MATLAB EXERCISE 1.16 **Fields due to line charges of finite and infinite lengths.** Write a MATLAB program that compares the electric field intensity E due to a straight line charge of finite length L to that due to an infinite line charge of the same uniform density Q' (charge per unit length) in free space, based on the following field expression, given for the situation in Fig.S1.10(a) and obtained applying Eq.(1.14) (from the book):

$$\mathbf{E} = \frac{Q'}{4\pi\varepsilon_0 D}\left[(\sin\theta_2 - \sin\theta_1)\,\hat{\mathbf{x}} + (\cos\theta_2 - \cos\theta_1)\,\hat{\mathbf{z}}\right] \quad \text{(line charge of finite length)}\,, \tag{S1.6}$$

where the geometry of the problem is defined by the perpendicular distance from the line charge to the field point, D, and angles θ_1 and θ_2 (for the particular position of the point P shown in Fig.S1.10(a), $\theta_1 < 0$ and $\theta_2 > 0$). Note that a numerical solution for this case is carried out in MATLAB Exercise 1.13. The field should be computed along the symmetry line of the finite line charge that is perpendicular to it, so that the vector $\mathbf{E}$ is radial with respect to the charge. The input data are L and Q'. As an output, the program plots the dependence of E on the distance D from each of the line charges, and finds the distance $D_{\max}$ beyond which the relative error when approximating the finite charge by an infinite one is above 5% or, equivalently, below which the line charge of length L appears, in terms of the electric field it produces, as infinite (within the 5% error margin). *(ME1_16.m on IR)*

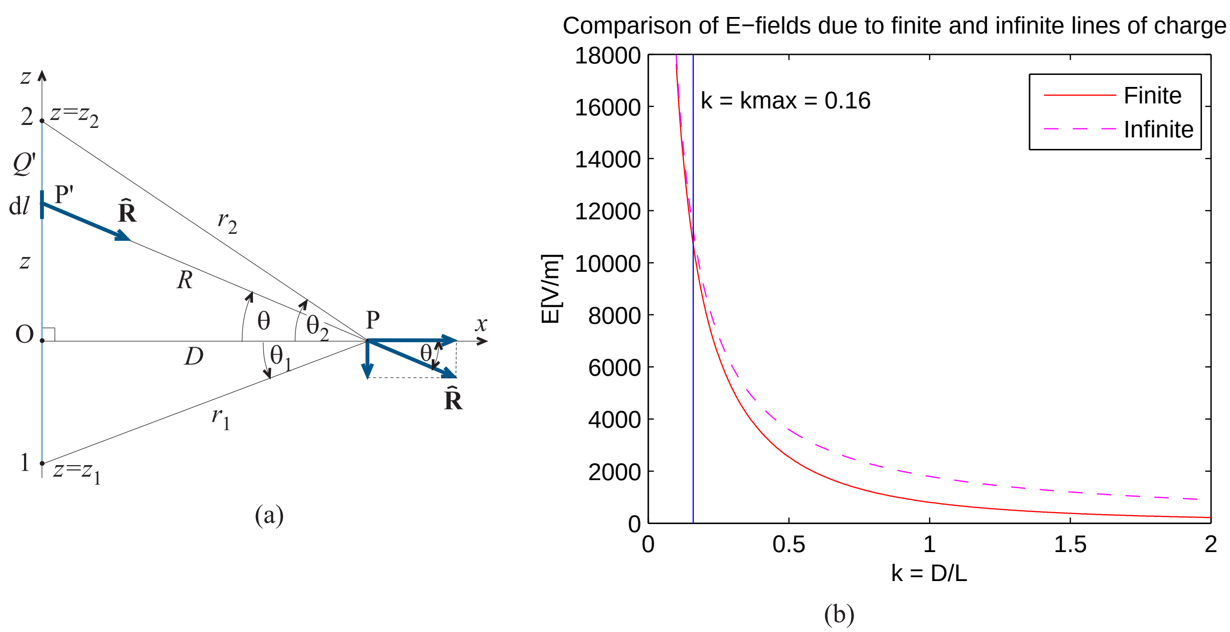

Figure S1.10 (a) Analytical evaluation of the electric field vector due to a straight line charge of finite length and (b) comparison of field intensities, calculated using MATLAB and Eq.(S1.6) (above) and Eq.(1.25) (from the book), due to line charges of finite ($L = 1$ cm) and infinite lengths and determination of the distance $D_{\max}$ ($k_{\max} = D_{\max}/L$) from the charges beyond which the relative error when approximating the finite charge by an infinite one is above 5%; for MATLAB Exercise 1.16.

SOLUTION:

The expression for the electric field vector due to an infinite line charge of uniform density Q' is given in Eq.(1.25) (in the book).

Upon implementing Eq.(S1.6) (above) and Eq.(1.25) (from the book) in MATLAB, Fig.S1.10(b) shows the respective field distributions and comparison for $L = 1$ cm and $Q' = 1$ nC/m, where it turns out that $D_{\max} = 0.16L = 1.6$ mm.

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for
%   educational purposes associated with the book
%
%
%


%   Fields  due to line charges of finite and infinite lengths


close all;
clear all;
EPS0 = 8.854*10^(-12);

L = input('Enter the length of line in cm: ');
L = L * 10^(-2);
dens = input('Enter the line charge in nC/m: ');
dens = dens * 10^(-9);
dD = 0.005*L;
D0 = 0.1*L;
D1 = 2*L;
D = D0:dD:D1;
k = D/L;
midpoint = L / 2;
theta2 = atan( midpoint./D);
theta1 = -theta2;

%Electric Field

EFinite = (dens./(4*pi*EPS0*D)).*(sin(theta2)-sin(theta1));

%Electric Field due to infinite line charge

EInfinite = dens./(2*pi*EPS0*D);

%relative error
Error = abs(EFinite - EInfinite)*100./EFinite;
ErrorMax = 5;

Emax = max(EInfinite);
[delta,n] = min(abs(Error - ErrorMax));
kmax = (D0+(n-1)*dD)/L;
disp('The Error of 5% is at D/L = kmax =');
disp(kmax);

figure(1);
```

```
plot(k,EFinite,'r',k,EInfinite,'m--');
line([kmax kmax],[0 Emax]);
legend('finite','infinite');
title('Comparison of E-fields due to finite and infinite lines of charge');
text(kmax,Emax*0.9,['k = kmax = ',num2str(kmax)]);
xlabel('k = D/L');
ylabel('E[V/m]');
```