1 Problem Solving

Exercise Set 1-1

1. Deductive reasoning makes conclusions from general rules; inductive reasoning arrives at general conclusions from specific examples.

2. A conjecture is a guess based on previous outcomes.

3. Answers may vary.

4. A counterexample is an example that contradicts a conjecture. It is used to disprove a conjecture.

5. In order for a conclusion to be "proven true," it needs to be shown to be true in every possible case. Inductive reasoning does not look at every possible case, but only at a few specific cases.

6. An arbitrary number is variable; a number selected at random is a specific value obtained by chance.

7. 1 2 4 7 11 16 22 29

+1 +2 +3 +4 +5 +6 +7 +8

The next number is 37.

8. 6 10 22 58 166 490

+4 +12 +36 +108 +324 +972

×3 ×3 ×3 ×3 ×3

The next number is 1,462.

9. 10 20 11 18 12 16 13 14 14 12 15

+10 +(–9) +7 +(–6) +4 +(–3) +1 +0 +(–2) +3 +(–5)

The next number is 10.

10. 2 3 8 63 3,968

×2 – 1 ×3 – 1 ×8 – 1 ×63 – 1 ×3968 – 1

The next number is 15,745,023.

11. 100 99 97 94 90 85 79

–1 –2 –3 –4 –5 –6 –7

The next number is 72.

12. 9 12 11 14 13 16 15 18

+3 –1 +3 –1 +3 –1 +3 –1

The next number is 17.

13. The line through the circle is horizontal, vertical, horizontal, vertical, and then horizontal. The first two circles have no shading, the third and fourth circles have dark shading on top and then on the left, and the fifth circle has light shading on top. We could reasonably expect the next figure to be  .

14. The outer shapes are square, triangle, square, triangle, square, triangle, and then square. The inner shapes are solid circle, solid circle, open square, open square, open circle, open circle, and then solid square. The triangles point up, then right, then down. We could reasonably expect the next figure to be  .

15. The eye shading and mouth shape is right-shaded smile, left-shaded frown, none shaded smile, both shaded smile, right-shaded frown. The eye pattern appears to be right-shaded, left-shaded, none shaded, both shaded. So, we can reasonably expect the next figure to have the left-shaded. The mouth pattern appears to be that the number of smiles between frowns increases by one each time. So, we could expect the next figure to be


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16. The shading is middle, upper left, lower left, middle, upper right. We could reasonable expect the next figure to be  .

17. 5 + 13 + 17 = 35, which is odd.

18. 7(15) + 4 = 109, which is odd.

19. $5^2 \div 2 = 12.5$

20. 58(6) = 348; 3 + 4 + 8 = 15, which is not divisible by 6.

21. Approach: Induction

Original number:	10	50
Double the number:	$10(2) = 20$	$50(2) = 100$
Subtract 20:	$20 - 20 = 0$	$100 - 20 = 80$
Divide by 2:	$0 \div 2 = 0$	$80 \div 2 = 40$
Subtract the original number:	$0 - 10 = -10$	$40 - 50 = -10$
Result:	-10	-10

Conjecture: The final answer is –10.

Approach: Deduction

Pick a number:	x
Double the number:	$2x$
Subtract 20:	$2x - 20$
Divide by 2:	$(2x - 20) \div 2 = x - 10$
Subtract the original number:	$x - 10 - x = -10$
Result:	-10

The final answer is always –10.

22. Approach: Induction

Original number:	1	11
Multiply by 9:	$1(9)=9$	$11(9)=99$
Add 21:	$9+21=30$	$99+21=120$
Divide by 3:	$30\div 3=10$	$120\div 3=40$
Subtract three times the original number:	$10-3(1)=7$	$40-3(11)=7$
Result:	7	7

Conjecture: The final answer is 7.

Approach: Deduction

Pick a number:	x
Multiply by 9:	$9x$
Add 21:	$9x+21$
Divide by 3:	$(9x+21)\div 3=3x+7$
Subtract three times the original number:	$3x+7-3x=7$
Result:	7

The final answer is always 7.

23. Approach: Induction

Original number:	4	13
Add 6:	$4+6=10$	$13+6=19$
Multiply by 9:	$10(9)=90$	$19(9)=171$
Divide by 3:	$90\div 3=30$	$171\div 3=57$
Subtract 3 times the original number:	$30-3(4)=18$	$57-3(13)=18$
Result:	18	18

Conjecture: The final answer is 18.

Approach: Deduction

Pick a number:	x
Add 6:	$x+6$
Multiply by 9:	$9(x+6)=9x+54$
Divide by 3:	$(9x+54)\div 3=3x+18$
Subtract three times the original number:	$3x+18-3x=18$
Result:	18

The final answer is always 18.

24. Approach: Induction

Original number:	4	10
Multiply by 4:	$4(4)=16$	$4(10)=40$
Add 8:	$16+8=24$	$40+8=48$
Divide by 2:	$24\div 2=12$	$48\div 2=24$
Subtract two times the original number:	$12-2(4)=4$	$24-2(10)=4$
Result:	4	4

Conjecture: The final answer is 4.

Approach: Deduction

Pick an even number:	x
Multiply by 4:	$4(x)=4x$
Add 8:	$4x+8$
Divide by 2:	$(4x+8)\div 2=2x+4$
Subtract 2 times the original number:	$2x+4-2(x)=4$
Result:	4

The final answer is always 4.

25. $12{,}345{,}679 \times 9 = 12{,}345{,}679 \times 9(1) = 111{,}111{,}111$

$12{,}345{,}679 \times 18 = 12{,}345{,}679 \times 9(2) = 222{,}222{,}222$

$12{,}345{,}679 \times 27 = 12{,}345{,}679 \times 9(3) = 333{,}333{,}333$

Therefore the last multiplication should be:

$12{,}345{,}679 \times 72 = 12{,}345{,}679 \times 9(8) = 888{,}888{,}888$

26. By inductive reasoning $5^2 + 11 = 6^2$.

27. By inductive reasoning

$999{,}999 \times 9 = 8{,}999{,}991$.

28. By inductive reasoning

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 7^2$.

29. By inductive reasoning

$99{,}999 \times 99{,}999 = 9{,}999{,}800{,}001$.

30. By inductive reasoning

$12{,}345 \times 8 + 5 = 98{,}765$.

31. By inductive reasoning

$11{,}111 \cdot 11{,}111 = 123{,}454{,}321$

32. By inductive reasoning $5 \cdot 91 = 455$.

33. $142{,}857 \times 2 = 285{,}714$

$142{,}857 \times 3 = 428{,}571$

$142{,}857 \times 4 = 571{,}428$

$142{,}857 \times 5 = 714{,}285$

$142{,}857 \times 6 = 857{,}142$

$142{,}857 \times 7 = 999{,}999$

$142{,}857 \times 8 = 1{,}142{,}856$

When multiplied by the numbers 1-6 the digits in the answer are a permutation (that's a fancy word for a scrambled version) of the original number. It might seem like this will always be true, but 7 and 8 are counterexamples of that conjecture.

34. $1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

So, using inductive reasoning,

$1 + 3 + 5 + 7 + 9 = 25$,

$1 + 3 + 5 + 7 + 9 + 11 = 36$, and

$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$.

35. By inductive reasoning you can conjecture that the next three sums are $\frac{9}{5}, \frac{11}{6}$, and $\frac{13}{7}$.

36. By inductive reasoning the sum should be

$7(8) = 56$. Add: $2 + 4 + 6 + 8 + 10 + 12 + 14 = 56$, so the result is verified.

37. By inductive reasoning you can conjecture that the next three letters are: g e h .

38. By inductive reasoning you can conjecture that the next letters are: j k u .

39. By inductive reasoning you can conjecture that the next letters are: M J J .

40. By inductive reasoning you can conjecture that the next letters are: J J M .

41. Inductive

42. Inductive

43. Deductive

44. Deductive

45. Inductive

46. Inductive

47. Deductive

48. Inductive

49. Deductive

50. Inductive

51. Deductive

52. Inductive

53. Deductive

54. Deductive

55. Deductive

56. Inductive

57. Deductive

58. Inductive

59. (a) If you use inductive reasoning, you'd be more likely to text while driving. (b) If you use deductive reasoning, you'd be less likely to text while driving.

60. You are using inductive reasoning and your parents are using deductive reasoning. Your parents win on this one, even if you would never actually admit it!

61. (a) The simplest answers are 16 and 32, since each new term is double the previous term. (b) The formula 2^n will give you those numbers if you plug the term number in for *n*. (c) You can get the answers 14 and 22 by checking the differences between the terms. You add 2 from the first term to the second term and 4 from the second term to the third term. If you add 6, you get a fourth term of 14 and then by adding 8 you get a fifth term of 22. (d) Since finding the formula is difficult, it helps to fill out a table. The formula $n^2 - n + 2$ works for each entry in the table.

n	1	2	3	4	5
$n^2 - n + 2$	2	4	8	14	22

If you only have the first few terms of a string of numbers, there may be more than one pattern and formula.

62. (a) The answers could be 81 and 243, since each new term is three times the previous term. (b) The formula 3^n will give you those numbers if you plug the term number in for *n*. (c) You can get the answers 57 and 99 by checking the differences between the terms. You add 6 to get from the first term to the second term and you add 18 to get from the second term to the third term. The difference of the differences is 12, so the next difference should be 18 + 12 = 30. The fourth term is 27 + 30 = 57. The difference from the fourth term to the fifth term is 30 + 12 = 42. The fifth term is 57 + 42 = 99. (d) Since finding the formula is difficult, it helps to fill out a table. The formula $6n^2 - 12n + 9$ works for each entry in the table.

n	1	2	3	4	5
$6n^2 - 12n + 9$	3	9	27	57	99

If you only have the first few terms of a string of numbers, there may be more than one pattern and formula.

63. Answers vary for parts (a) – (c).

64. Answers vary for parts (a) – (c).

65. (a) Answers will vary. One example is a distance of 120 miles. It will take 120 ÷ 20 = 6 hours to travel the distance at 20mph and it will take 120 ÷ 60 = 2 hours to travel back. In total, that is 8 hours of driving for 240 miles, so the average speed is 240 ÷ 8 = 30 mph. After doing a few more numeric examples, you can use inductive reasoning and conjecture that the average speed is always 30 mph. (b) If the distance is *x* miles, then the hours to travel at 20 mph is $\frac{x}{20}$. On return, the hours to travel will be $\frac{x}{60}$. The total hours driven is $\frac{x}{20} + \frac{x}{60}$ $\frac{x}{20} + \frac{x}{60} = \frac{3 \bullet x}{3 \bullet 20} + \frac{x}{60}$

$$\frac{3x}{60} + \frac{x}{60} = \frac{4x}{60}$$

$$\frac{4x}{60} = \frac{4 \bullet x}{15 \bullet 4}$$

$$\frac{\cancel{4} \bullet x}{15 \bullet \cancel{4}} = \frac{x}{15}$$

The average speed is the total distance driven divided by the total time.

$$2x \div \frac{x}{15} = \frac{2x}{1} \bullet \frac{15}{x}.$$

$\frac{2\cancel{x}}{1} \bullet \frac{15}{\cancel{x}} = 30$, so the average speed is always 30 mph.

66. (a) Answers will vary. One example is a distance of 120 miles. It will take 120 ÷ 40 = 3 hours to travel the distance at 40mph and it will take 240 ÷ 20 =12 hours to travel twice the distance after turning around. In total, that is 15 hours of driving for 360 miles, so the average speed is 360 ÷ 15 = 24 mph. After doing a few more examples, you can use inductive reasoning and conjecture that the average speed seems to always be 24 mph. (b) If the distance is x miles, then the hours to travel at 40 mph is $\frac{x}{40}$. On return, the hours to travel will be $\frac{2x}{20}$ or $\frac{x}{10}$. The total hours driven is $\frac{x}{40}+\frac{x}{10}$.

$$\frac{x}{40}+\frac{x}{10}=\frac{x}{40}+\frac{4\bullet x}{4\bullet 10}.$$

$$\frac{4x}{40}+\frac{x}{40}=\frac{5x}{40}$$

$$\frac{5x}{40}=\frac{5\bullet x}{5\bullet 8}$$

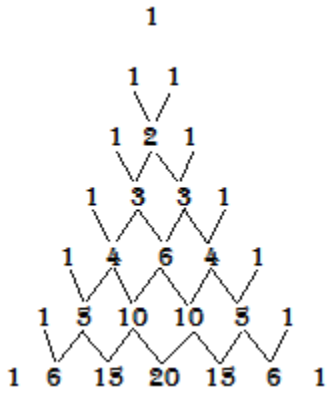
$$\frac{\cancel{5}\bullet x}{\cancel{5}\bullet 8}=\frac{x}{8}.$$

The average speed is the total distance driven divided by the total time.

$$3x\div\frac{x}{8}=\frac{3x}{1}\bullet\frac{8}{x}.$$

$\frac{3\cancel{x}}{1}\bullet\frac{8}{\cancel{x}}=24$, so the average speed is always 24 mph.

67. Each number in a row is the sum of the two numbers above it as shown below. The next line in the triangle is given.



68. Each number in a row is the product of the two numbers above it. The last line would be 2, 32, 1024, 1024, 32, 2.

69. This induction is weak because six games is really only a handful of observations.

70. This induction is weak because compared to the total number of air travelers, observing 52 travelers is not going to tell you too much.

71. This induction is strong both because the poll was likely taken scientifically by CNN and because the conclusion they make uses the phrase “majority of Americans” rather than specifically saying “70% of Americans.”

72. This induction is weak because the number of observations is relatively small.

73. This induction is strong, because we all know that there is no such thing as a person who lives in Michigan and cheers for OSU.

74. This induction is weak since it is based on only three observations. Besides, math teachers are not nerdy; we are merely “cool challenged.”

75. a) The triangular numbers are 1, 3 = 1+2, 6 = 3 + 3, 10 = 6 + 4, 15 = 10+5, so the next triangular number should be 15 + 6 = 21, then 28 and 36.

b) The square numbers are 1, 4 = 1+ 3, 9 = 4 + 5, 16 = 9 + 7, 25 = 16 + 9, so the next square number should be 25 + 11 = 36, then 49 and 64.

c) The pentagonal numbers are 1, 5 = 1 + 4, 12 = 5 + 7, 22 = 12 + 10, so the next pentagonal number should be 22 + 13 = 35, then 51 and 70.

d) All the types of numbers started at 1. The triangular numbers were formed by adding 2, 3, 4, 5, etc. to the previous number; the square numbers were formed by adding 3, 5, 7, 9, etc to the previous number; the pentagonal numbers were formed by adding 4, 7, 10, 13, etc to the previous number. So by inductive reasoning, the hexagonal numbers should be formed by adding 5, 9, 13, 17, etc. to the previous number. Therefore, the first four hexagonal numbers should be 1, 6, 15, and 28.

76. By inductive reasoning the formula for finding hexagonal numbers should be

$$\frac{n(4n-2)}{2} = n(2n-1).$$

Exercise Set 1-2

1. Answers may vary.

2. Answers may vary.

3. You can use estimation to see if the answer "makes sense."

4. If the next digit is 4 or less then leave the number as it is. If the next digit is 5 or more, then round the digit up.

5. There is no single correct answer because the answer depends on how the numbers were rounded.

6. Trace the top of the bar on a graph to either the horizontal or vertical axis to determine the quantity. The length of a bar on a bar graph may not fall even with an exact tick mark on the axis, so you will need to eyeball the length using the nearest tick mark on the axis and the scale used on the axis.

7. A pie chart shows all of the different categories that an entire quantity can be divided into. The size of a slice in the pie shows how much a category is worth compared to the entire quantity. Pie charts work well when a quantity can be broken into distinct categories, and comparing parts to a whole is helpful.

8. You can tell that a quantity is getting larger over time in a time series graph if the points and lines that connect the points are rising as you read the graph from left to right.

9. In the number 2,861, the 8 is the digit being rounded. Since the digit to the right is 6, 1 is added to the 8 and the digits 6 and 1 are replaced by zeros. The rounded number is 2,900.

10. In the number 732.6498, the 9 is the digit being rounded. Since the digit to the right is 8, 1 is added to the 9, which means the digit to the left is increased by 1 and the digit 9 is replaced by a zero and the digit 8 is dropped. The rounded number is 732.650.

11. In the number 3,261,437, the 6 is the digit being rounded. Since the digit to the right is 1, the digit 6 remains the same and the digits to the right are replaced by zeros. The rounded number is 3,260,000.

12. In the number 9,347, the 4 is the digit being rounded. Since the digit to the right is 7, 1 is added to the 4 and the 7 is replaced by zero. The rounded number is 9,350.

13. In the number 62.67, the 2 is the digit being rounded. Since the digit to the right is 6, 1 is added to the 2 and the digits to the right are dropped. The rounded number is 63.

14. In the number 45,371,999, the 5 is the digit being rounded. Since the digit to the right is 3, the digit 5 remains the same and the digits to the right are replaced by zeros. The rounded number is 45,000,000.

15. In the number 218,763, the 2 is the digit being rounded. Since the digit to the right is 1, the digit 2 remains the same and the digits to the right are replaced by zeros. The rounded number is 200,000.

16. In the number 923, the 9 is the digit being rounded. Since the digit to the right is 2, the digit 9 remains the same and the digits to the right are replaced by zeros. The rounded number is 900.

17. In the number 3.671, the 7 is the digit being rounded. Since the digit to the right is 1, the digit 7 remains the same and the digit 1 is

dropped. The rounded number is 3.67.

18. In the number 56.3, the 6 is the digit being rounded. Since the digit to the right is 3, the digit 6 remains the same and the digit 3 is dropped. The rounded number is 56.

19. In the number 327.146, the 1 is the digit being rounded. Since the digit to the right is 4, the digit 1 remains the same and the digits 4 and 6 are dropped. The rounded number is 327.1.

20. In the number 83,261,000, the 3 is the digit being rounded. Since the digit to the right is 2, the digit 3 remains the same and the digits to the right are replaced by zeros. The rounded number is 83,000,000.

21. In the number 5,462,371, the 6 is the digit being rounded. Since the digit to the right is 2, the digit 6 remains the same and the digits to the right are replaced by zeros. The rounded number is 5,460,000.

22. In the number 7.8662, the second 6 from the left is the digit being rounded. Since the digit to the right is 2, the digit 6 remains the same and the digit 2 is dropped. The rounded number is 7.866.

23. In the number 272,341, the first 2 from the left is the digit being rounded. Since the digit to the right is 7, 1 is added to the 2 and the digits to the right are replaced by zeros. The rounded number is 300,000.

24. In the number 63.715, the 7 is the digit being rounded. Since the digit to the right is 1, the digit 7 remains the same and the digits to the right are dropped. The rounded number is 63.7.

25. In the number 264.97348, the 4 is the digit being rounded. Since the digit to the right is 8, 1 is added to the 4 and the digit 8 is dropped. The rounded number is 264.9735.

26. In the number 1,655,432, the second 5from the left is the digit being rounded. Since the digit to the right is 4, the digit 5 remains the same and the digits to the right are replaced by zeros. The rounded number is 1,655,000.

27. In the number 482.6002, the first 0 is the digit being rounded. Since the digit to the right is 0, the digit 0 remains the same and the digits 0 and 2 are dropped. The rounded number is 563.60.

28. In the number 426.861356, the 5 is the digit being rounded. Since the digit to the right is 6, 1 is added to the 5 and the digit 6 is dropped. The rounded number is 426.86136.

29. Round –4.21 to –4. Round 7.38 to 7 and round 3.51 to 4. The expression using rounding is $-4(7 + 4) = -44$. The exact calculation is –45.8469. The error is 1.8469 and the percent error is $1.8469/45.8469 \approx 4\%$.

30. Round 10.24 to 10. Round –8.93 to –9 and 2.77 to 3. The expression using rounding is $10(-9 + 3) = -60$. The exact calculation is –63.0784. The error is 3.0784 and the percent error is $3.0784/63.0784 \approx 4.9\%$.

31. Round 9.36 to 9. Round 7.423 to 7 and 9.1 to 9. The expression using rounding is equal to $\frac{\sqrt{9}}{7-9} = -1.5$. The exact calculation is –1.8243. The error is 0.3243 and the percent error is $0.3243/1.8243 \approx 17.8\%$.

32. Round 47.256 to 47. Round 9.9 to 10 and 24.501 to 25. The expression using rounding is $\frac{47-10}{\sqrt{25}} = 7.4$. The exact calculation is –7.5469. The error is 0.1469 and the percent error is $0.1469/7.5469 \approx 1.9\%$.

33. Round \$16.99 to \$17, so $8 \times 17 = \$136$ is the estimated cost.

34. Round \$39.95 to \$40, so $40 \times 5 = \$200$ is the estimated cost.

35. Round the distance to 240 miles and the speed to 40 miles per hour. Then $240 \div 40 = 6$, so the estimated time is 6 hours.

36. Round the time to 3 hours and the speed to 40 miles per hour. Then $3 \times 40 = 120$, so the estimated distance is 120 miles.

37. Round the cost of the futon to 180. You will pay 40% so multiply $180 \times 0.4 = \$72$ is the estimated cost of the futon.

38. Round the cost of the Blu-ray player to \$43. Fifteen percent of 43 is $43 \times 0.15 = \$6.45$. So the cost of the Blu-ray player is about \$43 – 6.5 = \$36.50.

39. Round the Quarter Pounder with cheese to \$3, the fries to \$2 and the shake to \$1. The estimated cost of the meal is \$6.

40. Round the loft bed to \$160, the beanbag chair to \$50, the storage cubes to \$30 and the lava lamp to \$20. The estimated cost of the items is \$260.

41. Round the prize money to \$1000. Each student will receive about \$1000 / 5 = \$200.

42. Round the number of rats to 50 and round the pounds of food to 100. Each rat eats about 100 / 50 = 2 pounds of food per week.

43. Round \$48,300.00 to \$50,000.00. The person works 40 hours per week times 50 weeks per year or $40 \times 50 = 2000$ hours per year. Thus the person earns about \$50,000 ÷ 2000 = \$25 per hour.

44. Round \$8.75 to \$9.00. The person works 40 hours per week times 50 weeks per year or $40 \times 50 = 2000$ hours per year. Then the person earns about \$9.00 × 2000 = \$18,000 per year.

45. Round 24 ft to 25 ft and 18 ft to 20 ft. Round \$5.95 to \$6. There are approximately 25 + 25 + 20 + 20 = 90 feet at approximately \$6 per 10 feet. The estimated cost is 9 × 6 or \$54.

46. Round the dimensions of the float to 10 feet by 20 feet. The area to be painted is approximately 200 square feet. Round the amount a quart of paint covers to 50 square feet, and the cost of a quart of paint to \$12. Then it will take about four quarts of paint and will cost approximately \$48.

47. Five posters on a wall will take up 10 feet and will have 4 gaps in between. That's a total of $10 + (4 \times 5) = 30$ feet. With 2 walls, they will need 10 posters. The approximate total cost for the posters is 5 × 10 = \$50.

48. Round \$365.00 to \$360 and \$62.00 to \$60. Then monthly expenses are \$360 + \$60 = \$420, so yearly expenses are about \$420 × 12 = \$5,040.

49. The University of Michigan is approximately 3,200 acres.

50. Princeton University is approximately 500 acres

51. The largest campus shown is Harvard at approximately 4,900 acres and the smallest campus shown is Princeton at approximately 500 acres. The estimated difference is 4,900 – 500 = 4,400 acres.

52. The two closest in size are University of Florida and Ohio University at about 2,000 acres.

53. There are 1,385 office workers in the survey. The ones who are most productive outside of normal office hours are the "after office hours" and the "before office hours" people, which is approximately 34%. Multiply $1{,}385 \times 0.34 = 470.9$. Round up to an estimated 471 people who feel they are most productive outside of normal office hours.

54. There are 1,385 office workers in the survey. The percentage that feel most productive "before late morning" are those who are in the "before office hours" and the "first few working hours" categories, which is approximately 56%. So multiply $1{,}385 \times 0.56 = 775.6$. So an estimated 776 workers feel they are more productive before late morning.

55. There are 1,385 office workers in the survey. The percentage that feel most productive "in the first few working hours" is 31%. So, multiply $1{,}385 \times 0.31 = 429.35$, approximately 429. The percentage that feel most productive "in the last few office hours" is 13%. So multiply $1{,}385 \times 0.13 = 180.05$, approximately 180. Since 429 – 180 = 249, an estimated 249 workers feel more productive in the first few working hours than in the last few working hours.

56. There are 1,385 office workers in the survey. The percentage that feel most productive "before office hours" is 25%. So, multiply $1{,}385 \times 0.25 = 346.25$, approximately 346. The percentage that feel most productive "after office hours" is 9%. So multiply $1{,}385 \times 0.09 = 124.65$, approximately 125. Since 346/125 = 2.768, approximately 2.8 times more people are most productive before office hours than after office hours.

57. There were 21.4% that worked between 21 and 34 hours, and 18.6% that worked more than 34; this adds up to 40%.

58. There were approximately 19% who worked 35 or more hours a week, therefore about 81% worked less than 35 hours.

59. There were approximately 18.6% of students age 22 or under that work 35 or more hours, so the total number of students on the campus multiplied by 0.186 is equal to 620 students. To find the total number of students divide 620/0.186 = 3,333.33. There are approximately 3,333 students on this campus.

60. There were approximately 24.7% of students age 22 or under who do not work at all, so the total number of students surveyed multiplied by 0.247 is equal to 310 students. To find the total number of students surveyed, divide 310/0.247 = 1255.06. There are approximately 1,255 students surveyed.

61. Locate the year 1950 halfway between 1940 and 1960 on the horizontal axis and move up to the line on the graph. At this point, move horizontally to the vertical axis. We estimate that 350 billion cigarettes were smoked in 1950.

62. Locate the year 1985 slightly to the right of 1980 on the horizontal axis and move up to the line on the graph. At this point, move horizontally to the vertical axis. Estimate that 570 billion cigarettes were smoked in 1985.

63. Locate 200 billion on the vertical axis and move vertically to the horizontal axis. Estimate the year was approximately 1940.

64. Locate 400 billion on the vertical axis and move vertically to the horizontal axis. Estimate the years were approximately 1955 and 2003.

65. Cigarette consumption rose from 0 to an estimated 625 billion between 1900 and 1980, and there are 80 years between 1900 and 1980. The average rate of change in cigarette consumption is 625/80 = 7.8125, which is an approximate increase of 7.8 billion cigarettes per year.

66. Cigarette consumption fell from an estimated 625 billion to an estimated 320 billion between 1980 and 2010, and there are 30 years between 1980 and 2010. The average rate of change in cigarette consumption is 305/30 = 10.1667, which is an approximate decrease of 10.2 (–10.2) billion cigarettes per year.

67. Locate the bar representing Yahoo and then move horizontally across to the vertical axis. The average daily reach for Yahoo is approximately 23%.

68. Locate the bar representing YouTube and then move horizontally to the vertical axis. The average daily reach for YouTube is approximately 32%.

69. The average daily reach for Google is approximately 49% and the reach for Wikipedia is approximately 15%. So the combined average daily reach is 64%.

70. The most visited site is Google with 49% and the least visited site is Wikipedia with about 15%. The difference between these is 34%.

71. Locate the point on the graph that corresponds to 2002 on the horizontal axis then move across to the vertical axis. The approximate fees for 2002 were 10 billion. Locate the point on the graph that corresponds to 2005 on the horizontal axis then move across to the vertical axis. The approximate fees for 2005 were 16 billion. The approximate change in fees is 16 – 10 = $6 billion.

72. Locate the point on the graph that corresponds to 2004 on the horizontal axis then move across to the vertical axis. The approximate fees for 2004 were 14 billion. Locate the point on the graph that corresponds to 2009 on the horizontal axis then move across to the vertical axis. The approximate fees for 2009 were 22 billion. The approximate change in fees is 22 – 14 =$8 billion.

73. Locate the point on the graph that corresponds to 2002 on the horizontal axis then move across to the vertical axis. The approximate fees for 2002 were 10 billion. Locate the point on the graph that corresponds to 2005 on the horizontal axis then move across to the vertical axis. The approximate fees for 2005 were 16 billion. The approximate change in fees is 16 – 10 = 6 billion. To estimate the rate of change in penalty fees from 2002 to 2005 divide 6 billion/3 years = $2 billion per year.

74. Locate the point on the graph that corresponds to 2004 on the horizontal axis then move across to the vertical axis. The approximate fees for 2004 were 14 billion. Locate the point on the graph that corresponds to 2009 on the horizontal axis then move across to the vertical axis. The approximate fees for 2009 were 22 billion. The approximate change in fees is 22 – 14 = 8 billion. To estimate the rate of change in penalty fees from 2004 to 2009 divide 8 billion/5 years = $1.6 billion per year.

75. My guess was probably better than yours considering that I'm a math professor, but here is how to work the problem. I guessed that Jared would need to score a 76% on the final to ensure that he got the 'B' that he was shooting for. With a 76% on the final, Jared would have a sum of 493 for the six test grades. He would have an average of 82.17%. My guess made him a bit of an overachiever, but not too bad.

76. Your guess was probably much worse than mine, but I guessed that Marta would need to score 32 points in the remaining two games in order to reach her goal. With 32 points, she would have a point total of 84 and an average of 14 points per game. She didn't quite make it, but that's OK because I don't like ball hogs anyway.

77. Answers may vary depending on how you rounded. Round the cost of green beans to 75 cents for a total of $75 \times 4 = 3$ dollars. Round the cost of yogurt to 50 cents for a total of $0.50 \times 8 = 4$ dollars. Round the cost of the steak to 6 dollars per pound and the weight to 2 pounds for a total cost of 12 dollars. Round the cost of energy drinks to 1.25 each for a total cost of $4 \times 1.25 = 5$ dollars. Round the cost of water to 3 dollars per gallon and the amount of water to 1 gallon for a total cost of 3 dollars. Add the total cost of each item for a grand total of about 27 dollars.

78. Answers may vary depending on how you rounded. Round the cost of baking soda to 90 cents per box, so 8 boxes would be about 7 dollars. Since 96 ounces is $\frac{3}{4}$ of a gallon and the cost for $\frac{3}{4}$ of a gallon is \$1.29, round the cost of the bleach to \$1.50 per gallon. The approximate cost of the bleach is $20 \times 1.50 = 30$ dollars. Round the cost of chlorine stabilizer to 12 dollars for 4 pounds for a total of $12 \times 2 = 24$ dollars. Round the cans of soda to 100, so at a cost of 60 cents each, that is 60 dollars. Add the total cost of each item for a grand total of about 121 dollars.

79. The difference between the cost of milk in 1988 and the cost in 2006 is exaggerated by the fact that the picture changed in all three dimensions, rather than just vertically

80. The increase appears sharper because the vertical axis only covers values from 10 to 19 billion in the same space the previous graph covered values from 0 to 20 billion.

81. In the first hour, they traveled about 45 miles, so the average speed was 45 miles per hour. In the first two hours, they traveled 125 miles so the average speed was $125/ 2 = 62.5$ miles per hour. In the first three hours, they traveled about 175 miles so the average speed was $175/3 = 58.3$ miles per hour. In the first four hours, they traveled about 215 miles so the average speed was about $215/4 = 53.8$ miles per hour. The second hour was their fastest hour, so this was likely expressway driving after non-expressway driving when first leaving the campus. The third and fourth hours both showed a decrease in average speed, so either traffic or the posted speed limits slowed them down.

82. Beginning at about 4 hours, the students' distance from the home campus did not change implying that they had arrived at the other campus about 215 miles away. At about 16 hours, the students' distance from the home campus began to decrease, implying that they had left to return home. That gave them about 12 hours on the other campus to party.

83. At 16 hours, the roadtrippers were about 215 miles away from home. At about 17 hours, they were about 163 miles from home. Therefore they traveled a total of –52 miles in one hour for an average speed of –52 miles per hour. At hour 18, they were still 163 miles per home. They traveled no distance and therefore had a speed of 0 miles per hour. The sign of the speed in the first answer implies that they were on their return trip. An average speed of 0 miles per hour implies that they made a pit stop from hour 17 to hour 18.

84. The graph shows that the trip to the other campus took about 4 hours and the trip home took about 5 hours. Since they covered the same distance "going" in a shorter amount of time than the same distance "coming home," the average speed on the way back was less than the average speed on the way there.

85. The slope of the line that connects each portion of the trip indicates the average speed for that portion of the trip; the steeper the line, the faster the average speed.

86. Using the average speed formula, the total distance traveled according to the graph is 0 and the total time is 21 hours. Plugging these numbers into the average speed formula indicates that the average speed was 0 miles per hour, which means they never went anywhere and must have missed the party. Because they "returned" the same distance (–215) that they "went," (215), the two distances will add to be zero, when actually they covered 430 miles in their trip. This sum of zero makes the average speed formula deceiving and shows that it can't be used for trips when there is a change in direction.

Exercise Set 1-3

1. Understand the problem, devise a plan, carry out the plan, and check your answer.

2. Answers may vary.

3. Answers may vary.

4. There may be too many cases to try and finding a solution would be time consuming.

5. **Step 1** *Understand the problem*. If one number is given, a second number is 6 more than the given number, and their sum is 22. Find the two numbers.

Step 2 *Devise a plan to solve the problem*. Make a list of possible combinations of numbers and see if the sum is 22.

Step 3 *Carry out the plan to solve the problem.*

First Number	Second Number	Sum	
1	7	8	
5	11	16	
7	13	20	
8	14	22	← correct

Answer: The numbers are 8 and 14.

Step 4 *Check the answer*. 8 + 14 = 22

6. **Step 1** *Understand the problem*. Find two numbers for which one number plus 7 equals the other number and the sum of the two numbers is 23.

Step 2 *Devise a plan to solve the problem*. Make a list of numbers where one number is 7 more than the other number. Find the pair whose sum equals 23.

Step 3 *Carry out the plan to solve the problem*.

First Number	Second Number	Sum	
4	11	15	
6	13	19	
7	14	21	
8	15	23	← correct

Answer: The numbers are 8 and 15.

Step 4 *Check the answer*. 8 + 7 = 15; 8 + 15 = 23

7. **Step 1** *Understand the problem*. When I find the correct number, I'll be able to multiply it by 3 or add 24 to it and get the same answer.

Step 2 *Devise a plan to solve the problem*. Guess a number. Add 24 to the number in one column and multiply the number by 3 another column. Check to see if the result in the two columns is the same. Use that first guess to make a better educated second guess. Put the guesses in a table to keep things organized.

Step 3 *Carry out the plan to solve the problem*.

Number	Number times 3	Number plus 24	
4	12	28	
8	24	32	
10	30	34	
12	36	36	← correct

Answer: The number is 12.

Step 4 *Check the answer*. I can double check my calculations to make sure that I have the correct answer: 12 × 3 = 36 and 12 + 24 = 36.

8. **Step 1** *Understand the problem*. I need to find two numbers that add up to be 57. The larger number needs to be twice the smaller number.

Step 2 *Devise a plan to solve the problem*. Make a list of numbers where the second number is twice the first number. Find the sum of the numbers to determine if it is 57.

Step 3 *Carry out the plan to solve the problem*.

First Number	Second Number	Sum	
4	8	12	
10	20	30	
15	30	45	
17	34	51	
19	38	57	← correct

Answer: The numbers are 19 and 38.

Step 4 *Check the answer*. I can double check my calculations to make sure that I have the correct answer: $19 \times 2 = 38$ and $19 + 38 = 57$.

9. **Step 1** *Understand the problem*. In a two-digit number, the sum of the two digits is 7. The number minus 9 is the original number with the digits reversed. Find the number.

Step 2 *Devise a plan to solve the problem.* Make a list of two-digit numbers where the sum of the digits is 7. See which number minus 9 is the number with the digits reversed.

Step 3 *Carry out the plan to solve the problem.*

Original Number	Number minus 9	
16	7	
25	16	
34	25	
43	34	← correct
52	43	
61	52	
70	61	

Answer: The number is 43.

Step 4 *Check the answer*. I can double check my calculations to make sure that I have the correct answer: $4 + 3 = 7$ and $43 - 9 = 34$.

10. **Step 1** *Understand the problem*. I am looking for a two digit number whose digits add up to be seven. The tens digit has to be one more than the ones digit.

Step 2 *Devise a plan to solve the problem.* Make a list of 2 digit numbers that have digits that add to 7. Find the number which has a tens digit that is one more than the ones digit.

Step 3 *Carry out the plan to solve the problem.*

Original Number	
16	
61	
52	
25	
34	
43	← correct

Answer: The number is 43.

Step 4 *Check the answer*. I can double check my calculations to make sure that I have the correct answer: $4 + 3 = 7$ and 4 is one more than 3.

11. **Step 1** *Understand the problem*. If I start with the age of the house right now, I need to be able to add 28 to it or multiply it by 5 and get the same number.

Step 2 *Devise a plan to solve the problem.* Begin by guessing the age of the house. Add 28 to the guess in the first column. Multiply the guess by 5 in the second column. Check to see if I got the same answer in both columns.

Step 3 *Carry out the plan to solve the problem*.

Age	Age +28	Age × 5	
4	32	20	
8	36	40	
9	37	45	
7	35	35	← correct

Answer: The age of the house is 7 years.

Step 4 *Check the answer*. I can double check my calculations to make sure that I have the correct answer: $7 + 28 = 35$ and $7 \times 5 = 35$.

12. **Step 1** *Understand the problem*. I need to find the number of years that Hoang and Bill have worked at the hospital. Right now, Bill's time at the hospital needs to be 5 years less than Hoang's. If I subtract 4 from each of the numbers, then Hoang's number of years needs to double Bill's number of years.

Step 2 *Devise a plan to solve the problem*. Make a list of ages where Hoang's number of years is 5 more than Bill's number of years. Then in the same table, add 4 to each of these years. Check to find out which set of years in the third and fourth columns has Hoang's years double that of Bill's.

Step 3 *Carry out the plan to solve the problem.*

Hoang Now	**Bill Now**	**Hoang – 4**	**Bill – 4**
12	7	8	2
13	8	9	4
14	9	10	5 ↑ correct

Answer: Bill has worked at the hospital for 9 year and Hoang has worked at the hospital for 14 years.

Step 4 *Check the answer*. I can double check my calculations to make sure that I have the correct answer: $10 + 4 = 14$, $5 + 4 = 9$ and $5 \times 2 = 10$.

13. **Step 1** *Understand the problem*. 5 people are going to split an inheritance: 2 children and 3 grandchildren. The amount that each child gets needs to be double that of what each grandchild gets. The total amount given to all 5 must be \$140,000.

Step 2 *Devise a plan to solve the problem.* Make a list of dollar amounts where the children receive twice as much as the grandchildren. Find the total amount distributed by multiplying the amount per child by 2 and the amount per grandchild by 3 and adding the results. Find the set of numbers that results in a total of \$140,000.

Step 3 *Carry out the plan to solve the problem*.

Amount per Grandchild	**Amount per child**	**Total Distributed**
10,000	20,000	70,000
15,000	30,000	105,000
20,000	40,000	140,000 ↑ Correct

Answer: Each grandchild receives \$20,000 and each child receives \$40,000.

Step 4 *Check the answer*. I can double check my calculations to make sure that I have the correct answer: $20{,}000 \times 2 = 40{,}000$. $20{,}000 \times 3 + 40{,}000 \times 2 = 140{,}000$.

14. **Step 1** *Understand the problem.* At a dog park there are several dogs with their owners. Counting heads, there are 12; counting legs, there are 38. How many dogs are there?

Step 2 *Devise a plan to solve the problem.* Make a list of numbers of dogs and humans that total to 12 and count the number of legs.

Step 3 *Carry out the plan to solve the problem*.

Number of Dogs	**Number of Humans**	**Number of legs**
1	11	26
2	10	28
3	9	30
4	8	32
7	5	38 ← correct

Answer: There are 7 dogs and 5 owners.

Step 4 *Check the answer*. $7 + 5 = 12$;

$7 \times 4 + 5 \times 2 = 38$.

15. **Step 1** *Understand the problem.* Using only the digits 1, 2, and 3, I need to form 3 digit numbers. I need to identify the two numbers that are divisible by 6.

Step 2 *Devise a plan to solve the problem.* Make a list of all three digit numbers that can be formed using 1, 2, and 3. Check each number for divisibility by 6.

Step 3 *Carry out the plan to solve the problem*.

Three digit numbers	
123	
132	← divisible by 6
213	
231	
312	← divisible by 6
321	

Answer: 132 and 312 are both divisible by 6

Step 4 *Check the answer*: $22 \times 6 = 132$ and $52 \times 6 = 312$. Both 132 and 312 are divisible by 6.

16. **Step 1** *Understand the problem*. There are 32 pieces of pizza that were ordered. If the dog will get 5, and the coach ate 3, then the team ate 24 pieces. I need to find the number of men and women on the team knowing that there are nine total athletes and that they consumed a total of 24 pieces. Fortunately, I know that each man ate 4 pieces and each woman ate 2 pieces, otherwise I'd be here all day.

Step 2 *Devise a plan to solve the problem*. Make a list of men and women athletes that total 9. Calculate the total pizza consumed knowing each man ate 4 pieces and each woman ate 2 pieces. Locate the pair that equal 28 pieces.

Step 3 *Carry out the plan to solve the problem*.

Number of Men	Number of Women	Total Pizza Consumed	
1	8	20	
2	7	22	
3	6	24	← correct
4	5	26	
5	4	28	

Answer: There are 3 men and 6 women on the team.

Step 4 *Check the answer*. $6 + 3 = 9$ and $3 \times 4 + 6 \times 2 = 24$.

17. **Step 1** *Understand the problem.* Barney has 5 more dimes than Betty has quarters, and the total value of both the quarters and the dimes is \$5.05. Just wondering, since it is Barney and Betty we are talking about, shouldn't this problem be about "rubels"? You are probably not old enough to get that joke!

Step 2 *Devise a plan to solve the problem.* Make a list of possible coin amounts and corresponding dollar values. Find the combination that has a dollar value of \$5.05.

Step 3 *Carry out the plan to solve the problem*.

Betty's Quarters	Barney's Dimes	Dollar Value
11	16	\$4.35
12	17	\$4.70
13	18	\$5.05
		↑
		correct

Answer: Betty gets 13 quarters or \$3.25 and Barney gets 18 dimes or \$1.80.

Step 4 *Check the answer*.

$$\$0.25(13) + \$0.10(18) = \$3.25 + 1.80 = \$5.05$$

18. **Step 1** *Understand the problem.* A tip jar contains twice as many quarters as dollar bills. There is a total of $12. How many quarters and how many dollar bills does it contain?

Step 2 *Devise a plan to solve the problem.* Make a list of possible numbers of quarters and dollar bills where the number of quarters is twice the number of bills, and the corresponding dollar amounts.

Step 3 *Carry out the plan to solve the problem*.

Number Of Quarters	Number of Bills	Dollar Value
14	7	$10.50
16	8	$12.00
		↑
		correct

Answer: There are 16 quarters and 8 dollar bills.

Step 4 *Check the answer*.

$0.25(16) + $1.00(8) = $4.00 + $8.00 = $12.00

19. **Step 1** *Understand the problem.* The finals week bash tickets cost $2 for dudes and $1 for ladies. We know that 55 tickets brought in $75. How many ladies attended the party?

Step 2 *Devise a plan to solve the problem.* Make a list of tickets for dudes and ladies totaling 55 tickets and find the dollar amount of each combination.

Step 3 *Carry out the plan to solve the problem*.

Number of dudes	Number of ladies	Dollar Value
23	32	$78
22	33	$77
20	35	$75
		↑
		correct

Answer: There were 35 ladies at the party.

Step 4 *Check the answer*. 20 + 35 = 55; $2(20) + $1(35) = $40 + $35 = $75.

20. **Step 1** *Understand the problem.* There were a total of 9 people arrested. Each of those people now had 2 or 3 total arrests. All together, the 9 "slow learners" have 19 total arrests among them.

Step 2 *Devise a plan to solve the problem.* Make a list of possible numbers of two arrest and three arrest people where the number of people totals 9. Find the numbers that make for a total of 19 arrests.

Step 3 *Carry out the plan to solve the problem*.

Number with 2 arrests	Number with 3 arrests	Total Arrests
1	8	26
2	7	25
3	6	24
4	5	23
5	4	22
6	3	21
7	2	20
8	1	19
		↑
		correct

Answer: There were 8 people with 1 arrest and 1 person with 3 arrests.

Step 4 *Check the answer*.

2 (8) + 3(1) = 16 + 3 = 19

21. **Step 1** *Understand the problem*. You are given that Mae earns $87 for working 8 hours. Find how much Mae earned for working only 5 hours.

Step 2 *Devise a plan to solve the problem.* You first need to figure out how much money Mae earns for working 1 hour. Then multiply this amount by 5 to find how much she earns for working 5 hours.

Step 3 *Carry out the plan to solve the problem*. Divide the wages earned in 8 hours by 8 to find how much Mae earns per hour: $87 ÷ 8 = $10.875. Multiply this hourly amount by 5:
$10.875 × 5 = $54.375. Rounding to the nearest cent, Mae earned $54.38 for working 5 hours**.**

Step 4 *Check the answer*. We can find $\frac{5}{8}$ of the amount earned in 8 hours:

$\frac{5}{8} \times \$87 = \$54.375,$ which rounds to $54.38.

22. **Step 1** *Understand the problem*. You know that the manager wants to have 6 PCs that are 1.5 feet wide each on a table, with 2 feet on each end and three feet between each PC. How long should the table be?

Step 2 *Devise a plan to solve the problem*. You can find the total space needed by adding the space on each end, the space between each computer and the lengths of each computer.

Step 3 *Carry out the plan to solve the problem*. There are 2 feet on each end, 5 spaces of 3 feet each, and 6 PCs at 1.5 feet each. So add 2(2) + 5(3) + 6(1.5) = 28. The manager needs to order tables that are 28 feet long.

Step 4 *Check the answer*. 2(2) + 5(3) +6(1.5) = 28.

23. **Step 1** *Understand the problem*. Bob has posts that are 6 inches square and he will place them four feet apart on a 32-foot-long deck. How many posts will he use?

Step 2 *Devise a plan to solve the problem*. If the first post is on the edge, then from the edge to the next post will be 4.5 feet. Leaving 6 inches on the end for the last post we can divide 4.5 into 31.5 to see how many posts there are, then add one for the last post.

Step 3 *Carry out the plan to solve the problem*. 31.5/4.5 = 7. So there are 8 posts.

Step 4 *Check the answer*. There are eight posts and seven 4-foot spaces:

8 (0.5) +7(4) = 4 + 28 = 32.

24. **Step 1** *Understand the problem*. Suzie has 10 8½ inch wide pictures. She hangs them with 2 inches on each end, and 6 inches between each. How wide is the wall?

Step 2 *Devise a plan to solve the problem*. Ten pictures means nine spaces between. So multiply 10 by 8.5, 9 by 6 and 2 by 2 then add.

Step 3 *Carry out the plan to solve the problem*. 10(8.5) + 9(6) + 2(2) = 143. So the wall is 143 inches or about 12 feet wide.

Step 4 *Check the answer*. 10(8.5) + 9(6) + 2(2) = 85 + 54 + 4 = 143

25. **Step 1** *Understand the problem*. There are 6 pictures that are each 6 inches wide. You want a 1-inch border and 2 inches between pictures. How wide must the mat board be?

Step 2 *Devise a plan to solve the problem*. There is one inch on each end and five 2-inch spaces between pictures, and six 6-inch pictures. If we add up these values we will get the width of the mat board.

Step 3 *Carry out the plan to solve the problem*. Add 2(1) + 5(2) + 6(6) = 48. So the mat board should be 48 inches wide.

Step 4 *Check the answer*. 2(1) + 5(2) + 6(6) = 2 + 10 + 36 = 48

26. **Step 1** *Understand the problem*. There is 220 feet of border. We want to know the width of the pool if the length is 60 feet and the entire border is used.

Step 2 *Devise a plan to solve the problem*. Calculate how many feet of the tile is used for the two sides down the length of the pool, then see what is left to divide between the two sides along the width of the pool.

Step 3 *Carry out the plan to solve the problem*. Since the length of the pool is 60 feet and there are two sides this length, it will take up 2 × 60 = 120 feet of the border. That leaves 220 – 120 = 100 feet to divide between the remaining two sides. So, the width of the pool should be 100/2 or 50 feet.

Step 4 *Check the answer*. The border used will be 2(60) + 2(50) = 120 + 100 = 220 feet.

27. **Step 1** *Understand the problem*. The length of the nature area is twice the width. The distance around the area is 300 feet. We want to find the length and width of the area.

Step 2 *Devise a plan to solve the problem*. We know that the length is twice the width, therefore the distance around should be 2 times the width plus 2 times the length which is 4 times the width. The total distance around is then 6 times the width, so we can divide 300 by 6 to find the width.

Step 3 *Carry out the plan to solve the problem*. 300/6 = 50. So the width is 50 feet and the length is twice that or 100 feet.

Step 4 *Check the answer*. 50 + 50 + 100 + 100 = 300

28. **Step 1** *Understand the problem*. Manuel needs to caulk a total of 8 windows. The windows have various dimensions. We need to find the perimeter of each of the 8 windows to determine the total amount of caulk needed. Then we need to figure out how many tubes of caulk will be needed

Step 2 *Devise a plan to solve the problem*. Find the perimeter of the 2 × 3 windows, the 4 × 5 windows and the 6 × 4 windows. Add the perimeters together to find the total length needing caulk. Divide the total length by 50 to determine the number of tubes of caulk needed.

Step 3 *Carry out the plan to solve the problem*. Total perimeter for 2 × 3 windows is 3(2 + 2 + 3 + 3) = 36. Total perimeter for 4 × 5 windows is 3(4 + 4 + 5 + 5) = 54. Total perimeter for 4 × 6 windows is 2(4 + 4 + 6 + 6) = 40. Total perimeter for all windows is 36 + 54 + 40 = 130 feet. 130 feet ÷ 50 = 2.6, which should be rounded up to 3 tubes.

Step 4 *Check the answer*. 3(2 + 2 + 3 + 3) + 3(4 + 4 + 5 + 5) + 2(4 + 4 + 6 + 6) = 130. 130 ÷ 50 ≈ 3.

29. **Step 1** *Understand the problem*. Felicia needs to know how many boxes of lights to purchase for the gala. Each box contains 12 lights on a string 20 feet long. The room has a length of 70 feet and a width that is twice the length.

Step 2 *Devise a plan to solve the problem*. First, determine the distance around the outside of the room. Once you know this number, you can divide to find out the number of boxes needed. Using the number of boxes needed, you can determine the number of lights used.

Step 3 *Carry out the plan to solve the problem*. The length of the room is 70 and the width of the room is twice that or140. The distance around the room is 70 + 70 + 140 + 140 = 420. 420 ÷ 20 = 21 boxes. 21 boxes with 12 lights each is 21 × 12 = 252 lights.

Step 4 *Check the answer*. 2(70) + 2(140) = 420. 420 ÷ 20 = 21 .

30. **Step 1** *Understand the problem*. Two students earned a total of 60 dollars. One worked 1½ hours and the other worked only ½ hour. We want to know what their hourly wage was if they both earned the same per hour.

Step 2 *Devise a plan to solve the problem*. There was a total of two hours worked between the two students and they earned a total of $60. So the hourly wage can be calculated by dividing total hours worked into the amount earned, then multiply the hourly wage by the hours each worked to calculate their earnings.

Step 3 *Carry out the plan to solve the problem*. 60/2 = 30. So they earned 30 dollars an hour. Sam worked 1½ hours at $30 per hour so he earned 1½ (30) = $45 and Pete earned ½ (30) or $15.

Step 4 *Check the answer*. 1½ (30) + ½ (30) = 45 + 15 = 60.

31. **Step 1** *Understand the problem*. You have half a drink and you drink half of what's left. You need to figure out how much of the original drink you now have remaining.

Step 2 *Devise a plan to solve the problem*. You need to figure out what ½ of ½ is. This can be done by multiplying.

Step 3 *Carry out the plan to solve the problem*. ½ (½) = ¼. So you have ¼ of the original drink left.

Step 4 *Check the answer*. ½ (½) = ¼

32. **Step 1** *Understand the problem*. There are 832 bottles of water. Six bottles fit in a case. Find how many cases are needed to ship as many of the 832 bottles as possible. If a case is not full to ship, find how many bottles are left over.

Step 2 *Devise a plan to solve the problem*. Divide 832 by 6. The quotient is the number of cases needed and the remainder, if any, is the number of bottles left over.

Step 3 *Carry out the plan to solve the problem*. 832 ÷ 6 = 138 remainder 4, so 138 cases are needed, and 4 bottles will be left over.

Step 4 *Check the answer*. 138 × 6 + 4 = 832

33. **Step 1** *Understand the problem*. Kam has $1,624 per month to pay bills and cover living expenses. Some of those expenses are $256 for food, $125 for gasoline, and $150 for utilities. Find how much money is left for other expenses.

Step 2 *Devise a plan to solve the problem*. Add the known expenses and subtract the total from the amount of money available for the month. The result is the amount of money left to pay other bills.

Step 3 *Carry out the plan to solve the problem*. Add the costs for food, gasoline, and utilities:
$256 + $125 + $150 = $531. Subtract the total known expenses from the amount of money available:
$1,624 – $531 = $1,093. $1,093 is the amount left for other monthly expenses.

Step 4 *Check the answer*.
$1,093 + $150 + $125 + $256 = $1,624

34. **Step 1** *Understand the problem*. One protein bar has 15 grams of carbs and 20 grams of protein. If the trainer wants Cheryl to take in an extra 300 grams of carbs and 350 grams of protein per week, how many bars should Cheryl buy?

Step 2 *Devise a plan to solve the problem*. Since each bar contains 15 grams of carbs, we can divide 15 into 300 to find the number of bars needed for the carbs. Since each bar contains 20 grams of protein, we can divide 20 into 350 to find the number of bars needed for the protein.

Step 3 *Carry out the plan to solve the problem*. 300/15 = 20. She would need to eat 20 of the bars to get the required amount of carbs. 350/20 = 17.4. She would need to eat 18 bars to get the required amount of protein. 20 bars will cover both the carbs and the protein requirements.

Step 4 *Check the answer*. 20(15) = 300 and 20(20) =400. These numbers are both at or higher than her trainer recommends.

35. **Step 1** *Understand the problem*. There are three classes of seventh graders at Wilson Middle School. They contain 24, 26, and 21 students each. The PE teacher wants to divide them up into equal size groups of between 5 and 9 students for a mock Olympics. Find the number of students that should be in each group so that the groups are equal.

Step 2 *Devise a plan to solve the problem*. Find the total number of students in the seventh grade. Determine if the total is divisible by any of the numbers 5 through 9.

Step 3 *Carry out the plan to solve the problem*. Add 24 + 26 + 21 = 71 students. Check for divisibility: 71 ÷ 5 = 14.2; 71 ÷ 6 = 11.83; 71 ÷ 7 = 10.14; 71 ÷ 8 = 8.875; 71 ÷ 9 = 7.89. The seventh graders cannot be divided evenly into groups containing 5 through 9 students.

Step 4 *Check the answer*. 24 + 26 + 21 = 71. 71 is not divisible by 5, 6, 7, 8 or 9. Recognizing that 71 is a prime number and is only divisible by 1 and 71 would make this problem go much faster.

36. **Step 1** *Understand the problem*. Bart has scored 48, 45, 45, and 47 on his first four 50 point exams. He needs to be sure to have a 70% average after the last exam to pass the course. Determine the lowest that he can score on the last exam to pass the class.

Step 2 *Devise a plan to solve the problem*. Find the total number of points that Bart has earned so far, the total number of points that are available on all 5 exams, and the total number of points that he will need to have a 70% average.

Step 3 *Carry out the plan to solve the problem*. The total points earned so far is 48 + 45 + 45 + 47 = 185 points. The total points available for the five exams is 5(50) = 250 points. The total number of points needed to have at least a 70% average is 250 × .70 = 175. Since Bart already has 185 points, he could technically score a zero on the last exam and still pass the course. He would not do this though since he lives to please his math professor.

Step 4 *Check the answer*. 48 + 45 + 45 + 47 + 0 = 185. 185/250 = 74%.

37. **Step 1** *Understand the problem*. The loan is for a total of $12,381. The loan is to be paid once a month for 5 years. Find the amount of each monthly payment.

Step 2 *Devise a plan to solve the problem*. First we must find the total number of payments to be made over 5 years, using the fact that there are 12 months in a year. Then we can divide the loan amount by the number of payments that will be made to find the monthly payment amount.

Step 3 *Carry out the plan to solve the problem*. Find the total number of payments by multiplying 5 years by
12 months per year: 5 × 12 = 60, so
60 payments will be made. Then, divide the loan amount by the number of payments:
$12,381 ÷ 60 = $206.35. The monthly payment should be $206.35.

Step 4 *Check the answer*.
$206.35 × 12 months = $2476.20 per year;
$2476.20 × 5 years = $12,381.

38. **Step 1** *Understand the problem*. The temporary tire reduces the average speed by 15 miles per hour. With this reduced speed, how long will it take to drive the 39 miles home?

Step 2 *Devise a plan to solve the problem*. Before figuring out what the reduced speed is, we need to figure out what the normal average speed is. If the 39 mile trip at the average speed normally takes 45 minutes, then we can divide to find out the normal average speed. Next, subtract 15 from the normal average speed and divide again to figure out how long it will take to drive the 39 miles home.

Step 3 *Carry out the plan to solve the problem*. Find the normal average speed by dividing distance by time: 39/0.75 = 52, so the average speed without a damaged tire is 52 miles per hour. Then, subtract 15 from the average speed to get the speed with the temporary tire: 52 – 15 = 37. Finally, figure out how long the trip home will take by dividing distance by rate: 39/37 = 1.054 hours. 1.054 hours is the same as $1.054 \times 60 \approx 63$ minutes.

Step 4 *Check the answer*.
39 miles/0.75 hours = 52 mph; 52 mph – 15 mph – 37 mph; 39 miles/37 mph = 1.054 hours; 1.054 hours × 60 minutes/hour ≈ 63 minutes.

39. **Step 1** *Understand the problem*. Four people live together in an apartment. Each person pays a different amount of rent. Mary pays $\frac{1}{2}$ of the rent, Jean pays $\frac{1}{4}$ of the rent, Claire pays $\frac{1}{8}$ of the rent, and Margie pays the rest. The monthly rent is $2,375. Find how much each person pays.

Step 2 *Devise a plan to solve the problem*. Multiply each person's fractional responsibility for the rent by the total amount of rent. This gives the amounts Mary, Jean, and Claire pay. Add these amounts together and subtract the total from $2375 to find the amount Margie pays.

Step 3 *Carry out the plan to solve the problem*. Find the amounts Mary, Jean, and Claire pay by multiplying:

Mary: $\frac{1}{2} \times \$2,375 = \$1,187.50$

Jean: $\frac{1}{4} \times \$2,375 = \$593.75$

Claire: $\frac{1}{8} \times \$2,375 = \$296.875,$ or $296.88

Then Margie pays

$$\$2375 - (\$1,187.50 + \$593.75 + \$296.88)$$
$$= \$296.87$$

Step 4 *Check the answer*.
$$\$1,187.50 + \$593.75 + \$296.88 + \$296.87$$
$$= \$2,375$$

40. **Step 1** *Understand the problem*. At the first fill-up the odometer reading is 23,568.7 miles. At the second fill-up the odometer reading is 23,706.3 miles. The Jeep used 12.6 gallons of gasoline between fill-ups. Find the number of miles per gallon for the trip.

Step 2 *Devise a plan to solve the problem*. Subtract the initial odometer reading from the ending odometer reading to find the number of miles driven between fill-ups. Then divide the number of miles driven by the miles per gallon.

Step 3 *Carry out the plan to solve the problem*. Find the number of miles driven: 23,706.3 – 23,568.7 = 137.6 miles. Divide 137.6 miles by 12.6 gallons: 137.6 ÷ 12.6 = about 10.9. The Jeep got about 10.9 miles per gallon.

Step 4 *Check the answer*. 10.9 × 12.6 = 137.34, which is close to 137.6 miles.

41. **Step 1** *Understand the problem*. The clerk's regular hourly rate is \$9.50 per hour for working more than 0 but less than or equal to 40 hours. If the clerk works more than 40 hours the clerk's hourly rate for any hours more than 40 hours is one and one-half times the regular hourly rate. Find how much the clerk earned for working 46 hours in one week.

Step 2 *Devise a plan to solve the problem*. First find the clerk's hourly rate for working more than 40 hours by multiplying the regular hourly rate of \$9.50 by $1\frac{1}{2}$. Then, since the clerk worked 40 hours at the regular rate and 6 hours at the other rate, find how much the clerk earned by multiplying each rate by the number of hours worked at that rate. Add the results to get the total amount earned.

Step 3 *Carry out the plan to solve the problem*. Find the clerk's hourly rate for working more than 40 hours: $\$9.50 \times 1\frac{1}{2} = \$14.25$. The clerk earns \$14.25 per hour for any hours worked over 40 hours. Then the clerk earned \$9.50 × 40 = \$380 for the first 40 hours worked and \$14.25 × 6 = \$85.50 for the last 6 hours worked. The total amount earned was \$380 + \$85.50 = \$465.50.

Step 4 *Check the answer*.

$$40 \times \$9.50 + 6 \times 1\frac{1}{2} \times \$9.50 = \$465.50$$

42. **Step 1** *Understand the problem*. Sally went 32 minutes over her limit and her cell phone company will charge 35 cents if they are daytime minutes and only 10 cents if they are evening minutes. We are looking for the difference between the amount charged at the daytime rate and the evening rate.

Step 2 *Devise a plan to solve the problem*. Find the amount charged at the daytime rate by multiplying 32(35) and converting to dollars. Find the amount charged at the evening rate by multiplying 32 by 10 and converting to dollars.

Step 3 *Carry out the plan to solve the problem*. Daytime rate: 32(35) = 1120 cents or \$11.20. Evening rate: 32(10) = 320 cents or \$3.20. The difference between these is \$8. So Sally would save \$8.

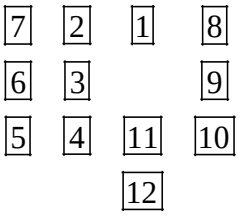
Step 4 *Check the answer*. 32(35) ÷100 – 32(10) ÷100 = 11.20 – 3.20 = 8.

43. **Step 1** *Understand the problem*. With a problem like this, understanding the problem involves some trial runs . . . or jumps. The trials help make clear the objective and the rules. They also can lead to some insights that will make finding the solution path easier.

Step 2 *Devise a plan to solve the problem*. Trial and Error is going to be our best strategy, but cutting down on the trials is important so that you don't ultimately decide to intentionally miss a lily pad and let the frog drown instead of finishing the problem. We can draw a diagram of the lily pads and number them 1 through 12 to represent the jumps.

Step 3 *Carry out the plan to solve the problem.* A few trial runs make a couple of things clear.

The last lily pad has to be at the bottom, since landing there before the end would make the frog return from the way he came. The second to last pad must be above the last, otherwise the jump to the last would have to be diagonal. The first lily pad has to be below the starting position so that the jump is not diagonal. The guesses for the second and tenth lily pads are also because of the no diagonal jumping rule. A few trials can also help you to realize that some of your jumps are going to be long ones. This frog has game! By making some initial guesses based on deductive reasoning, the following solution path can be found.



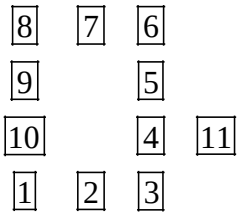
Step 4 *Check the answer*. Check the path to be sure that all rules have been followed.

44. **Step 1** *Understand the problem*. With a problem like this, understanding the problem involves some trial runs . . . or jumps. The trials help make clear the objective and the rules. They also can lead to some insights that will make finding the solution path easier.

Step 2 *Devise a plan to solve the problem.* Trial and Error is going to be our best strategy, but cutting down on the trials is important so that Kermit here can finish the path before he needs supplemental oxygen. We can draw a diagram of the lily pads and number them 1 through 11 to represent the jumps.

Step 3 *Carry out the plan to solve the problem.* A few trial runs make a couple of things clear.

The last lily pad has to be at the far right, since landing there before the end would make the frog return from the way he came. The second to last pad must be to the left of the last pad, otherwise the jump to the last would have to be diagonal. The first lily pad has to be to the right of the starting position so that it is not diagonal. The guesses for the second and tenth lily pads are also because of the no diagonal jumping rule. A few trials can also help you to realize that Kermit has skills and can make long jumps. By making some initial guesses based on deductive reasoning, the following solution path can be found.



Step 4 *Check the answer*. Check the path to be sure that all rules have been followed.

45. **Step 1** *Understand the problem*. The king pays the knight for 6 days of work using a 6-inch gold bar. The king pays the knight an equal amount of the bar each day. The king only makes 2 cuts in the bar. Find how the king was able to pay the knight daily.

Step 2 *Devise a plan to solve the problem.* The gold bar is 6 inches long, and the king pays the knight over 6 days so the king pays the knight in 1-inch pieces each day. Find a way the king can cut the bar using only two cuts so that the king can pay the knight with a 1-inch piece each day. Visualize the payments with a sketch of the gold bar.

Step 3 *Carry out the plan to solve the problem.* (Assume the knight doesn't spend any of the money until the end of six days). The king must pay the knight with 1 inch of the bar at the end of the first day, so the first cut must be made at the 1-inch mark on the bar. At the end of the second day the king must pay the knight with another 1-inch piece, but if he cuts the bar at the original 2-inch mark on the bar the remaining piece will be 4 inches and too big to pay the knight daily without making another cut. The king should cut the remaining 5 inches of the bar at the 3-inch mark so that there is a 3-inch piece and a 2-inch piece. The king will take the 1-inch piece back from the knight and give him the 2-inch piece at the end of the second day. Continuing in this manner, the king can pay the knight an additional 1-inch piece of gold bar for each of the 6 days.

Step 4 *Check the answer*. The king made only 2 cuts, and the knight was paid in 1-inch pieces each day for six days.

46. Fill the 3-gallon container; pour it into the 5-gallon container. Fill the 3-gallon container again and pour into the 5-gallon container until it's full. There will then be one gallon left in the 3-gallon container.

47. Three, since the car in front is in front of two cars and the car at the end is behind two cars.

48. **Step 1** *Understand the problem*. The cost of a computer is covered with $\frac{1}{3}$ of the money from the freshmen, $\frac{1}{2}$ of the money from the sophomores, and \$400 from the Student Government Association. Find the cost of the computer.

Step 2 *Devise a plan to solve the problem*. Let x represent the cost of the computer. Then the freshmen paid $\frac{1}{3}x$ and the sophomores paid $\frac{1}{2}x$. Add the amounts contributed by each group and solve for x.

Step 3 *Carry out the plan to solve the problem.*

$$\frac{1}{3}x+\frac{1}{2}x+\$400=x$$
$$\frac{2}{6}x+\frac{3}{6}x+\$400=x$$
$$\frac{5}{6}x+\$400=x$$
$$\$400=\frac{1}{6}x$$
$$6(\$400)=x$$
$$\$2,400=x$$

The computer cost \$2,400.

Step 4 *Check the answer*.

$$\frac{1}{3}(\$2,400)+\frac{1}{2}(\$2,400)+\$400$$
$$=\$800+\$1,200+\$400$$
$$=\$2,400$$

49. First of all, we need to assume that the hero is not stupid and doesn't drive half way across the desert and then walk the rest of the way. It's a desert! If four Jeeps are used, then once the Jeeps get ¼ of the way, each tank will be half full. Two of the Jeeps can transfer their fuel to the two other Jeeps so that they once again have a full tank. After another quarter of the way through the desert, the two remaining Jeeps will have a tank half full. One Jeep transfers its remaining fuel to the other so that it now has a full tank. Since the Jeep with a full tank is now half way through the desert, he will be able to make the rest of the trip without more fuel. Hopefully this plan also includes rescuing the guys who are now stranded in the desert with no fuel.

50. Each side of the pentagon can be used to create 4 unique triangles that do contain another other side of the pentagon. Since a pentagon has 5 sides, this is a total of 5 × 4 = 20 triangles. There are 5 isosceles triangles contained in the points of the star in the center of the figure. The total so far is 20 + 5 = 25. There are 5 unique triangles formed using 2 adjacent sides of the pentagon. Now the total is 25 + 5 = 30. There are 5 large triangles formed by using 2 non-adjacent "points" of the star. The total now is 30 + 5 = 35.

51. In order to make his average speed 160 mph, which is double the speed of the first lap, the driver would need to complete both laps in the same amount of time it already took him to complete one lap. Since the driver can't manufacture time, it's just not going to happen.

52. If you want to double your overall speed, you need to cover twice the distance in the same amount of time. Since you have already completed the first lap, and there are no "do overs," it is not possible.

53. If Maurice or Hani had 1 or 10, then they would have known that the other had either 2 or 9. Since both said "I don't know your number," we know that 1 and 10 are not one of the numbers.

When Hani says "I don't know your number," this led Maurice to go from not knowing to knowing. So Maurice must have had a number where Hani's answer of "And I don't know your number" was crucial.

If Maurice had 2, then Hani would have either 1 or 3 - and the crucial answer " And I don't know your number " would have ruled out 1, leaving 3 as the other number (Solution: Maurice has 2 and Hani has 3).

If Maurice had 3, then he would expect Hani to have either 2 or 4. But if Hani had 2 he would not say " And I don't know your number" because he would know that Maurice did not have 1 and must have had 3. With 2 ruled out, the other number is 4. (Solution: Maurice has 3 and Hani has 4).

The same arguments work at the other end of the range, providing the other two solutions: (Solution: Maurice has 9 and Hani has 8) and (Solution: Maurice has 8 and Hani has 7).

If Maurice says "I don't know your number," then he can't have 1 or 10. Otherwise, he would know for sure that Hani had 2 or 9. If Hani says "I don't know your number," then he also can't have 1 or 10 for the same reason.. This leaves for possible pairs of numbers: 3 and 4, 4 and 5, 5 and 6 or 6 and 7.

54. I'm glad I am solving the problem and NOT golfing with them, since they obviously don't have anything interesting to talk about. If the product of the ages of the children is 36, then there are a limited number of ages that the three children can have. Here is a list along with the sum of those three ages:

Ages	Sum of the Ages
1, 1, 36	38
1, 2, 18	21
1, 3, 12	16
1, 4, 9	14
1, 6, 6	13
2, 2, 9	13
2, 3, 6	11
3, 3, 4	10

There are two possible age combinations that also have a sum of 13, and 13 must be the number of holes of golf that they have played since any other number of holes would lead her to knowing the answer. After she is told that the oldest child usually wears a red hat, she realizes that there is an "oldest" child and that the ages 1, 6, and 6 are not correct, which leaves the ages 2, 2, and 9 as the correct ages.

Review Exercises

1. 3 4 6 7 9 10 12 13 15 16

+1 +2 +1 +2 +1 +2 +1 +2 +1 +2 +1 +2

The next three numbers are 18, 19, and 21.

2. 2 7 4 9 6 11 8 13

+5 −3 +5 −3 +5 −3 +5 −3 +5 −3

The next three numbers are 10, 15, and 12.

3. 4 z 16 w 64 t 256

×4 ×4 ×4 ×4

3 letters previous; 3 letters previous; 3 letters previous; 3 letters previous

The next three items in the sequence are q, 1,024, and n.

4. 20 A 18 C 15 F 11 J

−2 −3 −4 −5 −6

skip 1 letter; skip 2 letters; skip 3 letters; skip 4 letters

The next three items in the sequence are 64, O, and 0.

5. There are four circles with solid-circle centers then two squares with open-circle centers. On the outsides of the figures there are little lines at the top, right, bottom, left, top, and then right. We could reasonably expect the next figure to be  .

6. The figures are all stars, each with one "leg" of the star shaded. The shading starts at the top leg and moves clockwise around the legs of the stars. There are numbers above the stars in the sequence 1, 7, 13, and 19, so each number is 6 more than the last. We could reasonably expect the next figure to be 

.

7. 5(7)(11) = 385, which is odd.

8. 2(5) + 4(5) + 6(5) = 10 + 20 + 30 = 60, which does not end in a 5.

9. Approach: Induction

Original number:	2	12
Add 6:	$2+6=8$	$12+6=18$
Divide by 2:	$8\div 2=4$	$18\div 2=9$
Add 10:	$4+10=14$	$9+10=19$
Result:	$14=\frac{1}{2}(2)+13$	$19=\frac{1}{2}(12)+13$

Conjecture: The final answer is 13 more than $\frac{1}{2}$ of the original even number.

Approach: Deduction

Pick a number:	x
Add 6:	$x+6$
Divide by 2:	$\frac{1}{2}(x+6)=\frac{1}{2}x+3$
Add 10:	$\frac{1}{2}x+3+10=\frac{1}{2}x+13$
Result:	$\frac{1}{2}x+13$

The final answer is always 13 more than $\frac{1}{2}$ of the original even number.

10. Approach: Induction

Original number:	5	9
Multiply by 9:	$9(5) = 45$	$9(9) = 81$
Add 18:	$45 + 18 = 63$	$81 + 18 = 99$
Divide by 3:	$63 \div 3 = 21$	$99 \div 3 = 33$
Subtract 6:	$21 - 6 = 15$	$33 - 6 = 27$
Result:	$15 = 3(5)$	$27 = 3(9)$

Conjecture: The final answer equals 3 times the original number.

Approach: Deduction

Pick a number:	x
Multiply by 9:	$9x$
Add 18:	$9x + 18$
Divide by 3:	$(9x + 18) \div 3 = 3x + 6$
Subtract 6:	$3x + 6 - 6 = 3x$
Result:	$3x$

The final answer is always 3 times the original number.

11. Inductive reasoning suggests the next two equations are:

$337 \times 3(4) = 337 \times 12 = 4{,}044$

$337 \times 3(5) = 337 \times 15 = 5{,}055$

12. Inductive reasoning suggests the next two equations are:

$33{,}333 \cdot 33{,}333 = 1{,}111{,}088{,}889$

$333{,}333 \cdot 333{,}333 = 111{,}110{,}888{,}889$

13. Inductive reasoning suggests that $\sqrt{1+3+5+7+9+11+13+15+17} = 9$

14. I'll try some numbers for conjecture (a): I pick a few numbers that are divisible by 3: 15, 27 and 81. The squares of these numbers are 225, 729 and 6,561. Since 225, 729, and 6,561 are all divisible by 3, it appears as if conjecture (a) seems to be true. Even though 3 examples doesn't really prove anything for certain, I could stop now since the problem said that one conjecture is true and the other is false. But I'm pretty sure the guy who wrote this problem wants me to find a counter-example to (b) so that I can "prove" that (b) is false and that will prove that (a) is definitely true. $10^2 = 100$, $11^2 = 100$, $12^2 =$. . . wait a minute, I should probably start with the biggest two digit number. $99^2 = 9{,}801$. Since the square of 99 has four digits, conjecture (b) is false and I now know for certain that conjecture (a) is true.

15. Inductive

16. Deductive

17. Deductive

18. Inductive

19. In the number 132,356, the 2 is the digit being rounded. Since the digit to the right is 3, the digit 2 remains the same and the digits to the right are replaced by zeros. The rounded number is 132,000.

20. In the number 186.75, the 6 is the digit being rounded. Since the digit to the right is 7, 1 is added to the 6 and the digits to the right are dropped. The rounded number is 187.

21. In the number 14.63157, the 5 is the digit being rounded. Since the digit to the right is 7, 1 is added to the 5 and the 7 is dropped. The rounded number is 14.6316.

22. In the number 0.6314, the 6 is the digit being rounded. Since the digit to the right is 3, the digit 6 remains the same and the digits to the right are dropped. The rounded number is 0.6.

23. In the number 3,725.63, the 2 is the digit being rounded. Since the digit to the right is 5, 1 is added to the 2, the digit 5 to the right is replaced by zero and the digits 3 and 6 are dropped. The rounded number is 3,730.

24. Round the cost of each lawnmower to \$330.00. Then $4 \times \$330.00 = \$1{,}320.00$. The estimated cost of the lawnmowers is \$1,320.

25. Round the costs for the books as follows (for convenience in adding):

\$115.60	→	\$115
\$89.95	→	\$90
\$29.95	→	\$30
\$62.50	→	\$60
\$43.10	→	\$45
		\$340

The books cost an estimated \$340.

26. Round 19.7 miles per gallon to 20 and round 364 gallons of gas to 360. Then the tank will hold approximately 360/20 = 18 gallons of gas.

27. Round the cost of the tickets for those older than 12 to \$60 and round the cost of the tickets for those 12 and under to \$50. Then the cost for the family will be approximately 4(60) + 2(50) = \$340.

28. (a) Round the number of credits to 10, then the student's cost will be approximately 689(10) or $6,890.

(b) Round the student's pay to $10 per hour, so the student makes about $300 dollars per week. Round the cost of attending the university for the semester to $6,900 then divide 6,900 by 300. It will take her approximately 23 weeks to afford one semester.

29. Round the cost of the T-shirts to $20 and round the cost of the sweatpants to $17. Since my brain got fried during finals week, I better make an organized list to keep track of all of the possibilities spending as close to $130 as possible.

T-shirt	6	5	4	3	2	1	0
Pants	0	1	2	4	5	6	7

30. The percentage of people who plan to re-gift is 32% and the percentage of people who plan to sell is 14%. So, a total of 46% are going to use one of these options. 46% of 1000 people are $1000 \times 0.46 = 460$ people.

31. The percentage of people who plan to return the gift is 31%. The percentage of people who plan to donate the gift is 23%. So 8% more people plan to return than plan to donate. 8% of 1000 people is $1000 \times 0.08 = 80$ people.

32. The only people who will keep the gift are the 14% with a conscience who hide it until the gift-giver visits. If only 14% are going to keep it, then 86% are going to get rid of it. 86% of 1000 people ares $1000 \times 0.86 = 860$ people.

33. The total of all of the percentages on the graph is 114%. So, the number of responses given is greater than the number of people who took the survey, meaning that some people gave more than one response.

34. Locate the year 1988 between 1985 and 1990 on the horizontal axis and move up to the line on the graph. At this point, move horizontally to the vertical axis. We estimate that the average weekly salary in 1988 was about $350.

35. Locate $400 on the vertical axis and move left to the point of intersection on the graph. At this point, move vertically down to the horizontal axis. Estimate that the weekly salary went over $400 in 1995.

36. Locate 1970 on the horizontal axis and estimate that the weekly salary was $125. Locate 2010 on the horizontal axis and estimate that the weekly salary was $650. The change in salary is 650 – 110 = $540, and the change in years is 2010 – 1970 = 40 years. So the average rate of change in salary is 540/40 = $13.50 a week per year.

37. The graph looks steeper from the years 2000 to 2010 than it does from the years 1985 to 1995, so I would guess that the average rate of change is greater from 2000 to 2010. From 2000 to 2010, the salary went from about $475 to about $650, so the actual rate of change is about (650 – 475) / 10 = 17.5 or 18 dollars per year. From 1985 to 1995, the salary went from about $300 to about $400, so the actual rate of change is about (400 – 300)/10 = 10 dollars per year.

38. When I did problem 36, I found that the average rate of change per year was $13.50 per week. So, if the weekly salary in 2010 is 645, I would estimate that the weekly salary in 2012 is 625 + 2(13.50) = $652. Answers will vary on internet research.

39. Reread this one to see if it was a trick question. Nope. If she gave away all but nine, she has 9 left.

40. **Step 1** *Understand the problem*. The team played 40 matches, winning some and losing some. The team won 20 more matches than they lost. Find how many matches were lost.

Step 2 *Devise a plan to solve the problem*. The number of matches won plus the number of matches lost equals the total number of matches played. Let x represent the number of matches lost. Use subtraction to solve for x.

Step 3 *Carry out the plan to solve the problem*. If x represents the number of matches lost, then $x + 20$ is the number of matches won and

$$\begin{aligned} x + x + 20 &= 40 \\ 2x + 20 &= 40 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

The team lost 10 matches.

Step 4 *Check the answer*. The team lost 10 matches and won 10 + 20 = 30 matches. Then the number of matches lost plus the number of matches won equals the total number of matches played: 10 + 30 = 40.

41. Iesha's weight while standing on one foot is not supported by anything else, her weight on the scale does not change when the person steps on the scale with both feet. So the person still weighs 110 pounds.

42. **Step 1** *Understand the problem*. The total cost for the two items is $3.40 and the mocha latte cost $0.40 more than the biscotti. We need to find the cost of each item.

Step 2 *Devise a plan to solve the problem*. We'll let the cost of the biscotti be x, then the cost of the mocha latte is $x + 0.40$. Set the sum of the two equal to $3.40 and solve for x.

Step 3 *Carry out the plan to solve the problem*.

$$x + x + 0.40 = 3.40$$
$$2x + 0.40 = 3.40$$
$$3x = 3.00$$
$$x = 1.50$$

The cost of the biscotti is $1.50 and the cost of the mocha latte is $1.90.

Step 4 *Check the answer*. $1.50 + $1.90 = $3.40.

43. If the boards are laid across the width of the deck, you are going to maximize the number of gaps that you leave and that will also minimize the amount of wood used. Each board is going to take up 6 inches, including the gap. With 40 feet of length to span, that is going to require $0.5 \times 40 = 80$ boards. Each board is going to be 10 feet long, so that means the total length of board needed is $80 \times 10 = 800$ feet. Since each board purchased is 12 feet long, the number of boards is 800/12 = 66.67 or 67 boards will be needed to complete the job.

44. **Step 1** *Understand the problem*. Mary has $80 and spent $8. She then spent $\frac{1}{3}$ of the remainder on tickets for the dance and we want to know what she had left.

Step 2 *Devise a plan to solve the problem*. Subtract the $8 she spent on songs, then take $\frac{1}{3}$ of the remainder and subtract that value from the remainder to see what she had left.

Step 3 *Carry out the plan to solve the problem*. $(80 - 8) = 72$, $\frac{1}{3}(72) = 24$, $72 - 24 = 48$. So she had $48 left.

Step 4 *Check the answer*. $48 + \frac{1}{3}(72) + 8 = 48 + 24 + 8 = 80$.

45. **Step 1** *Understand the problem*. The total for your text and lab packet was $120. The text cost twice as much as the lab packet. Find how much the lab packet cost.

Step 2 *Devise a plan to solve the problem*. Let x represent the cost of the lab packet, then $2x$ is the cost of the text. Add these values and set them equal to $120, then solve for x.

Step 3 *Carry out the plan to solve the problem*.

$$x + 2x = 120$$
$$3x = 120$$
$$x = 40$$

The lab packet cost $40.

Step 4 *Check the answer*. $40 + 2(40) = 40 + 80 = 120$

46. A few possible answers are: $1{,}024 \times 38 = 38{,}912$; $2{,}024 \times 34 = 68{,}816$; $2{,}024 \times 39 = 78{,}936$.

47. **Step 1** *Understand the problem*. We want to find my house's age now, given that 10 years from now it will be five times as old as it was 10 years ago.

Step 2 *Devise a plan to solve the problem*. Make a list of possible combinations of ages for the house and see if a pair satisfies the conditions in the problem.

Step 3 *Carry out the plan to solve the problem*

House's Age Now	House's Age 10 Years Ago	House's Age 10 Years from Now
10	0	20
11	1	21
12	2	22
13	3	23
14	4	24
15	5	25
		↑ correct

My house is 15 years old now.

Step 4 *Check the answer*. 10 years ago my house was 15 – 10 = 5 years old. 10 years from now my house will be 15 + 10 = 25 years old, which is five times 5, my house's age 10 years ago.

48. **Step 1** *Understand the problem*. If there are 18 more hours of daylight than hours of darkness, find how many hours of daylight and hours of darkness there are in a 24-hour period.

Step 2 *Devise a plan to solve the problem.* Make a list of possible hours of daylight and darkness where the hours of daylight are 18 more than the hours of darkness. Find the combination that gives a total of 24 hours.

Step 3 *Carry out the plan to solve the problem*.

Hours of Darkness	Hours of Daylight	Total Hours
1	19	20
2	20	22
3	21	24 ← correct

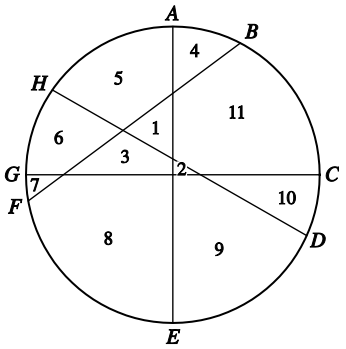
Answer: At times during the summer, Alaska has a 3-hour night and a 21-hour day.

Step 4 *Check the answer*. 3 + 18 = 21; 3 + 21 = 24

49. $2 \times 9 + 6 - 7 = 17$

50. Joe has done 2 problems and Tina has done 3. Since 2 + 1 = 3 if Joe does one more he'll have done the same numbers Tina; also if Tina does one more she will have done 3 + 1 = 4 which is twice the number Joe has done.

51.



The cuts are *AE*, *BF*, *CG*, and *DH*.

52. There are three sizes of triangles in the figure. There are 9 of the smallest triangles, 3 triangles each made up of 4 of the smallest triangles, and 1 triangle made up of all of the smallest triangles. Thus, there are 9 + 3 + 1 = 13 triangles in the figure.

53. I really doubt that a fly can fly faster than a train, but we will go with it. Since the trains are heading towards each other, they are covering 40 miles of the distance in between them each hour. So, it will take 200/40 = 5 hours before they collide. In that 5 hours, the fly can fly 60 × 5 = 300 miles.

54. **Step 1** *Understand the problem*. Find two numbers that when added equal 120 and when subtracted equal 15.

Step 2 *Devise a plan to solve the problem*. Make a list of pairs of numbers that have a difference of 15. Find the pair whose sum is 120.

Step 3 *Carry out the plan to solve the problem.*

First Number	Second Number	
40	55	
45	60	
50	65	
51	66	
52.5	67.5	← correct

The numbers are 52.5 and 67.5.

Step 4 *Check the answer*. 52.5 + 67.5 = 120; 67.5 – 52.5 = 15

55. Step 1 *Understand the problem*. The store charges $2.00 a pound for high-protein nature mix, and $2.75 a pound for low-carb soy medley. There are ten pounds total and the cost was $24.50. Find the amount of each type of mix.

Step 2 *Devise a plan to solve the problem.* Make a list of possible amounts of each type of mixture so that the total is 10 pounds then calculate the total cost.

Step 3 *Carry out the plan to solve the problem.*

Nature mix	Soy medley	Total cost	
6	4	$23.00	
5	5	$23.75	
4	6	$24.50	←correct

There are 4 pounds of the nature mix and 6 pounds of the soy medley in the mixture.

Step 4 *Check the answer*. 4(2) + 6(2.75) = 8 + 16.5 = $24.50.

56. Step 1 *Understand the problem*. Taylor invested part of $1,000 in something that earned 8% simple interest and the rest of the $1,000 in something that earned 6% simple interest. After 1 year, he had earned $76.00 in interest. Find the amount invested at each rate.

Step 2 *Devise a plan to solve the problem.* Make a list of possible combinations of investments that add up to $1,000. Find the combination that gives $76.00 in simple interest. Use the fact that simple interest equals the principal times the rate times the length of time.

Step 3 *Carry out the plan to solve the problem*

Amount Invested at 8%	Amount Invested at 6%	Interest Earned on 8% Investment	Interest Earned on 6% Investment
$200	$800	$16	$48
$400	$600	$32	$36
$600	$400	$48	$24
$700	$300	$56	$18
$800	$200	$64	$12← correct

Taylor invested $800 at 8% and $200 at 6%.

Step 4 *Check the answer*.
$800 + $200 = $1,000; ($800)(0.08)(1) + ($200)(0.06)(1) = $64 + $12 = $76.

Chapter Test

* When using estimation, other correct answers are possible.

1. 2 5 4 8 6 11 8

+3 –1 +4 –2 +5 –3 +6 –4 +7

The next three numbers are 14, 10, and 17.

2. Kansas, Louisiana, Maine, Nebraska, Ohio, . . .

The states begin with the letters: K, L, M, N, O. In the case where there is more than one state that begins with the letter, it is the state that comes first alphabetically that is listed. The next letter is P, and the only state that begins with P is Pennsylvania.

3. Following the pattern started:

$9{,}876 \cdot 9 + 4 = 88{,}888$

$98{,}765 \cdot 9 + 3 = 888{,}888$

$987{,}654 \cdot 9 + 2 = 8{,}888{,}888$

So by inductive reasoning, $9{,}876{,}543 \cdot 9 + 1 = 88{,}888{,}888$

4. By inductive reasoning $6{,}666 \times 6{,}667 = 44{,}442{,}222$

5. Approach: Induction

Original number:	2	7
Add 10:	$2+10=12$	$7+10=17$
Multiply by 5:	$12\times5=60$	$17\times5=85$
Add 15:	$60+15=75$	$85+15=100$
Divide by 5:	$75\div5=15$	$100\div5=20$
Result:	$15=2+13$	$20=7+13$

Conjecture: The final answer equals the original number plus 13.

Approach: Deduction

Pick a number:	x
Add 10:	$x+10$
Multiply by 5:	$5(x+10)=5x+50$
Add 15:	$5x+50+15=5x+65$
Divide by 5:	$(5x+65)\div5=x+13$
Result:	$x+13$

The final answer is equal to the original number plus 13.

6. If all but two changed their minds there were two left in line.

7. **Step 1** *Understand the problem*. The worker earns twice as much as the previous day for a total of 12 days of work. The last day the person earned $204.80. Find which day the workers received $25.00, how much they received on the first day, and the total amount of the bonuses.

Step 2 *Devise a plan to solve the problem*. Since the amount earned on a given day is 2 times the amount earned the previous day, the amount earned the previous day is half the amount earned the given day. Starting with $204.80 for the tenth day, work back in time dividing the day's earnings by 2, until you reach the first day. Use the table to answer all questions.

Step 3 *Carry out the plan to solve the problem.*

Day of Work	Amount Earned
12th	$204.80
11th	$102.40
10th	$51.20
9th	$25.60
8th	$12.80
7th	$6.40
6th	$3.20
5th	$1.60
4th	$0.80
3rd	$0.40
2nd	$0.20
1st	$0.10

Answers: (a) On the ninth day the workers earned about $25.00. (b) on the 1st day, the workers earned a measly dime – is this guy's name Ebeneezer Scrooge? and (c) by adding up all of the numbers in the second column of the table, the workers earned a total of $409.50.

Step 4 *Check the answer*. Check the table as well as calculations for accuracy.

8. The letters T, T, F, F, S, S, ... are the first letters of the words Two, Three, Four, Five, Six, Seven, ... so the next two letters are E (for eight) and N (for nine).

9. Move the coin second from the left to the right of the rightmost coin as illustrated. You can also more the upper left coin under the bottom left coin.

10. Since the boat is floating on the water, then length of the ladder above the waterline doesn't change – nine feet of the ladder are above the waterline.

11. Let x represent the age of the man when he died. Then

Boyhood lasted $\frac{1}{6}x$ years

Beard grew at age $\frac{1}{12}x+\frac{1}{6}x$

Married at age $\frac{1}{7}x+\frac{1}{12}x+\frac{1}{6}x$

Son born at man's age $5+\frac{1}{7}x+\frac{1}{12}x+\frac{1}{6}x$

Son died at man's age

$\frac{1}{2}x+5+\frac{1}{7}x+\frac{1}{12}x+\frac{1}{6}x$

Man died at age

$$x=4+\frac{1}{2}x+5+\frac{1}{7}x+\frac{1}{12}x+\frac{1}{6}x$$

$$x=\frac{25}{28}x+9$$

$$\frac{3}{28}x=9$$

$$x=84$$

The man was 84 when he died.

12. Let x represent the number. Then

$x\div(x-3)=\frac{8}{5}$. Solve for x: $\frac{x}{x-3}=\frac{8}{5}$

$x=8$

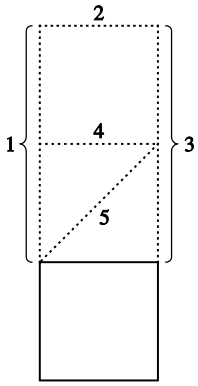
13. Let x represent the number. Then

$\frac{1}{2}x+\frac{1}{3}x=10$. Solve for x: $\frac{1}{2}x+\frac{1}{3}x=10$

$\frac{5}{6}x=10$

$x=12$

14. Add five lines where the dotted lines are shown.



15. **Step 1** *Understand the problem*. Two people work for 3 hours and 2 hours, respectively. The total amount earned is \$60.00. Find how much each person earned for the work.

Step 2 *Devise a plan to solve the problem*. The total time spent was 2 + 3 = 5 hours, and the total amount earned was \$60.00. Find the hourly rate by dividing, then find the amount earned by each person by multiplying by the number of hours worked by each person.

Step 3 *Carry out the plan to solve the problem*.

Hourly rate = \$60.00 ÷ 5 hours

= \$12 per hour

First person earns 3 × \$12 = \$36.

Second person earns 2 × \$12 = \$24.

Step 4 *Check the answer*. \$36 + \$24 = \$60

16. Let x and y be the two numbers. Then

$\frac{1}{x}+\frac{1}{y}=\frac{5}{6}$ and $\frac{1}{x}-\frac{1}{y}=\frac{1}{6}$. Solve for x and y.

Use trial-and-error, substituting different numbers for x and y until a solution is found.

Let x = 2 and y = 3. Then $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

and $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$.

17. **Step 1** *Understand the problem*. Sam has taken 2 exams and scored 72% and 78%. Find the score Sam needs on her third exam to have an average exam score of 80%. If she has an 80% average going into the last exam, what would she need to score to have a final average of between 90% and 92%?

Step 2 *Devise a plan to solve the problem*. Make a list of possible third exam scores. Find the average exam score by adding the three exam scores and dividing by three. Find the third exam score that gives an average of 80%. Repeat the problem with possible fourth exam scores. Find the average exam score by adding the fourth exam scores and dividing by four. Find the fourth exam score that gives at least a 90% average.

Step 3 *Carry out the plan to solve the problem.*

Third Exam	**Average**	
84%	78%	
86%	78.7%	
88%	79.3%	
90%	80%	← correct

Answer: Sam needs a 90% on the third exam.

Fourth Exam	**Average**	
90%	82.5%	
95%	83.75%	
100%	85%	
120%	90%	← correct

Step 4 *Check the answer*. 72 + 78 + 90 = 340; 342/ 3 = 80. Unless there is some serious extra credit on that exam, there is no way possible for Sam to get an A–.

18. Think of heights and depths above and below sea level as positive distances from a horizontal line. Add the positive distances to find the total distance. 20,300 feet + 280 feet = 20,580 feet. The adventurer would have an average rate of change of 20,580/3 = 6,860 feet per day.

19. Let x be Mark's age. Then Mark's mother is $x + 32$ years old and $x + x + 32 = 66$. Solve for x and find $x + 32$:

$$x + x + 32 = 66$$
$$2x = 34$$
$$x = 17$$
$$x + 32 = 49$$

Mark is 17 years old and his mother is 49.

20. In the number 1,674,253, the 6 is the digit being rounded. Since the digit to the right is 7, 1 is added to the 6 and the digits to the right are replaced by zeros. The rounded number is 1,700,000.

21. In the number 1.3752, the 7 is the digit being rounded. Since the digit to the right is 5, 1 is added to the 7 and the digits to the right are dropped. The rounded number is 1.38.

22. Round the blazer to \$70, the tie to \$33 and the pants to \$43. The cost is approximately 70 + 33 + 43 = \$146.

23. (a) Locate the year 1980 on the horizontal axis and move up to the corresponding point on the graph, then move horizontally to the vertical axis. Estimate that the average number of hours per week in 1980 was about 35.5 hours.

Locate the year 2005 on the horizontal axis and move up to the corresponding point on the graph, then, move horizontally to the vertical axis. We estimate that the average number of hours per week in 2005 was about 33.8 hours.

(b) Locate 35 hours worked on the vertical axis and move right to the corresponding point on the graph, then move vertically down to the horizontal axis. Estimate that the average hours worked dropped below 35 in about 1984.

(c) Estimate that the average hours worked per week in 1975 was 36. Estimate that the average hours worked per week in 2000 was 34.5. The number of years between 1975 and 2000 is 25. The rate of change is (34.5 – 36)/25 = –.06 hours per year.

24. Since 48.8% live in a residence hall, 51.2% of the students do not live in a residence hall: 51.2% of 3,646 is $3{,}646 \times 0.152 = 1{,}866.752$, so about 1,867 do not live in a residence hall. Since 8.5% live with their parents and 1.3% live in a frat or sorority house, there are 7.2% more students who live with their parents than in a frat or sorority house: $3{,}646 \times .072 = 262.5$, so about 263 students.

25. Detroit's population is the highest of the three cities. Since the rates are almost the same, this means that Detroit had the most homicides.

26. Calculate the total number of homicides for each city by using the rate provided on the bar graph and the population provided in the table. For example, the number of homicides in Baltimore is 639,929 people which is 639,929/100,000 = 6.4 hundred thousand people. If the number of homicides is 34 per 100,000 people, then the total number of homicides is 6.4 × 34 = 217.6 or 218.

City	Total Homicides
Baltimore	218
Detroit	306
New Orleans	174
Newark	90
St. Louis	142

Using the table, Detroit had the most homicides and Newark had the least homicides.

27. Since Chicago has 2,833,649 people, they have 2,833,649/100,000 = 28.34 hundred thousand people. So, to find the number of homicides per hundred thousand people, divide the 435 homicides by 28.34: 435/28.34 = 15.34. Create a bar on the graph for Chicago which has a bar height between the 10 and 20 on the vertical axis.