Chapter 2

2.1 $\mathbf{C} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} 8 & 17 & -3 \\ -1 & -1 & -1 \\ 1 & -7 & 1 \end{bmatrix}$; $\mathbf{D} = \mathbf{A} - \mathbf{B} = \begin{bmatrix} -2 & -1 & 1 \\ 17 & -13 & -7 \\ -3 & -1 & 9 \end{bmatrix}$

2.2 $\mathbf{C} = 2\mathbf{A} + \mathbf{B} = \begin{bmatrix} 19 & -14 & -9 \\ -2 & -1 & 0 \\ -10 & 2 & 4 \\ -5 & 20 & -7 \end{bmatrix}$; $\mathbf{D} = \mathbf{A} - 3\mathbf{B} = \begin{bmatrix} -1 & -12 & 6 \\ 13 & -11 & 0 \\ -12 & 29 & -19 \\ 1 & -4 & 21 \end{bmatrix}$

2.3 $\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 4 & -6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix} = 12 - 6 - 10 = -4$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix} \begin{bmatrix} 4 & -6 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -18 & 6 \\ 4 & -6 & 2 \\ -2 & 30 & -10 \end{bmatrix}$$

2.4 $\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 4 & 6 \\ -7 & -5 \\ 1 & -9 \\ -3 & 11 \end{bmatrix} \begin{bmatrix} -1 & 3 & -5 & 6 \\ -13 & -4 & 7 & 2 \end{bmatrix} = \begin{bmatrix} -82 & -12 & 22 & 44 \\ 72 & -1 & 0 & -44 \\ 116 & 39 & -68 & -52 \\ -140 & -53 & 92 & 60 \end{bmatrix}$

$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} -1 & 3 & -5 & 2 \\ -13 & -4 & 7 & 6 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ -7 & -5 \\ 1 & -9 \\ -3 & 11 \end{bmatrix} = \begin{bmatrix} -36 & 46 \\ -35 & -55 \end{bmatrix}$

2.5 $\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 4 & -6 & 1 \\ -6 & 5 & 7 \\ 1 & 7 & 8 \end{bmatrix} \begin{bmatrix} 3 & 5 & 0 \\ 5 & 7 & -2 \\ 0 & -2 & 9 \end{bmatrix} = \begin{bmatrix} -18 & -24 & 21 \\ 7 & -9 & 53 \\ 38 & 38 & 58 \end{bmatrix}$

$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 3 & 5 & 0 \\ 5 & 7 & -2 \\ 0 & -2 & 9 \end{bmatrix} \begin{bmatrix} 4 & -6 & 1 \\ -6 & 5 & 7 \\ 1 & 7 & 8 \end{bmatrix} = \begin{bmatrix} -18 & 7 & 38 \\ -24 & -9 & 38 \\ 21 & 53 & 58 \end{bmatrix}$

2.6 $\mathbf{C} = \mathbf{AB} = \begin{bmatrix} 12 & -11 & 10 \\ 0 & 2 & -4 \\ -7 & 9 & 8 \\ 6 & 15 & -5 \end{bmatrix} \begin{bmatrix} 13 & -1 & 5 \\ 16 & -9 & 0 \\ -3 & 20 & -7 \end{bmatrix} = \begin{bmatrix} -50 & 287 & -10 \\ 44 & -98 & 28 \\ 29 & 86 & -91 \\ 333 & -241 & 65 \end{bmatrix}$

2.8 $$\mathbf{AB} = \begin{bmatrix} 21 & 10 & 16 \\ -15 & 11 & 0 \\ 13 & 20 & -9 \\ 7 & -17 & 14 \end{bmatrix} \begin{bmatrix} 7 & -4 \\ -1 & 9 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 185 & -90 \\ -116 & 159 \\ 44 & 182 \\ 108 & -265 \end{bmatrix}$$

$$(\mathbf{AB})^T = \begin{bmatrix} 185 & -116 & 44 & 108 \\ -90 & 159 & 182 & -265 \end{bmatrix} \tag{1}$$

$$\mathbf{B}^T\mathbf{A}^T = \begin{bmatrix} 7 & -1 & 3 \\ -4 & 9 & -6 \end{bmatrix} \begin{bmatrix} 21 & -15 & 13 & 7 \\ 10 & 11 & 20 & -17 \\ 16 & 0 & -9 & 14 \end{bmatrix} = \begin{bmatrix} 185 & -116 & 44 & 108 \\ -90 & 159 & 182 & -265 \end{bmatrix} \tag{2}$$

From Eqs. (1) and (2), we can see that $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$.

2.9

$$[A][B][C]=\begin{bmatrix} -9 & 0 \\ 13 & 20 \\ 8 & -3 \\ -11 & -5 \end{bmatrix}\begin{bmatrix} 15 & -1 & -4 \\ 6 & 16 & 9 \end{bmatrix}\begin{bmatrix} -7 & 10 & 6 & 0 \\ -1 & 2 & -8 & -2 \\ 16 & 12 & 2 & 8 \end{bmatrix}$$

$$=\begin{bmatrix} -135 & 9 & 36 \\ 315 & 307 & 128 \\ 102 & -56 & -59 \\ -195 & -69 & -1 \end{bmatrix}\begin{bmatrix} -7 & 10 & 6 & 0 \\ -1 & 2 & -8 & -2 \\ 16 & 12 & 2 & 8 \end{bmatrix}$$

$$=\begin{bmatrix} 1512 & -900 & -810 & 270 \\ -464 & 5300 & -310 & 410 \\ -1602 & 200 & 942 & -360 \\ 1418 & -2100 & -620 & 130 \end{bmatrix}$$

$$\left([A][B][C]\right)^T=\begin{bmatrix} 1512 & -464 & -1602 & 1418 \\ -900 & 5300 & 200 & -2100 \\ -810 & -310 & 942 & -620 \\ 270 & 410 & -360 & 130 \end{bmatrix} \tag{1}$$

$$[C]^T[B]^T[A]^T=\begin{bmatrix} -7 & -1 & 16 \\ 10 & 2 & 12 \\ 6 & -8 & 2 \\ 0 & -2 & 8 \end{bmatrix}\begin{bmatrix} 15 & 6 \\ -1 & 16 \\ -4 & 9 \end{bmatrix}\begin{bmatrix} -9 & 13 & 8 & -11 \\ 0 & 20 & -3 & -5 \end{bmatrix}$$

$$=\begin{bmatrix} -168 & 86 \\ 100 & 200 \\ 90 & -74 \\ -30 & 40 \end{bmatrix}\begin{bmatrix} -9 & 13 & 8 & -11 \\ 0 & 20 & -3 & -5 \end{bmatrix}$$

$$=\begin{bmatrix} 1512 & -464 & -1602 & 1418 \\ -900 & 5300 & 200 & -2100 \\ -810 & -310 & 942 & -620 \\ 270 & 410 & -360 & 130 \end{bmatrix} \tag{2}$$

From Eqs. (1) and (2), we can see that

$$\left([A][B][C]\right)^T=[C]^T[B]^T[A]^T$$

2.10 $\mathbf{C} = \mathbf{B}^T\mathbf{A}\mathbf{B} = \begin{bmatrix} 5 & -7 & 3 \\ 7 & 8 & -4 \\ -3 & 4 & 9 \end{bmatrix} \begin{bmatrix} 40 & -10 & -25 \\ -10 & 15 & 12 \\ -25 & 12 & 30 \end{bmatrix} \begin{bmatrix} 5 & 7 & -3 \\ -7 & 8 & 4 \\ 3 & -4 & 9 \end{bmatrix}$

$$= \begin{bmatrix} 195 & -119 & -119 \\ 300 & 2 & -199 \\ -385 & 198 & 393 \end{bmatrix} \begin{bmatrix} 5 & 7 & -3 \\ -7 & 8 & 4 \\ 3 & -4 & 9 \end{bmatrix} = \begin{bmatrix} 1,451 & 889 & -2,132 \\ 889 & 2,912 & -2,683 \\ -2,132 & -2,683 & 5,484 \end{bmatrix}$$

2.11 $\mathbf{C} = \mathbf{B}^T\mathbf{A}\mathbf{B} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \\ -0.6 & 0.8 \\ -0.8 & -0.6 \end{bmatrix} \begin{bmatrix} 300 & -100 \\ -100 & 200 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 & -0.6 & -0.8 \\ -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix}$

$= \begin{bmatrix} 260 & -220 \\ 180 & 40 \\ -260 & 220 \\ -180 & -40 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 & -0.6 & -0.8 \\ -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix} = \begin{bmatrix} 332 & 76 & -332 & -76 \\ 76 & 168 & -76 & -168 \\ -332 & -76 & 332 & 76 \\ -76 & -168 & 76 & 168 \end{bmatrix}$

2.13 $$\frac{d\mathbf{A}}{dx} = \begin{bmatrix} -4x & 3\cos x & -7 \\ 3\cos x & -2\cos x \sin x & -9x^2 \\ -7 & -9x^2 & 6\sin x \cos x \end{bmatrix}$$

2.14 $\mathbf{C} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} 2x^2 - 3x & -x + 5 \\ 4x^2 - 12x & -x^3 + 8 \\ 2x^3 - 7 & -3x^2 + 5x \\ 2x^3 - 1 & -x^2 + 6x \end{bmatrix}$; $\dfrac{d\mathbf{C}}{dx} = \begin{bmatrix} 4x - 3 & -1 \\ 8x - 12 & -3x^2 \\ 6x^2 & -6x + 5 \\ 6x^2 & -2x + 6 \end{bmatrix}$

2.15 $$[A][B]=\begin{bmatrix} 12-30x^4 & -10x^2-20x^3 \\ -30x^3 & -2x-4x^2+9x^5 \\ -6x+14x^2+25x^5 & 28x+8x^3 \end{bmatrix}$$

$$\frac{d[A][B]}{dx}=\begin{bmatrix} -120x^3 & -20x-60x^2 \\ -90x^2 & -2-8x+45x^4 \\ -6+28x+125x^4 & 28+24x^2 \end{bmatrix}$$

2.16

$$\frac{\partial[A]}{\partial x}=\begin{bmatrix}2x & 0 & 0\\ 0 & 3y & 0\\ 0 & 0 & 4z\end{bmatrix}$$

$$\frac{\partial[A]}{\partial y}=\begin{bmatrix}0 & -2y & 0\\ -2y & 3x & -z\\ 0 & -z & 0\end{bmatrix}$$

$$\frac{\partial[A]}{\partial z}=\begin{bmatrix}0 & 0 & 4z\\ 0 & 0 & -y\\ 4z & -y & 4x\end{bmatrix}$$

2.17

$$\int_0^L \mathbf{A}\,dx = \begin{bmatrix} -5L & -L^3 \\ 2L^2 & -\dfrac{L^4}{4} \\ \dfrac{2L^5}{5} & 6L \\ \dfrac{5L^3}{3} & -\dfrac{L^2}{2} \end{bmatrix}$$

2.18 $$\int_0^L [A]\,dx = \begin{bmatrix} L^2 & \cos L - 1 & \sin L \cos L + L \\ \cos L - 1 & 5L & -L^4 \\ \sin L \cos L + L & -L^4 & L - \dfrac{L^3}{3} \end{bmatrix}$$

2.19 $[A][B] = \begin{bmatrix} 10x^2 + 9x^3 + 2x^4 & -9 - 4x^3 - x^5 \\ -9x^2 + 6x^6 & -2x^3 \end{bmatrix}$

$$\int_0^L \left([A][B]\right) dx = \begin{bmatrix} \frac{10L^3}{3} + \frac{9L^4}{4} + \frac{2L^5}{5} & -9L - L^4 - \frac{L^6}{6} \\ -3L^3 + \frac{6L^7}{7} & -\frac{L^4}{2} \end{bmatrix}$$

2.20 $$\mathbf{AA}^T = \begin{bmatrix} -0.28 & -0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & -0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix} \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ -0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & -0.96 & -0.28 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, **A** is orthogonal.

$$\mathbf{BB}^T = \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix} \begin{bmatrix} -0.28 & 0.96 & 0 & 0 \\ 0.96 & -0.28 & 0 & 0 \\ 0 & 0 & -0.28 & 0.96 \\ 0 & 0 & 0.96 & -0.28 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.5376 & 0 & 0 \\ -0.5376 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.5376 \\ 0 & 0 & -0.5376 & 1 \end{bmatrix}$$

Thus, **B** is not orthogonal.

2.21
$$\left[\begin{array}{ccc|c} 2 & -3 & 1 & -18 \\ -9 & 5 & 3 & 18 \\ 4 & 7 & -8 & 53 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1.5 & 0.5 & -9 \\ 0 & -8.5 & 7.5 & -63 \\ 0 & 13 & -10 & 89 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.82353 & 2.1176 \\ 0 & 1 & -0.88235 & 7.4118 \\ 0 & 0 & 1.4706 & -7.3529 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

Thus, $\underline{x_1 = -2 \quad x_2 = 3 \quad x_3 = -5}$

2.22
$$\left[\begin{array}{ccc|c} 20 & -9 & 15 & 354 \\ -9 & 16 & -5 & -275 \\ 15 & -5 & 18 & 307 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -0.45 & 0.75 & 17.7 \\ 0 & 11.95 & 1.75 & -115.7 \\ 0 & 1.75 & 6.75 & 41.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.8159 & 13.343 \\ 0 & 1 & 0.14644 & -9.682 \\ 0 & 0 & 6.4937 & 58.444 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

Thus, $x_1 = 6 \quad x_2 = -11 \quad x_3 = 9$

2.23
$$\left[\begin{array}{ccc|c} 4 & -2 & 3 & 37.2 \\ 3 & 5 & -1 & -7.2 \\ 1 & -4 & 2 & 30.3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -0.5 & 0.75 & 9.3 \\ 0 & 6.5 & -3.25 & -35.1 \\ 0 & -3.5 & 1.25 & 21 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.5 & 6.6 \\ 0 & 1 & -0.5 & -5.4 \\ 0 & 0 & -0.5 & 2.1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8.7 \\ 0 & 1 & 0 & -7.5 \\ 0 & 0 & 1 & -4.2 \end{array}\right]$$

Thus, $x_1 = 8.7 \quad x_2 = -7.5 \quad x_3 = -4.2$

2.24

$$\left[\begin{array}{rrrr|r} 6 & 15 & -24 & 40 & 190.9 \\ 15 & 9 & -13 & 0 & 69.8 \\ -24 & -13 & 8 & -11 & -96.3 \\ 40 & 0 & -11 & 5 & 119.35 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 2.5 & -4 & 6.6667 & 31.817 \\ 0 & -28.5 & 47 & -100 & -407.45 \\ 0 & 47 & -88 & 149 & 667.3 \\ 0 & -100 & 149 & -261.67 & -1153.3 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0.12281 & -2.1053 & -3.9246 \\ 0 & 1 & -1.6491 & 3.5088 & 14.296 \\ 0 & 0 & -10.491 & -15.912 & -4.6351 \\ 0 & 0 & -15.912 & 89.211 & 276.33 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & -2.2915 & -3.9788 \\ 0 & 1 & 0 & 6.01 & 15.025 \\ 0 & 0 & 1 & 1.5167 & 0.4418 \\ 0 & 0 & 0 & 113.35 & 283.36 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 0 & 1.75 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -3.35 \\ 0 & 0 & 0 & 1 & 2.5 \end{array}\right]$$

Thus, $\underline{x_1 = 1.75, \quad x_2 = 0, \quad x_3 = -3.35, \quad x_4 = 2.5}$

2.25

$$\left[\begin{array}{cccc|c} 2 & -5 & 8 & 11 & 39 \\ 10 & 7 & 4 & -1 & 127 \\ -3 & 9 & 5 & -6 & 58 \\ 1 & -4 & -2 & 9 & -14 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -2.5 & 4 & 5.5 & 19.5 \\ 0 & 32 & -36 & -56 & -68 \\ 0 & 1.5 & 17 & 10.5 & 116.5 \\ 0 & -1.5 & -6 & 3.5 & -33.5 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1.1875 & 1.125 & 14.188 \\ 0 & 1 & -1.125 & -1.75 & -2.125 \\ 0 & 0 & 18.688 & 13.125 & 119.69 \\ 0 & 0 & -7.6875 & 0.875 & -36.688 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0.29097 & 6.5819 \\ 0 & 1 & 0 & -0.95987 & 5.0803 \\ 0 & 0 & 1 & 0.70234 & 6.4047 \\ 0 & 0 & 0 & 6.2742 & 12.548 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

Thus, $\underline{x_1 = 6, \quad x_2 = 7, \quad x_3 = 5, \quad x_4 = 2}$

2.27
$$\left[\begin{array}{ccc|ccc} 5 & 3 & -4 & 1 & 0 & 0 \\ 3 & 8 & -2 & 0 & 1 & 0 \\ -4 & -2 & 7 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0.6 & -0.8 & 0.2 & 0 & 0 \\ 0 & 6.2 & 0.4 & -0.6 & 1 & 0 \\ 0 & 0.4 & 3.8 & 0.8 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -0.83871 & 0.25806 & -0.096774 & 0 \\ 0 & 1 & 0.064516 & -0.096774 & 0.16129 & 0 \\ 0 & 0 & 3.7742 & 0.83871 & -0.064516 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.44444 & -0.11111 & 0.22222 \\ 0 & 1 & 0 & -0.11111 & 0.16239 & -0.017094 \\ 0 & 0 & 1 & 0.22222 & -0.017094 & 0.26496 \end{array}\right]$$

Thus,

$$\mathbf{A}^{-1} = \begin{bmatrix} 0.44444 & -0.11111 & 0.22222 \\ -0.11111 & 0.16239 & -0.017094 \\ 0.22222 & -0.017094 & 0.26496 \end{bmatrix}$$

2.28
$$\left[\begin{array}{ccc|ccc} 6 & -4 & 1 & 1 & 0 & 0 \\ -1 & 9 & 3 & 0 & 1 & 0 \\ 4 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -0.66667 & 0.16667 & 0.16667 & 0 & 0 \\ 0 & 8.3333 & 3.1667 & 0.16667 & 1 & 0 \\ 0 & 4.6667 & 4.3333 & -0.66667 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0.42 & 0.18 & 0.08 & 0 \\ 0 & 1 & 0.38 & 0.02 & 0.12 & 0 \\ 0 & 0 & 2.56 & -0.76 & -0.56 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.30469 & 0.17188 & -0.16406 \\ 0 & 1 & 0 & 0.13281 & 0.20313 & -0.14844 \\ 0 & 0 & 1 & -0.29688 & -0.21875 & 0.39063 \end{array}\right]$$

Thus,

$$\mathbf{A}^{-1} = \begin{bmatrix} 0.30469 & 0.17188 & -0.16406 \\ 0.13281 & 0.20313 & -0.14844 \\ -0.29688 & -0.21875 & 0.39063 \end{bmatrix}$$

2.29

$$\left[\begin{array}{cccc|cccc} 7 & -6 & 3 & -2 & 1 & 0 & 0 & 0 \\ -6 & 4 & -1 & 5 & 0 & 1 & 0 & 0 \\ 3 & -1 & 8 & 9 & 0 & 0 & 1 & 0 \\ -2 & 5 & 9 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & -0.85714 & 0.42857 & -0.28571 & 0.14286 & 0 & 0 & 0 \\ 0 & -1.1429 & 1.5714 & 3.2857 & 0.85714 & 1 & 0 & 0 \\ 0 & 1.5714 & 6.7143 & 9.8571 & -0.42857 & 0 & 1 & 0 \\ 0 & 3.2857 & 9.8571 & 1.4286 & 0.28571 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -0.75 & -2.75 & -0.5 & -0.75 & 0 & 0 \\ 0 & 1 & -1.375 & -2.875 & -0.75 & -0.875 & 0 & 0 \\ 0 & 0 & 8.875 & 14.375 & 0.75 & 1.375 & 1 & 0 \\ 0 & 0 & 14.375 & 10.875 & 2.75 & 2.875 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -1.5352 & -0.43662 & -0.6338 & 0.084507 & 0 \\ 0 & 1 & 0 & -0.64789 & -0.6338 & -0.66197 & 0.15493 & 0 \\ 0 & 0 & 1 & 1.6197 & 0.084507 & 0.15493 & 0.11268 & 0 \\ 0 & 0 & 0 & -12.408 & 1.5352 & 0.64789 & -1.6197 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -0.62656 & -0.71396 & 0.2849 & -0.12372 \\ 0 & 1 & 0 & 0 & -0.71396 & -0.6958 & 0.2395 & -0.052213 \\ 0 & 0 & 1 & 0 & 0.2849 & 0.2395 & -0.098751 & 0.13053 \\ 0 & 0 & 0 & 1 & -0.12372 & -0.052213 & 0.13053 & -0.08059 \end{array}\right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} -0.62656 & -0.71396 & 0.2849 & -0.12372 \\ -0.71396 & -0.6958 & 0.2395 & -0.052213 \\ 0.2849 & 0.2395 & -0.098751 & 0.13053 \\ -0.12372 & -0.052213 & 0.13053 & -0.08059 \end{bmatrix}$$

2.30

$$\left[\begin{array}{cccc|cccc} 5 & -7 & -3 & 11 & 1 & 0 & 0 & 0 \\ 10 & -6 & -13 & 2 & 0 & 1 & 0 & 0 \\ -1 & 12 & 8 & -4 & 0 & 0 & 1 & 0 \\ -9 & 7 & -5 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & -1.4 & -0.6 & 2.2 & 0.2 & 0 & 0 & 0 \\ 0 & 8 & -7 & -20 & -2 & 1 & 0 & 0 \\ 0 & 10.6 & 7.4 & -1.8 & 0.2 & 0 & 1 & 0 \\ 0 & -5.6 & -10.4 & 25.8 & 1.8 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1.825 & -1.3 & -0.15 & 0.175 & 0 & 0 \\ 0 & 1 & -0.875 & -2.5 & -0.25 & 0.125 & 0 & 0 \\ 0 & 0 & 16.675 & 24.7 & 2.85 & -1.325 & 1 & 0 \\ 0 & 0 & -15.3 & 11.8 & 0.4 & 0.7 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1.4033 & 0.16192 & 0.029985 & 0.10945 & 0 \\ 0 & 1 & 0 & -1.2039 & -0.10045 & 0.055472 & 0.052474 & 0 \\ 0 & 0 & 1 & 1.4813 & 0.17091 & -0.07946 & 0.05997 & 0 \\ 0 & 0 & 0 & 34.463 & 3.015 & -0.51574 & 0.91754 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0.039153 & 0.050985 & 0.072084 & -0.040719 \\ 0 & 1 & 0 & 0 & 0.0048723 & 0.037456 & 0.084526 & 0.034933 \\ 0 & 0 & 1 & 0 & 0.041328 & -0.057293 & 0.020533 & -0.042981 \\ 0 & 0 & 0 & 1 & 0.087484 & -0.014965 & 0.026624 & 0.029016 \end{array}\right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} 0.039153 & 0.050985 & 0.072084 & -0.040719 \\ 0.0048723 & 0.037456 & 0.084526 & 0.034933 \\ 0.041328 & -0.057293 & 0.020533 & -0.042981 \\ 0.087484 & -0.014965 & 0.026624 & 0.029016 \end{bmatrix}$$