Chapter 2

Limits and Derivatives

2.1 Introduction to Derivatives

2.1.1. With Δ*t* = 1*.*0, Δ*f* = *f* (2*.*0) *− f* (1*.*0) = 3*.*0, soΔ*f*

Δ*t*=3*.*0.WithΔ*t*=0*.*5,Δ*f*=*f*(1*.*5)*−*

*f* (1*.*0) = 1*.*5, soΔ*f* = 3*.*0. With Δ*t* = 0*.*01,

Δ*t*=3*.*0.WithΔ*t*=0*.*1,Δ*f*=*f*(1*.*1)*−f*(1*.*0)=0*.*3,soΔ *t*

Δ*f* = *f*(1*.*01) *− f*(1*.*0) = 0*.*03, soΔ*f*

Δ*t*=3*.*0.

2.1.2. With Δ*t* = 1*.*0, Δ*g* = *g*(1*.*0) *− g*(0*.*0) = *−*3*.*0, soΔ*g*

Δ*t*=*−*3*.*0.WithΔ*t*=0*.*5,Δ*g*=*g*(0*.*5)*−*

*g*(0*.*0) = *−*1*.*5, soΔ*g* = *−*3*.*0. With

Δ*t*=*−*3*.*0.WithΔ*t*=0*.*1,Δ*g*=*g*(0*.*1)*−g*(0*.*0)=*−*0*.*3,soΔ*t*

Δ*t* = 0*.*01, Δ*g* = *g*(0*.*01) *− g*(0*.*0) = *−*0*.*03, soΔ*g*

Δ*t*=*−*3*.*0.

2.1.3. With Δ*t* = 1*.*0, Δ*h* = *h*(2*.*0) *− h*(1*.*0) = 6*.*0, soΔ*h*

Δ*t*=6*.*0.WithΔ*t*=0*.*5,Δ*h*=*h*(1*.*5)*−*

*h*(1*.*0) = 2*.*5, soΔ*h* = 4*.*2. With Δ*t* = 0*.*01,

Δ*t*=5*.*0.WithΔ*t*=0*.*1,Δ*h*=*h*(1*.*1)*−h*(1*.*0)=0*.*42,soΔ *t*

Δ*h* = *h*(1*.*01) *− h*(1*.*0) = 0*.*0402, soΔ*h*

Δ*t*=4*.*02.

2.1.4. With Δ*t* = 1*.*0, Δ*h* = *h*(1*.*0) *− h*(0*.*0) = 1*.*0, soΔ*h*

Δ*t*=1*.*0.WithΔ*t*=0*.*5,Δ*h*=*h*(0*.*5)*−*

*h*(0*.*0) = 0*.*25, soΔ*h* = 0*.*1. With

Δ*t*=0*.*5.WithΔ*t*=0*.*1,Δ*h*=*h*(0*.*1)*−h*(0*.*0)=0*.*01,soΔ *t*

Δ*t* = 0*.*01, Δ*h* = *h*(0*.*01) *− h*(0*.*0) = 0*.*0001, soΔ*h*

Δ*t*=0*.*01.

2.1.5. With Δ*t* = 1*.*0, Δ*G* = *G*(1*.*0) *− G*(0*.*0) = 6*.*389, soΔ*G*

= 6*.*389. With Δ*t* = 0*.*5, Δ*G* =

*G*(0*.*5) *− G*(0*.*0) = 1*.*718, soΔ*G*

Δ*t*= 3*.*436. With Δ*t* = 0*.*1, Δ*G* = *G*(0*.*1) *− G*(0*.*0) = 0*.*221, so

Δ*t*

Δ*G* = 2*.*02.

Δ*t*=2*.*21.WithΔ*t*=0*.*01,Δ*G*=*G*(0*.*01)*−G*(0*.*0)=0*.*0202,soΔ *t*

2.1.6. With Δ*t* = 1*.*0, Δ*G* = *G*(1*.*0) *− G*(0*.*0) = *−*0*.*632, soΔ*G*

Δ*t*=*−*0*.*632.WithΔ*t*=0*.*5,Δ*G*=

*G*(0*.*5) *− G*(0*.*0) = *−*0*.*393, soΔ*G*

Δ*t*=*−*0*.*787.WithΔ*t*=0*.*1,Δ*G*=*G*(0*.*1)*−G*(0*.*0)=*−*0*.*095,so

Δ*G* Δ*t*=*−*0*.*95.WithΔ*t*=0*.*01,Δ*G*=*G*(0*.*01)*−G*(0*.*0)=*−*0*.*00995,soΔ *t*

= *−*0*.*995.

2.1.7. Each secant line is *fs*(*t*) = 2 + 3*t*.

8.5

8

7.5

7

6.5

6

5.5

5 o

4.5

1

o

o

o

1.2 1.4 1.6 1.8 2

t

89

f(t)

90

2.1.8. Each secant line is *gs*(*t*) = 2 *−* 3*t*.

2.5

2

1.5

1

0.5

0

-0.5

-1

-1.5

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o

o

o

o

0 0.2 0.4 0.6 0.8 1

t

2.1.9. The coordinates of the base point are (1*,* 2), so the secant lines are: with Δ*t* = 1*.*0, *hs*(*t*) = 2+6(*t −*1), with Δ*t* = 0*.*5, *hs*(*t*) = 2+5(*t −*1), with Δ*t* = 0*.*1, *hs*(*t*) = 2+4*.*2(*t −*1), with Δ*t* = 0*.*01, *hs*(*t*) = 2 + 4*.*02(*t −* 1).

9

8

7

6

5

4

3

2 o

1

1

o

o

o

1.2 1.4 1.6 1.8 2

t

2.1.10. The coordinates of the base point are (0*,* 1), so the secant lines are: with Δ*t* = 1*.*0, *hs*(*t*) = 1+*t*,
with Δ*t* = 0*.*5, *hs*(*t*) = 1 + 0*.*5*t*, with Δ*t* = 0*.*1, *hs*(*t*) = 1 + 0*.*1*t*, with Δ*t* = 0*.*01, *hs*(*t*) = 1 + 0*.*01*t*.

2.4

2.2

2

1.8

1.6

1.4

1.2

1 o o

0.8

0 0.2 0.4

o

o

0.6 0.8 1
t

2.1.11. The coordinates of the base point are (0*,* 1), so the secant lines are: with Δ*t* = 1*.*0, *Gs*(*t*) =

1 + 6*.*389*t*, with Δ*t* = 0*.*5, *Gs*(*t*) = 1 + 3*.*436*t*, with Δ*t* = 0*.*1, *Gs*(*t*) = 1 + 2*.*21*t*, with Δ*t* = 0*.*01, *Gs*(*t*) = 1 + 2*.*02*t*.

10

9

8

7

6

5

4

3 o
2

1 o o
0

0 0.2 0.4 0.6 0.8

t

o

1

g(t)

h(t)

h(t)

G(t)

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2.1.12. The coordinates of the base point are (0*,* 1), so the secant lines are: with Δ*t* = 1*.*0, *Gs*(*t*) =

1 *−* 0*.*632*t*, with Δ*t* = 0*.*5, *Gs*(*t*) = 1 *−* 0*.*787*t*, with Δ*t* = 0*.*1, *Gs*(*t*) = 1 *−* 0*.*95*t*, with Δ*t* = 0*.*01, *Gs*(*t*) = 1 *−* 0*.*995*t*.

1.2

1 o

o

0.8

0.6 o

0.4 o

0.2
0

-0.2

0 0.2 0.4 0.6 0.8 1

t

2.1.13. The slope is 3, so the tangent line i *f* (*t*) = 2 + 3*t*.

2.1.14. The slope is -3, so the tangent line is *ĝ*(*t*) = 2 *−* 3*t*.

2.1.15. It looks like the slopes are getting close to 4.0, so the tangent line is*ĥ*(*t*) = 2 + 4(*t −* 1).

2.1.16. It looks like the slopes are getting close to 0.0, so the tangent line is*ĥ*(*t*) = 1.

2.1.17. It looks like the slopes are getting close to 2.0, so the tangent line is *Ĝ*(*t*) = 1 + 2*t*.

2.1.18. It looks like the slopes are getting close to -1.0, so the tangent line is *Ĝ*(*t*) = 1 *− t*.

2.1.19. The derivative of *g*(*t*), the slope of the tangent line.

2.1.20. *g′*(*t*),*dg*

*dt*,limΔ*t→* 0 Δ*t*

2.1.21.

2.4

2.2

2

1.8

1.6

1.4

1.2

1 o

0.8

0

a. *b*(0) = 1*.*0, *b*(1*.*0) = 1*.*5, *b*(2*.*0) = 2*.*25.

b. Δ*b* = 1*.*5 *−* 1*.*0 = 0*.*5, so Δ*b/*Δ*t* = 0*.*5.

c. Δ*b* = 2*.*25 *−* 1*.*5 = 0*.*75, so Δ*b/*Δ*t* = 0*.*75.

Secant line connecting t=0 and t=1

2.4

2.2

2

1.8

1.6

o

o

1 2
Time

Secant line connecting t=1 and t=2
4

o

3

o

2

1.4

1.2

1 o

0.8

0

2.1.22.

o

o

Δb = 0.5

1 o

Δ t = 1.0

0

0.5 1 1.5 2 0 0.5 1

Time

Δb = 0.75

Δt = 1.0

1.5 2 2.5 3
Time

G(t)

Population

Population

Population

92

1.4

1.3

1.2

1.1

1 o

0

a. *b*(0) = 1*.*0, *b*(1*.*0) = 1*.*2, *b*(2*.*0) = 1*.*44.

b. Δ*b* = 1*.*2 *−* 1*.*0 = 0*.*2, so Δ*b/*Δ*t* = 0*.*2.

c. Δ*b* = 1*.*44 *−* 1*.*2 = 0*.*24, so Δ*b/*Δ*t* = 0*.*24.

Secant line connecting t=0 and t=1

2

1.8

1.6

1.4

1.2 o

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o

o

1 2
Time

Secant line connecting t=1 and t=2

2

1.8

1.6

1.4 o

Δb = 0.24

1.2 o

1 o

0

2.1.23.

Δb = 0.2

Δt = 1.0

0.5 1 1.5 2 2.5 3

Time

Δt = 1.0

1

0 0.5 1 1.5 2 2.5 3

Time

a. Δ*b* = 1*.*51*.*0 *−* 1*.*0 = 0*.*5, and Δ*b/*Δ*t* = 0*.*5.

b. Δ*b* = 1*.*50*.*1 *−* 1*.*0 = 0*.*0413, and Δ*b/*Δ*t* = 0*.*414.

c. Δ*b* = 1*.*50*.*01 *−* 1*.*0 = 0*.*00406, and Δ*b/*Δ*t* = 0*.*406.

d. Δ*b* = 1*.*50*.*001 *−* 1*.*0 = 0*.*000405, and Δ*b/*Δ*t* = 0*.*405.

e. The limit looks like 0.405.

3

2

1

0

0 0.5 1 1.5 2

f. Time

2.1.24.

a. The slope is (2*.*01*.*0 *−* 1*.*0)*/*1*.*0 = 1*.*0.

b. The slope is (2*.*00*.*1 *−* 1*.*0)*/*0*.*1 = 0*.*718.

c. The slope is (2*.*00*.*01 *−* 1*.*0)*/*0*.*01 = 0*.*696.

d. The slope is (2*.*00*.*001 *−* 1*.*0)*/*0*.*001 = 0*.*693.

e. It looks like the slope of the tangent is 0.693.

f.

Population

Population

Population

Population

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3

2

1

0

0 0.5 1 1.5 2

Time

2.1.25.

a. The slope is (5 *·* 1*.*02 *−* 0*.*0)*/*1*.*0 = 5*.*0.

b. The slope is (5 *·* 0*.*12 *−* 0*.*0)*/*0*.*1 = 0*.*5.

c. The slope is (5 *·* 0*.*012 *−* 0*.*0)*/*0*.*01 = 0*.*05.

d. The slope is (5 *·* 0*.*0012 *−* 0*.*0)*/*0*.*001 = 0*.*005.

e. The slope gets close to 0.

f.

30

20

10

0

0 0.5 1 1.5 2

Time

2.1.26.

a. The slope is ((1 + 2 *·* 1*.*03) *−* (1*.*0 + 2*.*0 *·* 0*.*03))*/*1*.*0 = 26*.*0.

b. The slope is ((1 + 2 *·* 0*.*13) *−* (1*.*0 + 2*.*0 *·* 0*.*03))*/*0*.*1 = 7*.*28.

c. The slope is ((1 + 2 *·* 0*.*013) *−* (1*.*0 + 2*.*0 *·* 0*.*03))*/*0*.*01 = 6*.*1208.

d. The slope is ((1 + 2 *·* 0*.*0013 ) *−* (1*.*0 + 2*.*0 *·* 0*.*03))*/*0*.*001 = 6*.*012.

e. The slope seems to be approaching 6.0.

f.

20

15

10

5

0

0 0.2 0.4 0.6 0.8 1

Time

2.1.27. During the first hour, 3.0 bacteria/h. During the first half hour, 2.485 bacteria/h. During the second half hour, 3.515 bacteria/h. The population changes faster during the second half hour.

Population

Population

Population

94

6.5

6

5.5

5

4.5

4

3.5

3 o

2.5

2

0

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o

o

0.5 1

Time

2.1.28. During the first hour, 0.648 bacteria/h. During the first half hour, 0.568 bacteria/h. During
the second half hour, 0.729 bacteria/h. The population changes faster during the second half hour.

1.8

1.7

1.6

1.5

1.4

1.3 o

1.2

1.1

1 o

0.9

0 0.5

Time

o

1

2.1.29. During the first hour, -0.79 bacteria/h. During the first half hour, -0.88 bacteria/h. During

the second half hour, -0,69 bacteria/h. The population changes faster during the first half hour.

2 o

1.9

1.8

1.7

1.6

1.5

1.4

1.3

1.2

1.1

1

0

o

o

0.5 1

Time

2.1.30. During the first hour, -1.5 bacteria/h. During the first half hour, -1.757 bacteria/h. During

the second half hour, -1.243 bacteria/h. The population changes faster during the first half hour.

3 o

2.8

2.6

2.4

2.2

2

1.8

1.6

1.4

1.2

1

0

2.1.31.

o

o

0.5 1

Time

Population

Population

Population

Population