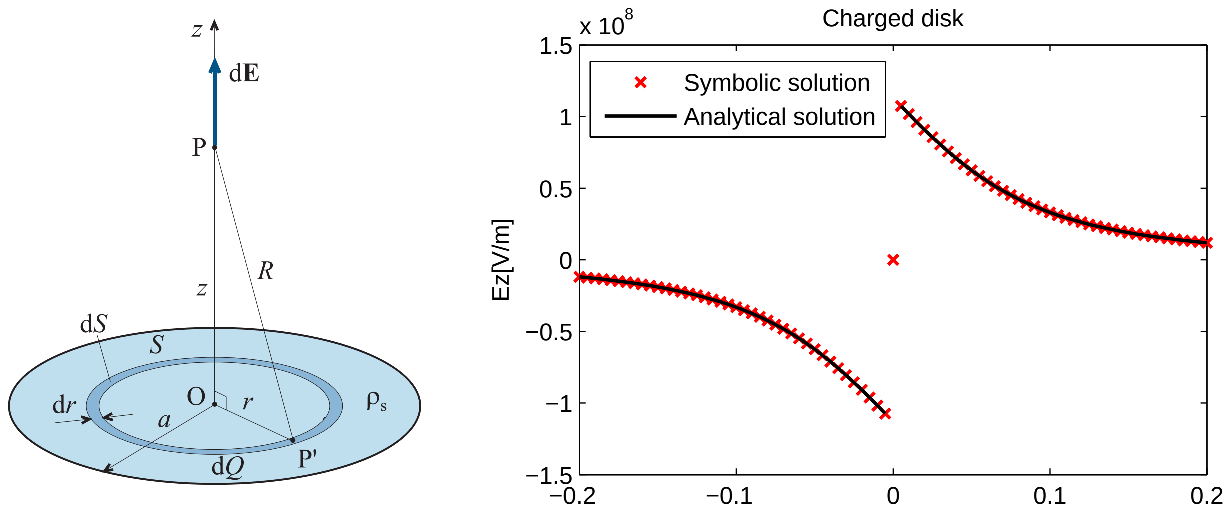
MATLAB EXERCISE 1.11 **Charged disk – symbolic and analytical solutions.** Consider a very thin charged disk (i.e., a circular sheet of charge), of radius a and a uniform surface charge density ρ_s, in free space, and the electric field it generates at a point P along the z-axis in Fig.S1.6(a). By subdividing the disk into elemental rings of width dr, as shown in Fig.S1.6(a), applying Eq.(S1.3) for the field at the point P due to a ring of radius r ($0 \le r \le a$) and charge $dQ = \rho_s \, dS$, with $dS = 2\pi r \, dr$ being the surface area of the ring (calculated as the area of a thin strip of length equal to the ring circumference, $2\pi r$, and width dr), and superposition, the total electric field vector is given by

$$\mathbf{E} = \int_S d\mathbf{E} = \int_S \frac{dQ z}{4\pi\varepsilon_0 R^3}\hat{\mathbf{z}} = \frac{\rho_s z}{2\varepsilon_0}\hat{\mathbf{z}} \int_{r=0}^{a} \frac{r\, dr}{R^3}, \quad R = \sqrt{r^2 + z^2}. \tag{S1.4}$$

Using this expression and function `integral` (written in the previous MATLAB exercise), compute $\mathbf{E}$ by symbolic integration. Also, solve the integral analytically and plot both solutions for $\rho_s = 2\ \mathrm{mC/m^2}$, $a = 10$ cm, and $-2a \le z \le 2a$. *(ME1_11.m on IR)*



(a) (b)

Figure S1.6 (a) Evaluation of the electric field due to a charged disk and (b) field intensity along the axis (z-axis) of the disk obtained by symbolic integration in Eq.(S1.4) using function `integral` (from the previous MATLAB exercise) and by analytical integration in Eq.(**??**), respectively; for MATLAB Exercise 1.11.

SOLUTION:

First, as explained in the TUTORIAL (in the book), we declare `r` [the radius of the elemental ring in Fig.S1.6(a)] and `z` (the range of z-coordinates along the disk axis at which the field is computed) as symbolic variables – in MATLAB, defining symbolic variables is done by command `syms`, so we type `syms r z` – and specify the values of constants ρ_s, a, and ε_0.

Then, we define the variable `R` in Fig.S1.6(a) and functions `f` and `t` constituting the field due to the elemental ring of charge, in Eq.(S1.4), to be integrated symbolically – invoking function `integral`. The result is the field `E`, which is then converted from the symbolic expression to a numerical value, by MATLAB function `double`, in order to plot it along the z-axis (the numerical variable is named `E1`, to distinguish it from its symbolic version).

The integral in Eq.(S1.4) is solved analytically in Eq.(1.18) (in the book). The field expression obtained by analytical integration is denoted in the program as `Ea`.

The two solutions and their mutual agreement are shown in Fig.S1.6(b).

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for educational purposes
%   associated with the book
%
%
%


%   Charged disk -- symbolic and analytical solutions


clear all;
close all;
syms r z
rho = 0.002;
a = 0.1;
EPS0 = 8.8542 * 10^(-12);
z = -2*a:0.05*a:2*a;
R = sqrt(r^2 + z.^2);
f = z./R.^3;
t = rho*2*r*pi/(4*pi*EPS0);
E = integral(f,t,r,0,a);
E1 = double(E);
Ea = rho/(2*EPS0)*(z./abs(z)- z./sqrt(a^2+z.^2));
Ea1 = double(Ea);
plot (z,E1,'rx',z,Ea1,'k','LineWidth',1.4);
xlabel('z[m]');
ylabel('Ez[V/m]');
legend('Symbolic solution','Analytical solution');
title('Charged disk');
```