MATLAB EXERCISE 1.9 **Charged ring and an equivalent point charge.** Consider a line charge of uniform charge density Q' distributed around the circumference of a ring of radius a in air. Due to symmetry, the electric field intensity vector along the axis of the ring normal to its plane, at the point P in Fig.S1.5(a), has a z-component only, which, applying Eq.(1.14) (from the book), comes out to be

$$E = E_z = \oint_C \mathrm{d}E_z = \oint_C \mathrm{d}E\cos\alpha = \oint_C \frac{Q'\,\mathrm{d}l}{4\pi\varepsilon_0 R^2}\frac{z}{R} = \frac{Q'z}{4\pi\varepsilon_0 R^3}\oint_C \mathrm{d}l = \frac{Q'az}{2\varepsilon_0 R^3}$$

$$= \frac{Qz}{4\pi\varepsilon_0\left(z^2+a^2\right)^{3/2}} \qquad (Q = Q'2\pi a)\ , \tag{S1.3}$$

where Q is the total charge of the ring. Plot in MATLAB the dependence of E on the coordinate z [in Fig.S1.5(a)], for $z > 0$. In addition, show that far away along the z-axis, the charged ring in Fig.S1.5(a) is equivalent to a point charge with the same amount of charge located at the coordinate origin. Finally, plot, along the z-axis, the electric field due to this equivalent point charge and compare the two graphs. *(ME1_9.m on IR)*

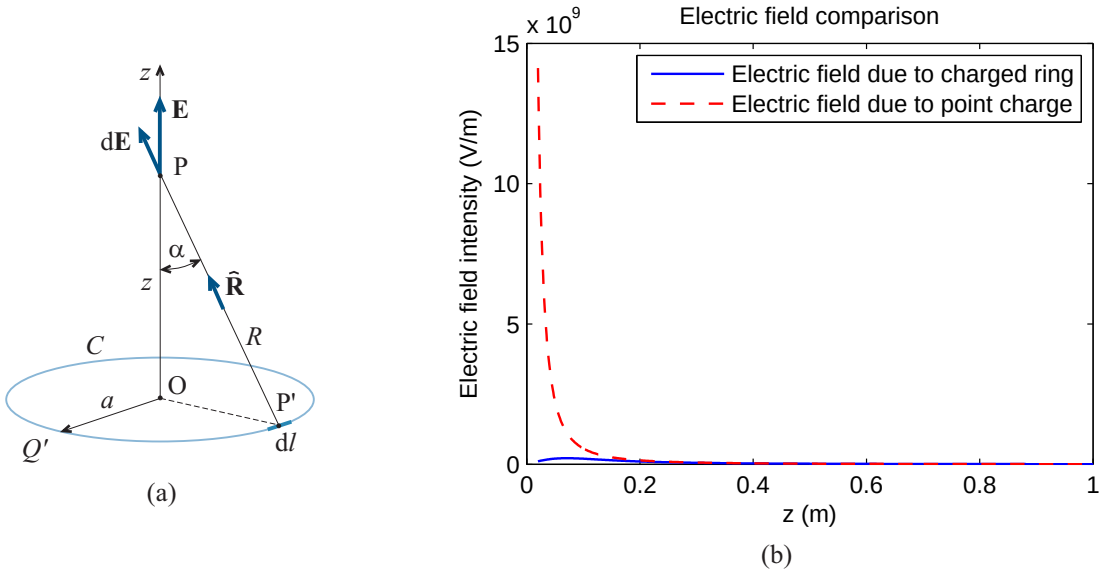

Figure S1.5 (a) Evaluation of the electric field along the axis of a charged ring normal to its plane, Eq.(S1.3), and (b) MATLAB comparison of electric field intensities along the z-axis due to the ring of charge and the equivalent (for $|z| \gg a$) point charge, respectively, for $Q' = 1$ mC/m and $a = 10$ cm; for MATLAB Exercise 1.9.

SOLUTION:

For $|z| \gg a$, $z^2 + a^2 \approx z^2$, Eq.(S1.3) becomes the approximate expression in Eq.(1.16) (from the book). As explained, far away from the ring, its charge is equivalent to a point charge Q located at its center.

The resulting graphical comparison of the expressions in Eq.(S1.3) above and Eq.(1.16) (in the book) using MATLAB is shown in Fig.S1.5(b).

```
%
%   Book: MATLAB-Based Electromagnetics (Pearson Prentice Hall)
%   Author: Branislav M. Notaros
%   Instructor Resources
%   (c) 2011
%
%   This MATLAB code or any part of it may be used only for educational purposes
%   associated with the book
%
%
%


%  Charged ring and an equivalent point charge


clear all;
close all;

Ql = input('Enter uniform charge per unit length of the ring in Coulombs per meter: ');
a = input('Enter radius of the ring in meters: ');
EPS0 = 8.854e-12;


Qtot = 2*pi*a*Ql; % total charge
dz = 0.01*a;
z = 20*dz:dz:10*a;
Ez =(z * Qtot)./(4*pi*EPS0*sqrt((z.^2 + a^2).^3));

% for z>>a
E = Qtot./(4*pi*EPS0*z.^2);

plot(z, Ez), hold on;
xlabel ('z (m)'), ylabel ('Electric field intensity (V/m)');
title ('Electric field comparison');
plot (z, E, 'r--');
legend ('Electric field due to charged ring','Electric field due to point charge',1);
hold off;
```