Chapter 0: Algebraic Concepts

Exercises 0.1

1. $12 \in \{1, 2, 3, 4, ...\}$

2. $5 \notin \{x: x \text{ is a natural number greater than } 5\}$

3. $6 \notin \{1, 2, 3, 4, 5\}$

4. $3 \notin \varnothing$

5. $\{1, 2, 3, 4, 5, 6, 7\}$

6. $\{7, 8, 9\}$

7. $\{x: x$ is a natural number greater than 2 and less than $8\}$

8. $\{x: x$ is a natural number greater than $6\}$

9. $\varnothing \subseteq A$ since $\varnothing$ is a subset of every set. $A \subseteq B$ since every element of A is an element of B. $B \subseteq B$ since a set is always a subset of itself.

10. $\varnothing \subseteq A$ since $\varnothing$ is a subset of every set. $A \subseteq B$ since every element of A is an element of B. $B \subseteq B$ since a set is always a subset of itself.

11. No. $c \in A$ but $c \notin B$.

12. No. $12 \in A$ but $12 \notin B$.

13. $D \subseteq C$ since every element of D is an element of C.

14. $E \subseteq F$ since every element of E is an element of F.

15. $A \subseteq B$ and $B \subseteq A$. (Also $A = B$.)

16. $D \subseteq F$ and $F \subseteq D$. (Also $D = F$.)

17. Yes. $A \subseteq B$ and $B \subseteq A$. Thus, $A = B$.

18. $A \neq D$

19. No. $D \neq E$ because $4 \in E$ and $4 \notin D$.

20. $F = G$

21. A and B are disjoint since they have no elements in common. B and D are disjoint since they have no elements in common. C and D are disjoint.

22. $\varnothing$

23. $A \cap B = \{4, 6\}$ since 4 and 6 are elements of each set.

24. $A \cap B = \{a, d, e\}$ since a, d, and e are elements of each set.

25. $A \cap B = \varnothing$ since they have no common elements.

26. $A \cap B = \{3\}$

27. $A \cup B = \{1, 2, 3, 4, 5\}$

28. $A \cup B = \{a, b, c, d, e, i, o, u\}$

29. $A \cup B = \{1, 2, 3, 4\}$ or $A \cup B = B$.

30. $A \cup B = \{x: x$ is a natural number not equal to $5\}$

For problems 31 - 42, we have $U = \{1, 2, 3, \ldots, 9, 10\}$.

31. $A' = \{4, 6, 9, 10\}$ since these are the only elements in U that are not elements of A.

32. $B' = \{1, 2, 5, 6, 7, 9\}$
since these are the only elements in U that are not elements of B.

33. $B' = \{1, 2, 5, 6, 7, 9\}$
$A \cap B' = \{1, 2, 5, 7\}$

34. $A' = \{4, 6, 9, 10\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$A' \cap B' = \{6, 9\}$

35. $A \cup B = \{1, 2, 3, 4, 5, 7, 8, 10\}$
$(A \cup B)' = \{6, 9\}$

36. $A \cap B = \{3, 8\}$
$(A \cap B)' = \{1, 2, 4, 5, 6, 7, 9, 10\}$

37. $A' = \{4, 6, 9, 10\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$A' \cup B' = \{1, 2, 4, 5, 6, 7, 9, 10\}$

38. $A' = \{4, 6, 9, 10\}$
$B = \{3, 4, 8, 10\}$
$A' \cup B = \{3, 4, 6, 8, 9, 10\}$
$(A' \cup B)' = \{1, 2, 5, 7\}$

39. $B' = \{1, 2, 5, 6, 7, 9\}$
$C' = \{1, 3, 5, 7, 9\}$
$A \cap B' = \{1, 2, 3, 5, 7, 8\} \cap \{1, 2, 5, 6, 7, 9\}$
$= \{1, 2, 5, 7\}$
$(A \cap B') \cup C' = \{1, 2, 3, 5, 7, 9\}$

40. $A = \{1, 3, 5, 8, 7, 2\}$
$B' = \{1, 2, 5, 6, 7, 9\}$
$C' = \{1, 3, 5, 7, 9\}$
$B' \cup C' = \{1, 2, 3, 5, 6, 7, 9\}$
$A \cap (B' \cup C') = \{1, 2, 3, 5, 7\}$

41. $B' = \{1, 2, 5, 6, 7, 9\}$
$A \cap B' = \{1, 2, 3, 5, 7, 8\} \cap \{1, 2, 5, 6, 7, 9\}$
$= \{1, 2, 5, 7\}$
$(A \cap B')' \cap C = \{3, 4, 6, 8, 9, 10\} \cap \{2, 4, 6, 8, 10\}$
$= \{4, 6, 8, 10\}$

42. $B \cup C = \{2, 3, 4, 6, 8, 10\}$
$A \cap (B \cup C) = \{2, 3, 8\}$

For problems 43 - 46, we have $U = \{1, 2, 3, \ldots, 8, 9\}$.

43. $A - B = \{1, 3, 7, 9\} - \{3, 5, 8, 9\} = \{1, 7\}$

44. $A - B = \{1, 2, 3, 6, 9\} - \{1, 4, 5, 6, 7\} = \{2, 3, 9\}$

45. $A - B = \{2, 1, 5\} - \{1, 2, 3, 4, 5, 6\} = \varnothing$ or $\{\ \}$

46. $A - B = \{1, 2, 3, 4, 5\} - \{7, 8, 9\} = \{1, 2, 3, 4, 5\}$

47. a. $L = \{2000, 2001, 2004, 2005, 2006, 2007, 2010, 2011, 2012\}$
$H = \{2000, 2001, 2006, 2007, 2008, 2010, 2011, 2012\}$
$C = \{2001, 2002, 2003, 2008, 2009\}$
b. no
c. C' is the set of all years when the percentage change from low to high was 35% or less.
d. $H' = \{2002, 2003, 2004, 2005, 2009\}$
$C' = \{2000, 2004, 2005, 2006, 2007, 2010, 2011, 2012\}$
$H' \cup C' = \{2000, 2002, 2003, 2004, 2005, 2006, 2007, 2009, 2010, 2011, 2012\}$. $H' \cup C'$ is the set of years when the high was less than or equal to 11,000 or the percent change was less than or equal to 35%.
e. $L' = \{2002, 2003, 2008, 2009\}$
$L' \cap C = \{2002, 2003, 2008, 2009\}$.
$L' \cap C$ is the set of years when the low was less than or equal to 8,000 and the percent change was more than 35%.

48. a. $A = \{\text{O, L, P}\}$
$B = \{\text{L, P}\}$
$C = \{\text{O, M, P}\}$
b. $B \subseteq A$
c. $A \cap C = \{\text{O, P}\}$; this is the set of cities with at least 2,000,000 jobs in 2000 or 2025 and projected annual growth rates of at least 2.5%.
d. B' is the set of cities with fewer than 1,500,000 jobs in 2000.

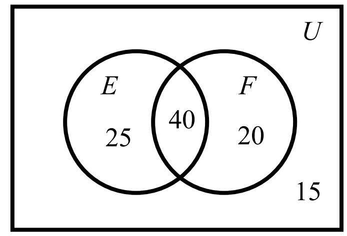
49. a. From the table, there are 100 white Republicans and 30 non-white Republicans who favor national health care, for a total of 130.
b. From the table, there are 350 + 40 Republicans, and 250 + 200 Democrats who favor national health care, for a total of 840.
c. From the table, there are 350 white Republicans, and 150 white Democrats and 20 non-whites who oppose national health care, for a total of 520.

50. **a.** From the table, 250 white Republicans and 150 white Democrats oppose national health care, for a total of 400.

b. From the table, there are 750 whites and there are 20 non-whites who oppose national health care. The total of this group is 770.

c. From the table, there are 200 non-white Democrats who favor national health care.

51. **a.** The key to solving this problem is to work from "the inside out". There are 40 aides in $E \cap F$. This leaves 65 – 40 = 25 aides who speak English but do not speak French. Also we have 60 – 40 = 20 aides who speak French but do not speak English. Thus there are 40 + 25 + 20 = 85 aides who speak English or French. This means there are 15 aides who do not speak English or French.



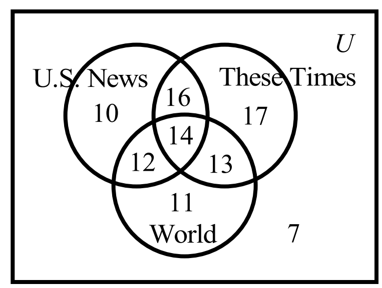
b. From the Venn diagram $E \cap F$ has 40 aides.

c. From the Venn diagram $E \cup F$ has 85 aides.

d. From the Venn diagram $E \cap F'$ has 25 aides.

52. **a.** There are 14 advertisers in the intersection of the sets. Since 30 advertised in *These Times* and *U.S. News* and we already have 14 in the center, 16 advertised in *These Times* and *U.S. News* and not in *World*. Since 26 advertised in *World* and *U.S. News* and we already have 14 in the center, 12 advertised in *World* and *U.S. News* and not in *These Times*. Since 27 advertised in *World* and *These Times* and we already have 14 in the middle, 13 advertised in *World* and *These Times* and not in *U.S. News*. 60 advertised in *These Times* and we have already accounted for 43, so 17 advertised in *These Times* only. 52 advertised in *U.S. News* and we have already accounted for 42, so 10 advertised in *U.S. News* only. 50 advertised in *World* and we have already accounted for 39, so 11 advertised in *World* only.

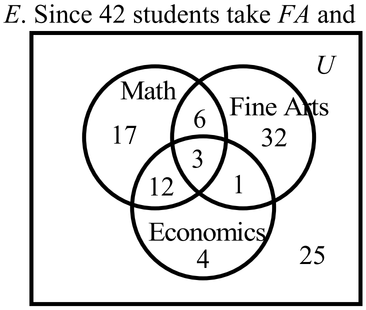
b. In the union of the 3 publications we have 10 + 16 + 17 + 14 + 12 + 13 + 11 = 93 advertisers. Thus, there are 100 – 93 = 7 who advertised in none of these publications.



c. There are 17 advertisers in the *These Times* circle that are not in an intersection.

d. In the union of *U.S. News* and *These Times* we have 10 + 12 + 16 + 14 + 17 + 13 = 82 advertisers.

53. Since 12 students take *M* and *E* but not *FA*, and 15 take *M* and *E*, 3 take all three classes. Since 9 students take *M* and *FA* and we have already counted 3, there are 6 taking *M* and *FA* which are not taking *E*. Since 4 students take *E* and *FA* and we have already counted 3, there is only 1 taking *E* and *FA* but not taking *M* also. Since 20 students take *E* and we already have 16 enrolled in *E*, this leaves 4 taking only *E*. Since 42 students take *FA* and we already have 10 enrolled in *FA*, this leaves 32 taking only *FA*. Since 38 students take *M* and we already have 21 enrolled in *M*, this leaves 17 taking only *M*.

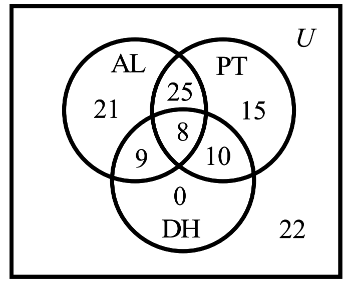


a. In the union of the 3 courses we have 17 + 12 + 3 + 6 + 32 + 1 + 4 = 75 students enrolled. Thus, there are 100 – 75 = 25 students who are not enrolled in any of these courses.

b. In $M \cup E$ we have 17 + 12 + 3 + 6 + 1 + 4 = 43 enrolled.

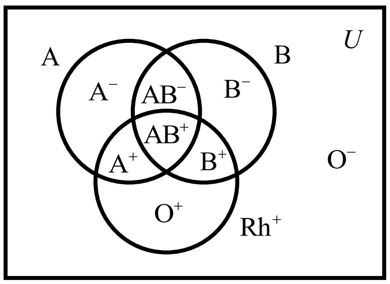
c. We have 17 + 32 + 4 = 53 students enrolled in exactly one of the courses.

54. Start by filling in the parts of the diagram for AL, since we have more information about it. 21 liked AL only. Since 30 liked AL but not PT, 9 liked AL or PT exclusively. 25 liked PT or AL but not DH, and 63 liked AL. That leaves 63 – (21 + 25 + 9) = 8 in the intersection of all 3. Since 18 liked PT and DH, only 10 liked PT and DH but not AL. Since 27 liked DH, 27 – (9 + 8 + 10) = 0 liked DH only. And since 58 liked PT, 58 – (25 + 8 + 10) = 15 liked PT only.



a. The number of students that liked PT or DH is 25 + 15 + 9 + 8 + 10 + 0 = 67.
b. The number that liked all three is 8.
c. The number that liked only DH is 0.

55. **a.** and **b.**



c. A^+ : 34%; B^+ : 9%; O^+ : 38%; AB^+ : 3%; O^- : 7%; A^- : 6%; B^- : 2%; AB^- : 1%

Exercises 0.2

1. **a.** Note that $-\frac{\pi}{10} = \pi \cdot \left(-\frac{1}{10}\right)$, where π is irrational and $-\frac{1}{10}$ is rational. The product of a rational number other than 0 and an irrational number is an irrational number.
b. –9 is rational and an integer.
c. $\frac{9}{3} = \frac{3}{1} = 3$. This is a natural number, an integer, and a rational number.
d. Division by zero is meaningless.

2. **a.** $\frac{0}{6} = 0$ is rational and an integer.
b. rational
c. rational
d. rational

3. **a.** Commutative
b. Distributive
c. Associative
d. Additive Identity

4. **a.** Multiplicative Identity
b. Additive Inverse
c. Multiplicative Inverse
d. Commutative

5. $-6 < 0$

6. $2 > -20$

7. $-14 < -3$

8. $\pi > 3.14$

9. $0.333 < \frac{1}{3}\left(\frac{1}{3} = 0.3333\cdots\right)$

10. $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

11. $|-3| + |5| > |-3+5|$

12. $|-9-3| = |-9| + |3|$ (12 = 12)

13. $-3^2 + 10 \cdot 2 = -3^2 + 20 = -9 + 20 = 11$

14. $(-3)^2 + 10 \cdot 2 = 9 + 20 = 29$

15. $\dfrac{4+2^2}{2}=\dfrac{4+4}{2}=\dfrac{8}{2}=4$

16. $\dfrac{(4+2)^2}{2}=\dfrac{6^2}{2}=\dfrac{36}{2}=18$

17. $\dfrac{16-(-4)}{8-(-2)}=\dfrac{16+4}{8+2}=\dfrac{20}{10}=2$

18. $\dfrac{(-5)(-3)-(-2)(3)}{-9+2}=\dfrac{15-(-6)}{-7}=\dfrac{15+6}{-7}$
$=\dfrac{21}{-7}=-3$

19. $\dfrac{|5-2|-|-7|}{|5-2|}=\dfrac{|3|-|-7|}{|3|}=\dfrac{3-7}{3}=-\dfrac{4}{3}$

20. $\dfrac{|3-|4-11||}{-|5^2-3^2|}=\dfrac{|3-|-7||}{-|25-9|}$
$=\dfrac{|3-7|}{-|16|}$
$=\dfrac{|-4|}{-16}$
$=\dfrac{4}{-16}=-\dfrac{1}{4}$

21. $\dfrac{(-3)^2-2\cdot 3+6}{4-2^2+3}=\dfrac{9-6+6}{4-4+3}=\dfrac{9}{3}=3$

22. $\dfrac{6^2-4(-3)(-2)}{6-6^2\div 4}=\dfrac{36-(-12)(-2)}{6-36\div 4}$
$=\dfrac{36-24}{6-9}$
$=\dfrac{12}{-3}$
$=-4$

23. $\dfrac{-4^2+5-2\cdot 3}{5-4^2}=\dfrac{-16+5-6}{5-16}=\dfrac{-17}{-11}=\dfrac{17}{11}$

24. $\dfrac{3-2(5-2)}{(-2)^2-2^2+3}=\dfrac{3-2\cdot 3}{4-4+3}=\dfrac{-3}{3}=-1$

25. The entire line

26. The interval notation corresponding to $x\ge 0$ is $[0,\infty)$.

27. (1, 3]; half-open interval

28. [–4, 3]; closed interval

29. (2, 10); open interval

30. $[2,\infty)$; half-open interval

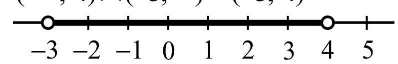
31. $-3\le x<5$

32. $x>-2$

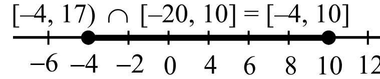
33. $x>4$

34. $0\le x<5$

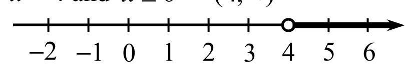
35. $(-\infty,4)\cap(-3,\infty)=(-3,4)$



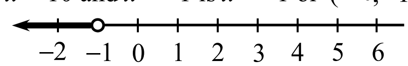
36. $[-4,17)\cap[-20,10]=[-4,10]$



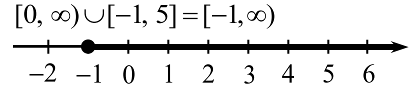
37. $x>4$ and $x\ge 0=(4,\infty)$



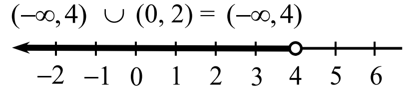
38. $x<10$ and $x<-1$ is $x<-1$ or $(-\infty,-1)$.



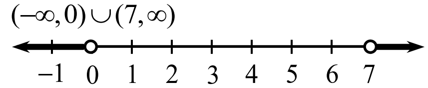
39. $[0,\infty)\cup[-1,5]=[-1,\infty)$



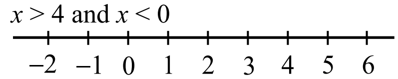
40. $(-\infty,4)\cup(0,2)=(-\infty,4)$



41. $(-\infty,0)\cup(7,\infty)$



42. $x>4$ and $x<0$



The intersection is the empty set.

43. –0.000038585

44. 0.404787025

45. 9122.387471

46. 11.80591621

47. $\dfrac{2500}{[(1.1^6)-1]}=\dfrac{2500}{0.771561}=3240.184509$

48. 1591.712652

49. **a.** $\$300.00+\$788.91=\$1088.91$
b. Federal withholding
= 0.25(1088.91 – 54.45) = $258.62
c. Retirement: 0.05(1088.91) = $54.45
State tax = Retirement = $54.45
Local tax = 0.01(1088.91) = $10.89
Federal tax = $258.62 (from **b.** above)
Social Security and Medicare tax
= 0.0765(1088.91) = $83.30
Total Withholding = $461.71
Take-home = 1088.91 – 461.71 = $627.20

50. **a.** $t=2010-2000=10$
b. $E=5.03(10)^2+100(10)+1380$
$=\$2883$ billion
c. $t=2015-2000=15$
$E=50.3(15)^2+100(15)+1380$
$=\$4011.75$ billion

51. **a.** Equation (1) is more accurate.
Equation (1) gives
$y=0.207(13)-0.000370$
$=2.69$ billion

Equation (2) gives
$y=0.00454(13)^2+0.126(13)+0.271$
$=2.68$ billion

b. For 2018, Equation (1) gives
$y=0.207(18)-0.000370$
$=3.73$ billion

For 2018, Equation (2) gives
$y=0.00454(18)^2+0.126(18)+0.271$
$=4.01$ billion

52. **a.** $H=2.31(10.5)+31.26=55.515$ inches
Upper: $1.05(55.515)=58.29$ inches
Lower: $0.95(55.515)=52.74$ inches
$52.74\le H\le 58.29$
b. $H=2.31(5.75)+31.26=44.5425$ inches
Upper: $1.05(44.5425)=46.77$ inches
Lower: $0.95(44.5425)=42.32$ inches
$42.32\le H\le 46.77$

53. **a.** $\$82,401\le I\le 171,850;$
$\$171,851\le I\le \$373,650;$
$I>\$373,650$
b. $T=\$4681.25$ for $I=\$34,000$
$T=\$16,781.25$ for $I=\$82,400$
c. $[4681.25,\ 16,781.25]$

Exercises 0.3

1. $(-4)^4=(-4)(-4)(-4)(-4)=256$

2. $-5^3=-1\cdot5\cdot5\cdot5=-125$

3. $-2^6=-1\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2=-64$

4. $(-2)^5=(-2)(-2)(-2)(-2)(-2)=-32$

5. $3^{-2}=\dfrac{1}{3^2}=\dfrac{1}{9}$

6. $6^{-1}=\dfrac{1}{6}$

7. $-\left(\dfrac{3}{2}\right)^2=(-1)\left(\dfrac{3}{2}\right)\left(\dfrac{3}{2}\right)=-\dfrac{9}{4}$

8. $\left(\dfrac{2}{3}\right)^3=\dfrac{2^3}{3^3}=\dfrac{8}{27}$

9. $1.2\ \boxed{y^x}\ 4\ \boxed{=}\ 2.0736$

10. $-3.7\ \boxed{y^x}\ 3\ \boxed{=}\ -50.653$

11. $1.5\ \boxed{y^x}\ -5\ \boxed{=}\ 0.1316872428$

12. $-0.8\ \boxed{y^x}\ -9\ \boxed{=}\ -7.450580597$

13. $6^5 \cdot 6^3 = 6^{5+3} = 6^8$

14. $8^4 \cdot 8^2 \cdot 8 = 8^{4+2+1} = 8^7$

15. $\frac{10^8}{10^9} = 10^{8-9} = 10^{-1} = \frac{1}{10}$

16. $\frac{7^8}{7^3} = 7^{8-3} = 7^5$

17. $\frac{9^4 \cdot 9^{-7}}{9^{-3}} = \frac{9^{4+(-7)}}{9^{-3}} = \frac{9^{-3}}{9^{-3}} = 9^{-3-(-3)} = 9^0 = 1$

18. $\frac{5^4}{\left(5^{-2} \cdot 5^3\right)} = \frac{5^4}{5^{-2+3}} = \frac{5^4}{5^1} = 5^{4-1} = 5^3$

19. $\left(3^3\right)^3 = 3^{3 \cdot 3} = 3^9$

20. $\left(2^{-3}\right)^{-2} = 2^{(-3)\cdot(-2)} = 2^6$

21. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

22. $\left(\frac{-2}{5}\right)^{-4} = \left(\frac{5}{-2}\right)^4 = \left(-\frac{5}{2}\right)^4$

23. $-x^{-3} = -1 \cdot x^{-3} = -1 \cdot \frac{1}{x^3} = -\frac{1}{x^3}$

24. $x^{-4} = \frac{1}{x^4}$

25. $xy^{-2}z^0 = x \cdot \frac{1}{y^2} \cdot 1 = \frac{x}{y^2}$

26. $4^{-1}x^0y^{-2} = \frac{1}{4} \cdot 1 \cdot \frac{1}{y^2} = \frac{1}{4y^2}$

27. $x^3 \cdot x^4 = x^{3+4} = x^7$

28. $a^5 \cdot a = a^{5+1} = a^6$

29. $x^{-5} \cdot x^3 = x^{-5+3} = x^{-2} = \frac{1}{x^2}$

30. $y^{-5} \cdot y^{-2} = y^{-5+(-2)} = y^{-7} = \frac{1}{y^7}$

31. $\frac{x^8}{x^4} = x^{8-4} = x^4$

32. $\frac{a^5}{a^{-1}} = a^{5-(-1)} = a^{5+1} = a^6$

33. $\frac{y^5}{y^{-7}} = y^{5-(-7)} = y^{12}$

34. $\frac{y^{-3}}{y^{-4}} = y^{-3-(-4)} = y^{-3+4} = y^1 = y$

35. $\left(x^4\right)^3 = x^{3 \cdot 4} = x^{12}$

36. $(y^3)^{-2} = y^{3\cdot(-2)} = y^{-6} = \frac{1}{y^6}$

37. $(xy)^2 = x^2y^2$

38. $(2m)^3 = 2^3m^3 = 8m^3$

39. $\left(\frac{2}{x^5}\right)^4 = \frac{2^4}{\left(x^5\right)^4} = \frac{16}{x^{5\cdot 4}} = \frac{16}{x^{20}}$

40. $\left(\frac{8}{a^3}\right)^3 = \frac{8^3}{(a^3)^3} = \frac{512}{a^{3\cdot 3}} = \frac{512}{a^9}$

41. $\left(2x^{-2}y\right)^{-4} = 2^{-4}x^8y^{-4} = \frac{x^8}{16y^4}$

42.
$$\begin{aligned}\left(-32x^5\right)^{-3} &= (-32)^{-3}\left(x^5\right)^{-3} \\ &= \frac{1}{(-32)^3}x^{5(-3)} \\ &= \frac{1}{-32768} \cdot x^{-15} \\ &= -\frac{1}{32768x^{15}}\end{aligned}$$

43. $\left(-8a^{-3}b^2\right)\left(2a^5b^{-4}\right) = -16a^{-3+5}b^{2-4}$
$$= -16a^2b^{-2}$$
$$= -\frac{16a^2}{b^2}$$

44. $\left(-3m^2y^{-1}\right)\left(2m^{-3}y^{-1}\right) = -6m^{2+(-3)}y^{-1+(-1)}$
$$= -6m^{-1}y^{-2}$$
$$= -6\left(\frac{1}{m}\right)\left(\frac{1}{y^2}\right)$$
$$= \frac{-6}{my^2}$$

45. $\left(2x^{-2}\right)\div\left(x^{-1}y^2\right) = \frac{2}{x^2}\div\frac{y^2}{x} = \frac{2}{x^2}\cdot\frac{x}{y^2} = \frac{2}{xy^2}$

46. $\left(-8a^{-3}b^2c\right)\div\left(2a^5b^4\right) = \frac{-8a^{-3}b^2c}{2a^5b^4}$
$$= \frac{-8}{2}\cdot\frac{a^{-3}}{a^5}\cdot\frac{b^2}{b^4}\cdot c$$
$$= \frac{-4c}{a^8b^2}$$

47. $\left(\frac{x^3}{y^{-2}}\right)^{-3} = \frac{x^{-9}}{y^6} = \frac{1}{x^9}\cdot\frac{1}{y^6} = \frac{1}{x^9y^6}$

48. $\left(\frac{x^{-2}}{y}\right)^{-3} = \frac{(x^{-2})^{-3}}{y^{-3}} = \frac{x^{(-2)(-3)}}{y^{-3}} = \frac{x^6}{y^{-3}} = x^6\cdot\frac{y^3}{1}$
$$= x^6y^3$$

49. $\left(\frac{a^{-2}b^{-1}c^{-4}}{a^4b^{-3}c^0}\right)^{-3} = \left(\frac{b^2}{a^6c^4}\right)^{-3} = \left(\frac{a^6c^4}{b^2}\right)^3 = \frac{a^{18}c^{12}}{b^6}$

50. $\left(\frac{4x^{-1}y^{-40}}{2^{-2}x^4y^{-10}}\right)^{-2} = \left(\frac{4}{(1/2)^2}\cdot x^{-1-4}\cdot y^{-40-(-10)}\right)^{-2}$
$$= (16x^{-5}y^{-30})^{-2}$$
$$= (16)^{-2}(x^{-5})^{-2}(y^{-30})^{-2}$$
$$= \frac{1}{(16)^2}\cdot x^{(-5)(-2)}y^{(-30)(-2)}$$
$$= \frac{1}{256}x^{10}y^{60}$$
$$= \frac{x^{10}y^{60}}{256}$$

51. a. $\frac{2x^{-2}}{(2x)^2} = 2\cdot\frac{1}{x^2}\cdot\frac{1}{(2x)^2} = 2\cdot\frac{1}{x^2}\cdot\frac{1}{4x^2} = \frac{1}{2x^4}$

b. $\frac{(2x)^{-2}}{(2x)^2} = \frac{1}{(2x)^2}\cdot\frac{1}{(2x)^2} = \frac{1}{4x^2}\cdot\frac{1}{4x^2} = \frac{1}{16x^4}$

c. $\frac{2x^{-2}}{2x^2} = 2\cdot\frac{1}{x^2}\cdot\frac{1}{2x^2} = \frac{1}{x^4}$

d. $\frac{2x^{-2}}{(2x)^{-2}} = 2\cdot\frac{1}{x^2}\cdot(2x)^2 = 2\cdot\frac{1}{x^2}\cdot 4x^2 = 8$

52. a. $\frac{2^{-1}x^{-2}}{(2x)^2} = \frac{2^{-1}x^{-2}}{2^2x^2} = 2^{-1-2}x^{-2-2}$
$$= 2^{-3}x^{-4} = \frac{1}{8x^4}$$

b. $\frac{2^{-1}x^{-2}}{2x^2} = 2^{-1-1}x^{-2-2} = 2^{-2}x^{-4} = \frac{1}{4x^4}$

c. $\frac{\left(2x^{-2}\right)^{-1}}{(2x)^{-2}} = \frac{2^{-1}x^{(-2)(-1)}}{2^{-2}x^{-2}} = \frac{2^{-1}x^2}{2^{-2}x^{-2}}$
$$= 2^{-1-(-2)}x^{2-(-2)} = 2x^4$$

d. $\frac{(2x^{-2})^{-1}}{2x^2} = \frac{2^{-1}x^{(-2)(-1)}}{2x^2} = \frac{2^{-1}x^2}{2x^2}$
$$= 2^{-1-1}x^{2-2} = \frac{1}{4}$$

53. $\frac{1}{x} = x^{-1}$

54. $\frac{1}{x^2} = x^{-2}$

55. $(2x)^3 = 2^3x^3 = 8x^3$

56. $(3x)^2 = 3^2x^2 = 9x^2$

57. $\dfrac{1}{(4x^2)} = \dfrac{1}{4} \cdot \dfrac{1}{x^2} = \dfrac{1}{4}x^{-2}$

58. $\dfrac{3}{(2x^4)} = \dfrac{3}{2} \cdot \dfrac{1}{x^4} = \dfrac{3}{2}x^{-4}$

59. $\left(\dfrac{-x}{2}\right)^3 = \dfrac{-x^3}{2^3} = -\dfrac{1}{8}x^3$

60. $\left(\dfrac{-x}{3}\right)^2 = \dfrac{(-x)^2}{3^2} = \dfrac{x^2}{9} = \dfrac{1}{9}x^2$

61. $P = 1200,\ i = 0.12,\ n = 5$

$$\begin{aligned} S &= P(1+i)^n \\ &= 1200(1+0.12)^5 \\ &= 1200(1.12)^5 \\ &= \$2114.81 \end{aligned}$$
$$I = S - P = 2114.81 - 1200 = \$914.81$$

62. $P = 1800,\ i = 0.10,\ n = 7$

$$\begin{aligned} S &= P(1+i)^n \\ &= 1800(1+0.10)^7 \\ &= 1800(1.10)^7 \\ &= 1800(1.9487171) \\ &= \$3507.69 \end{aligned}$$
$$I = S - P = 3507.69 - 1800 = \$1707.69$$

63. $P = 5000,\ i = 0.115,\ n = 6$

$$\begin{aligned} S &= P(1+i)^n \\ &= 5000(1+0.115)^6 \\ &= 5000(1.115)^6 \\ &= \$9607.70 \end{aligned}$$
$$I = S - P = 9607.70 - 5000 = \$4607.70$$

64. $P = 800,\ i = 0.105,\ n = 20$

$$\begin{aligned} S &= P(1+i)^n \\ &= 800(1+0.105)^{20} \\ &= 800(1.105)^{20} \\ &= 5892.99 \end{aligned}$$
$$I = S - P = 5892.99 - 800 = \$5092.99$$

65. $S = 15{,}000,\ n = 6,\ i = 0.115$

$$\begin{aligned} P &= S(1+i)^{-n} \\ &= 15{,}000(1+0.115)^{-6} \\ &= 15{,}000(1.115)^{-6} \\ &= \$7806.24 \end{aligned}$$

66. $S = 80{,}000,\ n = 20,\ i = 0.105$

$$\begin{aligned} P &= S(1+i)^{-n} \\ &= 80{,}000(1+0.105)^{-20} \\ &= 80{,}000(1.105)^{-20} \\ &= \$10{,}860.37 \end{aligned}$$

67. $I = 492.4(1.070)^t$

	Year	1980	2000	2008
a.	t-value	20	40	48
b.	Income (in billions)	\$1905	\$7373	\$12,669

c. $I = 492.4(1.070)^{58} \approx \$24{,}922$ billion

68. **a.** $t = 2019 - 2010 = 9$

b. $y = 0.012(1.75)^9 \approx 1.8$ billion cubic feet

c. $y = 0.012(1.75)^{12} \approx 9.9$ billion cubic feet

69. $y = \dfrac{1095}{1+10.12(1.212)^{-t}}$

a.

Year	1990	2003	2012
$t-$value	10	23	32
Predicted number of endangered species	442	976	1072

b. Year 2020: $t = 40$; $y = \dfrac{1095}{1+10.12(1.212)^{-40}} \approx 1090$

Increase between 2007 and 2020 is $1090 - 1037 = 53$ species

c. Two possibilities might be more environmental protections and the fact that there are only a limited number of species.

d. There are only a limited number of species. Also, below some threshold level the ecological balance might be lost, perhaps resulting in an environmental catastrophe (which the equation could not predict). To find the upper limit, which is 1095, compute y for large t-values:

Year	2040	2100	2200
$t-$value	60	120	220
Predicted number of endangered species	1094.9	1095	1095

70. $p = \dfrac{249.6}{1+1.915(1.028)^{-t}}$

a.

Year	1980	2000	2020
$t-$value	30	50	70
U.S. population (age 20 - 64) in millions	135.92	168.49	195.44

b. Year 2025: $t = 75$; $p = \dfrac{249.6}{1+1.915(1.028)^{-75}} \approx 201.07$ million

Year 2045: $t = 95$; $p = \dfrac{249.6}{1+1.915(1.028)^{-95}} \approx 219.15$ million

The increase from 2025 and 2045 is predicted to be $219.15 - 201.07 = 18.08$ million. This is less than the predicted 28.4 million increase from 2000 to 2020.

c. It is reasonable for a formula such as this to have an upper limit that cannot be exceeded because there are limited resources and space. To find the upper limit, which is 249.6 million, compute p for large t-values:

Year	2150	2350	2450
$t-$value	200	400	500
U.S. population (age 20 - 64) in millions	247.7	249.6	249.6

71. $H = 738.1(1.065)^t$

a. $t = 10$ corresponds to 2000.

b. $H = 738.1(1.065)^{10} \approx \$1385.5$ billion

c. $H = 738.1(1.065)^{20} \approx \$2600.8$ billion

d. $H = 738.1(1.065)^{28} \approx \$4304.3$ billion

Exercises 0.4

1. a. Since $\left(\frac{16}{3}\right)^2 = \frac{256}{9}$ we have $\sqrt{\frac{256}{9}} = \frac{16}{3} \approx 5.33$

b. $\sqrt{1.44} = 1.2$

2. a. $\sqrt[5]{-32^3} = \sqrt[5]{-1 \cdot 32^3} = -\sqrt[5]{32^3} = -(32)^{3/5}$
$= -\left(\sqrt[5]{32}\right)^3 = -(2)^3 = -8$

b. $\sqrt[4]{-16^5} = \sqrt[4]{-1048576}$ The square root of a negative number is not real.

3. a. $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

b. $(-16)^{-3/2} = \left(\sqrt{-16}\right)^{-3}$ The square root of a negative number is not real.

4. a. $-27^{-1/3} = -\left(27^{-1/3}\right) = -\frac{1}{\sqrt[3]{27}} = -\frac{1}{3}$

b. $32^{3/5} = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

5. $\left(\frac{8}{27}\right)^{-2/3} = \left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

6. $\left(\frac{4}{9}\right)^{3/2} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

7. a. $64^{2/3} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

b. $(-64)^{-2/3} = \frac{1}{(-64)^{2/3}} = \frac{1}{\left(\sqrt[3]{-64}\right)^2}$
$= \frac{1}{(-4)^2} = \frac{1}{16}$

8. a. $64^{-2/3} = \frac{1}{64^{2/3}} = \frac{1}{\left(\sqrt[3]{64}\right)^2} = \frac{1}{4^2} = \frac{1}{16}$

b. $-64^{2/3} = -\left(\sqrt[3]{64}\right)^2 = -(4)^2 = -16$

9. $\sqrt[9]{(6.12)^4} = (6.12)^{4/9} \approx 2.237$

10. $\sqrt[12]{4.96} = (4.96)^{1/12} \approx 1.1428$

11. $\sqrt{m^3} = m^{3/2}$

12. $\sqrt[3]{x^5} = x^{5/3}$

13. $\sqrt[4]{m^2 n^5} = \left(m^2 n^5\right)^{1/4} = m^{2/4} n^{5/4} = m^{1/2} n^{5/4}$

14. $\sqrt[5]{x^3} = x^{3/5}$

15. $2x^{\frac{1}{2}} = 2\sqrt{x}$

16. $12x^{\frac{1}{4}} = 12\sqrt[4]{x}$

17. $x^{7/6} = \sqrt[6]{x^7}$

18. $y^{11/5} = \sqrt[5]{y^{11}}$

19. $-\left(\frac{1}{4}\right)x^{-5/4} = -\frac{1}{4} \cdot \frac{1}{x^{5/4}} = \frac{-1}{4\sqrt[4]{x^5}}$

20. $-x^{-5/3} = -\sqrt[3]{x^{-5}} = \frac{-1}{\sqrt[3]{x^5}}$

21. $y^{1/4} \cdot y^{1/2} = y^{(1/4)+(1/2)} = y^{3/4}$

22. $x^{2/3} \cdot x^{1/5} = x^{(2/3)+(1/5)} = x^{(10/15)+(3/15)} = x^{13/15}$

23. $z^{3/4} \cdot z^4 = z^{(3/4)+(16/4)} = z^{19/4}$

24. $x^{-2/3} \cdot x^2 = x^{(-2/3)+2} = x^{(-2/3)+(6/3)} = x^{4/3}$

25. $y^{-3/2} \cdot y^{-1} = y^{(-3/2)-(2/2)} = y^{-5/2} = \frac{1}{y^{5/2}}$

26. $z^{-2} \cdot z^{5/3} = z^{-2+(5/3)} = z^{(-6/3)+(5/3)} = z^{-1/3} = \frac{1}{z^{1/3}}$

27. $\frac{x^{\frac{1}{3}}}{x^{\frac{-2}{3}}} = x^{\left(\frac{1}{3}\right)-\left(\frac{-2}{3}\right)} = x^{\frac{3}{3}} = x$

28. $\frac{x^{-1/2}}{x^{-3/2}} = x^{(-1/2)-(-3/2)} = x^{(-1/2)+(3/2)} = x^{2/2} = x$

29. $\frac{y^{-5/2}}{y^{-2/5}} = y^{(-5/2)-(-2/5)} = y^{(-25/10)+(4/10)}$

$= y^{-21/10} = \frac{1}{y^{21/10}}$

30. $\frac{x^{4/9}}{x^{1/12}} = x^{(4/9)-(1/12)} = x^{(16/36)-(3/36)} = x^{13/36}$

31. $(x^{2/3})^{3/4} = x^{(2/3)(3/4)} = x^{2/4} = x^{1/2}$

32. $(x^{4/5})^3 = x^{(4/5)(3)} = x^{12/5}$

33. $(x^{-1/2})^2 = x^{-1} = \frac{1}{x}$

34. $(x^{-2/3})^{-2/5} = x^{(-2/3)(-2/5)} = x^{4/15}$

35. $\sqrt{64x^4} = 8x^2$

36. $\sqrt[3]{-64x^6y^3} = \sqrt[3]{-64} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^3} = -4x^2y$

37. $\sqrt{128x^4y^5} = \sqrt{64x^4y^4 \cdot 2y}$

$= \sqrt{64} \cdot \sqrt{x^4} \cdot \sqrt{y^4} \cdot \sqrt{2y} = 8x^2y^2\sqrt{2y}$

38. $\sqrt[3]{54x^5z^8} = \sqrt[3]{54} \cdot \sqrt[3]{x^5} \cdot \sqrt[3]{z^8}$

$= 3\sqrt[3]{2} \cdot x\sqrt[3]{x^2} \cdot z^2\sqrt[3]{z^2} = 3xz^2\sqrt[3]{2x^2z^2}$

39. $\sqrt[3]{40x^8y^5} = \sqrt[3]{8x^6y^3 \cdot 5x^2y^2}$

$= \sqrt[3]{8} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^3} \cdot \sqrt[3]{5x^2y^2}$

$= 2x^2y\sqrt[3]{5x^2y^2}$

40. $\sqrt{32x^5y} = \sqrt{32} \cdot \sqrt{x^5} \cdot \sqrt{y} = 4\sqrt{2} \cdot x^2\sqrt{x} \cdot \sqrt{y}$

$= 4x^2\sqrt{2xy}$

41. $\sqrt{12x^3y} \cdot \sqrt{3x^2y} = \sqrt{36x^5y^2} = \sqrt{36} \cdot \sqrt{x^5} \cdot \sqrt{y^2}$

$= 6x^2y\sqrt{x}$

42. $\sqrt[3]{16x^2y} \cdot \sqrt[3]{3x^2y} = \sqrt[3]{48x^4y^2} = \sqrt[3]{48} \cdot \sqrt[3]{x^4} \cdot \sqrt[3]{y^2}$

$= 2\sqrt[3]{6} \cdot x\sqrt[3]{x} \cdot \sqrt[3]{y^2} = 2x\sqrt[3]{6xy^2}$

43. $\sqrt{63x^5y^3} \cdot \sqrt{28x^2y} = \sqrt{9x^4y^2 \cdot 7xy} \cdot \sqrt{4x^2 \cdot 7y}$

$= 3x^2y\sqrt{7xy} \cdot 2x\sqrt{7y}$

$= 42x^3y^2\sqrt{x}$

44. $\sqrt{10xz^{10}} \cdot \sqrt{30x^{17}z} = \sqrt{300x^{18}z^{11}}$

$= \sqrt{300} \cdot \sqrt{x^{18}} \cdot \sqrt{z^{11}}$

$= 10\sqrt{3} \cdot x^9 \cdot z^5\sqrt{z}$

$= 10x^9z^5\sqrt{3z}$

45. $\frac{\sqrt{12x^3y^{12}}}{\sqrt{27xy^2}} = \sqrt{\frac{4x^2y^{10}}{9}} = \frac{2xy^5}{3}$

46. $\frac{\sqrt{250xy^7z^4}}{\sqrt{18x^{17}y^2}} = \sqrt{\frac{250xy^7z^4}{18x^{17}y^2}} = \sqrt{\frac{125y^5z^4}{9x^{16}}}$

$= \frac{\sqrt{125}\sqrt{y^5}\sqrt{z^4}}{\sqrt{9}\sqrt{x^{16}}}$

$= \frac{5\sqrt{5} \cdot y^2\sqrt{y} \cdot z^2}{3x^8}$

$= \frac{5y^2z^2\sqrt{5y}}{3x^8}$

47. $\frac{\sqrt[4]{32a^9b^5}}{\sqrt[4]{162a^{17}}} = \sqrt[4]{\frac{16b^4}{81a^8} \cdot \frac{b}{1}} = \frac{2b}{3a^2}\sqrt[4]{b}$

48. $\dfrac{\sqrt[3]{-16x^3y^4}}{\sqrt[3]{128y^2}} = \sqrt[3]{\dfrac{-16x^3y^4}{128y^2}} = \dfrac{\sqrt[3]{x^3}\sqrt[3]{y^2}}{\sqrt[3]{-8}}$

$= \dfrac{x\sqrt[3]{y^2}}{-2} = \dfrac{-x\sqrt[3]{y^2}}{2}$

49. $(A^9)^x = A^{9x}$

$A^{9x} = A^1$

$9x = 1$

$x = \dfrac{1}{9}$

50. $(B^{20})^x = B$

$B^{20x} = B^1$

$20x = 1$

$x = \dfrac{1}{20}$

51. $\left(\sqrt[7]{R}\right)^x = R^{x/7}$

$R^{x/7} = R^1$

$\dfrac{x}{7} = 1$

$x = 7$

52. $\left(\sqrt{T^3}\right)^x = T$

$((T^3)^{1/2})^x = T^1$

$(T^{3/2})^x = T^1$

$T^{3x/2} = T^1$

$\dfrac{3x}{2} = 1$

$x = \dfrac{2}{3}$

53. $\sqrt{\dfrac{2}{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{\sqrt{6}}{3}$

54. $\sqrt{\dfrac{5}{8}} \cdot \dfrac{\sqrt{8}}{\sqrt{8}} = \dfrac{\sqrt{5}}{\sqrt{8}} \cdot \dfrac{\sqrt{8}}{\sqrt{8}} = \dfrac{\sqrt{40}}{\sqrt{64}} = \dfrac{2\sqrt{10}}{8} = \dfrac{\sqrt{10}}{4}$

55. $\dfrac{\sqrt{m^2x}}{\sqrt{mx^2}} = \dfrac{\sqrt{m}}{\sqrt{x}} = \dfrac{\sqrt{m}\cdot\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}} = \dfrac{\sqrt{mx}}{x}$

56. $\dfrac{5x^3w}{\sqrt{4xw^2}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{5x^3w\sqrt{x}}{\sqrt{4x^2w^2}} = \dfrac{5x^3w\sqrt{x}}{2xw} = \dfrac{5x^2\sqrt{x}}{2}$

57. $\dfrac{\sqrt[3]{m^2x}}{\sqrt[3]{mx^5}} = \dfrac{\sqrt[3]{m}}{\sqrt[3]{x^4}} = \dfrac{\sqrt[3]{m}}{\sqrt[3]{x^3}\cdot\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{mx^2}}{x\sqrt[3]{x^3}}$

$= \dfrac{\sqrt[3]{mx^2}}{x^2}$

58. $\dfrac{\sqrt[4]{mx^3}}{\sqrt[4]{y^2z^5}} \cdot \dfrac{\sqrt[4]{y^2z^3}}{\sqrt[4]{y^2z^3}} = \dfrac{\sqrt[4]{mx^3y^2z^3}}{\sqrt[4]{y^4z^8}} = \dfrac{\sqrt[4]{mx^3y^2z^3}}{yz^2}$

59. $\dfrac{-2}{3\sqrt[3]{x^2}} = \dfrac{-2}{3} \cdot \dfrac{1}{x^{2/3}} = -\dfrac{2}{3}x^{-2/3}$

60. $\dfrac{-2}{3\sqrt[4]{x^3}} = \dfrac{-2}{3x^{3/4}} = \dfrac{-2x^{-3/4}}{3} = -\dfrac{2}{3}x^{-3/4}$

61. $3x\sqrt{x} = 3x \cdot x^{1/2} = 3x^{3/2}$

62. $\sqrt{x}\cdot\sqrt[3]{x} = x^{1/2}x^{1/3} = x^{(1/2)+(1/3)} = x^{(3/6)+(2/6)}$

$= x^{5/6}$

63. $\dfrac{3}{2}x^{1/2} = \dfrac{3}{2}\sqrt{x}$

64. $\dfrac{4}{3}x^{1/3} = \dfrac{4}{3}\sqrt[3]{x} = \dfrac{4\sqrt[3]{x}}{3}$

65. $\dfrac{1}{2}x^{-1/2} = \dfrac{1}{2}\cdot\dfrac{1}{x^{1/2}} = \dfrac{1}{2\sqrt{x}}$

66. $\dfrac{-1}{2}x^{-3/2} = \dfrac{-1}{2x^{3/2}} = \dfrac{-1}{2\sqrt{x^3}}$

67. **a.** $R = 8.5 = \dfrac{17}{2} \qquad I = 10^{17/2} = \sqrt{10^{17}}$

b. $I = 10^{9.0} = 1{,}000{,}000{,}000$

c. $\dfrac{I_{2011}}{I_{1989}} = \dfrac{10^{9.0}}{10^{6.9}} = 10^{2.1} \approx 125.9$

68. **a.** $10^{D/10} = (10^D)^{1/10} = \sqrt[10]{10^D}$

b. $I_1 = \sqrt[10]{10^{32}} \approx 1584.89$

c. $\dfrac{I_2}{I_1} = \dfrac{\sqrt[10]{10^{140}}}{\sqrt[10]{10^{32}}} = \sqrt[10]{10^{140-32}} = \sqrt[10]{10^{108}}$

$= 10^{(108)(1/10)} = 10^{10.8}$

$\approx 6.31 \times 10^{10}$

69. a. $S = 1000\sqrt{\left(1 + \dfrac{r}{100}\right)^5}$

b. $S = 1000\sqrt{\left(1 + \dfrac{6.6}{100}\right)^5} \approx \$1173.26$

70. a. $0.21 = \dfrac{21}{100}$, so $L = 29\sqrt[100]{x^{21}}$

b. $L = 29(115)^{0.21} \approx 78.5$ years

71. a. $P = 0.924t^{13/100} = 0.924\sqrt[100]{t^{13}}$

b.

Year	t	Population
2005	5	1.1390
2010	10	1.2464
2045	45	1.5156
2050	50	1.5365

Change from 2005 to 2010: 0.1074 billion
Change from 2045 to 2050: 0.0209 billion
By 2045 and 2050 the population is much larger than earlier in the 21st century, and there is a limited number of people that any land can support—in terms of both space and food.

72. a. $p = 14.32t^{0.38} = 14.32t^{38/100} = 14.32t^{19/50} = 14.32\sqrt[50]{t^{19}}$

b.

Year	t	Percent of Roads Paved
1970	20	44.7
1980	30	52.1
2000	50	63.3
2010	60	67.9

Change from 1970 to 1980: 7.4%
Change from 2000 to 2010: 4.6%
The equation estimates a greater percent change from 1970 to 1980 than from 2000 to 2010. There were fewer roads left to be paved from 2000 to 2010.

c. When a t-value makes $p > 100\%$, you can be certain that the equation is no longer valid since you cannot pave more than 100% of the roads.

73. $k = 25, t = 10, q_0 = 98$

$q = q_0(2^{-t/k})$

$= 98(2^{-10/25})$

$= 98(2^{-2/5}) \approx 74$ kg

74. $k = 5600, t = 10{,}000, q_0 = 40$

$q = q_0(2^{-t/k})$

$= 40(2^{-10{,}000/5600})$

$= 40(2^{-25/14}) \approx 11.6$ g

75. $P = P_0(2.5)^{ht} = 30{,}000(2.5)^{0.03(10)}$

$= 30{,}000(2.5)^{0.3} \approx 39{,}491$

76. $x = 10$

$$S = 2000\left(2^{-0.1x}\right)$$
$$= 2000\left(2^{-0.1(10)}\right)$$
$$= 2000\left(2^{-1}\right)$$
$$= 2000 \cdot \frac{1}{2}$$
$$= \$1000$$

77. **a.** $N = 500(0.02)^{(0.7)^t}$; at $t = 0$ we have $(0.7)^0 = 1$. Thus, $N = 500(0.02)^1 = 10$.

b. $N = 500\left(0.02\right)^{(0.7)^5}$
$$= 500\left(0.02\right)^{0.16807}$$
$$= 259$$

Exercises 0.5

1. $10 - 3x - x^2$

- **a.** The largest exponent is 2. The degree of the polynomial is 2.
- **b.** The coefficient of x^2 is –1.
- **c.** The constant term is 10.
- **d.** It is a polynomial of one variable *x*.

2. $5x^4 - 2x^9 + 7$

- **a.** The largest exponent is 9. The degree of the polynomial is 9.
- **b.** The coefficient of x^9 is –2.
- **c.** The constant term is 7.
- **d.** It is a polynomial of one variable *x*.

3. $7x^2y - 14xy^3z$

- **a.** The sum of the exponents in each term is 3 and 5, respectively. The degree of the polynomial is 5.
- **b.** The coefficient of xy^3z is –14.
- **c.** The constant term is zero.
- **d.** It is a polynomial of several (three) variables: *x*, *y*, and *z*.

4. $2x^5 + 7x^2y^3 - 5y^6$

- **a.** The sum of the exponents of each term is 5, 5 and 6, respectively. The degree of the polynomial is 6.
- **b.** The coefficient of y^6 is –5.
- **c.** The constant term is 0.
- **d.** It is a polynomial of two variables; *x* and *y*.

5. $2x^5 - 3x^2 - 5$

- **a.** $a_n x^n$ means $2 = a_5$.
- **b.** $a_3 = 0$ (Term is $0x^3$)
- **c.** $-3 = a_2$
- **d.** $a_0 = -5$, the constant term.

6. $5x^3 - 4x - 17$

- **a.** $a_3 = 5$
- **b.** $a_1 = -4$ (Term is $-4x$)
- **c.** $a_2 = 0$
- **d.** $-17 = a_0$

7. $4x - x^2$

When $x = -2$,
$$4x - x^2 = 4(-2) - (-2)^2$$
$$= -8 - 4$$
$$= -12.$$

8. $10 - 6\left(4 - x\right)^2$

When $x = -1$,
$$10 - 6\left(4 - x\right)^2 = 10 - 6\left(4 - \left(-1\right)\right)^2$$
$$= 10 - 150$$
$$= -140.$$

9. $10xy-4(x-y)^2$

When $x=5$ and $y=-2$,

$$\begin{aligned}10xy-4(x-y)^2 &= 10(5)(-2)-4(5-(-2))^2\\ &= -100-196\\ &= -296.\end{aligned}$$

10. $3x^2-4y^2-2xy$

When $x=3$ and $y=-4$,

$$\begin{aligned}3x^2-4y^2-2xy &= 3\cdot 3^2-4(-4)^2-2\cdot 3(-4)\\ &= 27-64+24\\ &= -13.\end{aligned}$$

11. $\dfrac{2x-y}{x^2-2y}$

When $x=-5$ and $y=-3$,

$$\frac{2x-y}{x^2-2y}=\frac{2(-5)-(-3)}{(-5)^2-2(-3)}=\frac{-10+3}{25+6}=-\frac{7}{31}.$$

12. $\dfrac{16y}{1-y}$

When $y=-3$,

$$\frac{16y}{1-y}=\frac{16(-3)}{1-(-3)}=\frac{-48}{4}=-12$$

13. $1.98T-1.09(1-H)(T-58)-56.8$

$$\begin{aligned}&=1.98(74.7)-1.09(1-0.80)(74.7-58)-56.8\\ &=147.906-3.6406-56.8 = 87.4654\end{aligned}$$

14. $(100,000)\left[\dfrac{0.083(0.07)}{1-(1+0.083(0.07))^{-360}}\right]$

$$=(100,000)\left[\frac{0.00581}{0.87576}\right]\approx 663.4238$$

15. $(16pq-7p^2)+(5pq+5p^2)=21pq-2p^2$

16. $(3x^3+4x^2y^2)+(3x^2y^2-7x^3)$

$$\begin{aligned}&=(3x^3-7x^3)+(4x^2y^2+3x^2y^2)\\ &=-4x^3+7x^2y^2\end{aligned}$$

17. $(4m^2-3n^2+5)-(3m^2+4n^2+8)$

$$\begin{aligned}&=4m^2-3n^2+5-3m^2-4n^2-8\\ &=(4m^2-3m^2)-(3n^2+4n^2)+5-8\\ &=m^2-7n^2-3\end{aligned}$$

18. $(4rs-2r^2s-11rs^2)-(11rs^2-2rs+4r^2s)$

$$\begin{aligned}&=4rs-2r^2s-11rs^2-11rs^2+2rs-4r^2s\\ &=(4rs+2rs)-(2r^2s+4r^2s)-(11rs^2+11rs^2)\\ &=6rs-6r^2s-22rs^2\end{aligned}$$

19. $-[8-4(q+5)+q]=-[8-4q-20+q]$

$$\begin{aligned}&=-[-12-3q]\\ &=12+3q\end{aligned}$$

20. $x^3+\left[3x-(x^3-3x)\right]=x^3+\left[3x-x^3+3x\right]$

$$\begin{aligned}&=x^3+3x-x^3+3x\\ &=6x\end{aligned}$$

21. $x^2-\left[x-(x^2-1)+1-(1-x^2)\right]+x$

$$\begin{aligned}&=x^2-\left[x-x^2+1+1-1+x^2\right]+x\\ &=x^2-(x+1)+x\\ &=x^2-x-1+x\\ &=x^2-1\end{aligned}$$

22. $y^3-\left[y^2-(y^3+y^2)\right]-\left[y^3+(1-y^2)\right]$

$$\begin{aligned}&=y^3-\left[y^2-y^3-y^2\right]-\left[y^3+1-y^2\right]\\ &=y^3-y^2+y^3+y^2-y^3-1+y^2\\ &=y^3+y^2-1\end{aligned}$$

23. $(5x^3)(7x^2)=35x^{3+2}=35x^5$

24. $(-3x^2y)(2xy^3)(4x^2y^2)$

$$\begin{aligned}&=(-3)\cdot(2)\cdot(4)\cdot x^2\cdot x\cdot x^2\cdot y\cdot y^3\cdot y^2\\ &=-24x^5y^6\end{aligned}$$

25. $(39r^3s^2)\div(13r^2s)=3r^{3-2}s^{2-1}=3rs$

26. $(-15m^3n)\div(5mn^4)=\dfrac{-15m^3n}{5mn^4}=\dfrac{-3m^2}{n^3}$

27. $ax^2(2x^2+ax+ab)=2ax^4+a^2x^3+a^2bx^2$

28. $-3\left(3-x^2\right)=-9+3x^2=3x^2-9$

29. $$\begin{aligned}(3y+4)(2y-3)&=6y^2-9y+8y-12\\&=6y^2-y-12\end{aligned}$$

30. $$\begin{aligned}&(4x-1)(x-3)\\&=4x(x)+4x(-3)+(-1)(x)+(-1)(-3)\\&=4x^2-12x-x+3\\&=4x^2-13x+3\end{aligned}$$

31. $$\begin{aligned}&6\left(1-2x^2\right)\left(2-x^2\right)\\&=6\left(2-x^2-4x^2+2x^4\right)\\&=6\left(2-5x^2+2x^4\right)\\&=12x^4-30x^2+12\end{aligned}$$

32. $$\begin{aligned}2\left(x^3+3\right)\left(2x^3-5\right)&=2\left(2x^6-5x^3+6x^3-15\right)\\&=2\left(2x^6+x^3-15\right)\\&=4x^6+2x^3-30\end{aligned}$$

33. $\left(4x+3\right)^2=16x^2+2\left(4x\right)\left(3\right)+9=16x^2+24x+9$

34. $$\begin{aligned}(2y+5)^2&=(2y)^2+2(2y)(5)+(5)^2\\&=4y^2+20y+25\end{aligned}$$

35. $$\begin{aligned}(0.1-4x)(0.1+4x)&=(0.1)^2-(4x)^2\\&=0.01-16x^2\end{aligned}$$

36. $$\begin{aligned}\left(x^3y^3-0.3\right)^2&=\left(x^3y^3\right)^2+2\left(x^3y^3\right)\left(-0.3\right)+\left(-0.3\right)^2\\&=x^6y^6-0.6x^3y^3+0.09\end{aligned}$$

37. $$\begin{aligned}9(2x+1)(2x-1)&=9\left[(2x)^2-1^2\right]=9\left[4x^2-1\right]\\&=36x^2-9\end{aligned}$$

38. $$\begin{aligned}3(5y+2)(5y-2)&=3\left[(5y)^2-2^2\right]=3\left[25y^2-4\right]\\&=75y^2-12\end{aligned}$$

39. $$\begin{aligned}\left(x^2-\frac{1}{2}\right)^2&=x^4+2(x^2)\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)^2\\&=x^4-x^2+\frac{1}{4}\end{aligned}$$

40. $\left(\frac{2}{3}+x\right)\left(\frac{2}{3}-x\right)=\left(\frac{2}{3}\right)^2-(x)^2=\frac{4}{9}-x^2$

41. $$\begin{aligned}(0.1x-2)(x+0.05)&=0.1x^2+0.005x-2x-0.10\\&=0.1x^2-1.995x-0.10\end{aligned}$$

42. $$\begin{aligned}(6.2x+4.1)(6.2x-4.1)&=(6.2x)^2-(4.1)^2\\&=38.44x^2-16.81\end{aligned}$$

43. $$\begin{array}{r}x^2+2x+4\\ \underline{x-2}\\ -2x^2-4x-8\\ \underline{x^3+2x^2+4x}\\ x^3-8\end{array}$$

44. $$\begin{array}{l}a^2-ab+b^2\\ \underline{a+b}\\ a^3-a^2b+ab^2\\ \underline{\;a^2b-ab^2+b^3}\\ a^3+b^3\end{array}$$

45. $$\begin{array}{r}x^5-2x^3+5\\ \underline{x^3+5x}\\ 5x^6-10x^4+25x\\ \underline{x^8-2x^6+5x^3}\\ x^8+3x^6-10x^4+5x^3+25x\end{array}$$

46. $$\begin{array}{l}x^7-2x^4-5x^2+5\\ \underline{x^3-1}\\ x^{10}-2x^7-5x^5+5x^3\\ \underline{\phantom{x^{10}}-x^7+2x^4+5x^2-5}\\ x^{10}-3x^7-5x^5+2x^4+5x^3+5x^2-5\end{array}$$

47. $$\begin{aligned}&\frac{18m^2n+6m^3n+12m^4n^2}{6m^2n}\\&=\frac{18m^2n}{6m^2n}+\frac{6m^3n}{6m^2n}+\frac{12m^4n^2}{6m^2n}=3+m+2m^2n\end{aligned}$$

48. $$\begin{aligned}&\frac{16x^2+4xy^2+8x}{4xy}=\frac{16x^2}{4xy}+\frac{4xy^2}{4xy}+\frac{8x}{4xy}\\&=\frac{4x}{y}+y+\frac{2}{y}\end{aligned}$$

49. $\dfrac{24x^8y^4+15x^5y-6x^7y}{9x^5y^2}$

$$=\frac{24x^8y^4}{9x^5y^2}+\frac{15x^5y}{9x^5y^2}-\frac{6x^7y}{9x^5y^2}=\frac{8x^3y^2}{3}+\frac{5}{3y}-\frac{2x^2}{3y}$$

50. $\dfrac{27x^2y^2-18xy+9xy^2}{6xy}$

$$=\frac{27x^2y^2}{6xy}-\frac{18xy}{6xy}+\frac{9xy^2}{6xy}=\frac{9xy}{2}-3+\frac{3y}{2}$$

51. $(x+1)^3=x^3+3(x^2)(1)+3(x)(1)^2+1^3$

$=x^3+3x^2+3x+1$

52. $(x-3)^3=x^3-3(3)(x^2)+3(3)^2x-3^3$

$=x^3-9x^2+27x-27$

53. $(2x-3)^3=(2x)^3-3(2x)^2(3)+3(2x)(3)^2-3^3$

$=8x^3-36x^2+54x-27$

54. $(3x+4)^3=(3x)^3+3(4)(3x)^2+3(4)^2(3x)+(4)^3$

$=27x^3+108x^2+144x+64$

55.

$$\begin{array}{r@{}l}
 & \;x^2-2x+5 \\
x+2\,\big) & \overline{x^3\qquad\quad+x-1} \\
 & \underline{x^3+2x^2} \\
 & \quad -2x^2+\;x-1 \\
 & \quad \underline{-2x^2-4x} \\
 & \qquad\qquad 5x-\;1 \\
 & \qquad\qquad \underline{5x+10} \\
 & \qquad\qquad\quad -11
\end{array}$$

Quotient: $x^2-2x+5-\dfrac{11}{x+2}$

56.

$$\begin{array}{r@{}l}
 & \;x^4-x^3+x^2-x+6 \\
x+1\,\big) & \overline{x^5+0x^4+0x^3+0x^2+5x-7} \\
 & \underline{x^5+\;x^4} \\
 & \quad -x^4+0x^3 \\
 & \quad \underline{-x^4-\;x^3} \\
 & \qquad\quad x^3+0x^2 \\
 & \qquad\quad \underline{x^3+\;x^2} \\
 & \qquad\qquad -x^2+5x \\
 & \qquad\qquad \underline{-x^2-\;x} \\
 & \qquad\qquad\qquad 6x-7 \\
 & \qquad\qquad\qquad \underline{6x+6} \\
 & \qquad\qquad\qquad\; -13
\end{array}$$

Quotient: $x^4-x^3+x^2-x+6-\dfrac{13}{x+1}$

57.

$$\begin{array}{r@{}l}
 & \;x^2+3x-1 \\
x^2+1\,\big) & \overline{x^4+3x^3\qquad\;-x+1} \\
 & \underline{x^4\qquad\;+x^2\qquad\qquad} \\
 & \qquad 3x^3-x^2-x+1 \\
 & \qquad \underline{3x^3\qquad+3x\quad} \\
 & \qquad\quad -x^2-4x+1 \\
 & \qquad\quad \underline{-x^2\qquad\;-1} \\
 & \qquad\qquad\; -4x+2
\end{array}$$

Quotient: $x^2+3x-1+\dfrac{-4x+2}{x^2+1}$

58.

$$\begin{array}{r@{}l}
 & \;x\;+5 \\
x^2-2\,\big) & \overline{x^3+5x^2+0x-6} \\
 & \underline{x^3\qquad\;-2x\quad} \\
 & \qquad 5x^2+2x-6 \\
 & \qquad \underline{5x^2\qquad-10} \\
 & \qquad\qquad 2x+4
\end{array}$$

Quotient: $x+5+\dfrac{2x+4}{x^2-2}$

59. a. $(3x-2)^2-3x-2(3x-2)+5$

$=9x^2-12x+4-3x-6x+4+5$

$=9x^2-21x+13$

b. $(3x-2)^2-(3x-2)(3x-2)+5$

$=(3x-2)^2-(3x-2)^2+5$

$=5$

60. a. $(2x-3)(3x+2)-(5x-2)(x-3)$

$=6x^2-5x-6-(5x^2-17x+6)$

$=6x^2-5x-6-5x^2+17x-6$

$=x^2+12x-12$

b. $2x-3(3x+2)-5x-2(x-3)$

$=2x-9x-6-5x-2x+6$

$=-14x$

61. $x^{1/2}\left(x^{1/2}+2x^{3/2}\right)=x^{2/2}+2x^{4/2}=x+2x^2$

62. $x^{-2/3}\left(x^{5/3}-x^{-1/3}\right)=\left(x^{-2/3}\right)\left(x^{5/3}\right)+\left(x^{-2/3}\right)\left(-x^{-1/3}\right)$
$=x^{3/3}-x^{-3/3}$
$=x-\dfrac{1}{x}$

63. $\left(x^{1/2}+1\right)\left(x^{1/2}-2\right)=x-2x^{1/2}+x^{1/2}-2$
$=x-x^{1/2}-2$

64. $\left(x^{1/3}-x^{1/2}\right)\left(4x^{2/3}-3x^{3/2}\right)=\left(x^{1/3}\right)\left(4x^{2/3}\right)+\left(x^{1/3}\right)\left(-3x^{3/2}\right)+\left(-x^{1/2}\right)\left(4x^{2/3}\right)+\left(-x^{1/2}\right)\left(-3x^{3/2}\right)$
$=4x^{3/3}-3x^{11/6}-4x^{7/6}+3x^{4/2}$
$=4x-3x^{11/6}-4x^{7/6}+3x^{2}$

65. $\left(\sqrt{x}+3\right)\left(\sqrt{x}-3\right)=\left(\sqrt{x}\right)^2-(3)^2=x-9$

66. $\left(x^{1/5}+x^{1/2}\right)\left(x^{1/5}-x^{1/2}\right)=\left(x^{1/5}\right)^2-\left(x^{1/2}\right)^2=x^{2/5}-x$

67. $(2x+1)^{1/2}\left[(2x+1)^{3/2}-(2x+1)^{-1/2}\right]=(2x+1)^2-(2x+1)^0=4x^2+4x+1-1=4x^2+4x$

68. $(4x-3)^{-5/3}\left[(4x-3)^{8/3}+3(4x-3)^{5/3}\right]=(4x-3)^{-5/3}(4x-3)^{8/3}+(4x-3)^{-5/3}(3)(4x-3)^{5/3}$
$=(4x-3)^{3/3}+3(4x-3)^0=4x-3+3=4x$

69. $R=55x$

70. $R=215x$, $C=65x+15{,}000$
a. Profit $=P=215x-(65x+15{,}000)$
$=150x-15{,}000$
b. $x=1000$: $P=150(1000)-15{,}000$
$=150{,}000-15{,}000$
$=\$135{,}000$

71. a. $C=49.95+0.49x$
b. $C=49.95+0.49(132)=\$114.63$

72. a. $C=1500+18.50x$
b. $R=45.50x$
c. $P=45.50x-(1500+18.50x)$
$=27x-1500$

73. a. $4000-x$
b. $0.10x$
c. $0.08(4000-x)$
d. $0.10x+0.08(4000-x)$ or $320+0.02x$

74. a. y = 10 cc – amount of 20% solution = 10 – x
b. Amount of ingredient = (% concentration) $\cdot$ (#cc) = $0.20x$
c. Amount of ingredient in y = (% concentration) (#cc) = $0.05(10-x)$
d. Total amount of ingredient in mixture is (b) + (c).
Total amount:
$0.20x+0.05(10-x)=0.50+0.15x$

75. $V=x(15-2x)(10-2x)$

Exercises 0.6

1. $9ab-12a^2b+18b^2=3b\left(3a-4a^2+6b\right)$

2. $8a^2b-160x+4bx^2=4\left(2a^2b-40x+bx^2\right)$

3. $4x^2+8xy^2+2xy^3=2x\left(2x+4y^2+y^3\right)$

4. $12y^3z+4yz^2-8y^2z^3=4yz\left(3y^2+z-2yz^2\right)$

5. $\left(7x^3-14x^2\right)+\left(2x-4\right)=7x^2\left(x-2\right)+2\left(x-2\right)$
$=\left(x-2\right)\left(7x^2+2\right)$

6. $5y-20-x^2y+4x^2=(5y-20)+(-x^2y+4x^2)$
$=5(y-4)-x^2(y-4)$
$=(5-x^2)(y-4)$

7. $6x-6m+xy-my=(6x-6m)+(xy-my)$
$=6(x-m)+y(x-m)$
$=(x-m)(6+y)$

8. $x^3-x^2-5x+5=x^2(x-1)-5(x-1)$
$=(x-1)(x^2-5)$

9. $x^2+8x+12=(x+6)(x+2)$

10. $x^2-2x-8=(x-4)(x+2)$

11. $x^2-15x-16=(x-16)(x+1)$

12. $x^2-21x+20=(x-20)(x-1)$

13. $7x^2-10x-8$
$7x^2\cdot 8=56x^2$
The factors $-14x$ and $+4x$ give a sum of $-10x$.
$7x^2-10x-8=7x^2-14x+4x-8$
$=7x(x-2)+4(x-2)$
$=(x-2)(7x+4)$

14. $12x^2+11x+2$
Two expressions whose product is $(12x^2)(2)=24x^2$ and whose sum is $11x$ are $8x$ and $3x$.
So, $12x^2+11x+2=12x^2+3x+8x+2$
$=3x(4x+1)+2(4x+1)$
$=(4x+1)(3x+2)$.

15. $x^2-10x+25=x^2-2\cdot 5x+5^2=(x-5)^2$

16. $4y^2+12y+9$
Two expressions whose product is $(4y^2)(9)=36y^2$ and whose sum is 12y are $6y$ and $6y$.
So, $4y^2+12y+9=4y^2+6y+6y+9$
$=2y(2y+3)+3(2y+3)$
$=(2y+3)(2y+3)$
$=(2y+3)^2$.

17. $49a^2-144b^2=(7a)^2-(12b)^2$
$=(7a+12b)(7a-12b)$

18. $16x^2-25y^2=(4x)^2-(5y)^2$
$=(4x-5y)(4x+5y)$

19. **a.** $9x^2+21x-8$
$9x^2(-8)=-72x^2$
The factors $24x$ and $-3x$ give a sum of $21x$.
$9x^2+21x-8=9x^2+24x-3x-8$
$=3x(3x+8)-1(3x+8)$
$=(3x+8)(3x-1)$

b. $9x^2+22x+8$
$9x^2\cdot 8=72x^2$
The factors $18x$ and $4x$ give a sum of $22x$.
$9x^2+22x+8=9x^2+18x+4x+8$
$=9x(x+2)+4(x+2)$
$=(x+2)(9x+4)$

20. **a.** $10x^2-99x-63$
$10x^2\cdot(-63)=-630x^2$
The factors $-105x$ and $6x$ give a sum of $-99x$.
$10x^2-99x-63=10x^2-105x+6x-63$
$=5x(2x-21)+3(2x-21)$
$=(2x-21)(5x+3)$

b. $10x^2-27x-63$
Two expressions whose product is $(10x^2)(-63)=-630x^2$ and whose sum is $-27x$ are $-42x$ and $15x$. So,
$10x^2-27x-63=10x^2+15x-42x-63$
$=5x(2x+3)-21(2x+3)$
$=(2x+3)(5x-21)$.

c. $10x^2+61x-63$
$10x^2\cdot(-63)=-630x^2$
The factors $70x$ and $-9x$ give a sum of $61x$.
$$\begin{aligned}10x^2+61x-63&=10x^2+70x-9x-63\\&=10x(x+7)-9(x+7)\\&=(x+7)(10x-9)\end{aligned}$$

d. $10x^2+9x-63$
Two expressions whose product is $(10x^2)(-63)=-630x^2$ and whose sum is $9x$ are $30x$ and $-21x$. So,
$$\begin{aligned}10x^2+9x-63&=10x^2+30x-21x-63\\&=10x(x+3)-21(x+3)\\&=(x+3)(10x-21).\end{aligned}$$

21. $4x^2-x=x(4x-1)$

22. $2x^5+18x^3=2x^3(x^2+9)$

23. $$\begin{aligned}x^3+4x^2-5x-20&=x^2(x+4)-5(x+4)\\&=(x+4)(x^2-5)\end{aligned}$$

24. $$\begin{aligned}x^3-2x^2-3x+6&=(x^3-2x^2)+(-3x+6)\\&=x^2(x-2)-3(x-2)\\&=(x^2-3)(x-2)\end{aligned}$$

25. $x^2-x-6=(x-3)(x+2)$
Note that two numbers whose product is -6 and whose sum is -1 are -3 and 2.

26. $x^2+6x+8=(x+4)(x+2)$
Since two numbers whose product is 8 and whose sum is 6 are 4 and 2.

27. $2x^2-8x-42=2(x^2-4x-21)=2(x-7)(x+3)$

28. $3x^2-21x+36=3(x^2-7x+12)$
Two numbers whose product is 12 and whose sum is -7 are -3 and -4. So,
$3(x^2-7x+12)=3(x-3)(x-4)$.

29. $$\begin{aligned}2x^3-8x^2+8x&=2x(x^2-4x+4)\\&=2x(x^2-2\cdot 2x+2^2)\\&=2x(x-2)^2\end{aligned}$$

30. $x^3+16x^2+64x=x(x^2+16x+64)=x(x+8)^2$

31. $2x^2+x-6$
$2x^2\cdot(-6)=-12x^2$
The factors $4x$ and $-3x$ give a sum of x.
$$\begin{aligned}2x^2+x-6&=2x^2+4x-3x-6\\&=2x(x+2)-3(x+2)\\&=(2x-3)(x+2)\end{aligned}$$

32. $2x^2+13x+6$
Two expressions whose product is $(2x^2)(6)=12x^2$ and whose sum is $13x$ are $12x$ and x. So,
$$\begin{aligned}2x^2+13x+6&=2x^2+12x+x+6\\&=2x(x+6)+1(x+6)\\&=(x+6)(2x+1).\end{aligned}$$

33. $3x^2+3x-36=3(x^2+x-12)=3(x+4)(x-3)$

34. $4x^2-8x-60=4(x^2-2x-15)$
Two numbers whose product is -15 and whose sum is -2 are -5 and 3. So,
$4(x^2-2x-15)=4(x-5)(x+3)$.

35. $2x^3-8x=2x(x^2-4)=2x(x+2)(x-2)$

36. $16z^2-81w^2=(4z)^2-(9w)^2=(4z-9w)(4z+9w)$

37. $10x^2+19x+6$
$10x^2\cdot 6=60x^2$
The factors $4x$ and $15x$ give a sum of $19x$.
$$\begin{aligned}10x^2+19x+6&=10x^2+4x+15x+6\\&=2x(5x+2)+3(5x+2)\\&=(5x+2)(2x+3)\end{aligned}$$

38. $6x^2+67x-35$
Two expressions whose product is $(6x^2)(-35)=-210x^2$ and whose sum is $67x$ are $70x$ and $-3x$. So,

$$\begin{aligned}6x^2+67x-35&=6x^2-3x+70x-35\\&=3x(2x-1)+35(2x-1)\\&=(2x-1)(3x+35).\end{aligned}$$

39. $9-47x+10x^2$
$9\cdot 10x^2=90x^2$
The factors $-45x$ and $-2x$ give a sum of $-47x$.

$$\begin{aligned}9-47x+10x^2&=9-45x-2x+10x^2\\&=9(1-5x)-2x(1-5x)\\&=(1-5x)(9-2x)\\&\text{or }(5x-1)(2x-9)\end{aligned}$$

40. $10x^2+21x-10$
Two expressions whose product is $(10x^2)(-10)=-100x^2$ and whose sum is $21x$ are $25x$ and $-4x$. So,

$$\begin{aligned}10x^2+21x-10&=10x^2+25x-4x-10\\&=5x(2x+5)-2(2x+5)\\&=(2x+5)(5x-2)\end{aligned}$$

41. $$\begin{aligned}y^4-16x^4&=\left(y^2\right)^2-\left(4x^2\right)^2\\&=\left(y^2-4x^2\right)\left(y^2+4x^2\right)\\&=(y-2x)(y+2x)\left(y^2+4x^2\right)\end{aligned}$$

42. $$\begin{aligned}x^8-81&=\left(x^4\right)^2-9^2\\&=\left(x^4+9\right)\left(x^4-9\right)\\&=\left(x^4+9\right)\left(x^2+3\right)\left(x^2-3\right)\end{aligned}$$

43. $$\begin{aligned}x^4-8x^2+16&=\left(x^2\right)^2-2\cdot 4x^2+4^2=\left(x^2-4\right)^2\\&=\left[(x-2)(x+2)\right]^2\\&=(x-2)^2(x+2)^2\end{aligned}$$

44. $81-18x^2+x^4$
Two expressions whose product is $(81)(x^4)=81x^4$ and whose sum is $-18x^2$ are $-9x^2$ and $-9x^2$. So,

$$\begin{aligned}x^4-18x^2+81&=x^4-9x^2-9x^2+81\\&=x^2\left(x^2-9\right)-9\left(x^2-9\right)\\&=\left(x^2-9\right)\left(x^2-9\right)\\&=(x-3)(x+3)(x-3)(x+3)\\&=(x-3)^2(x+3)^2.\end{aligned}$$

45. $$\begin{aligned}4x^4-5x^2+1&=\left(4x^2-1\right)\left(x^2-1\right)\\&=(2x+1)(2x-1)(x+1)(x-1)\end{aligned}$$

46. x^4-3x^2-4
Two expression whose product is $(x^4)(-4)=-4x^4$ and whose sum is $-3x^2$ are $-4x^2$ and x^2. So,

$$\begin{aligned}x^4-3x^2-4&=x^4+x^2-4x^2-4\\&=x^2\left(x^2+1\right)-4\left(x^2+1\right)\\&=\left(x^2+1\right)(x+2)(x-2)\end{aligned}$$

47. $$\begin{aligned}x^{3/2}+x^{1/2}&=x^{1/2}(x^{2/2}+1)\\&=x^{1/2}(x+1)\\?&=x+1\end{aligned}$$

48. $$\begin{aligned}2x^{1/4}+4x^{3/4}&=2x^{1/4}\left(1+2x^{2/4}\right)\\&=2x^{1/4}\left(1+2x^{1/2}\right)\\?&=1+2x^{1/2}\end{aligned}$$

49. $$\begin{aligned}x^{-3}+x^{-2}&=x^{-3}\left(1+x^1\right)\\&=x^{-3}(1+x)\\?&=1+x\end{aligned}$$

50. $$\begin{aligned}x^{-1}-x&=x^{-1}\left(1-x^2\right)\\?&=1-x^2\end{aligned}$$

51. $x^3+3x^2+3x+1=(x+1)^3$

52. The expression $x^3+6x^2+12x+8$ is a perfect cube $(a+b)^3$ with $a=x$ and $b=2$. So,
$x^3+6x^2+12x+8=(x+2)^3$.

53. $x^3-12x^2+48x-64=x^3-3\left(4x^2\right)+3\left(16x\right)-4^3$
$=x^3-3x^2(4)+3x(4)^2-4^3$
$=\left(x-4\right)^3$

54. The expression $y^3-9y^2+27y-27$ is a perfect cube $(a-b)^3$ with $a=y$ and $b=3$. So, $y^3-9y^2+27y-27=(y-3)^3$.

55. $x^3-64=x^3-4^3=\left(x-4\right)\left(x^2+4x+16\right)$

56. $8x^3-1=(2x)^3-(1)^3=\left(2x-1\right)\left(4x^2+2x+1\right)$

57. $27+8x^3=3^3+(2x)^3=\left(3+2x\right)\left(9-6x+4x^2\right)$

58. $a^3+216=(a)^3+(6)^3=\left(a+6\right)\left(a^2-6a+36\right)$

59. $P+Prt=P\left(1+rt\right)$

60. $R=\dfrac{cm^2}{2}-\dfrac{m^3}{3}=m^2\left(\dfrac{c}{2}-\dfrac{m}{3}\right)$

61. $S=cm-m^2=m\left(c-m\right)$

62. $V=64x-32x^2+4x^3=4x\left(16-8x+x^2\right)$
$=4x\left(4-x\right)^2$

63. **a.** In the form px we have $p\left(10{,}000-100p\right)$.
$x=10{,}000-100p$
b. If $p=38$, then $x=10{,}000-100\cdot 38=6200$.

64. $\left(R+r\right)^2-2r\left(R+r\right)=\left(R+r\right)\left(R+r-2r\right)$
$=\left(R+r\right)\left(R-r\right)$

65. **a.** $R=x\left(300-x\right)$
b. $P=300-x$

66. $r^2-\left(r-x\right)^2$
$=\left[r+\left(r-x\right)\right]\left[r-\left(r-x\right)\right]$
$=\left[2r-x\right]\left[x\right]=x\left(2r-x\right)$

Exercises 0.7

1. $\dfrac{18x^3y^3}{9x^3z}=\dfrac{2x^3y^3}{x^3z}=\dfrac{2y^3}{z}$

2. $\dfrac{15a^4b^5}{30a^3b}=\dfrac{15a^3b(ab^4)}{15a^3b(2)}=\dfrac{ab^4}{2}$

3. $\dfrac{x-3y}{3x-9y}=\dfrac{1(x-3y)}{3(x-3y)}=\dfrac{1}{3}$

4. $\dfrac{x^2-6x+8}{x^2-16}=\dfrac{(x-4)(x-2)}{(x-4)(x+4)}=\dfrac{x-2}{x+4}$

5. $\dfrac{x^2-2x+1}{x^2-4x+3}=\dfrac{(x-1)(x-1)}{(x-3)(x-1)}=\dfrac{x-1}{x-3}$

6. $\dfrac{x^2-5x+6}{9-x^2}=\dfrac{(x-3)(x-2)}{(3-x)(3+x)}$
$=\dfrac{-(3-x)(x-2)}{(3-x)(3+x)}$
$=\dfrac{-(x-2)}{x+3}$

7. $\dfrac{6x^3}{8y^3}\cdot\dfrac{16x}{9y^2}\cdot\dfrac{15y^4}{x^3}=\dfrac{6}{y^3}\cdot\dfrac{2x}{9y^2}\cdot\dfrac{15y^4}{1}=\dfrac{2}{1}\cdot\dfrac{2x}{3y^2}\cdot\dfrac{15y}{1}$
$=\dfrac{2}{1}\cdot\dfrac{2x}{y}\cdot\dfrac{5}{1}$
$=\dfrac{20x}{y}$

8. $\dfrac{25ac^2}{15a^2c}\cdot\dfrac{4ad^4}{15abc^3}=\dfrac{100a^2c^2d^4}{225a^3bc^4}=\dfrac{25a^2c^2(4d^4)}{25a^2c^2(9abc^2)}$
$=\dfrac{4d^4}{9abc^2}$

9. $\dfrac{8x-16}{x-3}\cdot\dfrac{4x-12}{3x-6}=\dfrac{8(x-2)}{x-3}\cdot\dfrac{4(x-3)}{3(x-2)}=\dfrac{8\cdot 4}{3}=\dfrac{32}{3}$

10. $\dfrac{(x^2-4)}{1}\cdot\dfrac{(2x-3)}{(x+2)}=\dfrac{(x-2)(x+2)}{1}\cdot\dfrac{(2x-3)}{(x+2)}$
$=(x-2)(2x-3)$

11. $$\frac{x^2+7x+12}{3x^2+13x+4}\cdot\frac{9x+3}{1}$$
$$=\frac{(x+4)(x+3)}{(3x+1)(x+4)}\cdot\frac{3(3x+1)}{1}$$
$$=3(x+3)$$
$$=3x+9$$

12. $$\frac{4x+4}{x-4}\cdot\frac{x^2-6x+8}{8x^2+8x}=\frac{4(x+1)}{x-4}\cdot\frac{(x-4)(x-2)}{8x(x+1)}$$
$$=\frac{x-2}{2x}$$

13. $$\frac{x^2-x-2}{2x^2-8}\cdot\frac{18-2x^2}{x^2-5x+4}\cdot\frac{x^2-2x-8}{x^2-6x+9}$$
$$=\frac{(x-2)(x+1)}{2(x^2-4)}\cdot\frac{-2(x^2-9)}{(x-4)(x-1)}\cdot\frac{(x-4)(x+2)}{(x-3)(x-3)}$$
$$=\frac{(x-2)(x+1)}{2(x-2)(x+2)}\cdot\frac{-2(x-3)(x+3)}{(x-1)}\cdot\frac{(x+2)}{(x-3)(x-3)}$$
$$=-\frac{(x+1)(x+3)}{(x-1)(x-3)}$$

14. $$\frac{x^2-5x-6}{x^2-5x+4}\cdot\frac{x^2-x-12}{x^3-6x^2}\cdot\frac{x-x^3}{x^2-2x+1}$$
$$=\frac{(x-6)(x+1)}{(x-4)(x-1)}\cdot\frac{(x-4)(x+3)}{x^2(x-6)}\cdot\frac{x(1-x)(1+x)}{(x-1)(x-1)}$$
$$=\frac{(x+1)^2(x+3)(1-x)}{x(x-1)^3}$$
$$=\frac{-(x+1)^2(x+3)(x-1)}{x(x-1)^3}$$
$$=\frac{-(x+1)^2(x+3)}{x(x-1)^2}$$

15. $$\frac{15ac^2}{7bd}\div\frac{4a}{14b^2d}=\frac{15ac^2}{7bd}\cdot\frac{14b^2d}{4a}$$
$$=\frac{15c^2}{1}\cdot\frac{2b}{4}$$
$$=\frac{15bc^2}{2}$$

16. $$\frac{16}{x-2}\div\frac{4}{3x-6}=\frac{16}{x-2}\cdot\frac{3x-6}{4}$$
$$=\frac{16}{x-2}\cdot\frac{3(x-2)}{4}$$
$$=12$$

17. $$\frac{y^2-2y+1}{7y^2-7y}\div\frac{y^2-4y+3}{35y^2}$$
$$=\frac{y^2-2y+1}{7y(y-1)}\cdot\frac{35y^2}{y^2-4y+3}$$
$$=\frac{(y-1)(y-1)}{7y(y-1)}\cdot\frac{35y^2}{(y-3)(y-1)}$$
$$=\frac{5y}{y-3}$$

18. $$\frac{6x^2}{4x^2y-12xy}\div\frac{3x^2+12x}{x^2+x-12}$$
$$=\frac{6x^2}{4xy(x-3)}\cdot\frac{(x+4)(x-3)}{3x(x+4)}$$
$$=\frac{2}{4y}$$
$$=\frac{1}{2y}$$

19. $$\frac{x^2-x-6}{1}\div\frac{9-x^2}{x^2-3x}$$
$$=\frac{x^2-x-6}{1}\cdot\frac{x^2-3x}{-1(x^2-9)}$$
$$=\frac{(x-3)(x+2)}{1}\cdot\frac{x(x-3)}{-1(x-3)(x+3)}$$
$$=\frac{-x(x-3)(x+2)}{x+3}$$

20. $$\frac{2x^2+7x+3}{4x^2-1}\div(x+3)=\frac{(2x+1)(x+3)}{(2x-1)(2x+1)}\cdot\frac{1}{x+3}$$
$$=\frac{1}{2x-1}$$

21. $$\frac{2x}{x^2-x-2}-\frac{x+2}{x^2-x-2}=\frac{2x-x-2}{(x-2)(x+1)}$$
$$=\frac{x-2}{(x-2)(x+1)}$$
$$=\frac{1}{x+1}$$

22. $$\frac{4}{9-x^2}-\frac{x+1}{9-x^2}=\frac{4-(x+1)}{9-x^2}=\frac{4-x-1}{9-x^2}$$
$$=\frac{3-x}{(3+x)(3-x)}$$
$$=\frac{1}{3+x}$$

23. $$\frac{a}{a-2}-\frac{a-2}{a}=\frac{a}{a-2}\cdot\frac{a}{a}-\frac{a-2}{a}\cdot\frac{a-2}{a-2}$$
$$=\frac{a^2-(a^2-4a+4)}{a(a-2)}$$
$$=\frac{4a-4}{a(a-2)}$$
$$=\frac{4(a-1)}{a(a-2)}$$

24. $$x-\frac{2}{x-1}=\frac{(x-1)x}{x-1}-\frac{2}{x-1}$$
$$=\frac{(x-1)(x)-2}{x-1}$$
$$=\frac{x^2-x-2}{x-1}$$
$$=\frac{(x+1)(x-2)}{x-1}$$

25. $$\frac{x}{x+1}-x+1=\frac{x}{x+1}-\frac{x}{1}\cdot\frac{x+1}{x+1}+\frac{1}{1}\cdot\frac{x+1}{x+1}$$
$$=\frac{x-x^2-x+x+1}{x+1}$$
$$=\frac{-x^2+x+1}{x+1}$$

26. $$\frac{x-1}{x+1}-\frac{2}{x^2+x}=\frac{x-1}{x+1}-\frac{2}{x(x+1)}$$
$$=\frac{x(x-1)}{x(x+1)}-\frac{2}{x(x+1)}$$
$$=\frac{x^2-x-2}{x(x+1)}=\frac{(x+1)(x-2)}{x(x+1)}$$
$$=\frac{x-2}{x}$$

27. $$\frac{4a}{3x+6}+\frac{5a^2}{4x+8}=\frac{4a}{3(x+2)}+\frac{5a^2}{4(x+2)}=\frac{4a}{3(x+2)}\cdot\frac{4}{4}+\frac{5a^2}{4(x+2)}\cdot\frac{3}{3}=\frac{16a+15a^2}{12(x+2)}$$

28. $$\frac{b-1}{b^2+2b}+\frac{b}{3b+6}=\frac{b-1}{b(b+2)}+\frac{b}{3(b+2)}=\frac{3(b-1)}{3b(b+2)}+\frac{b^2}{3b(b+2)}=\frac{b^2+3b-3}{3b(b+2)}$$

29. $$\frac{3x-1}{2x-4}+\frac{4x}{3x-6}-\frac{x-4}{5x-10}=\frac{3x-1}{2(x-2)}+\frac{4x}{3(x-2)}-\frac{x-4}{5(x-2)}$$

$$= \frac{3x-1}{2(x-2)}\cdot\frac{3\cdot 5}{3\cdot 5} + \frac{4x}{3(x-2)}\cdot\frac{2\cdot 5}{2\cdot 5} - \frac{(x-4)}{5(x-2)}\cdot\frac{3\cdot 2}{3\cdot 2}$$
$$= \frac{(45x-15)+40x-6x+24}{30(x-2)}$$
$$= \frac{79x+9}{30(x-2)}$$

30. $\dfrac{2x+1}{4x-2} + \dfrac{5}{2x} - \dfrac{x+4}{2x^2-x} = \dfrac{2x+1}{2(2x-1)} + \dfrac{5}{2x} - \dfrac{x+4}{x(2x-1)}$

$$= \frac{x(2x+1)}{2x(2x-1)} + \frac{5(2x-1)}{2x(2x-1)} - \frac{2(x+4)}{2x(2x-1)}$$
$$= \frac{2x^2+x+10x-5-2x-8}{2x(2x-1)}$$
$$= \frac{2x^2+9x-13}{2x(2x-1)}$$

31. $\dfrac{x}{x^2-4} + \dfrac{4}{x^2-x-2} - \dfrac{x-2}{x^2+3x+2} = \dfrac{x}{(x+2)(x-2)} + \dfrac{4}{(x-2)(x+1)} - \dfrac{x-2}{(x+2)(x+1)}$

$$= \frac{x}{(x+2)(x-2)}\cdot\frac{x+1}{x+1} + \frac{4}{(x-2)(x+1)}\cdot\frac{x+2}{x+2} - \frac{x-2}{(x+2)(x+1)}\cdot\frac{x-2}{x-2}$$

$$= \frac{\left(x^2+x\right)+(4x+8)\left(x^2-4x+4\right)}{(x+2)(x+1)(x-2)} = \frac{9x+4}{(x+2)(x+1)(x-2)}$$

32. $\dfrac{3x^2}{x^2-4} + \dfrac{2}{x^2-4x+4} - 3 = \dfrac{3x^2}{(x+2)(x-2)} + \dfrac{2}{(x-2)^2} - 3$

$$= \frac{3x^2(x-2)}{(x-2)^2(x+2)} + \frac{2(x+2)}{(x-2)^2(x+2)} - \frac{3(x-2)^2(x+2)}{(x-2)^2(x+2)}$$
$$= \frac{3x^3-6x^2+2x+4-3\left(x^2-4x+4\right)(x+2)}{(x-2)^2(x+2)}$$
$$= \frac{3x^3-6x^2+2x+4-3\left(x^3+2x^2-4x^2-8x+4x+8\right)}{(x-2)^2(x+2)}$$
$$= \frac{3x^3-6x^2+2x+4-3x^3-6x^2+12x^2+24x-12x-24}{(x-2)^2(x+2)}$$
$$= \frac{14x-20}{(x-2)^2(x+2)}$$

33. $\dfrac{-x^3+x}{\sqrt{3-x^2}}+\dfrac{2x\sqrt{3-x^2}}{1}$

$=\dfrac{-x^3+x}{\sqrt{3-x^2}}+\dfrac{2x\sqrt{3-x^2}}{1}\cdot\dfrac{\sqrt{3-x^2}}{\sqrt{3-x^2}}$

$=\dfrac{-x^3+x+2x\left(3-x^2\right)}{\sqrt{3-x^2}}$

$=\dfrac{-x^3+x+6x-2x^3}{\sqrt{3-x^2}}$

$=\dfrac{7x-3x^3}{\sqrt{3-x^2}}$

34. $\dfrac{3x^2\left(x+1\right)}{\sqrt{x^3+1}}+\sqrt{x^3+1}$

$=\dfrac{3x^2\left(x+1\right)}{\sqrt{x^3+1}}+\dfrac{\left(\sqrt{x^3+1}\right)\left(\sqrt{x^3+1}\right)}{\sqrt{x^3+1}}$

$=\dfrac{3x^2\left(x+1\right)+x^3+1}{\sqrt{x^3+1}}$

$=\dfrac{3x^3+3x^2+x^3+1}{\sqrt{x^3+1}}$

$=\dfrac{4x^3+3x^2+1}{\sqrt{x^3+1}}$

35. $\dfrac{\frac{3}{1}-\frac{2}{3}}{\frac{14}{1}}\cdot\dfrac{3}{3}=\dfrac{9-2}{14(3)}=\dfrac{7}{14(3)}=\dfrac{1}{6}$

36. $\dfrac{4}{\frac{1}{4}+\frac{1}{4}}=\dfrac{4}{\frac{1}{2}}=8$

37. $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}}=\dfrac{(x+y)}{\frac{1}{x}+\frac{1}{y}}\cdot\dfrac{xy}{xy}=\dfrac{xy(x+y)}{y+x}=xy$

38. $\dfrac{\frac{5}{2y}+\frac{3}{y}}{\frac{1}{4}+\frac{1}{3y}}\cdot\dfrac{12y}{12y}=\dfrac{30+36}{3y+4}=\dfrac{66}{3y+4}$

39. $\dfrac{2-\frac{1}{x}}{2x-\frac{3x}{x+1}}=\dfrac{\frac{2}{1}-\frac{1}{x}}{\frac{2x}{1}-\frac{3x}{x+1}}\cdot\dfrac{x(x+1)}{x(x+1)}$

$=\dfrac{2x(x+1)-1(x+1)}{2x^2(x+1)-3x(x)}$

$=\dfrac{2x^2+x-1}{2x^3-x^2}$

$=\dfrac{(2x-1)(x+1)}{x^2(2x-1)}=\dfrac{x+1}{x^2}$

40. $\dfrac{1-\frac{2}{x-2}}{x-6+\frac{10}{x+1}}\cdot\dfrac{(x-2)(x+1)}{(x-2)(x+1)}$

$=\dfrac{(x-2)(x+1)-2(x+1)}{(x-6)(x-2)(x+1)+10(x-2)}$

$=\dfrac{x^2+x-2x-2-2x-2}{(x^2-2x-6x+12)(x+1)+10(x-2)}$

$=\dfrac{x^2-3x-4}{(x^2-8x+12)(x+1)+10x-20}$

$=\dfrac{x^2-3x-4}{x^3-8x^2+12x+x^2-8x+12+10x-20}$

$=\dfrac{(x-4)(x+1)}{x^3-7x^2+14x-8}$

$=\dfrac{(x-4)(x+1)}{(x-4)(x^2-3x+2)}$

$=\dfrac{x+1}{x^2-3x+2}=\dfrac{x+1}{(x-1)(x-2)}$

41. $\dfrac{\sqrt{a}-\frac{b}{\sqrt{a}}}{a-b}=\dfrac{\frac{\sqrt{a}}{1}-\frac{b}{\sqrt{a}}}{\frac{a-b}{1}}\cdot\dfrac{\sqrt{a}}{\sqrt{a}}=\dfrac{a-b}{\sqrt{a}(a-b)}$

$=\dfrac{1}{\sqrt{a}}$ or $\dfrac{\sqrt{a}}{a}$

42. $\dfrac{\sqrt{x-1}+\frac{1}{\sqrt{x-1}}}{x}\cdot\dfrac{\sqrt{x-1}}{\sqrt{x-1}}=\dfrac{(\sqrt{x-1})^2+1}{x\sqrt{x-1}}$

$=\dfrac{x-1+1}{x\sqrt{x-1}}=\dfrac{1}{\sqrt{x-1}}$

43. **a.** $(2^{-2}-3^{-1})^{-1}=\left(\dfrac{1}{2^2}-\dfrac{1}{3}\right)^{-1}=\left(-\dfrac{1}{12}\right)^{-1}=-12$

b. $(2^{-1}+3^{-1})^2=\left(\dfrac{1}{2}+\dfrac{1}{3}\right)^2=\left(\dfrac{5}{6}\right)^2=\dfrac{25}{36}$

Hint: Work inside ( ) first when adding or subtracting is involved.

44. a. $(3^2+4^2)^{-1/2}=(9+16)^{-1/2}=25^{-1/2}=\dfrac{1}{5}$

b. $(2^2+3^2)^{-1}=\dfrac{1}{2^2+3^2}=\dfrac{1}{4+9}=\dfrac{1}{13}$

45. $\dfrac{2a^{-1}-b^{-2}}{(ab^2)^{-1}}=\dfrac{\frac{2}{a}-\frac{1}{b^2}}{\frac{1}{ab^2}}\cdot\dfrac{ab^2}{ab^2}=\dfrac{2b^2-a}{1}$ or $2b^2-a$

46. $\dfrac{x^{-2}+xy^{-2}}{(x^2y)^{-2}}=\dfrac{\frac{1}{x^2}+\frac{x}{y^2}}{\frac{1}{x^4y^2}}\cdot\dfrac{x^4y^2}{x^4y^2}=\dfrac{x^2y^2+x^5}{1}$

$=x^2y^2+x^5=x^2(y^2+x^3)$

47. $\dfrac{1-\sqrt{x}}{1+\sqrt{x}}=\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\cdot\dfrac{1-\sqrt{x}}{1-\sqrt{x}}=\dfrac{1-2\sqrt{x}+x}{1-x}$

48. $\dfrac{x-3}{x-\sqrt{3}}\cdot\dfrac{x+\sqrt{3}}{x+\sqrt{3}}=\dfrac{x^2+\sqrt{3}x-3x-3\sqrt{3}}{x^2-3}$

$=\dfrac{x^2+(\sqrt{3}-3)x-3\sqrt{3}}{x^2-3}$

49. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}=\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$=\dfrac{(x+h)-(x)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\dfrac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$=\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

50. $\dfrac{\sqrt{9+2h}-3}{h}\cdot\dfrac{\sqrt{9+2h}+3}{\sqrt{9+2h}+3}=\dfrac{9+2h-9}{h(\sqrt{9+2h}+3)}$

$=\dfrac{2h}{h(\sqrt{9+2h}+3)}=\dfrac{2}{\sqrt{9+2h}+3}$

51. $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=\dfrac{1}{a}\cdot\dfrac{bc}{bc}+\dfrac{1}{b}\cdot\dfrac{ac}{ac}+\dfrac{1}{c}\cdot\dfrac{ab}{ab}=\dfrac{bc+ac+ab}{abc}$

52. a. $\dfrac{1}{p}+\dfrac{1}{q}-\dfrac{d}{pq}=\dfrac{q}{pq}+\dfrac{p}{pq}-\dfrac{d}{pq}=\dfrac{q+p-d}{pq}$

b. The reciprocal is $\dfrac{pq}{q+p-d}$.

53. a. Avg. cost $=\dfrac{4000}{x}+\dfrac{55}{1}+\dfrac{0.1x}{1}$

$=\dfrac{4000+55x+0.1x^2}{x}$

b. Total cost = (Avg. cost)(number of units)

$=4000+55x+0.1x^2$

54. a. $\dfrac{40,500}{x}+190+0.2x$

$=\dfrac{0.2x^2+190x+40,500}{x}$

b. Total cost = (Avg. cost)(number of units)

$=0.2x^2+190x+40,500$

55. $SV=1+\dfrac{3}{t+3}-\dfrac{18}{(t+3)^2}=\dfrac{(t+3)^2+3(t+3)-18}{(t+3)^2}=\dfrac{t^2+6t+9+3t+9-18}{(t+3)^2}=\dfrac{t^2+9t}{(t+3)^2}$

56. $\dfrac{(1+i)^{n+1}-1}{i}-1=\dfrac{(1+i)^{n+1}-1}{i}-\dfrac{i}{i}=\dfrac{(1+i)^{n+1}-1-i}{i}$

$=\dfrac{(1+i)^{n+1}-(1+i)^1}{i}=\dfrac{(1+i)\left[(1+i)^n-1\right]}{i}$

Chapter 0: Algebraic Concepts

Chapter 0 Review Exercises

1. Yes. $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Since every element of A is also an element of B, A is a subset of B.

2. No. $3 \notin \{x : x > 3\}$

3. No. A and B are not disjoint since each set contains the element 1.

4. $A = \{1, 2, 3, 9\}$ $B' = \{2, 4, 9\}$.
$A \cup B' = \{1, 2, 3, 4, 9\}$

5. $\{4, 5, 6, 7, 8, 10\} \cap \{1, 3, 5, 6, 7, 8, 10\}$
$= \{5, 6, 7, 8, 10\}$

6. $A = \{1, 2, 3, 9\}$ $B = \{1, 3, 5, 6, 7, 8, 10\}$
$A' = \{4, 5, 6, 7, 8, 10\}$
$A' \cap B = \{5, 6, 7, 8, 10\}$
$(A' \cap B)' = \{1, 2, 3, 4, 9\}$

7. $\{4, 5, 6, 7, 8, 10\} \cup \{2, 4, 9\}$
$= \{2, 4, 5, 6, 7, 8, 9, 10\}$
$(A' \cup B')' = \{2, 4, 5, 6, 7, 8, 9, 10\}' = \{1, 3\}$
$A \cap B = \{1, 2, 3, 9\} \cap \{1, 3, 5, 6, 7, 8, 10\}$
$= \{1, 3\}$ Yes.

8. **a.** $6 + \frac{1}{3} = \frac{1}{3} + 6$ illustrates the Commutative Property of Addition.
b. $2(3 \cdot 4) = (2 \cdot 3)4$ illustrates the Associative Property of Multiplication.
c. $\frac{1}{3}(6+9) = 2+3$ illustrates the Distributive Law.

9. **a.** irrational
b. rational, integer
c. undefined

10. **a.** $\pi > 3.14$
b. $-100 < 0.1$
c. $-3 > -12$

11. $|5-11| = |-6| = -(-6) = 6$

12. $44 \div 2 \cdot 11 - 10^2 = 22 \cdot 11 - 100 = 242 - 100 = 142$

13. $(-3)^2 - (-1)^3 = 9 - (-1) = 10$

14. $\dfrac{(3)(2)(15)-(5)(8)}{(4)(10)} = \dfrac{90-40}{40} = \dfrac{50}{40} = \dfrac{5}{4}$

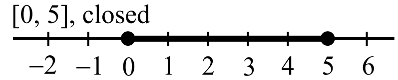
15.
$$\begin{aligned} 2-[3-(2-|-3|)]+11 &= 2-[3-(2-3)]+11 \\ &= 2-[3-(-1)]+11 \\ &= 2-[3+1]+11 \\ &= 2-4+11 \\ &= 9 \end{aligned}$$

16. $-4^2 - (-4)^2 + 3 = -16 - 16 + 3 = -32 + 3 = -29$

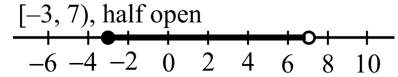
17. $\dfrac{4+3^2}{4} = \dfrac{4+9}{4} = \dfrac{13}{4}$

18. $\dfrac{(-2.91)^5}{\sqrt{3.29^5}} \approx \dfrac{-208.6724}{19.6331} \approx -10.62857888$

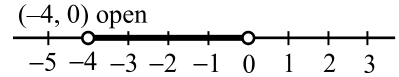
19. **a.** [0, 5], closed



b. [–3, 7), half open



c. (–4, 0) open



20. **a.** $(-1, 16)$
$-1 < x < 16$
b. $[-12, 8]$
$-12 \le x \le 8$
c. $x < -1$

21. **a.** $\left(\frac{3}{8}\right)^0 = 1$
b. $2^3 \cdot 2^{-5} = 2^{-2} = \dfrac{1}{2^2} = \dfrac{1}{4}$
c. $\dfrac{4^9}{4^3} = 4^6 = 4096$
d. $\left(\frac{1}{7}\right)^3 \left(\frac{1}{7}\right)^{-4} = \left(\frac{1}{7}\right)^{-1} = 7$

22. a. $x^5 \cdot x^{-7} = x^{5+(-7)} = x^{-2} = \dfrac{1}{x^2}$

b. $\dfrac{x^8}{x^{-2}} = x^{8-(-2)} = x^{10}$

c. $(x^3)^3 = x^{3\cdot 3} = x^9$

d. $(y^4)^{-2} = y^{(4)(-2)} = y^{-8} = \dfrac{1}{y^8}$

e. $(-y^{-3})^{-2} = y^{(-3)(-2)} = y^6$

There are other correct methods of working problems 23–28.

23. $\dfrac{-(2xy^2)^{-2}}{(3x^{-2}y^{-3})^2} = \dfrac{(-1)(2)^{-2}x^{-2}y^{-4}}{3^2x^{-4}y^{-6}}$

$= \dfrac{(-1)x^4y^6}{2^2\cdot 3^2x^2y^4}$

$= -\dfrac{x^2y^2}{36}$

24. $\left(\dfrac{2}{3}x^2y^{-4}\right)^{-2} = \left(\dfrac{2}{3}\right)^{-2}\left(x^2\right)^{-2}\left(y^{-4}\right)^{-2}$

$= \left(\dfrac{3}{2}\right)^2\left(x^{-4}\right)\left(y^8\right)$

$= \left(\dfrac{9}{4}\right)\left(\dfrac{1}{x^4}\right)\left(y^8\right)$

$= \dfrac{9y^8}{4x^4}$

25. $\left(\dfrac{x^{-2}}{2y^{-1}}\right)^2 = \left(\dfrac{y}{2x^2}\right)^2 = \dfrac{y^2}{4x^4}$

26. $\dfrac{\left(-x^4y^{-2}z^2\right)^0}{-\left(x^4y^{-2}z^2\right)^{-2}} = \dfrac{1}{-\left(x^4\right)^{-2}\left(y^{-2}\right)^{-2}\left(z^2\right)^{-2}}$

$= \dfrac{1}{-x^{-8}y^4z^{-4}} = \dfrac{-x^8z^4}{y^4}$

27. $\left(\dfrac{x^{-3}y^4z^{-2}}{3x^{-2}y^{-3}z^{-3}}\right)^{-1} = \left(\dfrac{y^{4-(-3)}z^{-2-(-3)}}{3x^{-2-(-3)}}\right)^{-1}$

$= \left(\dfrac{y^7z}{3x}\right)^{-1} = \dfrac{3x}{y^7z}$

28. $\left(\dfrac{x}{2y}\right)\left(\dfrac{y}{x^2}\right)^{-2} = \left(\dfrac{x}{2y}\right)\left(\dfrac{x^2}{y}\right)^2$

$= \left(\dfrac{x}{2y}\right)\left(\dfrac{(x^2)^2}{y^2}\right) = \left(\dfrac{x}{2y}\right)\left(\dfrac{x^4}{y^2}\right) = \dfrac{x^5}{2y^3}$

29. a. $-\sqrt[3]{-64} = -\sqrt[3]{(-4)^3} = -(-4) = 4$

b. $\sqrt{\dfrac{4}{49}} = \sqrt{\dfrac{2^2}{7^2}} = \dfrac{2}{7}$

c. $\sqrt[7]{1.9487171} = 1.1$

30. a. $\sqrt{x} = x^{1/2}$

b. $\sqrt[3]{x^2} = x^{2/3}$

c. $1/\sqrt[4]{x} = \dfrac{1}{x^{1/4}} = x^{-1/4}$

31. a. $x^{3/7} = \sqrt[7]{x^3}$

b. $x^{-1/2} = \dfrac{1}{\sqrt{x}} = \dfrac{\sqrt{x}}{x}$

c. $-x^{3/2} = -x\sqrt{x}$

32. a. $\dfrac{5xy}{\sqrt{2x}}\cdot\dfrac{\sqrt{2x}}{\sqrt{2x}} = \dfrac{5xy\sqrt{2x}}{2x} = \dfrac{5y\sqrt{2x}}{2}$

b. $\dfrac{y}{x\sqrt[3]{xy^2}}\cdot\dfrac{\sqrt[3]{x^2y}}{\sqrt[3]{x^2y}} = \dfrac{y\sqrt[3]{x^2y}}{x\sqrt[3]{x^3y^3}}$

$= \dfrac{y\sqrt[3]{x^2y}}{x(xy)}$

$= \dfrac{y\sqrt[3]{x^2y}}{x^2y}$

$= \dfrac{\sqrt[3]{x^2y}}{x^2}$

33. $x^{1/2}\cdot x^{1/3} = x^{(3/6)+(2/6)} = x^{5/6}$

34. $\dfrac{y^{-3/4}}{y^{-7/4}} = y^{-3/4-(-7/4)} = y^{4/4} = y$

35. $x^4\cdot x^{1/4} = x^{(16/4)+(1/4)} = x^{17/4}$

36. $\dfrac{1}{x^{-4/3}\cdot x^{-7/3}} = \dfrac{1}{x^{-11/3}} = x^{11/3}$

37. $(x^{4/5})^{1/2} = x^{(4/5)(1/2)} = x^{2/5}$

38. $(x^{1/2}y^2)^4 = (x^{1/2})^4(y^2)^4 = x^2y^8$

39. $\sqrt{12x^3y^5} = \sqrt{4x^2y^4 \cdot 3xy} = 2xy^2\sqrt{3xy}$

40. $\sqrt{1250x^6y^9} = \sqrt{625x^6y^8 \cdot 2y} = 25x^3y^4\sqrt{2y}$

41. $$\begin{aligned}\sqrt[3]{24x^4y^4} \cdot \sqrt[3]{45x^4y^{10}} &= \sqrt[3]{8x^3y^3 \cdot 3xy} \cdot \sqrt[3]{9x^3y^9 \cdot 5xy}\\ &= 2xy\sqrt[3]{3xy} \cdot xy^3\sqrt[3]{9 \cdot 5xy}\\ &= 2x^2y^4\sqrt[3]{27 \cdot 5x^2y^2}\\ &= 6x^2y^4\sqrt[3]{5x^2y^2}\end{aligned}$$

42. $$\begin{aligned}\sqrt{16a^2b^3} \cdot \sqrt{8a^3b^5} &= \sqrt{128a^5b^8}\\ &= \sqrt{64a^4b^8 \cdot 2a}\\ &= 8a^2b^4\sqrt{2a}\end{aligned}$$

43. $\dfrac{\sqrt{52x^3y^6}}{\sqrt{13xy^4}} = \sqrt{4x^2y^2} = 2xy$

44. $\dfrac{\sqrt{32x^4y^3}}{\sqrt{6xy^{10}}} = \sqrt{\dfrac{16x^3}{3y^7}} = \dfrac{4x\sqrt{x}}{y^3\sqrt{3y}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{4x\sqrt{3xy}}{3y^4}$

45. $(3x+5)-(4x+7) = 3x+5-4x-7 = -x-2$

46. $x(1-x)+x[x-(2+x)] = x-x^2+x(-2) = -x^2-x$

47. $(3x^3-4xy-3)+(5xy+x^3+4y-1)$
$= 4x^3+xy+4y-4$

48. $(4xy^3)(6x^4y^2) = 24x^{1+4}y^{3+2} = 24x^5y^5$

49. $(3x-4)(x-1) = 3x^2-3x-4x+4 = 3x^2-7x+4$

50. $(3x-1)(x+2) = 3x^2+6x-x-2 = 3x^2+5x-2$

51. $(4x+1)(x-2) = 4x^2-8x+x-2 = 4x^2-7x-2$

52. $$\begin{aligned}(3x-7)(2x+1) &= 6x^2+3x-14x-7\\ &= 6x^2-11x-7\end{aligned}$$

53. $(2x-3)^2 = (2x)^2-2(2x)(3)+3^2 = 4x^2-12x+9$

54. $(4x+3)(4x-3) = 16x^2-9$
Difference of two squares

55. $$\begin{array}{r} x^2+x-3\\ \underline{2x^2+1}\\ x^2+x-3\\ \underline{2x^4+2x^3-6x^2}\\ 2x^4+2x^3-5x^2+x-3\end{array}$$

56. $(2x-1)^3 = 8x^3-12x^2+6x-1$ Binomial cubed

57. $$\begin{array}{r} x^2+xy+y^2\\ \underline{x-y}\\ -x^2y-xy^2-y^3\\ \underline{x^3+x^2y+xy^2}\\ x^3-y^3\end{array}$$ Difference of two cubes

58. $\dfrac{4x^2y-3x^3y^3-6x^4y^2}{2x^2y^2} = \dfrac{2}{y}-\dfrac{3xy}{2}-3x^2$

59. $$\begin{array}{r} 3x^2+2x-3\\ x^2+1\overline{\smash{)}3x^4+2x^3-x+4}\\ \underline{3x^4+3x^2}\\ 2x^3-3x^2-x+4\\ \underline{2x^3+2x}\\ -3x^2-3x+4\\ \underline{-3x^2-3}\\ -3x+7\end{array}$$

Quotient is $3x^2+2x-3+\dfrac{7-3x}{x^2+1}$.

60. $$\begin{array}{r} x^3-x^2+2x+7\\ x-3\overline{\smash{)}x^4-4x^3+5x^2+x}\\ \underline{x^4-3x^3}\phantom{+5x^2+x}\\ -x^3+5x^2\phantom{+x}\\ \underline{-x^3+3x^2}\phantom{+x}\\ 2x^2+x\\ \underline{2x^2-6x}\\ 7x\\ \underline{7x-21}\\ 21\end{array}$$

Quotient is $x^3-x^2+2x+7+\dfrac{21}{x-3}$.

61. $x^{4/3}(x^{2/3}-x^{-1/3}) = x^{6/3}-x^{3/3} = x^2-x$

62. $\left(\sqrt{x}+\sqrt{a-x}\right)\left(\sqrt{x}-\sqrt{a-x}\right)=\left(\sqrt{x}\right)^2-\left(\sqrt{a-x}\right)^2$
$$\begin{aligned}&=x-(a-x)\\&=x-a+x\\&=2x-a\end{aligned}$$

63. $2x^4-x^3=x^3(2x-1)$

64. $4(x^2+1)^2-2(x^2+1)^3=2(x^2+1)^2[2-(x^2+1)]$
$$\begin{aligned}&=2(x^2+1)^2(2-x^2-1)\\&=2(x^2+1)^2(1-x^2)\\&=2(x^2+1)^2(1+x)(1-x)\end{aligned}$$

65. $4x^2-4x+1=(2x)^2-2(2x)+1^2=(2x-1)^2$

66. $16-9x^2=(4+3x)(4-3x)$

67. $2x^4-8x^2=2x^2(x^2-4)=2x^2(x+2)(x-2)$

68. $x^2-4x-21=(x-7)(x+3)$

69. $3x^2-x-2=(3x+2)(x-1)$

70. $x^2-5x+6=\left(x-3\right)\left(x-2\right)$

71. $x^2-10x-24=\left(x-12\right)\left(x+2\right)$

72. $12x^2-23x-24$
Two expressions whose product is $12x^2(-24)=-288x^2$ and whose sum is $-23x$ are $-32x$ and $9x$. So,
$$\begin{aligned}12x^2-23x-24&=12x^2+9x-32x-24\\&=3x(4x+3)-8(4x+3)\\&=(4x+3)(3x-8).\end{aligned}$$

73. $16x^4-72x^2+81=(4x^2)^2-2(4x^2\cdot 9)+9^2$
$$\begin{aligned}&=(4x^2-9)^2\\&=[(2x+3)(2x-3)]^2\\&=(2x+3)^2(2x-3)^2\end{aligned}$$

74. $x^{-2/3}+x^{-4/3}=x^{-4/3}(?)$
$x^{-2/3}+x^{-4/3}=x^{-4/3}(x^{2/3}+1)$
$?=x^{2/3}+1$

75. **a.** $\dfrac{2x}{2x+4}=\dfrac{2x}{2(x+2)}=\dfrac{x}{x+2}$

b. $\dfrac{4x^2y^3-6x^3y^4}{2x^2y^2-3xy^3}=\dfrac{2x^2y^3(2-3xy)}{xy^2(2x-3y)}$
$$=\frac{2xy(2-3xy)}{2x-3y}$$

76. $\dfrac{x^2-4x}{x^2+4}\cdot\dfrac{x^4-16}{x^4-16x^2}=\dfrac{x(x-4)}{x^2+4}\cdot\dfrac{(x^2-4)(x^2+4)}{x^2(x^2-16)}$
$$\begin{aligned}&=\frac{(x-4)(x+2)(x-2)}{x(x-4)(x+4)}\\&=\frac{(x+2)(x-2)}{x(x+4)}\\&=\frac{x^2-4}{x\left(x+4\right)}\end{aligned}$$

77. $\dfrac{x^2+6x+9}{x^2-7x+12}\cdot\dfrac{x^2-3x-4}{x^2+4x+3}$
$$=\frac{(x+3)(x+3)}{(x-4)(x-3)}\cdot\frac{(x-4)(x+1)}{(x+3)(x+1)}=\frac{x+3}{x-3}$$

78. $\dfrac{x^4-2x^3}{3x^2-x-2}\div\dfrac{x(x^2-4)}{9x^2-4}$
$$\begin{aligned}&=\frac{x^3(x-2)}{(3x+2)(x-1)}\cdot\frac{(3x+2)(3x-2)}{x(x+2)(x-2)}\\&=\frac{x^2(3x-2)}{(x-1)(x+2)}\end{aligned}$$

79. $1+\dfrac{3}{2x}-\dfrac{1}{6x^2}=\dfrac{1}{1}\cdot\dfrac{6x^2}{6x^2}+\dfrac{3}{2x}\cdot\dfrac{3x}{3x}-\dfrac{1}{6x^2}$
$$=\frac{6x^2+9x-1}{6x^2}$$

80. $\dfrac{1}{x-2}-\dfrac{x-2}{4}=\dfrac{1\cdot 4-(x-2)(x-2)}{4(x-2)}=\dfrac{4x-x^2}{4(x-2)}$

81. $\dfrac{x+2}{x(x-1)}-\dfrac{x^2+4}{(x-1)(x-1)}+\dfrac{1}{1}$

$=\dfrac{(x+2)(x-1)-(x^2+4)x+x(x-1)(x-1)}{x(x-1)(x-1)}$

$=\dfrac{x^2+x-2-x^3-4x+x^3-2x^2+x}{x(x-1)^2}$

$=\dfrac{-(x^2+2x+2)}{x(x-1)^2}$

82. $\dfrac{x-1}{x^2-x-2}-\dfrac{x}{x^2-2x-3}+\dfrac{1}{x-2}=\dfrac{x-1}{(x-2)(x+1)}-\dfrac{x}{(x-3)(x+1)}+\dfrac{1}{x-2}$

$=\dfrac{(x-1)(x-3)}{(x-2)(x+1)(x-3)}-\dfrac{x(x-2)}{(x-2)(x+1)(x-3)}+\dfrac{(x+1)(x-3)}{(x-2)(x+1)(x-3)}$

$=\dfrac{x^2-4x+3-x^2+2x+x^2-2x-3}{(x-2)(x+1)(x-3)}$

$=\dfrac{x^2-4x}{(x-2)(x+1)(x-3)}$

$=\dfrac{x(x-4)}{(x-2)(x+1)(x-3)}$

83. $\dfrac{\frac{x-1}{1}-\frac{x-1}{x}}{\frac{1}{x-1}+1}\cdot\dfrac{x(x-1)}{x(x-1)}=\dfrac{x(x-1)^2-(x-1)^2}{x+x(x-1)}$

$=\dfrac{(x-1)^2(x-1)}{x^2}$

$=\dfrac{(x-1)^3}{x^2}$

84. $\dfrac{x^{-2}-x^{-1}}{x^{-2}+x^{-1}}=\dfrac{\frac{1}{x^2}-\frac{1}{x}}{\frac{1}{x^2}+\frac{1}{x}}\cdot\dfrac{x^2}{x^2}=\dfrac{1-x}{1+x}$

85. $\dfrac{3x-3}{\sqrt{x}-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}=\dfrac{3(x-1)(\sqrt{x}+1)}{x-1}=3(\sqrt{x}+1)$

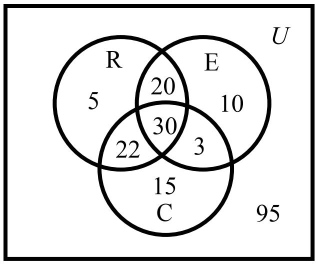
86. $\dfrac{\sqrt{x}-\sqrt{x-4}}{2}\cdot\dfrac{\sqrt{x}+\sqrt{x-4}}{\sqrt{x}+\sqrt{x-4}}=\dfrac{x-(x-4)}{2(\sqrt{x}+\sqrt{x-4})}$

$=\dfrac{x-x+4}{2(\sqrt{x}+\sqrt{x-4})}$

$=\dfrac{4}{2(\sqrt{x}+\sqrt{x-4})}$

$=\dfrac{2}{\sqrt{x}+\sqrt{x-4}}$

87. **a.** R: Recognized
C: Involved
E: Exercised



Numbered statement indicates solution for that question.

1. 30
2. 50 – 30 = 20
3. 52 – 30 = 22
4. 30 + 22 + 20 + 5 = 77
5. 37 – 22 = 15
6. 77 + 15 + 3 = 95

b. $200-(95+5+22+30+20+3+15)=10$ So, 10 exercised only.

c. $63+70-(3+30)=100$ So, 100 exercised or were involved in the community.

88. $-0.75(15)+63.8=52.55\%$

89. $5^2-(5-2)^2=25-3^2=25-9=16$

90. $S=100\left[\dfrac{(1.0075)^n-1}{0.0075}\right]$

a. $S(36)=100\left[\dfrac{(1.0075)^{36}-1}{0.0075}\right]$

$\approx 100\left[\dfrac{0.30865}{0.0075}\right]\approx \$4115.27$

b. $S(240)=100\left[\dfrac{(1.0075)^{240}-1}{0.0075}\right]$

$\approx 100\left[\dfrac{5.00915}{0.0075}\right]\approx \$66,788.69$

91. $C=31.9t+310$

a. $t=2021-2005=16$

b. $C=31.9(16)+310$

$=\$820.40$

c. $4(820.40)=3281.60$

A family of four can expect to pay $3281.60 for health insurance in 2021.

92. $h = 0.000595s^{1.922}$ or $s = 47.7h^{0.519}$

a. $h = 0.000595(50)^{1.922} \approx 1.1$ inch
(about quarter-sized)

b. $s = 47.7(4.5)^{0.519} \approx 104$ mph

93. **a.**
$$R = 10{,}000\left[\frac{0.0065}{1-(1.0065)^{-n}}\right]$$
$$= 10{,}000\left[\frac{0.0065}{1-\frac{1}{1.0065^n}}\right]\cdot\frac{1.0065^n}{1.0065^n}$$
$$= 10{,}000\left[\frac{0.0065(1.0065)^n}{1.0065^n - 1}\right]$$
$$= \frac{65(1.0065)^n}{1.0065^n - 1}$$

b.
$$R = 10{,}000\left[\frac{0.0065}{1-(1.0065)^{-48}}\right] \approx \$243.19$$
$$R = \frac{65(1.0065)^{48}}{1.0065^{48} - 1} \approx \$243.19$$

94. $S = kA^{1/3}$

a. $S = k\sqrt[3]{A}$

b. Let S_1 be the number of species on 20,000 acres. Then $S_1 = k\sqrt[3]{20{,}000}$. Let S_2 be the number of species on 45,000 acres. Then
$$S_2 = k\sqrt[3]{45000}$$
$$= \sqrt[3]{2.25\cdot 20{,}000}$$
$$= \sqrt[3]{2.25}\cdot k\sqrt[3]{20{,}000}$$
$$= \sqrt[3]{2.25}\cdot S_1$$
$$S_2 \approx 1.31S_1$$

95. $\text{Profit} = 30x - 0.001x^2 - (300 + 4x)$
$= -0.001x^2 + 26x - 300$

96. $\text{Value} = \$1{,}450{,}000 - 0.0025(1{,}450{,}000)x$
$= \$1{,}450{,}000 - 3625x$

97. $600 - 13x - 0.5x^2 = 0.5(1200 - 26x - x^2)$ or $0.5(50 + x)(24 - x)$ or $(25 + 0.5x)(24 - x)$ or $(50 + x)(12 - 0.5x)$

98. **a.** $C = \dfrac{1{,}200{,}000}{100-p} - \dfrac{12{,}000}{1} = \dfrac{1{,}200{,}000 - 12{,}000(100-p)}{100-p} = \dfrac{12{,}000p}{100-p}$

b. If $p = 0$, $C = \dfrac{12{,}000(0)}{100-0} = \dfrac{0}{100} = \$0$. The cost of removing no pollution is zero.

c. $C = \dfrac{12{,}000(98)}{100-98} = \$588{,}000$

d. The formula is not defined when $p = 100$. We are dividing by zero. The cost increases as p approaches 100. It is cost prohibitive (or maybe not feasible) to remove all of the pollution.

99. $\dfrac{1200}{1} + \dfrac{56x}{1} + \dfrac{8000}{x} = \dfrac{1200x}{x} + \dfrac{56x^2}{x} + \dfrac{8000}{x} = \dfrac{56x^2 + 1200x + 8000}{x}$

Chapter 0 Test

1. **a.** $A = \{6, 8\}$ $B' = \{3, 4, 6\}$
$A \cup B' = \{3, 4, 6, 8\}$

b. $\{3, 4\}$, $\{3, 6\}$, and $\{4, 6\}$ are disjoint from B.

c. $\{6\}$ and $\{8\}$ are non-empty subsets of A.

2. $(4 - 2^3)^2 - 3^4\cdot 0^{15} + 12 \div 3 + 1 = (-4)^2 - 0 + 4 + 1$
$= 16 + 4 + 1 = 21$

3. **a.** $x^4\cdot x^4 = x^8$

b. $x^0 = 1$, if $x \neq 0$

c. $\sqrt{x} = x^{1/2}$

d. $(x^{-5})^2 = x^{-10}$ or $\dfrac{1}{x^{10}}$

e. $a^{27} \div a^{-3} = a^{27-(-3)} = a^{30}$

f. $x^{1/2}\cdot x^{1/3} = x^{5/6}$

g. $\dfrac{1}{\sqrt[3]{x^2}} = \dfrac{1}{x^{2/3}}$

h. $\dfrac{1}{x^3} = x^{-3}$

4. **a.** $x^{1/5} = \sqrt[5]{x}$

b. $x^{-3/4} = \sqrt[4]{x^{-3}}$ or $\left(\sqrt[4]{x}\right)^{-3}$ or $\dfrac{1}{\sqrt[4]{x^3}}$

5. **a.** $x^{-5} = \dfrac{1}{x^5}$

b. $\left(\dfrac{x^{-8}y^2}{x^{-1}}\right)^{-3} = \dfrac{x^{24}y^{-6}}{x^3} = \dfrac{x^{21}}{y^6}$

6. **a.** $\dfrac{x}{\sqrt{5x}} \cdot \dfrac{\sqrt{5x}}{\sqrt{5x}} = \dfrac{x\sqrt{5x}}{5x} = \dfrac{\sqrt{5x}}{5}$

b. $\sqrt{24a^2b} \cdot \sqrt{a^3b^4} = 2a\sqrt{6b} \cdot ab^2\sqrt{a}$

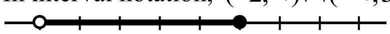
$= 2a^2b^2\sqrt{6ab}$

c. $\dfrac{1-\sqrt{x}}{1+\sqrt{x}} \cdot \dfrac{1-\sqrt{x}}{1-\sqrt{x}} = \dfrac{1-2\sqrt{x}+x}{1-x}$

7. $2x^3 - 7x^5 - 5x - 8$

a. Degree is 5.
b. Constant is –8.
c. Coefficient of x is –5.

8. In interval notation, $(-2,\infty) \cap (-\infty, 3] = (-2, 3]$



9. **a.** $8x^3 - 2x^2 = 2x^2(4x-1)$

b. $x^2 - 10x + 24 = (x-4)(x-6)$

c. $6x^2 - 13x + 6 = (2x-3)(3x-2)$

d. $2x^3 - 32x^5 = 2x^3(1-16x^2)$

$= 2x^3(1-4x)(1+4x)$

10. A quadratic polynomial has degree two.
(c) is the quadratic.
$4 - x - x^2 = 4 - (-3) - (-3)^2 = 4 + 3 - 9 = -2,$
when $x = -3$

11.

$$\begin{array}{r|l} & 2x+1 \\ x^2-1 & 2x^3 + x^2 \qquad -7 \\ & \underline{2x^3 \qquad -2x} \\ & x^2 + 2x - 7 \\ & \underline{x^2 \qquad -1} \\ & 2x - 6 \end{array}$$

Quotient: $2x + 1 + \dfrac{2x-6}{x^2-1}$

12. **a.** $4y - 5(9-3y) = 4y - 45 + 15y = 19y - 45$

b. $-3t^2(2t^4 - 3t^7) = -6t^6 + 9t^9$

c.

$$\begin{array}{r} x^2 - 5x + 2 \\ \underline{4x - 1} \\ -x^2 + 5x - 2 \\ \underline{4x^3 - 20x^2 + 8x} \\ 4x^3 - 21x^2 + 13x - 2 \end{array}$$

d. $(6x-1)(2-3x) = 12x - 18x^2 - 2 + 3x$

$= -18x^2 + 15x - 2$

e. $(2m-7)^2 = 4m^2 - 28m + 49$

f. $\dfrac{x^6}{x^2-9} \cdot \dfrac{x-3}{3x^2} = \dfrac{x^4}{(x+3)(x-3)} \cdot \dfrac{(x-3)}{3}$

$= \dfrac{x^4}{3(x+3)}$

g. $\dfrac{x^4}{9} \div \dfrac{9x^3}{x^6} = \dfrac{x^4}{9} \cdot \dfrac{x^6}{9x^3} = \dfrac{x^7}{81}$

h. $\dfrac{4}{x-8} - \dfrac{x-2}{x-8} = \dfrac{4-x+2}{x-8} = \dfrac{6-x}{x-8}$

i. $\dfrac{x-1}{x^2-2x-3} - \dfrac{3}{x^2-3x}$

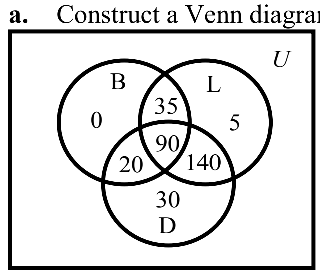
$= \dfrac{x-1}{(x-3)(x+1)} - \dfrac{3}{x(x-3)}$

$= \dfrac{x(x-1) - 3(x+1)}{x(x-3)(x+1)}$

$= \dfrac{x^2 - x - 3x - 3}{x(x-3)(x+1)} = \dfrac{x^2 - 4x - 3}{x(x-3)(x+1)}$

13. $\dfrac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + y} \cdot \dfrac{xy}{xy} = \dfrac{y-x}{y+xy^2}$ or $\dfrac{y-x}{y(1+xy)}$

14. **a.** Construct a Venn diagram:



b. 0 students ate only breakfast.
c. 320 – 145 = 175. 175 students skipped breakfast.

15. $S = 1000\left(1 + \dfrac{0.08}{4}\right)^{4x} = 1000\left(1 + \dfrac{0.08}{4}\right)^{4(20)}$

$= 1000(1+0.02)^{80} = 1000(1.02)^{80} \approx 4875.44$

In 20 years, the future value will be about \$4875.44.

Chapter 0: Algebraic Concepts

Chapter 0 Extended Applications & Group Projects

Campaign Management

1. $250,000(0.36) = 90,000$

$50,000(0.3) = 15,000$

$90,000 - 15,000 = 75,000$

So 75,000 voters read the newspaper but do not watch the local cable network news.

2.

75,000	newspaper
35,000	cable news
15,000	both
125,000	

125,000 read the newspaper or watch cable news or both.

3.

	Number of Voters Reached	Total Cost	Cost per Voter Reached
Pamphlet	125,000	$112,500	$0.90
Cable News	50,000	$40,000	$0.80
Newspaper	90,000	$27,000	$0.30

4. Since 125,000 voters are reached just through newspaper and cable network news advertising, and since reaching voters through each of these means is less expensive than advertising via pamphlet, one plan might be to pay $40,000 + $27,000 = $67,000 to reach voters through the cable network news and advertising alone (and thus not use a pamphlet).

Chapter 1: Linear Equations and Functions

Exercises 1.1

1.
$$\begin{aligned} 4x-7&=8x+2\\ 4x-7+7-8x&=8x+2+7-8x\\ -4x&=9\\ x&=-\frac{9}{4} \end{aligned}$$

2.
$$\begin{aligned} 3x+22&=7x+2\\ 22&=4x+2\\ 20&=4x\\ 5&=x \end{aligned}$$

3.
$$\begin{aligned} x+8&=8(x+1)\\ x+8&=8x+8\\ x-8x&=8-8\\ -7x&=0\\ x&=0 \end{aligned}$$

4.
$$\begin{aligned} x+x+x&=x\\ 3x&=x\\ 2x&=0\\ x&=0 \end{aligned}$$

5.
$$\begin{aligned} -\frac{3x}{4}&=24\\ -3x&=4(24)=96\\ x&=-32 \end{aligned}$$

6.
$$\begin{aligned} \frac{-1}{6}x&=12\\ -6\left(\frac{-1}{6}x\right)&=-6(12)\\ x&=-72 \end{aligned}$$

7.
$$\begin{aligned} 2(x-7)&=5(x+3)-x\\ 2x-14&=5x+15-x\\ 2x-5x+x&=15+14\\ -2x&=29\\ x&=-\frac{29}{2} \end{aligned}$$

8.
$$\begin{aligned} 3(x-4)&=4-2(x+2)\\ 3x-12&=4-2x-4\\ 3x-12&=-2x\\ 5x-12&=0\\ 5x&=12\\ x&=\frac{12}{5} \end{aligned}$$

9.
$$\begin{aligned} 8-2(3x+9)-6x&=50\\ 8-6x-18-6x&=50\\ -10-12x&=50\\ -12x&=50+10\\ -12x&=60\\ x&=-5 \end{aligned}$$

10.
$$\begin{aligned} 10x+6-2(1-5x)&=9\\ 10x+6-2+10x&=9\\ 20x+4&=9\\ 20x&=5\\ x&=\frac{5}{20}\\ x&=\frac{1}{4} \end{aligned}$$

11.
$$\begin{aligned} \frac{5x}{2}-4&=\frac{2x-7}{6}\\ 6\left(\frac{5x}{2}-4\right)&=6\left(\frac{2x-7}{6}\right)\\ 15x-24&=2x-7\\ 15x-2x&=24-7\\ 13x&=17\\ x&=\frac{17}{13} \end{aligned}$$

12.
$$\begin{aligned} \frac{2x}{3}-1&=\frac{x-2}{2}\\ 6\left(\frac{2x}{3}-1\right)&=6\left(\frac{x-2}{2}\right)\\ 4x-6&=3x-6\\ x&=0 \end{aligned}$$

13.
$$x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$$
$$x+\frac{1}{3}=2x-\frac{4}{3}-6x$$
$$3x+1=6x-4-18x$$
$$3x+18x-6x=-4-1$$
$$15x=-5$$
$$x=\frac{-5}{15}=-\frac{1}{3}$$

14.
$$\frac{3x}{4}-\frac{1}{3}=1-\frac{2}{3}\left(x-\frac{1}{6}\right)$$
$$\frac{3x}{4}-\frac{1}{3}=1-\frac{2x}{3}+\frac{2}{18}$$
$$36\left(\frac{3x}{4}-\frac{1}{3}\right)=36\left(1-\frac{2x}{3}+\frac{2}{18}\right)$$
$$27x-12=36-24x+4$$
$$27x-12=-24x+40$$
$$51x-12=40$$

15.
$$(5x)\left(\frac{33-x}{5x}\right)=5x(2)$$
$$33-x=10x$$
$$-x-10x=-33$$
$$-11x=-33$$
$$x=3$$

Check: $\frac{33-3}{5(3)}\stackrel{?}{=}2$
$$\frac{30}{15}\stackrel{?}{=}2$$
$$2=2$$
$x=3$ is the solution.

16.
$$\frac{3x+3}{x-3}=7$$
$$(x-3)\left(\frac{3x+3}{x-3}\right)=(x-3)(7)$$
$$3x+3=7x-21$$
$$-4x=-24$$
$$x=6$$

Check: $\frac{3(6)+3}{(6)-3}\stackrel{?}{=}7$
$$\frac{21}{3}=7$$

17. $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{2(2x+5)}$

Multiply each term by $6(2x+5)$.
$$12x=(8x+20)-15$$
$$12x-8x=20-15$$
$$4x=5 \text{ or } x=\frac{5}{4}$$

Check: $\frac{2\left(\frac{5}{4}\right)}{2\left(\frac{5}{4}\right)+5}\stackrel{?}{=}\frac{2}{3}-\frac{5}{4\left(\frac{5}{4}\right)+10}$
$$\frac{10}{10+20}\stackrel{?}{=}\frac{2}{3}-\frac{5}{15}$$
$$\frac{10}{30}=\frac{1}{3} \text{ and } \frac{2}{3}-\frac{5}{15}=\frac{1}{3}$$
$x=\frac{5}{4}$ is the solution.

18. $\frac{3}{x}+\frac{1}{4}=\frac{2}{3}+\frac{1}{x}$

LCD is $12x$.
$$(12x)\left(\frac{3}{x}\right)+(12x)\left(\frac{1}{4}\right)=(12x)\left(\frac{2}{3}\right)+(12x)\left(\frac{1}{x}\right)$$
$$36+3x=8x+12$$
$$-5x=-24$$
$$x=\frac{24}{5}$$

Check: $\frac{3}{\left(\frac{24}{5}\right)}+\frac{1}{4}\stackrel{?}{=}\frac{2}{3}+\frac{1}{\left(\frac{24}{5}\right)}$
$$\frac{5}{8}+\frac{1}{4}\stackrel{?}{=}\frac{2}{3}+\frac{5}{24}$$
$$24\left(\frac{5}{8}\right)+24\left(\frac{1}{4}\right)\stackrel{?}{=}24\left(\frac{2}{3}\right)+24\left(\frac{5}{24}\right)$$
$$15+6=16+5$$

19. $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$
$$\frac{2x-2}{x-1}+\frac{1}{3}=\frac{5}{6}$$
$$\frac{2(x-1)}{x-1}+\frac{1}{3}=\frac{5}{6}$$
$$2+\frac{1}{3}\neq\frac{5}{6}$$
There is no solution.

20. $\dfrac{2x}{x-3} = 4 + \dfrac{6}{x-3}$

$$(x-3)\left(\frac{2x}{x-3}\right) = (x-3)(4) + (x-3)\left(\frac{6}{x-3}\right)$$
$$2x = 4x - 12 + 6$$
$$-2x = -6$$
$$x = 3$$

Not defined for $x = 3$. No solution.

21. $3.259x - 8.638 = -3.8(8.625x + 4.917)$

$$3.259x - 8.638 = -32.775x - 18.6846$$
$$3.259x + 32.775x = 8.638 - 18.6846$$
$$36.034x = -10.0466$$
$$x = \frac{-10.0466}{36.034} \approx -0.279$$

22. $3.319(14.1x - 5) = 9.95 - 4.6x$

$$46.7979x - 16.595 = 9.95 - 4.6x$$
$$51.3979x - 16.595 = 9.95$$
$$51.3979x = 26.545$$
$$x \approx 0.516$$

23. $0.000316x + 9.18 = 2.1(3.1 - 0.0029x) - 4.68$

$$0.000316x + 9.18 = 6.51 - 0.00609x - 4.68$$
$$0.000316x + 0.00609x = 6.51 - 4.68 - 9.18$$
$$0.006406x = -7.35$$
$$x = \frac{-7.35}{0.006406}$$
$$x \approx -1147.362$$

24. $3.814x = 2.916(4.2 - 0.06x) + 5.3$

$$3.814x = 12.2472 - 0.17496x + 5.3$$
$$3.814x = 17.5472 - 0.17496x$$
$$3.98896x = 17.5472$$
$$x \approx 4.399$$

25. $3x - 4y = 15$

$$-4y = -3x + 15$$
$$y = \frac{-3x}{-4} + \frac{15}{-4}$$
$$y = \frac{3}{4}x - \frac{15}{4}$$

26. $3x - 5y = 25$

$$-5y = -3x + 25$$
$$y = \frac{3}{5}x - 5$$

27. $2\left(9x + \dfrac{3}{2}y\right) = 2(11)$

$$18x + 3y = 22$$
$$3y = -18x + 22$$
$$y = -6x + \frac{22}{3}$$

28. $\dfrac{3x}{2} + 5y = \dfrac{1}{3}$

LCD is 6.

$$6\left(\frac{3x}{2}\right) + 6(5y) = 6\left(\frac{1}{3}\right)$$
$$9x + 30y = 2$$
$$30y = -9x + 2$$
$$y = -\frac{3}{10}x + \frac{1}{15}$$

29. $S = P + Prt$

$$Prt = S - P$$
$$t = \frac{S-P}{Pr}$$

30. $\dfrac{y-b}{x-a} = \dfrac{m}{1}$

$$y - b = m(x-a)$$
$$y = mx - am + b$$

31. $3(x-1) < 2x - 1$

$$3x - 3 < 2x - 1$$
$$x - 3 < -1$$
$$x < 2$$

32. $2(x+1) > x - 1$

$$2x + 2 > x - 1$$
$$x + 2 > -1$$
$$x > -3$$

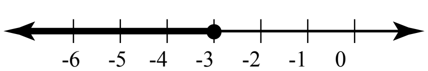
33.
$$\begin{aligned}1-2x&>9\\-2x&>8\\\left(-\tfrac{1}{2}\right)(-2x)&>8\left(-\tfrac{1}{2}\right)\\x&<-4\end{aligned}$$

34.
$$\begin{aligned}17-x&<-4\\-x&<-21\\(-1)(-x)&<(-1)(-21)\\x&>21\end{aligned}$$

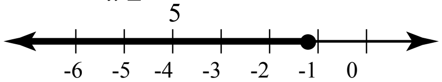
35.
$$\begin{aligned}\frac{3(x-1)}{2}&\le x-2\\3(x-1)&\le 2(x-2)\\3x-3&\le 2x-4\\x-3&\le -4\\x&\le -1\end{aligned}$$

36.
$$\begin{aligned}&\frac{x-1}{2}+1>x+1\\&\frac{x-1}{2}>x\\&x-1>2x\\&-1>x \text{ or } x<-1\end{aligned}$$

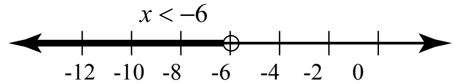
37.
$$\begin{aligned}2(x-1)-3&>4x+1\\2x-2-3&>4x+1\\2x-5&>4x+1\\-2x-5&>1\\-2x&>6\\\left(-\tfrac{1}{2}\right)(-2x)&>6\left(-\tfrac{1}{2}\right)\\x&<-3\end{aligned}$$



38.
$$\begin{aligned}7x+4&\le 2(x-1)\\7x+4&\le 2x-2\\5x+4&\le -2\\5x&\le -6\\x&\le -\frac{6}{5}\end{aligned}$$

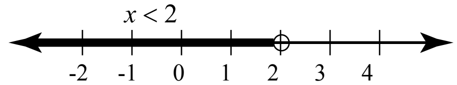


39.
$$\begin{aligned}&\frac{-3x}{2}>3-x\\&-3x>2(3-x)\\&-3x>6-2x\\&-x>6\\&(-1)(-x)>6(-1)\\&x<-6\end{aligned}$$

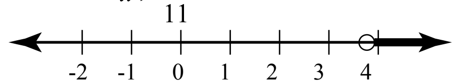


40.
$$\begin{aligned}&\frac{-2x}{5}\le -10-x\\&-2x\le 5(-10-x)\\&-2x\le -50-5x\\&3x\le -50\\&x\le -\frac{50}{3}\end{aligned}$$

41.
$$\begin{aligned}12\left(\frac{3x}{4}-\frac{1}{6}\right)&<12\left(x-\frac{2(x-1)}{3}\right)\\9x-2&<12x-8(x-1)\\9x-2&<12x-8x+8\\9x-2&<4x+8\\5x-2&<8\\5x&<10\\\left(\tfrac{1}{5}\right)(5x)&<\left(\tfrac{1}{5}\right)(10)\\x&<2\end{aligned}$$



42.
$$\begin{aligned}12\left(\frac{4x}{3}-3\right)&>12\left(\frac{1}{2}+\frac{5x}{12}\right)\\16x-36&>6+5x\\11x-36&>6\\11x&>42\\\left(\tfrac{1}{11}\right)(11x)&>\left(\tfrac{1}{11}\right)(42)\\x&>\frac{42}{11}\end{aligned}$$



43. $$y = 648,000 - 1800x$$
$$387,000 = 648,000 - 1800x$$
$$1800x = 648,000 - 387,000 = 261,000$$
$$x = \frac{261,000}{1800} = 145 \text{ months}$$
$$x = \frac{261,000}{1800} = 145 \text{ months}$$

44. Fully depreciated means
$$810,000 - 2250x = 0$$
$$2250x = 810,000$$
$$x = 360 \text{ months}$$

45. $$\frac{I}{175.393} + 0.663 = r$$
$$\frac{I}{175.393} + 0.663 = 19.8$$
$$\frac{I}{175.393} = 19.8 - 0.663 = 19.137$$
$$I = 19.137(175.393)$$
$$I = \$3356.50$$

46. **a.** $33p - 18d = 495$
$$p = \frac{18d + 495}{33} = \frac{6d + 165}{11}$$
b. When $d = 12,460$,
$$p = \frac{6(12,460) + 165}{11} = \frac{74,925}{11}$$
$p \approx 6811$ lbs/sq in.

47. $R = C$ for breakeven point
$$20x = 2x + 7920$$
$$18x = 7920$$
$x = 440$ packs or 220,000 CD's

48. $4P = 81x - 29970$
$P = 0$ if $81x - 29,970 = 0$
$$81x = 29,970$$
$x = 370$ systems

49. $170,500 = 5.76x$
$$x = \frac{170,500}{5.76} = \$29,600$$

50. Let the pre-tax price of the car be P. Then
$$P + 0.06P = 21,041$$
$$1.06P = 21,041$$
$$P = \frac{21,041}{1.06}$$
$$P = 19,850$$
Therefore, the tax on the car is 0.06.
$$0.06(19,850) = 1191$$
We could also find the tax by subtracting the pre-tax price from the total price:
$$21,041 - 19,850 = 1191$$
\$1191.00

51. **a.** $$25P - 34t = 1378$$
$$25P - 34(20) = 1378$$
$$25P - 680 = 1378$$
$$25P = 2058$$
$$P \approx 82.3\%$$
b. $$25P - 34t = 1378$$
$$25(90) - 34t = 1378$$
$$2250 - 34t = 1378$$
$$-34t = -872$$
$$t \approx 25.6$$
Since $1990 + 25.6 = 2015.6$, the year in which 90% of the population will have Internet access is 2016.

52. $y = 30.65x + 667.43$
$$1280.43 = 30.65x + 667.43$$
$$613 = 30.65x$$
$$x = 20$$
Since $2000 + 20 = 2020$, the year in which the expenses are predicted to be \$1280.43 is 2020.

53. $$\frac{93 + 69 + 89 + 97 + FE + FE}{6} = 90$$
$$2FE + 348 = 540$$
$$2FE = 192$$
$$FE = 96$$
A 96 is the lowest grade that can be earned on the final.

54. Let $x =$ the lowest score on the final.
If the 52 earned during the semester is not replaced,

$$\frac{x+83+67+52+90}{5}=80$$
$$x+292=400$$
$$x=108.$$

This indicates that a grade of 80 is not possible under these circumstances. If the grade of 52 is replaced with the final score x, then

$$\frac{x+83+67+x+90}{5}=80$$
$$2x+240=400$$
$$2x=160$$
$$x=80$$

55. $x =$ Amount in safe fund
$120,000-x =$ amount in risky fund
Yield: $0.09x+0.13(120,000-x)=12,000$
$$0.09x+15,600-0.13x=12,000$$
$$-0.04x=-3600$$
$$x=90,000$$
$x=\$90,000$ in 9% fund
$120,000-90,000=\$30,000$ in 13% fund.

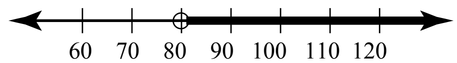
56. $x =$ Amount in safe fund
$145,600-x =$ amount in risky fund
Yield: $0.10x+0.18(145,600-x)=20,000$
$$0.10x+26,208-0.18x=20,000$$
$$-0.08x=-6208$$
$$x=77,600$$
$x=\$77,600$ in 10% fund
$145,600-77,600=\$68,000$ in 18% fund.

57. Reduced salary: $2000-0.10(2000)=\$1800$
Increased salary: $1800+0.20(1800)=\$2160$
$160=R\%$ of 2000
$$R=\frac{160}{2000}=\frac{8}{100}$$
\$160 is an 8% increase.

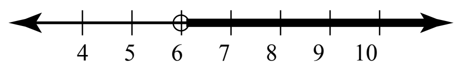
58. **a.** $\dfrac{3}{100}=\dfrac{100}{x}$
$$3x=100(100)$$
$$3x=10,000$$
$$x=3333 \text{ (rounded)}$$

b. $\dfrac{63}{1000}=\dfrac{1000}{x}$
$$63x=1000(1000)$$
$$63x=1,000,000$$
$$x=15,873 \text{ (rounded)}$$

59. $40x>20x+1600$
$$20x>1600$$
$$x>80$$



60. $20d+78<33d$
$$78<13d$$
$$6<d \text{ or } d>6$$



61. $695+5.75x\le 900$
$$5.75x\le 205$$
$$x\le 35.65$$
He could buy 35 or fewer memory sticks.

62. Let T be the tax and B be the amount of the monthly bill.
If $0\le B<60$, then $T=0.02B$.
If $60\le B<80$, then $T=0.04B$.
If $B\ge 80$, then $T=0.06B$.

63. **a.** $2018-1980=38$
b.
$$S=0.264t-2.57$$
$$10=0.264t-2.57$$
$$12.57=0.264t$$
$$t\approx 47.6$$
c. Since $1980+47.6=2027.6$, the year in which at least 10% of adults are predicted to be obese is 2028.

64. $y=486.48t+3486.84$
a. $t=2018-2010=8$
b. $y=486.48(8)+3486.84=\$7378.68$
c. $486.48t+3486.84>10000$
$$486.48t>6513.16$$
$$t>13.4$$
Since $2010+13.4=2023.4$. The first year in which the federal income tax per capita is expected to exceed \$10,000 is 2024.

65. $A = 90.2 + 41.3h$

a. $90.2 + 41.3h \geq 110$
$41.3h \geq 19.8$
$h \geq 0.48$

b. $90.2 + 41.3h < 100$
$41.3h < 9.8$
$h < 0.24$

66. $WC = 1.337t - 24.094$
$1.337t - 24.094 \leq t - 30$
$0.337t - 24.094 \leq -30$
$0.337t \leq -5.906$
$t \leq -17.53$

Exercises 1.2

1. **a.** For each value of x there is only one y.

b. $D = \{-7, -1, 0, 3, 4.2, 9, 11, 14, 18, 22\}$
$R = \{0, 1, 5, 9, 11, 22, 35, 60\}$

c. $f(0) = 1$, $f(11) = 35$

2. **a.** $f(9)$ is an output of f.

b. The table does not describe x as a function of y. For $y = 0$ there are two values of x.

3. This is a function, since for each x there is only one y. $D = \{1, 2, 3, 8, 9\}$, $R = \{-4, 5, 16\}$

4. No, the relation is not a function because the x-value 1 has two y-values, 4 and 9.
$D = \{-1,\ 0,\ 1,\ 3\}, R = \{0,\ 2,\ 4,\ 6,\ 9\}$

5. The vertical-line test shows that the graph represents y as a function of x.

6. The vertical-line test shows that the graph does not represent y as a function of x.

7. The vertical-line test shows that the graph does not represent y as a function of x.

8. The vertical-line test shows that the graph represents y as a function of x.

9. If $y = 3x^3$, then y is a function of x.

10. If $y = 6x^2$, then y is a function of x.

11. If $y^2 = 3x$, then y is not a function of x. If, for example, $x = 3$, then there are two possible values for y.

12. If $y^2 = 10x^2$, then y is not a function of x. For $x \neq 0$, there are two values of y.

13. $R(x) = 8x - 10$

a. $R(0) = 8(0) - 10 = -10$

b. $R(2) = 8(2) - 10 = 6$

c. $R(-3) = 8(-3) - 10 = -34$

d. $R(1.6) = 8(1.6) - 10 = 2.8$

14. $f(x) = 17 - 6x$

a. $f(-3) = 17 - 6(-3) = 17 + 18 = 35$

b. $f(1) = 17 - 6(1) = 17 - 6 = 11$

c. $f(10) = 17 - 6(10) = 17 - 60 = -43$

d. $f\left(\frac{2}{3}\right) = 17 - 6\left(\frac{2}{3}\right) = 17 - 4 = 13$

15. $C(x) = 4x^2 - 3$

a. $C(0) = 4(0)^2 - 3 = -3$

b. $C(-1) = 4(-1)^2 - 3 = 1$

c. $C(-2) = 4(-2)^2 - 3 = 13$

d. $C\left(-\frac{3}{2}\right) = 4\left(-\frac{3}{2}\right)^2 - 3 = 6$

16. $h(x) = 3x^2 - 2x$

a. $h(3) = 3(3)^2 - 2(3) = 27 - 6 = 21$

b. $h(-3) = 3(-3)^2 - 2(-3) = 27 + 6 = 33$

c. $h(2) = 3(2)^2 - 2(2) = 12 - 4 = 8$

d. $h\left(\frac{1}{6}\right) = 3\left(\frac{1}{6}\right)^2 - 2\left(\frac{1}{6}\right) = \frac{3}{36} - \frac{2}{6}$
$= \frac{3}{36} - \frac{12}{36} = -\frac{9}{36} = -\frac{1}{4}$

17. $h(x) = x - 2(4 - x)^3$

a. $h(-1) = -1 - 2(4 - (-1))^3$
$= -1 - 2(4 + 1)^3 = -1 - 2(5)^3$
$= -1 - 2(125) = -1 - 250 = -251$

b. $h(0) = 0 - 2(4 - (0))^3$
$= -2(4)^3 = -2(64) = 128$

c. $h(6) = 6 - 2(4-(6))^3$
$= 6 - 2(4-6)^3 = 6 - 2(-2)^3$
$= 6 - 2(-8) = 6 + 16 = 22$

d. $h(2.5) = 2.5 - 2(4-(2.5))^3$
$= 2.5 - 2(1.5)^3$
$= 2.5 - 2(3.375) = 2.5 - 6.75$
$= -4.25$

18. $R(x) = 100x - x^3$

a. $R(1) = 100(1) - 1^3 = 100 - 1 = 99$

b. $R(10) = 100(10) - (10)^3 = 1000 - 1000 = 0$

c. $R(2) = 100(2) - 2^3 = 200 - 8 = 192$

d. $R(-10) = 100(-10) - (-10)^3$
$= -1000 - (-1000) = 0$

19. $f(x) = x^3 - \dfrac{4}{x}$

a. $f\left(-\dfrac{1}{2}\right) = \left(-\dfrac{1}{2}\right)^3 - \dfrac{4}{-\frac{1}{2}} = -\dfrac{1}{8} + 8 = \dfrac{63}{8}$

b. $f(2) = 2^3 - \dfrac{4}{2} = 8 - 2 = 6$

c. $f(-2) = (-2)^3 - \dfrac{4}{-2} = -8 + 2 = -6$

20. $C(x) = \dfrac{x^2 - 1}{x}$

a. $C(1) = \dfrac{1^2 - 1}{1} = \dfrac{0}{1} = 0$

b. $C\left(\dfrac{1}{2}\right) = \dfrac{\left(\frac{1}{2}\right)^2 - 1}{\frac{1}{2}} = \dfrac{\frac{1}{4} - 1}{\frac{1}{2}} = \dfrac{-\frac{3}{4}}{\frac{1}{2}} = -\dfrac{3}{2}$

c. $C(-2) = \dfrac{(-2)^2 - 1}{-2} = \dfrac{4 - 1}{-2} = -\dfrac{3}{2}$

21. $f(x) = 1 + x + x^2$

a. $f(2+1) = f(3) = 1 + 3 + 3^2 = 13$
$f(2) + f(1) = 7 + 3 = 10$
$f(2) + f(1) \neq f(2+1)$

b. $f(x+h) = 1 + (x+h) + (x+h)^2$
$= 1 + x + h + x^2 + 2xh + h^2$

c. $f(x) + f(h) = 2 + x + h + x^2 + h^2$
No, $f(x+h) \neq f(x) + f(h)$.

d. $f(x) + h = 1 + x + x^2 + h$
No, $f(x+h) \neq f(x) + h$.

e. $f(x+h) = 1 + (x+h) + (x+h)^2$
$= 1 + x + h + x^2 + 2xh + h^2$
$f(x) = 1 + x + x^2$
$f(x+h) - f(x) = h + 2xh + h^2$
$= h(1 + 2x + h)$
$\dfrac{f(x+h) - f(x)}{h} = 1 + 2x + h$

22. $f(x) = 3x^2 - 6x$

a. $f(3+2) = f(5) = 3 \cdot 5^2 - 6 \cdot 5 = 75 - 30 = 45$
$f(3) + 2 = (3 \cdot 3^2 - 6 \cdot 3) + 2 = 27 - 18 + 2 = 11$
So, $f(3+2) \neq f(3) + 2$.

b. $f(x+h) = 3(x+h)^2 - 6(x+h)$
$= 3(x^2 + 2xh + h^2) - 6x - 6h$
$= 3x^2 + 6xh + 3h^2 - 6x - 6h$

c. $f(x) + h = 3x^2 - 6x + h$
So, $f(x+h) \neq f(x) + h$

d. $f(x) + f(h) = 3x^2 - 6x + 3h^2 - 6h$
So, $f(x+h) \neq f(x) + f(h)$

e. $f(x+h) = 3(x+h)^2 - 6(x+h)$
$= 3x^2 + 6xh + 3h^2 - 6x - 6h$
$f(x+h) - f(x)$
$= 3x^2 + 6xh + 3h^2 - 6x - 6h - (3x^2 - 6x)$
$= 6xh + 3h^2 - 6h$
$\dfrac{f(x+h) - f(x)}{h} = \dfrac{6xh + 3h^2 - 6h}{h}$
$= 6x + 3h - 6$

23. $f(x) = x - 2x^2$

a. $f(x+h) = (x+h) - 2(x+h)^2$
$= -2x^2 - 4xh - 2h^2 + x + h$

b. $f(x+h) - f(x)$
$= (x+h) - 2(x+h)^2 - (x - 2x^2)$
$= x + h - 2x^2 - 4xh - 2h^2 - x + 2x^2$
$= h - 4xh - 2h^2$

c. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h - 4xh - 2h^2}{h}$
$= 1 - 4x - 2h$

24. $f(x) = 2x^2 - x + 3$

a. $f(x+h) = 2(x+h)^2 - (x+h) + 3$
$= 2(x^2 + 2xh + h^2) - x - h + 3$
$= 2x^2 + 4xh + 2h^2 - x - h + 3$

b. $f(x+h) - f(x)$
$= 2x^2 + 4xh + 2h^2 - x - h + 3 - (2x^2 - x + 3)$
$= 2x^2 + 4xh + 2h^2 - x - h + 3 - 2x^2 + x - 3$
$= 4xh + 2h^2 - h$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{4xh + 2h^2 - h}{h}$
$= 4x + 2h - 1$

25. Since (9, 10) and (5, 6) are points on the graph:

a. $f(9) = 10$

b. $f(5) = 6$

26. **a.** From the figure, $g(0) = 0$.

b. There are three values of x that satisfy $g(x) = 0$.

27. **a.** The coordinates of $Q = (1, –3)$. Since the point is on the curve, the coordinates satisfy the equation.

b. The coordinates of $R = (3, –3)$. They satisfy the equation.

c. The ordered pair (a, b) satisfies the equation. Thus $b = a^2 - 4a$.

d. The x-values are 0 and 4. These values are also solutions of $x^2 - 4x = 0$.

28. **a.** The point (1, 1) does not lie on the graph. The coordinates do not satisfy the equation.

b. From the graph, the coordinates of point R are (1, 2). These coordinates do satisfy the equation.

c. If $P(a, b)$ is a point on the graph, then $b = 2a^2$.

d. The x-coordinate of the point whose y-coordinate is 0 is 0. This value of x does satisfy the equation $0 = 2x^2$.

29. $y = x^2 + 4$

There is no division by zero or square roots. Domain is all the reals, i.e., $\{x : x \in \text{Reals}\}$. Since $x^2 \ge 0, x^2 + 4 \ge 4$, the range is reals ≥ 4 or $\{y : y \ge 4\}$

30. Domain: all reals
Range: reals ≥ 1

31. $y = \sqrt{x+4}$

There is no division by zero. To get a real number y, we must have $x + 4 \ge 0$ or $x \ge -4$.
Domain: $x \ge -4$.
The square root is always nonnegative.
Thus, the range is $\{y : y \in \text{reals}, y \ge 0\}$.

32. Domain: all reals
Range: reals $y \ge 1$

33. Domain: $x \ge 1,\ x \ne 2$

34. Domain: $x > -3$

35. Domain: $-7 \le x \le 7$

36. Domain: $-3 \le x \le 3$

37. $f(x) = 3x,\quad g(x) = x^3$

a. $(f+g)(x) = 3x + x^3$

b. $(f-g)(x) = 3x - x^3$

c. $(f \cdot g)(x) = 3x \cdot x^3 = 3x^4$

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{3x}{x^3} = \dfrac{3}{x^2}$

38. $f(x)=\sqrt{x}, \quad g(x)=\frac{1}{x}$

a. $(f+g)(x)=f(x)+g(x)=\sqrt{x}+\frac{1}{x}$

b. $(f-g)(x)=\sqrt{x}-\frac{1}{x}$

c. $(f \cdot g)=f(x) \cdot g(x)=\sqrt{x} \cdot \frac{1}{x}=\frac{\sqrt{x}}{x}$

d. $(f/g)(x)=\frac{f(x)}{g(x)}=\frac{\sqrt{x}}{\frac{1}{x}}=x\sqrt{x}$

39. $f(x)=\sqrt{2x}$ **,** $g(x)=x^2$

a. $(f+g)(x)=\sqrt{2x}+x^2$

b. $(f-g)(x)=\sqrt{2x}-x^2$

c. $(f \cdot g)(x)=\sqrt{2x} \cdot x^2=x^2\sqrt{2x}$

d. $\left(\frac{f}{g}\right)(x)=\frac{\sqrt{2x}}{x^2}$

40. $f(x)=(x-1)^2, \quad g(x)=1-2x$

a. $(f+g)(x)=f(x)+g(x)$
$=(x-1)^2+1-2x$
$=x^2-2x+1+1-2x$
$=x^2-4x+2$

b. $(f-g)(x)=f(x)-g(x)$
$=(x-1)^2-(1-2x)$
$=x^2-2x+1-1+2x=x^2$

c. $(f \cdot g)(x)=f(x) \cdot g(x)=(x-1)^2(1-2x)$

d. $(f/g)(x)=\frac{f(x)}{g(x)}=\frac{(x-1)^2}{1-2x}$

41. $f(x)=(x-1)^3, \quad g(x)=1-2x$

a. $(f \circ g)(x)=f(1-2x)=(1-2x-1)^3=-8x^3$

b. $(g \circ f)(x)=g((x-1)^3)=1-2(x-1)^3$

c. $f(f(x))=f((x-1)^3)=[(x-1)^3-1]^3$

d. $(f \cdot f)(x)=(x-1)^3 \cdot (x-1)^3=(x-1)^6$
$\left[\left(f \cdot f\right)(x) \neq f\left(f(x)\right)\right]$

42. $f(x)=3x, \quad g(x)=x^3-1$

a. $(f \circ g)(x)=f(g(x))=f(x^3-1)$
$=3(x^3-1)=3x^3-3$

b. $(g \circ f)(x)=g(f(x))=g(3x)$
$=(3x)^3-1=27x^3-1$

c. $f(f(x))=f(3x)=3(3x)=9x$

d. $f^2(x)=(f \cdot f)(x)=f(x) \cdot f(x)$
$=3x \cdot 3x=9x^2$

43. $f(x)=2\sqrt{x}$ **,** $g(x)=x^4+5$

a. $(f \circ g)(x)=f(x^4+5)=2\sqrt{x^4+5}$

b. $(g \circ f)(x)=g(2\sqrt{x})=(2\sqrt{x})^4+5$
$=16x^2+5$

c. $f(f(x))=f(2\sqrt{x})=2\sqrt{2\sqrt{x}}$

d. $(f \cdot f)(x)=2\sqrt{x} \cdot 2\sqrt{x}=4x$
$\left[\left(f \cdot f\right)(x) \neq f\left(f(x)\right)\right]$

44. $f(x)=\frac{1}{x^3}, \quad g(x)=4x+1$

a. $(f \circ g)(x)=f(g(x))$
$=f(4x+1)=\frac{1}{(4x+1)^3}$

b. $(g \circ f)(x)=g(f(x))=g\left(\frac{1}{x^3}\right)$
$=4 \cdot \frac{1}{x^3}+1=\frac{4}{x^3}+1$

c. $f(f(x))=f\left(\frac{1}{x^3}\right)=\frac{1}{\left(\frac{1}{x^3}\right)^3}=\frac{1}{\frac{1}{x^9}}=x^9$

d. $f^2(x)=(f \cdot f)(x)=f(x) \cdot f(x)$
$=\frac{1}{x^3} \cdot \frac{1}{x^3}=\frac{1}{x^6}$

45. a. $f(20)=103{,}000$ means it will take 20 years to pay off a debt of \$103,000 (at \$800 per month and 7.5% compounded monthly.)

b. $f(5+5)=f(10)=69{,}000$;
$f(5)+f(5)=80{,}000$;
No.

c. It will take 15 years to pay off the debt, i.e., $89{,}000=f(15)$.

46. **a.** From the table, the monthly payment is \$775.30 if they refinance for 20 years, i.e $775.30 = f(20)$.

b. From the table, $f(10) = 1161.09$. The value of $f(10)$ is the monthly payment to repay a \$100,000 loan in 10 years when the interest rate is 7%.

c. $f(5+5) = f(10) = 1161.09$
$f(5) + f(5) = 1980.12 + 1980.12 = 3960.24$
$f(5+5) \neq f(5) + f(5)$

47. **a.** From the figure, $f(64) = \$866$ and $f(67) = \$1{,}080$.

b. $f(68) = \$1{,}160$. Starting benefits at age 68 provides \$1,160 per month.

c. $f(66) - f(62) = \$1000 - \$750 = \$250$. Starting benefits at age 66 gives \$250 more per month than starting benefits at age 62.

48. **a.** $f(0) \approx 11{,}225$ and $f(6.5) \approx 10{,}719.94$. These values represent the opening value and the closing value, respectively, for the Dow Jones average on August 10, 2011.

b. The domain is $0 \le t \le 6.5$. The range is approximately 10,600 to 11,300.

c. There are eleven t-values that satisfy $f(t) = 11{,}000$. Answers will vary.

49. **a.** $W(100) \approx 155$ million and $O(140) \approx 120$ million

b. $W(120) = 166.3$. In 2020 there are expected to be 166.3 million white, non-Hispanics in the civilian, non-institutional labor force (CN-ILF).

c. $O(90) = 42.6$. In 1990 there were 42.6 million non-Whites or Hispanics in the CN-ILF..

d.
$$\begin{aligned}(W-O)(120) &= W(120) - O(120)\\ &= 166.3 - 86.8\\ &= 79.5\end{aligned}$$
In 2020 there are expected to be 79.5 million more White non-Hispanics in the CN-ILF than others.

e. $(W+O)(150) = W(150) + O(150)$
$= 169.4 + 143.0$
$= 312.4$
In 2050 the total size of the CN-ILF is expected to be 312.4 million.

f. $(W-O)(100)$ is greater than $(W-O)(140)$ because the graphs are further apart at $t = 100$ than at $t = 140$.

50. **a.** $f(1970) = 30{,}000{,}000$

b. $f(1930) = 10{,}000{,}000$. There were 10,000,000 women in the labor force in 1930.

c. $f(2005) - f(1990) \approx 70 - 58 = 12$ million. There were approximately 12 million more women in the workforce in 2005 than in 1990.

51. $C = \frac{5}{9}F - \frac{160}{9}$

a. C is a function of F.

b. Mathematically, the domain is all reals.

c. Domain: $\{F : 32 \le F \le 212\}$
Range: $\{C : 0 \le C \le 100\}$

d.
$$\begin{aligned}C(40) &= \frac{5}{9}(40) - \frac{160}{9}\\ &= \frac{200-160}{9} = \frac{40}{9} = 4.44°C\end{aligned}$$

52. a. $P(2000)$

$= 47(2000) - 0.01(2000)^2 - 8000$
$= 94000 - 0.01(4,000,000) - 8000$
$= \$46,000$

b. $P(5000)$

$= 47(5000) - 0.01(5000)^2 - 8000$
$= 235,000 - 0.01(25,000,000) - 8000$
$= -\$23,000$

c. $P(5000)$ is negative, which means that it is not profitable for the company to produce 5000 units.

53. $C(x) = 300x + 0.1x^2 + 1200$

a. $C(10) = 300(10) + 0.1(10)^2 + 1200$
$= 3000 + 0.1(100) + 1200$
$= 3000 + 10 + 1200$
$= \$4210$

b. $C(100) = 300(100) + 0.1(100)^2 + 1200$
$= \$32,200$

c. The value $C(100)$ is the total cost of producing 100 items, which is $32,200.

54. $R(n) = \dfrac{0.6n}{0.4 + 0.6n}$

a. $R(1) = \dfrac{0.6(1)}{0.4 + 0.6(1)} = \dfrac{0.6}{1.0} = 0.6$

b. $R(2) = \dfrac{0.6(2)}{0.4 + 0.6(2)} = \dfrac{1.2}{1.6} = 0.75$

c. Improvement is 0.75 – 0.6 = 0.15. The percentage improvement is $\dfrac{0.15}{0.6} = 25\%$.

55. $C(p) = \dfrac{7300p}{100 - p}$

a. Domain: $\{p : 0 \le p < 100\}$

b. $C(45) = \dfrac{7300(45)}{100 - 45} = \dfrac{328,500}{55} = \$5972.73$

c. $C(90) = \dfrac{7300(90)}{100 - 90} = \dfrac{657,000}{10} = \$65,700$

d. $C(99) = \dfrac{7300(99)}{100 - 99} = \dfrac{722,700}{1} = \$722,700$

e. $C(99.6) = \dfrac{7300(99.6)}{100 - 99.6} = \dfrac{727,080}{0.4} = \$1,817,700$

In each case above, to remove p% of the particulate pollution would cost $C(p)$.

56. $V(x) = x^2(108 - 4x)$ is a function of x.

a. $V(10) = (10)^2(108 - 4(10))$
$= 100(108 - 40) = 100(68)$
$= 6800$ cubic inches

b. $V(20) = (20)^2(108 - 4(20))$
$= 400(108 - 80) = 400(28)$
$= 11,200$ cubic inches

c. The values for x must be such that $0 < x < 27$, otherwise the volume would be less than or equal to 0.

57. **a.** $P(q(t)) = P(1000 + 10t)$

$$= 180(1000 + 10t) - \frac{(1000 + 10t)^2}{100} - 200$$

$$= 169{,}800 + 1600t - t^2$$

b. $q(15) = 1000 + 10(15) = 1150$
$P(q(15)) = \$193{,}575$

58. $W(L) = kL^3$ When $k = 0.02$, we have

$W(L) = 0.02L^3$ and $L = L(t) = 50 - \dfrac{(t-20)^2}{10}$,

$0 \le t \le 20$.

So, $(W \circ L)(t) = W(L(t)) = 0.02\left[50 - \dfrac{(t-20)^2}{10}\right]^3$.

59. $R = f(C) \quad C = g(A)$

a. $(f \circ g)(x) = f(C) = R$

b. $(g \circ f)(x)$ is not defined.

c. A is the independent variable and R is the dependent variable. Revenue depends on money spent for advertising.

60. **a.** sanding the door
b. painting the door
c. sanding the door and then painting
d. painting the door and then sanding
e. painting the door with two coats

61. length $= x$ width $= y$ $L = 2x + 2y$

$1600 = xy$ or $y = \dfrac{1600}{x}$

$$L = 2x + 2\left(\frac{1600}{x}\right) = 2x + \frac{3200}{x}$$

62. Let x = the length of the base.

Then $\dfrac{1}{2}x$ = the height. Bottom: x^2 sq. ft.

Sides: $\dfrac{1}{2}x \cdot x = \dfrac{1}{2}x^2$ sq. ft. for each of 4 sides

for a total of $4 \cdot \dfrac{1}{2}x^2 = 2x^2$ sq. ft.

Top: x^2 sq. ft

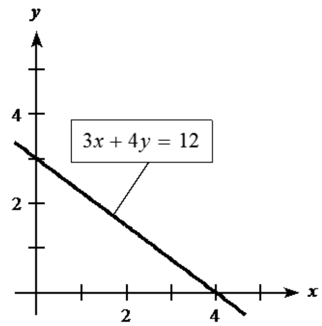
Cost: $C(x) = \underbrace{2x^2}_{\text{bottom}} + \underbrace{2(2x^2)}_{\text{sides}} + \underbrace{1.5x^2}_{\text{top}} = 7.5x^2$

63. $R = (30 + x)(100 - 2x)$

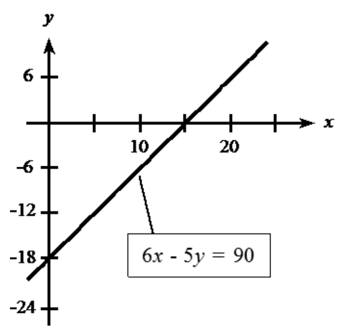
64. $R = (720 + 20x)(50 - x)$

Exercises 1.3

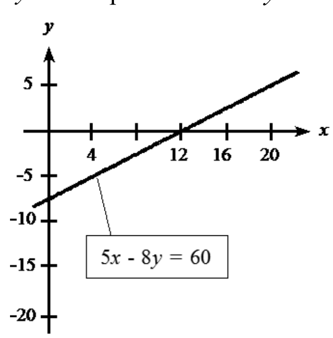
1. $3x + 4y = 12$
x-intercept: $y = 0$ then $x = 4$.
y-intercept: $x = 0$ then $y = 3$.



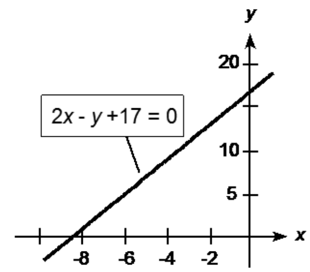
2. $6x - 5y = 90$
x-intercept: $y = 0$ then $x = 15$.
y-intercept: $x = 0$ then $y = -18$



3. $5x - 8y = 60$
x-intercept: $y = 0$ then $x = 12$.
y-intercept: $x = 0$ then $y = -7.5$.



4. $2x - y + 17 = 0$
x-intercept: $y = 0$ then $x = -8.5$.
y-intercept: $x = 0$ then $y = 17$.



5. $(22,11)$ and $(15,-17)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-17 - 11}{15 - 22} = \frac{-28}{-7} = 4$$

6. $(-6,-12)$ and $(-18,-24)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-24 - (-12)}{-18 - (-6)} = \frac{-12}{-12} = 1$$

7. $(3,-1)$ and $(-1,1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{-1 - 3} = \frac{2}{-4} = -\frac{1}{2}$$

8. $(-5,6)$ and $(1,-3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 6}{1 - (-5)} = \frac{-9}{6} = -\frac{3}{2}$$

9. (3, 2) and (–1, 2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{3 - (-1)} = \frac{0}{4} = 0$$

10. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{-4 - (-4)} = \frac{-4}{0}$, undefined

11. A horizontal line has a slope of 0 .

12. The slope of a vertical line is undefined.

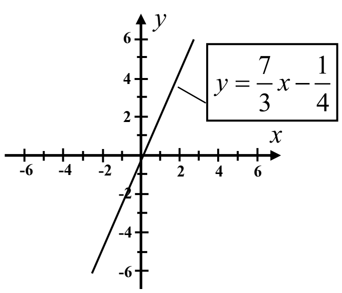
13. $(3,2)$ and $(-1,2)$
The rate of change is equivalent to the slope of the line.
$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-2}{-1-3}=\frac{0}{-4}=0$$

14. $(11,-5)$ and $(-9,-4)$
The rate of change is equivalent to the slope of the line.
$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-(-5)}{-9-11}=\frac{-1}{20}$$

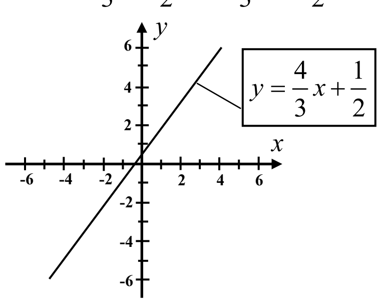
15. **a.** The slope is negative since the line slants downward toward the right.
b. The slope is undefined since the line is vertical.

16. **a.** The slope is zero since the line is horizontal.
b. The slope is positive since the line slants upward toward the right.

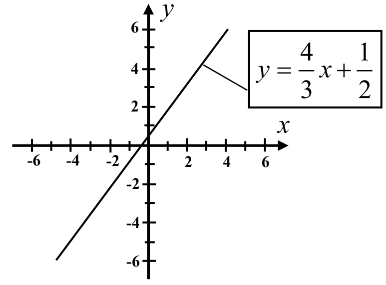
17. $y=\frac{7}{3}x-\frac{1}{4},\ m=\frac{7}{3},\ b=-\frac{1}{4}$



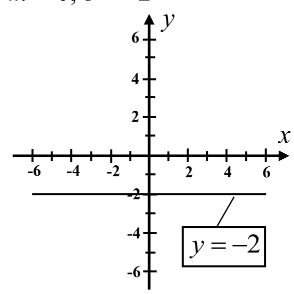
18. $y=\frac{4}{3}x+\frac{1}{2}, m=\frac{4}{3}, b=\frac{1}{2}$



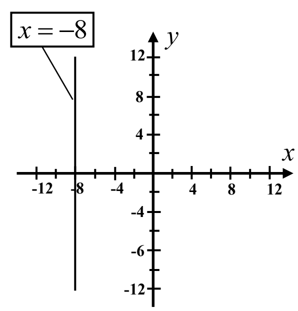
19. $y=3$ or $y=0x+3,\ m=0,\ b=3$



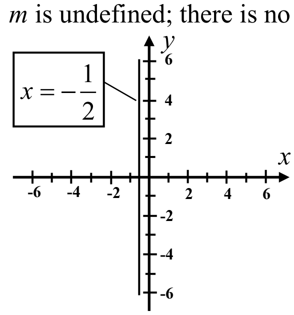
20. $y=-2$ horizontal line
$m=0$, $b=-2$



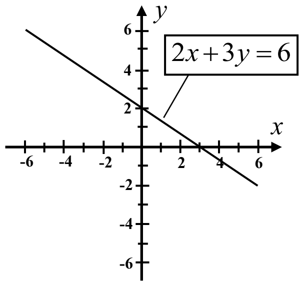
21. $x=-8$
Slope is undefined.
There is no y-intercept.



22. $x=-\frac{1}{2}$ vertical line
m is undefined; there is no y-intercept.



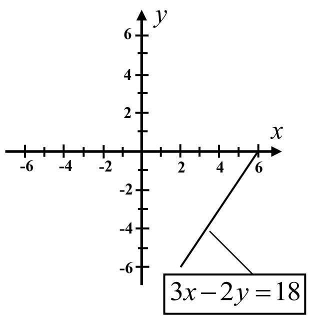
23. $2x + 3y = 6$ or $y = -\frac{2}{3}x + 2, m = -\frac{2}{3}, b = 2$.



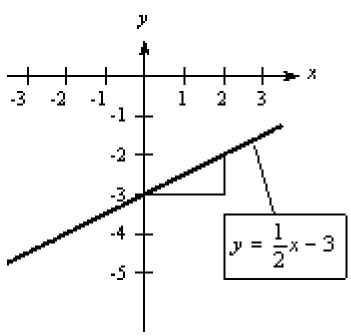
24. $3x - 2y = 18$

$-2y = -3x + 18$

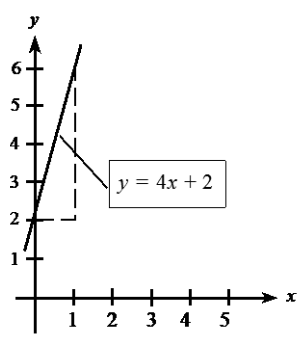
$y = \frac{3}{2}x - 9, \; m = \frac{3}{2}, b = -9$



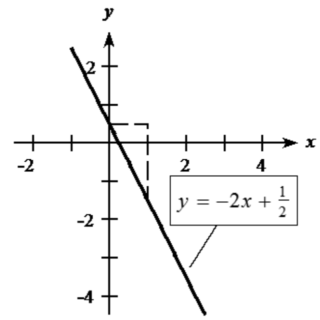
25. $m = \frac{1}{2}, b = -3; \; y = \frac{1}{2}x - 3$



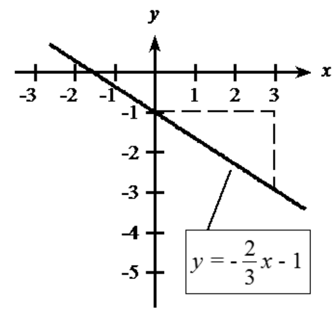
26. $m = 4$, $b = 2, \; y = 4x + 2$



27. $m = -2, \; b = \frac{1}{2}, \; y = -2x + \frac{1}{2}$



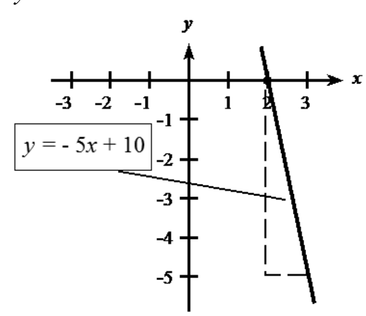
28. $m = -\frac{2}{3}, b = -1, y = -\frac{2}{3}x - 1$



29. $P(2, 0), \; m = -5$

$y - 0 = -5(x - 2)$

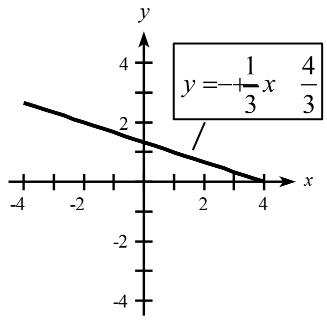
$y = -5x + 10$



30. $(1, 1), m = -\frac{1}{3}$

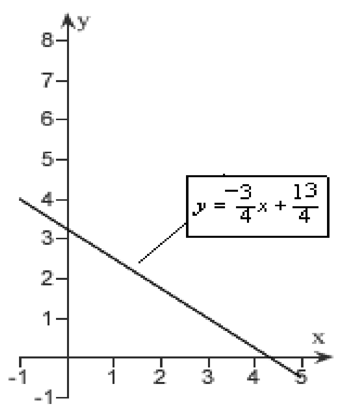
$y - 1 = -\frac{1}{3}(x - 1)$

$y = -\frac{1}{3}x + \frac{4}{3}$



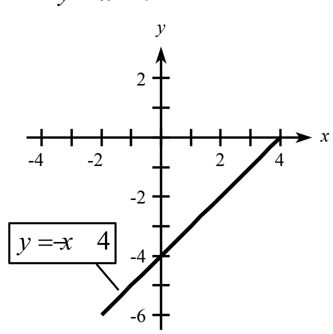
31. $P(-1,4),\ m=-\dfrac{3}{4}$

$$y-4=-\frac{3}{4}(x-(-1))$$
$$y=-\frac{3}{4}x+\frac{13}{4}$$

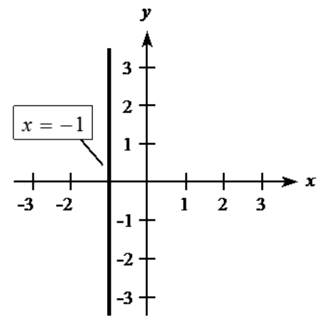


32. $(3,-1),\ m=1$

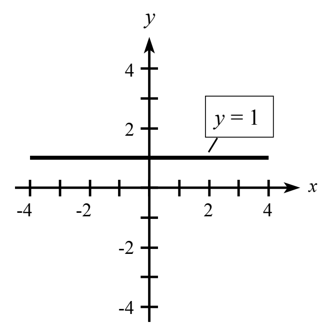
$$y-y_1=m(x-x_1)$$
$$y+1=1(x-3)$$
$$y+1=x-3$$
$$y=x-4$$



33. $P(-1, 1)$, m is undefined
$x=-1$



34. $(1, 1)$, $m = 0$; horizontal line, $y = 1$



35. $P_1=(3,2), P_2=(-1,-6)$

$$m=\frac{-6-2}{-1-3}=2$$
$$y-2=2(x-3)$$
$$y=2x-4$$

36. $(-4, 2), (2, 4),\ m=\dfrac{4-2}{2-(-4)}=\dfrac{2}{6}=\dfrac{1}{3}$

$$y-4=\frac{1}{3}(x-2)$$
$$y-4=\frac{1}{3}x-\frac{2}{3}$$
$$y=\frac{1}{3}x+\frac{10}{3}$$

37. $P_1=(7,3), P_2=(-6,2)$

$$m=\frac{2-3}{-6-7}=\frac{-1}{-13}=\frac{1}{13}$$
$$y-3=\frac{1}{13}(x-7)$$
$$y=\frac{1}{13}x-\frac{7}{13}+3$$
$$y=\frac{1}{13}x+\frac{32}{13}\ \text{ or }\ -x+13y=32$$

38. (10, 2), (8, 7), $m = \frac{7-2}{5-10} = -1$

$$y - 7 = -1(x - 8)$$
$$y - 7 = -x + 8$$
$$y = -x + 15$$

39. $P_1 = (-4, 0), P_2 = (0, -12)$

$$m = \frac{-12-0}{0-(-4)} = \frac{-12}{4} = -3$$
$$y = -3x - 12$$

40. $P_1 = (0, 3), P_2 = (9, 0)$

$$m = \frac{0-3}{9-0} = \frac{-3}{9} = -\frac{1}{3}$$
$$y = -\frac{1}{3}x + 3$$

41. $3x + 2y = 6$ and $2x - 3y = 6$

$y = -\frac{3}{2}x + 3$ $\qquad$ $y = \frac{2}{3}x - 2$

Lines are perpendicular since $\left(-\frac{3}{2}\right)\left(\frac{2}{3}\right) = -1$.

42. $5x - 2y = 8$ and $10x - 4y = 8$

$y = \frac{5}{2}x - 4$ $\qquad$ $y = \frac{5}{2}x - 2$

Lines are parallel since the slopes are equal.

43. $6x - 4y = 12$ and $3x - 2y = 6$

$y = \frac{6}{4}x - \frac{12}{4}$ $\qquad$ $y = \frac{3}{2}x - 3$

or $y = \frac{3}{2}x - 3$

Lines are the same.

44. $5x + 4y = 7$ and $y = \frac{4}{5}x + 7$

$$4y = -5x + 7$$
$$y = -\frac{5}{4}x + \frac{7}{4}$$

Lines are perpendicular since $\left(-\frac{5}{4}\right)\left(\frac{4}{5}\right) = -1$.

45. If $3x + 5y = 11$, then $y = -\frac{3}{5}x + \frac{11}{5}$. So, $m = -\frac{3}{5}$. A line parallel will have the same slope. Thus, $m = -\frac{3}{5}$ and $P = (-2, -7)$ gives $y - (-7) = -\frac{3}{5}(x - (-2))$ which simplifies to $y = -\frac{3}{5}x - \frac{41}{5}$.

46. Through (6, –4), parallel to $4x - 5y = 6$

Find the slope of $4x - 5y = 6$.

$$-5y = -4x + 6$$
$$y = \frac{4}{5}x - \frac{6}{5}$$
$$m = \frac{4}{5}$$

Use the same slope.

$$y - (-4) = \frac{4}{5}(x - 6)$$
$$y + 4 = \frac{4}{5}x - \frac{24}{5}$$
$$y = \frac{4}{5}x - \frac{44}{5}$$

47. If $5x - 6y = 4$, then $y = \frac{5}{6}x - \frac{4}{6}$. Slope of the perpendicular line is $-\frac{6}{5}$. Thus $m = -\frac{6}{5}$ and $P = (3, 1)$ gives $y - 1 = -\frac{6}{5}(x - 3)$ which simplifies to $y = -\frac{6}{5}x + \frac{23}{5}$.

48. (–2, –8), perpendicular to $x = 4y + 3$

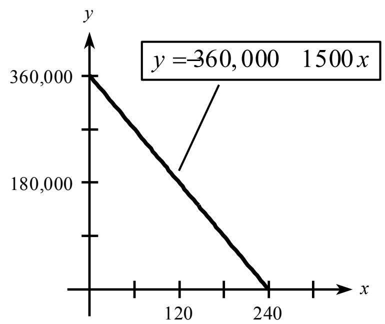
Find the slope of $x = 4y + 3$.

$$-4y = -x + 3$$
$$y = \frac{1}{4}x - \frac{3}{4},\ m = \frac{1}{4}$$

Slope of new line is $-\frac{1}{\frac{1}{4}} = -4$.

$$y - (-8) = -4(x - (-2))$$
$$y + 8 = -4x - 8$$
$$y = -4x - 16$$

49. a.



b. $0 = 360{,}000 - 1500x$

$$x = \frac{360000}{1500} = 240 \text{ months}$$

In 240 months, the building will be completely depreciated.

c. (60, 270,000) means that after 60 months the value of the building will be \$270,000.

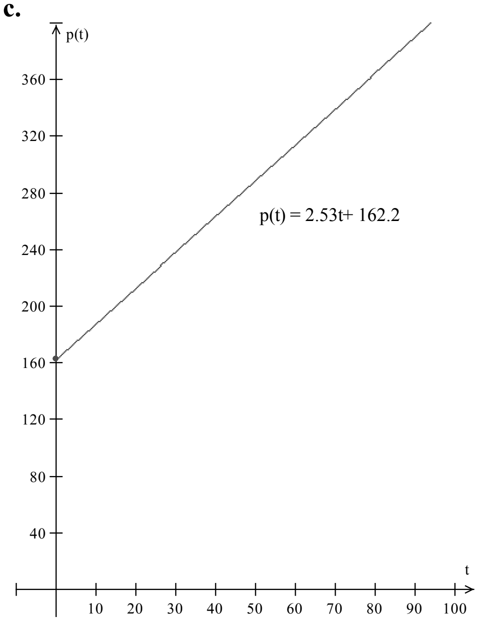
50. a. $p(t) = 2.53t + 162.2$

$p(0) = 162.2$

The U.S. population in 1950 was 162.2 million.

b. The slope, 2.53, means that the U.S. population increases at a rate of 2.53 million people per year.

c.

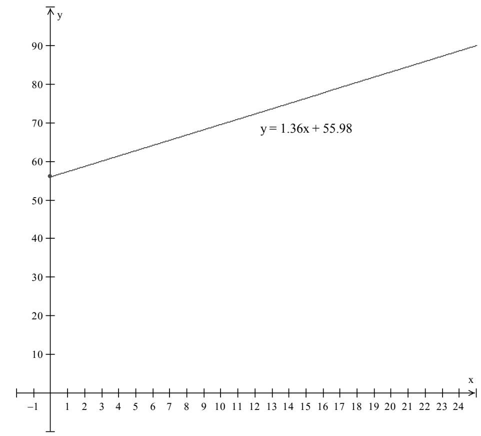


51. $y = 1.36x + 55.98$

a. $m = 1.36$; $b = 55.98$

b. The percent of the U.S. population with Internet service is changing at the rate of 1.36 percentage points per year.

c.



52. a. $m = -1.53$; $b = 38.56$

b. The percent of high school students who occasionally smoke cigarettes is changing at the rate of -1.53 percentage points per year.

c. The *p*-intercept represents the percent of the high school students who occasionally smoked cigarettes in 1995.

53. a. $m = -0.065$; $b = 31.39$

b. The *F*-intercept represents the percent of the world's land that was forest in 1990.

c. -0.065 percentage points per year. This means that after 1990, the world forest area as a percent of land area changes by -0.065 percentage points per year.

54. $S = 141.1 - 45.78(1 - H)$

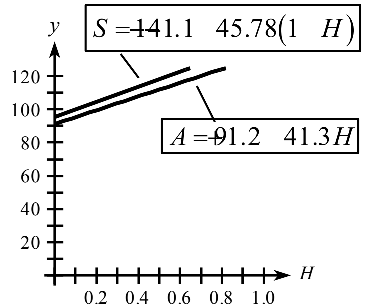
$A = 91.2 + 41.3H$

a. When $H = 0.40$,

$$S = 141.1 - 45.78(1 - 0.4)$$
$$= 141.1 - 45.78(0.6) \approx 113.6°$$
$$A = 91.2 + 41.3(0.4) \approx 107.7°\text{F}$$

b. According to the Summer Simmer Index, the effect of a relative humidity of 40% is to make 100°F seem like 113.6°F. The other model indicates that the 100°F seems like 107.7°F.

c.



55. $F = 0.78M - 1.316$

a. $m = 0.78$

b. For each \$1 increase in male earnings, the female's earning increases by only \$0.78.

c. $F = 0.78(60) - 1.316$

$= 45.484$ thousand

$= \$48{,}484$

56. $y = 977.8x + 13{,}643.2$

a. $m = 977.8$

b. The U.S. GDP is increasing at a rate of \$977.8 billion/year.

57. $y = 16.37 + 0.0838x$

58. $y = 9.19 + 0.9191x$

59. a. $(1950, 62.2)$ and $(2050, 191.8)$

$$m = \frac{191.8 - 62.2}{2050 - 1950} = \frac{129.6}{100} = 1.296$$

$$y - 62.2 = 1.296(x - 1950)$$

$$y = 1.296x - 2465$$

b. The slope, 1.296, indicates that the size of the U.S. civilian workforce changes at the rate of 1.296 million workers per year.

60. a. $p = 0.025(80{,}000)y = 2000y$

b. $p = 0.025 \cdot c \cdot 30 = 0.75c$

61. a. $(2020, 120.56)$ and $(2050, 276.05)$

$$m = \frac{276.05 - 120.56}{2050 - 2020} = \frac{155.49}{30} = 5.183$$

$$y - 120.56 = 5.183(x - 2020)$$

$$y = 5.183x - 10{,}349.10$$

b. The consumer price index increases at the rate of \$5.18 per year.

62. a. Yes

b. $m = \dfrac{0.13 - 0.11}{6 - 5} = 0.02$

$$y - 0.13 = 0.02(x - 6)$$

$$y = 0.02x + 0.01$$

c. The values in the table fit the model.

63. (x, p) is the reference. (0, 85000) is one point.

$$m = \frac{-1700}{1} = -1700$$

$p - 85{,}000 = -1700(x - 0)$ or

$p = -1700x + 85{,}000$

64. $P =$ (age, hours of sleep) $P_1 = (18, 8)$

Choose $P_2 = (14, 9)$

$$m = \frac{9 - 8}{14 - 18} = -\frac{1}{4}$$

$y - 8 = -\dfrac{1}{4}(x - 18)$ or $y - 8 = -\dfrac{1}{4}x + \dfrac{9}{2}$

or $y = -\dfrac{1}{4}x + \dfrac{25}{2}$

65. (t, R) is the ordered pair.

$P_1 = \left(\dfrac{7}{2}, 11\right)$, $P_2 = (6, 19)$

$$m = \frac{19 - 11}{6 - \frac{7}{2}} = \frac{8}{\frac{5}{2}} = \frac{16}{5} = 3.2$$

$R - 19 = 3.2(t - 6)$ or $R = 3.2t - 19.2 + 19$ or

$R = 3.2t - 0.2$

66. Pairs: (0, 960,000) and (240, 0)

$$m = \frac{0 - 960{,}000}{240 - 0} = -4000$$

$b = 960{,}000$

$y = 960{,}000 - 4000x$

67. $P_1 = (200, 25)$ $P_2 = (250, 49)$

$$m = \frac{49-25}{250-200} = \frac{24}{50} = 0.48$$

$y - 25 = 0.48(x - 200)$ or $y = 0.48x - 71$

Exercises 1.4

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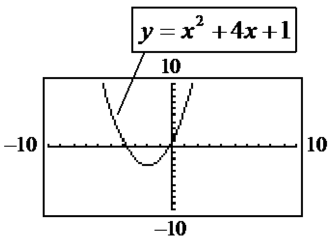

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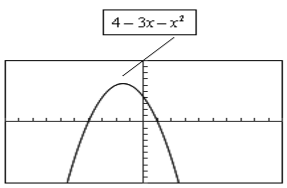

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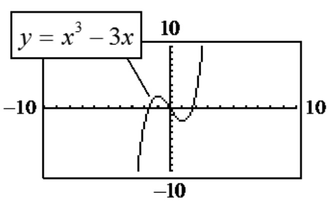

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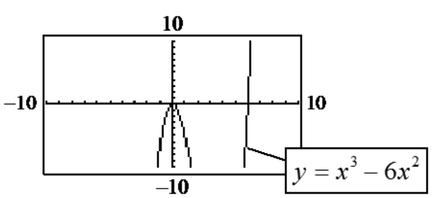

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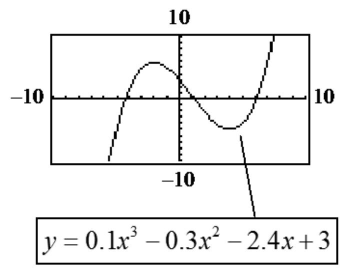

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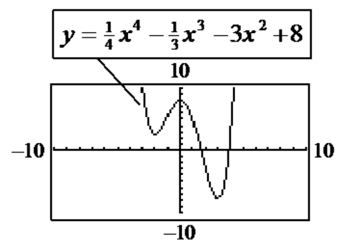

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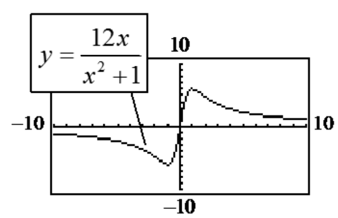

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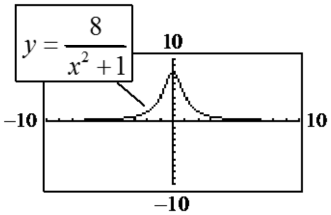

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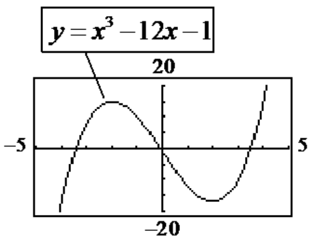
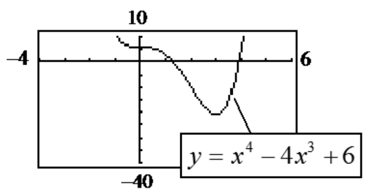
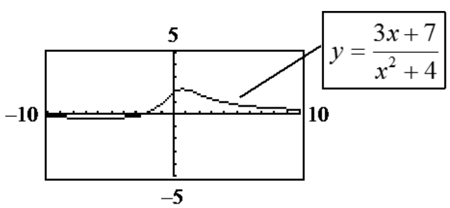


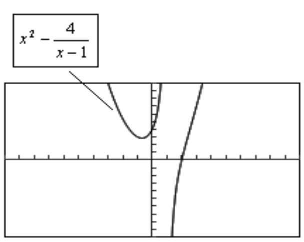
10.



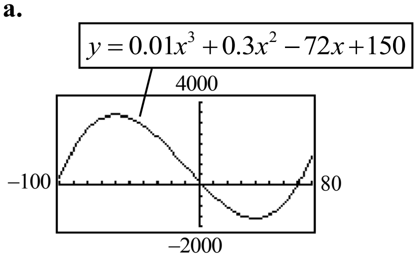
11.



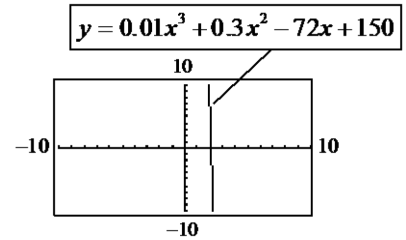
12.



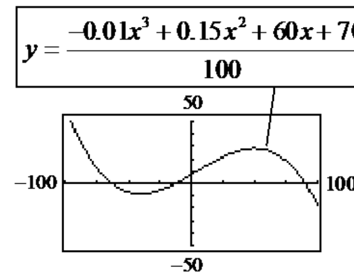
13. a.



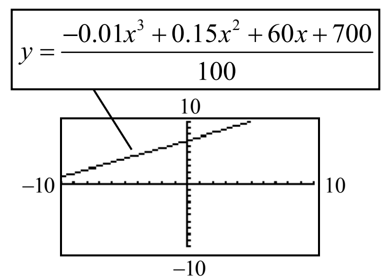
b. Standard viewing window



14. a.

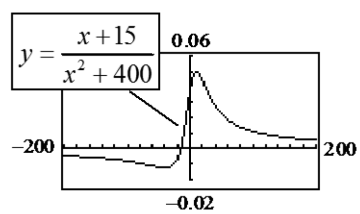


b. Standard viewing window

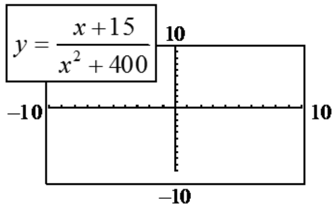


15.

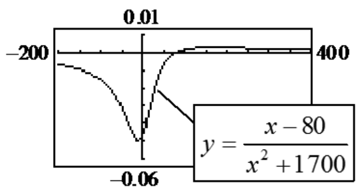
a.



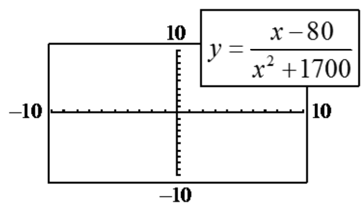
b. Standard Window



16. a.

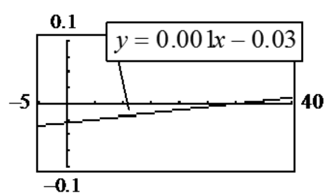


b. Standard viewing window



17. a. y-intercept $= -0.03$
x-intercept: $0.001x = 0.03$
$x = 30$

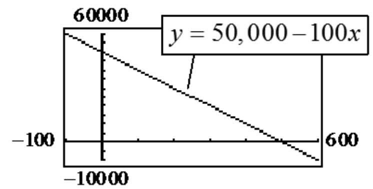
b.



18. $y = 50{,}000 - 100x$

a. The equation is linear, so use the x- and y-intercepts of the line graph to determine an appropriate range. (y-intercept: 50,000; x-intercept: 500)

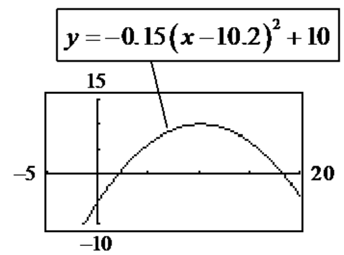
b. Window: x-min = –100 $\quad$ y-min = –10,000
x-max = 600 $\quad$ y-max = 60,000
Graph using the window in part (b).



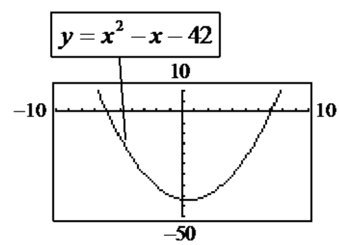
19. $y = -0.15(x-10.2)^2 + 10$

There is no min.
Max value of $y = 10$.
x-intercepts:

$$0 = -0.15(x-10.2)^2 + 10$$
$$(x-10.2)^2 = \frac{10}{0.15} = 66.66$$
$$x - 10.2 = \pm\sqrt{66.66} \approx \pm 8$$
$$x = 10.2 \pm 8 \text{ or } x = 2.2 \text{ or } 18.2$$

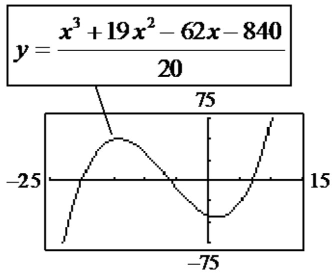


20. A suggested window is shown below.



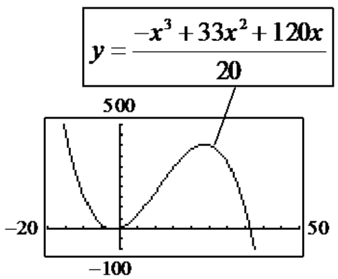
As the graph shows, more of the features of the graph are now visible.

21. If $x = 0$, $y = -42$.
A suggested window is shown below.



22. $y = \dfrac{-x^3 + 33x^2 + 120x}{20}$

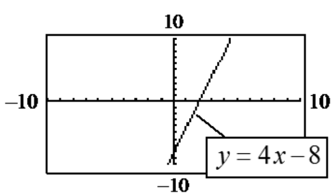
A suggested window is shown below.



As the graph shows, closer details of the features of the graph are now available.

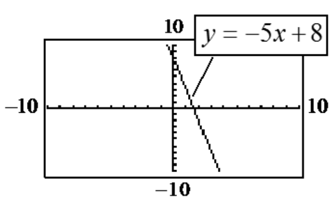
23. $4x - y = 8$

$y = 4x - 8$



24. $5x + y = 8$

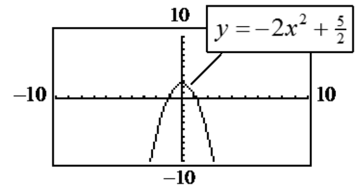
$y = -5x + 8$



25. $4x^2 + 2y = 5$

$2y = -4x^2 + 5$

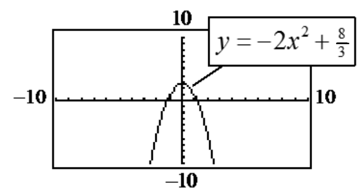
$y = -2x^2 + \dfrac{5}{2}$



26. $6x^2 + 3y = 8$

$3y = -6x^2 + 8$

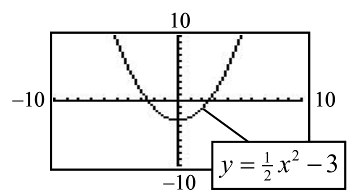
$y = -2x^2 + \dfrac{8}{3}$



27. $x^2 - 6 = 2y$

$2y = x^2 - 6$

$y = \dfrac{1}{2}x^2 - 3$

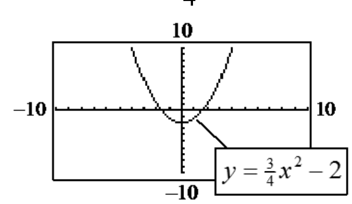


Not linear

28. $3x^2 - 4y = 8$

$-4y = -3x^2 + 8$

$y = \dfrac{3}{4}x^2 - 2$



29. $f(x) = x^3 - 3x^2 + 2$

$f(-2) = (-2)^3 - 3(-2)^2 + 2 = -8 - 12 + 2 = -18$

$f\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)^3 - 3\left(\frac{3}{4}\right)^2 + 2 = 0.734375$

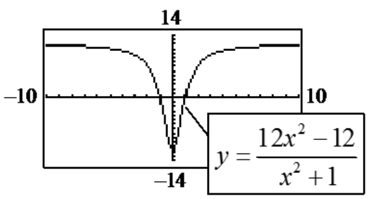
Use your graphing calculator, and evaluate the function at these two points. If either of your answers differ, can you explain the difference?

30. $f(x) = \dfrac{x^2 - 2x}{x - 1}$

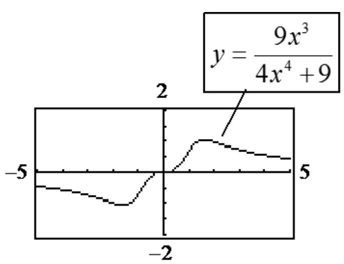
$f(3) = \dfrac{(3)^2 - 2(3)}{(3) - 1} = \dfrac{3}{2}$

$f(-4) = \dfrac{(-4)^2 - 2(-4)}{-4 - 1} = -4.8$

31. As x gets large, y approaches 12.
When $x = 0$, $y = -12$, x intercepts at $\pm$ 1.

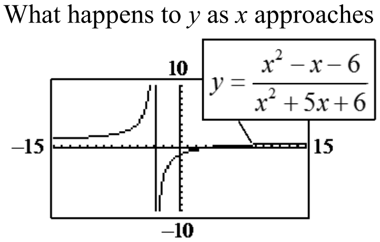


32.

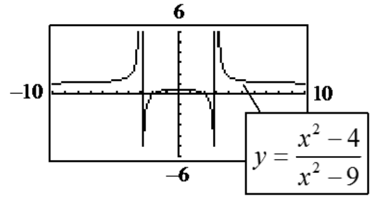


33. $y = \dfrac{x^2 - x - 6}{x^2 + 5x + 6} = \dfrac{(x-3)(x+2)}{(x+3)(x+2)}$

What happens to y as x approaches –3? –2?



34.



35. $6x - 21 = 0$

$6x = 21$

$x = \dfrac{21}{6} = \dfrac{7}{2}$

36. $12x + 28 = 0$

$x = -\dfrac{28}{12} = -\dfrac{7}{3}$

37. $x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x = -2$ or 5

38. $6x^2 + 4x = 4$

$6x^2 + 4x - 4 = 0$

The graphing calculator approximation is $x = -1.215$, $x = 0.549$.

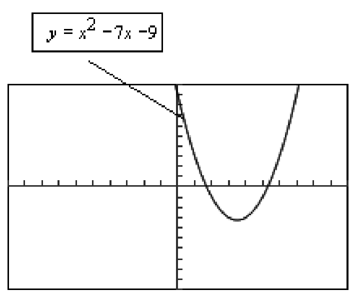
39. **a.**

$y = x^2 - 7x - 9$

$\Rightarrow x^2 - 7x - 9 = 0$

$\Rightarrow (x + 1.1098)(x - 8.1098) = 0$

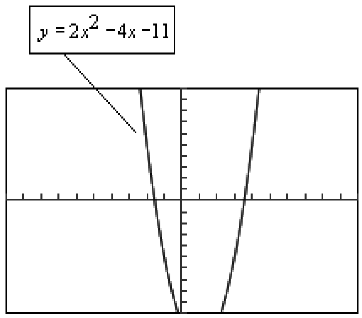
$\Rightarrow x = -1.1098$ or $x = 8.1098$



b. –1.1098, 8.1098

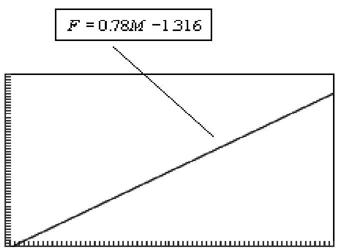
40. **a.** $y = 2x^2 - 4x - 11$

$\Rightarrow 2x^2 - 4x - 11 = 0$
$\Rightarrow (x + 1.55)(x - 3.55) = 0$
$\Rightarrow x = -1.55$ or $x = 3.55$



b. $x = -1.55$ or $x = 3.55$

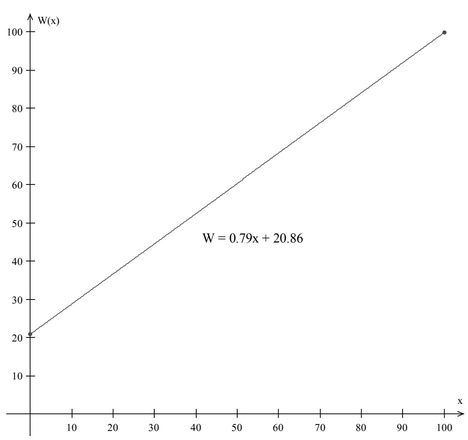
41. **a.** The graph of $F = 0.78M - 1.316$ is



b. When average annual earnings for males is \$50,000, average annual earnings for females is \$37,684.

c. $\$47,434$

42. **a.**

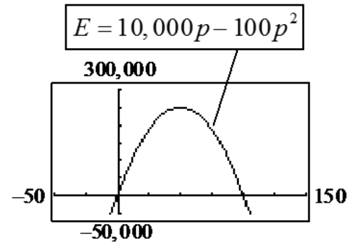


b. In 2036 (86 years past 1950), it is predicted that there will be 88.8 million women in the workforce.

c. $W = 0.79(90) + 20.86$

$= 91.96$ million women

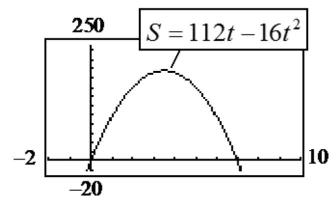
43. **a.**



b. $E \geq 0$ when $0 \leq p \leq 100$.

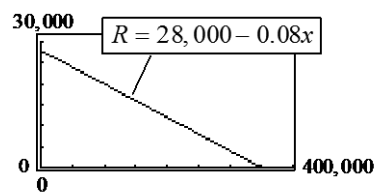
44. $S = 112t - 16t^2$

a.



b. From the graph above, the ball has an estimated maximum height of 196 feet when $t = 3.5$ seconds.

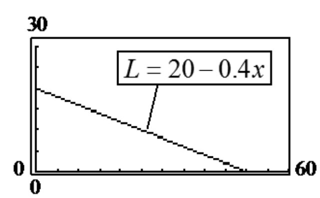
45. **a.**



b. The rate is -0.08 . As more people become aware of the product, there are fewer to learn about it

46. $L = 20 - 0.4x$

a. x-intercept: $x = 50$, y-intercept: $y = 20$

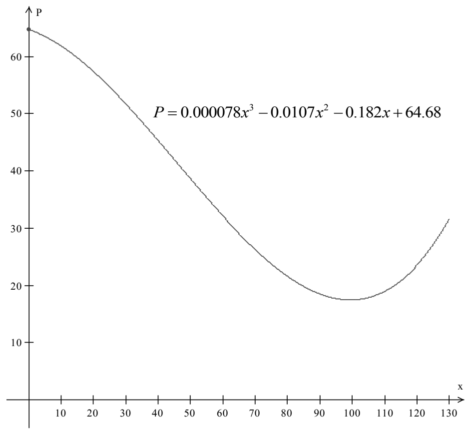


b. The rate is decreasing because the number of words learned is increasing.

47. **a.** x-min = 0, x-max = 130

b. P-max = 65 is reasonable

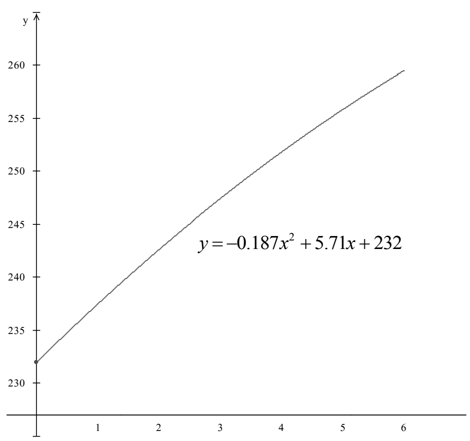
c.



d. The percentage decreases from 57.4% in 1920 to 17.5% in 2000, and then it increases to 31.4% in 2030.

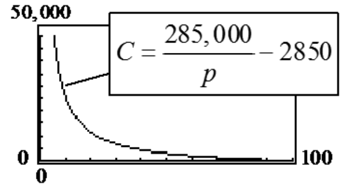
48. **a.** x-min = 0, x-max = 6
y-min = 230, y-max = 260

b.



c. It continues to increase but will eventually exceed the population of the U.S.

49. **a.**

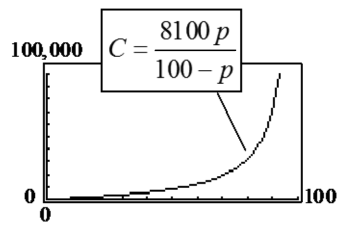


b. Near $p = 0$, cost grows without bound.

c. The coordinates of the point mean that the cost of obtaining stream water with 1% of the current pollution levels would cost \$282,150.

d. The p-intercept means that the cost of stream water with 100% of the current pollution levels would cost \$0.

50. **a.**

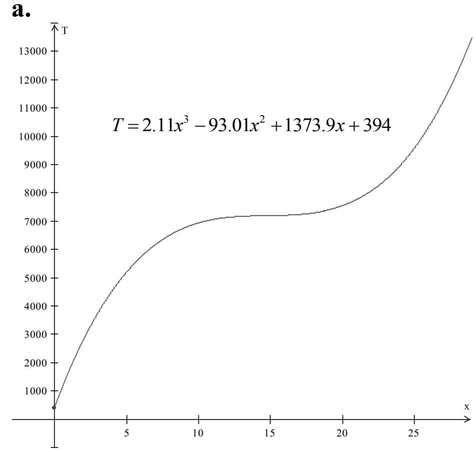


b. C increases rapidly as p gets close to 100.

c. The coordinates (98, 396900) indicate that the cost to remove 98% of the particulate pollution cost \$396,900.

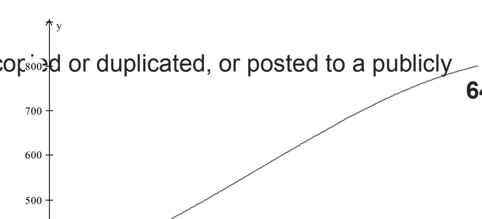
d. The p-intercept is (0, 0). The meaning is that it costs nothing to remove none of the particulate pollution

51. **a.**



b. The graph is increasing. The per capita federal tax burden is increasing.

52. **a.**



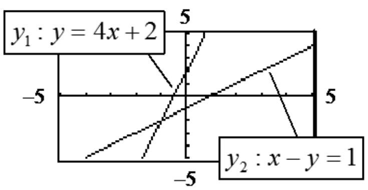
$y = -0.022x^3 + 0.86x^2 + 9.9x + 340$

b. The graph shows carbon dioxide emissions increasing over this time period.

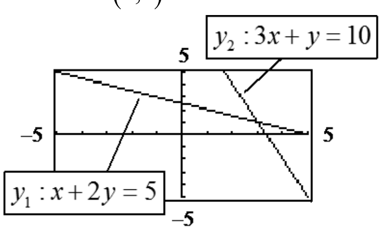
c. From 2010 to 2011 there is an increase of $\approx 350.74 - 340 = 10.74$ million metric tons. From 2029 to 2030 it is predicated that there will be an increase of $\approx 706 - 687.66$ $= 18.34$ million metric tons.

Exercises 1.5

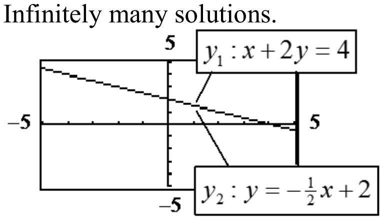
1. Solution: $(-1, -2)$



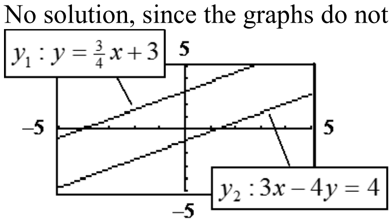
2. Solution: $(3, 1)$



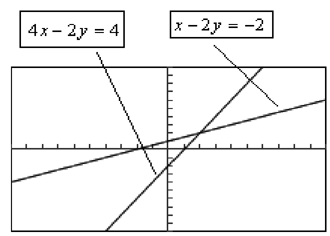
3. Infinitely many solutions.



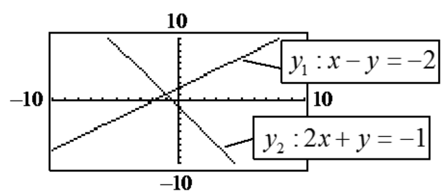
4. No solution, since the graphs do not intersect.



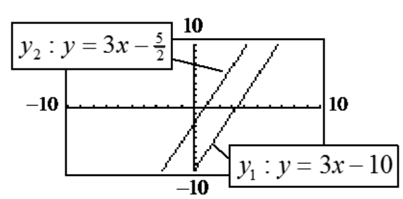
5. Solution: $(2, 2)$



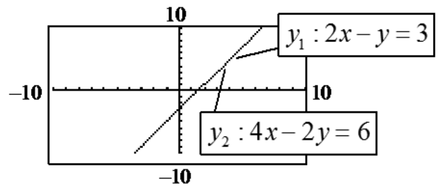
6. Solution: $(-1, 1)$



7. Solution: No solution, since the graphs do not intersect.



8. Solution: Infinitely many solutions.



9. $3x - 2y = 6$

$4y = 8$ Solve for *y*. $y = \frac{8}{4} = 2$

Substitute for this variable in first equation and solve for the other variable.

$3x - 2(2) = 6$
$3x = 6 + 4 = 10$
$x = \frac{10}{3}$

The solution of the system is $x = \frac{10}{3}$ and $y = 2$, or $\left(\frac{10}{3}, 2\right)$.

10. $\begin{cases} 3x = 6 \\ 4x - 3y = 5 \end{cases}$

Substitution: $x = 2$

$4(2) - 3y = 5$
$-3y = -3$
$y = 1$

The solution of the system is $x = 2$, $y = 1$ or $(2,1)$.

11. $2x - y = 2$

$3x + 4y = 6$ Solve for *y*. $y = 2x - 2$

Substitute for this variable in second equation and solve for the other variable.

$3x + 4(2x - 2) = 6$
$3x + 8x - 8 = 6$
$11x = 14$
$x = 14/11$

Solve for *y*: $y = 2\left(\frac{14}{11}\right) - 2 = \frac{6}{11}$

The solution of the system is $x = \frac{14}{11}$ and $y = \frac{6}{11}$, or $\left(\frac{14}{11}, \frac{6}{11}\right)$.

12. $\begin{cases} 4x - y = 3 \\ 2x + 3y = 19 \end{cases}$

Substitution: $y = 4x - 3$

$2x + 3(4x - 3) = 19$
$2x + 12x = 9 + 19$
$14x = 28$
$x = 2$
$y = 4(2) - 3 = 5$

The solution of the system is $x = 2$ and $y = 5$, or $(2,5)$.

13. $7x+2y=26$, $3x-4y=16$

Multiply 1st equation by 2.

$$\begin{aligned} 14x+4y&=52\\ 3x-4y&=16 \end{aligned}$$

Add the two equations. $17x=68$

Solve for the variable. $x=4$

Substitute for this variable in either original equation and solve for the other variable.

$$\begin{aligned} 3(4)-4y&=16\\ 4y&=4\\ y&=1 \end{aligned}$$

The solution of the system is $x=4$ and $y=1$, or $(4,1)$.

14. $2x+5y=24$, $-6x+2y=30$

Multiply 1st equation by 3.

$$\begin{aligned} 6x+15y&=72\\ -6x+2y&=30 \end{aligned}$$

Add the two equations. $17y=102$

Solve for the variable. $y=6$

Substitute for this variable in either original equation and solve for the other variable.

$$\begin{aligned} 2x+5(6)&=24\\ 2x&=-6\\ x&=-3 \end{aligned}$$

The solution of the system is $x=-3$ and $y=6$, or $(-3,6)$.

15. $3x+4y=1$, $2x-3y=12$

Multiply 1st equation by 3. $9x+12y=3$

Multiply 2nd equation by 4. $8x-12y=48$

Add the two equations. $17x=51$

Solve for the variable. $x=3$

Substitute for this variable in either original equation and solve for the other variable.

$$\begin{aligned} 3(3)+4y&=1\\ 4y&=-8\\ y&=-2 \end{aligned}$$

The solution of the system is $x=3$ and $y=-2$, or $(3,-2)$.

16.

$$\begin{cases} 5x-2y=4\\ 2x-3y=5 \end{cases} \rightarrow \begin{aligned} 10x-4y&=8\\ -10x+15y&=-25\\ 11y&=-17\\ y&=-\frac{17}{11} \end{aligned}$$

$$2x-3\left(-\frac{17}{11}\right)=5$$

$$\rightarrow \quad 2x+\frac{51}{11}=5$$

$$\rightarrow \quad 2x=\frac{4}{11}$$

$$\rightarrow \quad x=\frac{2}{11}$$

The solution of the system is $x=\frac{2}{11}$ and $y=-\frac{17}{11}$, or $\left(\frac{2}{11},-\frac{17}{11}\right)$.

17. $-4x+3y=-5$ Multiply first equation by 3. $-12x+9y=-15$

$3x-2y=4$ Multiply second equation by 4. $\underline{12x-8y=16}$

Add the two equations. $y=1$

Substitute for this variable in either original equation and solve for the other variable.

$$-4x+3(1)=-5$$
$$-4x=-8$$
$$x=2$$

The solution of the system is $x = 2$ and $y = 1$.

18.
$$\begin{cases} x+2y=3 \\ 3x+6y=6 \end{cases} \rightarrow \begin{array}{r} 3x+6y=9 \\ \underline{-3x-6y=-6} \\ 0\neq 3 \end{array}$$
No solution.

19. $0.2x-0.3y=4$ $\quad 0.20x-0.3y=4$

$2.3x- \quad y=1.2$ Multiply 2nd equation by 0.3. $\underline{0.69x-0.3y=0.36}$

Subtract the two equations. $-0.49x \quad =3.64$

Solve for the variable. $x=-\frac{52}{7}$

Substitute, solve for y. $y=-\frac{128}{7}$

The solution of the system is
$x=-\dfrac{52}{7}$ and $y=-\dfrac{128}{7}$, or $\left(-\dfrac{52}{7},-\dfrac{128}{7}\right)$.

20.
$$\begin{cases} 0.5x+y=3 \\ 0.3x+0.2y=6 \end{cases} \rightarrow \begin{array}{r} -0.5x-y=-3 \\ \underline{1.5x+y=30} \\ x \quad =27 \end{array}$$

$$0.5(27)+y=3$$
$$13.5+y=3$$
$$y=-10.5$$

The solution of the system is $x=27$ and $y=-10.5$, or $(27,-10.5)$.

21. $\frac{5}{2}x-\frac{7}{2}y=-1$ Multiply first equation by 6. $15x-21y=-6$

$8x+3y=11$ Multiply second equation by 7. $\underline{56x+21y=77}$

Add the two equations. $71x \quad =71$

Substitute for this variable in either original equation and solve for the other variable.

$$x=1$$
$$8(1)+3y=11$$
$$3y=3$$
$$y=1$$

The solution of the system is $x = 1$ and $y = 1$, or (1, 1).

22. $\begin{cases} x - \frac{1}{2}y = 1 \\ \frac{2}{3}x - \frac{1}{3}y = 1 \end{cases} \rightarrow \begin{array}{r} -2x + y = -2 \\ \underline{2x - y = 3} \\ 0 \neq 1 \end{array}$

No solution.

23. $4x + 6y = 4$ $\qquad\qquad\qquad\qquad\qquad\qquad 4x + 6y = 4$

$2x + 3y = 2$ Multiply second equation by -2. $\quad \underline{-4x - 6y = -4}$

Add the two equations: $\qquad 0 = 0$

There are infinitely many solutions.
The system is dependent. Solve for one
of the variables in terms of the remaining variable:

$y = \frac{2}{3} - \frac{2}{3}x$.

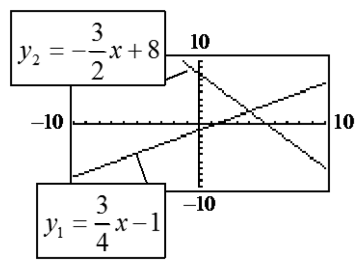
Then a general solution is $\left(c, \frac{2}{3} - \frac{2}{3}c\right)$,

where any value of c will give a particular solution.

24. $\begin{cases} 6x - 4y = 16 \\ 9x - 6y = 24 \end{cases} \rightarrow \begin{array}{r} -36x + 24y = -96 \\ \underline{36x - 24y = 96} \\ 0 = 0 \end{array}$

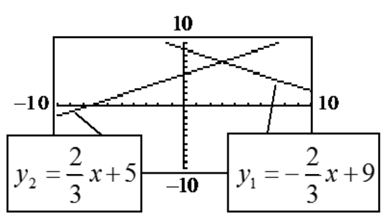
There are infinitely many solutions.
The system is dependent.

25. $\begin{cases} y = 8 - \dfrac{3x}{2} \\ y = \dfrac{3x}{4} - 1 \end{cases}$



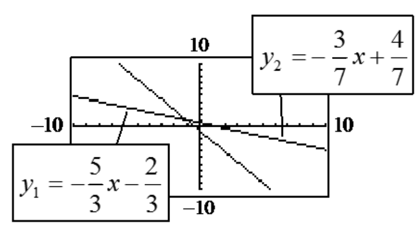
Solution: $(4, 2)$

26. $\begin{cases} y = 9 - \dfrac{2x}{3} \\ y = 5 + \dfrac{2x}{3} \end{cases}$



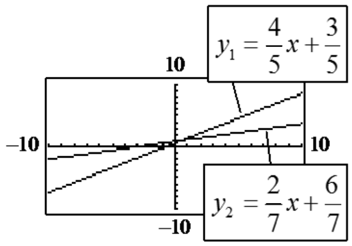
Solution: (3, 7)

27. $\begin{cases} y_1 : 5x + 3y = -2 \\ y_2 : 3x + 7y = 4 \end{cases}$



Solution: (–1, 1)

28. $\begin{cases} y_1: 4x-5y=-3 \\ y_2: 2x-7y=-6 \end{cases}$



Solution: $\left(\frac{1}{2}, 1\right)$

29. Eq. 1 $x+2y+z=2$ Steps 1, 2, and 3 of the systematic
Eq. 2 $-y+3z=8$ procedure are completed.
Eq. 3 $2z=10$ Step 4: $z=5$
From Eq. 2 $-y+3(5)=8$ or $y=7$
From Eq. 1 $x+2(7)+5=2$ or $x=-17$
The solution is $x=-17$, $y=7$, $z=5$.

30.
$$\begin{aligned} x-2y+2z&=-10 \\ y+4z&=-10 \\ -3z&=9 \\ z&=-3 \end{aligned} \qquad \begin{aligned} y+4(-3)&=-10 \\ y-12&=-10 \\ y&=2 \end{aligned} \qquad \begin{aligned} x-2(2)+2(-3)&=-10 \\ x-4-6&=-10 \\ x-10&=-10 \\ x&=0 \end{aligned}$$
Solution: (0, 2, –3)

31. Eq. 1 $x-y-8z=0$ Steps 1 and 2 of the systematic
Eq. 2 $y+4z=8$ procedure are completed.
Eq. 3 $3y+14z=22$
Step 3: $(-3)\times$ Eq 2 added to Eq. 3 gives $2z=-2$ or $z=-1$.
From Eq. 2 $y+4(-1)=8$ or $y=12$
From Eq. 1 $x-12-8(-1)=0$ or $x=4$
The solution is $x=4$, $y=12$, $z=-1$ or (4, 12, –1).

32.
$$\begin{aligned} x+3y-8z&=20 \\ y-3z&=11 \\ 2y+7z&=-4 \end{aligned} \;\to\; \begin{aligned} x+3y-8z&=20 \\ 2y-6z&=22 \\ 2y+7z&=-4 \end{aligned} \;\to\; \begin{aligned} x+3y-8z&=20 \\ 2y-6z&=22 \\ -13z&=26 \end{aligned} \qquad z=-2$$
$$\begin{aligned} 2y-6(-2)&=22 \\ 2y+12&=22 \\ 2y&=10 \\ y&=5 \end{aligned} \qquad \begin{aligned} x+3(5)-8(-2)&=20 \\ x+15+16&=20 \\ x+31&=20 \\ x&=-11 \end{aligned}$$
Solution: (–11, 5, –2)

33. Eq. 1 $x+4y-2z=9$ Step 1 is completed.
Eq. 2 $x+5y+2z=-2$
Eq. 3 $x+4y-28z=22$
Step 2:

$$\begin{array}{lll} & x+4y-2z=9 & \text{Eq. 1} \\ \text{Eq. 4} & y+4z=-11 & (-1)\times \text{ Eq. 1 added to Eq. 2} \\ \text{Eq. 5} & -26z=13 & (-1)\times \text{ Eq. 1 added to Eq. 3} \end{array}$$

Step 3 is also completed.

Step 4: $z=-\frac{1}{2}$ from Eq. 5.

From Eq. 4 $y+4\left(-\frac{1}{2}\right)=-11$ or $y=-9$

From Eq. 1 $x+4(-9)-2\left(-\frac{1}{2}\right)=9$ or $x=44$

The solution is $x = 44$, $y = -9$, $z=-\frac{1}{2}$ or $\left(44, -9, -\frac{1}{2}\right)$.

34.
$$\begin{array}{rcrcrcr} x-3y-z=0 & \to & x-3y-z=0 & \to & x-3y-z=0 & \to & x-3y-z=0 \\ x-2y+z=8 & \to & x-2y+z=8 & \to & y+2z=8 & \to & y+2z=8 \\ 2x-6y+z=6 & \to & -2y-z=-10 & \to & -2y-z=-10 & \to & 3z=6 \\ & & & & & \to & z=2 \end{array}$$

$$\begin{array}{rr} y+2(2)=8 & x-3(4)-2=0 \\ y+4=8 & x-12-2=0 \\ y=4 & x-14=0 \\ & x=14 \end{array}$$

Solution: $(14,4,2)$

35. a. If $F=2C+30$ and $F=1.8C+32$, then

$$2C+30=1.8C+32$$
$$0.2C=2$$
$$C=10$$

The formulas agree when the temperature is $10°$ Celsius and $50°$ Fahrenheit.

b. At temperatures above $10°$ Celsius, the tourist formula overestimates the actual Fahrenheit temperature. The slope of the tourist formula, 2, means that $2°$ F is added for every $1°$ C, but the actual change is $1.8°$ C.

36. a. $B(x)=0.057x+12.3,\ H(x)=0.224x+9.01$

$$0.057x+12.3=0.224x+9.01$$
$$0.167x=3.29$$
$$x\approx 19.7 \text{ during 2010}$$
$$B(19.7)=H(19.7)\approx \$13.422 \text{ billion}$$

b. Yes

37. a. $x+y=1800$ Total number of tickets

b. $20x=$ revenue from \$20 tickets

c. $30y=$ revenue from \$30 tickets

d. $20x+30y=42,000$ Total Revenue

e. Multiply equation from part (a) by -20.

$$\begin{array}{r} -20x-20y=-36000 \\ 20x+30y=42000 \\ \hline 10y=6000 \\ y=600 \end{array}$$

Substitution into equation from part (a) gives $x=1200$.

Sell 1200 of the \$20 tickets and 600 of the \$30 tickets.

38. $x=$ amount invested at 10%,
$y=$ amount invested at 12%

a. $x+y=500,000$

b. $0.10x$

c. $0.12y$

d. $0.10x+0.12y=53,000$

e.

$$\begin{aligned} -x - y &= -500,000 \\ x + 1.2y &= 530,000 \\ \hline 0.2y &= 30,000 \\ y &= \$150,000 \end{aligned}$$

$$\begin{aligned} x + 150,000 &= 500,000 \\ x &= \$350,000 \end{aligned}$$

Invest \$350,000 at 10% and \$150,000 at 12%.

39. x = amount of safe investment.
y = amount of risky investment.
$x + y = 145{,}600$ Total amount invested
$0.1x + 0.18y = 20{,}000$ Income from investments
The solution is the solution of the above system of equations.

$$\begin{aligned} x + \quad y &= 145,600 \\ x + 1.8y &= 200,000 && (10) \times \text{ second equation} \\ \hline 0.8y &= 54,400 && \text{Subtract equations} \\ y &= 68,000 && \text{Solve for } y \text{ or amount of risky investment.} \end{aligned}$$

Substituting $y = 68{,}000$ into one of the original equations we have $x + 68{,}000 = 145{,}600$ or $x = \$77{,}600$.
Solution: Put \$77,600 in a safe investment and \$68,000 in a risky investment.

40. Let A = loan for product A and
B = loan for product B.

$$\begin{aligned} A + B &= 237,000 &\rightarrow\quad A + B &= 237,000 \\ A &= 69,000 + B &\rightarrow\quad A - B &= 69,000 \\ && \hline 2A &= 306,000 \\ && A &= \$153,000 \end{aligned}$$

$$\begin{aligned} 153,000 + B &= 237,000 \\ B &= \$84,000 \end{aligned}$$

41. x = amount invested at 10%.
y = amount invested at 12%.

$$\begin{aligned} x + y &= 470,000 &\rightarrow\quad x + \quad y &= 470,000 \\ 0.10x + 0.12y &= 51,000 &\rightarrow\quad -x - 1.2y &= -510,000 \\ && \hline -0.2y &= -40,000 \\ && y &= \$200,000 \end{aligned}$$

$$\begin{aligned} x + 200,000 &= 470,000 \\ x &= 270,000 \end{aligned}$$

42. Let B = amount borrowed from bank and
L = amount borrowed from life insurance.

$$\begin{aligned} B + L &= 100,000 \\ B + 1.2L &= 109,000 \\ \hline 0.2L &= 9,000 \\ L &= \$45,000 \end{aligned}$$

$$B + 45,000 = 100,000$$

$$B = \$55,000$$

\$55,000 borrowed from the bank and \$45,000 borrowed from life insurance.

43. A = ounces of substance A.
B = ounces of substance B.

Required ratio $\frac{A}{B}=\frac{3}{5}$ gives $5A - 3B = 0$.

Required nutrition is $5\%A + 12\%B = 100\%$. This gives $5A + 12B = 100$.
The % notation can be trouble. Be careful! Now we can solve the system.

$$\begin{aligned} 5A - 3B &= 0 \\ \underline{5A + 12B} &\underline{= 100} \\ 15B &= 100 \quad \text{Subtract first equation from second.} \\ B &= \frac{100}{15} = \frac{20}{3} \end{aligned}$$

Substituting into the original equation gives $5A - 3\left(\frac{20}{3}\right) = 0$ or $A = 4$.

The solution is 4 ounces of substance A and $6\frac{2}{3}$ ounces of substance B.

44. x = number of glasses of milk
y = number of quarter-pound servings of meat

$$\begin{aligned} 0.1x + 3.4y &= 7.15 \quad \rightarrow \quad x + 34y = 71.5 \qquad \text{Substitution: } x = 71.5 - 34y \\ 8.5x + 22y &= 73.75 \quad \rightarrow \quad 8.5x + 22y = 73.75 \qquad 8.5(71.5 - 34y) + 22y = 73.75 \end{aligned}$$

$$\begin{aligned} 607.75 - 289y + 22y &= 73.75 \\ 607.75 - 267y &= 73.75 \\ -267y &= -534 \\ y &= 2 \end{aligned}$$

$x = 71.5 - 34(2) = 71.5 - 68 = 3.5$
The proper nutrition would be provided with 3.5 glasses of milk and 2 servings of meat.

45. x = population of species A.
y = population of species B.
$2x + y = 10{,}600$ units of first nutrient
$3x + 4y = 19{,}650$ units of second nutrient

$$\begin{aligned} 8x + 4y &= 42{,}400 \quad (4)\times \text{ first equation} \\ \underline{3x + 4y} &\underline{= 19{,}650} \\ 5x \quad &= 22{,}750 \quad \text{Subtract} \\ x \quad &= 4550 \quad \text{Solve for } x \end{aligned}$$

Substituting $x = 4550$ into an original equation we have $2(4550) + y = 10{,}600$.
So, $y = 1500$. Solution is 4550 of species A and 1500 of species B.

46. x = number of cubic centimeters of 40% solution
y = number of cubic centimeters of 10% solution

$$\begin{aligned} x + y &= 25 \quad \rightarrow \quad x + y = 25 \qquad \text{Substitution: } x = 25 - y \\ 0.40x + 0.10y &= 0.28(25) \quad \rightarrow \quad 0.40x + 0.10y = 7 \qquad 0.40(25 - y) + 0.10y = 7 \end{aligned}$$

$$\begin{aligned} 10 - 0.40y + 0.10y &= 7 \\ 10 - 0.30y &= 7 \\ -0.30y &= -3 \\ y &= 10 \end{aligned}$$

The biologist should mix 10 cc of 10% solution with 15 cc of 40% solution.

47. x = amount of 20% concentration.
y = amount of 5% concentration.
$x + y = 10$ amount of solution
$0.20x + 0.05y = 0.155(10)$ concentration of medicine
Solving this system of equations:

$$\begin{aligned} x + y &= 10 \\ \underline{x + 0.25y = 7.75} &\quad (5)\times \text{ second equation} \\ 0.75y &= 2.25 \quad \text{Subtract equations} \\ y &= 3 \quad \text{Solve for } y \end{aligned}$$

Substituting into the first equation we have $x + 3 = 10$ or $x = 7$.
The solution is 3 cc of 5% concentration and 7 cc of 20% concentration.

48. A = dosage of medication A
B = dosage of medication B

$$\begin{aligned} 8A = 5B \quad &\rightarrow \quad 8A - 5B = 0 \quad \rightarrow \quad 24A - 15B = 0 \\ 6A + 2B = 50.6 \quad &\rightarrow \quad 6A + 2B = 50.6 \quad \rightarrow \quad \underline{24A + 8B = 202.4} \\ & \qquad\qquad\qquad\qquad\qquad 23B = 202.4 \\ & \qquad\qquad\qquad\qquad\qquad B = 8.8 \end{aligned}$$

$$\begin{aligned} 8A - 5(8.8) &= 0 \\ 8A - 44 &= 0 \\ 8A &= 44 \\ A &= 5.5 \end{aligned}$$

Each dosage of medication should be 5.5 mg of A and 8.8 mg of B.

49. x = number of \$40 tickets.
y = number of \$60 tickets.

$$\begin{aligned} x + y = 16{,}000 \quad &\rightarrow \quad -40x - 40y = -640{,}000 \qquad x + 6{,}000 = 16{,}000 \\ 40x + 60y = 760{,}000 \quad &\rightarrow \quad \underline{40x + 60y = 760{,}000} \qquad x = 10{,}000 \\ & \qquad 20y = 120{,}000 \\ & \qquad y = 6{,}000 \end{aligned}$$

50. x = lbs of peanuts
y = lbs of cashews

$$\begin{aligned} x + y &= 100 & \text{Substitution: } x &= 100 - y \\ 2.80x + 5.30y &= 3.30(100) & 2.80(100 - y) + 5.30y &= 330 \\ & & 280 - 2.80y + 5.30y &= 330 \\ & & 2.50y &= 50 \\ & & y &= 20 \end{aligned}$$

The wholesaler should mix 20 pounds of cashews with 80 pounds of peanuts.

51. x = amount of 20% solution to be added.
$0.20x$ = concentration of nutrient in 20% solution.
$0.02(100) = 2$ is the concentration of nutrient in 2% solution.

$$\begin{aligned} 0.20x + 2 &= 0.10(x + 100) \\ 0.20x + 2 &= 0.10x + 10 \\ 0.1x &= 8 \text{ or } x = 80 \text{ cc of 20\% solution is needed.} \end{aligned}$$

52. Let x = the number of gallons of 13.5% washer fluid.

$$\begin{aligned} 0.135x + 0.11(200) &= 0.13(x + 200) \\ 0.135x + 22 &= 0.13x + 26 \\ 0.005x &= 4 \\ x &= 800 \text{ gallons} \end{aligned}$$

53. x = ounces of substance A,
y = ounces of substance B, and
z = ounces of substance C.

$5x+15y+12z=100$ Nutrition requirements

$x=z$ Digestive restrictions

$y=\frac{1}{5}z$ Digestive restrictions

Since both x and y are in terms of z, we can substitute in the first equation and solve for z.
So, $5z+3z+12z=100$ or $20z=100$. So, z = 5. Now, since $x=z$, we have x = 5.

Since $y=\frac{1}{5}z$, we have y = 1. The solution is 5 ounces of substance A, 1 ounce of substance B, and 5 ounces of substance C.

54. Let x = the number of glasses of skim milk
y = the number of $\frac{1}{4}$ lb servings of meat
z = the number of 2-slice servings of bread.

$$\begin{aligned} 0.1x+3.4y+2.2z=10.5 &\rightarrow & x+34y+22z=105 &\rightarrow & x+34y+22z=105 \\ 8.5x+22y+10z=94.5 &\rightarrow & 85x+220y+100z=945 &\rightarrow & 14y+10z=44 \\ x+20y+12z=61 &\rightarrow & x+20y+12z=61 &\rightarrow & -2670y-1770z=-7980 \end{aligned}$$

$$\begin{aligned} x+34y+22z=105 & & x+34y+22z=105 & & \\ y+\frac{5}{7}z=\frac{22}{7} &\rightarrow & y+\frac{5}{7}z=\frac{22}{7} &\rightarrow & z=3;\ y=\frac{22}{7}-\frac{5}{7}(3)=1;\ x=105-34-66=5 \\ -2670y-1770z=-7980 & & \frac{960}{7}z=\frac{2880}{7} & & \end{aligned}$$

The solution is: $(5,1,3)$.
The requirements will be met with 5 glasses of milk, 1 serving of meat and 3 servings of bread.

55. A = number of A type clients.
B = number of B type clients.
C = number of C type clients.

$A+B+C=500$ Total clients
$200A+500B+300C=150{,}000$ Counseling costs
$300A+200B+100C=100{,}000$ Food and shelter

To find the solution we must solve the system of equations.

Eq. 1 $A+B+C=500$

Eq. 2 $2A+5B+3C=1500$ Original equation divided by 100

Eq. 3 $3A+2B+C=1000$ Original equation divided by 100

$A+B+C=500$

Eq. 4 $3B+C=500$ $(-2)\times$ Eq. 1 added to Eq. 2

Eq. 5 $-B-2C=-500$ $(-3)\times$ Eq. 1 added to Eq. 3

$A+B+C=500$ Eq. 1

$3B+C=500$ Eq. 4

$-\frac{5}{3}C=\frac{-1000}{3}$ $\frac{1}{3}\times$ Eq. 4 added to Eq. 5

$C=\frac{1000}{3}\cdot\frac{3}{5}=200$

Substituting $C = 200$ into Eq. 4 gives $3B + 200 = 500$ or $3B = 300$. So, $B = 100$.
Substituting $C = 200$ and $B = 100$ into Eq. 1 gives $A + 100 + 200 = 500$. So, $A = 200$.
Thus, the solution is 200 type A clients, 100 type B clients, and 200 type C clients.

56. A = number of type A clients
B = number of type B clients
C = number of type C clients

$$200A+500B+300C=135,000$$
$$300A+200B+100C=90,000 \quad \begin{cases} A+B+C=450 \\ 300B+100C=45,000 \\ -100B-200C=-45,000 \end{cases} \quad \begin{cases} A+B+C=450 \\ B+2C=450 \\ -500C=-90,000 \end{cases}$$
$$A+B+C=450 \qquad \rightarrow \qquad C=180$$

Substitution gives 180 type A clients, 90 type B clients, and 180 type C clients.

Exercises 1.6

1. a. $P(x)=R(x)-C(x)$
$=68x-(34x+6800)$
$=34x-6800$

b. $P(3000)=34(3000)-6800=\$95,200$

2. a. $P(x)=R(x)-C(x)$
$=430x-(210x+3300)$
$=220x-3300$

b. $P(500)=220(500)-3300=\$106,150$

3. a. $P(x)=R(x)-C(x)$
$=80x-(43x+1850)$
$=37x-1850$

b. $P(30)=37(30)-1850=-\$740$
The total costs are more than the revenue.

c. $P(x)=0$ or $37x-1850=0$
So, $x=\frac{1850}{37}=50$ units is the break-even point.

4. a. $P(x)=R(x)-C(x)$
$=385x-(85x+3300)$
$=300x-3300$

b. $P(351)=300(351)-3300=\$102,000$

c. To avoid losing money, profit must be at least 0.
$0=300x-3300$
$3300=300x$
$x=11$

5. $C(x) = 5x + 250$

a. $m = 5$, C-intercept: 250

b. $\overline{MC}=5$ means that each additional unit produced costs \$5.

c. Slope = marginal cost.
C-intercept =fixed costs.

d. \$5, \$5 ($\overline{MC}=5$ at every point)

6. $C(x)=27.55x+5180$

a. $m = 27.55$, $b = 5180$ (C-intercept)

b. Marginal cost = \$27.55. The cost of each additional unit is \$27.55.

c. $m=\overline{MC}=27.55$
C-intercept = FC = \$5180

d. Regardless of the production level, the cost of each additional unit is \$27.55.

7. $R = 27x$

a. $m = 27$

b. 27; each additional unit sold yields \$27 in revenue.

c. In each case, one more unit yields \$27.

8. $R = 38.95x$

a. $m = 38.95$

b. $\overline{MR}=38.95$. Each additional unit sold adds \$38.95 to the total revenue.

c. The revenue from each additional unit sold is \$38.95 whether 50 are currently being sold or 100 are currently being sold.

9. $R(x) = 27x$, $C(x) = 5x + 250$

a. $P(x)=27x-(5x+250)=22x-250$

b. $m = 22$

c. Marginal profit is 22.

d. Each additional unit sold gives a profit of \$22. To maximize profit sell all that you can produce. Note that this is not always true.

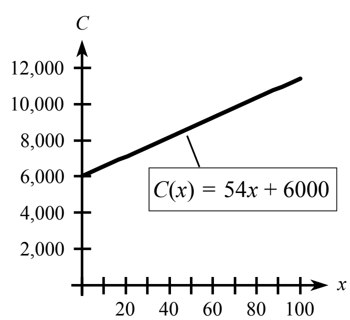
10. $P(x) = R(x) - C(x)$
$= 20x - (21.95x + 1400)$
$= -1.95x - 1400$

a. $\overline{MP} = -1.95$ so the company is losing money on every item produced and sold.

b. Stop production, $P(x)$ is never positive.

11. (x, P) is the correct form.
$P_1 = (200, 3100)$
$P_2 = (250, 6000)$
$$m = \frac{6000 - 3100}{250 - 200} = 58$$
$P - 3100 = 58(x - 200)$ or $P = 58x - 8500$
The marginal profit is 58.

12. $C = 54x + b$, use the fact that (50, 8700) is on the line to solve for b, the fixed costs.
$8700 = 54(50) + b$
$b = 6000$
The cost function is $C(x) = 54x + 6000$.



13. a. $TC = 35H + 6600$

b. $TR = 60H$

c. $P = R - C$
$= 60H - (35H + 6600)$
$= 25H - 6600$

d. $C(200) = 35(200) + 6600$
$= \$13,600$ cost of 200 helmets
$R(200) = 60(200)$
$= \$12,000$ revenue from 200 helmets
$P(200) = R(200) - C(200)$
$= \$12,000 - 13,600$
$= -\$1600$ loss from 200 helmets

e. $C(300) = 35(300) + 6600$
$= \$17,100$ cost of 300 helmets
$R(300) = 60(300)$
$= \$18,000$ revenue from 300 helmets
$P(300) = R(300) - C(300)$
$= 18,000 - 17,100$
$= \$900$ profit from 300 helmets

f. The marginal profit is \$25. Each additional helmet sold gives a profit of \$25.

14. a. $C(x) = 65x + 9800$

b. $R(x) = 100x$

c. $P(x) = 100x - (65x + 9800) = 35x - 9800$

d. $C(250) = 65(250) + 9800 = \$26,050$
$R(250) = \$25,000$
$P(250) = 35(250) - 9800 = -\$1050$
The sale of 250 units gives revenue of \$25,000 at a cost of \$26,050. This results in a loss of \$1050.

e. $C(400) = 65(400) + 9800 = \$35,800$
$R(400) = \$40,000$
$P(400) = 35(400) - 9800 = \$4200$
The sale of 400 units gives revenue of \$40,000 at a cost of \$35,800. This results in a profit of \$4200.

f. The marginal profit is \$35. Each additional unit sold increases the profit \$35.

15. a. The revenue function is the graph that passes through the origin.

b. At a production of zero the fixed costs are \$2000.

c. From the graph, the break-even point is 400 units and \$3000 in revenue or costs.

d. Marginal cost $= \dfrac{3000 - 2000}{400 - 0} = 2.5$

Marginal revenue $= \dfrac{3000 - 0}{400 - 0} = 7.5$

16. $R(x) = 81.50x$, $C(x) = 63x + 1850$
At the break-even point, $R(x) = C(x)$, so
$81.50x = 63x + 1850$
$18.50x = 1850$
$x = 100$ units

17. $R(x) = C(x) = 85x = 35x + 1650$ or $50x = 1650$ or $x = 33$.
Thus, 33 necklaces must be sold to break even.

18. $R(x) = 89x$, $C(x) = 1400 + 75x$
At the break-even point, $R(x) = C(x)$, so
$89x = 1400 + 75x$
$14x = 1400$
$x = 100$ sets of recaps

19. a. $R(x) = 12x$, $C(x) = 8x + 1600$

b. $R(x) = C(x)$ if $12x = 8x + 1600$ or $4x = 1600$ or $x = 400$.
It takes 400 units to break even.

20. a. $R(x) = 50x$
$C(x) = 30x + 10{,}000$

b. At the break-even point, $R(x) = C(x)$, so

$$50x = 30x + 10{,}000$$
$$20x = 10{,}000$$
$$x = 500 \text{ watches}$$

21. a.
$$P(x) = R(x) - C(x)$$
$$= 12x - (8x + 1600)$$
$$= 4x - 1600$$

b. By setting $P(x) = 0$ we get $x = 400$.
Same as 19(b).

22. a.
$$P(x) = R(x) - C(x)$$
$$= 50x - (30x + 10{,}000)$$
$$= 20x - 10{,}000$$

b.
$$20x - 10{,}000 = 0$$
$$20x = 10{,}000$$
$$x = 500$$
Same as 20(b).

23. a. $TC = 4.50x + 1045$

b. $TR = 10x$

c.
$$P = R - C$$
$$= 10x - (4.50x + 1045)$$
$$= 5.50x - 1045$$

d. Break even also means $P = 0$.
$$5.50x - 1045 = 0$$
$$5.50x = 1045$$
$$x = 190 \text{ units to break even}$$

24. a. $C(x) = 0.80x + 1245$

b. $R(x) = 4.95x$

c.
$$P(x) = 4.95x - (0.80x + 1245)$$
$$= 4.15x - 1245$$

d. From $4.95x = 0.80x + 1245$, we have $x = 300$ units to break even.

25. a. $R(x) = 54.90x$

b. $P_1 = (2000,\ 50000)$
$P_2 = (800,\ 32120)$
$$m = \frac{32{,}120 - 50{,}000}{800 - 2000} = \frac{-17{,}880}{-1200} = 14.90$$
$$y - 50{,}000 = 14.90(x - 2000) \text{ or}$$
$$y = 14.90x + 20{,}200 = C(x)$$

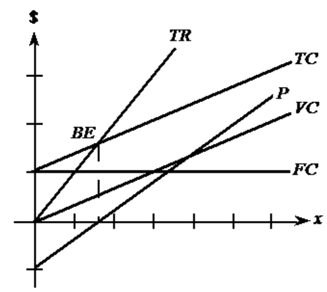
c. From $54.90x = 14.90x + 20{,}200$ we have $x = 505$ units to break even.

26. a. $R(x) = 50x$

b. We have points (100, 4360) and (250, 7060) on the cost function line.
$$m = \frac{7060 - 4360}{250 - 100} = \frac{2700}{150} = 18 = \overline{MC}$$
$$C(x) = mx + b$$
$$4360 = 18(100) + b$$
$$4360 = 1800 + b$$
$$2560 = b = \text{ fixed costs}$$
$$C(x) = 18x + 2560$$

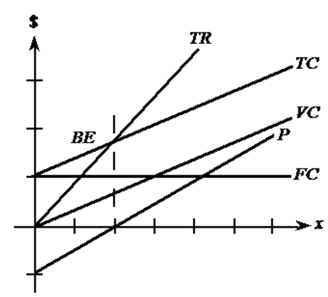
c. At the break-even point, $R(x) = C(x)$, so
$$50x = 18x + 2560$$
$$32x = 2560$$
$$x = 80$$

27. a.



b. TR starts at the origin and intersects TC at the break-even (BE). FC is a horizontal line from the vertical intercept of TC. VC starts at the origin and is parallel to TC.

28. a.



b. The upper line must be TC. The horizontal line for FC is drawn from the y-intercept of TC. VC starts at the origin and is parallel to TC. The lower line on the original graph must be P. The x-value of the BE occurs at the point where P crosses x-axis. You can then mark this point on R (using the same x-value) and use it to draw TR (going through the origin and BE).

29. If price increases, then the demand for the product decreases.

30. If the price increases, then the supply will increase.

31. **a.** If p = \$100, then q = 600 (approximately).
b. If p = \$100, then q = 300.
c. There is a shortage since more is demanded.

32. **a.** If p = \$200, then q = 400.
b. If p = \$200, then q = 700.
c. There will be a surplus since more is supplied.

33. Demand:
$$\begin{aligned} 2p+5q &= 200 \\ 2(60)+5q &= 200 \\ 5q &= 80 \\ q &= 16 \end{aligned}$$
Supply:
$$\begin{aligned} p-2q &= 10 \\ 60-2q &= 10 \\ 2q &= 50 \\ q &= 25 \end{aligned}$$
There will be a surplus of 9 units at a price of \$60.00.

34. Demand:
$$\begin{aligned} p+2q &= 100 \\ 14+2q &= 100 \\ 2q &= 86 \\ q &= 43 \end{aligned}$$
Supply:
$$\begin{aligned} 35p-20q &= 350 \\ 35(14)-20q &= 350 \\ -20q &= -140 \\ q &= 7 \end{aligned}$$
There will be a shortage at a price of \$14.

35. Remember that (q, p) is the correct form.
$P_1 = (240, 900)$
$P_2 = (315, 850)$
$$m = \frac{850-900}{315-240} = -\frac{50}{75} = -\frac{2}{3}$$
Note: $m < 0$ for demand equations.
$$p-900 = -\frac{2}{3}(q-240) \text{ or}$$
$$p = -\frac{2}{3}q+1060$$

36. (q, p) is the correct form.
$P_1 = (2500, 1)$
$P_2 = (3500, 0.90)$
$$m = \frac{0.90-1}{3500-2500} = \frac{-0.1}{1000} = -0.0001$$
$$\begin{aligned} p-1 &= -0.0001(q-2500) \\ p-1 &= -0.0001q+0.25 \\ p &= -0.0001q+1.25 \end{aligned}$$

37. (q, p) is the correct form.
$P_1 = (10000, 1.50)$
$P_2 = (5000, 1.00)$
$$m = \frac{1-1.50}{5000-10000} = \frac{-0.50}{-5000} = 0.0001$$
Note: $m > 0$ for supply equations.
$p - 1 = 0.0001(q - 5000)$ or $p = 0.0001q + 0.5$

38. (q, p) is the correct form.
$P_1 = (100{,}000, 30)$
$P_2 = (80{,}000, 25)$
$$m = \frac{25-30}{80{,}000-100{,}000} = 0.00025$$
$$\begin{aligned} p-30 &= 0.00025(q-100{,}000) \\ p-30 &= 0.00025q-25 \\ p &= 0.00025q+5 \end{aligned}$$

39. **a.** The decreasing function is the demand curve. The increasing function is the supply curve.
b. Reading the graph, we have equilibrium at q = 30 and p = 25.

40. **a.** 20
b. 40
c. Surplus of 20 (40 – 20 = 20)

41. **a.** Reading the graph, at p = 20 we have 20 units supplied.
b. Reading the graph, at p = 20 we have 40 units demanded.
c. At p = 20 there is a shortage of 20 units.

42. Surplus

43. By observing the graph in the figure, we see that a price below the equilibrium price results in a shortage.

44. At the market equilibrium point,
Demand = Supply, so

$$\begin{aligned}-2q+320&=8q+2\\318&=10q\\31.8&=q\end{aligned}$$

$$p=-2q+320$$

$$p=-2(31.8)+320=\$256.40$$

45. $-\frac{1}{2}q+28=\frac{1}{3}q+\frac{34}{3}$ Required condition.

$-3q+168=2q+68$ Multiply both sides by 6 to simplify.

$$\begin{aligned}-5q&=-100\\q&=20\end{aligned}$$

Substituting into one of the original equations gives $p=-\frac{1}{2}(20)+28=18$.

Thus, the equilibrium point is $(q, p) = (20, 18)$.

46. At the market equilibrium point,
Demand = Supply, so

$$\begin{aligned}480-3q&=17q+80\\400&=20q\\20&=q\end{aligned}$$

$$p=480-3q$$

$$p=480-3(20)=\$420$$

47. $-4q+220=15q+30$ Required condition.

$190=19q$

$q=10$ Solve for q.

Substituting $q = 10$ into one of the original equations gives $p = 180$.
Thus, the equilibrium point is $(q, p) = (10, 180)$.

48. Demand: (45, 10), (20, 60) Supply: (35, 30), (70, 50)

$$m=\frac{60-10}{20-45}=\frac{50}{-25}=-2 \qquad m=\frac{50-30}{70-35}=\frac{20}{35}=\frac{4}{7}$$

$$p-10=-2(q-45) \qquad p-30=\frac{4}{7}(q-35)$$

$$p-10=-2q+90$$

$$p=-2q+100 \qquad p=\frac{4}{7}q+10$$

Supply = Demand

$$\frac{4}{7}q+10=-2q+100 \qquad p=\frac{4}{7}(35)+10=30$$

$$q=35$$

Market equilibrium point: (35, 30)

49. Demand: (80, 350) and (120, 300) are two points. $m=\frac{350-300}{80-120}=-\frac{5}{4}$

$p-p_1=m(q-q_1)$ or $p-300=-\frac{5}{4}(q-120)$ or $p=-\frac{5}{4}q+450$

Supply: (60, 280) and (140, 370) are two points. $m=\frac{280-370}{60-140}=\frac{9}{8}$

$p-p_1=m(q-q_1)$ or $p-280=\frac{9}{8}(q-60)$ or $p=\frac{9}{8}q+212.5$

Now, set these two equations for p equal to each other and solve for q.

$\frac{9}{8}q+212.5=-\frac{5}{4}q+450$ Required for equilibrium.

$9q+1700=-10q+3600$ Multiply both sides by 8 to simplify.

$$\begin{aligned}19q&=1900\\q&=100\end{aligned}$$

Substituting $q = 100$ into one of the original equations gives $p = 325$.
Thus, the equilibrium point is $(q, p) = (100, 325)$.

50. Demand: (10,75), (30, 25)

$m=\frac{25-75}{30-10}=\frac{-50}{20}=-2.5$

$p-75=-2.5(q-10)$

$p-75=-2.5q+25$

$p=-2.5q+100$

Supply: (35, 80), (5, 20)

$m=\frac{20-80}{5-35}=\frac{-60}{-30}=2$

$p-20=2(q-5)$

$p-20=2q-10$

$p=2q+10$

Demand = Supply
$-2.5q + 100 = 2q + 10$
$p = 2(20) + 10 = 50$
$20 = q$
Market equilibrium point: (20, 50)

51. **a.** Reading the graph, we have that the tax is $15.
b. From the graph, the original equilibrium was (100, 100).
c. From the graph, the new equilibrium is (50, 110).
d. The supplier suffers because the increased price reduces the demand.

52. **a.** 0 (tax decreases units sold by 50)
b. Yes, because fewer units are demanded.

53. New supply price: $p = 15q + 30 + 38 = 15q + 68$
$15q+68=-4q+220$ Required condition

$19q=152$

$q=8$

Substituting $q = 8$ into one of the original equations gives $p = 188$.
Thus, the new equilibrium point is $(q, p) = (8, 188)$.

54. With the $56 tax/unit, supply becomes
$p=17q+80+56=17q+136$
At the equilibrium point, $480-3q=17q+136$

$344=20q$

$q=17.2$

$p = 17(17.2) + 136 = 428.40$. Market equilibrium point: (17.2, 428.40)

55. New supply price: $p=\frac{q}{20}+10+5=\frac{q}{20}+15$

$\frac{q}{20}+15=-\frac{q}{20}+65$ Required condition

$q+300=-q+1300$

$2q=1000$

$q=500$

Thus, $p=\frac{500}{20}+15=40$.

The new equilibrium point is (500, 40).

56. With the $15 tax/unit, supply becomes
$p=3q+35+15=3q+50$
At the equilibrium point, $3q+50=-8q+2800$

$11q=2750$

$q=250$

$p=3(250)+50=800$. Market equilibrium point: (250, 800).

57. Demand: $p=\frac{-q+2100}{60}$

Supply: $p=\frac{q+540}{120}$

New supply:

$p=\frac{q+540}{120}+\frac{1}{2}=\frac{q+540}{120}+\frac{60}{120}=\frac{q+600}{120}$

$\frac{q+600}{120}=\frac{-q+2100}{60}$ Required condition

$q+600=-2q+4200$ Multiply both sides by 120

$3q=3600$

$q=1200$ Thus, $p=\frac{1200+600}{120}=15$.

The new equilibrium quantity is 1200.
The new equilibrium price is $15.

58. With the \$2 tax/unit, supply becomes $p = \frac{1}{45}q + 8 + 2 = \frac{1}{45}q + 10$.

Demand: $p = -\frac{1}{10}q + 230$

$$\frac{1}{45}q + 10 = -\frac{1}{10}q + 230$$

$$\frac{11}{90}q = 220 \quad \rightarrow \quad q = 1800$$

$p = -\frac{1}{10}(1800) + 230 = 50$. Market equilibrium point: (1800, 50)

Chapter 1: Linear Equations and Functions

Chapter 1 Review Exercises

For this set of exercises we will not give reasons for any steps or list any formulas.

1. $3x-8=23$
$$3x=31$$
$$x=\frac{31}{3}$$

2. $2x-8=3x+5$
$$-x=13$$
$$x=-13$$

3. $\dfrac{6x+3}{6}=\dfrac{5(x-2)}{9}$
$$18\left(\frac{6x+3}{6}\right)=18\left(\frac{5(x-2)}{9}\right)$$
$$3(6x+3)=10(x-2)$$
$$18x+9=10x-20$$
$$8x=-29$$
$$x=-\frac{29}{8}$$

4. $2x+\dfrac{1}{2}=\dfrac{x}{2}+\dfrac{1}{3}$
$$12x+3=3x+2$$
$$9x=-1$$
$$x=-\frac{1}{9}$$

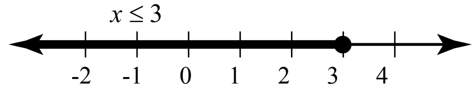
5. $\dfrac{6}{3x-5}=\dfrac{6}{2x+3}$
$$6(2x+3)=6(3x-5)$$
$$2x+3=3x-5$$
$$3+5=3x-2x$$
$$x=8$$

6. $\dfrac{2x+5}{x+7}=\dfrac{1}{3}+\dfrac{x-11}{2(x+7)}$
$$6(2x+5)=2(x+7)+3(x-11)$$
$$12x+30=2x+14+3x-33$$
$$12x-2x-3x=14-33-30$$
$$7x=-49$$
$$x=-7$$

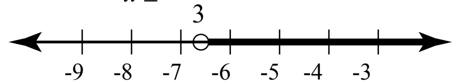
There is no solution since we have division by zero when $x=-7$.

7. $3y-6=-2x-10$
$$3y=-2x-4$$
$$y=\frac{-2x-4}{3}$$
$$y=-\frac{2}{3}x-\frac{4}{3}$$

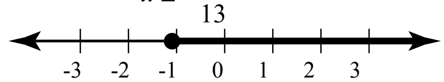
8. $3x-9\le 4(3-x)$
$$3x-9\le 12-4x$$
$$7x\le 21$$
$$x\le 3$$



9. $\dfrac{2}{5}x\le x+4$
$$5\left(\frac{2}{5}x\right)\le 5(x+4)$$
$$2x\le 5x+20$$
$$-3x\le 20$$
$$x\ge -\frac{20}{3}$$



10. $5x+1\ge \dfrac{2}{3}(x-6)$
$$3(5x+1)\ge 3\cdot\frac{2}{3}(x-6)$$
$$15x+3\ge 2(x-6)$$
$$15x+3\ge 2x-12$$
$$13x\ge -15$$
$$x\ge -\frac{15}{13}$$



11. Yes.

12. $y^2=9x$, is not a function of x. If $x=1$, then $y=\pm 3$.

13. Yes.

14. $y=\sqrt{9-x}$
Domain: $9-x\ge 0$ or $9\ge x$ or $x\le 9$.
Range: Nonnegative square root means $y\ge 0$.

15. $f(x)=x^2+4x+5$

a. $f(-3)=(-3)^2+4(-3)+5=9-12+5=2$

b. $f(4)=(4)^2+4(4)+5=16+16+5=37$

c. $f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)+5=\frac{1}{4}+2+5=\frac{29}{4}$

16. $g(x)=x^2+\frac{1}{x}$

a. $g(-1)=(-1)^2+\frac{1}{-1}=1-1=0$

b. $g\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2+\frac{1}{\frac{1}{2}}=\frac{1}{4}+2=2\frac{1}{4}$

c. $g(0.1)=(0.1)^2+\frac{1}{0.1}=0.01+10=10.01$

17. $f(x)=9x-x^2$

$$f(x+h)=9(x+h)-(x+h)^2$$
$$=9x+9h-x^2-2xh-h^2$$
$$f(x)=9x-x^2$$
$$f(x+h)-f(x)=9h-2xh-h^2$$
$$=h(9-2x-h)$$
$$\frac{f(x+h)-f(x)}{h}=9-2x-h$$

18. y is a function of x. (Use vertical-line test.)

19. No, the graph fails vertical-line test.

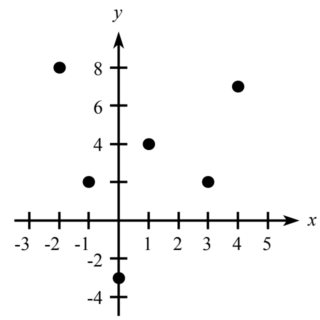
20. $f(2)=4$

21. $x=0, x=4$

22. a. $D=\{-2,-1,0,1,3,4\}$, $R=\{-3,2,4,7,8\}$

b. $f(4)=7$

c. $f(x)=2$ if $x=-1, 3$



d. No. For $y=2$, there are two values of x.

23. $f(x)=3x+5,\ g(x)=x^2$

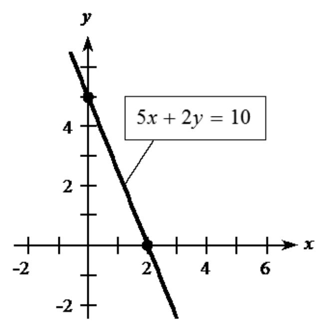
a. $(f+g)x=(3x+5)+x^2=x^2+3x+5$

b. $\left(\frac{f}{g}\right)x=\frac{3x+5}{x^2}$ or $\frac{3x}{x^2}+\frac{5}{x^2}=\frac{3}{x}+\frac{5}{x^2}$

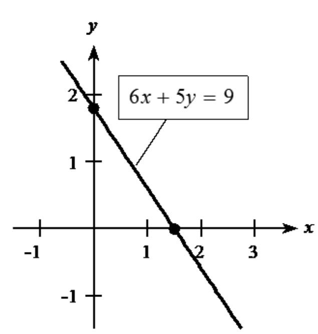
c. $f(g(x))=f(x^2)=3x^2+5$

d. $(f\circ f)x=f(3x+5)$
$$=3(3x+5)+5$$
$$=9x+20$$

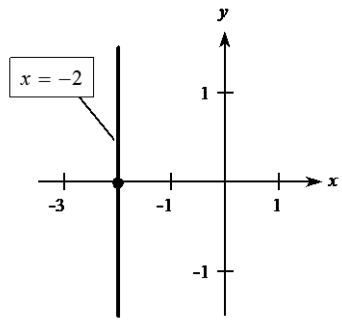
24. $5x+2y=10$
x-intercept: If $y=0, x=2$
y-intercept: If $x=0, y=5$



25. $6x+5y=9$
x-intercept: If $y=0$, $x=\frac{9}{6}=\frac{3}{2}$
y-intercept: If $x=0$ or $y=\frac{9}{5}$



26. $x = -2$
x-intercept: $x = -2$
There is no y-intercept.



27. $P_1(2, -1)$; $P_2(-1, -4)$

$$m = \frac{-4-(-1)}{-1-2} = \frac{-3}{-3} = 1$$

28. $(-3.8, -7.16)$ and $(-3.8, 1.16)$

$$m = \frac{-7.16-1.16}{-3.8-(-3.8)} = \frac{-8.32}{0}$$

Slope is undefined.

29. $2x + 5y = 10$

$$y = -\frac{2}{5}x + 2,\ \ m = -\frac{2}{5},\ \ b = 2$$

30. $x = -\frac{3}{4}y + \frac{3}{2}$ or $y = -\frac{4}{3}x + 2$

$$m = -\frac{4}{3},\ b = 2$$

31. $m = 4, b = 2, y = 4x + 2$

32. $m = -\frac{1}{2},\ b = 3,\ y = -\frac{1}{2}x + 3$

33. $P = (-2, 1),\ \ m = \frac{2}{5}$

$$y - 1 = \frac{2}{5}(x+2) \text{ or } y = \frac{2}{5}x + \frac{9}{5}$$

34. $(-2, 7)$ and $(6, -4)$

$$m = \frac{-4-7}{6-(-2)} = \frac{-11}{8}$$

$$y - 7 = \frac{-11}{8}(x-(-2)) \text{ or}$$

$$y = \frac{-11}{8}x + \frac{17}{4}$$

35. $P_1(-1, 8)$; $P_2(-1, -1)$
The line is vertical since the x-coordinates are the same. Equation: $x = -1$ $x = -1$

36. Parallel to $y = 4x - 6$ means $m = 4$.
$y - 6 = 4(x-1)$ or $y = 4x + 2$

37. $P(-1, 2)$; $\perp$ to $3x + 4y = 12$
or

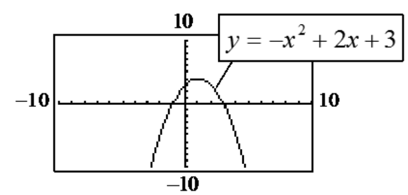
$$y = -\frac{3}{4}x + 3$$

$$m = \frac{4}{3}$$

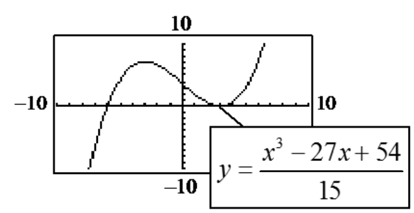
$$y - 2 = \frac{4}{3}(x+1) \text{ or}$$

$$y = \frac{4}{3}x + \frac{10}{3}$$

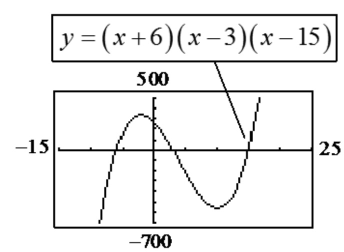
38. $x^2 + y - 2x - 3 = 0$; $y = -x^2 + 2x + 3$



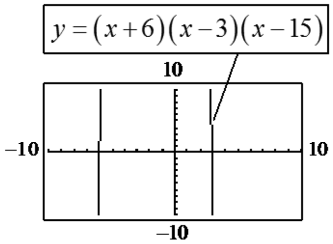
39.



40. a.



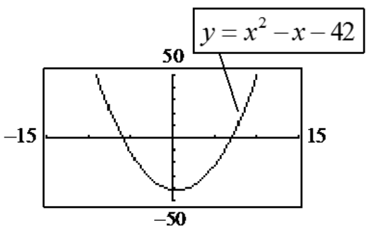
b.



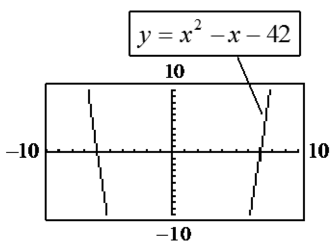
c. The graph in (a) shows the complete graph. The graph in (b) shows a piece that rises toward the high point and a piece between the high and low points.

41. $y=x^2-x-42$ is a parabola opening upward.

a.



b.



c. (a) shows the complete graph. The y-min is too large in absolute value for (b) to get a complete graph.

42. $y=\dfrac{\sqrt{x+3}}{x}$

$x\neq 0$; $x+3\geq 0$ or $x\geq -3$;

Domain: $x\neq 0$, $x\geq -3$

43. Trace approximates $x=-7.2749$, $x=0.2749$

44.

$$\begin{aligned}
4x-2y&=6\\
3x+3y&=9\\
\text{Then,}\quad 12x-6y&=18\\
\underline{6x+6y}&\underline{=18}\\
18x\qquad&=36\\
x&=2\\
4(2)-2y&=6\\
-2y&=-2\\
y&=1
\end{aligned}$$

Solution: $(2,1)$

45.

$$\begin{aligned}
2x+\ \ y&=19\\
x-2y&=12\\
\text{Then,}\quad 4x+2y&=38\\
\underline{x-2y}&\underline{=12}\\
5x\qquad&=50\\
x&=10\\
2(10)+y&=19\\
y&=-1
\end{aligned}$$

Solution: $(10,-1)$

46.

$$\begin{aligned}
3x+2y&=5\\
2x-3y&=12\\
\text{Then,}\quad 9x+6y&=15\\
\underline{4x-6y}&\underline{=24}\\
13x\qquad&=39\\
x&=3\\
3(3)+2y&=5\\
2y&=-4\\
y&=-2
\end{aligned}$$

Solution: $(3,-2)$

47.

$$\begin{aligned}
6x+3y&=1\\
y&=-2x+1\\
6x+3(-2x+1)&=1\\
6x-6x+3&=1\\
3&=1
\end{aligned}$$

No solution.

48.

$$\begin{array}{lll}
4x-3y=253 & 8x-6y=506 & 4(10)-3y=253\\
13x+2y=-12 & \underline{39x+6y=-36} & -3y=213\\
 & 47x\qquad=470 & y=-71\\
 & x\qquad=10 &
\end{array}$$

Solution: $(10,-71)$

49. $\begin{aligned} x+2y+3z&=5 \\ y+11z&=21 \\ 5y+9z&=13 \end{aligned}$

Steps 1 and 2: Nothing to be done.

Step 3: $\begin{aligned} x+2y+3z&=5 \\ y+11z&=21 \\ -46z&=-92 \end{aligned}$

Step 4: $z=2$ $\quad y+11(2)=21$, $y=-1$ $\quad x+2(-1)+3(2)=5$, $x=1$

Solution is $x=1,\ y=-1,\ z=2$.

50. $\begin{aligned} x+y-z&=12 \\ 2y-3z&=-7 \\ 3x+3y-7z&=0 \end{aligned}$

$\begin{aligned} x+y-z&=12 \\ 2y-3z&=-7 \\ -4z&=-36 \end{aligned}$

Thus $z=9$

$2y-27=-7$

$2y=20$ or $y=10$

$x+10-9=12$

$x=11$

Solution: (11, 10, 9)

51. **a.** $1950+70=2020$

b. $y=0.077(100)+13.8=21.5$

21.5 additional years of life expectancy (to age 86.5)

c. $20=0.077x+13.8$

$6.2=0.077x$

$x\approx 80.5$

$1950+80.5=2030.5$

In the year 2031, the function predicts the life expectancy to be 20 years after age 65.

52. Student has total points of $91+82+88+50+42+42=395$.

Total of possible points is $300+150+200=650$.

To earn an A students need at least $0.9(650)=585$ points.

Student must earn $585-395=190$ points on the final. This is the same as 95%.

53. Diesel: $C=0.76x+58{,}000$

Gas: $C=0.88x+53{,}200$

$0.76x+58{,}000=0.88x+53{,}200$

$0.12x=4800$

$x=40{,}000$

Costs are equal at $40{,}000$ miles.

He probably would drive more than 40,000 miles in 7 years, so he should buy the diesel.

54. **a.** Yes

b. No

c. $f(300)=4$

55. **a.** $f(80)=565.44$

b. The monthly payment on a loan is $\$494.75$.

56. $P(x)=330x-0.05x^2-5000$

$x=q(t)=100+10t$

a. $(P\circ q)(t)=P(100+10t)$

$=330(100+10t)-0.05(100+10t)^2-5000$

b. $x=q(15)=100+10(15)$

$=250$ units produced

$P(250)=330(250)-0.05(250)^2-5000$

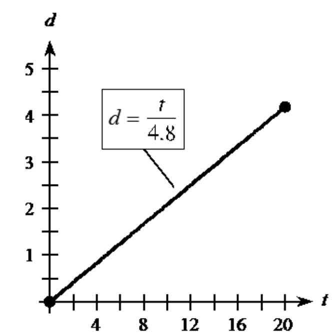
$=\$74{,}375$

57. $W(L)=kL^3,\ L(t)=65-0.1(t-25)^2,\ 0\le t\le 25$

$(W\circ L)(t)=W\left(65-0.1(t-25)^2\right)$

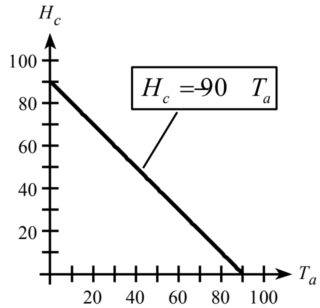
$=0.03\left(65-0.1(t-25)^2\right)^3$

58. **a.**



b. $(9.6, 2)$ means that the thunderstorm is two miles away if the flash and thunder are 9.6 seconds apart.

59.



60. **a.** (x, P) is the required form.

$P_1 = (200, 3100),\ P_2 = (250, 6000)$

$$m = \frac{6000-3100}{250-200} = \frac{2900}{50} = 58$$

$P - 3100 = 58(x-200)$ or

$P(x) = 58x - 8500$

b. For each additional unit sold the profit increases by $58.

61. $A = 427x + 4541$

a. Yes.

b. $m = 427$, A-intercept is 4541

c. In 2000, average annual health care costs were $4541 per customer.

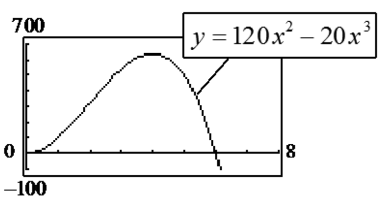
d. The average annual cost costs are changing at the rate of $427 per year.

62. (C, F): $(0, 32)$ and $(100, 212)$

$$m = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

Using $y = mx + b,\ F = \frac{9}{5}C + 32.$

63. **a.**



b. Algebraically, $y \ge 0$ if

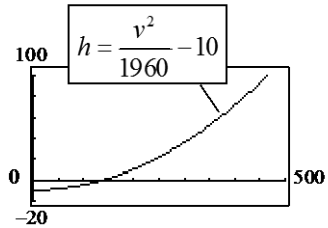
$120x^2 - 20x^3 = 20x^2(x-6) \ge 0.$

Answer: $0 \le x \le 6$

64. **a.** $v^2 = 1960(h+10)$

$$h + 10 = \frac{v^2}{1960}$$

$$h = \frac{v^2}{1960} - 10$$



b. $h(210) = \dfrac{210^2}{1960} - 10 = 12.5$ cm

65. x = amount of safer investment and y = amount of other investment.

$$\begin{aligned} x + y &= 150000 \\ 0.095x + 0.11y &= 15000 \end{aligned}$$

Solving the system:

$$\begin{aligned} 0.11x + 0.11y &= 16500 \\ 0.095x + 0.11y &= 15000 \\ \hline 0.015x \qquad &= 1500 \\ x \qquad &= 100000 \end{aligned}$$

Then $y = 50000$. Thus, invest $100,000 at 9.5% and $50,000 at 11%.

66. x = liters of 20% solution

y = liters of 70% solution

$$\begin{aligned} x + y &= 4 \\ 0.2x + 0.7y &= 1.4 \\ x + y &= 4 \\ x + 3.5y &= 7 \\ \hline 2.5y &= 3 \qquad y = 1.2 \\ x + 1.2 &= 4 \\ x &= 2.8 \end{aligned}$$

Answer: 2.8 liters of 20%, 1.2 of 70%.

67. $S: p = 4q + 5,\ D: p = -2q + 81$

a. $S: 53 = 4q + 5 \qquad D: 53 = -2q + 81$

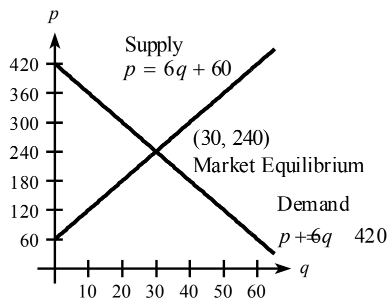
$4q = 48 \qquad 2q = 28$

$q = 12 \qquad q = 14$

b. Demand is greater.
There is a shortfall.

c. Price is likely to increase.

68. a. – c.



69. $C(x) = 38.80x + 4500, R(x) = 61.30x$

a. Marginal cost is $\$38.80$.
b. Marginal revenue is $\$61.30$.
c. Marginal profit is $\$61.30 - 38.80 = \$22.50$.
d. $61.30x = 38.80x + 4500$
$22.50x = 4500$
$x = 200$ units to break even.

70. $FC = \$1500, VC = \$22$ per unit, $R = \$52$ per unit

a. $C(x) = 22x + 1500$
b. $R(x) = 52x$
c. $P = R - C = 30x - 1500$
d. $\overline{MC} = 22$
e. $\overline{MR} = 52$
f. $\overline{MP} = 30$
g. Break even means $30x - 1500 = 0$ or $x = 0$.

71. Supply: $m = \dfrac{200-100}{150-125} = 4$ Demand: $m = \dfrac{200-100}{330-355} = -4$

$p - 100 = 4(q - 125)$ $p - 100 = -4(q - 355)$

$p = 4q - 400$ $p = -4q + 1520$

So, $4q - 400 = -4q + 1520$ or $q = 240$. With $q = 240$, $p = 4(240) - 400 = \$560$. Market equilibrium is achieved with a product quantity of 240 units at a price of \$560 per unit.

72. New supply equation: $p = \dfrac{q}{10} + 8 + 2 = \dfrac{q}{10} + 10$

Demand:
$$p = \frac{-q + 1500}{10} = -\frac{q}{10} + 150$$
$$\frac{q}{10} + 10 = -\frac{q}{10} + 150$$
$$\frac{2q}{10} = 140 \text{ or } q = 700$$
$$p = \frac{700}{10} + 10 = 80$$

Solution: $(700,\ 80)$

Chapter 1: Linear Equations and Functions

Chapter 1 Test

1. $10-2(2x-9)-4(6+x)=52$

$$\begin{aligned} 10-4x+18-24-4x&=52\\ 4-8x&=52\\ -8x&=48\\ x&=-6 \end{aligned}$$

2. $4x-3=\dfrac{x}{2}+6$

$$\begin{aligned} 8x-6&=x+12\\ 7x&=18\\ x&=\frac{18}{7} \end{aligned}$$

3. $\dfrac{3}{x}+4=\dfrac{4x}{x+1}$

$$\begin{aligned} 3(x+1)+4x(x+1)&=4x(x)\\ 3x+3+4x^2+4x&=4x^2\\ 7x&=-3\\ x&=-\frac{3}{7} \end{aligned}$$

4. $\dfrac{3x-1}{4x-9}=\dfrac{5}{7}$

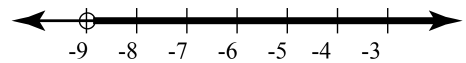
$$\begin{aligned} 7(3x-1)&=5(4x-9)\\ 21x-7&=20x-45\\ x&=-38 \end{aligned}$$

5. $f(x)=7+5x-2x^2$

$$\begin{aligned} f(x+h)&=7+5(x+h)-2(x+h)^2\\ &=7+5x+5h-2x^2-4xh-2h^2\\ f(x)&=7+5x-2x^2\\ f(x+h)-f(x)&=5h-4xh-2h^2\\ \frac{f(x+h)-f(x)}{h}&=5-4x-2h \end{aligned}$$

6. $1+\dfrac{2}{3}t\le 3t+22$

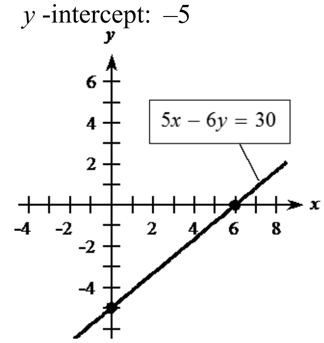
$$\begin{aligned} 3\left(1+\frac{2}{3}t\right)&\le 3(3t+22)\\ 3+2t&\le 9t+66\\ -7t&\le 63\\ t&\ge -9 \end{aligned}$$



7. $5x-6y=30$

x-intercept: 6

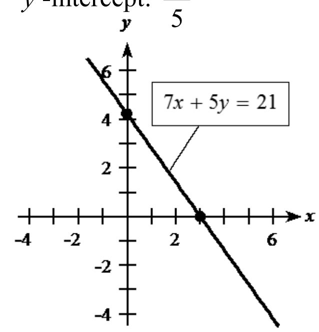
y-intercept: –5



8. $7x+5y=21$

x-intercept: 3

y-intercept: $\dfrac{21}{5}$



9. $f(x)=\sqrt{4x+16}$

a. $4x+16\ge 0$

$4x\ge -16$

Domain: $x\ge -4$; Range: $y\ge 0$

For range, note square root is positive.

b. $f(3)=\sqrt{12+16}=2\sqrt{7}$

c. $f(5)=\sqrt{20+16}=6$

10. $(-1,2)$ and $(3,-4)$

$$m=\frac{-4-2}{3-(-1)}=\frac{-6}{4}=\frac{-3}{2}$$

$$y-2=\frac{-3}{2}(x-(-1))$$

$$y=\frac{-3}{2}x+\frac{1}{2}$$

11. $5x+4y=15$

$$y=-\frac{5}{4}x+\frac{15}{4}$$

$$m=-\frac{5}{4},\ b=\frac{15}{4}$$

12. Point $(-3,-1)$

a. Undefined slope means vertical line. $x=-3$

b. $\perp$ to $y=\frac{1}{4}x+2$ means $m=-4$.

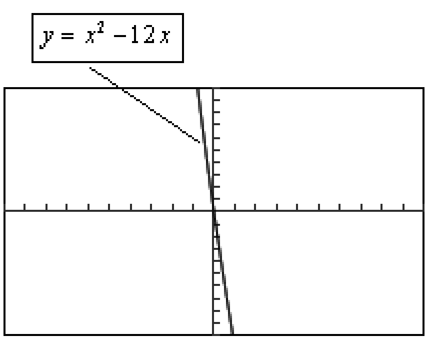
Thus, $y+1=-4(x+3)$ or $y=-4x-13$.

13. **a.** Is not a function since for some x-values there are two values of y.

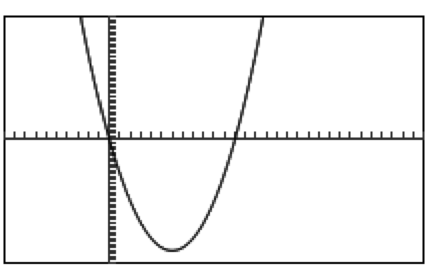
b. Is a function since for each x there is only one y.

c. Is not a function for same reason as (a).

14. **a.**



b.



15. $3x+2y=-2$

$$4x+5y=2$$

$$\begin{aligned}12x+\ \ 8y&=-8\\ \underline{12x+15y}&\underline{=6}\\ -7y&=-14\\ y&=2\\ 3x+2(2)&=-2\\ 3x&=-6\\ x&=-2\end{aligned}$$

Solution: $(-2,2)$

16. $f(x)=5x^2-3x,\ g(x)=x+1$

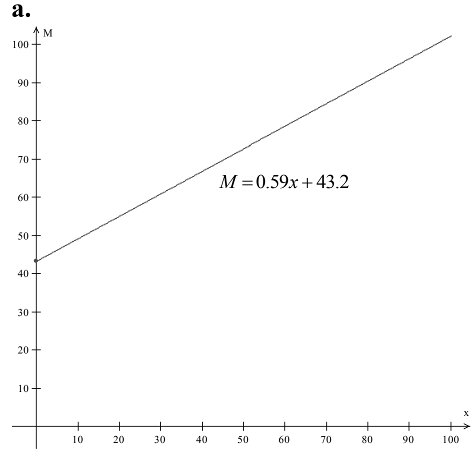
a. $(fg)(x)=(5x^2-3x)(x+1)$

b. $g(g(x))=g(x+1)=(x+1)+1=x+2$

c. $(f\circ g)(x)=f(x+1)$

$$\begin{aligned}&=5(x+1)^2-3(x+1)\\&=5x^2+10x+5-3x-3\\&=5x^2+7x+2\end{aligned}$$

17. **a.**



b. The model predicts that there will be 90.4 million men in the U.S. workforce in 2030.

c. $M=0.59(100)+43.2=102.2$

The model predicts that there will be 102.2 million men in the U.S. workforce in 2050.

18. **a.** $R(x)=38x, C(x)=30x+1200$

, $\overline{MC}=\$30$

b. $P(x)=38x-(30x+1200)$

$=8x-1200$

c. Break-even means $P(x)=0$.

$8x=1200$ or $x=150$ units

d. $\overline{MP}=\$8$. Each additional unit sold increases the profit by $8.

19. **a.** $R(x)=50x$

b. $C(100)=10(100)+18000$

$=\$19,000$

It costs $19,000 to make 100 units.

c. $50x=10x+18000$

$40x=18000$

$x=450$ units

20. $S: p=5q+1500, D: p=-3q+3100$

$5q+1500=-3q+3100$

$8q=1600$ or $q=200$

$p(200)=5(200)+1500=\$2500$

21. $y=720,000-2000x$

a. $b=720,000$

The original value is 720,000.

b. $m=-2000$.

The building is depreciating $2000 each month.

22. $x=$ number of reservations

$0.90x=360$

$x=400$

Accept 400 reservations.

23. $x=$ amount invested at 9%

$y=$ amount invested at 6%

$x+y=20000$ Amount

$0.09x+0.06y=1560$ Interest

$$\begin{array}{r} 0.09x+0.09y=1800 \\ \underline{0.09x+0.06y=1560} \\ 0.03y=240 \\ y=\$8000 \end{array}$$

Invest $8000 at 6% and $12000 at 9%.

Chapter 1: Linear Equations and Functions

Chapter 1 Extended Applications & Group Projects

I. Hospital Administration

1. Revenue per case = \$2000
Annual fixed costs = \$360,000 + \$540,000 = \$900,000
Annual variable costs = $\$\left(760+30+40\cdot\frac{1}{4}\right)x = \$800x$, where x is the number of operations per year.

2. Break-even occurs when Revenue = Total Costs

$$2000x = 900,000 + 800x$$
$$1200x = 900,000$$
$$x = 750$$

The hospital must perform 750 operations per year to break even.

3. We have (70 operations/month)(12 months/year) gives 840 operations/year with a savings of (840 operations)(\$100 savings) = \$84,000 on supplies. However, leasing the machine would cost \$100,000. Thus adding the machine would reduce the hospital's profits by \$16,000 a year at the current level of operations. (Note that 1000 operations must be performed each year to cover the cost of the machine: [(\$100)1000) = \$100,000].)

4. Profit = Revenue – Cost

$$P(x) = 2000x - (900,000 + 800x)$$
$$= 1200x - 900,000$$

At current level of operations, the annual profit is:

$$P(840) = 1200(840) - 900,000$$
$$= 1,008,000 - 900,000$$
$$= \$108,000$$

With (40 new operations/month)(12 months/year) = 480 new operations/year, the new level of operations is 840 + 480 = 1320. The advertising costs are (\$20,000/month)(12 months/year) = \$240,000/year.
At the new level of operations, the profit would be: $P(1320) = 1200(1320) - 900,000 - 240,000$

$$= 1,584,000 - 1,140,000$$
$$= \$444,000$$

The increase in profit is \$444,000 – \$108,000 = \$336,000.

5. Each extra operations adds \$2000 – \$800 = \$1200 of profit. If the ad campaign costs \$20,000 per month it must generate $\frac{\$20{,}000 \text{ per month}}{\$1200 \text{ per operation}} = 16\frac{2}{3}$ operations/month to cover its cost.

6. Recall that the break-even point for leasing the machine is 1000 operations per year. If the ad campaign meets its projections, 1320 operations per year will be performed, with a savings of (320)(\$100) = \$32,000 on medical supplies by leasing the machine. They should reconsider their decision. (Note that this example illustrates that if the assumptions on which a decision was made change, it may be time to take another look at the decision.)

II. Fundraising

(Answers will vary.)

Chapter 2: Quadratic and Other Special Functions

Exercises 2.1

1. $2x^2+3=x^2-2x+4$

$x^2+2x-1=0$

2. $x^2-2x+5=2-2x^2$

$3x^2-2x+3=0$

3. $(y+1)(y+2)=4$

$y^2+3y+2=4$

$y^2+3y-2=0$

4. $(z-1)(z-3)=1$

$z^2-4z+2=0$

5. $x^2-4x=12$

$x^2-4x-12=0$

$x^2-6x+2x-12=0$

$x(x-6)+2(x-6)=0$

$(x-6)(x+2)=0$

$x-6=0$ or $x+2=0$

Solution: x = -2, 6

6. $x^2=11x-10$

$x^2-11x+10=0$

$x^2-10x-x+10=0$

$(x-10)(x-1)=0$

$x-10=0$ or $x-1=0$

Solution: x = 1, 10

7. $9-4x^2=0$

$(3+2x)(3-2x)=0$

$3+2x=0$ or $3-2x=0$

Solution: $x=-\frac{3}{2}, \frac{3}{2}$

8. $25x^2-16=0$

$(5x-4)(5x+4)=0$

$5x-4=0$ or $5x+4=0$

Solution: $x=\frac{4}{5}, -\frac{4}{5}$

9. $x=x^2$

$x^2-x=0$

$x(x-1)=0$

Solution: x = 0, 1

Never divide by a variable. A root is lost if you divide.

10. $t^2-4t=3t^2$

$0=2t^2+4t$

$0=2t(t+2)$

$2t=0$ or $t+2=0$

Solution: t = 0, –2

11. $4t^2-4t+1=0$

$(2t-1)(2t-1)=0$

$2t-1=0$

Solution: $t=\frac{1}{2}$

12. $49z^2+14z+1=0$

$(7z+1)(7z+1)=0$

$7z+1=0$

$7z=-1$

Solution: $z=-\frac{1}{7}$

13. **a.** $x^2-4x-4=0$

$a = 1, b = –4, c = –4$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-4)}}{2(1)}$$

$$=\frac{4\pm\sqrt{32}}{2}=\frac{4\pm4\sqrt{2}}{2}=2\pm2\sqrt{2}$$

b. Since $\sqrt{2}\approx1.414$, the solutions are approximately 4.83, –0.83.

c. $x^2-6x+7=0$

$a = 1, b = –6, c = 7$

$$x=\frac{6\pm\sqrt{36-28}}{2}$$

$$=\frac{6\pm\sqrt{8}}{2}=\frac{6\pm2\sqrt{2}}{2}=3\pm\sqrt{2}$$

$\sqrt{2}\approx1.414$, the solutions are approximately 4.83, –0.83.

14. $x^2 - 6x + 7 = 0$
$a = 1,\ b = -6,\ c = 7$

$$x = \frac{6 \pm \sqrt{36-28}}{2}$$
$$= \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}$$

a. $3+\sqrt{2}, 3-\sqrt{2}$
b. 4.41, 1.59

15. $2w^2 + w + 1 = 0$
$a = 2,\ b = 1,\ c = 1$

$$w = \frac{-1 \pm \sqrt{1-8}}{4} = \frac{-1 \pm \sqrt{-7}}{4}$$

There are no real solutions.

16. $z^2 + 2z + 4 = 0$
$a = 1,\ b = 2,\ c = 4$

$$z = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm \sqrt{-12}}{2}$$

No real solutions.

17. $y^2 = 7$
$y = \pm\sqrt{7}$

18. $z^2 = 12$
$z = \pm\sqrt{12}$
$z = \pm 2\sqrt{3}$

19. $5x^2 = 80$
$x^2 = 16$
$x = \pm 4$

20. $3x^2 = 75$
$x^2 = 25$
$x = \pm 5$

21. $(x+4)^2 = 25$
$x + 4 = \pm 5$
$x = -4 \pm 5$
Solution: $x = 1, -9$

22. $(x+1)^2 = 2$
$x + 1 = \pm\sqrt{2}$
$x = -1 \pm \sqrt{2}$

23. $x^2 + 5x = 21 + x$
$x^2 + 4x - 21 = 0$
$(x+7)(x-3) = 0$
Solution: $x = -7, 3$

24. $x^2 + 17x = 8x - 14$
$x^2 + 9x + 14 = 0$
$(x+7)(x+2) = 0$
Solution: $x = -7, -2$

25. $\frac{w^2}{8} - \frac{w}{2} - 4 = 0$
$w^2 - 4w - 32 = 0$
$(w-8)(w+4) = 0$
$w - 8 = 0$ or $w + 4 = 0$
Solution: $w = 8, -4$

26. $\frac{y^2}{2} - \frac{11}{6}y + 1 = 0$
$3y^2 - 11y + 6 = 0$
$(3y-2)(y-3) = 0$
$3y - 2 = 0$ or $y - 3 = 0$
Solution: $y = \frac{2}{3}, 3$

27. $16z^2 + 16z - 21 = 0$
$a = 16,\ b = 16,\ c = -21$

$$z = \frac{-16 \pm \sqrt{256+1344}}{32}$$
$$= \frac{-16 \pm 40}{32} = \frac{3}{4} \text{ or } -\frac{7}{4}$$

Solution: $z = -\frac{7}{4}, \frac{3}{4}$

28. $10y^2 - y - 65 = 0$

$a = 10,\ b = -1,\ c = -65$

$$y = \frac{1 \pm \sqrt{1-(-2600)}}{20} = \frac{1 \pm \sqrt{2601}}{20} = \frac{1 \pm 51}{20} = -\frac{50}{20} \text{ or } \frac{52}{20}$$

Solution: $y = -\frac{5}{2}, \frac{13}{5}$

29. $(x-1)(x+5) = 7$

$x^2 + 4x - 5 = 7$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

Solution: $x = -6, 2$

30. $(x-3)(1-x) = 1$

$x - x^2 - 3 + 3x = 1$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x - 2 = 0$

Solution: $x = 2$

31. $5x^2 = 2x + 6$ or $5x^2 - 2x - 6 = 0$

$a = 5,\ b = -2,\ c = -6$

$$x = \frac{2 \pm \sqrt{4+120}}{10} = \frac{1 \pm \sqrt{31}}{5}$$

Solution: $x = \frac{1-\sqrt{31}}{5}, \frac{1+\sqrt{31}}{5}$

32. $3x^2 = -6x - 2$

$3x^2 + 6x + 2 = 0$

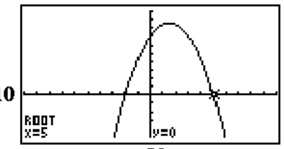
$a = 3,\ b = 6,\ c = 2$

$$x = \frac{-6 \pm \sqrt{36-24}}{6} = \frac{-6 \pm \sqrt{12}}{6} = \frac{-6 \pm 2\sqrt{3}}{6} = \frac{-3 \pm \sqrt{3}}{3}$$

Solution: $x = \frac{-3-\sqrt{3}}{3}, \frac{-3+\sqrt{3}}{3}$

33. $21x + 70 - 7x^2 = 0$

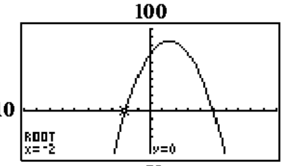
100



–10 10

–50

100



–10 10

–50

Divide by –7 and rearrange.

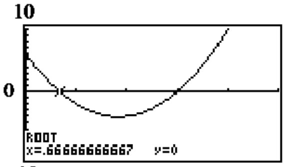
$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

Solution: $x = -2, 5$

34. $3x^2 - 11x + 6 = 0$

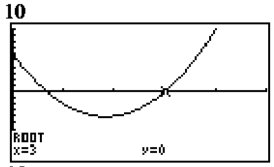
10



0 5

–10

10



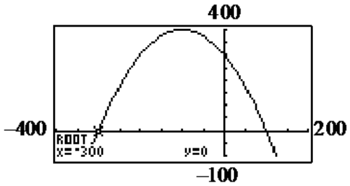
0 5

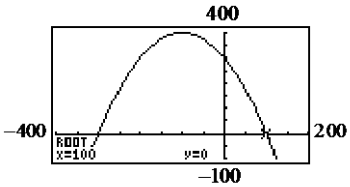
–10

$(3x - 2)(x - 3) = 0$

Solution: $x = \frac{2}{3}, 3$

35. $300-2x-0.01x^2=0$

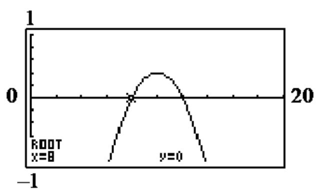


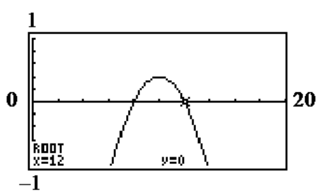


$a=-0.01$, $b=-2$, $c=300$

$$x=\frac{2\pm\sqrt{4+12}}{-0.02}=\frac{2\pm 4}{-0.02}$$
$$=-300 \text{ or } 100$$

36. $-9.6+2x-0.1x^2=0$

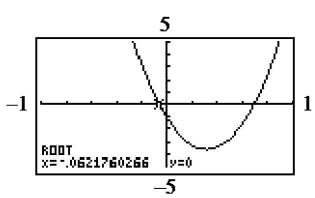


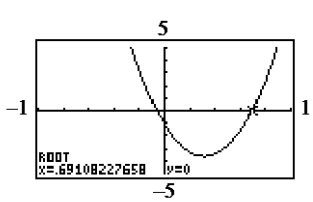


$$-9.6+2x-0.1x^2=0$$
$$x^2-20x+96=0$$
$$(x-12)(x-8)=0$$

Solution; $x=12, 8$

37. $25.6x^2-16.1x-1.1=0$

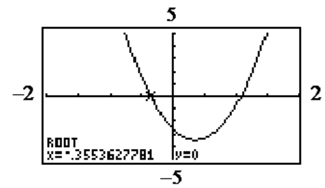


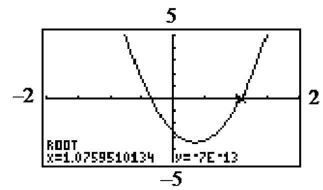


$a=25.6$, $b=-16.1$, $c=-1.1$

$$x=\frac{16.1\pm\sqrt{259.21+112.64}}{51.2}$$
$$=\frac{16.1\pm\sqrt{371.85}}{51.2}$$
$$\approx 0.69 \text{ or } -0.06$$

38. $6.8z^2-4.9z-2.6=0$





$$6.8z^2-4.9z-2.6=0$$
$$z=\frac{4.9\pm\sqrt{24.01+70.72}}{13.6}$$
$$=\frac{4.9\pm\sqrt{94.73}}{13.6}=\frac{4.9\pm 9.73}{13.6}$$
$$=1.08 \text{ or } -0.36$$

39.
$$x+\frac{8}{x}=9$$
$$x^2+8=9x$$
$$x^2-9x+8=0$$
$$(x-8)(x-1)=0$$

Solution: $x=1, 8$

40.
$$\frac{x}{x-2}-1=\frac{3}{x+1}$$
$$x(x+1)-1\cdot(x-2)(x+1)=3(x-2)$$
$$2x+2=3x-6$$
$$x=8$$

Solution: $x=8$

41. $\frac{x}{x-1} = 2x + \frac{1}{x-1}$

$x = (2x^2 - 2x) + 1$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

Solution: $x = \frac{1}{2}$

1 is not a root since division by zero is not defined.

42. $\frac{5}{z+4} - \frac{3}{z-2} = 4$

$5(z-2) - 3(z+4) = 4(z+4)(z-2)$

$2z - 22 = 4z^2 + 8z - 32$

$4z^2 + 6z - 10 = 0$

$2(2z+5)(z-1) = 0$

$2z + 5 = 0$ or $z - 1 = 0$

Solution: $z = -\frac{5}{2}, 1$

43. $(x+8)^2 + 3(x+8) + 2 = 0$

$\left[(x+8)+2\right]\left[(x+8)+1\right] = 0$

$(x+8)+2 = 0$ or $(x+8)+1 = 0$

Solution: $x = -10, -9$

44. $(s-2)^2 - 5(s-2) - 24 = 0$

$\left[(s-2)-8\right]\left[(s-2)+3\right] = 0$

$(s-2)-8 = 0$ or $(s-2)+3 = 0$

Solution: $s = 10, -1$

45. $P = -x^2 + 90x - 200$

$1200 = -x^2 + 90x - 200$

$0 = x^2 - 90x + 1400$

$0 = (x-20)(x-70)$

A profit of \$1200 is earned at $x = 20$ units or $x = 70$ units of production.

46. $P = 16x - 0.1x^2 - 100$

When $P = 180$ we have

$180 = 16x - 0.1x^2 - 100$ or $0.1x^2 - 16x + 280 = 0$

$x = \frac{16 \pm \sqrt{256 - 112}}{0.2} = \frac{16 \pm \sqrt{144}}{0.2}$

$= \frac{16 \pm 12}{0.2} = 140$ or 20 units

47. a. $P = -18x^2 + 6400x - 400$

$61,800 = -18x^2 + 6400x - 400$

$18x^2 - 6400x + 62,200 = 0$

Factoring appears difficult, so let us apply the quadratic formula.

$x = \frac{6400 \pm \sqrt{6400^2 - 4(18)(62,200)}}{36}$

$= \frac{6400 \pm \sqrt{36,481,600}}{36}$

$= \frac{6400 \pm 6040}{36} = 10$ or 345.56

So, a profit of \$61,800 is earned for 10 units or for 345.56 units.

b. Yes. Maximum profit occurs at vertex as seen using the graphing calculator.

48. a. $P = 50x - 300 - 0.01x^2$

When $P = 250$ we have

$250 = 50x - 300 - 0.01x^2$

or $0.01x^2 - 50x + 550 = 0.$

$x = \frac{50 \pm \sqrt{2500 - 22}}{0.02}$

$= \frac{50 \pm 49.78}{0.02} = 11$ or 4989 units

b. Yes. Try $P(4000)$ and $P(5000)$.

$P(4000) > \$250$.

49. $S = 100 + 96t - 16t^2$

$100 = 100 + 96t - 16t^2$

$0 = 96t - 16t^2 = 16t(6-t)$

The ball is 100 feet high 6 seconds later.

50. $D(t) = -16t^2 + 10t + 350$

$$0 = -16t^2 + 10t + 350$$
$$-16t^2 + 10t + 350 = 0$$
$$8t^2 - 5t - 175 = 0$$
$$(8t + 35)(t - 5) = 0$$

The answer $t = 5$ is the only one that makes sense in this case, so the ball hits the ground at 5 seconds.

51. $p = 25 - 0.01s^2$

a. $0 = 25 - 0.01s^2$
$= (5 + 0.1s)(5 - 0.1s)$
$p = 0$ if $5 - 0.1s = 0$ or $s = 50$.

b. $s \geq 0$. $p = 0$ means there is no particulate pollution.

52. $S = 100x - x^2$

a. $0 = x(100 - x)$
A dosage of 0 or 100 ml gives $S = 0$.

b. Dosage is effective if $0 < x < 100$.

53. $t = 0.001\left(0.732x^2 + 15.417x + 607.738\right)$

$$8.99 = 0.001\left(0.732x^2 + 15.417x + 607.738\right)$$
$$8990 = 0.732x^2 + 15.417x + 607.738$$
$$0 = 0.732x^2 + 15.417x - 8382.262$$
$$t = \frac{-15.417 \pm \sqrt{(15.417)^2 - 4(0.732)(-8382.262)}}{2(0.732)}$$

$t \approx 96.996$ or $t \approx -118.058$
The positive answer is the one that makes sense here, 97.0 mph.

54. $B = -0.0046t^2 - 0.033t + 6.05$

$$-5 = -0.0046t^2 - 0.033t + 6.05$$
$$0.0046t^2 + 0.033t - 11.05 = 0$$

Using the quadratic formula or a graphing utility gives the positive value $t \approx 45.6$. The fund is projected to be \$5 trillion in the red in the year 2046.

55. $p = 0.17t^2 - 2.61t + 52.64$

$$55 = 0.17t^2 - 2.61t + 52.64$$
$$0.17t^2 - 2.61t - 2.36 = 0$$

Using the quadratic formula or a graphing utility gives the positive value $t \approx 16.2$. In 2016 the percent of high school seniors who will have tried marijuana is predicted by the function to reach 55%.

56. a. $y = -0.0013x^2 + x + 10$

$$0.0013x^2 - x - 10 = 0$$
$$x \approx -9.873 \text{ or } x \approx 779.104$$

b. $y = -\frac{x^2}{81} + \frac{4}{3}x + 10$

$$\frac{x^2}{81} - \frac{4}{3}x - 10 = 0$$
$$x \approx 115.041 \text{ or } x \approx -7.041$$

Given that the distance x is not negative, the first projectile travels further (approximately 779 feet versus the second projectile's approximately 115 feet).

57. $P = \left(\frac{C}{100}\right) \cdot C$

We know that the selling price is \$144 and that the selling price equals the profit plus the cost C to the store.

$$144 = \frac{C^2}{100} + C$$
$$14400 = C^2 + 100C$$
$$C^2 + 100C - 14400 = 0$$
$$C = -180 \text{ or } C = 80$$

The cost C of the necklace to the store is not negative, so C = \$80 is the amount the store paid for the necklace.

58. $y = 0.787x^2 - 11.0x + 290$
$1000 = 0.787x^2 - 11.0x + 290$
$0.787x^2 - 11x - 710 = 0$

Using the quadratic formula or a graphing utility gives the positive value $x \approx 38$.

Spending is projected to reach $1000 billion in the year 2028.

59. $E = 7.94x^2 + 33.2x + 2190$
$5000 = 7.94x^2 + 33.2x + 2190$
$7.94x^2 + 33.2x - 2810 = 0$

Using the quadratic formula or a graphing utility gives the positive value $x \approx 16.8$.

The model predict these expenditures will reach $5 trillion in 2022.

60. $v = k\left(R^2 - r^2\right)$
$v = 2\left(0.01 - r^2\right)$

In each case below only nonnegative values of r are reported.

a. $0.02 = 2\left(0.01 - r^2\right)$
$0.01 = 0.01 - r^2$
$r^2 = 0$
$r = 0$

b. $0.015 = 2\left(0.01 - r^2\right)$
$0.0075 = 0.01 - r^2$
$r^2 = 0.0025$
$r = 0.05$

c. $0 = 2\left(0.01 - r^2\right)$
$r^2 = 0.01$
$r = 0.1$

In this case the corpuscle is at the wall of the artery.

61. $K^2 = 16v + 4$

In each case below only positive values of K are reported.

a. $K^2 = 16(20) + 4 = 324$
$K = 18$

b. $K^2 = 16(60) + 4 = 964$
$K \approx 31$

c. Speed triples, but K changes only by a factor of 1.72.

62. Given that $s = 16t_1^2$ and $s = 1090t_2$,
$t_1 + t_2 = 3.9 \Rightarrow t_2 = 3.9 - t_1$
$16t_1^2 = 1090t_2$
$= 1090(3.9 - t_1)$
$= 4251 - 1090t_1$
$16t_1^2 + 1090t_1 - 4251 = 0$

Using the quadratic formula or a graphing utility gives the positive value $t_1 \approx 3.70$.

$s = 16t_1^2$
$\approx 16(3.70)^2$
≈ 219

The depth of the fissure is about 219 ft.

Chapter 2: Quadratic and Other Special Functions

Exercises 2.2

Chapter 2: Quadratic and Other Special Functions

1. $y = \frac{1}{2}x^2 + x$

a. $x = \frac{-b}{2a} = \frac{-1}{2(1/2)} = -1$

$y = \frac{1}{2}(-1)^2 + (-1) = -\frac{1}{2}$

Vertex is at $\left(-1, -\frac{1}{2}\right)$.

b. $a > 0$, so vertex is a minimum.

c. -1

d. $-\frac{1}{2}$

2. $y = x^2 - 2x$

a. $x = -\frac{b}{2a} = \frac{2}{2} = 1$

When $x = 1$, $y = -1$. The vertex is $(1, -1)$.

b. $a > 0$, so vertex is a minimum.

c. 1

d. -1

3. $y = 8 + 2x - x^2$

a. $x = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

$y = 8 + 2(1) - (1)^2 = 9$

Vertex is at $(1, 9)$.

b. $a < 0$, so vertex is a maximum.

c. 1

d. 9

4. $y = 6 - 4x - 2x^2$

a. $x = \frac{-b}{2a} = \frac{4}{-4} = -1$

When $x = -1$, $y = 8$. The vertex is $(-1, 8)$.

b. $a < 0$, so vertex is a maximum.

c. -1

d. 8

5. $f(x) = 6x - x^2$

a. $x = \frac{-b}{2a} = \frac{-6}{-2} = 3.$

$f(3) = 6(3) - (3)^2 = 9$

Vertex is at $(3, 9)$.

b. $a < 0$, so vertex is a maximum.

c. 3

d. 9

6. $f(x) = x^2 + 2x - 3$

a. $x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$f(-1) = (-1)^2 + 2(-1) - 3 = -4$

Vertex is at $(-1, -4)$.

b. $a > 0$, so vertex is a minimum.

c. -1

d. -4

7. $y = -\frac{1}{4}x^2 + x$

Vertex is a maximum point since $a < 0$.

V: $x = \frac{-b}{2a} = \frac{-1}{2(-1/4)} = 2$

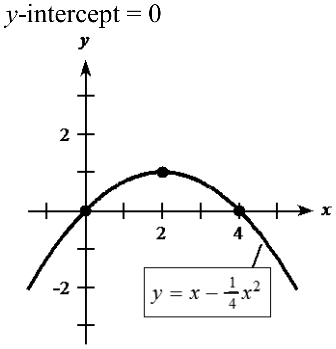
$y = -\frac{1}{4}(2)^2 + 2 = 1$

Zeros: $-\frac{1}{4}x^2 + x = 0$

$x\left(-\frac{1}{4}x + 1\right) = 0$

$x = 0, 4$

y-intercept = 0



8. $y = -2x^2 + 18x$

Vertex is a maximum since $a < 0$.

V: $x = \frac{-b}{2a} = \frac{-18}{-4} = \frac{9}{2}$

$y = -2\left(\frac{9}{2}\right)^2 + 18\left(\frac{9}{2}\right) = -\frac{81}{2} + \frac{162}{2} = \frac{81}{2}$

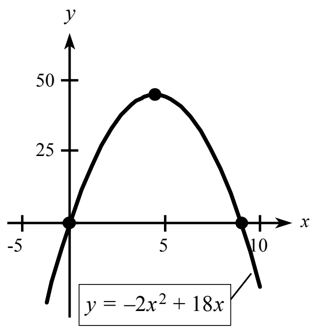
Zeros: $0 = -2x^2 + 18x$

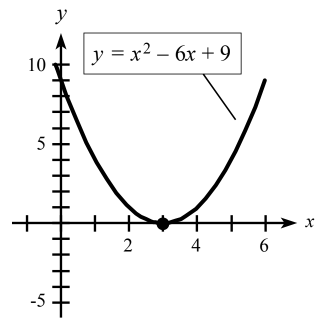
$0 = -2x(x - 9)$

$-2x = 0$ or $x - 9 = 0$

$x = 0 \qquad x = 9$

y-intercept = 0





9. $y = x^2 + 4x + 4$

Vertex is a minimum point since $a > 0$.

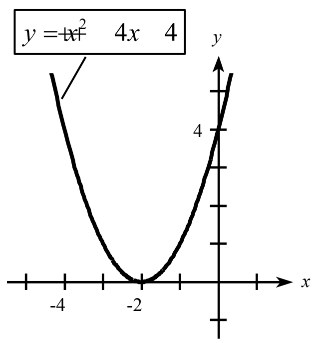
V: $x = \dfrac{-b}{2a} = \dfrac{-4}{2(1)} = -2$

$y = (-2)^2 + 4(-2) + 4 = 0$

Zeros: $x^2 + 4x + 4 = (x+2)(x+2) = 0$

$x = -2$

y-intercept = 4



10. $y = x^2 - 6x + 9$

Vertex is a minimum since $a > 0$.

V: $x = \dfrac{-b}{2a} = \dfrac{6}{2} = 3$

$y = 3^2 - 6(3) + 9 = 0$

Zeros: $0 = x^2 - 6x + 9$

$0 = (x-3)(x-3)$

$x - 3 = 0$

$x = 3$

y-intercept = 9

11. $y = \dfrac{1}{2}x^2 + x - 3$

Vertex is a minimum point since $a > 0$.

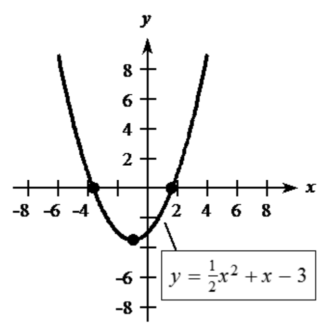
V: $x = \dfrac{-b}{2a} = \dfrac{-1}{2(1/2)} = -1$

$y = \dfrac{1}{2}(-1)^2 + (-1) - 3 = -\dfrac{7}{2}$

Zeros: $\dfrac{1}{2}x^2 + x - 3 = 0 \;\rightarrow\; x^2 + 2x - 6 = 0$

$x = \dfrac{-2 \pm \sqrt{4+24}}{2} = \dfrac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$

y-intercept = −3



12. $x^2 + x + 2y = 5$

$2y = -x^2 - x + 5$

$y = -\frac{1}{2}x^2 - \frac{1}{2}x + \frac{5}{2}$

Vertex is a maximum since $a < 0$.

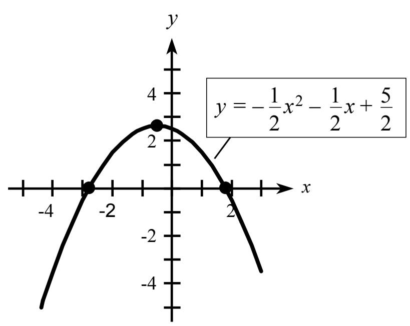
V: $x = \frac{-b}{2a} = -\frac{-\frac{1}{2}}{2\left(-\frac{1}{2}\right)} = -\frac{1}{2}$

$y = -\frac{1}{2}\left(-\frac{1}{2}\right)^2 - \frac{1}{2}\left(-\frac{1}{2}\right) + \frac{5}{2} = \frac{21}{8}$

Zeros: Using the quadratic formula,

$x = \frac{-1 \pm \sqrt{21}}{2}$.

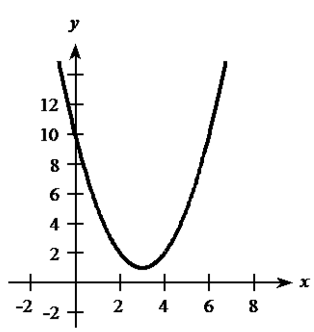
y-intercept $= \frac{5}{2}$



13. $y = (x - 3)^2 + 1$

a. Graph is shifted 3 units to the right and 1 unit up.

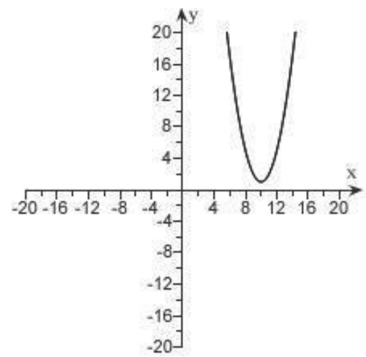
b.



14. $y = (x - 10)^2 + 1$

a. Graph is shifted 10 units to the right and 1 units up.

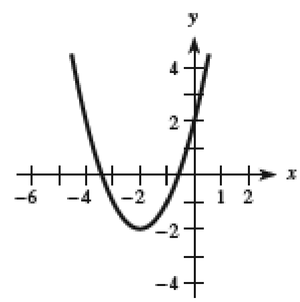
b.



15. $y = (x + 2)^2 - 2$

a. Graph is shifted 2 units to the left and 2 units down.

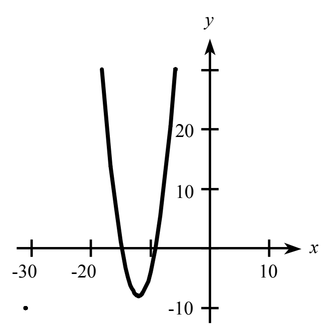
b.



16. $y = (x + 12)^2 - 8$

a. Graph is shifted 12 units to the left and 8 units down.

b.



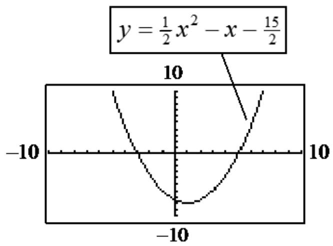
17. $y = \frac{1}{2}x^2 - x - \frac{15}{2}$

V: $x = \frac{-b}{2a} = \frac{-(-1)}{2(1/2)} = 1$

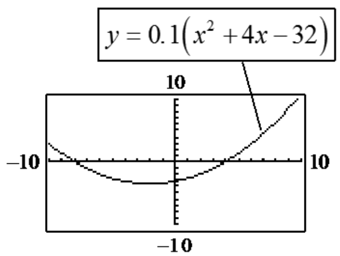
$y = \frac{1}{2}(1)^2 - 1 - \frac{15}{2} = -8$

Zeros: $x^2 - 2x - 15 = (x-5)(x+3) = 0$

$x = 5, -3$



18.



From the graph, the vertex is approximately $(-2, -3.5)$. The zeros are approximately –8 and 4. Algebraic check:

V: x-coordinate: $\frac{-b}{2a} = \frac{-4}{2} = -2$

y-coordinate: $0.1(4 - 8 - 32) = -3.6$

So, actual vertex is (–2, –3.6)

Zeros: $0 = x^2 + 4x - 32 = (x+8)(x-4)$

$x = -8, 4$

19. $y = \frac{1}{4}x^2 + 3x + 12$

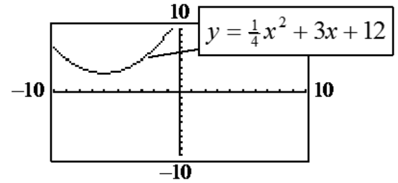
V: $x = \frac{-b}{2a} = \frac{-3}{2\left(\frac{1}{4}\right)} = -6$

$y = \frac{1}{4}(-6)^2 + 3(-6) + 12 = 3$

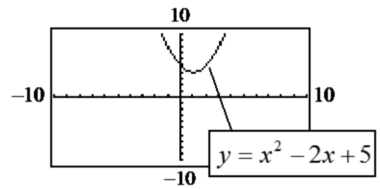
Zeros: $x^2 + 12x + 48 = 0$

$b^2 - 4ac = 144 - 192 < 0$

There are no zeros.



20. $y = x^2 - 2x + 5$



From the graph, the vertex is (1, 4).
There are no real zeros.
Algebraic check:

V: x-coordinate: $-\frac{b}{2a} = -\frac{-2}{2} = 1$

y-coordinate: $1^2 - 2(1) + 5 = 4$

The discriminant is negative, so no real zeros.

21. $f(x) = y = -5x - x^2$

Average Rate of Change $= \frac{f(1) - f(-1)}{1 - (-1)}$

$= \frac{-6 - 4}{2} = -\frac{10}{2} = -5$

22. $f(x) = y = 8 + 3x + 0.5x^2$

Average Rate of Change $= \frac{f(4) - f(2)}{4 - 2}$

$= \frac{28 - 16}{2} = \frac{12}{2} = 6$

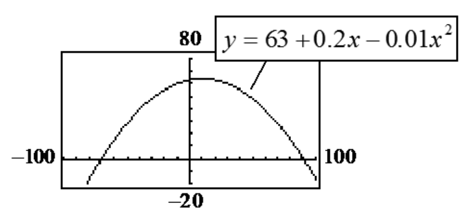
23. $y = 63 + 0.2x - 0.01x^2$

V: $x = \frac{-0.2}{-0.02} = 10$

$y = 63 + 2 - 1 = 64$

Zeros: $x^2 - 20x - 6300 = (x-90)(x+70) = 0$

$x = 90, -70$



24. $y = 0.2x^2 + 16x + 140$

V: x-coordinate: $\dfrac{-b}{2a} = \dfrac{-16}{2(0.2)} = -40$

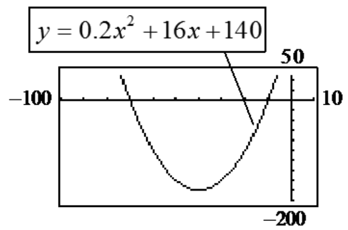
y-coordinate: $0.2(-40)^2 + 16(-40) + 140 = -180$

Zeros: $0 = 0.2(x^2 + 80x + 700)$

$= 0.2(x + 70)(x + 10)$

$x = -70, -10$

Graphing range: x-min = –100 y-min = –200
x-max = 0 y-max = 50



25. $y = 0.0001x^2 - 0.01$

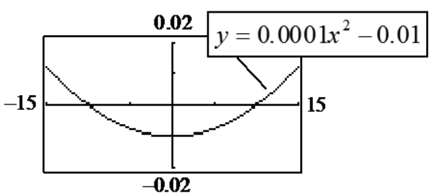
V: $x = \dfrac{-0}{2(0.0001)} = 0$

$y = 0 - 0.01 = -0.01$

Zeros: $0.0001x^2 - 0.01 = 0.01\left(0.01x^2 - 1\right) = 0$

$0.01(0.1x + 1)(0.1x - 1) = 0$

$x = -10, 10$



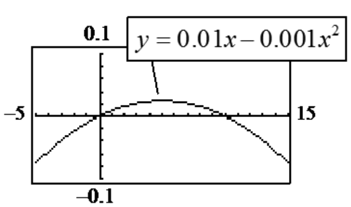
26. $y = 0.01x - 0.001x^2 = 0.001x(10 - x)$

Zeros: $x = 0$, $x = 10$

V: x-coordinate: $\dfrac{-b}{2a} = \dfrac{-0.01}{-0.002} = 5$

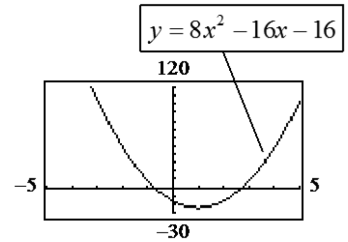
y-coordinate: 0.01(5) – 0.001(25) = 0.025

Graphing range: x-min = –5 y-min = –0.1
x-max = 15 y-max = 0.1



27. $f(x) = 8x^2 - 16x - 16$

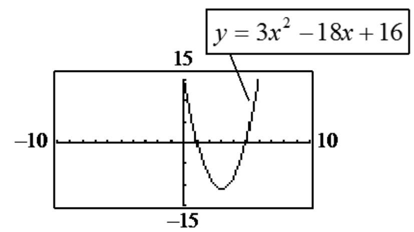
a. $x = \dfrac{-b}{2a} = \dfrac{16}{16} = 1$ and $f(1) = -24$



b. Graphical approximation gives
$x = -0.73,\ 2.73$

28. $f(x) = 3x^2 - 18x + 16$

a. $x = \dfrac{-b}{2a} = \dfrac{18}{6} = 3$ and $f(3) = -11$.



b. Graphical approximation gives
$x = 1.085,\ 4.915$

29. $f(x) = 3x^2 - 8x + 4$

a. The TRACE gives $x = 2$ as a solution.

b. $(x - 2)$ is a factor.

c. $3x^2 - 8x + 4 = (x - 2)(3x - 2)$

d. $(x - 2)(3x - 2) = 0$

$x - 2 = 0$ or $3x - 2 = 0$

Solution is $x = 2,\ 2/3$.

30. $f(x) = 5x^2 - 2x - 7$

a. The TRACE gives $x = -1$ as a solution.

b. $(x + 1)$ is the factor.

c. $5x^2 - 2x - 7 = (x + 1)(5x - 7)$

d. $(x + 1)(5x - 7) = 0$

$x + 1 = 0$ or $5x - 7 = 0$

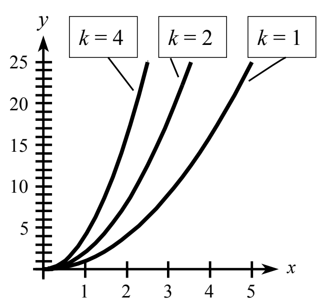
Solution is $x = -1,\ 7/5$.

31. $P = -0.1x^2 + 16x - 100$
The vertex coordinates are the answers to the questions.
a. $a = -0.1$, $b = 16$
$x = \frac{-b}{2a} = \frac{-16}{-0.2} = 80$
Profit is maximized at a production level of 80 units.
b. $P(80) = -0.1(80)^2 + 16(80) - 100 = \$540$
is the maximum profit.

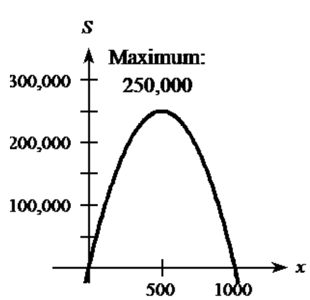
32. $P = 80x - 0.4x^2 - 200$
a. x-coordinate of vertex $= \frac{-b}{2a} = \frac{-80}{-0.8} = 100$
When $x = 100$, $P = 80(100) - 0.4(100)^2 - 200$
$= 8000 - 4000 - 200 = \$3800$

33. $Y = 800x - x^2$
Opens down so maximum Y is at vertex.
V: $x = \frac{-800}{-2} = 400$
Maximum yield occurs at $x = 400$ trees.

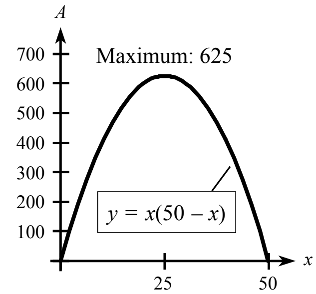
34. $y = kx^2$



35. $S = 1000x - x^2$
Maximum sensitivity occurs at vertex.
V: $x = \frac{-1000}{-2} = 500$
The dosage for maximum sensitivity is 500.

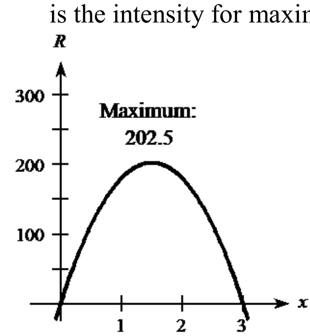


36. $A = 50x - x^2$



x-coordinate of the vertex $= \frac{-b}{2a} = \frac{-50}{-2} = 25$
A length of 25 feet and width of 25 feet gives a maximum area of 625 square feet.

37. $R = 270x - 90x^2$
Maximum rate occurs at vertex.
V: $x = \frac{-270}{2(-90)} = \frac{3}{2}$ (lumens)
is the intensity for maximum rate.



38. $s = 112t - 16t^2$
t-coordinate of the vertex
$= \frac{-b}{2a} = \frac{-112}{-32} = 3.5$ seconds
At $t = 3.5$, $s = 112(3.5) - 16(3.5)^2 = 196$ feet

39. **a.** $y = -0.0013x^2 + x + 10$
V: $x = \frac{-1}{-0.0026} = 384.62$;
$y = -0.0013(384.62)^2 + 384.62 + 10$
$= 202.31$

b. $y = -\frac{1}{81}x^2 + \frac{4}{3}x + 10$

V: $x = \frac{\frac{-4}{3}}{\frac{-2}{81}} = 54$;

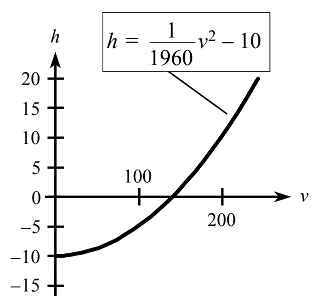
$y = -\frac{1}{81}(54)^2 + \frac{4}{3}(54) + 10 = 46$

Projectile **a.** goes 202.31 – 46 = 156.31 feet higher.

40. $v^2 = 1960(h+10)$

$\frac{v^2}{1960} = h + 10$

$h = \frac{1}{1960}v^2 - 10$



41. a. From b to c. The average rate of change is the same as the slope of the segment. The segment from b to c is steeper.

b. Needs to satisfy $d > b$ to make the segment from a to d have a greater slope.

42. a. From b to c. The average rate of change is the same as the slope of the segment. The segment from b to c has a negative slope.

b. Needs to satisfy $d < b$ to make the segment from a to d have a greater slope.

43. a.

No. of Apts	Rent	Total Revenue
50	\$600	\$30,000
49	\$620	\$30,380
48	\$640	\$30,720

b. Revenue increases \$720

c. $R = (50 - x)(600 + 20x)$

d. $R = -20x^2 + 400x + 30{,}000$

R is maximized at $x = \frac{-400}{2(-20)} = 10$.

Rent would be $\$600 + \$200 = \$800$.

44. a.

Price	No. of skaters	Total Revenue
12	50	\$600
11	60	\$660
10	70	\$700

b. The revenue increases.

c. $R(x) = (12 - 0.5x)(50 + 5x)$ where x is the number of each additional 5 skaters.

d. $R(x) = -2.5x^2 + 35x + 600$. Maximum revenue is at $x = \frac{-35}{-5} = 7$, or 85 skaters.

45. a. A quadratic function or parabola.

b. $a < 0$ because the graph opens downward.

c. The vertex occurs after 2004 (or when $x > 0$), so $-\frac{b}{2a} > 0$. Hence with $a < 0$ we must have $b > 0$. The value $c = f(0)$ or the y-value during 2004 which is positive.

46. $y = ax^2 + bx + c$

Zeros: (0, 0) and (40, 0)

Vertex: $\left(\frac{-b}{2a}, 40\right)$

(0, 0): $0 = a(0)^2 + b(0) + c$

$0 = c$

So, $y = ax^2 + bx$

(40, 0): $0 = 1600a + 40b$

$b = -40a$

So, $y = ax^2 - 40ax$

So, x-coordinate of the vertex = $-\frac{-40a}{2a} = 20$.

When $x = 20$, $y = 40$

$40 = a(20)^2 - 40(a)(20)$

$40 = 400a - 800a$

$-400a = 40$

$a = -\frac{1}{10}$ and $b = 4$

The equation is $y = -\frac{1}{10}x^2 + 4x$

47. $y = 20.61x^2 - 116.4x + 7406$

For 2010, $x = 10$ gives $y = 8303$.

For 2015, $x = 15$ gives $y = 10{,}297.25$.

For 2020, $x = 20$ gives $y = 13{,}322$.

Average rate of change from 2010 to 2015:

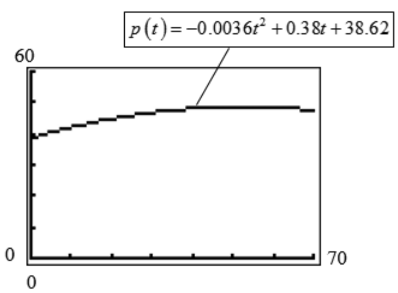
$$\frac{10{,}297.25 - 8303}{15 - 10} = 398.85$$

Average rate of change from 2015 to 2020:

$$\frac{13{,}322 - 10{,}297.25}{20 - 15} = 604.95$$

To the nearest dollar, the projected average rate of change of U.S. per capita health care costs from 2010 to 2015 will be \$399/year, and from 2015 to 2020 it will be \$605/year.

48. a.



b. Using the equation, we identify the maximum point by computing

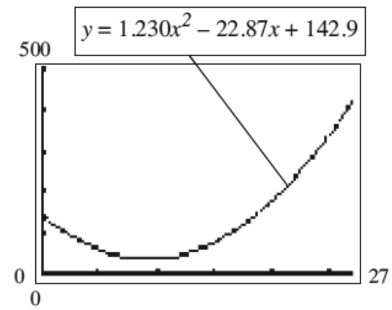
$$t = \frac{-b}{2a} = \frac{-0.38}{2(-0.0036)} \approx 52.78$$

$$p(52.78) \approx 48.65$$

The maximum point is (52.78, 48.65).

c. According to this model, the maximum percentage of women in the workforce occurs in the year $1970 + 53 = 2023$.

49.



50. The graphing calculator gives a minimum point at $(9.3, 36.6)$.

Exercises 2.3

1. $C(x) = x^2 + 40x + 2000$

$R(x) = 130x$

$x^2 + 40x + 2000 = 130x$

$x^2 - 90x + 2000 = 0$

$(x - 40)(x - 50) = 0$

$x = 40$ or $x = 50$

Break-even values are at $x = 40$ and 50 units.

2. At the break-even point, $R(x) = C(x)$.

$3600 + 25x + \frac{1}{2}x^2 = 175x - \frac{1}{2}x^2$

$x^2 - 150x + 3600 = 0$

$(x - 120)(x - 30) = 0$

$x = 120$ or $x = 30$ units

3. $C(x) = 15{,}000 + 35x + 0.1x^2$

$R(x) = 385x - 0.9x^2$

$15{,}000 + 35x + 0.1x^2 = 385x - 0.9x^2$

$x^2 - 350x + 15{,}000 = 0$

$(x - 300)(x - 50) = 0$

$x = 300$ or $x = 50$

4. At the break-even points, $R(x) = C(x)$.

$1600x - x^2 = 1600 + 1500x$

$0 = x^2 - 100x + 1600$

$0 = (x - 20)(x - 80)$

$x = 20$ or $x = 80$ units

5. $P(x) = -11.5x - 0.1x^2 - 150$

At the break-even points, $P(x) = 0$.

$$0 = 11.5x - 0.1x^2 - 150$$

$-0.1x^2 + 11.5x - 150 = 0$

$(x - 15)(x - 100) = 0$

Since production < 75 units, $x = 15$.

6. $P(x) = -1100 + 120x - x^2$

At the break-even points, $P(x) = 0$.

$$0 = -1100 + 120x - x^2$$

$x^2 - 120x + 1100 = 0$

$(x - 110)(x - 10) = 0$

Since production < 100 units, $x = 10$.

7. $R(x) = 385x - 0.9x^2$

$a = -0.9$, $b = 385$

Maximum revenue is at the vertex.

V: $x = \dfrac{-385}{-1.8} = 213.89$ or 214 total units

$R(214) = 385(214) - 0.9(214)^2 = \$41,173.60$

8. $R(x) = 1600x - x^2$

Maximum occurs at the vertex.

x-coordinate = $-\dfrac{1600}{-2} = 800$

$R(800) = 1600(800) - (800)^2 = \$640,000$

9. $R(x) = x(175 - 0.50x) = 175x - 0.5x^2$

$a = -0.50$, $b = 175$

Revenue is a maximum at $x = \dfrac{-175}{-1} = 175$.

Price that will maximize revenue is
$p = 175 - 87.50 = \$87.50$.

10. D: $p = 1600 - x \rightarrow x = 1600 - p$

Revenue: $R = px = p(1600 - p)$

$$R = 1600p - p^2$$

Max. revenue for $p = -\dfrac{1600}{-2} = \$800$.

11. $P(x) = -x^2 + 110x - 1000$

Maximum profit is at the vertex or when

$x = \dfrac{-110}{-2} = 55$.

$P(55) = \$2025$.

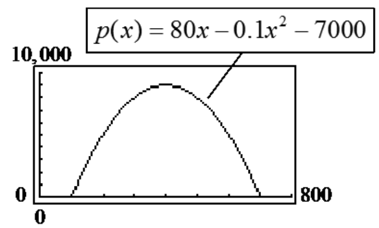
12. $P(x) = 88x - x^2 - 1200$

The x-coordinate giving the maximum profit is

$-\dfrac{b}{2a} = -\dfrac{88}{-2} = 44$.

$P(44) = 88(44) - (44)^2 - 1200 = \$736$

13. **a.**



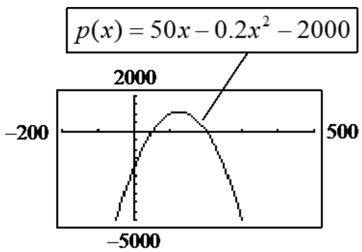
b. $(400, 9000)$ is the maximum

c. positive

d. negative

e. closer to 0

14. a.



b. $(125, 1125)$ is the maximum

c. positive

d. negative

e. closer to 0

15. $R(x) = 385x - 0.9x^2$

$C(x) = 15,000 + 35x + 0.1x^2$

a. $P(x) = 385x - 0.9x^2 - (15,000 + 35x + 0.1x^2)$

$= -x^2 + 350x - 15,000$

At the vertex we have $x = \frac{-350}{-2} = 175.$

So, $P(175) = \$15,625$.

b. No. More units are required to maximize revenue.

c. The break-even values and zeros of $P(x)$ are the same.

16. a. $P(x) = R(x) - C(x)$

$= 1600x - x^2 - (1600 + 1500x)$

$= 100x - x^2 - 1600$

x-coordinate of max is $-\frac{100}{-2} = 50$

$P(50) = 100(50) - (50)^2 - 1600 = \$900$

b. No. More units are required to maximize revenue.

c. $0 = 100x - x^2 - 1600$

$x^2 - 100x + 1600 = 0$

$(x - 80)(x - 20) = 0$

The x-coordinates are the same.

17. a. $C(x) = 28,000 + \left(\frac{2}{5}x + 222\right)x$

$= \frac{2}{5}x^2 + 222x + 28,000$

$R(x) = \left(1250 - \frac{3}{5}x\right)x = 1250x - \frac{3}{5}x^2$

(The key is “per unit x.”)

$R(x) = C(x)$

$1250x - \frac{3}{5}x^2 = \frac{2}{5}x^2 + 222x + 28,000$

$x^2 - 1028x + 28,000 = 0$

$(x - 1000)(x - 28) = 0$

Break-even values are at $x = 28$ and $x = 1000$.

b. Maximum revenue occurs at

$x = \frac{-1250}{-\frac{6}{5}} = 1042$ (rounded).

$R(1042) = \$651,041.60$ is the maximum revenue.

c. $P(x) = 1250x - \frac{3}{5}x^2 - \left(\frac{2}{5}x^2 + 222x + 28,000\right)$

$= -x^2 + 1028x - 28,000$

Maximum profit is at $x = \frac{-1028}{-2} = 514.$

$P(514) = \$236,196$ is the maximum profit.

d. Price that will maximize profit is

$p = 1250 - \frac{3}{5}(514) = \$941.60.$

18. a. $C(x) = 300 + \left(\frac{3}{4}x + 1460\right)x$

$$= 300 + \frac{3}{4}x^2 + 1460x$$

$$R(x) = \left(1500 - \frac{1}{4}x\right)x = 1500x - \frac{1}{4}x^2$$

At break-even points $C(x) = R(x)$.

$$300 + \frac{3}{4}x^2 + 1460x = 1500x - \frac{1}{4}x^2$$

$$x^2 - 40x + 300 = 0$$

$$(x - 30)(x - 10) = 0$$

$$x = 30 \text{ or } x = 10$$

b. Maximum revenue:

x-coordinate: $-\frac{b}{2a} = -\frac{1500}{-\frac{1}{2}} = 3000$

$$R(3000) = 1500(3000) - \frac{1}{4}(3000)^2$$

$$= \$2,250,000$$

c. $P(x) = R(x) - C(x)$

$$= 1500x - \frac{1}{4}x^2 - \left(300 + \frac{3}{4}x^2 + 1460x\right)$$

$$= 40x - x^2 - 300$$

Maximum profit:

x-coordinate: $-\frac{b}{2a} = -\frac{40}{-2} = 20$

$$P(20) = 40(20) - (20)^2 - 300 = \$100$$

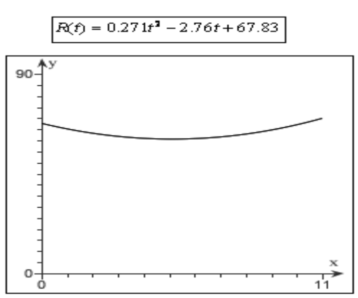
d. Selling price $= 1500 - \frac{1}{4}x$. When $x = 20$,

$$p = 1500 - \frac{1}{4}(20) = \$1495$$

19. a. $t \approx 5.1$, in 2012; $R \approx \$60.79$ billion

b. The data show a smaller revenue, R = \$60.27 billion in 2011.

c.



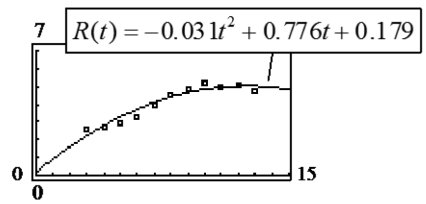
d. The model fits the data quite well.

20. $R(t) = -0.031t^2 + 0.776t + 0.179$

a. Maximum occurs at the vertex. The t-coordinate of the vertex is $-\frac{0.776}{-0.062} \approx 12.5$. Maximum revenue occurred during 2016. The maximum revenue predicted by the model is $R(12.5) \approx \$5.035$ million.

b. The entry in the table for 2016 is \$4.7489 million, so the values are close. However, the 2013 revenues were greater than this.

c.

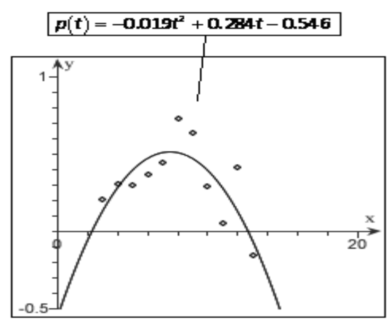


d. Although there are differences, the model appears to be a good quadratic fit for the data.

21. a. $p(t) = -0.019t^2 + 0.284t - 0.546$

b. 2011

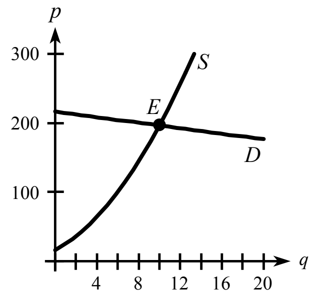
c.



d. The model projects decreasing profits, and, except for 2015, the data support this.

e. Management would be interested in increasing revenues or reducing costs (or both) to improve profit.

22. a. Supply: $p = q^2 + 8q + 16$ (see below)



Demand: $p = 216 - 2q$ (see below)

b. See E on the graph.

c. Supply = Demand

$$q^2 + 8q + 16 = 216 - 2q$$
$$q^2 + 10q - 200 = 0$$
$$(q - 10)(q + 20) = 0$$

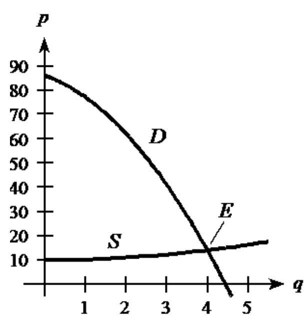
$q = 10$ (only positive value)

$p = 216 - 2(10) = 196$

$q = 10$, $p = \$196$

23. a. Supply: $p = \frac{1}{4}q^2 + 10$ (see below)

Demand: $p = 86 - 6q - 3q^2$ (see below)



b. See E on graph.

c. $\frac{1}{4}q^2 + 10 = 86 - 6q - 3q^2$

$$q^2 + 40 = 344 - 24q - 12q^2$$
$$0 = 13q^2 + 24q - 304$$
$$0 = (q - 4)(13q + 76)$$

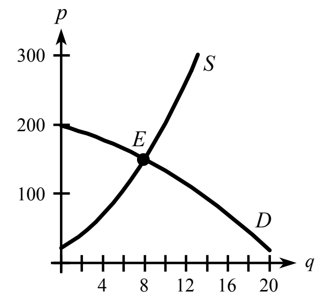
$q = 4$ must be positive.

$$p = \frac{1}{4}(4)^2 + 10 = 14$$

E: (4, 14)

24. a. Supply: $p = q^2 + 8q + 22$ (see below)

Demand: $p = 198 - 4q - \frac{1}{4}q^2$ (see below)



b. See E on the graph.

c. Supply = Demand

$$q^2 + 8q + 22 = 198 - 4q - \frac{1}{4}q^2$$
$$5q^2 + 48q - 704 = 0$$
$$(5q + 88)(q - 8) = 0$$

$q = 8$ (only positive value)

When $q = 8$, $p = (8)^2 + 8(8) + 22$

$p = 150$

So, E = (8, 150).

25. $p = q^2 + 8q + 16$

$p = -3q^2 + 6q + 436$

$$q^2 + 8q + 16 = -3q^2 + 6q + 436$$
$$4q^2 + 2q - 420 = 0$$
$$2q^2 + q - 210 = 0$$
$$(2q + 21)(q - 10) = 0$$
$$q = 10$$
$$p = 10^2 + 8(10) + 16 = 196$$

E: (10, 196)

26. S: $p = q^2 + 8q + 20$

D: $100 - 4q - q^2 = p$

$$q^2 + 8q + 20 = 100 - 4q - q^2$$
$$2q^2 + 12q - 80 = 0$$
$$2(q + 10)(q - 4) = 0$$

$q = 4$ (only positive value)

When $q = 4$, $p = 4^2 + 8(4) + 20 = \$68$

Equilibrium point: (4, 68)

27. $p^2 + 4q = 1600$

$300 - p^2 + 2q = 0$

$(300 + 2q) + 4q = 1600$

$6q = 1300$

$q = 216\frac{2}{3}$

$p^2 + 4\left(\frac{1300}{6}\right) = 1600$ or p^2

$= 733.33$ or $p = 27.08$

E: $\left(216\frac{2}{3}, 27.08\right)$

28. S: $4p - q = 42$ or $q = 4p - 42$

D: $(p+2)q = 2100$ or $q = \frac{2100}{p+2}$

$4p - 42 = \frac{2100}{p+2}$

$4p^2 - 34p - 84 = 2100$

$4p^2 - 34p - 2184 = 0$

$2(2p+39)(p-28) = 0$

$p = 28$ (only positive value)

When $p = \$28$, $q = 4(28) - 42 = 70$

Equilibrium point: (70, 28)

29. $p - q = 10$ or $q = p - 10$

$q(2p-10) = 2100$

$q = \frac{2100}{2p-10}$

$p - 10 = \frac{2100}{2p-10}$

$(p-10)(2p-10) = 2100$

$2p^2 - 30p + 100 = 2100$

$2p^2 - 30p - 2000 = 0$

$p^2 - 15p - 1000 = 0$

$(p-40)(p+25) = 0$

$p = 40$ or $p = -25$

(only the positive answer makes sense here)

$q = 40 - 10 = 30$

$E: (30, 40)$

30. S: $2p - q + 6 = 0$ or $q = 2p + 6$

D: $(p + q)(q + 10) = 3696$

Substitute $2p + 6$ for q in D and solve for p.

$(3p+6)(2p+16) = 3696$

$6p^2 + 60p - 3600 = 0$

$p^2 + 10p - 600 = 0$

$(p+30)(p-20) = 0$

$p = 20$ (only positive value)

When $p = 20$, $q = 2(20) + 6 = 46$.

Equilibrium point: (46, 20)

31. $2p - q - 10 = 0$

$(p + 10)(q + 30) = 7200$

So, $(p+10)(2p-10+30) = 7200$

$p^2 + 20p + 100 = 3600$

$p^2 + 20p - 3500 = 0$

$(p+70)(p-50) = 0$

$p = 50$

$q = 2(50) - 10 = 90$

E: $(q, p) = (90, 50)$

32. S: $2p - q = 50$ or $p = \frac{q+50}{2}$

D: $pq = 100 + 20q$ or $p = \frac{100+20q}{q}$

$\frac{q+50}{2} = \frac{100+20q}{q}$

$q^2 + 50q = 200 + 40q$

$q^2 + 10q - 200 = 0$

$(q+20)(q-10) = 0$

$q = 10$ (only positive value)

When $q = 10$, $p = 30$

Equilibrium point: (10, 30)

33. $p = \frac{1}{2}q + 5 + 22 = \frac{1}{2}q + 27$

So, $\left(\frac{1}{2}q + 27 + 10\right)(q + 30) = 7200$

$$(q + 74)(q + 30) = 14,400$$
$$q^2 + 104q - 12,180 = 0$$
$$(q + 174)(q - 70) = 0$$

$p = \frac{1}{2}(70) + 27 = 62$

E: (70, 62)

34. S: $p = \frac{q + 50}{2} + 12.50$

D: $p = \frac{100 + 20q}{q}$

$$\frac{q + 50}{2} + 12.50 = \frac{100 + 20q}{q}$$
$$q^2 + 75q = 200 + 40q$$
$$q^2 + 35q - 200 = 0$$
$$(q + 40)(q - 5) = 0$$

$q = 5$ (only positive value)

When $q = 5$, Equilibrium point: (5, 40)

Exercises 2.4

1. b

2. g

3. f

4. h

5. j

6. e

7. k

8. d

9. a

10. i

11. c

12. l

13. **a.** cubic
b. quartic

14. **a.** quartic
b. cubic

15. $y = x^3 - x = x(x + 1)(x - 1)$: e

16. $y = (x - 3)^2(x + 1)$: c

17. $y = 16x^2 - x^4 = x^2(4 + x)(4 - x)$: b

18. $y = x^4 - 3x^2 - 4 = (x^2 - 4)(x^2 + 1)$: h

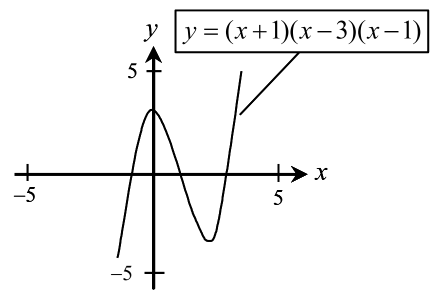
19. $y = x^2 + 7x = x(x + 7)$: d

20. $y = 7x - x^2 = x(7 - x)$: a

21. $y = \frac{x - 3}{x + 1}$: g

22. $y = \frac{1 - 3x}{2x + 5}$: f

23.



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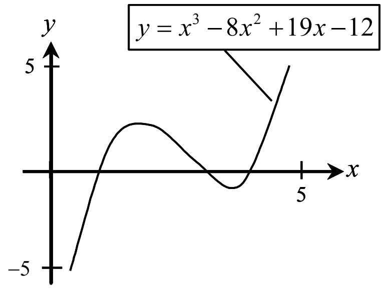

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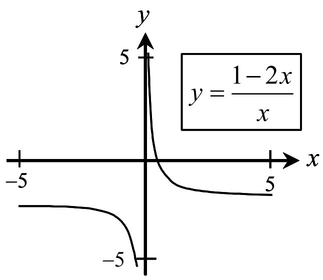
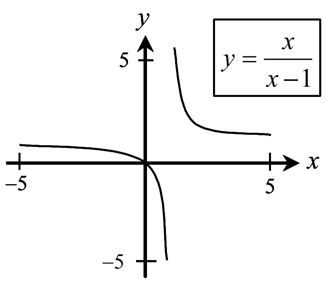
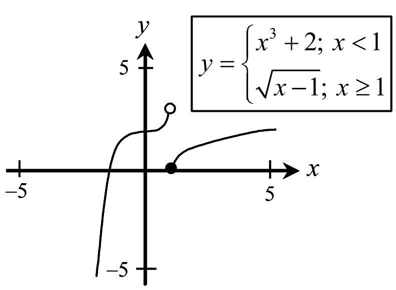


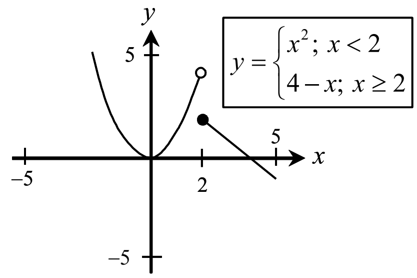
26.



27.



28.



29. $F(x) = \frac{x^2 - 1}{x}$

a. $F\left(-\frac{1}{3}\right) = \frac{\frac{1}{9} - 1}{-\frac{1}{3}} = \frac{8}{3}$

b. $F(10) = \frac{100 - 1}{10} = \frac{99}{10}$

$F(x) = \frac{x^2 - 1}{x}$

c. $F\left(-\frac{1}{3}\right) = \frac{\frac{1}{9} - 1}{-\frac{1}{3}} = \frac{8}{3}$

d. $F(10) = \frac{100 - 1}{10} = \frac{99}{10}$

e. $F(0.001) = \frac{0.000001 - 1}{0.001} = \frac{-0.999999}{0.001}$
$= -999.999$

f. $F(0)$ is not defined–division by zero.

30. $H(x) = |x - 1|$

a. $H(-1) = 2$
b. $H(1) = 0$
c. $H(0) = 1$
d. No

31. $f(x) = x^{3/2}$

a. $f(16) = (\sqrt{16})^3 = 64$

b. $f(1) = (\sqrt{1})^3 = 1$

c. $f(100) = (\sqrt{100})^3 = 1000$

d. $f(0.09) = (\sqrt{0.09})^3 = 0.027$

32. $k(x)=\begin{cases}4-2x & \text{if } x<0\\ |x-4| & \text{if } 0<x<4\end{cases}$

a. $k(-0.1)=4-2(-0.1)=4.2$

b. $k(0.1)=|0.1-4|=|-3.9|=3.9$

c. $k(3.9)=|3.9-4|=|-0.1|=0.1$

d. $k(4.1)$ is undefined

33. $k(x)=\begin{cases}2 & \text{if } x<0\\ x+4 & \text{if } 0\le x<1\\ 1-x & \text{if } x\ge 1\end{cases}$

a. $k(-5)=2$ since $x<0$.

b. $k(0)=0+4=4$

c. $k(1)=1-1=0$

d. $k(-0.001)=2$ since $x<0$.

34. $g(x)=\begin{cases}0.5x+4 & \text{if } x<0\\ 4-x & \text{if } 0\le x<4\\ 0 & \text{if } x>4\end{cases}$

a. $g(-4)=0.5(-4)+4=-2+4=2$

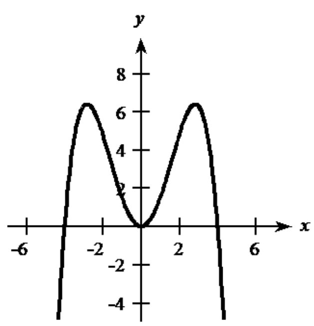
b. $g(1)=4-1=3$

c. $g(7)=0$

d. $g(3.9)=4-3.9=0.1$

35. $y=1.6x^2-0.1x^4$

a.



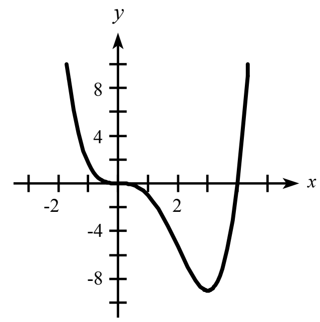
b. polynomial

c. no asymptotes

d. turning points at $x = 0$ and approximately $x = -2.8$ and $x = 2.8$

36. $f(x)=\dfrac{x^4-4x^3}{3}$

a.



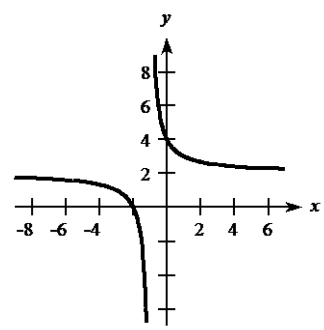
b. polynomial

c. no asymptotes

d. turning point at $x = 3$

37. $y=\dfrac{2x+4}{x+1}$

a.



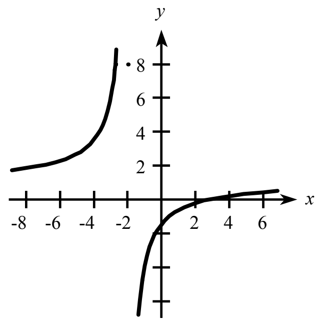
b. rational

c. vertical: $x = -1$
horizontal: $y = 2$

d. no turning points

38. $f(x)=\dfrac{x-3}{x+2}$

a.



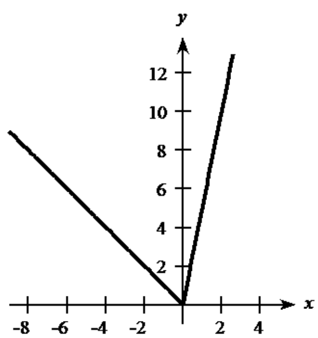
b. rational

c. vertical: $x=-2$
horizontal: $x=1$

d. no turning points

39. $f(x)=\begin{cases}-x & \text{if } x<0\\ 5x & \text{if } x\ge 0\end{cases}$

a.



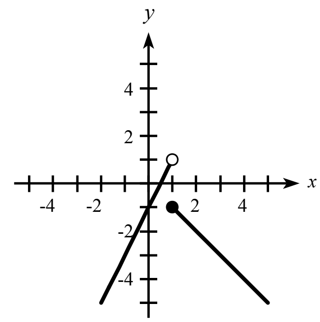
b. piecewise

c. no asymptotes

d. turning point at $x=0$.

40. $f(x)=\begin{cases}2x-1 & \text{if } x<1\\ -x & \text{if } x\ge 1\end{cases}$

a.



b. piecewise

c. no asymptotes

d. no turning point (there is a jump at $x=1$).

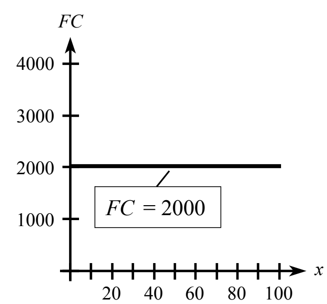
41. $V=V(x)=x^2(108-4x)$

a. $V(10)=100(68)=6800$ cubic inches
$V(20)=400(28)=11{,}200$ cubic inches

b. $108-4x>0$

$$-4x>-108$$
$$0<x<27$$

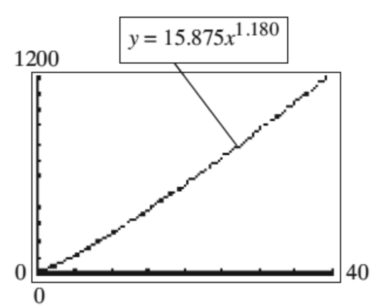
42.



43. $f(x)=15.875x^{1.18}$

a. upward

b.

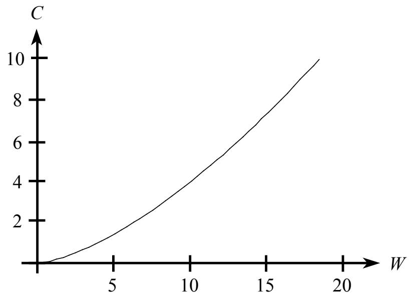


c. Intersecting the graphs of $y=15.875x^{1.18}$ and $y=1150$ gives $x\approx 37.7$. Global spending is expected to reach \$1,150,000,000,000 (\$1150 billion) in 1980 + 38 = 2018.

44. $C = 0.11W^{1.54}$

a. $W^{1.54}$ is "close" to W^2. The graph is turning up.

b.



c. $C(10) = 0.11(10)^{1.54} = 3.814$ grams

45. $C(p) = \dfrac{7300p}{100-p}$

a. $0 \le p < 100$

b. $C(45) = \dfrac{7300 \cdot 45}{100-45} = \$5972.73$

c. $C(90) = \dfrac{7300 \cdot 90}{100-90} = \$65,700$

d. $C(99) = \dfrac{7300 \cdot 99}{100-99} = \$722,700$

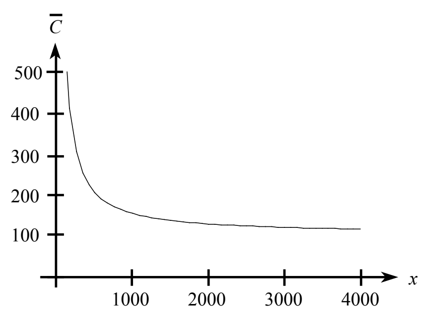
e. $C(99.6) = \dfrac{7300(99.6)}{100-99.6} = \$1,817,700$

f. To remove p% of the pollution would cost $C(p)$. Note how cost increases as p (the percent of pollution removed) increases.

46. $\overline{C} = \dfrac{50,000+105x}{x}$

a. $\overline{C}(3000) = \dfrac{50,000+105(3000)}{3000} = \$121.67$

b.



c. Yes. $\overline{C}(x) = \dfrac{50,000}{x} + 105$

47. $A = A(x) = x(50 - x)$

a. $A(2) = 2 \cdot 48 = 96$ square feet

$A(30) = 30 \cdot 20 = 600$ square feet

b. $0 < x < 50$ in order to have a rectangle.

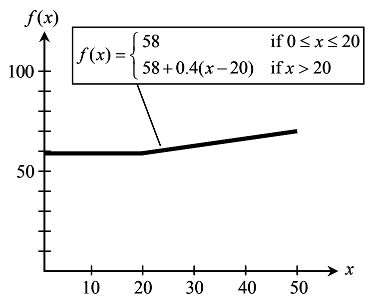
48. $f(x) = \begin{cases} 58 & \text{if } 0 \le x \le 20 \\ 58 + 0.4(x - 20) & \text{if } x > 20 \end{cases}$

a. $f(0.3) = \$58$

b. $f(30) = 58 + 0.4(30 - 20) = \$62$

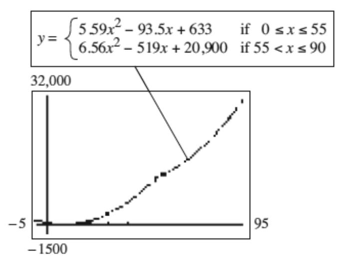
c. $f(40) = 58 + 0.4(40 - 20) = \$66$

d.



49. $y = \begin{cases} 5.59x^2 - 93.5x + 633 & \text{for } 0 \le x \le 55 \\ 6.56x^2 - 519x + 20{,}900 & \text{for } 55 < x \le 90 \end{cases}$

a.



b. $y(50) = 5.59(50)^2 - 93.5(50) + 633 = \$9933$ billion (\$9.933 trillion)

c. $y(75) = 6.56(75)^2 - 519(75) + 20{,}900 = \$18{,}875$ billion (\$18.875 trillion)

50. **a.** $C(5) = 7.52 + 0.1079(5) = \$8.06$

b. $C(6) = 19.22 + 0.1079(6) = \$19.87$

c. $C(3000) = 131.345 + 0.0321(3000) = \$227.65$

51. **a.** $P(x) = \begin{cases} 49 & \text{if } 0 < x \le 1 \\ 70 & \text{if } 1 < x \le 2 \\ 91 & \text{if } 2 < x \le 3 \\ 112 & \text{if } 3 < x \le 4 \end{cases}$

b. $P(1.2) = 70;$ it costs 70 cents to mail a 1.2-oz letter.

c. Domain: $0 < x \le 4$; Range: $\{49, 70, 91, 112\}$

d. The postage for a 2-ounce letter is 70 cents; for a 201-ounce letter, it is 91 cents.

52. **a.** $T(x)=\begin{cases}0.10x & \text{if } 0\le x\le 16,750\\ 0.15(x-16,750)+1,675 & \text{if } 16,750< x\le 68,000\\ 0.25(x-68,000)+9,362.50 & \text{if } 68,000< x\le 137,300\end{cases}$

b. $T(70,000)=0.25(70,000-68,000)+9,362.50=\$9,862.50$

c. $T(50,000)=0.15(50,000-16,750)+1,675=\$6,662.50$

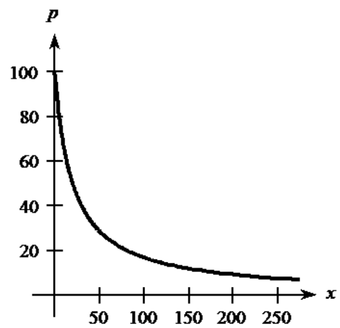
d. $T(68,000)=0.15(68,000-16,750)+1,675=\$9,362.50$

$T(68,001)=0.25(68,001-68,000)+9,362.50=\$9,362.75$

Jack's tax went up \$0.25 for the extra dollar earned. He is only charged 25% on the money he earns above \$68,000.

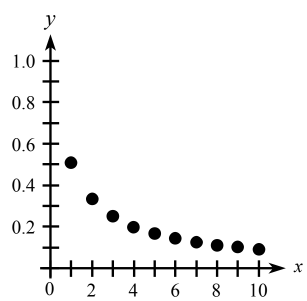
53. $p=\dfrac{200}{2+0.1x}$

a.



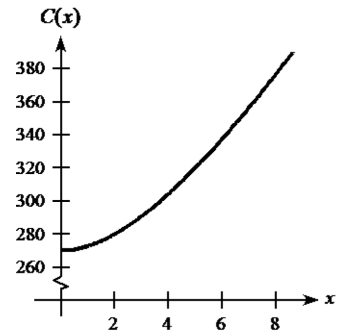
b. No

54. $y=\dfrac{1}{x+1}$, x positive integers



55. $C(x)=30(x-1)+\dfrac{3000}{x+10}$

a.



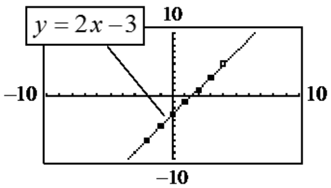
b. A turning point indicates a minimum or maximum cost.

c. This is the fixed cost of production.

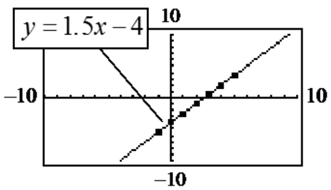
Exercises 2.5

1. Linear: The points are in a straight line.
2. Power
3. Quadratic: The points appear to fit a parabola.
4. Linear
5. Quartic: The graph crosses the x-axis four times. Also there are three bends.
6. Cubic
7. Quadratic: There is one bend. A parabola is the best fit.
8. Cubic

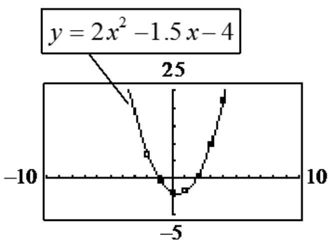
9. $y = 2x - 3$ is the best fit.



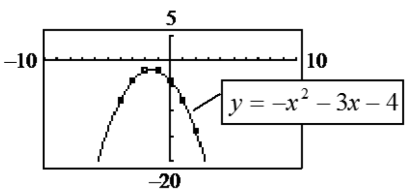
10. $y = 1.5x - 4$ is the best fit.



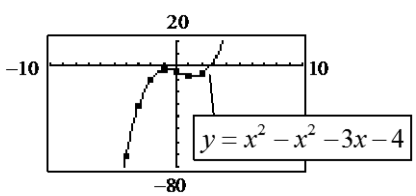
11. $y = 2x^2 - 1.5x - 4$ is the best fit.



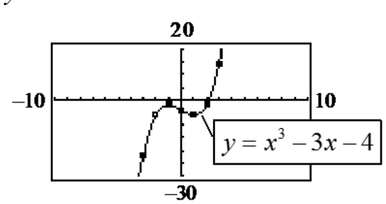
12. $y = -x^2 - 3x - 4$ is the best fit.



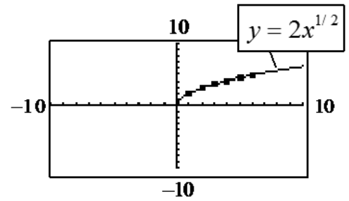
13. $y = x^3 - x^2 - 3x - 4$ is the best fit.



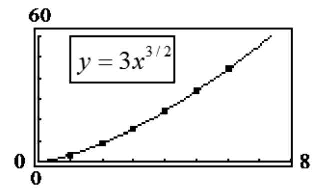
14. $y = x^3 - 3x - 4$ is the best fit



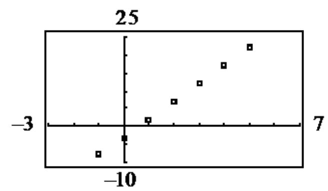
15. $y = 2x^{1/2}$ is the best fit.



16. $y = 3x^{3/2}$ is the best fit.



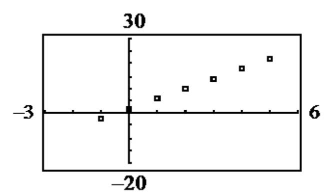
17. a.



b. linear

c. $y = 5x - 3$

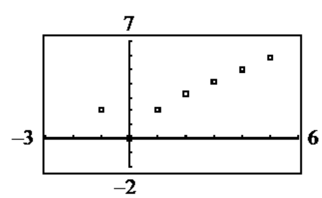
18. a.



b. linear

c. $y = 4x + 2$

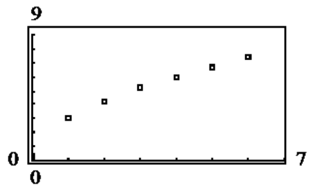
19. a.



b. quadratic

c. $y = 0.0959x^2 + 0.4656x + 1.4758$

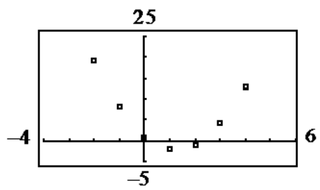
20. a.



b. power

c. $y = 3x^{1/2}$

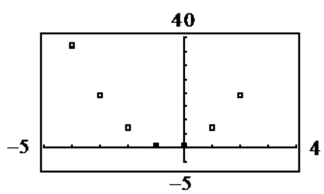
21. a.



b. quadratic

c. $y = 2x^2 - 5x + 1$

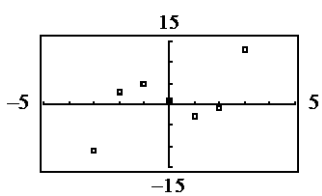
22. a.



b. quadratic

c. $3x^2 + 3x + 1$

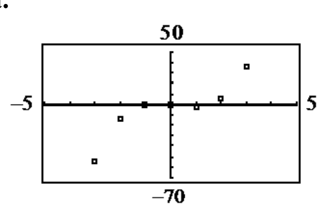
23. a.



b. cubic

c. $y = x^3 - 5x + 1$

24. a.



b. cubic

c. $y = 2x^3 - x^2 - 3x$

25. a. $y = 154.0x + 35,860$

b. $y(27) = 154.0(27) + 35,860 = 40,018$
The projected population of females under age 18 in 2037 is 40,018,000.

c. $45,000 = 154.0x + 35860 \Rightarrow x \approx 59.35$
This population will reach 45,000,000 in 2010 + 60 = 2070 according to this model.

26. a. $y = 18.96x + 321.5$

b. $y(14) = 18.96(14) + 321.5 \approx 586.9$ million metric tons

c. $m = 18.96;$ each year since 2010, carbon dioxide emissions in the U.S. are expected to change by 18.96 million metric tons.

27. a. A linear function is best; $y = 327.6x + 9591$

b. $y(17) = 327.6(17) + 9591 \approx \$15,160$ billion

c. $m = 327.6$ means the U.S. disposable income is increasing at the rate of about \$327.6 billion per year.

28. a. $y = 0.465x + 12.0$

b. $y(18) = 0.465(18) + 12.0 \approx 20.4\%$

c. $25 = 0.465x + 12.0 \Rightarrow x \approx 28$
This model predicts that the percent of U.S. adults with diabetes will each 25% in 2000 + 28 = 2028.

29. a. $y = 0.0052x^2 - 0.62x + 15$

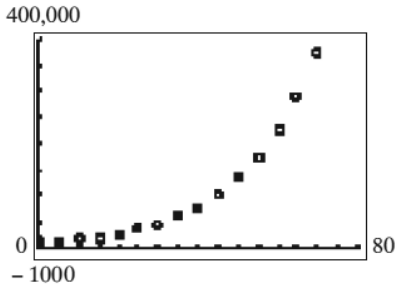
b. $x = \dfrac{-b}{2a} = \dfrac{0.62}{2(0.0052)} \approx 59.6$

c. No, it is unreasonable to feel warmer for winds greater than 60 mph.

30. a. $y = 0.0472x^2 + 2.64x + 12.1$

b. A maximum occurs at approximately $(28.0, 48.9)$. The model predicts that in the year 2000 + 28 = 2028, developing economies reach their maximum share, 48.9%, of the GDP.

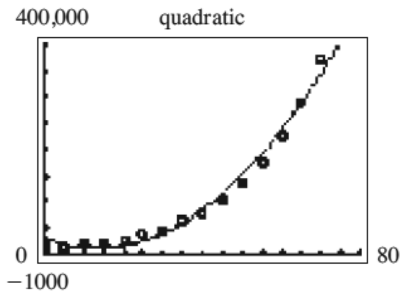
31. **a.**

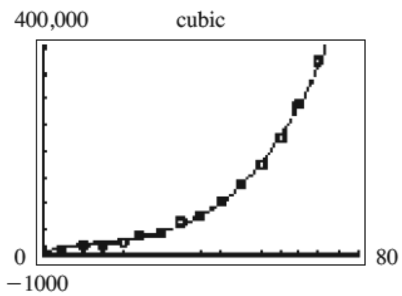


b. $y = 106x^2 - 2870x + 28,500$

c. $y = 1.70x^3 - 72.9x^2 + 1970x + 5270$

d.



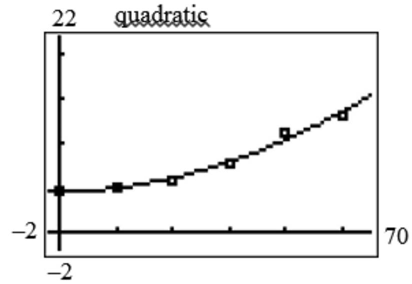


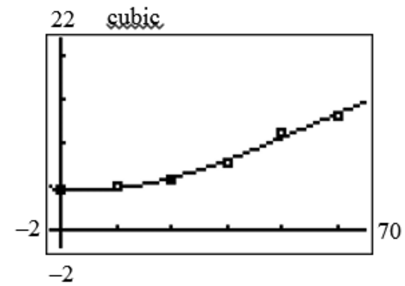
The cubic model fits better.

32. **a.** $y = 0.00336x^2 + 0.0127x + 4.47$

b. $y = -0.0000537x^3 + 0.00738x^2 - 0.0609x + 4.63$

c.



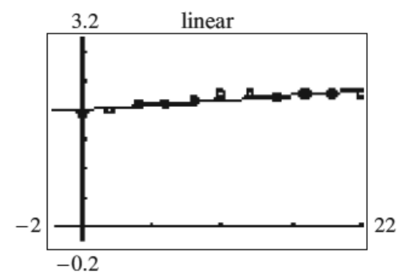


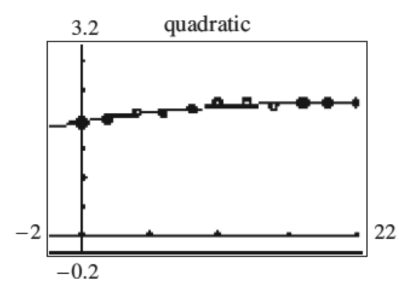
d. The fits look to be equally close.

33. **a.** $y = 0.0157x + 2.01$

b. $y = -0.00105x^2 + 0.367x + 1.94$

c.



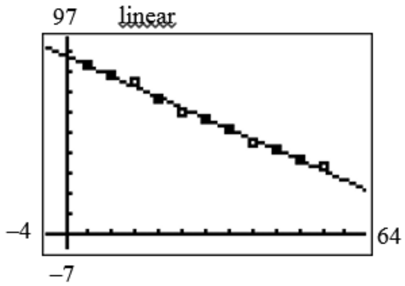


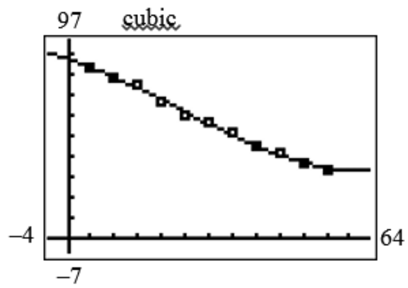
d. The quadratic model is a slightly better fit.

34. a. $y = -1.03x + 88.1$

b. $y = 0.000252x^3 - 0.0178x^2 - 0.0756x + 87.8$

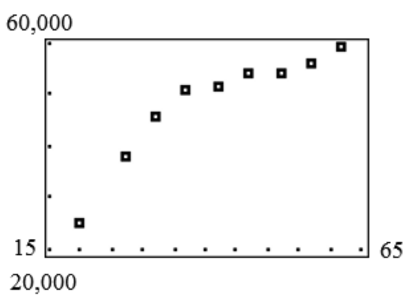
c.





d. The cubic model indicates that the percent of energy use may increase after 2035.

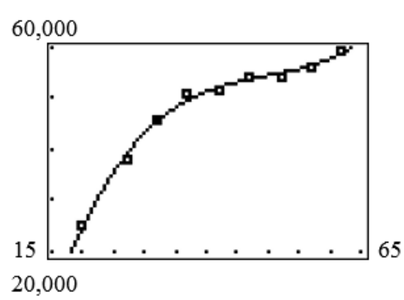
35. a.



A cubic model looks best because of the two bends.

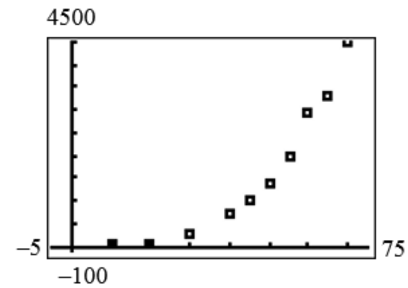
b. $y = 0.864x^3 - 128x^2 + 6610x - 62,600$

c.



d. Using the coefficient values reported by the calculator, the model estimates the median income to be \$56,250 at age 57.

36. a.



It appears that both quadratic and power functions would make good models for these data.

b. power: $y = 0.0315x^{2.74}$

quadratic: $y = 1.76x^2 - 71.0x + 679$

c. power: $y(70) \approx \$3661$ billion

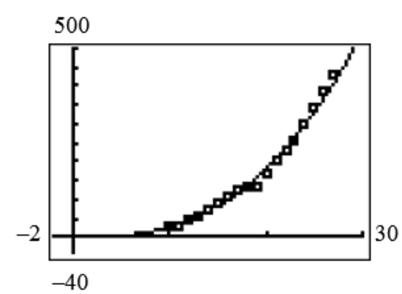
quadratic: $y(70) \approx \$4335$ billion

The quadratic model more accurately approximates the data point for 2020.

d. $y(75) \approx \$5257$ billion; \$5257 billion is the national health-care expenditure predicted by the model for 2025.

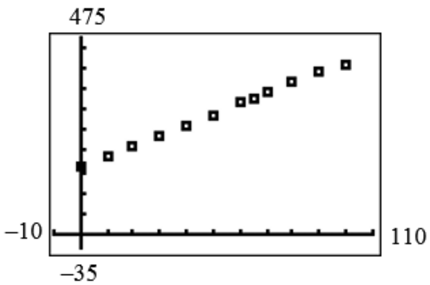
37. a. $y = 0.0514x^{2.73}$

b.



c. $y(30) \approx \$546$ billion

38. a.



b. Possible models are

linear: $y = 2.532x + 162.2$

quadratic: $y = -0.001020x^2 + 2.633x + 160.7$

cubic: $y = -0.00007456x^3 + 0.01030x^2 + 2.191x + 163.5$

c. linear: $y(90) \approx 390.08$

quadratic: $y(90) \approx 389.36$

cubic: $y(90) \approx 389.77$

The linear model most accurately approximates the data point for the year 2040.

d. Replacing y with 425 in the linear model gives $x \approx 103.8$. The U.S. population is predicted to reach 425 million in 1950 + 104 = 2054.

Chapter 2: Quadratic and Other Special Functions

Chapter 2 Review Exercises

1. $3x^2+10x=5x$

$3x^2+5x=0$

$x(3x+5)=0$

$x=0 \text{ or } x=-\dfrac{5}{3}$

2. $4x-3x^2=0$

$x(4-3x)=0$

$x=0 \text{ or } x=\dfrac{4}{3}$

3. $x^2+5x+6=0$

$(x+3)(x+2)=0$

$x=-3 \text{ or } x=-2$

4. $11-10x-2x^2=0$

$a=-2,\ b=-10,\ c=11$

$x=\dfrac{10\pm\sqrt{100+88}}{-4}=\dfrac{-5\pm\sqrt{47}}{2}$

5. $(x-1)(x+3)=-8$

$x^2+2x-3=-8$

$x^2+2x+5=0$

$b^2-4ac<0$

No real solution

6. $4x^2=3$

$x^2=\dfrac{3}{4}$

$x=\pm\sqrt{\dfrac{3}{4}}=\pm\dfrac{\sqrt{3}}{2}$

7. $20x^2+3x=20-15x^2$

$35x^2+3x-20=0$

$(7x-5)(5x+4)=0$

$x=\dfrac{5}{7} \text{ or } x=-\dfrac{4}{5}$

8. $8x^2+8x=1-8x^2$

$16x^2+8x-1=0$

$a=16,\ b=8,\ c=-1$

$x=\dfrac{-8\pm\sqrt{64+64}}{32}=\dfrac{-1\pm\sqrt{2}}{4}$

9. $7=2.07x-0.02x^2$

$0.02x^2-2.07x+7=0$

$a=0.02,\ b=-2.07,\ c=7$

$x=\dfrac{2.07\pm\sqrt{4.2849-0.56}}{0.04}=\dfrac{2.07\pm1.93}{0.04}$

$=100 \text{ or } 3.5$

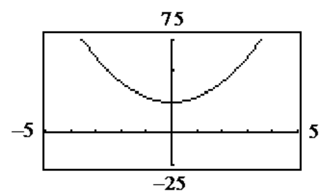
10. $46.3x-117-0.5x^2=0$

$a=-0.5,\ b=46.3,\ c=-117$

$x=\dfrac{-46.3\pm\sqrt{2143.69+(-234)}}{-1}=\dfrac{-46.3\pm43.7}{-1}$

$=90 \text{ or } 2.6$

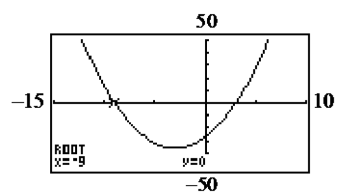
11. $4z^2+25=0$

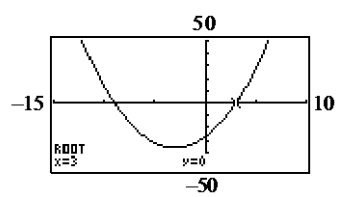


$4z^2+5^2=0$

The sum of 2 squares cannot be factored. There are no real solutions.

12. $f(z)=z^2+6z-27$





From the graph, the zeros are –9 and 3.

Algebraic solution:

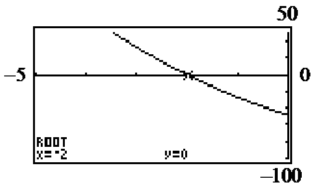
$z(z+6)=27$

$z^2+6z-27=0$

$(z+9)(z-3)=0$

$z=-9 \text{ or } z=3$

13. $3x^2 - 18x - 48 = 0$



$3(x^2 - 6x - 16) = 0$

$3(x-8)(x+2) = 0$

$x = -2,\ x = 8$

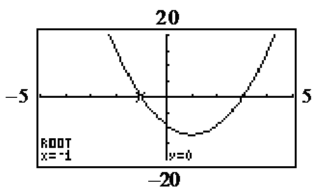
14. $f(x) = 3x^2 - 6x - 9$

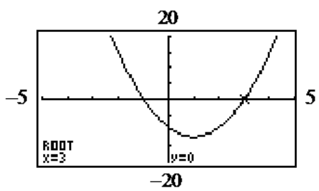
$3x^2 - 6x - 9 = 0$

$3\left(x^2 - 2x - 3\right) = 0$

$3\left(x-3\right)\left(x+1\right) = 0$

$x = 3,\ \ x = -1$





15. $x^2 + ax + b = 0$

To apply the quadratic formula we have *"a"* = 1, *"b"* = *a*, and *"c"* = *b*.

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

16. $xr^2 - 4ar - x^2c = 0$

To solve for *r*, use the quadratic formula with "*a*" = *x*, "*b*" = –4*a*, and "*c*" = $-x^2c$.

$$r = \frac{4a \pm \sqrt{16a^2 + 4x(x^2c)}}{2x} = \frac{4a \pm \sqrt{16a^2 + 4x^3c}}{2x}$$

$$= \frac{4a \pm 2\sqrt{4a^2 + x^3c}}{2x} = \frac{2a \pm \sqrt{4a^2 + x^3c}}{x}$$

17. $-0.002x^2 - 14.1x + 23.1 = 0$

$$x = \frac{14.1 \pm \sqrt{198.81 + 0.1848}}{-0.004} = \frac{14.1 \pm 14.107}{-0.004}$$

$= -7051.64,\ 1.64,\ \text{ or }\ 1.75\ \ (\text{using } 14.107)$

18. $1.03x^2 + 2.02x - 1.015 = 0$

$a = 1.03,\ b = 2.02,\ c = -1.015$

$$x = \frac{-2.02 \pm \sqrt{4.0804 + 4.1818}}{2.06} = \frac{-2.02 \pm 2.87}{2.06}$$

$= -2.38 \text{ or } 0.41$

19. $y = \frac{1}{2}x^2 + 2x$

$a > 0$, thus vertex is a minimum.

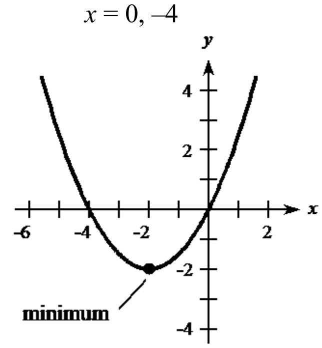
V: $x = \dfrac{-2}{2\left(\frac{1}{2}\right)} = -2$

$y = \frac{1}{2}(-2)^2 + 2(-2) = -2$

Zeros: $\frac{1}{2}x^2 + 2x = 0$

$x\left(\frac{1}{2}x + 2\right) = 0$

$x = 0, -4$



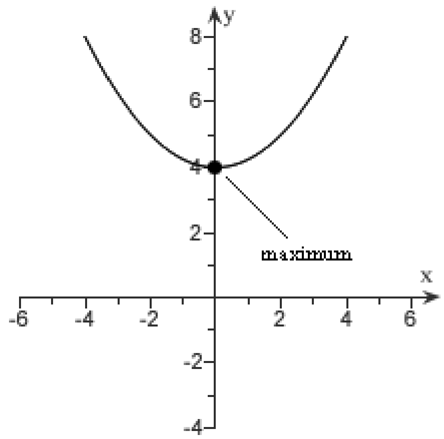
20. $y = 4 + \frac{1}{4}x^2$

V: *x*-coordinate = 0

y-coordinate = 4

(0, 4) is a maximum point

Zeros are $x = \pm 4$.



21. $y = 6 + x - x^2$

$a < 0$, thus vertex is a maximum.

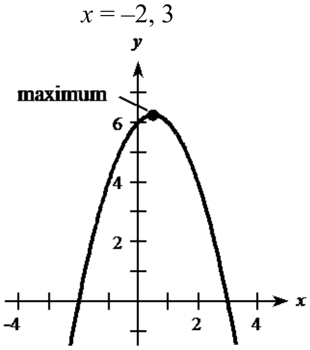
V: $x = \dfrac{-1}{2(-1)} = \dfrac{1}{2}$

$y = 6 + \dfrac{1}{2} - \left(\dfrac{1}{2}\right)^2 = \dfrac{25}{4}$

Zeros: $6 + x - x^2 = 0$

$(3 - x)(2 + x) = 0$

$x = -2, 3$



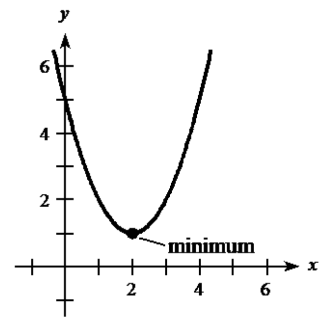
22. $y = x^2 - 4x + 5$

V: x-coordinate $= \dfrac{4}{2} = 2$

y-coordinate $= 2^2 - 4(2) + 5 = 1$

(2, 1) is a minimum point.

Zeros: Since the minimum point is above the x-axis, there are no zeros.



23. $y = x^2 + 6x + 9$

$a > 0$, thus vertex is a minimum.

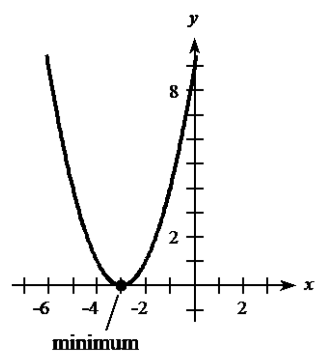
V: $x = \dfrac{-6}{2(1)} = -3$

$y = (-3)^2 + 6(-3) + 9 = 0$

Zeros: $x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

$x = -3$



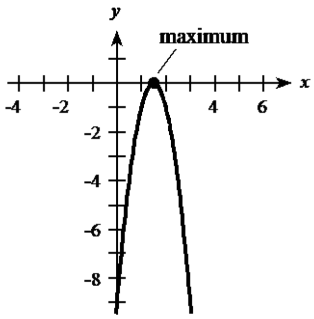
24. $y = 12x - 9 - 4x^2$

V: x-coordinate $= -\dfrac{12}{-8} = \dfrac{3}{2}$

y-coordinate $= 12\left(\dfrac{3}{2}\right) - 9 - 4\left(\dfrac{3}{2}\right)^2 = 0$

$\left(\frac{3}{2}, 0\right)$ is a maximum point.

Zeros: From the vertex we have that $x = \frac{3}{2}$ is the only zero.



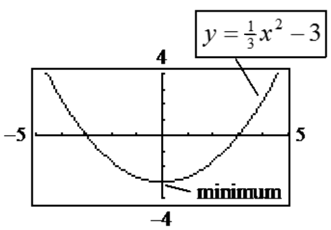
25. $y = \frac{1}{3}x^2 - 3$

V: (0, –3)

Zeros: $\frac{1}{3}x^2 - 3 = 0$

$$x^2 = 9$$

$$x = \pm 3$$

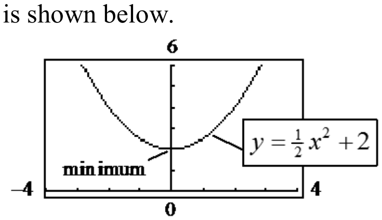


26. $y = \frac{1}{2}x^2 + 2$

Vertex: (0, 2) ← minimum

No zeros.

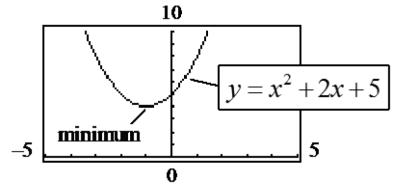
The graph using x-min = –4 y-min = 0
x-max = 4 y-max = 6
is shown below.



27. $y = x^2 + 2x + 5$

V: (–1, 4)

There are no real zeros.



28. $y = -10 + 7x - x^2$

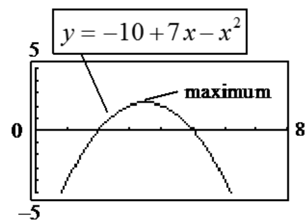
Vertex: $\left(\frac{7}{2}, \frac{9}{4}\right)$ ← maximum

Zeros: $x^2 - 7x + 10 = 0$

$$(x-5)(x-2) = 0$$

$$x = 5 \text{ or } x = 2$$

Graph using x-min = 0 y-min = –5
x-max = 8 y-max = 5



29. $y = 20x - 0.1x^2$

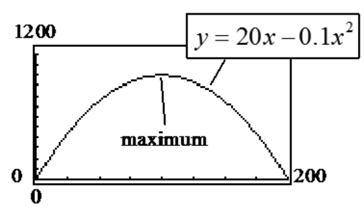
Zeros: $x(20 - 0.1x) = 0$

$x = 0, 200$

(This is an alternative method of getting the vertex.)

The x-coordinate of the vertex is halfway between the zeros.

V: (100, 1000)



30. $y = 50 - 1.5x + 0.01x^2$

Vertex: (75, –6.25) $\leftarrow$ minimum

Zeros: $0.01x^2 - 1.5x + 50 = 0$

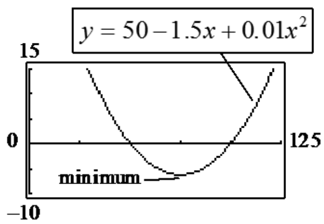
$$0.01(x^2 - 150x + 5000) = 0$$

$$0.01(x - 50)(x - 100) = 0$$

x = 50 or x =100

Graph using x-min = 0 y-min = –10

x-max = 125 y-max = 10



31. $\dfrac{f(50) - f(30)}{50 - 30} = \dfrac{2500 - 2100}{20} = \dfrac{400}{20} = 20$

32. $\dfrac{f(50) - f(10)}{50 - 10} = \dfrac{1022 + 178}{40} = \dfrac{1200}{40} = 30$

33. **a.** The vertex is halfway between the zeros. So, the vertex is $\left(1, -4\frac{1}{2}\right)$.

b. The zeros are where the graph crosses the x-axis. x = –2, 4.

c. The graph matches B.

34. From the graph,

a. Vertex is (0, 49)

b. Zeros are $x = \pm 7$.

c. Matches with D.

35. **a.** The vertex is halfway between the zeros. So, the vertex is (7, 24.5).

b. Zeros are x = 0, 14.

c. The graph matches A.

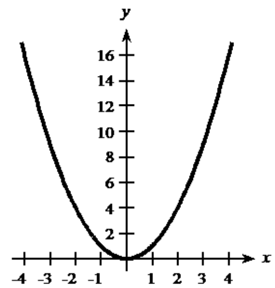
36. From the graph,

a. Vertex is (–1, 9).

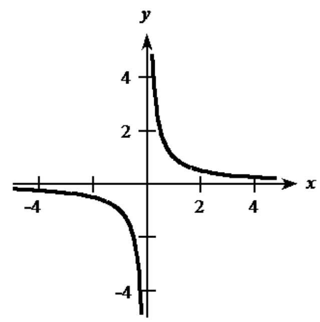
b. Zeros are x = –4 and x = 2.

c. Matches with C.

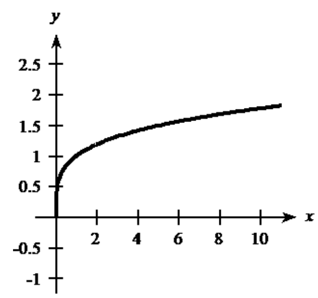
37. **a.** $f(x) = x^2$



b. $f(x) = \dfrac{1}{x}$



c. $f(x) = x^{1/4}$



38. $f(x) = \begin{cases} -x^2 & \text{if } x \le 0 \\ \dfrac{1}{x} & \text{if } x > 0 \end{cases}$

a. $f(0) = -(0^2) = 0$

b. $f(0.0001) = \dfrac{1}{0.0001} = 10{,}000$

c. $f(-5) = -(-5)^2 = -25$

d. $f(10) = \dfrac{1}{10} = 0.1$

39. $f(x) = \begin{cases} x & \text{if } x \le 1 \\ 3x - 2 & \text{if } x > 1 \end{cases}$

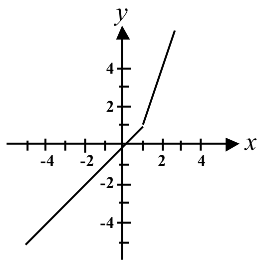
a. $f(-2) = -2$

b. $f(0) = 0$

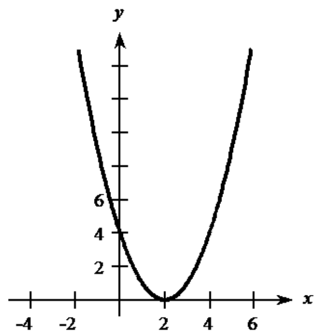
c. $f(1)=1$

d. $f(2)=3\cdot 2-2=4$

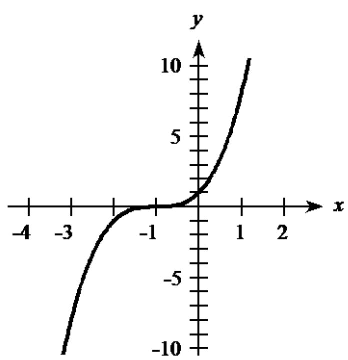
40. $f(x)=\begin{cases} x & \text{if } x\le 1 \\ 3x-2 & \text{if } x>1 \end{cases}$



41. a. $f(x)=(x-2)^2$

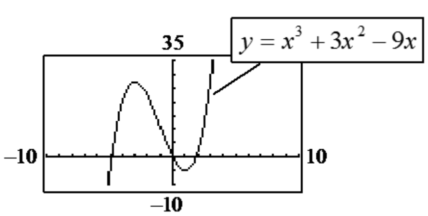


b. $f(x)=(x+1)^3$



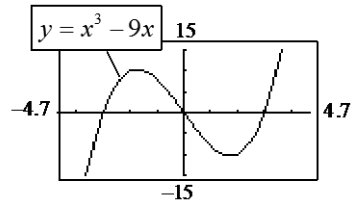
42. $y=x^3+3x^2-9x$

Using x-min = –10, x-max = 10, y-min = –10, y-max = 35, the turning points are at x = –3 and 1.



43. $y=x^3-9x$

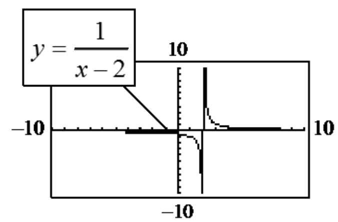
Using x-min = –4.7, x-max = 4.7, y-min = –15, y-max = 15, the turning points are at x = ±1.732.



Note: Your turning points in 42–43. may vary depending on your scale.

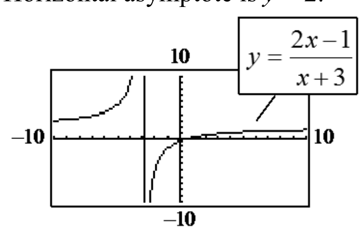
44. $y=\dfrac{1}{x-2}$

There is a vertical asymptote x = 2.
There is a horizontal asymptote y = 0.

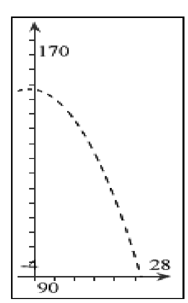


45. $y=\dfrac{2x-1}{x+3}=\dfrac{2-\frac{1}{x}}{1+\frac{3}{x}}$

Vertical asymptote is x = –3.
Horizontal asymptote is y = 2.



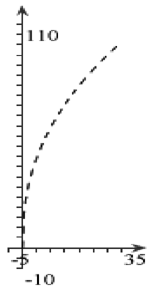
46. a.



b. $y=-2.1786x+159.8571$ is a good fit to the data.

c. $y=-0.0818x^2-0.2143x+153.3095$ is a slightly better fit.

47. a.



b. $y = 2.1413x + 34.3913$ is a good fit to the data.

c. $y = 22.2766x^{0.4259}$ is a slightly better fit.

48. $S=96+32t-16t^2$

a. $16(6+2t-t^2)=0$

$t=\dfrac{-2\pm\sqrt{4+24}}{-2}$

$t\approx-1.65$ or $t\approx3.65$

b. $t\geq0$ Use $t = 3.65$

c. After 3.65 seconds

49. $P(x)=-0.10x^2+82x-1600$

$(-0.10x+80)(x-20)=0$

Break-even at $x = 20$, 800

50. $E(t)=-0.0052t^2+0.080t+12$

a. The employment is a maximum at

$t=\dfrac{-b}{2a}=\dfrac{-0.080}{2(-0.0052)}\approx7.69$

$f(7.69)\approx12.3$; the maximum employment in manufacturing in the U.S. is predicted to be 12.3 million in 2010 + 8 = 2018.

b. $11.5=-0.0052t^2+0.080t+12$

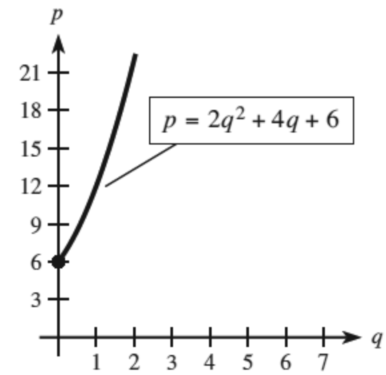
The quadratic formula gives $t\approx-4.8$ or $t\approx20.2$. The employment in manufacturing in the U.S. will be 11.5 million in 2010 + 21 = 2031.

51. $A=-\dfrac{3}{4}x^2+300x$

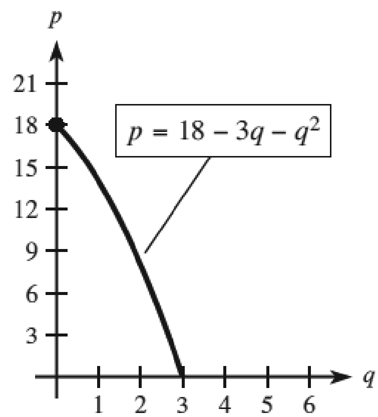
a. V: $x=\dfrac{-300}{-\frac{3}{2}}=200$ ft

b. $A=-\dfrac{3}{4}(200)^2+300(200)=30{,}000$ sq ft

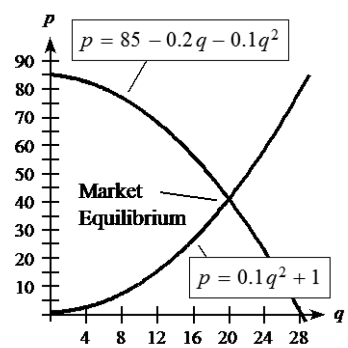
52.



53.



54. a.



b. $0.1q^2+1=85-0.2q-0.1q^2$

$0.2q^2+0.2q-84=0$

$0.2(q^2+q-420)=0$

$0.2(q-20)(q+21)=0$

$q = 20$ (only positive value)

$p=0.1(20)^2+1=41$

55. $p = q^2 + 300$
$p = -q + 410$
$q^2 + 300 = -q + 410$
$q^2 + q - 110 = 0$
$(q+11)(q-10) = 0$
$q = 10$
$p = -10 + 410 = 400$
So, E: (10, 400).

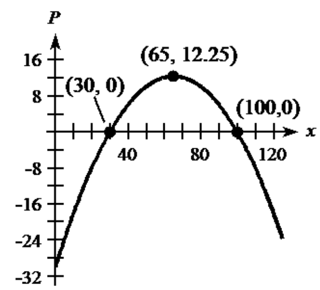
56. D: $p^2 + 5q = 200 \rightarrow p^2 = 200 - 5q$
S: $40 - p^2 + 3q = 0$
Substitute $200 - 5q$ for p^2 in the second equation and solve for q.
$40 - (200 - 5q) + 3q = 0$
$-160 = -8q$
$q = 20$
$p^2 = 200 - 5(20)$
$p^2 = 100$ or $p = 10$

57. $R(x) = 100x - 0.4x^2$
$C(x) = 1760 + 8x + 0.6x^2$
$100x - 0.4x^2 = 1760 + 8x + 0.6x^2$
$x^2 - 92x + 1760 = 0$
$x = \dfrac{92 \pm \sqrt{1424}}{2} = 46 \pm 2\sqrt{89} \approx 64.87, 27.13$
$(\sqrt{1424} = \sqrt{16 \cdot 89})$

58. $C(x) = 900 + 25x$
$R(x) = 100x - x^2$
$900 + 25x = 100x - x^2$
$x^2 - 75x + 900 = 0$
$(x-60)(x-15) = 0$
$x = 60$ or $x = 15$
$R(60) = 2400$; $R(15) = 1275$
(60, 2400) and (15, 1275)

59. $R(x) = 100x - x^2$
V: $x = \dfrac{-100}{-2} = 50$
$R(50) = 100(50) - 50^2$
$= \$2500$ max revenue
$P(x) = (100x - x^2) - (900 + 25x)$
$= -x^2 + 75x - 900$
V: $x = \dfrac{-75}{-2} = 37.5$
$P(37.5) = \$506.25$ max profit

60. $P(x) = 1.3x - 0.01x^2 - 30$
x-coordinate of the vertex $= \dfrac{1.3}{0.02} = 65$
$P(65) = 1.3(65) - 0.01(65)^2 - 30 = 12.25 \leftarrow$ max
Break-even points:
$0 = 1.3x - 0.01x^2 - 30$
$0 = -0.01(x^2 - 130x + 3000)$
$0 = -0.01(x-30)(x-100)$
$x = 30$ or $x = 100$



61. $P(x) = (50x - 0.2x^2) - (360 + 10x + 0.2x^2)$
$= -0.4x^2 + 40x - 360$
V: $x = \dfrac{-40}{-0.8} = 50$ units for maximum profit.
$P(50) = -0.4(50)^2 + 40(50) - 360$
$= \$640$ maximum profit.

62. **a.** $C(x) = 15{,}000 + (140 + 0.04x)x$
$= 15{,}000 + 140x + 0.04x^2$
$R(x) = (300 - 0.06x)x$
$= 300x - 0.06x^2$

b. $15{,}000 + 140x + 0.04x^2 = 300x - 0.06x^2$
$0.10x^2 - 160x + 15{,}000 = 0$
$0.1(x^2 - 1600x + 150{,}000) = 0$
$0.1(x-100)(x-1500) = 0$
$x = 100$ or $x = 1500$

c. Maximum revenue:

x-coordinate: $-\dfrac{300}{-0.12}=2500$

d. $P(x)=R(x)-C(x)$

$=-0.10x^2+160x-15,000$

x-coordinate of max $=-\dfrac{160}{-0.20}=800$

e. $P(2500)$ = \$240,000 loss
$P(800)$ = \$49,000 profit

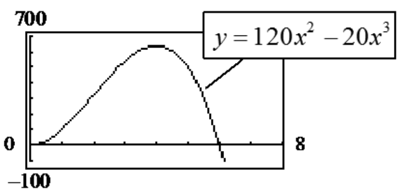
63. $D(t)=4.95t^{0.495}$

a. power function

b. $D(20)\approx 21.8\%$

c. 24.4; in 2025 about 24.4% of U.S. adults are expected to have diabetes.

64. **a.**



b. $y=20x^2(6-x)$
Domain: $0\le x\le 6$

65. $C(p)=\dfrac{4800p}{100-p}$

a. rational function

b. Domain: $0\le p<100$

c. $C(0)=0$ means that there is no cost if no pollution is removed.

d. $C(99)=\dfrac{4800(99)}{100-99}=\$475,200$

66. $C(x)=\begin{cases}2.557x & 0\le x\le 100\\ 255.70+2.04(x-100) & 100<x\le 1000\\ 2091.7+1.689(x-1000) & x>1000\end{cases}$

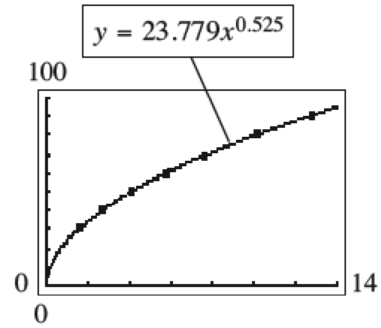
a. $C(12)=2.557(12)=\$30.68$

b. $C(825)=255.70+2.04(825-100)=$ \$1734.70

67. **a.** Linear, quadratic, cubic, and power functions are each reasonable.

b. $y=23.779x^{0.525}$

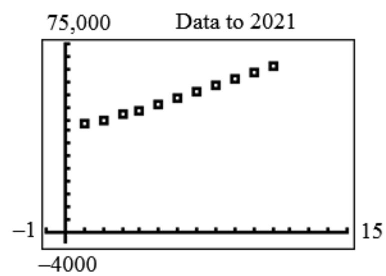
c.

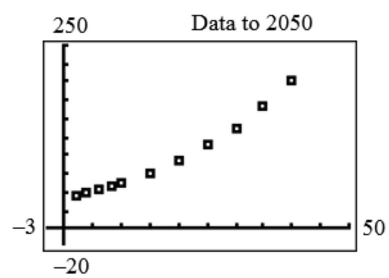


d. $f(5)=23.779(5)^{0.525}\approx 55$ mph

e. Use the TRACE KEY. It will take 9.9 seconds.

68. **a.**





b. A quadratic model could be used.
$a(x)=47.70x^2+1802x+40,870;$
$A(x)=0.07294x^2+0.9815x+44.45$

c. 2020 data: \$63,676
$a(10)\approx$ \$63,664–closer;
$A(10)\approx 61.564$ (\$61,564)
2050 data: 202.5 (\$202,500)
$a(40)\approx \$189,292;$
$A(40)\approx 200.413$ (\$200,413–closer)

d. $a=150,000$ when $x\approx 32.5$, in 2043;
$A=150,000$ when $x\approx 31.9$, in 2042

69. **a.** $O(x) = 0.743x + 6.97$

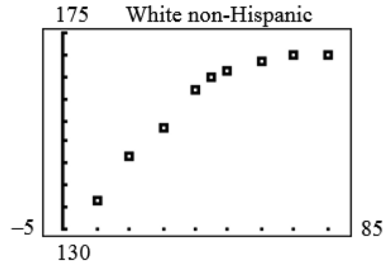
b. $S(x) = 0.264x - 2.57$

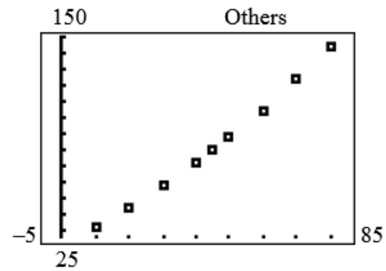
c. $F(x) = \dfrac{0.743x + 6.97}{0.264x - 2.57}$. This is called a rational function and measures the fraction of obese adults who are severely obese.

d. horizontal asymptote: $y \approx \dfrac{0.743}{0.264} \approx 0.355$.

This means that if this model remains valid far into the future, then the long-term projection is that about 0.355, or 35.5%, of obese adults will be severely obese.

70. **a.**





A quadratic function could be used to model each set of data.

$W(x) = -0.00903x^2 + 1.28x + 124$

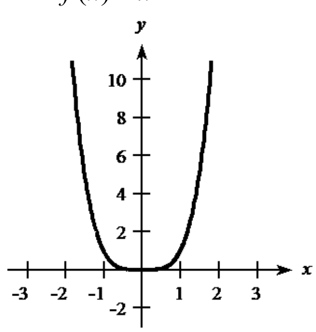
$O(x) = 0.00645x^2 + 1.02x + 20.0$

b. At $x \approx 91.1$, $W(x) = O(x) \approx 166.4$

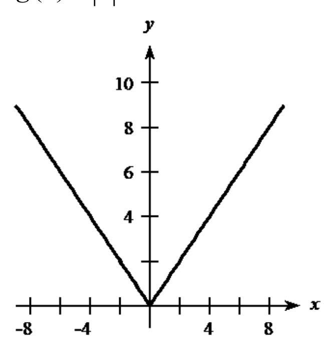
In 1970 + 92 = 2162, these population segments are predicted to be equal (at about 166.4 million each).

Chapter 2 Test

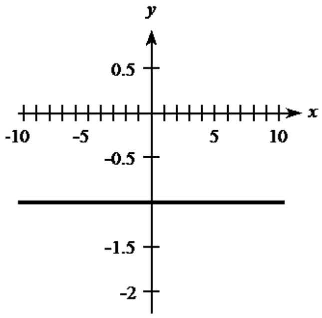
1. **a.** $f(x) = x^4$



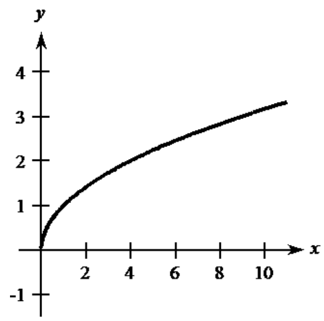
b. $g(x) = |x|$



c. $h(x) = -1$

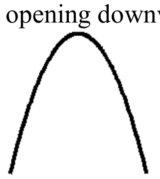


d. $k(x) = \sqrt{x}$

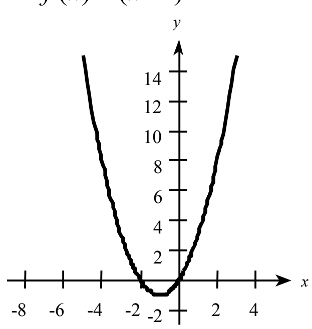


2. figure b is the graph for $b > 1$.
figure a is the graph for $0 < b < 1$.

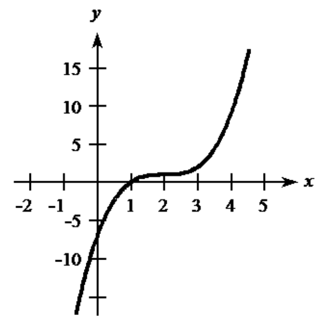
3. $f(x) = ax^2 + bx + c$ and a < 0 is a parabola opening downward.



4. **a.** $f(x) = (x+1)^2 - 1$



b. $f(x) = (x-2)^3 + 1$



5. $f(x) = x^3 - 4x^2 = x^2(x-4)$.
a. and **b.** are the cubic choices. $f(x) < 0$ if $0 < x < 4$. Answer: b

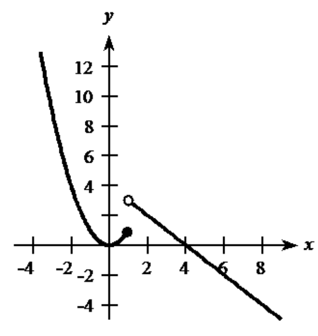
6. $$f(x) = \begin{cases} 8x + \dfrac{1}{x} & \text{if } x < 0 \\ 4 & \text{if } 0 \le x \le 2 \\ 6 - x & \text{if } x > 2 \end{cases}$$

a. $f(16) = 6 - 16 = -10$

b. $f(-2) = 8(-2) + \dfrac{1}{-2} = -16\dfrac{1}{2}$

c. $f(13) = 6 - 13 = -7$

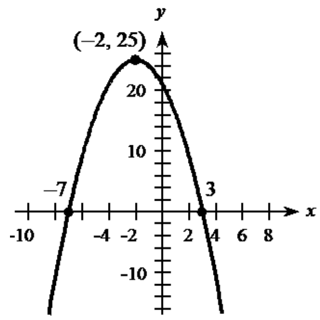
7. $$g(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 4 - x & \text{if } x > 1 \end{cases}$$



8. $f(x) = 21 - 4x - x^2 = (7+x)(3-x)$

Vertex: $x = \dfrac{-b}{2a} = \dfrac{-(-4)}{2(-1)} = -2$

Point: (–2, 25)
Zeros: $f(x) = 0$ at $x = -7$ or 3.



9. $3x^2+2=7x$

$3x^2-7x+2=0$

$(3x-1)(x-2)=0$

$3x-1=0$ or $x-2=0$

$x=\frac{1}{3}, 2$

10. $2x^2+6x-9=0$

$x=\frac{-6\pm\sqrt{36+72}}{4}=\frac{-6\pm6\sqrt{3}}{4}=\frac{-3\pm3\sqrt{3}}{2}$

11. $\left(\frac{1}{x}+2x=\frac{1}{3}+\frac{x+1}{x}\right)3x$

$3+6x^2=x+3x+3$

$6x^2-4x=0$

$2x(3x-2)=0$

$x=\frac{2}{3}$ is the only solution.

12. $g(x)=\frac{3(x-4)}{x+2}$

Vertical asymptote at $x=-2$.
$g(4)=0$
Answer: c

13. $f(x)=\frac{8}{2x-10}$

Horizontal: $y=0$

Vertical: $2x-10=0$

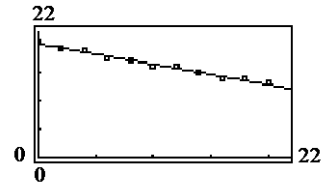
$2x=10$

$x=5$

14. $\frac{f(40)-f(10)}{40-10}=\frac{320-(-940)}{30}=\frac{1260}{30}=42$

15. a. quartic
b. cubic

16. a. $f(x)=-0.3577x+19.9227$



b. $f(40)=5.6$

c. $f(x)=0$ if $x=\frac{19.9227}{0.3577}\approx 55.7$

17. S: $p=\frac{1}{6}q+30$

D: $p=\frac{30{,}000}{q}-20$

$\left(\frac{1}{6}q+30=\frac{30{,}000}{q}-20\right)6q$

$q^2+180q=180{,}000-120q$

$q^2+300q-180{,}000=0$

$(q+600)(q-300)=0$

$E_q: q=300$

$E_p: p=50+30=80$

18. $R(x)=285x-0.9x^2$

$C(x)=15{,}000+35x+0.1x^2$

a. $P(x)=285x-0.9x^2-(15{,}000+35x+0.1x^2)$

$=-x^2+250x-15{,}000$

$=(100-x)(x-150)$

b. Maximum profit is at vertex.

$x=\frac{-250}{2(-1)}=125$

Maximum profit = $P(125)$ = \$625

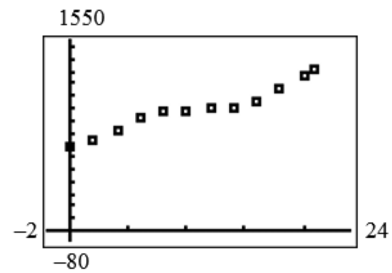
c. Break-even means $P(x)=0$.
From **a.**, $x=100, 150$.

19. a. Use middle rule for $s = 15$.
$f(15) = -19.5$ means that when the air temperature is 0ºF and the wind speed is 15 mph, then the air temperature feels like -19.5 ºF. In winter, the TV weather report usually gives the wind chill temperature.

b. $f(48) = -31.4$ºF

c. Break-even means $P(x) = 0$.
From **a.**, $x = 100, 150$.

20. a.



b. Linear: $y = 26.8x + 695$;
Cubic: $y = 0.175x^3 - 5.27x^2 + 65.7x + 654$

c. Linear: $y(21) \approx \$1258$;
Cubic: $y(21) \approx \$1326$
The cubic model is quite accurate, but both models are fairly close.

d. The linear model increases steadily, but the cubic model rises rapidly for years past 2021.

Chapter 2: Quadratic and Other Special Functions

Chapter 2 Extended Applications & Group Projects

I. Body Mass Index (Modeling)

1. Eight points in the table correspond to a BMI of 30. Converting heights to inches, we have:

Height (in.)	Weight (lb)
61	160
63	170
65	180
67	190
68	200
69	200
72	220
73	230

2. A linear model seems best as there appears to be roughly a constant rate of change of weight vs. height.
3. $y = 5.700x - 189.5$
4. We note that $y(61) \approx 158.1,$ lb close to the actual value of 160 lb, and $y(72) \approx 220.8,$ close to the actual value of 220 lb. The model seems to fit the data.
5. To test for obesity, substitute the person's height in inches for *x* in the model, computing *y*. If the person's weight is larger than *y*, then the person is considered obese. For a 5-foot-tall person, $y(60) \approx 152.4,$ so 152.4 lb is the obesity threshold for someone who is 5 feet tall. For a 6-feet-2-inches-tall person, $y(74) \approx 232.2,$ so 232.2 lb is the obesity threshold for someone who is 6 foot 2.
6. The Centers for Disease Control and Prevention (CDC) post the BMI formula
$$\text{BMI} = \frac{\text{weight (lb)}}{\left[\text{height (in)}\right]^2} \times 703$$
at their website (http://www.cdc.gov/healthyweight/assessing/bmi/adult_bmi/index.html, accessed September 15, 2014).
7. First solving the CDC formula for weight given a BMI of 30 gives $\text{weight} = \dfrac{30 \cdot \left[\text{height}\right]^2}{703}.$

Height (in.)	Weight (lb) from model	Weight (lb) from CDC definition	Weight (lb) from table
61	158	159	160
62	164	164	
63	170	169	170
64	175	175	
65	180	180	180
66	187	186	
67	192	192	190
68	198	197	200
69	204	203	200
70	209	209	
71	215	215	
72	221	221	220
73	227	227	230

Chapter 2: Quadratic and Other Special Functions

II. Operating Leverage and Business Risk

1. $R = xp$

2. **a.** $C = 100x + 10,000$

b. C is a linear function.

3. An equation that describes the break-even point is $xp = 100x + 10,000$

4. **a.** $xp = 100x + 10,000$

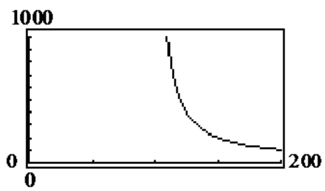
$$x = \frac{10,000}{p - 100}$$

b. The solution is **4.a.** is a rational function.

c. The domain is all real numbers, $p \neq 100$.

d. The domain in the context of this problem is $p > 100$.

5. **a.**



b. The function decreases as p increases.

6. A price of $1100 would increase the revenue for each unit but demand would decrease.

7. A price of $101 per unit would increase demand but perhaps such a demand could not be met.

8. **a.** Increasing fixed costs gives a higher operating leverage. Using modern equipment would give the higher operating leverage.

b. To find the break-even point with current costs we have $200x = 100x + 10,000$

$$x = 100.$$

To find the break-even point with modern equipment we have $200x = 50x + 30,000$

$$x = 200.$$

The higher the break-even point the greater the business risk. The cost with the modern equipment creates a higher business risk.

c. In this case, higher operating leverage and higher business event together. This higher risk might give greater profits for increases in sales. It might also give a greater loss of sales fall.