MATLAB EXERCISE 1.12 **Charged hemisphere – numerical integration.** The purpose of this MATLAB exercise is to introduce numerical integration as the third (often the only available) way to solve integrals, besides symbolic MATLAB integration and analytical solutions. Consider the uniformly charged hemispherical surface in Fig.S1.7 and use MATLAB and numerical integration to compute the electric field intensity vector at an arbitrary point (for any z) along the z-axis. The simplest numerical integration formula is

$$\int_a^b f(x)\,\mathrm{d}x \approx \sum_{i=1}^{N} f(x_i)\Delta x\ , \quad \Delta x = \frac{b-a}{N}\ , \tag{S1.5}$$

where N denotes the number of integration segments (increments) and x_i ($i = 1, 2, \ldots, N$) are coordinates of centers of segments. *(ME1_12.m on IR)*

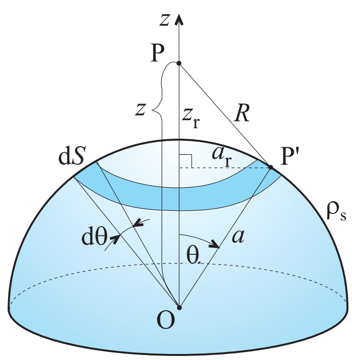

Figure S1.7 Evaluation of the electric field at an arbitrary point (P) along the axis of a hemispherical surface charge, carrying out numerical integration, in θ, in MATLAB; for MATLAB Exercise 1.12.

SOLUTION:

We solve the integral in θ in Eq.(1.22) (in the book) by numerical integration, according to Eq.(S1.5).

For instance, we compare the result for $\rho_s = 1\ \mu\text{C/m}^2$, $a = 1$ m, and the position of the field point along the hemisphere axis defined by $z = 2$ m (in Fig.S1.7), to the field expression obtained by analytical integration given in Eq.(1.23) (in the book). For the integration increment `dtheta` equal to $\pi/1000$ in the numerical integration `for` loop, the numerical result is $20,447.451209$ V/m, and the relative error of numerical integration, relative to the exact (analytical) solution, $20,431.635355$ V/m, amounts to 0.077409%.

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for educational purposes
%   associated with the book
%
%
%


%   Charged hemisphere -- numerical integration


clear all;
close all;
ros = input('Enter a uniform surface charge density in Coulombs per square meter: ');
a = input('Enter the radius of the hemisphere : ');
z = input('Enter the point position at z axis outside the hemisphere: ');
EPS0 = 8.854e-12; %Permittivity of Air

%Due to symmetry there is only z component of electric field
E = 0;
d_theta = pi/1000;

for theta = 0:d_theta:(pi/2)
    ds = 2*pi*a^2*sin(theta)*d_theta;
    dE = (ros/(4*pi*EPS0)*ds*(z - a*cos(theta)))/...
        (sqrt(z^2 + a^2 - 2*a*z*cos(theta)))^3;
    E = E + dE;
end
fprintf('Numerical result for the electric field at the point at z axis is %f V/m.\n', E);
% analytical formula
Ea = (ros*a^2)/(2*EPS0*z^2)*(a/(sqrt(z^2 + a^2)) + (z - a)/abs(z - a));
fprintf('Analytical result for the electric field at the same point is %f V/m.\n', Ea);
error = abs(Ea - E)*100/Ea;
fprintf(' Relative error in percent is %f\n', error);
```