Chapter 1

Exercise solutions

1. The radii of the Earth, Moon, and Sun are 6,371 km, 1,738 km, and 6.951×10^5 km, respectively. From Figures 1.1, 1.5, and 1.6, make a rough estimate of how long it takes a P-wave to traverse the diameter of each body.

   Crude estimates based on mean velocity follow:

   Earth: 2×6371 km / 11 km/s = 1160 s = 19.3 min

   Moon: 2×1738 km / 7.6 km/s = 460 s = 7.7 min

   Sun: $2 \times 7 \times 10^5$ km / 250 km/s = 5600 s = 93 min

2. The P to S velocity ratio for most common rocks is about 1.7 ($\sim \sqrt{3}$). What solid part of the Earth has a very different P/S velocity ratio? Hint: Look at Figure 1.1.

   The inner core, where the P/S velocity ratio is about 3.

3. Assume that the S velocity perturbations plotted at 150 km depth in Figure 1.7 extend throughout the uppermost 300 km of the mantle. Estimate how many seconds earlier a vertically upgoing S-wave will arrive at a seismic station in the middle of Canada, compared to a station in the eastern Pacific. Ignore any topographic or crustal thickness differences between the sites; consider only the integrated travel time difference through the upper mantle.

   S velocity is about 4.5 km/s, so wave takes $300/4.5 = 67$ s to go 300 km.

   Canada is about 1.4% fast; eastern Pacific is about 1.8% slow. The difference is 3.2% or 0.032.

   Thus, S-wave will arrive earlier at central Canadian station by about (0.032)67 = $\boxed{\text{2.1 s}}$. This may be an underestimate because the plot is saturated so the perturbations could exceed these values.

4. Assuming that the P velocity in the ocean is 1.5 km/s, estimate the minimum and maximum water depths shown in Figure 1.8. If the crustal P velocity is 5 km/s, what is the depth to the top of the magma chamber from the sea floor?

Minimum water depth = 3.5 × 1.5 / 2 = 2.62 km

Maximum water depth = 3.93 × 1.5 / 2 = 2.95 km

Top of magma chamber = (3.95 - 3.5) × 5 / 2 = 1.1 km below sea floor

5. Earthquake moment is defined as $M_0 = \mu DA$, where μ is the shear modulus, D is the average displacement on the fault, and A is the fault area that slipped. The *moment magnitude*, M_W, is defined as $M_W = \frac{2}{3}\left[\log_{10} M_0 - 9.1\right]$, where the moment M_0 is in N m.

   (a) The moment of the 2004 Sumatra-Andaman earthquake has been estimated to be about 1.0×10^{23} N m. What moment magnitude does this correspond to? Assuming that the fault is horizontal, crudely estimate the slip area from the image shown in Figure 1.9. Assuming that the shear modulus $\mu = 3.0 \times 10^{10}$ N/m^2, then compute the average displacement on the fault.

   From the equation given, we have

$$M_W = \frac{2}{3}\left[\log_{10}(10^{23}) - 9.1\right] = \frac{2}{3}(23 - 9.1) = 9.27$$

   The moment magnitude is $\boxed{9.27}$.

   The slip area is about 200 km wide and 1200 km long, thus A = 240,000 km^2 = 2.4×10^{11} m^2, and we have

$$D = \frac{M_0}{\mu A} = \frac{10^{23}}{3.0 \times 10^{10} \times 2.4 \times 10^{11}} = \frac{100}{7.2} = 14 \text{ m}$$

   The average displacement is about $\boxed{14 \text{ m}}$.

   (b) A Hollywood director wants to make a movie about a devastating magnitude 10 earthquake in California with 40 m of slip (displacement) along the San Andreas Fault. Given that the crust is about 35 km thick in California, how scientifically plausible is this scenario? Hint: you may assume that the shear modulus $\mu = 3 \times 10^{10}$ N/m^2

   We also need to know the length of the San Andreas Fault (SAF) that ruptures, which is not given, so some research is needed. The SAF is about 1200 km long, which is also roughly the N-S extent of California. So 1200 km is a reasonable upper bound on the maximum SAF rupture length. If we assume that the rupture extends throughout the crust (an assumption implied by the question wording, but which is probably an overestimate for typical SAF earthquakes), then the fault area is $A = 1.2 \times 10^6 \times 3.5 \times 10^4$ m^2 = 4.2×10^{10} m. We then have

$$M_0 = \mu DA = 3 \times 10^{10} \cdot 40 \cdot 4.2 \times 10^10 \approx 5 \times 10^{22} \text{ N m}$$

   The moment magnitude is

$$M_W = \frac{2}{3}\left[\log_{10}(5 \times 10^{22}) - 9.1\right] = \frac{2}{3}(22.7 - 9.1) = 9.07$$

The moment magnitude is about $\boxed{9.1}$, which is much less than 10. Thus the director's idea for a magnitude 10 earthquake is $\boxed{\text{not plausible}}$ for the SAF in California.

6. The 2004 Sumatra earthquake lasted about 10 minutes and radiated about 2×10^{17} joules of seismic energy. Compute the corresponding power output in terrawatts (TW). Then do some research on the web and compare this power to: (a) average rate of electricity consumption in the United States, (b) average dissipation rate of tidal energy in the world's oceans, and (c) total heat flow out of the Earth. Note that the total energy release (including heat generated on the fault, etc.) of the Sumatra earthquake may be significantly greater than the seismically radiated energy. This is discussed in Chapter 9.

   A watt is one joule per second. 10 minutes is 600 s. Thus, the radiated power during the earthquake was $2\times10^{17}/600 = 3.33\times10^{14}$ watts. A terrawatt (TW) is 10^{12} watts, so $\boxed{\text{the radiated power was about 330 TW}}$. For comparison,

   (a) 2017 US power consumption was about 4000 TW hours (source Wikipedia). Thus the average power usage is $4000/(365 \cdot 24) = \boxed{0.46 \text{ TW}}$.

   (b) Earth's tidal energy dissipation is about $\boxed{3.7 \text{ TW}}$, most of which occurs in the oceans (see p. 106–108 of Stacey and Davis, *Physics of the Earth*, 2008). The Wikipedia article on tidal acceleration cites 3.3 TW.

   (c) Earth's integrated heat flux is about $\boxed{44 \text{ TW}}$ (p. 337, Stacey and Davis, *Physics of the Earth*, 2008). The Wikipedia article on Earth's energy budget cites 47 TW from Davies and Davies, Earth's surface heat flux, *Solid Earth*, 1(1), 524, 2010.

Chapter 2

Exercise solutions

1. Assume that the horizontal components of the 2-D stress tensor are

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} \\ \tau_{yx} & \tau_{yy} \end{bmatrix} = \begin{bmatrix} -30 & -20 \\ -20 & -40 \end{bmatrix} \text{MPa}$$

   (a) Compute the normal and shear stresses on a fault that strikes 10° east of north.

   cos 10° = 0.9848, sin 10° = 0.1736, thus fault parallel vector $\hat{\mathbf{f}}$ = (0.1736, 0.9848), fault normal vector $\hat{\mathbf{n}}$ = (0.9848, -0.1736). The traction on the fault plane is given by

$$\mathbf{t}(\hat{\mathbf{n}}) = \boldsymbol{\tau}\hat{\mathbf{n}} = \begin{bmatrix} -30 & -20 \\ -20 & -40 \end{bmatrix} \begin{bmatrix} 0.9848 \\ -0.1736 \end{bmatrix} = \begin{bmatrix} -26.07 \\ -12.75 \end{bmatrix} \text{MPa.}$$

   and

$$\begin{aligned} \mathbf{t}_N &= \mathbf{t} \cdot \hat{\mathbf{n}} = (-26.07, -12.75) \cdot (0.9848, -0.1736) = \boxed{\text{-23.46 MPa}} \\ \mathbf{t}_S &= \mathbf{t} \cdot \hat{\mathbf{f}} = (-26.07, -12.75) \cdot (0.1736, 0.9848) = \boxed{\text{-17.08 MPa}} \end{aligned}$$

   The fault normal compression is 23.46 MPa. The shear stress is 17.08 MPa.

   (b) Compute the principal stresses, and give the azimuths (in degrees east of north) of the maximum and minimum compressional stress axes.

   $\lambda_1 = -55.61$ MPa, 38° E of N

   $\lambda_2 = -14.38$ MPa, 128° E of N

   (solution computed using Matlab)

2. The principal stress axes for a 2-D geometry are oriented at N45°E and N135°E, corresponding to principal stresses of -15 and -10 MPa. What are the 4 components of the 2-D stress tensor in a (x = east, y = north) coordinate system?

   The eigenvector matrix is

$$N = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

Note that $N = N^{\mathrm{T}} = N^{-1}$. The principal (diagonalized) stress tensor is

$$\boldsymbol{\tau}_P = \begin{bmatrix} -15 & 0 \\ 0 & -10 \end{bmatrix}$$

which we can rotate to the stress tensor in E-N coordinates as

$$\boldsymbol{\tau} = N\boldsymbol{\tau}_P N^{\mathrm{T}} = \begin{bmatrix} -12.5 & -2.5 \\ -2.5 & -12.5 \end{bmatrix} \quad \text{MPa}$$

3. Figure 2.5 shows a vertical-component seismogram of the 1989 Loma Prieta earthquake recorded in Finland.

(a) Estimate the dominant period, T, of the surface wave from its first ten cycles. Then compute the frequency $f = 1/T$.

There are about 8 peaks in 200 s so the period, T, is about $\boxed{25 \text{ s}}$. The frequency, f, is $1/25 = \boxed{0.04 \text{ Hz}}$.

(b) Make an estimate of the *maximum* surface-wave strain recorded at this site. Hints: 1 micron $= 10^{-6}$ m, assume the Rayleigh surface wave phase velocity at the dominant period is 3.9 km/s, remember that strain is $\partial u_z/\partial x$, Table 3.1 may be helpful.

The amplitude $A = 300$ microns $= 3 \times 10^{-4}$ m. The wavelength $\Lambda = cT =$ (3.9 km/s)(25 s) $= 97.5$ km $=$ about 100 km $= 1 \times 10^5$ m. The wavenumber $k = 2\pi/\Lambda = 0.0628$/km $= 6.28 \times 10^{-5}$/m. We can approximate displacement as a harmonic wave as $u_x = A\sin(kx)$. Strain $= \partial u_x/\partial x = kA\cos(kx)$ so the maximum strain occurs when cos $= 1$ and is $Ak = (3 \times 10^{-4})(6.28 \times 10^{-5}) = \boxed{1.9 \times 10^{-8}}$.

Note that this value is halved if we express this in terms of the strain tensor, i.e.,

$$\mathbf{e}_{\max} \approx \begin{bmatrix} 0 & 10^{-8} \\ 10^{-8} & 0 \end{bmatrix}$$

4. Using Equations (2.4), (2.23), and (2.30), show that the principal stress axes always coincide with the principal strain axes for isotropic media. In other words, show that if $\mathbf{x}$ is an eigenvector of $\mathbf{e}$, then it is also an eigenvector of $\boldsymbol{\tau}$.

We can show that the principal strain axes coincide if we prove that if $\mathbf{x}$ is an eigenvector of the strain tensor $\mathbf{e}$ then it also must be an eigenvector of the stress tensor τ, that is if

$$e_{ij}x_j = \alpha x_i$$

then

$$\tau_{ij}x_j = \alpha' x_i$$

Using the isotropic stress/strain relation $\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}$ (equation 2.29), we have

$$\begin{aligned}\tau_{ij}x_j &= \lambda e_{kk}\delta_{ij}x_j + 2\mu e_{ij}x_j \\ &= \lambda e_{kk}x_i + 2\mu\alpha x_i \\ &= (\lambda e_{kk} + 2\mu\alpha)x_i \\ &= \alpha' x_i \end{aligned} \tag{2.1}$$

where $\alpha' = \lambda e_{kk} + 2\mu\alpha$. This completes the proof.

5. From Equations (2.34) and (2.35) derive expressions for the Lamé parameters in terms of the seismic velocities and density.

   $\mu = \rho\beta^2$

   $\lambda = \rho(\alpha^2 - 2\beta^2)$

6. Seismic observations of S velocity can be directly related to the shear modulus μ. However, P velocity is a function of both the shear and bulk moduli. For this reason, sometimes seismologists will compute the *bulk sound speed*, defined as:

   $$V_c = \sqrt{\frac{\kappa}{\rho}} \tag{2.2}$$

   which isolates the sensitivity to the bulk modulus κ.

   (a) Derive an equation for V_c in terms of the P velocity, α, and the S velocity, β.

   We have

   $$V_c = \sqrt{\frac{\kappa}{\rho}}$$

   We need to solve for κ in terms of α and β. We have from the P and S velocity definitions:

   $$\begin{aligned}\lambda + 2\mu &= \rho\alpha^2 \\ \mu &= \rho\beta^2 \\ \lambda &= \rho(\alpha^2 - 2\beta^2)\end{aligned}$$

   The bulk modulus $\kappa = \lambda + 2/3\mu$ and thus

   $$\begin{aligned}\kappa &= \rho(\alpha^2 - 2\rho\beta^2 + 2/3\rho\beta^2) \\ &= \rho(\alpha^2 - 4/3\beta^2)\end{aligned}$$

   and thus

   $$V_c = \sqrt{\alpha^2 - \frac{4}{3}\beta^2}$$

(b) For the specific case of a Poisson solid, express V_c as a fraction of the P velocity.

For a Poisson solid, $\alpha = \sqrt{3}\beta$, so $\beta^2 = \frac{1}{3}\alpha^2$. Substituting into our result from (a), we have

$$V_c = \sqrt{\alpha^2 - \frac{4}{9}\alpha^2} = \alpha\sqrt{\frac{9}{9} - \frac{4}{9}} = \frac{\sqrt{5}}{3}\alpha = 0.745\,\alpha$$

7. What is the P/S velocity ratio for a rock with a Poisson's ratio of 0.30?

Starting with equation (2.36), we have

$$\begin{aligned}
\sigma &= \frac{(\alpha/\beta)^2 - 2}{2(\alpha/\beta)^2 - 2} \\
2\sigma(\alpha/\beta)^2 - 2\sigma &= (\alpha/\beta)^2 - 2 \\
2 - 2\sigma &= (\alpha/\beta)^2 - 2\sigma(\alpha/\beta)^2 = (\alpha/\beta)^2[1 - 2\sigma] \\
\frac{\alpha}{\beta} &= \sqrt{\frac{2 - 2\sigma}{1 - 2\sigma}} \\
\frac{\alpha}{\beta} &= \sqrt{\frac{2 - 0.6}{1 - 0.6}} = \sqrt{\frac{1.4}{0.4}} = \sqrt{3.5} = \boxed{1.87} \quad \text{for } \sigma = 0.30
\end{aligned}$$

8. A sample of granite in the laboratory is observed to have a P velocity of 5.5 km/s and a density of 2.6 Mg/m^3. Assuming it is a Poisson solid, obtain values for the Lamé parameters, Young's modulus, and the bulk modulus. Express your answers in pascals.

For a Poisson solid, $\beta = \alpha/\sqrt{3} = 3.1754$ km/s for $\alpha = 5.5$ km/s. From Exercise 2.5, we have

$$\begin{aligned}
\lambda = \rho(\alpha^2 - 2\beta^2) &= 2.6(5.5^2 - 2(3.1754)^2) \\
\lambda &= 26.22 \frac{\text{Mg}}{\text{m}^3}\frac{\text{km}^2}{\text{s}^2}\frac{10^6\text{m}^2}{\text{km}^2}\frac{10^3\text{kg}}{\text{Mg}} \\
\lambda &= 26.22 \times 10^9 \frac{\text{kg}}{\text{m s}^2} \\
\lambda &= 26.22 \text{ GPa}
\end{aligned}$$

and

$$\begin{aligned}
\mu &= \rho\beta^2 = 2.6(3.1754)^2 \\
\mu &= 26.22 \text{ GPa}
\end{aligned}$$

Note that $\lambda = \mu$ as should be the case for a Poisson solid. The Young's modulus is given by

$$E = \frac{(3\lambda + 2\mu)\mu}{\lambda + \mu} = 2.5\mu \qquad \text{since } \lambda = \mu$$

$$E = 65.55 \text{ GPa}$$

The bulk modulus is given by

$$\begin{aligned}\kappa &= \lambda + \frac{2}{3}\mu = \frac{5}{3}\mu \\ \kappa &= 43.7 \text{ GPa}\end{aligned}$$

Summarizing, we have: $\boxed{\lambda = \mu = 26.22 \text{ GPA}, E = 65.55 \text{ GPa}, \kappa = 43.7 \text{ GPa}}$.

9. Using values from the PREM model (Appendix 1), compute values for the bulk modulus on both sides of (a) the core–mantle boundary (CMB) and (b) the inner-core boundary (ICB). Express your answers in pascals.

   The bulk modulus is given by

$$\kappa = \lambda + \frac{2}{3}\mu$$

   From Exercise 2.5 we have $\mu = \rho\beta^2$ and $\lambda = \rho(\alpha^2 - 2\beta^2)$ and thus

$$\kappa = \rho(\alpha^2 - \frac{4}{3}\beta^2)$$

   From the PREM model in the Appendix, we can thus construct a table with the κ values:

	Vp	Vs	den	kappa
CMB top	13.72	7.26	5.57	657 Gpa
CMB bot	8.06	0	9.90	643 GPa
ICB top	10.36	0	12.17	1306 GPa
ICB bot	11.03	3.5	12.76	1344 GPa

10. Figure 2.6 shows surface displacement rates as a function of distance from the San Andreas Fault in California.

    (a) Consider this as a 2-D problem with the x-axis perpendicular to the fault and the y-axis parallel to the fault. From these data, estimate the yearly strain ($\mathbf{e}$) and rotation ($\mathbf{\Omega}$) tensors for a point on the fault. Express your answers as 2×2 matrices.

    Each year, $\partial y/\partial x \approx 43$ mm / 50 km = $(43 \times 10^{-3}$ m) / $(50 \times 10^3$ m) = $0.86 \times 10^{-6} = \boxed{8.6 \times 10^{-7}/\text{yr}}$. This is the only non-zero partial derivative in the $\mathbf{J}$ matrix. We thus have

$$\mathbf{J} = \begin{bmatrix} 0 & \theta \\ 0 & 0 \end{bmatrix} = \mathbf{e} + \mathbf{\Omega} = \begin{bmatrix} 0 & \theta/2 \\ \theta/2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \theta/2 \\ -\theta/2 & 0 \end{bmatrix}$$

    where $\theta = \partial y/\partial x = 8.6 \times 10^{-7}$ and thus

$$\mathbf{e} = \begin{bmatrix} 0 & 4.3 \times 10^{-7} \\ 4.3 \times 10^{-7} & 0 \end{bmatrix}$$

and

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & -4.3 \times 10^{-7} \\ 4.3 \times 10^{-7} & 0 \end{bmatrix}$$

(b) Assuming the crustal shear modulus is 27 GPa, compute the yearly change in the stress tensor. Express your answer as a 2×2 matrix with appropriate units.

$$\boldsymbol{\tau} = \begin{bmatrix} 0 & 2.3 \times 10^{4} \\ 2.3 \times 10^{4} & 0 \end{bmatrix} \text{ Pa}$$

(c) If the crustal shear modulus is 27 GPa, what is the shear stress across the fault after 200 years, assuming zero initial shear stress?

4.6×10^{6} Pa (4.6 MPa)

(d) If large earthquakes occur every 200 years and release all of the distributed strain by movement along the fault, what, if anything, can be inferred about the *absolute* level of shear stress?

The absolute shear stress across the fault before the earthquake must be at least 4.6 MPa, but it could also be much bigger, depending upon how large the stress drop is relative to the absolute stress. The involves the frictional properties of the fault, and is discussed in Chapter 9. (Note: this question is to get students to think or do research—the answer is not in the chapter)

(e) What, if anything, can be learned about the fault from the observation that most of the deformation occurs within a zone less than 50 km wide?

Vertical strike-slip faults like the San Andreas are typically modeled as a locked zone (where the earthquakes occur) above a creeping zone at depth. The width of the deformation zone is proportional in some sense to the depth of the locked zone. (Note: this question is to get students to think or do research—the answer is not in the chapter.)

11. Do some research on the observed density of the Sun. Are the high sound velocities in the Sun (see Fig. 1.6) compared to Earth's P velocities caused primarily by low solar densities compared to the Earth, a higher bulk modulus or some combination of these factors?

The average solar density is about 1.4 g/cc, compared to 4 to 5 g/cc in Earth's mantle. This would make solar velocities higher by about a factor of $\sqrt{3} \sim 1.7$. But the average solar P velocity is about 50 times higher than in Earth, so the more important difference is that the bulk modulus, κ, is much higher for the Sun. Students can make this more complicated by trying to take into account the velocity depth dependence, but I was just looking for a rough estimate based on average properties.

12. The University of California, San Diego, operates the Piñon Flat Observatory (PFO) in the mountains northeast of San Diego (near Anza). Instruments include high-quality strain meters for measuring crustal deformation.

(a) Assume, at 5 km depth beneath PFO, the seismic velocities are $\alpha = 6$ km/s and $\beta = 3.5$ km/s and the density is $\rho = 2.7$ Mg/m^3. Compute values for the Lamé parameters, λ and μ, from these numbers. Express your answer in units of pascals.

From the solution to Exercise 5, we have

$$\begin{aligned}
\lambda &= \rho(\alpha^2 - 2\beta^2) = 2.7(6.0^2 - 2(3.5)^2) \\
\lambda &= 31.05 \frac{\text{Mg}}{\text{m}^3} \frac{\text{km}^2}{\text{s}^2} \frac{10^6\text{m}^2}{\text{km}^2} \frac{10^3\text{kg}}{\text{Mg}} \\
\lambda &= 31.05 \times 10^9 \frac{\text{kg}}{\text{m}\,\text{s}^2} \\
\lambda &= 31.05 \text{ GPa}
\end{aligned}$$

$$\begin{aligned}
\mu &= \rho\beta^2 = 2.7(3.5)^2 \\
\mu &= 33.07 \text{ GPa}
\end{aligned}$$

Summarizing, $\boxed{\lambda = 31.05 \text{ GPa}, \ \mu = 33.07 \text{ GPa}}$.

(b) Following the 1992 Landers earthquake ($M_S = 7.3$), located in southern California 80 km north of PFO (Fig. 2.7), the PFO strain meters measured a large static change in strain compared to values before the event. Horizontal components of the strain tensor changed by the following amounts: $e_{11} = -0.26 \times 10^{-6}$, $e_{22} = 0.92 \times 10^{-6}$, $e_{12} = -0.69 \times 10^{-6}$. In this notation 1 is east, 2 is north, and extension is positive. You may assume that this strain change occurred instantaneously at the time of the event. Assuming these strain values are also accurate at depth, use the result you obtained in part (a) to determine the change in stress due to the Landers earthquake at 5 km, that is, compute the change in τ_{11}, τ_{22}, and τ_{12}. Treat this as a two-dimensional problem by assuming there is no strain in the vertical direction and no depth dependence of the strain.

From (2.30), we have

$$\boldsymbol{\tau} = \begin{bmatrix} \lambda\,\text{tr}[\mathbf{e}] + 2\mu e_{11} & 2\mu e_{12} \\ 2\mu e_{21} & \lambda\,\text{tr}[\mathbf{e}] + 2\mu e_{22} \end{bmatrix}$$

Given the Landers strain changes of $e_{11} = -0.26 \times 10^{-6}$, $e_{22} = 0.92 \times 10^{-6}$, $e_{12} = -0.69 \times 10^{-6}$, then $\text{tr}[\mathbf{e}] = 0.66 \times 10^{-6}$ and

$$\boldsymbol{\tau} = \begin{bmatrix} 3300 & -45600 \\ -45600 & 81300 \end{bmatrix} \text{Pa}$$

or $\boxed{\tau_{11} = 3.3 \times 10^3 \text{ Pa}, \ \tau_{22} = 8.13 \times 10^4 \text{ Pa, and } \tau_{12} = -4.56 \times 10^4 \text{ Pa}}$

(c) Compute the orientations of the principal strain axes (horizontal) for the response at PFO to the Landers event. Express your answers as azimuths (degrees east of north).

The strain tensor is

$$\begin{bmatrix} -0.26 & -0.69 \\ -0.69 & 0.92 \end{bmatrix} \times 10^{-6}$$

The solution to the eigenvalue problem for this matrix gives orientations of principal strains as $\boxed{\text{N65°E and N155°E (or N25°W)}}$.

(d) A steady long-term change in strain at PFO has been observed to occur in which the changes in one year are: $e_{11} = 0.101 \times 10^{-6}$, $e_{22} = -0.02 \times 10^{-6}$, $e_{12} = 0.005 \times 10^{-6}$. Note that the long-term strain change is close to simple E–W extension. Assuming that this strain rate has occurred steadily for the last 1,000 years, from an initial state of zero stress, compute the components of the stress tensor at 5 km depth. (Note: This is probably not a very realistic assumption!) Don't include the large hydrostatic component of stress at 5 km depth.

From (2.30), we have

$$\boldsymbol{\tau} = \begin{bmatrix} \lambda\,\mathrm{tr}[\mathbf{e}] + 2\mu e_{11} & 2\mu e_{12} \\ 2\mu e_{21} & \lambda\,\mathrm{tr}[\mathbf{e}] + 2\mu e_{22} \end{bmatrix}$$

Given the 1000 year strain changes of $e_{11} = 0.101 \times 10^{-3}$, $e_{22} = -0.02 \times 10^{-3}$, $e_{12} = 0.005 \times 10^{-3}$, then $\mathrm{tr}[\mathbf{e}] = 0.081 \times 10^{-3}$ and using $\lambda = 31.05$ GPa and $\mu = 33.07$ GPa from 2.6a we have

$$\boldsymbol{\tau} = \begin{bmatrix} 9.20 & 0.33 \\ 0.33 & 1.19 \end{bmatrix} \text{ MPa}$$

(e) Farmer Bob owns a 1 km^2 plot of land near PFO that he has fenced and surveyed with great precision. How much land does Farmer Bob gain or lose each year? How much did he gain or lose as a result of the Landers earthquake? Express your answers in m^2.

Use $\mathrm{tr}[\mathbf{e}]$ as a measure of area change. Each year, $\mathrm{tr}[\mathbf{e}] = 0.081 \times 10^{-6}$ so

$\boxed{\text{Farmer Bob gains 0.081 m}^2\text{ of land each year}}$.

For Landers $\mathrm{tr}[\mathbf{e}] = 0.66 \times 10^{-6}$ so

$\boxed{\text{Farmer Bob gained 0.66 m}^2\text{ of land following the Landers earthquake}}$.

(f) (COMPUTER) Write a computer program that computes the stress across vertical faults at azimuths between 0 and 170 degrees (east from north, at 10 degree increments). For the stress tensors that you calculated in (b) and (d), make a table that lists the fault azimuth and the corresponding shear stress and normal stress across the fault (for Landers these are the stress changes, not absolute stresses). At what azimuths are the maximum shear stresses for each case?

(g) (COMPUTER) Several studies (e.g., Stein et al., 1992, 1994; Harris and Simpson, 1992; Harris et al., 1995; Stein, 1999; Harris, 2002) have modeled the spatial distribution of events following large earthquakes by assuming that the likelihood of earthquake rupture along a fault is related to the *Coulomb failure function* (CFF). Ignoring the effect of pore fluid pressure, the change in CFF may be expressed as:

$$\Delta\text{CFF} = \Delta|\tau_s| + \mu_s \Delta\tau_n,$$

where τ_s is the shear stress (traction), τ_n is the normal stress, and μ_s is the coefficient of static friction (don't confuse this with the shear modulus!). Note that CFF increases as the shear stress increases, and as the compressional stress on the fault is reduced (recall in our sign convention that extensional stresses are positive and compressional stresses are negative). Assume that $\mu_s = 0.2$ and modify your computer program to compute ΔCFF for each fault orientation. Make a table of the yearly change in ΔCFF due to the long-term strain change at each fault azimuth.

(h) (COMPUTER) Now assume that the faults will fail when their long-term CFF reaches some critical threshold value. The change in time to the next earthquake may be expressed as

$$\Delta t = \frac{\text{CFF}_{1000+L} - \text{CFF}_{1000}}{\text{CFF}_a},$$

where CFF_a is the annual change in CFF, CFF_{1000} is the thousand year change in CFF, and CFF_{1000+L} is the thousand year + Landers change in CFF (note that $\text{CFF}_{1000+L} \neq \text{CFF}_{1000} + \text{CFF}_L$). Compute the effect of the Landers earthquake in terms of advancing or retarding the time until the next earthquake for each fault orientation. Express your answer in years, using the sign convention of positive time for advancement of the next earthquake and negative time for retardation.

Results for parts f–h are given in the following table:

	----Landers----		Annual (*1000)					
azi	tau-s	tau-n	tau-s	tau-n	CFF-L	CFF-a	dt	L/a
0	-45643.5	3294.0	330.7	9196.2	46302.3	2170.0	-20.7	21.3
10	-56239.4	21258.7	1679.6	8841.7	60491.1	3447.9	-15.1	17.5
20	-60052.0	41764.0	2825.9	8047.3	68404.8	4435.3	-11.7	15.4
30	-56621.4	62336.7	3631.3	6908.7	69088.8	5013.0	-8.8	13.8
40	-46361.5	80495.3	3998.7	5563.4	62460.5	5111.4	-5.9	12.2
50	-30509.7	94049.8	3883.8	4173.4	49319.6	4718.5	-2.5	10.5
60	-10977.9	101365.2	3300.5	2906.6	31251.0	3881.9	2.4	8.1
70	9877.9	101559.1	2319.1	1915.8	30189.8	2702.3	11.2	11.2
80	29542.3	94608.3	1058.0	1320.3	48464.0	1322.0	36.7	36.7
90	45643.5	81351.0	-330.8	1192.0	61913.7	569.2	-51.6	108.8
100	56239.4	63386.3	-1679.6	1546.5	68916.6	1988.9	-21.9	34.7
110	60052.0	42881.0	-2825.9	2341.0	68628.2	3294.0	-15.6	20.8
120	56621.4	22308.3	-3631.3	3479.5	61083.1	4327.2	-12.1	14.1
130	46361.5	4149.6	-3998.7	4824.9	47191.4	4963.7	-9.2	9.5
140	30509.6	-9404.8	-3883.8	6214.8	28628.7	5126.8	-6.3	5.6
150	10977.9	-16720.2	-3300.5	7481.6	7633.9	4796.8	-3.0	1.6
160	-9877.9	-16914.1	-2319.1	8472.5	6495.1	4013.6	1.6	1.6
170	-29542.3	-9963.3	-1058.0	9068.0	27549.7	2871.6	9.6	9.6

(i) No increase in seismicity (small earthquake activity) has been observed near PFO following the Landers event. Does this say anything about the validity of the threshold CFF model?

Maybe! (another food for thought question to get students thinking, with no simple answer)