Instructor's Solutions Manual for

MECHANICS OF MACHINES

Second Edition

William L. Cleghorn
University of Toronto

Nikolai Dechev
University of Victoria

Oxford University Press
2014

Acknowledgments

We express our sincere appreciation to the following students who assisted us in the completion of this manual: M. M. Alimardani, Dr. P. Honarmandi, Mr. H.S. Iu, Dr. Y. L. Kuo, Mr. V. P. K. Lee, Mr S. Momeni, Mr. G. Piras, Mr. M. Raessi, and other graduate and undergraduate students from the University of Toronto and the University of Manitoba.

William L. Cleghorn
Nikolai Dechev

P1.1

(a) $r_1 = 2.0 = s$; $r_2 = 6.5 = p$

$r_3 = 3.0 = q$; $r_4 = 7.0 = l$

(i) $l < s + p + q$, $\therefore$ links can form a mechanism

(ii) $s + l = 9.0$, $p + q = 9.5$

$s + l < p + q$, $\therefore$ Class I four-bar kinematic chain

since $r_1 = s$, drag link mechanism

(b) $r_1 = 2.0 = s$; $r_2 = 8.0 = p$

$r_3 = 3.0 = q$; $r_4 = 9.0 = l$

(i) $l < s + p + q$, $\therefore$ links can form a mechanism

(ii) $s + l = 11.0$; $p + q = 11.0$

$s + l = p + q$, $\therefore$ change point mechanism

P1.1 CONTINUED

(c) $r_1 = 2.5 = l$; $r_2 = 1.0 = s$

$r_3 = 2.5 = p$; $r_4 = 2.0 = q$

(i) $l < s + p + q$, $\therefore$ links can form a mechanism

(ii) $s + l = 3.5$, $p + q = 4.5$

$s + l < p + q$, $\therefore$ Class I four-bar kinematic chain

since $r_2 = s$, crank rocker four-bar mechanism

(d) $r_1 = 2.5 = p$; $r_2 = 3.0 = l$

$r_3 = 1.0 = s$; $r_4 = 2.0 = q$

(i) $l < s + p + q$, $\therefore$ links can form a mechanism

(ii) $s + l = 4.0$, $p + q = 4.5$

$s + l < p + q$, $\therefore$ Class I four-bar kinematic chain

since $r_3 = s$, rocker-rocker four-bar mechanism

P1.1 CONTINUED

(e) $r_1 = 2.0 = p$; $r_2 = 4.5 = q$

$r_3 = 1.5 = s$; $r_4 = 9.0 = l$

(i) $l > s + p + q$, $\therefore$ links cannot form a mechanism

(f) $r_1 = 1.5 = s$; $r_2 = 3.0 = p$

$r_3 = 2.5 = q$; $r_4 = 6.0 = l$

(i) $l < s + p + q$, $\therefore$ links can form a mechanism

(ii) $s + l = 7.5$, $p + q = 5.5$

$s + l > p + q$, rocker-rocker four-bar mechanism

P1.2

$r_1 = 1.0$; $r_3 = 2.5$; $r_4 = 2.0$

(a) to be a crank rocker mechanism, r_2 must be the shortest link (i.e., $r_2 < 1.0$)
also, we require $s + l < p + q$ ($s = r_2$; $l = r_3 = 2.5$; $p = r_1 = 1.0$; $q = r_4 = 2.0$)

$$r_2 + 2.5 < 1.0 + 2.0$$

$$r_2 < 0.5$$

(b) to be a drag link mechanism, r_1 must be the shortest link (i.e., $r_2 > 1.0$)
r_2 may be p, q or l.

Case 1 : if r_2 is l ($r_1 = s$, $r_3 = p$, $r_4 = q$)

$$s + l < p + q$$

$$1.0 + r_2 < 2.5 + 2.0 \quad , \quad r_2 < 3.5$$

Case 2 : if $r_2 = p$ or q ($r_1 = s$, $r_3 = l$, $r_4 = q$ or p)

$$s + l < p + q$$

$$1.0 + 2.5 < r_2 + 2.0 \, , \quad r_2 > 1.5$$

combining Cases 1 and 2: $1.5 < r_2 < 3.5$

P1.2 CONTINUED

(c) to be a change point mechanism, $s+l = p+q$

<u>Case 1 : if $r_2 = s$</u> $(r_1 = p,\ r_3 = l,\ r_4 = q)$

$s + l = p + q$

$r_2 + 2.5 = 1.0 + 2.0\ ,\quad r_2 = 0.5$

<u>Case 2 : if $r_2 = l$</u> $(r_1 = s,\ r_3 = p,\ r_4 = q)$

$s + l = p + q$

$1.0 + r_2 = 2.5 + 2.0,\quad r_2 = 3.5$

<u>Case 3 : if $r_2 = p$</u> $(r_1 = s,\ r_3 = l,\ r_4 = q)$

$s + l = p + q$

$1.0 + 2.5 = r_2 + 2.0,\quad r_2 = 1.5$

<u>combining</u> <u>Cases 1, 2 and 3</u> : $r_2 = 0.5$ OR
$r_2 = 1.5$ OR
$r_2 = 3.5$

(d) to be a rocker-rocker mechanism, $s+l > p+q$

<u>Case 1 : if $r_2 = s$</u>

$s + l > p + q$

$r_2 + 2.5 > 1.0 + 2.0,\quad r_2 > 0.5$ (i)

also, because r_2 is the shortest link, $r_2 < 1.0$ (ii)

P1.2 CONTINUED

combining (i) and (ii) : $0.5 < r_2 < 1.0$ (I)

Case 2 : if $r_2 = l$

$s + l > p + q$

$1.0 + r_2 > 2.0 + 2.5$, $r_2 > 3.5$ (iii)

also, for assembly of the kinematic chain,

$s + p + q > l$

$1.0 + 2.0 + 2.5 > r_2$, $r_2 < 5.5$ (iv)

combining (iii) and (iv) : $3.5 < r_2 < 5.5$ (II)

Case 3 : if $r_2 = p$

$s + l > p + q$

$1.0 + 2.5 > r_2 + 2.0$, $r_2 < 1.5$ (v)

but for this case, r_2 cannot be the shortest link, and therefore $r_2 \geq 1.0$ (vi)

combining (v) and (vi) : $1.0 \leq r_2 < 1.5$ (III)

combining (I), (II) and (III)

$0.5 < r_2 < 1.5$ OR $3.5 < r_2 < 5.5$

P1.3

$r_1 = 1.0$; $r_2 = 3.0$; $r_3 = 2.5$

(a) to be a crank rocker mechanism, the input link must be the shortest link
considering r_4 as the input link

$s + l < p + q$

$r_4 + 3.0 < 1.0 + 2.5, \quad r_4 < 0.5$

(b) to be a drag link mechanism, the base link length, r_1, must be the shortest link
thus, r_4 may be p, q or l

Case 1: if $r_4 = l$

$s + l < p + q$

$1.0 + r_4 < 3.0 + 2.5, \quad r_4 < 4.5$

also, for r_4 to be the longest link, $r_4 > 3.0$

$\therefore 3.0 < r_4 < 4.5$

Cas 2: if $r_4 = p$ or q

$s + l < p + q$

$1.0 + 3.0 < 2.5 + r_4, \quad r_4 > 1.5$

also, for r_4 to be p or q, $r_4 \leq 3.0$

$\therefore 1.5 < r_4 \leq 3.0$

P1.3 CONTINUED

(c) to be a change point mechanism: $s+l=p+q$

<u>Case 1: $r_4 = s$</u>

$s+l=p+q$

$r_4 + 3.0 = 1.0 + 2.5, \quad r_4 = 0.5$

<u>Case 2: $r_4 = l$</u>

$s+l=p+q$

$1.0 + r_4 = 2.5 + 3.0, \quad r_4 = 4.5$

<u>Case 3: $r_4 = p$ or q</u>

$1.0 + 3.0 = r_4 + 2.5, \quad r_4 = 1.5$

<u>combining Cases 1, 2 and 3</u>: $r_4 = 0.5$ OR
$r_4 = 1.5$ OR
$r_4 = 4.5$

(d) to be a rocker - rocker mechanism, $s+l > p+q$

<u>Case 1: $r_4 = s$</u>

$r_4 + 3.0 > 1.0 + 2.5, \quad r_4 > 0.5$

also, for r_4 to be the shortest link, $r_4 \leq 1.0$

$\therefore \; 0.5 < r_4 \leq 1.0 \quad (I)$

P1.3 CONTINUED

Case 2: $r_4 = p$ or q

$s + l > p + q$

$1.0 + 3.0 > r_4 + 2.5, \quad r_4 < 1.5 \quad (i)$

also, because r_4 cannot be the shortest link,
$r_4 \geq 1.0 \quad (ii)$

combining (i) and (ii): $1.0 \leq r_4 < 1.5 \quad (II)$

Case 3: $r_4 = l$

$s + l > p + q$

$1.0 + r_4 > 2.5 + 3.0, \quad r_4 > 4.5 \quad (iii)$

also, for assembly of the kinematic chain,

$s + p + q > l$

$1.0 + 2.5 + 3.0 > r_4 \quad , \quad r_4 < 6.5 \quad (iv)$

combining (iii) and (iv): $4.5 < r_4 < 6.5 \quad (III)$

combining (I), (II) and (III): $0.5 < r_4 < 1.5$ OR
$4.5 < r_4 < 6.5$

P1.4

$r_1 = 1.0\text{ cm}$; $r_3 = 2.5\text{ cm}$

(a) for full rotation of link 2:

$\overset{(i)}{r_2 < r_3}$ and $\overset{(ii)}{|r_1| \leqslant r_3 - r_2}$

$\therefore$ (i) $r_2 < 2.5\text{cm}$

(ii) $|r_1| \leqslant r_3 - r_2$

$1.0 \leqslant 2.5 - r_2$, $r_2 \leqslant 1.5\text{cm}$

combining (i) & (ii), conclude $\underline{r_2 \leqslant 1.5\text{cm}}$

(b) possible to assemble mechanism:

$|r_1| \leqslant r_2 + r_3$

$1.0 \leqslant r_2 + 2.5$, $r_2 \geqslant -1.5\text{cm}$

from which we conclude $\underline{r_2 \geqslant 0}$

P1.5

$r_1 = 1.0\text{cm}$; $r_2 = 2.5\text{cm}$

(a) for full rotation of link 2:

(i) $r_2 < r_3$ and (ii) $|r_1| \leqslant r_3 - r_2$

$\therefore$ (i) $2.5 < r_3$, $r_3 > 2.5\text{cm}$

(ii) $|r_1| \leqslant r_3 - r_2$

$1.0 \leqslant r_3 - 2.5$, $r_3 \geqslant 3.5\text{cm}$

combining (i) & (ii), conclude $\underline{r_3 \geqslant 3.5\text{cm}}$

(b) possible to assemble mechanism:

$|r_1| \leqslant r_2 + r_3$

$1.0 \leqslant 2.5 + r_3$, $r_3 \geqslant -1.5\text{cm}$

from which we conclude $\underline{r_3 \geqslant 0}$

P1.6

(a) $r_3 = 3.5\,cm$

$s = r_2 = 2.0\,cm$; $p = r_3 = 3.5\,cm$

$l = r_1 = 7.0\,cm$; $q = r_4 = 6.0\,cm$

$(s + l = 9.0\,cm) < (p + q = 9.5\,cm)$

$\therefore$ class I four-bar mechanism
since s is the input link, <u>crank rocker</u>

(b) $r_3 = 11.0\,cm$

$s = r_2 = 2.0\,cm$; $p = r_4 = 6.0\,cm$

$l = r_3 = 11.0\,cm$; $q = r_1 = 7.0\,cm$

$(s + l = 13.0\,cm) = (p + q = 13.0\,cm)$

$\therefore$ <u>change point</u>

(c) $r_3 = 11.5\,cm$

$s = r_2 = 2.0\,cm$; $p = r_4 = 6.0\,cm$

$l = r_3 = 11.5\,cm$; $q = r_1 = 7.0\,cm$

$(s + l = 13.5\,cm) > (p + q = 13.0\,cm)$

$\therefore$ class II four-bar mechanism
<u>rocker-rocker</u>

P1.7

crank rocker (a)

the input motion to the four-bar mechanism is the gear (i.e., crank) that executes full rotations

the output motion of the four-bar mechanism is the gear sector (i.e., rocker) that oscillates between two limit positions

the gear sector drives a smaller gear that is rigidly connected to the agitator that undergoes oscillatory motion and provides the washing action

P1.8

crank rocker (a)

the input motion to the four-bar mechanism executes full rotations

the output motion of the four-bar mechanism is the coupler point on the coupler that enters a perforation in the film. The coupler point then advances the film by one frame

P1.9

(a) $n = 6$

- turning pairs, 5 ;
- sliding pair, 1 ;
- rolling pair, 1 $\qquad (j_1 = 5 + 1 + 1 = 7)$
- 2 dof pairs, $\qquad (j_2 = 0)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(6-1) - 2 \times 7 = 1$$

(b) $n = 6$

- turning pairs, 5 ;
- sliding pairs, 2 ;
- rolling pairs, 0 $\qquad (j_1 = 5 + 2 + 0 = 7)$
- 2 dof pairs, 0 $\qquad (j_2 = 0)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(6-1) - 2 \times 7 = 1$$

P1.10

(a) $n = 4, \quad j_1 = 4, \quad j_2 = 0$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(4-1) - 2 \times 4 = 1$$

(b) $n = 6, \quad j_1 = 7, \quad j_2 = 0$

$$m = 3(6-1) - 2 \times 7 = 1$$

P1.11

(a) $n = 6$

- turning pairs, 7 ;
- sliding pairs, 0 ;
- rolling pairs, 0 $\quad (j_1 = 7 + 0 + 0 = 7)$
- 2 dof pairs, 0 $\quad (j_2 = 0)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(6-1) - 2 \times 7 = 1$$

(b) $n = 8$

- turning pairs, 10 ;
- sliding pairs, 0 ;
- rolling pairs, 0 $\quad (j_1 = 10 + 0 + 0 = 10)$
- 2 dof pairs, 0 $\quad (j_2 = 0)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(8-1) - 2 \times 10 = 1$$

P1.12

(a) $n = 4$

- turning pairs, 0 ;
- sliding pairs, 5 ;
- rolling pairs, 0 $\quad (j_1 = 0 + 5 + 0 = 5)$

- 2 dof pairs, 0 $\quad (j_2 = 0)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(4-1) - 2 \times 5 = -1$$

- however, there are two redundant constraints (i.e., $N_R = 2$)

- relative rotations between links 2 and 3, and between links 3 and 4 are accounted for twice (once by sliding pairs between links 2 and 3, and between links 3 and 4 ; and then again by sliding pairs between links 1 and 2, 1 and 3, and 1 and 4)

$$\therefore \quad m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.12 CONTINUED

(b) $n = 4$

- turning pairs, 0;
- sliding pairs, 3;
- rolling pairs, 0 $\quad (j_1 = 0 + 3 + 0 = 3)$
- 2 dof pairs, 2 $\quad (j_2 = 2)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(4-1) - 2 \times 3 - 2 = 1$$

P1.13

(a) $n = 4$

- turning pairs, 3;
- sliding pairs, 0;
- rolling pairs, 2 $\quad (j_1 = 3 + 0 + 2 = 5)$
- 2 dof pairs, 0 $\quad (j_2 = 0)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(4-1) - 2 \times 5 - 0 = -1$$

- however, there are two redundant constraints (i.e., $N_R = 2$)

- distances between base pivots of links 2 and 3, and between base pivots of links 3 and 4 are accounted for twice (once by rolling pairs between links 2 and 3, and between links 3 and 4; and then again by base pivots of link 2 and 3, and base pivots of links 3 and 4)

$$\therefore m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.13 CONTINUED

(b) $n = 8$

- turning pairs, 9;
- sliding pairs, 0;
- rolling pairs, 2 $\quad (j_1 = 9 + 0 + 2 = 11)$
- 2 dof pairs, 0 $\quad (j_2 = 0)$

$$m = 3(n-1) - 2j_1 - j_2$$
$$= 3(8-1) - 2 \times 11 = -1$$

- however, the above calculation incorporates two redundant constraints (same as for P1.5(a))

$N_R = 2$

$$\therefore m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.14

4 links, $n = 4$

4 turning pairs,

0 rolling pairs,

0 sliding pairs, $j_1 = 0 + 4 + 0 = 4$

0 DOF pairs, $j_2 = 0$

no redundant constraints, $N_R = 0$

$m = 3\ (n - 1) - 2\,j_1 - j_2 + N_R = 1$

P1.15

4 links, $n = 4$

4 turning pairs,

0 rolling pairs,

0 sliding pairs, $j_1 = 0 + 4 + 0 = 4$

0 DOF pairs, $j_2 = 0$

no redundant constraints, $N_R = 0$

$m = 3\ (n - 1) - 2\,j_1 - j_2 + N_R = 1$

$\boxed{\text{P1.16}}$

3 links, $n = 3$

1 turning pair,

1 rolling pair,

1 sliding pair, $j_1 = 1 + 1 + 1 = 3$

0 DOF pairs, $j_2 = 0$

one redundant constraints, $N_R = 1$

$m = 3\,(n - 1) - 2\,j_1 - j_2 + N_R = 1$

P1.17

4 links, $n = 4$

2 turning pairs,

1 rolling pairs,

1 sliding pairs, $j_1 = 2 + 1 + 1 = 4$

0 DOF pairs, $j_2 = 0$

no redundant constraints, $N_R = 0$

$m = 3\ (n - 1) - 2\,j_1 - j_2 + N_R = 1$

P1.18

(a) $n = 5; \quad j_1 = 6; \quad j_2 = 0$

$N_R = 1$ (center-to-center distance between O_5 & E accounted for length of link 5 AND lengths of $r_{O_4O_5}$, r_{O_4D} & r_{DE}

$$m = 3(n-1) - 2j_1 - j_2 + N_R$$
$$= 3(5-1) - 2 \times 6 - 0 + 1 = 1$$

(b) $n = 6; \quad j_1 = 8; \quad j_2 = 0$

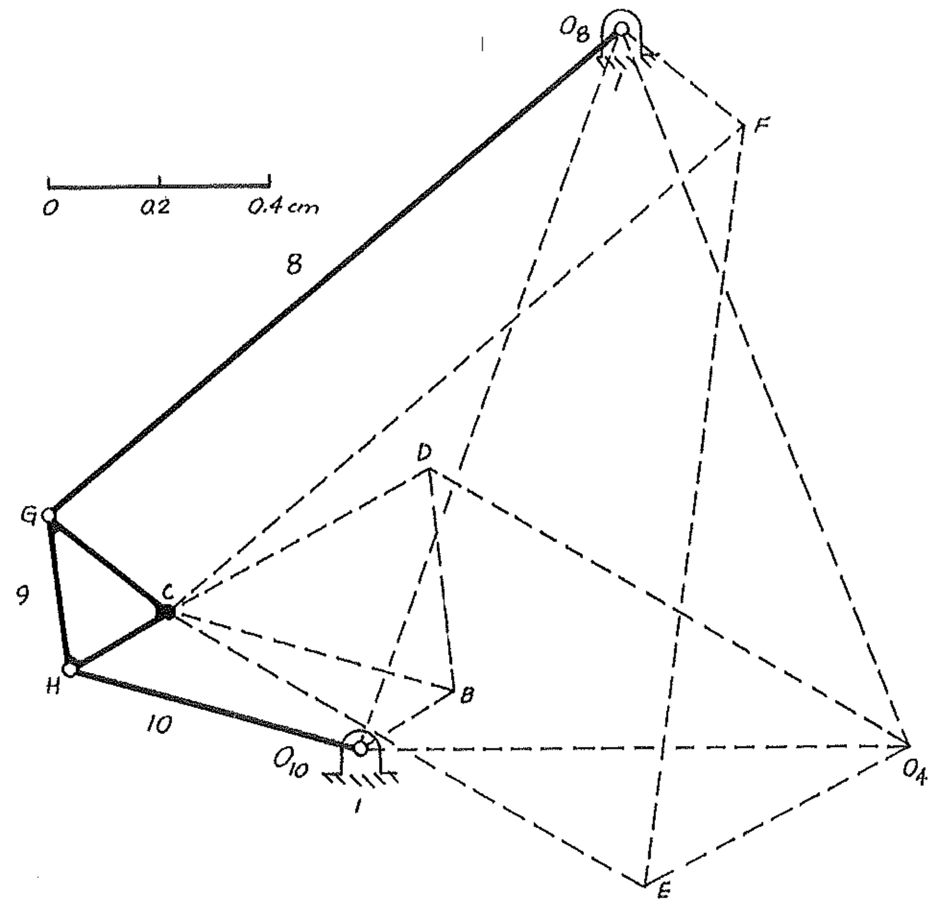
$N_R = 2$ (center-to-center distance between O_5 & E accounted for length of link 5 AND lengths of $r_{O_4O_5}$, r_{O_4D} & r_{DE};

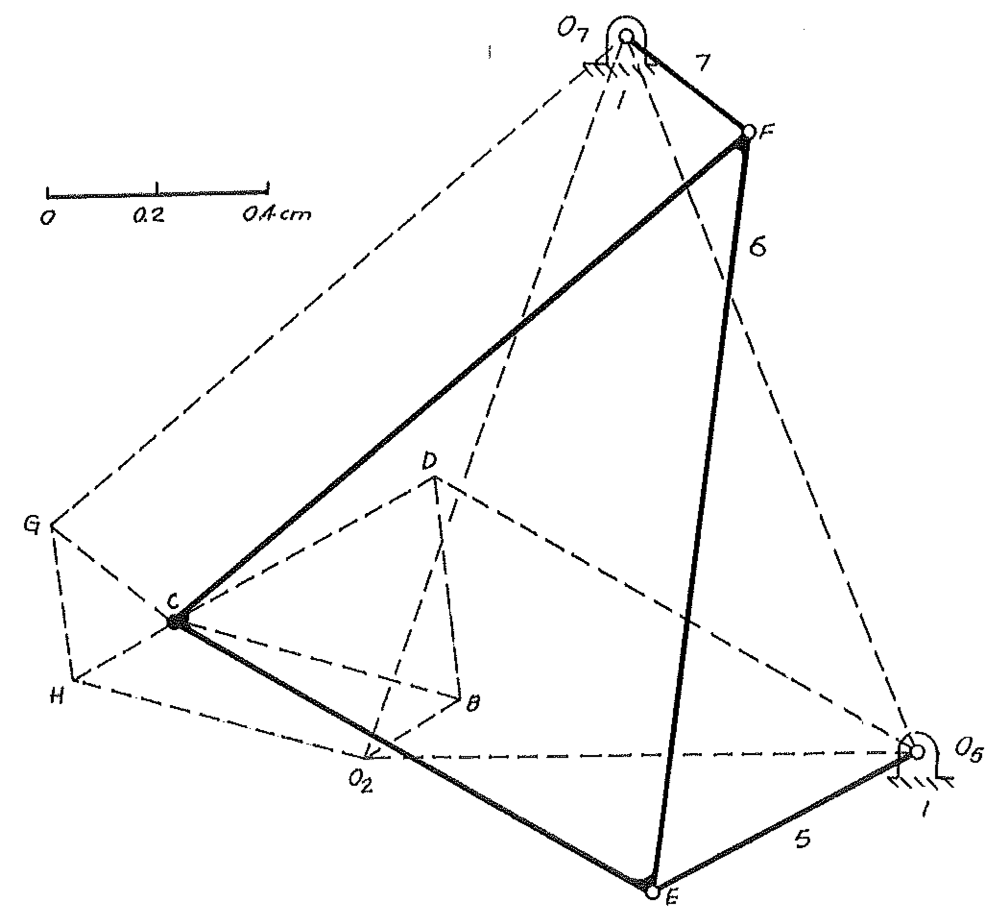
similar reasoning for length r_{O_6F})

$$m = 3(n-1) - 2j_1 - j_2 + N_R = 1$$

P1.19

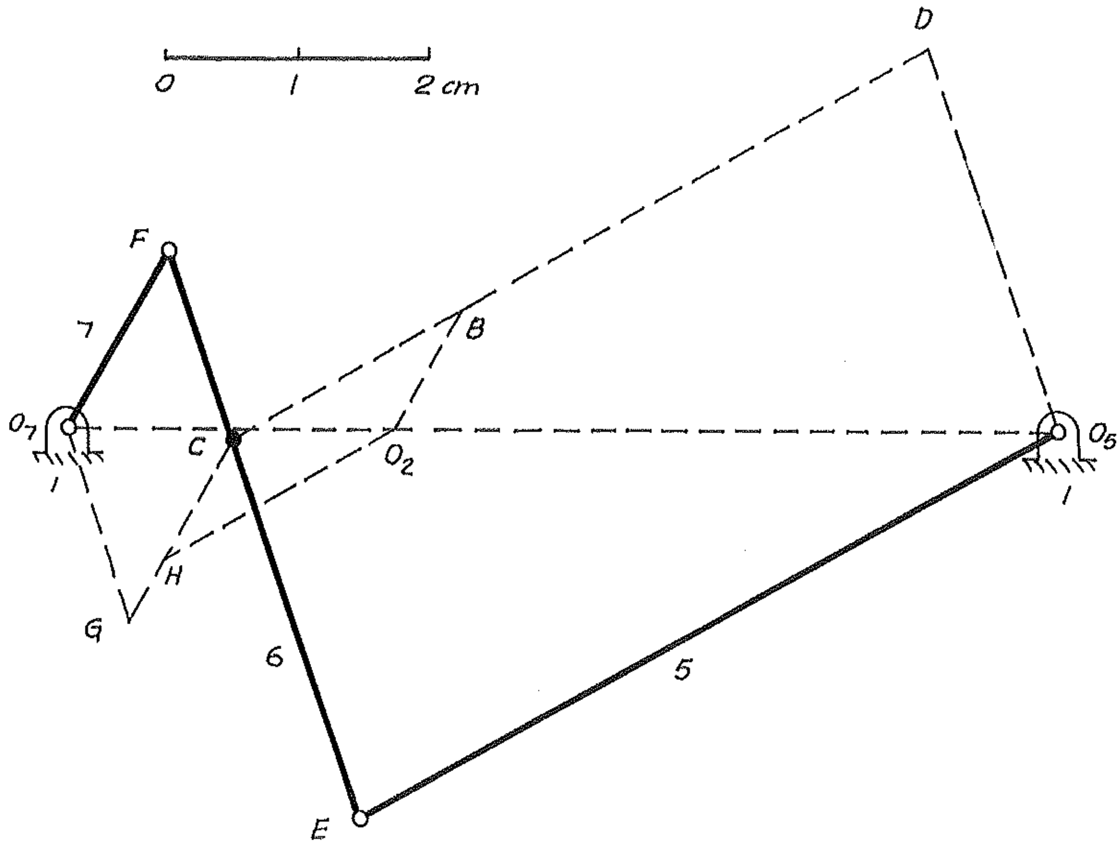
(a)

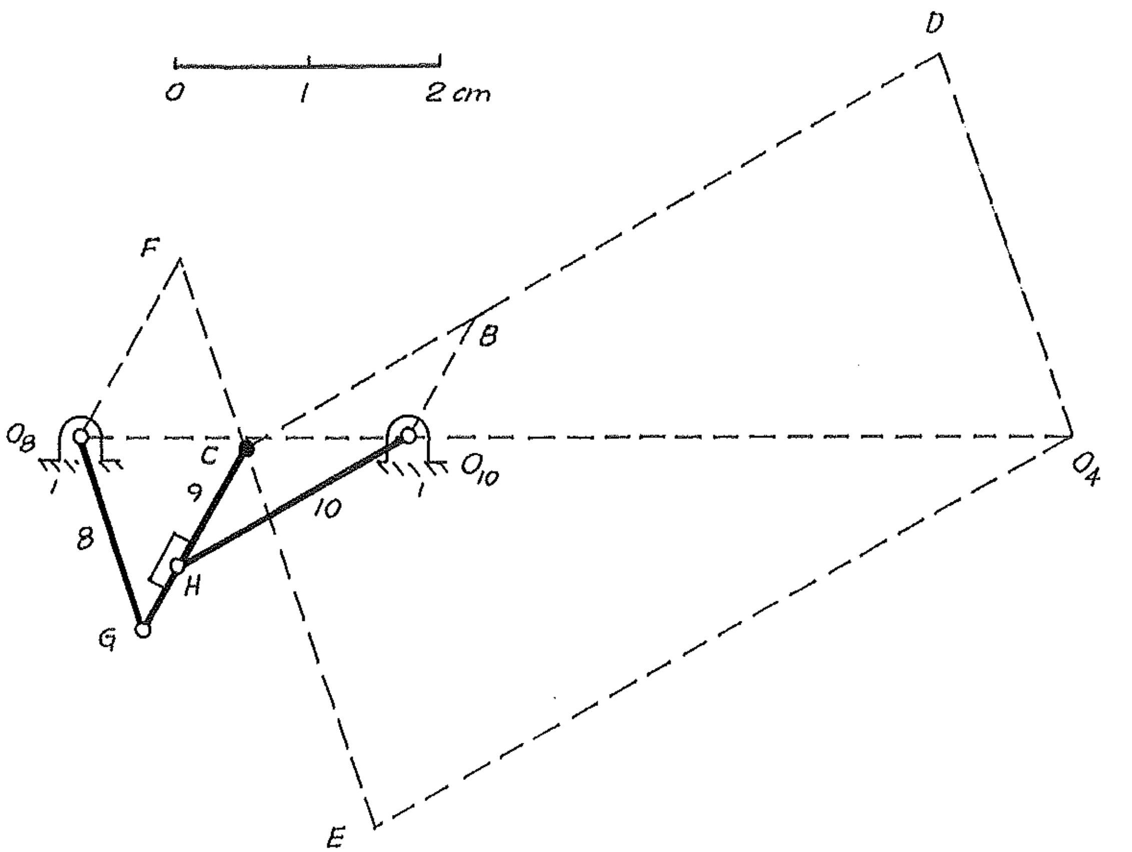

0
0.2
0.4 cm
O_8
F
8
D
G
9
C
H
B
10
O_{10}
1
O_4
E


O_7
7
F
0
0.2
0.4 cm
6
D
G
C
H
B
O_2
O_5
1
5
E

P1.19 CONTINUED

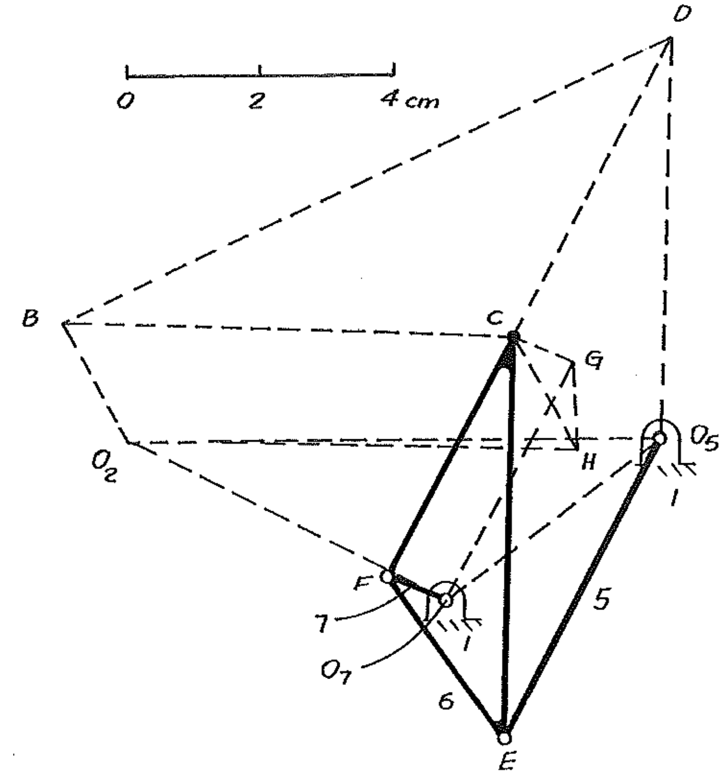
(b)

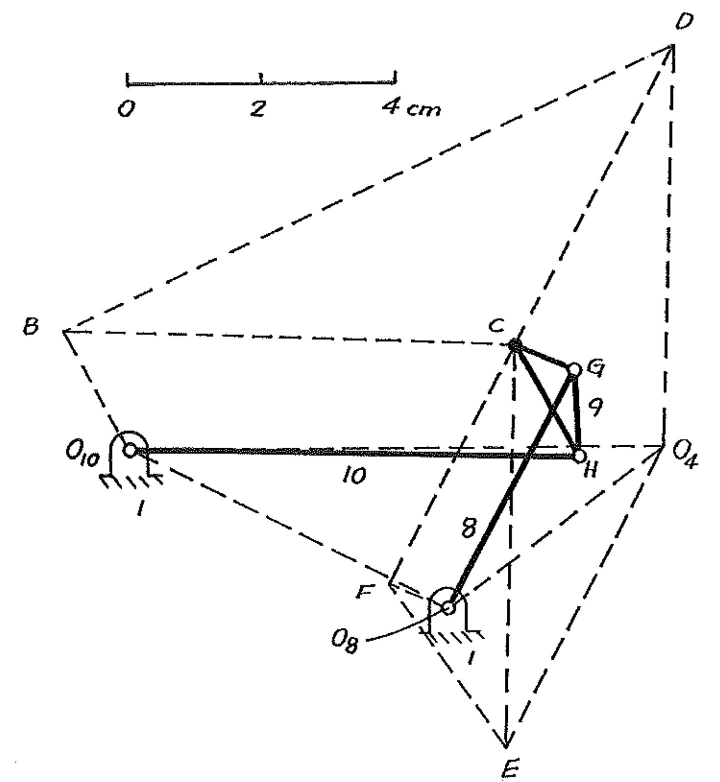




P1.19 CONTINUED

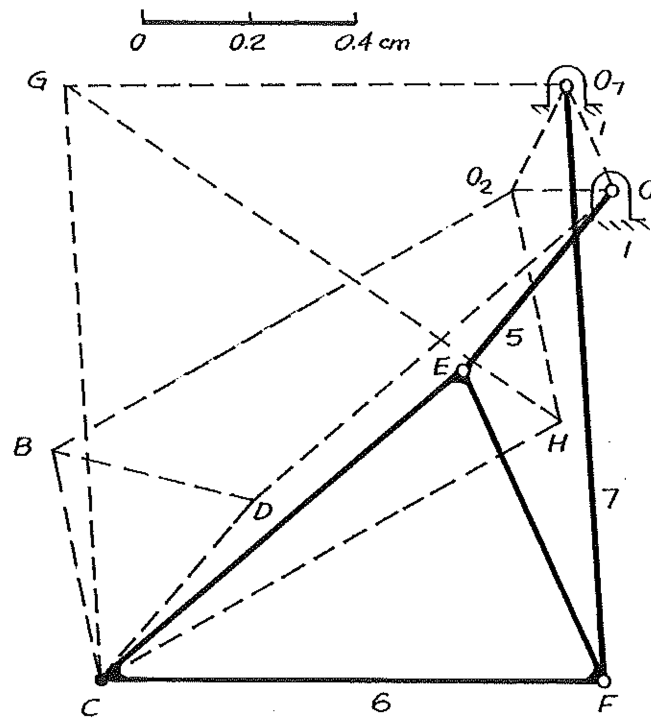
(c)

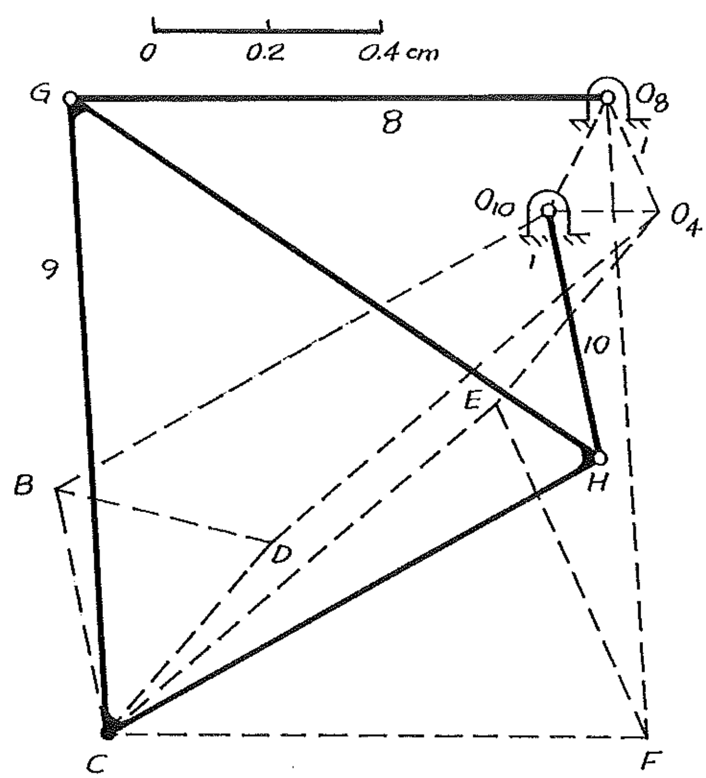




P1.19 CONTINUED

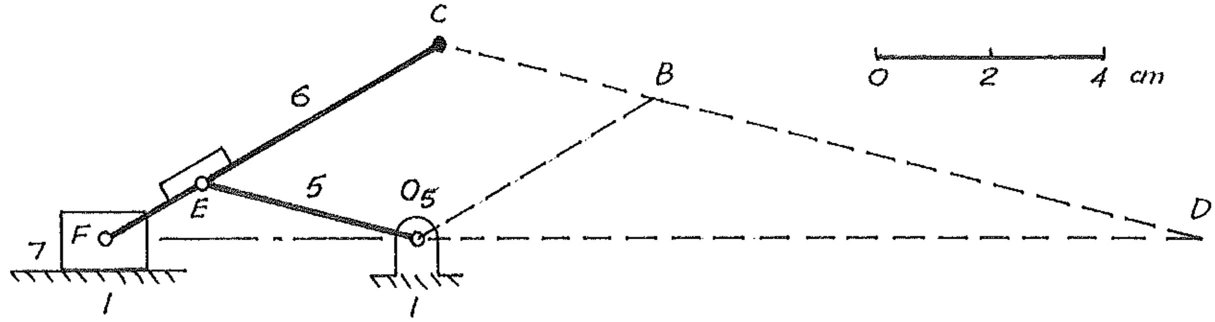
(d)



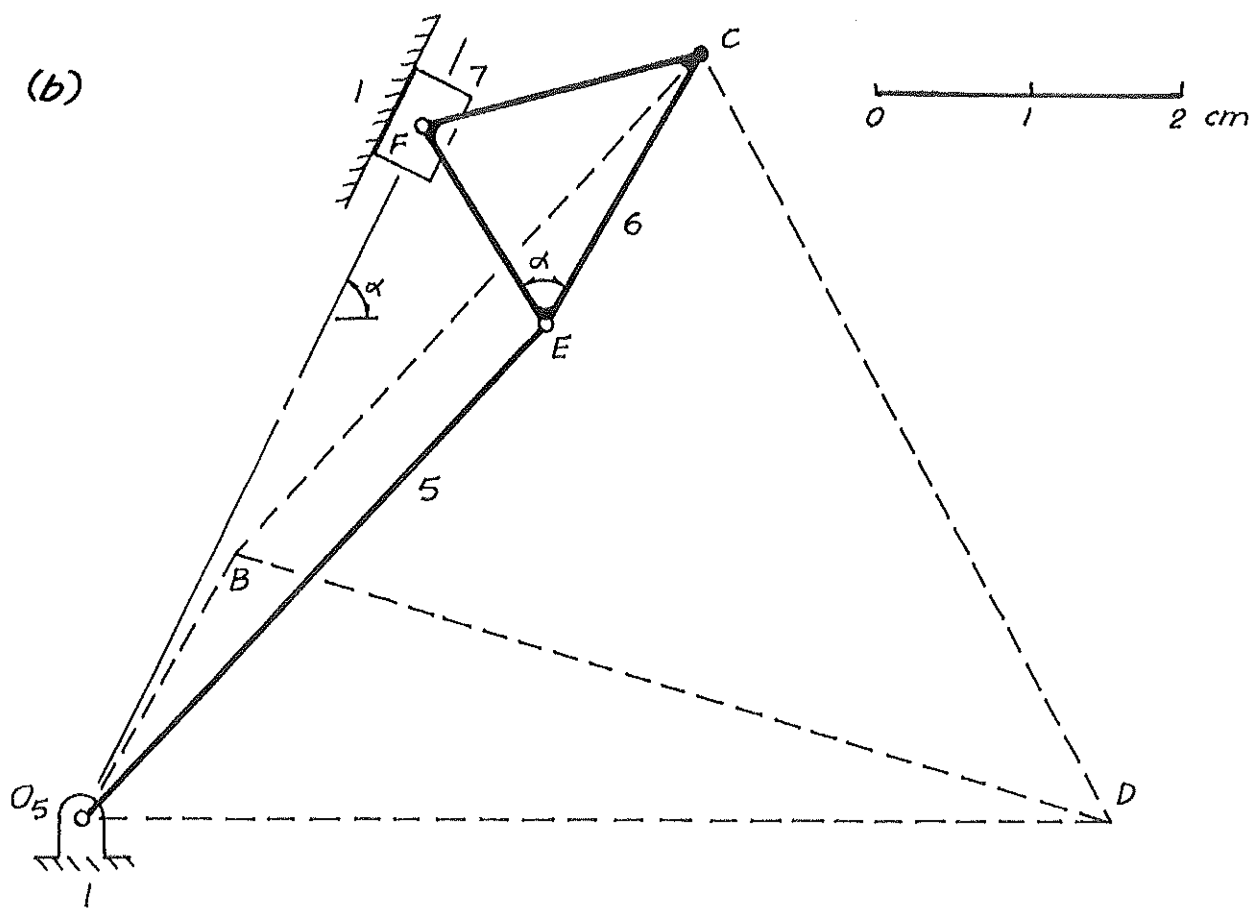


P1.20

(a)

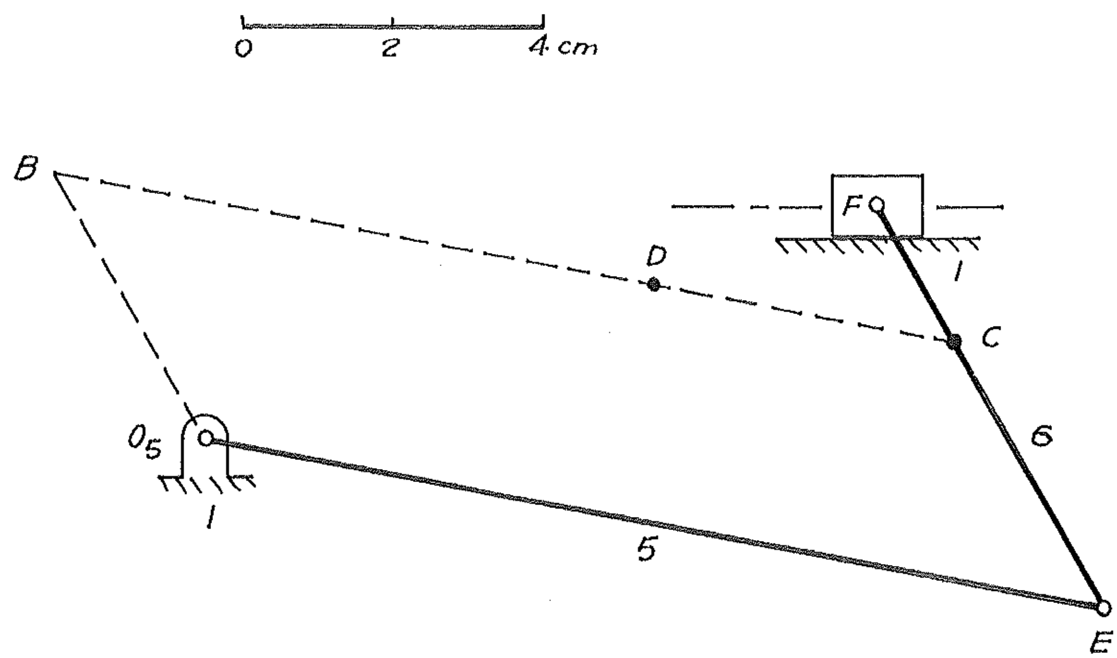


(b)

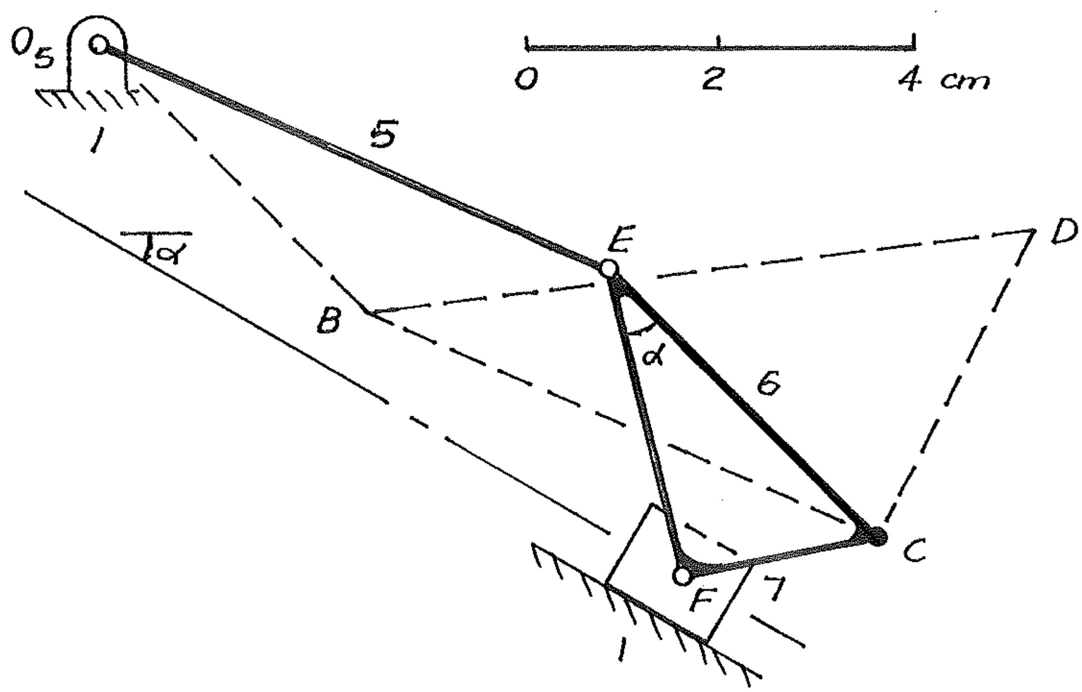


P1.20 CONTINUED

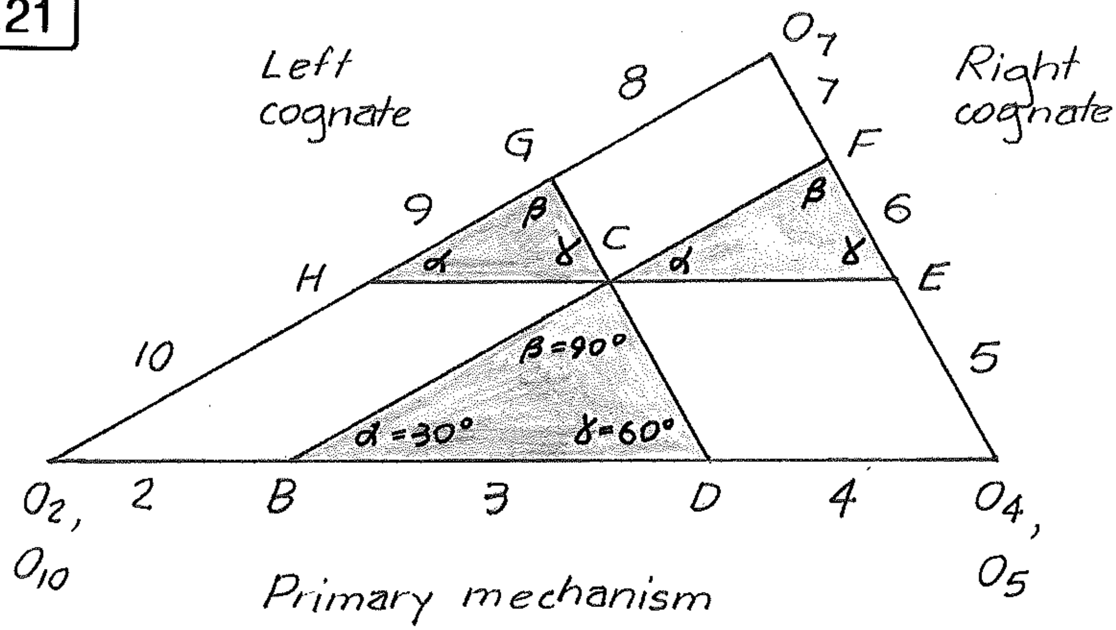
(c)



(d)



P1.21



__Left cognate:__

base link : $r_{O_8O_{10}} = 11.8$ cm ;

link 8 : $r_{O_8G} = 6.00$ cm ; link 9 : $r_{GH} = 4.33$ cm ;

link 10 : $r_{O_{10}H} = 7.79$ cm

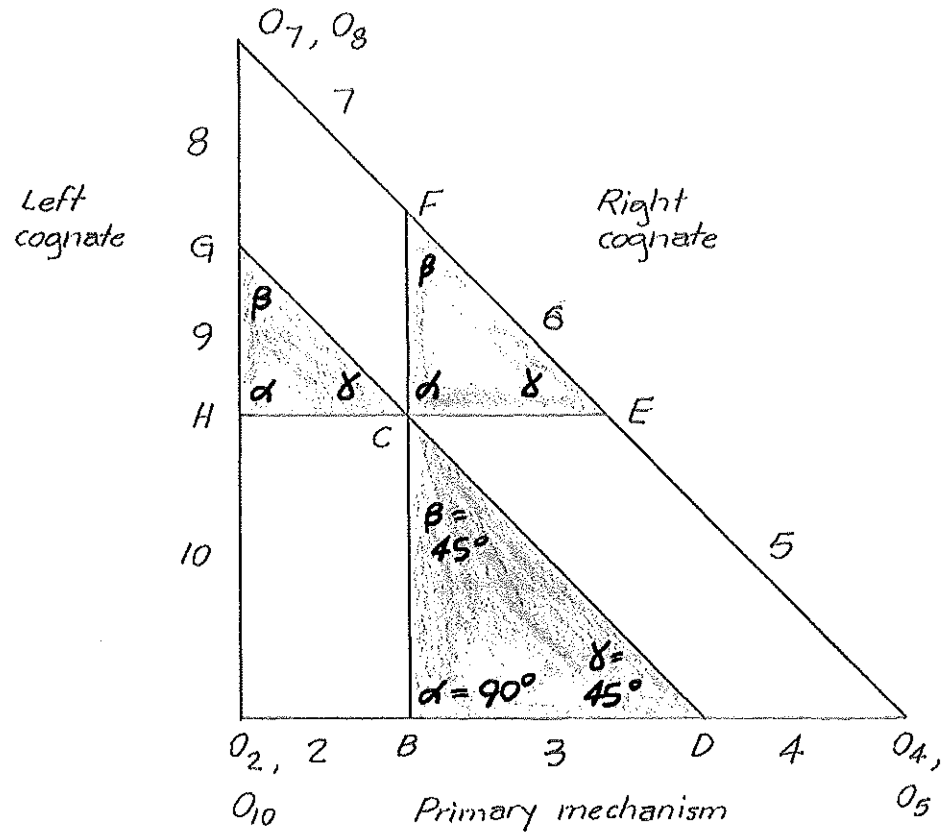
__Right cognate:__

base link : $r_{O_5O_7} = 6.50$ cm;

link 5 : $r_{O_5E} = 4.50$ cm ; link 6 : $r_{EF} = 3.00$ cm;

link 7 : $r_{O_7F} = 2.50$ cm

P1.22



Left cognate:

base link: $r_{O_8O_{10}} = 12.0\,cm$;

link 8: $r_{O_8G} = 6.00\,cm$; link 9: $r_{GH} = 5.00\,cm$;

link 10: $r_{O_{10}H} = 9.00\,cm$

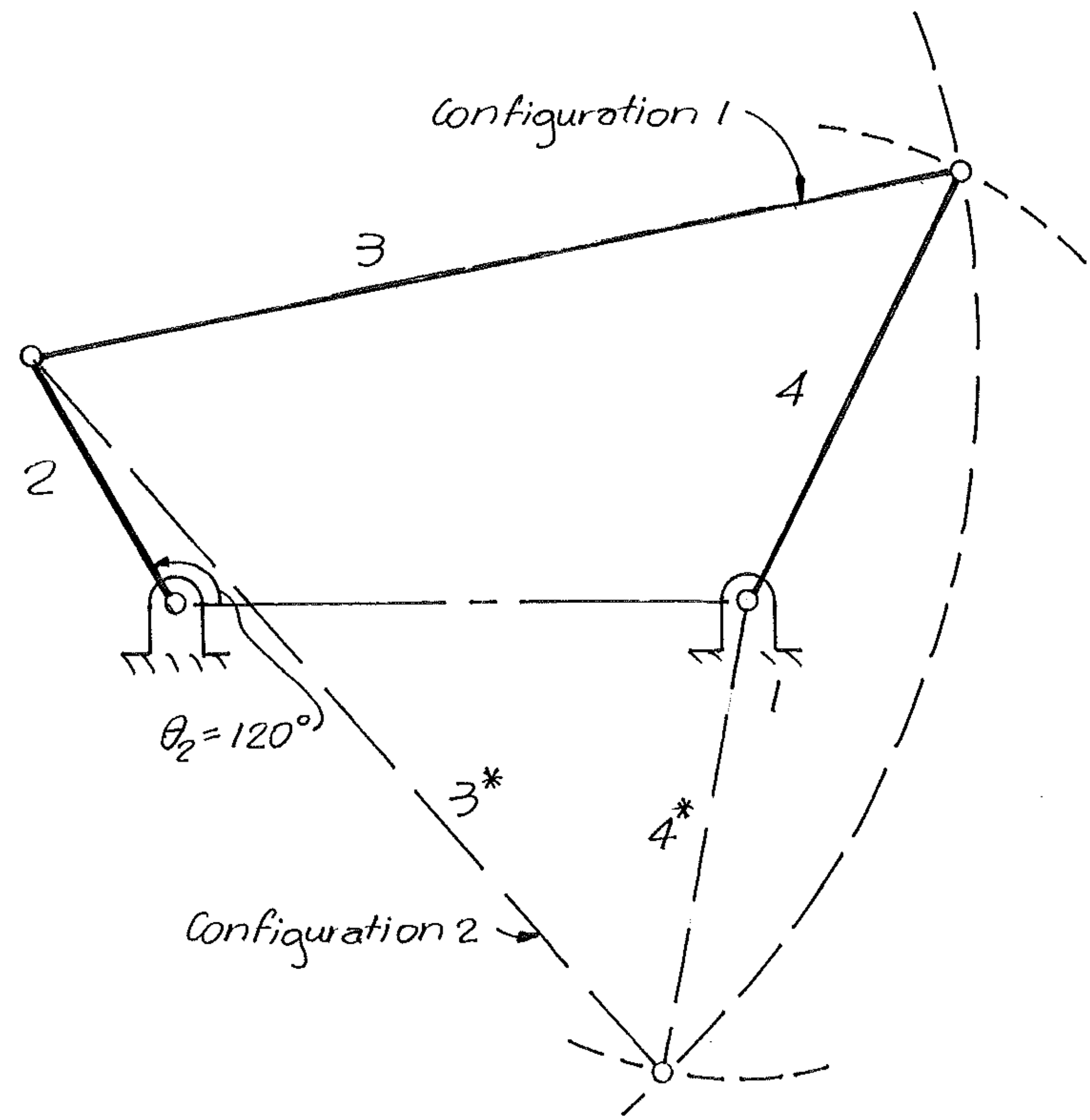
Right cognate:

base link: $r_{O_5O_7} = 17.0\,cm$;

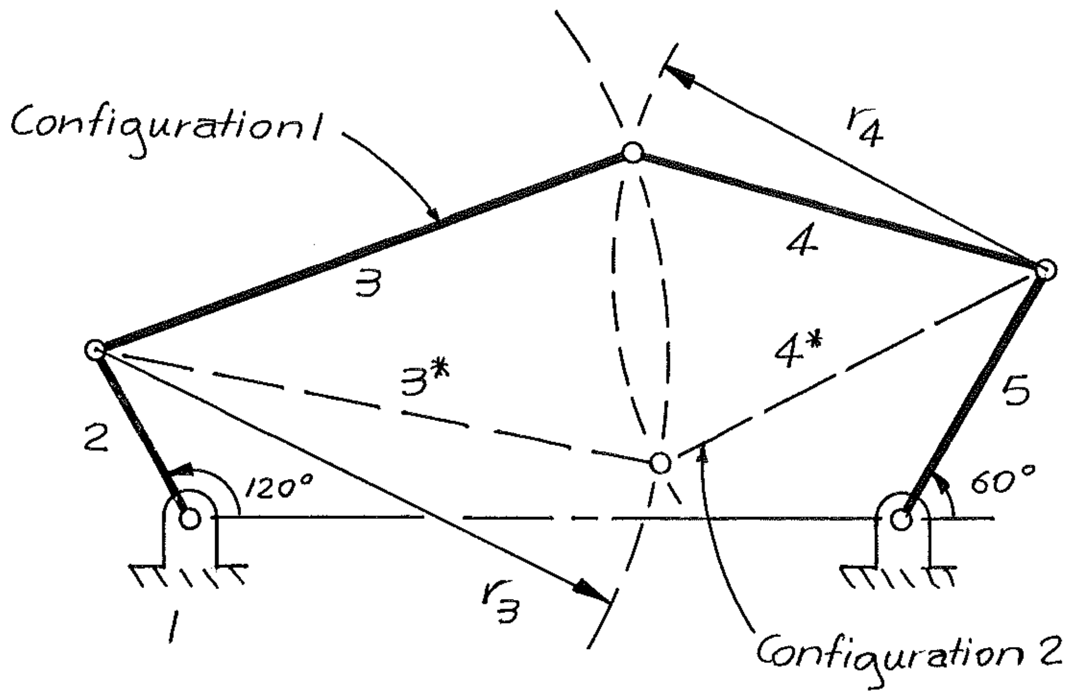
link 5: $r_{O_5E} = 12.7\,cm$; link 6: $r_{EF} = 8.49\,cm$;

link 7: $r_{O_7F} = 7.07\,cm$

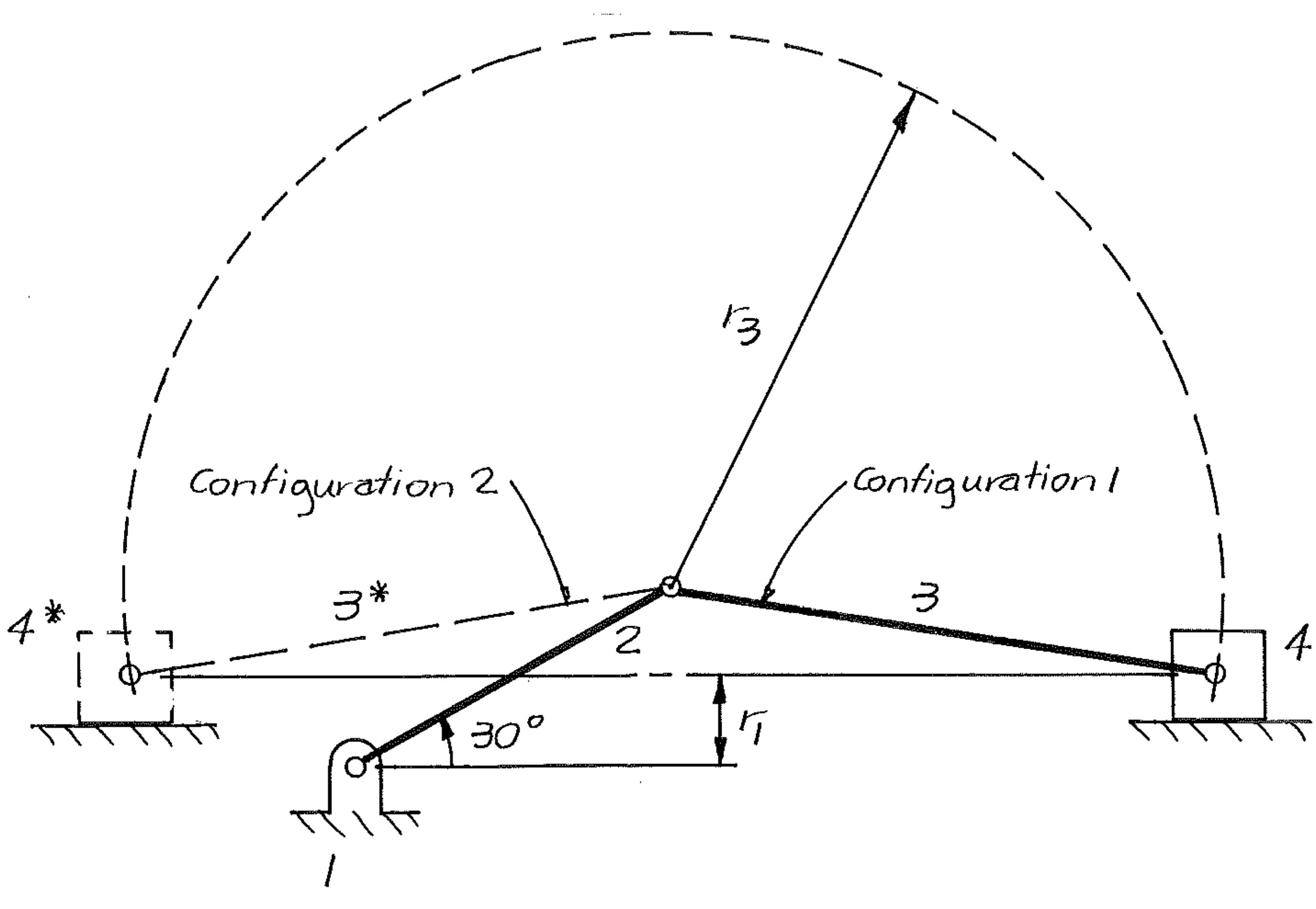
P2.1



P2.2

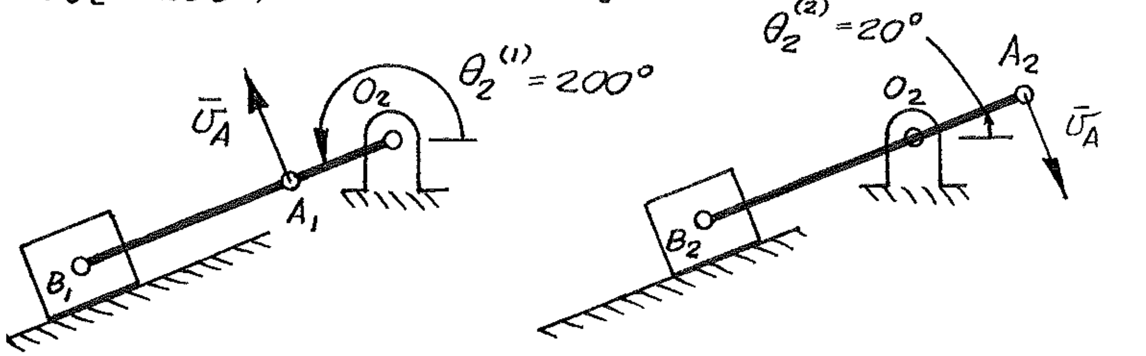


P2.3



P2.4

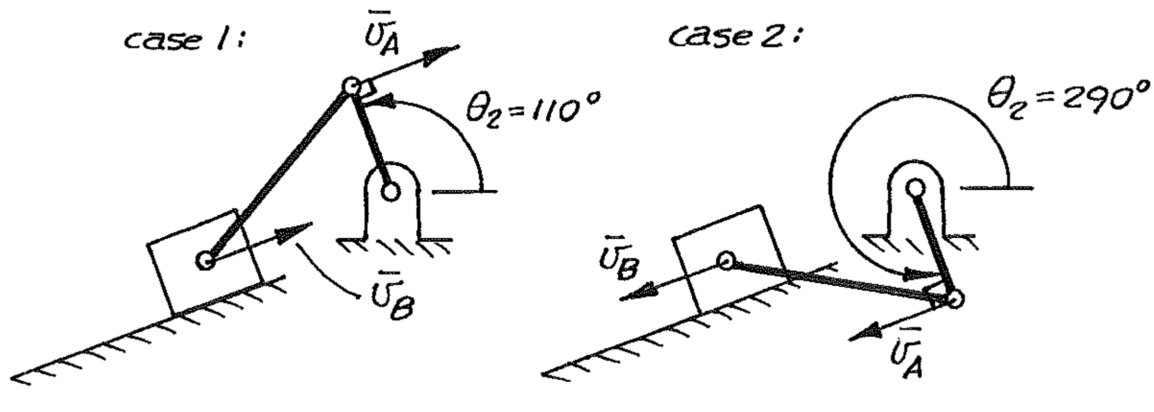
(a) limit position 1: ($\theta_2^{(1)} = 200°$) limit position 2: ($\theta_2^{(2)} = 20°$)



for both limit geometries

$$|\dot{\theta}_3| = \frac{v_A}{r_{AB}} = \frac{r_{AO_2}\,|\dot{\theta}_2|}{r_{AB}} = \frac{3.0 \times 100}{6.0} = 50\,rpm$$

(b) rotational speed of link 3 is zero when $\bar{v}_A$ and $\bar{v}_B$ have the same direction

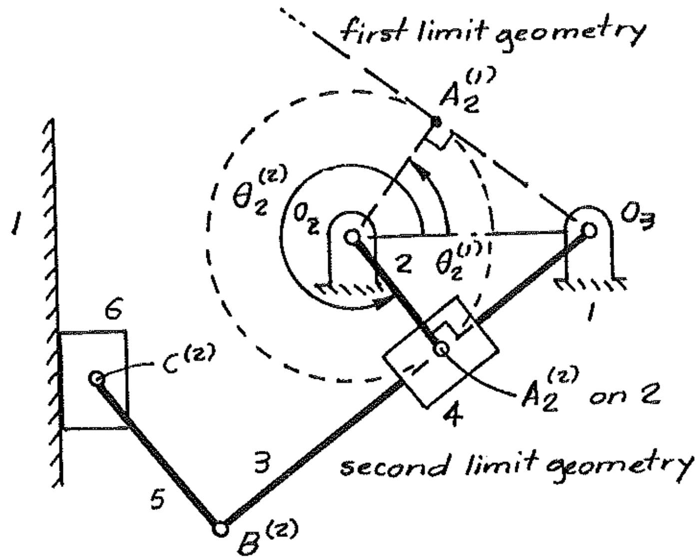


for both cases $\underline{v_B} = r_{AO_2}|\dot{\theta}_2| = 3.0 \times 100 \times \frac{2\pi}{60}$

$= \underline{31.4 \frac{cm}{sec}}$

P2.5

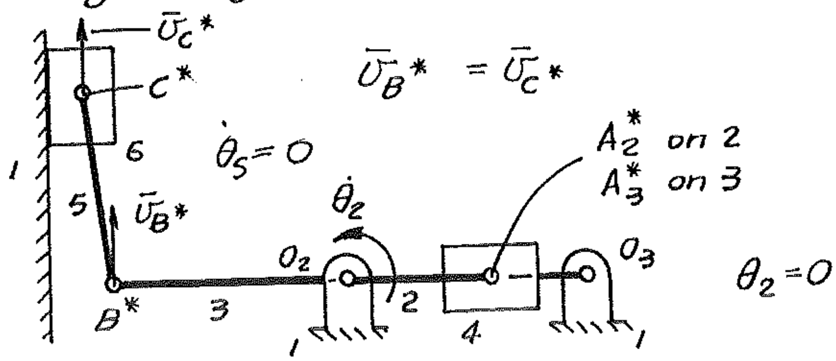
(a) geometries for which velocity of link 6 is zero (i.e., limit positions)



$$\theta_2^{(1)} = \cos^{-1}\left(r_{A_2O_2} / r_{O_2O_3} \right) = \cos^{-1}(3.0/5.0) = 53.1^\circ$$

$$\theta_2^{(2)} = 360^\circ - \theta_2^{(1)} = 306.9^\circ$$

(b) $\bar{v}_B$ and $\bar{v}_C$ have the same direction



$$v_{A_2^*} = r_{A_2O_2}\,|\dot\theta_2| = r_{A_3^*O_3}\,|\dot\theta_3|, \quad |\dot\theta_3| = \frac{r_{A_2O_2}}{r_{A_3^*O_3}}\,|\dot\theta_2|$$

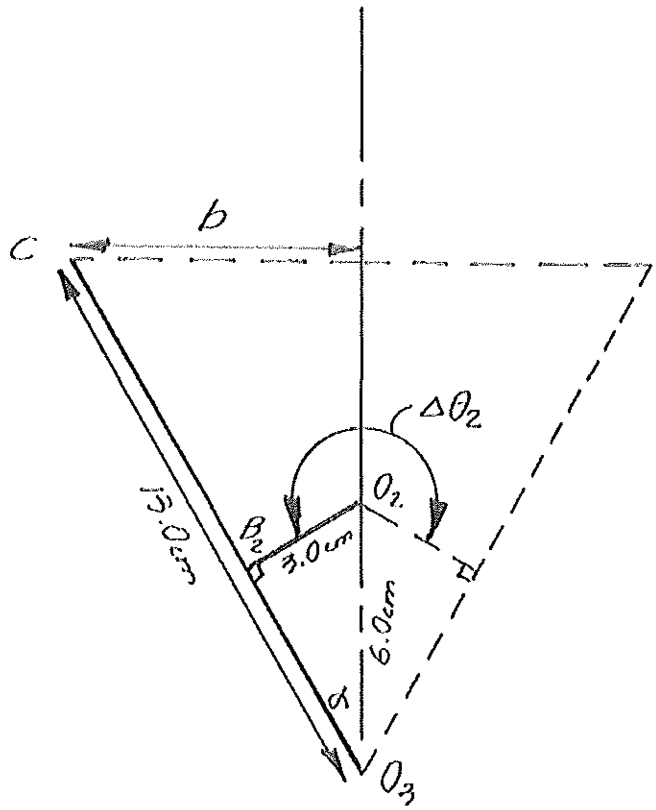
$$|\bar{v}_{C^*}| = |\bar{v}_{B^*}| = r_{B^*O_3}\,|\dot\theta_3| = r_{B^*O_3}\,\frac{r_{A_2O_2}}{r_{A_3^*O_3}}\,|\dot\theta_2|$$

$$= 10.0 \times \frac{3.0}{5.0 - 3.0} \times 10.0 = 150\ \frac{\text{cm}}{\text{sec}}$$

similarly at $\theta_2 = 180^\circ$, $|\bar{v}_C{}^*| = |\bar{v}_B{}^*| = 37.5\ \frac{\text{cm}}{\text{sec}}$

P2.6

limit geometries:



$$\alpha = \sin^{-1}\frac{3.0}{6.0} = 30°$$

$$b = 13.0 \times \sin 30° = 6.5\,cm$$

(a) stroke = $2b$ = 13.0 cm

(b) $\theta_2^{(1)} = 180° + \alpha = \underline{210°}$

$\theta_2^{(2)} = 360° - \alpha = \underline{330°}$

(c) $\theta_2 = \underline{90°}$ or $\underline{270°}$ when link 5 is in pure translation

also $\theta_2 = \underline{210°}$ or $\underline{330°}$ when $\dot{\theta}_3 = 0$ & $\dot{\theta}_5 = 0$

P2.6 CONTINUED

(d) motion to left requires 120° rotation of link 2

$$|\dot{\theta}_2| = 8.0\ \frac{\text{rad}}{\text{sec}} \times \frac{180°}{\pi\ \text{rad}} = 458°/\text{sec}$$

$$\therefore \text{ time taken: } \frac{120°}{458°/\text{sec}} = 0.262\ \text{sec}$$

$$\therefore \text{ average speed to left} = \frac{\text{stroke}}{\text{time taken}}$$

$$= \frac{13.0\ \text{cm}}{0.262\ \text{sec}} = \underline{49.7\ \frac{\text{cm}}{\text{sec}}}$$

(e) motion to right requires 240° rotation of link 2

$$\text{time taken } \frac{240°}{458°/\text{sec}} = 0.524\ \text{sec}$$

$$\therefore \text{ average speed to the right} = \frac{\text{stroke}}{\text{time taken}}$$

$$= \frac{13.0\ \text{cm}}{0.524\ \text{sec}} = \underline{24.8\ \frac{\text{cm}}{\text{sec}}}$$

P2.7

$r_1 = 8.0\,cm$; $r_2 = 2.0\,cm$; $r_3 = 4.0\,cm$; $r_4 = 7.0\,cm$

$\dot{\theta}_2 = 40.0 \frac{rad}{sec}$ CW

$$\alpha_1 = \cos^{-1}\left[\frac{r_1^2 + (r_2+r_3)^2 - r_4^2}{2(r_2+r_3)\,r_1}\right] = 57.9^\circ$$

$$\alpha_2 = \cos^{-1}\left[\frac{r_1^2 + (r_3-r_2)^2 - r_4^2}{2(r_3-r_2)\,r_1}\right] = 53.6^\circ$$

$$\beta_1 = \cos^{-1}\left[\frac{r_1^2 - (r_2+r_3)^2 + r_4^2}{2\,r_1\,r_4}\right] = 46.6^\circ$$

$$\beta_2 = \cos^{-1}\left[\frac{r_1^2 - (r_3-r_2)^2 + r_4^2}{2\,r_1\,r_4}\right] = 13.3^\circ$$

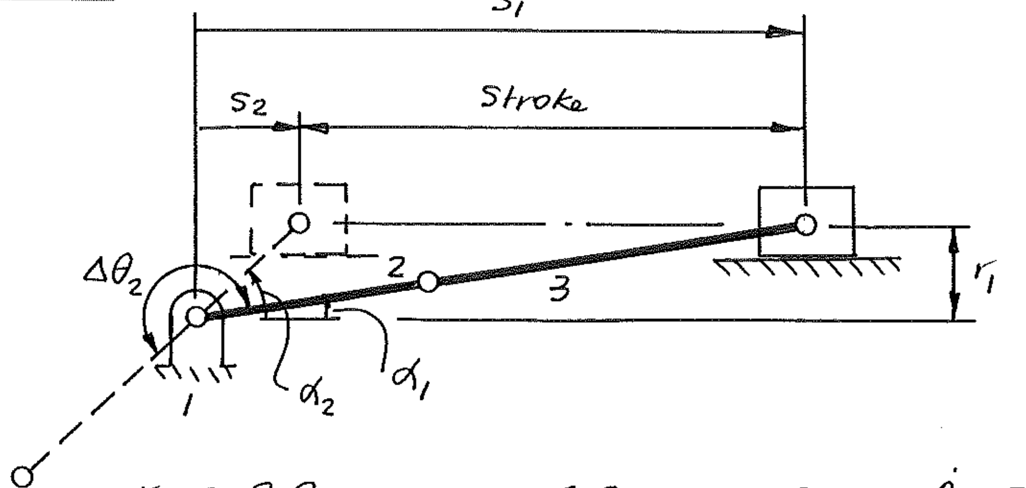
$$\Delta\theta_2 = 180^\circ + \alpha_2 - \alpha_1 = 175.7^\circ;\ \Delta\theta_4 = \beta_1 - \beta_2 = 33.3^\circ$$

(a) $$\underline{(\dot{\theta}_{4\,avg})_{CW}} = \frac{\Delta\theta_4}{(\Delta\theta_2 / \dot{\theta}_2)} = \underline{7.58 \frac{rad}{sec}}$$

(b) $$\underline{(\dot{\theta}_{4\,avg})_{CCW}} = \frac{\Delta\theta_4}{\left(\frac{2\pi - \Delta\theta_2}{\dot{\theta}_2}\right)} = \underline{7.22 \frac{rad}{sec}}$$

(c) $$\underline{T_R} = \frac{2\pi - \Delta\theta_2}{\Delta\theta_2} = \underline{1.05}$$

P2.8



$r_1 = 2.0\,cm;\ r_2 = 5.0\,cm,\ r_3 = 8.0\,cm,\ \dot{\theta}_2 = 30\,\frac{rad}{sec}$ CCW

$$\alpha_1 = \sin^{-1}\left(\frac{r_1}{r_2+r_3}\right) = \sin^{-1}\left(\frac{2.0}{5.0+8.0}\right) = 8.8^\circ$$

$$\alpha_2 = \sin^{-1}\left(\frac{r_1}{r_3-r_2}\right) = 41.8^\circ$$

$$\Delta\theta_2 = 180^\circ + \alpha_2 - \alpha_1 = 213^\circ = 3.72\ rad$$

$$s_1 = \left[(r_2+r_3)^2 - r_1^2\right]^{1/2} = \left[(5.0+8.0)^2 - 2.0^2\right]^{1/2} = 12.85\,cm$$

$$s_2 = \left[(r_3-r_2)^2 - r_1^2\right]^{1/2} = 2.24\,cm$$

$$\text{Stroke} = s_1 - s_2 = 12.85 - 2.24 = 10.61\,cm$$

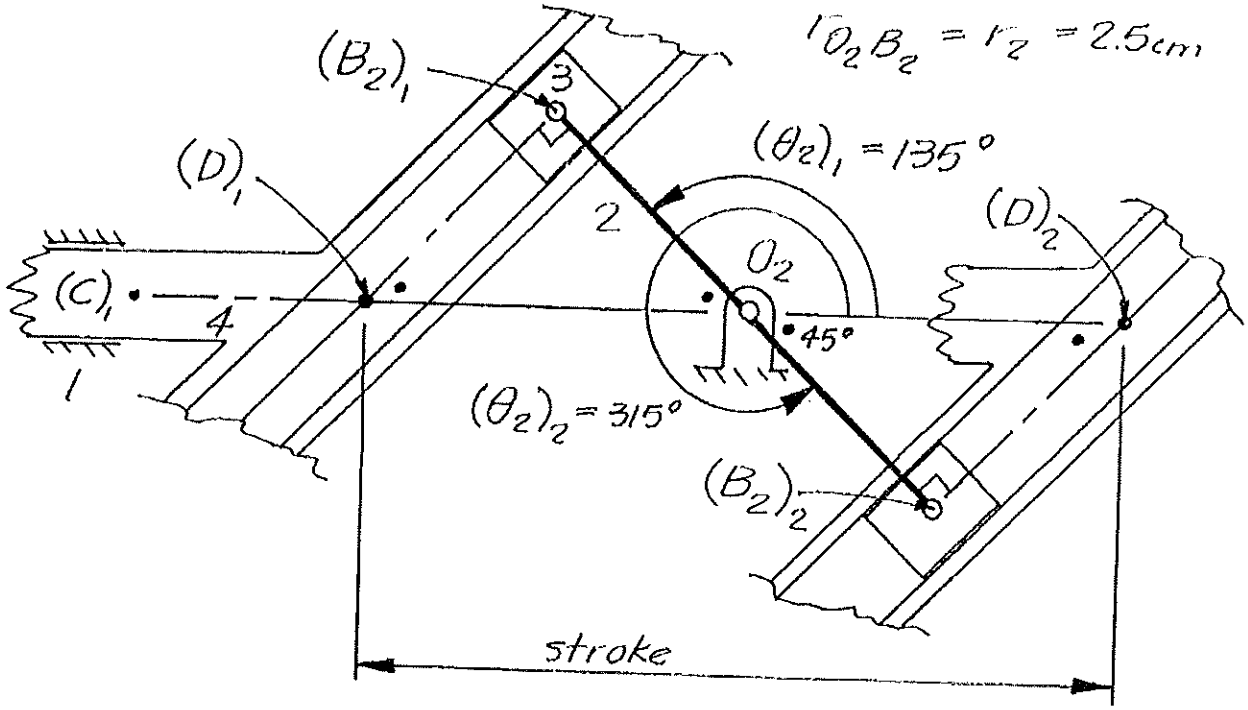
(a) $$\underline{(v_{4,avg})_{right}} = \frac{\text{Stroke}}{\left(\dfrac{2\pi - \Delta\theta_2}{\dot{\theta}_2}\right)} = \underline{124\ \frac{cm}{sec}}$$

(b) $$\underline{(v_{4,avg})_{left}} = \frac{\text{Stroke}}{\Delta\theta_2/\dot{\theta}_2} = \underline{85.6\ \frac{cm}{sec}}$$

(c) $$\underline{T_R} = \frac{\Delta\theta_2}{2\pi - \Delta\theta_2} = \underline{1.45}$$

P2.9

(a) limit geometries of mechanism (i.e., $v_C = 0$)



values of θ_2 when point C is stationary:

135° & 315°

(b) stroke of C = stroke of D

$$= 2\left\{2\left[r_2 \cos 45°\right]\right\} = 7.07cm$$

P2.9 CONTINUED

(c) time to move between limit positions :

$$\Delta t = \frac{\Delta \theta_2}{\dot{\theta}_2} = \frac{\pi \text{ rad}}{8 \frac{\text{rad}}{\text{sec}}} = 0.393 \text{ sec}$$

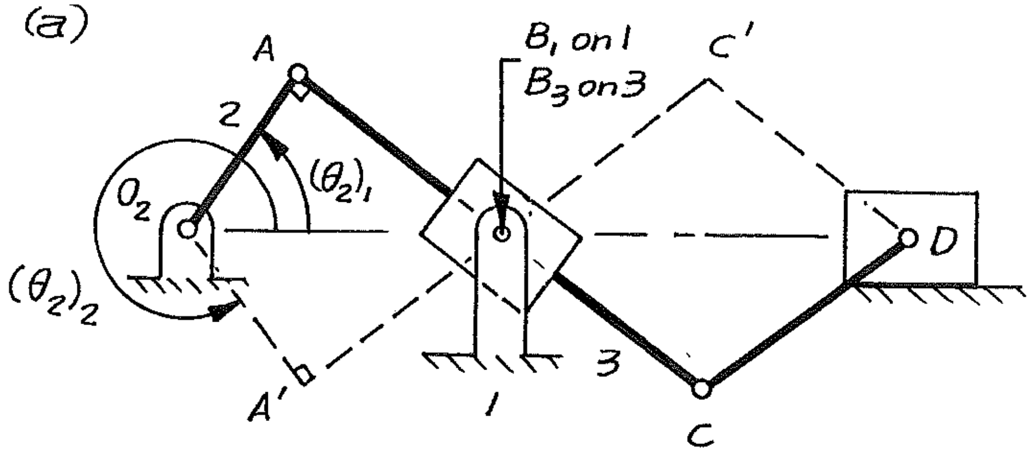
$\therefore$ average speed to the left $= \dfrac{\text{stroke}}{\Delta t}$

$$= \frac{7.07 \text{ cm}}{0.393 \text{sec}} = \underline{18.0 \ \frac{\text{cm}}{\text{sec}}}$$

P2.10

$r_{O_2A} = 3.0\,cm$; $r_{O_2B_1} = 5.0\,cm$; $r_{AC} = 8.0\,cm$; $r_{CD} = 4.0\,cm$

(a)



$\dot{\theta}_3 = 0$ when link 2 is perpendicular to link 3

$$(\theta_2)_1 = \tan^{-1}\left\{\frac{r_{AB_3}}{r_{O_2A}}\right\} = \tan^{-1}\left\{\frac{\left[(r_{O_2B_3})^2 - (r_{O_2A})^2\right]^{\frac{1}{2}}}{r_{O_2A}}\right\}$$

$$= \tan^{-1}\left\{\frac{[5.0^2 - 3.0^2]^{1/2}}{3.0}\right\} = 53.1^\circ$$

$$(\theta_2)_2 = 360^\circ - (\theta_2)_1 = 306.9^\circ$$

(b) $v_D = 0$ when mechanism is in limit positions (i.e., $\theta_2 = 0, 180^\circ$)

P2.10 CONTINUED

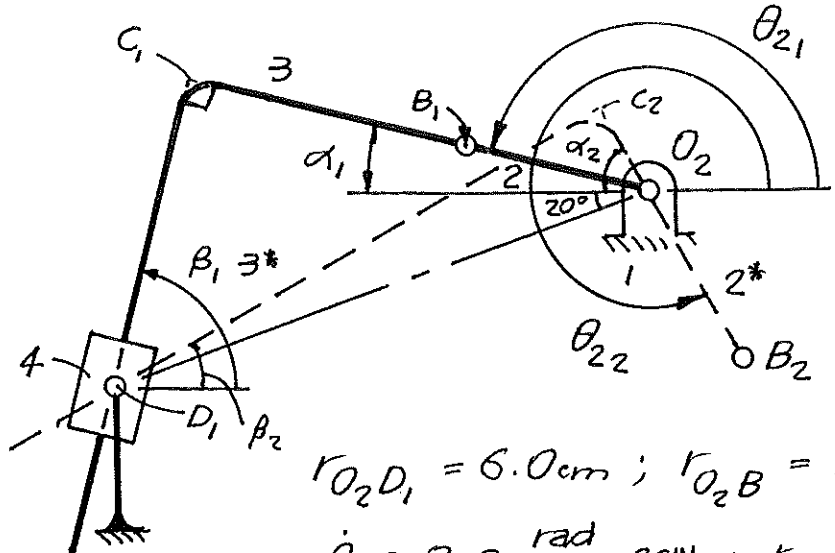
(c) $| a^{C}_{B_1 B_3} | = | 2 \dot{r}_{B_1 B_3} \dot{\theta}_3 |$

$a^{C}_{B_1 B_3} = 0$ when $\dot{r}_{B_1 B_3} = 0$ OR
$\dot{\theta}_3 = 0.$

$\therefore$ the corresponding values of θ_2 are
$0,\ 53.1°,\ 180°,\ 306.9°$

P2.11

(a)



$r_{O_2D_1} = 6.0cm$; $r_{O_2B} = 2.0cm$

$\dot{\theta}_2 = 8.0\ \frac{rad}{sec}$ CCW ; $r_{BC} = 3.0cm$

$$\alpha_1 + 20° = \cos^{-1}\left(\frac{5.0}{6.0}\right) = 33.6°,$$

$$\therefore\ \underline{\theta_{2_1}} = 180° - 33.6° + 20° = \underline{166.4°}$$

$$\alpha_2 + 20° = \cos^{-1}\left(\frac{1.0}{6.0}\right) = 80.4°$$

$$\therefore\ \underline{\theta_{2_2}} = 360° - 80.4° + 20° = \underline{299.6°}$$

(b) triangle $D_1C_1O_2$:

$$(\beta_1 - 20°) + \underbrace{\cancel{\alpha_1 + 20°}}_{33.6°} + 90° = 180° \ ; \ \beta_1 = 76.4°$$

triangle $D_1C_2O_2$:

$$(\beta_2 - 20°) + \underbrace{\cancel{\alpha_2 + 20°}}_{80.4°} + 90° = 180° \ ; \ \beta_2 = 29.6°$$