ELE/COS 381: Assignment 1

Solutions

1. *Q1.1: Distributed power control* ⋆

(a) Consider three pairs of transmitters and receivers in a cell, with the following channel gain matrix **G** and noise of 0.1 mW for all the receivers. The target SIRs are also shown below.

$$\mathbf{G} = \begin{bmatrix} 1 & 0.1 & 0.3 \\ 0.2 & 1 & 0.3 \\ 0.2 & 0.2 & 1 \end{bmatrix}, \qquad \gamma = \begin{bmatrix} 1 \\ 1.5 \\ 1 \end{bmatrix}.$$

With an initialization of all transmit powers at 1 mW, run DPC for ten iterations and plot the evolution of transmit powers and received SIRs. You can use any programming language, or even write the steps out by hand.

(b) Now suppose the power levels for logical links 1, 2, and 3 have converged to the equilibrium in (a). A new pair of transmitter and receiver, labeled as logical link 4, shows up in the same cell, with an initial transmit power of 1 mW and demands a target SIR of 1. The new channel gain matrix is shown below.

$$\mathbf{G} = \begin{bmatrix} 1 & 0.1 & 0.3 & 0.1 \\ 0.2 & 1 & 0.3 & 0.1 \\ 0.2 & 0.2 & 1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 1 \end{bmatrix}.$$

Similarly to what you did in (a), show what happens in the next ten timeslots. What happens at the new equilibrium?

Solution: (a) At equilibrium, the power levels are $p_1^* = 0.19, p_2^* = 0.30, p_3^* = 0.20$. The SIR levels are $SIR_1 = 1.00, SIR_2 = 1.50, SIR_3 = 1.00$. This is shown in Fig. 1.

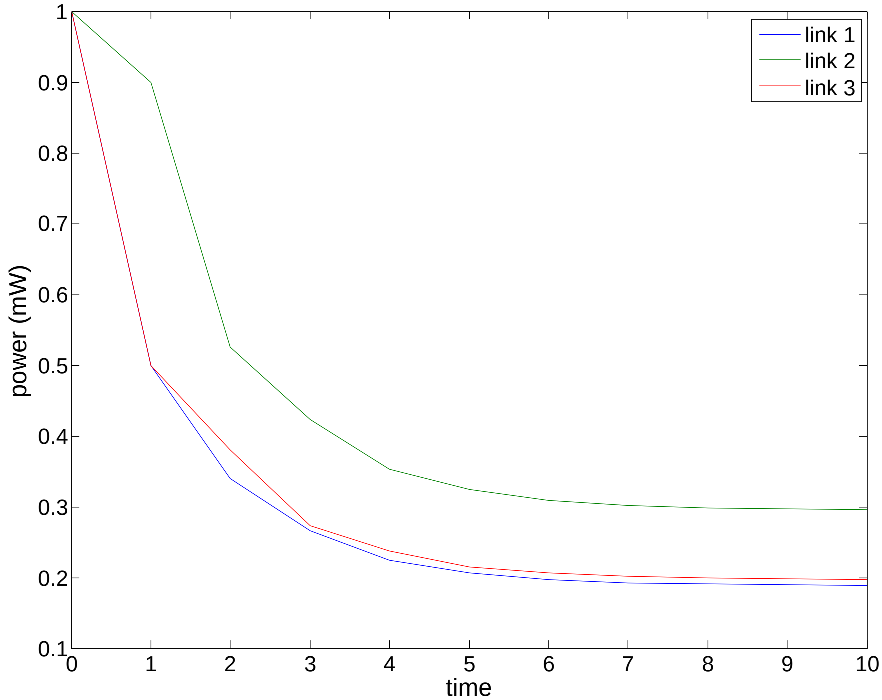
```
function [P SIR] = DPC( gamma )

iterations = 11;
n = 3;
noise = 0.1;

G = [1 0.1 0.3; 0.2 1 0.3; 0.2 0.2 1];
P = zeros(n,iterations);
SIR = zeros(n,iterations);

P(:,1) = ones(n,1);
SIR(1,1) = G(1,1)*P(1,1) / (G(1,2)*P(2,1) + G(1,3)*P(3,1) + noise);
SIR(2,1) = G(2,2)*P(2,1) / (G(2,1)*P(1,1) + G(2,3)*P(3,1) + noise);
SIR(3,1) = G(3,3)*P(3,1) / (G(3,1)*P(1,1) + G(3,2)*P(2,1) + noise);

for j=2:iterations
    P(1,j) = gamma(1)/SIR(1,j-1)*P(1,j-1);
    P(2,j) = gamma(2)/SIR(2,j-1)*P(2,j-1);
```



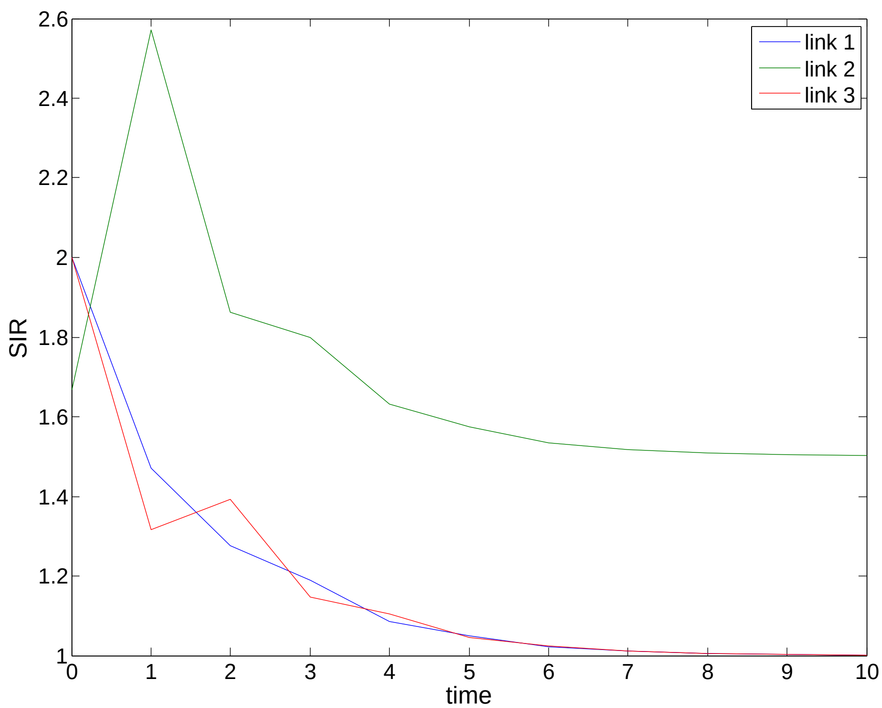


Figure 1: Power and SIR plots for simple transmit power control.

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    P(3,j) = gamma(3)/SIR(3,j-1)*P(3,j-1);

    SIR(1,j) = G(1,1)*P(1,j) / (G(1,2)*P(2,j) + G(1,3)*P(3,j) + noise);
    SIR(2,j) = G(2,2)*P(2,j) / (G(2,1)*P(1,j) + G(2,3)*P(3,j) + noise);
    SIR(3,j) = G(3,3)*P(3,j) / (G(3,1)*P(1,j) + G(3,2)*P(2,j) + noise);
end
```

(b) Initially, p_1, p_2, p_3 are the equilibrium values from part (a), and $p_4 = 1$ (in Fig. 2, $p_1[11] = 0.19, p_2[11] = 0.30, p_3[11] = 0.20, p_4[11] = 1$). The new power levels at convergence are $p_1^* = 0.22, p_2^* = 0.35, p_3^* = 0.23, p_4^* = 0.18$ mW. The SIR values dip initially at $t = 11$, then converge again to $SIR_1 = 1.00, SIR_2 = 1.50, SIR_3 = 1.00, SIR_4 = 1.00$. The equilibrium power levels are slightly higher now to account for the interference generated by link 4.

```
function [P SIR] = DPC_newUser( gamma )

iterations = 11;
n = 4;
noise = 0.1;

G=[1 0.1 0.3 0.1; 0.2 1 0.3 0.1; 0.2 0.2 1 0.1; 0.1 0.1 0.1 1];

P = ones(n,iterations);
P(:,1) = [0.1890 0.2959 0.1972 1];

SIR = zeros(n,iterations);
SIR(1,1) = G(1,1)*P(1,12) / (G(1,2)*P(2,12) + G(1,3)*P(3,12) + G(1,4)*P(4,12) + noise);
SIR(2,1) = G(2,2)*P(2,12) / (G(2,1)*P(1,12) + G(2,3)*P(3,12) + G(2,4)*P(4,12) + noise);
SIR(3,1) = G(3,3)*P(3,12) / (G(3,1)*P(1,12) + G(3,2)*P(2,12) + G(3,4)*P(4,12) + noise);
SIR(4,1) = G(4,4)*P(4,12) / (G(4,1)*P(1,12) + G(4,2)*P(2,12) + G(4,3)*P(3,12) + noise);

for j=2:iterations
    P(1,j) = gamma(1)/SIR(1,j-1)*P(1,j-1);
    P(2,j) = gamma(2)/SIR(2,j-1)*P(2,j-1);
    P(3,j) = gamma(3)/SIR(3,j-1)*P(3,j-1);
    P(4,j) = gamma(4)/SIR(4,j-1)*P(4,j-1);

    SIR(1,j) = G(1,1)*P(1,j) / (G(1,2)*P(2,j) + G(1,3)*P(3,j) + G(1,4)*P(4,j) + noise);
    SIR(2,j) = G(2,2)*P(2,j) / (G(2,1)*P(1,j) + G(2,3)*P(3,j) + G(2,4)*P(4,j) + noise);
    SIR(3,j) = G(3,3)*P(3,j) / (G(3,1)*P(1,j) + G(3,2)*P(2,j) + G(3,4)*P(4,j) + noise);
    SIR(4,j) = G(4,4)*P(4,j) / (G(4,1)*P(1,j) + G(4,2)*P(2,j) + G(4,3)*P(3,j) + noise);
end
```

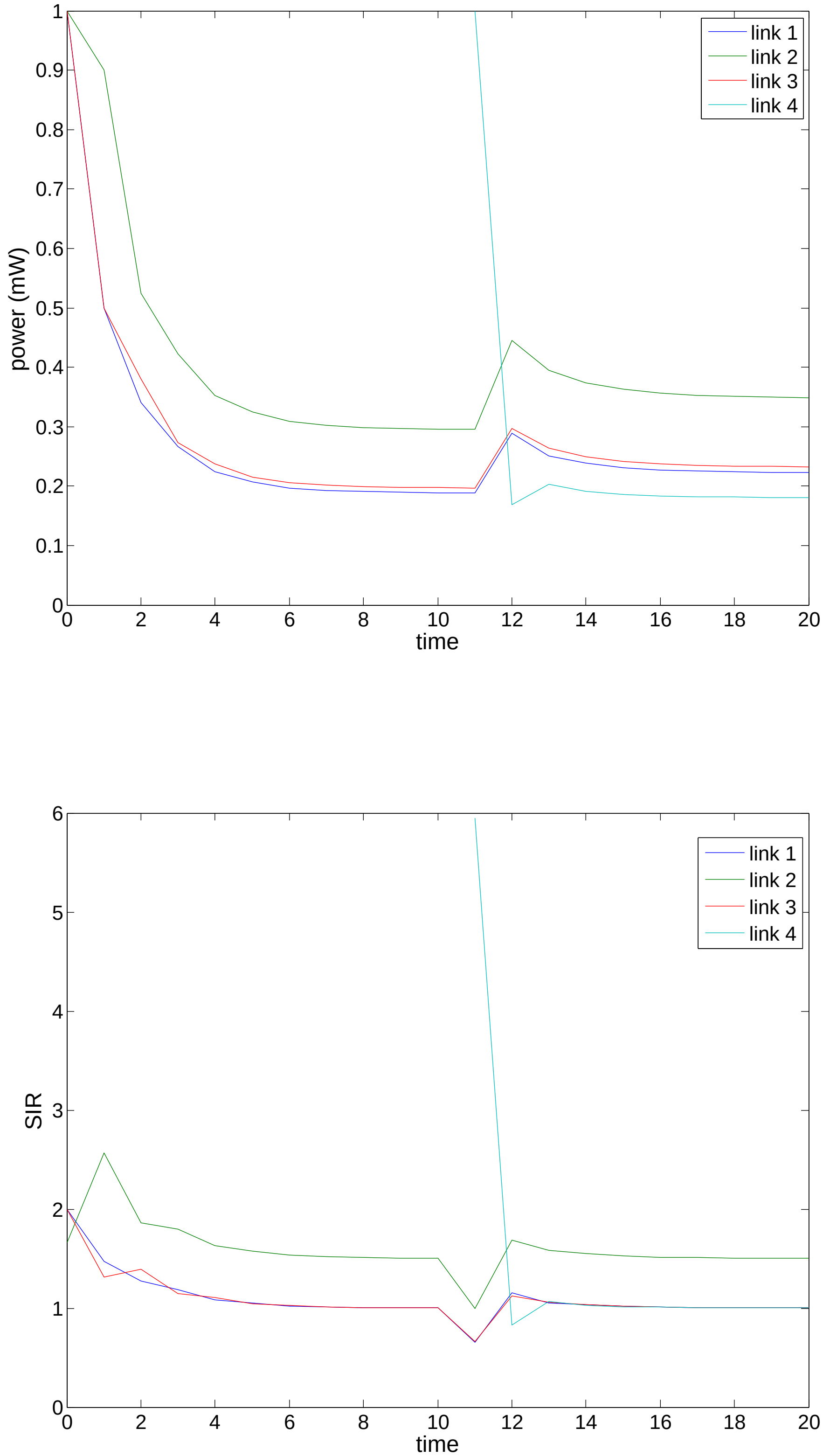

Figure 2: Power and SIR plots for transmit power control with population dynamics.

2. *Q1.2: Power control infeasibility* ⋆⋆

Consider a three-link cell with the link gains G_{ij} shown below. The receivers request $\gamma_1 = 1, \gamma_2 = 2$, and $\gamma_3 = 1$. The noise $n_i = 0.1$ for all i.

$$\mathbf{G} = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}.$$

Show this set of target SIRs is infeasible.

Solution: From the SIR formula, we know:

$$\begin{aligned} SIR_1 &= \frac{G_{11}p_1}{G_{12}p_2 + G_{13}p_3 + n_1} = \frac{p_1}{0.5p_2 + 0.5p_3 + 0.1} \geq 1 = \gamma_1 \\ SIR_2 &= \frac{G_{22}p_2}{G_{21}p_1 + G_{23}p_3 + n_2} = \frac{p_2}{0.5p_1 + 0.5p_3 + 0.1} \geq 2 = \gamma_2 \\ SIR_3 &= \frac{G_{33}p_3}{G_{31}p_1 + G_{32}p_2 + n_3} = \frac{p_3}{0.5p_1 + 0.5p_2 + 0.1} \geq 1 = \gamma_3 \end{aligned}$$

Rearranging, this gives:

$$\begin{aligned} p_1 - 0.5p_2 - 0.5p_3 &\geq 0.1 \\ -p_1 + p_2 - p_3 &\geq 0.2 \\ -0.5p_1 - 0.5p_2 + p_3 &\geq 0.1 \end{aligned}$$

Solving this system of linear equations, we find that $p_1 \leq -0.4, p_2 \leq -0.6, p_3 \leq -0.4$, which is infeasible since the powers are negative.

This problem can also be solved using the Perron-Frobenius theorem discussed in the Advanced Material. Letting $\mathbf{D}$ be a diagonal matrix with γ_i on the diagonal, and defining the matrix $\mathbf{F}$ where $F_{ij} = G_{ij}/G_{ii}$ for $i \neq j$ and 0 otherwise, we have:

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix},$$

and hence

$$\mathbf{DF} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 1 \\ 0.5 & 0.5 & 0 \end{bmatrix}.$$

The eigenvalues of the $\mathbf{DF}$ matrix are 1.28, -0.5, and -0.78, and hence $\rho = 1.28$. Since the spectral radius ρ is not less than 1, the set of target SIRs is infeasible.

```
gamma = [1 2 1]';

G = [1 0.5 0.5; 0.5 1 0.5; 0.5 0.5 1];

N = size(G,1);
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```
D = diag(gamma);

F = zeros(N);
for i=1:N
    for j=1:N
        if(i ~= j)
            F(i,j) = G(i,j)/G(i,i);
        end
    end
end

[~,E] = eig(D*F);
E = diag(E);
```

3. *Q1.3: A zero-sum game* ⋆

   In the following two-user game, the payoffs of users Alice and Bob are exactly negative of each other in all the combinations of strategies (a,a), (a,b), (b,a), (b,b). This models an extreme case of competition, and is called a **zero-sum game**. Is there any pure strategy equilibrium? How many are there?

	a	b
a	$(2, -2)$	$(3, -3)$
b	$(3, -3)$	$(4, -4)$

Solution: There is one equilibrium point at (3,-3). Nash equilibrium holds because $3 = U_1(b, a) \geq U_1(a, a) = 2$ and $-3 = U_2(b, a) \geq U_2(b, b) = -4$.