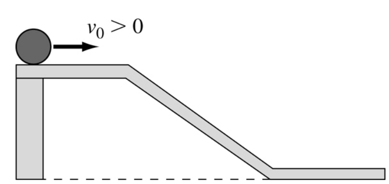
Confusion between position and velocity, and difficulty interpreting slopes, is seen with a simple example. Here is a graph that shows the motion of two objects A and B. Students are asked: Do A and B ever have the same speed? If so, at what time? A significant fraction will answer that A and B have the same speed at $t = 2$ s, confusing a common height with common slope.

In another exercise, students are shown the following position-versus-time graph and asked at which lettered point or points is the object moving fastest, at rest, slowing down, etc. Students initially have difficulty with such exercises because they can't interpret the meaning of the graph. Fortunately, most students can master questions similar to these with a small amount of instruction and practice.



A much more difficult problem for most students, and one that takes more practice, is changing from one type of graph to another. For example, students might be given the x-versus-t graph shown below on the left and asked to draw the corresponding v_x-versus-t graph. When first presented with such a problem, almost no students can generate the correct velocity graph shown on the right. Many feel that a "conservation of shape" law applies and redraw the position graph—perhaps translated up or down—as a velocity graph. They need a careful explanation, through several examples, of how the *slope* of the position graph becomes the *value* of the velocity graph at the same t. Changing from a velocity graph back to a position graph is even more difficult.





These examples require giving physical meaning to the slope of and area under curves, but they are still somewhat removed from the actual *situation* in which the motion occurs. To tie all aspects of a student's understanding of kinematics together, McDermott and her group presented students with situations of a ball rolling along a series of level and inclined tracks. The students are then asked to draw x-versus-t, v_x-versus-t, and a_x-versus-t graphs of the motion, with the graphs stacked vertically so that a vertical line connects equal values of t on each of the three graphs. Students in a conventional physics class were presented—after kinematics instruction—with the simple track shown in the figure. Only 1 student of 118 gave a completely correct response. Many students draw wildly incorrect graphs for questions like these, indicating an inability to translate from a visualization of the motion to a graphical description of the motion.

Difficulties with terminology: Arons (1990) has written about student difficulties with the term *per.* Many students have difficulty giving a verbal explanation of what "20 meters per second" *means*—especially for an instantaneous velocity that is only "20 meters per second" for "an instant." Students will often say things such as "acceleration is delta v over delta t," but they frequently *don't* use the word "over" in the sense of a ratio but rather to mean "during the interval."

Another difficult terminology issue for students is our use of the words *initial* and *final.* Sometimes we use *initial* to mean the initial conditions with which the problem starts, and *final* refers to the end of the problem. But then we use $\Delta x = x_{\text{final}} - x_{\text{initial}}$ and $\Delta v = v_{\text{final}} - v_{\text{initial}}$ when we're looking at how position and velocity change over *small* intervals Δt. Students often don't recognize the distinction between these uses.

Finally, students often don't make the same *assumptions* we do about the beginning and ending points of a problem. We interpret "Bob throws a ball at 20 m/s..." as a problem that starts with Bob releasing the ball. Students often want to include his throw as part of the problem. Similarly, a question to "find the final speed of a ball dropped from a height of 10 m" will get many answers of "zero," because that really is the *final* speed. These are not insurmountable issues, but you need to be aware that students don't always interpret a problem statement as a physicist would.

Difficulties with mathematics: Many students, especially if they are starting calculus concurrently, are not sure what a *function* is. They don't really understand the notation $x(t)$ or our discussion of "position as a function of time." A not insignificant fraction of students interpret $x(t)$ as meaning *x times t*, as it would in an expression such as $a(b + \mathrm{c})$. Instructors need to give explicit attention to this issue.

Students are easily confused with changes in notation. Math courses tend to work with functions $y(x)$, with x the independent variable. This includes graphing y-versus-x and taking derivatives dy/dx. In physics, we use functions $x(t)$, with x the dependent variable. We make x-versus-t graphs and take derivatives dx/dt. Despite how trivial this seems, instructors should be aware that many students are confused by the different notation and need assistance with this.

Finally, students at this stage often lack an operational understanding of differentials and integrals. They're not perturbed by writing expressions such as $dx = x^2$, in which they equate an infinitesimal to a finite expression. When faced with an integral such as $\int v dt$, students are likely to pull the v out of the integral, as if it were as constant, rather than recognize that v is an implicit function of t. Physics can help them solidify their understanding and use of calculus, but you should be cautious about assuming that students have a working knowledge of calculus.

Student Learning Objectives

- To differentiate clearly between the concepts of position, velocity, and acceleration.
- The interpret kinematic graphs.
- To translate kinematic information between verbal, pictorial, graphical, and algebraic representations.
- To learn the basic ideas of calculus (differentiation and integration) and to utilize these ideas both symbolically and graphically.
- To understand free-fall motion.
- To begin the development of a robust problem-solving strategy.
- To solve quantitative kinematics problems and to interpret the results.

Pedagogical Approach

This chapter treats one-dimensional motion only. Although the basic kinematic quantities x, v_x, and a_x (or y, v_y, and a_y) are components of vectors, a full discussion of vectors is not needed for one-dimensional motion. Indeed, the term *component* is not introduced until Chapter 3. The major issue is whether each of these quantities is positive or negative, and that only depends on whether the vector $\vec{r}, \vec{v},$ or $\vec{a}$ points in the positive or the negative direction. This is easily determined with a motion diagram. Tactics Box 1.4 summarized the signs of these quantities, but students made minimal use of this information in Chapter 1. They now need practice associating a verbal description of the motion with the proper signs, especially for acceleration.

Note: In this textbook, $v = |\vec{v}|$ is the magnitude of the velocity vector, or speed, and $a = |\vec{a}|$ is the magnitude of the acceleration. Component of vectors, such as v_x or a_y, always use explicit x- and y-subscripts. Not surprisingly, students can easily be confused by the rather common practice in one-dimensional motion of using v both for velocity (a signed quantity) and for speed.

We want students to recognize vertical motion, horizontal motion, and even motion along an incline as just variations of "one-dimensional motion." Consequently, the text often uses a generic symbol s to represent position. Examples then use x for horizontal motion and y for vertical motion, but instructors are encouraged to use s when writing kinematic equations that don't refer to a specific situation or direction.

This chapter introduced two important models: uniform motion and motion with constant acceleration. It's important to emphasize—especially when working example problems—where you're making simplifying assumptions. Few objects exhibit true uniform motion or constant acceleration, but it's often reasonable to *model* their motion this way. Not many students are familiar with the crucial role that assumptions and modeling play in physics, so it's important to be explicit about this rather than hoping that students will pick it up on their own.

A major goal of this chapter is to provide both the conceptual foundations of kinematics and a systematic approach to analyzing problems. To this end, the text emphasizes *multiple representations of knowledge.* In particular, motion has the following descriptions:

- **Verbal**, as presented in typical end-of-chapter problems.
- **Pictorial,** including (1) motion diagrams and (2) a sketch showing beginning and ending points as well as coordinates and symbols.
- **Graphical**, as shown in position-, velocity-, and acceleration-versus-time graphs.
- **Mathematical**, through the relevant equations of kinematics.

To acquire an accurate, intuitive sense of motion, students must learn to move back and forth between these different representations. Much of this chapter is focused on learning the different representations of kinematic knowledge.

The connection between motion diagrams and graphs is strongly emphasized. Students learned motion diagrams in Chapter 1, and they should now be able to draw a correct motion diagram for nearly any one-dimensional motion. This is a good intermediate stage in the process of interpreting a verbal description of motion. Students can see where velocities are big or small and where the motion speeds up or slows down. As they proceed into the less familiar territory of drawing graphs, you can keep calling their attention to whether or not the graph is consistent with the motion diagram. This approach is particularly useful for establishing correct signs.

The ultimate goal, of course, is for students to be able to work kinematics problems. There is now good evidence that initial attention to these conceptual issues leads students to become *better* problem solvers.

Using Class Time

A minimum of three days is needed to cover this chapter if students are to have an adequate opportunity to practice the many ideas. A fourth day of additional practice problem solving can really help to cement these important ideas that will be used throughout the course. The fourth day is highly recommended if your students are starting calculus concurrently with physics.

Day 1: The Chapter Preview introduces the "Looking Back" feature that recommends specific previous sections for review. New to the 4th edition, Looking Back references are also given in the body of the text on an as-needed basis; examples are in Signs and Units on p. 44. It's worth calling attention to this feature and recommending its use. Although it seems like extra work, suggest to students that a brief review will actually *save* them time by making the current chapter easier to understand.

Chapter 2 is the first serious test of an instructor's intent to use an active-learning teaching style. The temptation to start lecturing about slopes and derivatives is strong, but I urge you to jump right in with questions and problems for the students to work on. You can make the necessary points about slopes, derivatives, and other matters as you go over the answers and underlying reasoning of the questions.

A particularly important point to make as you go along is the role of Δ. Students tend to make no distinction between position and displacement (x and Δx) or between velocity and change of velocity (v and Δv). Half-remembered formulas from high school, such as $v = d/t$, are often more hindrance than help for coming to a solid understanding of kinematics. Even many college texts don't distinguish between t, an instant of time, and Δt, an interval of time. Equations such as $x = x_0 + v_x t$ are actually using t to represent an interval, not an instant. This text consistently uses expressions such as $x = x_0 + v_x \Delta t$ to make the meaning of symbols clear.

Kinematics gets off to a faster start if students have *already* had the opportunity to measure the motion of their own bodies in a microcomputer-based laboratory. Otherwise, it's good to start with a number of examples in which you ask students to draw a position-versus-time graph for an object they see moving, then draw the corresponding velocity-versus-time graph.

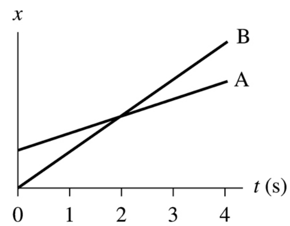
It's good to establish a coordinate system across the front of the class, with a well-defined origin and with the "x-axis" pointing to the students' right. Ask a student to start at the origin, then walk across the room (left to right) at *constant* speed. Have the students first draw a motion diagram, then an x-vs-t graph, and finally a v_x-vs-t graph. This will give you an opportunity to talk about slopes and to note that the velocity vectors in the motion diagram are all equal length, pointing to the right. Then repeat the process, with the student

- walking right to left at constant speed, ending at the origin.
- starting at a negative value of x, then walking to the right (or left) at constant speed.

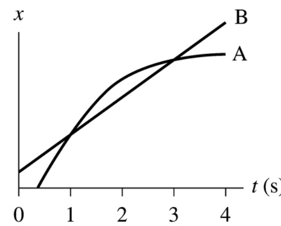
These will provide an opportunity to discuss the role of signs for both x and v_x.

Next have a student, starting at the origin, *slowly* speed up until moving very fast at the far side of the room. (At this time, it's best to talk about *speeding up* and *slowing down* rather than to introduce the term *acceleration*.) Again, use motion diagrams, position graphs, and velocity graphs to illustrate the idea of instantaneous velocity. (For simplicity, consider the position graph to be parabolic and the velocity graph to be linear.) A good analogy is to ask what a speedometer would read at different points in the motion, if the student were carrying one.

Finally, have a student start very slowly on the far side of the room, gradually speed up while moving to the left, and reach the origin at top speed. Students find this one *much* more difficult, especially the proper shape of the position-versus-time graph. Focusing on the motion diagram helps. Time permitting, you can also demonstrate slowing down.

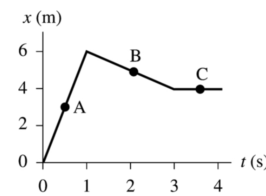


Do A and B ever have the same speed? If so, at what time or times?



Do A and B ever have the same speed? If so, at what time or times?

Once students seem to have the basic idea, the two questions shown above are effective. For each, the issue is whether A and B ever have the same speed, and if so, when? Students who haven't practiced graph interpretation tend to confuse the crossing points (equal position) with points of equal speeds. The practice they've just completed should have most of them thinking about slopes, so error rates shouldn't be too high, but this exercise reinforces the message and catches a few more who are still confusing height with slope.



Rank in order the speeds at points A, B, and C, from the fastest to the slowest.

Another good question to pose is shown in the figure at the right. First, ask students to give a *verbal* description of the motion. Then, ask them to rank in order the speeds at points A, B, and C, from fastest to slowest. Finally, ask them to draw a velocity-versus-time graph—*with a proper numerical scale.* Computing the slope at B will prove to be difficult for many students.

The following exercise illustrates the meaning of *per*, and it is a prelude to a similar acceleration exercise on day 2.

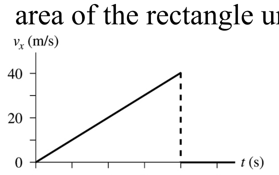
> A train is moving at a steady 30 m/s. At $t = 0$ s, the engine passes a signal light at $x = 0$ m. *Without using any formulas*, find the engine's position at $t = 1$ s. Also at $t = 2$ s and $t = 3$ s.

The objective is for students to realize the *meaning* of "30 meters per second" to be that "x increases by 30 meters during each second." The position increases by 30 m during the first second, to $x_1 = 30$ m, by 30 m more during the next second, to $x_2 = 60$ m, and so on. Some students will find this so obvious as to be trivial, but others will find this a difficult way to reason.

Have students graph both position and velocity for the train, then call their attention to the fact that the displacement Δx is exactly the same as the area under the velocity curve. The constant-velocity equation $s = s_0 + v_s \Delta t$ is merely giving algebraic expression to their observation that

$$\Delta s = \text{area under the velocity-versus-time curve.}$$

You can direct them to the text for a proof that the graphical result Δs = area is true for *any* velocity, not just constant velocity. But $\Delta s = v_s \Delta t$ applies only to *constant* velocity since $v_s \times \Delta t$ is clearly the area of the rectangle under a horizontal line.



Find the position at times $t = 1, 2, 3, 4$, and 5 seconds. Assume $x_0 = 0$ m.

For a nonconstant velocity, you can give them a graph like the one shown on the left and ask them to find—using the graph, not a kinematics equation—the position at $t = 1, 2, 3, 4$, and 5 s. Then have them draw a position-versus-time graph. Now they've practiced going forward, from position to velocity, and backward, from velocity to position.

Day 2: An excellent exercise to start day 2, and review the ideas of day 1, is the following exercise taken from the *Student Workbook*:

> Trucker Bob starts the day 120 miles west of Denver. He drives east for 3 hours at a steady 60 miles/hour before stopping for his coffee break. Draw a position-versus-time graph for Bob,

> including appropriate numerical scales along both axes. Let Denver be located at $x = 0$ and assume that the x-axis points to the east.

Although this seems straightforward, I've found that only a small fraction of students can draw an appropriate graph. Seeing the types of errors they make and responding to their questions and concerns can lead into an excellent class discussion about the graphical representation of motion.

It's then a good opportunity for an example problem that requires all the steps in the problem-solving strategy. Encourage students to use a Dynamics Worksheet from the back of the Student Workbook as they work along with you. The Workbook has only a few sample worksheets; more copies can be downloaded via the "Resources" tab in the textbook's Instructor Resource Center (www.pearsonhighered.com/educator/catalog/index.page) or from the textbook's Instructor Resource Area in MasteringPhysics® (www.masteringphysics.com). If you intend to use worksheets—highly recommended for developing problem-solving skills—either have students photocopy more, download and print worksheets for them, or provide them with the PDF to print their own. A good first problem might be.

> Sally opens her parachute at an altitude of 1500 m. She then descends slowly to earth at a steady speed of 5 m/s. How long does it take her to touch down?

The goal is to illustrate the problem-solving strategy, hence the problem itself is so simple that all students can easily do the numerical part. Start with a pictorial representation that establishes a coordinate system and defines symbols. Then draw a motion diagram. Finally—and for the first time—use the Mathematical Representation section of the worksheet to solve the problem. Call attention to the fact that all the symbols used in the mathematical solution, such as y_0 or t_1, were identified and defined in the pictorial representation. End by having them assess whether or not the result is "reasonable."

Note: Physicists often like to use a coordinate system for vertical-motion problems with the y-axis pointing down. This avoids a few negative signs. However, many students find this more confusing than helpful. The text consistently uses an upward-pointing y-axis for kinematics, matching the coordinate system we'll need later for gravitational potential energy.

After spending about a day and a half on velocity, it's time to explore acceleration. To set the stage, toss a ball straight up and down a few times. First ask about the velocity. As the ball rises, is v_y positive, negative, or zero? As it falls? Then focus on the turning point at the top. Nearly all students will now agree that $v_y = 0$ at the top, but it allows you to reinforce the idea of an instantaneous velocity. This is a good place to define a *turning point* and note that the instantaneous velocity is always zero at a turning point.

Then ask if the *acceleration* at the top point is positive, negative, or zero. After giving them a minute to think about it, and perhaps discuss it with a neighbor, ask for a show of hands (or make this a clicker question). In nearly all classes, a large majority thinks that the acceleration is zero.

Note: An especially important aspect of having students make a prediction is that they now have a vested interest in the outcome. This is a much better learning experience than simply seeing you demonstrate and explain something.

Rather than directly discussing the answer, tell the class you're going to let the question be answered experimentally, but that you'll need to build up to it in several steps. Then turn to a demonstration of a ball or a cart rolling down a small incline in the positive x-direction. It's important to keep the speed slow so students can observe that the velocity increases continuously. (It's best to start with the object moving to the right, so that v_x is positive.) An ultrasonic motion detector (at the top of the ramp) interfaced to a computer is an especially useful tool for showing that the velocity is increasing linearly with time.

On day 1, students associated velocity with a changing position and found that velocity is the slope of the position graph. Now, by analogy, you can associate acceleration with a changing velocity and the slope of the velocity graph. Remind students that they can judge velocity fairly easily when observing an object, but that it's much more difficult to judge acceleration. That's why the motion diagrams and graphical tools are so important.

After rolling the ball or cart down and making graphs, roll it *up* the same incline, now moving in the negative x-direction, but catch it at the top before it reverses. Although many students will now recognize that this is a negative v_x that "increases" toward zero as it slows, you'll want to be quite explicit about the reasoning for those students who are still struggling with the proper signs. After you've drawn position and velocity graphs (or had them produced by the computer), ask *them* if the acceleration a_x is positive, negative, or zero. Although you've just talked about the fact that acceleration is the slope of the velocity graph, and they're looking at a velocity graph, a large fraction of the class is likely to respond that the acceleration is negative because of their belief that positive and negative accelerations mean speeding up and slowing down.

You can use the slope of the velocity graph to draw an acceleration graph, appeal to the logical argument that v_x is becoming more positive as the object slows, and use motion diagrams to show that $\vec{a}$ points in the positive x-direction. It's worthwhile to look for two or three other opportunities to have students consider the sign of the acceleration in situations where their speeding up/slowing down reasoning will fail. This is not a belief that is quickly or easily changed.

Finally, roll the object up the incline and let it roll back down. Note that the turning point is just like the turning point of the ball you had tossed in the air, and that now you're ready to answer the question. This is very nicely done with a motion detector by measuring the velocity and seeing that it linearly increases from negative to positive values, passing through zero (the turning point) with no change of slope. (Actually, carts on a track often do have a small change of slope due to the friction force changing direction. You'll need to explain this, but you can easily note that the slope never becomes zero.) An acceleration graph then shows that the acceleration is uniform throughout the entire motion, with nothing to distinguish the turning point. You'll also want to note that the object wouldn't be able to move away from the turning point if both v_x and a_x were zero.

Motion detectors are especially good for showing that objects fall with a *constant* acceleration and that the acceleration is *independent of the mass.* You can show the mass independence with carts of different mass on an incline. (First ask them to predict whether the acceleration of a heavier cart will be less than, greater than, or equal to the mass of a lighter cart.)

You can also demonstrate free fall with a motion detector placed face up on the floor or on the lecture table if you build a protective cage around the probe. Dropping a ball onto the probe is easy. With a little more care, you can toss a ball upward over the probe and follow the motion up and down. Without a motion detector, there's not enough time in lecture to make the measurements that would be required to demonstrate that free-fall motion is one of constant acceleration, so you're forced to assert this without proof.

Students, for some reason, have a strong tendency to call g by the name "gravity." It is worth emphasizing that g is "the free-fall acceleration" and requiring them to use the term correctly. You'll also want to emphasize that g is always a *positive* value. The acceleration is negative, given by $a_y = -g$, but g itself is positive.

Day 3: Two full days have been used on conceptual and graphical topics. Although this seems an inefficient use of time, since you're ultimately going to test students on their problem-solving skill, these two days are extremely important for building the conceptual foundations that underlie good problem-solving ability. Most students cannot move beyond simple plug-and-chug problems until they develop a better conceptual understanding of motion.

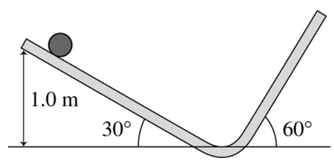
But now that the conceptual foundations have been laid, it's time to start kinematics problems. Deriving kinematics equations is not an effective use of class time; students should have read the derivations in the textbook. However, you can use examples to reinforce the textbook derivations. A good exercise is to tell students that a jet plane accelerates at 3 m/s^2 during take-off, then ask them *without using any equations or their calculators* the plane's velocity at $t = 1, 2, 3$, and

4 s. You want them to reason that 3 m/s^2 = (3 m/s) per second, so the velocity increases by 3 m/s every second. (This is analogous to the 30 m/s train exercise on day 1.) The velocity graph is linear, and you can then use area under the curve to find (and graph) the position at t = 1, 2, 3, and 4 s. You want students to recognize that $\Delta s = \frac{1}{2} a_s (\Delta t)^2$ for constant acceleration can be understood from the geometry of the graphs, that it's not "just" a result derived from calculus.

Here are four good examples. In working them, I encourage you to use the full step-by-step approach of the Dynamics Worksheets and to be very explicit about all the small steps in your reasoning. In other words, think out loud about the various assumptions that are being made and the reasons for your choices. These problems are so second nature to an experienced physicist that we're usually not aware of our assumptions or reasoning, but this "hidden problem solving" is the information most needed by beginners.

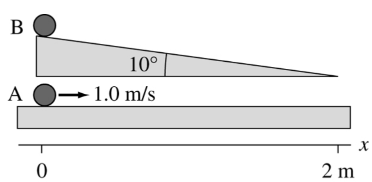
Example 1: Bob throws a ball straight up at 20 m/s, releasing the ball 1.5 m above the ground. What is the maximum height of the ball? What is the ball's impact speed as it hits the ground?

Example 2: A ball is released at a height of 1.0 m on a frictionless 30° slope. At the bottom, it turns smoothly onto a 60° slope going back up. What maximum height does it reach on the right side? (This is a two-part problem. Most students will be surprised that the answer is 1.0 m, and this gives you an opportunity to say a few initial words about energy.)



Example 3: A sprinter accelerates at 2.5 m/s^2 until reaching his top speed of 15 m/s. He then continues to run at top speed. How long does it take him to run the 100-m dash? (It's worth including a graphical analysis with this problem.)

Example 4: Ball A rolls along a frictionless, horizontal surface at a speed of 1.0 m/s. Ball B is released from rest at the top of a 2.0-m-long, 10° ramp at the exact instant ball A passes by. Will B overtake A before reaching the bottom of the ramp? If so, at what position? (The answer is *yes* at x = 1.193 m. This problem is considerably more difficult and allows you to point out that a simple plug-and-chug approach will not succeed. Students really do need the pictorial representation and good conceptual understanding of the motion in order to devise a strategy for solving it. Before doing the mathematics, it's worth sketching position graphs and showing that you're trying to find where the two graphs intersect.)



At some point, after starting calculations, you'll want to discuss significant figures. Most students are aware of these rules, except perhaps for subtle points such as whether a 0 is significant or not, but likely they've never been required to follow them. We all know the students who write down ten digits from their calculator display. An equally serious problem is the student who keeps his or her calculator set to display two decimal points, leading them to give the one-significant-figure answer 0.02 when computing 2.87/123. It's worth urging students to keep their calculator set in scientific notation mode, with two decimal places, so as to always be displaying three significant figures.

I think it's counterproductive to be overly rigid on significant figures. Although I emphasize that two or three significant figures is usually appropriate, depending on the information in the problem, I'm willing to accept up to four. I'm more concerned with getting students to recognize that less than two or more than four is clearly inappropriate. I try to enforce proper usage with an automatic one-point deduction on homework problems and an automatic two-point deduction on exam problems for improper significant figures. Alas, even with repeated penalties it is hard to get some students to pay attention to significant figures.

Day 4: A fourth day, if you have one, allows more practice problem solving. You can take the time to allow students to work in small groups rather than your presenting the solution as a worked example. Problems with turning points and with accelerations opposite in sign from what students might guess are well worthwhile. Problems involving two moving objects are particularly challenging because they can't be solved by equation hunting.

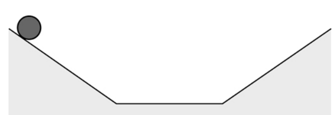
Another good exercise for day 4 is to have students work through several examples similar to those on pages 53–54 of balls rolling along multi-segment tracks. These exercises require students to *visualize* the motion and to relate it to graphs. Most students find these difficult, even after the exercises of days 1 and 2. But quite a few "get it" after a few such examples, and their ability to relate visualized motion to graphs suddenly takes a quantum leap.

Most instructors will want to cover Section 2.7 on instantaneous acceleration, but this is an optional section that is easily omitted if you're pressed for time. Instantaneous acceleration isn't seen again until simple harmonic motion, and by then students will be farther along in calculus and will know (or readily accept) that instantaneous acceleration is the time-derivative of velocity.

Sample Exam Questions

These questions cover the material of Chapters 1–2. If you've been having students do homework on the Dynamics Worksheets, you'll probably want to require their use on the quantitative problems on an exam. These are not "kinematics problems," for which borrowing unassigned Chapter 2 homework problems is recommended, but problems to assess whether students are acquiring a more sophisticated understanding of motion.

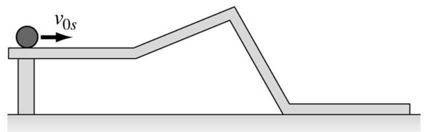
1. A ball released from rest rolls down a ramp, across a horizontal floor, and up the other side. Draw a complete motion diagram of the ball until it reaches its highest point on the right side.



2. Mike falls out of a tree and lands on a trampoline. The trampoline sags 2 feet before launching Mike back into the air. At the very bottom, where the sag is the greatest, is Mike's acceleration upward, downward, or zero? Use the tools that you've learned in these first chapters to give a convincing explanation of your answer.

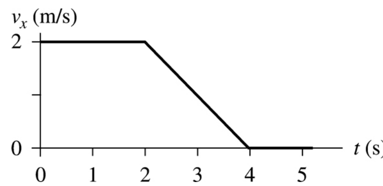
3. Is it possible for an object with a negative acceleration to be speeding up? If so, give an explicit example. If not, explain why not.

4. The figure below shows a ball rolling along a smooth frictionless track. Each segment of the track is straight, and the ball can move from segment to segment with no loss of speed. The ball starts from the left edge with an initial velocity v_{0s} that is large enough to make it over the top. Draw position-, velocity-, and acceleration-versus-time graphs for the ball until it rolls off the right edge of the track. (Position s is measured along the track.) Your three graphs should have the same time scale.

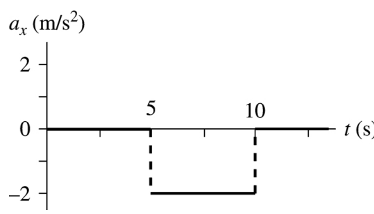


(It's good on a question like this to supply them with three empty sets of axes stacked one above the other.)

5. Draw the position graph and the acceleration graph that go with the velocity graph shown below. The initial position is $x_0 = -2.0$ m.



6. An object moving horizontally has the acceleration-versus-time graph shown below. At $t = 0$ s, the object has $x_0 = 0$ m. and velocity $v_{0x} = 10$ m/s.



   a. Draw a velocity-versus-time graph for the object. Include a numerical scale on the vertical axis.
   b. Draw a motion diagram of the object's motion.
   c. Write a description of a real object for which this is a realistic motion.