Section 2-4   Resistors

P 2.4-1   A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that i_s = 3 A and R = 7 Ω. Calculate the voltage v across the resistor and the power absorbed by the resistor.

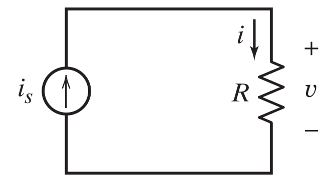
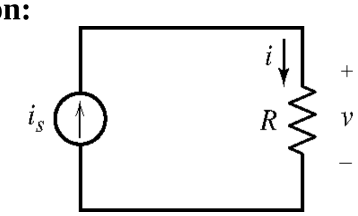


Figure P 2.4-1

Answer: v = 21 V and the resistor absorbs 63 W.

Solution:



$i = i_s = 3$ A and $v = Ri = 7 \times 3 = \underline{21\ \text{V}}$

v and i adhere to the passive convention

$\therefore P = v\,i = 21 \times 3 = \underline{63\ \text{W}}$

is the power absorbed by the resistor.

P 2.4-2   A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that i = 3 mA and v = 24 V. Calculate the resistance R and the power absorbed by the resistor.

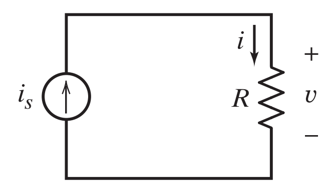
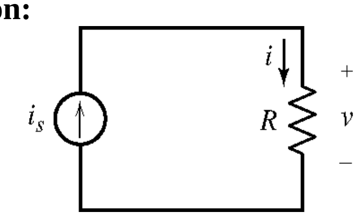


Figure P 2.4-1

Answer: R = 8 kΩ and the resistor absorbs 72 mW.

Solution:



$i = i_s = 3$ mA and $v = 48$ V

$$R = \frac{v}{i} = \frac{48}{0.003} = 16000 = \underline{16\ \text{K}\Omega}$$

$$P = (3\times10^{-3})\times48 = 144\times10^{-3} = \underline{144\ \text{mW}}$$

P 2.4-3 A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that v_s = 10 V and R = 5 Ω. Calculate the current i in the resistor and the power absorbed by the resistor.

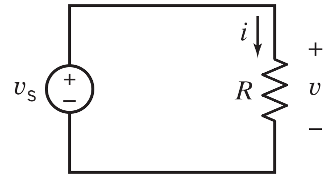
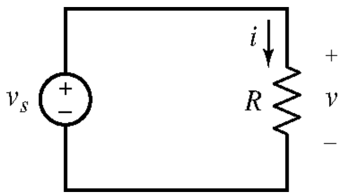


Figure P 2.4-3

Answer: i = 2 A and the resistor absorbs 20 W.

Solution:



$v = v_s = 10$ V and $R = 5\ \Omega$

$$i = \frac{v}{R} = \frac{10}{5} = \underline{2\ \text{A}}$$

v and i adhere to the passive convention

$\therefore\ p = v\,i = 2 \cdot 10 = \underline{20\ \text{W}}$

is the power absorbed by the resistor

P 2.4-4 A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that v_s = 24 V and i = 2 A. Calculate the resistance R and the power absorbed by the resistor.

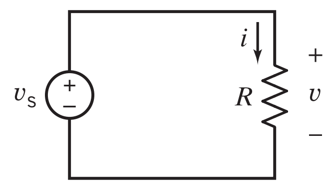
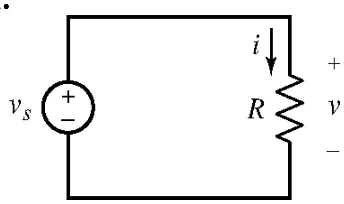


Figure P 2.4-3

Answer: R = 12 Ω and the resistor absorbs 48 W.

Solution:



$v = v_s = 24$ V and $i = 3$ A

$$R = \frac{v}{i} = \frac{24\ \text{V}}{3\ \text{A}} = \underline{8\ \Omega}$$

$$p = v\,i = 24(3) = \underline{72\ \text{W}}$$

P 2.4-5 A voltage source and two resistors are connected in parallel in the circuit shown in Figure P 2.4-5. Elements connected in parallel have the same voltage, so $v_1 = v_s$ and $v_2 = v_s$ in this circuit. Suppose that $v_s = 150$ V, $R_1 = 50\ \Omega$, and $R_2 = 25\ \Omega$. Calculate the current in each resistor and the power absorbed by each resistor.

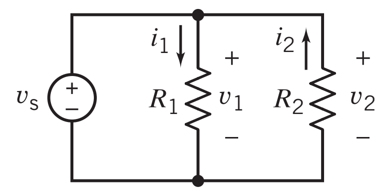
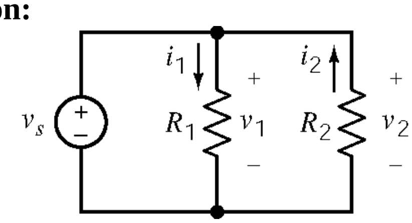


Figure P 2.4-5

Hint: Notice the reference directions of the resistor currents.
Answer: $i_1 = 3$ A and $i_2 = -6$ A. R_1 absorbs 450 W and R_2 absorbs 900 W.

Solution:



$v_1 = v_2 = v_s = 150$ V;

$R_1 = 50\ \Omega$; $R_2 = 25\ \Omega$

v_1 and i_1 adhere to the passive convention so

$$i_1 = \frac{v_1}{R_1} = \frac{150}{50} = \underline{3\ \text{A}}$$

v_2 and i_2 do not adhere to the passive convention so $i_2 = -\frac{v_2}{R_2} = -\frac{150}{25} = \underline{-6\ \text{A}}$

The power absorbed by R_1 is $P_1 = v_1 i_1 = 150 \cdot 3 = \underline{450\ \text{W}}$

The power absorbed by R_2 is $P_2 = -v_2 i_2 = -150(-6) = \underline{900\ \text{W}}$

P 2.4-6 A current source and two resistors are connected in series in the circuit shown in Figure P 2.4-6. Elements connected in series have the same current, so $i_1 = i_s$ and $i_2 = i_s$ in this circuit. Suppose that $i_s = 2$ A, $R_1 = 4\ \Omega$, and $R_2 = 8\ \Omega$. Calculate the voltage across each resistor and the power absorbed by each resistor.

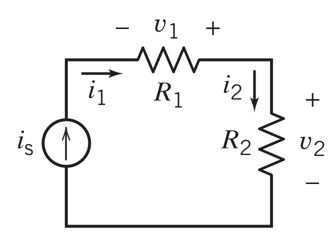
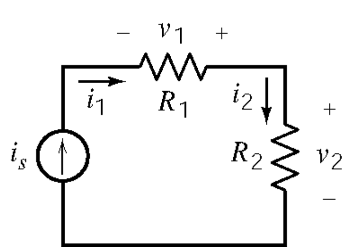


Figure P 2.4-6

Hint: Notice the reference directions of the resistor voltages.
Answer: $v_1 = -8$ V and $v_2 = 16$ V. R_1 absorbs 16 W and R_2 absorbs 32 W.

Solution:



$i_1 = i_2 = i_s$=25 mA and $R_1 = 4\ \Omega$ and $R_2 = 8\ \Omega$

v_1 and i_1 do <u>not</u> adhere to the passive convention so

$v_1 = -R_1\ i_1 = -4(0.025) = \underline{-0.1\ \text{V}}.$

The power absorbed by R_1 is

$P_1 = -v_1 i_1 = -(-0.1)(0.025) = \underline{2.5\ \text{mW}}.$

v_2 and i_2 do adhere to the passive convention so $v_2 = R_2\ i_2 = 8(0.025) = \underline{0.2\ \text{V}}.$

The power absorbed by R_2 is $P_2 = v_2 i_2 = (0.2)(0.025) = \underline{5\ \text{mW}}.$

P 2.4-7 An electric heater is connected to a constant 250-V source and absorbs 1000 W. Subsequently, this heater is connected to a constant 210-V source. What power does it absorb from the 210-V source? What is the resistance of the heater?

Hint: Model the electric heater as a resistor.

Solution:

Model the heater as a resistor, then from $P = \dfrac{v^2}{R} \;\Rightarrow\; R = \dfrac{v^2}{P} = \dfrac{(250)^2}{1000} = 62.5\ \Omega$

with a 220 V source $\qquad P = \dfrac{v^2}{R} = \dfrac{(220)^2}{62.5} = \underline{774.4\ \text{W}}$

P 2.4-8 The portable lighting equipment for a mine is located 100 meters from its dc supply source. The mine lights use a total of 5 kW and operate at 120 V dc. Determine the required cross-sectional area of the copper wires used to connect the source to the mine lights if we require that the power lost in the copper wires be less than or equal to 5 percent of the power required by the mine lights.

Hint: Model both the lighting equipment and the wire as resistors.

Solution:

The current required by the mine lights is: $i = \dfrac{P}{v} = \dfrac{5000}{120} = \dfrac{125}{3}$ A

Power loss in the wire is : $i^2\ R$

Thus the maximum resistance of the copper wire allowed is

$$R = \frac{0.05P}{i^2} = \frac{0.05 \times 5000}{(125/3)^2} = 0.144\ \Omega$$

now since the length of the wire is $L = 2 \times 100 = 200$ m= 20,000 cm

thus $R = \rho L / A$ with $\rho = 1.7 \times 10^{-6}\,\Omega \cdot \text{cm}$ from Table 2.5–1

$$A = \frac{\rho L}{R} = \frac{1.7 \times 10^{-6} \times 20{,}000}{0.144} = \underline{0.236\ \text{cm}^2}$$

***P 2.4-9** The resistance of a practical resistor depends on the nominal resistance and the resistance tolerance as follows:

$$R_{\text{nom}}\left(1-\frac{t}{100}\right) \le R \le R_{\text{nom}}\left(1+\frac{t}{100}\right)$$

where R_{nom} is the nominal resistance and t is the resistance tolerance expressed as a percentage. For example, a 100-Ω, 2 percent resistor will have a resistance given by

$$98\,\Omega \le R \le 102\,\Omega$$

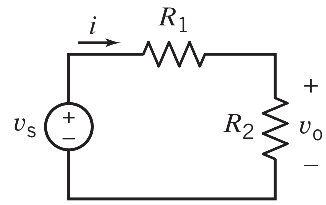


Figure P 2.4-9

The circuit shown in Figure P 2.4-9 has one input, v_s, and one output, v_o. The gain of this circuit is given by

$$\text{gain} = \frac{v_0}{v_s} = \frac{R_2}{R_1 + R_2}$$

Determine the range of possible values of the gain when R_1 is the resistance of a 100-Ω, 2 percent resistor and R_2 is the resistance of a 400-Ω, 5 percent resistor. Express the gain in terms of a nominal gain and a gain tolerance.

Solution:

$$0.7884 = \frac{380}{102+380} \le gain \le \frac{420}{98+420} = 0.8108$$

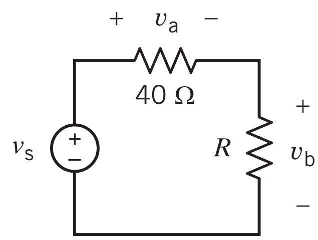
$$nominal\ gain = \frac{0.7884+0.8108}{2} = 0.7996$$

$$gain\ tolerance = \frac{0.7996-0.7884}{0.7996}\times 100 = \frac{0.8108-0.7996}{0.7996}\times 100 = 1.40\%$$

So

$$gain = 0.7996 \pm 1.40\%$$

P 2.4-10 The voltage source shown in Figure P 2.4-10 is an adjustable dc voltage source. In other words, the voltage v_s is a constant voltage, but the value of that constant can be adjusted. The tabulated data were collected as follows. The voltage, v_s, was set to some value, and the voltages across the resistor, v_a and v_b, were measured and recorded. Next, the value of v_s was changed, and the voltages across the resistors were measured again and recorded. This procedure was repeated several times. (The values of v_s were not recorded.) Determine the value of the resistance, R.



v_a, V	v_b, V
11.75	7.05
7.5	4.5
5.625	3.375
10	6
4.375	2.625

Figure P 2.4-10

Solution:

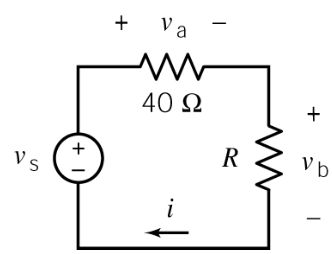
Label the current i as shown. That current is the element current in both resistors. First

$$i = \frac{v_a}{40}$$

Next $\quad v_b = R\, i = R\, \frac{v_a}{40} \quad \Rightarrow \quad R = 40\, \frac{v_b}{v_a}$

For example,

$$R = 40\, \frac{7.05}{11.75} = 24\ \Omega$$



v_a, V	v_b, V
11.75	7.05
7.5	4.5
5.625	3.375
10	6
4.375	2.625

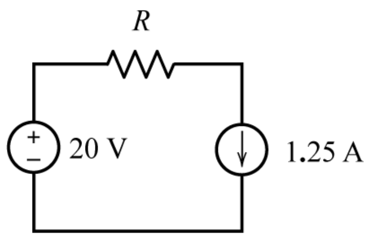


Figure P2.4-11

P2.4-11 Consider the circuit shown in Figure P2.4-11.

(a) Suppose the current source supplies 3.125 W of power. Determine the value of the resistance *R*.

(b) Suppose instead the resistance is $R = 12\ \Omega$. Determine the value of the power supplied by the current source.

Solution:

(a) Suppose the current source supplies 3.125 W of power.

$$1.25\, v_3 = 3.125 \quad \Rightarrow \quad v_3 = 2.5\ \text{V}$$

Then, using KVL

$$R = \frac{20 + 2.5}{1.25} = 18\ \Omega$$

(b) Suppose instead the resistance is $R = 12\ \Omega$. From KVL

$$1.25(12) - v_3 - 20 = 0 \quad \Rightarrow \quad v_3 = -5\ \text{V}$$

The value of the power supplied by the current source is

$$1.25\, v_3 = 1.25(-5) = -6.25\ \text{W}$$

P2.4-12. We will encounter "ac circuits" in Chapter 10. Frequently we analyze ac circuits using "phasors" and "impedances". Phasors are complex numbers that represents currents and voltages in an ac circuit. Impedances are complex numbers that describe ac circuit elements. (See Appendix B for a discussion of complex numbers.) Figure P2.4-11 shows a circuit element in an ac circuit. **I** and **V** are complex numbers representing the element current and voltage. **Z** is a complex number describing the element itself. "Ohm's law for ac circuits" indicates that

$$\mathbf{V} = \mathbf{Z}\,\mathbf{I}$$

(a) Suppose $\mathbf{V} = 12\angle 45°$ V, $\mathbf{I} = B\angle\theta$ A and $\mathbf{Z} = 18 + j8\ \Omega$. Determine the values of B and θ.

(b) Suppose $\mathbf{V} = 48\angle 135°$ V, $\mathbf{I} = 3\angle 15°$ A and $\mathbf{Z} = R + jX\ \Omega$. Determine the values of R and X.

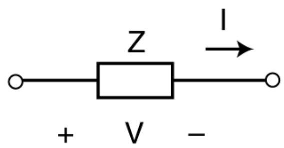


Figure P2.4-12

Solution:

(a)
$$12\angle 45° = (18 + j8)(B\angle\theta)$$

$$B\angle\theta = \frac{12\angle 45°}{18 + j8} = \frac{12\angle 45°}{19.7\angle 24°} = 0.609\angle 21°$$

so
$$B = 0.609 \text{ A} \quad \text{and} \quad \theta = 21°$$

(b)
$$48\angle 135° = (R + jX)(3\angle 15°)$$

$$R + jX = \frac{48\angle 75°}{3\angle 15°} = 18\angle 60° = 9 + j15.6\ \Omega$$

so
$$R = 9\ \Omega \quad \text{and} \quad X = 15.6\ \Omega$$