Questions and Solutions at End of Chapter 2

2.1 Describe active and passive, null type and deflection type, analog and digital, indicating and signal-output types. Explain differences between these. Give examples of each (See Section 2.2 in the book for full details).

2.2 **Sample answer:**
(a) Quantity being measured modulates the magnitude of some external power source.
(b) Instrument output is entirely produced by the quantity being measured.
Give examples (see Section 2.2 in the book).
Relative merits:
Active: better measurement resolution (but limited by heating effect due to power source and also by safety considerations—small voltage is necessary).
Passive: simpler construction, no power supply needed, and limited measurement resolution.

2.3 Null type is more accurate but tedious to use.
Deflection type is less accurate but easier to use.
Explain the difference by means of an example, e.g., null and deflection types of pressure gauge (See Section 2.2 in the book for full details).
Null type normally reserved for calibration duties where best accuracy is needed.

2.4 An analog instrument gives an output that varies continuously as the quantity being measured changes. The output can have an infinite number of values within the range that the instrument is designed to measure, but the number of different positions that the eye can discriminate between is strictly limited according to how large the scale is and how finely it is divided.
A digital instrument has an output that varies in discrete steps and so can only have a finite number of values.
Give examples of each.
Digital instrument is best for computer control systems since analog-to-digital (A/D) conversion is needed for an analog instrument. Explain problems with A/D conversion—cost and conversion time (see Section 2.2 in the book).

2.5 **Static characteristics:** These are their steady-state attributes (when the output measurement value has settled to a constant reading after any initial varying output)

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such as accuracy, measurement sensitivity, and resistance to errors caused by variations in their operating environment.
Define and explain the various static characteristics (see Section 2.3 in the book).
Dynamic characteristics: These describe the behavior of a measuring instrument between the time a measured quantity changes value and the time when the instrument output attains a steady value in response.
Depending on the time allowed, student may be expected to sketch the main types of a dynamic response (zero, first, and second order)—see Section 2.4 in the book.

2.6 Definition and explanation expected for accuracy (or inaccuracy or measurement uncertainty), precision (or repeatability or reproducibility), tolerance, range (or span), linearity (or nonlinearity), sensitivity of measurement, threshold, resolution, sensitivity to disturbance—explaining zero drift (or bias) and sensitivity drift, hysteresis, and dead space. Student should draw sketches to illustrate as appropriate.

2.7 **Accuracy** is more usually expressed as inaccuracy or measurement uncertainty. The latter quantifies the extent to which a measurement may be incorrect and is usually expressed as a percentage of the full-scale instrument output reading.
Precision is a term that describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high precision instrument, then the spread of readings will be very small.
Precision is often, though incorrectly, confused with accuracy. High precision does not imply anything about measurement accuracy. A high precision instrument may have a low accuracy. Low accuracy measurements from a high precision instrument are normally caused by a bias in the measurements, which are removable by recalibration.
Depending on the time allowed, student should be expected to illustrate with an example of high precision but low accuracy (e.g., Figure 2.5 in the book).

2.8 See Section 2.4 in the book for appropriate sketches.

2.9 Dynamic characteristics describe the behavior of an instrument following the time that the measured quantity changes value up until the time when the output reading attains a steady value. Various kinds of dynamic behavior can be observed in different instruments ranging from an output that varies slowly until it reaches a final constant value to an output that oscillates about the final value until a steady reading is obtained. The dynamic characteristics are a very important factor in deciding on the suitability of an instrument for a particular measurement application.
A zero-order instrument responds instantaneously (or effectively so) to a change in measured quantity and so is suitable for all measurement situations.

A large number of instruments have a first-order characteristic. This limits their use in control systems because it is necessary to take account of the time lag that occurs between a measured quantity changing in value and the measuring instrument indicating the change. Fortunately, the time constant of many first-order instruments is small relative to the dynamics of the process being measured and so no serious problems are created in such cases. However, if there is a need to sample the output of a measurement system at a high frequency, the time lag before the instrument responds to a change in the value of the measured quantity may preclude the use of the instruments with a first-order characteristic.

A second-order instrument has an oscillatory output unless damping is applied. When damped, the output response resembles that of a first-order instrument and the above arguments apply.

(See discussion on first- and second-order instruments in Section 2.4 in the book for full explanation).

2.10 The *accuracy* of an instrument is a measure of how close the output reading of the instrument is to the correct value. In practice, it is more usual to quote the *inaccuracy* or *measurement uncertainty* value rather than the accuracy value for an instrument. The inaccuracy or measurement uncertainty is the extent to which a reading might be wrong and is often quoted as a percentage of the full-scale reading of an instrument.

Likely error is 1.5% of 1100 °C (the full-scale reading), i.e., 16.5 °C.

2.11 Tolerance is a term that is closely related to accuracy. It describes the maximum deviation of a manufactured component from some specified value. For instance, crankshafts are machined with a diameter tolerance quoted as so many microns (10^{-6} m), and electric circuit components such as resistors have tolerances of perhaps 5%.

Expected shortest rod is 5000 mm − 2% = 4900 mm. Expected longest rod is 5000 mm + 2% = 5100 mm.

2.12 Maximum error is 1.5% of the full-scale reading, i.e., 1.5% × 20,000 = 100 bar.

2.13 Tolerance is a term that is closely related to accuracy. It describes the maximum deviation of a manufactured component from some specified value. For instance, crankshafts are machined with a diameter tolerance quoted as so many microns (10^{-6} m), and electric circuit components such as resistors have tolerances of perhaps 5%.

Maximum deviation in length (given by the tolerance) is 1.5% of the nominal length, i.e., 1.5% × 250 mm = 3.75 mm. Thus, the shortest and longest bricks likely are 250 − 3.75 and 250 + 3.75, i.e., 246.25 and 253.75.

2.14 Range is 7.5−5.0 = 2.5 cm.

2.15 The minimum likely value is 5000 Ω − 3% = 5000 − 150 = 4850 Ω.
The maximum likely value is 5000 Ω + 3% = 5000 + 150 = 5150 Ω.

2.16 (a) The maximum error expected in any measurement reading is 0.5% of the full-scale reading, which is 30 bar for this particular instrument. Hence, the maximum likely error is 0.5% × 30 bar = 0.15 bar. [30%]
(b) The maximum measurement error is a constant value related to the full-scale reading of the instrument, irrespective of the magnitude of the quantity that the instrument is actually measuring. In this case, as worked out above, the magnitude of the error is 0.15 bar. Thus, when measuring a pressure of 5 bar, the maximum possible error of 0.15 bar is 3% of the measurement value. [30%]
(c) If the measurement error is deemed to be too high, you could either use a higher quality instrument with better accuracy (usually more expensive) or use a pressure sensor with a similar quality (and price) but that has a smaller range, for example, 0−10 bar. The recommended option is the latter (because it is cheaper). [40%]

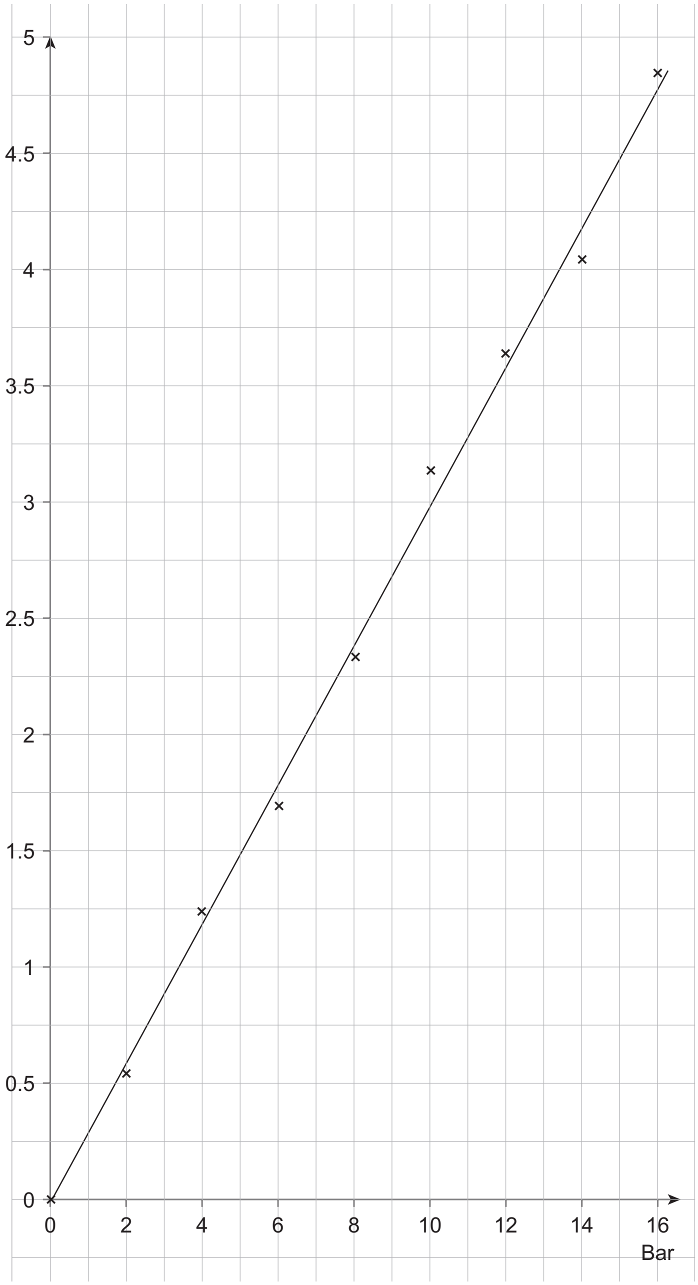
2.17 (a) The mean (average) value of the 10 measurements made with the ultrasonic rule is 4.293 m.
The maximum deviation below this mean value is −0.003 m and the maximum deviation above the mean value is +0.003 m. Thus, the precision of the ultrasonic rule can be expressed as ±0.003 m (±3 mm). [50%]
(b) The correct value of the room width has been measured as 4.276 m by the calibrated steel rule. All ultrasonic rule measurements are above this, with the largest value being 4.296 m. This last measurement is the one that exhibits the largest measurement error. This maximum measurement error can be calculated as 4.296 − 4.276 = 0.020 m (20 mm). Thus, the maximum measurement inaccuracy can be expressed as +20 mm. [50%]

2.18 Given table of data values is:

mV	4.37	8.74	13.11	17.48
°C	250	500	750	1000

Students may make a graph of these data points and fit a straight line in order to determine the sensitivity. This is acceptable. However, there is a quicker analytical solution in this case since the output emf increments by exactly 4.37 mV for each 250 °C rise in temperature. The sensitivity can therefore be calculated as 4.37/250 = 0.0175 mV/°C (N.B. The sensitivity is expressed in units of mV/°C since this is what the question asked for, but it would be more appropriate in practice to express the sensitivity as 17.5 μV/°C).

2.19 (a)



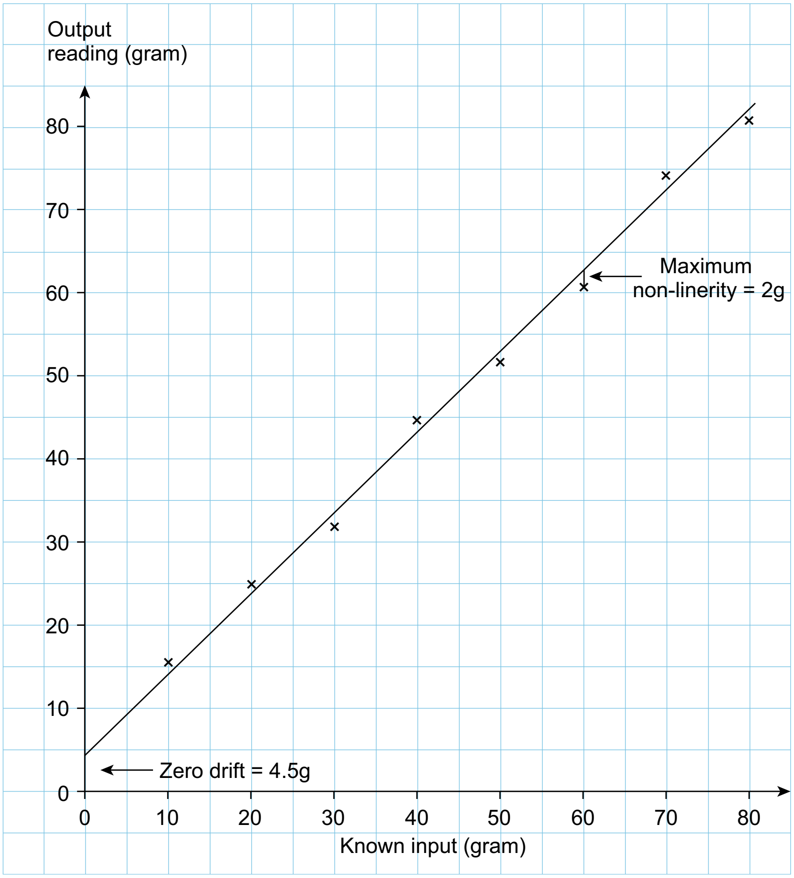
(b) From graph, output is 4.75 V for input of 16 bar. Thus, the sensitivity is 4.75/16 = 0.297 V/bar.

(c) Maximum nonlinearity: On the graph drawn, this is apparently the data point for input of 10 bar.
Measured output for data point at 10 bar input is 3.15 V (taken from the table of measured input–output data).
Output read from the graph for input of 10 bar is 3.00 V.
Thus, nonlinearity is $3.15 - 3.00 = 0.15$ V.
Full-scale deflection is 4.85 V.
Hence, maximum nonlinearity expressed as a percentage $= \frac{0.15}{4.85} \times 100 = 3.09\%$

(Marking: 40% for part (a), 30% for part (b), and 30% for part (c))

2.20 Sensitivity drift (also known as scale factor drift) defines the amount by which an instrument's sensitivity of measurement varies as environmental conditions (e.g., temperature and pressure) change.
Zero drift (also known as bias) describes the effect where the zero reading of an instrument is modified by a change in environmental conditions. This causes a constant error that exists over the full range of measurement of the instrument.
Sensitivity drift and zero drift are collectively known as the sensitivity to disturbance of an instrument. As variations occur in the temperature, pressure, etc., in the environment surrounding a measurement system, certain static instrument characteristics change, and the sensitivity to disturbance is a measure of the magnitude of this change. This change can cause either zero drift or sensitivity drift, or sometimes both of these. See Section 2.3 in the book for a full explanation.

2.21 (a)



(b) The three measurement characteristics evident in the data plotted on the graph are nonlinearity, zero drift, and sensitivity drift.

(c) *Maximum nonlinearity*: On the graph drawn, this is apparently the data point for input of 60 g.
Measured output for data point at 60 g input is 61 g (taken from the table of measured input–output data).
Output read from the graph for input of 60 g is 63 g.
Thus, nonlinearity is $63 - 61 = 2$ g.
Zero drift: This is the output when the input is zero. From the graph, zero drift $= 4.5$ g.
Sensitivity drift: Correct measurement sensitivity is 1 (output should equal input).
Sensitivity calculated from graph is

$$\left(\text{gradient of straight line fitted to the data } \frac{(82.0-4.5)}{80} = \frac{77.5}{80} = 0.97\right)$$

Hence, sensitivity drift $= 0.97 - 1.00 = -0.03$.

(Marking: 40% for part (a), 20% for part (b), and 40% for part (c))

2.22 Zero drift (also known as bias) describes the effect where the zero reading of an instrument is modified by a change in environmental conditions (e.g., environmental temperature and pressure). This causes a constant error that exists over the full range of measurement of the instrument.
The zero drift coefficient is the amount of change in the instrument output for a given change in an environmental parameter such as temperature or pressure. If the characteristic of an instrument is sensitive to several environmental parameters, then it will have several zero drift coefficients, one for each environmental parameter.
If an instrument has a voltage output, the zero drift due to environmental temperature change would be expressed in units of volts/°C. [40%]
The zero drift at the temperature of 30 °C is the constant difference between the pairs of output readings, i.e., 0.19 Pa. [20%]
The zero drift coefficient is the magnitude of the drift (0.19 Pa) divided by the magnitude of the temperature change causing the drift (10 °C). Thus, the zero drift coefficient is 0.19/10 = 0.019 Pa/°C. [30%]

2.23 (a) Given table of data values at a temperature of 20 °C is:

y	13.1	26.2	39.3	52.4	65.5	78.6
x	5	10	15	20	25	30

Students may make a graph of these data points and fit a straight line in order to determine the sensitivity. This is acceptable. However, there is a quicker analytical solution in this case since the value of *y* increments by exactly 13.1 for each increment of 5 in the value of *x*. The sensitivity, expressed as the ratio *y*/*x* can therefore be calculated as 13.1/5 = 2.62. [40%]

(b) When the instrument is subsequently used in an environment at a temperature of 50 °C, the table of data values changes to the following:

y	14.7	29.4	44.1	58.8	73.5	88.2
x	5	10	15	20	25	30

As before, graphical analysis of the data is unnecessary, since this time the value of *y* increments by exactly 14.7 for each increment of 5 in the value of *x*. The new sensitivity, expressed as the ratio *y*/*x* can therefore be calculated as 14.7/5 = 2.94. The sensitivity of 2.62 at 20 °C has changed to a sensitivity of 2.94 at 50 °C. Thus, the sensitivity changes by 0.32 as the temperature increases by 30 °C. Hence, the sensitivity drift (change in sensitivity per °C) can be expressed as 0.32/30 = 0.01067/°C. [60%]

2.24 At 20 °C, deflection/load characteristic is a straight line. Sensitivity = 90 degrees/kg.
At 27 °C, deflection/load characteristic is still a straight line.
Sensitivity $= \frac{(191-6)}{2} = 92.5$ degrees/kg.

Sensitivity drift = 2.5 degrees/kg [20%]
Zero drift (bias) = 6 degrees(the no-load deflection) [20%]
Sensitivity drift/°C = 2.5/7 = 0.357 degrees/kg/°C [30%]
Zero drift/°C = 6/7 = 0.857 degrees/°C. [30%]

2.25 The table of measurements given is:

Values measured by uncalibrated instrument (°C)	Correct value of temperature (°C)
20	21.5
35	36.5
50	51.5
65	66.5

Bias is the difference between the correct value and the measured value. This difference is 1.5 °C for each pair of measurements. Hence, the bias due to the instrument being out of calibration is 1.5 °C.

2.26 The measurements in an environment at a temperature of 21 °C are:

Load(kg)	0	50	100	150	200
Deflection(mm)	0.0	1.0	2.0	3.0	4.0

At 35 °C, the measurements change to:

Load(kg)	0	50	100	150	200
Deflection(mm)	0.2	1.3	2.4	3.5	4.6

(a) At 21 °C, the deflection increases by 1.0 mm for each 50 kg increase in load. Therefore, the sensitivity is 1.0/50 = 0.020 mm/kg = 20 μm/kg.
At 35 °C, the deflection increases by 1.1 mm for each 50 kg increase in load. Therefore, the sensitivity is 1.1/50 = 0.022 mm/kg = 22 μm/kg. [40%]

(b) The total zero drift due to the increase in temperature is the change in deflection when the load is zero, i.e., 0.2 mm.
The total sensitivity drift due to the increase in temperature is the change in sensitivity, i.e., (22 − 20) = 2.0 μm/kg. [30%]

(c) The zero drift of 0.2 mm is caused by a temperature increase of 14 °C. Thus, the zero drift coefficient can be expressed as 0.2/14 = 0.0143 mm/°C = 14.3 μm/°C.
The sensitivity drift caused by the temperature increase of 14 °C is 2.0 μm/kg. Thus, the sensitivity drift coefficient can be expressed as 2/14 = 0.143 μm/kg/°C. [30%]

2.27 Let the temperature reported by the balloon at some general time t be T_r. Then, T_h is related to T_r by the relation:

$$T_r = \frac{T_h}{1 + \tau D} = \frac{T_o - 0.012h}{1 + \tau D} = \frac{20 - 0.012h}{1 + 10D}$$

It is given that $h = 6t$ (velocity is 6 m/s), thus:

$$T_r = \frac{20 - 0.072t}{1 + 10D}$$

The transient or complementary function part of the solution ($T_h = 0$) is given by:

$$T_{r_{cf}} = Ce^{-t/10}$$

The particular integral part of the solution is given by:

$$T_{r_{pi}} = 20 - 0.072(t - 10)$$

Thus, the whole solution is given by:

$$T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/10} + 20 - 0.072(t - 10)$$

Applying initial conditions: At $t = 0$, $T_r = 20$, i.e., $20 = Ce^{-0} + 20 - 0.072(-10)$
Thus, $C = -0.72$ and the solution can be written as:

$$T_r = 20 - 0.72e^{-t/10} - 0.072(t - 10)$$

Using the above expression to calculate T_r for various values of t, the following table is constructed:

Time	Altitude	Temperature Reading	Temperature Error
0	0	0.00	0.00
10	60	19.28	0.46
20	120	18.56	0.62
30	180	17.84	0.68
40	240	17.12	0.71
50	300	16.40	0.72
60	360	15.68	0.72
70	420	14.96	0.72
80	480	14.24	0.72
90	540	13.52	0.72
100	600	12.80	0.72

[50%]

(b) At 8000 m, $t = 1333.3$ s. Calculating T_r from the above expression:

$$T_r = 20 - 0.72e^{-1333.3/10} - 0.072(1333.3 - 10)$$

The exponential term approximates to zero and so T_r can be written as:

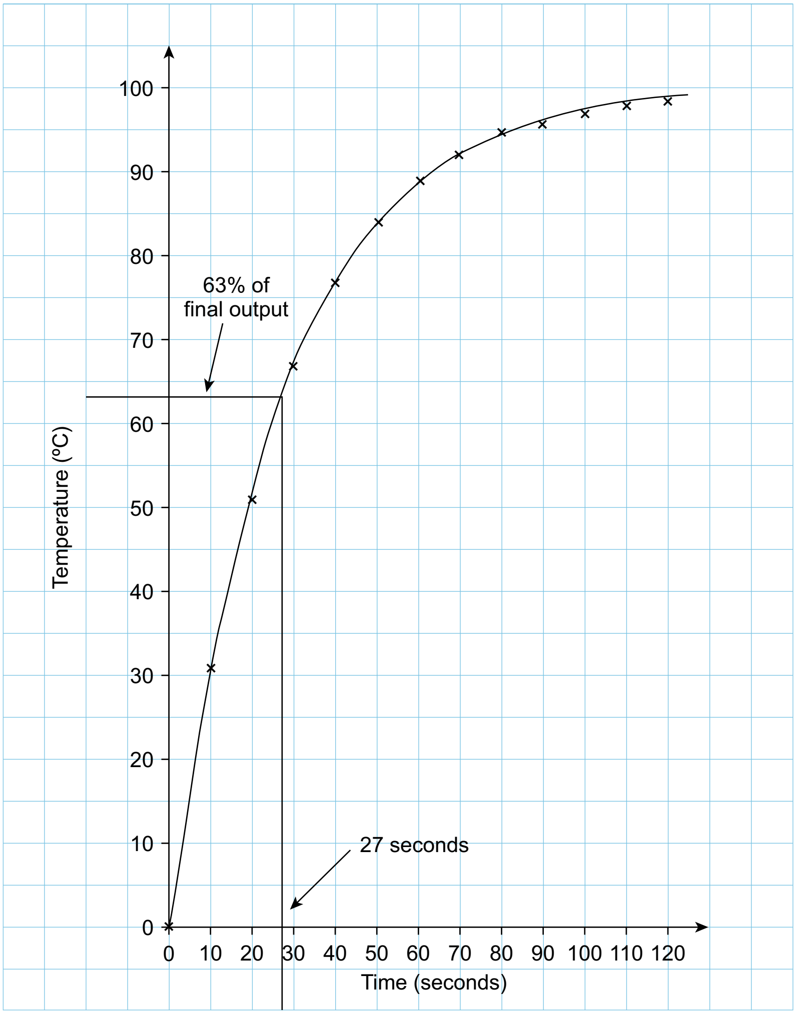
$$T_r \approx 20 - 0.072(1323.3) = -75.28°\text{C}$$

[30%]

(c) Since the temperature falls according to the relation: $T_h = T_0 - 0.012h$, the true temperature at an altitude of 8000 m is $T_h = T_0 - 0.012h = 20 - (0.012 \times 8000) = 20 - 96 = -76$.

Thus, the temperature error is 0.72. [20%]
This result might have been inferred from the table above where it can be seen that the error has converged to a value of 0.72. For large values of t, the transducer reading lags the true temperature value by a period of time equal to the time constant of 10 s. In this time, the balloon travels a distance of 60 m and the temperature falls by 0.72°. Thus, for large values of t, the output reading is always 0.72° less than it should be.

2.28



[50%]

The time constant is the time taken for the output reading to rise to 63% of its final value. Since the output reading is rising from 0 to 100 °C, this means the time when the output has risen to 63 °C. Using the graph of temperature readings, this point is reached after 27 s. Thus, the time constant of the thermometer is 27 s. [50%]

2.29 Note: It is assumed that students are familiar with the solution of a first-order differential equation, as commonly covered in mathematics courses.
The water temperature on the sea surface, T_0, is 20 °C.
The temperature T_x at a depth of x meters is given by the relation:

$$T_x = T_0 - 0.01x$$

The temperature-measuring instrument characteristic is approximately first order with a time constant (τ) of 50 s.
It is also given that the submarine is diving at a rate of 0.5 m/s, i.e., $x = 0.5t$.

Part (a) Let the temperature reported by the temperature sensor at some general time (t) be T_r.
Thus, the temperature reading T_r is related to T_x by the expression:

$$T_r = \frac{T_x}{1+\tau D} = \frac{T_0 - 0.01x}{1+50D} = \frac{20-(0.01\times[0.5t])}{1+50D} = \frac{20-0.005t}{1+50D}$$

The transient or complementary function part of the solution ($T_x = 0$) is given by:

$$T_{r_{cf}} = Ce^{-t/50}$$

The particular integral part of the solution is given by:

$$T_{r_{pi}} = 20 - 0.005(t-50)$$

Thus, the whole solution is given by:

$$T_r = T_{r_{cf}} + T_{r_{pi}} = Ce^{-t/50} + 20 - 0.005(t-50)$$

Applying initial conditions: At $t = 0$, $T_r = 20$, i.e., $20 = Ce^{-0} + 20 - 0.005(-50)$
Thus, $C = 0.25$ and therefore, $T_r = 0.25\, e^{-t/50} + 20 - 0.005(t-50)$
Using the above expression to calculate T_r, for various values of t, the following table can be constructed:

Time (t)	Depth (x) in m	Actual Temperature (T_x) in °C	Temperature Reading (T_r) in °C	Temperature Error in °C
0	0	20.0	20.0	0.0
100	50	19.5	19.716	0.216
200	100	19.0	19.245	0.245
300	150	18.5	18.749	0.249
400	200	18.0	18.250	0.250
500	250	17.5	17.750	0.250

[70%]

Part (b) At 1000 m, $t = 2000$ s.
Calculating T_r from the above expression:

$$T_r = 0.25\, e^{-t/50} + 20 - 0.005(t - 50)$$

The exponential term approximates to zero and so T_r can be written as:
$T_r \approx 20 - 0.005(1950) = 20 - 9.75 = 10.25\,°\text{C}.$ [30%]

2.30 For a step input, the general differential equation describing the behavior of a second-order system can be written as:

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \tag{1}$$

where q_i is the measured quantity, q_o is the instrument output reading, and a_o, a_1, a_2, and b_o are constants.
Applying the D operator:

$$a_2 D^2 q_o + a_1 D q_o + a_0 q_o = b_0 q_i,$$

and rearranging:

$$q_o = \frac{b_0 q_i}{a_0 + a_1 D + a_2 D^2} \tag{2}$$

It is convenient to reexpress the variables a_0, a_1, a_2, and b_0 in Eqn (2) in terms of three parameters K (static sensitivity), ω (undamped natural frequency), and ξ (damping ratio), where:

$$K = b_0/a_0; \quad \omega = \sqrt{a_0/a_2} \quad ; \quad \xi = a_1/2\sqrt{a_0 a_2}$$

ξ can be written as $\xi = \frac{a_1}{2a_0\sqrt{a_2/a_0}} = \frac{a_1\omega}{2a_0}$

If Eqn (2) is now divided through by a_0 we get:

$$q_o = \frac{(b_0/a_0)q_i}{1 + (a_1/a_0)D + (a_2/a_0)D^2} \tag{3}$$

The terms in Eqn (3) can be written in terms of ω and ξ as follows:

$$\frac{b_0}{a_0} = K; \quad \left(\frac{a_1}{a_2}\right)D = \frac{2\xi D}{\omega}; \quad \left(\frac{a_2}{a_0}\right)D^2 = \frac{D^2}{\omega^2}$$

Hence, dividing Eqn (3) through by q_i and substituting for a_0, a_1, and a_2 gives:

$$\frac{q_0}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1} \tag{4}$$

See Figure 2.12 in Section 2.4.3 for sketches of instrument response in heavy damped, critically damped, and lightly damped cases.
Student should identify the critically damped case ($\xi = 0.707$) as the usual design target.