Solutions for Chapter 1 Activity Manual

Activity 1.1.1

1. Fill in the blanks

   a. Pattern

   b. Conjecture

   c. Inductive reasoning

   d. Sequence

   e. Term

2. Consecutive numbers

   a. $4+5=9$, $5+6=11$, and $6+7=13$

   b. The sum of two consecutive numbers is an odd number.

   c. Inductive reasoning

3. Odds

   a. $9=4+5$, $11=5+6$, and $13=6+7$

   b. An odd number is the sum of two consecutive numbers.

   c. Inductive reasoning

4. Repeating sequence

   a. Answers will vary. For example, the terms Q, 7, X, Y are repeating such that every 4th term is a Y.

   b. Y

   c. Y

   d. $441 \div 4 = 110$ R1. The remainder is 1, and the first term after Y is Q. So the 441st term is Q.

5. 111,111

   a. Answers vary. For example, dividing 21 by 3 and multiplying 5291 by 3 leads to the numbers 7 and 15,873.

   b. Answers vary. For example, $222,222 = 2 \cdot 111,111 = 2 \cdot (21 \cdot 5291) = (2 \cdot 21) \cdot 5291 = 42 \cdot 5291$.

   c. Answers vary. For example, $555,555 = 5 \cdot 111,111 = 5 \cdot (21 \cdot 5291) = (5 \cdot 21) \cdot 5291 = 105 \cdot 5291$.

6. $35^2 = 1225$, $335^2 = 112225$, $3335^2 = 11122225$ and $33335^2 = 1111222225$. The number of 1's equals the number of 3's, the number of 2's equals one more than the number of 3's, and the ones digit will be a 5. Then $3333335^2 = 11111122222225$.

7. 3, 7, 11, 15, 19, …

8. Arithmetic sequence

    a. 7

    b. 3

    c. 16

    d. 7; 3

    e. 22, 25, 28

    f. Table:

n, position	1	2	3	4	5
y, term	7	10	13	16	19

    g. $y = 3n + 4 = 3 \cdot 115 + 4 = 349$. The 115th term in the sequence is 349.

9. Arithmetic sequence

    a. 9

    b. 4

    c. 405

Activity 1.1.2

a. Table:

n, number of points	2	3	4	5
Number of non-overlapping regions	2	4	8	16

b. Prediction: 32, since the number of regions appears to double

c. Actual result: 31

d. No, what a surprise.

e. Inductive reasoning is reliable, but conclusions based on inductive reasoning may be incorrect.

Activity 1.2.1

$1 = \dfrac{4+4}{4+4}$
$2 = \dfrac{4}{4} + \dfrac{4}{4}$
$3 = \dfrac{4+4+4}{4}$
$4 = \sqrt{4} + \dfrac{2}{2} \cdot \sqrt{4}$
$5 = 4 + \sqrt{4} - \dfrac{4}{4}$
$6 = \dfrac{4+4}{4} + 4$
$7 = 4 + 4 - \dfrac{4}{4}$
$8 = 4 \cdot \dfrac{4}{4} + 4$
$9 = 4 + 4 + \dfrac{4}{4}$
$10 = 4 \cdot 4 - 4 - \sqrt{4}$
$11 = \dfrac{4! + 4}{4} + 4$
$12 = 4 \cdot 4 - \sqrt{4} - \sqrt{4}$
$13 = \dfrac{4! + 4! + 4}{4}$
$14 = 4 + 4 + 4 + \sqrt{4}$
$15 = 4 \cdot 4 - \dfrac{4}{4}$
$16 = 4 \cdot 4 \cdot \dfrac{4}{4}$
$17 = 4 \cdot 4 + \dfrac{4}{4}$
$18 = 4 \cdot 4 + 4 - \sqrt{4}$
$19 = 4! - 4 - \dfrac{4}{4}$
$20 = 4 \cdot 4 + \sqrt{4} + \sqrt{4}$
$21 = 4! - \dfrac{4}{4} - \sqrt{4}$
$22 = 4 + 4 \cdot 4 + \sqrt{4}$
$23 = 4! + \dfrac{4}{4} - \sqrt{4}$
$24 = 4 \cdot 4 + 4 + 4$
$25 = 4! - \dfrac{4}{4} + \sqrt{4}$

Answers may vary,
but the table gives some possibilities

Activity 1.2.2

1. There are possible 11 solutions to this riddle, they are:
   213, 235, 257, 279, 303, 325, 347, 369, 415, 437 and 459.

2. Animal farm

   a. Horses have four legs; $4 \cdot 24$ tells the teacher that there are 24 horses. Similarly ducks have 2 feet; $18 \cdot 2$ tells the teacher that there are 18 ducks.

   b. Answers will vary. For example, $50 \cdot 4 + 30 \cdot 2 = 260$ leads to the following barnyard problem: *A farmer wants to know how many donkeys and ostriches are on the farm. He counted 260 legs and 80 animals. How many of each animal is there?*

3. Digit sum

   a. 19

   b. 31

   c. Use a pattern to conclude $533333^2 = 284444088889$. Then the digit sum of 533333^2 is 67.

4. Assume a pile of 1, 7, 7 will be the same configuration as 7, 1, 7 and 7, 7, 1.
   There are 7 solutions: 1, 1, 13; 1, 3, 11; 1, 5, 9; 1, 7, 7; 3, 3, 9; 3, 5, 7; 5, 5, 5.

5. Magic square

4	3	8
9	5	1
2	7	6

   a. The magic number is 15.

8	1	6
3	5	7
4	9	2

   b. The magic number is 15.

   c. In any magic square, the sum of the nine numbers is $1 + 2 + ... + 9 = 45$. There are three rows (or columns), so each row (or column) will have the sum $45 \div 3 = 15$. The sum of each row or column will always be 15.

6. Cupcakes

   a. 18

   b. Answers will vary. We used the Work Backward strategy: $(2 + 4 + 3) \cdot 2 = 18$.

7. Dividing the class into groups of 4 with 3 leftover means there could be 3, 7, 11, 15, **19**, 23, 27, 31, 35, **39**, 43 students.   Dividing the class into groups of 5 with 4 leftover means there could be 4, 9, 14, **19**, 24, 29, 34, **39**, 44 students.   Some possible solutions include 19 and 39.   There are more possibilities, but we assumed that the teacher has less than 45 students.

8. Susie pays for 9 sandwiches in every group of 11 sandwiches. $634 \div 11 = 57$ R7 . There are 57 groups of 11 in 634. Then Susie has to pay for $7 + 57 \cdot 9 = 520$ sandwiches.

9. Diagram.
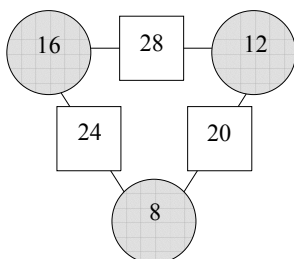
10. Dart board

   a. Lowest possible score $= 2 \cdot 5 = 10$ points
      highest possible score $= 75 + 30 + 50 + 20 + 10 + 5 + 100 + 40 + 60 + 100 = 590$.

   b. Lowest possible score $= 3 \cdot 5 = 15$ points
      highest possible score $= 2(75 + 30 + 50 + 20 + 10 + 5 + 100 + 40 + 60) + 100 = 1080$

11. Dots in the figure. $d = 3n + 2$

n, position	1	2	3
d, number of dots	5	8	11

Activity 1.3.1

1. Coin relationship

   a. The number of coins Cody has

   b. The arrow indicates that altogether they have 143 coins.

   c. 148 coins

   d. $143 + 5 = 148$; $148 \div 4 = 37$; One box represents 37 coins.

   e. $3 \cdot 37 - 5 = 106$. Cody has 37 coins and Hannah has 106 coins.

2. Coin relationship

   a. The number of coins Cody has.

   b. The arrow indicates that Hannah has 121 coins.

   c. $121 - 7 = 114$, so the three boxes represent 114 coins.

   d. $114 \div 3 = 38$, so each box represents 38 coins.

   e. 38 coins

3.

G ☐

R ☐ ☐

4.

B ☐

K ☐ ☐ $+3$

5.

I ☐

S ☐ ☐ ☐ ☐ ☐ -2

6. $17-3=14$; $14 \div 2 = 7$. Whitney has 7 apples.

W ☐

A ☐ $+3$ $\updownarrow$ 17

7. $83-2=81$; $81 \div 3 = 27$. Collin has 27 songs on his MP3 player.

W ☐

A ☐ ☐ ☐ $+2$

$\longleftrightarrow$ 83

8. $89-11=78$; $78 \div 3 = 26$. Lucas has 26 baseball cards and Clint has $2 \cdot 26 + 11 = 63$ baseball cards.

L ☐

C ☐ ☐ $+11$ $\updownarrow$ 89

9. Maria has $3k+6$ books

10. Shaun has $4m-3$ coins.

11. Let *n* represent the number of friends that Carlo has. Then Krista has $3n - 2$ friends.
The equation is $n + 3n - 2 = 94$.

12. Let *w* represent the number of apples that Whitney has. Then Annie has $w + 3$ apples. Then

$w + w + 3 = 17$

$2w + 3 = 17$

$2w = 14$

$w = 7$

Whitney has 7 apples.

13. Let *c* represent the number of songs Collin has. Then

$3c + 2 = 83$

$3c = 81$

$c = 27$

Then Collin has 27 songs on his MP3 player.

14. Let *b* represent the number of baseball cards Lucas has. Then Clint has $2b + 11$ baseball cards. Then

$b + 2b + 11 = 89$

$3b + 11 = 89$

$3b = 78$

$b = 26$

Lucas has 26 baseball cards and Clint has $2 \cdot 26 + 11 = 63$ baseball cards.

15. Context of the word problem may vary, and the question asked many vary. For example:

a. Benny has three fewer than four times as many coins as Annie. Altogether they have 337 coins.
How many coins does Benny have?

b. Diana has 217 coins. She has five more than three times as many coins as Cindy.
How many coins does Cindy have?

16. Context of the word problem may vary, and the question asked many vary. For example:

a. Samantha has 10 more than three times as many coins as Mitchell. Altogether they have 366 coins.
How many coins does Samantha have?

b. Ralph has 225 coins. He has three fewer than four times as many as Dorothy.
How many coins does Dorothy have?

Activity 1.3.2

1. Magic trick

   a. Answers will vary

   b. The original number is six less than the answer from the calculator. All members of the group should have seen the same relationship.

   c. Step 1. n

   Step 2. $n+7$

   Step 3. $6(n+7)$

   Step 4. $6(n+7)-4$

   Step 5. $\dfrac{6(n+7)-4}{2}$

   Step 6. $\dfrac{6(n+7)-4}{2}-1$

   Step 7. $\dfrac{\dfrac{6(n+7)-4}{2}-1}{3}$

   d. $n+6$

   e. Yes

   f. The original number was 61.

2. More magic

   a. According to the computations, the final result is nine more than the original number.

   b. Sharing

   c. Step 1. n, Step 2. $2n$, Step 3. $2n+30$, Step 4. $n+15$, Step 5. $n+9$

3. Step 1. x, Step 2. $x+7$, Step 3. $2x+14$, Step 4. $2x+4$, Step 5. $x+2$.
   The magician will only have to subtract two from the announced number to obtain the original number.

4. Each day is one day apart from the next and each week is 7 days apart. Let day 1 be n. Then day 2 will be $n+1$. The third day would be seven days from day 1, so we get $n+7$. The final day is one day more so we get $n+8$. The sum is $n+n+1+n+7+n+8 = 4n+16$. Then the magician subtracts 16, and then divides by 4.

5. Answers will vary.

Solutions for Chapter 2 Activity Manual

Activity 2.1.1

1.

    a. 5

    b. −5 and 5

    c. No solutions

2. 11 divided by 4

    a. $11 \div 4 = 2\ R3$

    b. $11 \div 4 = \dfrac{11}{4}$

    c. 2.75

3. Set

    a. {1,2,4,5,7,8}

    b. {0,2,3,5,9}

    c. {0,1,2,3,4,5,6,7,8,9,10}

    d. {0,1,2,3,4,5,7,8,9}

    e. {2,5}

    f. {1,4,7,8}

    g. {0,3,9}

    h. {0,3,6,9,10}

    i. {1,4,6,7,8,10}

4. Shading diagrams
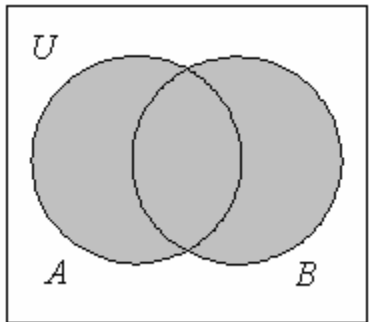
a.   **b.**
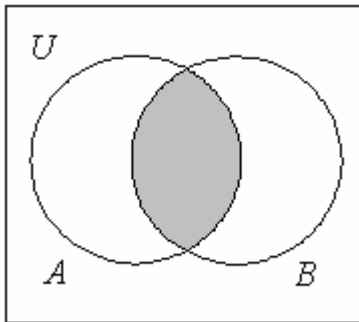
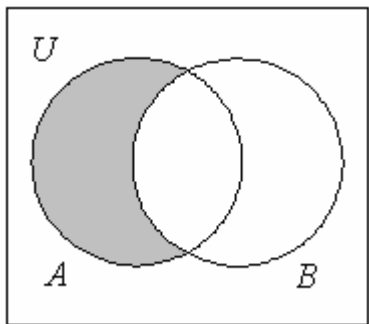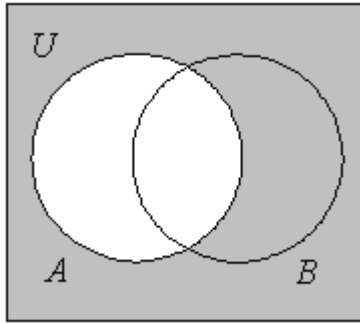**c.**   **d.**

5. Sets

a. $A \cup B = \{1,3,5,6,7\}$, $A \cap B = \{3,5\}$, $A - B = \{1,7\}$, $B - A = \{6\}$, $\sim A = \{0,2,4,6,8\}$, $\sim B = \{0,1,2,4,7,8\}$

b. $A \cup B = \{1,3,6,7\}$, $A \cap B = \{1\}$, $A - B = \{3,6\}$, $B - A = \{7\}$, $\sim A = \{0,2,4,5,7,8\}$, $\sim B = \{0,2,3,4,5,6,8\}$

6. $A \cup B$ represents the set of all students that either completed their assignments or passed the class, inc. both.
   $A \cap B$ represents the set of all students that completed their assignments and passed the class.
   $A - B$ represents the set of all students that completed their assignments but did not pass the class.
   $B - A$ represents the set of all students that passed the class but did not complete all their assignments.

7. The corresponding element of B would be $3n$.

8. Counting formula

a. $n(A) = 4$, $n(B) = 2$, $n(A \cup B) = 5$, $n(A \cap B) = 1$. $5 \neq 4 + 2$, so $n(A \cup B) \neq n(A) + n(B)$.
   But $5 = 4 + 2 - 1$, so $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

b. $n(A) = 3$, $n(B) = 5$, $n(A \cup B) = 6$, $n(A \cap B) = 2$. $6 \neq 3 + 5$, so $n(A \cup B) \neq n(A) + n(B)$.
   But $6 = 3 + 5 - 2$, so $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
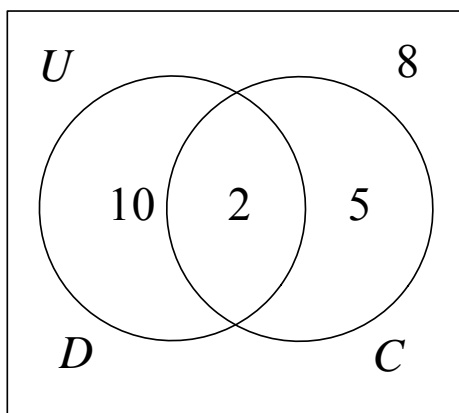
9. 3

10. Subsets

    a. Yes

    b. No

    c. Yes

Activity 2.1.2

1. Venn

    a. 16

    b. 19

    c. 4

    d. 12

    e. 15

    f. 27

    g. 3

    h. 34

2. Venn

    a. Venn Diagram

    b. 15

    c. 8

3. Eclipse or meteor

   a. 25
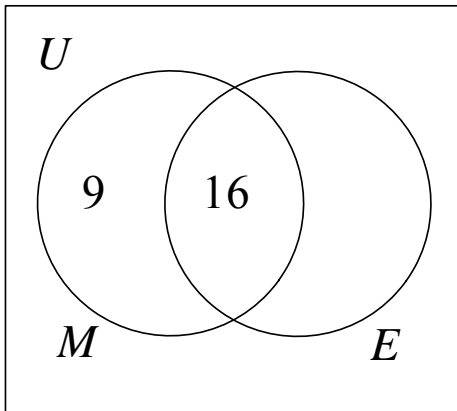
   b. 41
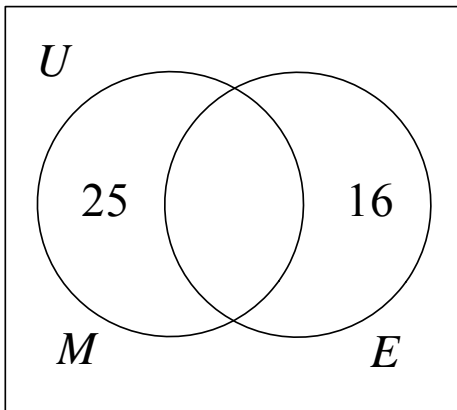
4. (i) ↔ (e), (ii) ↔ (d), (iii) ↔ (b), (iv) ↔ (c), (v) ↔ (f), (vi) ↔ (a)

5. hop scotch, jump rope, or frisbee

   a. 3

   b. 22

   c. 14

   d. 37

   e. 7

   f. 44

   g. 15

6. $4 + 6 + 3 = 13$
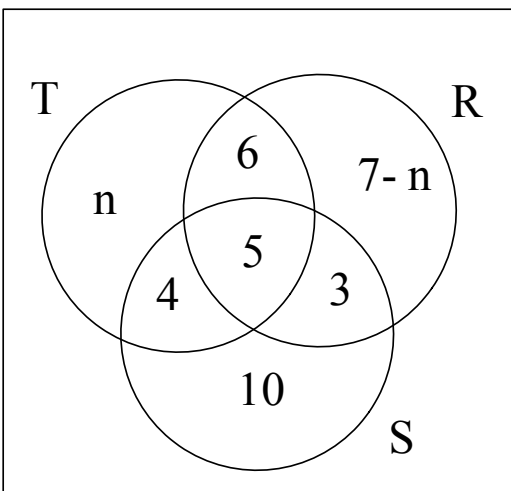
7. Venn

  a. 14

  b. 18

8. Answers will vary.

Activity 2.2.1

1. Place value

  a. Sketch of 32 ones or sketch of 1 ten and 22 ones
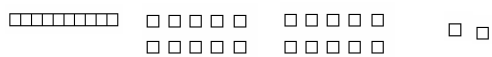
  b. 3 tens and 2 ones; 5 pieces altogether

  c. 32 ones, 32 pieces altogether

  d. 4 (3 tens and 2 ones, 2 tens and 12 ones, 1 ten and 22 ones, and 32 ones)

2. Place value

  a. 4 tens and 6 ones; 10 pieces altogether

  b. 46 ones; 46 pieces altogether

  c. 5 (4 tens and 6 ones, 3 tens and 16 ones, 2 tens and 26 ones, 1 ten and 35 ones, 46 ones)

3. Fill in the blanks

    a. 345 $=$ 34 tens and 5 ones

    b. 2468 $=$ 24 hundreds, 6 tens, 8 ones

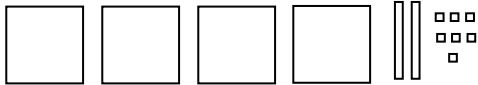   c. 4731 $=$ 4 thousands, 731 ones

    d. 18,054 $=$ 180 hundreds, 5 tens, 4 ones

4. Sketch of 427: 4 hundreds, 2 tens, 7 ones

5. Place value and value of digits from left to right:

    a. 353

digit	3	5	3
place value of digit	hundreds	tens	ones
value of digit	300	50	3

    b. 72,029

digit	7	2	0	2	9
place value of digit	ten thousands	thousands	hundreds	tens	ones
value of digit	70,000	2000	0	20	9

6. Short word form

    a. 45 thousand, 83

    b. 6 million, 507 thousand, 362

7. Expanded form

    a. $5000 + 600 + 70 + 8$

    b. $6000 + 600 + 0 + 4$

8. Expanded form again.

   a. $5 \cdot 1000 + 6 \cdot 100 + 7 \cdot 10 + 8$

   b. $6 \cdot 1000 + 6 \cdot 100 + 0 \cdot 10 + 4$

9. Answers will vary. These results came from the fact that $39 \cdot 259 = 10101$, this number allows for your age to be displayed three consecutive times.

10. Calculator

    a. Answers will vary

    b. The calculator is displaying my birthday as mmdd.

    c. Step 1. m

       Step 2. $2m$

       Step 3. $2m + 40$

       Step 4. $20m + 400$

       Step 5. $20m + 393$

       Step 6. $100m + 1965$

       Step 7. $100m + d + 1965$

       Step 8. $100m + d$

    d. The month displays in the thousands and hundreds places, the day displays in the tens and ones place.

Activity 2.3.1

1. Classify

   a. Comparison

   b. Missing addend

   c. Take away

2. Key words

   a. More than

   b. Answers will vary

3. Key words again

    a. Fewer than

    b. Answers will vary

4. Definition of subtraction

    a.   **b.**   **c.**

$$13 - n = 8$$

$$n - 26 = 64 \qquad 28 + n = 72$$
$$13 = 8 + n$$
$$n = 64 + 26 \qquad n = 72 - 28$$
$$13 - 8 = n$$
$$n = 90 \qquad\qquad n = 44$$
$$5 = n$$

5. Calculator

    a. 56   **b.** 75   **c.** a

    d. 83   **e.** 52   **f.** a

    g. 18   **h.** 21   **i.** b

    j. It proves that addition and subtraction are inverse operations.

6. Inverse operations

    a.
$$n + 23 = 47$$
$$n + 23 - 23 = 47 - 23$$
$$n = 47 - 23$$
$$n = 24$$

    b.
$$k - 42 = 75$$
$$k - 42 + 42 = 75 + 42$$
$$k = 75 + 42$$
$$k = 117$$

    c.
$$w + 30 = 62$$
$$w + 30 - 30 = 62 - 30$$
$$w = 62 - 30$$
$$w = 32$$

    d.
$$m - 20 = 55$$
$$m - 20 + 20 = 55 + 20$$
$$m = 55 + 20$$
$$m = 75$$

    e.
$$12 - n = 5$$
$$12 - n + n = 5 + n$$
$$12 = 5 + n$$
$$12 - 5 = 5 + n - 5$$
$$12 - 5 = n$$
$$7 = n$$

Activity 2.3.2

1. Adding strategies

   a. Say 7, 8, 9, 10, 11, 12. The answer is 12.

   b. Say 9, 10, 11, 12, 13. The answer is 13.

2. Since the commutative property says $3 + 8 = 8 + 3$, then start by saying 8, 9, 10, 11. The answer is 11.

3. Doubles

   a. $4 + 4 = 8$ and $5 + 5 = 10$ **b.** : : : : :

4. Doubles

   a. $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$

   b. $9 + 7 = 2 + 7 + 7 = 2 + 14 = 16$

5. Decomposing

   a. $9 + 8 = 9 + 1 + 7 = 10 + 7 = 17$

   b. $7 + 5 = 7 + 3 + 2 = 10 + 2 = 12$

6. $n + 9 = n + 10 - 1$. So add 10 to n, and then subtract 1. For example, $6 + 9 = 6 + 10 - 1 = 16 - 1 = 15$.

7. Take away

   a. Model with nine objects and take away 4, leaving 5

   b. Model with 8 objects and take away 6, leaving 2

8. Missing addend

   a. 13-7 is the same as $13 = 7 + ?$, since 7+6=13, the answer is 6

   b. 15-8 is the same as $15 = 8 + ?$, since 8+7=15, the answer is 7

9. Patterns

   a. Answers will vary

   b. Answers will vary

10. Decomposition

    a. $12 - 7 = 12 - 2 - 5 = 10 - 5 = 5$

    b. $15 - 8 = 15 - 5 - 3 = 10 - 3 = 7$

11. Subtraction using the empty number line and counting forward. Answers vary. For example:

   a. Start at 8, add **2** to get to 10, add **5** to get to 15.
      $2 + 5 = 7$, so $15 - 8 = 7$.

   b. Start at 36, add **4** to get to 40, add **20** to get to 60, add **4** more to get to 64.
      Add $4 + 20 + 4 = 28$, so $64 - 36 = 28$.

   c. Start at 18, add **2** to get to 20, add **10** to get to 30, add **2** to get to 32.
      $2 + 10 + 2 = 14$, so $32 - 18 = 14$.

12. Addition table

   a. Addition Property of Zero

   g. The answer to the question is the commutative property of addition.

   b, c, d, e, f, g, h require shading.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

   i. Yes, there are 8 unshaded squares.

Activity 2.4.1

1. Add 285 and 79 using base ten blocks.

    a. 5 ones + 9 ones = 14 ones = 1 ten, 4 ones.
    The one above the 8 represents the 1 ten from regrouping 14 ones as 1 ten, 4 ones.

    b. 1 ten + 8 tens + 7 tens = 16 tens = 1 hundred, 6 tens
    The one above the 2 represents the 1 hundred from regrouping 16 tens as 1 hundred, 6 tens

2. Add 674 and 269 using base ten blocks.

    a. 4 ones + 9 ones = 13 ones = 1 ten, 3 ones.
    The one above the 7 represents the 1 ten from regrouping 13 ones as 1 ten, 3 ones.

    b. 1 ten + 7 tens + 6 tens = 14 tens = 1 hundred, 4 tens
    The one above the 6 represents the 1 hundred from regrouping 14 tens as 1 hundred, 4 tens.

3. Regroupings

    a. 543

    b. 943 (4 hundreds + 9 hundreds = 13 hundreds, regroup as 1 thousand, 3 hundreds)

    c. 993 (5 tens + 9 tens = 14 tens, regroup as 1 hundred, 4 tens; 1 hundred + 4 hundreds + 9 hundreds = 14 hundreds, regroup as 1 thousand, 4 hundreds)

4. Find a, b, and k

    a. $a = 6$ and $b = 3$

    b. 1

5. Answers will vary.

6. $a = 8$, $b = 9$, and $c = 6$.

7. Missing digits

    ```
        8   8   3
    +   4   4   9
    1   3   3   2
    ```

8. Student

    a. regroup

    b. ones

    c. tens

Activity 2.4.2

1. Students will use base ten blocks to subtract. They begin by building the minuend. Then they take-away blocks to subtract the subtrahend. They should use the word "regroup" rather than the word "carry."

   a. $75 - 38 = 37$

   b. $437 - 59 = 378$

   c. $523 - 367 = 156$

2. More base ten blocks.

   a. $374 = 3$ hundreds, 7 tens, 4 ones. There are 4 ones and we need to take away 5 ones. There are not enough ones, so we regroup one of the tens as 10 ones. Then 6 tens remain.

   b. 1 ten is regrouped as 10 ones. Then 10 ones + 4 ones $= 14$ ones.

3. More base ten blocks.

   a. $653 = 6$ hundreds, 5 tens, 3 ones. There are 3 ones and we need to take away 5 ones. There are not enough ones, so we regroup one of the tens as 10 ones. Then 4 tens remain.

   b. 1 ten is regrouped as 10 ones. Then 10 ones + 3 ones $= 13$ ones.

4.
```
    6  [4]  7
 -  2   5  [8]
   ─────────────
   [3]  8   9
```

5.
```
    6  [2]  8
 - [1]  5  [3]
   ─────────────
    4   7   5
```

6. Regroupings

   a. 675

   b. 669 (regroup 1 ten as 10 ones)

   c. 599 (regroup 1 ten as 10 ones, and regroup 1 hundred as 10 tens)

7. $a = 0$, $b = 8$, and $c = 4$.

8. Answers will vary. For example, $34 - 17$.