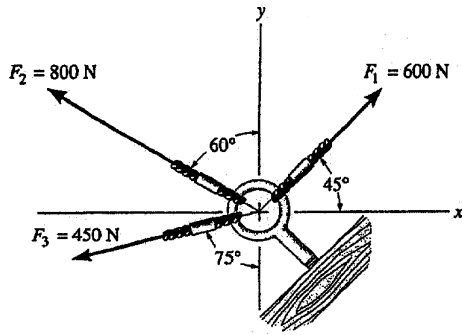


2-1. Determine the magnitude of the resultant force $F_R = F_1 + F_3$ and its direction, measured counterclockwise from the positive x axis.

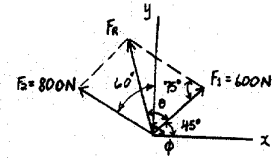


$$F_R = \sqrt{(600)^2 + (800)^2 - 2(600)(800)\cos 75^\circ} = 866.91 = 867 \text{ N} \quad \text{Ans.}$$

$$\frac{866.91}{\sin 75^\circ} = \frac{800}{\sin \theta}$$

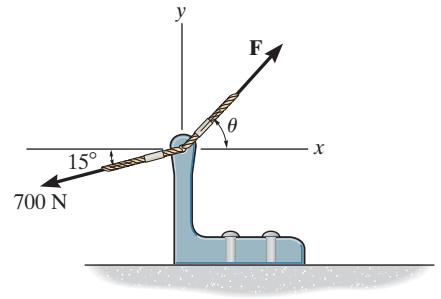
$$\theta = 63.05^\circ$$

$$\phi = 63.05^\circ + 45^\circ = 108^\circ \quad \text{Ans.}$$



2-2.

If $\theta = 60^\circ$ and $F = 450\text{ N}$, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450) \cos 45^\circ}$$
$$= 497.01\text{ N} = 497\text{ N}$$

Ans.

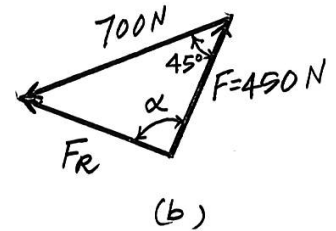
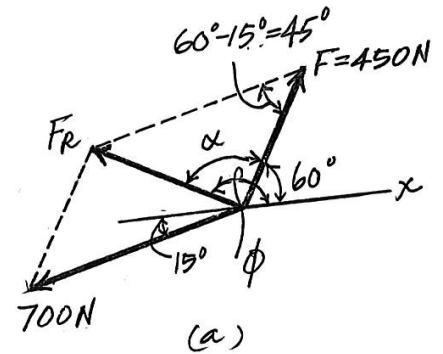
This yields

$$\frac{\sin \alpha}{700} = \frac{\sin 45^\circ}{497.01} \quad \alpha = 95.19^\circ$$

Thus, the direction of angle ϕ of \mathbf{F}_R measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60^\circ = 95.19^\circ + 60^\circ = 155^\circ$$

Ans.



2-3.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ .

SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

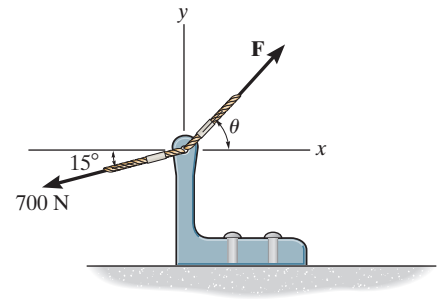
Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 700^2 - 2(500)(700) \cos 105^\circ}$$
$$= 959.78 \text{ N} = 960 \text{ N}$$

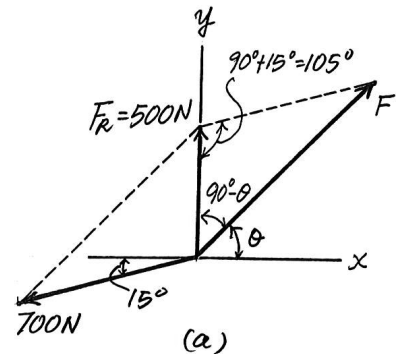
Applying the law of sines to Fig. *b*, and using this result, yields

$$\frac{\sin(90^\circ + \theta)}{700} = \frac{\sin 105^\circ}{959.78}$$

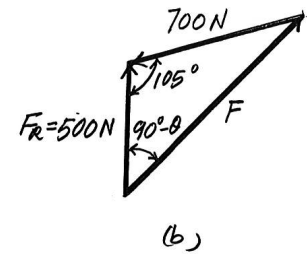
$$\theta = 45.2^\circ$$



Ans.

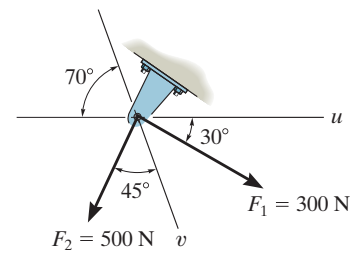


Ans.



2-4.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

$$F_R = \sqrt{(300)^2 + (500)^2 - 2(300)(500) \cos 95^\circ} = 605.1 = 605 \text{ N}$$

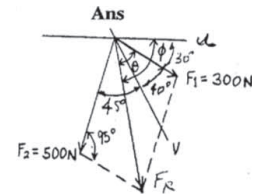
Ans.

$$\frac{605.1}{\sin 95^\circ} = \frac{500}{\sin \theta}$$

$$\theta = 55.40^\circ$$

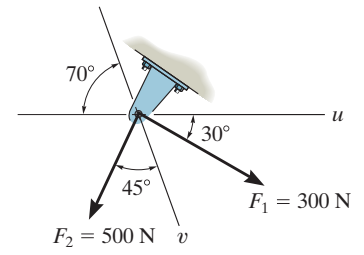
$$\phi = 55.40^\circ + 30^\circ = 85.4^\circ$$

Ans.



2-5.

Resolve the force \mathbf{F}_1 into components acting along the u and v axes and determine the magnitudes of the components.



SOLUTION

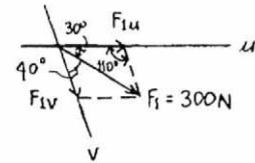
$$\frac{F_{1u}}{\sin 40^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1u} = 205\text{ N}$$

$$\frac{F_{1v}}{\sin 30^\circ} = \frac{300}{\sin 110^\circ}$$

$$F_{1v} = 160\text{ N}$$

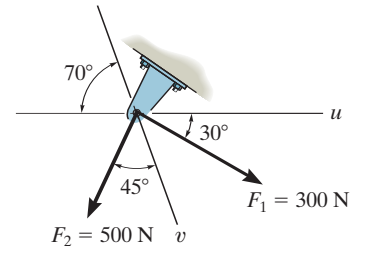
Ans.



Ans.

2-6.

Resolve the force \mathbf{F}_2 into components acting along the u and v axes and determine the magnitudes of the components.



SOLUTION

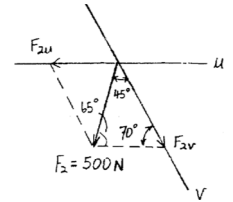
$$\frac{F_{2u}}{\sin 45^\circ} = \frac{500}{\sin 70^\circ}$$

$$F_{2u} = 376\text{ N}$$

$$\frac{F_{2v}}{\sin 65^\circ} = \frac{500}{\sin 70^\circ}$$

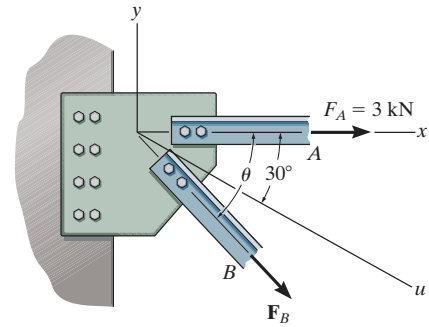
$$F_{2v} = 482\text{ N}$$

Ans.



Ans.

2-7. If $F_B = 2$ kN and the resultant force acts along the positive u axis, determine the magnitude of the resultant force and the angle θ .



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of sines to Fig. *b*, yields

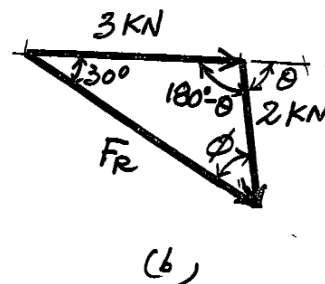
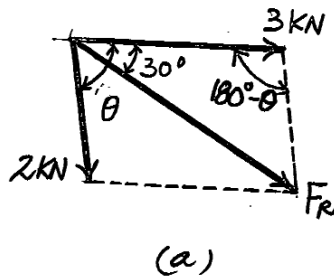
$$\frac{\sin \phi}{3} = \frac{\sin 30^\circ}{2} \quad \phi = 48.59^\circ$$

Thus,

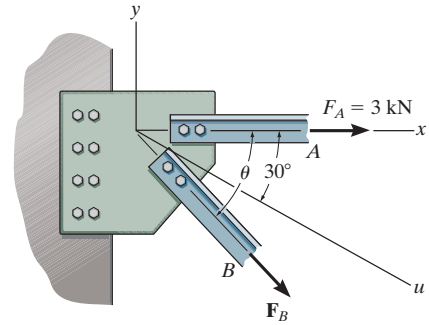
$$\theta = 30^\circ + \phi = 30^\circ + 48.59^\circ = 78.59^\circ = 78.6^\circ \quad \text{Ans.}$$

With the result $\theta = 78.59^\circ$, applying the law of sines to Fig. *b* again, yields

$$\frac{F_R}{\sin(180^\circ - 78.59^\circ)} = \frac{2}{\sin 30^\circ} \quad F_R = 3.92 \text{ kN} \quad \text{Ans.}$$



2-8. If the resultant force is required to act along the positive u axis and have a magnitude of 5 kN, determine the required magnitude of F_B and its direction θ .



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{3^2 + 5^2 - 2(3)(5)\cos 30^\circ} \\ = 2.832 \text{ kN} = 2.83 \text{ kN}$$

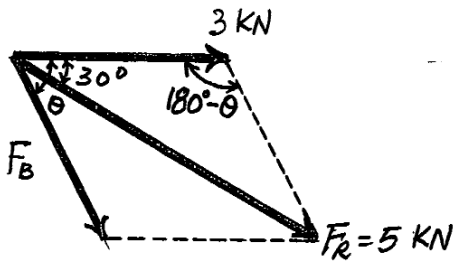
Ans.

Using this result and realizing that $\sin(180^\circ - \theta) = \sin\theta$, the application of the sine law to Fig. *b*, yields

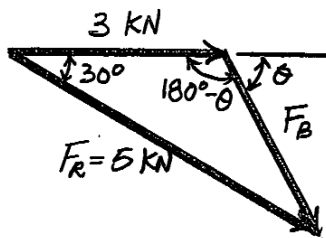
$$\frac{\sin\theta}{5} = \frac{\sin 30^\circ}{2.832}$$

$$\theta = 62.0^\circ$$

Ans.



(a)



(b)

2-9.

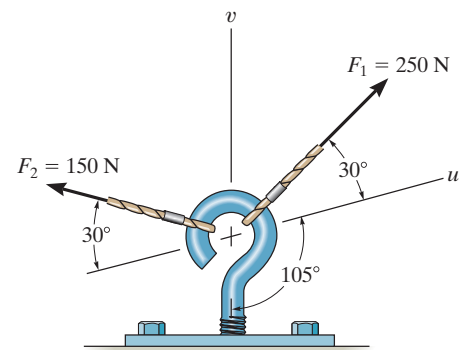
Resolve \mathbf{F}_1 into components along the u and v axes and determine the magnitudes of these components.

SOLUTION

Sine law:

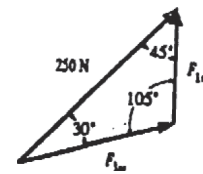
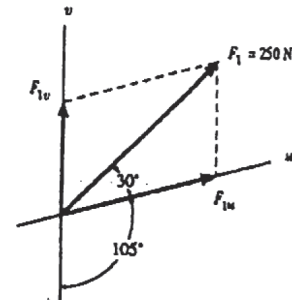
$$\frac{F_{1v}}{\sin 30^\circ} = \frac{250}{\sin 105^\circ} \quad F_{1v} = 129 \text{ N}$$

$$\frac{F_{1u}}{\sin 45^\circ} = \frac{250}{\sin 105^\circ} \quad F_{1u} = 183 \text{ N}$$



Ans.

Ans.



2-10.

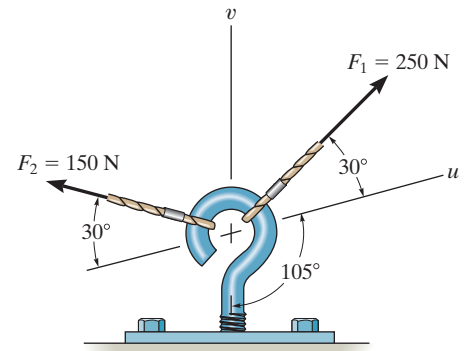
Resolve \mathbf{F}_2 into components along the u and v axes and determine the magnitudes of these components.

SOLUTION

Sine law:

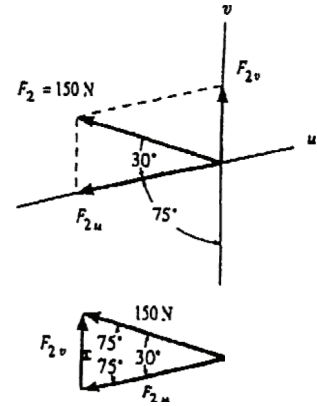
$$\frac{F_{2v}}{\sin 30^\circ} = \frac{150}{\sin 75^\circ} \quad F_{2v} = 77.6 \text{ N}$$

$$\frac{F_{2u}}{\sin 75^\circ} = \frac{150}{\sin 75^\circ} \quad F_{2u} = 150 \text{ N}$$

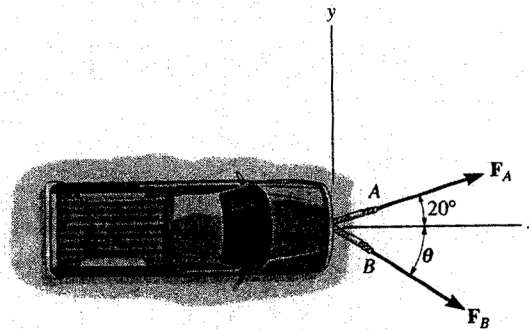


Ans.

Ans.



2-11. The truck is to be towed using two ropes. Determine the magnitude of forces F_A and F_B acting on each rope in order to develop a resultant force of 950 N directed along the positive x axis. Set $\theta = 50^\circ$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 50^\circ} = \frac{950}{\sin 110^\circ}$$

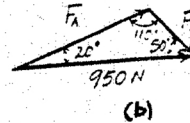
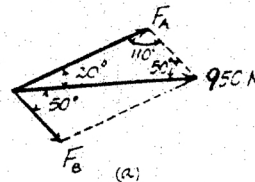
$$F_A = 774 \text{ N}$$

Ans.

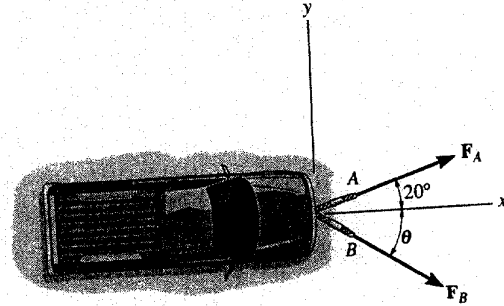
$$\frac{F_B}{\sin 20^\circ} = \frac{950}{\sin 110^\circ}$$

$$F_B = 346 \text{ N}$$

Ans.



2-12. The truck is to be towed using two ropes. If the resultant force is to be 950 N, directed along the positive x axis, determine the magnitudes of forces F_A and F_B acting on each rope and the angle of θ of F_B so that the magnitude of F_B is a *minimum*. F_A acts at 20° from the x axis as shown.



Parallelogram Law : In order to produce a *minimum* force F_B , F_B has to act perpendicular to F_A . The parallelogram law of addition is shown in Fig. (a).

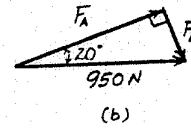
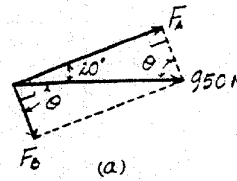
Trigonometry : Fig. (b).

$$F_B = 950 \sin 20^\circ = 325 \text{ N} \quad \text{Ans.}$$

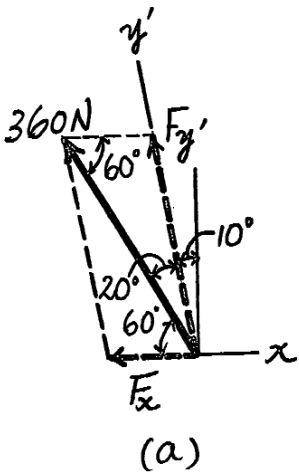
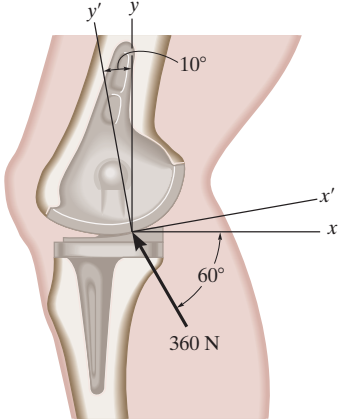
$$F_A = 950 \cos 20^\circ = 893 \text{ N} \quad \text{Ans.}$$

The angle θ is

$$\theta = 90^\circ - 20^\circ = 70.0^\circ \quad \text{Ans.}$$

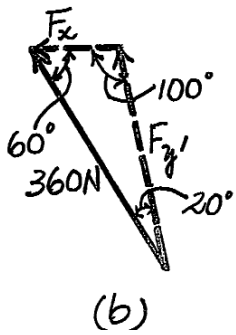


2-13. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x and y' axes.

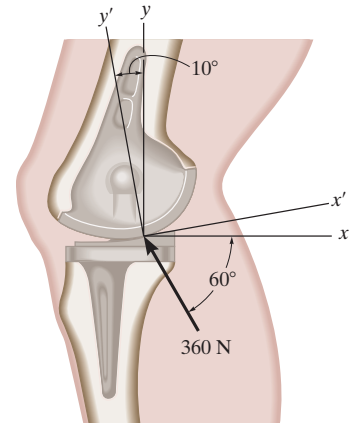


$$\frac{-F_x}{\sin 20^\circ} = \frac{360}{\sin 100^\circ} ; F_x = -125 \text{ N} \quad \text{Ans.}$$

$$\frac{F_{y'}}{\sin 60^\circ} = \frac{360}{\sin 100^\circ} ; F_{y'} = 317 \text{ N} \quad \text{Ans.}$$

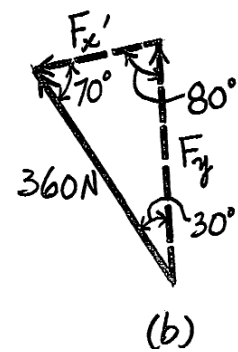
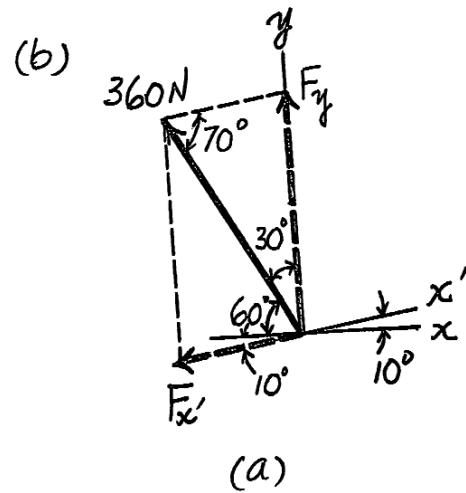


2-14. The device is used for surgical replacement of the knee joint. If the force acting along the leg is 360 N, determine its components along the x' and y axes.



$$\frac{-F_{x'}}{\sin 30^\circ} = \frac{360}{\sin 80^\circ}; \quad F_{x'} = -183 \text{ N} \quad \text{Ans.}$$

$$\frac{F_y}{\sin 70^\circ} = \frac{360}{\sin 80^\circ}; \quad F_y = 344 \text{ N} \quad \text{Ans.}$$



2-15.

The plate is subjected to the two forces at A and B as shown. If $\theta = 60^\circ$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.

SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. a .

Trigonometry: Using law of cosines (Fig. b), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6) \cos 100^\circ}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

The angle θ can be determined using law of sines (Fig. b).

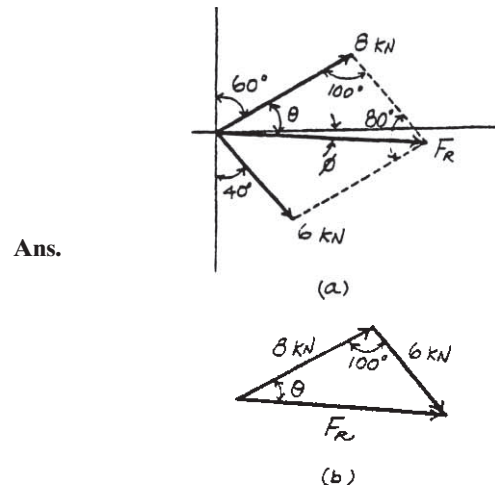
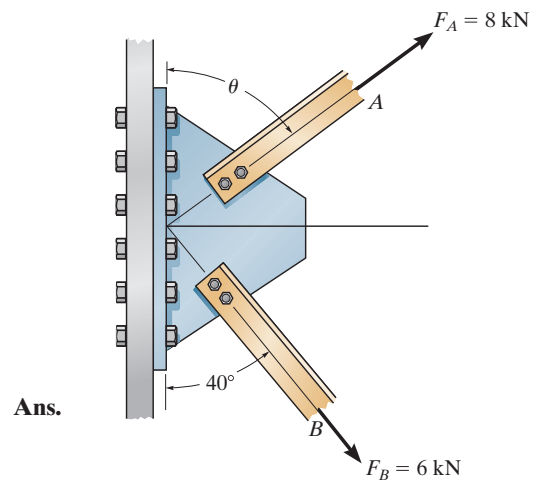
$$\frac{\sin \theta}{6} = \frac{\sin 100^\circ}{10.80}$$

$$\sin \theta = 0.5470$$

$$\theta = 33.16^\circ$$

Thus, the direction ϕ of F_R measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ$$



2-16.

Determine the angle of θ for connecting member A to the plate so that the resultant force of \mathbf{F}_A and \mathbf{F}_B is directed horizontally to the right. Also, what is the magnitude of the resultant force?

SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

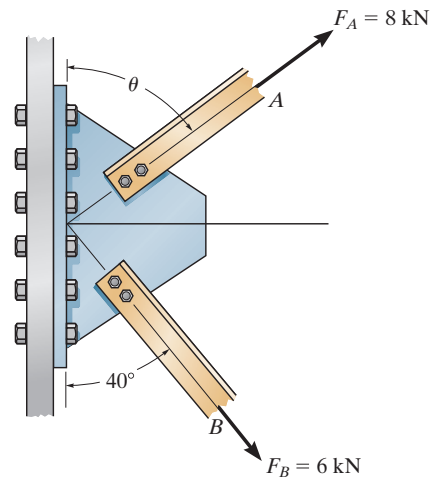
$$\sin(90^\circ - \theta) = 0.5745$$

$$\theta = 54.93^\circ = 54.9^\circ$$

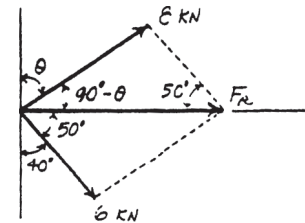
From the triangle, $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$. Thus, using law of cosines, the magnitude of \mathbf{F}_R is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6) \cos 94.93^\circ}$$

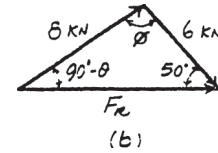
$$= 10.4 \text{ kN}$$



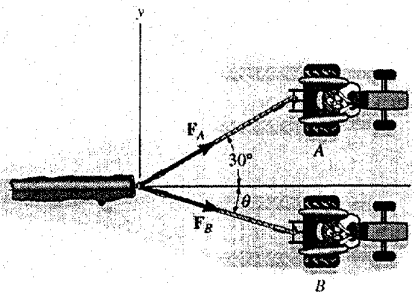
Ans.



Ans.



2-17. The log is being towed by two tractors A and B . Determine the magnitude of the two towing forces F_A and F_B if it is required that the resultant force have a magnitude $F_R = 10$ kN and be directed along the x axis. Set $\theta = 15^\circ$.



Parallelogram Law: The parallelogram law of addition is shown in Fig. (a).

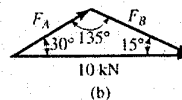
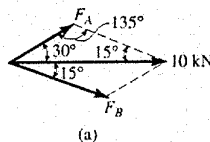
Trigonometry: Using law of sines [Fig. (b)], we have

$$\frac{F_A}{\sin 15^\circ} = \frac{10}{\sin 135^\circ}$$

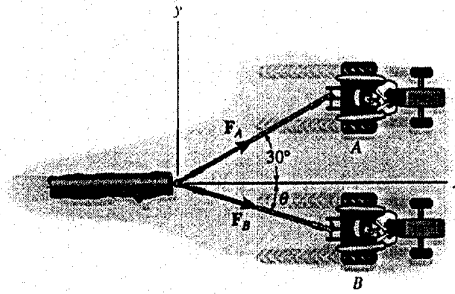
$$F_A = 3.66 \text{ kN} \quad \text{Ans.}$$

$$\frac{F_B}{\sin 30^\circ} = \frac{10}{\sin 135^\circ}$$

$$F_B = 7.07 \text{ kN} \quad \text{Ans.}$$



2-18. If the resultant F_R of the two forces acting on the log is to be directed along the positive x axis and have a magnitude of 10 kN, determine the angle θ of the cable, attached to B such that the force F_B in this cable is minimum. What is the magnitude of the force in each cable for this situation?



Parallelogram Law: In order to produce a *minimum* force F_B , F_B has to act perpendicular to F_A . The parallelogram law of addition is shown in Fig. (a).

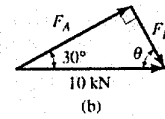
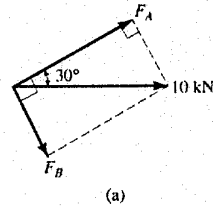
Trigonometry: Fig. (b).

$F_B = 10 \sin 30^\circ = 5.00 \text{ kN}$ Ans.

$F_A = 10 \cos 30^\circ = 8.66 \text{ kN}$ Ans.

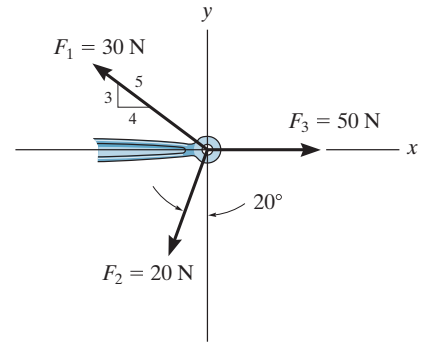
The angle θ is

$\theta = 90^\circ - 30^\circ = 60.0^\circ$ Ans.



2-19.

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



SOLUTION

$$F' = \sqrt{(20)^2 + (30)^2 - 2(20)(30) \cos 73.13^\circ} = 30.85 \text{ N}$$

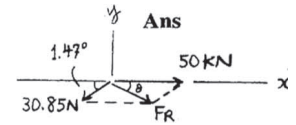
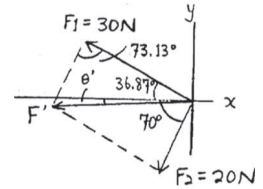
$$\frac{30.85}{\sin 73.13^\circ} = \frac{30}{\sin (70^\circ - \theta')}; \quad \theta' = 1.47^\circ$$

$$F_R = \sqrt{(30.85)^2 + (50)^2 - 2(30.85)(50) \cos 1.47^\circ} = 19.18 = 19.2 \text{ N}$$

$$\frac{19.18}{\sin 1.47^\circ} = \frac{30.85}{\sin \theta}; \quad \theta = 2.37^\circ \swarrow$$

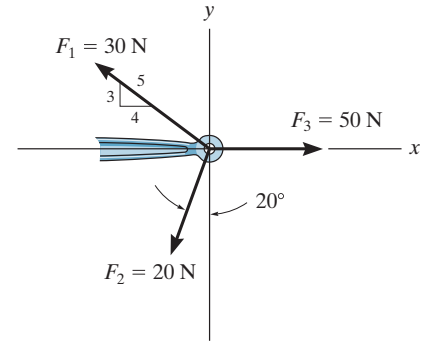
Ans.

Ans.



2-20.

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



SOLUTION

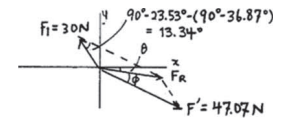
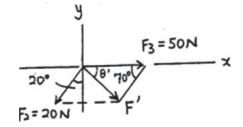
$$F' = \sqrt{(20)^2 + (50)^2 - 2(20)(50) \cos 70^\circ} = 47.07 \text{ N}$$

$$\frac{20}{\sin \theta'} = \frac{47.07}{\sin 70^\circ}; \quad \theta' = 23.53^\circ$$

$$F_R = \sqrt{(47.07)^2 + (30)^2 - 2(47.07)(30) \cos 13.34^\circ} = 19.18 = \mathbf{19.2 \text{ N}} \quad \text{Ans.}$$

$$\frac{19.18}{\sin 13.34^\circ} = \frac{30}{\sin \phi}; \quad \phi = 21.15^\circ$$

$$\theta = 23.53^\circ - 21.15^\circ = \mathbf{2.37^\circ \searrow} \quad \text{Ans.}$$



2-21.

Two forces act on the screw eye. If $F_1 = 400\text{ N}$ and $F_2 = 600\text{ N}$, determine the angle θ ($0^\circ \leq \theta \leq 180^\circ$) between them, so that the resultant force has a magnitude of $F_R = 800\text{ N}$.

SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

$$800 = \sqrt{400^2 + 600^2 - 2(400)(600) \cos(180^\circ - \theta)}$$

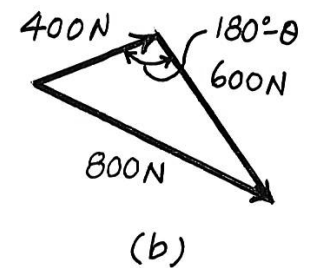
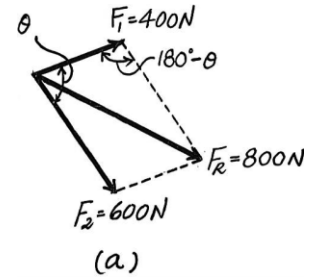
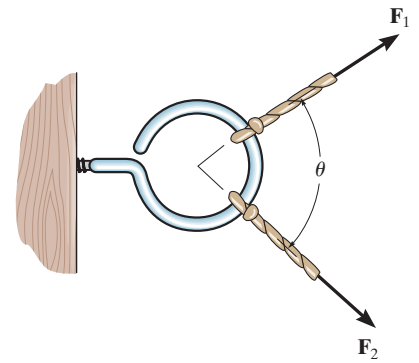
$$800^2 = 400^2 + 600^2 - 480000 \cos(180^\circ - \theta)$$

$$\cos(180^\circ - \theta) = -0.25$$

$$180^\circ - \theta = 104.48$$

$$\theta = 75.52^\circ = 75.5^\circ$$

Ans.



2-22.

Two forces \mathbf{F}_1 and \mathbf{F}_2 act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force \mathbf{F}_R and the angle between \mathbf{F}_R and \mathbf{F}_1 .

SOLUTION

$$\frac{F}{\sin \phi} = \frac{F}{\sin (\theta - \phi)}$$

$$\sin (\theta - \phi) = \sin \phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2}$$

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F) \cos (180^\circ - \theta)}$$

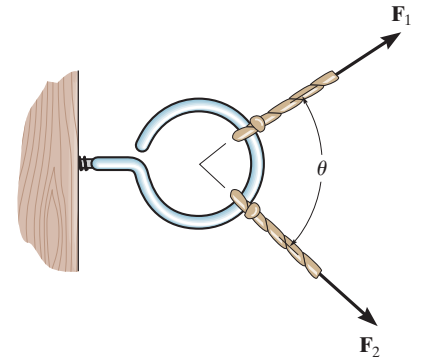
Since $\cos (180^\circ - \theta) = -\cos \theta$

$$F_R = F(\sqrt{2})\sqrt{1 + \cos \theta}$$

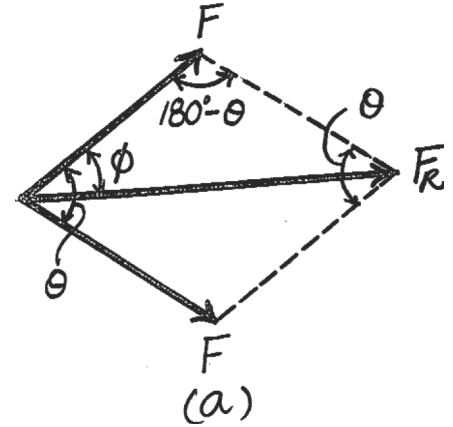
Since $\cos \left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos \theta}{2}}$

Then

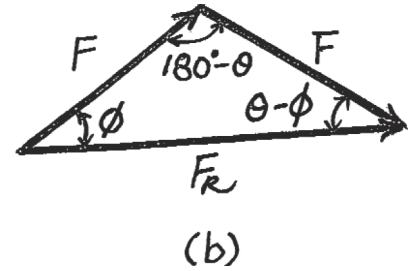
$$F_R = 2F \cos \left(\frac{\theta}{2}\right)$$



Ans.



Ans.



2-23.

Two forces act on the screw eye. If $F = 600\text{ N}$, determine the magnitude of the resultant force and the angle θ if the resultant force is directed vertically upward.

SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b* respectively. Applying law of sines to Fig. *b*,

$$\frac{\sin \theta}{600} = \frac{\sin 30^\circ}{500}; \quad \sin \theta = 0.6 \quad \theta = 36.87^\circ = 36.9^\circ$$

Ans.

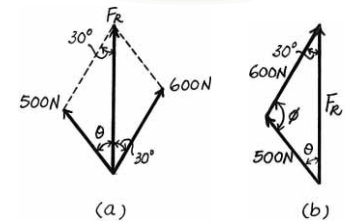
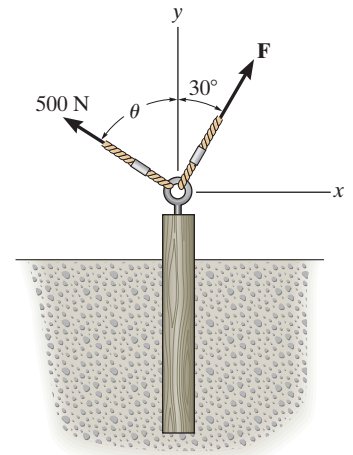
Using the result of θ ,

$$\phi = 180^\circ - 30^\circ - 36.87^\circ = 113.13^\circ$$

Again, applying law of sines using the result of ϕ ,

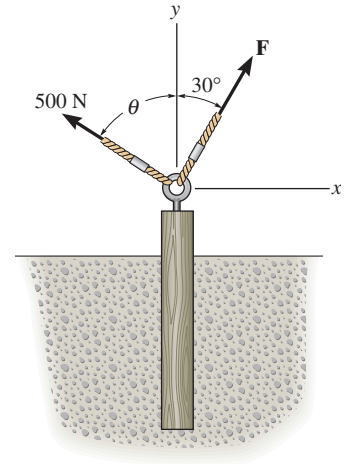
$$\frac{F_R}{\sin 113.13^\circ} = \frac{500}{\sin 30^\circ}; \quad F_R = 919.61\text{ N} = 920\text{ N}$$

Ans.



2-24.

Two forces are applied at the end of a screw eye in order to remove the post. Determine the angle θ ($0^\circ \leq \theta \leq 90^\circ$) and the magnitude of force \mathbf{F} so that the resultant force acting on the post is directed vertically upward and has a magnitude of 750 N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin \phi}{750} = \frac{\sin 30^\circ}{500}$$

$$\sin \phi = 0.750$$

$$\phi = 131.41^\circ \text{ (By observation, } \phi > 90^\circ \text{)}$$

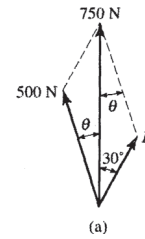
Thus,

$$\theta = 180^\circ - 30^\circ - 131.41^\circ = 18.59^\circ = 18.6^\circ$$

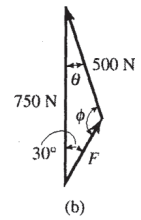
$$\frac{F}{\sin 18.59^\circ} = \frac{500}{\sin 30^\circ}$$

$$F = 319 \text{ N}$$

Ans.

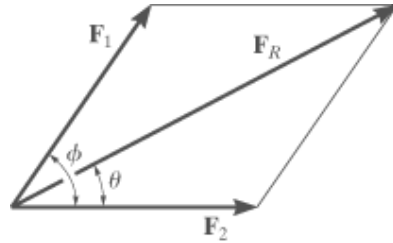


Ans.



2-25.

Determine the magnitude and direction of the resultant force \mathbf{F}_R . Express the result in terms of the magnitudes of the component \mathbf{F}_2 and resultant \mathbf{F}_R and the angle θ .



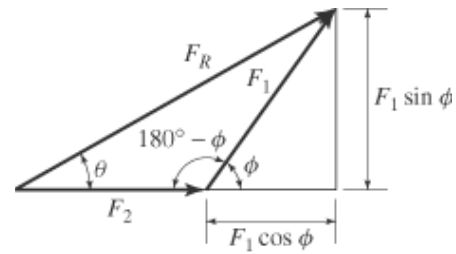
Solution:

$$F_1^2 = F_R^2 + F_2^2 - 2F_R F_2 \cos(\theta)$$

Since $\cos(180^\circ - \phi) = -\cos(\phi)$,

$$F_1 = \sqrt{F_R^2 + F_2^2 - 2 F_R F_2 \cos(\theta)}$$

Ans.



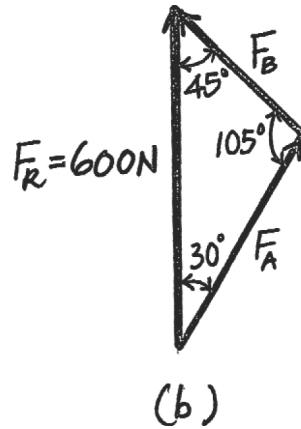
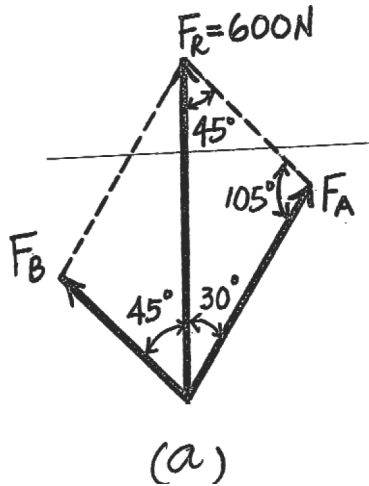
2-26.

The beam is to be hoisted using two chains. Determine the magnitudes of forces F_A and F_B acting on each chain in order to develop a resultant force of 600 N directed along the positive y axis. Set $\theta = 45^\circ$.

SOLUTION

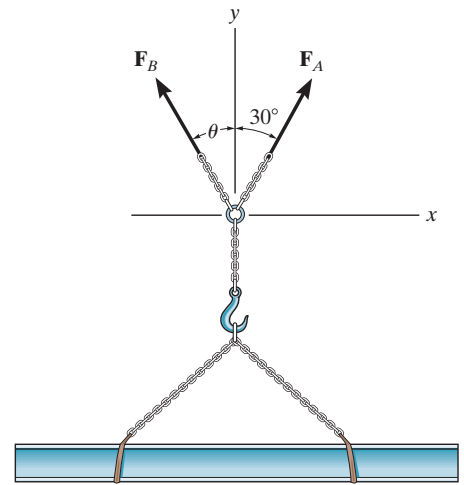
$$\frac{F_A}{\sin 45^\circ} = \frac{600}{\sin 105^\circ}; \quad F_A = 439 \text{ N}$$

$$\frac{F_B}{\sin 30^\circ} = \frac{600}{\sin 105^\circ}; \quad F_B = 311 \text{ N}$$



Ans.

Ans.



2-27.

The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive y axis, determine the magnitudes of forces F_A and F_B acting on each chain and the angle θ of F_B so that the magnitude of F_B is a *minimum*. F_A acts at 30° from the y axis, as shown.

SOLUTION

For minimum F_B , require

$\theta = 60^\circ$

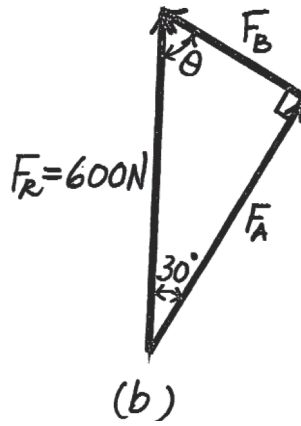
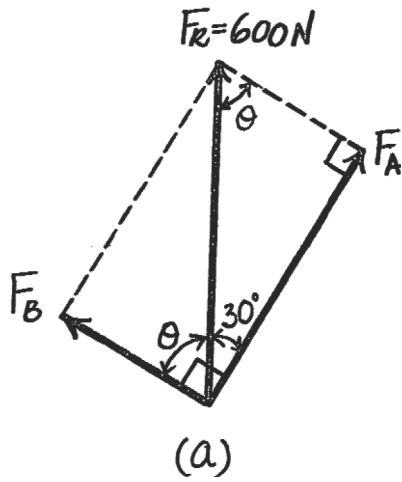
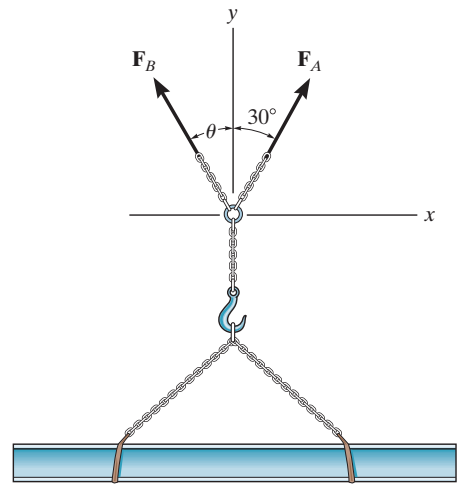
$F_A = 600 \cos 30^\circ = 520 \text{ N}$

$F_B = 600 \sin 30^\circ = 300 \text{ N}$

Ans.

Ans.

Ans.



2-28.

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force F_B and its direction θ .

SOLUTION

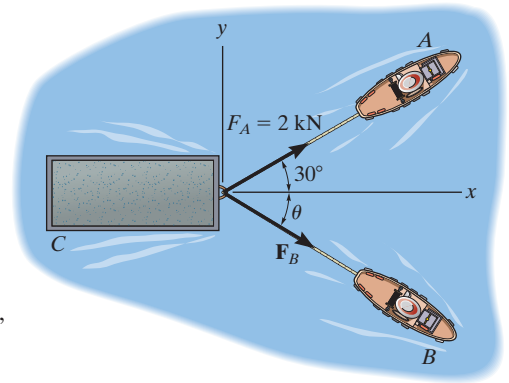
The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

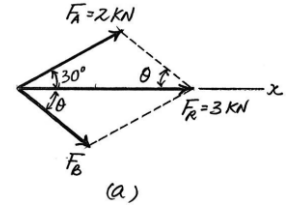
$$\begin{aligned} F_B &= \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ} \\ &= 1.615\text{kN} = 1.61\text{ kN} \end{aligned}$$

Using this result and applying the law of sines to Fig. *b*, yields

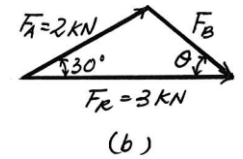
$$\frac{\sin \theta}{2} = \frac{\sin 30^\circ}{1.615} \quad \theta = 38.3^\circ$$



Ans.



Ans.



2-29.

If $F_B = 3 \text{ kN}$ and $\theta = 45^\circ$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.

SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

$$\begin{aligned} F_R &= \sqrt{2^2 + 3^2 - 2(2)(3) \cos 105^\circ} \\ &= 4.013 \text{ kN} = 4.01 \text{ kN} \end{aligned}$$

Ans.

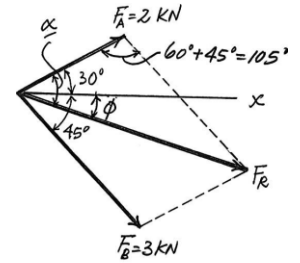
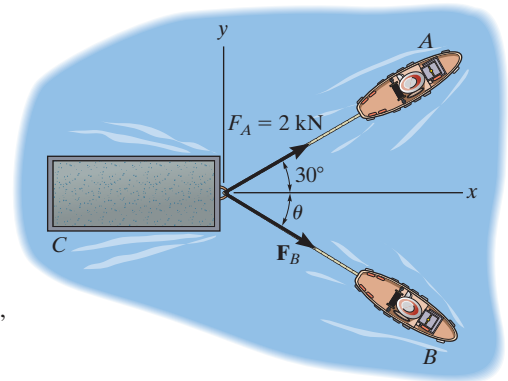
Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin \alpha}{3} = \frac{\sin 105^\circ}{4.013} \quad \alpha = 46.22^\circ$$

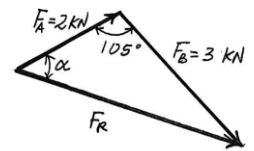
Thus, the direction angle ϕ of \mathbf{F}_R , measured clockwise from the positive x axis, is

$$\phi = \alpha - 30^\circ = 46.22^\circ - 30^\circ = 16.2^\circ$$

Ans.



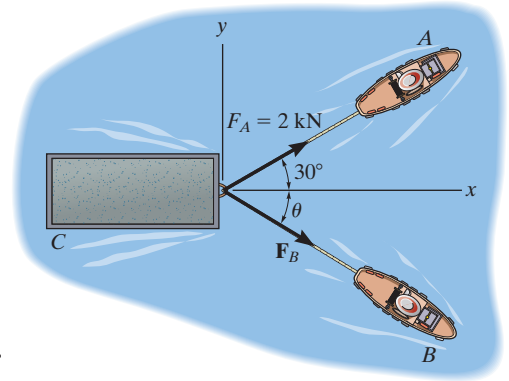
(a)



(b)

2-30.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and F_B is to be a minimum, determine the magnitude of F_R and F_B and the angle θ .



SOLUTION

For F_B to be minimum, it has to be directed perpendicular to F_R . Thus,

$$\theta = 90^\circ$$

Ans.

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

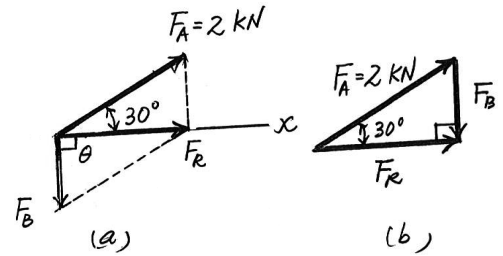
By applying simple trigonometry to Fig. *b*,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN}$$

Ans.

$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN}$$

Ans.



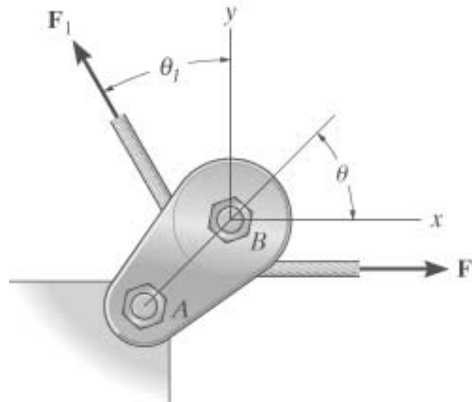
2-31.

If the tension in the cable is F_1 , determine the magnitude and direction of the resultant force acting on the pulley. This angle defines the same angle θ of line AB on the tailboard block.

Given:

$$F_1 = 400 \text{ N}$$

$$\theta_1 = 30 \text{ deg}$$



Solution:

$$F_R = \sqrt{F_1^2 + F_1^2 - 2F_1F_1 \cos(90 \text{ deg} - \theta_1)}$$

$$F_R = 400 \text{ N}$$

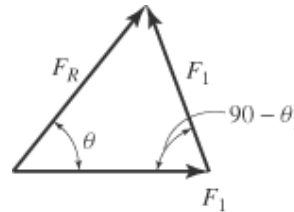
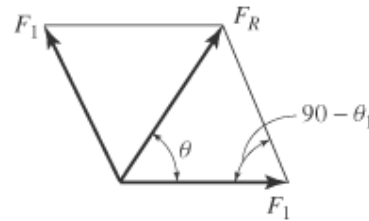
Ans.

$$\frac{\sin(90 \text{ deg} - \theta)}{F_R} = \frac{\sin(\theta_1)}{F_1}$$

$$\theta = 90 \text{ deg} - \text{asin}\left(\frac{F_R}{F_1} \sin(\theta_1)\right)$$

$$\theta = 60 \text{ deg}$$

Ans.



2-32.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.

Given:

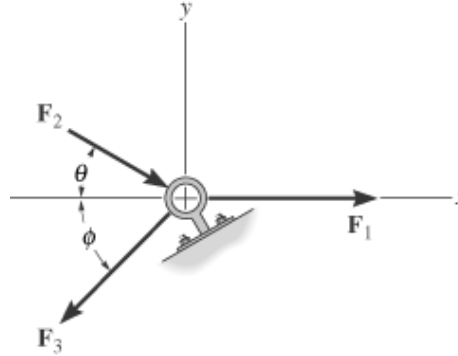
$$F_1 = 70 \text{ N}$$

$$F_2 = 50 \text{ N}$$

$$F_3 = 65 \text{ N}$$

$$\theta = 30 \text{ deg}$$

$$\phi = 45 \text{ deg}$$



Solution:

$$\begin{aligned} \rightarrow \quad F_{Rx} = \Sigma F_x; \quad F_{RX} &= F_1 + F_2 \cos(\theta) - F_3 \cos(\phi) \end{aligned}$$

$$\begin{aligned} \uparrow \quad F_{Ry} = \Sigma F_y; \quad F_{RY} &= -F_2 \sin(\theta) - F_3 \sin(\phi) \end{aligned}$$

$$F_R = \sqrt{F_{RX}^2 + F_{RY}^2}$$

$$\theta = \text{atan}\left(\frac{|F_{RY}|}{|F_{RX}|}\right)$$

$$F_R = 97.8 \text{ N}$$

Ans.

$$\theta = 46.5 \text{ deg}$$

Ans.

2-33.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

SOLUTION

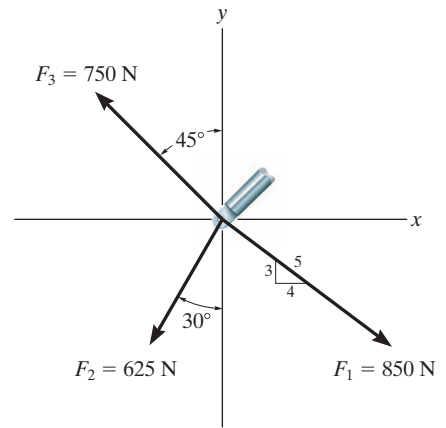
$$\pm \rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} = \frac{4}{5}(850) - 625 \sin 30^\circ - 750 \sin 45^\circ = -162.8 \text{ N}$$

$$+ \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -\frac{3}{5}(850) - 625 \cos 30^\circ + 750 \cos 45^\circ = -520.9 \text{ N}$$

$$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546 \text{ N} \quad \text{Ans.}$$

$$\phi = \tan^{-1} \left[\frac{-520.9}{-162.8} \right] = 72.64^\circ$$

$$\theta = 180^\circ + 72.64^\circ = 253^\circ \quad \text{Ans.}$$



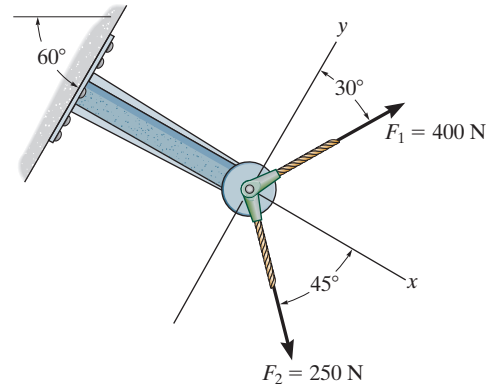
2-34.

Resolve \mathbf{F}_1 and \mathbf{F}_2 into their x and y components.

SOLUTION

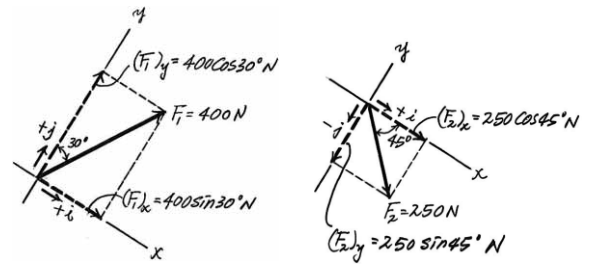
$$\begin{aligned}\mathbf{F}_1 &= \{400 \sin 30^\circ(+\mathbf{i})+400 \cos 30^\circ(+\mathbf{j})\} \text{ N} \\ &= \{200\mathbf{i}+346\mathbf{j}\} \text{ N}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_2 &= \{250 \cos 45^\circ(+\mathbf{i})+250 \sin 45^\circ(-\mathbf{j})\} \text{ N} \\ &= \{177\mathbf{i}-177\mathbf{j}\} \text{ N}\end{aligned}$$



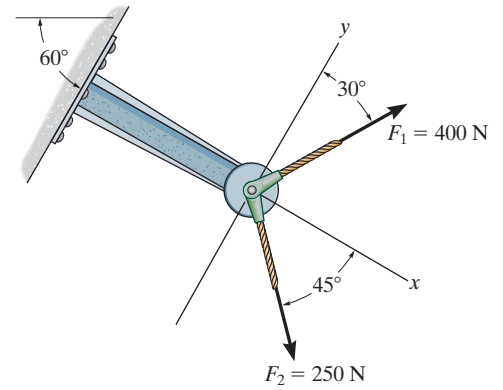
Ans.

Ans.



2-35.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of \mathbf{F}_1 and \mathbf{F}_2 can be written as

$$(F_1)_x = 400 \sin 30^\circ = 200 \text{ N} \quad (F_1)_y = 400 \cos 30^\circ = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45^\circ = 176.78 \text{ N} \quad (F_2)_y = 250 \sin 45^\circ = 176.78 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\rightarrow \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 200 + 176.78 = 376.78 \text{ N}$$

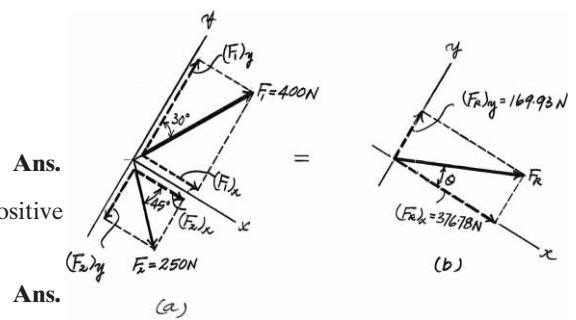
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N } \uparrow$$

The magnitude of the resultant force \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N}$$

The direction angle θ of \mathbf{F}_R , Fig. *b*, measured counterclockwise from the positive axis, is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{169.63}{376.78} \right) = 24.2^\circ$$



2-36.

Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\} \text{ N}$$

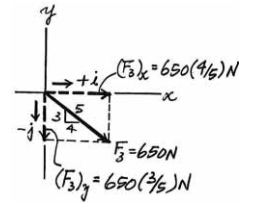
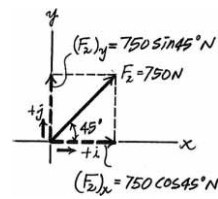
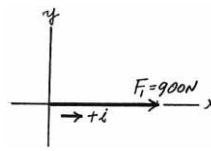
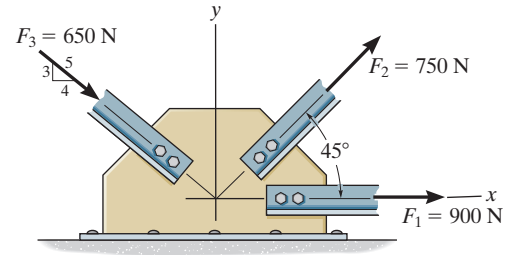
$$\begin{aligned} \mathbf{F}_2 &= \{750 \cos 45^\circ(+\mathbf{i}) + 750 \sin 45^\circ(+\mathbf{j})\} \text{ N} \\ &= \{530\mathbf{i} + 530\mathbf{j}\} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_3 &= \left\{ 650 \left(\frac{4}{5} \right) (+\mathbf{i}) + 650 \left(\frac{3}{5} \right) (-\mathbf{j}) \right\} \text{ N} \\ &= \{520\mathbf{i} - 390\mathbf{j}\} \text{ N} \end{aligned}$$

Ans.

Ans.

Ans.



2-37.

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.

SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 can be written as

$$\begin{aligned} (F_1)_x &= 900 \text{ N} & (F_1)_y &= 0 \\ (F_2)_x &= 750 \cos 45^\circ = 530.33 \text{ N} & (F_2)_y &= 750 \sin 45^\circ = 530.33 \text{ N} \\ (F_3)_x &= 650 \left(\frac{4}{5}\right) = 520 \text{ N} & (F_3)_y &= 650 \left(\frac{3}{5}\right) = 390 \text{ N} \end{aligned}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

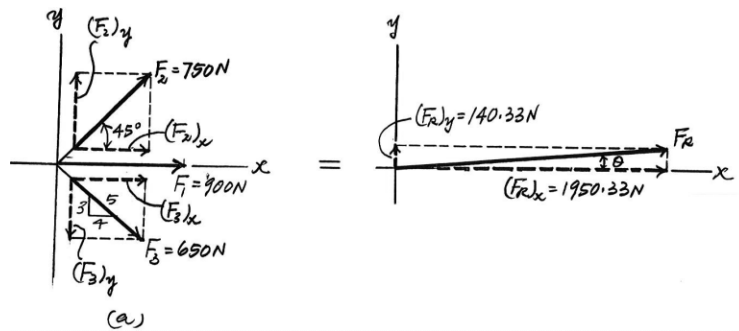
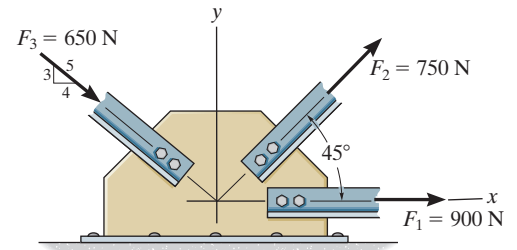
$$\begin{aligned} \rightarrow \Sigma(F_R)_x &= \Sigma F_x; & (F_R)_x &= 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow \\ + \uparrow \Sigma(F_R)_y &= \Sigma F_y; & (F_R)_y &= 530.33 - 390 = 140.33 \text{ N} \uparrow \end{aligned}$$

The magnitude of the resultant force \mathbf{F}_R is

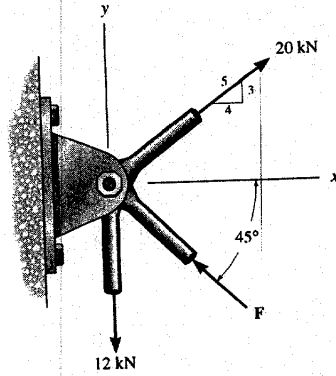
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \text{ Ans.}$$

The direction angle θ of \mathbf{F}_R , measured clockwise from the positive x axis, is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{140.33}{1950.33} \right) = 4.12^\circ \text{ Ans.}$$



2-38. Determine the magnitude of force F so that the resultant F_R of the three forces is as small as possible.



Scalar Notation : Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} &= 20\left(\frac{4}{5}\right) - F \cos 45^\circ \\ &= 16.0 - 0.7071F \rightarrow \end{aligned}$$

$$\begin{aligned} + \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} &= 20\left(\frac{3}{5}\right) - 12 + F \sin 45^\circ \\ &= 0.7071F \uparrow \end{aligned}$$

The magnitude of the resultant force F_R is

$$\begin{aligned} F_R &= \sqrt{F_{R_x}^2 + F_{R_y}^2} \\ &= \sqrt{(16.0 - 0.7071F)^2 + (0.7071F)^2} \\ &= \sqrt{F^2 - 22.63F + 256} \end{aligned} \quad [1]$$

$$\begin{aligned} F_R^2 &= F^2 - 22.63F + 256 \\ 2F_R \frac{dF_R}{dF} &= 2F - 22.63 \end{aligned} \quad [2]$$

$$\left(F_R \frac{d^2 F_R}{dF^2} + \frac{dF_R}{dF} \times \frac{dF_R}{dF} \right) = 1 \quad [3]$$

In order to obtain the *minimum* resultant force F_R , $\frac{dF_R}{dF} = 0$. From Eq.[2]

$$2F_R \frac{dF_R}{dF} = 2F - 22.63 = 0$$

$$F = 11.31 \text{ kN} = 11.3 \text{ kN} \quad \text{Ans.}$$

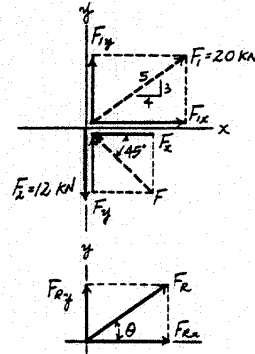
Substitute $F = 11.31 \text{ kN}$ into Eq.[1], we have

$$F_R = \sqrt{11.31^2 - 22.63(11.31) + 256} = \sqrt{128} \text{ kN}$$

Substitute $F_R = \sqrt{128} \text{ kN}$ with $\frac{dF_R}{dF} = 0$ into Eq.[3], we have

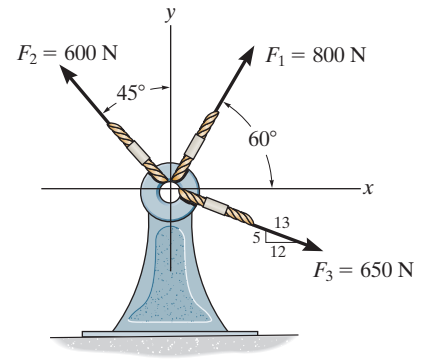
$$\begin{aligned} \left(\sqrt{128} \frac{d^2 F_R}{dF^2} + 0 \right) &= 1 \\ \frac{d^2 F_R}{dF^2} &= 0.0884 > 0 \end{aligned}$$

Hence, $F = 11.3 \text{ kN}$ is indeed producing a minimum resultant force.



2-39.

Resolve each force acting on the support into its x and y components, and express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \{800 \cos 60^\circ(+\mathbf{i}) + 800 \sin 60^\circ(+\mathbf{j})\} \text{ N}$$

$$= \{400\mathbf{i} + 693\mathbf{j}\} \text{ N}$$

Ans.

$$\mathbf{F}_2 = \{600 \sin 45^\circ(-\mathbf{i}) + 600 \cos 45^\circ(+\mathbf{j})\} \text{ N}$$

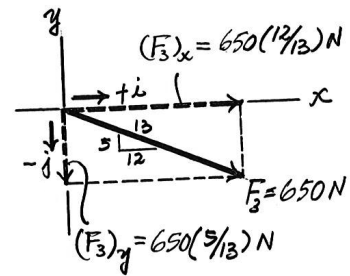
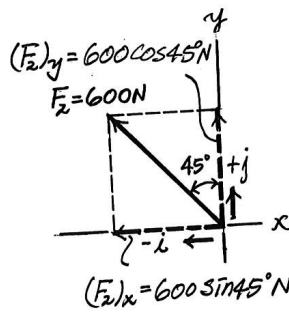
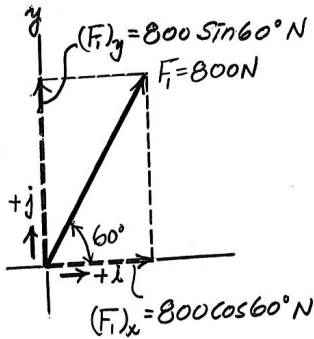
$$= \{-424\mathbf{i} + 424\mathbf{j}\} \text{ N}$$

Ans.

$$\mathbf{F}_3 = \left\{ 650 \left(\frac{12}{13} \right) (+\mathbf{i}) + 650 \left(\frac{5}{13} \right) (-\mathbf{j}) \right\} \text{ N}$$

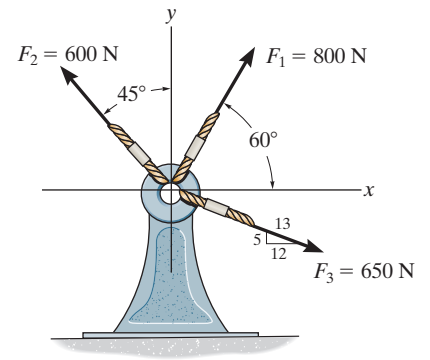
$$= \{600\mathbf{i} - 250\mathbf{j}\} \text{ N}$$

Ans.



2-40.

Determine the magnitude of the resultant force and its direction θ , measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 can be written as

$$\begin{aligned} (F_1)_x &= 800 \cos 60^\circ = 400 \text{ N} & (F_1)_y &= 800 \sin 60^\circ = 692.82 \text{ N} \\ (F_2)_x &= 600 \sin 45^\circ = 424.26 \text{ N} & (F_2)_y &= 600 \cos 45^\circ = 424.26 \text{ N} \\ (F_3)_x &= 650 \left(\frac{12}{13}\right) = 600 \text{ N} & (F_3)_y &= 650 \left(\frac{5}{13}\right) = 250 \text{ N} \end{aligned}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

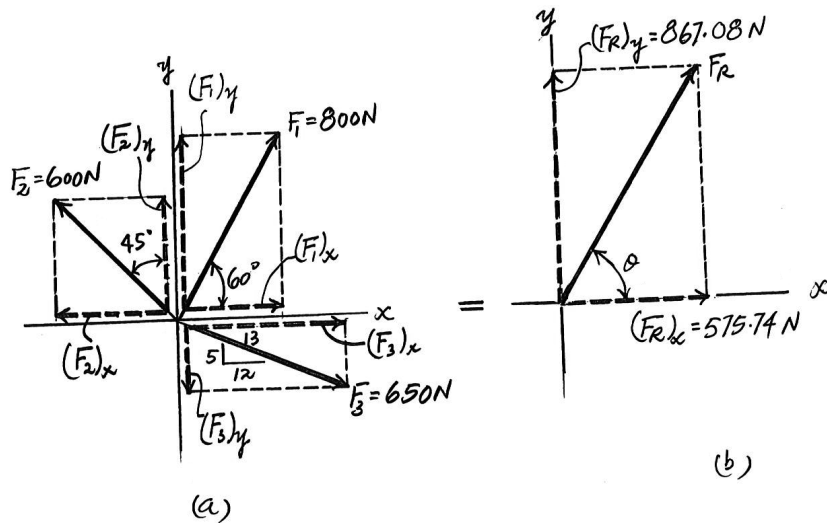
$$\begin{aligned} \rightarrow \Sigma(F_R)_x &= \Sigma F_x; & (F_R)_x &= 400 - 424.26 + 600 = 575.74 \text{ N} \rightarrow \\ + \uparrow \Sigma(F_R)_y &= \Sigma F_y; & (F_R)_y &= - 692.82 + 424.26 - 250 = 867.08 \text{ N} \uparrow \end{aligned}$$

The magnitude of the resultant force \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{575.74^2 + 867.08^2} = 1041 \text{ N} = 1.04 \text{ kN} \text{ Ans.}$$

The direction angle θ of \mathbf{F}_R , Fig. *b*, measured counterclockwise from the positive x axis, is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{867.08}{575.74} \right) = 56.4^\circ \text{ Ans.}$$



2-41.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

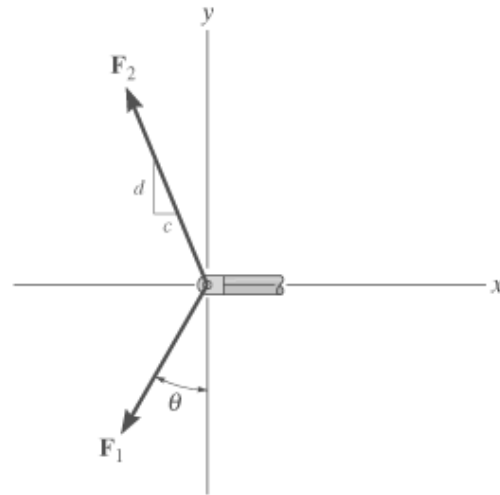
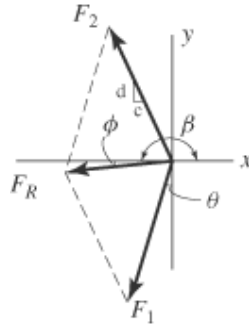
$$F_1 = 30 \text{ kN}$$

$$F_2 = 26 \text{ kN}$$

$$\theta = 30 \text{ deg}$$

$$c = 5$$

$$d = 12$$



Solution:

$$\begin{aligned} \rightarrow F_{Rx} = \Sigma F_x; & \quad F_{Rx} = -F_1 \sin(\theta) - \left(\frac{c}{\sqrt{c^2 + d^2}} \right) F_2 & \quad F_{Rx} = -25 \text{ kN} \end{aligned}$$

$$\begin{aligned} \uparrow F_{Ry} = \Sigma F_y; & \quad F_{Ry} = -F_1 \cos(\theta) + \left(\frac{d}{\sqrt{c^2 + d^2}} \right) F_2 & \quad F_{Ry} = -2 \text{ kN} \end{aligned}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad F_R = 25.1 \text{ kN} \quad \text{Ans.}$$

$$\phi = \text{atan}\left(\frac{F_{Ry}}{F_{Rx}}\right) \quad \phi = 4.5 \text{ deg}$$

$$\beta = 180 \text{ deg} + \phi \quad \beta = 184.5 \text{ deg} \quad \text{Ans.}$$

2-42.

Determine the magnitude and orientation θ of \mathbf{F}_B so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.

SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\rightarrow F_{R_x} = \Sigma F_x; \quad 0 = 700 \sin 30^\circ - F_B \cos \theta$$

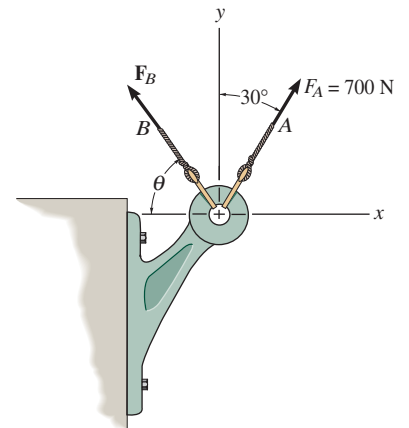
$$F_B \cos \theta = 350 \quad (1)$$

$$+ \uparrow F_{R_y} = \Sigma F_y; \quad 1500 = 700 \cos 30^\circ + F_B \sin \theta$$

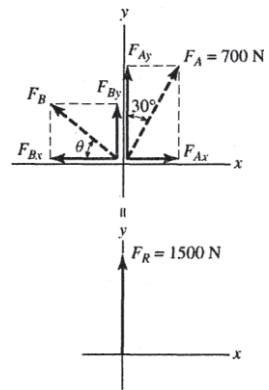
$$F_B \sin \theta = 893.8 \quad (2)$$

Solving Eq. (1) and (2) yields

$$\theta = 68.6^\circ \quad F_B = 960 \text{ N}$$

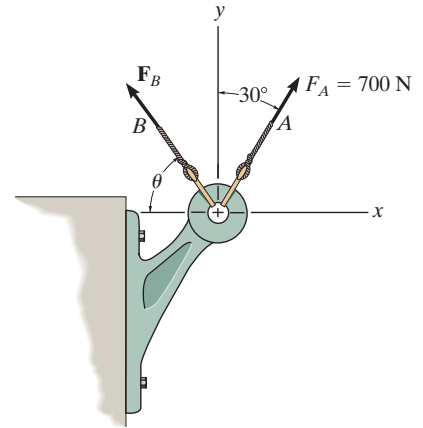


Ans.



2-43.

Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20^\circ$.



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} &= 700 \sin 30^\circ - 600 \cos 20^\circ \\ &= -213.8 \text{ N} = 213.8 \text{ N} \leftarrow \end{aligned}$$

$$\begin{aligned} +\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} &= 700 \cos 30^\circ + 600 \sin 20^\circ \\ &= 811.4 \text{ N} \uparrow \end{aligned}$$

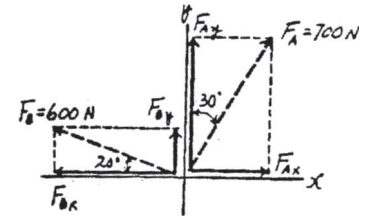
The magnitude of the resultant force \mathbf{F}_R is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N}$$

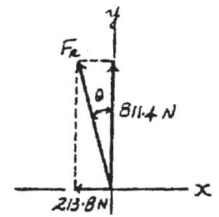
The direction angle θ measured counterclockwise from the positive y axis is

$$\theta = \tan^{-1} \frac{F_{R_x}}{F_{R_y}} = \tan^{-1} \left(\frac{213.8}{811.4} \right) = 14.8^\circ$$

Ans.

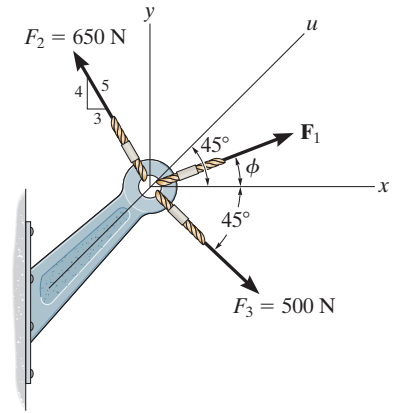


Ans.



2-44.

The magnitude of the resultant force acting on the bracket is to be 400 N. Determine the magnitude of \mathbf{F}_1 if $\phi = 30^\circ$.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 can be written as

$$(F_1)_x = F_1 \cos 30^\circ = 0.8660F_1 \quad (F_1)_y = F_1 \sin 30^\circ = 0.5F_1$$

$$(F_2)_x = 650\left(\frac{3}{5}\right) = 390 \text{ N} \quad (F_2)_y = 650\left(\frac{4}{5}\right) = 520 \text{ N}$$

$$(F_3)_x = 500 \cos 45^\circ = 353.55 \text{ N} \quad (F_3)_y = 500 \sin 45^\circ = 353.55 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\begin{aligned} \rightarrow \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x &= 0.8660F_1 - 390 + 353.55 \\ &= 0.8660F_1 - 36.45 \end{aligned}$$

$$\begin{aligned} + \uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y &= 0.5F_1 + 520 - 353.55 \\ &= 0.5F_1 + 166.45 \end{aligned}$$

Since the magnitude of the resultant force is $F_R = 400 \text{ N}$, we can write

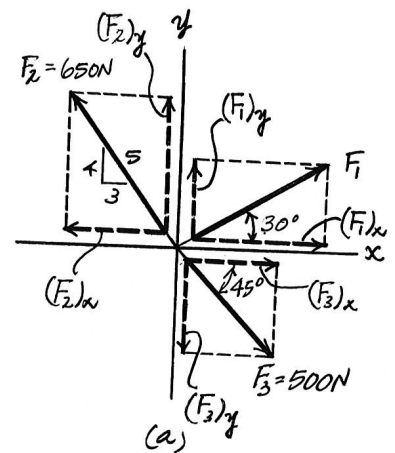
$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2} \\ 400 &= \sqrt{(0.8660F_1 - 36.45)^2 + (0.5F_1 + 166.45)^2} \\ F_1^2 + 103.32F_1 - 130967.17 &= 0 \end{aligned}$$

Ans.

Solving,

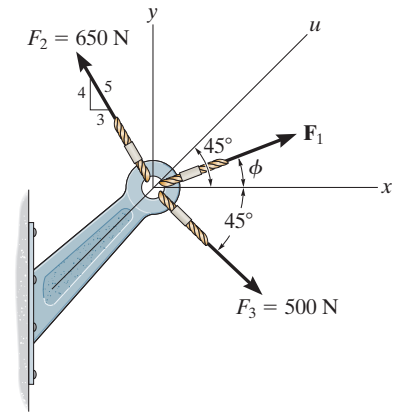
$$F_1 = 314 \text{ N} \quad \text{or} \quad F_1 = -417 \text{ N} \quad \text{Ans.}$$

The negative sign indicates that $\mathbf{F}_1 = 417 \text{ N}$ must act in the opposite sense to that shown in the figure.



2-45.

If the resultant force acting on the bracket is to be directed along the positive u axis, and the magnitude of \mathbf{F}_1 is required to be *minimum*, determine the magnitudes of the resultant force and \mathbf{F}_1 .



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of \mathbf{F}_1 , \mathbf{F}_2 , \mathbf{F}_3 , and \mathbf{F}_R can be written as

$$\begin{aligned} (F_1)_x &= F_1 \cos \phi & (F_1)_y &= F_1 \sin \phi \\ (F_2)_x &= 650 \left(\frac{3}{5}\right) = 390 \text{ N} & (F_2)_y &= 650 \left(\frac{4}{5}\right) = 520 \text{ N} \\ (F_3)_x &= 500 \cos 45^\circ = 353.55 \text{ N} & (F_3)_y &= 500 \sin 45^\circ = 353.55 \text{ N} \\ (F_R)_x &= F_R \cos 45^\circ = 0.7071 F_R & (F_R)_y &= F_R \sin 45^\circ = 0.7071 F_R \end{aligned}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\begin{aligned} \rightarrow \Sigma(F_R)_x &= \Sigma F_x; & 0.7071 F_R &= F_1 \cos \phi - 390 + 353.55 & (1) \\ + \uparrow \Sigma(F_R)_y &= \Sigma F_y; & 0.7071 F_R &= F_1 \sin \phi + 520 - 353.55 & (2) \end{aligned}$$

Eliminating F_R from Eqs. (1) and (2), yields

$$F_1 = \frac{202.89}{\cos \phi - \sin \phi} \quad (3)$$

The first derivative of Eq. (3) is

$$\frac{dF_1}{d\phi} = \frac{\sin \phi + \cos \phi}{(\cos \phi - \sin \phi)^2} \quad (4)$$

The second derivative of Eq. (3) is

$$\frac{d^2 F_1}{d\phi^2} = \frac{2(\sin \phi + \cos \phi)^2}{(\cos \phi - \sin \phi)^3} + \frac{1}{\cos \phi - \sin \phi} \quad (5)$$

For \mathbf{F}_1 to be minimum, $\frac{dF_1}{d\phi} = 0$. Thus, from Eq. (4)

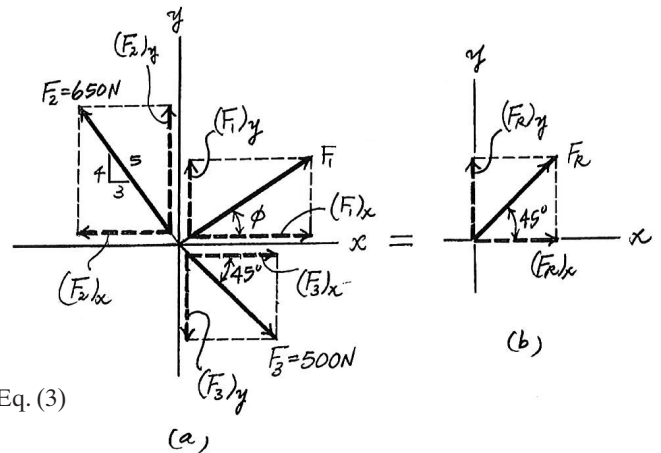
$$\begin{aligned} \sin \phi + \cos \phi &= 0 \\ \tan \phi &= -1 \\ \phi &= -45^\circ \end{aligned}$$

Substituting $\phi = -45^\circ$ into Eq. (5), yields

$$\frac{d^2 F_1}{d\phi^2} = 0.7071 > 0$$

This shows that $\phi = -45^\circ$ indeed produces minimum F_1 . Thus, from Eq. (3)

$$F_1 = \frac{202.89}{\cos(-45^\circ) - \sin(-45^\circ)} = 143.47 \text{ N} = 143 \text{ N}$$



Ans.

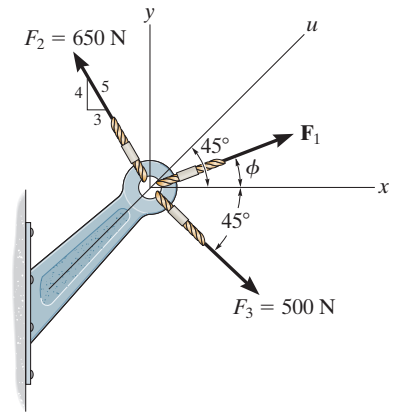
Substituting $\phi = -45^\circ$ and $F_1 = 143.47 \text{ N}$ into either Eq. (1) or Eq. (2), yields

$$F_R = 91.9 \text{ N}$$

Ans.

2-46.

If the magnitude of the resultant force acting on the bracket is 600 N, directed along the positive u axis, determine the magnitude of \mathbf{F} and its direction ϕ .



SOLUTION

Rectangular Components: By referring to Figs. a and b , the x and y components of \mathbf{F}_1 , \mathbf{F}_2 , \mathbf{F}_3 , and \mathbf{F}_R can be written as

$$\begin{aligned} (F_1)_x &= F_1 \cos \phi & (F_1)_y &= F_1 \sin \phi \\ (F_2)_x &= 650 \left(\frac{3}{5}\right) = 390 \text{ N} & (F_2)_y &= 650 \left(\frac{4}{5}\right) = 520 \text{ N} \\ (F_3)_x &= 500 \cos 45^\circ = 353.55 \text{ N} & (F_3)_y &= 500 \sin 45^\circ = 353.55 \text{ N} \\ (F_R)_x &= 600 \cos 45^\circ = 424.26 \text{ N} & (F_R)_y &= 600 \sin 45^\circ = 424.26 \text{ N} \end{aligned}$$

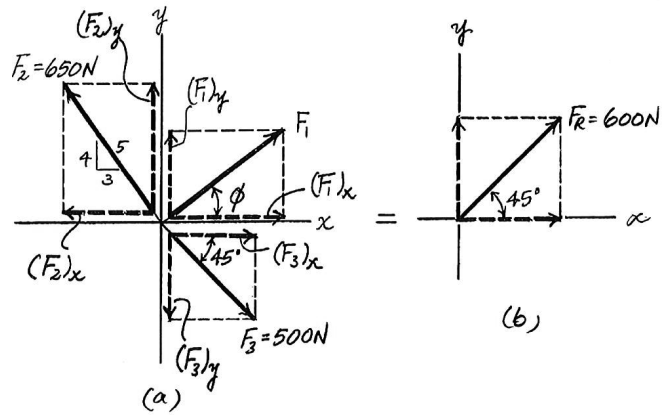
Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\begin{aligned} \rightarrow \Sigma(F_R)_x &= \Sigma F_x; & 424.26 &= F_1 \cos \phi - 390 + 353.55 & (1) \\ & & F_1 \cos \phi &= 460.71 & \\ + \uparrow \Sigma(F_R)_y &= \Sigma F_y; & 424.26 &= F_1 \sin \phi + 520 - 353.55 & (2) \\ & & F_1 \sin \phi &= 257.82 & \end{aligned}$$

Solving Eqs. (1) and (2), yields

$\phi = 29.2^\circ$ $F_1 = 528 \text{ N}$

Ans.



2-47.

Determine the magnitude and direction θ of the resultant force \mathbf{F}_R . Express the result in terms of the magnitudes of the components \mathbf{F}_1 and \mathbf{F}_2 and the angle ϕ .

SOLUTION

$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos(180^\circ - \phi)$$

Since $\cos(180^\circ - \phi) = -\cos \phi$,

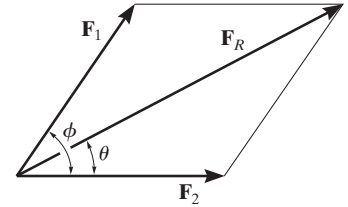
$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \phi}$$

From the figure,

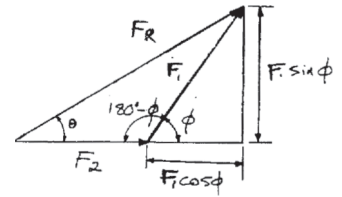
$$\tan \theta = \frac{F_1 \sin \phi}{F_2 + F_1 \cos \phi}$$

$$\theta = \tan^{-1} \left(\frac{F_1 \sin \phi}{F_2 + F_1 \cos \phi} \right)$$

Ans.



Ans.



2-48.

If $F_1 = 600\text{ N}$ and $\phi = 30^\circ$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.

SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of each force can be written as

$$(F_1)_x = 600 \cos 30^\circ = 519.62\text{ N} \quad (F_1)_y = 600 \sin 30^\circ = 300\text{ N}$$

$$(F_2)_x = 500 \cos 60^\circ = 250\text{ N} \quad (F_2)_y = 500 \sin 60^\circ = 433.01\text{ N}$$

$$(F_3)_x = 450 \left(\frac{3}{5}\right) = 270\text{ N} \quad (F_3)_y = 450 \left(\frac{4}{5}\right) = 360\text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\rightarrow \Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 519.62 + 250 - 270 = 499.62\text{ N} \rightarrow$$

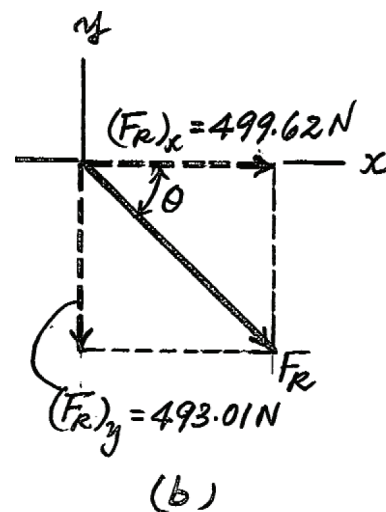
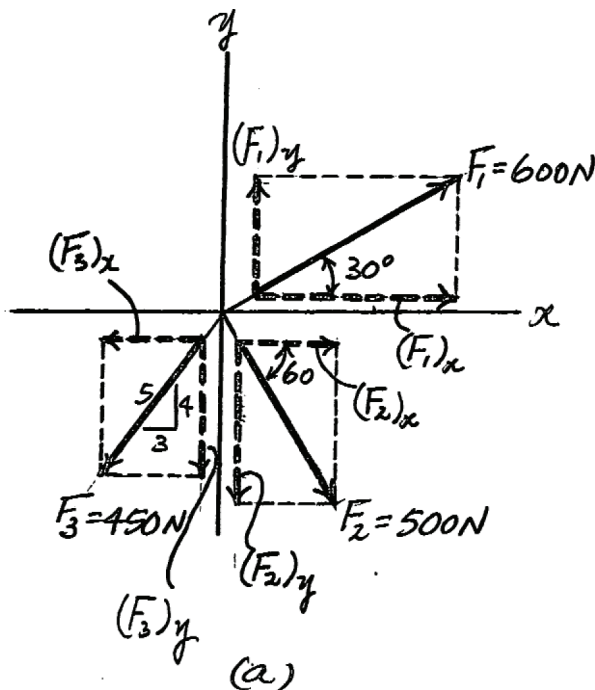
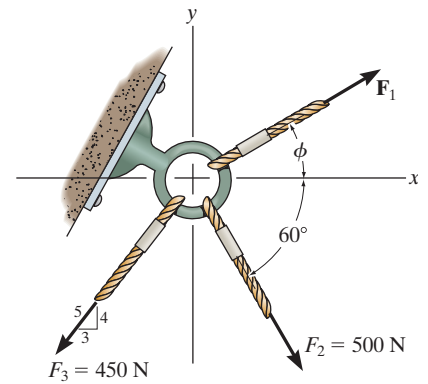
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad (F_R)_y = 300 - 433.01 - 360 = -493.01\text{ N} = 493.01\text{ N} \downarrow$$

The magnitude of the resultant force \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{499.62^2 + 493.01^2} = 701.91\text{ N} = 702\text{ N} \quad \text{Ans.}$$

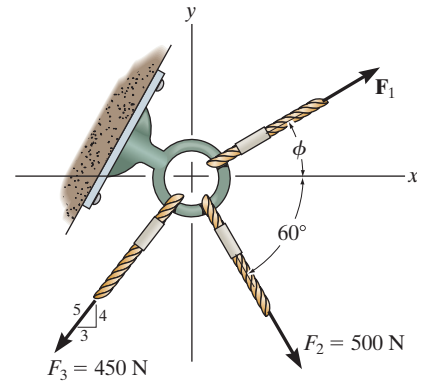
The direction angle θ of \mathbf{F}_R , Fig. *b*, measured clockwise from the x axis, is

$$\theta = \tan^{-1} \left[\frac{(F_R)_y}{(F_R)_x} \right] = \tan^{-1} \left(\frac{493.01}{499.62} \right) = 44.6^\circ \quad \text{Ans.}$$



2-49.

If the magnitude of the resultant force acting on the eyebolt is 600 N and its direction measured clockwise from the positive x axis is $\theta = 30^\circ$, determine the magnitude of F_1 and the angle ϕ .



SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the x and y components of F_1 , F_2 , F_3 , and F_R can be written as

$$\begin{aligned} (F_1)_x &= F_1 \cos \phi & (F_1)_y &= F_1 \sin \phi \\ (F_2)_x &= 500 \cos 60^\circ = 250 \text{ N} & (F_2)_y &= 500 \sin 60^\circ = 433.01 \text{ N} \\ (F_3)_x &= 450 \left(\frac{3}{5}\right) = 270 \text{ N} & (F_3)_y &= 450 \left(\frac{4}{5}\right) = 360 \text{ N} \\ (F_R)_x &= 600 \cos 30^\circ = 519.62 \text{ N} & (F_R)_y &= 600 \sin 30^\circ = 300 \text{ N} \end{aligned}$$

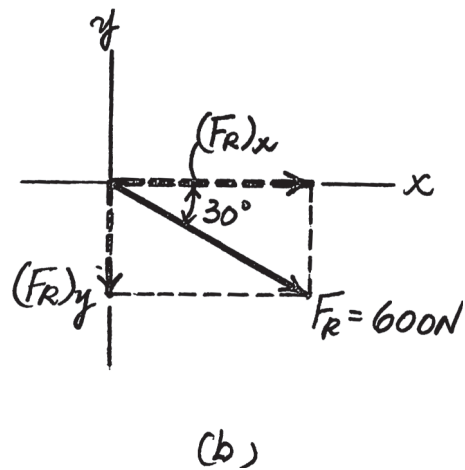
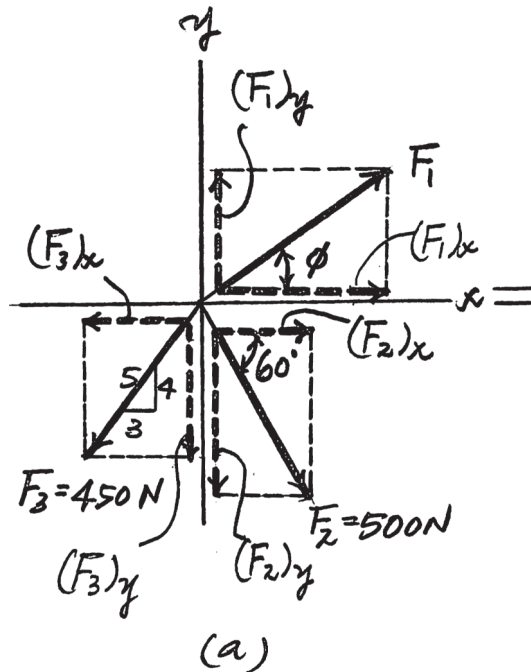
Resultant Force: Summing the force components algebraically along the x and y axes,

$$\begin{aligned} \rightarrow \Sigma(F_R)_x &= \Sigma F_x; & 519.62 &= F_1 \cos \phi + 250 - 270 \\ & & F_1 \cos \phi &= 539.62 \end{aligned} \tag{1}$$

$$\begin{aligned} +\uparrow \Sigma(F_R)_y &= \Sigma F_y; & -300 &= F_1 \sin \phi - 433.01 - 360 \\ & & F_1 \sin \phi &= 493.01 \end{aligned} \tag{2}$$

Solving Eqs. (1) and (2), yields

$\phi = 42.4^\circ$ $F_1 = 731 \text{ N}$ **Ans.**



2-50.

Determine the magnitude of \mathbf{F}_1 and its direction θ so that the resultant force is directed vertically upward and has a magnitude of 800 N.

SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} = 0 = F_1 \sin \theta + 400 \cos 30^\circ - 600 \left(\frac{4}{5}\right)$$

$$F_1 \sin \theta = 133.6$$

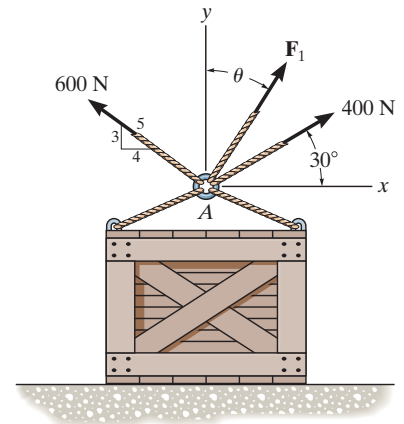
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 800 = F_1 \cos \theta + 400 \sin 30^\circ + 600 \left(\frac{3}{5}\right)$$

$$F_1 \cos \theta = 240$$

Solving Eqs. (1) and (2) yields

$$\theta = 29.1^\circ$$

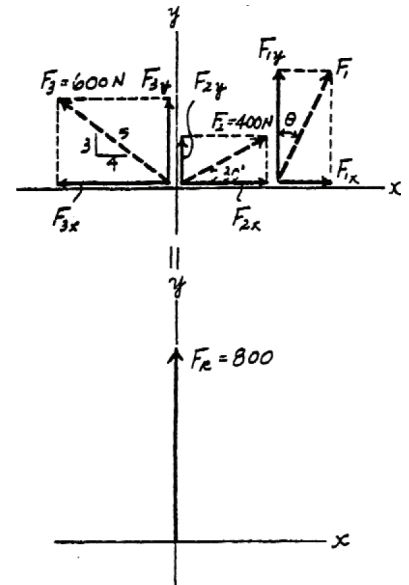
$$F_1 = 275 \text{ N}$$



(1)

(2)

Ans.



2-51.

Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force of the three forces acting on the ring A . Take $F_1 = 500\text{ N}$ and $\theta = 20^\circ$.

SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\begin{aligned} \pm F_{R_x} &= \Sigma F_x; & F_{R_x} &= 500 \sin 20^\circ + 400 \cos 30^\circ - 600 \left(\frac{4}{5}\right) \\ & & &= 37.42\text{ N} \rightarrow \end{aligned}$$

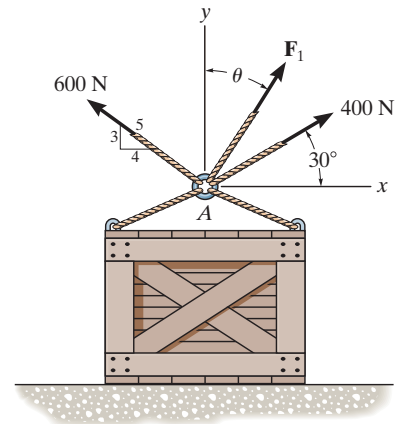
$$\begin{aligned} +\uparrow F_{R_y} &= \Sigma F_y; & F_{R_y} &= 500 \cos 20^\circ + 400 \sin 30^\circ + 600 \left(\frac{3}{5}\right) \\ & & &= 1029.8\text{ N} \uparrow \end{aligned}$$

The magnitude of the resultant force \mathbf{F}_R is

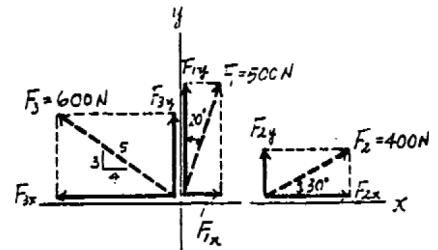
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{37.42^2 + 1029.8^2} = 1030.5\text{ N} = 1.03\text{ kN}$$

The direction angle θ measured counterclockwise from positive x axis is

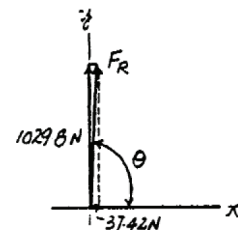
$$\theta = \tan^{-1} \frac{F_{R_y}}{F_{R_x}} = \tan^{-1} \left(\frac{1029.8}{37.42} \right) = 87.9^\circ$$



Ans.

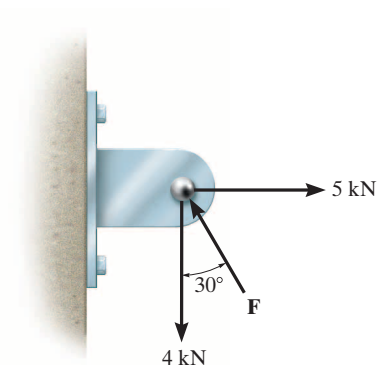


Ans.



2-52.

Determine the magnitude of force \mathbf{F} so that the resultant \mathbf{F}_R of the three forces is as small as possible. What is the minimum magnitude of \mathbf{F}_R ?



SOLUTION

Scalar Notation: Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow F_{R_x} &= \Sigma F_x; & F_{R_x} &= 5 - F \sin 30^\circ \\ & & &= 5 - 0.50F \rightarrow \\ +\uparrow F_{R_y} &= \Sigma F_y; & F_{R_y} &= F \cos 30^\circ - 4 \\ & & &= 0.8660F - 4 \uparrow \end{aligned}$$

The magnitude of the resultant force \mathbf{F}_R is

$$\begin{aligned} F_R &= \sqrt{F_{R_x}^2 + F_{R_y}^2} \\ &= \sqrt{(5 - 0.50F)^2 + (0.8660F - 4)^2} \\ &= \sqrt{F^2 - 11.93F + 41} \end{aligned} \tag{1}$$

$$F_R^2 = F^2 - 11.93F + 41 \tag{2}$$

$$2F_R \frac{dF_R}{dF} = 2F - 11.93 \tag{2}$$

$$\left(F_R \frac{d^2 F_R}{dF^2} + \frac{dF_R}{dF} \times \frac{dF_R}{dF} \right) = 1 \tag{3}$$

In order to obtain the *minimum* resultant force \mathbf{F}_R , $\frac{dF_R}{dF} = 0$. From Eq. (2)

$$2F_R \frac{dF_R}{dF} = 2F - 11.93 = 0$$

$$F = 5.964 \text{ kN} = 5.96 \text{ kN} \tag{Ans.}$$

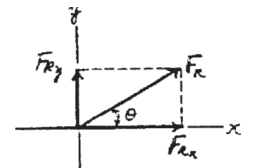
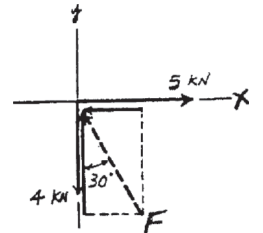
Substituting $F = 5.964 \text{ kN}$ into Eq. (1), we have

$$\begin{aligned} F_R &= \sqrt{5.964^2 - 11.93(5.964) + 41} \\ &= 2.330 \text{ kN} = 2.33 \text{ kN} \tag{Ans.} \end{aligned}$$

Substituting $F_R = 2.330 \text{ kN}$ with $\frac{dF_R}{dF} = 0$ into Eq. (3), we have

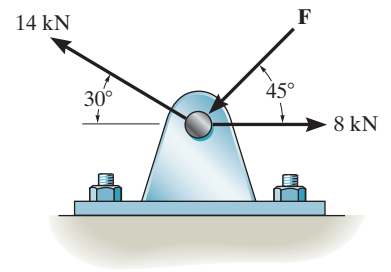
$$\begin{aligned} \left[(2.330) \frac{d^2 F_R}{dF^2} + 0 \right] &= 1 \\ \frac{d^2 F_R}{dF^2} &= 0.429 > 0 \end{aligned}$$

Hence, $F = 5.96 \text{ kN}$ is indeed producing a minimum resultant force.



2-53.

Determine the magnitude of force \mathbf{F} so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



SOLUTION

$$\begin{aligned} \rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} &= 8 - F \cos 45^\circ - 14 \cos 30^\circ \\ &= -4.1244 - F \cos 45^\circ \end{aligned}$$

$$\begin{aligned} + \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} &= -F \sin 45^\circ + 14 \sin 30^\circ \\ &= 7 - F \sin 45^\circ \end{aligned}$$

$$F_R^2 = (-4.1244 - F \cos 45^\circ)^2 + (7 - F \sin 45^\circ)^2 \quad (1)$$

$$2F_R \frac{dF_R}{dF} = 2(-4.1244 - F \cos 45^\circ)(-\cos 45^\circ) + 2(7 - F \sin 45^\circ)(-\sin 45^\circ) = 0$$

$$F = 2.03 \text{ kN}$$

Ans.

From Eq. (1);

$$F_R = 7.87 \text{ kN}$$

Ans.

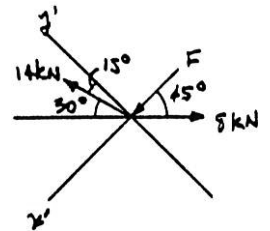
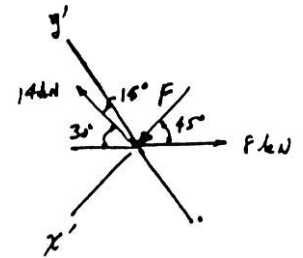
Also, from the figure require

$$\begin{aligned} (F_R)_{x'} = 0 = \Sigma F_{x'}; \quad F + 14 \sin 15^\circ - 8 \cos 45^\circ &= 0 \\ F &= 2.03 \text{ kN} \end{aligned}$$

Ans.

$$\begin{aligned} (F_R)_{y'} = \Sigma F_{y'}; \quad F_R &= 14 \cos 15^\circ - 8 \sin 45^\circ \\ F_R &= 7.87 \text{ kN} \end{aligned}$$

Ans.



2-54.

Three forces act on the bracket. Determine the magnitude and direction θ of \mathbf{F}_1 so that the resultant force is directed along the positive x' axis and has a magnitude of 1 kN.

SOLUTION

$$\rightarrow F_{Rx} = \Sigma F_x; \quad 1000 \cos 30^\circ = 200 + 450 \cos 45^\circ + F_1 \cos(\theta + 30^\circ)$$

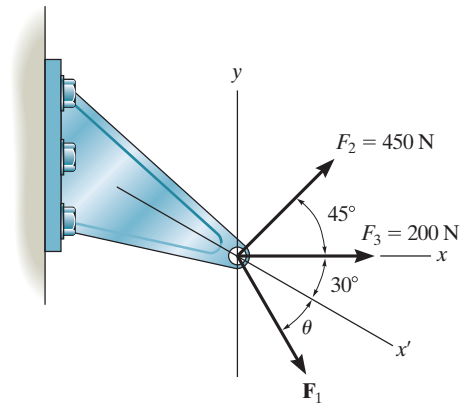
$$+ \uparrow F_{Ry} = \Sigma F_y; \quad -1000 \sin 30^\circ = 450 \sin 45^\circ - F_1 \sin(\theta + 30^\circ)$$

$$F_1 \sin(\theta + 30^\circ) = 818.198$$

$$F_1 \cos(\theta + 30^\circ) = 347.827$$

$$\theta + 30^\circ = 66.97^\circ, \quad \theta = 37.0^\circ$$

$$F_1 = 889 \text{ N}$$



Ans.

Ans.

2-55.

If $F_1 = 300\text{ N}$ and $\theta = 20^\circ$, determine the magnitude and direction, measured counterclockwise from the x' axis, of the resultant force of the three forces acting on the bracket.

SOLUTION

$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = 300 \cos 50^\circ + 200 + 450 \cos 45^\circ = 711.03\text{ N}$$

$$+ \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -300 \sin 50^\circ + 450 \sin 45^\circ = 88.38\text{ N}$$

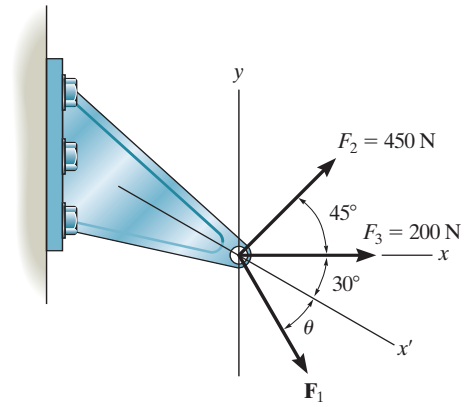
$$F_R = \sqrt{(711.03)^2 + (88.38)^2} = 717\text{ N}$$

$$\phi' \text{ (angle from } x \text{ axis)} = \tan^{-1} \left[\frac{88.38}{711.03} \right]$$

$$\phi' = 7.10^\circ$$

$$\phi \text{ (angle from } x' \text{ axis)} = 30^\circ + 7.10^\circ$$

$$\phi = 37.1^\circ$$



Ans.

Ans.

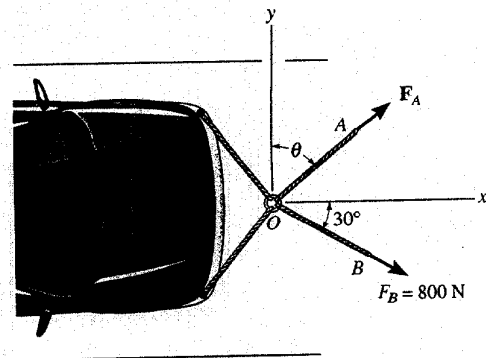
2-56. Determine the magnitude and direction θ of \mathbf{F}_A so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.

$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = F_A \sin \theta + 800 \cos 30^\circ = 1250$$

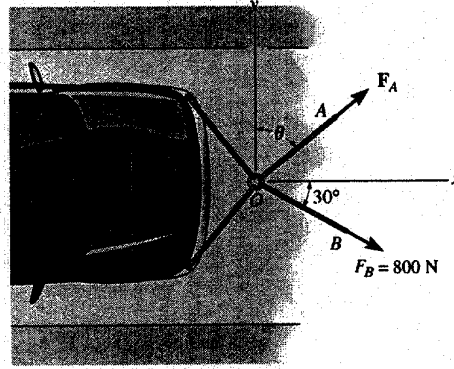
$$+ \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = F_A \cos \theta - 800 \sin 30^\circ = 0$$

$$\theta = 54.3^\circ \quad \text{Ans.}$$

$$F_A = 686 \text{ N} \quad \text{Ans.}$$



2-57. Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O , if $F_A = 750$ N and $\theta = 45^\circ$.



Scalar Notation : Summing the force components algebraically, we have

$$\begin{aligned} \rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} &= 750 \sin 45^\circ + 800 \cos 30^\circ \\ &= 1223.15 \text{ N } \rightarrow \end{aligned}$$

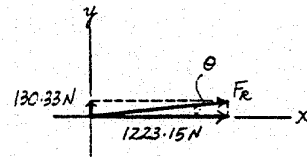
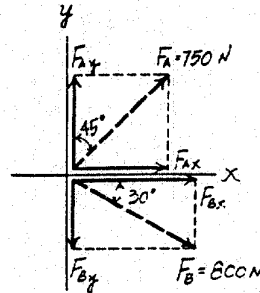
$$\begin{aligned} + \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} &= 750 \cos 45^\circ - 800 \sin 30^\circ \\ &= 130.33 \text{ N } \uparrow \end{aligned}$$

The magnitude of the resultant force F_R is

$$\begin{aligned} F_R &= \sqrt{F_{R_x}^2 + F_{R_y}^2} \\ &= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \quad \text{Ans.} \end{aligned}$$

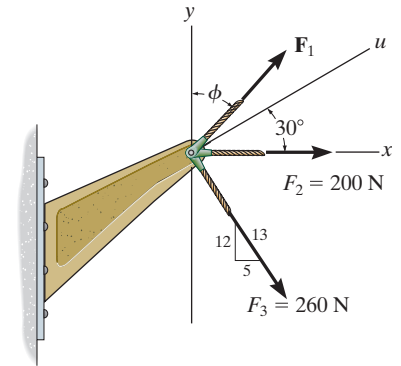
The directional angle θ measured counterclockwise from positive x axis is

$$\theta = \tan^{-1} \frac{F_{R_y}}{F_{R_x}} = \tan^{-1} \left(\frac{130.33}{1223.15} \right) = 6.08^\circ \quad \text{Ans.}$$



2-58.

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of \mathbf{F}_1 and its direction ϕ .



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of \mathbf{F}_1 , \mathbf{F}_2 , \mathbf{F}_3 , and \mathbf{F}_R can be written as

$$\begin{aligned} (F_1)_x &= F_1 \sin \phi & (F_1)_y &= F_1 \cos \phi \\ (F_2)_x &= 200\text{ N} & (F_2)_y &= 0 \\ (F_3)_x &= 260 \left(\frac{5}{13} \right) = 100\text{ N} & (F_3)_y &= 260 \left(\frac{12}{13} \right) = 240\text{ N} \\ (F_R)_x &= 450 \cos 30^\circ = 389.71\text{ N} & (F_R)_y &= 450 \sin 30^\circ = 225\text{ N} \end{aligned}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\rightarrow \Sigma(F_R)_x = \Sigma F_x; \quad 389.71 = F_1 \sin \phi + 200 + 100$$

$$F_1 \sin \phi = 89.71$$

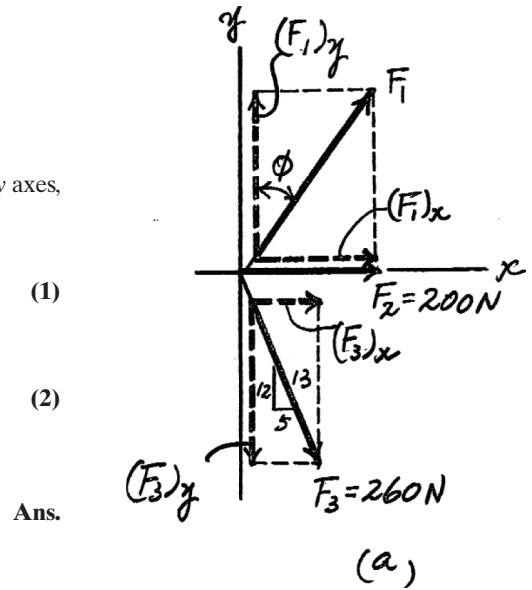
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 225 = F_1 \cos \phi - 240$$

$$F_1 \cos \phi = 465$$

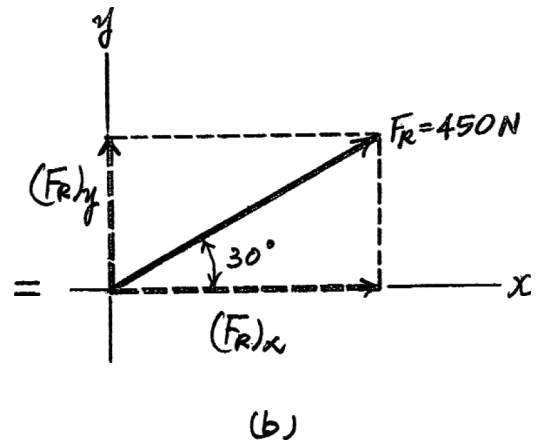
Solving Eqs. (1) and (2), yields

$$\phi = 10.9^\circ$$

$$F_1 = 474\text{ N}$$



Ans.



2-59.

If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of F_1 and the resultant force. Set $\phi = 30^\circ$.

SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of F_1 , F_2 , and F_3 can be written as

$$\begin{aligned} (F_1)_x &= F_1 \sin 30^\circ = 0.5F_1 & (F_1)_y &= F_1 \cos 30^\circ = 0.8660F_1 \\ (F_2)_x &= 200 \text{ N} & (F_2)_y &= 0 \\ (F_3)_x &= 260\left(\frac{5}{13}\right) = 100 \text{ N} & (F_3)_y &= 260\left(\frac{12}{13}\right) = 240 \text{ N} \end{aligned}$$

Resultant Force: Summing the force components algebraically along the x and y axes,

$$\begin{aligned} \rightarrow \Sigma(F_R)_x &= \Sigma F_x; & (F_R)_x &= 0.5F_1 + 200 + 100 = 0.5F_1 + 300 \\ + \uparrow \Sigma(F_R)_y &= \Sigma F_y; & (F_R)_y &= 0.8660F_1 - 240 \end{aligned}$$

The magnitude of the resultant force F_R is

$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2} \\ &= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2} \\ &= \sqrt{F_1^2 - 115.69F_1 + 147\,600} \end{aligned} \tag{1}$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147\,600 \tag{2}$$

The first derivative of Eq. (2) is

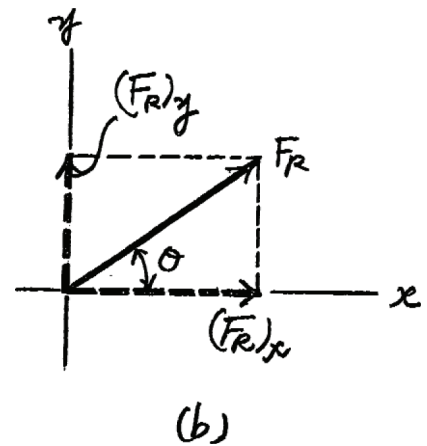
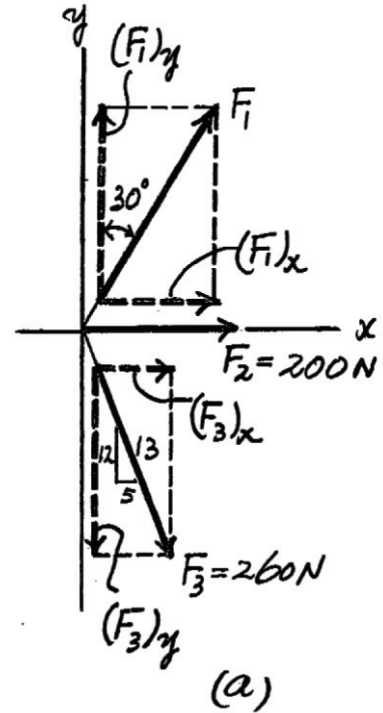
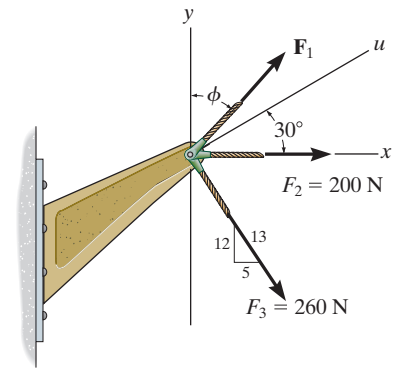
$$2F_R \frac{dF_R}{dF_1} = 2F_1 - 115.69 \tag{3}$$

For F_R to be minimum, $\frac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$\begin{aligned} 2F_R \frac{dF_R}{dF_1} &= 2F_1 - 115.69 = 0 \\ F_1 &= 57.846 \text{ N} = 57.8 \text{ N} \end{aligned}$$

from Eq. (1),

$$F_R = \sqrt{(57.846)^2 - 115.69(57.846) + 147\,600} = 380 \text{ N}$$



Ans.

Ans.

2-60.

The stock mounted on the lathe is subjected to a force of 60 N. Determine the coordinate direction angle β and express the force as a Cartesian vector.

SOLUTION

$$1 = \sqrt{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma}$$

$$1 = \cos^2 60^\circ + \cos^2 \beta + \cos^2 45^\circ$$

$$\cos \beta = \pm 0.5$$

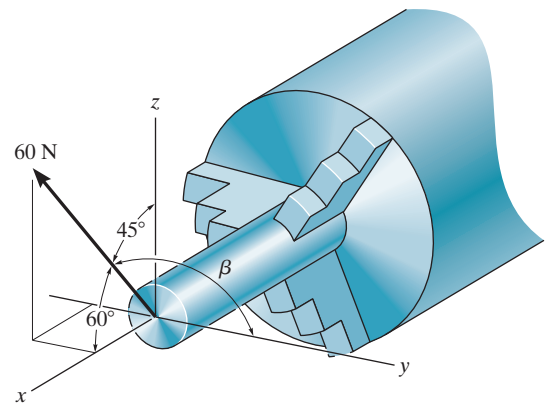
$$\beta = 60^\circ, 120^\circ$$

Use

$$\beta = 120^\circ$$

$$F = 60 \text{ N}(\cos 60^\circ \mathbf{i} + \cos 120^\circ \mathbf{j} + \cos 45^\circ \mathbf{k})$$

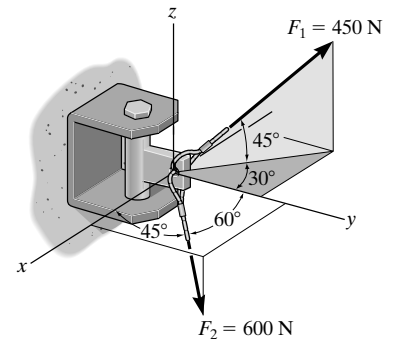
$$= \{30\mathbf{i} - 30\mathbf{j} + 42.4\mathbf{k}\} \text{ N}$$



Ans.

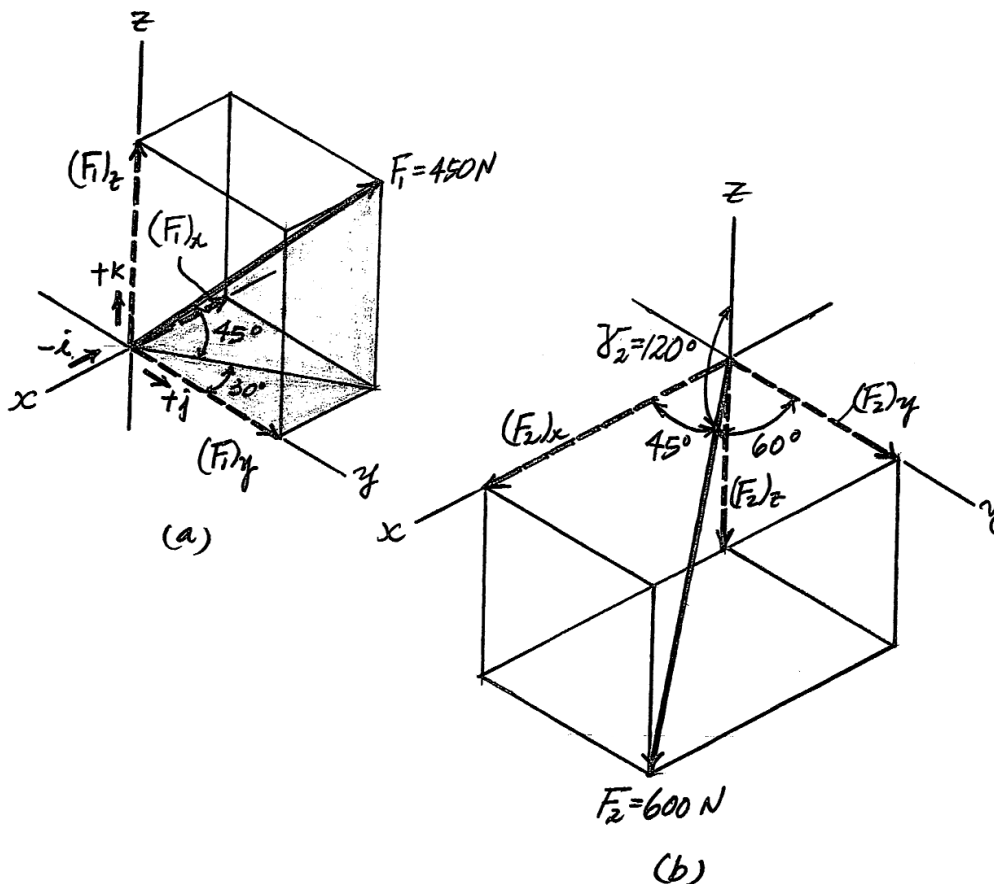
Ans.

2-61. Determine the coordinate angle γ for \mathbf{F}_2 and then express each force acting on the bracket as a Cartesian vector.

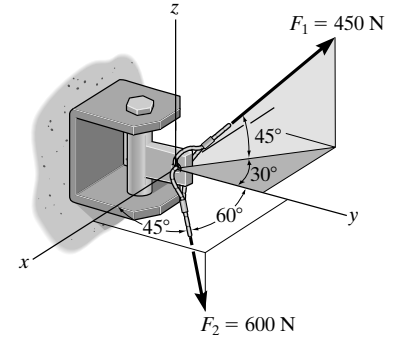


Rectangular Components: Since $\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$, then $\cos \gamma_2 = \pm \sqrt{1 - \cos^2 45^\circ - \cos^2 60^\circ} = \pm 0.5$.
 However, it is required that $\gamma_2 > 90^\circ$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120^\circ$. By resolving \mathbf{F}_1 and \mathbf{F}_2 into their x , y , and z components, as shown in Figs. *a* and *b*, respectively \mathbf{F}_1 and \mathbf{F}_2 can be expressed in Cartesian vector form as

$$\begin{aligned} \mathbf{F}_1 &= 450 \cos 45^\circ \sin 30^\circ (-\mathbf{i}) + 450 \cos 45^\circ \cos 30^\circ (+\mathbf{j}) + 450 \sin 45^\circ (+\mathbf{k}) \\ &= \{-159\mathbf{i} + 276\mathbf{j} + 318\mathbf{k}\} \text{ N} \quad \text{Ans.} \\ \mathbf{F}_2 &= 600 \cos 45^\circ \mathbf{i} + 600 \cos 60^\circ \mathbf{j} + 600 \cos 120^\circ \mathbf{k} \\ &= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\} \text{ N} \quad \text{Ans.} \end{aligned}$$



2-62. Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket.



Rectangular Components: Since $\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$, then $\cos \gamma_2 = \pm \sqrt{1 - \cos^2 45^\circ - \cos^2 60^\circ} = \pm 0.5$.

However, it is required that $\alpha_2 > 90^\circ$, thus, $\gamma_2 = \cos^{-1}(-0.5) = 120^\circ$. By resolving F_1 and F_2 into their x , y , and z components, as shown in Figs. *a* and *b*, respectively, F_1 and F_2 , can be expressed in Cartesian vector form, as

$$\begin{aligned} F_1 &= 450 \cos 45^\circ \sin 30^\circ (-\mathbf{i}) + 450 \cos 45^\circ \cos 30^\circ (+\mathbf{j}) + 450 \sin 45^\circ (+\mathbf{k}) \\ &= \{-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}\} \text{ N} \end{aligned} \quad \text{Ans.}$$

$$\begin{aligned} F_2 &= 600 \cos 45^\circ \mathbf{i} + 600 \cos 60^\circ \mathbf{j} + 600 \cos 120^\circ \mathbf{k} \\ &= \{424\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}\} \text{ N} \end{aligned} \quad \text{Ans.}$$

Resultant Force: By adding F_1 and F_2 vectorally, we obtain F_R .

$$\begin{aligned} F_R &= F_1 + F_2 \\ &= (-159.10\mathbf{i} + 275.57\mathbf{j} + 318.20\mathbf{k}) + (424.26\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}) \\ &= \{265.16\mathbf{i} + 575.57\mathbf{j} + 18.20\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of F_R is

$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{265.16^2 + 575.57^2 + 18.20^2} = 633.97 \text{ N} = 634 \text{ N} \end{aligned} \quad \text{Ans.}$$

The coordinate direction angles of F_R are

$$\alpha = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{265.16}{633.97} \right) = 65.3^\circ \quad \text{Ans.}$$

$$\beta = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{575.57}{633.97} \right) = 24.8^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{18.20}{633.97} \right) = 88.4^\circ \quad \text{Ans.}$$

2-63.

The bolt is subjected to the force \mathbf{F} , which has components acting along the x , y , z axes as shown. If the magnitude of \mathbf{F} is 80 N, and $\alpha = 60^\circ$ and $\gamma = 45^\circ$, determine the magnitudes of its components.

SOLUTION

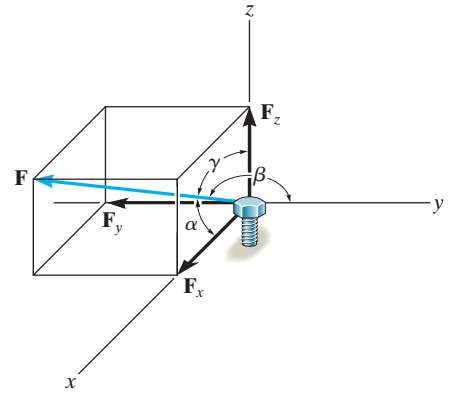
$$\begin{aligned}\cos\beta &= \sqrt{1 - \cos^2\alpha - \cos^2\gamma} \\ &= \sqrt{1 - \cos^2 60^\circ - \cos^2 45^\circ}\end{aligned}$$

$$\beta = 120^\circ$$

$$F_x = |80 \cos 60^\circ| = 40 \text{ N}$$

$$F_y = |80 \cos 120^\circ| = 40 \text{ N}$$

$$F_z = |80 \cos 45^\circ| = 56.6 \text{ N}$$



Ans.

Ans.

Ans.

2-64.

Determine the magnitude and coordinate direction angles of the force \mathbf{F} acting on the stake.

Given:

$$F_h = 40 \text{ N}$$

$$\theta = 70 \text{ deg}$$

$$c = 3$$

$$d = 4$$

Solution:

$$F = F_h \left(\frac{\sqrt{c^2 + d^2}}{d} \right)$$

$$F = 50 \text{ N}$$

Ans.

$$F_x = F_h \cos(\theta)$$

$$F_y = F_h \sin(\theta)$$

$$F_z = \left(\frac{c}{\sqrt{c^2 + d^2}} \right) F$$

$$F_x = 13.7 \text{ N}$$

$$F_y = 37.6 \text{ N}$$

$$F_z = 30 \text{ N}$$

$$\alpha = \cos^{-1} \left(\frac{F_x}{F} \right)$$

$$\beta = \cos^{-1} \left(\frac{F_y}{F} \right)$$

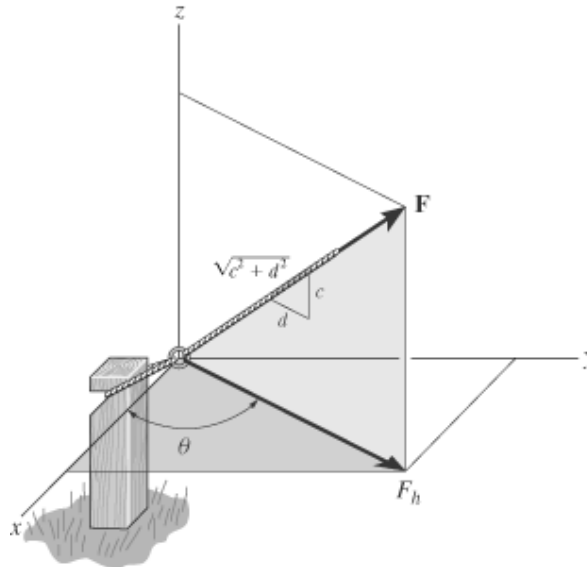
$$\gamma = \cos^{-1} \left(\frac{F_z}{F} \right)$$

$$\alpha = 74.1 \text{ deg}$$

$$\beta = 41.3 \text{ deg}$$

$$\gamma = 53.1 \text{ deg}$$

Ans.



2-65.

Determine the magnitude and coordinate direction angles of the force \mathbf{F} acting on the stake.

Given:

$$F_h := 40\text{N}$$

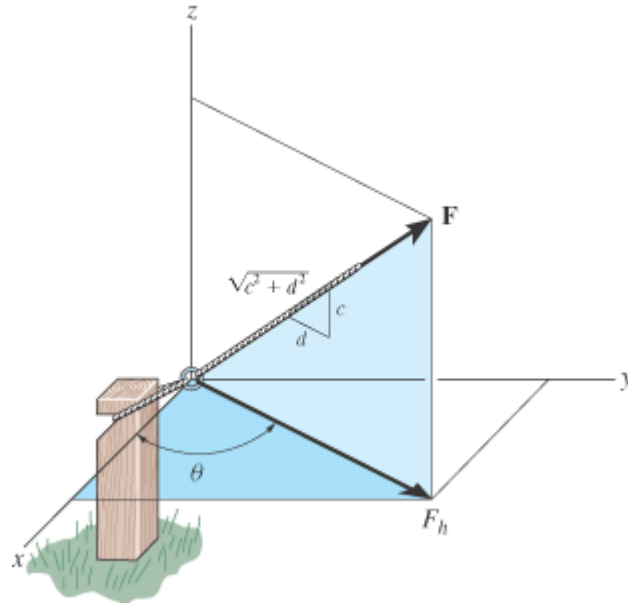
$$\theta := 50\text{deg}$$

$$c := 3$$

$$d := 4$$

Solution:

$$F := F_h \cdot \frac{\sqrt{c^2 + d^2}}{d}$$



$$F = 50\text{N}$$

Ans.

$$F_x := F_h \cdot \cos(\theta)$$

$$F_y := F_h \cdot \sin(\theta)$$

$$F_z := \frac{c}{\sqrt{c^2 + d^2}} \cdot F$$

$$F_x = 25.7\text{N}$$

$$F_y = 30.6\text{N}$$

$$F_z = 30\text{N}$$

$$\alpha := \arccos\left(\frac{F_x}{F}\right)$$

$$\beta := \arccos\left(\frac{F_y}{F}\right)$$

$$\gamma := \arccos\left(\frac{F_z}{F}\right)$$

$$\alpha = 59.1\text{deg}$$

Ans.

$$\beta = 52.2\text{deg}$$

Ans.

$$\gamma = 53.1\text{deg}$$

Ans.

2-66.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_1 = \{60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}\}$ N and $\mathbf{F}_2 = \{-40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}\}$ N. Sketch each force on an x, y, z reference frame.

SOLUTION

$$\mathbf{F}_1 = 60\mathbf{i} - 50\mathbf{j} + 40\mathbf{k}$$

$$F_1 = \sqrt{(60)^2 + (-50)^2 + (40)^2} = 87.7496 = 87.7 \text{ N}$$

$$\alpha_1 = \cos^{-1}\left(\frac{60}{87.7496}\right) = 46.9^\circ$$

$$\beta_1 = \cos^{-1}\left(\frac{-50}{87.7496}\right) = 125^\circ$$

$$\gamma_1 = \cos^{-1}\left(\frac{40}{87.7496}\right) = 62.9^\circ$$

$$\mathbf{F}_2 = -40\mathbf{i} - 85\mathbf{j} + 30\mathbf{k}$$

$$F_2 = \sqrt{(-40)^2 + (-85)^2 + (30)^2} = 98.615 = 98.6 \text{ N}$$

$$\alpha_2 = \cos^{-1}\left(\frac{-40}{98.615}\right) = 114^\circ$$

$$\beta_2 = \cos^{-1}\left(\frac{-85}{98.615}\right) = 150^\circ$$

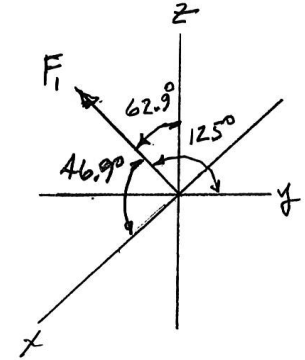
$$\gamma_2 = \cos^{-1}\left(\frac{30}{98.615}\right) = 72.3^\circ$$

Ans.

Ans.

Ans.

Ans.

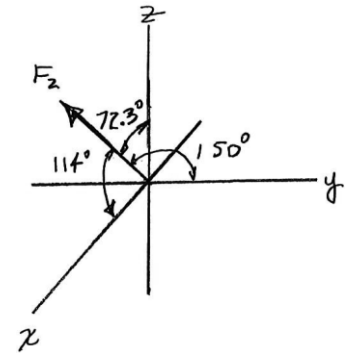


Ans.

Ans.

Ans.

Ans.



2-67. Express each force in Cartesian vector form.

$$(F_1)_x = 0$$

$$(F_1)_y = 2 \text{ kN}$$

$$(F_1)_z = 0$$

$$\text{Thus, } \mathbf{F}_1 = 0(\mathbf{i}) + 2(-\mathbf{j}) + 0(\mathbf{k}) = \{-2\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = 2.5(\mathbf{i}) + 3.54(\mathbf{j}) + 2.5(\mathbf{k})$$

$$= \{2.5\mathbf{i} + 3.54\mathbf{j} + 2.5\mathbf{k}\} \text{ N}$$

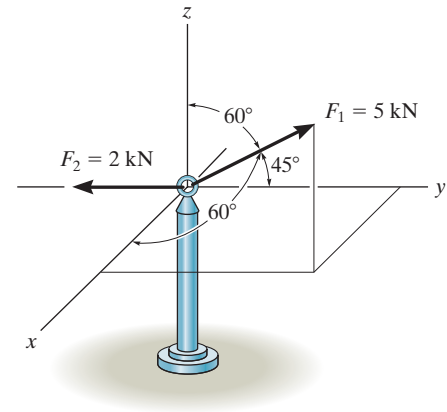
$$(F_2)_x = 5 \cos 60^\circ = 2.5 \text{ kN}$$

$$(F_2)_y = 5 \cos 45^\circ = 3.54 \text{ kN}$$

$$(F_2)_z = 5 \cos 60^\circ = 2.5 \text{ kN}$$

Ans.

Ans.



2-68.

Express each force as a Cartesian vector.

SOLUTION

Rectangular Components: By referring to Figs. *a* and *b*, the x , y , and z components of \mathbf{F}_1 and \mathbf{F}_2 can be written as

$$(F_1)_x = 300 \cos 30^\circ = 259.8 \text{ N} \quad (F_2)_x = 500 \cos 45^\circ \sin 30^\circ = 176.78 \text{ N}$$

$$(F_1)_y = 0 \quad (F_2)_y = 500 \cos 45^\circ \cos 30^\circ = 306.19 \text{ N}$$

$$(F_1)_z = 300 \sin 30^\circ = 150 \text{ N} \quad (F_2)_z = 500 \sin 45^\circ = 353.55 \text{ N}$$

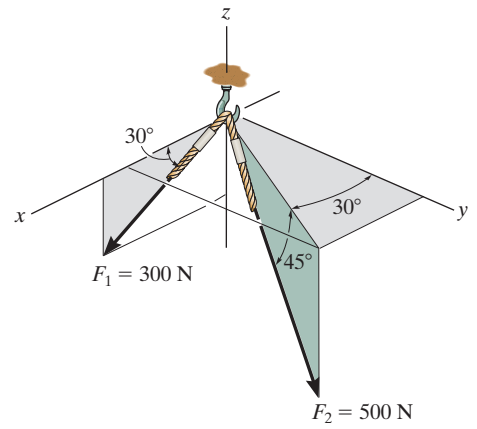
Thus, \mathbf{F}_1 and \mathbf{F}_2 can be written in Cartesian vector form as

$$\mathbf{F}_1 = 259.81(+\mathbf{i}) + 0\mathbf{j} + 150(-\mathbf{k})$$

$$= \{260\mathbf{i} - 150\mathbf{k}\} \text{ N}$$

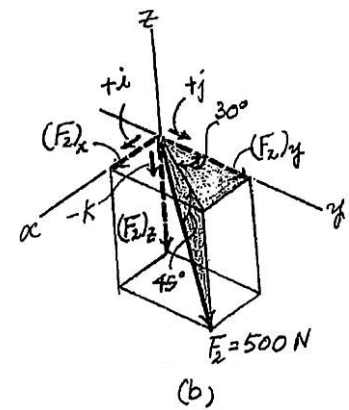
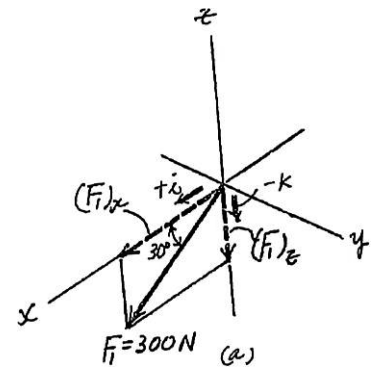
$$\mathbf{F}_2 = 176.78(+\mathbf{i}) + 306.19(+\mathbf{j}) + 353.55(-\mathbf{k})$$

$$= \{177\mathbf{i} + 306\mathbf{j} - 354\mathbf{k}\} \text{ N}$$



Ans.

Ans.



2-69.

Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.

SOLUTION

Force Vectors: By resolving \mathbf{F}_1 and \mathbf{F}_2 into their x , y , and z components, as shown in Figs. *a* and *b*, respectively, \mathbf{F}_1 and \mathbf{F}_2 can be expressed in Cartesian vector form as

$$\begin{aligned}\mathbf{F}_1 &= 300 \cos 30^\circ(+\mathbf{i}) + 0\mathbf{j} + 300 \sin 30^\circ(-\mathbf{k}) \\ &= \{259.81\mathbf{i} - 150\mathbf{k}\} \text{ N}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_2 &= 500 \cos 45^\circ \sin 30^\circ(+\mathbf{i}) + 500 \cos 45^\circ \cos 30^\circ(+\mathbf{j}) + 500 \sin 45^\circ(-\mathbf{k}) \\ &= \{176.78\mathbf{i} - 306.19\mathbf{j} - 353.55\mathbf{k}\} \text{ N}\end{aligned}$$

Resultant Force: The resultant force acting on the hook can be obtained by vectorally adding \mathbf{F}_1 and \mathbf{F}_2 . Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= (259.81\mathbf{i} - 150\mathbf{k}) + (176.78\mathbf{i} + 306.19\mathbf{j} - 353.55\mathbf{k}) \\ &= \{436.58\mathbf{i} + 306.19\mathbf{j} - 503.55\mathbf{k}\} \text{ N}\end{aligned}$$

The magnitude of \mathbf{F}_R is

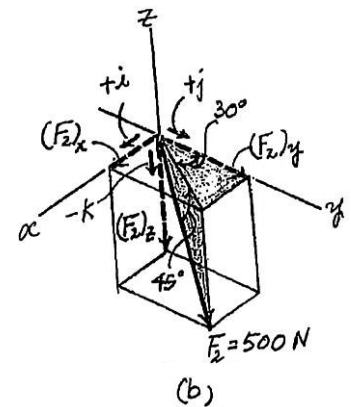
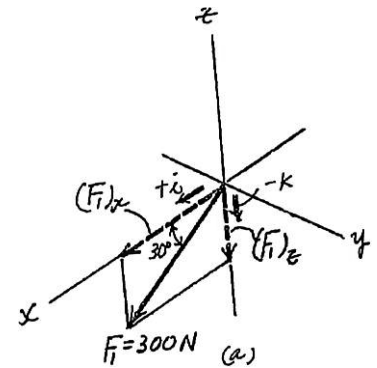
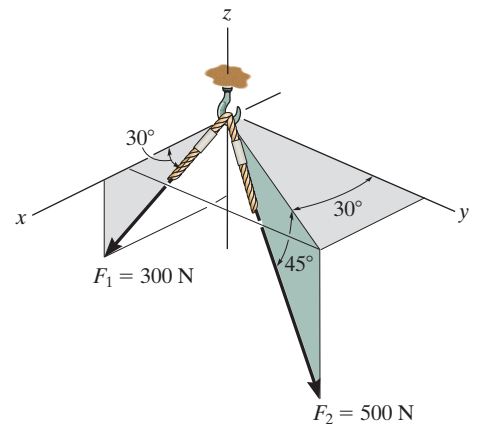
$$\begin{aligned}F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{(436.58)^2 + (306.19)^2 + (-503.55)^2} = 733.43 \text{ N} = \mathbf{733 \text{ N}} \quad \text{Ans.}\end{aligned}$$

The coordinate direction angles of \mathbf{F}_R are

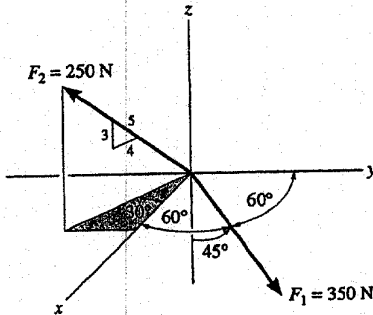
$$\theta_x = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{436.58}{733.43} \right) = \mathbf{53.5^\circ} \quad \text{Ans.}$$

$$\theta_y = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{306.19}{733.43} \right) = \mathbf{65.3^\circ} \quad \text{Ans.}$$

$$\theta_z = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{-503.55}{733.43} \right) = \mathbf{133^\circ} \quad \text{Ans.}$$



2-70. Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = 350 \cos 60^\circ \mathbf{i} + 350 \cos 60^\circ \mathbf{j} - 350 \cos 45^\circ \mathbf{k} + 250 \left(\frac{4}{5}\right) \cos 30^\circ \mathbf{i} - 250 \left(\frac{4}{5}\right) \sin 30^\circ \mathbf{j} + 250 \left(\frac{3}{5}\right) \mathbf{k}$$

$$\mathbf{F}_R = \{348.21\mathbf{i} + 75.0\mathbf{j} - 97.487\mathbf{k}\} \text{ N}$$

$$F_R = \sqrt{(348.21)^2 + (75.0)^2 + (-97.487)^2}$$

$$= 369.29 = 369 \text{ N}$$

Ans.

$$\alpha = \cos^{-1}\left(\frac{348.21}{369.29}\right) = 19.5^\circ$$

Ans.

$$\beta = \cos^{-1}\left(\frac{75.0}{369.29}\right) = 78.3^\circ$$

Ans.

$$\gamma = \cos^{-1}\left(\frac{-97.487}{369.29}\right) = 105^\circ$$

Ans.

2-71.

If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of \mathbf{F} so that $\beta < 90^\circ$.

SOLUTION

Force Vectors: By resolving \mathbf{F}_1 and \mathbf{F} into their x, y, and z components, as shown in Figs. a and b, respectively, \mathbf{F}_1 and \mathbf{F} can be expressed in Cartesian vector form as

$$\begin{aligned} \mathbf{F}_1 &= 600 \cos 30^\circ \sin 30^\circ(+\mathbf{i}) + 600 \cos 30^\circ \cos 30^\circ(+\mathbf{j}) + 600 \sin 30^\circ(-\mathbf{k}) \\ &= \{259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}\} \text{ N} \end{aligned}$$

$$\mathbf{F} = 500 \cos \alpha \mathbf{i} + 500 \cos \beta \mathbf{j} + 500 \cos \gamma \mathbf{k}$$

Since the resultant force \mathbf{F}_R is directed towards the positive y axis, then

$$\mathbf{F}_R = F_R \mathbf{j}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}$$

$$F_R \mathbf{j} = (259.81\mathbf{i} + 450\mathbf{j} - 300\mathbf{k}) + (500 \cos \alpha \mathbf{i} + 500 \cos \beta \mathbf{j} + 500 \cos \gamma \mathbf{k})$$

$$F_R \mathbf{j} = (259.81 + 500 \cos \alpha)\mathbf{i} + (450 + 500 \cos \beta)\mathbf{j} + (500 \cos \gamma - 300)\mathbf{k}$$

Equating the \mathbf{i} , \mathbf{j} , and \mathbf{k} components,

$$0 = 259.81 + 500 \cos \alpha$$

$$\alpha = 121.31^\circ = 121^\circ$$

$$F_R = 450 + 500 \cos \beta$$

$$0 = 500 \cos \gamma - 300$$

$$\gamma = 53.13^\circ = 53.1^\circ$$

However, since $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, $\alpha = 121.31^\circ$, and $\gamma = 53.13^\circ$,

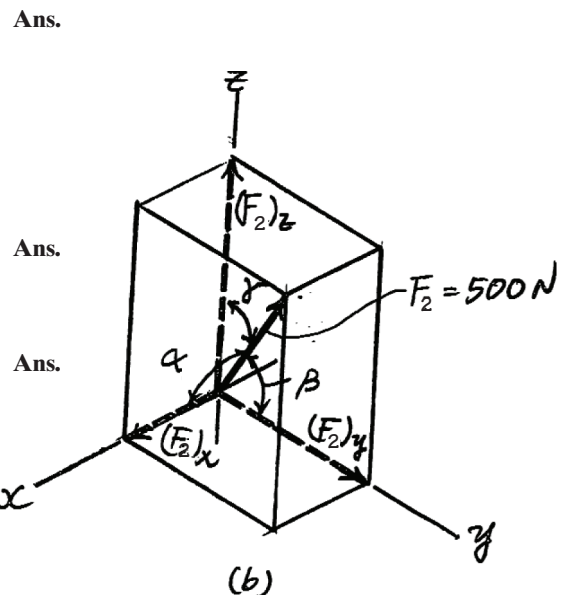
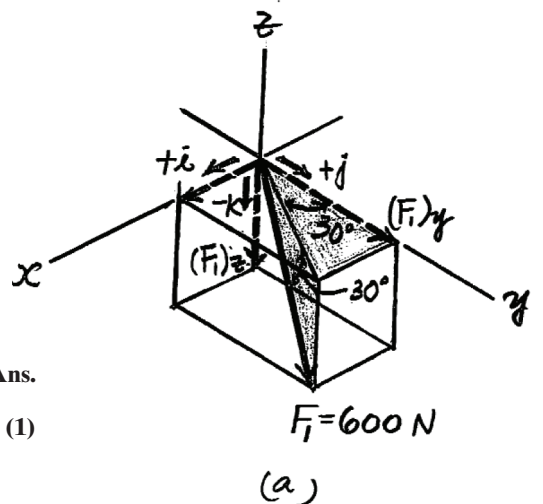
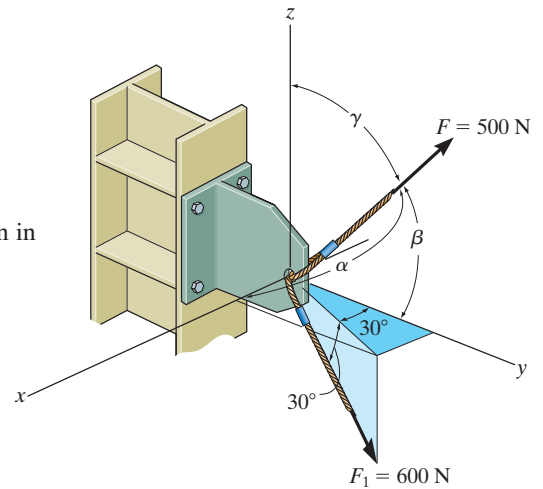
$$\cos \beta = \pm \sqrt{1 - \cos^2 121.31^\circ - \cos^2 53.13^\circ} = \pm 0.6083$$

If we substitute $\cos \beta = 0.6083$ into Eq. (1),

$$F_R = 450 + 500(0.6083) = 754 \text{ N}$$

and

$$\beta = \cos^{-1}(0.6083) = 52.5^\circ$$



2-72.

Specify the magnitude F_3 and directions α_3 , β_3 , and γ_3 of \mathbf{F}_3 so that the resultant force of the three forces is \mathbf{F}_R .

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

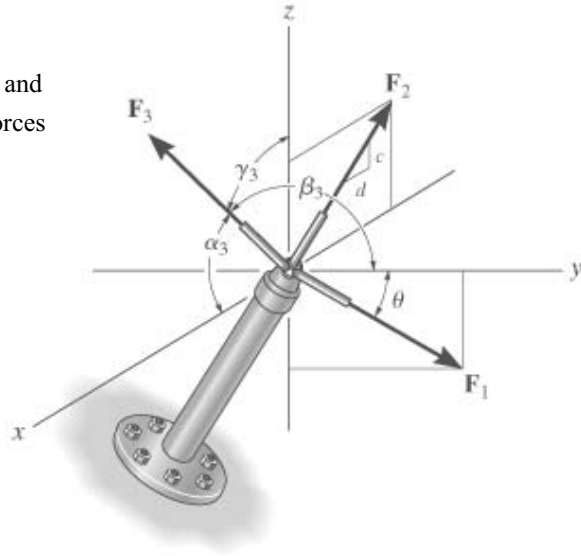
Given:

$$F_1 = 12 \text{ kN} \quad c = 5$$

$$F_2 = 10 \text{ kN} \quad d = 12$$

$$\theta = 30 \text{ deg}$$

$$\mathbf{F}_R = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} \text{ kN}$$



Solution:

Initial Guesses: $F_{3x} = 1 \text{ kN} \quad F_{3y} = 1 \text{ kN} \quad F_{3z} = 1 \text{ kN}$

Given
$$\mathbf{F}_R = \begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} + F_1 \begin{pmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{pmatrix} + \frac{F_2}{\sqrt{c^2 + d^2}} \begin{pmatrix} -d \\ 0 \\ c \end{pmatrix}$$

$$\begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} = \text{Find}(F_{3x}, F_{3y}, F_{3z}) \quad \mathbf{F}_3 = \begin{pmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \end{pmatrix} \quad \mathbf{F}_3 = \begin{pmatrix} 9.2 \\ -1.4 \\ 2.2 \end{pmatrix} \text{ kN} \quad |\mathbf{F}_3| = 9.6 \text{ kN} \quad \text{Ans.}$$

$$\begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{pmatrix} = \text{acos}\left(\frac{\mathbf{F}_3}{|\mathbf{F}_3|}\right) \quad \begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{pmatrix} = \begin{pmatrix} 15.5 \\ 98.4 \\ 77.0 \end{pmatrix} \text{ deg} \quad \text{Ans.}$$

2-73.

Determine the magnitude and coordinate direction angles of \mathbf{F}_3 so that the resultant of the three forces acts along the positive y axis and has magnitude F .

Given:

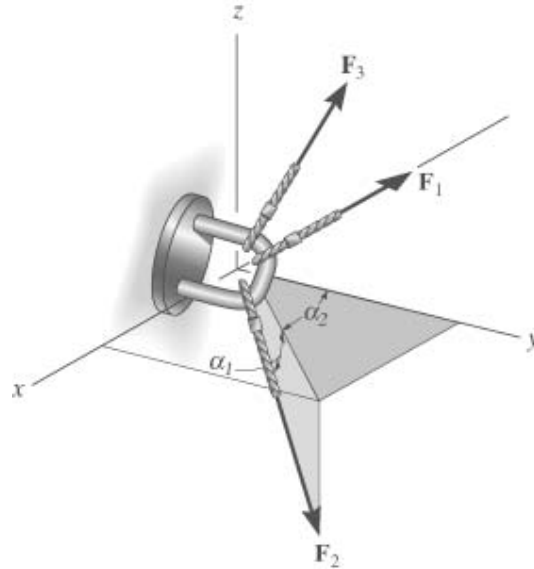
$$F = 600 \text{ N}$$

$$F_1 = 180 \text{ N}$$

$$F_2 = 300 \text{ N}$$

$$\alpha_1 = 30^\circ$$

$$\alpha_2 = 40^\circ$$



Solution:

Initial guesses:

$$\alpha = 40^\circ \quad \gamma = 50^\circ$$

$$\beta = 50^\circ \quad F_3 = 45 \text{ N}$$

Given

$$F_{Rx} = \Sigma F_x; \quad 0 = -F_1 + F_2 \cos(\alpha_1) \sin(\alpha_2) + F_3 \cos(\alpha)$$

$$F_{Ry} = \Sigma F_y; \quad F = F_2 \cos(\alpha_1) \cos(\alpha_2) + F_3 \cos(\beta)$$

$$F_{Rz} = \Sigma F_z; \quad 0 = -F_2 \sin(\alpha_1) + F_3 \cos(\gamma)$$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$

$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma)$$

$$F_3 = 428 \text{ N}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 88.3 \\ 20.6 \\ 69.5 \end{pmatrix} \text{ deg}$$

Ans.

2-74.

Determine the magnitude and coordinate direction angles of \mathbf{F}_3 so that the resultant of the three forces is zero.

Given:

$$F_1 = 180 \text{ N} \quad \alpha_1 = 30 \text{ deg}$$

$$F_2 = 300 \text{ N} \quad \alpha_2 = 40 \text{ deg}$$

Solution:

Initial guesses:

$$\alpha = 40 \text{ deg} \quad \gamma = 50 \text{ deg}$$

$$\beta = 50 \text{ deg} \quad F_3 = 45 \text{ N}$$

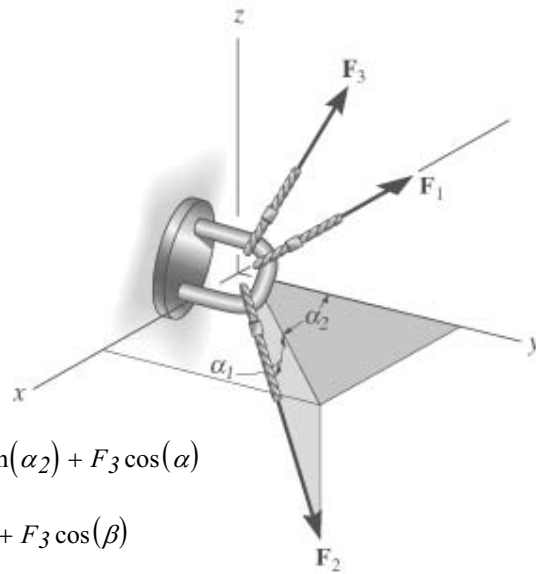
Given

$$F_{Rx} = \Sigma F_x; \quad 0 = -F_1 + F_2 \cos(\alpha_1) \sin(\alpha_2) + F_3 \cos(\alpha)$$

$$F_{Ry} = \Sigma F_y; \quad 0 = F_2 \cos(\alpha_1) \cos(\alpha_2) + F_3 \cos(\beta)$$

$$F_{Rz} = \Sigma F_z; \quad 0 = -F_2 \sin(\alpha_1) + F_3 \cos(\gamma)$$

$$\cos(\alpha)^2 + \cos(\beta)^2 + \cos(\gamma)^2 = 1$$



$$\begin{pmatrix} F_3 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{Find}(F_3, \alpha, \beta, \gamma)$$

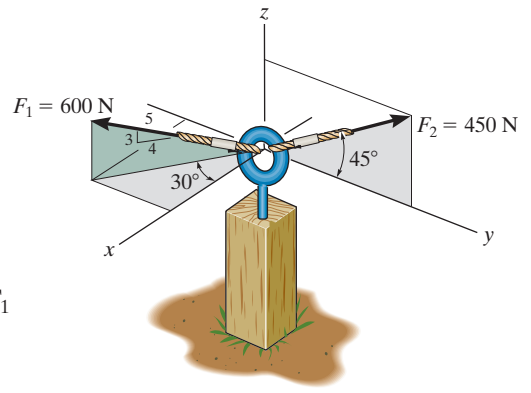
$$F_3 = 250 \text{ N}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 87.0 \\ 142.9 \\ 53.1 \end{pmatrix} \text{ deg}$$

Ans.

2-75.

Determine the coordinate direction angles of force \mathbf{F}_1 .



SOLUTION

Rectangular Components: By referring to Figs. *a*, the x , y , and z components of \mathbf{F}_1 can be written as

$$(F_1)_x = 600\left(\frac{4}{5}\right) \cos 30^\circ \text{ N} \quad (F_1)_y = 600\left(\frac{4}{5}\right) \sin 30^\circ \text{ N} \quad (F_1)_z = 600\left(\frac{3}{5}\right) \text{ N}$$

Thus, \mathbf{F}_1 expressed in Cartesian vector form can be written as

$$\begin{aligned} \mathbf{F}_1 &= 600\left\{\frac{4}{5} \cos 30^\circ(+\mathbf{i}) + \frac{4}{5} \sin 30^\circ(-\mathbf{j}) + \frac{3}{5}(+\mathbf{k})\right\} \text{ N} \\ &= 600[0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}] \text{ N} \end{aligned}$$

Therefore, the unit vector for \mathbf{F}_1 is given by

$$\mathbf{u}_{F_1} = \frac{\mathbf{F}_1}{F_1} = \frac{600(0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k})}{600} = 0.6928\mathbf{i} - 0.4\mathbf{j} + 0.6\mathbf{k}$$

The coordinate direction angles of \mathbf{F}_1 are

$$\alpha = \cos^{-1}(u_{F_1})_x = \cos^{-1}(0.6928) = 46.1^\circ$$

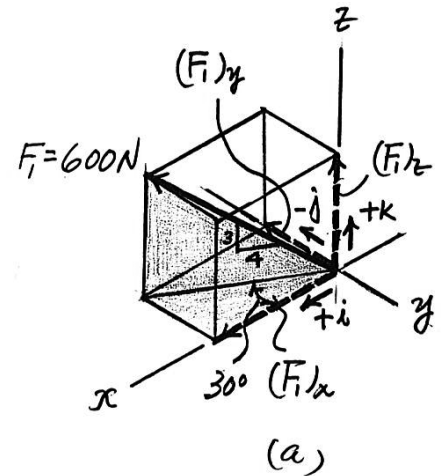
Ans.

$$\beta = \cos^{-1}(u_{F_1})_y = \cos^{-1}(-0.4) = 114^\circ$$

Ans.

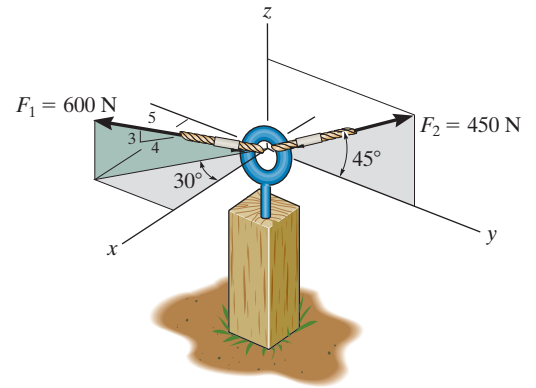
$$\gamma = \cos^{-1}(u_{F_1})_z = \cos^{-1}(0.6) = 53.1^\circ$$

Ans.



2-76.

Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

Force Vectors: By resolving \mathbf{F}_1 and \mathbf{F}_2 into their x , y , and z components, as shown in Figs. *a* and *b*, respectively, they are expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\cos 30^\circ(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\sin 30^\circ(-\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 450 \cos 45^\circ(+\mathbf{j}) + 450 \sin 45^\circ(+\mathbf{k})$$

$$= \{318.20\mathbf{j} + 318.20\mathbf{k}\} \text{ N}$$

Resultant Force: The resultant force acting on the eyebolt can be obtained by vectorally adding \mathbf{F}_1 and \mathbf{F}_2 . Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}) + (318.20\mathbf{j} + 318.20\mathbf{k})$$

$$= \{415.69\mathbf{i} + 78.20\mathbf{j} + 678.20\mathbf{k}\} \text{ N}$$

The magnitude of \mathbf{F}_R is given by

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

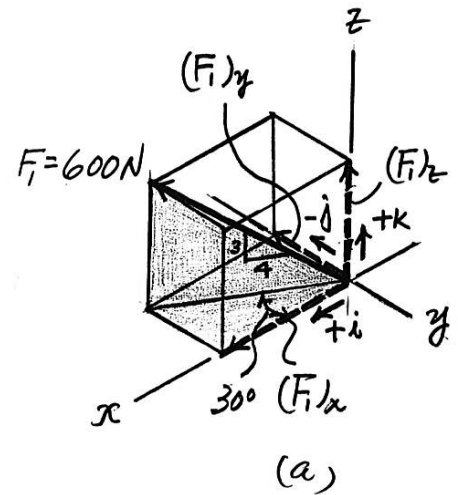
$$= \sqrt{(415.69)^2 + (78.20)^2 + (678.20)^2} = 799.29 \text{ N} = 799 \text{ N}$$

The coordinate direction angles of \mathbf{F}_R are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{415.69}{799.29}\right) = 58.7^\circ$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{78.20}{799.29}\right) = 84.4^\circ$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{678.20}{799.29}\right) = 32.0^\circ$$

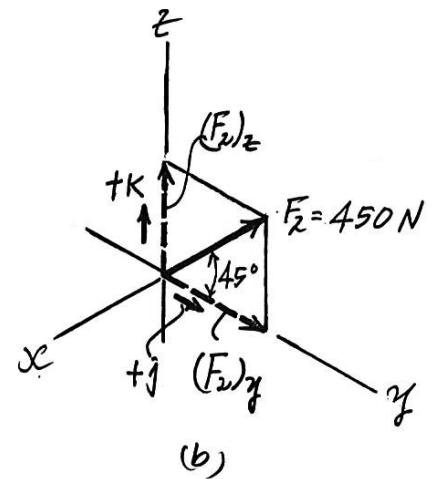


Ans.

Ans.

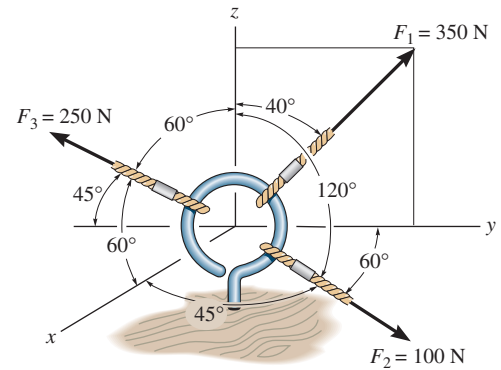
Ans.

Ans.



2-77.

The cables attached to the screw eye are subjected to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$$\begin{aligned} \mathbf{F}_1 &= 350\{\sin 40^\circ \mathbf{j} + \cos 40^\circ \mathbf{k}\} \text{ N} \\ &= \{224.98\mathbf{j} + 268.12\mathbf{k}\} \text{ N} \\ &= \{225\mathbf{j} + 268\mathbf{k}\} \text{ N} \end{aligned}$$

Ans.

$$\begin{aligned} \mathbf{F}_2 &= 100\{\cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k}\} \text{ N} \\ &= \{70.71\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\} \text{ N} \\ &= \{70.7\mathbf{i} + 50.0\mathbf{j} - 50.0\mathbf{k}\} \text{ N} \end{aligned}$$

Ans.

$$\begin{aligned} \mathbf{F}_3 &= 250\{\cos 60^\circ \mathbf{i} + \cos 135^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}\} \text{ N} \\ &= \{125.0\mathbf{i} - 176.78\mathbf{j} + 125.0\mathbf{k}\} \text{ N} \\ &= \{125\mathbf{i} - 177\mathbf{j} + 125\mathbf{k}\} \text{ N} \end{aligned}$$

Ans.

Resultant Force:

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= \{(70.71 + 125.0)\mathbf{i} + (224.98 + 50.0 - 176.78)\mathbf{j} + (268.12 - 50.0 + 125.0)\mathbf{k}\} \text{ N} \\ &= \{195.71\mathbf{i} + 98.20\mathbf{j} + 343.12\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of the resultant force is

$$\begin{aligned} F_R &= \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2} \\ &= \sqrt{195.71^2 + 98.20^2 + 343.12^2} \\ &= 407.03 \text{ N} = 407 \text{ N} \end{aligned}$$

Ans.

The coordinate direction angles are

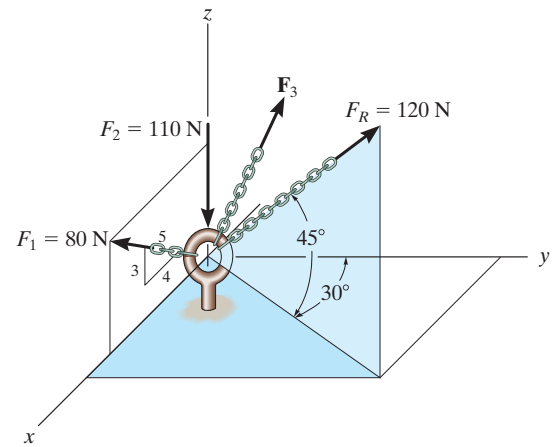
$$\cos \alpha = \frac{F_{R_x}}{F_R} = \frac{195.71}{407.03} \quad \alpha = 61.3^\circ \quad \text{Ans.}$$

$$\cos \beta = \frac{F_{R_y}}{F_R} = \frac{98.20}{407.03} \quad \beta = 76.0^\circ \quad \text{Ans.}$$

$$\cos \gamma = \frac{F_{R_z}}{F_R} = \frac{343.12}{407.03} \quad \gamma = 32.5^\circ \quad \text{Ans.}$$

2-78.

Three forces act on the ring. If the resultant force \mathbf{F}_R has a magnitude and direction as shown, determine the magnitude and the coordinate direction angles of force \mathbf{F}_3 .



SOLUTION

Cartesian Vector Notation:

$$\begin{aligned}\mathbf{F}_R &= 120\{\cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k}\} \text{ N} \\ &= \{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\} \text{ N}\end{aligned}$$

$$\mathbf{F}_1 = 80\left\{\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right\} \text{ N} = \{64.0\mathbf{i} + 48.0\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = \{-110\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_3 = \{F_{3_x}\mathbf{i} + F_{3_y}\mathbf{j} + F_{3_z}\mathbf{k}\} \text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$\{42.43\mathbf{i} + 73.48\mathbf{j} + 84.85\mathbf{k}\} = \{(64.0 + F_{3_x})\mathbf{i} + F_{3_y}\mathbf{j} + (48.0 - 110 + F_{3_z})\mathbf{k}\}$$

Equating \mathbf{i} , \mathbf{j} and \mathbf{k} components, we have

$$64.0 + F_{3_x} = 42.43 \qquad F_{3_x} = -21.57 \text{ N}$$

$$F_{3_y} = 73.48 \text{ N}$$

$$48.0 - 110 + F_{3_z} = 84.85 \qquad F_{3_z} = 146.85 \text{ N}$$

The magnitude of force \mathbf{F}_3 is

$$\begin{aligned}F_3 &= \sqrt{F_{3_x}^2 + F_{3_y}^2 + F_{3_z}^2} \\ &= \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2} \\ &= 165.62 \text{ N} = \mathbf{166 \text{ N}}\end{aligned}$$

Ans.

The coordinate direction angles for \mathbf{F}_3 are

$$\cos \alpha = \frac{F_{3_x}}{F_3} = \frac{-21.57}{165.62} \qquad \alpha = 97.5^\circ \qquad \mathbf{Ans.}$$

$$\cos \beta = \frac{F_{3_y}}{F_3} = \frac{73.48}{165.62} \qquad \beta = 63.7^\circ \qquad \mathbf{Ans.}$$

$$\cos \gamma = \frac{F_{3_z}}{F_3} = \frac{146.85}{165.62} \qquad \gamma = 27.5^\circ \qquad \mathbf{Ans.}$$

2-79.

Determine the coordinate direction angles of \mathbf{F}_1 and \mathbf{F}_R .

SOLUTION

Unit Vector of \mathbf{F}_1 and \mathbf{F}_R :

$$\mathbf{u}_{F_1} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k} = 0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\begin{aligned}\mathbf{u}_R &= \cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k} \\ &= 0.3536\mathbf{i} + 0.6124\mathbf{j} + 0.7071\mathbf{k}\end{aligned}$$

Thus, the coordinate direction angles \mathbf{F}_1 and \mathbf{F}_R are

$$\cos \alpha_{F_1} = 0.8 \quad \alpha_{F_1} = 36.9^\circ$$

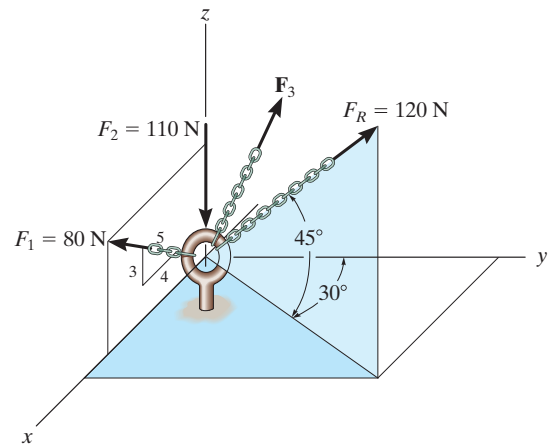
$$\cos \beta_{F_1} = 0 \quad \beta_{F_1} = 90.0^\circ$$

$$\cos \gamma_{F_1} = 0.6 \quad \gamma_{F_1} = 53.1^\circ$$

$$\cos \alpha_R = 0.3536 \quad \alpha_R = 69.3^\circ$$

$$\cos \beta_R = 0.6124 \quad \beta_R = 52.2^\circ$$

$$\cos \gamma_R = 0.7071 \quad \gamma_R = 45.0^\circ$$



Ans.

Ans.

Ans.

Ans.

Ans.

Ans.

2-80. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of \mathbf{F}_1 so that the resultant force acting on the mast is $\mathbf{F}_R = \{350\mathbf{i}\}$ N.

$$\mathbf{F}_1 = 500 \cos \alpha_1 \mathbf{i} + 500 \cos \beta_1 \mathbf{j} + 500 \cos \gamma_1 \mathbf{k}$$

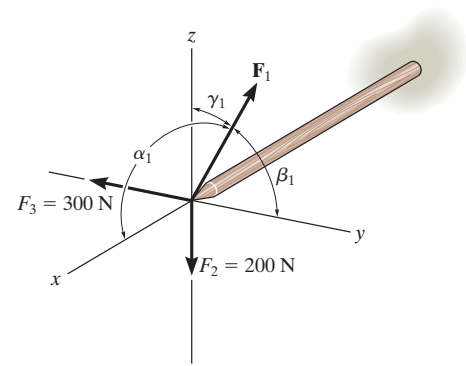
$$\mathbf{F}_R = \mathbf{F}_1 + (-300\mathbf{j}) + (-200\mathbf{k})$$

$$350\mathbf{i} = 500 \cos \alpha_1 \mathbf{i} + (500 \cos \beta_1 - 300)\mathbf{j} + (500 \cos \gamma_1 - 200)\mathbf{k}$$

$$350 = 500 \cos \alpha_1; \quad \alpha_1 = 45.6^\circ \quad \text{Ans.}$$

$$0 = 500 \cos \beta_1 - 300; \quad \beta_1 = 53.1^\circ \quad \text{Ans.}$$

$$0 = 500 \cos \gamma_1 - 200; \quad \gamma_1 = 66.4^\circ \quad \text{Ans.}$$



2-81. The mast is subjected to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of \mathbf{F}_1 so that the resultant force acting on the mast is zero.

$$\mathbf{F}_1 = \{ 500 \cos \alpha_1 \mathbf{i} + 500 \cos \beta_1 \mathbf{j} + 500 \cos \gamma_1 \mathbf{k} \} \text{ N}$$

$$\mathbf{F}_2 = \{-200\mathbf{k}\} \text{ N}$$

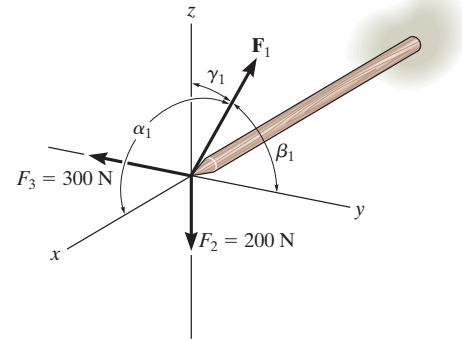
$$\mathbf{F}_3 = \{-300\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$$

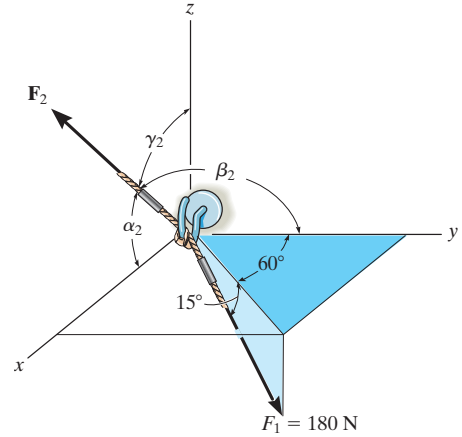
$$500 \cos \alpha_1 = 0; \quad \alpha_1 = 90^\circ \quad \text{Ans.}$$

$$500 \cos \beta_1 = 300; \quad \beta_1 = 53.1^\circ \quad \text{Ans.}$$

$$500 \cos \gamma_1 = 200; \quad \gamma_1 = 66.4^\circ \quad \text{Ans.}$$



2-82. Determine the magnitude and coordinate direction angles of \mathbf{F}_2 so that the resultant of the two forces acts along the positive x axis and has a magnitude of 500 N.



$$\mathbf{F}_1 = (180 \cos 15^\circ) \sin 60^\circ \mathbf{i} + (180 \cos 15^\circ) \cos 60^\circ \mathbf{j} - 180 \sin 15^\circ \mathbf{k}$$

$$= 150.57 \mathbf{i} + 86.93 \mathbf{j} - 46.59 \mathbf{k}$$

$$\mathbf{F}_2 = F_2 \cos \alpha_2 \mathbf{i} + F_2 \cos \beta_2 \mathbf{j} + F_2 \cos \gamma_2 \mathbf{k}$$

$$\mathbf{F}_R = (500 \mathbf{i}) \text{ N}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

i components :

$$500 = 150.57 + F_2 \cos \alpha_2$$

$$F_{2x} = F_2 \cos \alpha_2 = 349.43$$

j components :

$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_{2y} = F_2 \cos \beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_{2z} = F_2 \cos \gamma_2 = 46.59$$

Thus,

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{(349.43)^2 + (-86.93)^2 + (46.59)^2}$$

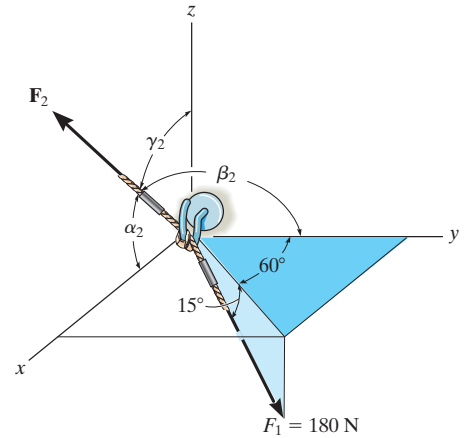
$$F_2 = 363 \text{ N} \quad \text{Ans.}$$

$$\alpha_2 = 15.8^\circ \quad \text{Ans.}$$

$$\beta_2 = 104^\circ \quad \text{Ans.}$$

$$\gamma_2 = 82.6^\circ \quad \text{Ans.}$$

2-83. Determine the magnitude and coordinate direction angles of F_2 so that the resultant of the two forces is zero.



$$F_1 = (180 \cos 15^\circ) \sin 60^\circ \mathbf{i} + (180 \cos 15^\circ) \cos 60^\circ \mathbf{j} - 180 \sin 15^\circ \mathbf{k}$$

$$= 150.57 \mathbf{i} + 86.93 \mathbf{j} - 46.59 \mathbf{k}$$

$$F_2 = F_2 \cos \alpha_2 \mathbf{i} + F_2 \cos \beta_2 \mathbf{j} + F_2 \cos \gamma_2 \mathbf{k}$$

$$F_R = \mathbf{0}$$

i components :

$$0 = 150.57 + F_2 \cos \alpha_2$$

$$F_2 \cos \alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_2 \cos \beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_2 \cos \gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving.

$$F_2 = 180 \text{ N} \quad \text{Ans.}$$

$$\alpha_2 = 147^\circ \quad \text{Ans.}$$

$$\beta_2 = 119^\circ \quad \text{Ans.}$$

$$\gamma_2 = 75.0^\circ \quad \text{Ans.}$$

2-84.

The pole is subjected to the force \mathbf{F} , which has components acting along the x , y , z axes as shown. If the magnitude of \mathbf{F} is 3 kN, $\beta = 30^\circ$, and $\gamma = 75^\circ$, determine the magnitudes of its three components.

SOLUTION

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

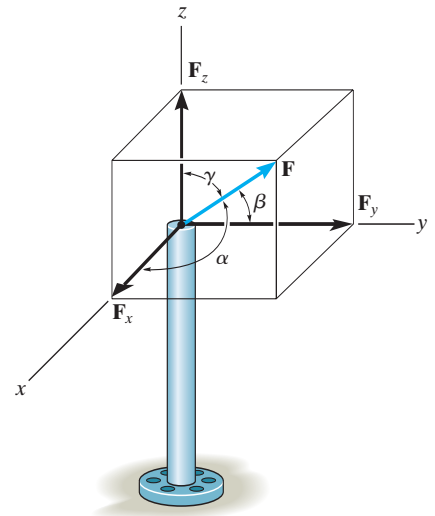
$$\cos^2 \alpha + \cos^2 30^\circ + \cos^2 75^\circ = 1$$

$$\alpha = 64.67^\circ$$

$$F_x = 3 \cos 64.67^\circ = 1.28 \text{ kN}$$

$$F_y = 3 \cos 30^\circ = 2.60 \text{ kN}$$

$$F_z = 3 \cos 75^\circ = 0.776 \text{ kN}$$

Ans.**Ans.****Ans.**

2-85.

The pole is subjected to the force \mathbf{F} which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75^\circ$, determine the magnitudes of \mathbf{F} and \mathbf{F}_y .

SOLUTION

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

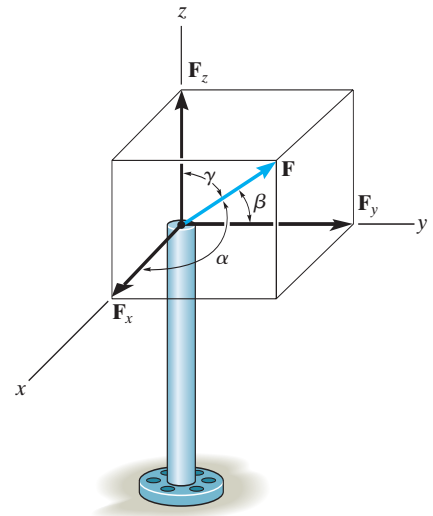
$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75^\circ + \left(\frac{1.25}{F}\right)^2 = 1$$

$$F = 2.02 \text{ kN}$$

$$F_y = 2.02 \cos 75^\circ = 0.523 \text{ kN}$$

Ans.

Ans.



2-87.

Determine the lengths of wires AD , BD , and CD . The ring at D is midway between A and B .

SOLUTION

$$D\left(\frac{2+0}{2}, \frac{0+2}{2}, \frac{1.5+0.5}{2}\right) \text{ m} = D(1, 1, 1) \text{ m}$$

$$\begin{aligned}\mathbf{r}_{AD} &= (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1.5)\mathbf{k} \\ &= -1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}\end{aligned}$$

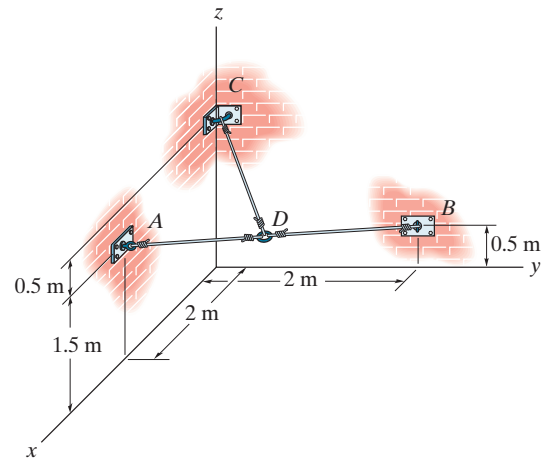
$$\begin{aligned}\mathbf{r}_{BD} &= (1-0)\mathbf{i} + (1-2)\mathbf{j} + (1-0.5)\mathbf{k} \\ &= 1\mathbf{i} - 1\mathbf{j} + 0.5\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{r}_{CD} &= (1-0)\mathbf{i} + (1-0)\mathbf{j} + (1-2)\mathbf{k} \\ &= 1\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}\end{aligned}$$

$$r_{AD} = \sqrt{(-1)^2 + 1^2 + (-0.5)^2} = 1.50 \text{ m}$$

$$r_{BD} = \sqrt{1^2 + (-1)^2 + 0.5^2} = 1.50 \text{ m}$$

$$r_{CD} = \sqrt{1^2 + 1^2 + (-1)^2} = 1.73 \text{ m}$$



Ans.

Ans.

Ans.

2-88.

Determine the length of member AB of the truss by first establishing a Cartesian position vector from A to B and then determining its magnitude.

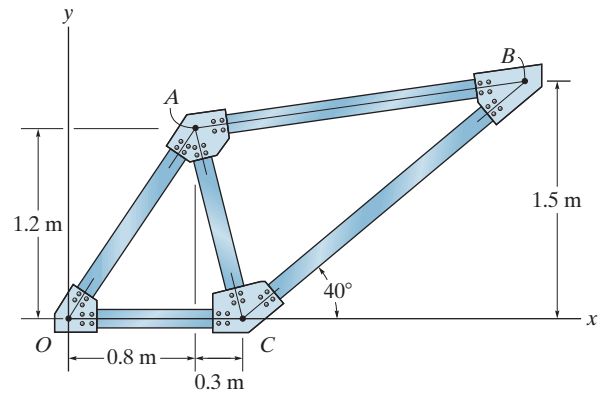
SOLUTION

$$\mathbf{r}_{AB} = (1.1) = \frac{1.5}{\tan 40^\circ} - 0.80\mathbf{i} + (1.5 - 1.2)\mathbf{j}$$

$$\mathbf{r}_{AB} = \{2.09\mathbf{i} + 0.3\mathbf{j}\} \text{ m}$$

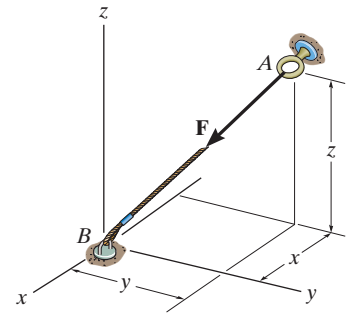
$$r_{AB} = \sqrt{(2.09)^2 + (0.3)^2} = 2.11 \text{ m}$$

Ans.



2-89.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x , y , z coordinates of point A .



SOLUTION

Position Vector: The position vector \mathbf{r}_{AB} , directed from point A to point B , is given by

$$\begin{aligned}\mathbf{r}_{AB} &= [0 - (-x)]\mathbf{i} + [0 - (-y)]\mathbf{j} + [0 - (-z)]\mathbf{k} \\ &= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}\end{aligned}$$

Unit Vector: Knowing the magnitude of \mathbf{r}_{AB} is 9 m, the unit vector for \mathbf{r}_{AB} is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force \mathbf{F} is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force \mathbf{F} is also directed from point A to point B , then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the \mathbf{i} , \mathbf{j} , and \mathbf{k} components,

$$\frac{x}{9} = 0.5623 \qquad x = 5.06 \text{ m} \qquad \text{Ans.}$$

$$\frac{-y}{9} = -0.4016 \qquad y = 3.61 \text{ m} \qquad \text{Ans.}$$

$$\frac{-z}{9} = 0.7229 \qquad z = 6.51 \text{ m} \qquad \text{Ans.}$$

2-90.

Express \mathbf{F}_B and \mathbf{F}_C in Cartesian vector form.

SOLUTION

Force Vectors: The unit vectors \mathbf{u}_B and \mathbf{u}_C of \mathbf{F}_B and \mathbf{F}_C must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

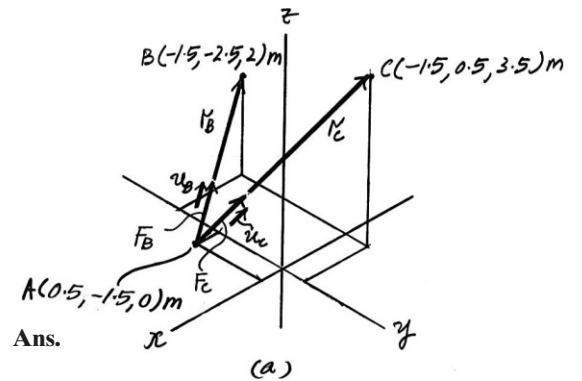
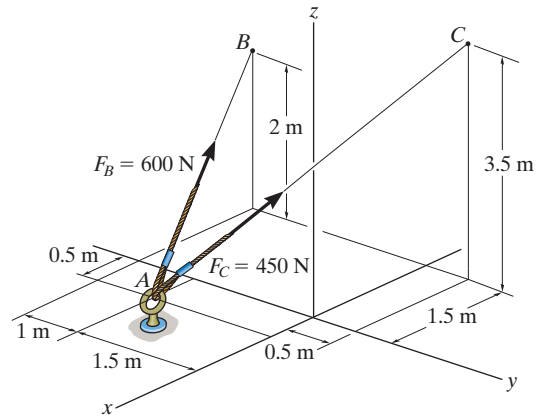
$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B and \mathbf{F}_C are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 600 \left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k} \right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 450 \left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k} \right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\} \text{ N}$$

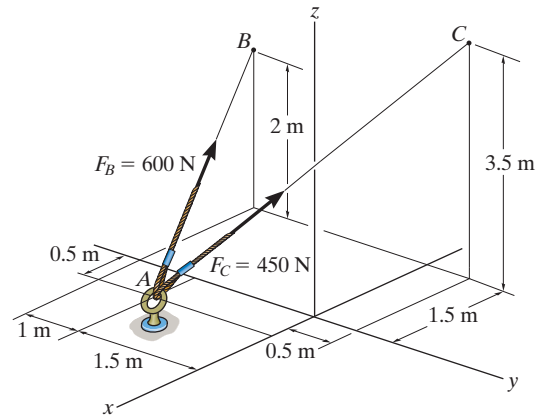


Ans.

Ans.

2-91.

Determine the magnitude and coordinate direction angles of the resultant force acting at A.



SOLUTION

Force Vectors: The unit vectors \mathbf{u}_B and \mathbf{u}_C of \mathbf{F}_B and \mathbf{F}_C must be determined first. From Fig. a

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B and \mathbf{F}_C are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 600 \left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k} \right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 450 \left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k} \right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\} \text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}) + (-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k})$$

$$= \{-600\mathbf{i} + 750\mathbf{k}\} \text{ N}$$

The magnitude of \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{(-600)^2 + 0^2 + 750^2} = 960.47 \text{ N} = 960 \text{ N}$$

The coordinate direction angles of \mathbf{F}_R are

$$\alpha = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{-600}{960.47} \right) = 129^\circ$$

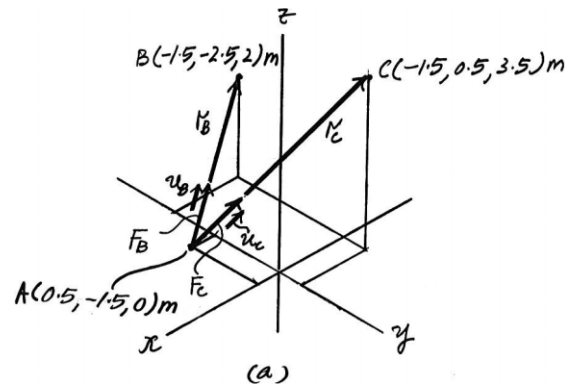
Ans.

$$\beta = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{0}{960.47} \right) = 90^\circ$$

Ans.

$$\gamma = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{750}{960.47} \right) = 38.7^\circ$$

Ans.



2-92.

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

SOLUTION

Force Vectors: The unit vectors \mathbf{u}_B and \mathbf{u}_C of \mathbf{F}_B and \mathbf{F}_C must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2 - 0)\mathbf{i} + (-3 - 0)\mathbf{j} + (0 - 6)\mathbf{k}}{\sqrt{(2 - 0)^2 + (-3 - 0)^2 + (0 - 6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 6)\mathbf{k}}{\sqrt{(3 - 0)^2 + (2 - 0)^2 + (0 - 6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B and \mathbf{F}_C are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 560 \left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 700 \left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\} \text{ N}$$

Resultant Force:

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}) \\ &= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of \mathbf{F}_R is

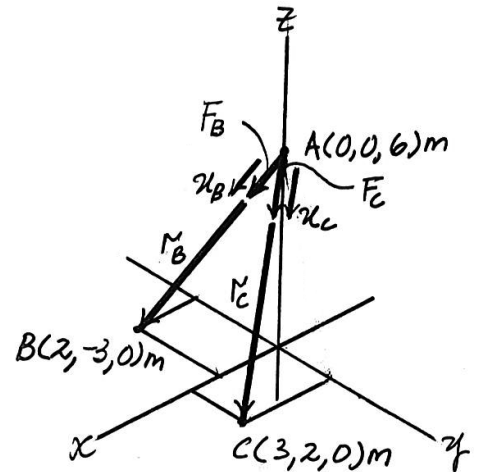
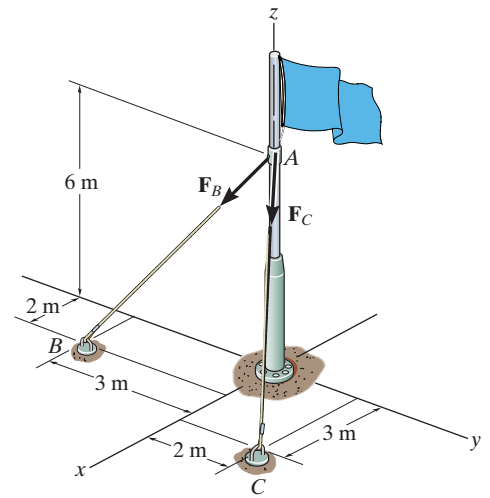
$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{(460)^2 + (-40)^2 + (1080)^2} = 1174.56 \text{ N} = 1.17 \text{ kN} \end{aligned}$$

The coordinate direction angles of \mathbf{F}_R are

$$\alpha = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{460}{1174.56} \right) = 66.9^\circ$$

$$\beta = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{-40}{1174.56} \right) = 92.0^\circ$$

$$\gamma = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{1080}{1174.56} \right) = 157^\circ$$



Ans.

Ans.

Ans.

Ans.

2-93.

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

SOLUTION

Force Vectors: The unit vectors \mathbf{u}_B and \mathbf{u}_C of \mathbf{F}_B and \mathbf{F}_C must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B and \mathbf{F}_C are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 700 \left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 560 \left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\} \text{ N}$$

Resultant Force:

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}) \\ &= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\} \text{ N} \end{aligned}$$

The magnitude of \mathbf{F}_R is

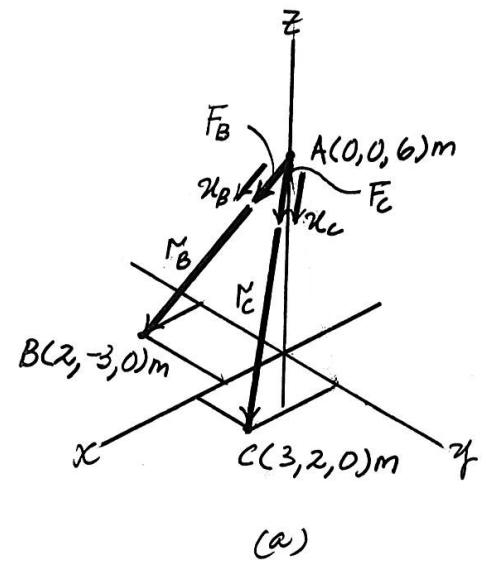
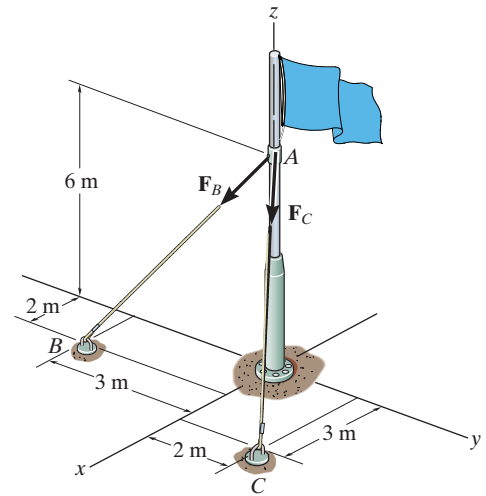
$$\begin{aligned} F_R &= \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} \\ &= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56 \text{ N} = 1.17 \text{ kN} \end{aligned}$$

The coordinate direction angles of \mathbf{F}_R are

$$\alpha = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{440}{1174.56} \right) = 68.0^\circ$$

$$\beta = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{-140}{1174.56} \right) = 96.8^\circ$$

$$\gamma = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{-1080}{1174.56} \right) = 157^\circ$$



Ans.

Ans.

Ans.

Ans.

2-94.

The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α, β, γ of the resultant force. Take $x = 15 \text{ m}, y = 20 \text{ m}$.

SOLUTION

$$\mathbf{F}_{DA} = 400 \left(\frac{15}{34.66} \mathbf{i} + \frac{20}{34.66} \mathbf{j} - \frac{24}{34.66} \mathbf{k} \right) \text{ N}$$

$$\mathbf{F}_{DB} = 800 \left(\frac{-6}{25.06} \mathbf{i} + \frac{4}{25.06} \mathbf{j} - \frac{24}{25.06} \mathbf{k} \right) \text{ N}$$

$$\mathbf{F}_{DC} = 600 \left(\frac{16}{34} \mathbf{i} - \frac{18}{34} \mathbf{j} - \frac{24}{34} \mathbf{k} \right) \text{ N}$$

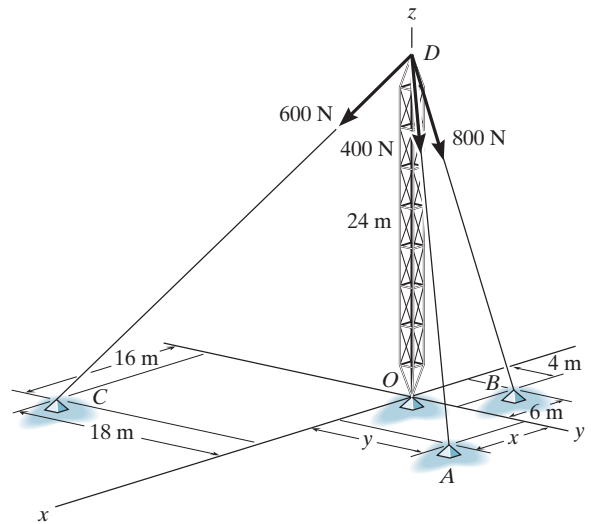
$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC} \\ &= \{263.92\mathbf{i} + 40.86\mathbf{j} - 1466.71\mathbf{k}\} \text{ N} \end{aligned}$$

$$\begin{aligned} F_R &= \sqrt{(263.92)^2 + (40.86)^2 + (-1466.71)^2} \\ &= 1490.83 \text{ N} = 1.49 \text{ kN} \end{aligned}$$

$$\alpha = \cos^{-1} \left(\frac{263.92}{1490.83} \right) = 79.8^\circ$$

$$\beta = \cos^{-1} \left(\frac{40.86}{1490.83} \right) = 88.4^\circ$$

$$\gamma = \cos^{-1} \left(\frac{-1466.71}{1490.83} \right) = 169.7^\circ$$



Ans.

Ans.

Ans.

Ans.

2-95.

At a given instant, the position of a plane at A and a train at B are measured relative to a radar antenna at O . Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B , and then determine its magnitude.

SOLUTION

Position Vector: The coordinates of points A and B are

$$A(-5 \cos 60^\circ \cos 35^\circ, -5 \cos 60^\circ \sin 35^\circ, 5 \sin 60^\circ) \text{ km}$$

$$= A(-2.048, -1.434, 4.330) \text{ km}$$

$$B(2 \cos 25^\circ \sin 40^\circ, 2 \cos 25^\circ \cos 40^\circ, -2 \sin 25^\circ) \text{ km}$$

$$= B(1.165, 1.389, -0.845) \text{ km}$$

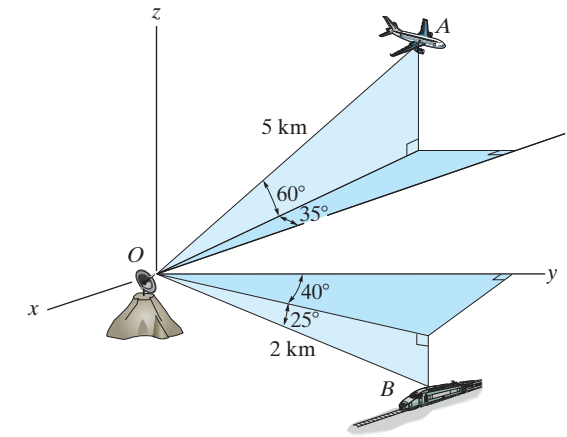
The position vector \mathbf{r}_{AB} can be established from the coordinates of points A and B .

$$\mathbf{r}_{AB} = \{[1.165 - (-2.048)]\mathbf{i} + [1.389 - (-1.434)]\mathbf{j} + (-0.845 - 4.330)\mathbf{k}\} \text{ km}$$

$$= \{3.213\mathbf{i} + 2.822\mathbf{j} - 5.175\mathbf{k}\} \text{ km}$$

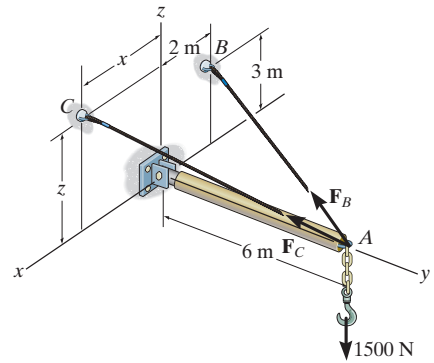
The distance between points A and B is

$$d = r_{AB} = \sqrt{3.213^2 + 2.822^2 + (-5.175)^2} = 6.71 \text{ km}$$



Ans.

2-96. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O , determine the magnitudes of the resultant force and forces F_B and F_C . Set $x = 3$ m and $z = 2$ m.



Force Vectors: The unit vectors \mathbf{u}_B and \mathbf{u}_C must be determined first. From Fig. a ,

$$\mathbf{u}_B = \frac{\mathbf{r}_{B}}{r_B} = \frac{(-2-0)\mathbf{i} + (0-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-0)^2 + (3-0)^2}} = -\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (0-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(3-0)^2 + (0-0)^2 + (2-0)^2}} = \frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B and \mathbf{F}_C are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = -\frac{2}{7}F_B \mathbf{i} - \frac{6}{7}F_B \mathbf{j} + \frac{3}{7}F_B \mathbf{k}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = \frac{3}{7}F_C \mathbf{i} - \frac{6}{7}F_C \mathbf{j} + \frac{2}{7}F_C \mathbf{k}$$

Since the resultant force \mathbf{F}_R is directed along the negative y axis, and the load \mathbf{W} is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R \mathbf{j} \quad \text{and} \quad \mathbf{W} = [-1500\mathbf{k}] \text{ N}$$

Resultant Force: The vector addition of \mathbf{F}_B , \mathbf{F}_C , and \mathbf{W} is equal to \mathbf{F}_R . Thus,

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R \mathbf{j} = \left(-\frac{2}{7}F_B \mathbf{i} - \frac{6}{7}F_B \mathbf{j} + \frac{3}{7}F_B \mathbf{k}\right) + \left(\frac{3}{7}F_C \mathbf{i} - \frac{6}{7}F_C \mathbf{j} + \frac{2}{7}F_C \mathbf{k}\right) + (-1500\mathbf{k})$$

$$-F_R \mathbf{j} = \left(-\frac{2}{7}F_B + \frac{3}{7}F_C\right)\mathbf{i} + \left(-\frac{6}{7}F_B - \frac{6}{7}F_C\right)\mathbf{j} + \left(\frac{3}{7}F_B + \frac{2}{7}F_C - 1500\right)\mathbf{k}$$

Equating the \mathbf{i} , \mathbf{j} , and \mathbf{k} components,

$$0 = -\frac{2}{7}F_B + \frac{3}{7}F_C \quad (1)$$

$$-F_R = -\frac{6}{7}F_B - \frac{6}{7}F_C \quad (2)$$

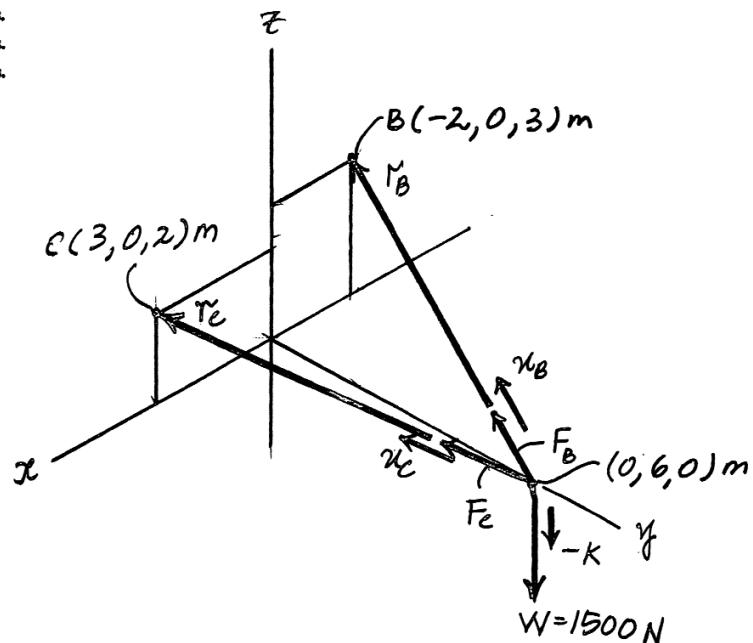
$$0 = \frac{3}{7}F_B + \frac{2}{7}F_C - 1500 \quad (3)$$

Solving Eqs. (1), (2), and (3) yields

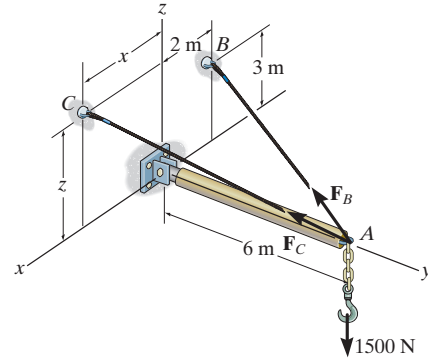
$$F_C = 1615.38 \text{ N} = 1.62 \text{ kN} \quad \text{Ans.}$$

$$F_B = 2423.08 \text{ N} = 2.42 \text{ kN} \quad \text{Ans.}$$

$$F_R = 3461.53 \text{ N} = 3.46 \text{ kN} \quad \text{Ans.}$$



2-97. Two cables are used to secure the overhang boom in position and support the 1500-N load. If the resultant force is directed along the boom from point A towards O , determine the values of x and z for the coordinates of point C and the magnitude of the resultant force. Set $F_B = 1610$ N and $F_C = 2400$ N.



Force Vectors: From Fig. a ,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-2-0)\mathbf{i} + (0-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-0)^2 + (3-0)^2}} = -\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(x-0)\mathbf{i} + (0-0)\mathbf{j} + (z-0)\mathbf{k}}{\sqrt{(x-0)^2 + (0-0)^2 + (z-0)^2}} = \frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i} - \frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j} + \frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Thus,

$$\mathbf{F}_B = F_B \mathbf{u}_B = 1610 \left(-\frac{2}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{3}{7}\mathbf{k} \right) = [-460\mathbf{i} - 1380\mathbf{j} + 690\mathbf{k}] \text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 2400 \left(\frac{x}{\sqrt{x^2+z^2+36}}\mathbf{i} - \frac{6}{\sqrt{x^2+z^2+36}}\mathbf{j} + \frac{z}{\sqrt{x^2+z^2+36}}\mathbf{k} \right)$$

$$= \frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i} - \frac{14400}{\sqrt{x^2+z^2+36}}\mathbf{j} + \frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k}$$

Since the resultant force \mathbf{F}_R is directed along the negative y axis, and the load is directed along the z axis, these two forces can be written as

$$\mathbf{F}_R = -F_R \mathbf{j} \quad \text{and} \quad \mathbf{W} = [-1500\mathbf{k}] \text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{W}$$

$$-F_R \mathbf{j} = (-460\mathbf{i} - 1380\mathbf{j} + 690\mathbf{k}) + \left(\frac{2400x}{\sqrt{x^2+z^2+36}}\mathbf{i} - \frac{14400}{\sqrt{x^2+z^2+36}}\mathbf{j} + \frac{2400z}{\sqrt{x^2+z^2+36}}\mathbf{k} \right) + (-1500\mathbf{k})$$

$$-F_R \mathbf{j} = \left(\frac{2400x}{\sqrt{x^2+z^2+36}} - 460 \right) \mathbf{i} - \left(\frac{14400}{\sqrt{x^2+z^2+36}} + 1380 \right) \mathbf{j} + \left(690 + \frac{2400z}{\sqrt{x^2+z^2+36}} - 1500 \right) \mathbf{k}$$

Equating the \mathbf{i} , \mathbf{j} , and \mathbf{k} components,

$$0 = \frac{2400x}{\sqrt{x^2+z^2+36}} - 460 \quad \frac{2400x}{\sqrt{x^2+z^2+36}} = 460 \quad (1)$$

$$-F_R = - \left(\frac{14400}{\sqrt{x^2+z^2+36}} + 1380 \right) \quad F_R = \frac{14400}{\sqrt{x^2+z^2+36}} + 1380 \quad (2)$$

$$0 = 690 + \frac{2400z}{\sqrt{x^2+z^2+36}} - 1500 \quad \frac{2400z}{\sqrt{x^2+z^2+36}} = 810 \quad (3)$$

Dividing Eq. (1) by Eq. (3), yields

$$x = 0.5679z \quad (4)$$

Substituting Eq. (4) into Eq. (1), and solving

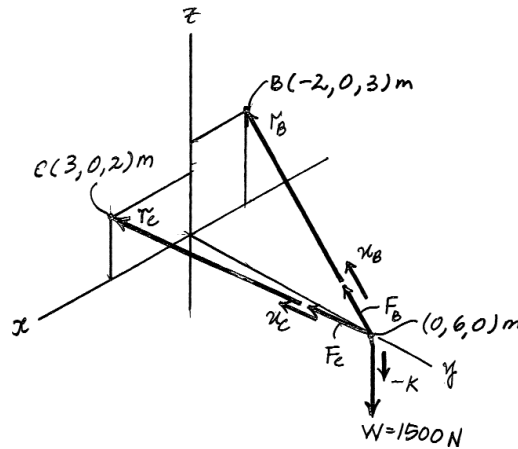
$$z = 2.197 \text{ m} = 2.20 \text{ m} \quad \text{Ans.}$$

Substituting $z = 2.197$ m into Eq. (4), yields

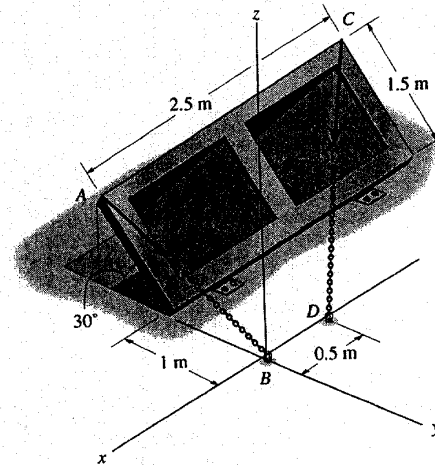
$$x = 1.248 \text{ m} = 1.25 \text{ m} \quad \text{Ans.}$$

Substituting $x = 1.248$ m and $z = 2.197$ m into Eq. (2), yields

$$F_R = 3591.85 \text{ N} = 3.59 \text{ kN} \quad \text{Ans.}$$



2-98. The door is held opened by means of two chains. If the tension in AB and CD is $F_A = 300$ N and $F_C = 250$ N, respectively, express each of these forces in Cartesian vector form.



Unit Vector: First determine the position vector r_{AB} and r_{CD} . The coordinates of points A and C are

$$A(0, -(1 + 1.5\cos 30^\circ), 1.5\sin 30^\circ) \text{ m} = A(0, -2.299, 0.750) \text{ m}$$

$$C(-2.50, -(1 + 1.5\cos 30^\circ), 1.5\sin 30^\circ) \text{ m} = C(-2.50, -2.299, 0.750) \text{ m}$$

Then

$$r_{AB} = \{(0-0)i + [0 - (-2.299)]j + (0-0.750)k\} \text{ m}$$

$$= \{2.299j - 0.750k\} \text{ m}$$

$$r_{AB} = \sqrt{2.299^2 + (-0.750)^2} = 2.418 \text{ m}$$

$$u_{AB} = \frac{r_{AB}}{r_{AB}} = \frac{2.299j - 0.750k}{2.418} = 0.9507j - 0.3101k$$

$$r_{CD} = \{[-0.5 - (-2.5)]i + [0 - (-2.299)]j + (0-0.750)k\} \text{ m}$$

$$= \{2.00i + 2.299j - 0.750k\} \text{ m}$$

$$r_{CD} = \sqrt{2.00^2 + 2.299^2 + (-0.750)^2} = 3.138 \text{ m}$$

$$u_{CD} = \frac{r_{CD}}{r_{CD}} = \frac{2.00i + 2.299j - 0.750k}{3.138} = 0.6373i + 0.7326j - 0.2390k$$

Force Vector:

$$F_A = F_A u_{AB} = 300\{0.9507j - 0.3101k\} \text{ N}$$

$$= \{285.21j - 93.04k\} \text{ N}$$

$$= \{285j - 93.0k\} \text{ N} \quad \text{Ans.}$$

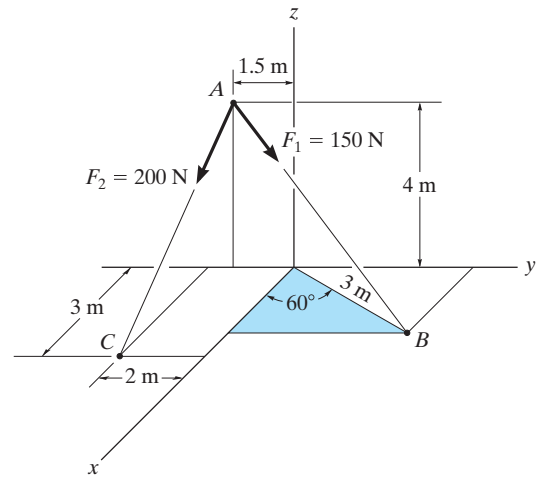
$$F_C = F_C u_{CD} = 250\{0.6373i + 0.7326j - 0.2390k\} \text{ N}$$

$$= \{159.33i + 183.15j - 59.75k\} \text{ N}$$

$$= \{159i + 183j - 59.7k\} \text{ N} \quad \text{Ans.}$$

2-99.

Determine the magnitude and coordinate direction angles of the resultant force acting at point A.



SOLUTION

$$\mathbf{r}_{AC} = \{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}\} \text{ m}$$

$$|\mathbf{r}_{AC}| = \sqrt{3^2 + (-0.5)^2 + (-4)^2} = \sqrt{25.25} = 5.02494$$

$$\mathbf{F}_2 = 200 \left(\frac{3\mathbf{i} - 0.5\mathbf{j} - 4\mathbf{k}}{5.02494} \right) = (119.4044\mathbf{i} - 19.9007\mathbf{j} - 159.2059\mathbf{k})$$

$$\mathbf{r}_{AB} = (3 \cos 60^\circ \mathbf{i} + (1.5 + 3 \sin 60^\circ) \mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_{AB} = (1.5\mathbf{i} + 4.0981\mathbf{j} + 4\mathbf{k})$$

$$|\mathbf{r}_{AB}| = \sqrt{(1.5)^2 + (4.0981)^2 + (-4)^2} = 5.9198$$

$$\mathbf{F}_1 = 150 \left(\frac{1.5\mathbf{i} + 4.0981\mathbf{j} - 4\mathbf{k}}{5.9198} \right) = (38.0079\mathbf{i} + 103.8396\mathbf{j} - 101.3545\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (157.4124\mathbf{i} + 83.9389\mathbf{j} - 260.5607\mathbf{k})$$

$$F_R = \sqrt{(157.4124)^2 + (83.9389)^2 + (-260.5604)^2} = 315.7786 = \mathbf{316 \text{ N}} \quad \text{Ans.}$$

$$\alpha = \cos^{-1} \left(\frac{157.4124}{315.7786} \right) = 60.100^\circ = \mathbf{60.1^\circ} \quad \text{Ans.}$$

$$\beta = \cos^{-1} \left(\frac{83.9389}{315.7786} \right) = 74.585^\circ = \mathbf{74.6^\circ} \quad \text{Ans.}$$

$$\gamma = \cos^{-1} \left(\frac{-260.5607}{315.7786} \right) = 145.60^\circ = \mathbf{146^\circ} \quad \text{Ans.}$$

2-100.

The guy wires are used to support the telephone pole. Represent the force in each wire in Cartesian vector form. Neglect the diameter of the pole.

SOLUTION

Unit Vector:

$$\mathbf{r}_{AC} = \{(-1 - 0)\mathbf{i} + (4 - 0)\mathbf{j} + (0 - 4)\mathbf{k}\} \text{ m} = \{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}\} \text{ m}$$

$$r_{AC} = \sqrt{(-1)^2 + 4^2 + (-4)^2} = 5.745 \text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-1\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}}{5.745} = -0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}$$

$$\mathbf{r}_{BD} = \{(2 - 0)\mathbf{i} + (-3 - 0)\mathbf{j} + (0 - 5.5)\mathbf{k}\} \text{ m} = \{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}\} \text{ m}$$

$$r_{BD} = \sqrt{2^2 + (-3)^2 + (-5.5)^2} = 6.576 \text{ m}$$

$$\mathbf{u}_{BD} = \frac{\mathbf{r}_{BD}}{r_{BD}} = \frac{2\mathbf{i} - 3\mathbf{j} - 5.5\mathbf{k}}{6.576} = 0.3041\mathbf{i} - 0.4562\mathbf{j} - 0.8363\mathbf{k}$$

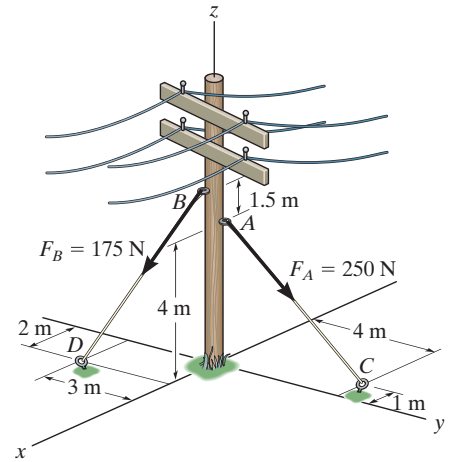
Force Vector:

$$\begin{aligned} \mathbf{F}_A &= F_A \mathbf{u}_{AC} = 250\{-0.1741\mathbf{i} + 0.6963\mathbf{j} - 0.6963\mathbf{k}\} \text{ N} \\ &= \{-43.52\mathbf{i} + 174.08\mathbf{j} - 174.08\mathbf{k}\} \text{ N} \\ &= \{-43.5\mathbf{i} + 174\mathbf{j} - 174\mathbf{k}\} \text{ N} \end{aligned}$$

Ans.

$$\begin{aligned} \mathbf{F}_B &= F_B \mathbf{u}_{BD} = 175\{0.3041\mathbf{i} - 0.4562\mathbf{j} - 0.8363\mathbf{k}\} \text{ N} \\ &= \{53.22\mathbf{i} - 79.83\mathbf{j} - 146.36\mathbf{k}\} \text{ N} \\ &= \{53.2\mathbf{i} - 79.8\mathbf{j} - 146\mathbf{k}\} \text{ N} \end{aligned}$$

Ans.



2-101.

The force acting on the man, caused by his pulling on the anchor cord, is \mathbf{F} . If the length of the cord is L , determine the coordinates $A(x, y, z)$ of the anchor.

Given:

$$\mathbf{F} = \begin{pmatrix} 40 \\ 20 \\ -50 \end{pmatrix} \text{ N}$$

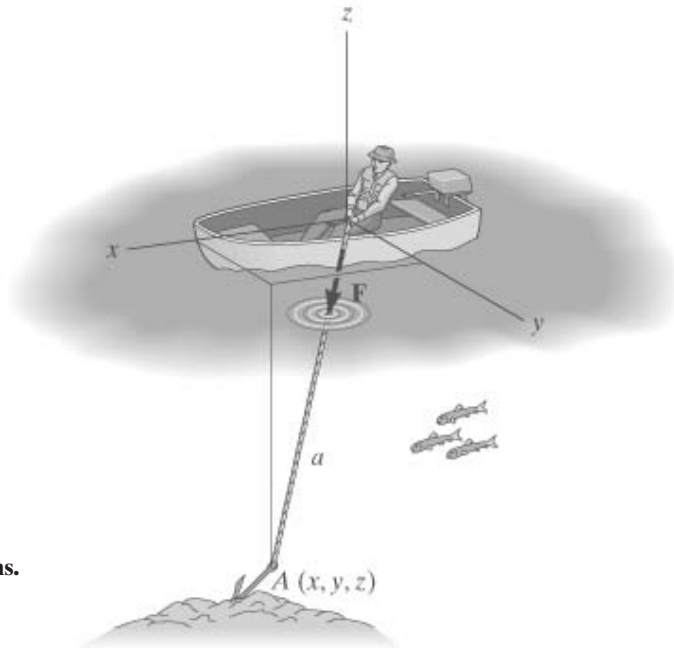
$$L = 25 \text{ m}$$

Solution:

$$\mathbf{r} = L \frac{\mathbf{F}}{|\mathbf{F}|}$$

$$\mathbf{r} = \begin{pmatrix} 14.9 \\ 7.5 \\ -18.6 \end{pmatrix} \text{ m}$$

Ans.



2-102.

Each of the four forces acting at E has a magnitude of 28 kN. Express each force as a Cartesian vector and determine the resultant force.

SOLUTION

$$\mathbf{F}_{EA} = 28 \left(\frac{6}{14} \mathbf{i} - \frac{4}{14} \mathbf{j} - \frac{12}{14} \mathbf{k} \right)$$

$$\mathbf{F}_{EA} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_{EB} = 28 \left(\frac{6}{14} \mathbf{i} + \frac{4}{14} \mathbf{j} - \frac{12}{14} \mathbf{k} \right)$$

$$\mathbf{F}_{EB} = \{12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_{EC} = 28 \left(\frac{-6}{14} \mathbf{i} + \frac{4}{14} \mathbf{j} - \frac{12}{14} \mathbf{k} \right)$$

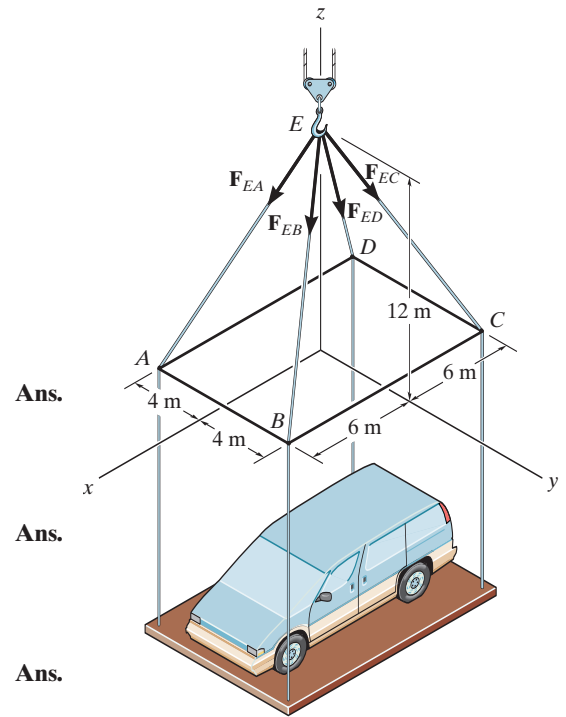
$$\mathbf{F}_{EC} = \{-12\mathbf{i} + 8\mathbf{j} - 24\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_{ED} = 28 \left(\frac{-6}{14} \mathbf{i} - \frac{4}{14} \mathbf{j} - \frac{12}{14} \mathbf{k} \right)$$

$$\mathbf{F}_{ED} = \{-12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\} \text{ kN}$$

$$\mathbf{F}_R = \mathbf{F}_{EA} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED}$$

$$= \{-96\mathbf{k}\} \text{ kN}$$



Ans.

Ans.

Ans.

Ans.

Ans.

2-103.

The cord exerts a force \mathbf{F} on the hook. If the cord is length L , determine the location x, y of the point of attachment B , and the height z of the hook.

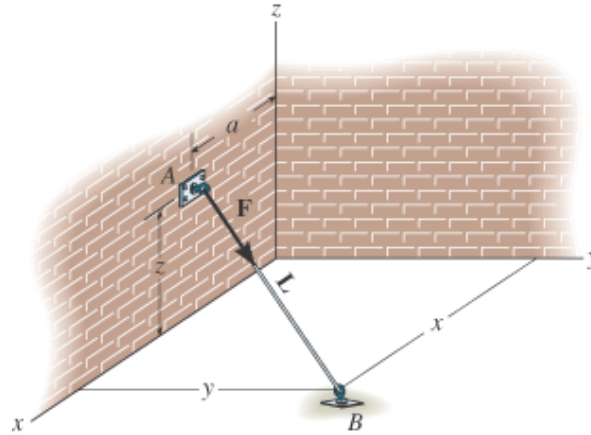
Units Used:

Given:

$$\mathbf{F} := \begin{pmatrix} 12 \\ 9 \\ -8 \end{pmatrix} \text{ N}$$

$$L := 4 \text{ m}$$

$$a := 1 \text{ m}$$



Solution :

Initial guesses $x := 1 \text{ m}$ $y := 1 \text{ m}$ $z := 1 \text{ m}$

$$\text{Given } \begin{pmatrix} x - a \\ y \\ -z \end{pmatrix} = L \cdot \frac{\mathbf{F}}{|\mathbf{F}|} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} := \text{Find}(x, y, z) \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3.82 \\ 2.12 \\ 1.88 \end{pmatrix} \text{ m} \quad \text{Ans.}$$

2-104.

The cord exerts a force of magnitude F on the hook. If the cord length L , the distance z , and the x component of the force F_x are given, determine the location x, y of the point of attachment B of the cord to the ground.

Given:

$$F := 30\text{kN}$$

$$L := 4\text{m}$$

$$z := 2\text{m}$$

$$F_x := 25\text{kN}$$

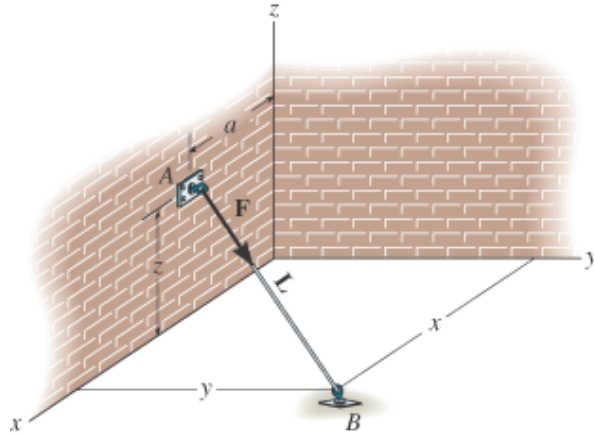
$$a := 1\text{m}$$

Solution :

Guesses

$$x := 1\text{m}$$

$$y := 1\text{m}$$

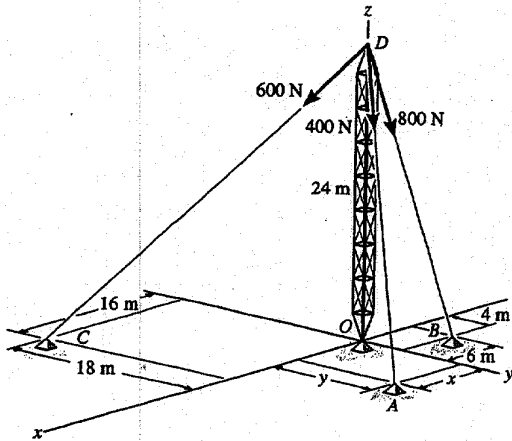


Given

$$F_x = \frac{x-a}{L} \cdot F \quad L^2 = (x-a)^2 + y^2 + z^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} := \text{Find}(x, y) \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4.33 \\ 0.94 \end{pmatrix} \text{m} \quad \text{Ans.}$$

2-105. The tower is held in place by three cables. If the force of each cable acting on the tower is shown, determine the magnitude and coordinate direction angles α , β , γ of the resultant force. Take $x = 20$ m, $y = 15$ m.



$$F_{DA} = 400\left(\frac{20}{34.66}i + \frac{15}{34.66}j - \frac{24}{34.66}k\right) \text{ N}$$

$$F_{DB} = 800\left(\frac{-6}{25.06}i + \frac{4}{25.06}j - \frac{24}{25.06}k\right) \text{ N}$$

$$F_{DC} = 600\left(\frac{16}{34}i - \frac{18}{34}j - \frac{24}{34}k\right) \text{ N}$$

$$\begin{aligned} F_R &= F_{DA} + F_{DB} + F_{DC} \\ &= \{321.66i - 16.82j - 1466.71k\} \text{ N} \end{aligned}$$

$$\begin{aligned} F_R &= \sqrt{(321.66)^2 + (-16.82)^2 + (-1466.71)^2} \\ &= 1501.66 \text{ N} = 1.50 \text{ kN} \quad \text{Ans.} \end{aligned}$$

$$\alpha = \cos^{-1}\left(\frac{321.66}{1501.66}\right) = 77.6^\circ \quad \text{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{-16.82}{1501.66}\right) = 90.6^\circ \quad \text{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{-1466.71}{1501.66}\right) = 168^\circ \quad \text{Ans.}$$

2-106.

The chandelier is supported by three chains which are concurrent at point O . If the force in each chain has magnitude F , express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.

Given:

$$F := 300\text{N}$$

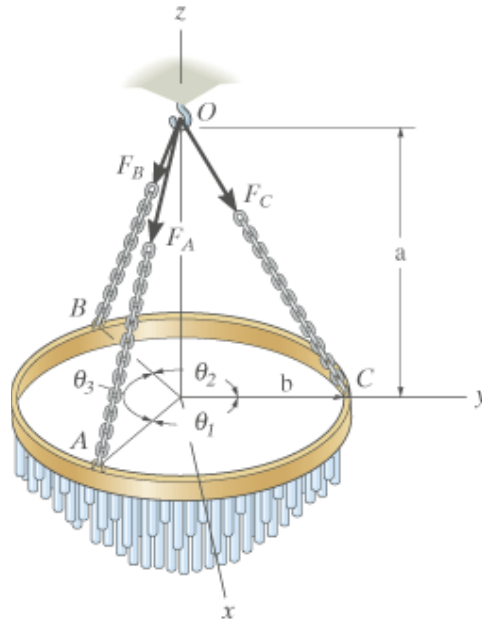
$$a := 1.8\text{m}$$

$$b := 1.2\text{m}$$

$$\theta_1 := 120\text{deg}$$

$$\theta_2 := 120\text{deg}$$

Solution:



$$\theta_3 := 360\text{deg} - \theta_1 - \theta_2$$

$$\mathbf{r}_{OA} := \begin{pmatrix} b \cdot \sin(\theta_1) \\ b \cdot \cos(\theta_1) \\ -a \end{pmatrix}$$

$$\mathbf{F}_A := F \cdot \frac{\mathbf{r}_{OA}}{|\mathbf{r}_{OA}|}$$

$$\mathbf{F}_A = \begin{pmatrix} 144.12 \\ -83.21 \\ -249.62 \end{pmatrix} \text{ N}$$

Ans.

$$\mathbf{r}_{OB} := \begin{pmatrix} b \cdot \sin(\theta_1 + \theta_2) \\ b \cdot \cos(\theta_1 + \theta_2) \\ -a \end{pmatrix}$$

$$\mathbf{F}_B := F \cdot \frac{\mathbf{r}_{OB}}{|\mathbf{r}_{OB}|}$$

$$\mathbf{F}_B = \begin{pmatrix} -144.12 \\ -83.21 \\ -249.62 \end{pmatrix} \text{ N}$$

Ans.

$$\mathbf{r}_{OC} := \begin{pmatrix} 0 \\ b \\ -a \end{pmatrix}$$

$$\mathbf{F}_C := F \cdot \frac{\mathbf{r}_{OC}}{|\mathbf{r}_{OC}|}$$

$$\mathbf{F}_C = \begin{pmatrix} 0.00 \\ 166.41 \\ -249.62 \end{pmatrix} \text{ N}$$

Ans.

$$\mathbf{F}_R := \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$

$$|\mathbf{F}_R| = 748.85 \text{ N}$$

Ans.

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} := \text{acos} \left(\frac{\mathbf{F}_R}{|\mathbf{F}_R|} \right)$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 90.00 \\ 90.00 \\ 180.00 \end{pmatrix} \text{deg} \quad \text{Ans.}$$

2-107.

The chandelier is supported by three chains which are concurrent at point O . If the resultant force at O has magnitude F_R and is directed along the negative z axis, determine the force in each chain assuming $F_A = F_B = F_C = F$.

Given:

$$a := 1.8\text{m}$$

$$b := 1.2\text{m}$$

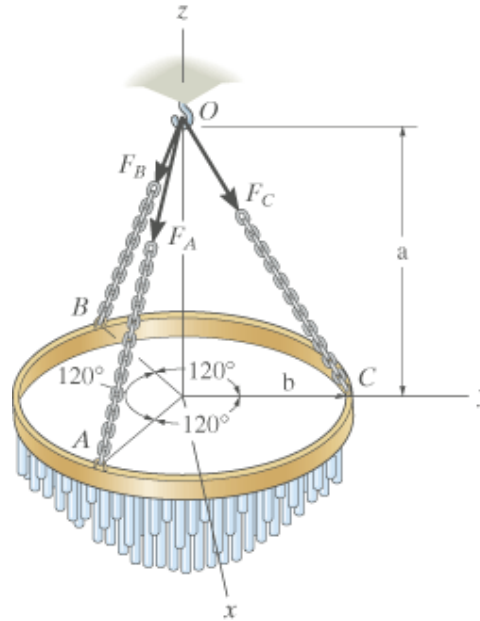
$$F_R := 650\text{N}$$

Solution:

$$F := \frac{\sqrt{a^2 + b^2}}{3a} F_R$$

$$F = 260.4\text{ N}$$

Ans.

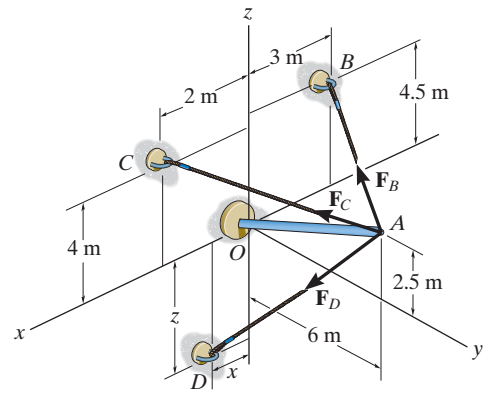


2-108.

Determine the magnitude and coordinate direction angles of the resultant force. Set $F_B = 630\text{ N}$, $F_C = 520\text{ N}$ and $F_D = 750\text{ N}$, and $x = 3\text{ m}$ and $z = 3.5\text{ m}$.

SOLUTION

Force Vectors: The unit vectors \mathbf{u}_B , \mathbf{u}_C , and \mathbf{u}_D of \mathbf{F}_B , \mathbf{F}_C , and \mathbf{F}_D must be determined first. From Fig. a,



$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (4.5 - 2.5)\mathbf{k}}{\sqrt{(-3 - 0)^2 + (0 - 6)^2 + (4.5 - 2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (4 - 2.5)\mathbf{k}}{\sqrt{(2 - 0)^2 + (0 - 6)^2 + (4 - 2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

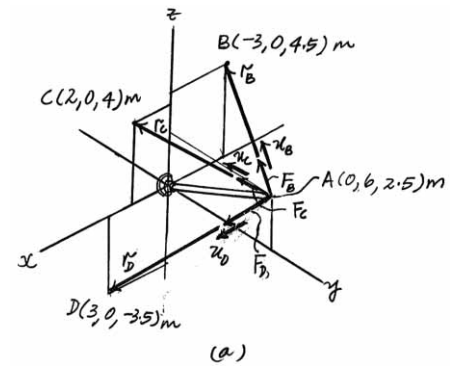
$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(3 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (-3.5 - 2.5)\mathbf{k}}{\sqrt{(0 - 3)^2 + (0 - 6)^2 + (-3.5 - 2.5)^2}} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Thus, the force vectors \mathbf{F}_B , \mathbf{F}_C , and \mathbf{F}_D are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = 630 \left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k} \right) = \{-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = 520 \left(\frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k} \right) = \{160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_D = F_D \mathbf{u}_D = 750 \left(\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \right) = \{250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k}\}\text{ N}$$



Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (-270\mathbf{i} - 540\mathbf{j} + 180\mathbf{k}) + (160\mathbf{i} - 480\mathbf{j} + 120\mathbf{k}) + (250\mathbf{i} - 500\mathbf{j} - 500\mathbf{k})$$

$$= [140\mathbf{i} - 1520\mathbf{j} - 200\mathbf{k}]\text{ N}$$

The magnitude of \mathbf{F}_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$

$$= \sqrt{140^2 + (-1520)^2 + (-200)^2} = 1539.48\text{ N} = 1.54\text{ kN}$$

Ans.

The coordinate direction angles of \mathbf{F}_R are

$$\alpha = \cos^{-1} \left[\frac{(F_R)_x}{F_R} \right] = \cos^{-1} \left(\frac{140}{1539.48} \right) = 84.8^\circ$$

Ans.

$$\beta = \cos^{-1} \left[\frac{(F_R)_y}{F_R} \right] = \cos^{-1} \left(\frac{-1520}{1539.48} \right) = 171^\circ$$

Ans.

$$\gamma = \cos^{-1} \left[\frac{(F_R)_z}{F_R} \right] = \cos^{-1} \left(\frac{-200}{1539.48} \right) = 97.5^\circ$$

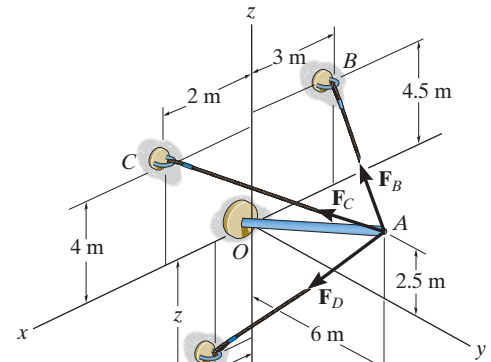
Ans.

2-109.

If the magnitude of the resultant force is 1300 N and acts along the axis of the strut, directed from point A towards O , determine the magnitudes of the three forces acting on the strut. Set $x = 0$ and $z = 5.5$ m.

SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B, \mathbf{u}_C, \mathbf{u}_D$, and \mathbf{u}_{F_R} of $\mathbf{F}_B, \mathbf{F}_C, \mathbf{F}_D$, and \mathbf{F}_R must be determined first. From Fig. a ,



$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-3 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (4.5 - 2.5)\mathbf{k}}{\sqrt{(-3 - 0)^2 + (0 - 6)^2 + (4.5 - 2.5)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(2 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (4 - 2.5)\mathbf{k}}{\sqrt{(2 - 0)^2 + (0 - 6)^2 + (4 - 2.5)^2}} = \frac{4}{13}\mathbf{i} - \frac{12}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(0 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (-5.5 - 2.5)\mathbf{k}}{\sqrt{(0 - 0)^2 + (0 - 6)^2 + (-5.5 - 2.5)^2}} = -\frac{3}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$$

$$\mathbf{u}_{F_R} = \frac{\mathbf{r}_{AO}}{r_{AO}} = \frac{(0 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (0 - 2.5)\mathbf{k}}{\sqrt{(0 - 0)^2 + (0 - 6)^2 + (0 - 2.5)^2}} = -\frac{12}{13}\mathbf{j} + \frac{5}{13}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B, \mathbf{F}_C, \mathbf{F}_D$, and \mathbf{F}_R are given by

$$\mathbf{F}_B = F_B \mathbf{u}_B = -\frac{3}{7}F_B \mathbf{i} - \frac{6}{7}F_B \mathbf{j} + \frac{2}{7}F_B \mathbf{k}$$

$$\mathbf{F}_C = F_C \mathbf{u}_C = \frac{4}{13}F_C \mathbf{i} - \frac{12}{13}F_C \mathbf{j} + \frac{3}{13}F_C \mathbf{k}$$

$$\mathbf{F}_D = F_D \mathbf{u}_D = -\frac{3}{5}F_D \mathbf{j} - \frac{4}{5}F_D \mathbf{k}$$

$$\mathbf{F}_R = F_R \mathbf{u}_{F_R} = 1300 \left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k} \right) = [-1200\mathbf{j} - 500\mathbf{k}] \text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B \mathbf{i} - \frac{6}{7}F_B \mathbf{j} + \frac{2}{7}F_B \mathbf{k} \right) + \left(\frac{4}{13}F_C \mathbf{i} - \frac{12}{13}F_C \mathbf{j} + \frac{3}{13}F_C \mathbf{k} \right) + \left(-\frac{3}{5}F_D \mathbf{j} - \frac{4}{5}F_D \mathbf{k} \right)$$

$$-1200\mathbf{j} - 500\mathbf{k} = \left(-\frac{3}{7}F_B + \frac{4}{13}F_C \right) \mathbf{i} + \left(-\frac{6}{7}F_B - \frac{12}{13}F_C - \frac{3}{5}F_D \right) \mathbf{j} + \left(\frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D \right) \mathbf{k}$$

Equating the \mathbf{i}, \mathbf{j} , and \mathbf{k} components,

$$0 = -\frac{3}{7}F_B + \frac{4}{13}F_C \tag{1}$$

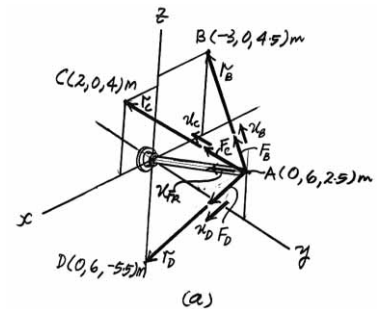
$$-1200 = -\frac{6}{7}F_B - \frac{12}{13}F_C - \frac{3}{5}F_D \tag{2}$$

$$-500 = \frac{2}{7}F_B + \frac{3}{13}F_C - \frac{4}{5}F_D \tag{3}$$

Solving Eqs. (1), (2), and (3), yields

$$F_C = 442 \text{ N} \quad F_B = 318 \text{ N} \quad F_D = 866 \text{ N}$$

Ans.



2-110.

The positions of point A on the building and point B on the antenna have been measured relative to the electronic distance meter (EDM) at O . Determine the distance between A and B . *Hint:* Formulate a position vector directed from A to B ; then determine its magnitude.

Given:

$$a = 460 \text{ m}$$

$$b = 653 \text{ m}$$

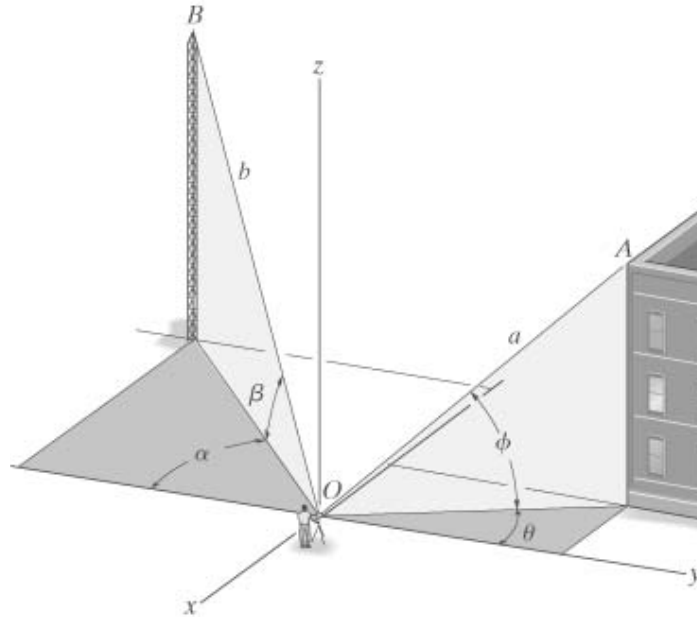
$$\alpha = 60 \text{ deg}$$

$$\beta = 55 \text{ deg}$$

$$\theta = 30 \text{ deg}$$

$$\phi = 40 \text{ deg}$$

Solution:

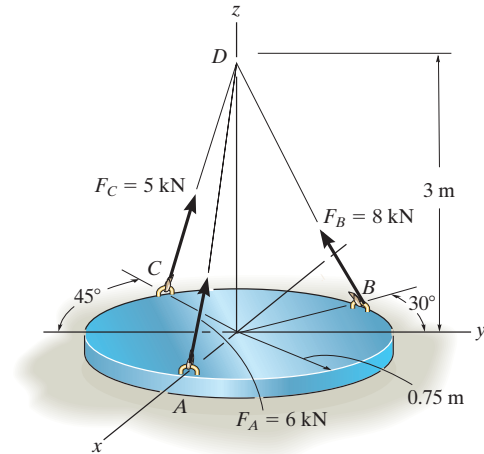


$$\mathbf{r}_{OA} = \begin{pmatrix} -a \cos(\phi) \sin(\theta) \\ a \cos(\phi) \cos(\theta) \\ a \sin(\phi) \end{pmatrix}$$

$$\mathbf{r}_{OB} = \begin{pmatrix} -b \cos(\beta) \sin(\alpha) \\ -b \cos(\beta) \cos(\alpha) \\ b \sin(\beta) \end{pmatrix}$$

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA} \quad \mathbf{r}_{AB} = \begin{pmatrix} -148.2 \\ -492.4 \\ 239.2 \end{pmatrix} \text{ m} \quad |\mathbf{r}_{AB}| = 567.2 \text{ m} \quad \text{Ans.}$$

2-111. The cylindrical plate is subjected to the three cable forces which are concurrent at point D . Express each force which the cables exert on the plate as a Cartesian vector, and determine the magnitude and coordinate direction angles of the resultant force.



$$r_A = (0 - 0.75)i + (0 - 0)j + (3 - 0)k = \{-0.75i + 0j + 3k\} \text{ m}$$

$$r_A = \sqrt{(-0.75)^2 + 0^2 + 3^2} = 3.0923 \text{ m}$$

$$F_A = F_A \left(\frac{r_A}{r_A} \right) = 6 \left(\frac{-0.75i + 3k}{3.0923} \right)$$

$$= \{-1.4552i + 5.8209k\} \text{ kN}$$

$$= \{-1.46i + 5.82k\} \text{ kN}$$

Ans.

$$r_C = [0 - (-0.75 \sin 45^\circ)]i + [0 - (-0.75 \cos 45^\circ)]j + (3 - 0)k$$

$$= \{0.5303i + 0.5303j + 3k\} \text{ m}$$

$$r_C = \sqrt{(0.5303)^2 + (0.5303)^2 + 3^2} = 3.0923 \text{ m}$$

$$F_C = F_C \left(\frac{r_C}{r_C} \right) = 5 \left(\frac{0.5303i + 0.5303j + 3k}{3.0923} \right)$$

$$= \{0.8575i + 0.8575j + 4.8507k\} \text{ kN}$$

$$= \{0.857i + 0.857j + 4.85k\} \text{ kN}$$

Ans.

$$r_B = [0 - (-0.75 \sin 30^\circ)]i + (0 - 0.75 \cos 30^\circ)j + (3 - 0)k$$

$$= \{0.375i - 0.6495j + 3k\} \text{ m}$$

$$r_B = \sqrt{(0.375)^2 + (-0.6495)^2 + 3^2} = 3.0923 \text{ m}$$

$$F_B = F_B \left(\frac{r_B}{r_B} \right) = 8 \left(\frac{0.375i - 0.6495j + 3k}{3.0923} \right)$$

$$= \{0.9701i - 1.6803j + 7.7611k\} \text{ kN}$$

$$= \{0.970i - 1.68j + 7.76k\} \text{ kN}$$

Ans.

$$F_R = F_A + F_B + F_C$$

$$= \{-1.4552i + 5.8209k\} + \{0.9701i - 1.6803j + 7.7611k\}$$

$$+ \{0.8575i + 0.8575j + 4.8507k\}$$

$$= \{0.3724i - 0.8228j + 18.4327k\} \text{ kN}$$

$$F_R = \sqrt{(0.3724)^2 + (-0.8228)^2 + (18.4327)^2}$$

$$= 18.4548 \text{ kN} = 18.5 \text{ kN}$$

Ans.

$$u_R = \frac{F_R}{F_R} = \frac{0.3724i - 0.8228j + 18.4327k}{18.4548}$$

$$= 0.02018i - 0.04459j + 0.9988k$$

$$\cos \alpha = 0.02018$$

$$\alpha = 88.8^\circ$$

Ans.

$$\cos \beta = -0.04458$$

$$\beta = 92.6^\circ$$

Ans.

$$\cos \gamma = 0.9988$$

$$\gamma = 2.81^\circ$$

Ans.

2-112.

Given the three vectors \mathbf{A} , \mathbf{B} , and \mathbf{D} , show that $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$.

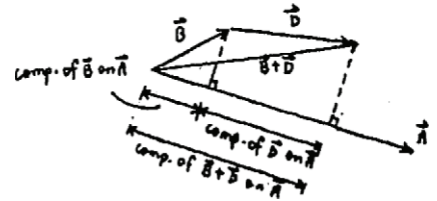
SOLUTION

Since the component of $(\mathbf{B} + \mathbf{D})$ is equal to the sum of the components of \mathbf{B} and \mathbf{D} , then

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{D} \quad (\text{QED})$$

Also,

$$\begin{aligned} \mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) &= (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \cdot [(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}] \\ &= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z) \\ &= (A_x B_x + A_y B_y + A_z B_z) + (A_x D_x + A_y D_y + A_z D_z) \\ &= (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D}) \quad (\text{QED}) \end{aligned}$$



2-113.

Determine the angle θ between the edges of the sheet-metal bracket.

SOLUTION

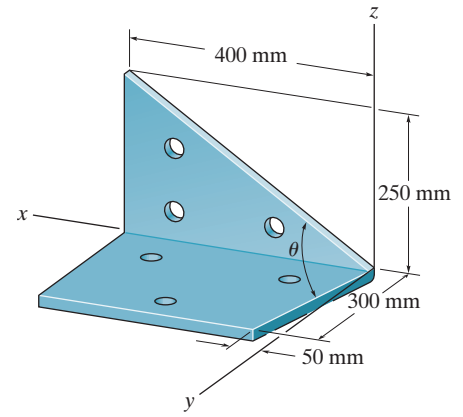
$$\mathbf{r}_1 = \{400\mathbf{i} + 250\mathbf{k}\} \text{ mm}; \quad r_1 = 471.70 \text{ mm}$$

$$\mathbf{r}_2 = \{50\mathbf{i} + 300\mathbf{j}\} \text{ mm}; \quad r_2 = 304.14 \text{ mm}$$

$$\mathbf{r}_1 \cdot \mathbf{r}_2 = (400)(50) + 0(300) + 250(0) = 20\,000$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}\right)$$

$$= \cos^{-1}\left(\frac{20\,000}{(471.70)(304.14)}\right) = 82.0^\circ$$

**Ans.**

2-114.

Determine the angle θ between the sides of the triangular plate.

SOLUTION

$$\mathbf{r}_{AC} = \{3\mathbf{i} + 4\mathbf{j} - 1\mathbf{k}\} \text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990 \text{ m}$$

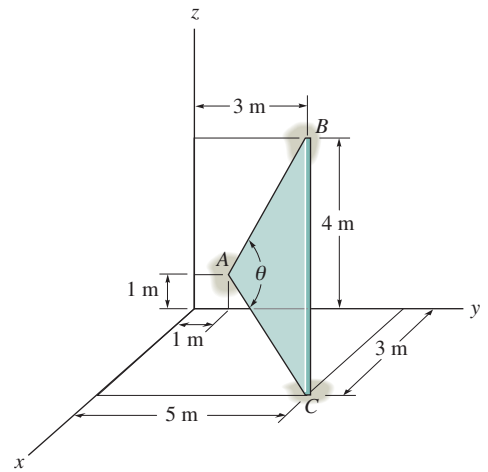
$$\mathbf{r}_{AB} = \{2\mathbf{j} + 3\mathbf{k}\} \text{ m}$$

$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056 \text{ m}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

$$\theta = \cos^{-1} \left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC} r_{AB}} \right) = \cos^{-1} \frac{5}{(5.0990)(3.6056)}$$

$$\theta = 74.219^\circ = 74.2^\circ$$



Ans.

2-115.

Determine the length of side BC of the triangular plate. Solve the problem by finding the magnitude of \mathbf{r}_{BC} ; then check the result by first finding θ , r_{AB} , and r_{AC} and then using the cosine law.

SOLUTION

$$\mathbf{r}_{BC} = \{3 \mathbf{i} + 2 \mathbf{j} - 4 \mathbf{k}\} \text{ m}$$

$$r_{BC} = \sqrt{(3)^2 + (2)^2 + (-4)^2} = 5.39 \text{ m}$$

Also,

$$\mathbf{r}_{AC} = \{3 \mathbf{i} + 4 \mathbf{j} - 1 \mathbf{k}\} \text{ m}$$

$$r_{AC} = \sqrt{(3)^2 + (4)^2 + (-1)^2} = 5.0990 \text{ m}$$

$$\mathbf{r}_{AB} = \{2 \mathbf{j} + 3 \mathbf{k}\} \text{ m}$$

$$r_{AB} = \sqrt{(2)^2 + (3)^2} = 3.6056 \text{ m}$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = 0 + 4(2) + (-1)(3) = 5$$

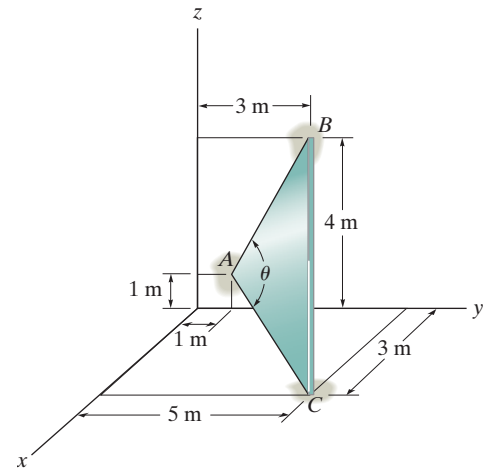
$$\theta = \cos^{-1} \left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC} r_{AB}} \right) = \cos^{-1} \frac{5}{(5.0990)(3.6056)}$$

$$\theta = 74.219^\circ$$

$$r_{BC} = \sqrt{(5.0990)^2 + (3.6056)^2 - 2(5.0990)(3.6056) \cos 74.219^\circ}$$

$$r_{BC} = 5.39 \text{ m}$$

Ans.



Ans.

2-116.

Determine the angle θ between the tails of the two vectors.

Given:

$$r_1 = 9 \text{ m}$$

$$r_2 = 6 \text{ m}$$

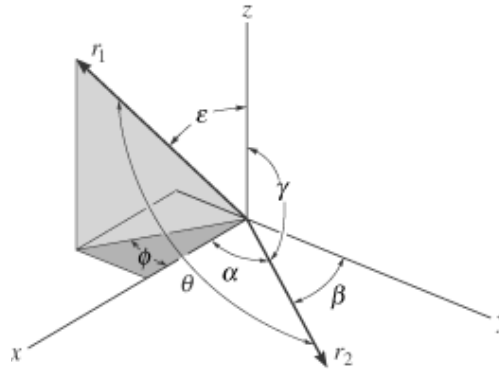
$$\alpha = 60 \text{ deg}$$

$$\beta = 45 \text{ deg}$$

$$\gamma = 120 \text{ deg}$$

$$\phi = 30 \text{ deg}$$

$$\varepsilon = 40 \text{ deg}$$



Solution:

Determine the two position vectors and use the dot product to find the angle

$$\mathbf{r}_{1\mathbf{v}} = r_1 \begin{pmatrix} \sin(\varepsilon) \cos(\phi) \\ -\sin(\varepsilon) \sin(\phi) \\ \cos(\varepsilon) \end{pmatrix} \quad \mathbf{r}_{2\mathbf{v}} = r_2 \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix}$$

$$\theta = \text{acos} \left(\frac{\mathbf{r}_{1\mathbf{v}} \cdot \mathbf{r}_{2\mathbf{v}}}{|\mathbf{r}_{1\mathbf{v}}| |\mathbf{r}_{2\mathbf{v}}|} \right)$$

$$\theta = 109.4 \text{ deg}$$

Ans.

2-117.

Determine the magnitude of the projected component of \mathbf{r}_1 along \mathbf{r}_2 , and the projection of \mathbf{r}_2 along \mathbf{r}_1 .

Given:

$$r_1 = 9 \text{ m}$$

$$r_2 = 6 \text{ m}$$

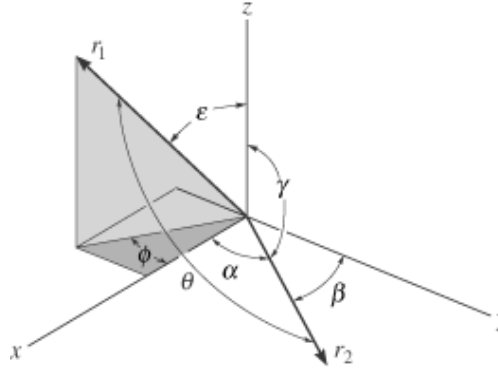
$$\alpha = 60 \text{ deg}$$

$$\beta = 45 \text{ deg}$$

$$\gamma = 120 \text{ deg}$$

$$\phi = 30 \text{ deg}$$

$$\varepsilon = 40 \text{ deg}$$



Solution:

Write the vectors and unit vectors

$$\mathbf{r}_{1v} = r_1 \begin{pmatrix} \sin(\varepsilon) \cos(\phi) \\ -\sin(\varepsilon) \sin(\phi) \\ \cos(\varepsilon) \end{pmatrix} \quad \mathbf{r}_{1v} = \begin{pmatrix} 5.01 \\ -2.89 \\ 6.89 \end{pmatrix} \text{ m}$$

$$\mathbf{r}_{2v} = r_2 \begin{pmatrix} \cos(\alpha) \\ \cos(\beta) \\ \cos(\gamma) \end{pmatrix} \quad \mathbf{r}_{2v} = \begin{pmatrix} 3 \\ 4.24 \\ -3 \end{pmatrix} \text{ m}$$

$$\mathbf{u}_1 = \frac{\mathbf{r}_{1v}}{|\mathbf{r}_{1v}|} \quad \mathbf{u}_2 = \frac{\mathbf{r}_{2v}}{|\mathbf{r}_{2v}|} \quad \mathbf{u}_1 = \begin{pmatrix} 0.557 \\ -0.321 \\ 0.766 \end{pmatrix} \quad \mathbf{u}_2 = \begin{pmatrix} 0.5 \\ 0.707 \\ -0.5 \end{pmatrix}$$

The magnitude of the projection of \mathbf{r}_1 along \mathbf{r}_2 .

$$|\mathbf{r}_{1v} \cdot \mathbf{u}_2| = 2.99 \text{ m}$$

Ans.

The magnitude of the projection of \mathbf{r}_2 along \mathbf{r}_1 .

$$|\mathbf{r}_{2v} \cdot \mathbf{u}_1| = 1.99 \text{ m}$$

Ans.

2-118.

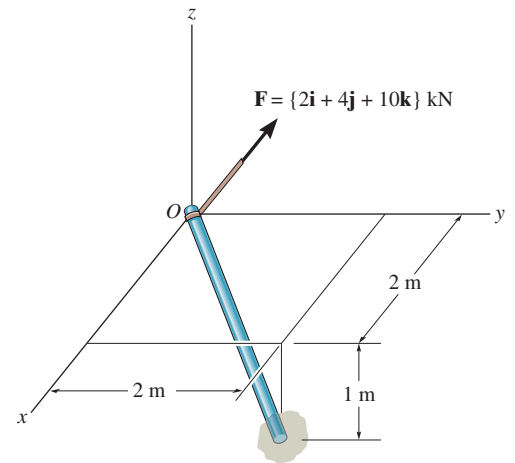
Determine the projection of the force \mathbf{F} along the pole.

SOLUTION

$$\text{Proj } F = \mathbf{F} \cdot \mathbf{u}_a = (2\mathbf{i} + 4\mathbf{j} + 10\mathbf{k}) \cdot \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k} \right)$$

$$\text{Proj } F = 0.667 \text{ kN}$$

Ans.



2-119.

Determine the angle θ between the two cords.

Given:

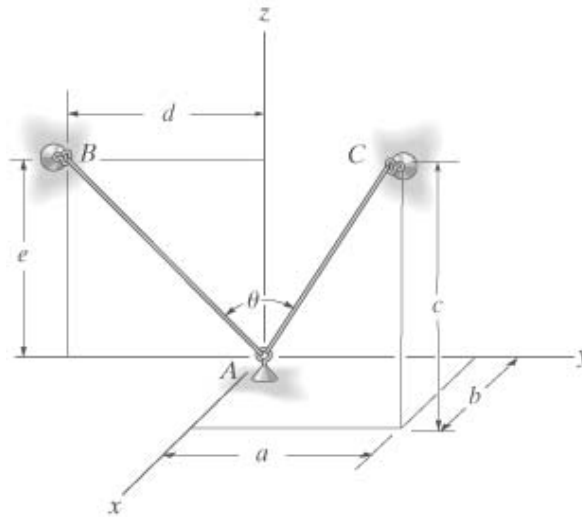
$$a = 3 \text{ m}$$

$$b = 2 \text{ m}$$

$$c = 6 \text{ m}$$

$$d = 3 \text{ m}$$

$$e = 4 \text{ m}$$



Solution:

$$\mathbf{r}_{AC} = \begin{pmatrix} b \\ a \\ c \end{pmatrix} \text{ m}$$

$$\mathbf{r}_{AB} = \begin{pmatrix} 0 \\ -d \\ e \end{pmatrix} \text{ m}$$

$$\theta = \arccos\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{|\mathbf{r}_{AC}| |\mathbf{r}_{AB}|}\right)$$

$$\theta = 64.6 \text{ deg}$$

Ans.

2-120. Two forces act on the hook. Determine the angle θ between them. Also, what are the projections of \mathbf{F}_1 and \mathbf{F}_2 along the y axis?

$$\mathbf{F}_1 = 600 \cos 120^\circ \mathbf{i} + 600 \cos 60^\circ \mathbf{j} + 600 \cos 45^\circ \mathbf{k}$$

$$= -300 \mathbf{i} + 300 \mathbf{j} + 424.3 \mathbf{k}; \quad F_1 = 600 \text{ N}$$

$$\mathbf{F}_2 = 120 \mathbf{i} + 90 \mathbf{j} - 80 \mathbf{k}; \quad F_2 = 170 \text{ N}$$

$$\mathbf{F}_1 \cdot \mathbf{F}_2 = (-300)(120) + (300)(90) + (424.3)(-80) = -42\,944$$

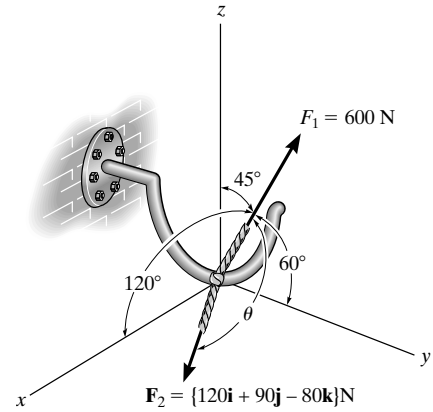
$$\theta = \cos^{-1} \left(\frac{-42\,944}{(170)(600)} \right) = 115^\circ \quad \text{Ans.}$$

$$\mathbf{u} = \mathbf{j}$$

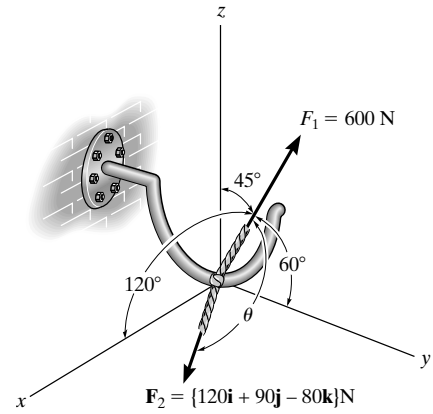
So,

$$F_{1y} = \mathbf{F}_1 \cdot \mathbf{j} = (300)(1) = 300 \text{ N} \quad \text{Ans.}$$

$$F_{2y} = \mathbf{F}_2 \cdot \mathbf{j} = (90)(1) = 90 \text{ N} \quad \text{Ans.}$$



2-121. Two forces act on the hook. Determine the magnitude of the projection of \mathbf{F}_2 along \mathbf{F}_1 .

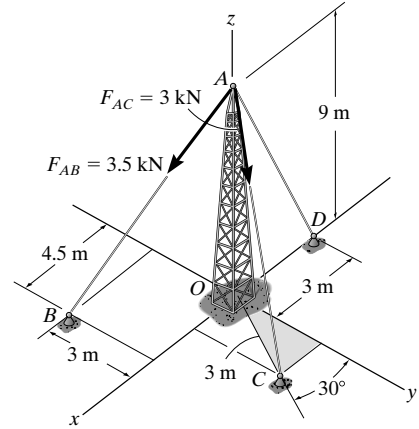


$$\mathbf{u}_1 = \cos 120^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 45^\circ \mathbf{k}$$

$$\text{Proj } \mathbf{F}_2 = \mathbf{F}_2 \cdot \mathbf{u}_1 = (120)(\cos 120^\circ) + (90)(\cos 60^\circ) + (-80)(\cos 45^\circ)$$

$$|\text{Proj } \mathbf{F}_2| = 71.6 \text{ N} \quad \text{Ans.}$$

2-122. Determine the magnitude of the projected component of force \mathbf{F}_{AB} acting along the z axis.



Unit Vector: The unit vector \mathbf{u}_{AB} must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(4.5-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-9)\mathbf{k}}{\sqrt{(4.5-0)^2 + (-3-0)^2 + (0-9)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vector \mathbf{F}_{AB} is given by

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 3.5\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{1.5\mathbf{i} - 1\mathbf{j} - 3\mathbf{k}\} \text{ kN}$$

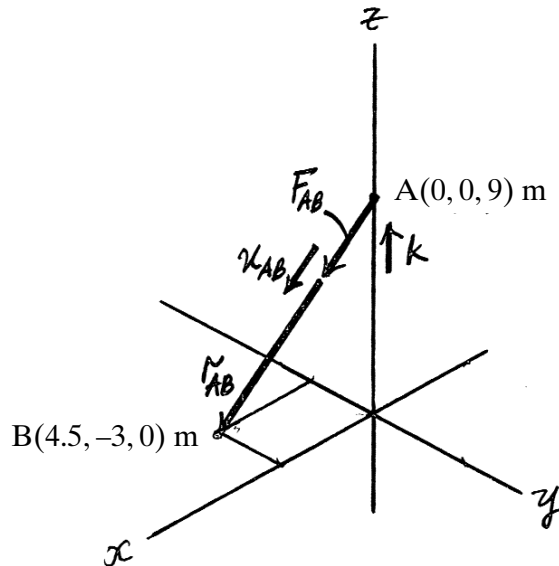
Vector Dot Product: The projected component of \mathbf{F}_{AB} along the z axis is

$$\begin{aligned} (F_{AB})_z &= \mathbf{F}_{AB} \cdot \mathbf{k} = (1.5\mathbf{i} - 1\mathbf{j} - 3\mathbf{k}) \cdot \mathbf{k} \\ &= -3 \text{ kN} \end{aligned}$$

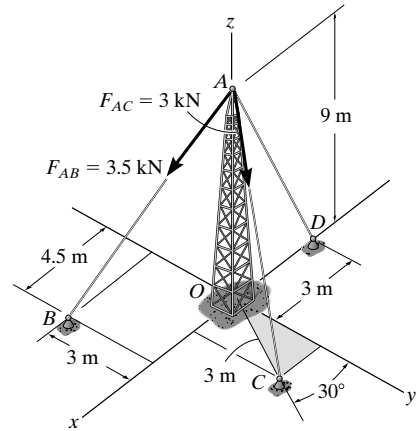
The negative sign indicates that $(F_{AB})_z$ is directed towards the negative z axis. Thus

$$(F_{AB})_z = 3 \text{ kN}$$

Ans.



2-123. Determine the magnitude of the projected component of force \mathbf{F}_{AC} acting along the z axis.



Unit Vector: The unit vector \mathbf{u}_{AC} must be determined first. From Fig. *a*,

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(3 \sin 30^\circ - 0)\mathbf{i} + (3 \cos 30^\circ - 0)\mathbf{j} + (0 - 9)\mathbf{k}}{\sqrt{(3 \sin 30^\circ - 0)^2 + (3 \cos 30^\circ - 0)^2 + (0 - 9)^2}} = 0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}$$

Thus, the force vector \mathbf{F}_{AC} is given by

$$\mathbf{F}_{AC} = F_{AC} \mathbf{u}_{AC} = 3(0.1581\mathbf{i} + 0.2739\mathbf{j} - 0.9487\mathbf{k}) = \{0.4743\mathbf{i} + 0.8217\mathbf{j} - 2.8461\mathbf{k}\} \text{ kN}$$

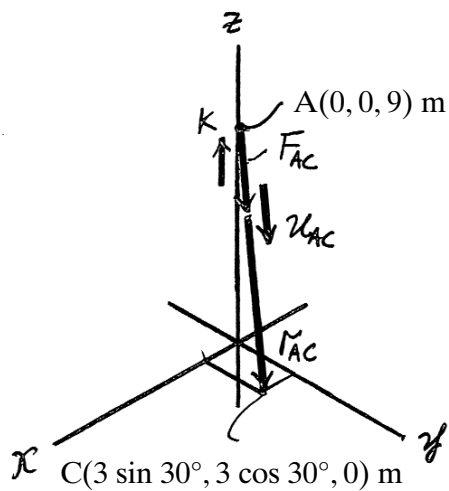
Vector Dot Product: The projected component of \mathbf{F}_{AC} along the z axis is

$$\begin{aligned} (F_{AC})_z &= \mathbf{F}_{AC} \cdot \mathbf{k} = (0.4743\mathbf{i} + 0.8217\mathbf{j} - 2.8461\mathbf{k}) \cdot \mathbf{k} \\ &= -2.8461 \text{ kN} \end{aligned}$$

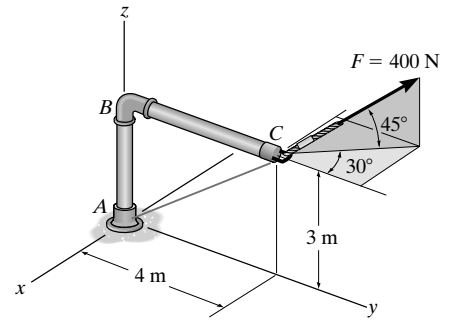
The negative sign indicates that $(F_{AC})_z$ is directed towards the negative z axis. Thus

$$(F_{AC})_z = 2.846 \text{ kN}$$

Ans.



2-124. Determine the projection of force $F = 400$ N acting along line AC of the pipe assembly. Express the result as a Cartesian vector.



Force and unit Vector: The force vector F and unit vector u_{AC} must be determined first.

From Fig. (a)

$$F = 400(-\cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k})$$

$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\}$$

$$u_{AC} = \frac{r_{AC}}{r_{AC}} = \frac{(0-0)\mathbf{i} + (4-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-0)^2 + (4-0)^2 + (3-0)^2}} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of F along line AC is

$$F_{AC} = F \cdot u_{AC} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right)$$

$$= (-141.42)(0) + 244.95\left(\frac{4}{5}\right) + 282.84\left(\frac{3}{5}\right)$$

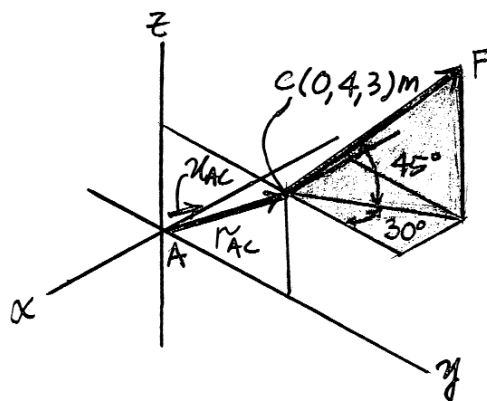
$$= 365.66 \text{ N}$$

Ans.

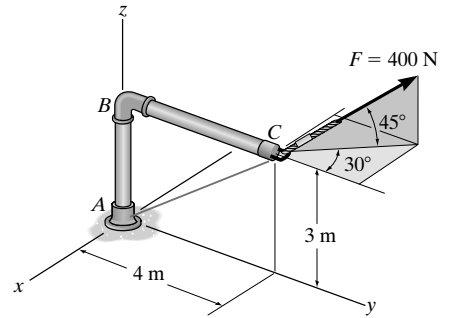
Thus, F_{AC} written in Cartesian vector form is

$$F_{AC} = F_{AC} u_{AC} = 365.66 \left(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}\right) = \{293\mathbf{j} + 219\mathbf{k}\} \text{ N}$$

Ans.



2-125. Determine the magnitudes of the components of force $F = 400$ N acting parallel and perpendicular to segment BC of the pipe assembly.



Force Vector: The force vector \mathbf{F} must be determined first. From Fig. *a*,

$$\mathbf{F} = 400(\cos 45^\circ \sin 30^\circ \mathbf{i} + \cos 45^\circ \cos 30^\circ \mathbf{j} + \sin 45^\circ \mathbf{k})$$

$$= \{-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}\} \text{ N}$$

Vector Dot Product: By inspecting Fig. (*a*) we notice that $u_{BC} = \mathbf{j}$. Thus, the magnitude of the component of \mathbf{F} parallel to segment BC of the pipe assembly is

$$(F_{BC})_{\text{paral}} = \mathbf{F} \cdot \mathbf{j} = (-141.42\mathbf{i} + 244.95\mathbf{j} + 282.84\mathbf{k}) \cdot \mathbf{j}$$

$$= -141.42(0) + 244.95(1) + 282.84(0)$$

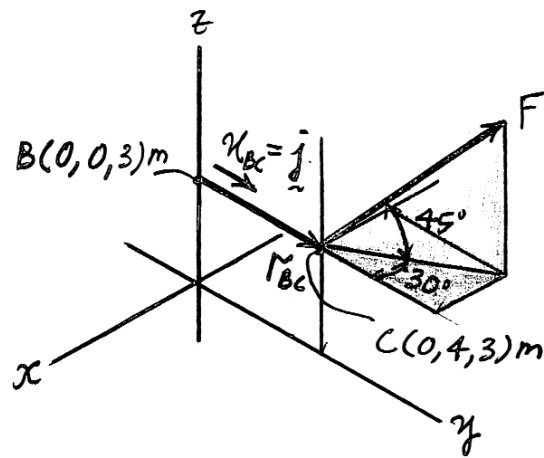
$$= 244.95 \text{ N} = 245 \text{ N}$$

Ans.

The magnitude of the component of \mathbf{F} perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{\text{per}} = \sqrt{F^2 - (F_{BC})_{\text{paral}}^2} = \sqrt{400^2 - 244.95^2} = 316 \text{ N}$$

Ans.



2-126. Cable OA is used to support column OB . Determine the angle θ it makes with beam OC .

Unit Vector :

$$\mathbf{u}_{OC} = \mathbf{i}$$

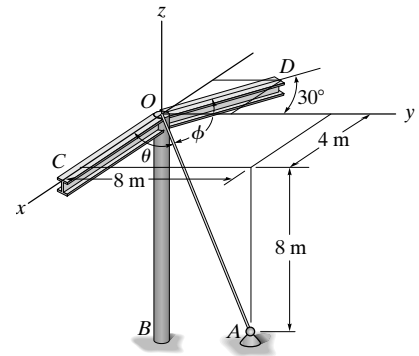
$$\begin{aligned} \mathbf{u}_{OA} &= \frac{(4-0)\mathbf{i} + (8-0)\mathbf{j} + (-8-0)\mathbf{k}}{\sqrt{(4-0)^2 + (8-0)^2 + (-8-0)^2}} \\ &= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \end{aligned}$$

The Angle^{ns} Between Two Vectors θ :

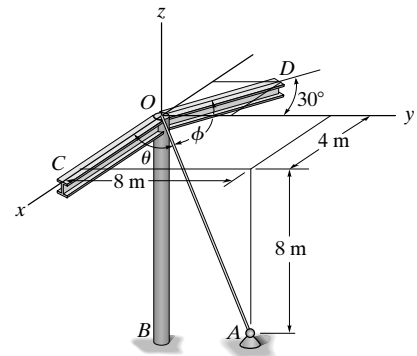
$$\mathbf{u}_{OC} \cdot \mathbf{u}_{OA} = (\mathbf{i}) \cdot \left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \right) = 1\left(\frac{1}{3}\right) + (0)\left(\frac{2}{3}\right) + 0\left(-\frac{2}{3}\right) = \frac{1}{3}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{OC} \cdot \mathbf{u}_{OA}) = \cos^{-1}\frac{1}{3} = 70.5^\circ \quad \text{Ans.}$$



2-127. Cable OA is used to support column OB . Determine the angle ϕ it makes with beam OD .



Unit Vector :

$$\mathbf{u}_{OD} = -\sin 30^\circ \mathbf{i} + \cos 30^\circ \mathbf{j} = -0.5\mathbf{i} + 0.8660\mathbf{j}$$

$$\begin{aligned} \mathbf{u}_{OA} &= \frac{(4-0)\mathbf{i} + (8-0)\mathbf{j} + (-8-0)\mathbf{k}}{\sqrt{(4-0)^2 + (8-0)^2 + (-8-0)^2}} \\ &= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \end{aligned}$$

The Angles Between Two Vectors ϕ :

$$\begin{aligned} \mathbf{u}_{OD} \cdot \mathbf{u}_{OA} &= (-0.5\mathbf{i} + 0.8660\mathbf{j}) \cdot \left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) \\ &= (-0.5)\left(\frac{1}{3}\right) + (0.8660)\left(\frac{2}{3}\right) + 0\left(-\frac{2}{3}\right) \\ &= 0.4107 \end{aligned}$$

Then,

$$\phi = \cos^{-1}(\mathbf{u}_{OD} \cdot \mathbf{u}_{OA}) = \cos^{-1} 0.4107 = 65.8^\circ \quad \text{Ans.}$$

2-128.

Determine the angles θ and ϕ between the axis OA of the pole and each cable, AB and AC .

Given:

$$F_1 = 50 \text{ N}$$

$$F_2 = 35 \text{ N}$$

$$a = 1 \text{ m}$$

$$b = 3 \text{ m}$$

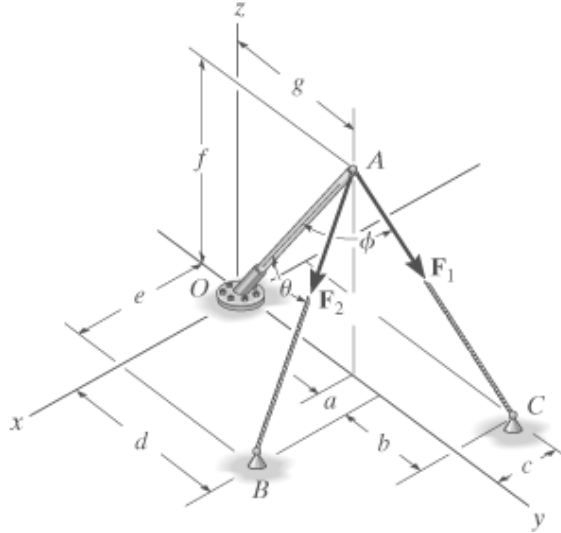
$$c = 2 \text{ m}$$

$$d = 5 \text{ m}$$

$$e = 4 \text{ m}$$

$$f = 6 \text{ m}$$

$$g = 4 \text{ m}$$



Solution:

$$\mathbf{r}_{AO} = \begin{pmatrix} 0 \\ -g \\ -f \end{pmatrix} \quad \mathbf{r}_{AB} = \begin{pmatrix} e \\ a \\ -f \end{pmatrix} \quad \mathbf{r}_{AC} = \begin{pmatrix} -c \\ a+b \\ -f \end{pmatrix}$$

$$\theta = \text{acos} \left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{|\mathbf{r}_{AO}| |\mathbf{r}_{AB}|} \right) \quad \theta = 52.4 \text{ deg} \quad \text{Ans.}$$

$$\phi = \text{acos} \left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AC}}{|\mathbf{r}_{AO}| |\mathbf{r}_{AC}|} \right) \quad \phi = 68.2 \text{ deg} \quad \text{Ans.}$$

2-129.

The two cables exert the forces shown on the pole. Determine the magnitude of the projected component of each force acting along the axis OA of the pole.

Given:

$$F_1 = 50 \text{ N}$$

$$F_2 = 35 \text{ N}$$

$$a = 1 \text{ m}$$

$$b = 3 \text{ m}$$

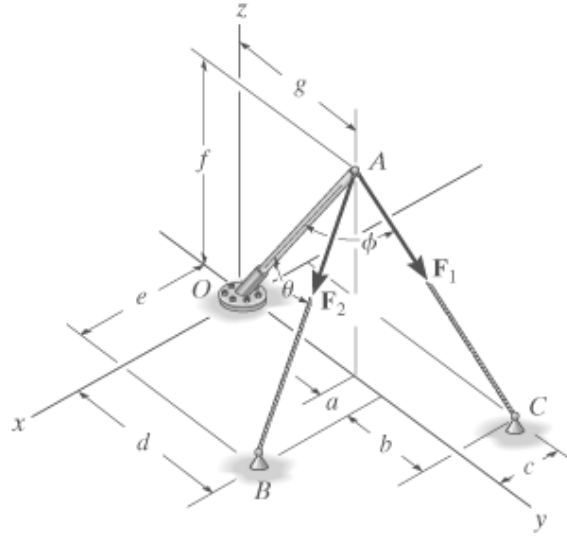
$$c = 2 \text{ m}$$

$$d = 5 \text{ m}$$

$$e = 4 \text{ m}$$

$$f = 6 \text{ m}$$

$$g = 4 \text{ m}$$



Solution:

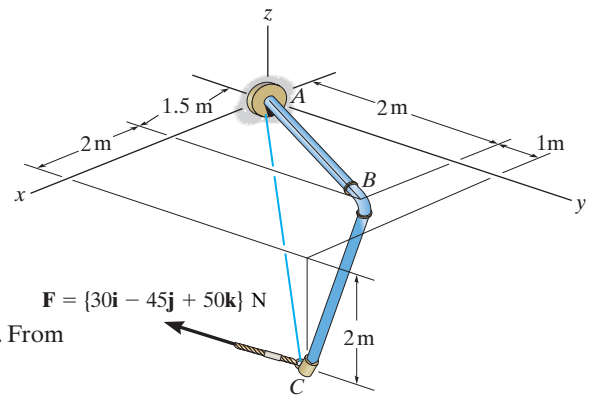
$$\mathbf{r}_{AB} = \begin{pmatrix} e \\ a \\ -f \end{pmatrix} \quad \mathbf{r}_{AC} = \begin{pmatrix} -c \\ a+b \\ -f \end{pmatrix} \quad \mathbf{r}_{AO} = \begin{pmatrix} 0 \\ -g \\ -f \end{pmatrix} \quad \mathbf{u}_{AO} = \frac{\mathbf{r}_{AO}}{|\mathbf{r}_{AO}|}$$

$$\mathbf{F}_{1v} = F_1 \frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} \quad F_{1AO} = \mathbf{F}_{1v} \cdot \mathbf{u}_{AO} \quad F_{1AO} = 18.5 \text{ N} \quad \text{Ans.}$$

$$\mathbf{F}_{2v} = F_2 \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} \quad F_{2AO} = \mathbf{F}_{2v} \cdot \mathbf{u}_{AO} \quad F_{2AO} = 21.3 \text{ N} \quad \text{Ans.}$$

2-130.

Determine the angle θ between the pipe segments BA and BC .



SOLUTION

Position Vectors: The position vectors \mathbf{r}_{BA} and \mathbf{r}_{BC} must be determined first. From Fig. *a*,

$$\mathbf{r}_{BA} = (0 - 1.5)\mathbf{i} + (0 - 2)\mathbf{j} + (0 - 0)\mathbf{k} = \{-1.5\mathbf{i} - 2\mathbf{j}\} \text{ m}$$

$$\mathbf{r}_{BC} = (3.5 - 1.5)\mathbf{i} + (3 - 2)\mathbf{j} + (-2 - 0)\mathbf{k} = \{2\mathbf{i} + 1\mathbf{j} - 2\mathbf{k}\} \text{ m}$$

The magnitude of \mathbf{r}_{BA} and \mathbf{r}_{BC} are

$$r_{BA} = \sqrt{(-1.5)^2 + (-2)^2} = 2.5 \text{ m}$$

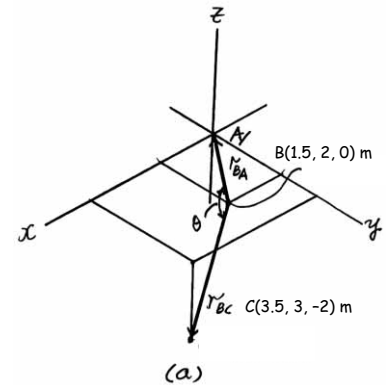
$$r_{BC} = \sqrt{2^2 + 1^2 + (-2)^2} = 3 \text{ m}$$

Vector Dot Product:

$$\begin{aligned} \mathbf{r}_{BA} \cdot \mathbf{r}_{BC} &= (-1.5\mathbf{i} - 2\mathbf{j}) \cdot (2\mathbf{i} + 1\mathbf{j} - 2\mathbf{k}) \\ &= (-1.5)(2) + (-2)(1) + 0(-2) \\ &= -5 \text{ m}^2 \end{aligned}$$

Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA} r_{BC}}\right) = \cos^{-1}\left[\frac{-5}{2.5(3)}\right] = 132^\circ$$



Ans.

2-131.

Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC , respectively, of each cable.

SOLUTION

$$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\} \text{ m}; \quad r_{AC} = 4.58 \text{ m}$$

$$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\} \text{ m}; \quad r_{AB} = 5.22 \text{ m}$$

$$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\} \text{ m}; \quad r_{AO} = 5.00 \text{ m}$$

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

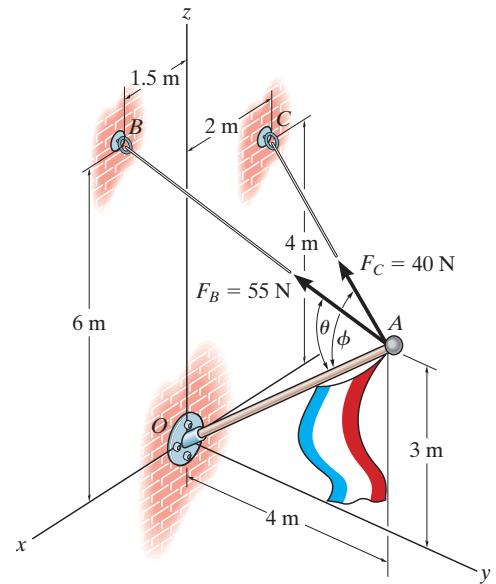
$$\theta = \cos^{-1} \left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB} r_{AO}} \right)$$

$$= \cos^{-1} \left(\frac{7}{5.22(5.00)} \right) = 74.4^\circ$$

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

$$\phi = \cos^{-1} \left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC} r_{AO}} \right)$$

$$= \cos^{-1} \left(\frac{13}{4.58(5.00)} \right) = 55.4^\circ$$



Ans.

Ans.

2-132.

The cables each exert a force of 400 N on the post. Determine the magnitude of the projected component of \mathbf{F}_1 along the line of action of \mathbf{F}_2 .

SOLUTION

Force Vector:

$$\begin{aligned} \mathbf{u}_{F_1} &= \sin 35^\circ \cos 20^\circ \mathbf{i} - \sin 35^\circ \sin 20^\circ \mathbf{j} + \cos 35^\circ \mathbf{k} \\ &= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_1 &= F_1 \mathbf{u}_{F_1} = 400(0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \text{ N} \\ &= \{215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}\} \text{ N} \end{aligned}$$

Unit Vector: The unit vector along the line of action of \mathbf{F}_2 is

$$\begin{aligned} \mathbf{u}_{F_2} &= \cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k} \\ &= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k} \end{aligned}$$

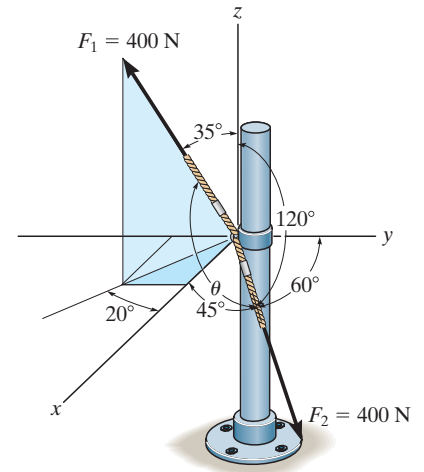
Projected Component of \mathbf{F}_1 Along Line of Action of \mathbf{F}_2 :

$$\begin{aligned} (F_1)_{F_2} &= \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (215.59\mathbf{i} - 78.47\mathbf{j} + 327.66\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}) \\ &= (215.59)(0.7071) + (-78.47)(0.5) + (327.66)(-0.5) \\ &= -50.6 \text{ N} \end{aligned}$$

Negative sign indicates that the force component $(\mathbf{F}_1)_{F_2}$ acts in the opposite sense of direction to that of \mathbf{u}_{F_2} .

Thus the magnitude is $(F_1)_{F_2} = 50.6 \text{ N}$

Ans.



2-133.

Determine the angle θ between the two cables attached to the post.

SOLUTION

Unit Vector:

$$\begin{aligned}\mathbf{u}_{F_1} &= \sin 35^\circ \cos 20^\circ \mathbf{i} - \sin 35^\circ \sin 20^\circ \mathbf{j} + \cos 35^\circ \mathbf{k} \\ &= 0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{u}_{F_2} &= \cos 45^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 120^\circ \mathbf{k} \\ &= 0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}\end{aligned}$$

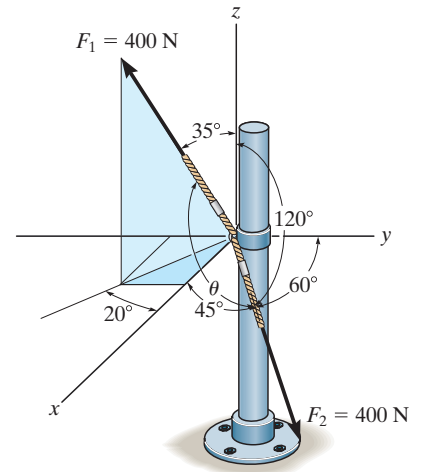
The Angle Between Two Vectors θ : The dot product of two unit vectors must be determined first.

$$\begin{aligned}\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} &= (0.5390\mathbf{i} - 0.1962\mathbf{j} + 0.8192\mathbf{k}) \cdot (0.7071\mathbf{i} + 0.5\mathbf{j} - 0.5\mathbf{k}) \\ &= 0.5390(0.7071) + (-0.1962)(0.5) + 0.8192(-0.5) \\ &= -0.1265\end{aligned}$$

Then,

$$\theta = \cos^{-1}(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}) = \cos^{-1}(-0.1265) = 97.3^\circ$$

Ans.



2-134.

Force \mathbf{F} is applied to the handle of the wrench. Determine the angle θ between the tail of the force and the handle AB .

Given:

$$a = 300 \text{ mm}$$

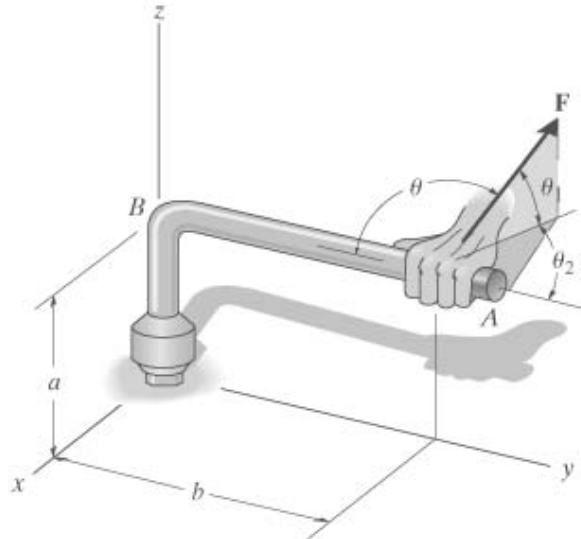
$$b = 500 \text{ mm}$$

$$F = 80 \text{ N}$$

$$\theta_1 = 30 \text{ deg}$$

$$\theta_2 = 45 \text{ deg}$$

Solution:



$$\mathbf{F}_v = F \begin{pmatrix} -\cos(\theta_1) \sin(\theta_2) \\ \cos(\theta_1) \cos(\theta_2) \\ \sin(\theta_1) \end{pmatrix}$$

$$\mathbf{u}_{ab} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\theta = \text{acos} \left(\frac{\mathbf{F}_v \cdot \mathbf{u}_{ab}}{F} \right)$$

$$\theta = 127.8 \text{ deg}$$

Ans.

2-135.

Determine the projected component of the force \mathbf{F} acting along the axis AB of the pipe.

Given:

$$F = 80 \text{ N}$$

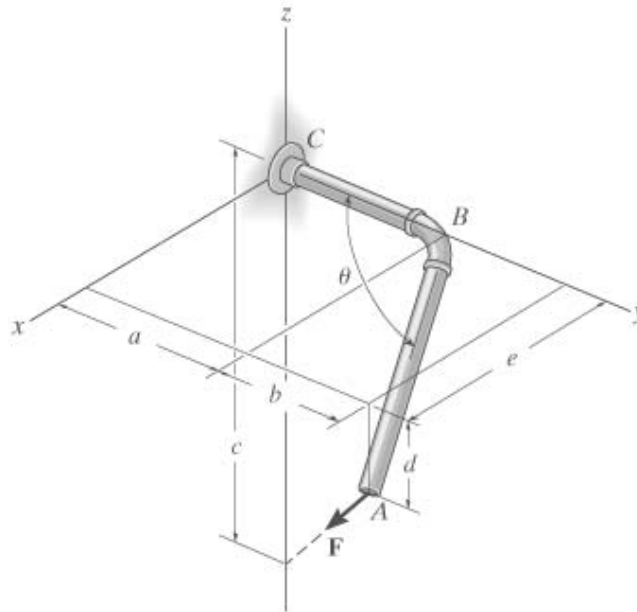
$$a = 4 \text{ m}$$

$$b = 3 \text{ m}$$

$$c = 12 \text{ m}$$

$$d = 2 \text{ m}$$

$$e = 6 \text{ m}$$



Solution:

Find the force and the unit vector

$$\mathbf{r}_A = \begin{pmatrix} -e \\ -a-b \\ d-c \end{pmatrix} \quad \mathbf{r}_A = \begin{pmatrix} -6 \\ -7 \\ -10 \end{pmatrix} \text{ m} \quad \mathbf{F}_v = F \frac{\mathbf{r}_A}{|\mathbf{r}_A|} \quad \mathbf{F}_v = \begin{pmatrix} -35.3 \\ -41.2 \\ -58.8 \end{pmatrix} \text{ N}$$

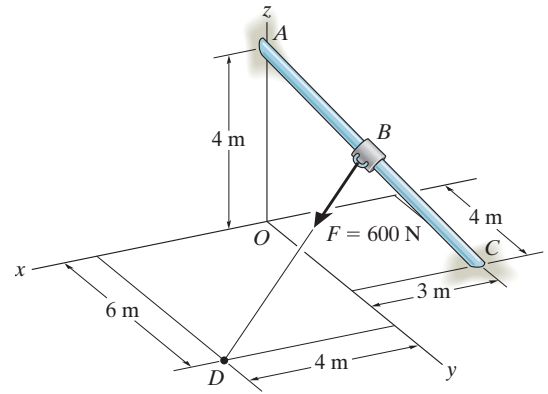
$$\mathbf{r}_{AB} = \begin{pmatrix} -e \\ -b \\ d \end{pmatrix} \quad \mathbf{r}_{AB} = \begin{pmatrix} -6 \\ -3 \\ 2 \end{pmatrix} \text{ m} \quad \mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} \quad \mathbf{u}_{AB} = \begin{pmatrix} -0.9 \\ -0.4 \\ 0.3 \end{pmatrix}$$

Now find the projection using the Dot product.

$$F_{AB} = \mathbf{F}_v \cdot \mathbf{u}_{AB} \quad F_{AB} = 31.1 \text{ N} \quad \text{Ans.}$$

2-136.

Determine the components of \mathbf{F} that act along rod AC and perpendicular to it. Point B is located at the midpoint of the rod.



SOLUTION

$$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \quad r_{AC} = \sqrt{(-3)^2 + 4^2 + (-4)^2} = \sqrt{41} \text{ m}$$

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{AC}}{2} = \frac{-3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$$

$$\begin{aligned} \mathbf{r}_{BD} &= \mathbf{r}_{AD} - \mathbf{r}_{AB} \\ &= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \\ &= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\} \text{ m} \end{aligned}$$

$$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887 \text{ m}$$

$$\mathbf{F} = 600 \left(\frac{\mathbf{r}_{BD}}{r_{BD}} \right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$$

Component of \mathbf{F} along \mathbf{r}_{AC} is $F_{||}$

$$F_{||} = \frac{\mathbf{F} \cdot \mathbf{r}_{AC}}{r_{AC}} = \frac{(465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}) \cdot (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$$F_{||} = 99.1408 = 99.1 \text{ N}$$

Ans.

Component of F perpendicular to \mathbf{r}_{AC} is F_{\perp}

$$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$$

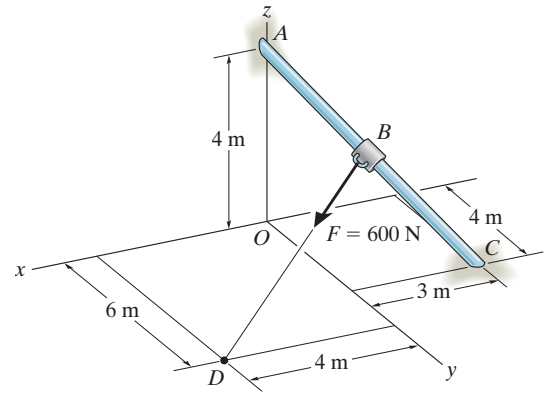
$$F_{\perp}^2 = 600^2 - 99.1408^2$$

$$F_{\perp} = 591.75 = 592 \text{ N}$$

Ans.

2-137.

Determine the components of \mathbf{F} that act along rod AC and perpendicular to it. Point B is located 3 m along the rod from end C .



SOLUTION

$$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$$

$$r_{CA} = 6.403124$$

$$\mathbf{r}_{CB} = \frac{3}{6.403124}(\mathbf{r}_{CA}) = 1.40556\mathbf{i} - 1.874085\mathbf{j} + 1.874085\mathbf{k}$$

$$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$$

$$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$$

$$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$$

$$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$$

$$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$$

$$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$$

$$r_{BD} = \sqrt{(5.5944)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$$

$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$$

$$\mathbf{r}_{AC} = (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}), \quad r_{AC} = \sqrt{41}$$

Component of \mathbf{F} along \mathbf{r}_{AC} is $\mathbf{F}_{||}$

$$F_{||} = \frac{\mathbf{F} \cdot \mathbf{r}_{AC}}{r_{AC}} = \frac{(475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}) \cdot (-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k})}{\sqrt{41}}$$

$$F_{||} = 82.4351 = 82.4 \text{ N}$$

Ans.

Component of \mathbf{F} perpendicular to \mathbf{r}_{AC} is \mathbf{F}_{\perp}

$$F_{\perp}^2 + F_{||}^2 = F^2 = 600^2$$

$$F_{\perp}^2 = 600^2 - 82.4351^2$$

$$F_{\perp} = 594 \text{ N}$$

Ans.

2-138.

Determine the magnitudes of the projected components of the force $F = 300 \text{ N}$ acting along the x and y axes.

SOLUTION

Force Vector: The force vector \mathbf{F} must be determined first. From Fig. *a*,

$$\begin{aligned} \mathbf{F} &= -300 \sin 30^\circ \sin 30^\circ \mathbf{i} + 300 \cos 30^\circ \mathbf{j} + 300 \sin 30^\circ \cos 30^\circ \mathbf{k} \\ &= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N} \end{aligned}$$

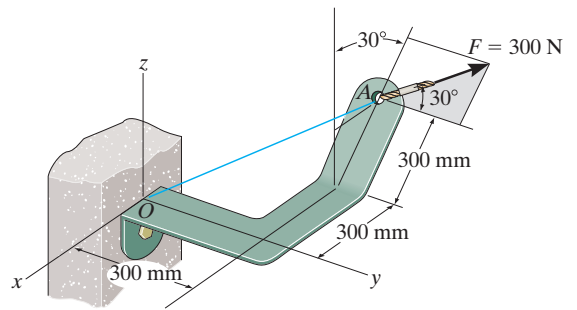
Vector Dot Product: The magnitudes of the projected component of \mathbf{F} along the x and y axes are

$$\begin{aligned} F_x &= \mathbf{F} \cdot \mathbf{i} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{i} \\ &= -75(1) + 259.81(0) + 129.90(0) \\ &= -75 \text{ N} \end{aligned}$$

$$\begin{aligned} F_y &= \mathbf{F} \cdot \mathbf{j} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot \mathbf{j} \\ &= -75(0) + 259.81(1) + 129.90(0) \\ &= 260 \text{ N} \end{aligned}$$

The negative sign indicates that \mathbf{F}_x is directed towards the negative x axis. Thus

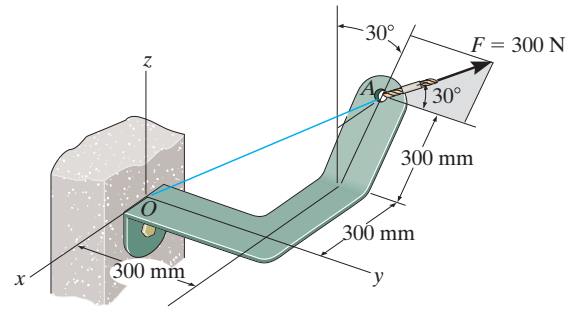
$$F_x = 75 \text{ N}, \quad F_y = 260 \text{ N}$$



Ans.

2-139.

Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA .



SOLUTION

Force and Unit Vector: The force vector \mathbf{F} and unit vector \mathbf{u}_{OA} must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30^\circ \sin 30^\circ \mathbf{i} + 300 \cos 30^\circ \mathbf{j} + 300 \sin 30^\circ \cos 30^\circ \mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

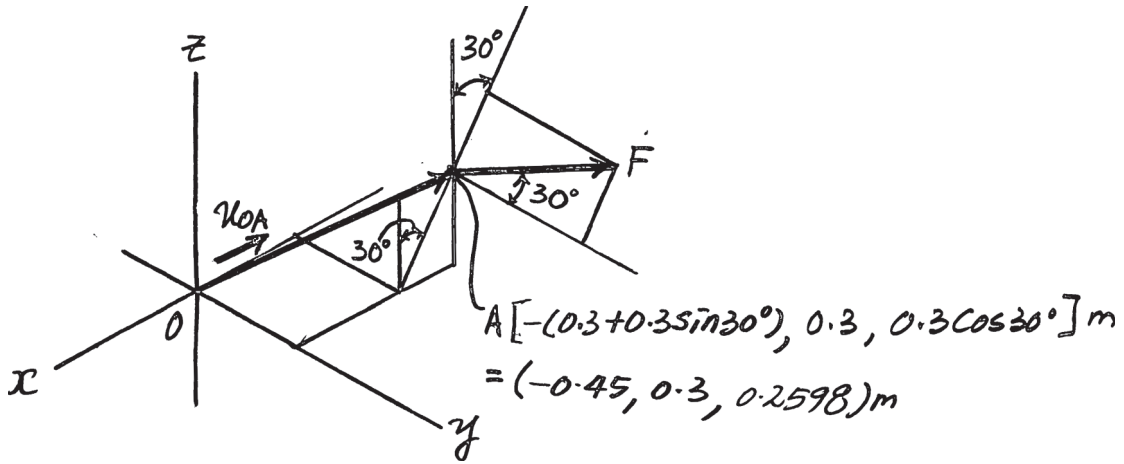
Vector Dot Product: The magnitude of the projected component of \mathbf{F} along line OA is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = (-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}) \cdot (-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k})$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

$$= 242 \text{ N}$$

Ans.



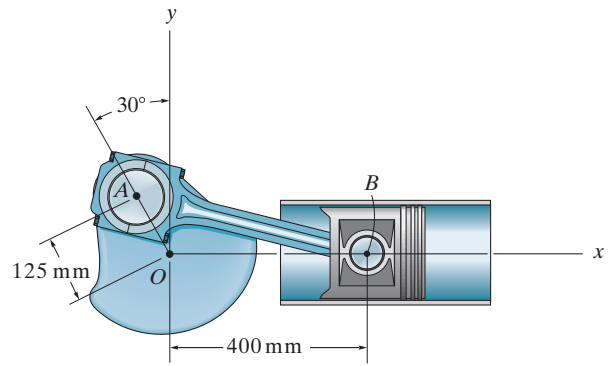
2-140.

Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.

SOLUTION

$$\begin{aligned}\mathbf{r}_{AB} &= [400 - (-125 \sin 30^\circ)]\mathbf{i} + (0 - 125 \cos 30^\circ)\mathbf{j} \\ &= \{462.5 \mathbf{i} - 108.25 \mathbf{j}\} \text{ mm}\end{aligned}$$

$$r_{AB} = \sqrt{(462.5)^2 + (108.25)^2} = 475 \text{ mm}$$

**Ans.**

2-141.

Determine the x and y components of \mathbf{F}_1 and \mathbf{F}_2 .

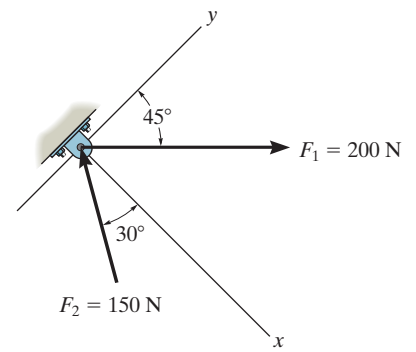
SOLUTION

$$F_{1x} = 200 \sin 45^\circ = 141 \text{ N}$$

$$F_{1y} = 200 \cos 45^\circ = 141 \text{ N}$$

$$F_{2x} = -150 \cos 30^\circ = -130 \text{ N}$$

$$F_{2y} = 150 \sin 30^\circ = 75 \text{ N}$$



Ans.

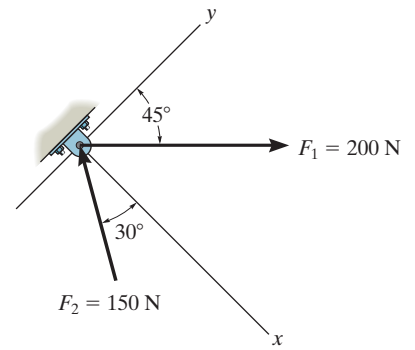
Ans.

Ans.

Ans.

2-142.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

**SOLUTION**

$$+\searrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = -150 \cos 30^\circ + 200 \sin 45^\circ = 11.518 \text{ N}$$

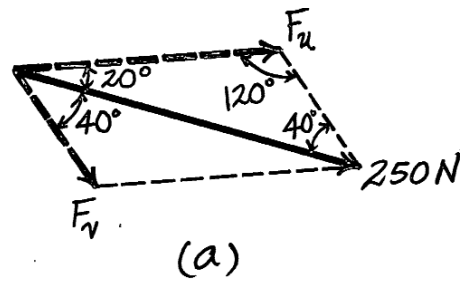
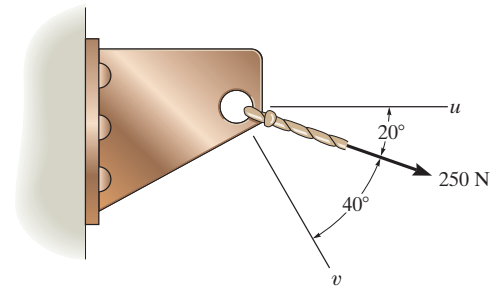
$$\nearrow + F_{Ry} = \Sigma F_y; \quad F_{Ry} = 150 \sin 30^\circ + 200 \cos 45^\circ = 216.421 \text{ N}$$

$$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0^\circ$$

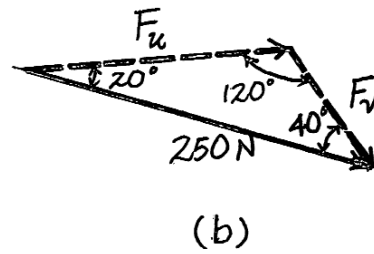
Ans.**Ans.**

2-143. Resolve the 250-N force into components acting along the u and v axes and determine the magnitudes of these components.



$$\frac{250}{\sin 120^\circ} = \frac{F_u}{\sin 40^\circ} : F_u = 186 \text{ N} \quad \text{Ans.}$$

$$\frac{250}{\sin 120^\circ} = \frac{F_v}{\sin 20^\circ} : F_v = 98.7 \text{ N} \quad \text{Ans.}$$



2-144.

Express \mathbf{F}_1 and \mathbf{F}_2 as Cartesian vectors.

SOLUTION

$$\mathbf{F}_1 = -30 \sin 30^\circ \mathbf{i} - 30 \cos 30^\circ \mathbf{j}$$

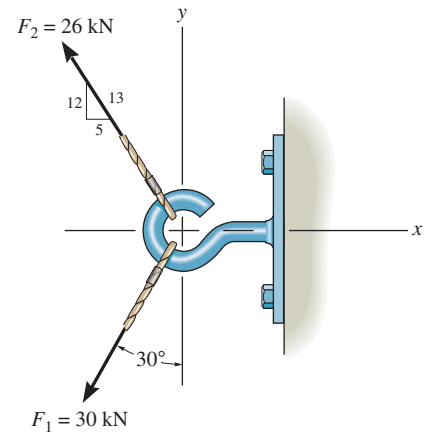
$$= \{-15.0 \mathbf{i} - 26.0 \mathbf{j}\} \text{ kN}$$

$$\mathbf{F}_2 = -\frac{5}{13}(26) \mathbf{i} + \frac{12}{13}(26) \mathbf{j}$$

$$= \{-10.0 \mathbf{i} + 24.0 \mathbf{j}\} \text{ kN}$$

Ans.

Ans.



2-145.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.

SOLUTION

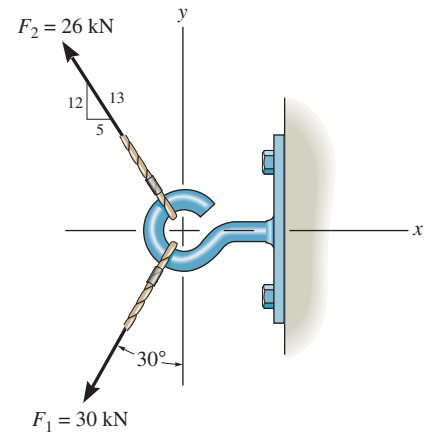
$$\pm \rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = -30 \sin 30^\circ - \frac{5}{13}(26) = -25 \text{ kN}$$

$$+ \uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -30 \cos 30^\circ + \frac{12}{13}(26) = -1.981 \text{ kN}$$

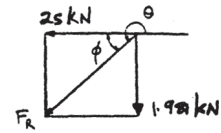
$$F_R = \sqrt{(-25)^2 + (-1.981)^2} = 25.1 \text{ kN}$$

$$\phi = \tan^{-1}\left(\frac{1.981}{25}\right) = 4.53^\circ$$

$$\theta = 180^\circ + 4.53^\circ = 185^\circ$$



Ans.



Ans.

2-146. Cable AB exerts a force of 80 N on the end of the 3-m-long boom OA . Determine the magnitude of the projection of this force along the boom.

Vector Analysis :

$$\mathbf{F} = 80 \left(\frac{\mathbf{r}_{AB}}{r_{AB}} \right) = 80 \left(-\frac{3 \cos 60^\circ}{5} \mathbf{i} - \frac{3 \sin 60^\circ}{5} \mathbf{j} + \frac{4}{5} \mathbf{k} \right)$$

$$= -24 \mathbf{i} - 41.57 \mathbf{j} + 64 \mathbf{k}$$

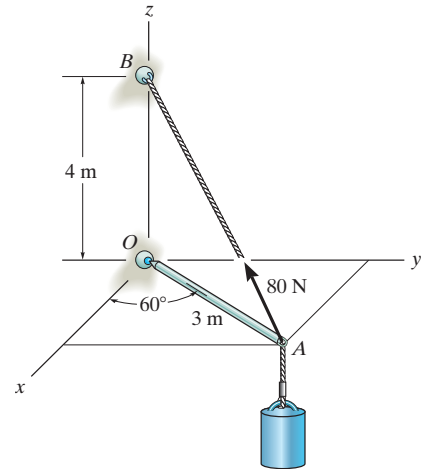
$$\mathbf{u}_{AO} = -\cos 60^\circ \mathbf{i} - \sin 60^\circ \mathbf{j} = -0.5 \mathbf{i} - 0.866 \mathbf{j}$$

$$\text{Proj } \mathbf{F} = \mathbf{F} \cdot \mathbf{u}_{AO} = (-24)(-0.5) + (-41.57)(-0.866) + (64)(0) = 48.0 \text{ N} \quad \text{Ans.}$$

Scalar Analysis :

$$\text{Angle } OAB = \tan^{-1} \left(\frac{4}{3} \right) = 53.13^\circ$$

$$\text{Proj } F = 80 \cos 53.13^\circ = 48.0 \text{ N} \quad \text{Ans.}$$



2-147.

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_2$. Specify its direction measured counterclockwise from the positive x axis.

SOLUTION

$$F' = \sqrt{(80)^2 + (50)^2 - 2(80)(50) \cos 105^\circ} = 104.7 \text{ N}$$

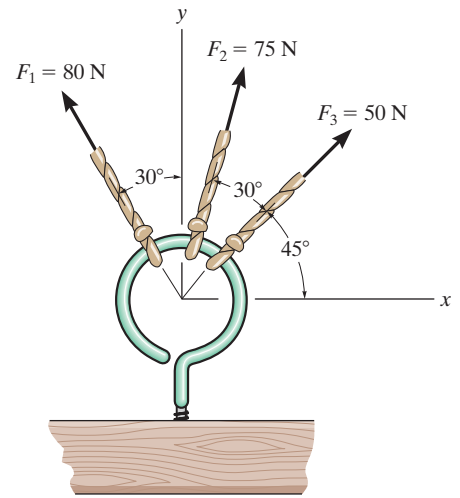
$$\frac{\sin \phi}{80} = \frac{\sin 105^\circ}{104.7}; \quad \phi = 47.54^\circ$$

$$F_R = \sqrt{(104.7)^2 + (75)^2 - 2(104.7)(75) \cos 162.46^\circ}$$

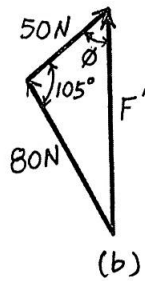
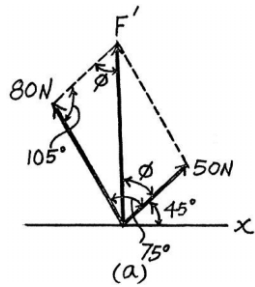
$$F_R = 177.7 = 178 \text{ N}$$

$$\frac{\sin \beta}{104.7} = \frac{\sin 162.46^\circ}{177.7}; \quad \beta = 10.23^\circ$$

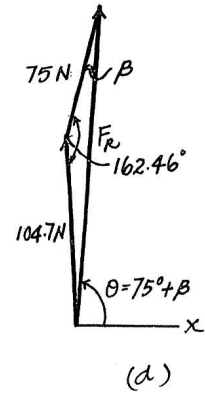
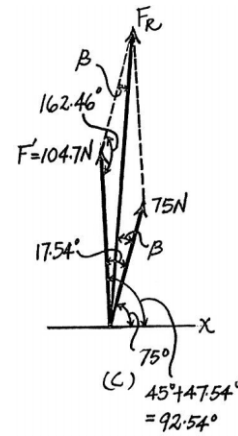
$$\theta = 75^\circ + 10.23^\circ = 85.2^\circ$$



Ans.



Ans.



2-148.

If $\theta = 60^\circ$ and $F = 20$ kN, determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.

SOLUTION

$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = 50\left(\frac{4}{5}\right) + \frac{1}{\sqrt{2}}(40) - 20 \cos 60^\circ = 58.28 \text{ kN}$$

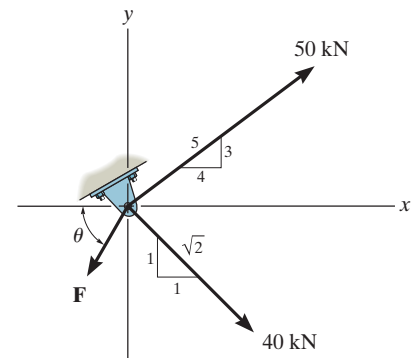
$$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = 50\left(\frac{3}{5}\right) - \frac{1}{\sqrt{2}}(40) - 20 \sin 60^\circ = -15.60 \text{ kN}$$

$$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3 \text{ kN}$$

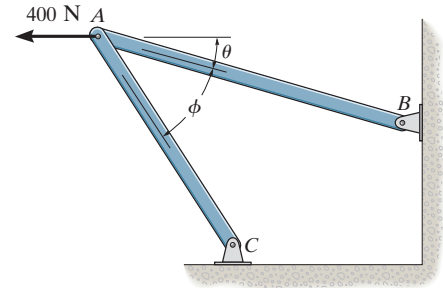
$$\phi = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0^\circ$$

Ans.

Ans.



2-149. Determine the design angle θ ($0^\circ \leq \theta \leq 90^\circ$) for strut AB so that the 400-N horizontal force has a component of 500 N directed from A towards C . What is the component of force acting along member AB ? Take $\phi = 40^\circ$.



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have

$$\frac{\sin \theta}{500} = \frac{\sin 40^\circ}{400}$$

$$\sin \theta = 0.8035$$

$$\theta = 53.46^\circ = 53.5^\circ \quad \text{Ans.}$$

Thus, $\psi = 180^\circ - 40^\circ - 53.46^\circ = 86.54^\circ$

Using law of sines [Fig. (b)]

$$\frac{F_{AB}}{\sin 86.54^\circ} = \frac{400}{\sin 40^\circ}$$

$$F_{AB} = 621 \text{ N} \quad \text{Ans.}$$

