Solutions to the exercises.

1.1 In a patient study for a new test for multiple sclerosis (MS), thirty-two of the one hundred patients studied actually have MS. For the data given below, complete the two-by-two matrices and construct an ROC. The number of lesions (50, 40, 30, 20 or 10) corresponds to the threshold value for designating MS as the diagnosis.

50 lesions

2	0

40 lesions

8	1

30 lesions

16	3

20 lesions

22	6

10 lesions

28	12

Solution. The sum of the left hand column (true positives plus false negatives) must add up to thirty-two. Therefore the bottom left square is filled in this way. This leaves only the bottom right which has to fulfill the requirement that the total number of patients is one hundred. So the completed tables and corresponding ROC are:

50 lesions

2	0
30	68

40 lesions

8	1
24	67

30 lesions

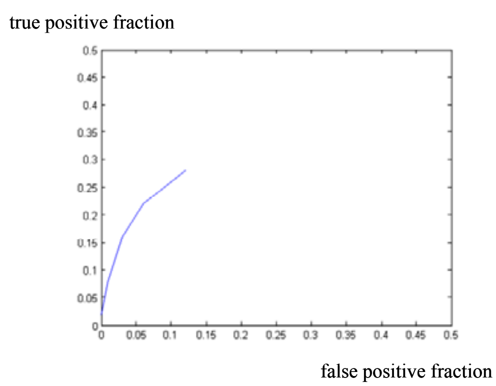
16	3
16	65

20 lesions

22	6
10	62

10 lesions

28	12
4	56



The ROC lies above the random line, but clearly needs some extra points corresponding to a smaller number of lesions for full characterization.

1.2 Choose a medical condition and suggest a clinical test which would have:
(a) High sensitivity but low specificity,
(b) Low sensitivity but high specificity.

Solution. (a) the sensitivity is defined as the number of true positives divided by the sum of the true positives and the false negatives, whereas the specificity is the number of true negatives divided by the sum of the true negatives and false positives. So a high sensitivity but low specificity suggests a diagnosis with a very low number of false negatives but significant number of false positives, i.e. a diagnosis which doesn't often miss the disease but often suggests that there is a disease present whereas in fact there isn't. An example might be mammography in which small lesions are very well visualized, but often subsequent biopsies result in the lesions being found not to be malignant.

(b) A low sensitivity but high specificity implies a relatively high number of false negatives and low number of false positives, i.e. a diagnosis that often misses the disease but almost never gives a false impression that the disease is present when it isn't. An example might be cognitive and behavioural tests in the early progression of neurodegenerative diseases such as Alzheimers. Signs are often missed, but very poor test results are very certain indicators of brain disfunction.

1.3. What does an ROC curve that lies below the random line suggest? Could this be diagnostically useful?

Solution. This suggests that the criteria that are being used to provide a clinical assessment are being used to support a hypothesis that is directly opposite to the truth. For example, cardiac disease is being diagnosed due to a low heart rate, whereas in fact the low heart rate occurs due to the patients being more fit and therefore these patients have a lower level of cardiac disease. Since the ROC curve is not random, there is useful information in the analysis, but the interpretation needs to be reversed with respect to the original hypothesis in order to take advantage of this information.

1.4 For the one-dimensional objects and PSF's shown in Figure 1.29, draw the resulting projections I(x). Write down whether each object contains high, low, or both high and low spatial frequencies, and which projection is affected most by h(x).

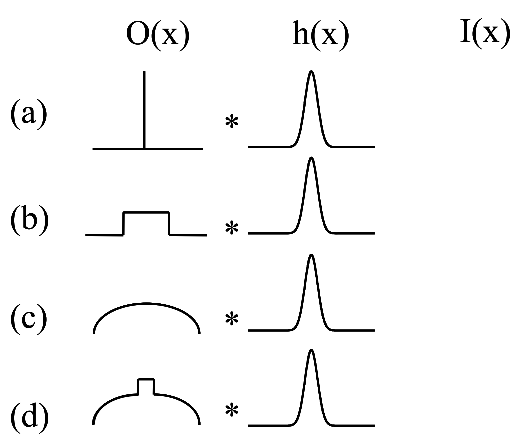
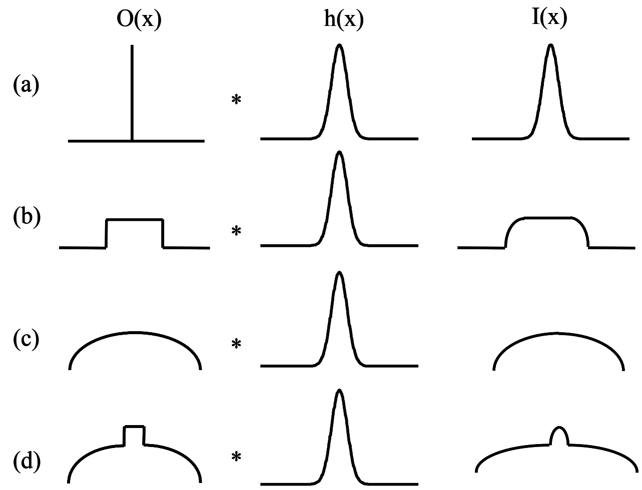


Figure 1.29.

Solution.



In (a) the object contains only very high spatial frequencies, (b) has both very high (at the edges) and very low (the flat parts of the profile) spatial frequencies, (c) has predominantly low spatial frequencies, and (d) has both low and high spatial frequencies. Since h(x) contains mainly low spatial frequencies, it will affect the object with the greatest proportion of high spatial frequencies, i.e. (a), to the greatest degree

1.5 Show mathematically that the FWHM of a gaussian function is given by:

$$\mathrm{FWHM} = \left(2\sqrt{2\ln 2}\right)\sigma \cong 2.36\sigma$$

Solution. The equation for a gaussian function is given by:

$$h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - x_0)^2}{2\sigma^2}\right)$$

The maximum value of h(x) occurs when $x=x_0$, and the exponent equals unity. Therefore, the FWHM corresponds to h(x)=0.5. Solving for the exponential term:

$$\exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right)=\frac{1}{2}$$

$$\Rightarrow -\frac{(x-x_0)^2}{2\sigma^2}=-\ln 2$$

$$\Rightarrow x-x_0=\pm\sigma\sqrt{2\ln 2}$$

Since the gaussian function is two-sided and symmetric, the FWHM is given by:

$$\text{FWHM}=\left(2\sqrt{2\ln 2}\right)\sigma$$

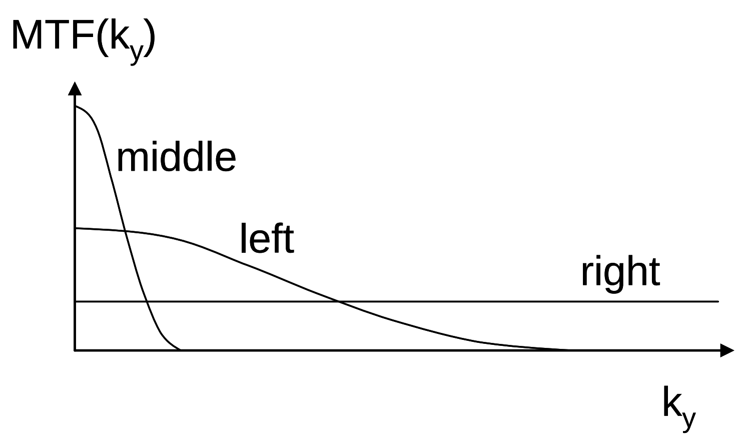
1.6 Plot the MTF on a single graph for each of the convolution filters shown below.

1	1	1
1	4	1
1	1	1

1	1	1
1	12	1
1	1	1

1	1	1
1	1	1
1	1	1

Solution. The filter on the left is a low-pass filter, the one in the middle is also low-pass with a greater degree of smoothing than the one on the left, and the filter on the right leaves the image unchanged. The MTF's are sketched below.



1.7 What type of filter is represented by the following kernel?

1	0	-1
1	0	-1
1	0	-1

Solution. This is a one-dimensional edge filter in the left/right direction which effectively produces a one-dimensional derivative of the image, i.e. it will emphasize the edges in the image. This is possible to see since the positive and negative numbers in the horizontal direction will emphasize any differences around the central pixel.

1.8 Using the filter in exercise 1.7 calculate the filtered image using the original image from Figure 1.11.

original image

1	5	3	5	4	6
4	3	32	5	6	9
6	10	4	8	8	7

*

filter

1	0	-1
1	0	-1
1	0	-1

=

filtered image

1	5	3	5	4	6
4	a	b	c	d	9
6	10	4	8	8	7

$$\left\{\begin{array}{l}
a=(1)(1)+(5)(0)+(3)(-1)+(4)(1)+(3)(0)+(32)(-1)+(6)(1)+(10)(0)+(4)(-1)=-28 \\
b=(5)(1)+(3)(0)+(5)(-1)+(3)(1)+(32)(0)+(5)(-1)+(10)(1)+(4)(0)+(8)(-1)=0 \\
c=(3)(1)+(5)(0)+(4)(-1)+(32)(1)+(5)(0)+(6)(-1)+(4)(1)+(8)(0)+(8)(-1)=21 \\
d=(5)(1)+(4)(0)+(6)(-1)+(5)(1)+(6)(0)+(9)(-1)+(8)(1)+(8)(0)+(7)(-1)=-4
\end{array}\right\}$$

1	5	3	5	4	6
4	-28	0	21	-4	9
6	10	4	8	8	7

filtered image

1.9 An ultrasound signal is digitized using a 16-bit ADC at a sampling rate of 3 MHz. If the image takes 20 ms to acquire, how much data (in Mbytes) are there in each ultrasound image. If images are acquired for 20 seconds continuously, what is the total data output of the scan?

Solution. Since 8-bits are equivalent to 1 byte, a 16-bit ADC produces 2 bytes of data every time that it samples. Sampling at 3 MHz produces 6 megabytes every second. Each image therefore contains 0.12 megabytes. For the continuous imaging 120 megabytes are produced over 20 seconds.

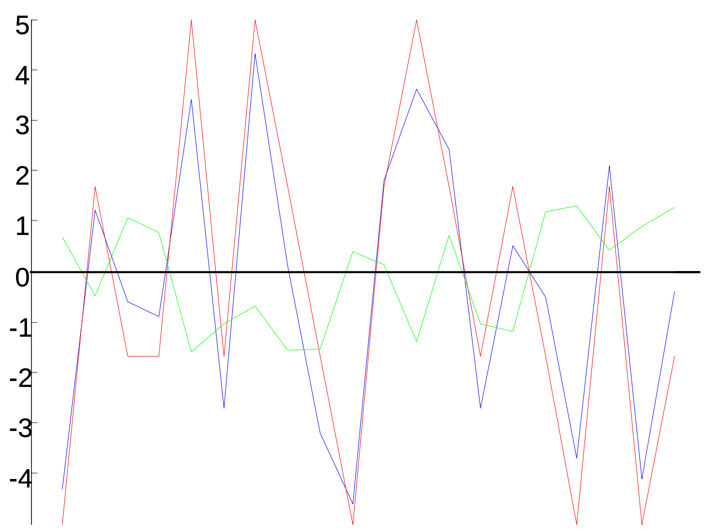
1.10 If a signal is digitized at a sampling rate of 20 kHz centred at 10 kHz, at what frequency would a signal at 22 kHz appear?

Solution. The frequency span is 0 to 20 kHz, centred at 10 kHz, and so a signal at 22 kHz is identical to one at 2 kHz, which is the frequency at which it appears in the spectrum.

1.11 A signal is sampled every 1 ms for 20 ms, with the following "true" values of the analogue voltage at successive sampling times. Plot the values of the voltage recorded by a 5 volt, 4-bit ADC assuming that the noise level is much lower than the signal and so can be neglected. On the same graph, plot the quantization error.

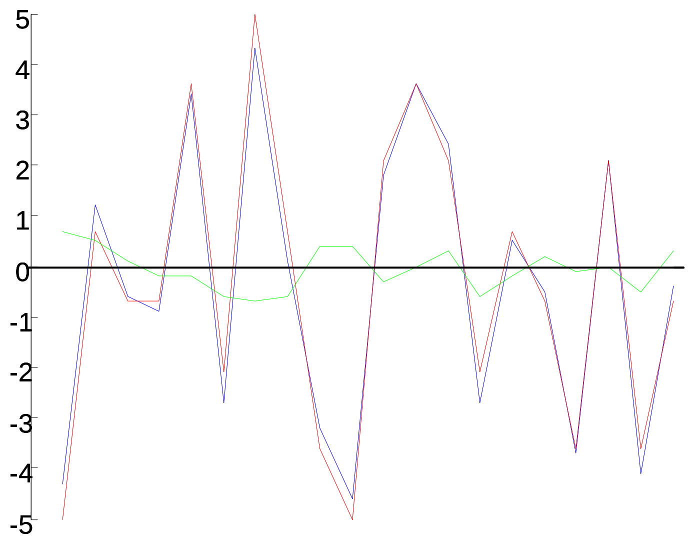
Signal (volts) = -4.3, +1.2, -0.6, -0.9, +3.4, -2.7, +4.3, +0.1, -3.2, -4.6, +1.8, +3.6, +2.4, -2.7, +0.5, -0.5, -3.7, +2.1, -4.1, -0.4

Solution. A 5 volt ADC has a range of -5 to +5 volts. Since it is a 4-bit, it can record values of -5, -1.67, + 1.67 and +5 volts. Each voltage is rounded either up or down to the nearest of these four values. The graph is shown below (blue)-true values, (red) 4-bit ADC, (green) quantization error.



1.12 Using the same signal as in exercise 1.11, plot the values of the voltage and the quantization error recorded by a 5 volt, 8-bit ADC.

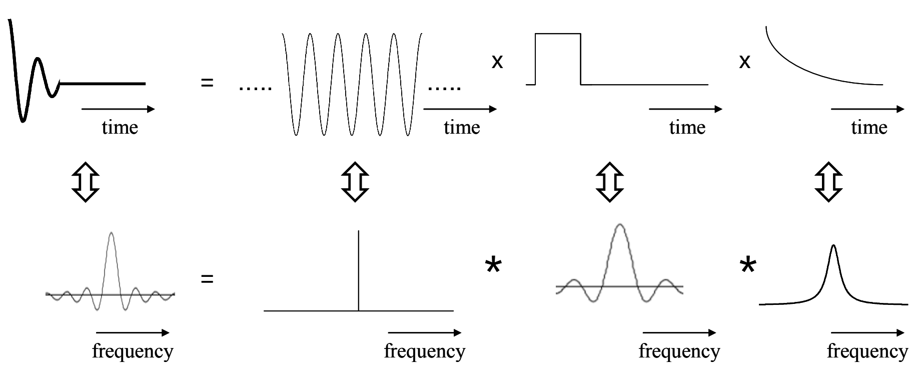
Solution. The recorded values can now be: -5, -3.6, -2.1, -0.7, +0.7, +2.1, +3.6 and +5. The graph is shown below (blue)-true values, (red) 8-bit ADC, (green) quantization error.



Fourier transforms

1.13 In Figure 1.20, plot the time and frequency domain signals for the case that the sampling time becomes very short.

Solution. If the sampling time is short, then the sinc function in the frequency domain becomes very broad, and the signal in the time domain is truncated. This leads to so-called "ringing artifacts" in the frequency spectrum as shown below.



1.14 Figure 1.30 shows two different two-dimensional PSFs in the (x,y) spatial domain. Draw the corresponding two-dimensional MTFs in the (k_x, k_y) spatial frequency dimension, and plot separately the one-dimensional MTF vs. k_x and MTF vs. k_y.

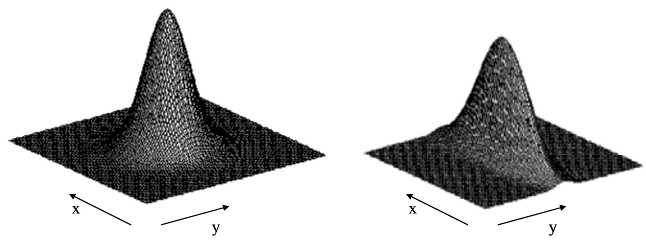
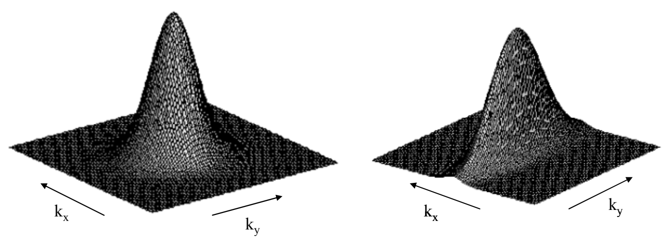
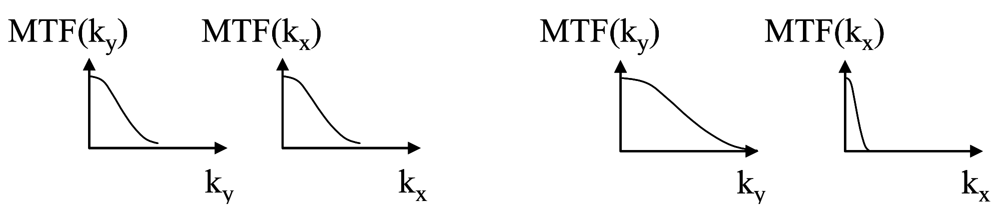


Figure 1.30.

Solution. Since a broad function in the spatial domain (PSF(x,y)) corresponds to a narrow function in the spatial frequency domain (MTF(k_x,k_y)), the plots in the spatial frequency domain are:

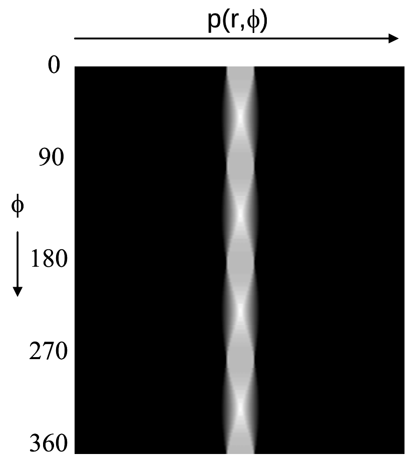


The corresponding one-dimensional MTFs are shown below:



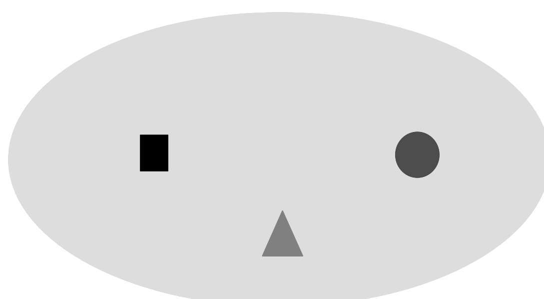
Backprojection

1.15 For Figure 1.25, suggest one possible shape that could have produced the sinogram.

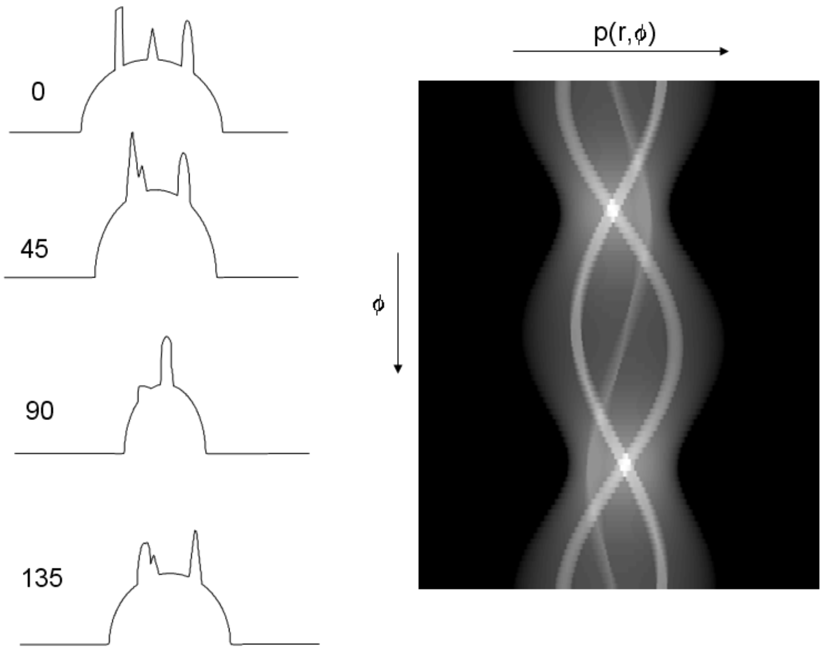


Solution. There are four repeating structures in the sinogram as the angle goes through 360 degrees, suggesting two-fold symmetry. One possible solution, therefore, is a square.

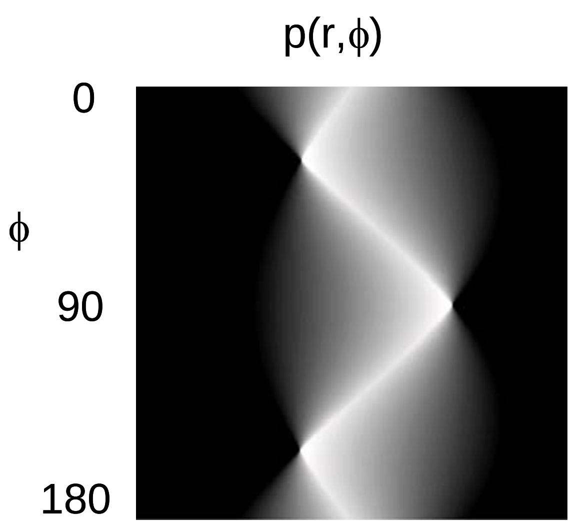
1.16 For the object shown in Figure 1.31: (a) draw the projections at angles of 0, 45, 90, and 135^o, and (b) draw the sinogram from the object. Assume that a dark area corresponds to an area of high signal.



Solution.



1.17 A scan is taken of a patient, and an area of radioactivity is found. The sinogram is shown in Figure 1.32. Assuming that the radioactivity is uniform within the targeted area, what is the possible shape of the area of radioactivity?



Solution. There are three bright "apices" which suggest that the object is triangular in shape. The three maxima occur at ~30, 90 and 150 degrees and so the triangle must be equilateral to produce these equally spaced angles.