Chapter 2

1. For a particle Newton's second law says $\vec{F} = m\vec{a} = m\left[\frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}\right]$.

   Take the second derivative of each of the expressions in Equation (2.1):
   $\frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2}$ $\frac{d^2y'}{dt^2} = \frac{d^2y}{dt^2}$ $\frac{d^2z'}{dt^2} = \frac{d^2z}{dt^2}$. Substitution into the previous equation gives
   $\vec{F} = m\vec{a} = m\left[\frac{d^2x'}{dt^2}\hat{i} + \frac{d^2y'}{dt^2}\hat{j} + \frac{d^2z'}{dt^2}\hat{k}\right] = \vec{F}'$.

2. From Equation (2.1) $\vec{p} = m\left[\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}\right]$.

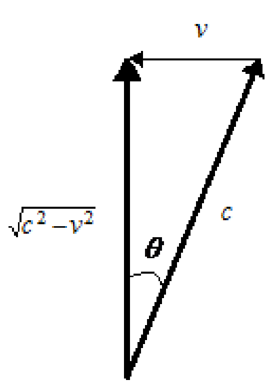
   In a Galilean transformation $\frac{dx'}{dt} = \frac{dx}{dt} - v$ $\frac{dy'}{dt} = \frac{dy}{dt}$ $\frac{dz'}{dt} = \frac{dz}{dt}$.

   Substitution into Equation (2.1) gives $\vec{p} = m\left[\frac{dx'}{dt} + v\right]\hat{i} + \frac{dy'}{dt}\hat{j} + \frac{dz'}{dt}\hat{k} \neq \vec{p'}$.

   However, because $\vec{p'} = m\left[\frac{dx'}{dt}\hat{i} + \frac{dy'}{dt}\hat{j} + \frac{dz'}{dt}\hat{k}\right]$ the same form is clearly retained, given the velocity transformation $\frac{dx'}{dt} = \frac{dx}{dt} - v$.

3. Using the vector triangle shown, the speed of light coming toward the mirror is $\sqrt{c^2 - v^2}$ and the same on the return trip. Therefore the total time is $t_2 = \frac{\text{distance}}{\text{speed}} = \frac{2\ell_2}{\sqrt{c^2 - v^2}}$.

   Notice that $\sin\theta = \frac{v}{c}$, so $\theta = \sin^{-1}\left(\frac{v}{c}\right)$.



4. As in Problem 3, $\sin\theta = v_1 / v_2$, so $\theta = \sin^{-1}(v_1 / v_2) = \sin^{-1}\left[\dfrac{0.350 \text{ m/s}}{1.25 \text{ m/s}}\right] = 16.3°$ and

   $v = \sqrt{v_2^2 - v_1^2} = \sqrt{(1.25 \text{ m/s})^2 - (0.35 \text{ m/s})^2} = 1.20 \text{ m/s}$.

5. When the apparatus is rotated by 90°, the situation is equivalent, except that we have effectively interchanged ℓ_1 and ℓ_2. Interchanging ℓ_1 and ℓ_2 in Equation (2.3) leads to Equation (2.4).

6. Let *n* = the number of fringes shifted; then $n = \dfrac{\Delta d}{\lambda}$. Because $\Delta d = c\left(\Delta t' - \Delta t\right)$, we have

   $n = \dfrac{c\left(\Delta t' - \Delta t\right)}{\lambda} = \dfrac{v^2\left(\ell_1 + \ell_2\right)}{c^2 \lambda}$. Solving for *v* and noting that $\ell_1 + \ell_2 = 22$ m,

   $$v = c\sqrt{\frac{n\lambda}{\ell_1 + \ell_2}} = \left(3.00 \times 10^8 \text{ m/s}\right)\sqrt{\frac{(0.005)\left(589 \times 10^{-9} \text{ m}\right)}{22 \text{ m}}} = 3.47 \text{ km/s.}$$

7. Letting $\ell_1 \to \ell_1\sqrt{1-\beta^2}$ (where $\beta = v / c$) the text equation (not currently numbered) for t_1 becomes

   $$t_1 = \frac{2\ell_1\sqrt{1-\beta^2}}{c\left(1-\beta\right)} = \frac{2\ell_1}{c\sqrt{1-\beta^2}}$$

   which is identical to t_2 when $\ell_1 = \ell_2$ so $\Delta t = 0$ as required.

8. Since the Lorentz transformations depend on *c* (and the fact that *c* is the same constant for all inertial frames), different values of *c* would necessarily lead two observers to different conclusions about the order or positions of two spacetime events, in violation of postulate 1.

9. Let an observer in K send a light signal along the + *x*-axis with speed *c*. According to the Galilean transformations, an observer in K′ measures the speed of the signal to be $\dfrac{dx'}{dt} = \dfrac{dx}{dt} - v = c - v$. Therefore the speed of light cannot be constant under the Galilean transformations.

10. From the Principle of Relativity, we know the correct transformation must be of the form (assuming $y = y'$ and $z = z'$):

    $x = ax' + bt'$; $x' = ax - bt$.

    The spherical wave front equations (2.9a) and (2.9b) give us:

    $ct = (ac + b)t'$; $ct' = (ac - b)t$.

    Solve the second wave front equation for t' and substitute into the first:

$$ct = \left[\frac{(ac+b)(ac-b)t}{c}\right] \text{ or } c^2 = (ac+b)(ac-b) = a^2c^2 - b^2.$$

Now v is the speed of the origin of the x'-axis. We can find that speed by setting $x' = 0$ which gives $0 = ax - bt$, or $v = x/t = b/a$, or equivalently $b = av$. Substituting this into the equation above for c^2 yields $c^2 = a^2c^2 - a^2v^2 = a^2\left(c^2 - v^2\right)$. Solving for a:

$$a = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma.$$

This expression, along with $b = av$, can be substituted into the original expressions for x and x' to obtain:

$$x = \gamma\left(x' + vt'\right); \qquad x' = \gamma\left(x - vt\right)$$

which in turn can be solved for t and t' to complete the transformation.

11. When $v << c$ we find $1 - \beta^2 \to 1$, so:

$$x' = \frac{x - \beta ct}{\sqrt{1-\beta^2}} \to x - \beta ct = x - vt\,;$$

$$t' = \frac{t - \beta x/c}{\sqrt{1-\beta^2}} \to t - \beta x/c \approx t\,;$$

$$x = \frac{x' + \beta ct'}{\sqrt{1-\beta^2}} \to x' + \beta ct' = x' + vt'\,;$$

$$t = \frac{t' + \beta x'/c}{\sqrt{1-\beta^2}} \to t' + \beta x'/c \approx t'\,.$$

12. (a) First we convert to SI units: 95 km/h = 26.39 m/s, so

$$\beta = v/c = (26.39 \text{ m/s})/\left(3.00\times10^8 \text{ m/s}\right) = 8.8\times10^{-8}$$

(b) $\beta = v/c = (240\,\text{m/s})/\left(3.00\times10^8 \text{ m/s}\right) = 8.0\times10^{-7}$

(c) $v = 2.3v_{\text{sound}} = \left(2.3\times330 \text{ m/s}\right)$ so

$$\beta = v/c = \left(2.3\times330 \text{ m/s}\right)/\left(3.00\times10^8 \text{ m/s}\right) = 2.5\times10^{-6}$$

(d) Converting to SI units, 27,000 km/h = 7500 m/s, so

$$\beta = v/c = (7500 \text{ m/s})/\left(3.00\times10^8 \text{m/s}\right) = 2.5\times10^{-5}$$

(e) (25 cm)/(2 ns) = 1.25×10^8 m/s so $\beta = v/c = (1.25\times10^8 \text{m/s})/\left(3.00\times10^8 \text{ m/s}\right) = 0.42$

(f) $\left(1\times10^{-14} \text{m}\right)/\left(0.35\times10^{-22} \text{s}\right) = 2.857\times10^8 \text{m/s}$, so

$$\beta = v/c = (2.857\times10^8 \text{ m/s})/\left(3.00\times10^8 \text{m/s}\right) = 0.95$$

13. From the Lorentz transformations $\Delta t' = \gamma\left[\Delta t - v\Delta x / c^2\right]$. But $\Delta t' = 0$ in this case, so solving for v we find $v = c^2\Delta t / \Delta x$. Inserting the values $\Delta t = t_2 - t_1 = -a/2c$ and $\Delta x = x_2 - x_1 = a$, we find $v = \frac{c^2(-a/2c)}{a} = -c/2$. We conclude that the frame K' travels at a speed $c/2$ in the $-x$-direction. Note that there is no motion in the transverse direction.
14. Try setting $\Delta x' = 0 = \gamma(\Delta x - v\Delta t)$. Thus $0 = \Delta x - v\Delta t = a + va/2c$. Solving for v we find $v = -2c$, which is impossible. There is no such frame K'.
15. For the smaller values of β we use the binomial expansion $\gamma = (1-\beta^2)^{-1/2} \approx 1 + \beta^2/2$.
    (a) $\gamma \approx 1 + \beta^2/2 = 1 + 3.87\times10^{-15}$
    (b) $\gamma \approx 1 + \beta^2/2 = 1 + 3.2\times10^{-13}$
    (c) $\gamma \approx 1 + \beta^2/2 = 1 + 3.1\times10^{-12}$
    (d) $\gamma \approx 1 + \beta^2/2 = 1 + 3.1\times10^{-10}$
    (e) $\gamma = (1-\beta^2)^{-1/2} = (1-0.42^2)^{-1/2} = 1.10$
    (f) $\gamma = (1-\beta^2)^{-1/2} = (1-0.95^2)^{-1/2} = 3.20$

16. There is no motion in the transverse direction, so $y = z = 3.5$ m.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.8^2}} = 5/3$$

$$x = \gamma(x' + vt') = \frac{5}{3}\left(2\text{m} + 0.8c(0)\right) = 10/3 \text{ m}$$

$$t = \gamma\left(t' + vx'/c^2\right) = \frac{5}{3}\left(0 + (0.8c)(2\text{ m})/c^2\right) = 8.9\times10^{-9}\text{ s}$$

17. (a) $t = \frac{\sqrt{x^2+y^2+z^2}}{c} = \frac{\sqrt{(3\text{ m})^2 + (5\text{ m})^2 + (10\text{ m})^2}}{3.00\times10^8\text{ m/s}} = 3.86\times10^{-8}\text{ s}$
    (b) With $\beta = 0.8$ we find $\gamma = 5/3$. Then $y' = y = 5$ m, $z' = z = 10$ m,

$$x' = \gamma(x - vt) = \frac{5}{3}\left[3\text{m} - (2.40\times10^8\text{ m/s})(3.86\times10^{-8}\text{ s})\right] = -10.4\text{ m}$$

$$t' = \gamma\left(t - vx/c^2\right) = \frac{5}{3}\left[(3.86\times10^{-8}\text{ s}) - (2.40\times10^8\text{ m/s})(3\text{ m})/(3.00\times10^8\text{ m/s})^2\right] = 51.0\text{ ns}$$

(c) $\dfrac{\sqrt{x'^2+y'^2+z'^2}}{t'}=\dfrac{\sqrt{(-10.4\text{ m})^2+(5\text{ m})^2+(10\text{ m})^2}}{51.0\times10^{-9}\text{ s}}=2.994\times10^8\text{ m/s}$ which equals c to within rounding errors.

18. At the point of reflection the light has traveled a distance $L+v\Delta t_1=c\Delta t_1$. On the return trip it travels $L-v\Delta t_2=c\Delta t_2$. Then the total time is $\Delta t=\Delta t_1+\Delta t_2=\dfrac{2Lc}{c^2-v^2}=\dfrac{2L/c}{1-v^2/c^2}$.

But from time dilation we know (with $\Delta t'=\text{proper time}=2L_0/c$) that

$\Delta t=\gamma\Delta t'=\dfrac{2L_0/c}{\sqrt{1-v^2/c^2}}$. Comparing these two results for Δt we get $\dfrac{2L/c}{1-\dfrac{v^2}{c^2}}=\dfrac{2L_0/c}{\sqrt{1-v^2/c^2}}$

which reduces to $L=L_0\sqrt{1-v^2/c^2}=\dfrac{L_0}{\gamma}$. This is Equation (2.21).

19. (a) With a contraction of 1%, $L/L_0=0.99=\sqrt{1-v^2/c^2}$. Thus $1-\beta^2=(0.99)^2=0.9801$. Solving for β, we find $\beta=0.14$ or $v=0.14c$.

(b) The time for the trip in the Earth-based frame is

$\Delta t=\dfrac{d}{v}=\dfrac{5.00\times10^6\text{m}}{0.14\times3.00\times10^8\text{m/s}}=1.19\times10^{-1}\text{s}$. With the relativistic factor $\gamma=1.01$ (corresponding to a 1% shortening of the ship's length), the elapsed time on the rocket ship is 1% less than the Earth-based time, or a difference of

$(0.01)1.2\times10^{-1}\text{s}=1.2\times10^{-3}\text{s}$.

20. The round-trip distance is d = 40 ly. Assume the same constant speed $v=\beta c$ for the entire round trip. In the rocket's reference frame the distance is only $d'=d\sqrt{1-\beta^2}$. Then in the rocket's frame of reference $v=\dfrac{\text{distance}}{\text{time}}=\dfrac{d'}{40\text{ y}}=\dfrac{40\text{ly}\sqrt{1-\beta^2}}{40y}=c\sqrt{1-\beta^2}$.

Rearranging $\beta=\dfrac{v}{c}=\sqrt{1-\beta^2}$. Solving for β we find $\beta=\sqrt{0.5}$, or $v=\sqrt{0.5}c\approx0.71c$.

To find the elapsed time t on Earth, we know $t'=40$ y, so $t=\gamma t'=\dfrac{1}{\sqrt{1-\beta^2}}40\text{y}=56.6$ y.

21. In the muon's frame $T_0=2.2$ μs. In the lab frame the time is longer; see Equation (2.19): $T'=\gamma T_0$. In the lab the distance traveled is $9.5\text{cm}=vT'=v\gamma T_0=\beta c\gamma T_0$, since $v=\beta c$.

Therefore $\beta = \dfrac{9.5\text{ cm}\left(\sqrt{1-\beta^2}\right)}{cT_0}$, so $\beta = \dfrac{v}{c} = \dfrac{9.5\text{ cm}\left(\sqrt{1-\beta^2}\right)}{c(2.2\mu\text{s})}$. Now all quantities are known except β. Solving for β we find $\beta = 1.4\times10^{-4}$ or $v = 1.4\times10^{-4}c$.

22. Converting the speed to m/s we find 25,000 mi/h = 11,176 m/s. From tables the distance is 3.84×10^8 m. In the earth's frame of reference the time is the distance divided by speed, or $t = \dfrac{d}{v} = \dfrac{3.84\times10^8\text{ m}}{11{,}176\text{ m/s}} = 34{,}359\text{ s}$. In the astronauts' frame the time elapsed is $t' = t/\gamma = t\sqrt{1-\beta^2}$. The time difference is $\Delta t = t - t' = t - t\sqrt{1-\beta^2} = t\left[1-\sqrt{1-\beta^2}\right]$.

Evaluating numerically $\Delta t = 34{,}359\text{ s}\left[1-\sqrt{1-\left(\dfrac{11{,}176\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2}\right] = 2.4\times10^{-5}\text{ s}$.

23. $T' = \gamma T_0$, so we know that $\gamma = 5/3 = \dfrac{1}{\sqrt{1-v^2/c^2}}$. Solving for v we find $v = 4c/5$.

24. $L = L_0/\gamma$ so clearly $\gamma = 2$ in this case. Thus $2 = \dfrac{1}{\sqrt{1-v^2/c^2}}$ and solving for v we find $v = \dfrac{\sqrt{3}c}{2}$.

25. The clocks' rates differ by a factor of $\gamma = 1/\sqrt{1-v^2/c^2}$. Because β is very small we will use the binomial theorem approximation $\gamma \approx 1+\beta^2/2$. Then the time difference is $\Delta t = t - t' = t - \gamma t = t(\gamma - 1)$. Using $\gamma - 1 \approx \beta^2/2$ and the fact that the time for the trip equals distance divided by speed,

$$\Delta t = t\left(\beta^2/2\right) = \frac{8\times10^6\text{ m}}{375\text{ m/s}}\,\frac{\left(\dfrac{375\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2}{2}$$

$$\Delta t = 1.67\times10^{-8}\text{ s} = 16.7\text{ ns.}$$

26. (a) $L' = L/\gamma = L\sqrt{1-v^2/c^2} = \left(3.58\times10^4\text{ km}\right)\sqrt{1-0.94^2} = 1.22\times10^4\text{ km}$

(b) Earth's frame: $t = L/v = \dfrac{3.58\times10^7\text{ m}}{(0.94)\left(3.00\times10^8\text{ m/s}\right)} = 0.127\text{ s}$

Golf ball's frame: $t' = t/\gamma = 0.127\text{ s}\,\sqrt{1-0.94^2} = 0.0433\text{ s}$

27. Spacetime invariant (see Section 2.9): $c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 - \Delta x'^2$. We know $\Delta x = 4$ km, $\Delta t = 0$, and $\Delta x' = 5$ km. Thus $\Delta t'^2 = \dfrac{\Delta x'^2 - \Delta x^2}{c^2} = \dfrac{(5000\text{ m})^2 - (4000\text{ m})^2}{(3.00\times 10^8\text{ m/s})^2} = 1.0\times 10^{-10}\text{ s}^2$ and $\Delta t' = 1.0\times 10^{-5}$ s.

28. (a) Converting v = 120 km/h = 33.3 m/s. Now with c = 100 m/s, we have $\beta = v/c = 0.333$ and $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = \dfrac{1}{\sqrt{1-0.333^2}} = 1.061$. We conclude that the moving person ages 6.1% slower.
(b) $L' = L/\gamma = (1\text{ m})/(1.061) = 0.942$ m.

29. Converting v = 300 km/h = 83.3 m/s. Now with c =100 m/s, we have $\beta = v/c = 0.833$ and $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = \dfrac{1}{\sqrt{1-0.833^2}} = 1.81$. So the length is $L = L_0/\gamma = 40/1.81 = 22.1$ m.

30. Let subscript 1 refer to firing and subscript 2 to striking the target. Therefore we can see that $x_1 = 1$ m, $x_2 = 121$ m, and $t_1 = 3$ ns.

$$t_2 = t_1 + \frac{\text{distance}}{\text{speed}} = 3\text{ ns} + \frac{120\text{ m}}{0.98c} = 3\text{ ns} + 408\text{ ns} = 411\text{ ns.}$$

To find the four primed quantities we can use the Lorentz transformations with the known values of x_1, x_2, t_1, and t_2. Note that with $v = 0.8c$, $\gamma = \sqrt{1 - v^2/c^2} = 5/3$.

$$t_1' = \gamma\left(t_1 - vx_1/c^2\right) = 0.56\text{ ns}$$
$$t_2' = \gamma\left(t_2 - vx_2/c^2\right) = 147\text{ ns}$$
$$x_1' = \gamma\left(x_1 - vt_1\right) = 0.47\text{ m}$$
$$x_2' = \gamma\left(x_2 - vt_2\right) = 37.3\text{ m}$$

31. Start from the formula for velocity addition, Equation (2.23a): $u_x = \dfrac{u_x' + v}{1 + vu_x'/c^2}$.

(a) $u_x = \dfrac{0.62c + 0.84c}{1 + (0.62c)(0.84c)/c^2} = \dfrac{1.46c}{1.52} = 0.96c$

(b) $u_x = \dfrac{-0.62c + 0.84c}{1 + (-0.62c)(0.84c)/c^2} = \dfrac{0.22c}{0.48} = 0.46c$

32. Velocity addition, Equation (2.24): $u_x' = \dfrac{u_x - v}{1 - vu_x/c^2}$ with $v = -0.8c$ and $u_x = 0.8c$.

$$u_x' = \frac{0.8c - (-0.8c)}{1 - (-0.8c)(0.8c)/c^2} = \frac{1.6c}{1.64} = 0.976c$$

33. Conversion: 110 km/h = 30.556 m/s and 140 km/h = 38.889 m/s. Let $u_x = 30.556$ m/s and $v = -38.889$ m/s. Our premise is that $c = 100$ m/s. Then by velocity addition,

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} = \frac{30.556 \text{ m/s} - (-38.889 \text{ m/s})}{1 - (-38.889 \text{ m/s})(30.556 \text{ m/s})/(100 \text{ m/s})^2} = 62.1 \text{ m/s}.$$

By symmetry each observer sees the other one traveling at the same speed.

34. From Example 2.5 we have $u = \frac{c}{n}\left[\frac{1 + nv/c}{1 + v/nc}\right]$. For light traveling in opposite directions $\Delta u = \frac{c}{n}\left[\frac{1 + nv/c}{1 + v/nc} - \frac{1 - nv/c}{1 - v/nc}\right]$. Because v/c is very small, use the binomial expansion: $\frac{1 + nv/c}{1 + v/nc} = (1 + nv/c)(1 + v/nc)^{-1} \approx (1 + nv/c)(1 - v/nc) \approx 1 + nv/c - v/nc$, where we have dropped terms of order v^2/c^2. Similarly $\frac{1 - nv/c}{1 - v/nc} \approx 1 - nv/c + v/nc$. Thus

$$\Delta u \approx \frac{c}{n}\left[(1 + nv/c - v/nc) - (1 - nv/c + v/nc)\right] = \frac{2v}{n}(1 - 1/n) = 2v(1 - 1/n^2).$$

Evaluating numerically we find $\Delta u \approx 2(5 \text{ m/s})\left(1 - \frac{1}{1.33^2}\right) = 4.35$ m/s.

35. Clearly the speed of B is just $0.60c$. To find the speed of C use $u_x = 0.60c$ and $v = -0.60c$: $u'_x = \frac{u_x - v}{1 - vu_x/c^2} = \frac{0.60c - (-0.60c)}{1 - (-0.60c)(0.60c)/c^2} = 0.88c.$

36. We can ignore the 400 km, which is small compared with the Earth-to-moon distance 3.84×10^8 m. The rotation rate is $\omega = 2\pi \text{ rad} \times 100 \text{ s}^{-1} = 2\pi \times 10^2$ rad/s. Then the speed across the moon's surface is $v = \omega R = (2\pi \times 10^2 \text{ rad/s})(3.84 \times 10^8 \text{ m}) = 2.41 \times 10^{11}$ m/s.

37. Classical: $t = \frac{4205 \text{ m}}{0.98c} = 1.43 \times 10^{-5}$ s. Then $N = N_0 \exp\left[\frac{-(\ln 2)t}{t_{1/2}}\right] = 14.6$ or about 15 muons.

Relativistic: $t' = t/\gamma = \frac{1.43 \times 10^{-5} \text{ s}}{5} = 2.86 \times 10^{-6}$ s so

$N = N_0 \exp\left[\frac{-(\ln 2)t}{t_{1/2}}\right] = 2710$ muons. Because of the exponential nature of the decay curve, a factor of five (shorter) in time results in many more muons surviving.

38. The circumference of the fixed point's rotational path is $2\pi R_E \cos(39^\circ)$, where R_E = Earth's radius = 6378 km. Thus the circumference of the path is 31,143 km. The

rotational speed of that point is $v = (31{,}143\text{ km})/24\text{ h} = 1298\text{ km/h} = 360.5\text{ m/s}$. The observatory clock runs slow by a factor of $\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 1+\beta^2/2 = 1+7.22\times10^{-13}$. In 41.2 h the observatory clock is slow by $(41.2\text{ h})(7.22\times10^{-13}) = 2.9746\times10^{-11}$ h = 107 ns. In 48.6 h it is slow by $(48.6\text{ h})(7.22\times10^{-13}) = 3.5089\times10^{-11}$ h = 126 ns. The Eastward-moving clock has a ground speed of 31,143 km/41.2 h = 755.9 km/h = 210.0 m/s and thus has a net speed of 210.0 m/s + 360.5 m/s = 570.5 m/s. For this clock $\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 1+\beta^2/2 = 1+1.81\times10^{-12}$ and in 41.2 hours it runs slow by $(41.2\text{ h})(1.81\times10^{-12}) = 7.4572\times10^{-11}$ h = 268 ns. The Westward-moving clock has a ground speed of 31,143 km/48.6 h = 640.8 km/h = 178.0 m/s and thus has a net speed of 360.5 m/s − 178.0 m/s = 182.5 m/s. For this clock $\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx 1+\beta^2/2 = 1+1.85\times10^{-13}$ and in 48.6 hours it runs slow by $(48.6\text{ h})(1.85\times10^{-13}) = 8.991\times10^{-12}$ h = 32 ns. So our prediction is that the Eastward-moving clock is off by 107 ns – 269 ns $= -162$ ns, while the Westward-moving clock is off by 126 ns − 32 ns = 94 ns. These results are correct for special relativity but do not reconcile with those in the table in the text, because general relativistic effects are of the same order of magnitude.

39. The derivations of Equations (2.31) and (2.32) in the beginning of Section 2.10 will suffice. Mary receives signals at a rate f' for t_1' and a rate f'' for t_2'. Frank receives signals at a rate f' for t_1 and a rate f'' for t_2.

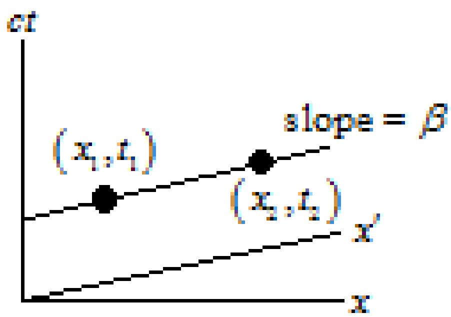
40. $T = t_1 + t_2 = \frac{L}{v} + \frac{L}{c} + \frac{L}{v} - \frac{L}{c} = \frac{2L}{v}$; Frank sends signals at rate *f*, so Mary receives $f\ T = 2f\ L/v$ signals.

$T' = t_1' + t_2' = \frac{2L}{\gamma v}$; Mary sends signals at rate *f*, so Frank receives $fT' = 2f\ L/\gamma v$ signals.

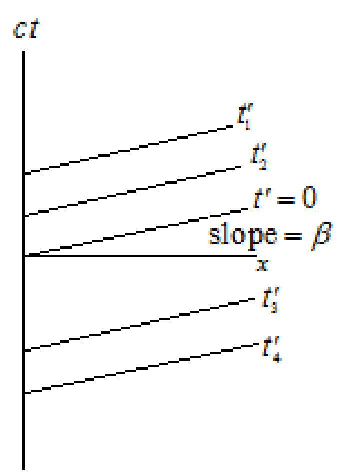
41. $s^2 = x^2 + y^2 + z^2 - c^2t^2$; Using the Lorentz transformation

$$\begin{aligned} s^2 &= \gamma^2(x'+vt')^2 + y'^2 + z'^2 - c^2\gamma^2(t'+vx'/c^2)^2 \\ &= x'^2\gamma^2\left(1-v^2/c^2\right) + y'^2 + z'^2 - c^2t'^2\gamma^2\left(1-v^2/c^2\right) \\ &= x'^2 + y'^2 + z'^2 - c^2t'^2 = s'^2 \end{aligned}$$

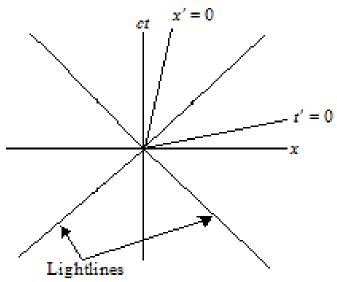
42. For a timelike interval $\Delta s^2 < 0$ so $\Delta x^2 < c^2\Delta t^2$. We will prove by contradiction. Suppose that there is a frame K' is which the two events were simultaneous, so that $\Delta t' = 0$. Then by the spacetime invariant $\Delta x^2 - c^2\Delta t^2 = \Delta x'^2 - c^2\Delta t'^2 = \Delta x'^2$. But because $\Delta x^2 < c^2\Delta t^2$, this implies $\Delta x'^2 < 0$ which is impossible because $\Delta x'$ is real.
43. As in Problem 42, we know that for a spacelike interval $\Delta s^2 > 0$ so $\Delta x^2 > c^2\Delta t^2$. Then in a frame K' in which the two events occur in the same place, $\Delta x' = 0$ and $\Delta x^2 - c^2\Delta t^2 = \Delta x'^2 - c^2\Delta t'^2 = -c^2\Delta t'^2$. But because $\Delta x^2 > c^2\Delta t^2$ we have $c^2\Delta t'^2 < 0$, which is impossible because $\Delta t'$ is real.
44. In order for two events to be simultaneous in K', the two events must lie along the x' axis, or along a line parallel to the x' axis. The slope of the x' axis is $\beta = v/c$, so $v/c = \text{slope} = \dfrac{c\Delta t}{\Delta x}$. Solving for *v*, we find $v = c^2\Delta t/\Delta x$. Since the slope of the x' axis must be less than one, we see that $\Delta x > c\Delta t$ so $s^2 = \Delta x^2 - c^2\Delta t^2 > 0$ is required.



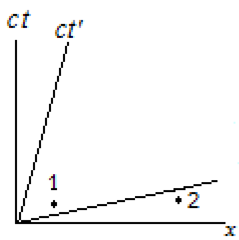
45. parts (a) and (b) To find the equation of the line use the Lorentz transformation. With $t' = 0$ we have $t' = 0 = \gamma\left(t - vx/c^2\right)$ or, rearranging, $ct = vx/c = \beta c$. Thus the graph of *ct* vs. *x* is a straight line with a slope β.
(c) Now with t' constant, the Lorentz transformation gives $t' = \gamma\left(t - vx/c^2\right)$. Again we solve for *ct* : $ct = \beta x + ct'/\gamma = \beta x + \text{constant}$. This line is parallel to the $t' = 0$ line we found earlier but shifted by the constant.
(d) Here both the x' and ct' axes are shifted from their normal (*x*, *ct*) orientation and they are not perpendicular.



46. The diagram is shown here. Note that there is only one worldline for light, and it bisects both the x, ct axes and the x', ct' axes. The x' and ct' axes are not perpendicular. This can be seen as a result of the Lorentz transformations, since $x' = 0$ defines the ct' axis and $t' = 0$ defines the x' axis.



47. The diagram shows that the events A and B that occur at the same time in K occur at different times in K′.



48. The Doppler shift gives $\lambda = \lambda_0\sqrt{\dfrac{1-\beta}{1+\beta}}$. With numerical values $\lambda_0 = 650$ nm and $\lambda = 540$ nm, solving this equation for β gives $\beta = 0.183$. The astronaut's speed is $v = \beta c = 5.50\times10^7$ m/s. In addition to a red light violation, the astronaut gets a speeding ticket.

49. According to the fixed source (K) the signal and receiver move at speeds c and v, respectively, in opposite directions, so their relative speed is $c+v$. The time interval between receipt of signals is $\Delta t = \lambda/(c+v) = 1/f_0$. By time dilation $\Delta t' = \dfrac{\Delta t}{\gamma} = \dfrac{\lambda}{\gamma(c+v)}$.

Using $\lambda = c/v_0$ and $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$ we find $\Delta t' = \dfrac{c\sqrt{1-v^2/c^2}}{f_0(c+v)} = \dfrac{\sqrt{1-\beta^2}}{f_0(1+\beta)}$ and

$$f' = \frac{1}{\Delta t'} = \frac{f_0(1+\beta)}{\sqrt{1-\beta^2}} = f_0\sqrt{\frac{1+\beta}{1-\beta}}.$$

50. For a fixed source and moving receiver, the length of the wave train is $cT + vT$. Since n waves are emitted during time T, $\lambda = \dfrac{cT + vT}{n}$ and the frequency $f = c/\lambda$ is $f = \dfrac{cn}{cT + vT}$.

As in the text $n = f_0 T_0'$ and $T_0' = T/\gamma$. Therefore $f = \dfrac{cf_0 T/\gamma}{cT + vT} = \dfrac{f_0\sqrt{1-\beta^2}}{1+\beta} = f_0\sqrt{\dfrac{1-\beta}{1+\beta}}$.

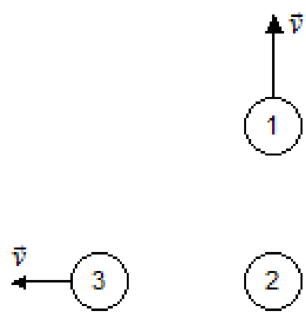
51. $f = f_0\sqrt{\dfrac{1-\beta}{1+\beta}} = (1400\,\text{kHz})\sqrt{\dfrac{1-0.95}{1+0.95}} = 224\text{ kHz}$

52. The Doppler shift function $f' = f_0\sqrt{\dfrac{1-\beta}{1+\beta}}$

is the rate at which #1 and #2 receive signals from each other and the rate at which #2 and #3 receive signals from each other. But for signals between #1 and #3 the rate is

$$f'' = f'\sqrt{\frac{1-\beta}{1+\beta}} = f_0\frac{1-\beta}{1+\beta}.$$

53. The Doppler shift function $f' = f_0\sqrt{\dfrac{1-\beta}{1+\beta}}$



is the rate at which #1 and #2 receive signals from each other and the rate at which #2 and #3 receive signals from each other. As for #1 and #3 we will assume that these plumbing vans are non-relativistic $(v << c)$. Otherwise it would be necessary to use the velocity addition law and apply the transverse Doppler shift. From the figure we see that $f' = \dfrac{1}{t_0 + (t_2 - t_1)}$. Now $f_0 = 1/t_0$ and

$t_2 - t_1 = \dfrac{2x}{c} = \dfrac{2vt_0\cos\theta}{c}$. With an angle of 45°, $\cos(45) = 1/\sqrt{2}$ and

$$f' = \frac{1}{1/f_0 + (2v\cos\theta)/cf_0} = \frac{f_0}{1 + (2v\cos\theta)/c} = \frac{f_0}{1 + \sqrt{2}v/c}.$$

54. The Doppler shift to higher wavelengths is (with $\lambda_0 = 589\text{ nm}$) $\lambda = 700\text{ nm} = \lambda_0\sqrt{\dfrac{1+\beta}{1-\beta}}$.

Solving for β we find $\beta = 0.171$. Then $t = \dfrac{v}{a} = \dfrac{(0.171)(3.00\times10^8\text{ m/s})}{29.4\text{ m/s}^2} = 1.75\times10^6\text{ s}$ which is 20.25 days. One problem with this analysis is that we have only computed the time as measured by Earth. We are not prepared to handle the non-inertial frame of the spaceship.

55. Let the instantaneous momentum be in the *x*-direction and the force be in the *y*-direction. Then $d\vec{p} = \vec{F}\,dt = \gamma m d\vec{v}$ and $d\vec{v}$ is also in the *y*-direction. So we have $\vec{F} = \gamma m \dfrac{d\vec{v}}{dt} = \gamma m \vec{a}$.

56. The magnitude of the centripetal force is $\gamma m a = \gamma m \dfrac{v^2}{r}$ for circular motion. For a charged particle $F = qvB$, so $qvB = \gamma m \dfrac{v^2}{r}$ or, rearranging $qBr = \gamma m v = p$. Therefore

$$r = \frac{p}{qB}.$$

When the speed increases the momentum increases, and thus for a given value of *B* the radius must increase.

57. $\vec{p} = \gamma m \vec{v} = \dfrac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$ and $\vec{F} = \dfrac{d\vec{p}}{dt}$. The momentum is the product of two factors that contain the velocity, so we apply the product rule for derivatives:

$$\begin{aligned}
\vec{F} &= m\frac{d}{dt}\left[\frac{m\vec{v}}{\sqrt{1-v^2/c^2}}\right] \\
&= m\left[\frac{d\vec{v}/dt}{\sqrt{1-v^2/c^2}} + \vec{v}\,\frac{d}{dt}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right)\right] \\
&= \gamma m\vec{a} + m\vec{v}\left(-\frac{1}{2}\right)\left(-\frac{2v}{c^2}\right)\gamma^3\frac{dv}{dt} \\
&= \gamma m\vec{a} + \gamma^3 m\vec{a}\left(\frac{v^2}{c^2}\right) \\
&= \gamma^3 m\vec{a}\left[1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}\right] \\
&= \gamma^3 m\vec{a}
\end{aligned}$$

58. From the preceding problem $F = \gamma^3 ma$. We have $a = 10^{19}\ \text{m/s}^2$ and $m = 1.67\times10^{-27}$ kg.

(a) $$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.01^2}} = 1.00005$$

$$F = (1.00005)^3\left(1.67\times10^{-27}\ \text{kg}\right)\left(10^{19}\ \text{m/s}^2\right) = 1.67\times10^{-8}\ \text{N}$$

(b) As in (a) $\gamma = 1.005$ and $F = 1.70\times10^{-8}$ N

(c) As in (a) $\gamma = 2.294$ and $F = 2.02\times10^{-7}$ N

(d) As in (a) $\gamma = 7.0888$ and $F = 5.95\times10^{-6}$ N

59. $p = \gamma mv$ with $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = \dfrac{1}{\sqrt{1-0.92^2}} = 2.5516$;

$$m = \frac{p}{\gamma v} = \frac{10^{-16}\ \text{kg·m/s}}{(2.5516)(0.92)(3.00\times10^8\ \text{m/s})} = 1.42\times10^{-25}\ \text{kg}$$

60. The initial momentum is $p_0 = \gamma mv = \dfrac{1}{\sqrt{1-(0.5)^2}}m(0.5c) = 0.57735mc$.

(a) $p/p_0 = 1.01$

$$1.01 = \frac{\gamma mv}{0.57735mc}$$

$$\gamma v = (1.01)(0.57735c) = 0.58312c$$

Substituting for γ and solving for v, $v = \left[\dfrac{1}{(.58312c)^2} + \dfrac{1}{c^2}\right]^{-1/2} = 0.504c.$

(b) Similarly $v = \left[\dfrac{1}{(.63509c)^2} + \dfrac{1}{c^2}\right]^{-1/2} = 0.536c$

(c) Similarly $v = \left[\dfrac{1}{(1.1547c)^2} + \dfrac{1}{c^2}\right]^{-1/2} = 0.756c$

61. 6.3 GeV protons have $K = 6.3\times10^3$ MeV and $E = K + E_0 = 7238$ MeV. Then

$p = \dfrac{\sqrt{E^2 - E_0^2}}{c} = 7177$ MeV/c. Converting to SI units

$$p = 7177\ \text{MeV}/c\left(\frac{1.60\times10^{-13}\ \text{J}}{\text{MeV}}\right)\left(\frac{c}{3.00\times10^8\ \text{m/s}}\right) = 3.83\times10^{-18}\ \text{kg·m/s}$$

From Problem 56 we have $B = \dfrac{p}{qr} = \dfrac{3.83\times10^{-18}\ \text{kg}\cdot\text{m/s}}{(1.60\times10^{-19}\ \text{C})(15.2\ \text{m})} = 1.57\,\text{T}$.

62. Initially Mary throws her ball with velocity (primes showing the measurements are in Mary's frame): $u'_{M_x} = 0 \quad u'_{M_y} = -u_0$. After the elastic collision, the signs on the above expressions are reversed, so the change in momentum as measured by Mary is

$$\Delta p'_M = \frac{mu_0}{\sqrt{1-u_0^2/c^2}} - \frac{-mu_0}{\sqrt{1-u_0^2/c^2}} = \frac{2mu_0}{\sqrt{1-u_0^2/c^2}}.$$

Now for Frank's ball, we know $u_{F_x} = 0$ and $u_{F_y} = u_0$. The velocity transformations give for Frank's ball as measured by Mary: $u'_{F_x} = -v \qquad u'_{F_y} = u_0\sqrt{1-v^2/c^2}$.

To find γ for Frank's ball, note that $\left(u'_{F_x}\right)^2 + \left(u'_{F_y}\right)^2 = v^2 + u_0^2\left(1-v^2/c^2\right)$. Then

$$\gamma = \frac{1}{\sqrt{1-u_F'^2/c^2}} = \frac{1}{\sqrt{1-v^2/c^2-u_0^2\left(1-v^2/c^2\right)/c^2}} = \frac{1}{\sqrt{\left(1-u_0^2/c^2\right)\left(1-v^2/c^2\right)}}$$

. Using $p' = \gamma m u'$ along with the reversal of velocities in an elastic collision, we find

$$\Delta p'_F = \gamma m\left(-u_0\right)\sqrt{1-v^2/c^2} - \gamma m u_0\sqrt{1-v^2/c^2} = -2\gamma m u_0\sqrt{1-v^2/c^2}$$

$$= \frac{-2mu_0\sqrt{1-v^2/c^2}}{\sqrt{\left(1-u_0^2/c^2\right)\left(1-v^2/c^2\right)}} = \frac{-2mu_0}{\sqrt{\left(1-u_0^2/c^2\right)}}$$

Finally $\Delta p' = \Delta p'_F + \Delta p'_M = \dfrac{-2mu_0 + 2mu_0}{\sqrt{\left(1-u_0^2/c^2\right)}} = 0$ as required.

63. To prove by contradiction, suppose that $K = \frac{1}{2}\gamma m v^2$. Then

$K = E - E_0 = \gamma mc^2 - mc^2 = \left(\gamma - 1\right)mc^2 = \frac{1}{2}\gamma m v^2$. This implies $\gamma - 1 = v^2/2c^2$, or

$\gamma = 1 + v^2/2c^2$, which is clearly false.

64. The source of the energy is the internal energy associated with the change of state, commonly called that latent heat of fusion L_f. Let m be the mass equivalent of 2 grams and M be the mass of ice required. $m = \dfrac{E}{c^2} = \dfrac{L_f M}{c^2}$ Rearranging

$$M = \frac{mc^2}{L_f} = \frac{\left(0.002\ \text{kg}\right)\left(3.00\times10^8\ \text{m/s}\right)^2}{334\times10^3\ \text{J/kg}} = 5.39\times10^8\ \text{kg}$$

65. In general $K = \left(\gamma - 1\right)mc^2$, so $\gamma = 1 + \dfrac{K}{mc^2}$. For 9 GeV electrons:

$\gamma = 1 + \dfrac{9000\ \text{MeV}}{0.511\ \text{MeV}} = 1.76\times10^4$ Then from the definition of γ we have

$\beta = \sqrt{1-\dfrac{1}{\gamma^2}} = \sqrt{1-\dfrac{1}{\left(1.76\times10^4\right)^2}} = 1 - 1.6\times10^{-9}$. Thus

$v = \left(1-1.6\times10^{-9}\right)c = 0.9999999984c$.

For 3.1 GeV positrons: $\gamma = 1 + \frac{3100 \text{ MeV}}{0.511 \text{ MeV}} = 6068$ and $\beta = \sqrt{1 - \frac{1}{(6068)^2}} = 1 - 1.4 \times 10^{-8}$.

Thus $v = (1 - 1.4 \times 10^{-8})c = 0.999999986c$.

66. Note that the proton's mass is 938 MeV/c^2. In general $K = (\gamma - 1)mc^2$, so $\gamma = 1 + \frac{K}{mc^2}$.

Then from the definition of γ we have $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$. For the first section $K = 0.750$ MeV, and $\gamma = 1 + \frac{0.750 \text{ MeV}}{938 \text{ MeV}} = 1.00080$ with $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \beta = \sqrt{1 - \frac{1}{(1.00080)^2}} = 0.040$.

Thus $v = 0.04c$ at the end of the first stage. For the other stages the computations are similar, and we tabulate the results:

K (GeV)	γ	β
0.400	1.43	0.71
8	9.53	0.994
150	160.9	0.99998
1000	1067	0.9999996

67. (a) $p = \gamma m u = \frac{(511 \text{ keV}/c^2)(0.020c)}{\sqrt{1 - 0.020^2}} = 10.22 \text{ keV}/c$;

$E = \gamma mc^2 = \frac{(511 \text{ keV}/c^2)(c^2)}{\sqrt{1 - 0.02^2}} = 511.102 \text{ keV}$;

$K = E - E_0 = 511.102 \text{ keV} - 511.00 \text{ keV} = 102 \text{ eV}$

The results for (b) and (c) follow with similar computations and are tabulated:

β	p (keV/c)	E (keV)	K (keV)
0.20	104.3	521.5	10.5
0.90	1055	1172	661

68. $E = 2E_0 = \gamma E_0$ so $\gamma = 2$. Then $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \frac{\sqrt{3}}{2}$ and $v = \frac{\sqrt{3}c}{2}$.

69. For a constant force, work = change in kinetic energy = $Fd = mc^2/4,$ because

$$25\% = 1/4. \quad d = \frac{mc^2}{4F} = \frac{(80\text{ kg})(3.00\times10^8\text{ m/s})^2}{4(8\text{ N})} = 2.25\times10^{17}\text{ m} = 23.8\text{ ly.}$$

70. $E = K + E_0 = 2E_0 + E_0 = 3E_0 = \gamma E_0$ so $\gamma = 3$. Then

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \beta = \sqrt{1-\frac{1}{3^2}} = \frac{2\sqrt{2}}{3} = 0.943.$$ Thus $v = 0.943c$.

71. (a) $E = K + E_0 = 0.1E_0 + E_0 = 1.1E_0 = \gamma E_0$, so $\gamma = 1.1$. Then

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \beta = \sqrt{1-\frac{1}{1.1^2}} = 0.417$$ and $v = 0.417c$.

(b) As in (a) $\gamma = 2$ and $v = \sqrt{3}c/2$.

(c) As in (a) $\gamma = 11$ and $v = 0.996c$.

72. $K = E - E_0 = \dfrac{E_0}{\sqrt{1-\beta^2}} - E_0$ so $K + E_0 = \dfrac{E_0}{\sqrt{1-\beta^2}}$. Rearranging $1-\beta^2 = \left(\dfrac{E_0}{K+E_0}\right)^2$ which

can be written as $\beta^2 = 1-\left(\dfrac{E_0}{K+E_0}\right)^2$ or $\beta = \sqrt{1-\left(\dfrac{E_0}{K+E_0}\right)^2}$.

73. Using $E = \gamma mc^2$ along with $p = \gamma mv$ we see that $\gamma = E/mc^2 = p/mv$. Solving for *v/c* we find $\beta = v/c = pc/E$.

74. The speed is the same for protons, electrons, or any particle.

$$K = (\gamma-1)mc^2 = 1.01\left(\frac{1}{2}mv^2\right) = 0.505mc^2\beta^2 \text{ so } \gamma - 1 = \frac{1}{\sqrt{1-\beta^2}} - 1 = 0.505\beta^2.$$

Rearranging and solving for β, we find $\beta = 0.114$ or $v = 0.114c$.

75. Converting 0.1 ounce = 2.835×10^{-3} kg.

$$E = mc^2 = (2.835\times10^{-3}\text{ kg})(3.00\times10^8\text{ m/s})^2 = 2.55\times10^{14}\text{ J}.$$

Eating 10 ounces results in a factor of 100 greater mass-energy increase, or 2.55×10^{16} J. This is a small increase compared with your original mass-energy, but it will tend to increase your weight; depending on how they are prepared, peanuts generally contain about 100 kcal of food energy per ounce.

76. The energy needed equals the kinetic energy of the spaceship.

$$K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)mc^2$$
$$= \left(\frac{1}{\sqrt{1-0.3^2}} - 1\right)\left(10^4 \text{ kg}\right)\left(3.00\times10^8 \text{ m/s}\right)^2 = 4.35\times10^{19} \text{ J}$$

or 4.35% of 10^{21} J.

77. Up to Equation (2.57) the derivation in the text is complete. Then using the integration by parts formula, $\int x dy = xy - \int y dx$ and noting that in this case $x = u$ and $y = \gamma u$, we have $\int u d(\gamma u) = \gamma u^2 - \int \gamma u du$. Thus

$$K = m\int_0^{\gamma u} u\, d(\gamma u) = \gamma m u^2 - m\int \gamma u\, du$$
$$= \gamma m u^2 - m\int \frac{u}{\sqrt{1-u^2/c^2}} du$$

Using integral tables or simple substitution:

$$K = \gamma m u^2 + mc^2\sqrt{1-u^2/c^2}\Big|_0^u$$
$$= \gamma m u^2 + mc^2\sqrt{1-u^2/c^2} - mc^2$$
$$= \frac{mc^2 + mc^2\left(1-u^2/c^2\right)}{\sqrt{1-u^2/c^2}} - mc^2$$
$$= \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$$

78. Converting 0.11 $\text{cal}\cdot\text{g}^{-1}\cdot{}^\circ\text{C}^{-1} = 460\ \text{J}\cdot\text{kg}^{-1}\cdot{}^\circ\text{C}^{-1} = c_\text{V}$

(specific heat at constant volume). From thermodynamics the energy ΔE used to change the temperature by ΔT is $mc_\text{V}\Delta T$. Thus

$\Delta E = (1000 \text{ kg})\left[\left(460 \text{ J/(kg}\cdot{}^\circ\text{C}\right)\right]\left(0.5^\circ\text{C}\right) = 2.30\times10^5 \text{J}$ and

$\Delta m = \dfrac{\Delta E}{c^2} = \dfrac{2.30\times10^5 \text{ J}}{\left(3.00\times10^8 \text{ m/s}\right)^2} = 2.56\times10^{-12} \text{ kg}$. The source of this energy is the internal energy of the arrangement of atoms and molecules prior to the collision.

79.

$$E_b = \left[2m_p + 2m_n - m(\text{He})\right]c^2$$

$$= \left[2(1.007276\text{ u}) + 2(1.008665\text{ u}) - 4.001505\text{ u}\right]c^2\left(\frac{931.494\text{ MeV}}{c^2\cdot\text{u}}\right) = 28.3\text{ MeV}$$

80.

$$\Delta E = \left[m_n - m_p - m_e\right]c^2$$

$$= \left[1.008665\text{ u} - 1.007276\text{ u} - 0.000549\text{ u}\right]c^2\left(\frac{931.494\text{ MeV}}{c^2\cdot\text{u}}\right) = 0.782\text{ MeV}$$

81. $E = K + E_0 = 1\text{ TeV} + 938\text{ MeV} \approx 1\text{ TeV}$;

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(1\text{ TeV} + 938\text{MeV})^2 - (938\text{ MeV})^2}}{c} = 1.000938\text{ TeV}/c$$

$$\gamma = \frac{E + E_0}{E_0} = \frac{1.000938\text{ TeV}}{0.000938\text{ TeV}} = 1067; \qquad \beta^2 = 1 - \frac{1}{\gamma^2} = 1 - 8.78\times10^{-7}$$

$$\beta = \sqrt{1 - 8.78\times10^{-7}} \approx 1 - 4.39\times10^{-7}; \text{ and } v = \beta c \approx 0.999999561c$$

82. (a) $E = \sqrt{p^2c^2 + E_0^2} = \sqrt{(40\text{GeV})^2 + (511\text{ keV})^2} \approx 40.0\text{ GeV}$

$$K = E - E_0 = 40.0\text{ GeV}$$

(b) $E = \sqrt{p^2c^2 + E_0^2} = \sqrt{(40\text{ GeV})^2 + (0.938\text{ GeV})^2} = 40.011\text{GeV}$

$$K = E - E_0 = 40.011\text{ GeV} - 0.938\text{ GeV} = 39.07\text{ GeV}$$

83. $E = K + E_0 = 200\text{ MeV} + 106\text{ MeV} = 306\text{ MeV}$

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(306\text{ MeV})^2 - (106\text{ MeV})^2}}{c} = 287.05\text{ MeV}/c$$

$$\gamma = \frac{E}{E_0} = \frac{306\text{ MeV}}{106\text{ MeV}} = 2.887$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.938$$

so $v = 0.938c$

84. (a) The mass-energy imbalance occurs because the helium-3 $({}^3\text{He})$ nucleus is more tightly bound than the two separate deuterium nuclei $({}^2\text{H})$. (Masses from Appendix 8.)

$$\Delta E = \left([2m({}^2\text{H})] - [m_n + m({}^3\text{He})]\right)c^2$$

$$= \left[2(2.014102\text{ u}) - (1.008665\text{ u} + 3.016029\text{ u})\right]c^2\left(\frac{931.494\text{ MeV}}{c^2 \cdot \text{u}}\right) = 3.27\text{ MeV}$$

(b) The initial rest energy is $2m({}^2\text{H}) = 2(2.014102\text{ u})\left(\frac{931.494\text{ MeV}}{\text{c}^2 \cdot \text{u}}\right) = 3752\text{ MeV}$.

Thus the answer in (a) is about 0.09% of the initial rest energy.

85. (a) The mass-energy imbalance occurs because the helium-4 $({}^4\text{He})$ is more tightly bound than the deuterium $({}^2\text{H})$ and tritium nuclei $({}^3\text{H})$.

$$\Delta E = \left([m({}^2\text{H}) + m({}^3\text{H})] - [m_n + m({}^4\text{He})]\right)c^2$$

$$= \left[(2.014102\text{ u} + 3.016029\text{ u}) - (1.008665\text{ u} + 4.002603\text{ u})\right]c^2\left(\frac{931.494\text{ MeV}}{c^2 \cdot \text{u}}\right)$$

$$= 17.6\text{ MeV}$$

(b) The initial rest energy is $\left[m({}^2\text{H}) + m({}^3\text{H})\right] \cdot c^2 = \left[(5.030131\text{ u})\right] \cdot c^2\left(\frac{931.494\text{ MeV}}{c^2 \cdot \text{u}}\right) =$ 4686 MeV. Thus the answer in (a) is about 0.37% of the initial rest energy.

86. (a) In the inertial frame moving with the negative charges in wire 1, the negative charges in wire 2 are stationary, but the positive charges are moving. The density of the positive charges in wire 2 is thus greater than the density of negative charges, and there is a net attraction between the wires.

(b) By the same reasoning as in (a), note that the positive charges in wire 2 will be stationary and have a normal density, but the negative charges are moving and have an increased density, causing a net attraction between the wires.

(c) There are two facts to be considered. First, (a) and (b) are consistent with the physical result being independent of inertial frame. Second, we know from classical physics that two parallel wires carrying current in the same direction attract each other. That is, the same result is achieved in the "lab" frame.

87. As in the solution to Problem 21 we have $\beta = \frac{v}{c} = \frac{d\sqrt{1-\beta^2}}{ct'}$ where d is the length of the particle track and t' the particle's lifetime in its rest frame. In this problem $t' = 8.2 \times 10^{-11}$ s

and d = 24 mm. Solving the above equation we find $\beta = 0.698$. Then

$$E = \frac{E_0}{\sqrt{1-\beta^2}} = \frac{1672 \text{ MeV}}{\sqrt{1-0.698^2}} = 2330 \text{ MeV}$$

88.

$$F = \frac{dp}{dt} = \frac{d}{dt}(\gamma m v) = \frac{d}{dt}\left[mv\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= m\gamma\frac{dv}{dt} - \frac{mv}{2}\gamma^3\left(-\frac{2v}{c^2}\right)\frac{dv}{dt}$$

$$= m\gamma\frac{dv}{dt}\left[1+\frac{v^2\gamma^2}{c^2}\right] = m\gamma\frac{dv}{dt}\left[1+\frac{v^2/c^2}{1-v^2/c^2}\right]$$

$$= m\gamma\frac{dv}{dt}\left[\frac{1}{1-v^2/c^2}\right] = m\gamma^3\frac{dv}{dt} = m\frac{dv}{dt}\frac{1}{\left(1-v^2/c^2\right)^{3/2}}$$

89. (a) The number n received by Frank at f' is half the number sent by Mary at that rate, or $fL/\gamma v$. The detected time of turnaround is

$$t = \frac{n}{f'} = \frac{fL/\gamma v}{v\sqrt{(1-\beta)/(1+\beta)}} = \frac{L\sqrt{1+\beta}}{\gamma v\sqrt{1-\beta}} = \frac{L(1+\beta)}{v} = \frac{L}{v} + \frac{L}{c}.$$

(b) Similarly, the number n' received by Mary at f' is

$$n' = f'\frac{T'}{2} = f\sqrt{\frac{1-\beta}{1+\beta}}\frac{L}{\gamma v} = \frac{f\,L(1-\beta)}{v}.$$ Her turnaround time is $T'/2 = L/\gamma v$.

(c) For Frank, the time t_2 for the remainder of the trip is $t_2 = T - t_1 = L/v - L/c$.

Number of signals = $f''t_2 = f\sqrt{\dfrac{1+\beta}{1-\beta}}\left(L/v - L/c\right) = \dfrac{fL}{\gamma v}$.

Total number received = $\dfrac{fL}{\gamma v} + \dfrac{fL}{\gamma v} = \dfrac{2fL}{\gamma v}$.

Mary's age = $\dfrac{\text{Total number received}}{f} = \dfrac{2L}{\gamma v}$.

(d) For Mary, $t_2' = T' - t_1' = L/\gamma v$.

Number of signals = $f''t_2' = f\sqrt{\dfrac{1+\beta}{1-\beta}}\dfrac{L}{\gamma v} = \dfrac{fL}{v}(1+\beta)$.

Total number received = $\dfrac{fL}{v}(1-\beta+1+\beta) = \dfrac{2fL}{v}$.

$$\text{Frank's age} = \frac{\text{Total number received}}{f} = \frac{2L}{v}.$$

90. In the fixed frame, the distance is $\Delta x = 8$ ly and the elapsed time is $\Delta t = 10$ y, so the interval is $s^2 = \Delta x^2 - c^2\Delta t^2 = 64 \text{ ly}^2 - 100 \text{ ly}^2 = -36 \text{ ly}^2$. In the moving frame, Mary's clock is at rest, so $\Delta x' = 0$, and the time interval is $\Delta t' = 6$ y. Thus the interval is $s'^2 = \Delta x'^2 - c^2\Delta t'^2 = 0 \text{ ly}^2 - 36 \text{ ly}^2 = -36 \text{ ly}^2$. The results are the same, as they should be, because the spacetime interval is the same for all inertial frames.

91. (a) From Table 2.1 number $= \dfrac{fL}{v}(1-\beta) = \dfrac{(52\text{y}^{-1})(4.3 \text{ ly})}{0.8c}(1-0.8) = 55.9 \approx 56$

(b) $t_1 = \dfrac{L}{v} + \dfrac{L}{c} = \dfrac{4.3 \text{ ly}}{0.8c} + \dfrac{4.3 \text{ ly}}{c} = 9.68 \text{ y}$; number $= f' t_1 = \dfrac{fL}{v}\sqrt{1-\beta^2} = 167.7 \approx 168$

(c) Frank: $f'' t_2' = \dfrac{f\,L}{v}\sqrt{1-\beta^2} \approx 168$ so the total is 168 + 168 = 336.

Mary: number $= 2f\,L/v = 559$

(d) Frank: $T = 2L/v = 10.75 \text{ y}$ Mary: $T' = 2L/\gamma v = 6.45 \text{ y}$

(e) From part (c), 559 weeks = 10.75 years and 336 weeks = 6.46 years, which agrees with part (d).

92. Notice that the radar is shifted twice, once upon receipt by the speeding car and again upon reemission (reflection) of the beam. For this double shift, the received frequency f is

$$\frac{f}{f_0} = \sqrt{\frac{1-\beta}{1+\beta}}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{1-\beta}{1+\beta}$$

For speeds much less than c, a Taylor series approximation gives an excellent result:

$$\frac{f}{f_0} = \frac{1-\beta}{1+\beta} = (1-\beta)(1+\beta)^{-1} \approx 1-2\beta.$$

Converting 80 mph to 35.8 m/s gives $\beta = 1.19 \times 10^{-7}$, so the received frequency is approximately

$f = f_0(1-2\beta) = 10.4999975 \text{ GHz}$, which is 2.5 kHz less than the original frequency.

93. For this transformation $v = 0.8c$ (so $\gamma = 5/3$), $u_x' = 0$ and $u_y' = 0.8c$. Applying the transformations,

$$u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}} = \frac{0 + 0.8c}{1 + 0} = 0.8c\ ; \qquad u_y = \frac{u'_y}{\gamma\left(1 + \frac{vu'_x}{c^2}\right)} = \frac{0.8c}{\frac{5}{3}(1+0)} = 0.48c$$

The speed is $u = \sqrt{u_x^2 + u_y^2} = 0.93c$, safely under the speed of light.

94. (a) $K = E + E_0 = 250E_0$. Thus $K = 249E_0 = 249(511\text{keV}) = 127.2\text{ MeV}$

(b) $\gamma = 250$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.999992$$

$v = 0.999992c$

(c) $p = \dfrac{\sqrt{E^2 - E_0^2}}{c} = \dfrac{\sqrt{(250 \times 511\text{ keV})^2 - (511\text{ keV})^2}}{c} = 128\text{ MeV}/c$

95. (a) For the proton: $p = \gamma m u = \dfrac{1}{\sqrt{1 - 0.9^2}}(938\text{ MeV}/c^2)(0.9c) = 1940\text{ MeV}/c$.

For the electron: $E = \sqrt{p^2c^2 + E_0^2} = \sqrt{(1940\text{ MeV})^2 + (0.511\text{ MeV})^2} = 1940\text{ MeV}$.

$$\gamma = \frac{E}{E_0} = \frac{1940\text{ MeV}}{0.511\text{ MeV}} = 3797$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{3797^2}} = \sqrt{1 - 6.94 \times 10^{-8}} \approx 1 - 3.97 \times 10^{-8}$$

$$v = (1 - 3.97 \times 10^{-8})c$$

(b) For the proton: $K = (\gamma - 1)E_0 = \left(\dfrac{1}{\sqrt{1 - 0.9^2}} - 1\right)(938\text{ MeV}) = 1214\text{ MeV}$.

For the electron: $\gamma = \dfrac{K + E_0}{E_0} = \dfrac{1214\text{ MeV} + 0.511\text{ MeV}}{0.511\text{ MeV}} = 2377$.

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{2377^2}} = \sqrt{1 - 1.77 \times 10^{-7}} \approx 1 - 8.85 \times 10^{-8}$$

$$v = (1 - 8.85 \times 10^{-8})c$$

96. In the frame of the decaying K^0 meson, the pi mesons must recoil with equal speeds in opposite directions in order to conserve momentum. In that reference frame the available kinetic energy is $498\text{ MeV} - 2(140\text{ MeV}) = 218\text{ MeV}$. The pi mesons share this equally,

so each one has a kinetic energy of 109 MeV in that frame. The speed of each pi meson can be found:

$$\gamma = \frac{K+E_0}{E_0} = \frac{109 \text{ MeV } + 140 \text{ MeV}}{140 \text{ MeV}} = 1.779 \text{ so } u = \sqrt{1-\frac{1}{\gamma^2}}c = 0.827c.$$

The greatest and least speeds in the lab frame are obtained when the pi mesons are released in the forward and backward directions. Then by the velocity addition laws:

$$v_{max} = \frac{0.9c + 0.827c}{1+(0.9)(0.827)} = 0.990c$$

$$v_{min} = \frac{0.9c - 0.827c}{1-(0.9)(0.827)} = 0.285c$$

97. (a) The round-trip distance is $L_0 = 8.60 \text{ ly}$. Assume the same constant speed $v = \beta c$ for the entire trip. In the rocket's frame, the distance is only $L = L_0\gamma^{-1} = L_0\sqrt{1-\beta^2}$. Mary will age in the rocket's reference frame a total of 22 y, and in that frame

$$v = \frac{\text{distance}}{\text{time}} = \frac{L}{22\text{y}} = \frac{8.60 \text{ ly}\sqrt{1-\beta^2}}{22\text{y}} = 0.39c\sqrt{1-\beta^2}.$$ Therefore, $\beta = \frac{v}{c} = 0.39\sqrt{1-\beta^2}$.

Solving for β we find $\beta = \sqrt{\frac{0.39^2}{1.00+0.39^2}}$, or $\beta = 0.36$ so Mary's speed is $v = 0.36c$.

(b) To find the elapsed time on Earth, we know that $T_0 = 22\text{y}$, so

$T = \gamma T_0 = \frac{1}{\sqrt{1-\beta^2}}22\text{y} = 23.6 \text{ y}$. Frank will be 53.6 years old when Mary returns at the age of 52 y.

98. With $v = 0.995c$ then $\beta = 0.995$ and $\gamma = 10.01$. The distance out to the star is $L_0 = 5.98 \text{ ly}$. In the rocket's reference frame, the distance is only

$$L = L_0\gamma^{-1} = \frac{5.98 \text{ ly}}{10.01} = 0.597 \text{ ly}.$$

(a) The time out to the star in Mary's frame is $time = \frac{\text{distance}}{\text{speed}} = \frac{0.597 \text{ ly}}{0.995c} = 0.6 \text{ y}$. The time for the return journey will be the same. When you include the 3 years she spends at the star, her total journey will take 4.2 y.

(b) To find the elapsed time on Earth for the outbound journey, we know that $T_0 = 0.6\text{y}$ so $T = \gamma T_0 = (10.01)0.6 \text{ y} = 6.006 \text{ y}$. The return journey will take an equal time. The 3 years the spaceship orbits the star will be equivalent for both observers. Therefore Frank will measure a total elapsed time of 15.012 y, which makes him 10.8 years older than her.

99. (a) The Earth to moon distance is 3.84×10^{8} m. The rotation rate is $\omega = 2\pi \text{ rad}\times 0.030 \text{ s}^{-1} = 1.885\times10^{-1}$ rad/s. Then the speed across the moon's surface is $v = \omega R = \left(1.885\times10^{-1}\text{ rad/s}\right)\left(3.82\times10^{8}\text{ m}\right) = 7.24\times10^{7}$ m/s.

(b) The required speed can be found using $c = \omega R = 2\pi f\, R$ which requires the frequency to be $f = \dfrac{c}{2\pi R} = \dfrac{3.00\times10^{8}\text{m/s}}{2\pi\left(3.84\times10^{8}\text{m/s}\right)} = 0.124$ Hz.

100. With the data given, $\beta = 3.28\times10^{-6}$, which is very small. We will use the binomial approximation theorem. From Equation (2.21), we know that $L = L_0\gamma^{-1}.\gamma^{-1} = \sqrt{1-\beta^2} \approx 1-\beta^2/2$.

(a) The percentage of length contraction would be:

$$\%\text{ change} = \frac{\left[L_0 - L\right]}{L_0}\times100\% = \frac{\left[L_0 - L_0\gamma^{-1}\right]}{L_0}\times100\%$$

$$= \left[1-\gamma^{-1}\right]\times100\% = \left[1-(1-\beta^2/2)\right]\times100\%$$

$$= \frac{\beta^2}{2}\times100\% = 5.37\times10^{-10}$$

(b) The clocks' rates differ by a factor $\gamma = 1/\sqrt{1-\beta^2}$. The clock on the SR-71 measures the proper time and Equation 2.19 tells us that $T' = \gamma T_0$ so the time difference is $\Delta t = T' - T_0 = \gamma T_0 - T_0 = T_0(\gamma - 1)$. Using $\gamma - 1 \approx \beta^2/2$ and the fact that the time for the trip in the SR-71 equals the distance divided by the speed

$$\Delta t = t\left(\beta^2/2\right) = \frac{3.2\times10^{6}\text{ m}}{983\text{ m/s}}\,\frac{\left(\dfrac{983\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)^2}{2}$$

$$= 1.75\times10^{-8}\text{s} = 17.5\text{ ns}$$

101. As the spaceship is approaching the observer, we will make use of Equation (2.32), $f' = \dfrac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$ with the prime indicating the Doppler shifted frequency (or wavelength).

This equation indicates that the Doppler shifted frequency will be larger than the frequency measured in a frame where the observer is at rest with respect to the source. Since $c = \lambda f$, this means the Doppler shifted wavelengths will be lower. As given in the problem, we see that the difference in wavelengths for an observer at rest with respect to the source is $\Delta\lambda_0 = 0.5974\text{nm}$. We want to find a speed so that the Doppler shifted difference is reduced to $\Delta\lambda' = 0.55\text{nm}$. We have

$$\Delta\lambda' = \lambda_2' - \lambda_1' = \frac{c}{f_2'} - \frac{c}{f_1'} = c\left(\frac{f_1' - f_2'}{f_1' f_2'}\right)$$

$$= c\left[\frac{\left(\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}\right) f_{0,1} - \left(\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}\right) f_{0,2}}{\frac{1+\beta}{1-\beta} f_{0,1}\, f_{0,2}}\right] = c\left[\left(\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}\right)\left(\frac{f_{0,1} - f_{0,2}}{f_{0,1}\, f_{0,2}}\right)\right]$$

$$= \left[\left(\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}\right)\left(\frac{c}{f_{0,2}} - \frac{c}{f_{0,1}}\right)\right] = \left[\left(\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}\right)\left(\lambda_{0,2} - \lambda_{0,1}\right)\right]$$

$$= \left[\left(\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}\right)\left(\Delta\lambda_0\right)\right]$$

We can complete the algebra to show that $\beta = \dfrac{\Delta\lambda_0^2 - (\Delta\lambda')^2}{\Delta\lambda_0^2 + (\Delta\lambda')^2} = 0.0825$ so that

$v = 2.47 \times 10^7$ m/s.

102. As we know that the quasars are moving away at high speeds, we make use of Equation (2.33) and the equation $c = \lambda f$. Using a prime to indicate the Doppler shifted frequency (or wavelength), Equation (2.33) indicates that frequency is given by $f' = \dfrac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$, or

$\dfrac{f_0}{f'} = \dfrac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$ so

$$z = \frac{(\lambda' - \lambda_0)}{\lambda_0} = \left(\frac{\lambda'}{\lambda_0} - 1\right) = \left(\frac{c/f'}{c/f_0} - 1\right)$$

$$= \left(\frac{f_0}{f'} - 1\right) = \left(\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} - 1\right)$$

Therefore $(z+1)^2 = \dfrac{1+\beta}{1-\beta}$. We can complete the algebra to show that

$\beta = \dfrac{(z+1)^2 - 1}{(z+1)^2 + 1}$ and thus $v = \left[\dfrac{(z+1)^2 - 1}{(z+1)^2 + 1}\right]c$. For the values of z given, $v = 0.787c$ for $z = 1.9$ and $v = 0.944c$ for $z = 4.9$.

103. (a) In the frame of the decaying K^0 meson, the pi mesons must recoil with equal momenta in opposite directions in order to conserve momentum. In that reference frame the available kinetic energy is $498\text{ MeV} - 2(135\text{ MeV}) = 228\text{ MeV}$.

(b) The pi mesons share this equally, so each one has a kinetic energy of 114 MeV in that frame. The energy of each pi meson is $E = K + E_0 = 114\ \text{MeV} + 135\ \text{MeV} = 249\ \text{MeV}$. The momentum of each pi meson can be found:

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(249\ \text{MeV})^2 - (135\ \text{MeV})^2}}{c} = 209.2\ \text{MeV}/c$$

104. (a) For each second, the energy used is 3.9 × 10^{26} J. The mass used in each second is

$$m = \frac{E}{c^2} = \frac{3.9\times10^{26}\ \text{J}}{\left(3.0\times10^{8}\ \text{m/s}\right)^2} = 4.3\times10^{9}\ \text{kg}$$

(b) The time to use this mass is

$$\Delta t = \frac{m_{\text{sun}}}{\Delta m/\Delta t}\times\text{efficiency} = \frac{2.0\times10^{30}\ \text{kg}}{4.3\times10^{9}\ \text{kg/s}}\times0.007 = 3.3\times10^{18}\ \text{s}\ \approx 1.0\times10^{11}\ \text{y}.$$

That is 20 times longer than the expected lifetime of the sun.

105. a) The planet's orbital speed is $v_\text{p} = \dfrac{2\pi r}{T} = 1.30\times10^{4}\ \text{m/s}$. If the system's center of mass is at rest, conservation of momentum gives the star's speed:

$$m_\text{p}v_\text{p} = m_\text{s}v_\text{s},\ \text{so}\ v_\text{s} = \frac{m_\text{p}v_\text{p}}{m_\text{s}} = \frac{1.90\times10^{27}}{1.99\times10^{30}}1.30\times10^{4}\ \text{m/s} = 12.4\ \text{m/s}$$

(b) The redshifted wavelength is $\lambda = \lambda_0\sqrt{\dfrac{1+\beta}{1-\beta}} \approx 550\ \text{nm} + 2.3\times10^{-5}\ \text{nm}$

Similarly, the blueshifted wavelength is $\lambda = \lambda_0\sqrt{\dfrac{1-\beta}{1+\beta}} \approx 550\ \text{nm} - 2.3\times10^{-5}\ \text{nm}$

These small differences can be detected using modern spectroscopic tools.

106. To a good approximation, the shift is the same in each direction, so the redshift for the approaching light is half the difference, or 0.0045 nm. Using this in the equation $\lambda = \lambda_0\sqrt{\dfrac{1-\beta}{1+\beta}}$ leads to a speed parameter β = 6.9 × 10^{-6}, or $v = \beta c$ = 2070 m/s. The speed is equal to the circumference $2\pi R$ divided by the period, so the period is

$$T = \frac{2\pi R}{v} = \frac{2\pi\left(6.96\times10^{8}\ \text{m}\right)}{2070\ \text{m/s}} = 2.11\times10^{6}\ \text{s} = 24.5\ \text{days}.$$