MATLAB EXERCISE 1.28 **Direction of the steepest ascent.** The terrain elevation in a region is given by a function $h(x, y) = 100x \ln y$ [m] (x, y in km), where x and y are coordinates in the horizontal plane and 1 km $\leq x, y \leq$ 10 km. Write a code in MATLAB [use function `gradCar` (from MATLAB Exercise 1.26)] to answer the following questions: What is the direction of the steepest ascent at (3 km, 3 km)? How steep, in degrees, is that ascent? *(ME1_28.m on IR)*

SOLUTION:

As the gradient of a scalar function at a location in space points in the direction in which the function increases the most at that location, the direction of the steepest ascent (maximum increase of the terrain elevation, h) at the location (x, y) is given by $\text{grad}\, h = \nabla h$, for which we use function `gradCar` (from MATLAB Exercise 1.26). We first transform the expression for $h(x, y)$ such that x and y are entered in meters:

$$h(x, y) = 0.1x \ln(0.001y) \text{ m} \quad (x, y \text{ in m}) . \tag{S1.11}$$

Note that MATLAB uses "log" in place of "ln" to denote the natural logarithm.

The MATLAB code gives, symbolically,

$$\nabla h(x, y) = 0.1 \ln(0.001y)\ \hat{\mathbf{x}} + 0.1 \frac{x}{y}\ \hat{\mathbf{y}} \quad (x, y \text{ in m}) . \tag{S1.12}$$

At the specified location, (3000 m, 3000 m), the unit vector $\hat{\mathbf{l}}$ ($|\hat{\mathbf{l}}| = 1$) defining this direction (of the steepest ascent) turns out to be, from the code,

$$\hat{\mathbf{l}} = \frac{\nabla h}{|\nabla h|} = 0.7395\ \hat{\mathbf{x}} + 0.6731\ \hat{\mathbf{y}} . \tag{S1.13}$$

The maximum space rate of increase in the function h per unit distance is equal to the magnitude of the vector ∇h at that location [$(\text{d}h/\text{d}l)_{\max} = |\nabla h|$]. Hence, the ascent at the point (3000 m, 3000 m), expressed as an angle α (in degrees), amounts to

$$\left.\frac{\Delta h}{\Delta l}\right|_{\max} = \left.\frac{\text{d}h}{\text{d}l}\right|_{\max} = |\nabla h| \quad \longrightarrow \quad \alpha = \arctan \frac{\Delta h}{\Delta l} = \arctan |\nabla h| \tag{S1.14}$$

($\arctan \equiv \tan^{-1}$, `atan` in MATLAB), which results in $\alpha = 8.45°$.

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for
%   educational purposes associated with the book
%
%
%


%  Direction of the steepest ascent


clear all;
close all;
syms x y

h = 0.1*x*log(0.001*y);  % x, y in meters
[dirx,diry] = gradCar(h);
dir = [dirx,diry];
fprintf('\nExpression for grad h (x, y in meters) is: ');
pretty(dir);
%a, b in meters
a = 3000;
b = 3000;
direction = double(limit(limit(dir,x,a),y,b));
unit = direction/vectorMag(direction);
fprintf('\nDirection of steepest ascent (unit vector) is: [%.4f %.4f]',unit);
%b
angle = atan(vectorMag(direction))*180/pi;
fprintf('\nAscent angle in degrees is: %.2f',angle);
```