MATLAB EXERCISE 1.20 **Numerical proof that E-field is conservative – movie.** By numerical integration in MATLAB, prove that the circulation (line integral) of the electric field intensity vector due to a point charge Q in free space along a square contour with the charge at its center is zero. *(ME1_20.m on IR)*

SOLUTION:

A snapshot (frame) of the MATLAB movie is shown in Fig.S1.12.

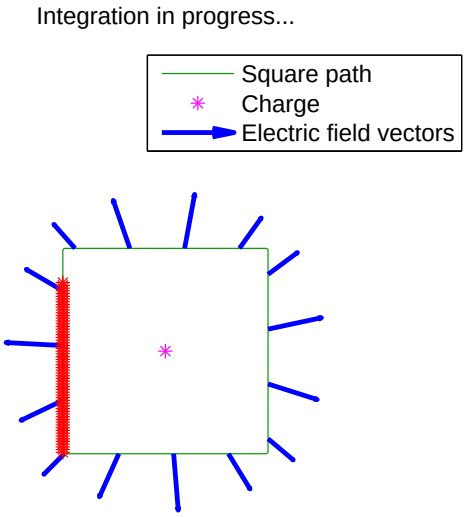

Figure S1.12 Snapshot of the MATLAB movie visualizing the process of numerical integration (line integral) of the electric field vector due to a point charge along a square contour with the charge at its center; for MATLAB Exercise 1.20.

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for
%   educational purposes associated with the book
%
%
%


%   Numerical proof that E-field is conservative -- movie


clear all;
close all;
EPS0 = 8.8542*10^(-12);

Q = input('\nEnter the charge in Coulombs for point at the center of the square: ');
a = input('Enter the lenght in meters of square side : ');

% Division of square sides into N segments
N = 300; % It should be 4*k, where k is integer
n = N/4; % each side
delta = a/n;

% Coordinates of patch centers
temp = -a/2+delta/2 : delta : a/2-delta/2;
x = [-a/2*ones(1,n),temp,a/2*ones(1,n),-temp];
y = [temp,a/2*ones(1,n),-temp,-a/2*ones(1,n)];

    r = sqrt(x.^2 + y.^2);  % Distance of the center of the patch from the charge point
    ux = x./r;  % X component of unit vector of radius vector
    uy = y./r;  % Y component of unit vector of radius vector

% dl vector for integration
dx = [zeros(1,n),delta * ones(1,n),zeros(1,n),-delta * ones(1,n)] ;
dy = [delta * ones(1,n),zeros(1,n),-delta * ones(1,n),zeros(1,n)];
dl = [dx;dy];

% Integration
Int = 0;
plot(x,y,'g');
hold on;
plot(0,0,'m*')
hold on;
for i = 1:N
    E(:,i) = Q/r(i)^2/(4*pi*EPS0)*[ux(i),uy(i)];  % Electric field vector
    if mod(i,20)==0
    vecPlot2D([x(i),y(i)],[x(i)+E(1,i),y(i)+E(2,i)],abs(1/E(1,1))/10,'b',0);
```

```
    hold on;
    end;
end;
legend('Square path' ,'Charge', 'Electric field vectors');
title('Integration in progress...');
for i = 1:N
    Int = Int + dotProduct(E(:,i),dl(:,i));
    %movie
    plot(x(i),y(i),'r*');
    hold on;
    axis equal;
    axis off;
    if i==N
        title(['Result of integral of electric field'...
            'along closed contour is 0!']);
    end;
    xlim([-1.5*a,1.5*a]);
    ylim([-1.5*a,1.5*a]);
    M(i)= getframe();
end
hold off;

Vvertex = Q/(4*pi*EPS0*a/2*sqrt(2));

fprintf(['\nPotential at square vertex due to charge Q'...
    'is %e V while circulation of E is %e V'],Vvertex,Int);
fprintf(['\nBeing aware of some numerical errors, we conclude that the'...
    'obtained result of integration is negligibly small -- zero' ]);
```