2–1.

An air-filled rubber ball has a diameter of 6 in. If the air pressure within the ball is increased until the diameter becomes 7 in., determine the average normal strain in the rubber.

SOLUTION

$d_0 = 6$ in.

$d = 7$ in.

$$\epsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.}$$ **Ans.**

Ans:
$\epsilon = 0.167$ in./in.

2–2.

A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

SOLUTION

$L_0 = 15 \text{ in.}$

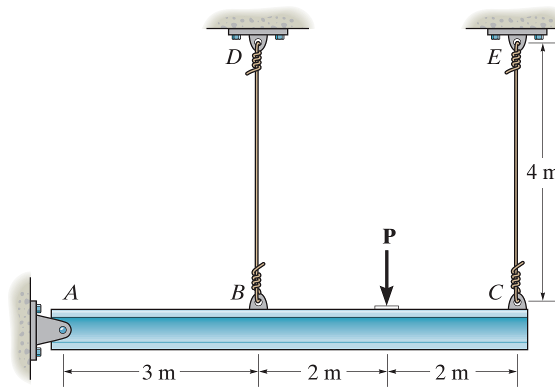
$L = \pi(5 \text{ in.})$

$$\epsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.} \qquad \textbf{Ans.}$$

Ans:
$\epsilon = 0.0472 \text{ in./in.}$

2–3.

If the load **P** on the beam causes the end *C* to be displaced 10 mm downward, determine the normal strain in wires *CE* and *BD*.



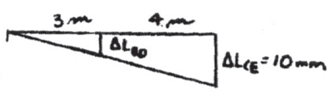
SOLUTION

$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

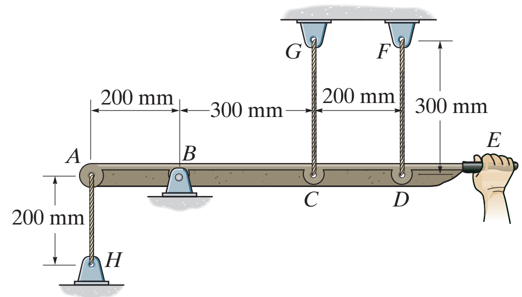
$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_{CE} = 0.00250 \text{ mm/mm}, \epsilon_{BD} = 0.00107 \text{ mm/mm}$

***2–4.**

The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin B through an angle of 2°. Determine the average normal strain in each wire. The wires are unstretched when the lever is in the horizontal position.



SOLUTION

Geometry: The lever arm rotates through an angle of $\theta = \left(\frac{2°}{180}\right)\pi \text{ rad} = 0.03491 \text{ rad}$.

Since θ is small, the displacements of points A, C, and D can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

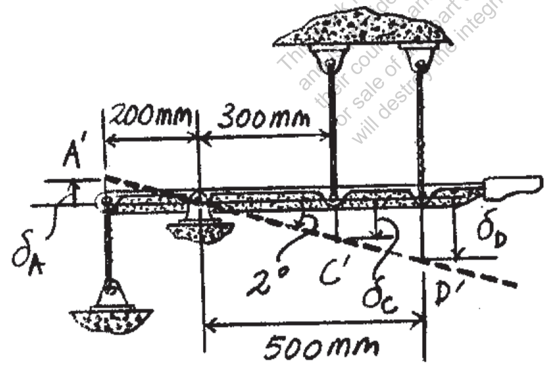
$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

Average Normal Strain: The unstretched length of wires AH, CG, and DF are $L_{AH} = 200$ mm, $L_{CG} = 300$ mm, and $L_{DF} = 300$ mm. We obtain

$$(\epsilon_{avg})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm}$$ **Ans.**

$$(\epsilon_{avg})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm}$$ **Ans.**

$$(\epsilon_{avg})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm}$$ **Ans.**

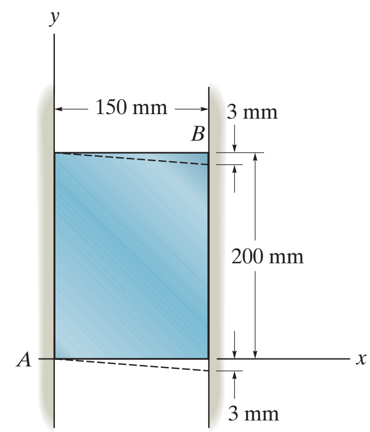


(a)

Ans:
$(\epsilon_{avg})_{AH} = 0.0349$ mm/mm
$(\epsilon_{avg})_{CG} = 0.0349$ mm/mm
$(\epsilon_{avg})_{DF} = 0.0582$ mm/mm

2–5.

The rectangular plate is subjected to the deformation shown by the dashed line. Determine the average shear strain γ_{xy} in the plate.



SOLUTION

Geometry:

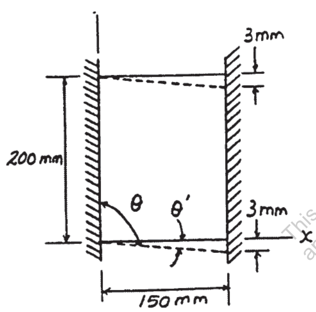
$$\theta' = \tan^{-1}\frac{3}{150} = 0.0200 \text{ rad}$$

$$\theta = \left(\frac{\pi}{2} + 0.0200\right) \text{rad}$$

Shear Strain:

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \left(\frac{\pi}{2} + 0.0200\right)$$

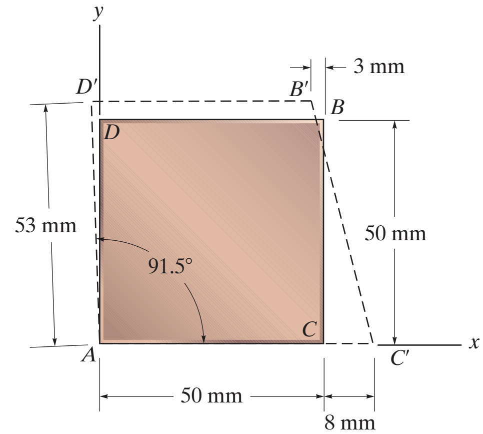
$$= -0.0200 \text{ rad} \qquad \textbf{Ans.}$$



Ans:
$\gamma_{xy} = -0.0200$ rad

2–6.

The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners, A, B, C, and D, relative to the x, y axes. Side $D'B'$ remains horizontal.



SOLUTION

Geometry:

$$B'C' = \sqrt{(8 + 3)^2 + (53 \sin 88.5°)^2} = 54.1117 \text{ mm}$$

$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58) \cos 91.5°}$$

$$= 79.5860 \text{ mm}$$

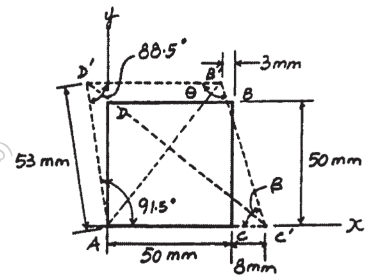
$$B'D' = 50 + 53 \sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$\cos \theta = \frac{(B'D')^2 + (B'C')^2 - (C'D')^2}{2(B'D')(B'C')}$$

$$= \frac{48.3874^2 + 54.1117^2 - 79.5860^2}{2(48.3874)(54.1117)} = -0.20328$$

$$\theta = 101.73°$$

$$\beta = 180° - \theta = 78.27°$$



Shear Strain:

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \pi\left(\frac{91.5°}{180°}\right) = -0.0262 \text{ rad}$$ **Ans.**

$$(\gamma_B)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \pi\left(\frac{101.73°}{180°}\right) = -0.205 \text{ rad}$$ **Ans.**

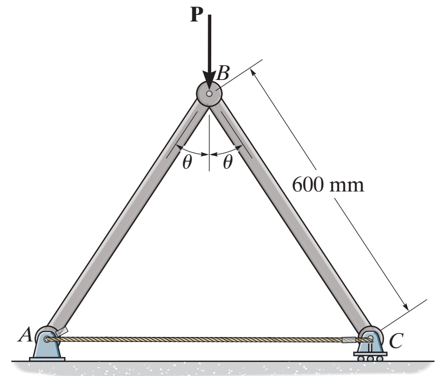
$$(\gamma_C)_{xy} = \beta - \frac{\pi}{2} = \pi\left(\frac{78.27°}{180°}\right) - \frac{\pi}{2} = -0.205 \text{ rad}$$ **Ans.**

$$(\gamma_D)_{xy} = \pi\left(\frac{88.5°}{180°}\right) - \frac{\pi}{2} = -0.0262 \text{ rad}$$ **Ans.**

Ans:
$(\gamma_A)_{xy} = -0.0262$ rad
$(\gamma_B)_{xy} = -0.205$ rad
$(\gamma_C)_{xy} = -0.205$ rad
$(\gamma_D)_{xy} = -0.0262$ rad

2–7.

The pin-connected rigid rods AB and BC are inclined at $\theta = 30°$ when they are unloaded. When the force **P** is applied θ becomes 30.2°. Determine the average normal strain in wire AC.



SOLUTION

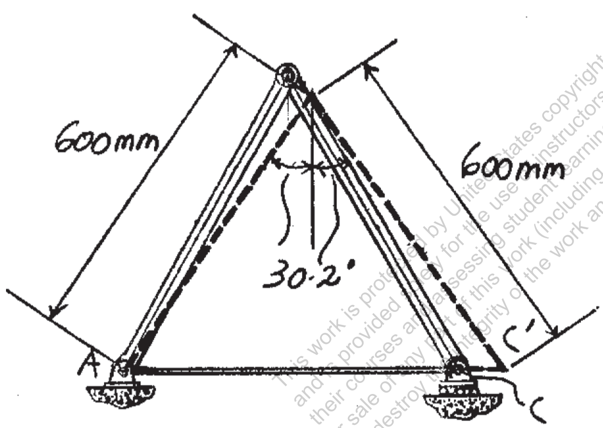
Geometry: Referring to Fig. *a*, the unstretched and stretched lengths of wire AD are

$$L_{AC} = 2(600 \sin 30°) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2°) = 603.6239 \text{ mm}$$

Average Normal Strain:

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$

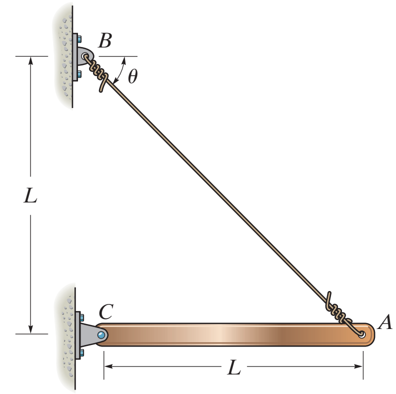


(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3}) \text{ mm/mm}$

***2–8.**

The wire AB is unstretched when $\theta = 45°$. If a load is applied to the bar AC, which causes θ to become $47°$, determine the normal strain in the wire.



SOLUTION

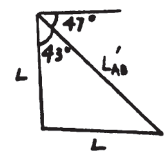
$$L^2 = L^2 + L'^2_{AB} - 2LL'_{AB}\cos 43°$$

$$L'_{AB} = 2L\cos 43°$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{2L\cos 43° - \sqrt{2}L}{\sqrt{2}L}$$

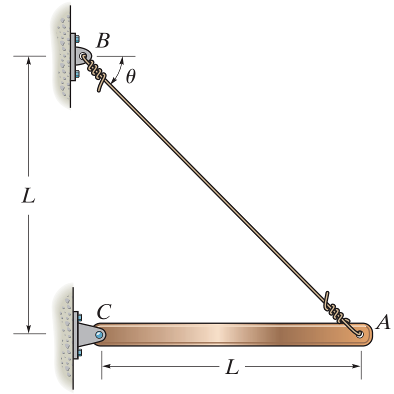
$$= 0.0343$$ **Ans.**



Ans:
$\epsilon_{AB} = 0.0343$

2–9.

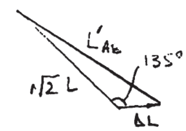
If a horizontal load applied to the bar AC causes point A to be displaced to the right by an amount ΔL, determine the normal strain in the wire AB. Originally, $\theta = 45°$.



SOLUTION

$$L'_{AB} = \sqrt{(\sqrt{2}L)^2 + \Delta L^2 - 2(\sqrt{2}L)(\Delta L)\cos 135°}$$

$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$



$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}L}{\sqrt{2}L}$$

$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$

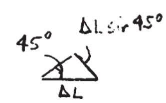
Neglecting the higher-order terms,

$$\epsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots.. - 1 \qquad \text{(binomial theorem)}$$

$$= \frac{0.5\Delta L}{L} \qquad \textbf{Ans.}$$

Also,



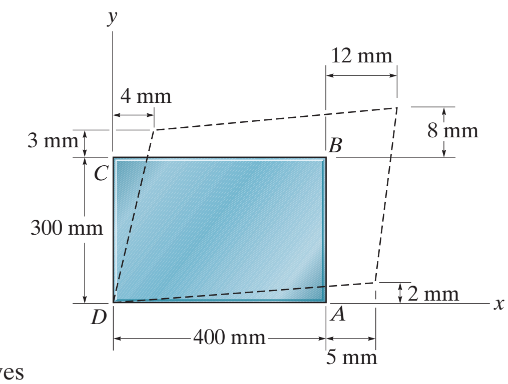
$$\epsilon_{AB} = \frac{\Delta L \sin 45°}{\sqrt{2}L} = \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans.}$$

Ans:

$$\epsilon_{AB} = \frac{0.5\Delta L}{L}$$

2–10.

Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{7}{306} = 0.022876 \text{ rad}$$

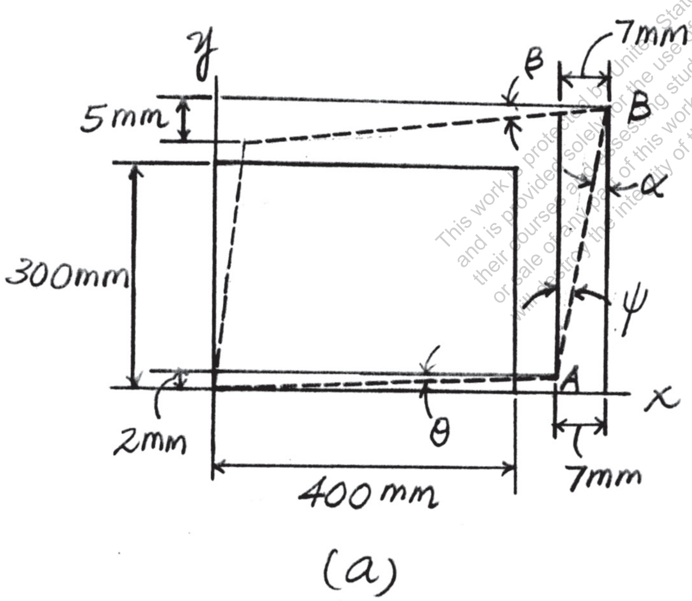
$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

Shear Strain: By definition,

$$(\gamma_A)_{xy} = \theta + \psi = 0.02781 \text{ rad} = 27.8(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_B)_{xy} = \alpha + \beta = 0.03513 \text{ rad} = 35.1(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

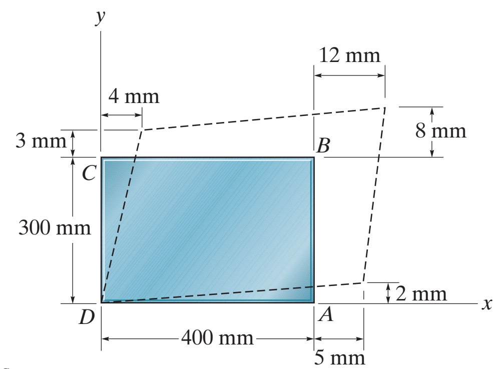


(a)

Ans:
$(\gamma_A)_{xy} = 27.8(10^{-3})$ rad
$(\gamma_B)_{xy} = 35.1(10^{-3})$ rad

2–11.

Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{4}{303} = 0.013201 \text{ rad}$$

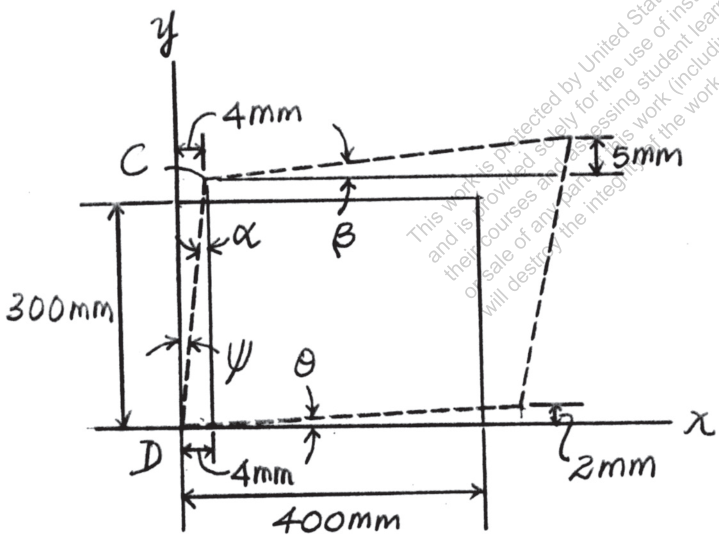
$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

Shear Strain: By definition,

$$(\gamma_{xy})_C = \alpha + \beta = 0.02546 \text{ rad} = 25.5(10^{-3}) \text{ rad}$$ **Ans.**

$$(\gamma_{xy})_D = \theta + \psi = 0.01814 \text{ rad} = 18.1(10^{-3}) \text{ rad}$$ **Ans.**

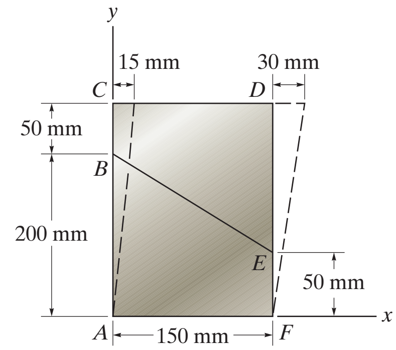


(a)

Ans:
$(\gamma_{xy})_C = 25.5(10^{-3})$ rad
$(\gamma_{xy})_D = 18.1(10^{-3})$ rad

***2–12.**

The material distorts into the dashed position shown. Determine the average normal strains ϵ_x, ϵ_y and the shear strain γ_{xy} at A, and the average normal strain along line BE.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$\tan\theta = \frac{15}{250}; \qquad \theta = (3.4336°)\left(\frac{\pi}{180°}\text{ rad}\right) = 0.05993 \text{ rad}$$

$$L_{AC'} = \sqrt{15^2 + 150^2} = \sqrt{62725} \text{ mm}$$

$$\frac{BB'}{15} = \frac{200}{250}; \qquad BB' = 12 \text{ mm} \qquad \frac{EE'}{30} = \frac{50}{250}; \qquad EE' = 6 \text{ mm}$$

$$x' = 150 + EE' - BB' = 150 + 6 - 12 = 144 \text{ mm}$$

$$L_{BE} = \sqrt{150^2 + 150^2} = 150\sqrt{2} \text{ mm} \quad L_{B'E'} = \sqrt{144^2 + 150^2} = \sqrt{43236} \text{ mm}$$

Average Normal and Shear Strain: Since no deformation occurs along x axis,

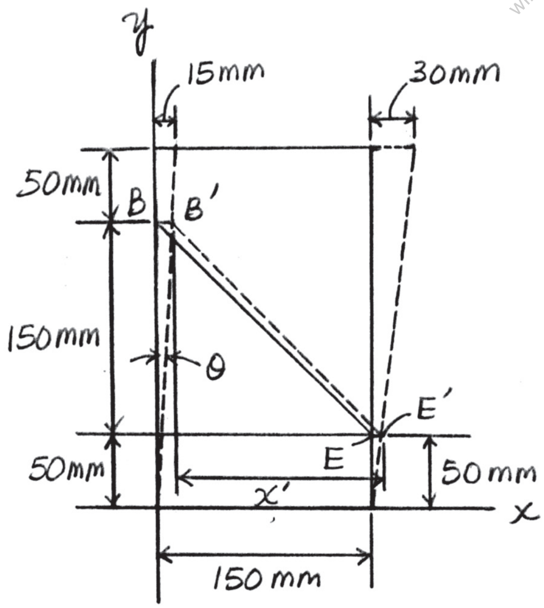
$$(\epsilon_x)_A = 0 \qquad \textbf{Ans.}$$

$$(\epsilon_y)_A = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{62725} - 250}{250} = 1.80(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$

By definition,

$$(\gamma_{xy})_A = \theta = 0.0599 \text{ rad} \qquad \textbf{Ans.}$$

$$\epsilon_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{43236} - 150\sqrt{2}}{150\sqrt{2}} = -0.0198 \text{ mm/mm} \qquad \textbf{Ans.}$$

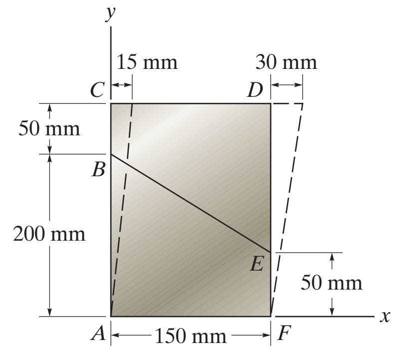


(a)

Ans:
$(\epsilon_x)_A = 0$
$(\epsilon_y)_A = 1.80(10^{-3})$ mm/mm
$(\gamma_{xy})_A = 0.0599$ rad
$\epsilon_{BE} = -0.0198$ mm/mm

2–13.

The material distorts into the dashed position shown. Determine the average normal strains along the diagonals *AD* and *CF*.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$L_{AD} = L_{CF} = \sqrt{150^2 + 250^2} = \sqrt{85000}\ \text{mm}$$

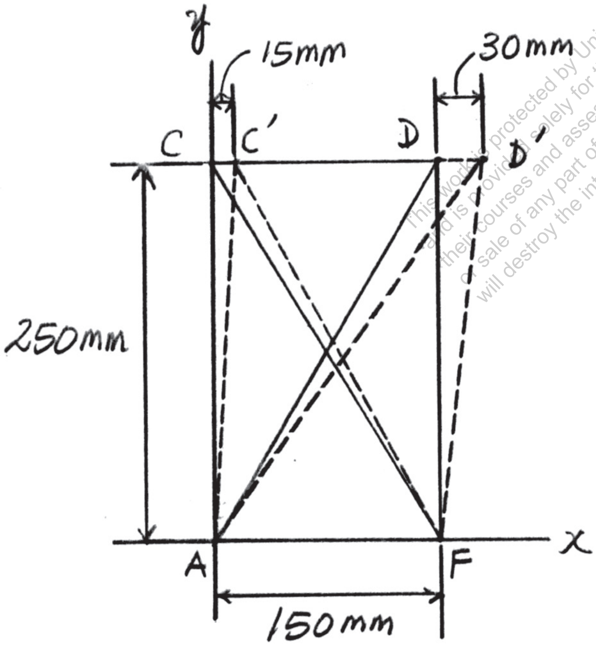
$$L_{AD'} = \sqrt{(150 + 30)^2 + 250^2} = \sqrt{94900}\ \text{mm}$$

$$L_{C'F} = \sqrt{(150 - 15)^2 + 250^2} = \sqrt{80725}\ \text{mm}$$

Average Normal Strain:

$$\epsilon_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{94900} - \sqrt{85000}}{\sqrt{85000}} = 0.0566\ \text{mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{80725} - \sqrt{85000}}{\sqrt{85000}} = -0.0255\ \text{mm/mm} \qquad \textbf{Ans.}$$

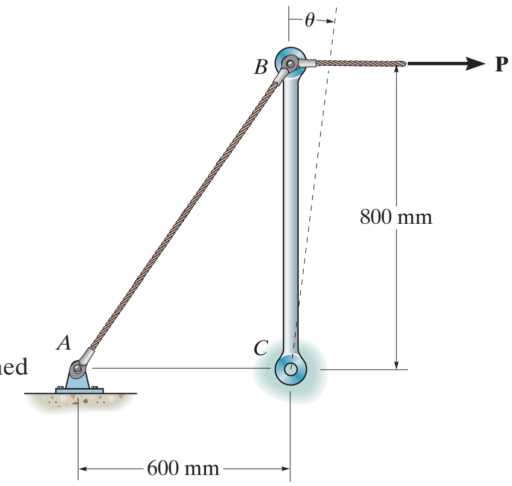


(a)

Ans:
$\epsilon_{AD} = 0.0566\ \text{mm/mm}$
$\epsilon_{CF} = -0.0255\ \text{mm/mm}$

2–14.

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes it to rotate by $\theta = 0.5°$, determine the normal strain in the cable. Originally the cable is unstretched.



SOLUTION

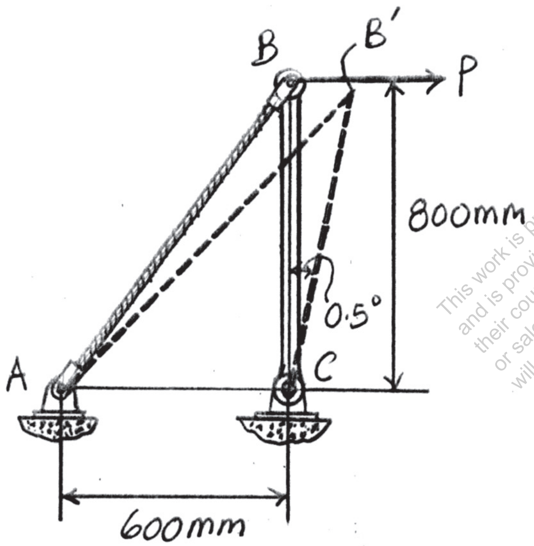
Geometry: Referring to the geometry shown in Fig. *a*, the unstretched and stretched lengths of cable AB are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos 90.5°} = 1004.18 \text{ mm}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{1004.18 - 1000}{1000} = 0.00418 \text{ mm/mm} \qquad \textbf{Ans.}$$

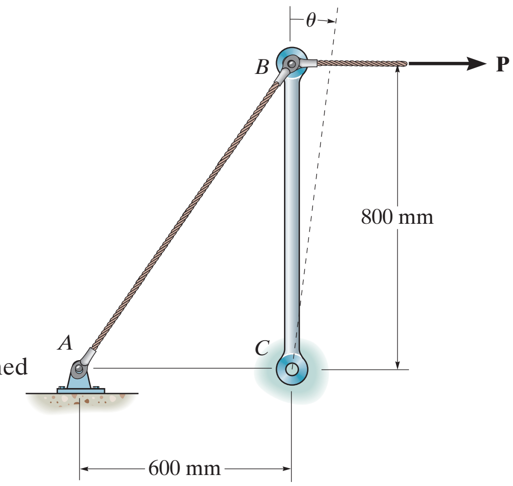


(a)

Ans:
$\epsilon_{AB} = 0.00418$ mm/mm

2–15.

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes a normal strain in the cable of 0.004 mm/mm, determine the displacement of point B. Originally the cable is unstretched.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the unstretched and stretched lengths of cable AB are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos(90^\circ + \theta)}$$

$$L_{AB'} = \sqrt{1(10^6) - 0.960(10^6)\cos(90^\circ + \theta)}$$

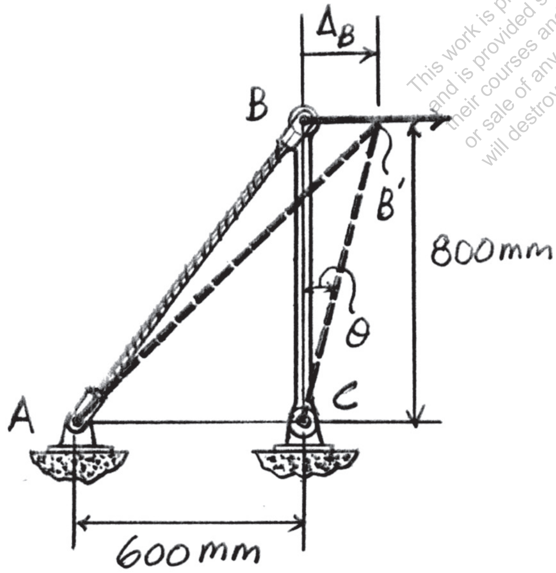
Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}}; \qquad 0.004 = \frac{\sqrt{1(10^6) - 0.960(10^6)\cos(90^\circ + \theta)} - 1000}{1000}$$

$$\theta = 0.4784^\circ\left(\frac{\pi}{180^\circ}\right) = 0.008350 \text{ rad}$$

Thus,

$$\Delta_B = \theta L_{BC} = 0.008350(800) = 6.68 \text{ mm} \qquad \textbf{Ans.}$$

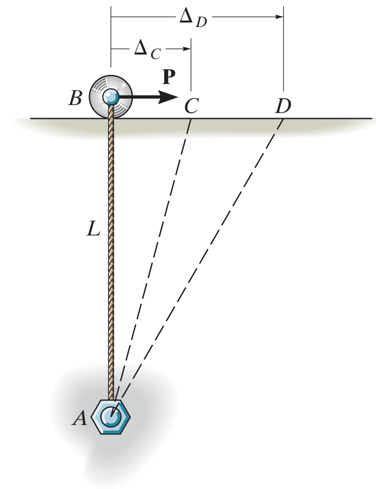


(a)

Ans:
$\Delta_B = 6.68$ mm

***2–16.**

The nylon cord has an original length L and is tied to a bolt at A and a roller at B. If a force **P** is applied to the roller, determine the normal strain in the cord when the roller is at C, and at D. If the cord is originally unstrained when it is at C, determine the normal strain ϵ'_D when the roller moves to D. Show that if the displacements Δ_C and Δ_D are small, then $\epsilon'_D = \epsilon_D - \epsilon_C$.



SOLUTION

$$L_C = \sqrt{L^2 + \Delta_C^2}$$

$$\epsilon_C = \frac{\sqrt{L^2 + \Delta_C^2} - L}{L}$$

$$= \frac{L\sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - L}{L} = \sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - 1$$

For small Δ_C,

$$\epsilon_C = 1 + \frac{1}{2}\left(\frac{\Delta_C^2}{L^2}\right) - 1 = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$ **Ans.**

In the same manner,

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$ **Ans.**

$$\epsilon_D{}' = \frac{\sqrt{L^2 + \Delta_D^2} - \sqrt{L^2 + \Delta_C^2}}{\sqrt{L^2 + \Delta_C^2}} = \frac{\sqrt{1 + \frac{\Delta_D^2}{L^2}} - \sqrt{1 + \frac{\Delta_C^2}{L^2}}}{\sqrt{1 + \frac{\Delta_C^2}{L^2}}}$$

For small Δ_C and Δ_D,

$$\epsilon_D{}' = \frac{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right) - \left(1 + \frac{1}{2}\frac{\Delta_D^2}{L^2}\right)}{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right)} = \frac{\frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right)}{\frac{1}{2L^2}\left(2L^2 + \Delta_C^2\right)}$$

$$\epsilon_D{}' = \frac{\Delta_C^2 - \Delta_D^2}{2L^2 - \Delta_C^2} = \frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right) = \epsilon_C - \epsilon_D$$ **QED**

Also this problem can be solved as follows:

$$A_C = L\sec\theta_C\,; \qquad A_D = L\sec\theta_D$$

$$\epsilon_C = \frac{L\sec\theta_C - L}{L} = \sec\theta_C - 1$$

$$\epsilon_D = \frac{L\sec\theta_D - L}{L} = \sec\theta_D - 1$$

Expanding $\sec\theta$

$$\sec\theta = 1 + \frac{\theta^2}{2\,!} + \frac{5\,\theta^4}{4\,!}\ \ldots\ldots$$

***2–16. Continued**

For small θ neglect the higher order terms

$$\sec\theta = 1 + \frac{\theta^2}{2}$$

Hence,

$$\epsilon_C = 1 + \frac{{\theta_C}^2}{2} - 1 = \frac{{\theta_C}^2}{2}$$

$$\epsilon_D = 1 + \frac{{\theta_D}^2}{2} - 1 = \frac{{\theta_D}^2}{2}$$

$$\epsilon_D{}' = \frac{L\sec\theta_D - L\sec\theta_C}{L\sec\theta_C} = \frac{\sec\theta_D}{\sec\theta_C} - 1 = \sec\theta_D\cos\theta_C - 1$$

$$\text{Since } \cos\theta = 1 - \frac{\theta^2}{2\,!} + \frac{\theta^4}{4\,!}\ldots\ldots$$

$$\sec\theta_D\cos\theta_C = \left(1 + \frac{{\theta_D}^2}{2}\ldots\ldots\right)\left(1 - \frac{{\theta_C}^2}{2}\ldots\ldots\right)$$

$$= 1 - \frac{\theta_C^2}{2} + \frac{\theta_D^2}{2} - \frac{\theta_C^2\,\theta_D^2}{4}$$

Neglecting the higher order terms

$$\sec\theta_D\cos\theta_C = 1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$\epsilon_D{}' = \left[1 + \frac{{\theta_2}^2}{2} - \frac{{\theta_1}^2}{2}\right] - 1 = \frac{{\theta_D}^2}{2} - \frac{{\theta_C}^2}{2}$$

$$= \epsilon_D - \epsilon_C$$

QED

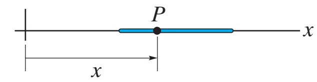
Ans:

$$\epsilon_C = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$

2–17.

A thin wire, lying along the x axis, is strained such that each point on the wire is displaced $\Delta x = kx^2$ along the x axis. If k is constant, what is the normal strain at any point P along the wire?



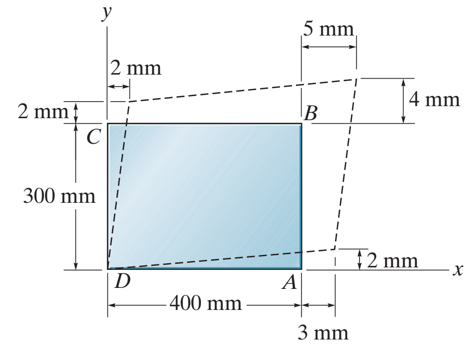
SOLUTION

$$\epsilon = \frac{d(\Delta x)}{dx} = 2\,k\,x$$ **Ans.**

Ans:
$\epsilon = 2kx$

2–18.

Determine the shear strain γ_{xy} at corners A and B if the plate distorts as shown by the dashed lines.

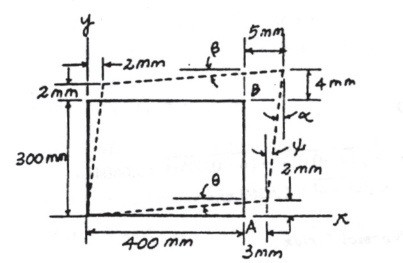


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain:

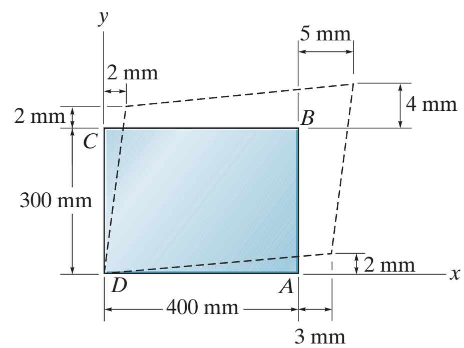
$$(\gamma_B)_{xy} = \alpha + \beta$$
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_A)_{xy} = \theta + \psi$$
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_B)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_A)_{xy} = 11.6(10^{-3})$ rad

2–19.

Determine the shear strain γ_{xy} at corners D and C if the plate distorts as shown by the dashed lines.

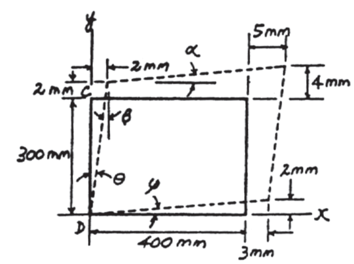


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain:

$$(\gamma_C)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

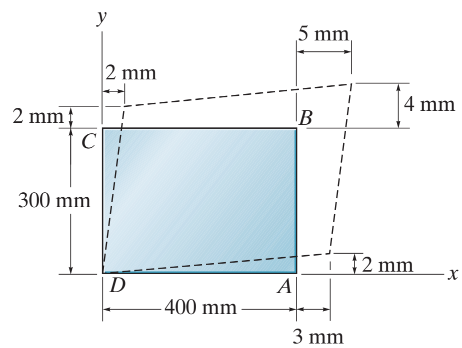
$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_C)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_D)_{xy} = 11.6(10^{-3})$ rad

***2–20.**

Determine the average normal strain that occurs along the diagonals AC and DB.



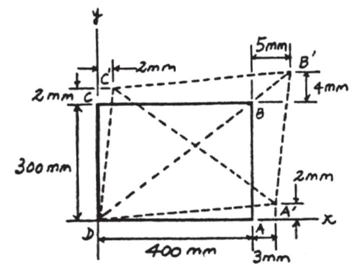
SOLUTION

Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$



Average Normal Strain:

$$\epsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

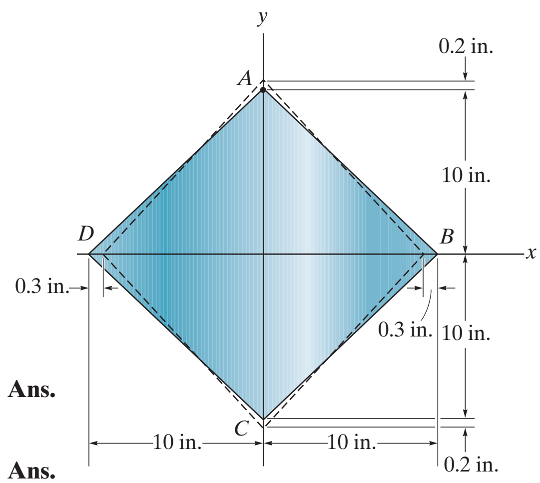
$$\epsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

Ans:
$\epsilon_{AC} = 1.60\left(10^{-3}\right)$ mm/mm
$\epsilon_{DB} = 12.8\left(10^{-3}\right)$ mm/mm

2–21.

The corners of the square plate are given the displacements indicated. Determine the average normal strains ϵ_x and ϵ_y along the x and y axes.



SOLUTION

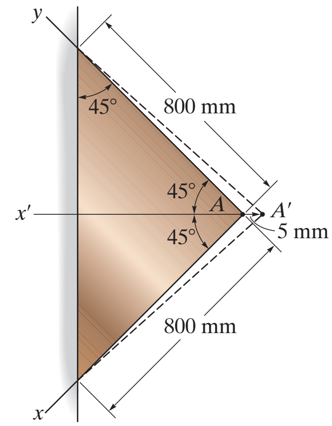
$$\epsilon_x = \frac{-0.3}{10} = -0.03 \text{ in./in.}$$ **Ans.**

$$\epsilon_y = \frac{0.2}{10} = 0.02 \text{ in./in.}$$ **Ans.**

Ans:
$\epsilon_x = -0.03$ in./in.
$\epsilon_y = 0.02$ in./in.

2–22.

The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain, γ_{xy}, at A.



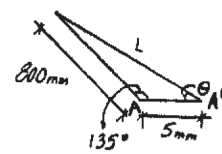
SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\frac{\sin 135^\circ}{803.54} = \frac{\sin \theta}{800}; \qquad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

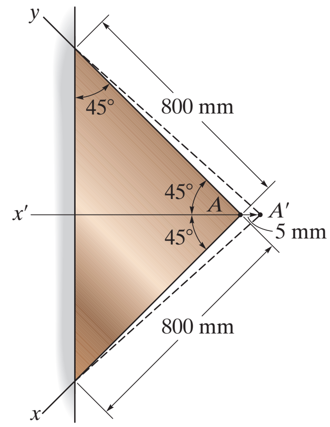
$$= 0.00880 \text{ rad}$$ **Ans.**



Ans:
$\gamma_{xy} = 0.00880$ rad

2–23.

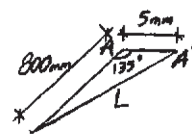
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.



SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

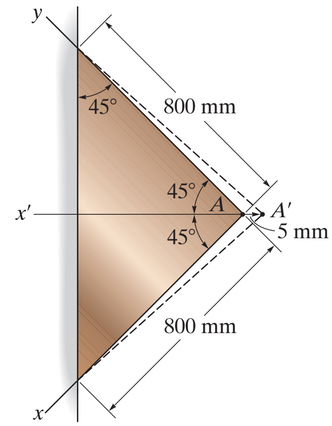
$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_x = 0.00443$ mm/mm

***2–24.**

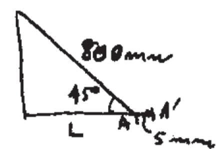
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.



SOLUTION

$L = 800 \cos 45° = 565.69 \text{ mm}$

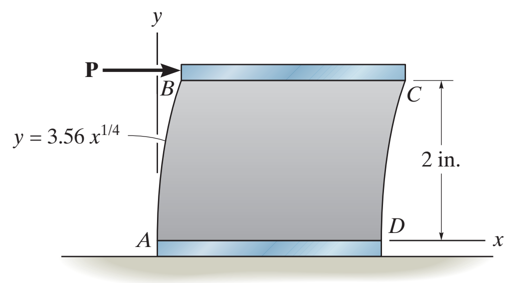
$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm} \qquad \textbf{Ans.}$$



Ans:
$\epsilon_{x'} = 0.00884 \text{ mm/mm}$

2–25.

The polysulfone block is glued at its top and bottom to the rigid plates. If a tangential force, applied to the top plate, causes the material to deform so that its sides are described by the equation $y = 3.56\,x^{1/4}$, determine the shear strain at the corners A and B.



SOLUTION

$$y = 3.56\,x^{1/4}$$

$$\frac{dy}{dx} = 0.890\,x^{-3/4}$$

$$\frac{dx}{dy} = 1.123\,x^{3/4}$$

At A, $x = 0$

$$\gamma_A = \frac{dx}{dy} = 0$$ **Ans.**

At B,

$$2 = 3.56\,x^{1/4}$$

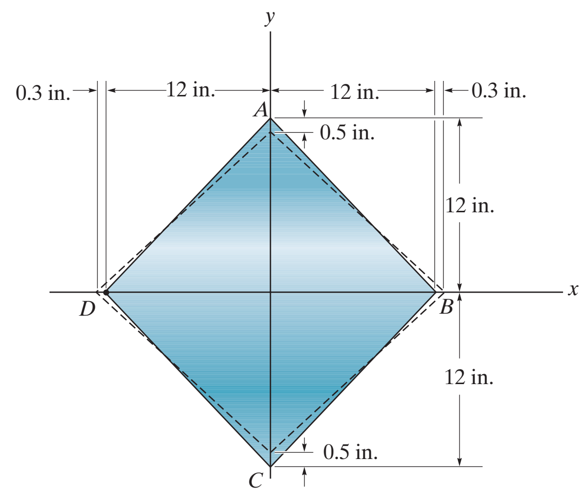
$$x = 0.0996 \text{ in.}$$

$$\gamma_B = \frac{dx}{dy} = 1.123(0.0996)^{3/4} = 0.199 \text{ rad}$$ **Ans.**

Ans:
$\gamma_A = 0$
$\gamma_B = 0.199$ rad

2–26.

The corners of the square plate are given the displacements indicated. Determine the shear strain at A relative to axes that are directed along AB and AD, and the shear strain at B relative to axes that are directed along BC and BA.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

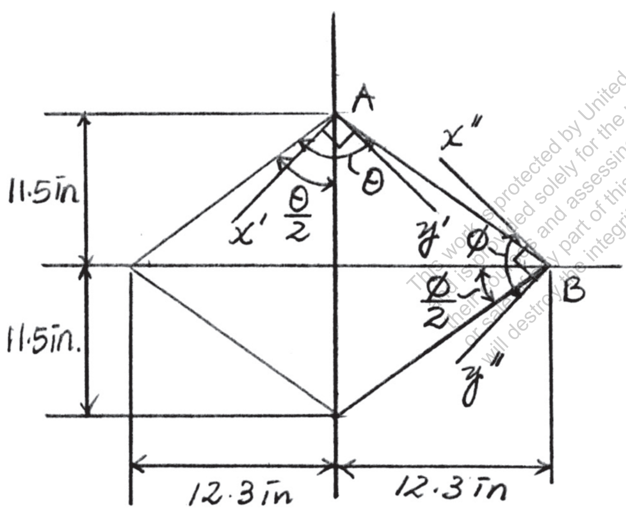
$$\tan\frac{\theta}{2} = \frac{12.3}{11.5} \qquad \theta = (93.85°)\left(\frac{\pi}{180°}\text{ rad}\right) = 1.6380 \text{ rad}$$

$$\tan\frac{\phi}{2} = \frac{11.5}{12.3} \qquad \phi = (86.15°)\left(\frac{\pi}{180°}\text{ rad}\right) = 1.5036 \text{ rad}$$

Shear Strain: By definition,

$$(\gamma_{x'y'})_A = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.6380 = -0.0672 \text{ rad}$$ **Ans.**

$$(\gamma_{x''y''})_B = \frac{\pi}{2} - \phi = \frac{\pi}{2} - 1.5036 = 0.0672 \text{ rad}$$ **Ans.**

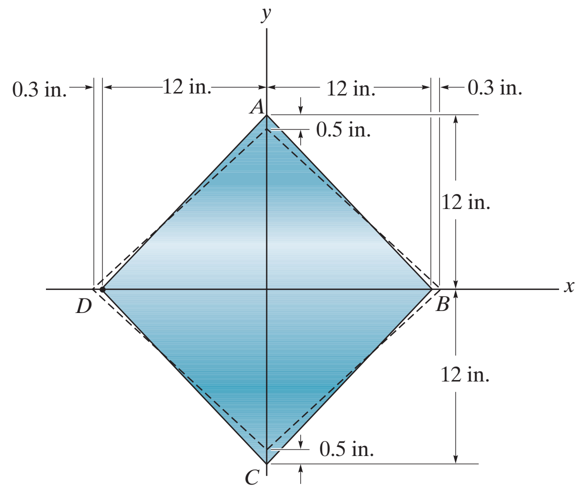


(a)

Ans:
$(\gamma_{x'y'})_A = -0.0672$ rad
$(\gamma_{x''y''})_B = 0.0672$ rad

2–27.

The corners of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonals AC and BD.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$L_{AB} = \sqrt{12^2 + 12^2} = 12\sqrt{2} \text{ in.}$$

$$L_{A'B'} = \sqrt{12.3^2 + 11.5^2} = \sqrt{283.54} \text{ in.}$$

$$L_{BD} = 2(12) = 24 \text{ in.}$$

$$L_{B'D'} = 2(12 + 0.3) = 24.6 \text{ in.}$$

$$L_{AC} = 2(12) = 24 \text{ in.}$$

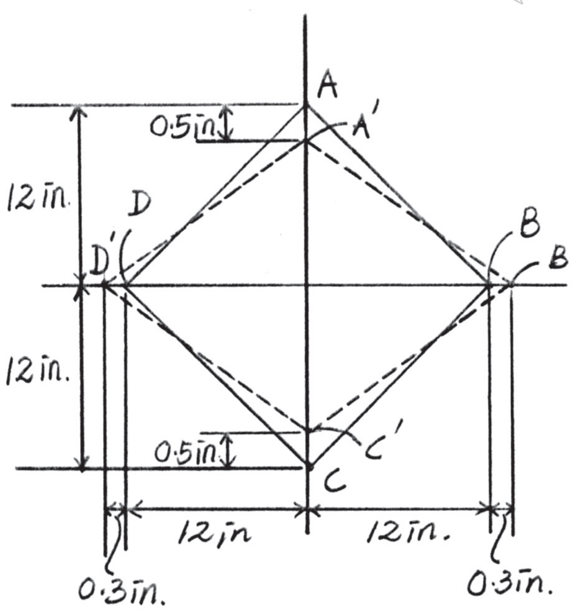
$$L_{A'C'} = 2(12 - 0.5) = 23 \text{ in.}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{A'B'} - L_{AB}}{L_{AB}} = \frac{\sqrt{283.54} - 12\sqrt{2}}{12\sqrt{2}} = -7.77(10^{-3}) \text{ in./in.} \qquad \textbf{Ans.}$$

$$\epsilon_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{24.6 - 24}{24} = 0.025 \text{ in./in.} \qquad \textbf{Ans.}$$

$$\epsilon_{AC} = \frac{L_{A'C'} - L_{AC}}{L_{AC}} = \frac{23 - 24}{24} = -0.0417 \text{ in./in.} \qquad \textbf{Ans.}$$

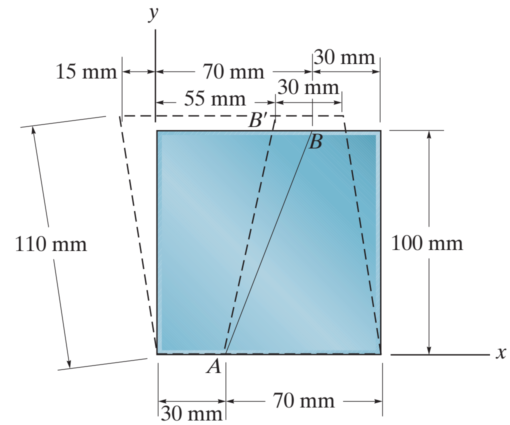


(a)

Ans:
$\epsilon_{AB} = -7.77(10^{-3})$ in./in.
$\epsilon_{BD} = 0.025$ in./in.
$\epsilon_{AC} = -0.0417$ in./in.

***2–28.**

The block is deformed into the position shown by the dashed lines. Determine the average normal strain along line AB.



SOLUTION

Geometry:

$$AB = \sqrt{100^2 + (70 - 30)^2} = 107.7033 \text{ mm}$$

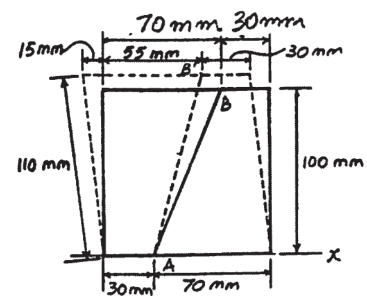
$$AB' = \sqrt{(70 - 30 - 15)^2 + \left(110^2 - 15^2\right)} = 111.8034 \text{ mm}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{111.8034 - 107.7033}{107.7033}$$

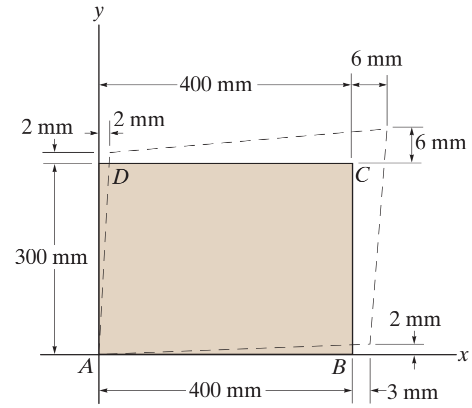
$$= 0.0381 \text{ mm/mm} = 38.1\left(10^{-3}\right) \text{ mm} \qquad \textbf{Ans.}$$



Ans:
$\epsilon_{AB} = 38.1\left(10^{-3}\right)$ mm

2–29.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal *AC*, and the average shear strain at corner *A* relative to the *x*, *y* axes.



SOLUTION

Geometry: The unstretched length of diagonal *AC* is

$$L_{AC} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. *a*, the stretched length of diagonal *AC* is

$$L_{AC'} = \sqrt{(400 + 6)^2 + (300 + 6)^2} = 508.4014 \text{ mm}$$

Referring to Fig. *a* and using small angle analysis,

$$\phi = \frac{2}{300 + 2} = 0.006623 \text{ rad}$$

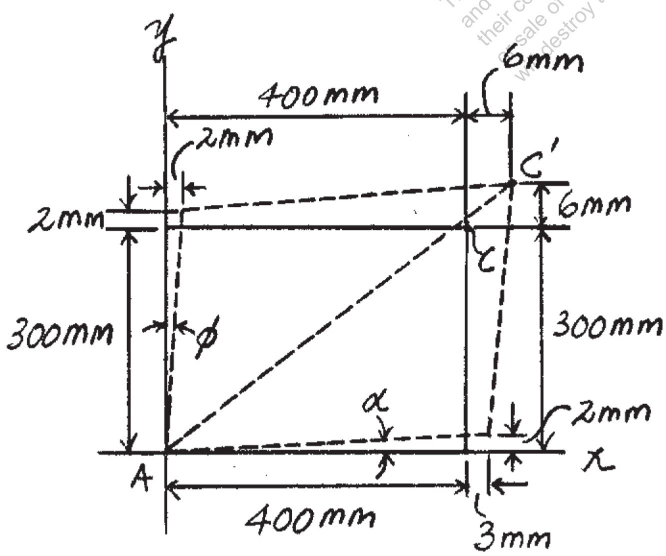
$$\alpha = \frac{2}{400 + 3} = 0.004963 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{508.4014 - 500}{500} = 0.0168 \text{ mm/mm} \qquad \textbf{Ans.}$$

Shear Strain: Referring to Fig. *a*,

$$(\gamma_A)_{xy} = \phi + \alpha = 0.006623 + 0.004963 = 0.0116 \text{ rad} \qquad \textbf{Ans.}$$

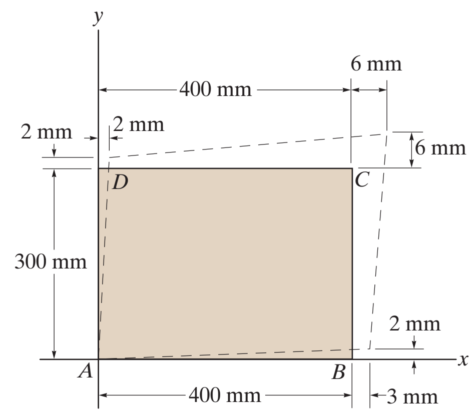


(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 0.0168 \text{ mm/mm}$, $(\gamma_A)_{xy} = 0.0116 \text{ rad}$

2–30.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal BD, and the average shear strain at corner B relative to the x, y axes.



SOLUTION

Geometry: The unstretched length of diagonal BD is

$$L_{BD} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. a, the stretched length of diagonal BD is

$$L_{B'D'} = \sqrt{(300 + 2 - 2)^2 + (400 + 3 - 2)^2} = 500.8004 \text{ mm}$$

Referring to Fig. a and using small angle analysis,

$$\phi = \frac{2}{403} = 0.004963 \text{ rad}$$

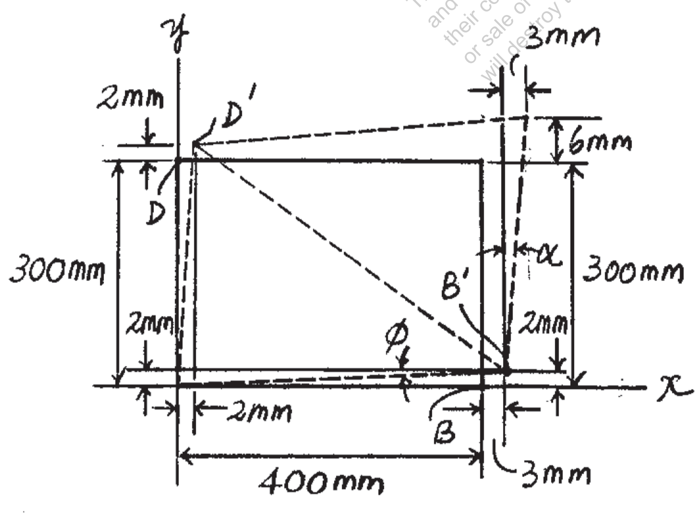
$$\alpha = \frac{3}{300 + 6 - 2} = 0.009868 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{500.8004 - 500}{500} = 1.60(10^{-3}) \text{ mm/mm} \quad \textbf{Ans.}$$

Shear Strain: Referring to Fig. a,

$$(\gamma_B)_{xy} = \phi + \alpha = 0.004963 + 0.009868 = 0.0148 \text{ rad} \quad \textbf{Ans.}$$

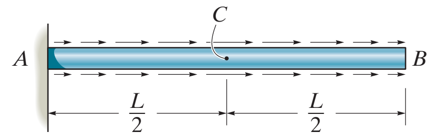


(a)

Ans:
$(\epsilon_{\text{avg}})_{BD} = 1.60(10^{-3})$ mm/mm,
$(\gamma_B)_{xy} = 0.0148$ rad

2–31.

The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$, where k is a constant. Determine the displacement of the center C and the average normal strain in the entire rod.



SOLUTION

$$\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$$

$$(\Delta x)_C = \int_0^{L/2} \epsilon_x \, dx = \int_0^{L/2} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right)\cos\left(\frac{\pi}{L}x\right)\Big|_0^{L/2} = -k\left(\frac{L}{\pi}\right)\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= \frac{kL}{\pi}$$ **Ans.**

$$(\Delta x)_B = \int_0^{L} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right)\cos\left(\frac{\pi}{L}x\right)\Big|_0^{L} = -k\left(\frac{L}{\pi}\right)(\cos \pi - \cos 0) = \frac{2kL}{\pi}$$

$$\epsilon_{avg} = \frac{(\Delta x)_B}{L} = \frac{2k}{\pi}$$ **Ans.**

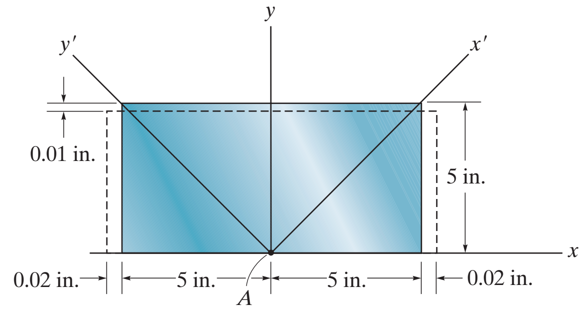
Ans:

$(\Delta x)_C = \frac{kL}{\pi}$

$\epsilon_{avg} = \frac{2k}{\pi}$

***2–32.**

The rectangular plate undergoes a deformation shown by the dashed lines. Determine the shear strain γ_{xy} and $\gamma_{x'y'}$ at point A.

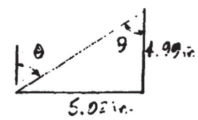


SOLUTION

Since the right angle of an element along the x,y axes does not distort, then

$\gamma_{xy} = 0$ **Ans.**

$$\tan\theta = \frac{5.02}{4.99}$$



$$\theta = 45.17° = 0.7884 \text{ rad}$$

$$\gamma_{x'y'} = \frac{\pi}{2} - 2\theta$$

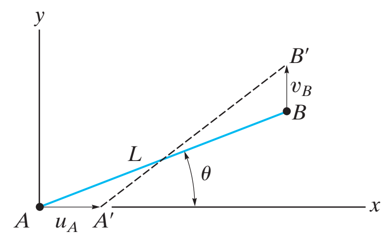
$$= \frac{\pi}{2} - 2(0.7884)$$

$$= -0.00599 \text{ rad}$$ **Ans.**

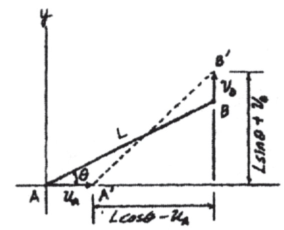
Ans:
$\gamma_{xy} = 0$
$\gamma_{x'y'} = -0.00599$ rad

2–33.

The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B respectively, determine the normal strain in the fiber when it is in position $A'B'$.



SOLUTION



Geometry:

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

Neglecting higher terms u_A^2 and v_B^2

$$\epsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\epsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ \ldots \ -1$$

$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$ **Ans.**

Ans:

$$\epsilon_{AB} = \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$

2–34.

If the normal strain is defined in reference to the final length $\Delta s'$, that is,

$$\epsilon' = \lim_{\Delta s' \to 0} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon - \epsilon' = \epsilon \epsilon'$.

SOLUTION

$$\epsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon - \epsilon' = \frac{\Delta s' - \Delta s}{\Delta s} - \frac{\Delta s' - \Delta s}{\Delta s'}$$

$$= \frac{\Delta s'^2 - \Delta s \Delta s' - \Delta s' \Delta s + \Delta s^2}{\Delta s \Delta s'}$$

$$= \frac{\Delta s'^2 + \Delta s^2 - 2\Delta s' \Delta s}{\Delta s \Delta s'}$$

$$= \frac{(\Delta s' - \Delta s)^2}{\Delta s \Delta s'} = \left(\frac{\Delta s' - \Delta s}{\Delta s} \right) \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

$$= \epsilon \epsilon'$$

(Q.E.D)

Ans:
N/A