Instructor's Manual

Mathematics for Economics and Business

Ninth edition

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SOLUTIONS TO PROBLEMS

CHAPTER 1

Linear Equations

Section 1.1 Introduction to algebra

Practice Problems

1. **(a)** −30 **(b)** 2 **(c)** −5

(d) 5 **(e)** 36 **(f)** −1

2. **(a)** −1 **(b)** −7 **(c)** 5

(d) 0 **(e)** −91 **(f)** −5

3. **(a)** 19 **(b)** 1500 **(c)** 32

(d) 35

4. **(a)** $x + 9y$ **(b)** $2y + 4z$

(c) not possible

(d) $8r^2 + s + rs - 3s^2$ **(e)** $-4f$

(f) not possible **(g)** 0

5. **(a)** $5z - 2z^2$

(b) $6x - 6y + 3y - 6x = -3y$

(c) $x - y + z - x^2 - x + y = z - x^2$

6. **(a)** $7(d+3)$ **(b)** $4(4w - 5q)$

(c) $3(2x - y + 3z)$ **(d)** $5Q(1 - 2Q)$

7. **(a)** $x^2 - 2x + 3x - 6 = x^2 + x - 6$

(b) $x^2 - xy + yx - y^2 = x^2 - y^2$

(c) $x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$

(d) $5x^2 - 5xy + 5x + 2yx - 2y^2 + 2y$
$= 5x^2 - 3xy + 5x - 2y^2 + 2y$

8. **(a)** $(x + 8)(x - 8)$

(b) $(2x + 9)(2x - 9)$

Exercise 1.1 (p. 18)

1. **(a)** -20 **(b)** 3 **(c)** -4 **(d)** 1

(e) -12 **(f)** 50 **(g)** -5 **(h)** 3

(i) 30 **(j)** 4.

2. **(a)** -1 **(b)** -3 **(c)** -11 **(d)** 16

(e) -1 **(f)** -13 **(g)** 11 **(h)** 0

(i) -31 **(j)** -2

3. **(a)** -3 **(b)** 2 **(c)** 18 **(d)** -15

(e) -41 **(f)** -3 **(g)** 18 **(h)** -6

(i) -25 **(j)** -6

4. **(a)** $2PQ$ **(b)** $8I$ **(c)** $3xy$ **(d)** $4qwz$

(e) b^2 **(f)** $3k^2$

5. **(a)** $19w$ **(b)** $4x - 7y$ **(c)** $9a + 2b - 2c$

(d) $x^2 + 2x$ **(e)** $4c - 3cd$ **(f)** $2st + s^2 + t^2 + 9$.

6. **(a)** 10 **(b)** 18 **(c)** 2000

(d) 96 **(e)** 70

7. **(a)** 1 **(b)** 5 **(c)** -6 **(d)** -6

(e) -30 **(f)** 44

8. **(a)** 16

(b) Presented with the calculation, -4^2, your calculator uses BIDMAS, so squares first to get 16 and then subtracts from zero to give a final answer, -16. To obtain the correct answer you need to use brackets:

[( ] [-] [4] [ )] [x^2] [=]

9. **(a)** 9 **(b)** 21 no.

10. **(a)** 43.96 **(b)** 1.13 **(c)** 10.34 **(d)** 0.17

(e) 27.38 **(f)** 3.72 **(g)** 62.70 **(h)** 2.39

11. **(a)** $7x-7y$ **(b)** $15x-6y$ **(c)** $4x+12$ **(d)** $21x-7$

(e) $3x+3y+3z$ **(f)** $3x^2-4x$ **(g)** $y+2z-2x-6y+2z=-2x-5y+4z$

12. **(a)** $5(5c+6)$ **(b)** $9(x-2)$ **(c)** $x(x+2)$ **(d)** $4(4x-3y)$

(e) $2x(2x-3y)$ **(f)** $5(2d-3e+10)$

13. **(a)** $x^2+7x+10$ **(b)** a^2+3a-4 **(c)** $d^2-5d-24$

(d) $6s^2+23s+21$ **(e)** $2y^2+5y+3$ **(f)** $10t^2-31t-14$

(g) $9n^2-4$ **(h)** $a^2-2ab+b^2$

14. **(a)** $6x+2y$ **(b)** $11x^2-3x-3$ **(c)** $14xy+2x$

(d) $6xyz+2xy$ **(e)** $10a-2b$ **(f)** $17x+22y$

(g) $11-3p$ **(h)** x^2+10x

15. **(a)** $(x+2)(x-2)$ **(b)** $(Q+7)(Q-7)$ **(c)** $(x+y)(x-y)$

(d) $(3x+10y)(3x-10y)$

16. **(a)** $4x^2+8x-2$ **(b)** $3x^2+2x-3x^2-15x=-13x$

17. $S=1.2N+3000E+1000(A-21)$; \$204000

18. **(a)** $C=80+60L+K$ **(b)** $C=10+1.25x$ **(c)** $H=5a+10b$ **(d)** $X=Cd+cm$

Section 1.2 Further algebra

Practice Problems

1. **(a)** $\frac{3}{5}$ **(b)** $\frac{4}{5}$ **(c)** $\frac{1}{2y}$ **(d)** $\frac{1}{2+3x}$ **(e)** $\frac{1}{x-4}$

2. **(a)** $\frac{1}{2}\times\frac{3}{4}=\frac{1\times 3}{2\times 4}=\frac{3}{8}$

(b) $\frac{7}{1}\times\frac{1}{4}=\frac{7}{4}$

(c) $\frac{2}{3}\div\frac{8}{9}=\frac{{}^{1}\cancel{2}}{{}_{1}\cancel{3}}\times\frac{\cancel{9}^{3}}{\cancel{8}_{4}}=\frac{3}{4}$

(d) $\frac{8}{9}\div 16=\frac{{}^{1}\cancel{8}}{9}\times\frac{1}{\cancel{16}_{2}}=\frac{1}{18}$

3. **(a)** $\frac{3}{7}-\frac{1}{7}=\frac{2}{7}$

(b) $\frac{1}{3}+\frac{2}{5}=\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$

(c) $\frac{7}{18}-\frac{1}{4}=\frac{14}{36}-\frac{9}{36}=\frac{5}{36}$

4. **(a)** $\frac{5}{\cancel{x-1}}\times\frac{\cancel{x-1}}{x+2}=\frac{5}{x+2}$

(b) $\frac{x^2}{x+10}\div\frac{x}{x+1}=\frac{x^2}{x+10}\times\frac{x+1}{x}=\frac{x(x+1)}{x+10}$

(c) $\frac{4}{x+1}+\frac{1}{x+1}=\frac{4+1}{x+1}=\frac{5}{x+1}$

(d) $\frac{2}{x+1}-\frac{1}{x+2}$

$$=\frac{2(x+2)}{(x+1)(x+2)}-\frac{(1)(x+1)}{(x+1)(x+2)}$$

$$=\frac{(2x+4)-(x+1)}{(x+1)(x+2)}=\frac{(x+3)}{(x+1)(x+2)}$$

5. **(a)** $4x+1=25$

$4x=24$ (subtract 1 from both sides)

$x=6$ (divide both sides by 4)

(b) $4x+5=5x-7$

$5=x-7$ (subtract $4x$ from both sides)

$12=x$ (add 7 to both sides)

(c) $3(3-2x)+2(x-1)=10$

$9-6x+2x-2=10$ (multiply out brackets)

$7-4x=10$ (collect like terms)

$-4x=3$ (subtract 7 from both sides)

$x=-\frac{3}{4}$ (divide both sides by −4)

(d) $\frac{4}{x-1}=5$

$4=5(x-1)$ (multiply both sides by $x-1$)
$4=5x-5$ (multiply out brackets)
$9=5x$ (add 5 to both sides)
$\frac{9}{5}=x$ (divide both sides by 5)

(e) $\frac{3}{x}=\frac{5}{x-1}$

$3(x-1)=5x$ (cross-multiplication)
$3x-3=5x$ (multiply out brackets)
$-3=2x$ (subtract $3x$ from both sides)
$-\frac{3}{2}=x$ (divide both side by 2)

6. **(a)** $12 > 9$ (true) **(b)** $12 > 6$ (true)

(c) $3 > 0$ (true) **(d)** same as (c)

(e) $2 > 1$ (true) **(f)** $-24 > -12$ (false)

(g) $-6 > -3$ (false) **(h)** $-2 > -1$ (false)

(i) $-4 > -7$ (true).

7. **(a)** $2x < 3x + 7$

$-x < 7$ (subtract $3x$ from both sides)

$x > -7$ (divide both sides by −1 changing sense because $-1 < 0$)

(b) $21x - 19 \geq 4x + 15$

$17x - 19 \geq 15$ (subtract $4x$ from both sides)

$17x \geq 34$ (add 19 to both sides)

$x \geq 2$ (divide both sides by 17, leaving inequality unchanged because $17 > 0$)

Exercise 1.2 (p. 36)

1. **(a)** $\frac{1}{2}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{3}{5}$ **(d)** $\frac{1}{3}$ **(e)** $\frac{4}{3} = 1\frac{1}{3}$

2. **(a)** $\frac{35}{100} = \frac{7}{20}$; $\frac{56}{100} = \frac{14}{25}$ **(b)** $\frac{56}{35} = 1\frac{3}{5}$

3. **(a)** $\frac{2x}{3}$ **(b)** $\frac{1}{2x}$ **(c)** $\frac{1}{ac}$ **(d)** $\frac{2}{3xy}$ **(e)** $\frac{3a}{4b}$.

4. **(a)** $\frac{2p}{2(2q+3r)} = \frac{p}{2q+3r}$ **(b)** $\frac{x}{x(x-4)} = \frac{1}{x-4}$ **(c)** $\frac{3ab}{3a(2a+1)} = \frac{b}{2a+1}$

(d) $\frac{14d}{7d(3-e)} = \frac{2}{3-e}$ **(e)** $\frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2}$ (using the difference of two squares for the denominator)

5. $\frac{x-1}{2x-2} = \frac{x-1}{2(x-1)} = \frac{1}{2}$; other two have no common factors on top and bottom.

6. **(a)** $\frac{3}{7}$ **(b)** $-\frac{1}{3}$ **(c)** $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

(d) $\frac{15}{20} - \frac{8}{20} = \frac{7}{20}$ **(e)** $\frac{3}{18} + \frac{4}{18} = \frac{7}{18}$ **(f)** $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

(g) $\frac{5}{{}_2\cancel{6}} \times \frac{\cancel{3}^1}{4} = \frac{5}{8}$ **(h)** $\frac{{}^2\cancel{4}}{{}_5\cancel{15}} \times \frac{\cancel{3}^1}{\cancel{2}_1} = \frac{2}{5}$ **(i)** $\frac{7}{{}_4\cancel{8}} \times \frac{\cancel{2}^1}{3} = \frac{7}{12}$

(j) $\frac{{}^1\cancel{2}}{{}_{15}\cancel{75}} \times \frac{\cancel{5}^1}{\cancel{4}_2} = \frac{1}{30}$ **(k)** $\frac{2}{9} \times \frac{1}{3} = \frac{2}{27}$ **(l)** $\frac{3}{1} \times \frac{7}{2} = \frac{21}{2} = 10\frac{1}{2}$

7. $47\frac{1}{2} \div 1\frac{1}{4} = \frac{95}{2} \div \frac{5}{4} = \frac{\cancel{95}^{19}}{\cancel{2}_1} \times \frac{\cancel{4}^2}{\cancel{5}_1} = 38$

8. **(a)** $\frac{2}{3x} + \frac{1}{3x} = \frac{3}{3x} = \frac{1}{x}$ **(b)** $\frac{2}{{}_1\cancel{x}} \times \frac{\cancel{x}^1}{5} = \frac{2}{5}$ **(c)** $\frac{3}{x} - \frac{2}{x^2} = \frac{3x}{x^2} - \frac{2}{x^2} = \frac{3x-2}{x^2}$

(d) $\frac{7}{x}+\frac{2}{y}=\frac{7y}{xy}+\frac{2x}{xy}=\frac{7y+2x}{xy}$

(e) $\frac{a}{2}\div\frac{a}{6}=\frac{{}^{1}\cancel{a}}{{}_{1}\cancel{2}}\times\frac{\cancel{6}^{3}}{\cancel{a}_{1}}=3$

(f) $\frac{5c}{12}+\frac{5d}{18}=\frac{15c}{36}+\frac{10d}{36}=\frac{15c+10d}{36}$

(g) $\frac{x+2}{{}_{1}\cancel{y-5}}\times\frac{\cancel{y-5}^{1}}{x+3}=\frac{x+2}{x+3}$

(h) $\frac{4gh}{7}\div\frac{2g}{9h}=\frac{{}^{2}\cancel{4}\cancel{g}h}{7}\times\frac{9h}{\cancel{2}_{1}\cancel{g}}=\frac{18h^{2}}{7}$

(i) $\frac{t}{4}\div 5=\frac{t}{4}\times\frac{1}{5}=\frac{t}{20}$

(j) $\frac{{}^{1}\cancel{P}}{{}_{1}\cancel{Q}}\times\frac{\cancel{Q}^{1}}{\cancel{P}_{1}}=1$

9. **(a)** $x+2=7$

$x=5$ (subtract 2 from both sides)

(b) $3x=18$

$x=6$ (divide both sides by 3)

(c) $\frac{x}{9}=2$

$x=18$ (multiply both sides by 9)

(d) $x-4=-2$

$x=2$ (add 4 to both sides)

(e) $2x-3=17$

$2x=20$ (add 3 to both sides)

$x=10$ (divide both sides by 2)

(f) $3x+4=1$

$3x=-3$ (subtract 4 from both sides)

$x=-1$ (divide both sides by 3)

(g) $\frac{x}{6}-7=3$

$\frac{x}{6}=10$ (add 7 to both sides)

$x=60$ (multiply both sides by 6)

(h) $3(x-1)=2$

$3x-3=2$ (multiply out brackets)

$3x=5$ (add 3 to both sides)

$x=\frac{5}{3}=1\frac{2}{3}$ (divide both sides by 3)

(i) $4-x=9$

$-x=5$ (subtract 4 from both sides)

$x=-5$ (divide both sides by −1)

(j) $6x+2=5x-1$

$x+2=-1$ (subtract $5x$ from both sides)

$x=-3$ (subtract 2 from both sides)

(k) $5(3x+8)=10$

$15x+40=10$ (multiply out brackets)

$15x=-30$ (subtract 40 from both sides)

$x=-2$ (divide both sides by 15)

(l) $2(x-3)=5(x+1)$

$2x-6=5x+5$ (multiply out brackets)

$-3x-6=5$ (subtract $5x$ from both sides)

$-3x=11$ (add 6 to both sides)

$x=\frac{-11}{3}=-3\frac{2}{3}$ (divide both sides by -3)

(m) $\frac{4x-7}{3}=2$

$4x-7=6$ (multiply both sides by 3)

$4x=13$ (add 7 to both sides)

$x=\frac{13}{4}=3\frac{1}{4}$ (divide both sides by 4)

(n) $\frac{4}{x+1}=1$

$4=x+1$ (multiply both sides by $x + 1$)

$3=x$ (subtract 1 from both sides)

(o) $5-\frac{1}{x}=1$

$5=1+\frac{1}{x}$ (add $\frac{1}{x}$ to both sides)

$4=\frac{1}{x}$ (subtract 1 from both sides)

$4x=1$ (multiply both sides by x)

$x=\frac{1}{4}$ (divide both sides by 4)

10. (a), (d), (e), (f)

11. (a) $2x > x + 1$

$x > 1$ (subtract x from both sides)

(b) $7x + 3 \le 9 + 5x$

$2x + 3 \le 9$ (subtract $5x$ from both sides)

$2x \le 6$ (subtract 3 from both sides)

$x \le 3$ (divide both sides by 2)

(c) $x - 5 > 4x + 4$

$-3x - 5 > 4$ (subtract $4x$ from both sides)

$-3x > 9$ (add 5 to both sides)

$x < -3$ (divide both sides by -3)

(d) $x - 1 < 2x - 3$

$-x - 1 < -3$ (subtract $2x$ form both sides)

$-x < -2$ (add 1 to both sides)

$x > 2$ (divide both sides by -1)

12.

$$\frac{4}{x^2 y} \div \frac{2x}{y} = \frac{{}^2\not{4}}{x^2 \not{y}} \times \frac{\not{y}}{\not{2}_1 x} = \frac{2}{x^3}$$

13. (a) $6(2 + x) = 5(1 - 4x)$

$12 + 6x = 5 - 20x$ (multiply out brackets)

$12 + 26x = 5$ (add $20x$ to both sides)

$26x = -7$ (subtract 12 from both sides)

$x = -\frac{7}{26}$ (divide both sides by 26)

(b) $3x + 6 \ge 5x - 14$

$-2x + 6 \ge -14$ (subtract $5x$ from both sides)

$-2x \ge -20$ (subtract 6 from both sides)

$x \le 10$ (divide both sides by -2)

Section 1.3 Graphs of linear equations

Practice Problems

1. From Figure S1.1, note that all five points lie on a straight line.

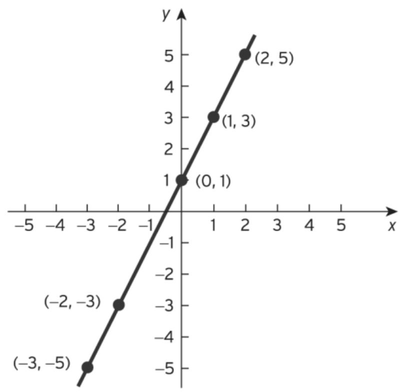


Figure S1.1

2.

Point	Check	
(−1,2)	2(−1) + 3(2) = −2 + 6 = 4	✓
(−4,4)	2(−4) + 3(4) = −8 + 12 = 4	✓
(5, −2)	2(5) + 3(−2) = 10 − 6 = 4	✓
(2,0)	2(2) + 3(0) = 4 + 0 = 4	✓

The graph is sketched in Figure S1.2.

The graph shows that (3, −1) does not lie on the line. This can be verified algebraically:

$$2(3) + 3(-1) = 6 - 3 = 3 \neq 4$$

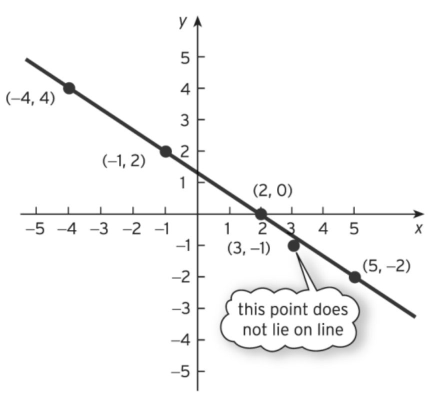


Figure S1.2

3.

$$3x - 2y = 4$$
$$3(2) - 2y = 4 \quad \text{(substitute } x = 2\text{)}$$
$$6 - 2y = 4$$
$$-2y = -2 \quad \text{(subtract 6 from both sides)}$$
$$y = 1 \quad \text{(divide both sides by } -2\text{)}$$

Hence (2, 1) lies on the line.

$$3x - 2y = 4$$
$$3(-2) - 2y = 4$$
$$-6 - 2y = 4 \quad \text{(substitute } x = -2\text{)}$$
$$-2y = 10 \quad \text{(add 6 to both sides)}$$
$$y = -5 \quad \text{(divide both sides by } -2\text{)}$$

Hence (−2, −5) lies on the line.

The line is sketched in Figure S1.3.

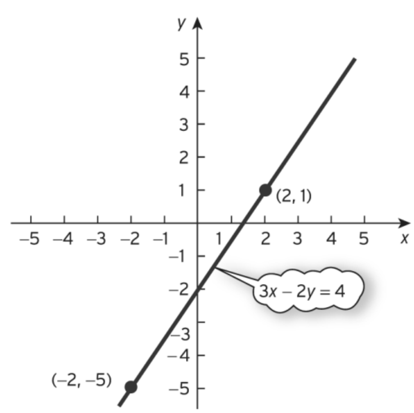


Figure S1.3

4.

$$x - 2y = 2$$
$$0 - 2y = 2 \quad \text{(substitute } x = 0\text{)}$$
$$-2y = 2$$
$$y = -1 \quad \text{(divide both sides by } -2\text{)}$$

Hence (0, –1) lies on the line.

$$x - 2y = 2$$
$$x - 2(0) = 2 \quad \text{(substitute } y = 0\text{)}$$
$$x - 0 = 2$$
$$x = 2$$

Hence (2, 0) lies on the line.

The graph is sketched in Figure S1.4.

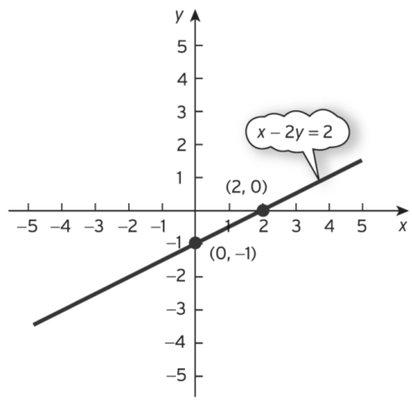


Figure S1.4

5. From Figure S1.5, the point of intersection is (1, −½).

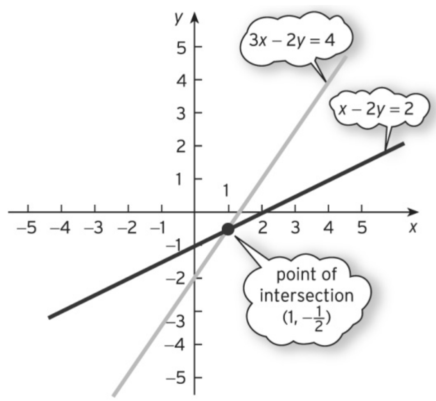


Figure S1.5

6. **(a)** a = 1, b = 2. The graph is sketched in Figure S1.6.

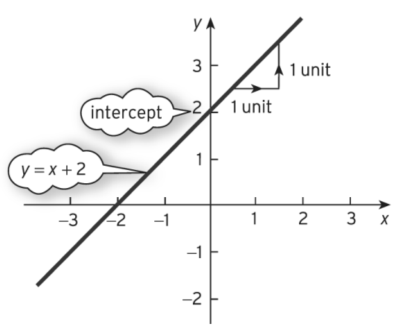


Figure S1.6

(b) $4x+2y=1$

$2y=1-4x$ (subtract $4x$ from both sides)

$y=\frac{1}{2}-2x$ (divide both sides by 2)

so $a=-2,\ b=\frac{1}{2}$. The graph is sketched in Figure S1.7.

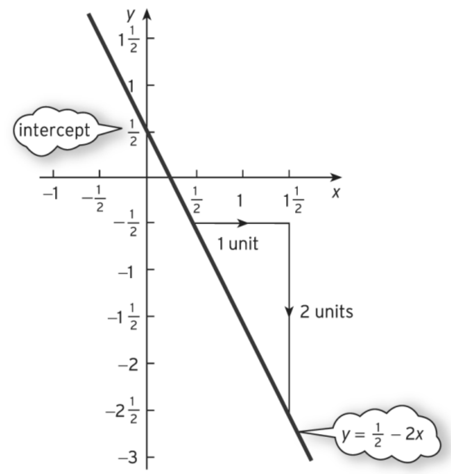


Figure S1.7

Exercise 1.3 (p. 52)

1. From Figure S1.8, the point of intersection is (2, 3).

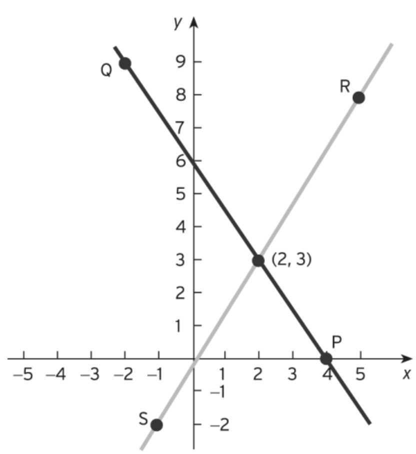


Figure S1.8

2 **(a)** The graph is sketched in Figure S1.9.

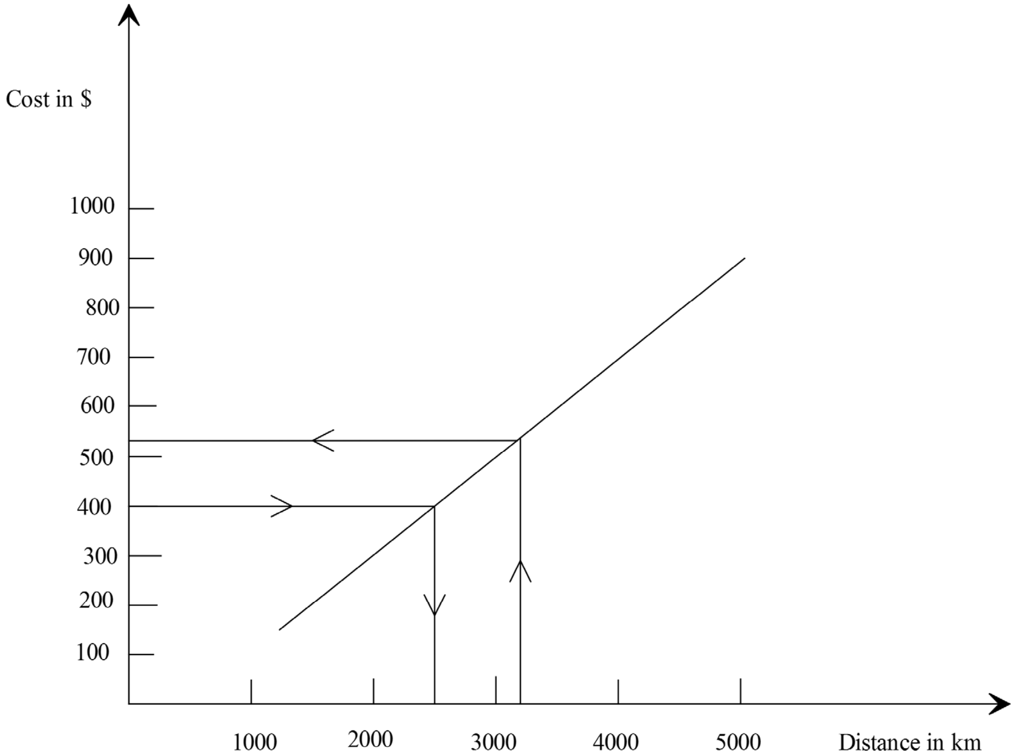


Figure S1.9

(b) **(i)** \$540 **(ii)** 2500 km

3. *A, C, D, E*

4. **(a)** $3x-10=8$

$3x=18$ (add 10 to both sides)

$x=6$ (divide both sides by 3)

(b) $3-5y=8$

$-5y=5$ (subtract 3 from both sides)

$y=-1$ (divide both sides by -5)

$(6,2)$, $(1,-1)$.

5.

x	y
0	8
6	0
3	4

The graph is sketched in Figure S1.10.

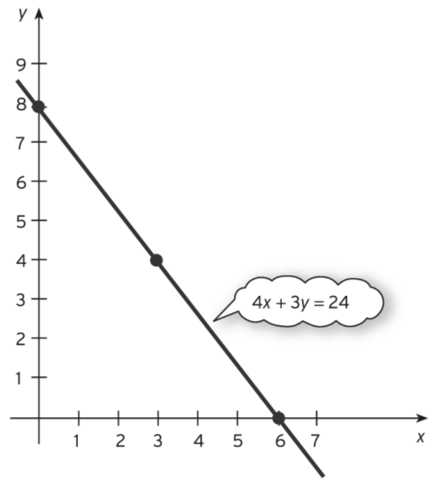


Figure S1.10

6. **(a)** The line $-2x+y=2$ passes through (0, 2) and (−1, 0)

The line $2x+y=-6$ passes through (0, −6) and (−3, 0)

The point of intersection has coordinates (–2, –2)

(b) The line $3x+4y=12$ passes through (0, 3) and (4, 0)

The line $x+4y=8$ passes through (0, 2) and (8, 0)

The point of intersection has coordinates (2, 1 $\frac{1}{2}$)

(c) The line $2x+y=4$ passes through (0, 4) and (2, 0)

The line $4x-3y=3$ passes through (0, −1) and (3/4, 0)

The point of intersection has coordinates (1 $\frac{1}{2}$, 1)

(d) The line $x+y=1$ passes through (0, 1) and (1, 0)

The line $6x+5y=15$ passes through (0, 3) and (5/2, 0)

The point of intersection has coordinates (10, –9)

7. **(a)** 5, 9 **(b)** 3, -1 **(c)** -1, 13

(d) $-x+y=4$

$y=x+4$ (add x to both sides)

so the slope is 1 and the y-intercept is 4

(e) $4x+2y=5$

$2y=-4x+5$ (subtract $4x$ from both sides)

$y=-2x+\frac{5}{2}$ (divide both sides by 2)

so the slope is -2 and the y-intercept is $\frac{5}{2}$

(f) $5x-y=6$

$-y=-5x+6$ (subtract $5x$ from both sides)

$y=5x-6$ (divide both sides by -1)

so the slope is 5 and the y-intercept is -6

8. **(a)** The line has a slope of -1 and a y-intercept of 0.

The line passes through the origin and for every one unit along the graph goes down 1.

The graph is sketched in Figure S1.11.

(b) $x-2y=6$

$-2y=-x+6$ (subtract x from both sides)

$y=\frac{1}{2}x-3$ (divide both sides by –2)

so the line has a slope of 1/2 and a y-intercept of -3.

The line passes through the point $(0, -3)$ and for every unit along the graph goes up by 1/2 unit, or equivalently, for every 2 units along it goes up by 1 unit.

The graph is sketched in Figure S1.12.

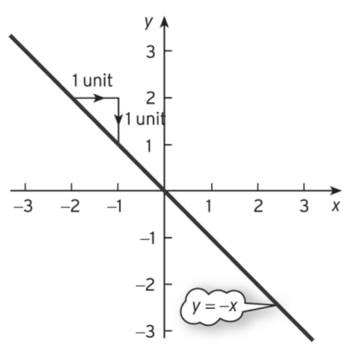


Figure S1.11

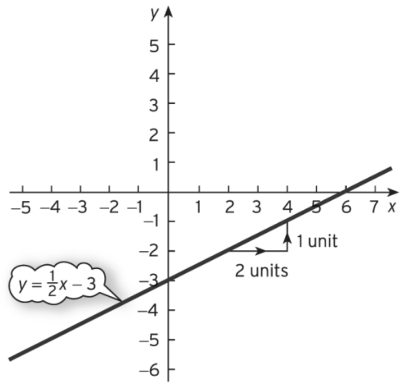


Figure S1.12

9. **(a)** $C = 4 + 2.5x$

(b) The graph is sketched in Figure S1.13.

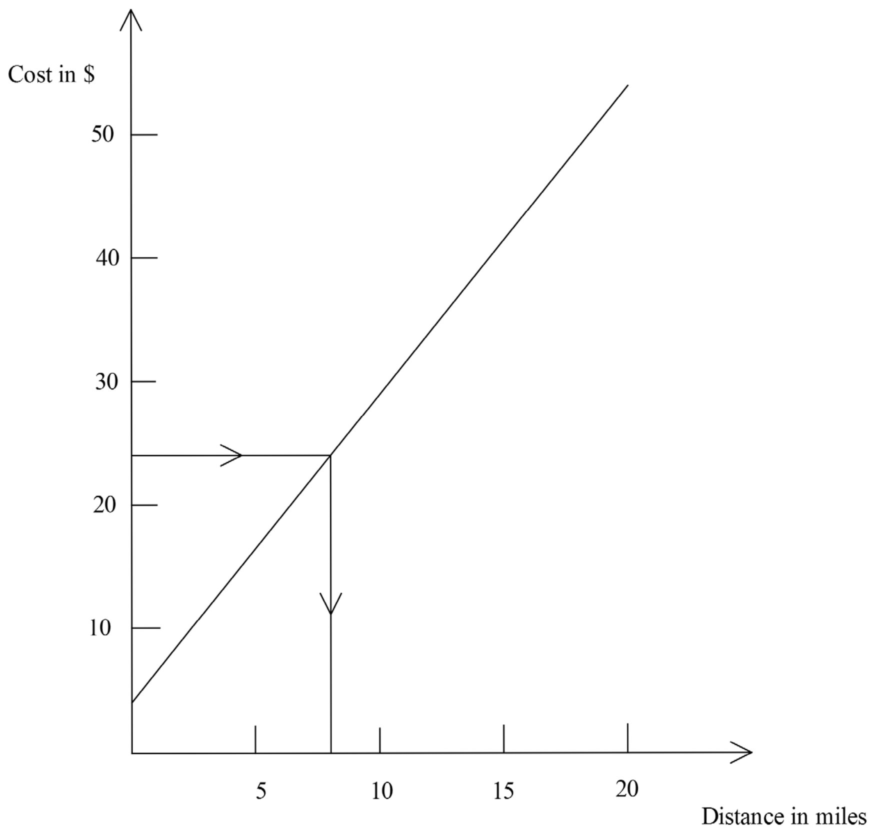


Figure S1.13

(c) The diagram shows that the distance is 8 miles. Alternatively, using algebra:

$$4 + 2.5x = 24$$
$$2.5x = 20$$
$$x = 8$$

10. (a) The graph is sketched in Figure S1.14.

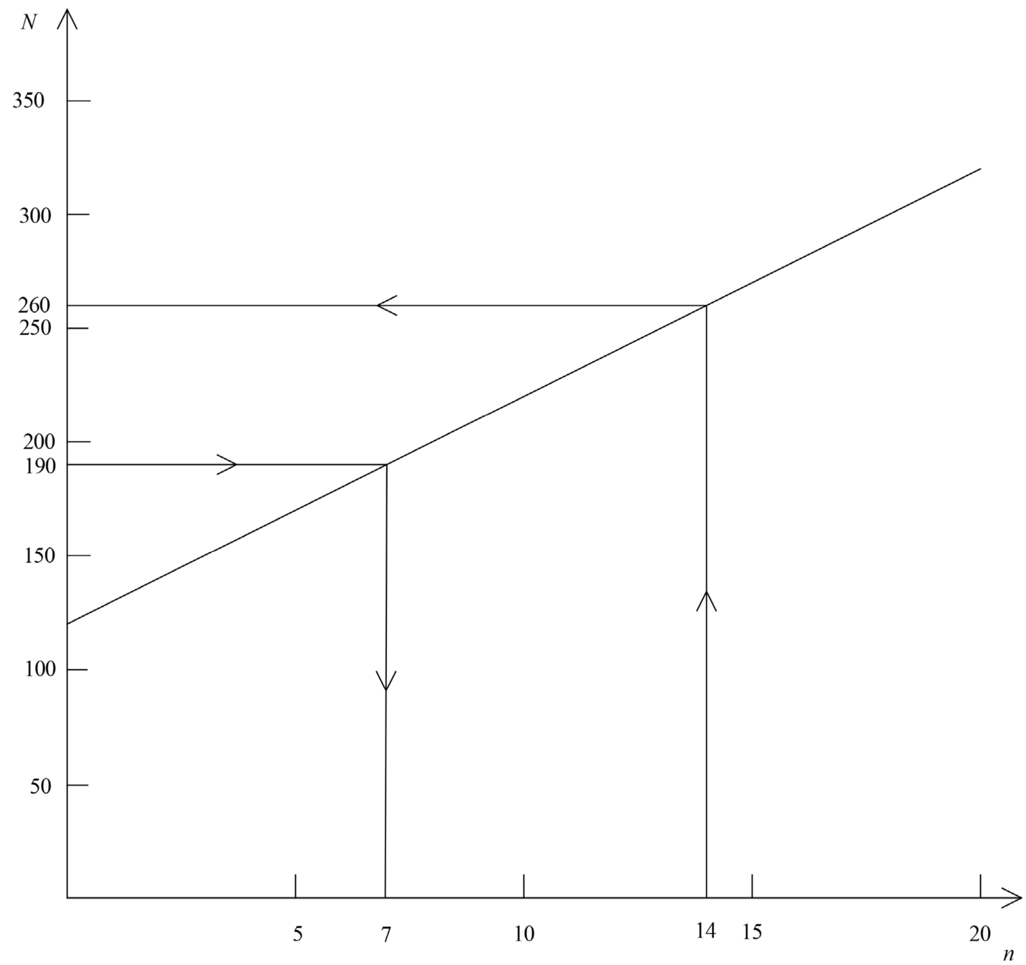


Figure S1.14

(b) (i) From the graph, N = 260. Alternatively, from the formula, $N = 10 \times 14 + 120 = 260$.

(ii) From the graph, n = 7. Alternatively, using algebra,

$$10n + 120 = 190$$
$$10n = 70$$
$$n = 7$$

(c) Slope = 10; this is the number of staff employed in each café.
Intercept = 120; number of staff employed managing the company.

11. **(a)** When $A = 0$, $S = \$9000$

(b) $S = 9000 + 12 \times 800 = \$18,600$

(c)

$$9000 + 12A = 15,000$$
$$12A = 6000$$
$$A = \$500$$

(d) The slope of the graph is \$12.

Section 1.4 Algebraic solution of simultaneous linear equations

Practice Problems

1. **(a)** ***Step 1***

It is probably easiest to eliminate y. This can be done by subtracting the second equation from the first:

$$\begin{array}{l} 3x - 2y = 4 \\ \underline{\;\; x - 2y = 2 -} \\ 2x \qquad\; = 2 \end{array}$$

Step 2

The equation $2x = 2$ has solution $x = 2/2 = 1$.

Step 3

If this is substituted into the first equation, then

$$\begin{aligned} 3(1) - 2y &= 4 \\ 3 - 2y &= 4 \\ -2y &= 1 \quad \text{(subtract 3 from both sides)} \\ y &= -\frac{1}{2} \quad \text{(divide both sides by} -2) \end{aligned}$$

Step 4

As a check the second equation gives

$$\begin{aligned} x - 2y &= 1 - 2(-1/2) \\ &= 1 - (-1) = 2 \checkmark \end{aligned}$$

Hence the solution is $x = 1$, $y = -1/2$.

If you decide to eliminate x, then the corresponding steps are as follows:

Step 1

Triple the second equation and subtract from the first:

$$\begin{array}{r} 3x-2y=4 \\ \underline{3x-6y=6-} \\ 4y=-2 \end{array}$$

Step 2

The equation $4y = -2$ has solution $y = -2/4 = -1/2$.

Step 3

If this is substituted into the first equation, then

$$\begin{aligned} 3x-2(-\tfrac{1}{2}) &= 4 \\ 3x+1 &= 4 \\ 3x &= 3 \quad \text{(subtract 1 from both sides)} \\ x &= 1 \quad \text{(divide both sides by 3)} \end{aligned}$$

(b) ***Step 1***

It is immaterial which variable is eliminated. To eliminate x multiply the first equation by 5, multiply the second by 3 and add:

$$\begin{array}{r} 15x+25y=95 \\ \underline{-15x+6y=-33+} \\ 31y=62 \end{array}$$

Step 2

The equation $31y = 62$ has solution $y = 62/31 = 2$.

Step 3

If this is substituted into the first equation, then

$$\begin{aligned} 3x+5(2) &= 19 \\ 3x+10 &= 19 \quad \text{(subtract 10 from both sides)} \\ 3x &= 9 \\ x &= 3 \quad \text{(divide both sides by 3)} \end{aligned}$$

Step 4

As a check the second equation gives

$$\begin{aligned} -5x+2y &= -5(3)+2(2) \\ &= -15+4=-11 \quad \checkmark \end{aligned}$$

Hence the solution is $x = 3$, $y = 2$.

2. **(a)** ***Step** 1*

To eliminate x multiply the first equation by 4, multiply the second equation by 3 and add:

$$\begin{array}{r} 12x - 24y = -8 \\ \underline{-12x + 24y = -3 +} \\ 0y = -11 \end{array}$$

Step 2

This is impossible, so there are no solutions.

(b) ***Step 1***

To eliminate x multiply the first equation by 2 and add to the second:

$$\begin{array}{r} -10x + 2y = 8 \\ \underline{10x - 2y = -8 +} \\ 0y = 0 \end{array}$$

Step 2

This is true for any value of y, so there are infinitely many solutions.

3. ***Step 1***

To eliminate x from the second equation multiply equation (2) by 2 and subtract from equation (1):

$$\begin{array}{r} 2x + 2y - 5z = -5 \\ \underline{2x - 2y + 2z = 6 -} \\ 4y - 7z = -11 \end{array} \qquad (4)$$

To eliminate x from the third equation multiply equation (1) by 3, multiply equation (3) by 2 and add:

$$\begin{array}{r} 6x + 6y - 15z = -15 \\ \underline{-6x + 2y + 4z = -4 +} \\ 8y - 11z = -19 \end{array} \qquad (5)$$

The new system is

$$\begin{array}{r} 2x + 2y - 5z = -5 \\ 4y - 7z = -11 \\ 8y - 11z = -19 \end{array}$$

Step 2

To eliminate y from the third equation multiply equation (4) by 2 and subtract equation (5):

$$\begin{array}{r} 8y - 14z = -22 \\ \underline{8y - 11z = -19} \\ -3z = -3 \end{array} \qquad (6)$$

The new system is

$$2x + 2y - 5z = -5 \qquad (1)$$
$$4y - 7z = -11 \qquad (4)$$
$$-3z = -3 \qquad (6)$$

Step 3

Equation (6) gives $z = -3/-3 = 1$. If this is substituted into equation (4), then

$$4y - 7(1) = -11$$
$$4y - 7 = -11$$
$$4y = -4 \quad \text{(add 7 to both sides)}$$
$$y = -1 \quad \text{(divide both sides by 4)}$$

Finally, substituting $y = -1$ and $z = 1$ into equation (1) produces

$$2x + 2(-1) - 5(1) = -5$$
$$2x - 7 = -5$$
$$2x = 2 \quad \text{(add 7 to both sides)}$$
$$x = 1 \quad \text{(divide both sides by 2)}$$

Step 4

As a check, the original equations (1), (2) and (3) give

$$2(1) + 2(-1) - 5(1) = -5 \quad ✓$$
$$1 - (-1) + 1 = 3 \quad ✓$$
$$-3(1) + (-1) + 2(1) = -2 \quad ✓$$

Hence the solution is $x = 1$, $y = -1$, $z = 1$.

Exercise 1.4 (p. 65)

1. **(a)** ***Step 1***

Add the two equations to eliminate x:

$$\begin{array}{r} -2x + y = 2 \\ 2x + y = -6 + \\ \hline 2y = -4 \end{array}$$

Step 2

The equation $2y = -4$ has solution $y = -4/2 = -2$.

Step 3

If this is substituted into the first equation, then

$$\begin{aligned} -2x + (-2) &= 2 \\ -2x - 2 &= 2 \\ -2x &= 4 \end{aligned}$$

so $x = \dfrac{4}{-2} = -2$

Step 4

As a check the second equation gives

$$\begin{aligned} 2x + y &= 2(-2) + (-2) \\ &= -4 - 2 = -6 \qquad \checkmark \end{aligned}$$

Hence the solution is $x = -2$, $y = -2$

(b) ***Step 1***

It is probably easiest to eliminate y. This can be done by subtracting the second equation from the first:

$$\begin{array}{l} 3x + 4y = 12 \\ \;\;x + 4y = 8 - \\ \hline 2x \qquad = 4 \end{array}$$

Step 2

The equation $2x = 4$ has solution $x = 4/2 = 2$.

Step 3

If this is substituted into the first equation, then

$$\begin{aligned} 3(2)+4y&=12 \\ 6+4y&=12 \\ 4y&=6 \qquad \text{(subtract 6 from both sides)} \\ y&=\frac{6}{4}=3/2 \qquad \text{(divide both sides by 4)} \end{aligned}$$

Step 4

As a check the second equation gives

$$\begin{aligned} x+4y &= 2+4(3/2) \\ &= 2+6=8 \quad \checkmark \end{aligned}$$

Hence the solution is $x = 2$, $y = 3/2$

(c) ***Step 1***

To eliminate x multiply the first equation by 2 and subtract the second:

$$\begin{array}{r} 4x+2y=8 \\ \underline{4x-3y=3-} \\ 5y=5 \end{array}$$

Step 2

The equation $5y = 5$ has solution $y = 5/5 = 1$.

Step 3

If this is substituted into the first equation, then

$$\begin{aligned} 2x+1&=4 \\ 2x&=3 \end{aligned}$$

so $x = \dfrac{3}{2}$

Step 4

As a check the second equation gives

$$\begin{aligned} 4x-3y &= 4\left(\frac{3}{2}\right)-3(1) \\ &= 6-3=3 \quad \checkmark \end{aligned}$$

Hence the solution is $x = 3/2$, $y = 1$

(d) ***Step 1***

To eliminate x multiply the first equation by 6, and subtract the second equation:

$$\begin{array}{r} 6x + 6y = 6 \\ \underline{6x + 5y = 15 -} \\ y = -9 \end{array}$$

Step 2

$y = -9$

Step 3

If this is substituted into the first equation, then

$$\begin{aligned} x - 9 &= 1 \\ x &= 10 \end{aligned}$$

Step 4

As a check the second equation gives

$$\begin{aligned} 6x + 5y &= 6(10) + 5(-9) \\ &= 60 - 45 = 15 \quad \checkmark \end{aligned}$$

Hence the solution is $x = 10$, $y = -9$

2. **(a)**

$$\begin{aligned} x + y &= 3500 \\ 30x + 25y &= 97500 \end{aligned}$$

(b) Multiply the first equation by 30 and subtract the second:

$$\begin{aligned} 5y &= 7500 \\ y &= 1500 \end{aligned}$$

3. The lines are sketched in Figure S1.15.

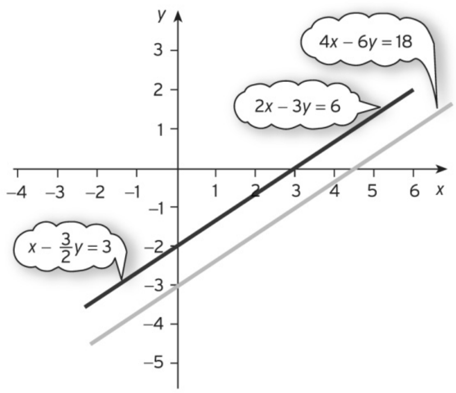


Figure S1.15

(a) The two lines are on top of each other so intersect throughout their length. There are infinitely many solutions.

(b) The lines are parallel, so do not intersect anywhere. There is no solution.

4. **(a)** ***Step 1***

To eliminate x multiply the first equation by 3, and add:

$$\begin{array}{r} -9x+15y=12 \\ \underline{9x-15y=-12-} \\ 0y=0 \end{array}$$

Step 2

The equation $0y = 0$ is true for all values of y so there are infinitely many solutions

(b) ***Step 1***

To eliminate x multiply the first equation by 5, multiply the second by 2 and subtract :

$$\begin{array}{r} 30x-10y=15 \\ \underline{30x-10y=8-} \\ 0y=7 \end{array}$$

Step 2

The equation $0y = 7$ is not true for any value of y, so there are no solutions.

5. ***Step 1***

To eliminate x multiply the second equation by 2, and add to the first:

$$\begin{array}{r} 6x - 4y = 2 \\ \underline{-6x + 4y = 2k +} \\ 0y = 2k + 2 \end{array}$$

Step 2

The equation $0y = 2k + 2$ only has solutions (when there will be infinitely many) if the right-hand side is zero so that

$2k + 2 = 0$

$2k = -2$ (subtract 2 from both sides)

$k = -1$

Section 1.5 Supply and demand analyses

Practice Problems

1. **(a)** 0 **(b)** 48 **(c)** 16 **(d)** 25 **(e)** 1 **(f)** 17

The function g reverses the effect of f and takes you back to where you started. For example, if 25 is put into the function f, the outgoing number is 0; and when 0 is put into g, the original number, 25, is produced. We describe this by saying that g is the inverse of f (and vice versa).

2. The demand curve that passes through (0,75) and (25,0) is sketched in Figure S1.16. From this diagram we see that

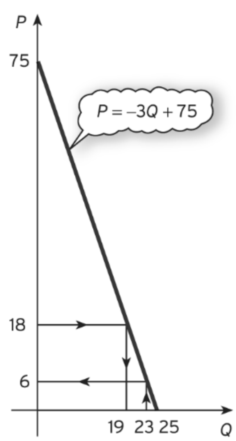


Figure S1.16

(a) $P = 6$ when $Q = 23$

(b) $Q = 19$ when $P = 18$

Alternatively, using algebra:

(a) Substituting $Q = 23$ gives

$P = -3(23) + 75 = 6$

(b) Substituting $P = 18$ gives $18 = -3Q + 75$ with solution $Q = 19$

3. **(a)** In equilibrium, $Q_S = Q_D = Q$, so

$$P = -4Q + 120$$
$$P = 1/3Q + 29$$

Hence

$$-4Q + 120 = \frac{1}{3}Q + 29 \qquad \text{(since both sides equal } P\text{)}$$

$$-4\frac{1}{3}Q + 120 = 29 \qquad \text{(subtract } \frac{1}{3}Q \text{ from both sides)}$$

$$-4\frac{1}{3}Q = -91 \qquad \text{(subtract 120 from both sides)}$$

$$Q = 21 \qquad \text{(divide both sides by } -4\frac{1}{3}\text{)}$$

Substituting this value into either the demand or supply equations gives $P = 36$.

(b) After the imposition of a \$13 tax the supply equation becomes

$$P - 13 = \frac{1}{3}Q_S + 29$$
$$P = \frac{1}{3}Q_S + 42$$

The demand equation remains unchanged, so, in equilibrium,

$$P = -4Q + 120$$
$$P = \frac{1}{3}Q + 42$$

Hence

$$-4Q + 120 = \frac{1}{3}Q + 42$$

This equation can now be solved as before to get $Q = 18$ and the corresponding price is $P = 48$. The equilibrium price rises from \$36 to \$48, so the consumer pays an additional \$12. The remaining \$1 of the tax is paid by the firm.

4. For good 1, $Q_{D_1} = Q_{S_1} = Q_1$ in equilibrium, so the demand and supply equations become

$$Q_1 = 40 - 5P_1 - P_2$$
$$Q_1 = -3 + 4P_1$$

Hence

$40 - 5P_1 - P_2 = -3 + 4P_1$	(since both sides equal Q_1)
$40 - 9P_1 - P_2 = -3$	(subtract $4P_1$ from both sides)
$-9P_1 - P_2 = -43$	(subtract 40 from both sides)

For good 2, $Q_{D_2} = Q_{S_2} = Q_2$ in equilibrium, so the demand and supply equations become

$$Q_2 = 50 - 2P_1 - 4P_2$$
$$Q_2 = -7 + 3P_2$$

Hence

$50 - 2P_1 - 4P_2 = -7 + 3P_2$	(since both sides equal Q_2)
$50 - 2P_1 - 7P_2 = -7$	(subtract $3P_2$ from both sides)
$-2P_1 - 7P_2 = -57$	(subtract 50 from both sides)

The equilibrium prices therefore satisfy the simultaneous equations

$$-9P_1 - P_2 = -43 \quad (1)$$
$$-2P_1 - 7P_2 = -57 \quad (2)$$

Step 1

Multiply equation (1) by 2 and (2) by 9 and subtract to get

$$61P_2 = 427 \quad (3)$$

Step 2

Divide both sides of equation (3) by 61 to get $P_2 = 7$.

Step 3

Substitute P_2 into equation (1) to get $P_1 = 4$.

If these equilibrium prices are substituted into either the demand or the supply equations then $Q_1 = 13$ and $Q_2 = 14$.

The goods are complementary because the coefficient of P_2 in the demand equation for good 1 is negative, and likewise for the coefficient of P_1 in the demand equation for good 2.

Exercise 1.5 (p. 80)

1. **(a)** 21 **(b)** 45 **(c)** 15 **(d)** 2 **(e)** 10 **(f)** 0

The answers to parts (a) and (b) show that putting 2 into f gives 21 and putting 21 into g takes us back to 2.

A similar property holds for parts (b) and (e) as well as (c) and (f).

If one function undoes the result of another and takes you back to number you first thought of then we say that they are inverses of each other.

2. The supply curve is sketched in Figure S1.17.

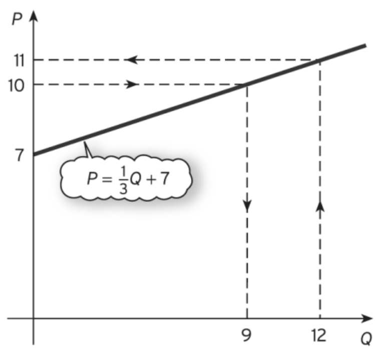


Figure S1.17

(a) 11; this value can either be found by substituting $Q = 12$ into $P = \frac{1}{3}Q + 7$ or by reading values off the graph.

(b) 9; this value can either be found by solving the equation, $\frac{1}{3}Q + 7 = 10$ or by reading values off the graph.

(c) 0; once the price falls below 7 the graph shows that the firm does not plan to produce any goods.

3. **(a)** $Q = 100 - 10 + 2 \times 40 + \frac{1}{2} \times 6 = 173$.

To work out the new advertising expenditure you could solve the equation

$$100 - 10 + 2 \times 40 + \frac{1}{2}A = 179$$

which gives:

$$170 + \frac{1}{2}A = 179$$

$\frac{1}{2}A = 9$ (subtract 170 from both sides)

$A = 18$ (multiply both sides by 2)

Hence additional advertising expenditure is $18 - 6 = 12$.

Alternatively notice that the demand increases by $179 - 173 = 6$. In the demand function, the term involving advertising expenditure is $\frac{1}{2}A$ so that for every one unit increase in *A,* demand increases by $\frac{1}{2}$. Hence expenditure will need to rise by 12 to achieve a change of 6 in the value of *Q*.

(b) Normal; the coefficient of *Y* is positive so an increase in *Y* leads to an increase in *Q*.

4. (a) $Q = 30 - 3 \times 4 + 5 = 23$

(b) Substitutable; the coefficient of P_A is positive so a rise in P_A leads to an increase in Q

(c) The demand equation is:

$30 - 3P + 11 = 23$

$41 - 3P = 23$

$-3P = -18$ (subtract 41 from both sides)

$P = 6$ (divide both sides by -3)

5.

$$50a + b = 420$$
$$80a + b = 240$$

Subtract to get

$$-30a = 180$$
$$a = -6$$

Substituting this into either of the original equations gives, $b = 720$.

6. (a) Substituting $Q = 0$ and $Q = 50$ into the supply function give $P = 20$ and 45, respectively.

To find the value of Q when $P = 25$, solve the equation:

$\frac{1}{2}Q + 20 = 25$

$\frac{1}{2}Q = 5$ (subtract 20 from both sides)

$Q = 10$ (multiply both sides by 2)

Line passes through (0, 20), (10, 25) and (50, 45)

(b) Line passing through (50, 0) and (0, 50)

The lines intersect at the point (20, 30) so the equilibrium values are $Q = 20$ and $P = 30$

(c) As income rises demand increases so the demand curve moves to the right. As it does so the point of intersection moves both to the right and upwards on the page.

Hence price and quantity both increase.

7. The new supply equation is

$$P - 4 = \frac{1}{2}Q_S + 23$$

which rearranges as $P = \frac{1}{2}Q_S + 27$.

In equilibrium, $Q_S = Q_D = Q$ so we need to solve the equations:

$$P = -3Q + 48$$

$$P = \frac{1}{2}Q + 27$$

Hence

$$-3Q + 48 = \frac{1}{2}Q + 27$$

$$-3\frac{1}{2}Q + 48 = 27 \qquad \text{(subtract } \tfrac{1}{2}Q \text{ from both sides)}$$

$$-3\frac{1}{2}Q = -21 \qquad \text{(subtract 48 from both sides)}$$

$$Q = 6 \qquad \text{(divide both sides by } -3.5\text{)}$$

8. In equilibrium, $Q_{D_1} = Q_{S_1} = Q_1$ and $Q_{D_2} = Q_{S_2} = Q_2$

For good 1 we have

$$100 - 2P_1 + P_2 = -10 + P_1$$

$$100 - 3P_1 + P_2 = -10 \qquad \text{(subtract } P_1 \text{ from both sides)}$$

$$-3P_1 + P_2 = -110 \qquad \text{(subtract 100 from both sides)} \qquad (1)$$

For good 2 we have

$$5 + 2P_1 - 3P_2 = -5 + 6P_2$$

$$5 + 2P_1 - 9P_2 = -5 \qquad \text{(subtract } 6P_2 \text{ from both sides)}$$

$2P_1 - 9P_2 = -10$ (subtract 5 from both sides) (2)

It is probably simplest to eliminate P_2. This can be done by multiplying equation (1) by 9 and adding equation (2).

This gives $-25P_1 = -1000$ and so $P_1 = 40$.

Substituting this into equation (1):

$-120 + P_2 = -110$

$P_2 = 10$ (add 120 to both sides)

The easiest way of finding the equilibrium quantities is to use the supply equations which give:

$Q_1 = -10 + 40 = 30$

$Q_2 = -5 + 60 = 55$

9. **(a)** $Q = -20\times8 + 0.04\times1000 + 4\times15 + 3\times30 = 30$

(b) Substitutable; e.g. since coefficient of P_r is positive

(c) $-20P + 0.04\times8000 + 4\times30 + 3\times25 = 235$

$-20P + 515 = 235$

$-20P = -280$ (subtract 515 from both sides)

$P = 14$ (divide both sides by -20)

(d) **(i)** $Q = -20P + 0.04\times2000 + 4\times10 + 3\times5$

$Q = -20P + 135$

slope = -20, intercept = 135

(ii) $Q + 20P = 135$ (add $20P$ to both sides)

$20P = -Q + 135$ (subtract Q from both sides)

$P = -0.05Q + 6.75$ (divide both sides by 20)

slope = -0.05, intercept = 6.75

Section 1.6 Transposition of formulae

Practice Problems

1. **(a)** $\frac{1}{2}Q = 4$ (subtract 13 from both sides)

$Q = 8$ (multiply both sides by 2)

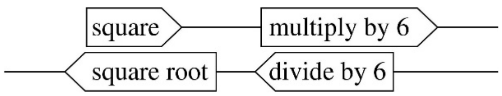
(b) $\frac{1}{2}Q = P - 13$ (subtract 13 from both sides)

Q = 2(P – 13) (multiply both sides by 2)

Q = 2P – 26 (multiply out brackets)

(c) $Q = 2 \times 17 - 26 = 8$

2. **(a)**

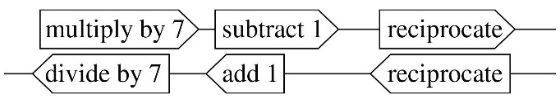


$6x^2 = y$

$x^2 = \frac{y}{6}$ (divide both sides by 6)

$x = \sqrt{\frac{y}{6}}$ (square root both sides)

(b)



$\frac{1}{7x-1} = y$

$7x - 1 = \frac{1}{y}$ (reciprocate both sides)

$7x = \frac{1}{y} + 1$ (add 1 to both sides)

$x = \frac{1}{7}\left(\frac{1}{y} + 1\right)$ (divide both sides by 7)

3. **(a)** $x - ay = cx + y$

$x = cx + y + ay$ (add *ay* to both sides)

$x - cx = y + ay$ (subtract *cx* from both sides)

$(1-c)x = (1+a)y$ (factorise both sides)

$x = \left(\frac{1+a}{1-c}\right)y$ (divide both sides by $1 - c$)

(b) $y = \frac{x-2}{x+4}$

$(x+4)y = x - 2$ (multiply both sides by $x + 4$)

$xy + 4y = x - 2$ (multiply out the brackets)

$xy = x - 2 - 4y$ (subtract $4y$ from both sides)

$xy - x = -2 - 4y$ (subtract x from both sides)

$(y-1)x = -2 - 4y$ (factorise left-hand side)

$x = \dfrac{-2-4y}{y-1}$ (divide both sides by $y - 1$)

Exercise 1.6 (p. 91)

1. $2Q + 8 = P$

$2Q = P - 8$ (subtract 8 from both sides)

$Q = \frac{1}{2}(P-8) = \frac{1}{2}P - 4$ (divide both sides by 2)

Substituting $P = 52$ into this formula gives $Q = \frac{1}{2} \times 52 - 4 = 26 - 4 = 22$

2. **(a)** $y = 2x + 5$; **(b)** $y = 2(x+5)$; **(c)** $y = \dfrac{5}{x^2}$;

(d) $y = 2(x+4)^2 - 3$.

3. **(a)** multiply by 5 add 3

(b) add 3 multiply by 5

(c) multiply by 6 subtract 9

(d) square multiply by 4 subtract 6

(e) divide by 2 add 7

(f) reciprocate multiply by 2

(g) add 3 reciprocate

4. **(a)** $9x - 6 = y$

$9x = y + 6$ (add 6 to both sides)

$x = \frac{1}{9}(y+6)$ (divide both sides by 9)

(b) $\dfrac{x+4}{3} = y$

$x+4=3y$ (multiply both sides by 3)

$x=3y-4$ (subtract 4 from both sides)

(c) $\frac{x}{2}=y$

$x = 2y$ (multiply both sides by 2)

(d) $\frac{x}{5}+8=y$

$\frac{x}{5}=y-8$ (subtract 8 from both sides)

$x=5(y-8)$ (multiply both sides by 5)

(e) $y=\frac{1}{x+2}$

$y(x+2)=1$ (multiply both sides by $x + 2$)

$x+2=\frac{1}{y}$ (divide both sides by y)

$x=\frac{1}{y}-2$; (subtract 2 from both sides)

(f) $y=\frac{4}{3x-7}$

$y(3x-7)=4$ (multiply both sides by $3x-7$)

$3x-7=\frac{4}{y}$ (divide both sides by y)

$3x=\frac{4}{y}+7$ (add 7 to both sides)

$x=\frac{1}{3}\left(\frac{4}{y}+7\right)$ (divide both sides by 3)

5. **(a)** $aP+b=Q$

$aP=Q-b$ (subtract b from both sides)

$P=\frac{1}{a}(Q-b)=\frac{Q}{a}-\frac{b}{a}$ (divide both sides by a)

(b) $Y=aY+b+I$

$Y-aY=b+I$ (subtract aY from both sides)

$(1-a)Y = b + I$ (factorise by taking out a common factor of Y)

$Y = \dfrac{b+I}{1-a}$ (divide both sides by $1-a$)

(c) $Q = \dfrac{1}{aP+b}$

$Q(aP+b) = 1$ (multiply both sides by $aP+b$)

$aP + b = \dfrac{1}{Q}$ (divide both sides by Q)

$aP = \dfrac{1}{Q} - b$ (subtract b from both sides)

$P = \dfrac{1}{a}\left(\dfrac{1}{Q} - b\right) = \dfrac{1}{aQ} - \dfrac{b}{a}$ (divide both sides by a)

6. $y = \dfrac{3}{x} - 2$

$y + 2 = \dfrac{3}{x}$ (add 2 to both sides)

$(y+2)x = 3$ (multiply both sides by x)

$x = \dfrac{3}{y+2}$ (divide both sides by $y+2$)

7. $\sqrt{\dfrac{2DR}{H}} = Q$

Square both sides:

$$\frac{2DR}{H} = Q^2$$

Multiply both sides by H:

$$2DR = HQ^2$$

(a) Divide both sides by $2R$:

$$D = \frac{HQ^2}{2R}$$

(b) Divide both sides by Q^2:

$$H = \frac{2DR}{Q^2}$$

Section 1.7 National income determination

Practice Problems

1. $S = Y - C$

$= Y - (0.8Y + 25)$ (substitute expression for C)

$= Y - 0.8Y - 25$ (multiply out brackets)

$= 0.2Y - 25$ (collect terms)

2. $Y = C + I$ (from theory)

$C = 0.8Y + 25$ (given in question)

$I = 17$ (given in question)

Substituting the given value of I into the first equation gives

$$Y = C + 17$$

and if the expression for C is substituted into this then

$$Y = 0.8Y + 42$$

$0.2Y = 42$ (subtract $0.8Y$ from both sides)

$Y = 210$ (divide both sides by 0.2)

Repeating the calculations with $I = 18$ gives $Y = 215$, so a 1 unit increase in investment leads to a 5 unit increase in income. The scale factor, 5, is called the investment multiplier. In general, the investment multiplier is given by $1/(1 - a)$, where a is the marginal propensity to consume. The foregoing is a special case of this with $a = 0.8$.

3. $Y = C + I + G$ (1)

$G = 40$ (2)

$I = 55$ (3)

$C = 0.8Y_d + 25$ (4)

$T = 0.1Y + 10$ (5)

$Y_d = Y - T$ (6)

Substituting equations (2) and (3) into equation (1) gives

$Y = C + 95$ (7)

Substituting equation (5) into (6) gives

$Y_d = Y - (0.1Y + 10)$

$= 0.9Y - 10$

so from equation (4),

$C = 0.8(0.9Y - 10) + 25$

$= 0.72Y + 17 \quad (8)$

Finally, substituting equation (8) into (7) gives

$Y = 0.72Y + 112$

which has solution $Y = 400$.

4. The commodity market is in equilibrium when

$Y = C + I$

so we can substitute the given expressions for consumption ($C = 0.7Y + 85$) and investment ($I = 50r + 1200$) to deduce that

$Y = 0.7Y - 50r + 1285$

which rearranges to give the IS schedule,

$0.3Y + 50r = 1285 \qquad (1)$

The money market is in equilibrium when

$M_S = M_D$

Now we are given that $M_S = 500$ and that total demand,

$M_D = L_1 + L_2 = 0.2Y - 40r + 230$

so that

$500 = 0.2Y - 40r + 230$

which rearranges to give the LM schedule,

$0.2Y - 40r = 270 \qquad (2)$

We now solve equations (1) and (2) as a pair of simultaneous equations.

Step 1

Multiply equation (1) by 0.2 and (2) by 0.3 and subtract to get

$22r = 176$

Step 2

Divide through by 22 to get $r = 8$.

Step 3

Substitute $r = 8$ into equation (1) to give $Y = 2950$.

The IS and LM curves shown in Figure S1.18 confirm this, since the point of intersection has coordinates (8, 2950). A change in I does not affect the LM schedule. However, if the autonomous level of investment increases from its current level of 1200 the IS curve moves upwards, causing both r and Y to increase.

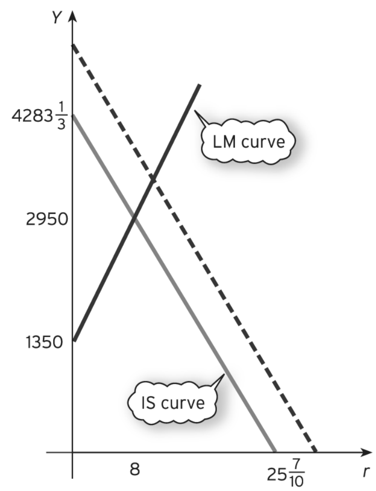


Figure S1.18

Exercise 1.7 (p. 105)

1. MPC = 0.75; MPS = $1 - 0.75 = 0.25$

2. **(a)** MPC = 70/100 = 0.7; MPS $1 - 0.7 = 0.3$

(b) From part (a), MPC = 0.7 so $C = 0.7Y + b$
Substituting, $Y = 1000$ and $C = 800$ gives $800 = 0.7 \times 1000 + b$ so $b = 100$.
Hence $C = 0.7Y + 100$.

3. **(a)** 40; **(b)** 0.7

$0.7Y + 40 = C$

$0.7Y = C - 40$ (subtract 40 from both sides)

$\frac{7}{10}Y = C - 40$

$7Y = 10(C - 40)$ (multiply both sides by 10)

$Y = \frac{10}{7}(C - 40)$ (divide both sides by 7)

$Y = \frac{10}{7}(110 - 40) = 100$

4. **(a)** $S = Y - C = Y - (0.9Y + 72) = Y - 0.9Y - 72 = 0.1Y - 72$

(b) $S = Y - C = Y - (0.8Y + 100) = Y - 0.8Y - 100 = 0.2Y - 100$

5. **(a)** $Y = C + I$

$Y = 0.6Y + 30 + 100$

$Y = 0.6Y + 130$

$0.4Y = 130$ (subtract $0.6Y$ from both sides)

$Y = 325$ (divide both sides 0.4)

(b) $C = 0.6Y + 30 = 0.6 \times 325 + 30 = 225$

(c) $S = Y - C = 325 - 225 = 100$

6. $10a + b = 28$
$30a + b = 44$

Subtracting these equations gives:

$20a = 16$ so $a = 0.8$

From the first equation,

$10 \times 0.8 + b = 28$

$8 + b = 28$

$b = 20$

Finding the equilibrium level of national income:

$Y = C + I = 0.8Y + 20 + 13 = 0.8Y + 33$

$0.2Y = 33$

$Y = 165$

7. $Y = C + I + G$

$Y = 0.75Y_d + 45 + 40 + 50$

$Y = 0.75Y_d + 135$

Now $Y_d = Y - T = Y - (0.2Y + 80) = Y - 0.2Y - 80 = 0.8Y - 80$

Substituting gives

$$Y = 0.75(0.8Y - 80) + 135 = 0.6Y - 60 + 135 = 0.6Y + 75$$

Hence

$0.4Y = 75$ (subtract $0.6Y$ from both sides)

$Y = 187.5$ (divide both sides by 0.4)

Examination Questions

1 (**a**) The graphs are sketched in Figure S1.19.

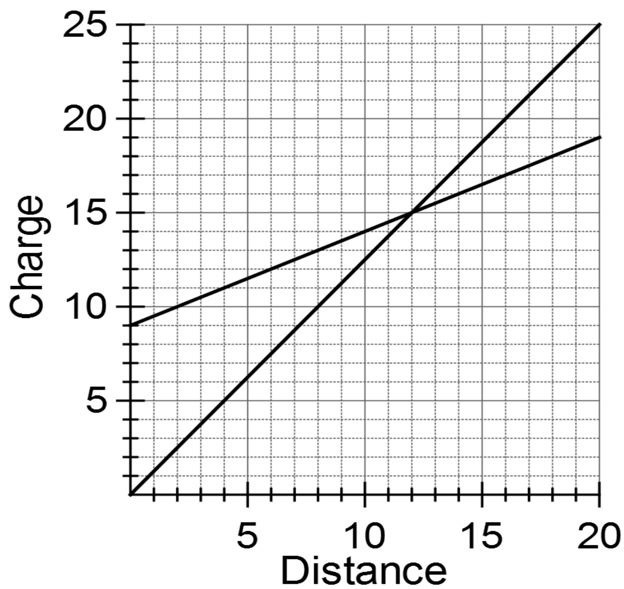


Figure S1.19

(**b**) 12 miles

(**c**) EatMeNow: $y = 0.5x + 9$ Deliver4U: $y = 1.25x$

$$1.25x = 0.5x + 9$$

$$0.75x = 9$$

Hence $x = 12$.

2 (**a**) The graphs are sketched in Figure S1.20 based on the following table of values.

P	5	10	15	20	25
Q_D	325	250	175	100	25
Q_S	0	50	100	150	200

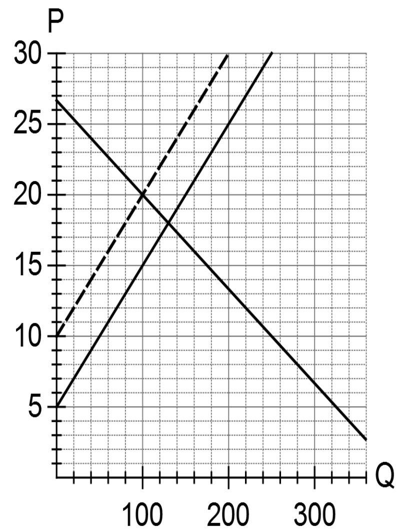


Figure S1.20

(**b**) $400 - 15P = -50 + 10P$

$25P = 450$

$P = 18$

Substitute into either equation to get $Q = 130$.

(**c**) Demand curve above that of supply for $P < 18$.

(**d**) Demand curve unchanged and supply curve shifts upwards by 5 units, as shown by the dashed line in Figure S1.20.

$400 - 15P = -50 + 10(P - 5)$

$400 - 15P = -100 + 10P$

$25P = 500$

Hence $P = 20$ and $Q = 100$.

3 (**a**) For (1) an increase in P causes a decrease in Q so this must be the demand curve and so (2) is the supply curve.

(**b**) $Q + 3P = 48$

$$3P = 48 - Q$$

$$P = 16 - \frac{1}{3}Q$$

Hence the slope is $-1/3$ and the intercept is 16.

(**c**) Subtract (2) from (1) to get $5P = 18$ so $P = 3.6$.

Substituting this into (2) gives $Q = 30 + 2 \times 3.6 = 37.2$.

4 (**a**)

$$C = 0.75(Y - 50) + 150$$
$$= 0.75Y + 112.5$$

Substitute this into $Y = C + I + G$ to get

$$Y = 0.75Y + 112.5 + 100 + 250$$
$$= 0.75Y + 462.5$$

Hence

$$0.25Y = 462.5$$
$$Y = 1850$$

(**b**) Substitute $Y = 1870$ into $Y = 0.75Y + 112.5 + I + G$ to get $I + G = 355$

Hence the increase is

$$355 - 350 = 5.$$

5 (**a**) 12

(**b**) $4 = \frac{10L}{10 + 2L}$

$$4(10 + 2L) = 10L$$
$$40 + 8L = 10L$$
$$40 = 2L$$
$$L = 20$$

(**c**) $Q(K + 2L) = KL$

$$QK + 2QL = KL$$
$$KL - QK = 2QL$$
$$K(L - Q) = 2QL$$
$$K = \frac{2QL}{L - Q}$$

6

Y	0	100	200
C + *I*	40	115	190

The graph is sketched in Figure S1.21.

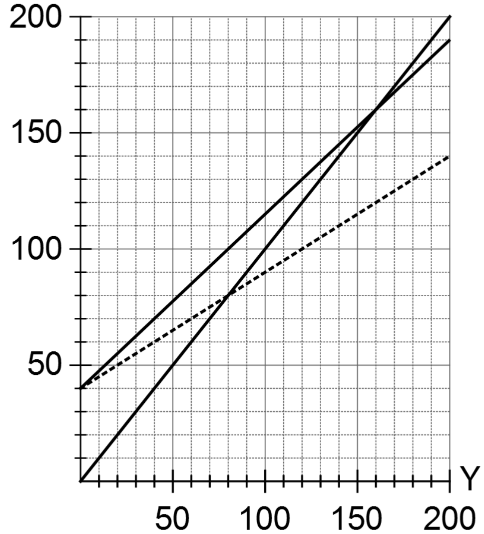


Figure S1.21

The intersection of the 45° line and the aggregate expenditure line occurs at $Y = 160$.

Check:
$$Y = 0.75Y + 18 + 22$$
$$0.25Y = 40$$
$$Y = 160$$

If MPC decreases the slope of the aggregate expenditure line decreases as shown by the dashed line in Figure S1.21. The point of intersection shifts down the 45° line indicating that the equilibrium value of Y decreases.

7 (**a**) The coefficient of P_2 in the demand equation for good 1 is positive indicating that demand for good 1 rises as the price of good 2 goes up. Similarly for good 2; as the price of good 1 goes up, the demand for good 2 rises. Hence the goods are substitutable.

(**b**)

$$120 - 2P_1 + P_2 = -7 + 7P_1 \Rightarrow 9P_1 - P_2 = 127 \quad (1)$$

$$168 + 3P_1 - 7P_2 = -3 + 20P_2 \Rightarrow 3P_1 - 27P_2 = -171 \quad (2)$$

Subtract 3 × (2) from (1) to get $80P_2 = 640$ so $P_2 = 8$.

Substituting this into (1) gives $9P_1 - 8 = 127 \Rightarrow 9P_1 = 135 \Rightarrow P_1 = 15$

The prices can now be substituted into the supply equations to get

$$Q_1 = 98, \quad Q_2 = 157$$

(**c**) The supply equation for good 1 is now

$$Q_{S_1} = -7 + 7(P_1 + 4) = 21 + 7P_1$$

The new equation (1) is

$$120 - 2P_1 + P_2 = 21 + 7P_1 \quad \Rightarrow \quad 9P_1 - P_2 = 99 \qquad (1)'$$

Equations (1)' and (2) can be solved as before to get

$$80P_2 = 612 \quad \Rightarrow \quad P_2 = 7.65 \text{ and } P_1 = 11.85$$

The price of good 1 goes down by \$3.15 and the price of good 2 goes down by \$0.35.

8

(a) $3f + 4s \leq 3000$

$$3f + 1080 \leq 3000$$

$$3f \leq 1920$$

$$f \leq 640$$

(b) (i) $P = 3$

(ii) $2.4 = \frac{36}{2Q+5}$

$$2.4(2Q+5) = 36$$

$$2Q + 5 = 15$$

$$2Q = 10$$

$$Q = 5$$

(iii) $P(2Q+5) = 36$

$$2Q + 5 = \frac{36}{P}$$

$$2Q = \frac{36}{P} - 5$$

$$Q = \frac{18}{P} - \frac{5}{2} = \frac{36 - 5P}{2P}$$

Check **(ii)**: $Q = \frac{36 - 5 \times 2.4}{2 \times 2.4} = 5$

(c)

$$\begin{aligned}S &= Y - C\\ &= Y - \frac{Y^2+100}{Y+2}\\ &= \frac{Y(Y+2)-(Y^2+100)}{Y+2}\\ &= \frac{Y^2+2Y-Y^2-100}{Y+2}\\ &= \frac{2Y-100}{Y+2}\end{aligned}$$

9

(a)

$$2x + 6y = 1 \quad (1)$$

$$3x + my = n \quad (2)$$

(1) × 3: $\quad 6x + 18y = 3$

(2) × 2: $\quad 6x + 2my = 2n$

Subtract to get

$$(18-2m)y = 3-2n \qquad (**)$$

Hence

$$y = \frac{3-2n}{18-2m} = \frac{3-2n}{2(9-m)}$$

$$\begin{aligned}2x &= 1-6y\\ &= 1 - \frac{6(3-2n)}{2(9-m)}\\ &= 1 - \frac{3(3-2n)}{9-m}\\ &= \frac{(9-m)-3(3-2n)}{9-m}\\ &= \frac{6n-m}{9-m}\end{aligned}$$

Hence

$$x = \frac{6n - m}{2(9 - m)}$$

Denominator is zero when $m = 9$.

If $m = 9$ equation (**) becomes $0y = 3 - 2n$

(i) $n = 1.5 \Rightarrow 0y = 0$ so infinitely many solutions

(ii) $n \neq 1.5 \Rightarrow 0y \neq 0$ so no solutions

(b)

$$5P_1 - 3P_2 = 39 \quad (1)$$

$$2P_1 + 4P_2 - 3P_3 = 28 \quad (2)$$

$$3P_2 + P_3 = 29 \quad (3)$$

Use equation (1) to eliminate P_1 from equation (2):

$2 \times (1)$: $10P_1 - 6P_2 = 78$

$5 \times (2)$: $10P_1 + 20P_2 - 15P_3 = 140$

Subtract to get: $-26P_2 + 15P_3 = -62$ (4)

Now eliminate P_2 from (3) using (4):

$3 \times (4)$: $-78P_2 + 45P_3 = -186$

$26 \times (3)$: $78P_2 + 26P_3 = 754$

Add to get: $71P_3 = 568 \Rightarrow P_3 = 8$

Substitute into (3) to get: $3P_2 + 8 = 29 \Rightarrow P_2 = 7$

Substitute into (1) to get: $5P_1 - 21 = 39 \Rightarrow P_1 = 12$

10

If the commodity market is in equilibrium,

$$Y = C + I$$
$$Y = 0.6Y + 60 + (-40r + 1300)$$
$$Y = 0.6Y - 40r + 1360$$

Hence $0.4Y + 40r = 1360$ (1)

For the money market, $M_D = L_1 + L_2 = 0.2Y - 30r + 40$

If the money market is in equilibrium,

$$M_S = M_D$$
$$600 = 0.2Y - 30r + 40$$

Hence $0.2Y - 30r = 560$ (2)

Subtract $2 \times$ (2) from (1): $100r = 240 \quad \Rightarrow \quad r = 2.4$

Substituting $r = 2.4$ into either (1) or (2) gives $Y = 3160$.

Figure S1.22 (not drawn to scale) shows the IS and LM curves. The equilibrium is determined by the point of intersection (2.4, 3160).

If MPC decreases from its current level of 0.6, the effect on equation (1) is to increase the coefficient of *Y* from its current level of 0.4. The r-intercept is unchanged and the *Y*-intercept decreases. This is illustrated by the dashed line which shows that the equilibrium values of *Y* and *r* both decrease.

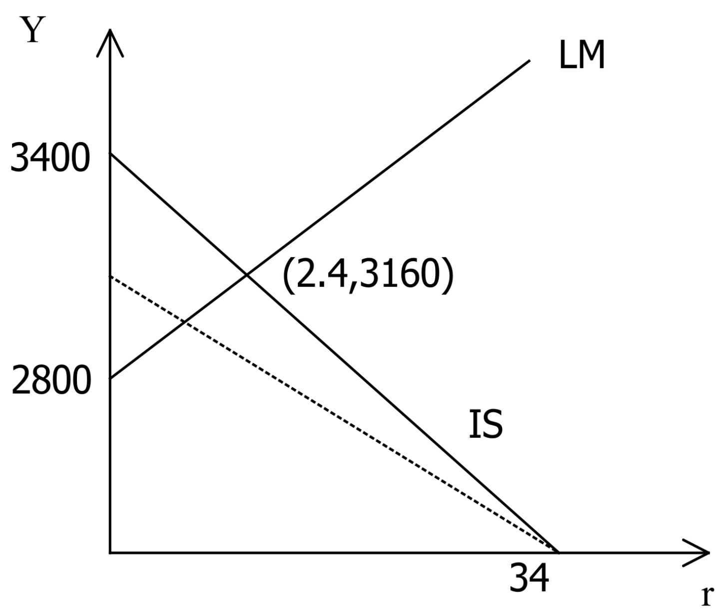


Figure S1.22

CHAPTER 2

Non-linear equations

Section 2.1 Quadratic functions

Practice Problems

1. **(a)** $x^2 - 100 = 0$

$x^2 = 100$

$x = \pm\sqrt{100}$

$x = \pm 10$

(b) $2x^2 - 8 = 0$

$2x^2 = 8$

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

(c) $x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x = \pm 1.73$(to 2 decimal places)

(d) $x^2 = 5.72 = 0$

$x^2 = 5.72$

$x = \pm\sqrt{5.72}$

$x = \pm 2.39$ (to 2 decimal places)

(e) $x^2 + 1 = 0$

$x^2 = -1$

This equation does not have a solution, because the square of a number is always positive. Try using your calculator to find $\sqrt{(-1)}$. An error message should be displayed.

(f) $3x^2 + 6.21 = 0$

$3x^2 = -6.21$

$x^2 = -2.07$

This equation does not have a solution, because it is impossible to find the square root of a negative number.

(g) $x^2 = 0$

This equation has exactly one solution, $x = 0$.

2. **(a)** $a = 2$, $b = -19$, $c = -10$

$$x = \frac{-(-19) \pm \sqrt{\left((-19)^2 - 4(2)(-10)\right)}}{2(2)}$$

$$= \frac{19 \pm \sqrt{(361 + 80)}}{4}$$

$$= \frac{19 \pm \sqrt{441}}{4} = \frac{19 \pm 21}{4}$$

This equation has two solutions:

$$x = \frac{19 + 21}{4} = 10$$

$$x = \frac{19 - 21}{4} = -\frac{1}{2}$$

(b) $a = 4$, $b = 12$, $c = 9$.

$$x = \frac{-12 \pm \sqrt{\left((12)^2 - 4(4)(9)\right)}}{2(4)}$$

$$= \frac{-12 \pm \sqrt{(144 - 144)}}{8}$$

$$= \frac{-12 \pm 0}{8}$$

This equation has one solution, $x = -3/2$.

(c) $a = 1$, $b = 1$, $c = 1$.

$$x = \frac{-1 \pm \sqrt{\left((1)^2 - 4(1)(1)\right)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{(1 - 4)}}{2}$$

$$= \frac{-1 \pm \sqrt{(-3)}}{2}$$

This equation has no solutions, because $\sqrt{(-3)}$ does not exist.

(d) We first need to collect like terms to convert

$x^2 - 3x + 10 = 2x + 4$

into the standard form

$ax^2 + bx + c = 0$

Subtracting $2x + 4$ from both sides gives

$x^2 - 5x + 6 = 0$

$$a = 1, b = -5, c = 6.$$

$$x = \frac{-(-5) \pm \sqrt{\left((5)^2 - 4(1)(6)\right)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{(25 - 24)}}{2}$$

$$= \frac{5 \pm \sqrt{1}}{2}$$

$$= \frac{5 \pm 1}{2}$$

This equation has two solutions:

$x = \frac{5+1}{2} = 3$

$x = \frac{5-1}{2} = 2$

3. **(a)**

x	−1	0	1	2	3	4
$f(x)$	21	5	−3	−3	5	21

The graph is sketched in Figure S2.1.

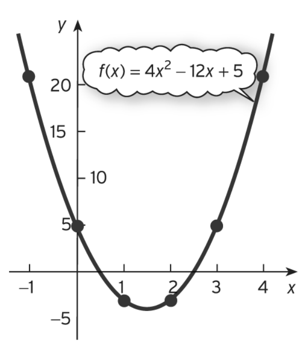


Figure S2.1

(b)

x	0	1	2	3	4	5	6
$f(x)$	–9	–4	–1	0	–1	–4	–9

The graph is sketched in Figure S2.2.

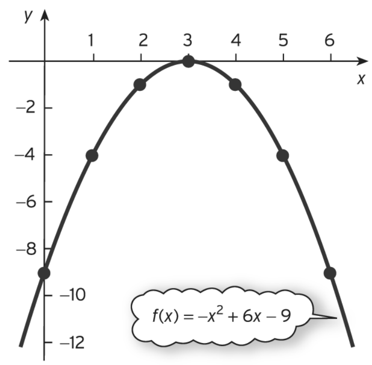


Figure S2.2

(c)

x	–2	–1	0	1	2	3	4
$f(x)$	–22	–12	–6	–4	–6	–12	–22

The graph is sketched in Figure S2.3.

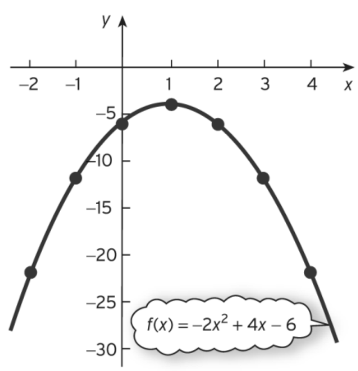


Figure S2.3

4. **(a)** ***Step 1***

The coefficient of x^2 is 2, which is positive, so the graph is U-shaped.

Step 2

The constant term is −6, so the graph crosses the vertical axis at $y = -6$.

Step 3

The quadratic equation

$$2x^2 - 11x - 6 = 0$$

has solution

$$x = \frac{-(-11) \pm \sqrt{\left((-11)^2 - 4(2)(6)\right)}}{2(2)}$$

$$= \frac{11 \pm \sqrt{(121+48)}}{4}$$

$$= \frac{11 \pm \sqrt{169}}{4}$$

$$= \frac{11 \pm 13}{4}$$

so the graph crosses the horizontal axis at $x = -1/2$ and $x = 6$.

In fact, we can use symmetry to locate the coordinates of the turning point on the curve. The x coordinate of the minimum occurs halfway between $x = -1/2$ and $x = 6$ at

$$x = \frac{1}{2}\left(-\frac{1}{2} + 6\right) = \frac{11}{4}$$

The corresponding y coordinate is

$$2\left(\frac{11}{4}\right)^2 - 11\left(\frac{11}{4}\right) - 6 = -\frac{169}{8}$$

The graph is sketched in Figure S2.4.

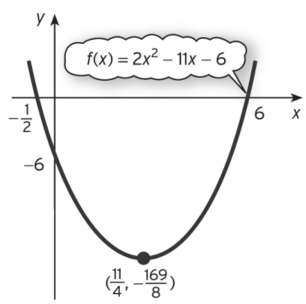


Figure S2.4

(b) ***Step 1***

The coefficient of x is 1, which is positive, so the graph is U-shaped.

Step 2

The constant term is 9, so the graph crosses the vertical axis at $y = 9$.

Step 3

The quadratic equation

$$x^2 - 6x + 9 = 0$$

has solution

$$x = \frac{-(-6) \pm \sqrt{\left((-6)^2 - 4(1)(9)\right)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{(36-36)}}{2}$$
$$= \frac{6 \pm \sqrt{0}}{2} = 3$$

so the graph crosses the x axis at $x = 3$.

The graph is sketched in Figure S2.5.

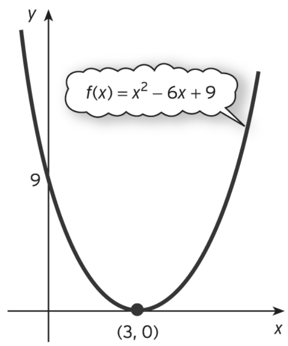
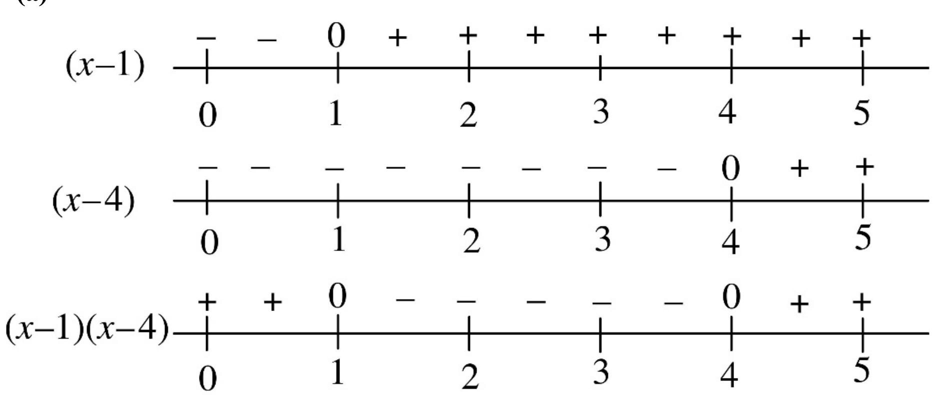


Figure S2.5

5. **(a)** Figure S2.4 shows that the graph is on or below the x-axis for values of x between $-\frac{1}{2}$ and 6 with the function taking the value 0 at the end-points so the solution is $-\frac{1}{2} \le x \le 6$.

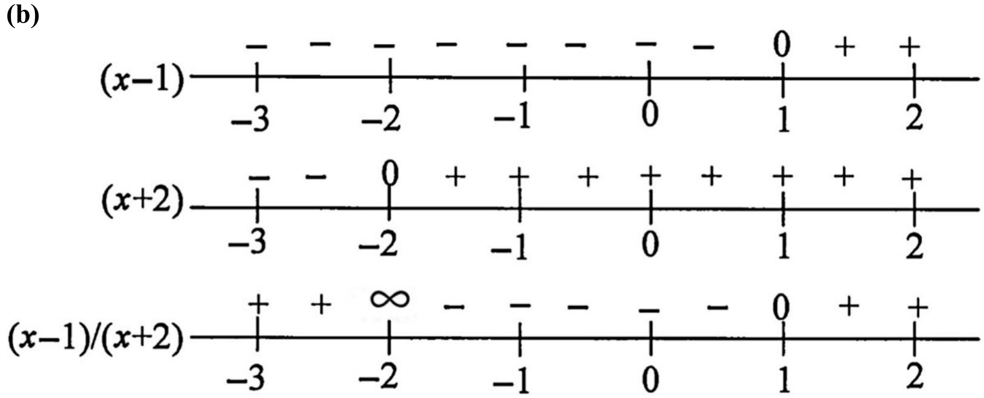
(b) Figure S2.5 shows that the graph is always on or above the x-axis taking the value 0 at the one point, $x = 3$. However, because the inequality is strict we need to exclude this point so he solution consists of all values of x except for $x = 3$.

6. **(a)**



$$1 \le x \le 4$$

(b)



$x < -2, \quad x \geq 1$

7. In equilibrium, $Q_S = Q_D = Q$, so the supply and demand equations become

$$P = 2Q^2 + 10Q + 10$$
$$P = Q^2 - 5Q + 52$$

Hence

$$2Q^2 + 10Q + 10 = -Q^2 - 5Q + 52$$
$$3Q^2 + 15Q - 42 = 0$$
$$Q^2 + 5Q - 14 = 0 \qquad \text{(dividing both sides by 3)}$$
$$Q = \frac{-5 \pm \sqrt{\left((5)^2 - 4(1)(-14)\right)}}{2(1)}$$
$$= \frac{-5 \pm \sqrt{81}}{2}$$
$$= \frac{-5 \pm 9}{2}$$

so $Q = -7$ and $Q = 2$. Ignoring the negative solution gives $Q = 2$. From the supply equation, the corresponding equilibrium price is

$$P = 2(2)^2 + 10(2) + 10 = 38$$

As a check, the demand equation gives

$$P = -(2)^2 - 5(2) + 52 = 38$$

Exercise 2.1 (p. 137)

1. **(a)** ± 9; **(b)** ± 6;

(c) $2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

(d) $(x-1)^2 = 9$

$x - 1 = \pm 3$

$x = 1 \pm 3$

$x = -2, 4$

(e) $(x+5)^2 = 16$

$x + 5 = \pm 4$

$x = -5 \pm 4$

$x = -9, -1.$

2. **(a)** 1, −3 **(b)** 1/2, −10 **(c)** 0, −5

(d) −5/3, 9/4 **(e)** 5/4, 5

3. **(a)** $x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{5 \pm \sqrt{25 - 8}}{2}$

$x = \dfrac{5 \pm \sqrt{17}}{2}$

$x = 0.44, 4.56$

(b) $x = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{2(2)}$

$x = \dfrac{-5 \pm \sqrt{25 - 8}}{4}$

$x = \dfrac{-5 \pm \sqrt{17}}{4}$

$x = -2.28, -0.22$

(c) $x = \frac{-7 \pm \sqrt{7^2 - 4(-3)(2)}}{2(-3)}$

$x = \frac{-7 \pm \sqrt{49 + 24}}{-6}$

$x = \frac{-7 \pm \sqrt{73}}{-6}$

$x = -0.26, 2.59$

(d) $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{3 \pm \sqrt{9 + 4}}{2}$

$x = \frac{3 \pm \sqrt{13}}{2}$

$x = -0.30, 3.30$

(e) $x = \frac{-8 \pm \sqrt{8^2 - 4(2)(8)}}{2(2)}$

$x = \frac{-8 \pm \sqrt{64 - 64}}{4}$

$x = \frac{-8 \pm \sqrt{0}}{4}$

$\mathrm{x} = -2$

(f) $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 - 40}}{2}$

$x = \frac{6 \pm \sqrt{-4}}{2}$

It is impossible to find the square root of a negative number so there are no solutions.

4. **(a)** $x^2 - 16 = 0$

$x^2 = 16$

$x = -4, 4$

(b) 0, 100

(c) $x = \dfrac{-22 \pm \sqrt{22^2 - 4(-1)(-85)}}{2(-1)}$

$x = \dfrac{-22 \pm \sqrt{484 - 340}}{-2}$

$x = \dfrac{-22 \pm \sqrt{144}}{-2}$

$x = 5, 17$

(d) $x = \dfrac{-18 \pm \sqrt{18^2 - 4(1)(81)}}{2(1)}$

$x = \dfrac{18 \pm \sqrt{324 - 324}}{2}$

$x = \dfrac{18 \pm \sqrt{0}}{2}$

$x = 9$

(e) $x = \dfrac{-4 \pm \sqrt{4^2 - 4(2)(3)}}{2(2)}$

$x = \dfrac{-4 \pm \sqrt{16 - 24}}{4}$

$x = \dfrac{-4 \pm \sqrt{-8}}{4}$

It is impossible to find the square root of a negative number so there is no solution.

5. The graphs are sketched in Figure S2.6.

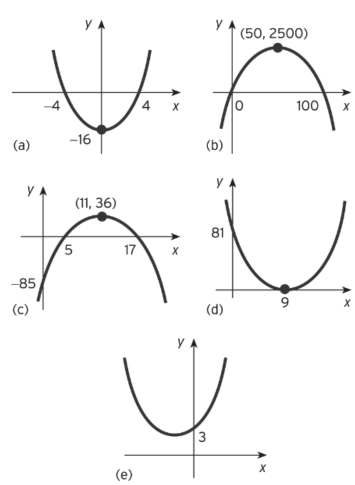


Figure S2.6

6. **(a)** $x \le -4,\ \ x \ge 4$ **(b)** $0 < x < 100$; **(c)** $5 \le x \le 17$;

(d) $x = 9$ **(e)** all values of x

7. **(a)**

T	23	24	25	26	27	28	29	30
Q	56.166	56.304	56.35	56.304	56.166	55.936	55.614	55.2

The graph is sketched in Figure S2.7.

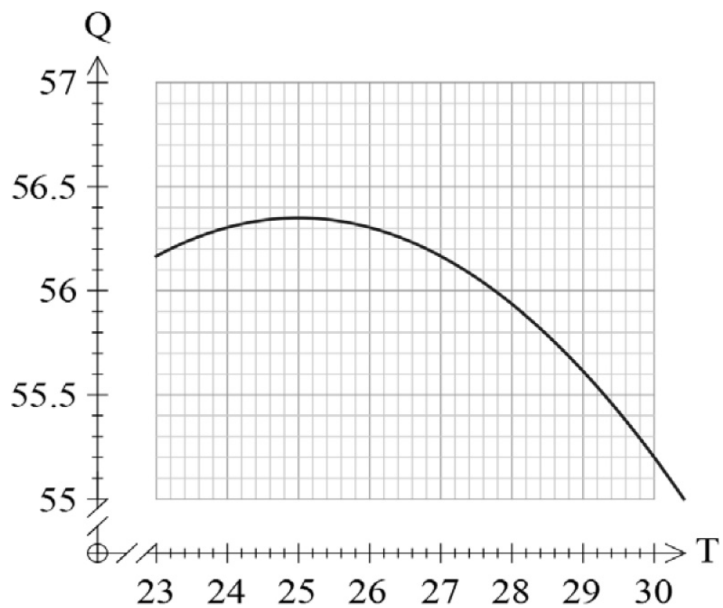
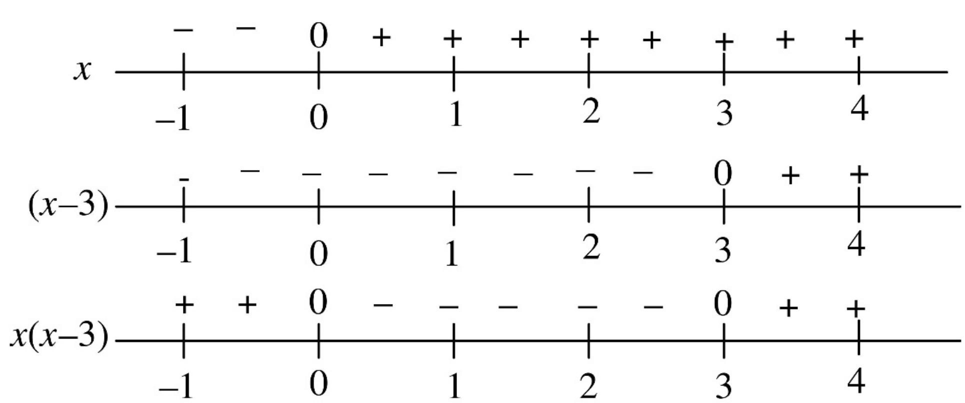


Figure S2.7

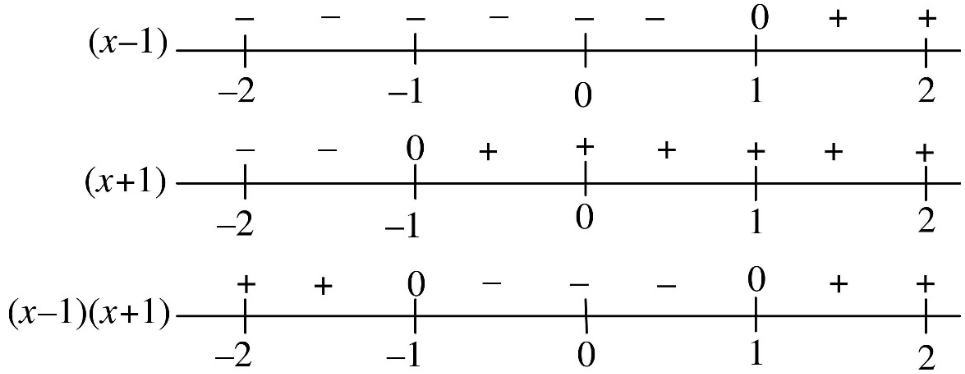
(b) The production level is a maximum at $25°C$ so as temperature increases output will fall.

8. **(a)**



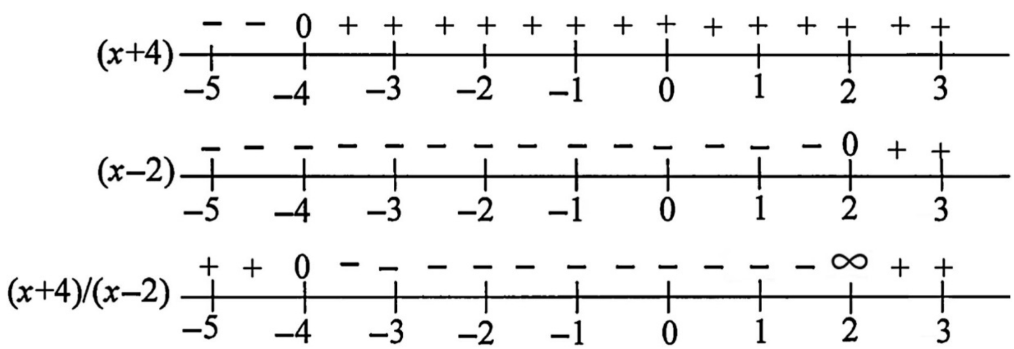
$x < 0,\ x > 3$

(b)



$x \le -1$, $x \ge 1$

(c)



$-4 < x < 2$

9. In equilibrium, $Q_S = Q_D = Q$ so the supply and demand equations become:

$P = Q^2 + 2Q + 12$ and $P = -Q^2 - 4Q + 68$

Hence

$Q^2 + 2Q + 12 = -Q^2 - 4Q + 68$

$2Q^2 + 6Q - 56 = 0$

$Q^2 + 3Q - 28 = 0$ (divide by 2 to make the use of the quadratic formula slightly easier)

$$Q = \frac{-3 \pm \sqrt{3^2 - 4(1)(-28)}}{2(1)}$$

$$Q = \frac{-3 \pm \sqrt{9 + 112}}{2}$$

$$Q = \frac{-3 \pm \sqrt{121}}{2}$$

$Q = 4$ or -7

Ignoring the negative solution gives $Q = 4$ and substituting this value into the supply equation gives

$$P = 4^2 + 2 \times 4 + 12 = 36$$

10. In equilibrium, $Q_S = Q_D = Q$ so the supply and demand equations become:

$P = Q^2 + 2Q + 7$ and $P = -Q + 25$

Hence

$$Q^2 + 2Q + 7 = -Q + 25$$

$$Q^2 + 3Q - 18 = 0$$

$$Q = \frac{-3 \pm \sqrt{3^2 - 4(1)(-18)}}{2(1)}$$

$$Q = \frac{-3 \pm \sqrt{9 + 72}}{2}$$

$$Q = \frac{-3 \pm \sqrt{81}}{2}$$

$Q = 3$ or -6

Ignoring the negative solution gives $Q = 3$ and substituting this value into the demand equation gives

$$P = -3 + 25 = 22$$

11. (a) $30 \times 7 + 10(7 - 0.03 \times 10) = \$277$

(b) The first 30 shirts cost \$210 so the remainder costs $504.25 - 210 = \$294.25$

$$x(7 - 0.03x) = 294.25$$

$$7x - 0.03x^2 = 294.25$$

$$0.03x^2 - 7x + 294.25 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 0.03 \times 294.25}}{2 \times 0.03} = \frac{7 \pm \sqrt{13.69}}{0.06} = 55,\ 178\frac{1}{3}$$

But $x \le 100$ so 85 shirts.

Section 2.2 Revenue, cost and profit

Practice Problems

1. TR = PQ = (1000 – Q)Q = 1000Q – Q^2

Step 1

The coefficient of Q^2 is negative, so the graph has an inverted U shape.

Step 2

The constant term is zero, so the graph crosses the vertical axis at the origin.

Step 3

From the factorisation

TR = (1000 – Q)Q

the graph crosses the horizontal axis at Q = 0 and Q = 1000.

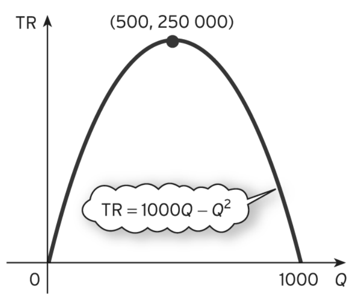


Figure S2.8

The graph is sketched in Figure S2.8. By symmetry, the parabola reaches its maximum halfway between 0 and 1000 at Q = 500. The corresponding value of TR is

TR = 1000(500) − (500)2 = 250 000

From the demand equation, when Q = 500,

$$P = 1000 - 500 = 500$$

2. $\text{TC} = 100 + 2Q$

$$\text{AC} = \frac{100+2Q}{Q} = \frac{100}{Q} + 2$$

The graph of the total cost function is sketched in Figure S2.9.

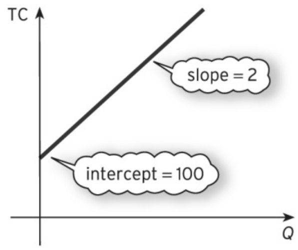


Figure S2.9

One possible table of function values for the average cost function is

Q	10	25	50	100	200
AC	12	6	4	3	2.5

The graph of the average cost function is sketched in Figure S2.10.

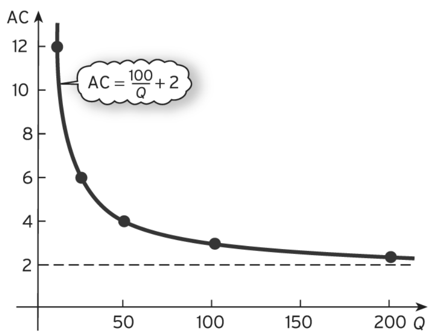


Figure S2.10

In fact, it is not necessary to plot the tabulated values if all that is required is a rough sketch. It is obvious that if a very small number is put into the AC function then a very large number is produced because of the term $100/Q$. For example, when $Q = 0.1$

$$\text{AC} = \frac{100}{0.1} + 2 = 1002$$

It should also be apparent that if a very large number is put into the average cost function then the term $100/Q$ is insignificant, so AC is approximately 2. For example, when $Q = 10\,000$

$$\text{AC} = \frac{100}{10000} + 2 = 2.01$$

The graph of AC therefore 'blows up' near $Q = 0$ but settles down to a value just greater than 2 for large Q. Consequently, the general shape of the graph shown in Figure S2.10 is to be expected.

3. TC = 25 + 2Q

TR = PQ = (20 – Q)Q = 20Q – Q^2

Hence

$$\begin{aligned}\pi &= \text{TR} - \text{TC}\\ &= (20Q - Q^2) - (25 + 2Q)\\ &= 20Q - Q^2 - 25 - 2Q\\ &= -Q^2 + 18Q - 25\end{aligned}$$

Step 1

The coefficient of Q^2 is negative, so the graph has an inverted U shape.

Step 2

The constant term is −25, so the graph crosses the vertical axis at −25.

Step 3

The quadratic equation

$$-Q^2 + 18Q - 25 = 0$$

has solutions

$$\begin{aligned}Q &= \frac{-18 \pm \sqrt{(324 - 100)}}{-2}\\ &= \frac{-18 \pm 14.97}{-2}\end{aligned}$$

so the graph crosses the horizontal axis at Q = 1.52 and Q = 16.48.

The graph of the profit function is sketched in Figure S2.11.

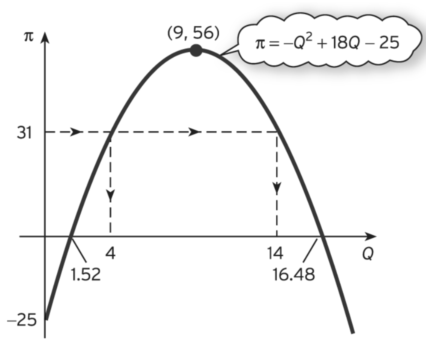


Figure S2.11

(a) If π = 31, then we need to solve

$$-Q^2 + 18Q - 25 = 31$$

that is,

$$-Q^2 + 18Q - 56 = 0$$

$$Q = \frac{-18 \pm \sqrt{(324 - 224)}}{-2} = \frac{18 \pm 10}{-2}$$

so $Q = 4$ and $Q = 14$.

These values can also be found by drawing a horizontal line π = 31 and then reading off the corresponding values of Q from the horizontal axis as shown on Figure S2.11.

(b) By symmetry, the parabola reaches its maximum halfway between 1.52 and 16.48: that is, at

$$Q = \tfrac{1}{2}(1.52 + 16.48) = 9$$

The corresponding profit is given by

$$\pi = -(9)^2 + 18(9) - 25 = 56$$

Exercise 2.2 (p. 148)

1. **(a)** $P = 80 - 3 \times 10 = 50$; TR = $50 \times 10 = 500$

(b) TC = $100 + 5 \times 10 = 150$

(c) $\pi = 500 - 150 = 350$

2. **(a)** TR = $PQ = 4Q$

(b) TR = $PQ = \frac{7}{Q} \times Q = 7$

(c) TR = $PQ = (10 - 4Q)Q = 10Q - 4Q^2$.

The graphs are sketched in Figures S2.12, S2.13 and S2.14.

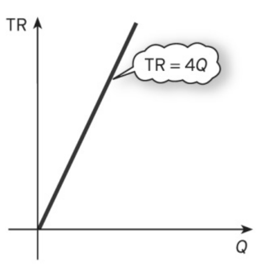


Figure S2.12