Optical Fiber Communications

Principles and Practice

Third Edition

SOLUTIONS MANUAL

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Chapter 2 Optical Fiber Waveguides

2.1 (a) In Eq. (2.8) the acceptance angle is given in terms of the NA as

$$\sin \theta_a = \frac{NA}{n_o} = \frac{0.20}{1.33} = 0.15$$

Hence $\theta_a = \underline{8.6^o}$.

(b) Using Eq. (2.8) for the definition of the NA, $NA = (n_1^2 - n_2^2)^{\frac{1}{2}}$

Hence $0.2 = (n_1^2 - 1.59^2)^{\frac{1}{2}}$

and $n_1^2 = 0.2^2 + 1.59^2 = 0.04 + 2.53 = 2.57$.

Therefore $n_1 = 1.60$

The critical angle is given by Eq. (2.2) as

$$\sin \phi_c = \underline{83.6^o}$$

It has been assumed that the fiber core diameter is large enough for ray theory analysis to hold (> 10 μm).

2.2 Considering the light propagating in the fiber core:

From Eq. (2.39) the velocity

$$v = \frac{c}{n_1}, \text{ hence } n_1 = \frac{2.998 \times 10^3}{2.01 \times 10^3} = 1.492$$

Using Eq. (2.2) for the critical angle, $\sin \phi_c = \frac{n_2}{n_1}$, then

$$n_2 = 1.492 \sin 80^o = 1.469.$$

The NA is given by Eq. (2.8) as

$$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} = (1.492^2 - 1.469^2)^{\frac{1}{2}}$$

$$= (0.069)^{\frac{1}{2}} = \underline{0.263}$$

Using Eq. (2.8), the acceptance angle is given by

$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.263 = \underline{15.2^o}$$

2.3 From Eq. (2.8),

$$NA = \sin \phi_a = \sin 22^{\circ} = \underline{0.374(6)}$$

Using Eq. (2.10) for the NA,

$$NA = n_1 \ (2\,\Delta)^{\frac{1}{2}}$$

$$\text{Therefore,}\ 0.375 \simeq n_1 \ (0.06)^{\frac{1}{2}}$$

$$\text{Hence}\ n_1 \simeq \frac{0.375}{0.245} = 1.531$$

Using the approximate definition for the relative index difference of Eq. (2.9),

$$\Delta \simeq \frac{n_1 - n_2}{n_1} \quad \text{and}\ 0.03 \simeq \frac{1.531 - n_2}{1.531}$$

$$\text{Hence}\ n_2 \simeq 1.531 - (1.531 \times 0.03)$$

$$= 1.531 - 0.046 = 1.485$$

The critical angle is given by Eq. (2.2) as

$$\phi_c = \sin^{-1} \frac{1.485}{1.531} = \sin^{-1} 0.970 = \underline{75.9^{\circ}}$$

2.4 For small angles the solid acceptance angle in air is given by,

$$\zeta \simeq \sin^2 \theta_a \simeq \pi(NA)^2.$$

Hence $(NA)^2 \simeq \frac{0.115}{\pi} = 3.65 \times 10^{-2}$

Therefore $NA \simeq 0.191$

Using Eq. (2.10), the numerical aperture is also given by,

$$NA \simeq n_1 (2\Delta)^{\frac{1}{2}}, \quad \text{Hence } n_1 \simeq \frac{NA}{\sqrt{2\Delta}} = \frac{0.191}{\sqrt{0.018}} = 1.424$$

Thus the speed of light in the fiber core is,

$$V = \frac{c}{n_1} = \frac{2.998 \times 10^8}{1.424} = \underline{2.11 \times 10^8 \text{ m s}^{-1}}$$

2.5 The numerical aperture can be obtained from Eq. (2.8) as

$$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} = (1.144^2 - 1.42^2)^{\frac{1}{2}} = 0.2$$

Using Eq. (2.17) for skew rays,

$$NA = \sin\theta_{as} \cos\gamma, \quad \text{Hence } \sin\theta_{as} = \frac{0.24}{\cos 65^0} = 0.568$$

Therefore $\theta_{as} = \underline{34.6^0}$

2.6 From Eq. (2.17) for skew rays,

$$NA = \sin\theta_{as} \cos\gamma = \sin 42^0 \cos 45^0 = 0.473$$

Using Eq. 2.8 the acceptance angle in air is

$$\theta_{as} = \sin^{-1} NA = \sin^{-1} 0.473 = \underline{28.2^0}$$

2.9 The normalized frequency may be obtained from Eq. (2.69),

$$V = \frac{2\pi}{\lambda} a \text{ (NA)} = 2\pi \times \frac{30 \times 10^{-6} \times 0.16}{0.9 \times 10^{-6}} = \underline{33.5}$$

The number of modes within a step index fiber is:

given by Eq. (2.74):

$$M_s \simeq \frac{V^2}{2} = \frac{(33.5)^2}{2} \simeq \underline{561 \text{ modes}}$$

2.11 Using Eq. (2.74), the mode volume

$$M_s \simeq \frac{V^2}{2}, \quad \text{Hence } V^2 \simeq 2 \times 1100 = 2200$$

Therefore V = 46.90

From Eq. (2.70), the core radius is given by,

$$a \simeq \frac{V\lambda}{2\pi n_1 (2\Delta)^{\frac{1}{2}}} = \frac{46.90 \times 1.3 \times 10^{-6}}{2\pi \times 1.5 \times (0.02)^{\frac{1}{2}}}$$

$$\simeq 46 \ \mu\text{m}$$

Hence the diameter of the fiber core is approximately $\underline{92 \ \mu\text{m}}$

2.13 (a) From Eq. (2.10) the NA is given by,

$NA \simeq n_1 \ (2\Delta)^{\frac{1}{2}}$, and using refractive index at the core axis for n_1, then

$$NA \simeq 1.45 \ (0.014)^{\frac{1}{2}} = \underline{0.172}$$

(b) The relative index difference is given by Eq. (2.9), as

$\Delta \simeq \frac{n_1 - n_2}{n_1}$. Again assuming n_1 is the value at the core

and $n_2 \simeq 1.45 - (1.45 \times 0.007) = 1.45 - 0.01 = 1.44$

Considering a triangular index profile then n_1 is the mean between 1.45 and 1.44 (i.e. 1.445).

When this value is used for n_1 in Eq. (2.10),

$$NA \simeq 1.445 \ (0.014)^{\frac{1}{2}} = \underline{0.171}$$

The two values obtained for the numerical aperture are almost identical, only differing slightly in the third decimal place. Although very crude approximations have been made in this instance, it is apparent that graded index fibers may be treated in a similar manner to step index fibers when regarding the numerical aperture with little loss in accuracy.

2.14 Using Eq. (2.8) for the NA in air, NA = sin 8° = 0.14

From Eq. (2.10) the NA is also given by,

$$NA \simeq n_1 (2\Delta)^{\frac{1}{2}} \text{ and } (2\Delta)^{\frac{1}{2}} \simeq \frac{NA}{n_1} = \frac{0.14}{1.52}$$

$$\text{Hence, } \Delta \simeq \frac{\left(\frac{0.14}{1.52}\right)^2}{2} = 4.24 \times 10^{-3} = \underline{0.42\ \%}$$

2.15 In a practical fiber the parabolic index profile is truncated at the fiber core boundarv. r = a. Therefore for a confined mode Eq. (2.88) becomes: $n^2(r) = n_1^2 (1 - 2\Delta)$

Hence the mode propagation constant must be greater than $n_1 (1 - 2\Delta)^{\frac{1}{2}} k$ or in the range

$$n_1 (1 - 2\Delta)^{\frac{1}{2}} k \leq \beta \leq n_1 k$$

From Eq. (2.91), the limitation on the mode numbers m and l is

$$n_1 k \left[1 - \frac{2\sqrt{(2\Delta)}}{n_1 ka} (2m + l + 1)\right]^{\frac{1}{2}} \leq n_1 (1 - 2\Delta)^{\frac{1}{2}} k$$

$$\text{and } 1 - \frac{2\sqrt{(2\Delta)}}{n_1 ka} (2m + l + 1) \leq (1 - 2\Delta)$$

$$\text{Hence } 2(2m + l + 1) \leq \frac{n_1 ka (2\Delta)}{\sqrt{(2\Delta)}}$$

$$\text{giving } 2(2m + l + 1) \leq n_1 ka \sqrt{(2\Delta)}$$

$$\text{From Eq. (2.9), } \sqrt{(2\Delta)} = \frac{(n_1^2 - n_2^2)^{\frac{1}{2}}}{n_1}$$

Therefore the limiting condition is $\underline{2(2m + l + 1) \leq ka(n_1^2 - n_2^2)^{\frac{1}{2}}}$

2.16 Using Eq. (2.95) for the mode volume in a graded index fiber with a parabolic refractive index profile,

$$M_g \simeq \frac{V^2}{4} . \text{ Hence } V^2 \simeq 4 \times 742 = 2968$$

Therefore, V = 54.5

To obtain the wavelength of the light propagating in the fiber, Eq. (2.69) may be used, where

$$\lambda = \frac{2\pi a\,(NA)}{V} = \frac{2\pi \times 35 \times 10^{-6} \times 0.3}{54.5}$$

$$= \underline{1.2\ \mu m}$$

For single-mode operation the maximum value of normalized frequency may be obtained from Eq. (2.97),

$$V_c = 2.4\left(1 + \frac{2}{\alpha}\right)^{\frac{1}{2}} = 2.4\sqrt{2} \text{ for a parabolic profile.}$$

The maximum core radius is given by Eq. (2.69) where

$$a \simeq \frac{V\lambda}{2\pi\,(NA)} = \frac{2.4\sqrt{2} \times 1.2 \times 10^{-6}}{2\pi \times 0.3} \simeq 2.2\ \mu m$$

Hence the maximum diameter of the fiber for single-mode operation is approximately $\underline{4.4\ \mu m}$

2.17 Initially the V value for the fiber must be obtained using Eq.(2.70):

$$V = \frac{2\pi a n_1 (2\Delta)^{\frac{1}{2}}}{\lambda} = \frac{2\pi \times 20 \times 1.5 \times 10^{-6}(0.026)^{\frac{1}{2}}}{1.55 \times 10^{-6}}$$

$$= 19.6$$

From Eq.(2.95) the mode volume is given by

$$M_g \simeq \frac{\alpha}{(\alpha + 2)}\left(\frac{V^2}{2}\right) \simeq \frac{1.9}{3.9}\left(\frac{V^2}{2}\right) \simeq \underline{94}$$

The cutoff value of the normalized frequency for single-mode operation is given by Eq.(2.97), where

$$V_c = 2.405\ (1 + \frac{2}{\alpha})^{\frac{1}{2}} = 2.405\ (2 + \frac{2}{1.9})^{\frac{1}{2}}$$

$$= \underline{3.45}$$

2.18 Using Eq.(2.98) for a single-mode step index fiber,

$$\lambda_1 \simeq \frac{2\pi n_1 a (2\Delta)^{\frac{1}{2}}}{V_c}$$

For single-mode operation the maximum V_c value is 2.405.

Hence the shortest wavelength which will allow single-mode operation is given by,

$$\lambda_c \simeq \frac{2\pi \times 1.49 \times 3.5 \times 10^{-6} \times (0.01)^{\frac{1}{2}}}{2.405} = \underline{1.36\ \mu m}$$

2.19 Using Eq. (2.98) for the limit of single-mode operation, then,

$$(2\Delta)^{\frac{1}{2}} \simeq \frac{\lambda\, 2.405}{2\pi n_1 a} = \frac{1.36 \times 10^{-6} \times 2.405}{2\pi \times 1.49 \times 5 \times 10^{-6}}$$

$$= 0.0698$$

Therefore,

$$\Delta \simeq \frac{(0.0698)^2}{2} = \frac{48.7 \times 10^{-4}}{2} = \underline{0.24\ \%}$$

2.20 The normalized frequency for a step index fiber is given in Eq. (2.69) as

$$V = \frac{2\pi}{\lambda} a\ (NA)$$

and $\frac{a}{\lambda} = \frac{V}{2\pi(NA)}$

The cutoff value of the normalized frequency for single-mode propagation occurs at 2.405. Hence the maximum value of a/λ for a step index fiber is

$$\frac{a}{\lambda} = \frac{2.405}{2\pi(NA)} = \frac{0.383}{(NA)}$$

The cutoff value of the normalized frequency for a single-mode parabolic profile fiber is given by Eq. (2.97) as

$$V_c = 2.405(1 + 2/\alpha)^{\frac{1}{2}} = 2.405(2)^{\frac{1}{2}} = 3.401$$

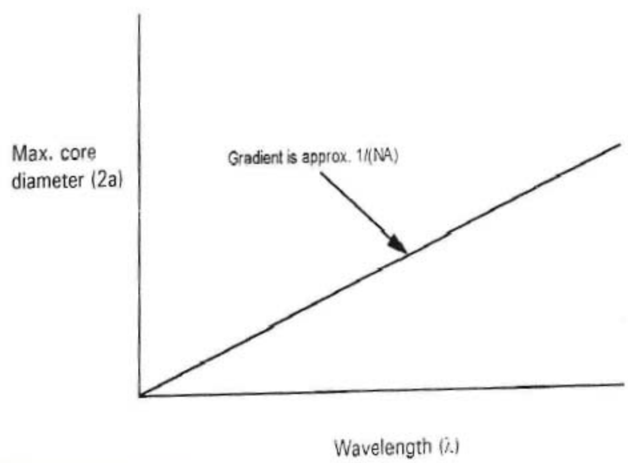
This gives a maximum value of a/λ for the parabolic profile single-mode fiber of

$$\frac{a}{\lambda} = \frac{3.401}{2\pi(NA)} = \frac{0.541}{(NA)}$$

Therefore the maximum value of a/λ is $0.541/0.383 \simeq 1.4$ times larger for a parabolic profile single-mode fiber than for a single-mode step index fiber.

The maximum core diameter (2a) to enable single-mode transmission in the parabolic profile fiber is related to the propagating wavelength following,

$$2a = \frac{1.082\ \lambda}{(NA)}$$



2.21 The relative refractive index difference for an optical fiber is given in Eq. (2.9) as

$$\Delta \simeq \frac{n_1 - n_2}{n_1} = \frac{0.005}{1.447} = 3.5 \times 10^{-3}$$

Furthermore the cutoff wavelength for single-mode operation is provided by Eq. (2.98) where

$$\lambda_c = \frac{2\pi\ an_1}{V_c}(2\Delta)^{\frac{1}{2}}$$

Hence,

$$\lambda_c = \frac{2\pi \times 3.6 \times 10^{-6} \times 1.447(0.007)^{\frac{1}{2}}}{2.405}$$

$$= 1.139\ \mu m\ (1139\ nm)$$

The fiber will permit single-mode transmission down to a cutoff wavelength of 1139 nm.

2.22 The relative refractive index difference for the fiber can be obtained from Eq. (2.9) as

$$\Delta \simeq \frac{n_1 - n_2}{n_1} = \frac{0.003}{1.498} = 2 \times 10^{-3}$$

For operation over the range 1.48 to 1.60 μm, then the cutoff wavelength is 1.48 μm. Hence the core radius to provide this cutoff wavelength is given by Eq. (2.98) as

$$a = \frac{\lambda_c V_c}{2\pi n_1 (2\Delta)^{\frac{1}{2}}} = \frac{2.405 \times 1.48 \times 10^{-6}}{2\pi \times 1.498\ (0.004)^{\frac{1}{2}}}$$

$$a = 5.98\ \mu m$$

Hence the maximum core diameter which will permit operation over the wavelength range is 12 μm

To enable single-mode transmission at a wavelength of 1.30 μm, the core radius is again provided by Eq. (2.98) where

$$a = \frac{2.405 \times 1.30 \times 10^{-6}}{2\pi \times 1.498\ (0.004)^{\frac{1}{2}}} = 5.25\ \mu m$$

Therefore the new fiber core diameter is 10.5 μm

2.23 The relative refractive index difference can be expressed in relation to the fiber core radius using Eq. (2.70) as

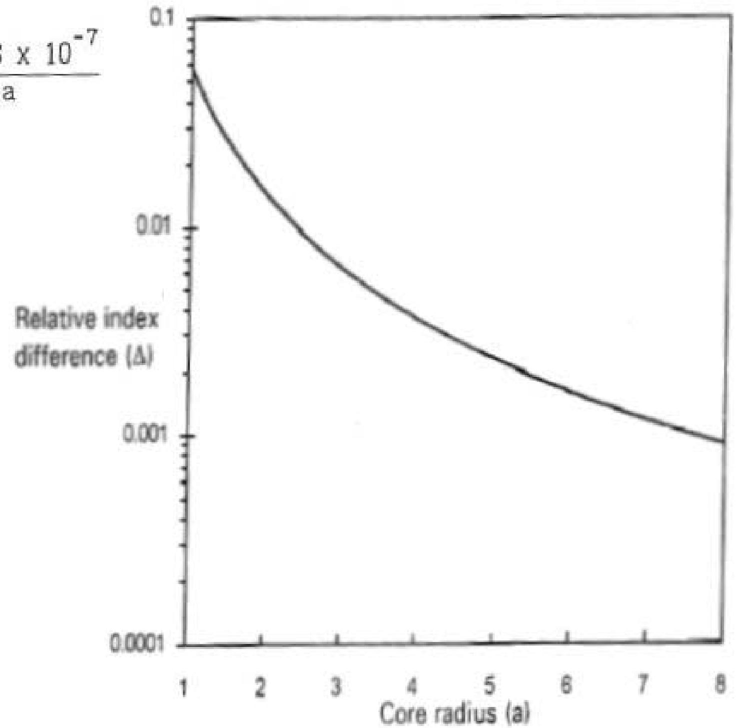
$$(2\Delta)^{\frac{1}{2}} = \frac{V\lambda}{2\pi\ a\ n_1}$$

Hence to maintain single-mode operation at 1.3 μm,

$$(2\Delta)^{\frac{1}{2}} = \frac{2.4 \times 1.3 \times 10^{-6}}{2\pi \times 1.47\ a} = \frac{3.38 \times 10^{-7}}{a}$$

and

$$\Delta = \frac{5.71 \times 10^{-14}}{a^2} \quad \text{or} \quad \Delta = \frac{5.7 \times 10^{-2}}{a^2\,(\mu m)}$$



For a = 4.5 μm, then $\Delta \simeq 0.003$ and at a wavelength of 0.85 μm the normalized frequency is

$$V = \frac{2\pi a}{\lambda} n_1 (2\Delta)^{\frac{1}{2}} = \frac{2\pi \times 4.5 \times 10^{-6}}{0.85 \times 10^{-6}} \times 1.47(0.006)^{\frac{1}{2}}$$

$$= 3.788$$

Hence the fiber is no longer single-mode at 0.85 μm.

2.24 Considering a single-mode step index fiber, then Eq.(2.100) can be written as

$$V = 2.405 \frac{\lambda_c}{\lambda}$$

Using the approximation for the normalized propagation constant obtained in Example 2.9

$$b(V) \simeq \left(1.1428 - \frac{0.9960}{V}\right)^2$$

Substituting for V gives:

$$b(V) \simeq \left(1.1428 - 0.9960 \frac{\lambda}{\lambda_c}\right)^2$$

The relative error in the approximation is less than 0.2 % for $1.5 \simeq V \simeq 2.5$. Hence the similar range for λ/λ_c is

$$\frac{2.405}{2.5} \simeq \frac{\lambda}{\lambda_c} \simeq \frac{2.405}{1.5} \quad \text{or} \quad 1.0 \simeq \frac{\lambda}{\lambda_c} \simeq 1.6 ,$$

and the relative error in the approximation is less than 2 % for $1.0 \simeq V \simeq 3.0$. The similar range for λ/λ_c is therefore

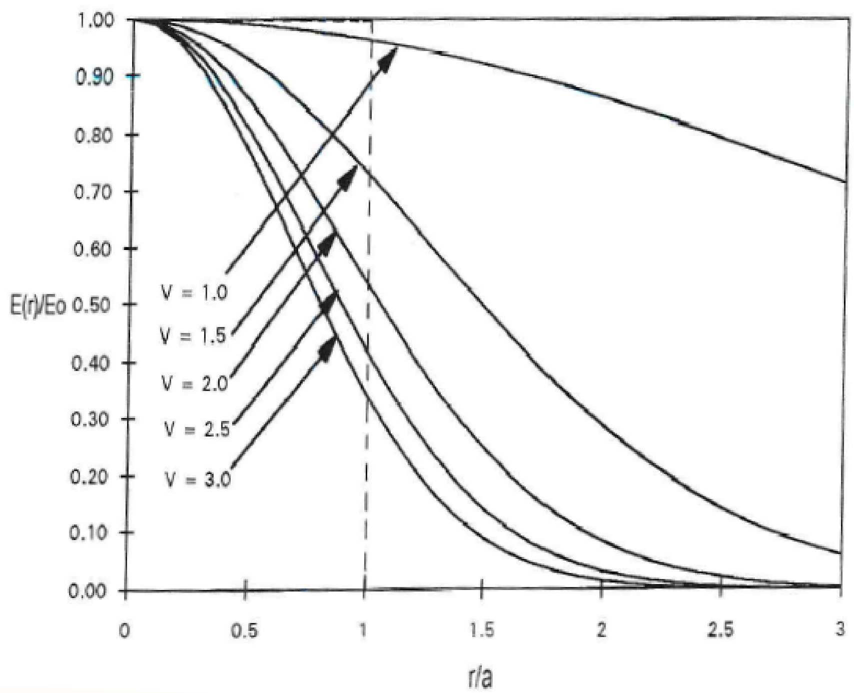
$$0.8 \leq \frac{\lambda}{\lambda_c} \leq 2.4$$

Thus the range of values over which the relative error in the approximation is between 0.2 % and 2 % is

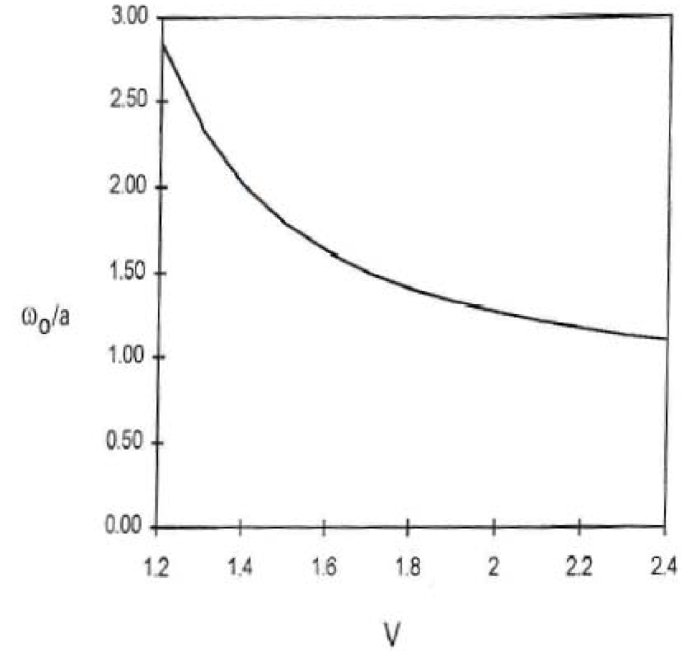
$$\underline{0.8 \leq \frac{\lambda}{\lambda_c} \leq 1.0 \text{ and } 1.6 \leq \frac{\lambda}{\lambda_c} \leq 2.4}$$

2.25 Inserting the approximation of Eq. (2.125) into the relationship provided gives

$$\frac{E(r)}{E_o} = \exp \frac{-r^2}{(0.65 + 1.619\, V^{-\frac{3}{2}} + 2.879\, V^{-6})^2\, a^2}$$



2.26 Using the approximate expression of Eq. (2.125):



The magnitude of ω_0/a increases gradually and then more rapidly as the normalized frequency, V, is reduced below 2.4. Hence the light initially (at V = 2.4) occupies the whole of the fiber core and as V is reduced the spot size expands further into the cladding region such that below a normalized frequency of 1.4 it occupies an area which is greater than four times the core region.

2.27 Using the ESI technique defined by Eq.(2.130), then the effective normalized frequency is given by Eq.(2.131) as

$$V_{eff} = 2.405\left(\frac{\lambda_c}{\lambda}\right) = 2.405\left(\frac{1.22}{1.55}\right) = 1.89$$

Hence the fiber effective core radius may be obtained from Eq.(2.132) following

$$a_{eff} = \frac{\omega_0}{0.6043 + 1.755\,V_{eff}^{-\frac{3}{2}} + 2.78\,V_{eff}^{-6}}$$

$$= \frac{5.5 \times 10^{-6}}{0.6043 + 1.755(1.89)^{-\frac{3}{2}} + 2.78(1.89)^{-6}}$$

$$= \underline{3.0\ \mu m}$$

The angle at which the first minimum of the diffraction pattern occurs is given by Eq.(2.130) as

$$\sin\theta_{min} = \frac{3.832\,\lambda}{a_{eff}\,2\pi} = \frac{3.832 \times 1.55 \times 10^{-6}}{3.0 \times 10^{-6} \times 2\pi}$$

$$= 0.315$$

Hence $\theta_{min} = \underline{18.4^\circ}$

2.28 In the cutoff method, the ESI fiber core radius may be obtained from Eq. (2.134) as

$$a_{ESI} = \frac{\omega_o}{1.099} = \frac{4.6}{1.099} = 4.186\ \mu m$$

The maximum refractive index of the fiber core is therefore given by rearranging Eq. (2.135)

$$n_1 = \left(\frac{0.293}{\Delta_{ESI}}\right)^{\frac{1}{2}} \left(\frac{\lambda_c}{2a_{ESI}}\right)$$

$$= \left(\frac{0.293}{3 \times 10^{-3}}\right)^{\frac{1}{2}} \left(\frac{1.29}{2 \times 4.186}\right)$$

$$= \underline{1.523}$$