Intermediate Financial Theory

Danthine and Donaldson

Solutions to Problems

**CHAPTER I.**

1.1. U is a utility function, i.e., U(x) > U(y) ⇔ x  y

f(.) is an increasing monotone transformation, f(a) > f(b) ⇔ a > b;

then f(U(x)) > f(U(y)) ⇔ U(x) > U(y) ⇔ x  y

* 1. Utility function U():

FOC: U1/U2=p1/p2

Let f=f(U(.)) be a monotone transformation.

Apply the chain rule for derivatives:

FOC: f1/f2=f 'U1/f 'U2=p1/p2 (prime denotes derivation).

Economic interpretation: f and U represent the same preferences, they must lead to the same choices.

* 1. When an agent has very little of one given good, he is willing to give up a big quantity of another good to obtain a bit more of the first.

MRS is constant when the utility function is linear additive (that is, the indifference curve is also linear):





Not very interesting; for example, the optimal choice over 2 goods for a consumer is always to consume one good only (if the slope of the budget line is different from the MRS) or an indefinite quantity of the 2 goods (if the slopes are equal).

Convex preferences can exhibit indifference curves with flat spots, strictly convex preferences cannot. The utility function is not strictly quasi-concave here.

Pareto set: 2 cases.

* Indifference curves for the agents have the same slope: Pareto set is the entire box;

* Indifference curves have not the same slope: Pareto set is the lower side and the right side of the box, or the upper side and the left side, depending on which MRS is higher.

1.4. a) U1 =  = 4.90

U2 =  = 14.97



with α = 0.5, 





MRS1 ≠ MRS2, not PO; it is possible to reallocate the goods and make one agent (at least) better off without hurting the other.

b) PS = { }, the Pareto set is a straight line (diagonal from lower-left to upper-right corner).

c) The problem of the agents is

.

The Lagrangian and the FOC's are given by



Rearranging the FOC's leads to . Now we insert this ratio into the budget constraints of agent 1  and after rearranging we get . This expression can be interpreted as a demand function. The remaining demand functions can be obtained using the same steps.



To determine market equilibrium, we use the market clearing condition .

Finally we find .

The after-trade MRS and utility levels are:

U1 =  = 5

U2 =  = 15





Both agents have increased their utility level and their after-trade MRS is equalized.

d) Uj()=,



Same condition as that obtained in a). This is not a surprise since the new utility function is a monotone transformation (logarithm) of the utility function used originally.

U1 =  = 1.59

U2 =  = 2.71

MRS's are identical to those obtained in a), but utility levels are not. The agents will make the same maximizing choice with both utility functions, and the utility level has no real meaning, beyond the statement that for a given individual a higher utility level is better.

e) Since the maximizing conditions are the same as those obtained in a)-c) and the budget constraints are not altered, we know that the equilibrium allocations will be the same too (so is the price ratio).

The after-trade MRS and utility levels are:

U1 =  = 1.61

U2 =  = 2.71





1.5. Recall that in equilibrium there should not be excess demand or excess supply for any good in the economy. If there is, then prices change accordingly to restore the equilibrium. The figure shows excess demand for good 2 and excess supply for good 1, a situation which requires p2 to increase and p1 to decrease to restore market clearing. This means that p1/p2 should decrease and the budget line should move counter-clockwise.

* 1. Consider a two agent –two good economy. Assume well-behaved utility functions (in particular, indifference curves don't exhibit flat spots). At a competitive equilibrium, both agents maximize their utility given their budget constraints. This leads each of them to select a bundle of goods corresponding to a point of tangency between one of his or her indifference curves and the price line. Tangency signifies that the slope of the IC and the slope of the budget line (the price ratio) are the same. But both agents face the same market prices. The slope of their indifference curves are thus identical at their respective optimal point.

Now consider the second requirement of a competitive equilibrium: that market clear. This means that the respective optimal choices of each of the two agents correspond to the same point of the Edgeworth-Bowley box.

Putting the two elements of this discussion together, we have that a competitive equilibrium is a point in the box corresponding to a feasible allocation where both agents’ indifference curves are tangent to the same price line, have the same slope, and, consequently, are tangent to one another. Since the contract curve is the locus of all such points in the box at which the two agents’ indifference curves are tangent, the competitive equilibrium is on the contract curve.

Of course, we could have obtained this result simply by invoking the First Welfare Theorem.

1.7. Indifference curves of agent 2 are non-convex.

Point A is a PO : the indifference curves of the two agents are tangent. This PO cannot be obtained as a Competitive Equilibrium, however. Let a price line tangent to I1 at point A. It is also tangent to I2, but “in the wrong direction”: it corresponds to a local minimum for agent 2 who, at those prices, can reach higher utility levels. The difficulty is generic when indifference curves have such a shape. The geometry is inescapable, underlining the importance of the assumption that preferences should be convex.