2 Set Theory

Exercise Set 2-1

1. A set is a well-defined collection of objects.

2. A set is well-defined when there is no "gray area" when deciding whether an object belongs to a set. There is no such thing as "kind of" or "sort of" in a well-defined set.

3. Example of well-defined set: The set of countries that competed in the 2012 Summer Olympics. Example of a set that is not well-defined: The set of "hot" math professors at your school.

4. (a) Roster method: an actual list of the elements in a set, separated by commas, and put inside {}; (b) Descriptive method: using a short sentence to describe the elements in a set ; (c) Set-builder notation: also a descriptive method, but instead of using words, it uses variables and other symbols.

5. Equal sets have exactly the *same elements*. Equivalent sets have exactly the *same number* of elements.

6. In a *finite* set, the number of elements is either zero or can be counted. In an *infinite* set, the number of elements cannot be counted.

7. Each element of one set can be associated (paired) with exactly one element of the other set, and no element in either set is left alone.

8. The empty set admits no elements. Two examples:

(a) $\{\text{5-leg horses}\} = \varnothing$

(b) $\left\{\text{integers between } \frac{1}{10} \text{ and } \frac{9}{10}\right\} = \varnothing$

9. T= {t, h, i, n, k, g}

10. A = {A, L, B, M}

11. Natural numbers are the counting numbers so:

P = {51, 52, 53, 54, 55, 56, 57, 58, 59}

12. R = {12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38}

13. C = {1, 2, 3, 4, 5, 6, 7, 8}

14. F = {101, 102, 103, 104, 105, ...}

15. G = {11, 12, 13, ... }

16. B = {101, 102, 103, ... }

17. Y = {2,001, 2,002, 2,003, ... , 2,999}

18. Z = {501, 502, 503, ... , 5,999}

19. Z = {white, red, blue, green, gray, brown, black, yellow}

20. This set usually changes with each senatorial election.

21. L = {medial collateral, lateral collateral, anterior cruciate, posterior cruciate}

22. A = {Belmopan, San José, San Salvador, Guatemala City, Tegucigalpa, Managua, Panama City}

23. True; 5 is contained in the set.

24. False; 8 is an even number and is an element of the set, so to say that it is not an element of the set is a false statement.

25. True; $\frac{1}{2}$ is not a natural number.

26. False; 0.6 is not a natural number.

27. True; unless there is a "land of the lost" that we just don't know about.

28. False; last time I checked, Cleveland was a city.

29. The set of multiples of 5.

30. The set of multiples of 4 from 4 to 16.

31. The set of multiplies of 13 from 13 to 52.

32. The set of multiples of 7.

33. Answers may vary. Examples: the letters in the name Steven, the letters in the word vents.

34. Answers may vary. Example: the letters in the word August.

35. This is the set of natural numbers from 100 to 199.

36. This is the set of natural numbers from 21 to 30.

For Exercises 37 – 42 answers can vary for the alternate descriptions

37. $\{x \mid x$ is a multiple of $10\}$; The set of all multiples of 10

38. $\{x \mid x$ is a multiple of $3\}$; The set of all multiples of 3

39. $\{x \mid x$ is odd and less than $16\}$; X is the set of odd numbers from 1 to 15

40. $\{x \mid x$ is natural number and x is between 70 and $76\}$; Z is the set of natural numbers from 71 to 75

41. Answers may vary. Example: $\{x \mid x$ is a color in the American flag$\}$; The set of colors in an American flag

42. Answers may vary. Example: $\{x \mid x$ is a primary color$\}$; The set of colors in a black and white photo

43. There are no natural numbers less than zero so $H = \varnothing$.

44. {71, 72, 73, 74, 75, 76, 77, 78, 79}

45. {Spring, Summer, Fall, Winter}

46. {y}

47. {102, 104, 106, 108, 110, 112, 114, 116, 118}

48. {91, 93, 95, 97, 99}

49. This set is well-defined since if you were given a name, you would know for sure if they had won a season of Survivor.

50. This is set is well-defined since if you were given a name, you would know if they were on death row or not.

51. This set is not well-defined. If you and a friend were given a name of an NBA player that dunked last week, you may think his dunk was awesome while someone else might think his dunk was not just ordinary.

52. This set is not well-defined. If two doctors were given the name of a person who needed a heart transplant, one doctor may decide the patient is deserving and the other may decide that the patient is not.

53. This set is not well-defined. To a small child, 156 might be a large number. To a high school student, 156 might be a small number.

54. This set is well-defined. The number of people in the United States is a known quantity, so if you were given a number, you could decide if it was greater or not.

55. False; 35 is not a multiple of 10.

56. False; the guy is on the $100 bill, but he was never a President.

57. True; Philadelphia is NOT a capital, so it is NOT an element of set A.

58. True; 350 is a multiple of 10.

59. True; Cheyenne is the capital of Wyoming

60. True; James Madison was the fourth U.S. President.

61. Infinite: There is not a fixed number of even numbers

62. Finite: There are a fixed number of elements, 1000.

63. There are 26 letters in the English alphabet, therefore K is finite.

64. Finite: The set of past presidents in the United States has a fixed number of elements, so the set of years in which they were born has a fixed number of elements as well.

65. Infinite: There is not a fixed number of natural numbers that end with a zero.

66. Finite: There are a fixed number of elements, zero.

67. Finite: There are a fixed number of television programs that are currently airing.

68. Infinite: A fraction is the quotient of two integers. Since there isn't a fixed number of integers, there isn't a fixed number of fractions.

69. Equal

70. Equivalent

71. Neither

72. Neither

73. Equivalent

74. Equal

75. {10, 20, 30, 40}
↕ ↕ ↕ ↕
{40, 10, 20, 30}

76. {w, x, y, z}
↕ ↕ ↕ ↕
{1, 2, 3, 4}

77. {1, 2, 3, 4, 5, ...}
↕ ↕ ↕ ↕ ↕
{4, 8, 12, 16, 20,...}

78. {1, 3, 5, 7, 9}
↕ ↕ ↕ ↕ ↕
{2, 4, 6, 8, 10}

79. $n(A) = 4$ (since there are 4 elements in the set)

80. $n(B) = 11$ (since there are 11 elements in the set)

81. $n(C) = 7$ (since there are seven days in a week)

82. $n(D) = 12$ (since there are 12 months in a year)

83. $n(E) = 1$ (the word "three" is the only element in the set)

84. $n(F) = 4$ (though 5 elements are listed, the letter e is repeated so it is not counted twice)

85. $n(G) = 0$ (there are no negative natural numbers so G is the empty set)

86. $n(H) = 0$ (since there are no elements in the empty set)

87. True, for two sets to be equal they must have exactly the same elements. To be equivalent, two sets must have the same cardinal number. A set that has exactly the same elements is going to have the same cardinal number.

88. False, some sets are equivalent and also equal.

89. False, there is one element in the set {∅}, namely ∅.

90. False, the set continues on to include all even natural numbers and does not have a fixed number of elements.

91. True, the elements in the second set are in one to one correspondence with the elements in the first set so they must contain the same number of elements.

92. True, the empty set contains zero elements.

93. a) {California, New York, Florida}.

b) {New Jersey, Illinois, Massachusetts, Virginia, Georgia, Pennsylvania}

c) {California, New York, Florida, Texas, New Jersey}

d) {Texas, New Jersey, Illinois}

94. a) {California, Texas, New York, Florida}

b) {New York, Florida, New Jersey, Virginia}

c) Answers vary. One example: List the set of states in both of the top tens that have a percentage of legal immigrants and illegal immigrants that are greater than 30.

d) One example: The set of states in the top four in both categories

95. a){Drunk driving, Assault, Injury}

b) {Injury, Unsafe sex, Health problems}

c) {Injury, Assault, Drunk driving}

d) {97,000, 1,825, 150,000}

e) No. We don't know how many students were in more than one group, so adding the numbers together probably won't be correct.

96. a) {Business, health professions, social sciences and history, psychology, biological and biomedical sciences, visual and performing arts, engineering, communications, homeland security/law enforcement/firefighting}

b) {Education}

c) {Engineering, visual and performing arts, education, biological and biomedical sciences, psychology}

d) {Business, health professions, social sciences and history, psychology, biological and biomedical sciences, visual and performing arts, engineering, communications, homeland security/law enforcement/firefighting}

e)

Major	Increase
Business	2.9%
Social sciences and history	2.7%
Health professions	65.0%
Psychology	24.4%
Visual and performing arts	9.1%
Biological and biomedical sciences	26.3%
Communication	12.3%
Engineering	33.7%
Homeland security, law enforcement, and firefighting	49.3%

The set of majors that increased at least 30% is {Health professions, homeland/security/law enforcement/firefighting, engineering}.

97. a) {Employment, loan}

b) {40 – 49, 50 – 59, 60 and over}

c) {Government documents/benefits, credit card, other}

d) {20%, 24%, 21%}

e) {Credit card, other}

f) 106.8%; some reports might fall into two categories, or information may have come from different sources

98. a) {2010, 2011, 2012, 2013, 2014, 2015}

b) {2014, 2015}

c) {2012, 2014}

d) {2011, 2012, 2013}

99. a) {2013, 2014, 2015, 2016}

b) {2009, 2012}

c) {2012, 2013, 2014, 2015, 2016}

d) {2010, 2011}

100. Yes; $A \cong B$ means A and B have the same number of elements. $A \cong C$ means A and C have the same number of elements. Then B and C have the same number of elements, so $B \cong C$.

101. No; ∅ contains no elements, but {0} contains one element: zero. So ∅ and {0} do not have the same number of elements.

102. $A = \{2, 4, 6, 8\}$ and $B = \{7, 9, 11, 13\}$. A and B are equivalent because they both have 4 elements. If two sets are equal, then they have the exact same elements. Having the exact same elements implies that they have the exact same number of elements.

103. a) A has 6 elements listed and B has only 5 elements listed.

b)

$$\begin{array}{ccccccc} \{1, & 2, & 3, & 4, & 5, & 6, & \ldots\} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ \{2, & 4, & 6, & 8, & 10, & 12, & \ldots\} \end{array}$$

The one to one correspondence shows that each number in A can be paired up with a number in B, which means that A and B are equivalent. A and B have the same number of elements if they are equivalent.

104. a) {2, 4, 6}, {2, 4}, {2, 6}, {4, 6}, {2}, {4}, {6}, ∅

b) If you only found 7 sets, then you probably forgot the empty set.

105. a) What it means to be an American is "open to interpretation," so deciding who belongs in the set is also "open to interpretation."

b) Well, to me, a poor math professor, a new Hyundai is a luxury car. But I doubt that the University president would consider it a luxury car.

c) The phrase "legitimate chance" is subjective and some colleges who might actually have a legitimate chance might not even be invited to the big dance.

d) Salaries can be fluid and also are not always limited to just what is in your normal weekly paycheck. Benefits contribute to salary and overtime can change weekly paychecks.

e) If it were the set of biological mothers, this would be a well-defined set. However, if you are talking about people who take on the role of "mothering" then the phrase is open to interpretation.

106. Answers will vary. Some sample answers:

a) The set of all legal citizens of the United States

b) The set of 2017 cars that cost more than $50,000

c) The set of all colleges with teams seeded 8 or higher in the NCAA tournament

d) The set of jobs with a base salary of more than $50,000 per year, not counting benefits

e) The set of women that have given birth

Exercise Set 2-2

1. If every element of set A is also in set B, then A is a *subset* of B.

2. A *subset* of (say) set M can equal M, but a *proper subset* of M cannot equal M.

3. A subset is a set in its own right, hence a well-defined *collection* of objects. An element of a set is just an *individual member* of the set.

4. $\varnothing \subseteq \varnothing$ because the requirement for being a subset is trivially true, there being no elements to be tested for inclusion. But $\varnothing \not\subset \varnothing$ because $\varnothing = \varnothing$ always.

5. The union of sets A and B consists of all elements that are in *at least one* of A and B. The intersection of A and B consists of all elements that are *in both* A and B.

6. When they have no elements in common, two sets are said to be disjoint.

7. The set of all elements used in a particular problem or situation is called a universal set.

8. The *complement* of a set (say) A is the set of elements that belong to the universal set but not to A.

9. Answers vary, for instance, the set of math students in your class who are male or over the age of 23. It represents a union because to be in this set you are in at least one of the categories "male" or "over 23." Answers vary, for instance, the set of math students in your class who are male and over the age of 23. It represents an intersection because to be in this set you are in both of the categories.

10. Answers vary, for instance, the set of students in your math class who are liberal arts majors but not freshman. This is a difference because it's the subset of liberal arts majors with those who are freshman taken out.

11. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in A:

U = {2, 3, ~~5, 7, 11, 13~~, 17, 19}

A' = {2, 3, 17, 19}

12. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in B:

U = {~~2~~, 3, 5, 7, 11, 13, 17, 19}

B' = {3, 5, 7, 11, 13, 17, 19}

13. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in C:

U = {2, 3, 5, 7, 11, ~~13, 17, 19~~}

C' = {2, 3, 5, 7, 11}

14. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in D:

U = {~~2, 3, 5~~, 7, 11, 13, 17, 19}

D' = {7, 11, 13, 17, 19}

15. U = {1, 2, 3, 4, 5, 6, 7, 8, ...}

Cross off those elements in A:

U = {1, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ...}

A' = {1, 2, 3, 5, 7, ...}

16. U = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, ...}

Cross off those elements in B:

U = {1, 3, 5, 7, 9, 11, ~~13, 15, 17, 19, ...~~}

B' = {1, 3, 5, 7, 9, 11}

17. {OVI, theft, fraud}, {OVI, theft}, {OVI, fraud}, {theft, fraud}, {OVI}, {theft}, {fraud}, $\varnothing$

18. {assault, manslaughter, battery}, {assault, manslaughter}, {assault, battery}, {manslaughter, battery}, {assault}, {manslaughter}, {battery}, ∅

19. {radio, TV}, {radio}, {TV}, ∅

20. {online, print}, {online}, {print}, ∅

21. ∅

22. { }

23. {fever, chills, nausea, headache}, {fever, chills, nausea}, {fever, chills, headache], {fever, nausea, headache}, {chills, nausea, headache}, {fever, chills}, {fever, nausea}, {fever, headache}, {chills, nausea}, {chills, headache}, {nausea, headache}, {fever}, {chills}, {nausea}, {headache}, ∅

24. {seizures, numbness, paralysis, pain}, {seizures, numbness, paralysis}, {seizures, numbness, pain], {seizures, paralysis, pain}, {numbness, paralysis, pain}, {seizures, numbness}, {seizures, paralysis}, {seizures, pain}, {numbness, paralysis}, {numbness, pain}, {paralysis, pain}, {seizures}, {numbness}, {paralysis}, {pain}, ∅

25. True

26. False, since a proper subset cannot equal the original set.

27. False; the second set contains one element, 123.

28. False; a proper subset cannot be equal to itself.

29. False; {   } has no elements.

30. True

31. False; {3} is a set, not an element. $\subset$ or $\subseteq$ should be used with subsets.

32. True

33. True

34. False; Since {7, 11, 13, 17} contains more elements than {7, 13, 11}, it cannot possibly be a subset.

35. Subsets: $2^3 = 8$; Proper Subsets: $8 - 1 = 7$

36. Subsets: $2^{26} = 67{,}108{,}864$; Proper Subsets: $67{,}108{,}864 - 1 = 67{,}108{,}863$

37. Subsets: $2^0 = 1$; Proper Subsets: $1 - 1 = 0$

38. Subsets: $2^1 = 2$; Proper Subsets: $2 - 1 = 1$

39. Subsets: $2^2 = 4$; Proper Subsets: $4 - 1 = 3$

40. Subsets: $2^{15} = 32{,}768$; Proper Subsets: 32,767

41. $U = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

42. $A = \{1, 5, 9, 11, 17\}$

43. $B = \{5, 11, 13, 15\}$

44. $A \cap B = \{5, 11\}$

45. $A \cup B = \{1, 5, 9, 11, 13, 15, 17\}$

46. $A' = \{3, 7, 13, 15, 19\}$

47. $B' = \{1, 3, 7, 9, 17, 19\}$

48. $(A \cup B)' = \{3, 7, 9\}$

49. $(A \cap B)' = \{1, 3, 7, 9, 13, 15, 17, 19\}$

50. $A \cap B' = \{1, 9, 17\}$

51. $A \cup C = \{12, 14, 15, 16, 17, 19, 20\}$

52. $A \cap B = \{15, 17\}$

53. $A' = \{11, 12, 13, 18, 19, 20\}$

54. $A \cap B = \{15, 17\}$
$(A \cap B) \cup C = \{12, 14, 15, 17, 19, 20\}$

55. $B \cup C = \{11, 12, 13, 14, 15, 17, 19, 20\}$
$A' = \{11, 12, 13, 18, 19, 20\}$
$A' \cap (B \cup C) = \{11, 12, 13, 19, 20\}$

56. $A \cap B = \{15, 17\}$
$(A \cap B) \cap C = \{15\}$

57. $A \cup B = \{11, 13, 14, 15, 16, 17, 19\}$
$(A \cup B)' = \{12, 18, 20\}$
$(A \cup B)' \cap C = \{12, 20\}$

58. $B' = \{12, 14, 16, 18, 20\}$
$A \cap B' = \{14, 16\}$

59. $B \cup C = \{11, 12, 13, 14, 15, 17, 19, 20\}$
$A' = \{11, 12, 13, 18, 19, 20\}$
$(B \cup C) \cap A' = \{11, 12, 13, 19, 20\}$

60. $A' = \{11, 12, 13, 18, 19, 20\}$
$(A' \cup B) = \{11, 12, 13, 15, 17, 18, 19, 20\}$
$(A' \cup B)' = \{14, 16\}$
$C' = \{11, 13, 16, 17, 18\}$
$(A' \cup B)' \cup C' = \{11, 13, 14, 16, 17, 18\}$

61. $W \cap Y = \varnothing$

62. $X \cup Z = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11\}$

63. $W \cup X = \{2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

64. $X \cap Y = \varnothing$

$(X \cap Y) \cap Z = \varnothing$

65. $W \cap X = \{6, 8\}$

66. $Y \cup Z = \{1, 3, 5, 7, 9, 11, 21, 22, 23, 24\}$
$(Y \cup Z)' = \{2, 4, 6, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

67. $X \cup Y = \{2, 4, 6, 8, 21, 22, 23, 24\}$
$(X \cup Y) \cap Z = \varnothing$

68. $Z \cap Y = \varnothing$
$(Z \cap Y) \cup W = W = \{6, 7, 8, 9, 10, 11, 12, 13, 14\}$

69. $W' = \{1, 2, 3, 4, 5, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$
$X' = \{1, 3, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$
$W' \cap X' = \{1, 3, 5, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$

70. $Z \cup X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13\}$
$(Z \cup X)' = \{12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$
$(Z \cup X)' \cap Y = \{21, 22, 23, 24\}$

71. $A \cap B = B$ since $B \subset A$.

72. $A' = \{$*natural numbers that are not multiples of 3*$\}$
$A' \cap C = \{$*all even natural numbers that are not multiples of 3*$\}$
$= \{2, 4, 8, 10, 14, \ldots\}$

73. $C' = \{$*all odd natural numbers*$\}$
$B \cup C' = \{$*odd natural numbers or multiples of 9*$\}$

$A \cap (B \cup C') = \{x \mid x$ *is an odd multiple of 3 or an even multiple of 9*$\}$
$= \{3, 9, 15, 18, 21, 27, 33, 36, 39, \ldots\}$

74. $A \cup B = A$ since $B \subset A$.

75. List the members of C: {p, r, t, v}

Cross off those that are in B: {p~~, r, t, v~~}

$C - B = \{p\}$

76. List the elements in A: {p, q, r, s, t}

Cross off those in C: {~~p~~, q, ~~r~~, s, ~~t~~}

$A - C = \{q, s\}$

77. List the elements in B: {r, s, t, u, v}

Cross of those in C: {~~r~~, s, ~~t,~~ u, ~~v~~}

$B - C = \{s, u\}$

78. List the elements in B: {r, s, t, u, v}

Cross off those in A: {~~r, s, t~~, u, v}

$B - A = \{u, v\}$

79. $B \cap C' = B - C$ so, list the elements in B: {r, s, t, u, v}

Cross of those in C: {~~r~~, s, ~~t,~~ u, ~~v~~}

$B \cap C' = \{s, u\}$

80. $C \cap A' = C - A$ so, List the elements in C: {p, r, t, v}

Cross off those in A: {~~p, r, t~~, v}

$C \cap A' = \{v\}$

81. $D - M = \{11, 13, 15, 17, \ldots\}$

82. $T - D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

83. $D - M = \{11, 13, 15, 17, ...\}$

$(D - M) - T = \varnothing$

84. $T - D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(T - D) - M = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

85. B'

86. $A \cup B$

87. $(A \cup B) - (A \cap B)$

88. $(A \cup B)'$

89. {tablet, laptop, smartphone}, {tablet, laptop}, {tablet, smartphone}, {laptop, smartphone}, {tablet}, {laptop}, {smartphone}, $\varnothing$

90. $2^5 = 32$

91. $2^7 - 1 = 127$

92. $2^6 = 64$

93. $2^4 = 16$

94. {treadmill, cycle, stair stepper}; {treadmill, cycle}; {treadmill, stair stepper}; {cycle, stair stepper}; {treadmill}; {cycle}; {stair stepper}

95. Set A is "strong management skills," Set B is "good at working as part of a team," and Set C is "5 years experience with a similar project."

96. Set A is "women who can dig and clean worms," Set B is "women who can clean fish," and Set C is "women who have a boat with a motor." To be considered a partner for this "catch," women must be lucky enough to be included in the set $(A \cup B) \cap C$.

97. a) $B \cup C$ is the set of people that have been convicted of a felony.

b) $C \cup D$ is the set of people that have been convicted of a felony and have been released, or charged with a felony and found not guilty.

c) $D \cup E$ is the set of people that were charged with a felony and either found not guilty or had charges dropped before standing trial.

98. a) A' is the set of people that have been charged with a felony and have already stood trial, or had the charges dropped before trial.

b) C' is the set of people that have been charged with a felony who were not convicted, or were convicted and are still in prison.

c) E' is the set of people that have been charged with a felony and are either on trial, awaiting trial, or have already been tried.

99. a) $B \cap C$ is the set of people that have been convicted of a felony and have been released from prison.

b) $A \cap B$ is the set of people that have previously been convicted of a felony, and are currently awaiting trial on another felony charge.

c) B' is the set of people that have been charged with, but not convicted of a felony. $C \cap B'$ is an empty set and contains no people.

100. a) $A \cup B$ is the set of people that are on trial or awaiting trial on felony charges or that have been convicted of a felony. $(A \cup B)'$ is the set of people that were charged with a felony and found not guilty or were charged with a felony and had the charges dropped before trial.

b) $B \cup D$ is the set of people that have been convicted of a felony or who were charged with a felony and found not guilty. $(B \cup D)'$ is the set of people that have been charged with a felony but have not stood trial.

c) $B \cap C$ is the set of people that have been convicted of a felony and have been released from prison. $A - (B \cap C)$ is the set of people that are on trial or awaiting trial on felony charges that have not previously been convicted of a felony and released from prison.

101. $A \times B$ is the set of possible finished cakes that can be created using the three kinds of cakes and the two kinds of icing given. $A \times B$ = {(chocolate, fudge icing) – in my opinion no other combinations are worth considering, but – (chocolate, cream cheese icing), (yellow, fudge icing), (yellow, cream cheese icing), (angel food, fudge icing), (angel food, cream cheese icing)}.

102. $C \times D$ is the set of possible outcomes for criminal cases for three different crimes. $C \times D$ = {(guilty, possession with intent), (guilty, DUI), (guilty, assault), (not guilty, possession with intent), (not guilty, DUI), (not guilty, assault)}.

103. $n(A \times B)$ is $n \times m$. Think of how you write all the ordered pairs in a cross product. You start with the first element of the first set and pair it with each element of the second set. This will give you m ordered pairs. Then you use the second element of the first set and pair it with each element of the second set. This gives you m more ordered pairs. You will repeat for each of the n elements of the first set, so you get $n \times m$ ordered pairs.

104. Look at the answer for 103 again. A Cartesian product is a way to model multiplication because it shows. the repeated addition perfectly.

105. Answers may vary.

106. Answers may vary.

107. Answers may vary .

108. Answers may vary .

109. If $A \cap B = A$, then that means that every element of A is also an element of B. A more elegant way to say this is that A is a subset of B. If $A \cap B = B$, then that means that every element of B is also an element of A. A more elegant way to say this is that B is a subset of A.

110. The only way that both $A \cap B = A$ and $A \cap B = B$ is if A and B are equal.

111. Given n elements, each has two choices: in the subset or not in the subset. When we build a subset, each time we consider an element, it doubles the possibilities that we have so far. So, there are $2 \cdot 2 \cdot 2 \cdot \cdots \cdot 2$ ways to choose where the number of twos is the number of elements in the set. It's much easier to say 2^n than $2 \cdot 2 \cdot 2 \cdot \cdots \cdot 2$.

Exercise Set 2-3

1. Answers may vary.

2. Answers may vary.

3. Answers may vary.

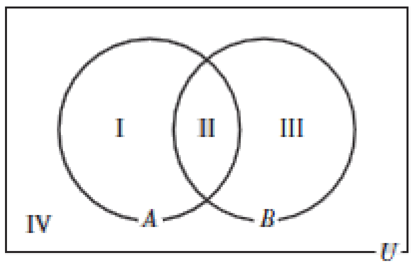
4. Create a Venn diagram for each set. If the Venn diagrams are the same, then the sets are actually equal.

5. The complement of the union is the intersection of the complements and the complement of the intersection is the union of the complements.

6. Add the cardinal numbers of the two sets then subtract the cardinal number of their intersection.

For Exercises 7-12, all solutions have the same first two steps, given here.

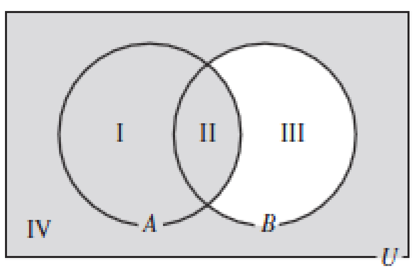
Step 1 Draw the Venn diagram and label each area.



Step 2 From the diagram, list the regions in each set. $U = \{\text{I, II, III, IV}\}$ $A = \{\text{I, II}\}$ $B = \{\text{II, III}\}$

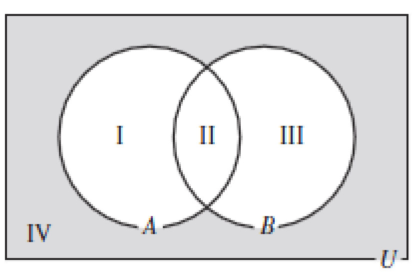
7. **Step 3** $B' = \{\text{I, IV}\}$, so $A \cup B' = \{\text{I, II, IV}\}$

Step 4 Shade regions I, II, and IV



8. **Step 3** $A \cup B = \{\text{I, II, III}\}$, so $(A \cup B)' = \{\text{IV}\}$

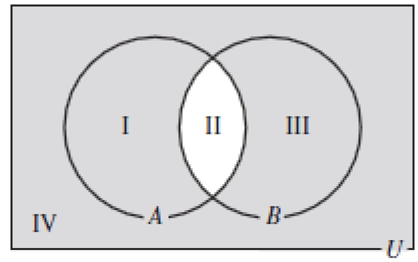
Step 4 Shade region IV



9. **Step 3** $A' = \{\text{III, IV}\}$ and $B' = \{\text{I, IV}\}$,

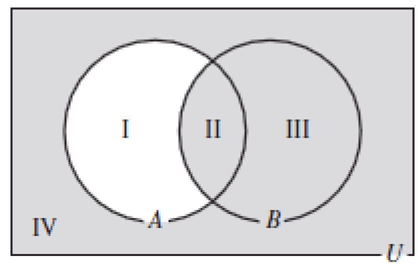
so $A' \cup B' = \{\text{I, III, IV}\}$

Step 4 Shade regions I, III, and IV



10. **Step 3** $A' = \{\text{III, IV}\}$ so $A' \cup B = \{\text{II, III, IV}\}$

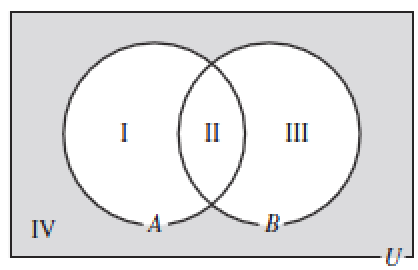
Step 4 Shade regions II, III, and IV



11. **Step 3** $A' = \{\text{III, IV}\}$ and $B' = \{\text{I, IV}\}$,

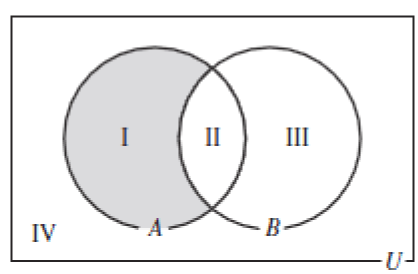
so $A' \cap B' = \{\text{IV}\}$

Step 4 Shade region IV



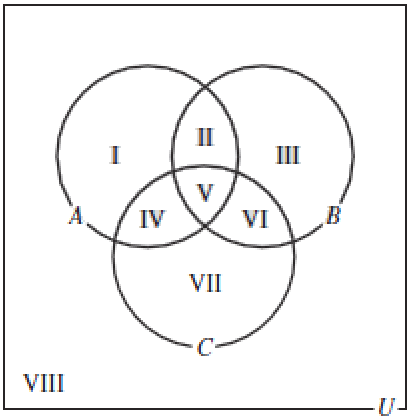
12. **Step 3** $B' = \{\text{I, IV}\}$, so $A \cap B' = \{\text{I}\}$

Step 4 Shade region I



For Exercises 13-30, all solutions have the same first two steps, given here.

Step 1 Draw and label the diagram as shown



Step 2 From the diagram, list the regions in each set.

$U = \{I, II, III, IV, V, VI, VII, VIII\}$

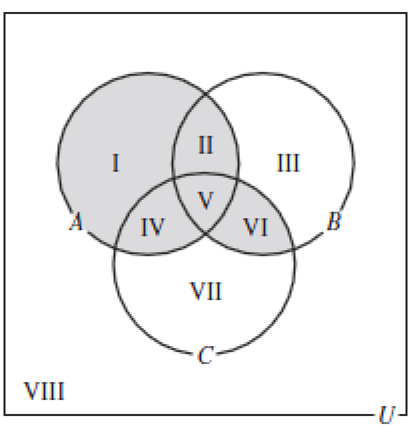
$A = \{I, II, IV, V\}$

$B = \{II, III, V, VI\}$

$C = \{IV, V, VI, VII\}$

13. **Step 3** $B \cap C = \{V, VI\}$, so $A \cup (B \cap C) = \{I, II, IV, V, VI\}$.

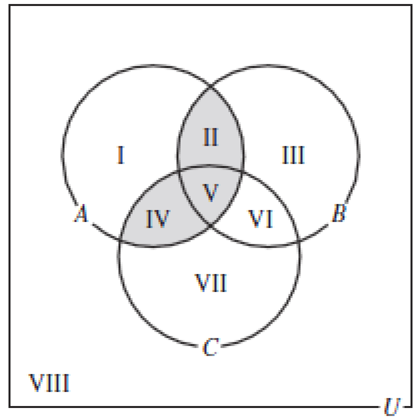
Step 4 Shade regions I, II, IV, V, and VI



14. **Step 3** $B \cup C = \{II, III, IV, V, VI, VII\}$,

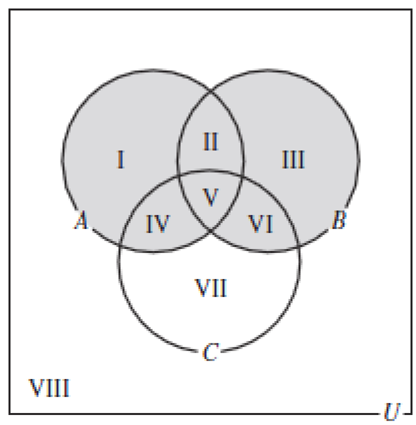
so $A \cap (B \cup C) = \{II, IV, V\}$.

Step 4 Shade regions II, IV, and V



15. **Step 3** $A \cup B = \{I, II, III, IV, V, VI\}$ and $A \cap C = \{IV, V\}$, so $(A \cup B) \cup (A \cap C) = \{I, II, III, IV, V, VI\}$.

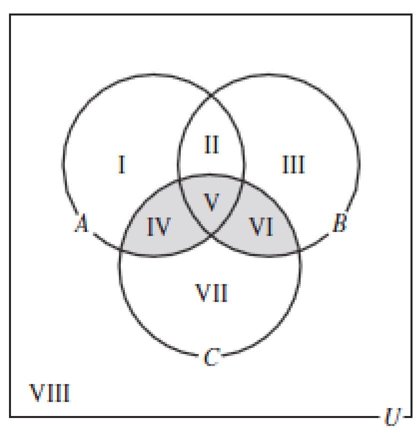
Step 4 Shade regions I, II, III, IV, V, and VI



16. **Step 3** $A \cup B = \{I, II, III, IV, V, VI\}$, so

$(A \cup B) \cap C = \{ IV, V, VI \}$.

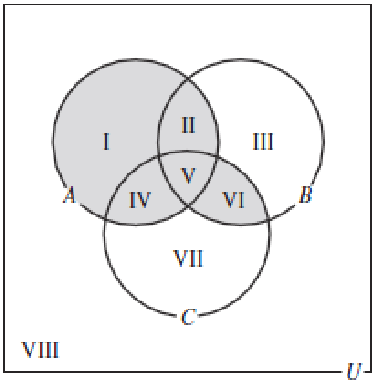
Step 4 Shade regions IV, V, and VI



17. **Step 3** $A \cup B = \{I, II, III, IV, V, VI\}$ and $A \cup C = \{I, II, IV, V, VI, VII\}$, so

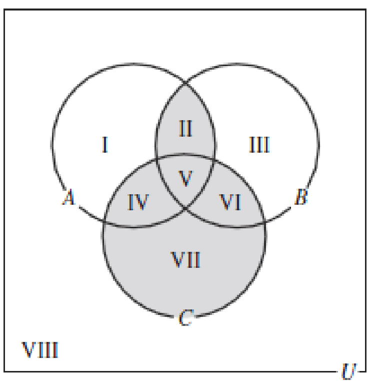
$(A \cup B) \cap (A \cup C) = \{ I, II, III, IV, V, VI\}$.

Step 4 Shade regions I, II, IV, V, and VI



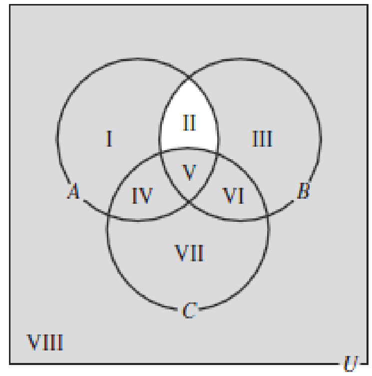
18. **Step 3** $A \cap B = \{II, V\}$, so $(A \cap B) \cup C = \{II, IV, V, VI, VII\}$.

Step 4 Shade regions II, IV, V, VI, and VII



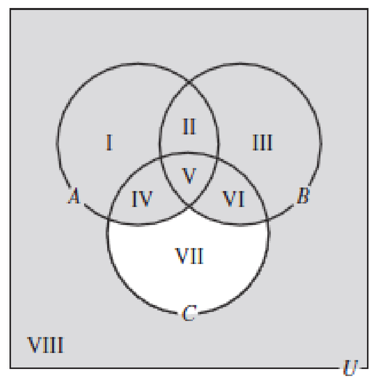
19. **Step 3** $A \cap B = \{II, V\}$, so $(A \cap B)' = \{I, III, IV, VI, VII, VIII\}$ and $(A \cap B)' \cup C = \{ I, III, IV, V, VI, VII, VIII \}$.

Step 4 Shade regions I, III, IV, V, VI, VII, and VIII



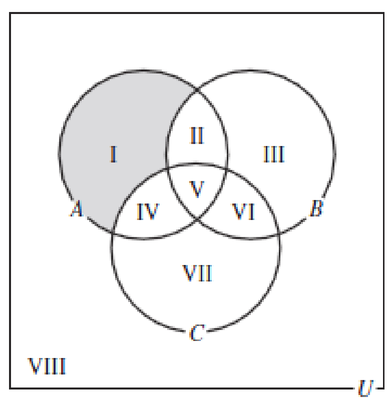
20. **Step 3** $A \cup B = \{I, II, III, IV, V, VI\}$ and $C' = \{I, II, III, VIII\}$, so $(A \cup B) \cup C' = \{I, II, III, IV, V, VI, VIII\}$

Step 4 Shade regions I, II, III, IV, V, VI, and VIII



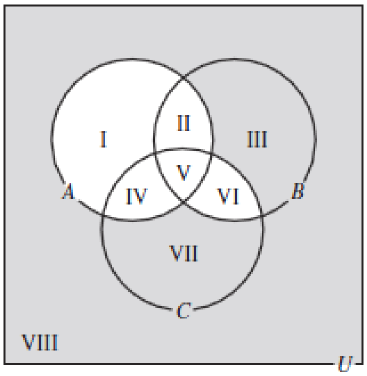
21. **Step 3** $B \cup C = \{II, III, IV, V, VI, VII\}$, so $(B \cup C)' = \{I, VIII\}$ and $A \cap (B \cup C)' = \{I\}$.

Step 4 Shade region I



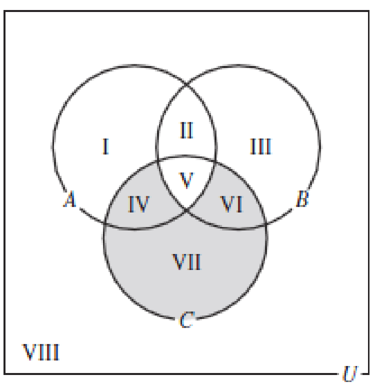
22. **Step 3** $B' \cup C' = (B \cap C)'$ (by DeMorgan's law) = {I, II, III, IV, VII, VIII} and A' = {III, VI, VII, VIII}, so $A' \cap (B' \cup C')$ = {III, VII, VIII}

Step 4 Shade regions III, VII, and VIII



23. **Step 3** $A' \cup B' = (A \cap B)'$ (by DeMorgan's law) = {I, III, IV, VI, VII, VIII} , so $(A' \cup B') \cap C$ = {IV, VI, VII}

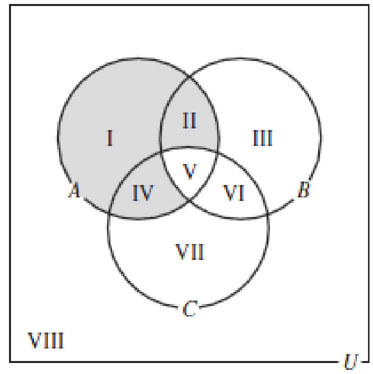
Step 4 Shade regions IV, VI, and VII



24. **Step 3** $(B \cap C)'$ = {I, II, III, IV, VII, VIII}, so

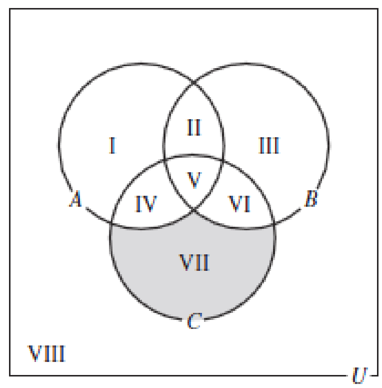
$A \cap (B \cap C)'$ = {I, II, IV}

Step 4 Shade regions I, II, and IV



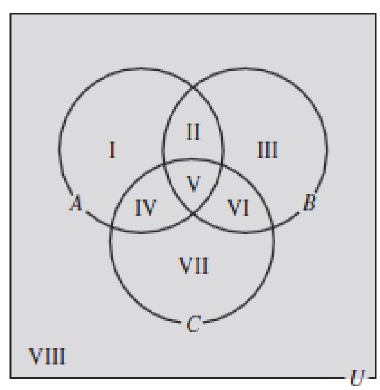
25. **Step 3** $(A \cup B)'$ = {VII, VIII} and $A \cup C$ = {I, II, IV, V, VI, VII}, so $(A \cup B)' \cap (A \cup C)$ = {VII}

Step 4 Shade region VII



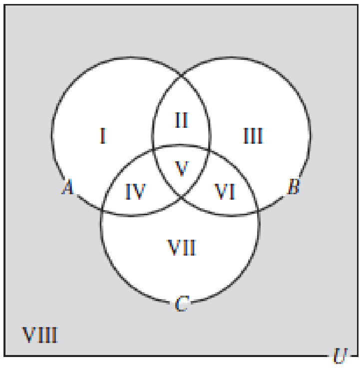
26. **Step 3** $B \cup C$ = {II, III, IV, V, VI, VII} and C'={I, II, III, VIII}, so $(B \cup C) \cup C'$ = {I, II, III, IV, V, VI, , VII, VIII} = U

Step 4 Shade all regions



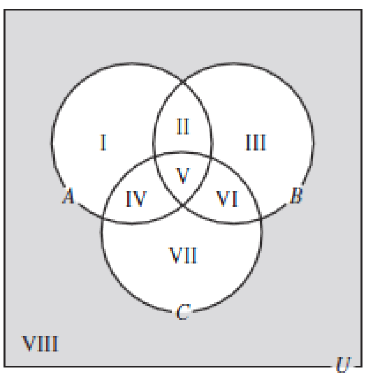
27. **Step 3** $B' \cap C' = (B \cup C)'$ ={I, VIII} and A' = {III, VI, VII, VIII}, so $A' \cap (B' \cap C')$={VIII}

Step 4 Shade region VIII



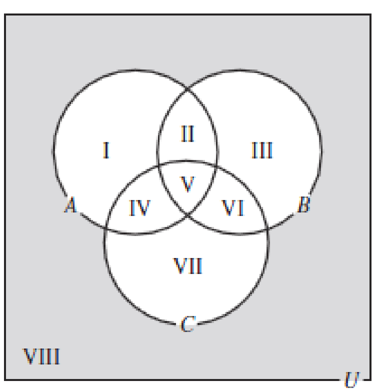
28. **Step 3** $(A \cup B)'$ = {VII, VIII} and C' = {I, II, III, VIII}, so $(A \cup B)' \cap C'$ = {VIII}

Step 4 Shade region VIII



29. **Step 3** $(B \cup C)'$ ={I, VIII} and A' = {III, VI, VII, VIII}, so $A' \cap (B \cup C)'$={VIII}

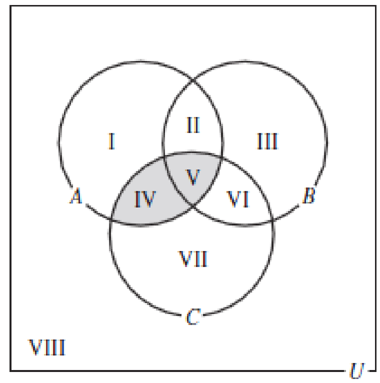
Step 4 Shade region VIII



30. **Step 3** $A \cup B$ = {I, II, III, IV, V, VI} and

$A \cap C$ = {IV, V}, so $(A \cup B) \cap (A \cap C)$ = {IV, V}

Step 4 Shade regions IV, and V



For Exercises 31-38, use the labeled Venn diagram from Step 1 of the solutions for Exercises 13-30.

31. $A \cap B$ = {II, V}

$(A \cap B)'$ = {I, III, IV, VI, VII, VIII}
A' = {III, VI, VII, VIII}
B' = {I, IV, VII, VIII}

$A' \cup B'$ = {I, III, IV, VI, VII, VIII}

Yes, $(A \cap B)'$ is equal to $A' \cup B'$.

32. $A \cup B$ = {I, II, III, IV, V, VI}
$(A \cup B)'$ = {VII, VIII}
A' = {III, VI, VII, VIII}
B' = {I, IV, VII, VIII}

$A' \cup B'$ = {I, III, IV, VI, VII, VIII}
No, $(A \cup B)'$ is not equal to $A' \cup B'$.

33. $(A \cup B) \cup C$ = {I, II, III, IV, V, VI, VII}
$A \cup (B \cup C)$ = {I, II, III, IV, V, VI, VIII}
Yes, $(A \cup B) \cup C$ is equal to $A \cup (B \cup C)$.

34. $B \cup C = \{II, III, IV, V, VI, VII\}$
$A \cap (B \cup C) = \{II, IV, V\}$
$A \cap B = \{II, V\}$
$A \cap C = \{VI, V\}$
$(A \cap B) \cup (A \cap C) = \{II, VI, V\}$
Yes, $A \cap (B \cup C)$ is equal to
$(A \cap B) \cup (A \cap C)$.

35. $C' = \{I, II, III, VIII\}$
$B \cap C' = \{II, III\}$
$A' = \{III, VI, VII, VIII\}$
$A' \cup (B \cap C') = \{II, III, VI, VII, VIII\}$
$A' \cup B = \{II, III, V, VI, VII, VIII\}$
$(A' \cup B) \cap C' = \{II, III, VIII\}$
No, $A' \cup (B \cap C')$ is not equal to
$(A' \cup B) \cap C'$.

36. $A \cap B = \{II, V\}$
$C' = \{I, II, III, VIII\}$
$(A \cap B) \cup C' = \{I, II, III, V, VIII\}$
$B \cap C' = \{II, III\}$
$(A \cap B) \cup (B \cap C') = \{II, III, V\}$
No, $(A \cap B) \cup C'$ is not equal to
$(A \cap B) \cup (B \cap C')$.

37. $A \cap B = \{II, V\}$
$(A \cap B)' = \{I, III, IV, VI, VII, VIII\}$
$(A \cap B)' \cup C = \{I, III, IV, V, VI, VII, VIII\}$
$A' = \{III, VI, VII, VIII\}$
$B' = \{I, IV, VII, VIII\}$
$A' \cup B' = \{I, III, IV, VI, VII, VIII\}$
$(A' \cup B') \cap C = \{IV, VI, VII\}$
No, $(A \cap B)' \cup C$ is not equal to $(A' \cup B') \cap C$.

38. $A' = \{III, VI, VII, VIII\}$
$B' = \{I, IV, VII, VIII\}$
$C = \{IV, V, VI, VII\}$
$(A' \cup B') \cup C = \{I, III, IV, V, VI, VII, VIII\}$
$A \cap B = \{II, V\}$
$(A \cap B)' = \{I, III, IV, VI, VII, VIII\}$
$C' = \{I, II, III, VIII\}$
$(A \cap B)' \cap C' = \{I, III, VIII\}$
No, $(A' \cup B') \cup C$ is not equal to
$(A \cap B)' \cap C'$.

39. $n(A) = 10$

40. $n(B) = 11$

41. $n(A \cap B) = 4$

42. $n(A \cup B) = 17$

43. $n(A') = 13$

44. $n(B') = 12$

45. $n(A' \cap B') = 6$

46. $n(A' \cup B') = 19$

47. $n(A - B) = 6$

48. $n(B - A) = 7$

49. $n(A \cap (B - A)) = 0$

50. $n(B' \cup (B - A)) = 19$

51. $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$
$n(A) = 8$

52. $B = \{7, 11, 13, 17, 19\}$
$n(B) = 5$

53. $A \cap B = \{7, 11, 13\}$
$n(A \cap B) = 3$

54. $A \cup B = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$
$n(A \cup B) = 10$

55. $B' = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$
$A \cap B' = \{1, 3, 5, 9, 15\}$
$n(A \cap B') = 5$

56. $A' = \{2, 4, 6, 8, 10, 12, 14, 16, 17, 18, 19\}$

$A' \cup B = \{2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19\}$

$n(A' \cup B) = 14$

57. $A' = \{2, 4, 6, 8, 10, 12, 14, 16, 17, 18, 19\}$

$n(A') = 11$

58. $B' = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$n(B') = 14$

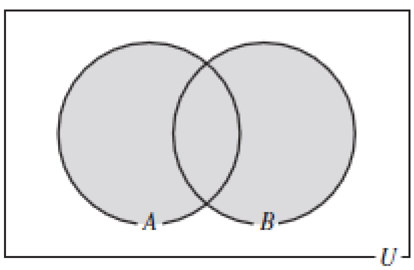
59. $A - B = \{1, 3, 5, 9, 15\}$

$n(A - B) = 5$

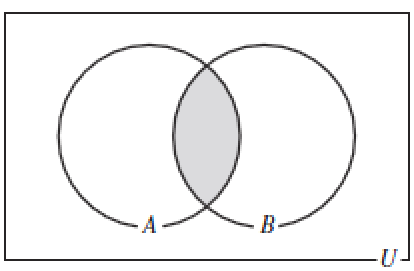
60. $B' - A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$n(B' - A) = 9$

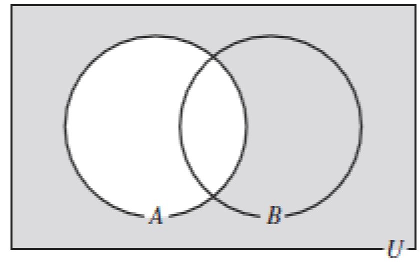
61. People who drive an SUV or a hybrid vehicle



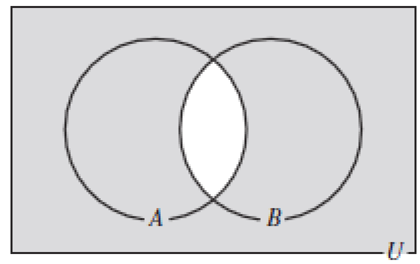
62. People who drive a hybrid SUV



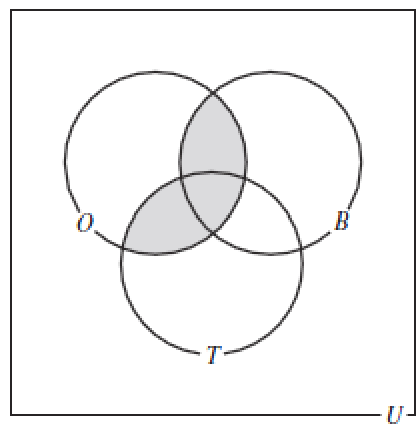
63. People who do not drive an SUV



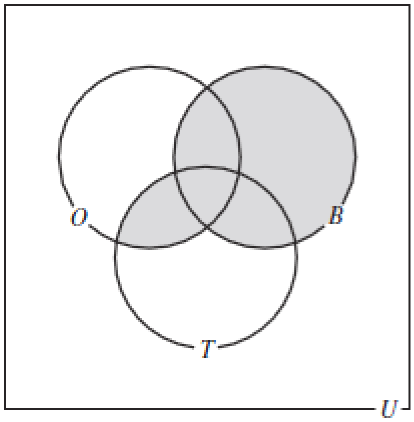
64. People who do not drive a hybrid SUV



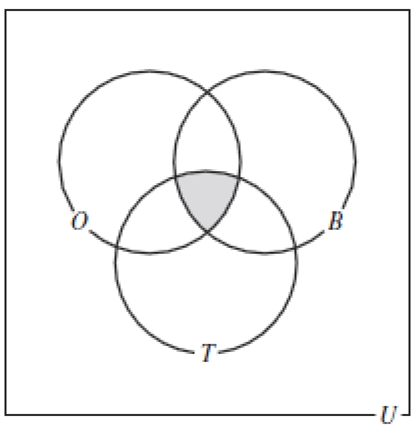
65. Students in online courses and blended or traditional courses



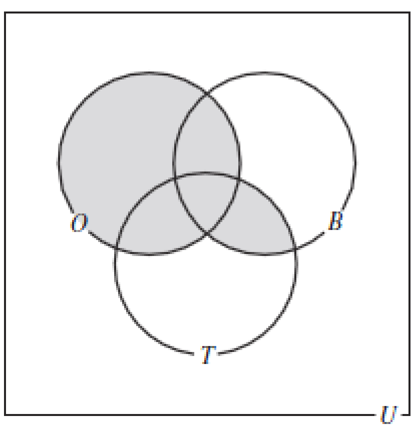
66. Students who are in blended courses or online and traditional courses



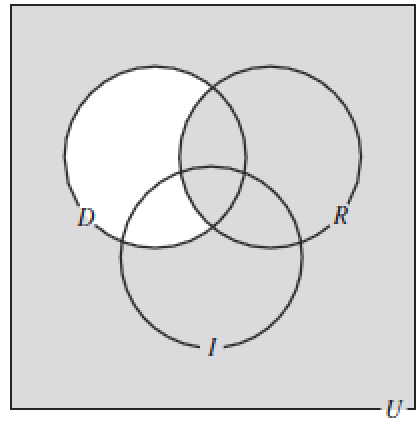
67. Students who are in blended, online, and traditional courses



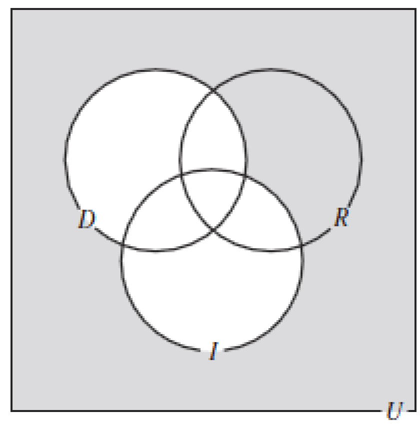
68. Students in blended and traditional courses or online courses



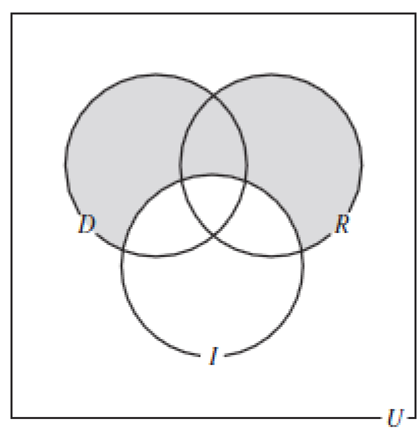
69. Students not voting Democrat or voting Republican



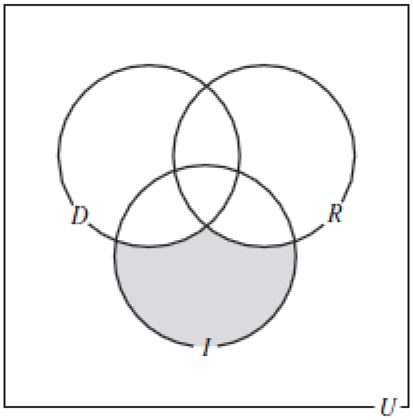
70. Students not voting Democrat or Independent



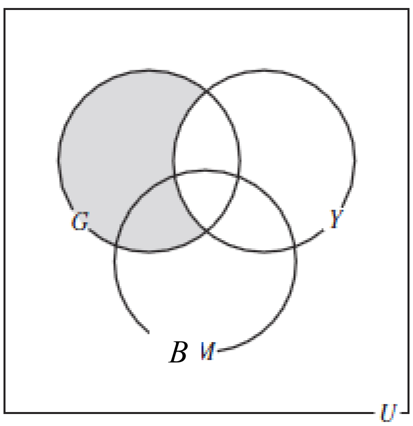
71. Students voting Democrat or Republican but not voting Independent



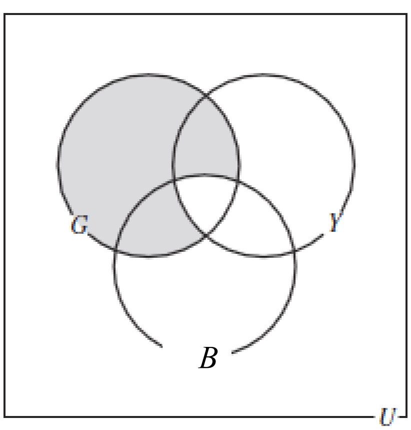
72. Students voting Independent but not voting Democrat or Republican



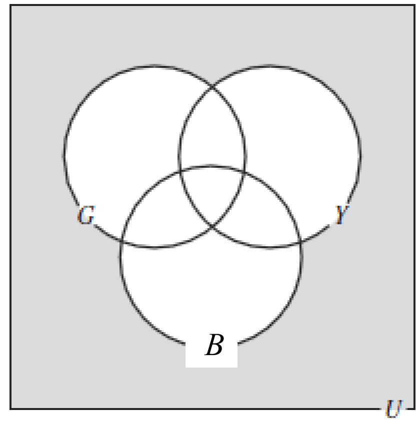
73. People who regularly use Google but not Yahoo!



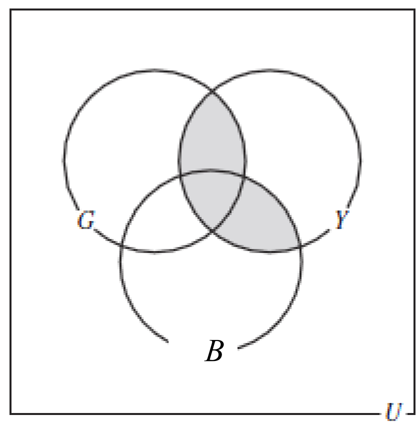
74. People who regularly use Google and either Yahoo! or Bing but not all three



75. People who do not regularly use Google, Yahoo!, or Bing



76. People who regularly use Yahoo! and Bing or Yahoo! and Google



77. Indianapolis was in the playoffs in 2013 and 2014, but not in 2015. So, they are in set A and B, but not C, which is region II.

78. Kansas City was in the playoffs in 2013 and 2015, but not in 2014. So they are in *A* and C, but not B, which is region IV.

79. San Diego was in the playoffs in only 2013, which is region I.

80. Baltimore was in the playoffs in only 2014, which is region III..

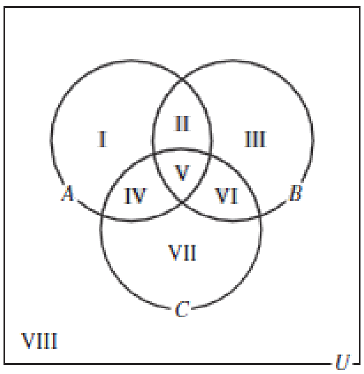
81. Pittsburgh was in the playoffs in 2014 and 2015, but not in 2013. So, they are in Set B and C, but not in A, which is region VI.

82. Miami was never in the playoffs, so they are not inside any of the circles, which is region VIII.

83. No; $n(A - B) = n(A) - n(A \cap B)$

84. There is no formula for $n(A \cap B)$ in terms of $n(A)$ and $n(B)$ only. Answers vary for possible formulas, one example is $n(A \cap B) = n(A \cup B) - n(A - B) - n(B - A)$.

85. $(A \cup B \cup C)' = A' \cap B' \cap C$



$A \cup B \cup C = \{\text{I, II, III, IV, V, VI, VII}\}$ so

$(A \cup B \cup C)' = \{\text{VIII}\}$.

$A' = \{\text{III, VI, VII, VIII}\}$

$B' = \{\text{I, IV, VII, VIII}\}$

$C' = \{\text{I, II, III, VIII}\}$ so $A' \cap B' \cap C' = \{\text{VIII}\}$.

Therefore $(A \cup B \cup C)' = A' \cap B' \cap C'$.

86. $(A \cap B \cap C)' = A' \cup B' \cup C'$

(Using the Venn diagram in the previous solution)

$A \cap B \cap C = \{\text{V}\}$ so $(A \cap B \cap C)' = \{\text{I, II, III, IV, VI, VII, VIII}\}$

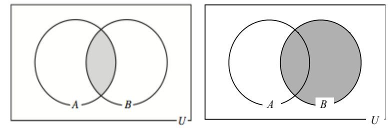
$A' = \{\text{III, VI, VII, VIII}\}$

$B' = \{\text{I, IV, VII, VIII}\}$

$C' = \{\text{I, II, III, VIII}\}$ so $A' \cup B' \cup C' = \{\text{I, II, III, IV, VI, VII, VIII}\}$

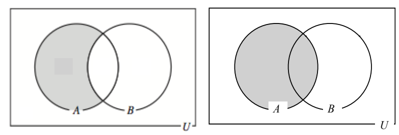
Therefore $(A \cap B \cap C)' = A' \cup B' \cup C'$

87. a)



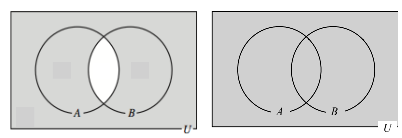
b) For $A \cap B$ and B to be equal, then all elements of B would also have to be elements of A. Symbolically, this is $B \subseteq A$.

88. a)



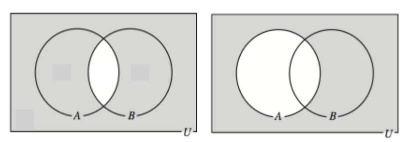
b) For $A - B$ and A to be equal, there are no elements of B that are also elements of A, so A and B are disjoint.

89. a)



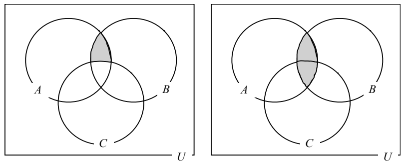
b) For $(A \cap B)'$ to be equal to U, there are no elements of B that are also elements of A, so A and B are disjoint.

90. a)



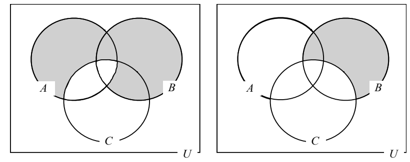
b) For $(A \cap B)'$ to be equal to A', there are no elements of A that are not also elements of B. Symbolically, this is $A \subseteq B$.

91. a)



b) For $(A - C) \cap B$ to be equal to $B \cap A$, there are no elements in the set $B \cap C$, so B and C are disjoint.

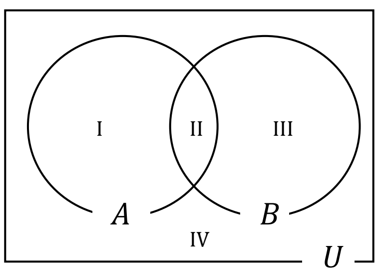
92. a)

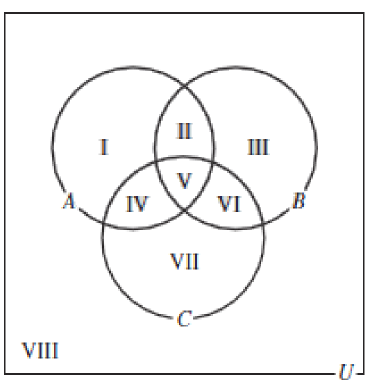


b) For $(A - C) \cup (B - A)$ to be equal to $B - C$, then all elements of A and C are also elements of B. Symbolically, this is $C \subseteq A \subseteq B$.

Exercise Set 2-4

The solutions to Exercises 1-16 refer to the regions labeled one of the following Venn diagram.





1. **Draw a Venn diagram, where:**
   U = universal set
   A = people who use Instagram
   B = people who use Facebook

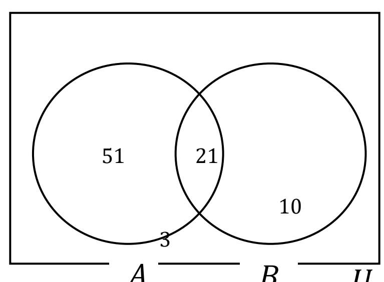
   Place given data: Since 21 people use both Facebook and Instagram, place 21 in region II.

Place calculated data: Since 72 people use Instagram and 21 people use both, subtract $72 - 21 = 51$ to get the number of people who use Instagram only. Place 51 in region I.

Subtract $31 - 21 = 10$ to find the number of people who use Facebook only. Place 10 in region III.

Find the number of people who used neither Facebook nor Instagram by adding, $51 + 21 + 10 = 82$, and subtracting that number from the total number of students, 85; $85 - 82 = 3$. Place 3 in region IV.

Answer questions using Venn diagram:



a) The number of students that use Facebook only is 10.

b) The number of students that use Instagram only is 51.

c) The number of students that use neither is 3.

2. **Draw a Venn diagram, where:**
   U = universal set
   M = math majors
   C = computer science majors

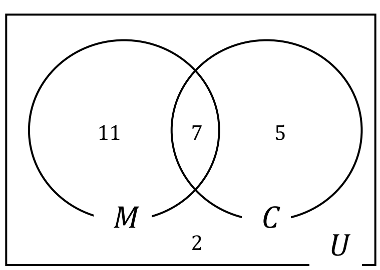
   Place given data: Since 7 students were dual majors in math and computer science, place 7 in region II.

   Place calculated data: Since 18 students were math majors and 7 students were dual majors, subtract 18 – 7 = 11 to get the number of students who were majoring in math only. Place 11 in region I.

   By subtracting, find the number of students majoring in computer science only; 12 – 7 = 5.
   Place 5 in region III.

   Find the number of students who were not math or computer science majors by adding, 7 + 11 + 5 = 23, and subtracting that number from the total number of students, 25; 25 – 23 = 2. Place 2 in region IV.

   Answer questions using Venn diagram:



   a) The number of students majoring in math only is 11.

   b) The number of students not majoring in computer science is 11+ 2 = 13.

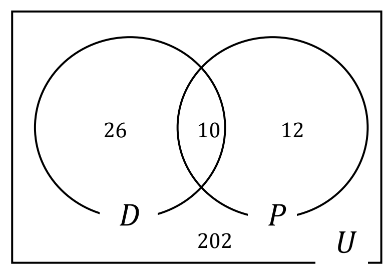
   c) The number of students who were not math or computer science majors is 2.

3. **Draw a Venn diagram, where:**
   U = universal set
   D = students arrested for underage drinking
   P = students arrested for drug possession

   Place given data: Since 26 have been arrested for underage drinking and not drug possession, place 26 in region I. Since 12 have been arrested for drug possession, and not underage drinking, place 12 in region III. Since 202 have been arrested for neither, place 202 in region IV.

   Place calculated data: To find the number of students in region II, subtract the total students placed in the diagram so far (202 + 26 + 12 = 240) from the total number of incoming freshman; 250 – 240 = 10.

   Answer questions using Venn diagram:



   a) The number of students that have been arrested for drug possession is the sum of regions II and III; 10 + 12 = 22.

   b) The number of students that have been arrested for underage drinking is the sum of regions I and II; 26 + 10 = 36.

4. **Draw a Venn diagram, where**:
U = universal set
T = mice that developed tumors
R = mice that developed respiratory failure

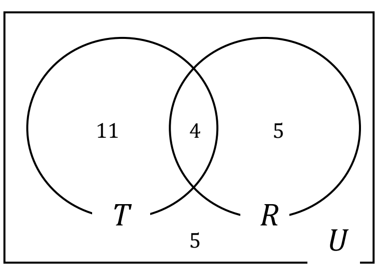
Place given data: Since there were 4 mice that developed tumors and had respiratory failure, place 4 in region II.

Place calculate data: Since 15 mice developed at least one tumor, the number of mice who developed tumors only is 15 – 4 = 11. Place 11 in region I.

Since 9 mice developed respiratory failure, the number of mice who developed respiratory failure only is 9 – 4 = 5. Place 5 in region III.

To find the number of mice who didn't develop tumors or respiratory failure, subtract the sum of the mice placed in the diagram so far (11 + 4 + 5 = 20) from the number of mice in the study; 25 – 20 = 5. Place 5 in region IV.

Answer questions using Venn diagram:



a) The number of mice who got only tumors is 11.

b) The number of mice who didn't get a tumor is 5 + 5 = 10.

c) The number of mice who suffered from at least one of these effects is 11 + 4 + 5 = 20.

5. **Draw a Venn diagram, where:**
U = universal set
F = students that answered the first question
S = students that answered the second question

Place given data: Since 2 students didn't bother with either question, place 2 in region IV.

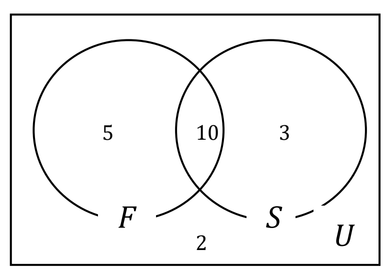
Place calculated data: Since 2 students didn't bother with either question, 20 – 2 = 18 students attempted one or both problems. 15 students attempted the first question and 13 students attempted the second question.

Since only 18 students attempted one or both questions, the number of students that attempted both questions is 28 – 18 = 10. Place 10 in region II.

Since 15 students attempted question 1, subtract to find the number of students who attempted only question 1; 15 – 10 = 5. Place 5 in region I.

Since 13 students attempted question 2, subtract to find the number of students who attempted only question 2; 13 – 10 = 3. Place 3 in region III.

Answer questions using Venn diagram:



a) The percentage of students who tried both questions is 10/20 = 50%.

b) The percentage of students who tried at least one question is 18/20 = 90%.

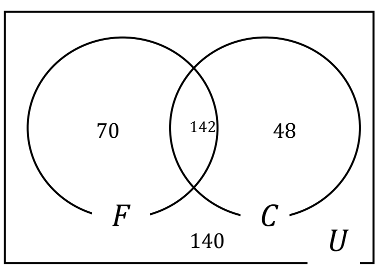
6. **Draw a Venn diagram, where:**

   U = universal set
   F = entrees that had less than 10 grams of fat
   C = entrees that had less than 350 calories

   Place given data: Since 70 entrees had less than 10 grams of fat but not less than 350 calories, place 70 in region I. Since 48 entrees had less than 350 but not less than 10 grams of fat, place 48 in region III. Since 140 entrees had over 350 calories and over 10 grams of fat, place 140 in region IV.

   Place calculated data: Find the number of entrees that had less than 350 calories and less than 10 grams of fat by subtracting the number of entrees in regions I, II and IV from the total number of entrees studied, 400 – (70 + 48 + 140) = 400 – 256 = 142. Place 142 in region II.

   Answer questions using Venn diagram:



   a) The percentage of entrees that had less than 10 grams of fat is (70 + 142)/400 = 190/400 = 53%.

   b) The percentage of entrees that had less than 350 calories is (142 + 48)/400 = 190/400 = 47.5%.

7. **Draw a Venn diagram, where:**

   U = universal set
   S = scholarships
   L = student loans
   G = private grants

   Place given data: Since 2 students receive all three types of aid, place 2 in region V.

   Place calculated data: Find the number of students who had scholarships and loans but did not receive a grant. Subtract the number of students who had all three types of aid (2) from the number of students who had scholarships and grants (9), 9 – 2 = 7. Place 7 in region II.

   Find the number of students that had loans and grants but did not have scholarships by subtracting 11 – 2 = 9. Place 9 in region VI.

   Find the number of students who had scholarships and grants but did have student loans; 7 – 2 = 5. Place 5 in region IV.

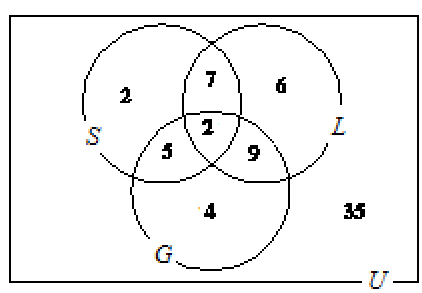
   By subtracting, find the number of students that had only scholarships; 16 – (7 + 5 + 2) = 2. Place 2 in region I.

   By subtracting, find the number of that had only student loans; 24 – (7 + 9 + 2) = 6. Place 6 in region III.

   By subtracting, find the number of students that had only grants;
   20 – (9 + 5 + 2) = 4. Place 4 in region VII.

   Find the number of students who did not receive any financial aid by adding all the numbers,
   2 + 7 + 6 + 5 + 2 + 9 + 4 = 35, and subtracting that number from the total number of students, 70; 70 – 35 = 35. Place 35 in region VIII.

   Answer questions using Venn diagram:



a) The number of students who had only scholarships is 2.

b) The number of students who received grants and loans but not scholarships is 9.

c) The number of students who did not receive any of these types of aid is 35.

8. Draw a Venn diagram, where:

U = Universal set
Y = Yoga
P = Pilates
S = Spinning

Place given data: Since 2 students were interested in all three, place 2 in region V.

Place calculated data: Find the number of students that were interested in yoga and Pilates but not spinning. Subtract the number of students interested in all three (2) from the number of students that were interested in yoga and Pilates (9), 9 – 2 = 7. Place 7 in region II.

Find the number of students that were interested in Pilates and spinning but not yoga by subtracting 3 – 2 = 1. Place 1 in region VI.

Find the number of students that were interested in yoga and spinning but not Pilates by subtracting 5 – 2 = 3. Place 3 in region IV.

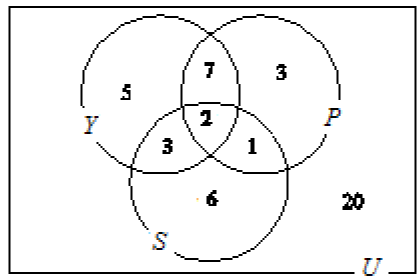
By subtracting, find the number of students interested in yoga only;
17 – (2 + 7 + 3) = 5. Place 5 in region I.

By subtracting, find the number of students interested in Pilates only;
13 – (2 + 7 + 1) = 3. Place 3 in region III.

By subtracting, find the number of students interested in spinning only;
12 – (2 + 1 + 3) = 6. Place 6 in region VII.

Find the number of students that had interest in none of the classes by adding all the numbers,
5 + 7 + 3 + 3 + 2 + 1 + 6 = 27, and subtracting that number from the total number of students, 47; 47 – 27 = 20. Place 20 in region VIII.

Answer questions using Venn diagram:



a) The number of students who are interested in yoga or spinning but not Pilates is 5 + 3 + 6 = 14.

b) The number of students who are interested in exactly two of the classes is 7 + 3 + 1 = 11.

c) The number of students who are interested in yoga but not Pilates is 5 + 3 = 8.

9. Draw a Venn diagram, where:

U = Universal set
AT = Poor attendance
S = Not studying
AS = Not turning in assignments

Place given data: Since 2 students failed because of all three reasons, place 2 in region V.

Place calculated data: Find the number of students who failed because of poor attendance and not studying but not because of not turning in assignments. Subtract the number of students that failed because of all three conditions (2) from the number of students that failed because of poor attendance and not studying (9), 9 – 2 = 7. Place 7 in region II.

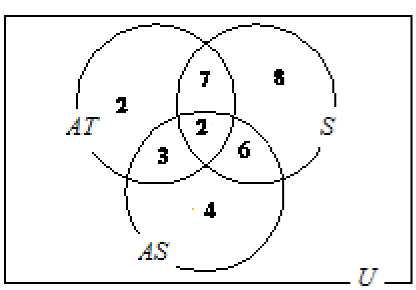
Find the number of students who failed because of not studying and not turning in assignments but not because of poor attendance by subtracting 8 – 2 = 6. Place 6 in region VI.

Find the number of students who failed because poor attendance and not turning in assignments but not because of not studying by subtracting 5 – 2 = 3. Place 3 in region IV.

By subtracting, find the number of students that failed because of poor attendance only; 14 – (2 + 7 + 3) = 2. Place 2 in region I.
By subtracting, find the number of students that failed because of not studying only; 23 – (2 + 7 + 6) = 8. Place 8 in region III.

By subtracting, find the number of students that failed because of not turning in assignments only; 15 – (2 + 6 + 3) = 4. Place 4 in region VII.

Answer questions using Venn diagram:



a) The number of students who failed for exactly two of the three reasons is 7 + 3 + 6 = 16.

b) The number of students who failed because of poor attendance and not studying but not because of not turning in assignments is 7.

c) The number of students who failed because of exactly one of the three reasons is 2 + 8 + 4 = 14.

d) The number of students who failed because of poor attendance and not turning in assignments but not because of not studying is 2 + 3 + 4 = 9.

10. Step 1 Draw a Venn diagram, where

U = universal set
A = people who ordered pizza with pepperoni
B = people who ordered pizza with sausage
C = people who ordered pizza with onions

Place given data: Since 32 preferred pizza with just pepperoni, place 32 in region I. Since 40 ordered just sausage, place 40 in region III. Since 18 ordered just onion, place 18 in region VII. Since 7 customers ordered all three, place 7 in region V.

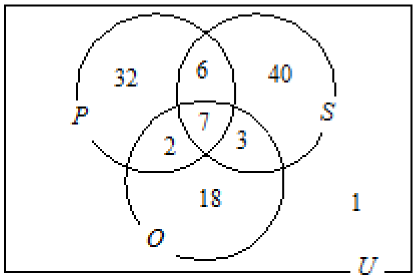
Place calculated data: By subtracting 13 – 7 = 6 we find the number of customers who ordered pepperoni and sausage but not onions. Place 6 in region II.

By subtracting 10 – 7 = 3 we find the number of customers who ordered sausage and onions but not pepperoni. Place 3 in region VI.

By subtracting 9 – 7 = 2 we find the number of customers who ordered pepperoni and onions but not sausage. Place 2 in region IV.

Add all the numbers 32 + 6 + 40 + 2 + 7 + 3 + 18 = 108 subtract from 109 to get 1 customer in region VIII.

Answer questions using Venn diagram:



a) There were 32 + 6 + 40 + 2 + 7 + 3 = 90 customers who ordered their pizzas with pepperoni or sausage or pepperoni and sausage with no onions.

b) There were 6 + 40 + 2 + 7 + 3 + 18 = 76 customers who ordered sausage or onions or sausage and onions with no pepperoni.

c) There was 1 boring customer that ordered their pizza without sausage, pepperoni, or onions.

11. Draw a Venn diagram, where:

U = universal set
Z = percentage of patients who took Zoloft
L = percentage of patients who took Lexapro
P = percentage of patients who took Prozac

Place given data: Since 4% of patients took all 3, place 4 in region V.

Placed calculated data: By subtracting 13 – 4 = 9 we get the percentage of patients who took Zoloft and Lexapro but not all three. Place 9 in region II.

By subtracting 11.5 – 4 = 7.5 we get the percentage of patients who took Lexapro and Prozac but not all three. Place 7.5 in region VI.

By subtracting 7 – 4 = 3 we get the number of Patients who were given Zoloft and Prozac but all three. Place 3 in region IV.

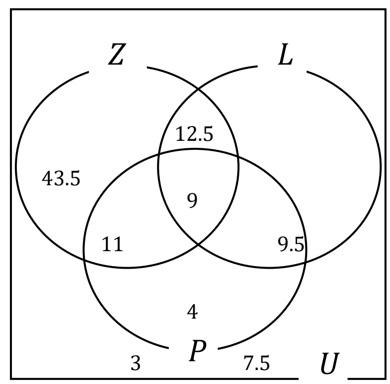
Subtract 27 – (3 + 4 + 9) = 11 to get the percentage of patients who took Zoloft only. Place 11 in region I.

Subtract 30 – (9 + 4 + 7.5) = 9.5 to get the percentage of patients who took Lexapro only. Place 9.5 in region III.

Subtract 27 – (3 + 4 + 7.5) = 12.5 to get the percentage of patients who took only Zoloft. Place 12.5 in region VII.

Subtract 100 – (11 + 9 + 9.5 + 3 + 4 + 7.5 + 12.5) = 43.5 to get the percentage of patients who were given none of these drugs.

Answer questions using Venn diagram:



a) At most two means zero, one, or two so add 43.5 + 11 + 9 + 9.5 + 3 + 7.5 + 12.5 = 96%. So, the number of students who took two or more was 96% of 200, or 192 students. Another way of looking at this is that this would be all students expcept those who took three drugs; since there were 4% of 200, 8 students who took all three, 192 must take at most two.

b) For Zoloft or Prozac but not Lexpro, there is only 3%. So, the number of students who took Zoloft or Prozac but not Lexpro was 3% of 200, or 6 students.

c) 43.5% of 200 took none of the drugs, so 87 students took none.

12. **Draw a Venn diagram, where:**

Place given data: Since 1 student read all three news sources, place 1 in region V.

Place calculated data: By subtracting 8 – 1 = 7 we get the number of students who read the *Campus Observer* and the Internet news, but not the local paper. Place 7 in region II.

By subtracting 4 – 1 = 3 we get the number of students who read the Internet news and the local paper, but not the *Campus Observer*. Place 3 in region VI.

By subtracting 7 – 1 = 6 we get the number of students who read the *Campus Observer* and the local paper, but not the Internet news. Place 6 in region IV.

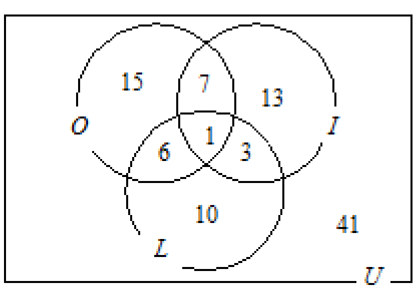
By subtracting 29 – (6+ 7 + 1) = 15 we get the number of students who read only the *Campus Observer*. Place 15 in region I.

By subtracting 24 – (7 + 3 + 1) = 13 we get the number of students who read only the Internet news. Place 13 in region III.

By subtracting 20 – (6 + 3 + 1) = 10 we get the number of students who read only the local paper. Place 10 in region VII.

By subtracting 96 – (15 + 7 + 13 + 6 + 1 + 3 + 10) = 41 we get the number of students who read none of the three sources. Place 41 in region VIII.

Answer questions using the Venn diagram:



a) 7 + 13 + 6 + 10 = 36 students read the Internet news or local paper but not both.

b) There were 3 students who read the Internet news and local paper but not the *Campus Observer.*

c) 15 + 7 + 13 + 6 + 1 + 3 = 45 students read the *Campus Observer* or the Internet news.

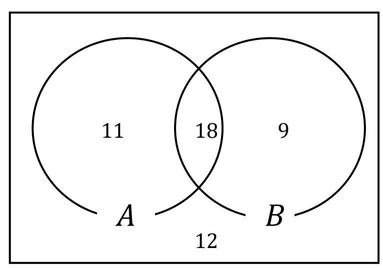
13. **Draw a Venn diagram, where:**

U = universal set
A = cities that have a team in the NBA
B = cities that have a MLB team

Place given data: Since 11 cities have an NBA team but not a MLB team, place 11 in region I. Since 9 cities have an MLB team but not a NBA team, place 9 in region III. Since 12 cities have neither, place 12 in region IV.

Place calculated data: By subtracting 50 – (11+ 9 + 12) = 18, we get the number of cities that have both an NBA team and a MLB team. Place 18 in region II.

Answer questions using Venn diagram:



a) The number of cities that have both a NBA team and a MLB team is 18.

b) The number of NBA teams is 11 + 18 + 1 = 30. The number of MLB teams is 18 + 9 + 3 = 30.

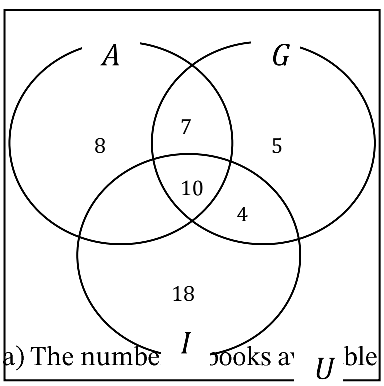
14. Draw a Venn diagram where,

U = universal set
A = books only available on Amazon
G = books only available on Google books
I = books only available on iTunes

Place given data: Since 8 books were only available on Amazon, place 8 in region I. Since 5 books were only available on Google books, place 5 in region III. Since 18 were available only on iTunes, place 18 in region VII. Since 7 could be found on both Amazon and Google but not iTunes, place 7 in region II. Since 4 could be found on both iTunes and Google but not Amazon, place 4 in region VI.

Step 3 By subtracting 26 – (5 +7 + 4) = 10 we get the number of books that were available on all three. Place 10 in region V.

Answer questions using Venn diagram:



a) The number of books available on all three services was 10.

b) There is not enough information to determine the number that belongs in region IV. Without knowing that number, we can't figure out region VIII either.

c) If all books were available on at least one, then the number in region VIII is 0. The number of books available on Amazon and iTunes but not Google is 100 – (8 + 7 + 5 + 10 + 4 + 18) = 48.

d) The number of books available on exactly 2 services is 48 + 7 + 4 = 59.

15. Draw a Venn diagram, where:

U = universal set
F = people who listen to FM radio
A = people who listen to AM radio
S = people who listen to satellite radio

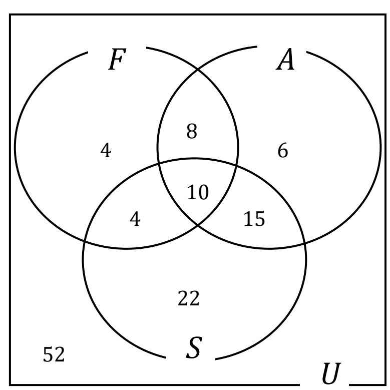
Place given data: Since 4 people listen to only FM radio, place 4 in region I. Since 8 listen to FM and AM only, place 8 in region II. Since 4 listen to FM and satellite only, place 4 in region IV. Since 6 people listen to AM only, place 6 in region III. Since 22 listen to satellite only, place 22 in region VII.

Place calculated data: By subtracting 26 – (4 + 8 + 4) = 10 we get the number of people who listen to all three. Place 10 in region V.

By subtracting 69 – (4 + 8 + 6 + 4 + 10 + 22) = 15 we get the number of people who listen to AM and satellite only. Place 15 in region VI.

By subtracting 121 – 69 = 52 we get the number of who do not listen to none of these 3 types of radio. Place 52 in region VIII.

Answer questions using Venn diagram:



a) The number of people who listen to satellite radio is 4 + 10 + 15 + 22 = 51, which is less than the 52 who listen to none of the three.

b) The total number who listen to FM is 26. The total number who listen to AM is 39. Thirteen more people listen to AM than listen to FM.

c) The number of people who listen to some form of radio is 69, while the number who listen to AM is 39. Thirty people listen to some form of radio but not AM.

16. Draw a Venn diagram, where:

U = universal set
A = cities with an art museum
S = cities with an orchestra
B = cities with a ballet

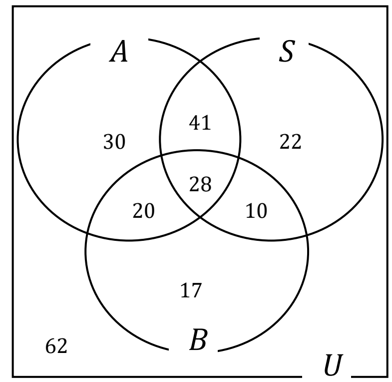
Place given data: Since 20 cities have an art museum and ballet but no orchestra, place 20 in region IV. Since 41 cities have an art museum and orchestra but no ballet, place 41 in region II. Since 30 cities have an art museum but neither an orchestra nor a ballet, place 30 in region I. Since 10 cities have a ballet and an orchestra, but no art museum, place 10 in region VI.

Place calculated data: By subtracting 119 – (30 + 41 + 20) = 28 we get the number of cities that have all three. Place 28 in region V.

By subtracting 75 – (20 + 28 + 10) = 17 we get the number of cities that have ballet only. Place 17 in region VII.

By subtracting 230 – (30 + 41 + 22 + 20 + 28 + 10 + 17) = 62 we get the number of cities that have none of these three attractions. Place 62 in region VIII.

Answer questions using Venn diagram:

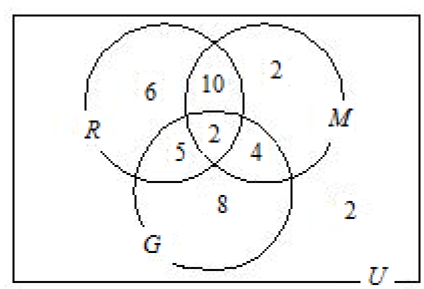


a) The number of cities that have an orchestra is (41 + 22 + 28 + 10)/230 = 101/230 = 43.9%.

b) There are 62 – 28 = 34 more cities than have none of the three than have all three.

c) The percentage chance that you will be disappointed is (17 + 62)/230 = 79/230 = 34.3%.

17. The Venn diagram that summarizes the researcher's results would be as follows.



The total of the eight regions is 39 but the researcher surveyed 40 people.

18. Answers vary; one possible answer is: of the 40 email recipients, only 3 looked at all three advertisements.

19. Consider a Venn diagram, where:

U = universal set
A = people who baseball
B = people who watch basketball
C = people who watch football

a) If all the given data were used, we would know that region I is 205, region IV and V together total 90, and that regions VII and VI together equal 145. Although we know that regions II, III and IV together would equal 35, the exact value of regions II, III and IV would remain unknown. Since the total number of people surveyed is the total of all 8 regions, we are out of luck.

We could determine the sum of regions II, III and IV as well as the total number of people surveyed if we knew the number of people who watched only baseball.

b) When we find out that every single person who watches baseball also watches basketball, and that none of the people who watch baseball also watch football, we can determine the number of people in regions II, III and VI. Regions II and III would both contain 0, and region VI would contain 35. Knowing that region VI contained 35, we would now know that the number of people who only watched basketball is 235 – (90 + 35) = 110. The total number of people surveyed is now: 205 + 0 + 0 + 90 + 35 + 110 + 560 = 1000.

c) The number of people who watch only basketball is 110. The number of people who watch only football is 295 – (0 + 90) = 205.

20. a) A 2 circle Venn diagram has 4 regions. A 3 circle Venn diagram has 8 regions. Since $2^2 = 4$, and $2^3 = 8$, my conjecture is that a 4 circle Venn diagram will have $2^4 = 16$ regions.

b) It's impossible because you can't get enough regions where only 2 of the circles intersect because 2 circles can only intersect in at most 2 points.

c) By using another shape, like ellipses, there are 16 regions, just as expected.

Exercise Set 2-5

1. An *infinite set* is one that does not have a fixed number of elements. Cantor's definition: A set is infinite if it can be put into a one-to-one correspondence with a subset of itself.

2. A general term of an infinite set is written in terms of n, such that when 1 is substituted for n, one gets the first term of the set. When 2 is substituted for n, one gets the second term of the set, etc.

3. A countable set is one that can be put into one-to-one correspondence with a subset of the natural numbers.

4. Natural numbers $N = \{1, 2, 3, \ldots\}$ can be put into one-to-one correspondence with the even numbers $\{0, 2, -2, 4, -4, 6, -6, \ldots\}$ as follows. For every odd number n in N let n correspond to $n - 1$. For every even number n in N let n correspond to $-n$. Since the two sets can be put into a one-to-one correspondence with each other they have the same cardinality.

5. Use inductive reasoning.

$7(1) = 7$
$7(2) = 14$
$7(3) = 21$
$7(4) = 28$
$7(5) = 35$
etc.

A general term is $7n$.

6. Use inductive reasoning.

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
etc.

A general term is n^3.

7. Use inductive reasoning.

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$
etc.

A general term is 4^n.

8. Use inductive reasoning.

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
etc.

A general term is n^2.

9. Use inductive reasoning.

$-3(1) = -3$
$-3(2) = -6$
$-3(3) = -9$
$-3(4) = -12$
$-3(5) = -15$
etc.

A general term is $-3n$.

10. Use inductive reasoning.

$22(1) = 22$
$22(2) = 44$
$22(3) = 66$
$22(4) = 88$
$22(5) = 110$
etc.

A general term is $22n$.

11. Use inductive reasoning.

$\frac{1}{4} = \frac{1}{4}$
$\frac{2}{4} = \frac{1}{2}$
$\frac{3}{4} = \frac{3}{4}$
$\frac{4}{4} = 1$
$\frac{5}{4} = \frac{5}{4}$
etc.

A general term is $\frac{n}{4}$.

12. Use inductive reasoning.

$\frac{1}{6} = \frac{1}{6}$
$\frac{2}{6} = \frac{1}{3}$
$\frac{3}{6} = \frac{1}{2}$
$\frac{4}{6} = \frac{2}{3}$
$\frac{5}{6} = \frac{5}{6}$ etc.

A general term is $\frac{n}{6}$.

13. Use inductive reasoning.

$4(1) - 2 = 2$
$4(2) - 2 = 6$
$4(3) - 2 = 10$
$4(4) - 2 = 14$
$4(5) - 2 = 18$
etc.

A general term is $4n - 2$.

14. Use inductive reasoning.

$3(1)-2=1$

$3(2)-2=4$

$3(3)-2=7$

$3(4)-2=10$

$3(5)-2=13$

etc.

A general term is $3n-2$.

15. Use inductive reasoning

$\frac{2}{3}=\frac{1+1}{1+2}$

$\frac{3}{4}=\frac{2+1}{2+2}$

$\frac{4}{5}=\frac{3+1}{3+2}$

$\frac{5}{6}=\frac{4+1}{4+2}$

$\frac{6}{7}=\frac{5+1}{5+2}$

etc.

A general term is $\frac{n+1}{n+2}$.

16. Use inductive reasoning

$\frac{1}{1}=\frac{1}{1^3}$

$\frac{1}{8}=\frac{1}{2^3}$

$\frac{1}{27}=\frac{1}{3^3}$

$\frac{1}{64}=\frac{1}{4^3}$

$\frac{1}{125}=\frac{1}{5^3}$

etc.

A general term is $\frac{1}{n^3}$.

17. Use inductive reasoning

$100=100(1)$

$200=100(2)$

$300=100(3)$

$400=100(4)$

$500=100(5)$

etc.

A general term is $100n$.

18. Use inductive reasoning

$50=50(1)$

$100=50(2)$

$150=50(3)$

$200=50(4)$

$250=50(5)$

etc.

A general term is $50n$.

19. Use inductive reasoning

$-4=-3(1)-1$

$-7=-3(2)-1$

$-10=-3(3)-1$

$-13=-3(4)-1$

$-16=-3(5)-1$

etc.

A general term is $-3n-1$.

20. Use inductive reasoning

$-3=-2(1)-1$

$-5=-2(2)-1$

$-7=-2(3)-1$

$-9=-2(4)-1$

$-11=-2(5)-1$

A general term is $-2n-1$.

For 21 through 30 we will show each set is infinite by putting it into a one-to-one correspondence with a proper subset of itself.

21. $\{3, 6, 9, 12, 15, \ldots, 3n, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{6, 12, 18, 24, 30, \ldots, 6n, \ldots\}$

22. $\{10, 15, 20, 25, 30, \ldots, 5n+5, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{15, 25, 35, 45, 55, \ldots, 10n+5, \ldots\}$

23. $\{9, 18, 27, 36, 45, \ldots, 9n, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{18, 36, 54, 72, 90, \ldots, 18n, \ldots\}$

24. $\{4, 10, 16, 22, 28, \ldots, 6n-2, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{10, 22, 34, 46, 58, \ldots, 12n-2, \ldots\}$

25. $\{2, 5, 8, 11, \ldots, 3n-1, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{5, 11, 17, 23, \ldots, 6n-1, \ldots\}$

26. $\{20, 24, 28, \ldots, 16+4n, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{24, 28, 32, \ldots, 20+4n, \ldots\}$

27. $\{10, 100, \ldots, 10^n, \ldots\}$
$\updownarrow \; \updownarrow \qquad \updownarrow$
$\{100, 10{,}000, \ldots, 10^{2n}, \ldots\}$

28. $\{100, 200, 300, 400, \ldots, 100n, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{200, 400, 600, 800, \ldots, 200n, \ldots\}$

29. $\left\{\frac{5}{1}, \frac{5}{2}, \frac{5}{3}, \ldots, \frac{5}{n}, \ldots\right\}$
$\updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\left\{\frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \ldots, \frac{5}{n+1}, \ldots\right\}$

30. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^n}, \ldots\right\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\left\{\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots, \frac{1}{2^{n+1}}, \ldots\right\}$

31. The set {5, 10, 15, 20, 25, ...} is countable because it can be put into one-to-one correspondence with the set of natural numbers.

$\{1, 2, 3, 4, 5\ldots, n, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{5, 10, 15, 20, 25\ldots, 5n, \ldots\}$

32. The set is countable because it can be put into one-to-one correspondence with the set of natural numbers.

$\{1, 2, 3, 4, 5, 6, \ldots, n, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{-3, -6, -9, -12, -15, -18\ldots, -3n, \ldots\}$

33. The set is countable because it can be put into one-to-one correspondence with the set of natural numbers.

$\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \; \updownarrow \qquad \updownarrow$
$\{0, 1, 4, 9, 16, \ldots, (n-1)^2, \ldots\}$

34. The set is countable because it can be put into one-to-one correspondence with the set of natural numbers.

$$n \to \begin{cases} -\dfrac{(n+1)/2}{5} = -\dfrac{n+1}{10} & \text{if } n \text{ is odd} \\ -\dfrac{n/2}{7} = -\dfrac{n}{14} & \text{if } n \text{ is even} \end{cases}$$

35. Answers may vary: Though it may seem that there are more rational numbers than natural numbers there are not. The rational numbers can be put into a one-to-one correspondence with the natural numbers.

36. Answers may vary: Although the idea of infinity extends beyond what is tangible.

37. In example 2, we showed that every term in the set {4, 7, 10, 13, 16, . . .} matched up to a natural number. 3(1) + 1 = 4, 3(2) + 1 = 7, 3(3) = 7, etc. Because each natural number is used to get a unique term in the sequence, we proved that the set {4, 7, 10, 13, 16, ...} is countable.

38. a) False. If an infinite set is countable, then a subset of it will be countable.

b) This is true. If a subset of A is infinite, it means that A is not countable and is therefore infinite.

39. (a)
$$\begin{array}{ccccccc} \{1, & 2, & 3, & 4, & 5\ldots, & n, & \ldots\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ \{0, & 1, & 2, & 3, & 4\ldots, & n-1, & \ldots\} \end{array}$$

(b) $\aleph_0 + 1 = \aleph_0$

40. (a)
$$\begin{array}{ccccccc} \{1, & 2, & 3, & 4, & 5, & 6\ldots, & \} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ \{1, & -1, & 2, & -2, & 3, & -3\ldots, & \} \end{array}$$

(b) $\aleph_0 + \aleph_0 = \aleph_0$ or $2\aleph_0 = \aleph_0$

41. The set {10, 11, 12, 13, 14, . . .} is infinite, but it can be put into one-to-one correspondence with the natural numbers with $n \rightarrow n+9$. Since the cardinal number of the natural number is $\aleph$, it is also the cardinal number for this set.

42. The set {–1, –2, –3, –4, –5, ...} is infinite, but it can be put into on- to-one correspondence with the natural numbers with $n \rightarrow -n$. Since the cardinal number of the natural number is $\aleph$, it is also the cardinal number for this set.

43. The set {1, 3, 5, 7, . . ., 29} is finite and has a cardinal number of 15.

44. The set {2, 4, 6, 8, 10, . . . , 24} is finite and has a cardinal number of 12.

45. The set of odd natural numbers is infinite, but it can be put into one to one correspondence with the natural numbers with $n \rightarrow 2n-1$. Since the cardinal number of the natural number is $\aleph$, it is also the cardinal number for this set.

46. The set of even negative integers is infinite, but it can be put into one to one correspondence with the natural numbers with $n \rightarrow -2n$. Since the cardinal number of the natural number is $\aleph$, it is also the cardinal number for this set.

Review Exercises

1. $D = \{52, 54, 56, 58\}$

2. $F = \{5, 7, 9, ..., 39\}$

3. $L = \{$l, e, t, r$\}$

4. $A = \{$a, r, k, n, s$\}$

5. $B = \{501, 502, 503, ...\}$

6. $C = \{6, 7, 8, 9, 10, 11\}$

7. {Buzz Aldrin, Neil Armstrong, Alan Bean, Gene Cernan, Pete Conrad, Charles Duke, James Irwin, Edgar Mitchell, Harrison Schmitt, David Scott, Alan B. Shepard, John Young}

8. { }

9. $\{x \mid x$ is even and $16 < x < 26\}$

10. $\{x \mid x$ is a multiple of 5 between 0 and 25$\}$

11. $\{x \mid x$ is an odd natural number greater than 100$\}$

12. $\{x \mid x$ is a positive multiple of 8 less than 73$\}$

13. Infinite

14. Infinite

15. Finite

16. Finite

17. Finite

18. Finite

19. Finite

20. The set of annoying commercials is not well defined because there is no exact definition of annoying. The set of people with red hair is not well defined because there are some "shades" that may or may not be classified as red.

21. False since 100 is in the first set but not the second.

22. True since every element of {6} is also in {6, 12, 18} and the sets are not equal.

23. False since 6 is in the first set but not the second.

24. False because proper subsets cannot be equal.

25. ∅; {r}; {s}; {t}; {r, s}; {r, t}; {s, t}; {r, s, t}

26. $2^6 = 64$ subsets; 64 – 1 = 63 proper subsets

27. $A \cap B$ = {Toyota, Honda, Lexus}

28. $B \cup C$ = {Mercedes, Toyota, Honda, Lexus, Acura, Hyundai, Tesla, Dodge}

29. $A \cap B$ = {Toyota, Honda, Lexus}
$(A \cap B) \cap C = \varnothing$

30. B' = {Chevy, Ford, BMW, Mercedes, Acura, Dodge}

31. List the elements in A = {Chevy, BMW, Toyota, Honda, Lexus}.

Cross off those elements of A that are in B: {Chevy, BMW, ~~Toyota, Honda, Lexus~~} so $A - B$ = {Chevy, BMW}.

32. List the elements in B = {Toyota, Honda, Lexus, Hyundai, Tesla}.

Cross off those in B which are also in A: {~~Toyota, Honda, Lexus~~, Hyundai, Tesla} so $B - A$ = {Hyundai, Tesla}.

33. $A \cup B$ = {Chevy, BMW, Toyota, Honda, Lexus, Hyundai, Tesla}
$(A \cup B)'$ = {Ford, Mercedes, Acura, Dodge}
$(A \cup B)' \cap C$ = {Mercedes, Acura, Dodge}

34. B' = {Chevy, Ford, BMW, Mercedes, Acura, Dodge}
C' = {Chevy, Ford, BMW, Toyota, Honda, Lexus, Hyundai, Tesla}
$B' \cap C'$ = {Chevy, Ford, BMW}

35. $B \cup C$ = {Mercedes, Toyota, Honda, Lexus, Acura, Hyundai, Tesla, Dodge}
A' = {Ford, Mercedes, Acura, Hyundai, Tesla, Dodge}
$(B \cup C) \cap A'$ = {Mercedes, Acura, Hyundai, Tesla, Dodge}

36. $A \cup B$ = {Chevy, BMW, Toyota, Honda, Lexus, Hyundai, Tesla}
C' = {Chevy, Ford, BMW, Toyota, Honda, Lexus, Hyundai, Tesla}
$(A \cup B) \cap C'$ = {Chevy, BMW, Toyota, Honda, Lexus, Hyundai, Tesla}

37. B' = {Chevy, Ford, BMW, Mercedes, Acura, Dodge}

C' = {Chevy, Ford, BMW, Toyota, Honda, Lexus, Hyundai, Tesla}

$B' \cap C'$ = {Chevy, Ford, BMW}

A' = {Ford, Mercedes, Acura, Hyundai, Tesla, Dodge}
$(B' \cap C') \cup A'$ = {Chevy, Ford, BMW, Mercedes, Acura, Hyundai, Tesla, Dodge}

38. A' = {Ford, Mercedes, Acura, Hyundai, Tesla, Dodge}
$A' \cap B$ = {Hyundai, Tesla}
$(A' \cap B) \cup C$ = {Mercedes, Acura, Hyundai, Tesla, Dodge}

39. Set K is {26, 27, 28, 29, . . .} and set L is {12, 14, 16, 18, . . .}.

$K \cap L = \{x \mid x \in E, x > 25\}$.
$K \cup L = \{12, 14, 16, 18, 20, 22, 24, \ldots\}$.
$L - K = \{12, 14, 16, 18, 20, 22, 24\}$

40. $A \times B$ = {(play video games, go to class), (watch TV, go to class), (go for a run, go to class), (play video games, go to work), (watch TV, go to work), (go for a run, go to work)}; $A \times B$ is all the possible pairings of a leisure activity followed by a responsibility.

$B \times A$ = {(go to class, play video games), (go to class, watch TV), (go to class, go for a run), (go to work, play video games,), (go to work, watch TV), (go to work, go for a run)}; $B \times A$ is all the possible pairings of a responsibility followed by a leisure activity.

41. Region I contains the elements in A which are not also in B: $A - B$

42. Region II contains the elements that are in both A and B: $A \cap B$

43. Region III contains the elements in B which are not also in A: $B - A$

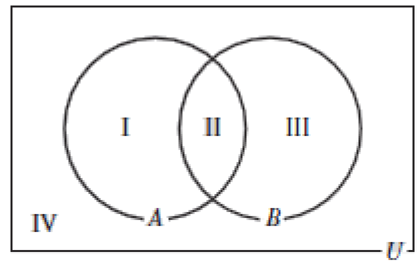
44. Region IV contains the elements which are neither in A nor B: $(A \cup B)'$

45. Regions I and III contain the elements which are in A or B but not both A and B:

$(A \cup B) - (A \cap B)$

46. Regions I and IV contain the elements which are not in B: B'

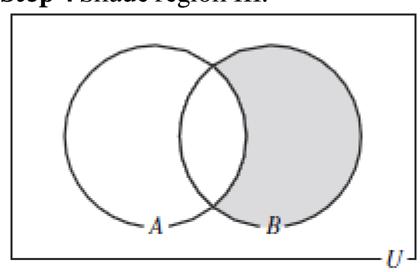
47. **Step 1** Draw the Venn diagram and label each region



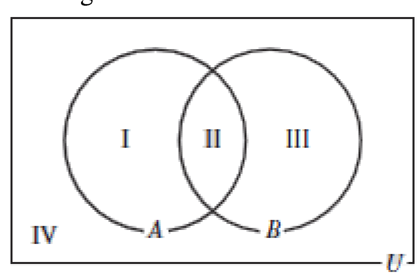
Step 2 From the diagram, list the regions in each set.
U = {I, II, III, IV}
A = {I, II}
B = {II, III}

Step 3 A' = {III, IV}; $A' \cap B$ = {III}

Step 4 Shade region III.



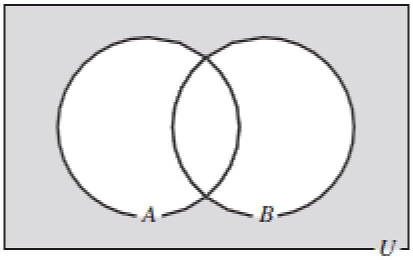
48. **Step 1** Draw the Venn diagram and label each region.



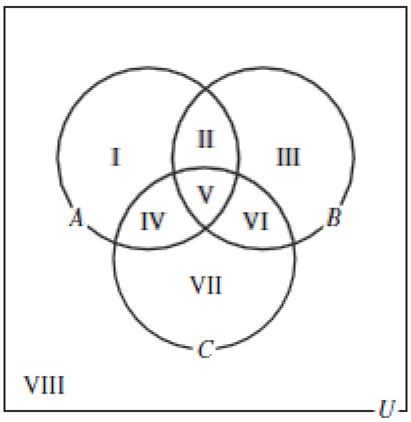
Step 2 From the Venn diagram, list the regions in each set.
U = {I, II, III, IV}
A = {I, II}
B = {II, III}

Step 3 $A \cup B$ = {I, II, III}; $(A \cup B)'$ = {IV}

Step 4 Shade region IV.



49. **Step 1**Draw and label the diagram as shown.



Step 2 From the diagram, list the regions in each set.

$U = \{I, II, III, IV, V, VI, VII, VIII\}$

$A = \{I, II, IV, V\}$

$B = \{II, III, V, VI\}$

$C = \{IV, V, VI, VII\}$

Step 3 Find the solution to

$(A' \cap B') \cup C$

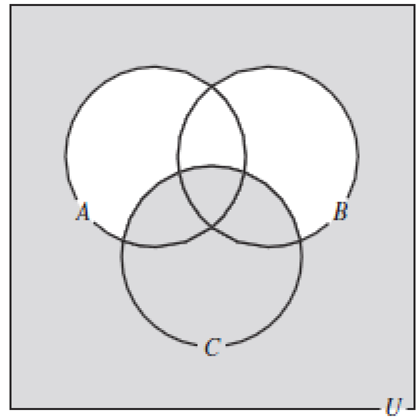
$A' = \{III, VI, VII, VIII\}$

$B' = \{I, IV, VII, VIII$

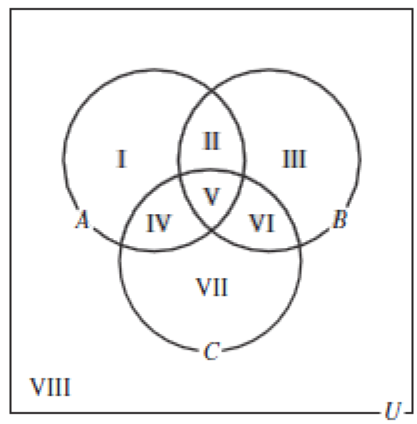
$A' \cap B' = \{VII, VIII\}$

$(A' \cap B') \cup C = \{IV, V, VI, VII, VIII\}$

Step 4 Shade regions IV, V, VI, VII, and VIII.



50. **Step 1** Draw and label the diagram as shown.



Step 2 From the diagram, list the regions in each set.

$U = \{I, II, III, IV, V, VI, VII, VIII\}$

$A = \{I, II, IV, V\}$

$B = \{II, III, V, VI\}$

$C = \{IV, V, VI, VII\}$

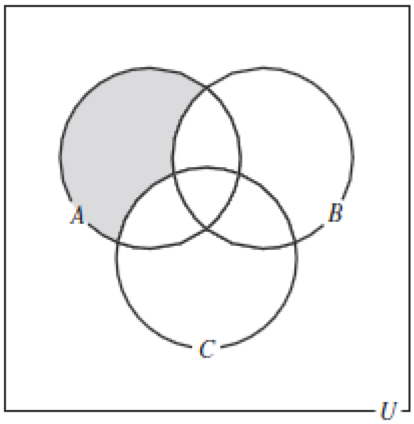
Step 3

$B \cup C = \{II, III, IV, V, VI, VII\}$

$(B \cup C)' = \{I, VIII\}$

$A \cap (B \cup C)' = \{I\}$

Step 4 Shade region I.



51. The cardinal number formula says $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, so $n(A \cup B) = 15 + 9 - 4 = 20$.

52. $n(A \cup B) = 24 + 20 - 14 = 30$.

53. Since Kentucky appears in set *A* and set *B*, but not in set *C*, it would be in region II.

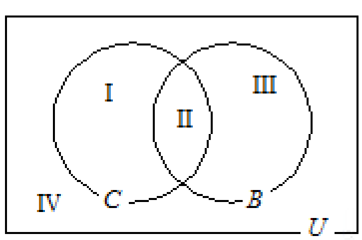
54. Since Mississippi appears in set *A*, set *B*, and set *C*, it would be in region V.

55. Since Louisiana appears in set *B* and set *C*, but not in set *A*, it would be in region VI..

56. Since Ohio does not appear in any of the three sets, it would be in region VIII.

57. **Draw a Venn diagram** with regions I, II, III, and IV as follows, where

U = Universal set
C = Clinton supporters
B = Sanders supporters

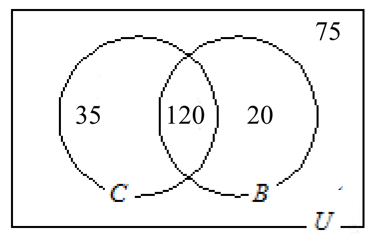


Place given data: Since 120 voters supported both candidates, place 120 in region II.

Place calculated data: By subtracting 155 – 120 = 35 we get the number of voters who supported Clinton only. Place 35 in region I.

By subtracting 140 – 120 = 20 we get the number of voters who support Sanders only. Place 20 in region III. By subtraction 250 – (120 + 20 + 35) = 75 we get the number of voters who supported neither of these candidates. Place 75 in region IV.

Answer questions using Venn diagram:

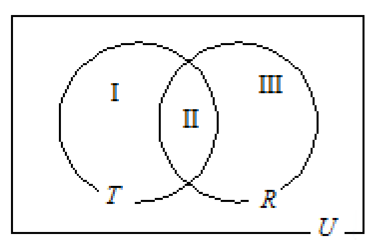


a) The number of voters who supported neither candidate is 75.

b) The number of voters who supported only Bernie Sanders is 20.

58. **Draw a Venn diagram** with regions I, II, III, and IV indicated as follows, where

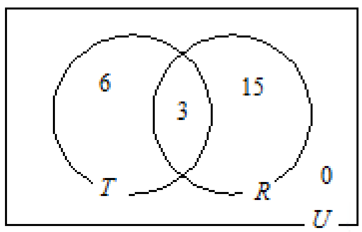
U = Universal set
T = low frequency hearing loss
R = high frequency hearing los



Place given data: Since 10 residents had significant hearing loss at low frequencies but not high, place 10 in region I. Since 40 residents had significant hearing loss at high frequencies but not low, place 40 in region III. Since 26 show no significant hearing loss at all, place 26 in region IV.

Place calculated data: By subtracting 94 – (10 + 40 + 26) = 18 we get the number of residents who had significant hearing loss at both low and high frequencies. Place 18 in region II.

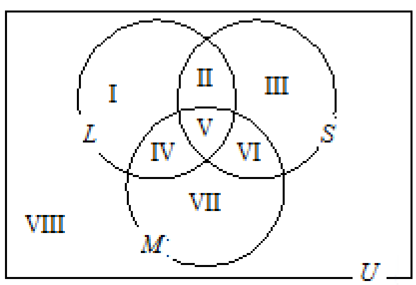
Answer questions using Venn diagram:



a) The number of residents that had hearing loss at both low and high frequencies is 18.

b) The percentage who suffer from hearing loss at high frequencies is (18 + 40)/94 = 58/94 = 61.7%.

59. Draw and label the diagram as shown



Place given data: Since there are 6 callers who listen to all three, place 6 in region V.

Place calculated data: By subtracting 8 – 6 = 2 we get the number of callers that listen to local radio and satellite radio but not MP3 players. Place 2 in region II.

By subtracting 13 – 6 = 7 we get the number of callers who listen to satellite and MP3 but not local radio. Place 7 in region VI.

By subtracting 11 – 6 = 5 we get the number of callers that listen to local radio and MP3 but not satellite. Place 5 in region IV.

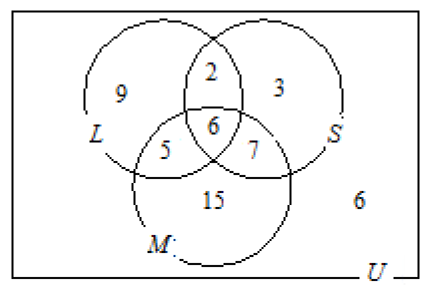
By subtracting 22 – (5 + 6 + 2) = 9 we get the number of callers who listened to local radio only. Place 9 in region I.

By subtracting 18 – (6 + 2 + 7) = 3 we get the number of callers who listened to satellite radio only. Place 3 in region III.

By subtracting 33 – (5 + 6 + 7) = 15 we get the number of callers who listened to MP3 players only. Place 15 in region VII.

By subtracting 53 – (9 + 2 + 3 + 5 + 6 + 7 + 15) = 6 we get the number of callers who listened to none of the three. Place 6 in region VIII.

Answer questions using Venn diagram:



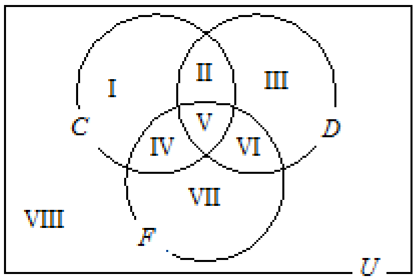
a) There were 3 callers that listened to satellite radio only.

b) There were 5 callers that listened to local radio and MP3 but not satellite.

c) There were six callers that listened to none of the three.

60. **Draw and label the Venn diagram** as shown where,

C = cash
D = debit card
F = financial aid

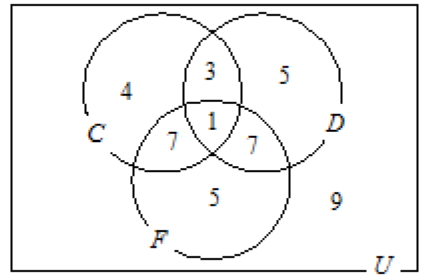


Place given data: Since 4 used cash only, place 4 in region I. Since 5 used financial aid voucher only, place 5 in region VII. Since 5 used a debit card only, place 5 in region III. Since 7 used debit and financial aid voucher and no cash, place 7 in region VI. Since 3 used cash and debit but no financial aid assistance, place 3 in region II. Since 9 used none of these payment forms, place 9 in region VIII.

Place calculated data: By subtracting 16 – (3 + 5 + 7) = 1 we get the number of students who used all three forms of payment. Place 1 in region V.

By subtracting 41 – (4 + 3 + 5 + 1 + 7 + 5 + 9) = 7 we get the number of students that used case and financial aid but not debit. Place 7 in region IV.

Answer questions using Venn diagram:



a) The number of students who used all three forms of payment is 1.

b) The number of students who used cash is 4 + 3 + 7 + 1 = 15. The number of students who used voucher is 7 + 1 + 7 + 5 = 20, so more students used voucher than used debit.

c) The percentage of students who did not use a financial aid voucher is (41 – 20)/41 = 51.2%.

61. Use inductive reasoning.
$-3 - 2(1) = -5$
$-3 - 2(2) = -7$
$-3 - 2(3) = -9$
$-3 - 2(4) = -11$
$-3 - 2(5) = -13$
etc.
A general term is $-3 - 2n$.

62. We will show the set is infinite by putting it into a one-to-one correspondence with a subset of itself:

$$\begin{array}{ccccc} \{12, & 24, & 36, & \ldots, & 12n, \ldots\} \\ \updownarrow & \updownarrow & \updownarrow & & \updownarrow \\ \{24, & 48, & 72, & \ldots, & 24n, \ldots\} \end{array}$$

63. Since the set in Exercise 62 can be put into one-to-one correspondence with the natural numbers, it is countable.

$$\begin{array}{ccccc} \{1, & 2, & 3, & \ldots, & n, \ldots\} \\ \updownarrow & \updownarrow & \updownarrow & & \updownarrow \\ \{12, & 24, & 36, & \ldots, & 12n, \ldots\} \end{array}$$

Chapter Test

1. P = {92, 94, 96, 98}

2. K = {e, n, v, l, o, p}

3. X = {1, 2, 3, 4, ... , 79}

4. J = {January, June, July}

5. $\{x \mid x \in E \text{ and } 10 < x < 20\}$

6. $\{x \mid x = 2^{n+1}$ when n is a natural number less than 7$\}$

7. Infinite

8. Finite

9. Finite

10. The set of people with awesome hair is not well defined, because "awesome hair" is subjective (except for in the case of the author, who clearly has awesome hair), so it is not cut and dry (get it?) who belongs in the set and who does not.

11. There are 3 states that border California, so there are $2^3 = 8$ subsets. The subsets are: $\varnothing$, {Arizona}, {Nevada}, {Oregon}, {Arizona, Nevada}, {Arizona, Oregon}, {Nevada, Oregon}, {Arizona, Nevada, Oregon}; The proper subsets are all but the last one since a proper subset cannot be equal to the set.

12. $A \cap B = \{a\}$; $(A \cap B) \cup C = \{a, e, h, j\}$

13. $A \cup B = \{a, b, d, e, f, g, i, j, k\}$
$(A \cup B)' = \{c, h\}$

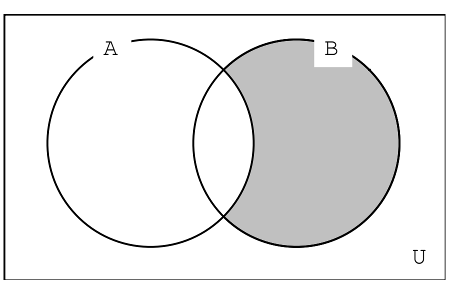
14. List elements in A and cross off those which are also in B: {~~a~~, b, d, e, f}, so,

$A - B = \{b, d, e, f\}$

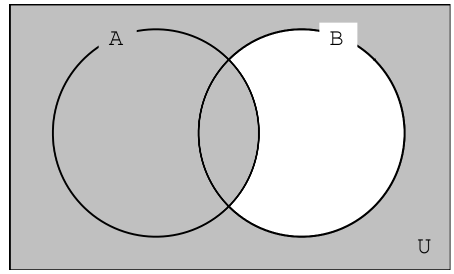
15. From Exercise 14, $A - B = \{b, d, e, f\}$, cross off those also in C: {b, d, ~~e~~, f}, so

$(A - B) - C = \{b, d, f\}$

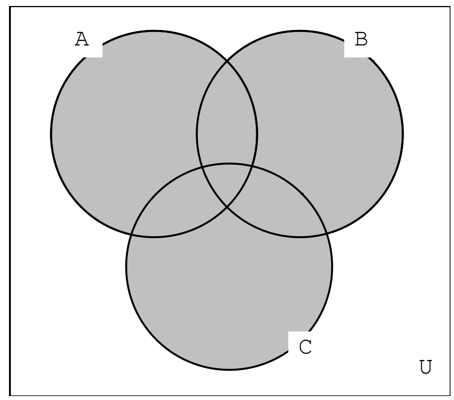
16. $B - A$



$B' \cup A$

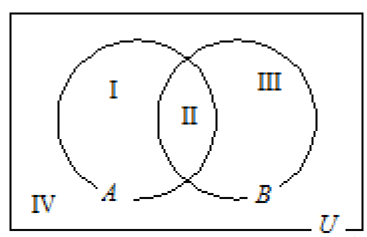


$A \cup B \cup C$



17. {(Arizona, e), (Nevada, e), (Oregon, e), (Arizona, h), (Nevada, h), (Oregon, h), (Arizona, j), (Nevada, j), (Oregon, j)} and {(e, Arizona), (h, Arizona), (j, Arizona), (e, Nevada), (h, Nevada), (j, Nevada), (e, Oregon), (h, Oregon), (j, Oregon)}

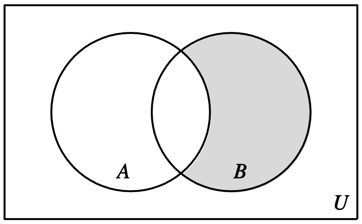
18. **Step 1** Draw the Venn diagram and label each region.



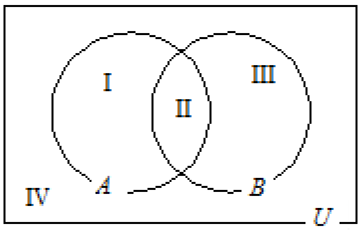
Step 2 From the diagram, list the regions in each set.
$U = \{I, II, III, IV\}$
$A = \{I, II\}$
$B = \{II, III\}$

Step 3 $A' = \{III, IV\}$
$A' \cap B = \{III\}$

Step 4 Shade region III.



19. Draw the Venn diagram and label each region. **Step 1**

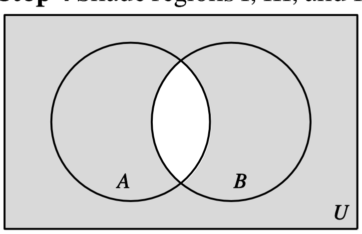


Step 2 From the diagram, list the regions in each set.

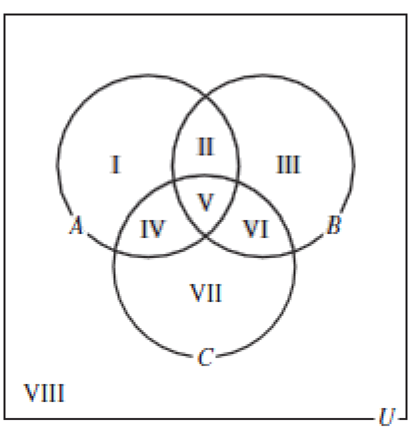
$U = \{I, II, III, IV\}$
$A = \{I, II\}$
$B = \{II, III\}$

Step 3 $A \cap B = \{II\}$
$(A \cap B)' = \{I, III, IV\}$

Step 4 Shade regions I, III, and IV.



20. **Step 1** Draw and label the Venn diagram as shown.

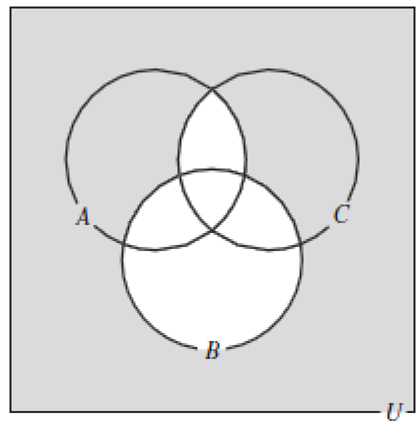


Step 2 From the diagram, list the regions in each set.

$U = \{I, II, III, IV, V, VI, VII, VIII\}$
$A = \{I, II, IV, V\}$
$B = \{II, III, V, VI\}$
$C = \{IV, V, VI, VII\}$

Step 3 $A' = \{III, VI, VII, VIII\}$
$B' = \{I, IV, VII, VIII\}$
$A' \cup B' = \{I, III, IV, VI, VII, VIII\}$
$C' = \{I, II, III, VIII\}$
$(A' \cup B') \cap C' = \{I, III, VIII\}$

Step 4 Shade regions I, III, and VIII.

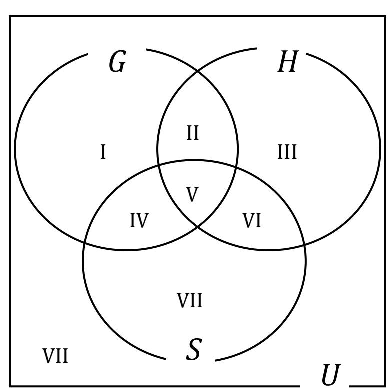


21. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 1{,}500 + 1{,}150 - 350 = 2{,}300$

22. **Step 1** Draw a Venn diagram as follows where,

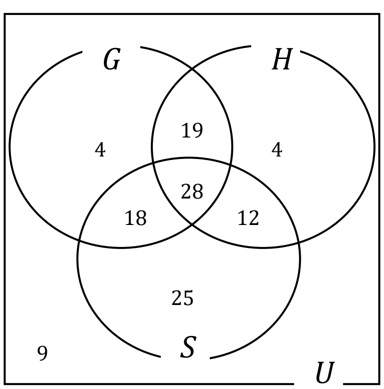
U = Universal Set
G = schools with women's golf
H = schools with field hockey
S = schools with women's swimming



Place Given Data: Since there are 28 schools that field teams in all three sports, place 28 in region V.

Place Calculate Data: By subtracting 46 – 28 = 18, we get the number of schools that have golf and women's swimming, but no field hockey. Place 18 in region IV. By subtraction 40 – 28 = 12, we get the number of schools that have women's swimming and field hockey, but not golf. Place 12 in region VI. By subtraction 47 – 28 = 19, we get the number of schools that have golf and field hockey, but swimming. Place 19 in region II. By subtracting 69 – (19 + 18 + 28) = 4, we get the number of schools that have only a women's golf team. Place 4 in region I. By subtracting 63 – (19 + 28 + 12) = 4, we get the number of schools that have only a field hockey team. Place 4 in region III. By subtraction 83 – (18 + 28 + 12) = 25, we get the number of schools that only have women's swimming. Place 25 in region VII. By subtraction 119 – (4 + 19 + 4 + 18 + 28 + 12 + 25) = 9, we get the number of schools that have none of the three sports. Place 9 in region VIII.

Step 4



(a) The number of schools that have women's golf but no women's swimming or field hockey is 4.

(b) The percentage of teams that have at least two of the three sports is (19 + 18+ 28 + 12)/119 = 77/119 = 64.7%.

(c) The percent chance that a team chosen at random from the student has none of the three sports is 9/119 = 7.6%.

23. Use inductive reasoning.
15(1) = 15
15(2) = 30
15(3) = 45
15(4) = 60
15(5) = 75
A general term is $15n$.

24. Place the set into a one-to-one correspondence with a subset of itself:

$$\begin{array}{cccccccccc} \{1, & -1, & 2, & -2, & 3, & -3, & \ldots, & n, & -n\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \updownarrow \\ \{1, & 2, & 3, & 4, & 5, & 6, & \ldots, & 2n-1, & 2n\} \end{array}$$

This one-to-one correspondence between the set and a proper subset shows that it is an infinite set. It is also a correspondence between the natural numbers and the set which shows that it is a countable infinite set.

25. True

26. True

27. True

28. True

29. False; y is an element of the first set but not the second set.

30. False; $12 \in \{12, 24, 36, \ldots\}$ but $\{12\} \notin \{12, 24, 36, \ldots\}$.