MATLAB EXERCISE 1.5 **Four equal point charges at tetrahedron vertices.** Figure 1.2(a) shows four point charges $Q = 1$ nC positioned in free space at four vertices of a regular (equilateral) tetrahedron with the side length $a = 1$ cm. Using MATLAB, find the resultant electric force on one of the charges – for example, on the charge at the top of the tetrahedron. *(ME1_5.m on IR)*

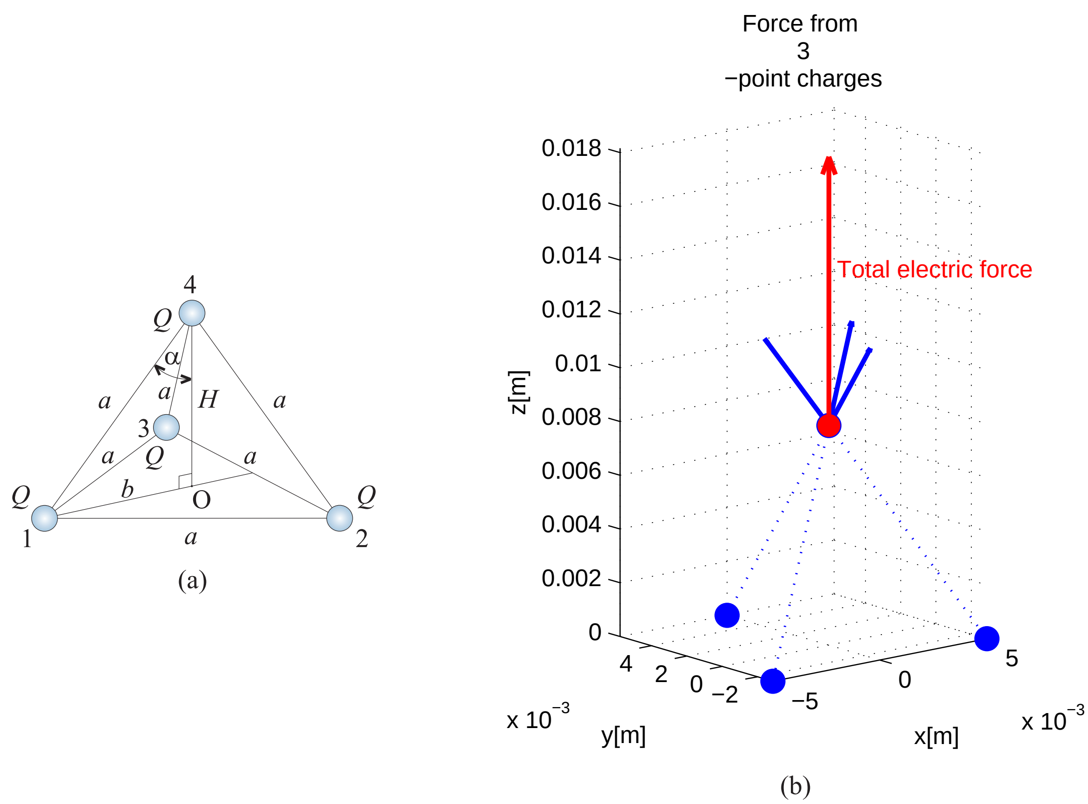

Figure S1.2 (a) Four point charges at tetrahedron vertices and (b) 3-D visualization of a MATLAB solution for individual forces on charge 4 due to charges 1, 2, and 3, respectively, and the resultant force – obtained using the program from MATLAB Exercise 1.4; for MATLAB Exercise 1.5.

SOLUTION:

We apply the program written in the previous MATLAB exercise, with `N` = 3, charges 1, 2, and 3 in Fig.S1.2(a) as the "source" charges, and charge 4 as the "target" charge. In particular, we place the origin of the Cartesian coordinate system at the center of the base of the tetrahedron [point O in Fig.S1.2(a)]. To determine the coordinates of the charges, and implement the lines of MATLAB code to input data, we use the relationships in Eq.(1.6) (from the book).

As a result of computation, the plot of individual forces on the top charge due to the three charges at the tetrahedron base and the total force ($\mathbf{F}_{e4} = \mathbf{F}_{e14} + \mathbf{F}_{e24} + \mathbf{F}_{e34}$) looks like the one in Fig.S1.2(b). By means of the rotation button in the MATLAB figure, we rotate the 3-D visualization of the solution to better see the forces and the geometrical representation of the problem.

The computed magnitude of the resultant force on the top charge is $F_{\rm e4} = 0.220149$ mN. The unit vector of the resultant force is $(0, 0, 1) = \hat{\mathbf{z}}$.

Finally, invoking Coulomb's law and the superposition principle, the analytical expression for $\mathbf{F}_{\rm e4}$ is obtained to be:

$$F_{\rm e4} = 3F_{\rm e14}\frac{\sqrt{6}}{3} = \frac{\sqrt{6}Q^2}{4\pi\varepsilon_0 a^2}\ , \tag{S1.2}$$

which may be used to validate the MATLAB result ($F_{\rm e4} = 0.220149$ mN).

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for
%   educational purposes associated with the book
%
%
%


%  Four charges at tetrahedron vertices


clear all;
close all;
EPS0 = 8.8542*10^(-12);

 N = 3;
 x = [0 -0.5 0.5];
 y = [sqrt(3)/3  -sqrt(3)/6 -sqrt(3)/6];
 z = [0 0 0];
 xp = 0;
 yp = 0;
 zp = sqrt(2)/sqrt(3);
 Q = [1 1 1];
 Qp = 1;


x = x * 10^(-2);
y = y * 10^(-2);
z = z * 10^(-2);
Q = Q * 10^(-9);

xp = xp * 10^(-2);
yp = yp * 10^(-2);
zp = zp * 10^(-2);
Qp = Qp * 10^(-9);

% Compute distance and direction between observation point and each charge

r = sqrt((xp - x).^2 + (yp - y).^2 + (zp - z).^2);
ux = (xp - x)./r;
uy = (yp - y)./r;
uz = (zp - z)./r;
uVec = [ux; uy; uz];

% Electric force computation

F = (ones(3,1)*(Qp*Q./(4*pi*EPS0*r.^2))).*uVec;
```

```
Ftot = sum(F,2);
Fmag = vectorMag(Ftot);
Fuv = (Ftot/Fmag)';

% Output

fprintf('Magnitude of resultant force at point P is %f mN.\n',Fmag*1000 );
disp('Unit vector of resultant force :');
disp(Fuv);

figure(1);
plot3(0,0,0,'k');
hold on;
for i=1:N
plot3(x(i),y(i),z(i),'o','MarkerSize',10,'MarkerFaceColor','b');
line ([xp , x(i)],[yp,y(i)], [zp,z(i)],'LineStyle',':');
hold on;
vecPlot3D([xp yp zp],[xp yp zp] + F(:,i)',1/Fmag/100,'b',0);
hold on;
end;
vecPlot3D([xp yp zp],[xp yp zp] + Ftot',1/Fmag/100,'r',1);
text(1.4*xp,1.4*yp,1.4*zp,'Total electric force','Color','r');

hold off;
axis equal;
xlabel('x[m]');
ylabel('y[m]');
zlabel('z[m]');
title({'Force from ',int2str(N),'-point charges'});
```