Instructor's Manual
for Modeling Monetary Economies
Third Edition

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1 Tips for Instructors

This book has evolved over two decades of teaching money and banking and monetary economics to undergraduates. Although it is slimmer than most textbooks, its emphasis on explicit models will take more time per page than the standard. An instructor will find it a full semester's course.

2 Book Organization

The book is organized into three parts of increasing complexity. The first examines money in isolation. Here we take up the questions of the demand for fiat money, a comparison of fiat and commodity monies, inflation, and exchange rates. In the second part, we add capital, to study money's interaction with other assets, and banking, the intermediation of these assets into substitutes for fiat money. In the third section we look at money's effects on saving, investment, and output through its effect on nonmonetary government debt.

Because the models grow in complexity, building on the same basic framework, we recommend that the topics be taken up in the order presented. There is no need, however, to take up each topic presented. Chapters 2, 4, 5, 9, 10, 11, 13, and 17 and the appendices are the endpoints of a line of analysis. Each may be dropped without major consequence to the teaching of later chapters. This is not to say that these topics are of lesser importance or that they themselves do not depend on material from earlier chapters. However, we find that most instructors are unable to cover the entire book in a single semester, especially when teaching the course for the first time. These instructors should choose among the chapters above when looking for material to leave out. Alternatively, an instructor using this book as a supplementary text may drop either all of Part II (except for Chapter 6) or all of Part III.

3 Level of Book

This book is written for undergraduates. Its mathematical requirements are no more advanced than the understanding of basic graphs and algebra. Calculus is not required. (Those who want to teach using calculus can find a simple exposition of this approach in the appendix to Chapter 1.) While the book may also prove useful as a primer for graduate students in monetary theory, the main text is pitched at the undergraduate level. This pitch has held us back from a few demanding topics, such as nonstationary equilibria, but we hope the reader will be satisfied by the large number of topics that we have been able to present in simple, clear models within a single basic framework. Material that is difficult but within the grasp of advanced undergraduates is set apart in appendices and thus easily skipped or inserted. The appendices also contain nonmonetary extensions, like the models covering credit or

Social Security, that some instructors may wish to use but were not essential to the exposition of the main topics.

4 References

The references display the most tension between the needs of undergraduates and the technical base in which this approach originated. Whenever possible we reference related material written for undergraduates or general audiences. Asterisks mark these references in the list. We also reference the works from which our models and data have been drawn. Finally, when undergraduate-level references are not available we have inserted references to a few academic articles and surveys to offer graduate and advanced undergraduate students some places to start with more advanced work. These are not intended as a full survey of the advanced literature.

5 Choices of Topics

The choice of topics to be covered was also difficult. We make no claim to encyclopedic coverage of every topic or opinion related to monetary economics. We limited coverage to the topics most directly linked to money, covering banking (but not finance in general) and government debt (but not macroeconomics in general). We insisted on ideas consistent with fully rational people operating in explicitly specified environments—models of unexplained sticky prices or people with irrational expectations are readily available elsewhere. To promote the unity and consistency of our approach across topics, we also selected topics tractably teachable in the basic framework of the overlapping generations model. Finally, we offer what we best know and understand. Where this leaves gaps, we hope that individual instructors can build on our foundations to fill them.

6 Suggestions for Teaching

We offer some other teaching suggestions, chapter by chapter:

6.1 Chapter 1: A Simple Model of Money

Students need repetition and practice to learn the technical material in early chapters. Since they use these tools throughout the book, this material will be worth a careful presentation even if progress seems slow to the teacher. This is why the book covers separately the nearly redundant cases of constant and growing population. One way to make this seem less repetitious is to ask the students to do the growing population case as a homework assignment, then go over it in class. Students often confuse the budget and feasible sets because both use many of the same variables and in this chapter, they turn out to be identical. Take care to differentiate the two by distinguishing their origins.

The appendix using calculus is a minimal exposition of how calculus can be used to solved the models. It is not required for any material taught in the book but is inserted for those teachers who wish to teach using calculus.

6.2 Chapter 2: Barter and Commodity Money

This chapter deals with barter and commodity money exchange and may be skipped. It might also be covered after Chapter 3 or 4 if you find the barter model too distracting from the standard overlapping generations model of Chapters 1, 3 and 4. Also, one might teach one of the two topics without teaching the other, although we found that the topics go well together.

6.3 Chapter 3: Inflation

This is the key chapter of the book, of great importance for future chapters. Present it carefully, assigning exercises and going over them in class. Students often seem to understand this material until they are asked to do it themselves.

6.4 Chapter 4: International Monetary Systems

Another endpoint chapter, all or part of which can be skipped or moved. These international topics are taught earlier than usual because only the basic model is used. The material on speculative attacks can be tricky, but we find that students have great interest in this topic.

6.5 Chapter 5: Price Surprises

Students may find this material difficult. we have added it to the second edition nevertheless because of the revolutionary effect of Lucas model on macroeconomic thought. Related ideas involving rational expectations and policy advice are covered in Chapters 9 and 16 so this chapter may be skipped if the students find it too difficult. The emphasis on methodology makes this chapter better suited to advanced students.

6.6 Chapter 6: Capital

This chapter contains all the theory of finance needed for subsequent chapters. It is not intended as a self-contained finance course. To cover the material in the second part of the book we needed only one asset as an alternative to fiat money. We chose capital because of its direct link to output. Many instructors and students, however, enjoy a more thorough treatment of private debt. The appendix is for them.

6.7 Chapter 7: Liquidity and Financial Intermediation

The material in this chapter is important for the next three chapters but the appendix stands alone as a foray into a more sophisticated model of banking as intermediation. The material in this chapter is important for the next three chapters but the appendix stands alone as a foray into a more sophisticated model of banking as intermediation.

6.8 Chapter 8: Central Banking and the Money Supply

Attention paid to the monetary aggregates here will be rewarded in Chapter 9. if desired, the section on central bank lending can be skipped.

6.9 Chapter 9: Money Stock Fluctuations

The money/output link studied in this chapter involves a long train of argument (see Figure 8.4). Students nevertheless master the material when taught each step of the argument before studying it all together. We feel it is an important chapter, the cumulation of Chapters 6 and 7 but it is not required for later chapters. The appendix, which is separated from the main body of the chapter because its topic is different, may serve as a useful review and consolidation of material taught in Chapter 7.

The money/output link studied in this chapter involves a train of argument with multiple steps. Students nevertheless master the material when taught each step of the argument before studying it all together. We feel it is an important chapter, the culmination of Chapters 7 and 8, but it is not required for later chapters. The appendix is separated from the main body of the chapter because its topic is not essential to the chapter's central theme. However, it may serve as a consolidation and extension of material taught in Chapter 8.

6.10 Chapter 10: Fully Backed Central Bank Money

Although the math is not difficult, this can be a puzzling topic for students because economic behavior in this case is distinctly different from that of the fiat money economies of other chapters. The section on currency boards presumes a familiarity with Chapter 4.

6.11 Chapter 11: The Payments System

This new chapter stands alone and so may be skipped if desired. The value of the chapter is its illustration of a nontrivial role for central banking, public or private. The island structure may take some exposition time, but the essential market, that for reserves, is easy to present.

6.12 Chapter 12: Bank Risk

This chapter stands apart building mostly on the banking models of Chapter 7. It could be taught directly after that chapter. It is not required for later chapters.

6.13 Chapter 13: Liquidity Risk and Bank Panics

This new chapter builds on the concepts developed in Chapter 12. The primary step is to add monetary factors into a banking model, reflecting the facts on the role that currency plays in bank panics. It can be taught in addition to or substituted for the material in Chapter 12. It is not required for later chapters.

6.14 Chapter 14: Deficits and the National Debt\

Chapters 14–17 stand together as a group on the national debt. as such, many instructors have used this group as a supplement in nonmonetary macroeconomics courses. They require elements of Chapters 1, 3, and 6 as a background.

6.15 Chapter 15: Savings and Investment

This chapter introduces the formal modeling of saving and investment used in Part III. It is worth taking slowly because it is essential for understanding the complicated effects of government debt studied in Chapters 16 and 17. The model of saving and investment invites the study of a number of issues (like social security) that are interesting but are not essential elements of monetary economics. To maintain the book's focus, these nonmonetary issues are introduced only in exercises and the appendix from which the instructor may pick and choose. The appendix does not require calculus but would benefit more than most topics from its use.

6.16 Chapter 16: The Effect of the National Debt on Capital and Savings

Instructors short of time or leery of more abstract topics may wish to skip the section on fiat money and the crowding out of capital. The appendix about infinitely lived agents also cannot be taught quickly and should not be taught if a large block of time is not available.

6.17 Chapter 17: The Temptation of Inflation

The chain of reasoning that links unexpected inflation to its real effects involves a number of advanced concepts. The instructor should make sure that students understand the effects and time inconsistency of a default on the national debt before teaching that an unexpected inflation works like a default. the concepts of rational expectations and the Lucas critique can be studied in the context of this chapter as a substitute for or complement to the Lucas model of Chapter 5.

7 Chapter 1

8 A Simple Model of Money

Example 1.1 The answer is summarized in the following table

Period	**Young Alive**	**Old Alive**	**Total Population**
1	120	100	220
2	144	120	264

Calculating from the table, we can see that the total population also grows at a net rate of 20% [=(264-220)/220]. In general, we can prove that the total population grows at the rate n:

$$\begin{aligned} (\text{Total population})_t &= N_t + N_{t-1} \\ &= nN_{t-1} + nN_{t-2} \\ &= (N_{t-1} + N_{t-2}) \\ &= n\,(\text{Total population})_{t-1} \end{aligned}$$

Exercise 1.1 (a) Feasible set: $100c_{1,t} + 100c_{2,t} \leq 100y = 100\,(20) \Rightarrow c_{1,t} + c_{2,t} \leq 20$.

The graph is easy. The horizontal and vertical intercepts equal 20. Note that until you know more about preferences, you cannot find exact values for c_1 and c_2, but you can draw a general graph. An example with properly drawn indifference curves is in Figure 1A.

(b) First period: $c_{1,t} + v_t m_t \leq y$.

Second period: $c_{2,t+1} \leq v_{t+1} m_t$.

Lifetime: $c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq y \Rightarrow c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq 20$.

(c) The money market clearing condition is:

$$v_t M_t = N_t\,(y - c_{1,t}) \Rightarrow 400 v_t = 100\,(y - c_{1,t}) \Rightarrow v_t = \frac{100\,(y - c_{1,t})}{400}.$$

You could substitute for y here, but this form is good enough for our purposes. We want to find $\frac{v_{t+1}}{v_t}$.

$$\frac{v_{t+1}}{v_t} = \frac{\frac{100(y - c_{1,t+1})}{400}}{\frac{100(y - c_{1,t})}{400}} = 1.$$

where the last equality follows from the cancellation and imposing stationarity ($c_{1,t} = c_{1,t+1}$ for all t).

(d) Since the rate of return on fiat money is 1, we find that the real demand for fiat money is

$$v_t m_t = \frac{y}{1+\frac{v_t}{v_{t+1}}} = \frac{20}{1+1} = 10.$$

Note from the first-period budget constraint that $v_t m_t$ is equal to $y - c_1$, so that $y - c_1 = 10$. Since $v_t m_t = 10$, $c_1 = y - y - v_t m_t = 20 - 10 = 10/$ Half the endowment is consumed and half is sold for real money balances. Using $y - c_1$ in the expression derived in part c,

$$v_t = \frac{100\,(10)}{400} = 2.5 \Rightarrow p_t = \frac{1}{v_t} = \frac{1}{2.5} = 0.4.$$

(Question to answer on your own: What will c_2 be?)

(e) We saw in this chapter that the rate of return on fiat money is n in an economy with a constant fiat money stock and a changing population. So an increase in n will cause an increase in the rate of return on fiat money.

An increase in the rate of return on fiat money will increase the real money balances. This should make intuitive sense and is easy to see by plugging a few numbers into the money demand function (*e.g.* suppose that v_{t+1}/v_t increases to 2, re-solve for real money demand).

Given that the real demand for money increases, the money market-clearing condition tells us that the value of money will increase in the initial period. This also should be intuitive–an increase in the demand for apples increases the value of apples. Since the value of money increases in the initial period, the initial old (the initial holders of money) are made better off. (The initial old are better off whenever the real value of money in period 1 increases.)

(f) Following part c, we get $v_t = \frac{100(10)}{800} = 1.25$. The value of money is cut in half (the price level doubles to $1/v_t = 0.8$). However, the rate of return on fiat money is still one if the population is held constant. Notice that the total real value of the fiat money stock, which is initially held by the initial old, does not change (v_t is cut in half whereas M doubles). This implies that the welfare of the initial old does not change. Their holdings of money will not buy any more (or less) of the consumption good.

Exercise 1.2 (a) Since N and M are constant, the rate of return on fiat money in a stationary equilibrium will be one in each country. Intuitively, the economies are identical in the sense of how they change over time. They do not change. Even if each country has a different value of money, v_t, that value does not change over time; therefore, the rate of return on fiat money, v_{t+1}/v_t, equals 1.

(b) The value of money in economies A and B are, respectively

$$\frac{N\left(y - c_1^A\right)}{M} \quad \text{and} \quad \frac{N\left(y - c_1^B\right)}{M},$$

where c_1^A and c_1^B are first-period consumption in economies A and B, respectively. The assumption on preferences implies that $c_1^A > c_1^B$ so that $\left(y - c_1^A\right) < \left(y - c_1^B\right)$.

This, in turn, implies that the value of money in economy A will be lower than the value of money in economy B. Intuitively, the demand for money will be larger in economy B than in economy A. This is because individuals in economy B want to hold relatively more money to finance their higher second-period consumption. Since all else is equal between the two economies (importantly, the supply of money and population), money will have a higher value in economy B than in economy A.

Exercise 1.3 (a) The total amount of the consumption good in period t is $N_t y_1 + N_{t-1} y_2$. This is the total endowment of the economy at time t (young and old). Hence, the feasible set with a stationary allocation is

$$N_t c_1 + N_{t-1} c_2 \leq N_t y_1 + N_{t-1} y_2.$$

Dividing through both sides of this equation by N_t,

$$c_1 + \frac{N_{t-1}}{N_t} c_2 \leq y_1 + \frac{N_{t-1}}{N_t} y_2.$$

Noting that $N_t = n N_{t-1}$, the feasible set is:

$$c_1 + \left[\frac{1}{n}\right] c_2 \leq y_1 + \left[\frac{1}{n}\right] y_2.$$

(b) Plotting the feasible set and superimposing arbitrary indifference curves as in Figure 1B, we find the consumption allocation that maximizes the welfare of future generations, (c_1^*, c_2^*).

(c) The constraints facing individuals are

$$c_{1,t} + v_t m_t \leq y_1 \qquad \text{and} \qquad c_{2,t+1} \leq v_{t+1} m_t + y_2.$$

From the first-period constraint, we see that individual real demand for money is $(y_1 - c_{1,t})$. So, aggregate real money demand is $N_t (y_1 - c_{1,t})$. Setting this equal to the total real supply of money,

$$v_t M_t = N_t (y_1 - c_{1,t}).$$

(d) From part c, we find that

$$v_t = \frac{N_t (y_1 - c_{1,t})}{M_t} \qquad \text{and} \qquad v_{t+1} = \frac{N_{t+1} (y_1 - c_{1,t+1})}{M_{t+1}},$$

so that the rate of return on fiat money is

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y_1 - c_{1,t+1})}{M_{t+1}}}{\frac{N_t (y_1 - c_{1,t})}{M_t}} = n,$$

due to stationarity and a constant money supply. Work the algebra out completely so you can see this.

(e) To draw the individual's budget set, we need to construct the lifetime budget constraint. Use the first- and second-period constraints found at the beginning of part c. Also impose stationarity. From the second-period constraint at equality, we see that

$$m_t = \frac{c_2 - y_2}{v_{t+1}}.$$

Substituting this into the first-period budget constraint, we get

$$c_1 + \left[\frac{v_t}{v_{t+1}}\right](c_2 - y_2) \leq y_1$$

$$c_1 + \left[\frac{v_t}{v_{t+1}}\right] c_2 \leq y_1 + \left[\frac{v_t}{v_{t+1}}\right] y_2$$

$$c_1 + \left[\frac{1}{n}\right] c_2 \leq y_1 + \left[\frac{1}{n}\right] y_2 \qquad \text{(from part d).}$$

Reference to part a shows that this individual budget constraint is the same as the feasible set (the constraint faced by a central planner). This implies that individuals in the monetary equilibrium will choose the same (c_1^*, c_2^*) combination as the one which maximizes the utility of all future generations. This shows that the basic results of Chapter 1 do not change with this alteration in the environment. The monetary equilibrium here can attain the golden rule allocation.

Exercise 1.4 (a) First-period constraint: $c_{1,t} + v_t m_t \leq y_t$ or $c_{1,t} + \frac{y_t}{2} \leq y_t \Rightarrow c_{1,t} \leq \frac{y_t}{2}$. Second-period constraint: $c_{2,t+1} \leq v_{t+1} m_t$. Lifetime constraint: $c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq y_t$.

(b) By assumption, individual real money holdings are $y - c_{1,t} = v_t m_t = \frac{y_t}{2}$. In such a case, money market-clearing becomes:

$$v_t M = N(y - c_{1,t}) \qquad \Rightarrow \qquad v_t M = \frac{N y_t}{2} \qquad \Rightarrow \qquad v_t = \frac{N y_t}{2M}.$$

The rate of return of fiat money will be

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N y_{t+1}}{2M}}{\frac{N y_t}{2M}} = \frac{y_{t+1}}{y_t} = \frac{\alpha y_t}{y_t} = \alpha.$$

Note the similarity of this case to that found in Chapter 1. In that chapter, we modeled growth in the economy by growth in the number of young people born each period ($N_t = nN_{t-1}$). We found that in that case, the rate of return of fiat money equal to n, the growth rate of the economy. In this example, α is the growth rate of the economy (it is the gross rate of change of the total endowment). We discover that even in this more complicated setup, the rate of return of fiat money is equal to the growth rate of the economy when the money supply is fixed.

Appendix Exercise 1.1 (a) Follow equations (1.33) to (1.35) of the text. Using the notation of the appendix of Chapter 1, utility is

$$\ln\left(y-q_t\right)+\beta\ \ln\left(\frac{v_{t+1}}{v_t}\left[q_t\right]\right).$$

Differentiating this expression with respect to q_t and setting the result equal to zero:

$$\frac{-1}{y-q_t^*}+\beta\frac{\frac{v_{t+1}}{v_t}}{\frac{v_{t+1}}{v_t}\left[q_t^*\right]}=0.$$

Canceling terms and rearranging, we get

$$\frac{\beta}{q_t^*}=\frac{1}{y-q_t^*}\Rightarrow\beta\left(y-q_t^*\right)=q_t^*\Rightarrow q_t^*=\frac{\beta y}{1+\beta}.$$

(b) $c_{1,t}^*=y-q_t^*=\frac{y}{1+\beta}$. $c_{2,t+1}^*=\frac{v_{t+1}}{v_t}\left[q_t^*\right]=\frac{v_{t+1}}{v_t}\left[\frac{\beta y}{1+\beta}\right]$

(c) The analytical way to solve this is to differentiate the expressions for q_t^*, $c_{1,t}^*$, and $c_{2,t+1}^*$ with respect to β and sign the results. (Try this on your own, taking money rate of return as given.) However, a little thought can save us the hassle. It is easy to see that as β increases, $c_{1,t}^*$ falls. A smaller value for first-period consumption more of the endowment to trade for money so money holdings rise. Furthermore, an increase in q_t^* will then imply larger second-period consumption.

An increase in β increases the "weight" put on second-period consumption in the expression for utility. Intuitively, an individual with a large β places a larger importance on second-period consumption than someone with a small β. And, as we see, a "large" β will cause first-period consumption to be reduced. Money holdings will be "large" to finance a "large" among of second period consumption.

9 Chapter 2

10 Barter and Commodity Money

Exercise 2.1 (a) $\frac{1}{J^2-J} = \frac{1}{100^2-100} = \frac{1}{9900} \approx 0.0001$.

(b) $\alpha \left(J^2 - J\right) = (2)\left(100^2 - 100\right) = 19,800$.

(c) $2\alpha J = 2\,(2)\,(100) = 400$. Lifetime search costs are much higher with barter.

(d) The person only has to make one exchange where the quality of goods has to be verified. Total exchange costs for barter = 4.

(e) The person has to make two exchanges—once where the quality of money is verified and once where the quality of goods is verified. Total exchange costs for money $= 1 + 4 = 5$. However, note that total average lifetime costs (search and exchange) with barter are 19,804; with money, they are 405.

Exercise 2.2 (a) The exercise assumes that an individual's demand for money half his endowment, or $10/2 = 5$. This implies that $v_t^g m_t^g = y - c_{1,t} = 5$. The market for gold clears when the total real supply of gold equals the total real demand for gold:

$$\begin{aligned} v_t^g M_t^g &= N\left(y - c_{1,t}\right) \\ &\Rightarrow v_t^g = \frac{N\left(y - c_{1,t}\right)}{M_t^g} \Rightarrow v_t^g = \frac{100\,(5)}{100} = \frac{500}{100} = 5. \end{aligned}$$

Since $v_t^g m_t^g = 5$ and $v_t^g = 5$, each young person holds 1 unit of gold ($m_t^g = 1$). This accounts for the 5 ($= 5 \cdot 1$) units of the consumption good that an individual does not consume.

(b) Given the assumptions of this problem, $\tilde{v} = 2$. In other words, consuming gold gives an individual utility equivalent to consuming 2 units of the consumption good. Since the initial old can obtain 5 (> 2)units of the consumption good for every unit of gold traded, they will choose to trade their gold (i.e., use it as money) rather than consume it. The entire gold stock will be used as a medium of exchange.

(c) This change matters. Now $M^g = 800$. If the initial old traded their gold for the consumption good, gold market-clearing would imply (as in part a),

$$v_t^g = \frac{N\left(y - c_{1,t}\right)}{M^g} \Rightarrow v_t^g = \frac{100\,(5)}{800} = \frac{500}{800} = 0.625.$$

Now the initial old will be better consuming some of their gold. For each unit consumed they receive utility equal to 2 units of the consumption good. If they were to trade the gold, they would only receive 0.625 units of the consumption good for each unit of gold traded.

As the initial old consume their gold, the total gold stock falls and the value of gold begins to rise. This continues until the stock of gold is sufficiently small so that its values equals its intrinsic value. This occurs when the value of gold is such that

$$v_t^g = \frac{N\left(y - c_{1,t}\right)}{M^{g*}} = \tilde{v},$$

where $M^{g*}$is the final stock of gold that is used for monetary purposes (not consumed by the initial old).We can solve the above equation for M^{g*}:

$$M^{g*} = \frac{N\left(y - c_{1,t}\right)}{\tilde{v}} = \frac{100\,(5)}{2} = \frac{500}{2} = 250.$$

Only 250 units of gold will be used as money. The initial old will consume 550 $(= 800 - 250 = M^g - M^{g*})$ units of gold. Note that the final value of gold, after the initial old consume what they want, will be $\tilde{v} = 2$.

Note in this case the quantity theory of money does not hold. The stock of gold increased eightfold (from 100 to 800). However, the value of gold only fell from 5 to 2, not by a factor of 8.

(d) We know from part (a) that in period $t^* - 1$, the price of gold will be $v_{t-1}^g = 5$. In period t^*, setting the supply of gold equal to the demand for gold, we get

$$v_{t*}^g = \frac{N\left(y - c_{1,t}\right)}{M_{t*}^g} \Rightarrow v_{t*}^g = \frac{100\,(5)}{200} = \frac{500}{200} = 2.5.$$

Note v_{t*}^g is still above $\tilde{v}$ so the entire stock of gold is still traded. The rate of return of gold from period $t^* - 1$ to period t^* is

$$\frac{v_{t*}^g}{v_{t*-1}^g} = \frac{2.5}{5} = 0.5.$$

Gold loses 50 of its value (the net rate of return is the gross rate of return minus $1 = 0.5 - 1 = 0.5 = -50\%$) from period $t^* - 1$ to period t^*. Here there has been an increase in the (commodity) money stock. We will study increases in the money more thoroughly in Chapter 3.

Exercise 2.3 (a) In this problem, we can think of χ as the opportunity costs of a unit of gold. In other words, χ stands the ratio at which a person gives up units of the consumption good to obtain a unit of gold. In the text, $\tilde{v}$ is the intrinsic value of gold. The intrinsic value of gold is declining in its quantity. So, in equilibrium, the intrinsic value is also equal to χ. Since the intrinsic value is the quantity of the consumption goods per unit of gold. In other words, χ takes on exactly the same role as $\tilde{v}$ in the model economy described in the text. The monetary trading value of gold will not be higher than χ. Consider a case in which $v_t^g > \chi$. Miners will see an arbitrage opportunity, extracting gold at cost χ units of the consumption good in order to obtain $v_t^g > \chi$ units of the consumption good when used as money. For this problem, therefore, the equilibrium occurs when $v_t^g = \chi$. We also know that $v_t^g < \chi$ because the marginal cost of obtaining gold cannot exceed the monetary value of gold. Mining would stop at this point.

The value of gold in monetary exchange is determined by setting the demand for money equal to the supply of money. Thus

$$v_t^g M_t^g = N_t (y - c_{1,t})$$

or

$$v_t^g = \frac{N_t (y - c_{1,t})}{M_t^g}.$$

If there is an increase in the use of gold as money, then M_t^g increases, resulting in a decrease in the value of gold as money; that is, v_t^g declines. Since v_t^g is equal to χ in equilibrium, a decline in v_t^g sets up where money is highest valued use is its commodity use. Thus, gold money would be taken out of circulation until $v_t^g = \chi$.

(b) If the marginal mining costs increases and the marginal benefit decreases, there exists a point at which the marginal benefit of gold is equal to its marginal cost. This intersection represents the equilibrium commodity, or intrinsic, value of gold. Let $\tilde{v}$ represent the equilibrium intrinsic value of gold in this economy. As in part a, the value of gold in monetary exchange is determined by the equation

$$v_t^g = \frac{N_t (y - c_{1,t})}{M_t^g}.$$

With mining costs increasing at an increasing rate, we know that $v_t^g \geq \tilde{v}$. If more gold is used as money, we know that v_t^g falls. For $v_t^g > \tilde{v}$, the decline in the monetary value of gold will not disrupt anything. However, for $v_t^g < \tilde{v}$, the outcome has people effectively burying the gold. The intrinsic value cannot be affected by finding more gold because the marginal cost of mining exceeds the marginal benefit of any extra gold mined. So any increased use of gold will drive down the monetary value of gold, making the intrinsic value greater than the monetary value. People will choose to shift the use of gold from its monetary use to its intrinsic use, provided the amount is small. If too large, the shift to the intrinsic use will affect the market value and it would be more efficient to just leave the gold idle.

Exercise 2.4 (a) We denote the value of silver at time t by v_t^s and the supply of silver at time t by M_t^s. The demand for money must equal the total supply of money (gold and silver):

$$\begin{aligned} N (y - c_1) &= v_t^g M_t^g + v_t^s M_t^s \qquad (2A) \\ 100 (5) &= v_t^g 100 + v_t^s 50 \end{aligned}$$

If $v_t^s = 1.5$, equation (2A) reveals that $v_t^g = 4.25$. Since this is greater than the intrinsic value of gold (2), both gold and silver serve as money; neither is consumed.

If $v_t^s = 2$, equation (2A) reveals that $v_t^g = 4$. Since this is also greater than the intrinsic value of gold (2), both gold and silver serve as money; neither is consumed.

However, we can see that as v_t^s rises, v_t^g falls. There is a cutoff value for v_t^s , above which the value of gold will be less than its intrinsic value. To find that cutoff value, we merely substitute a value of 2 (the intrinsic value of gold) for v_t^g in equation (2A).

$$500 = 2\,(100) + v_t^s\,(50) \Rightarrow 50v_t^s = 500 - 200 \Rightarrow v_t^s = \frac{300}{50} = 6.$$

If v_t^s were greater than 6, the trading value of gold calculated in equation (2A) would be less than the intrinsic value of gold. Then some gold would be consumed.

Both gold and silver have trading values greater than their intrinsic values when v_t^s lies between 1 and 6.

(b) If silver alone were used as money, the equality of money's supply and demand would be

$$N\,(y - c_1) = v_t^s M_t^s \qquad \text{or} \qquad 100\,(5) = v_t^s\,(50) \Rightarrow v_t^s = 10.$$

(c) The sole owner of the entire stock of silver would want silver alone to be used as money because this increases the value of the silver.

(d) Since money cannot be consumed, you want to tie up as money that with the smallest total intrinsic value–in this case, silver. This frees up the more useful gold to be consumed. Total consumption of bread will be identical regardless of which metal is used as money since the metal's value will adjust to clear the market. [For each member of the initial old, the total intrinsic value of their silver holdings is $\tilde{v}^s m_0^s = (1)\,(1/2) = 0.5$, and the total intrinsic value of their gold holdings is $\tilde{v}^g m_0^g = (2)\,(2) = 4$. Indeed, the total intrinsic value of gold is greater than for silver. The initial old will want to consume the more intrinsically valuable gold and use the silver as money.