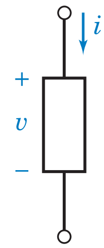
Section 2-2 Engineering and Linear Models

P 2.2-1 An element has voltage *v* and current *i* as shown in Figure P 2.2-1*a*. Values of the current *i* and corresponding voltage *v* have been tabulated as shown in Figure P 2.2-1*b*. Determine if the element is linear.



(*a*)

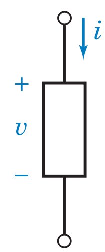
v, V	*i*, A
−3	−3
−4	−2
0	0
12	2
32	4
60	6

(*b*)

Figure P 2.2-1

Solution:
The element is not linear. For example, doubling the current from 2 A to 4 A does not double the voltage. Hence, the property of homogeneity is not satisfied.

P 2.2-2 A linear element has voltage *v* and current *i* as shown in Figure P 2.2-2*a*. Values of the current *i* and corresponding voltage *v* have been tabulated as shown in Figure P 2.2-2*b*. Represent the element by an equation that expresses *v* as a function of *i*. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of *v* corresponding to a current of $i = 40$ mA. (c) Use the model to predict the value of *i* corresponding to a voltage of $v = 4$ V.



(*a*)

v, V	*i*, A
−3.6	−30
2.4	20
6.0	50

(*b*)

Figure P 2.2-2*a*

Hint: Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.

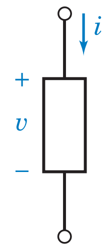
Solution:

(a) The data points do indeed lie on a straight line. The slope of the line is 0.12 V/A and the line passes through the origin so the equation of the line is $v = 0.12\,i$. The element is indeed linear.

(b) When $i = 40$ mA, v = (0.12 V/A)×(40 mA) = (0.12 V/A)×(0.04 A) = 4.8 mV

(c) When $v = 3\ \text{V},\ i = \dfrac{3}{0.12} = 25\ \ \text{A}$

P 2.2-3 A linear element has voltage v and current i as shown in Figure P 2.2-3*a*. Values of the current i and corresponding voltage v have been tabulated as shown in Figure P 2.2-3*b*. Represent the element by an equation that expresses v as a function of i. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of v corresponding to a current of $i = 4$ mA. (c) Use the model to predict the value of i corresponding to a voltage of $v = 12$ V.



(*a*)

v, V	i, mA
3.078	12
5.13	20
12.825	50

(*b*)

Figure P 2.2-3

Hint: Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.

Solution:

(a) The data points do indeed lie on a straight line. The slope of the line is 256.5 V/A and the line passes through the origin so the equation of the line is $v = 256.5\,i$. The element is indeed linear.

(b) When $i = 6$ mA, $v = (256.5 \text{ V/A}) \times (6 \text{ mA}) = (256.5 \text{ V/A}) \times (0.006 \text{ A}) = 1.054 \text{ V}$

(c) When $v = 12\ \text{V},\ i = \dfrac{12}{256.5} = 0.04678\ \text{A} = 46.78\ \text{mA}.$

P 2.2-4 An element is represented by the relation between current and voltage as

$$v = 3i + 5$$

Determine whether the element is linear.

Solution:

Let $i = 1$ A , then $v = 3i + 5 = 8$ V. Next $2i = 2$A but $16 = 2v \neq 3(2i) + 5 = 11$.. Hence, the property of homogeneity is not satisfied. The element is not linear.

P 2.2-5 The circuit shown in Figure P 2.3-5 consists of a current source, a resistor, and element A.

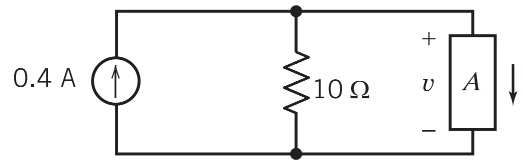


Figure P 2.3-5

Consider three cases.

(a) When element A is a 40-Ω resistor, described by $i = v / 40$, then the circuit is represented by

$$0.4=\frac{v}{10}+\frac{v}{40}$$

Determine the values of v and i. Notice that the above equation has a unique solution.

(b) When element A is a nonlinear resistor described by $i = v^2 / 2$, then the circuit is represented by

$$0.4=\frac{v}{10}+\frac{v^2}{2}$$

Determine the values of v and i. In this case there are two solutions of the above equation. Nonlinear circuits exhibit more complicated behavior than linear circuits.

(c) When element A is a nonlinear resistor described by $i = 0.8+\frac{v^2}{2}$, then the circuit is described by

$$0.4=\frac{v}{10}+0.8+\frac{v^2}{2}$$

Show that this equation has no solution. This result usually indicates a modeling problem. At least one of the three elements in the circuit has not been modeled accurately.

Solution:

(a)

$$0.4=\frac{v}{10}+\frac{v}{40}=\frac{v}{8} \quad \Rightarrow \quad v=3.2 \text{ V}$$

$$i=\frac{v}{40}=0.08 \text{ A}$$

(b)

$$0.4=\frac{v}{10}+\frac{v^2}{2} \quad \Rightarrow \quad v^2+\frac{v}{5}-0.8=0$$

Using the quadratic formula $v=\frac{-0.2\pm 1.8}{2}=0.8,\ -1.0 \text{ V}$

When $v = 0.8$ V then $i=\frac{0.8^2}{2}=0.32 \text{ A}$. When $v = -1.0$ V then $i=\frac{(-1)^2}{2}=0.5 \text{ A}$.

(c)

$$0.4=\frac{v}{10}+0.8+\frac{v^2}{2} \quad \Rightarrow \quad v^2+\frac{v}{5}+0.8=0$$

Using the quadratic formula $$v = \frac{-0.2 \pm \sqrt{0.04 - 3.2}}{2}$$

So there is no real solution to the equation.