**MATLAB EXERCISE 2.21** **Breakdown in a spherical capacitor with a multilayer dielectric.** Consider a spherical capacitor with $N$ concentric dielectric layers. The inner radius, relative permittivity, and the dielectric strength of the $i$th layer are $a_i$, $\varepsilon_{ri}$, and $E_{cri}$, respectively ($i = 1, 2, \ldots, N$). The inner radius of the outer conductor of the capacitor is $b$. Write a MATLAB program to find which dielectric layer would break down first after a voltage of critical value is applied across the capacitor electrodes and to compute the breakdown voltage of the capacitor. Test the program for $N = 3$, $a_1 = 2.5$ cm, $a_2 = 5$ cm, $a_3 = 7$ cm, and $b = 9$ cm, if the dielectrics constituting layers 1, 2, and 3 are polystyrene, quartz, and silicon, respectively (use GUI's from MATLAB Exercises 2.1 and 2.3 to get the values of material parameters). *(ME2_21.m on IR)*

**SOLUTION:**

For the test example ($N = 3$, $a_1 = 2.5$ cm, $a_2 = 5$ cm, $a_3 = 7$ cm, and $b = 9$ cm), the material parameters for layers 1, 2, and 3 are, respectively: polystyrene ($\varepsilon_r = 2.56$, $E_{cr} = 20$ MV/m), quartz ($\varepsilon_r = 5$, $E_{cr} = 1000$ MV/m), and silicon ($\varepsilon_r = 11.9$, $E_{cr} = 30$ MV/m). The breakdown occurs in layer 1. The critical value of the capacitor charge for breakdown amounts to $Q_{cr} = 3.5605$ $\mu$C and the breakdown voltage of the capacitor is $V_{cr} = 295.108$ kV.

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for
%   educational purposes associated with the book
%
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%

% This program calculates breakdown voltage in spherical capacitor with
% arbitrary number of concentric dielectric layers.

clear all;
close all;
EPS0 = 8.8542*10^(-12);

% Number of dielectric layers
N = input('Enter the number of concentric dielectric layers: ');

% Dielectric permittivity constant, inner radius of dielectric layer and
% dielectric strength for each dielectric layer.
for i = 1:N
    EPSR(i) = input(['Enter dielectric rel. permittivity in ',int2str(i),'. layer: ']);
    a(i) = input(['Enter inner radius (in mm) of ',int2str(i),'. layer: ']);
    a(i) = a(i)/1000;
    Ecr(i) = input(['Enter dielectric strength (in MV/m) for ',int2str(i),'. layer: ']);
    Ecr(i) = Ecr(i)*10^6;
end;
b = input(['Enter outer radius (in mm) of ',int2str(N),'. layer: ']);
b = b/1000;

% Maximum of line charge before breakdown - calculation for each layer
Q = 4*pi*EPS0.*EPSR.*Ecr.*a.^2;
% In which layer will happen breakdown - minimum of the line charge
[Qmin,index] = min(Q);
fprintf(['Breakdown will happen in ',int2str(index),'. dielectric layer for Q = ']);
fprintf('%f nC/m.\n',Qmin*10^9);

% Breakdown voltage
Vcr = 0;   % Initialization
if N>1
    for i=1:N-1  % Integral E*dl in the first N-1 layers
        Vcr = Vcr + Qmin/4/pi/EPS0/EPSR(i)*(a(i+1)-a(i))/a(i+1)/a(i);
    end
end
% Addition of integral in the last layer
Vcr = Vcr + Qmin/4/pi/EPS0/EPSR(N)*(b-a(N))/b/a(N);
fprintf('Breakdown voltage is: %f V.\n',Vcr);
```