Chapter 1 Project

Pollution in Clear Lake

The Toxic Waste Disposal Company (TWDC) specializes in the disposal of a particularly dangerous pollutant, Agent Yellow (AY). Unfortunately, instead of safely disposing of this pollutant, the company simply dumped AY in (formerly) Clear Lake. Fortunately, they have been caught and are now defending themselves in court.

The facts below are not in dispute. As a result of TWDC's activity, the current concentration of AY in Clear Lake is now 10 ppm (parts per million). Clear Lake is part of a chain of rivers and lakes. Fresh water flows into Clear Lake and the contaminated water flows downstream from it. The Department of Environmental Protection estimates that the level of contamination in Clear Lake will fall by 20% each year. These facts can be modeled as

$$p(0) = 10 \qquad p(t + 1) = 0.80p(t)$$

where $p = p(t)$, measured in ppm, is the concentration of pollutants in the lake at time t, in years.

1. Explain how the above equations model the facts.
2. Create a table showing the values of t for $t = 0, 1, 2, \ldots, 20$.
3. Show that $p(t) = 10(0.8)^t$
4. Use graphing technology to graph $p = p(t)$.
5. What is $\lim_{x\to\infty} p(t)$?

Lawyers for TWDC looked at the results in 1–5 above and argued that their client has not done any real damage. They concluded that Clear Lake would eventually return to its former clear and unpolluted state. They even called in a mathematician, who wrote the following on a blackboard:

$$\lim_{x\to\infty} p(t) = 0$$

and explained that this bit of mathematics means, descriptively, that after many years the concentration of AY will, indeed, be close to zero.

Concerned citizens booed the mathematician's testimony. Fortunately, one of them has taken calculus and knows a little bit about limits. She noted that, although "after many years the concentration of AY will approach zero," the townspeople like to swim in Clear Lake and state regulations prohibit swimming unless the concentration of AY is below 2 ppm. She proposed a fine of $100,000 per year for each full year that the lake is unsafe for swimming. She also questioned the mathematician, saying, "Your testimony was correct as far as it went, but I remember from studying calculus that talking about the eventual concentration of AY after many, many years is only a small part of the story. The more precise meaning of your statement $\lim_{x\to\infty} p(t) = 0$ is that given some tolerance T

for the concentration of AY, there is some time N (which may be very far in the future) so that for all $t > N$, $p(t) < T$."

6. Using the table or the graph for $p = p(t)$, find N so that if $t > N$, then $p(t) < 2$.
7. How much is the fine?

Her words were greeted by applause. The town manager sprang to his feet and noted that although a tolerance of 2 ppm was fine for swimming, the town used Clear Lake for its drinking water and until the concentration of AY dropped below 0.5 ppm, the water would be unsafe for drinking. He proposed a fine of $200,000 per year for each full year the water was unfit for drinking.

8. Using the table or the graph for $p = p(t)$, find N so that if $t > N$, then $p(t) < 0.5$.
9. How much is the fine?
10. How would you find if you were on the jury trying TWDC? If the jury found TWDC guilty, what fine would you recommend? Explain your answers

Solutions for Chapter 1 Project

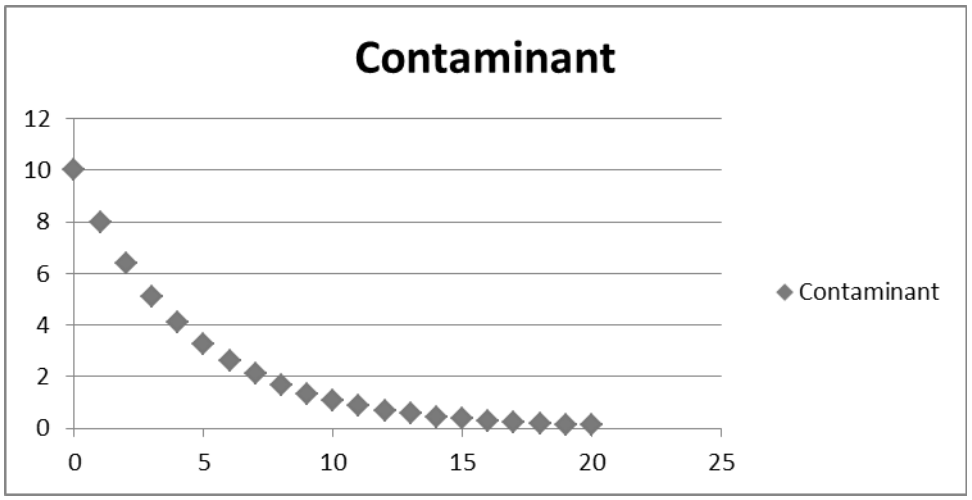
1. Since we are given that the level of contamination in Clear Lake falls by 20% each year, this means 80% of the previous year's contamination remains. But if $p(t)$ is the amount of contamination in year t, then 80% of $p(t)$, or $0.8p(t)$, is the amount that remains in the following year. So $p(t + 1) = 0.8p(t)$, as desired.

2.

t = year	$p(t)$ = amount of contaminant (in ppm) in year t, with initial $t = 0$		
0	10		
1	8	11	0.858993
2	6.4	12	0.687195
3	5.12	13	0.549756
4	4.096	14	0.439805
5	3.2768	15	0.351844
6	2.62144	16	0.281475
7	2.097152	17	0.22518
8	1.677722	18	0.180144
9	1.342177	19	0.144115
10	1.073742	20	0.115292

3. Formal proof can be established using simple induction, but the pattern is clear. In general, the "next" term is equal to 0.8 times the "current" term, with $p(0) = 10$. So $p(1) = 100(.8)$, $p(2) = (10(.8))(.8) = 10(.8)^2$, $p(3) = \left(10(.8)^2\right)(.8) = 10(.8)^3$, and so on.

4.



5. Since this is a simple exponential function of the form $f(x) = b^x$ with $b = 0.8$ between 0 and 1, successive powers of the function approach zero as a limit. This is supported by both the data set and the graph above.

6. $p(t)$ is first less than 2 when $t = 8$, so $N = 7$.

7. Since the fine is \$100,000/year, and the answer to part 6 establishes that fines must be paid for 7 years, the total fine is \$700,000.

8. $p(t)$ is first less than 0.5 when $t = 14$, so $N = 13$.

9. Since the fine is \$200,000/year, and the answer to part 8 establishes that fines must be paid for 13 years, the total fine is \$2,600,000.

10. Either side could be argued, although this solution's author is really partial to clean drinking water.