Mini-Lecture 1.1

Study Skill Tips for Success in Mathematics

Learning Objectives:

1. Get ready for this course.
2. Understand some general tips for success.
3. Know how to use the text.
4. Know how to use text resources.
5. Get help as soon as you need it.
6. Learn how to prepare for and take an exam.
7. Develop good time management.

Examples:

1. Get ready for this course.
   a) Positive attitude b) Allow adequate time for class arrival c) Bring all required material

2. Understand some general tips for success.
   a) Organize class materials b) Form study groups; exchange names and email addresses
   c) Attend <u>all</u> classes d) Do your homework
   e) Check your work; learn from mistakes f) Know how to get help
   g) Ask questions h) Hand in all assignments on time

3. Know how to use the text.
   a) Each example in every section has a Practice Problem associated with it.
   b) Review the meaning of icons used in text.
   c) Each chapter ends with Chapter Highlights, Reviews, Chapter Tests, and Cumulative Reviews.
   d) Student Resources section at the back of the text book contents Study Skill Builders, Study Guide Outline, a Practice Final, and Answers to Selected Exercises.

4. Know how to use text resources.
   a) Refer to the Lecture Videos and Chapter Test Prep Videos that accompany this text
   b) Use the Video Organizer and Student Organizer that accompany this text.

5. Get help as soon as you need it.
   a) Try your instructor, a tutoring center, or a math lab, or you may want to form as study group with fellow classmates.

6. Learn how to prepare for and take an exam.
   a) Review previous homework assignments, class notes, quizzes, etc.
   b) Read Chapter Highlights to review concepts and definitions.
   c) Practice working out exercises in the end-of-the-chapter Review and Test.
   d) When taking a test, read directions and problems carefully.
   e) Pace yourself. Use all available time. Check your work and answers.

7. Develop good time management.
   a) Make a list of all weekly commitments with estimated time needed.
   b) Be sure to schedule study time. Don't forget eating, sleeping, and relaxing!

Teaching Notes:

- Most developmental students have a high anxiety level with mathematics.
- Many developmental students are hesitant to ask questions and seek extra help.
- Be sure to include <u>your</u> individual expectations. Keep your expectations clear and concise.

Mini-Lecture 1.2

Algebraic Expressions and Sets of Numbers

Learning Objectives:

1. Identify natural numbers, whole numbers, integers, rational, and irrational real numbers.
2. Write phrases as algebraic expressions.
3. Key vocabulary: *the number sets, subset, element, variable, algebraic expression*.

Examples:

1. Define the number sets using set builder notation.

   a) natural numbers b) whole numbers c) integers
   d) rational numbers e) irrational numbers f) real numbers

2. Write each set in roster form.

   a) $\{x \mid x$ is an odd natural number$\}$ b) $\{x \mid x$ is an integer less than 2$\}$

   List the elements of the set $\{5, 0, \sqrt{3}, \sqrt{49}, \frac{1}{9}, -112\}$ that are also elements of the given set.

   c) natural numbers d) whole numbers e) integers
   f) rational numbers g) irrational numbers h) real numbers

   Place $\in$ or $\notin$ in the space provided to make each statement true.

   i) -9 $\{x \mid x$ is an integer$\}$ j) $\frac{2}{5}$ $\{x \mid x$ is a rational number$\}$ k) -3 $\{1, 3, 5, \ldots\}$

3. Write each phrase as an algebraic expression. Use the variable x to represent each unknown number.

   a) three times a number b) a number minus 2
   c) the quotient of a number and 5 d) ten and one-tenth plus a number
   e) five more than twelve times a number f) four less than six times a number

Teaching Notes:

- Remind students that "rational" starts with "ratio", and any number that can be expressed as a ratio of integers is a rational number.
- Many students have trouble identifying irrational numbers. Remind them that irrational numbers have non-terminating or non-repeating decimals.
- Some students are very confused by writing phrases into algebraic expressions and need additional examples.
- Refer students to the ***Real Numbers Diagram*** and the ***Selected Key Words/Phrases and Their Translations*** chart in the textbook.

<u>*Answers*</u>*: 1a) {1,2,3,…}, b) {0,1,2,3,…}, c) {…-3,-2,-1,0,1,2,3,…}, d)* $\left\{\frac{a}{b} \mid a \text{ and } b \text{ are integers and } b \neq 0\right\}$*, e) {x|x is a real number and x is not a rational number}, f) {x|x corresponds to a point on the number line}; 2a) {1,3,5,…}, b) {…-3,-2,-1,0,1}, c)* $\{5, \sqrt{49}\}$*, d)* $\{5, 0, \sqrt{49}\}$*, e)* $\{5, 0, \sqrt{49}, -112\}$*, f)* $\left\{5, 0, \sqrt{49}, \frac{1}{9}, -112\right\}$*, g)* $\{\sqrt{3}\}$*, h)* $\left\{5, 0, \sqrt{3}, \sqrt{49}, \frac{1}{9}, -112\right\}$*, i)* $\in$*, j)* $\in$*, k)* $\notin$*; 3a) 3x, b) x-2, c)* $\frac{x}{5}$*, d)* $10\frac{1}{10} + x$*, e) 12x+5, f) 6x-4*

Mini-Lecture 1.3

Equations, Inequalities, and Properties of Real Numbers

Learning Objectives:

1. Write sentences as equations.
2. Use inequality symbols.
3. Find the opposite, or additive inverse, and the reciprocal, or multiplicative inverse, of a number.
4. Identify and use the commutative, associative, and distributive properties.
5. Key vocabulary: *equation, inequality symbolism* ($<, >, =, \leq, \geq, \neq$)*, identity, inverse, commutative, associative, distributive*.

Examples:

1. Write each sentence as an equation.

   a) the difference of x and 4 amounts to 15
   b) five more than the product of 3 and b is 7
   c) the quotient of 9 and y is 2 more than y
   d) three added to one-half t is the same as eight more that t

2. Insert <, >, or = between each pair of numbers to form a true statement.

   a) -2 0 b) $\frac{36}{9} \quad \frac{36}{6}$ c) 2.5 -6.7 d) $\frac{3}{5} \quad \frac{9}{15}$

3. Write the opposite of each number, and then write the reciprocal of each number.

   a) 8 b) $-\frac{2}{3}$ c) 0 d) $\frac{45}{7}$

4. Use a commutative property to write an equivalent expression.

   a) $5x+3y$ b) $m \cdot n$ c) $\frac{x}{5} \cdot \frac{3}{11}$

   Use an associative property to write an equivalent expression.

   d) $4 \cdot (15x)$ e) $9p+(3q+r)$ f) $(3.5x) \cdot y$

   Use the distributive property to multiply.

   g) $3(x+5)$ h) $2(m-7)$ i) $3(5-y)$

   j) $10y(z-4)$ k) $0.2(2x+6y)$ l) $\frac{1}{3}(9x-5y)$

Teaching Notes:

- Some students find number 1 very difficult and need more examples.
- In number 2, also discuss how the $\leq$, $\geq$, and $\neq$ symbols could be used between the numbers.
- Many students confuse the associative and commutative properties.
- Many students have trouble using the distributive property when fractions are involved.

Answers: 1a) x-4=15, b) 3b+5=7, c) $\frac{9}{y} = y+2$*, d)* $\frac{1}{2}t+3=t+8$*; 2a) <, b) <, c) >, d) =; 3a) -8,* $\frac{1}{8}$*, b)* $\frac{2}{3}, -\frac{3}{2}$*, c) 0, undefined, d)* $-\frac{45}{7}, \frac{7}{45}$*; 4a) 3y+5x, b)* $n \cdot m$*, c)* $\frac{3}{11} \cdot \frac{x}{5}$*, d)* $(4 \cdot 15)x$*, e) (9p+3q)+r, f)* $3.5(x \cdot y)$*, g) 3x+15, h) 2m-14, i) 15-3y, j) 10yz-40y, k) 0.4x+1.2y, l)* $3x-\frac{5}{3}y$

Mini-Lecture 1.4

Operations on Real Numbers

Learning Objectives:

1. Find the absolute value of a number.
2. Add and subtract real numbers.
3. Multiply and divide real numbers.
4. Simplify expressions containing exponents.
5. Find roots of numbers.
6. Key vocabulary: *absolute value, exponent, square root, principal square root.*

Examples:

1. Find each absolute value.

   a) $|6|$ b) $|-2|$ c) $-|12|$ d) $-|-14|$

2. Add, subtract, multiply, or divide as indicated.

   a) $-6+(-3)$ b) $6+(-3)$ c) $-6+3$ d) $-\frac{5}{12}+\frac{3}{24}$

   e) $2-5$ f) $2-(-5)$ g) $-2-5$ h) $-9.7-(-4.2)$

   i) $4\cdot 5$ j) $-4\cdot 5$ k) $0(-9)$ l) $-\frac{2}{3}\left(-\frac{9}{12}\right)$

   m) $6\div(-2)$ n) $\frac{-12}{-4}$ o) $-4.6\div 2.3$ p) $\frac{-3}{0}$

3. Find the value of each expression, each of which contains an exponent or a root.

   a) -5^2 b) $(-5)^2$ c) $\left(-\frac{1}{2}\right)^3$ d) $-\left(\frac{1}{3}\right)^4$

   e) $\sqrt{25}$ f) $\sqrt{81}$ g) $\sqrt{\frac{1}{100}}$ h) $\sqrt[3]{8}$

4. Mixed practice.

   a) $-\frac{6}{11}\div 2$ b) $-12-5+7$ c) $-3(4)(-7)$ d) -3^4

Teaching Notes:

- Some students try to distribute negative signs through an absolute value symbol.
- In number 2, students need to master the integer examples before trying fractions and decimals.
- Many students make sign errors when evaluating expressions with exponents.
- Refer students to the ***Adding Real Numbers*** and ***Subtracting Real Numbers*** charts in the text.

Answers: 1a) 6, b) 2, c) -12, d) -14; 2a) -9, b) 3, c) -3, d) $-\frac{7}{24}$*, e) -3, f) 7, g) -7, h) -5.5, i) 20, j) -20, k)0, l)* $\frac{1}{2}$*, m) -3, n) 3, 0) -2, p) undefined; 3a) -25, b) 25, c)* $-\frac{1}{8}$*, d)* $-\frac{1}{81}$*, e) 5, f) 9, g)* $\frac{1}{10}$*, h) 2; 4a)* $-\frac{3}{11}$*, b) -10, c) 84, d) -81*

Mini-Lecture 1.5

Order of Operations and Algebraic Expressions

Learning Objectives:

1. Use the order of operations.
2. Identify and evaluate algebraic expressions.
3. Identify like terms and simplify algebraic expressions.
4. Key vocabulary: *expression, term, like terms, evaluate, simplify*.

Examples:

1. Simplify each expression using order of operations.

   a) $6(2-5)^2$ b) $(-4)^2+3^3$ c) $5[9-(1-3)]$

   d) $-8\left(-\frac{3}{4}\right)-6$ e) $\frac{(-8+5)(-2^2)}{-3-3}$ f) $(\sqrt[3]{8})(-4)-(\sqrt{16})(-2)$

   g) $\frac{-\sqrt{25}-(5-3.4)}{-3}$ h) $\frac{|-12|-|3-8|}{-12}$ i) $\frac{\frac{1}{4}\cdot 28-5}{6+\frac{1}{4}\cdot 16}$

2. Evaluate each expression when $x = 5$ and $y = -3$.

   a) $3x-7y$ b) $-9y^2+x$ c) $\frac{6+4|y+x|}{x+4y}$

3. Simplify by distributing, if necessary, and combining like terms.

   a) $4x+5x$ b) $12y-y$ c) $3x-9-12x$

   d) $2.5y-8.6+3.4y-12.3$ e) $3(6y+9)$ f) $2k-(5k-4)$

   g) $-(9-t)+(3t-6)$ h) $\frac{1}{2}(12x-4)-\frac{1}{6}(30x-y)$ i) $7.3b+8.1-2(3.2b-0.4)$

4. If C is degrees Celsius, the algebraic expression $1.8C+32$ represents the equivalent temperature in degrees Fahrenheit. Calculate the degrees Fahrenheit for C = -20, 0, and 40. As degrees Celsius increase, do degrees Fahrenheit increase or decrease?

Teaching Notes:

- Many students have trouble with order of operations. Encourage them to be neat and organized.
- When collecting like terms, some students think $4x+5x=9x^2$.
- Some students find it helpful to write a "1" in front of the parenthesis in 3f and 3g.
- Refer students to the ***Order of Operations*** chart in the text.

Answers: 1a) 54, b) 43, c) 55, d) 0, e) -2, f) 0, g) 2.2, h) $-\frac{7}{12}$*, i)* $\frac{1}{5}$*; 2a) 36, b) -76, c) -2; 3a) 9x, b) 11y, c) -9x-9, d) 5.9y-20.9, e) 18y+27, f) -3k+4, g) 4t-15, h)* $x+\frac{1}{6}y-2$*, i) 0.9b+8.9; 4) -4°F, 32°F, 104°F, increase*

Mini-Lecture 1.6

Exponents and Scientific Notation

Learning Objectives:

1. Use the product rule for exponents.
2. Simplify expressions raised to the zero power.
3. Use the quotient rule for exponents.
4. Simplify expressions raised to negative powers.
5. Simplify exponential expressions containing variables in the exponent.
6. Convert between scientific notation and standard notation.
7. Key vocabulary: *exponential expressions, scientific notation*.

Examples:

1. Use the product rule or the quotient rule to simplify each expression.

   a) $2^3 \cdot 2^4$ b) $m \cdot m^9 \cdot m^7$ c) $(-6xy)(6y)$ d) $(-3a^3b^2)(-5a^3b)$

   e) $\dfrac{x^8}{x^3}$ f) $-\dfrac{10y^{11}}{2y^7}$ g) $\dfrac{15x^6y^5}{9xy^3}$ h) $\dfrac{36a^2b^3c^{12}}{-4abc^9}$

2. Evaluate or simplify each expression. Write final answers using positive exponents only.

   a) 2^0 b) -5^0 c) $(-10)^0$ d) $(2x+1)^0$

   e) 2^{-4} f) $(-3)^{-2}$ g) $\dfrac{x^2}{x^4}$ h) $2a^{-3}$

3. Simplify and write using positive exponents only.

   a) $\dfrac{y^{-3}}{y^6}$ b) $\dfrac{x^{-5}x^4}{x^{-2}}$ c) $\dfrac{12ab^{-3}}{4a^{-3}b^3}$ d) $-4y \cdot -6y^3$

   e) $2x^0 - (2x)^0$ f) $2^{-3} - 4^{-2}$ g) $\dfrac{20x^{-8}yz^{-13}}{2xyz}$ h) $(4a^5b)(3b^2)(-2a^7)$

4. Simplify. Assume that variables in the exponents represent nonzero integers, and $x, y, z \neq 0$.

   a) $x^{2m} \cdot x^{7m}$ b) $\dfrac{y^{3p-2}}{y^{2p}}$ c) $x^{5y} \cdot x^{-7y}$ d) $\dfrac{y^{3x} \cdot y^{x-2}}{y^x}$

5. Write each number in scientific notation or in standard notation.

   a) 645,000 b) 0.005621 c) 3.6×10^{-4} d) 9.5×10^5

Teaching Notes:

- Students need a lot of repetition and practice in order to master these objectives.
- Students often move constants along with a variable that has a negative exponent. For example, in 2h, a common {incorrect} answer is $2a^{-3} = 1/(2a^3)$.
- Refer students to the exponent rule charts and the ***Writing a Number in Scientific Notation*** chart in the text.

Answers: 1a) 128, b) m^{17}, c) $-36xy^2$, d) $15a^6b^3$, e) x^5, f) $-5y^4$, g) $\dfrac{5x^5y^2}{3}$, h) $-9ab^2c^3$; 2a) 1, b) -1, c) 1, d) 1, e) $\dfrac{1}{16}$, f) $\dfrac{1}{9}$, g) $\dfrac{1}{x^2}$, h) $\dfrac{2}{a^3}$; 3a) $\dfrac{1}{y^9}$, b) x, c) $\dfrac{3a^4}{b^6}$, d) $24y^4$, e) 1, f) $\dfrac{1}{16}$, g) $\dfrac{10}{x^9z^{14}}$, h) $-24a^{12}b^3$; 4a) x^{9m}, b) y^{p-2}, c) $\dfrac{1}{x^{2y}}$, d) y^{3x-2}; 5a) 6.45×10^5, b) 5.621×10^{-3}, c) 0.00036, d) 950,000

Mini-Lecture 1.7

More Work with Exponents and Scientific Notation

Learning Objectives:

1. Use the power rules for exponents.
2. Use exponent rules and definitions to simplify exponential expressions.
3. Simplify exponential expressions containing variables in the exponent.
4. Use scientific notation to compute.
5. Key vocabulary: *exponential expression, scientific notation*.

Examples:

1. Simplify using the product rules for exponents. Write each answer using positive exponents only.

   a) $(x^3)^2$ b) $(x^2y^3)^2$ c) $\left(\dfrac{x^2}{y^3}\right)^2$ d) $(m^3)^{-4}$

   e) $(2x^2yz^3)^2$ f) $\left(\dfrac{3x^4}{y^{-2}}\right)^5$ g) $(4x^{-5}y^3z^0)^{-3}$ h) $(-2^{-3}y^{-3})^{-4}$

2. Simplify using exponent rules and definitions. Write each answer using positive exponents only.

   a) $\left(\dfrac{a^{-3}b^{-4}}{c^{-9}}\right)^{-2}$ b) $(-4x^2)^3$ c) $\left(\dfrac{n^6}{2m^{-3}}\right)^{-5}$ d) $\dfrac{7^{-2}x^{-2}y^{10}}{x^3y^{-4}}$

   e) $(-2x^0y^2)^{-3}$ f) $x^3(x^3by)^{-3}$ g) $\left(\dfrac{2z^{-3}}{y}\right)\left(\dfrac{7y^{-5}}{z^{-2}}\right)^{-1}$ h) $(3x^4y^2)^{-3}(2x^8y^3)$

3. Simplify. Assume that variables in the exponents represent nonzero integers and that all other variables are not zero.

   a) $(x^{3b+5})^2$ b) $\dfrac{x^{-2y+9}x^y}{x}$ c) $\dfrac{(y^{3a})^6}{y^{2a-1}}$ d) $\dfrac{12x^{4a-b}y^{3a+b}}{4x^{a-3b}y^{a+2b}}$

4. Perform each indicated operation using the properties of exponents. Write each answer in scientific notation.

   a) $(4.9\times10^{-9})(6\times10^7)$ b) $(4\times10^{-6})^5$ c) $\dfrac{1.2\times10^8}{3\times10^{-4}}$

Teaching Notes:

- Some students are confused by when to add exponents versus when to multiply exponents.
- Encourage students to write the exponent rules on an index card to view while doing homework.
- Refer students to the ***Summary of Rules for Exponents*** chart in the text.

Answers: 1a) x^6, b) x^4y^6, c) $\dfrac{x^4}{y^6}$, d) $\dfrac{1}{m^{12}}$, e) $4x^4y^2z^6$, f) $243x^{20}y^{10}$, g) $\dfrac{x^{15}}{64y^9}$, h) $4096y^{12}$; 2a) $\dfrac{a^6b^8}{c^{18}}$, b) $-64x^6$, c) $\dfrac{32}{m^{15}n^{30}}$, d) $\dfrac{y^{14}}{49x^5}$, e) $-\dfrac{1}{8y^6}$, f) $\dfrac{1}{b^3x^6y^3}$, g) $\dfrac{2y^4}{7z^5}$, h) $\dfrac{2}{27x^4y^3}$; 3a) x^{6b+10}, b) x^{-y+8}, c) y^{16a+1}, d) $3x^{3a+2b}y^{2a-b}$; 4a) 2.94×10^{-1}, b) 1.024×10^{-27}, c) 4.0×10^{11}

Mini-Lecture 2.1

Linear Equations in One Variable

Learning Objectives:

1. Decide whether a number is a solution of an equation.
2. Solve linear equations using properties of equality.
3. Solve linear equations that can be simplified by combining like terms.
4. Solve linear equations containing fractions or decimals.
5. Recognize identities and equations with no solution.
6. Key vocabulary: *equation, solution, equivalent equation, contradiction, identity*.

Examples:

1. Determine whether each number is a solution of the given equation.

   a) $2;\ x+3=5$ b) $-6;\ x-3=9$ c) $-36;\ \frac{x}{-6}=12$

2. Solve each equation and check.

   a) $x-5=7$ b) $x+3=15$ c) $-6=x+4$

   d) $2x=10$ e) $-3x=15$ f) $\frac{x}{4}=3$

   g) $4x-2=6+3x$ h) $5y-4=10+3y$ i) $9.3-4x=-2.3$

   j) $3(2x+4)=9x-3$ k) $-2(3n-1)-n=-5(n-4)$

3. Solve each equation and check.

   a) $\frac{x}{3}+\frac{x}{2}=\frac{1}{4}$ b) $\frac{2x}{5}-\frac{x}{3}=5$ c) $\frac{2r}{5}-3=\frac{r}{10}$

   d) $\frac{28-4x}{3}=x$ e) $\frac{2y-6}{5}=1-2y$ f) $3.4(2x+5)=-0.2(2x+5)$

4. Solve each equation.

   a) $2(x+6)=12+2x$ b) $4(x+5)+3=5(x+2)-x$

Teaching Notes:

- Encourage students to check their solutions.
- Some students prefer to always end up with the variable on the left, while others prefer to always end up with a positive coefficient in front of the variable.
- Some students try to subtract the coefficient from a variable instead of dividing it off.
- Refer students to the ***Addition/Multiplication Property*** and ***Solving a Linear Equation in One Variable*** charts in the text.

Answers: 1a) yes, b) no, c) no; 2a) {12}, b) {12}, c) {-10}, d) {5}, e) {-5}, f) {12}, g) {8}, h) {7}, i) {2.9}, j) {5}, k) {-9}; 3a) $\left\{\frac{3}{10}\right\}$*, b) {75}, c) {10}, d) {4}, e)* $\left\{\frac{11}{12}\right\}$*, f) {-2.5}; 4a) {x|x is a real number}, b)* $\varnothing$

Mini-Lecture 2.2

An Introduction to Problem Solving

Learning Objectives:

1. Write algebraic expressions that can be simplified.
2. Apply the steps for problem solving.
3. Key vocabulary: *consecutive integers, complementary angles, supplementary angles*.

Examples:

1. Write the following as algebraic expressions. Then simplify.

   a) The sum of three consecutive integers if the first integer is x.
   b) The perimeter of a rectangle with length x and width $x - 7$.
   c) The total amount of money (in cents) in x quarters, $5x$ dimes, and $(3x\text{-}1)$ nickels.

2. Solve using the General Strategy for Problem Solving.

   a) ***Number Problem*** One number is two times another number. The sum of the numbers is 90. What are the two numbers?

   b) ***Number Problem*** Three times the difference of a number and 5 is the same as 1 increased by five times the number plus twice the number.

   c) ***Age Problem*** Today Henry is 7 years older than twice his age of 23 years ago. Find Henry's age today.

   d) ***Car Rental*** A car rental agency advertised renting a luxury, full-size car for $19.95 per day and $0.29 per mile. If you rent this car for 5 days, how many whole miles can you drive if you only have $200 to spend?

   e) ***Carpentry*** A 7-ft. board is cut into 2 pieces so that one piece is 3 feet longer than 3 times the shorter piece. If the shorter piece is x feet long, find the lengths of both pieces.

   f) ***Unknown Sides*** A triangle has sides measuring $2.5x$ cm, $3x$ cm, and $(2x + 3)$ cm. It's perimeter measures 60 cm. Find the measures of the sides.

   g) ***Unknown Angles*** Two angles are complementary if their sum is 90°. If the measure of the first angle is x°, and the measure of the second angle is $(3x - 2)$°, find the measure of each angle.

   h) ***Lay-offs*** A major car manufacturer announced it would lay off 17,000 employees worldwide. This is equivalent to 20% of its work force. Find the size of the work force prior to lay-offs.

Teaching Notes:

- Many students have difficulty with word problems.
- Encourage students to draw and label diagrams when appropriate.
- Some students need to see several examples of consecutive or consecutive odd/even integers.
- Refer students to the ***General Strategy for Problem Solving*** chart in the text.

Answers: 1a) x+x+1+x+2=3x+3, b) 4x-14, c) 90x-5; 2a) 30,60, b) -4, c) 39, d) 345 miles, e) 1 foot, 6 feet, f) 19 cm, 22.8 cm, 18.2 cm, g) 23°, 67°, h) 85,000 employees

Mini-Lecture 2.3

Formulas and Problem Solving

Learning Objectives:

1. Solve a formula for a specified variable.
2. Use formulas to solve problems.
3. Key vocabulary: *formula*.

Examples:

1. Solve each equation for the specified variable.

a) $M = kt$ for t b) $C = 2\pi r$ for r c) $a^2 + b^2 = c^2$ for a^2

d) $4x + 5y = 16$ for y e) $P = 2l + 2w$ for l f) $C = \frac{5}{9}(F - 32)$ for F

2. Solve. Round all dollar amounts to two decimal places.

a) ***Volume*** Find the volume of a rectangular crate with dimensions 3 ft by 4 ft by 8 ft.

b) ***Distance*** Sheranda drives at a constant 65 miles per hour. How far will she travel in 4 hours?

c) ***Compound Interest*** Emmanuel puts $5010 at 9% compounded semiannually for 12 years. What is the value of his account at the end of the 12 years?

d) ***Circle*** Crystal is making a cover for a round table that has a diameter of 46 inches. How much fabric will she need if she wants the cover to fit exactly, with no material hanging off? (Use 3.14 for π and round to two decimal places.)

e) ***Office Rental*** An accountant rents office space. He is charged $2040 per month for a rectangular office that measures 17 ft by 20 ft. How much is he paying each month in rent per square foot?

f) ***Temperature*** Michael's cousin Luke was visiting from Montreal during the summer. On a news report Luke heard that the temperature in Montreal that day was 98°F. He was used to hearing temperature in degrees Celsius. What is 98°F in degrees Celsius?

g) ***Triangle*** A triangular piece of wood needs to be varnished. The base of the triangle is 3 meters and the height is 13 meters. How many cans of varnish will be needed if each can covers 10 square meters?

Teaching Notes:

- Some students are very confused by solving for a variable when other variables are present.
- Many students benefit from seeing a parallel example with numbers instead of variables. For example, next to (1a) solve: 6 = 3*t*
- Encourage students to draw and label diagrams when appropriate.
- Refer students to the ***Formula*** and ***Solving an Equation for a Specified Variable*** charts in the text.

Answers: 1a) $t = \frac{M}{k}$*, b)* $r = \frac{C}{2\pi}$*, c)* $a^2 = c^2 - b^2$*, d)* $y = \frac{16 - 4x}{5}$*, e)* $l = \frac{P - 2w}{2}$*, f)* $F = \frac{9}{5}C + 32$*; 2a) 96 cubic feet, b) 260 miles, c) $14,408.83, d) 1,661.06 square inches, e) $6.00 per square foot, f) 36.67°C, g) 2 cans*

Mini-Lecture 2.4

Linear Inequalities and Problem Solving

Learning Objectives:

1. Use interval notation.
2. Solve linear inequalities using the addition property of inequality.
3. Solve linear inequalities using the multiplication property of inequality.
4. Solve linear inequalities using both properties of inequality
5. Solve problems that can be modeled by linear inequalities.
6. Key vocabulary: *greater than (or equal to), less than (or equal to), solution set, interval notation*.

Examples:

1. Graph the solution set of each inequality on a number line and then write it in interval notation.

   a) $\{x \mid x > 3\}$ b) $\{x \mid x < -2\}$ c) $\{x \mid -4.2 \geq x\}$ d) $\{x \mid -3 < x \leq 0\}$

2. Solve. Graph the solution set and write it in interval notation.

   a) $x + 2 \leq 6$ b) $10x < 9x + 3$ c) $5x - 5 \geq 4x - 5$

3. Solve. Graph the solution set and write it in interval notation.

   a) $\frac{1}{2}x \geq 2$ b) $2x > -7.2$ c) $-3x \leq 6$

4. Solve. Show your answer as an inequality.

   a) $x - 6 < -9$ b) $-7x \leq 2.8$ c) $\frac{5}{6} - \frac{3}{4} > \frac{x}{3}$ d) $\frac{3}{4}(x+2) \geq x + 2$

   e) $0.3(6x-1) < 1.4(x-3) - 0.1$ f) $\frac{4}{5} + \frac{2}{3}x \geq \frac{3}{10}x + \frac{1}{4}$

Teaching Notes:

- Some students are very confused by solving for a variable when other variables are present.
- Many students forget to reverse the direction of the inequality symbol when necessary.
- Some students prefer to move the variable in such a way that it has a positive coefficient if possible.
- Refer to the end-of-section exercises for application problems.
- Refer students to the ***Addition/Multiplication Property of Inequality*** and ***Solving a Linear Inequality in One Variable*** charts in the text.

Answers: (graph answers at end of mini-lectures) 1a) $(3,\infty)$ *, b)* $(-\infty,-2)$ *, c)* $(-\infty,-4.2]$ *, d)* $(-3,0]$ *; 2a)* $(-\infty,4]$ *, b)* $(-\infty,3)$ *, c)* $[0,\infty)$ *; 3a)* $[4,\infty)$ *, b)* $(-3.6,\infty)$ *, c)* $[-2,\infty)$ *; 4a)* $x < -3$ *, b)* $x \geq -0.4$ *, c)* $x < \frac{1}{4}$ *, d)* $x \leq -2$ *, e)* $x < -10$ *, f)* $x \geq -\frac{3}{2}$

Mini-Lecture 2.5

Sets and Compound Inequalities

<u>Learning Objectives</u>:

1. Find the intersection of two sets.
2. Solve compound inequalities containing "*and*".
3. Find the union of two sets.
4. Solve compound inequalities containing "*or*".
5. Key vocabulary: *and, or, intersection, union*.

<u>Examples</u>:

1. If $A=\{x\,|\,x \text{ is an even integer}\}$, $B=\{x\,|\,x \text{ is an odd integer}\}$, $C=\{1,2,3,4\}$, and $D=\{3,4,5,6\}$, list the elements of each set.

   a) $C\cap D$ b) $C\cup D$ c) $A\cup D$ d) $B\cap C$ e) $A\cap B$

2. Solve each compound inequality by graphing the solution on a number line.

   a) $x\le 1$ and $x\ge -3$ b) $x<1$ and $x>4$ c) $x\ge -3$ and $x>2$

   Solve each compound inequality. Write solutions in interval notation.

   d) $x+3\ge 4$ and $5x-2\ge 8$ e) $-5x<-15$ and $x-15<-10$

   f) $-4\le x+1\le -2$ g) $-3<\frac{2}{3}x-1<1$ h) $-1\le \frac{-3x+4}{5}\le 1$

3. Solve each compound inequality by graphing the solution on a number line.

   a) $x\ge -3$ or $x\le 3$ b) $x<-1$ or $x<1$ c) $x\ge -2$ or $x\le -3$

   Solve each compound inequality. Write solutions in interval notation.

   d) $-10x\le 20$ or $3x-4\ge 2$ e) $x+8<-1$ or $5x>-15$ f) $6(x-2)\ge -12$ or $4-x\le 10$

4. Solve each compound inequality. Write solutions in interval notation.

   a) $x<\frac{2}{3}$ and $x<1$ b) $x<\frac{2}{3}$ or $x<1$ c) $1<5x-1<9$

   d) $2x-3\ge 1$ and $-x>3$ e) $-3<\frac{-2x-1}{2}<3$ f) $-x<6$ or $4x+4<-20$

<u>Teaching Notes</u>:

- In problems 2a-c) and 3a-c), show students how each inequality can be graphed separately on its own number line. Then the solution graph is the intersection (or union) of the individual graphs.

<u>Answers</u>: (graph answers at end of mini-lectures) 1a) {3,4}, b) {1,2,3,4,5,6}, c) {x|x is an even integer, x=3, x=5}, d) {1,3}, e) $\varnothing$; *2a) [-3,1], b) no solution, c)* $(2,\infty)$, *d)* $[2,\infty)$, *e) (3,5), f) [-5,-3], g) (-3,3), h)* $\left[-\frac{1}{3},3\right]$; *3a) all real numbers, b)* $(-\infty,1)$, *c)* $(-\infty,-3]\cup[-2,\infty)$, *d)* $[-2,\infty)$, *e)* $(-\infty,-9)\cup(-3,\infty)$, *f)* $[-6,\infty)$; *4a)* $\left(-\infty,\frac{2}{3}\right)$, *b)* $(-\infty,1)$, *c)* $\left(\frac{2}{5},2\right)$, *d)* $\varnothing$, *e) (-3.5,2.5), f)* $(-\infty,-6)\cup(-6,\infty)$

Mini-Lecture 2.6

Absolute Value Equations

Learning Objectives:

1. Solve absolute value equations.

Examples:

1. Solve.

a) $|x| = 6$　　b) $|x| = -6$　　c) $|3m| = 9.3$　　d) $6|x| - 7 = 5$

e) $|x+4| = 9$　　f) $\left|\frac{x}{3} - 2\right| = 1$　　g) $|5x| = 0$　　h) $|2n+3| + 9 = 4$

i) $2|x-1| + 15 = 20$　　j) $|5x+9| = |x+4|$　　k) $\left|\frac{1}{2}x + 3\right| = \left|\frac{2}{3}x - 1\right|$

Teaching Notes:

- Refer students to the ***Solving Equations of the Form |X| = a*** and ***Absolute Value Equations*** charts in the text.

Answers: 1a) {6,-6}, b) $\varnothing$, c) {3.1,-3.1}, d) {-2,2}, e) {-13,5}, f) {3,9}, g) {0}, h) $\varnothing$, i) $\left\{-\frac{3}{2}, \frac{7}{2}\right\}$, j) $\left\{-\frac{13}{6}, -\frac{5}{4}\right\}$, k) $\left\{24, -\frac{12}{7}\right\}$

Mini-Lecture 2.7

Absolute Value Inequalities

Learning Objectives:

1. Solve absolute value inequalities of the form $|X| < a$.
2. Solve absolute value inequalities of the form $|X| > a$.

Examples:

1. Solve. Graph the solution set.

   a) $|x| \le 3$ b) $|x| \ge 3$ c) $|x| < -3$ d) $|x| > -3$

   e) $|x+3| < 7$ f) $|x| + 4 \le 8$ g) $\left|\frac{x-3}{5}\right| < 1$ h) $|6-3x| < 4$

   i) $|x-5| \ge 8$ j) $|x| + 6 > 7$ k) $|9+4x| - 3 > -2$ l) $\left|\frac{11+x}{7}\right| \ge 2$

2. Solve each equation or inequality for x.

   a) $|x| < 4$ b) $|x| = 2$ c) $|3x| = 15$ d) $|8+2x| \ge 0$

   e) $|x| - 3 = -7$ f) $|x-2| \ge 8$ g) $\left|\frac{1}{3}x - 3\right| < 2$ h) $|4x-1| + 9 = 11$

Teaching Notes:

- Most students need to see the solutions to 1a-d) on a number line in order to visualize the solution set. For the rest of the problems in #1 they can go right to the method shown in the solving Absolute Value Equations and Inequalities chart in the text.

Answers: (graph answers at end of mini-lectures) 1a) [-3,3], b) $(-\infty,-3]\cup[3,\infty)$ *, c)* $\varnothing$ *, d) {all real numbers}, e) (-10,4), f) [-4,4], g) (-2,8), h)* $\left(\frac{2}{3},\frac{10}{3}\right)$*, i)* $(\infty,-3]\cup[13,\infty)$*, j)* $(-\infty,-1)\cup(1,\infty)$*, k)* $\left(-\infty,-\frac{5}{2}\right)\cup(-2,\infty)$*, l)* $(-\infty,-25]\cup[3,\infty)$*; 2a) (-4,4), b) {-2,2}, c) {5,-5}, d) {all real numbers}, e)* $\varnothing$*, f)* $(-\infty,-6]\cup[10,\infty)$*, g) (3,15),*

Mini-Lecture 3.1

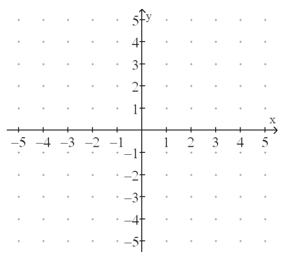
Graphing Linear Equations

Learning Objectives:

1. Plot ordered pairs on a rectangular coordinate system.
2. Determine whether an ordered pair of numbers is a solution of an equation in two variables.
3. Graph linear equations.
4. Graph vertical and horizontal lines.
5. Key vocabulary: *rectangular coordinate system, Cartesian, axis, origin, quadrant, ordered pair, coordinate, point, solution, intercept, standard form.*

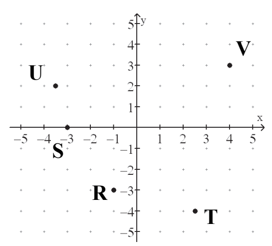
Examples:

1. Identify and label the following features on the rectangular coordinate system shown.



a) x-axis
b) y-axis
c) x values –5 to 5
d) y values –5 to 5
e) origin (0,0)
f) ordered pair (3,4)
g) ordered pair (x,y)
h) quadrant I,II,III,IV

2. Determine the ordered pairs, or, plot the points. Name the quadrant in which each point lies.

a) 

b) (4,2) ; (-3,5) ; (-2,-4) ; (3,-4) ; (0,5) ; (-2.5,0)

3. Determine whether each ordered pair is a solution of the given equation.

a) $x + y = 7$; (1,6), (-3,10) b) $y = -3x + 2$; (0,2), (-2,10) c) $4x - 3y = 1$; $\left(\frac{1}{2}, \frac{2}{3}\right)$, (0,1)

4. Graph each linear equation by finding any three ordered pairs that are solutions to the equation.

a) $x + y = 2$ b) $2x - 4y = 8$ c) $y = \frac{2}{3}x + 3$ d) $x = 3$ e) $y = -2$

Teaching Notes:

- In problem 4, some students do not realize that they can choose any x-value at all and solve for y, or vice versa.
- Be sure to show students how to plot using x- and y-intercepts too.
- Refer to the end of section exercises for scatter diagram problems and word problems.
- Refer students to the ***Linear Equation in Two Variables***, ***Finding x- and y-Intercepts***, and ***Graphing Vertical and Horizontal Lines*** charts in the text.

Answers: (graph answers at end of mini-lectures) 2a) R(-1,-3), S(-3,0), T(2.5,-4), U(-3.5,2), V(4,3); 3a) yes, yes, b) yes, no, c) no, no

Mini-Lecture 3.2

The Slope of a Line

Learning Objectives:

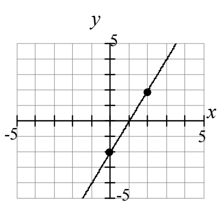
1. Find the slope of a line given two points on the line.
2. Find the slope of a line given the equation of the line.
3. Find the slopes of horizontal and vertical lines.
4. Find the slope of a line given the graph of the line.
5. Compare the slopes of parallel and perpendicular lines.
6. Key vocabulary: *rise, run, slope, slope-intercept form, perpendicular, parallel.*

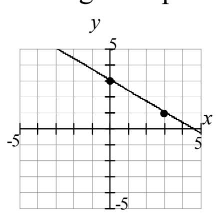
Examples:

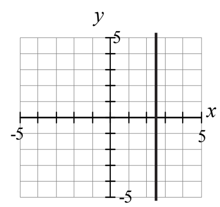
1. Fill in the missing information and draw an example of each line.

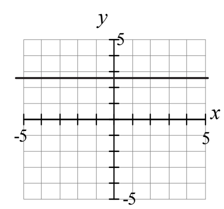
   a) If the height of a line increases as the x-value increases, the slope is __________.
   b) If the height of a line decreases as the x-value increases, the slope is __________.

2. Find the slope of the line containing each pair of points.

   a) 

   b) 

   c) 

   d) 

   e) (1, 5) , (6, 11) f) (3, 6) , (-2, 9) g) (3, -1) , (4, -5)

3. Find the slope and the y-intercept of each line.

   a) $y = x + 3$ b) $y = -4x - 1$ c) $-3x + y = 9$

   d) $x = 3.4$ e) $y = -\frac{1}{3}x$ f) $2x - 9y = 36$ g) $y - 8 = 0$

4. Determine whether each pair of lines is parallel, perpendicular, or neither.

   a) $\begin{array}{l} y = 3x - 4 \\ y = 3x + 2 \end{array}$ b) $\begin{array}{r} -2x + 4y = 1 \\ 6x + 3y = 3 \end{array}$ c) $\begin{array}{l} y = 3x + 4 \\ y = -3x + 4 \end{array}$

Teaching Notes:

- Some students need to see many numeric examples of m = rise/run shown on a graph before trying to use the slope formula.
- Many students make sign errors with the slope formula.
- Some students consistently put the change in x instead of the change in y in the numerator.
- Some students are confused by the slopes of horizontal and vertical lines.
- Some students understand objective 5 better if it is introduced using a discovery activity.
- Refer students to the ***Slope of a Line***, ***Slope-Intercept Form***, ***Slopes of Vertical and Horizontal Lines***, and ***Parallel/Perpendicular Lines*** charts in the text.

Answers: (graph answers at end of mini-lectures) 1a) positive, b) negative; 2a) m=2, b) $m = -\frac{2}{3}$*, c) undefined, d) m=0, e)* $m = \frac{6}{5}$*, f)* $m = -\frac{3}{5}$*, g) m=-4; 3a) m=1, (0,3), b) m=-4, (0,-1), c) m=3, (0,9), d) undefined, no y-intercept, e)* $m = -\frac{1}{3}$*, (0,0), f)* $m = \frac{2}{9}$*, (0,-4), g) m=0, (0,8); 4a) parallel, b) perpendicular, c) neither*

Mini-Lecture 3.3

The Slope-Intercept Form

Learning Objectives:

1. Graph a line using its slope and y-intercept.
2. Use the slope-intercept form to write an equation of the line.
3. Interpret the slope-intercept form in an application.

Examples:

1. Graph each line passing through the given point with the given slope.

   a) through (1,2); slope $\frac{2}{3}$ b) through (-2,3); slope $-\frac{5}{2}$ c) through (0,0); slope 2

   Graph each linear equation using the slope and y-intercept.

   d) $y = 2x$ e) $y = 2x + 3$ f) $y = -2x + 1$

   g) $y = \frac{1}{2}x - 2$ h) $x + 2y = 6$ i) $3x - 2y = 12$

2. Use the slope-intercept form of a linear equation to write the equation of each line with the given slope and y-intercept.

   a) slope -1; y-intercept (0,4) b) slope $\frac{1}{3}$; y-intercept (0,-7) c) slope $-\frac{5}{2}$; y-intercept (0,0)

3. Solve.

   a) When a road-side service truck is called, the cost of the service is given by the linear function $y = 2x + 60$, where y is in dollars and x is the number of hours the car is worked on. Find and interpret the slope and y-intercept of the linear equation.

   b) The amount of water in a leaky water jug is given by the linear function $y = 117 - 10x$, where y is in ounces and x is in minutes. Find and interpret the slope and y-intercept of the linear function.

Teaching Notes:

- Some students need a lot of practice using the slope to graph a line.
- Some students are uncertain how to use the slope to graph a line when the slope is an integer.
- Emphasize to students how the sign of the slope is built into the direction you go when using the slope to graph a line.
- Emphasize the importance of units when interpreting the meaning of slope for applied problems.

Answers: (graph answers at end of mini-lectures) 2a) $y = -x + 4$, b) $y = \frac{1}{3}x - 7$, c) $y = -\frac{5}{2}x$; 3a) m=2…cost increases 2 dollars for every hour of work, (0,60)…there is a minimum basic charge of $60, b) m=-10…the jug loses 10 ounces per minute, (0,117)…the jug started with 117 ounces in it

Mini-Lecture 3.4

More Equations of Lines

Learning Objectives:

1. Use the point-slope form to write the equation of a line.
2. Write equations of vertical and horizontal lines.
3. Write equations of parallel and perpendicular lines.
4. Use the point-slope form in real-world applications.
5. Key vocabulary: *point-slope form, standard form*.

Examples:

1. Derive the point-slope form of the equation of a line by starting with $m=\dfrac{y-y_1}{x-x_1}$ and multiplying both sides by ($x-x_1$).

2. Write an equation of each line with the given slope and containing the given point. Write the final equation in slope-intercept form.

   a) slope 3; through (6,2) b) slope $-\dfrac{2}{3}$; through (1,-5) c) slope $\dfrac{3}{2}$; through (-2,-7)

   Write an equation of the line passing through the given points. Write the final equation in standard form.

   d) (3,0) and (5,4) e) (8,-4) and (5,5) f) $\left(-\dfrac{1}{2},\dfrac{1}{3}\right)$ and $\left(\dfrac{5}{2},-\dfrac{2}{3}\right)$

3. Write an equation of each line.

   a) vertical; through (2,4) b) horizontal; through (-1,-3)
   c) undefined slope; through (0,3) d) slope 0; through (-6,4)

4. Write an equation of each line. Write the equation in the form $x = a$, $y = b$, or $y = mx+b$.

   a) through (0,3); parallel to $y = 2x - 1$ b) through (1,4); parallel to $2x - 3y = 1$
   c) through (0,-2); perpendicular to $y = -4x + 2$ d) through (-6,4); perpendicular to $2x + 5y = 10$

5. A long distance phone company charges \$0.75 for a call lasting 3 minutes, and \$1.75 for a call lasting 13 minutes. Let y be the cost of making a call lasting x minutes. Write a linear equation that models the cost of making a call lasting x minutes.

Teaching Notes:

- Most students understand the point-slope form better if they see that it is just a re-arranging of the slope formula.
- Some students struggle with the fractions that arise when solving the problems in number 4.
- Refer students to the ***Point-Slope Form of the Equation of a Line*** chart in the text.

Answers: 1) $m=\dfrac{y-y_1}{x-x_1}\rightarrow y-y_1=m(x-x_1)$; *2a) y=3x-16, b)* $y=-\dfrac{2}{3}x-\dfrac{13}{3}$, *c)* $y=\dfrac{3}{2}x-4$, *d) 2x-y=6, e) 3x+y=20, f) 2x+6y=1; 3a) x=2, b) y=-3, c) x=0, d) y=44a) y=2x+3, b)* $y=\dfrac{2}{3}x+\dfrac{10}{3}$ *c)* $y=\dfrac{1}{4}x-2$, *d)* $y=\dfrac{5}{2}x+19$; *5) y=0.1x+0.45*

Mini-Lecture 3.5

Graphing Linear Inequalities

Learning Objectives:

1. Graph linear inequalities.
2. Graph the intersection or union of two linear inequalities.
3. Key vocabulary: *boundary line, half planes, solution region, test point.*

Examples:

1. Determine whether the ordered pair satisfies $y \geq x + 2$.

   a) (0,2) b) (1,4) c) (-4,2) d) (-1,-2)

2. Graph each inequality. Use a test point to check the solution region.

   a) $y < x$ b) $y \geq x + 2$ c) $y \leq -x - 3$

   d) $x + 2y > -2$ e) $-2x - 5y \geq 10$ f) $2x < -3y$

   g) $y > \frac{1}{2}x$ h) $y \leq 2$ i) $x \geq -2\frac{1}{3}$

3. Graph each union or intersection.

   a) The intersection of $x \leq 2$ and $y \geq -3$
   b) The union of $x \leq 2$ or $y \geq -3$
   c) The intersection of $x - y < 2$ and $x + y \geq 3$
   d) The union of $2x - 3y < 6$ or $2x + y \geq 3$

Teaching Notes:

- Most students who are good at graphing linear equations find this section easy.
- Although students do not fully understand the region they are testing in problem 1 until they graph it in 2b), many of them need to practice testing ordered pairs before they use it within a graphing problem.
- Remind students to always use a test point from the solution region, and not from the boundary line, to check their graph.
- Remind students to use a dashed line for < or > and a solid line for ≤ or ≥.
- Refer students to the ***Graphing a Linear Inequality in Two Variables*** chart in the text.

Answers: (graph answers at end of mini-lectures) 1a) yes, b) yes, c) yes, d) no

Mini-Lecture 3.6
Introduction to Functions

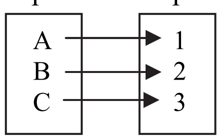
Learning Objectives:

1. Define relation, domain, and range.
2. Identify functions.
3. Use the vertical line test for functions.
4. Use function notation.
5. Graph a linear function.
6. Key vocabulary: *relation, domain, range, function, dependent/independent variable*.

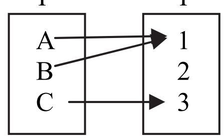
Examples:

1. Find the domain and range of each relation. Also determine whether the relation is a function.

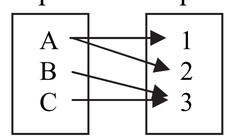
a) Input: Output:



b) Input: Output:



c) Input: Output:

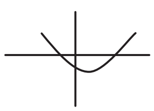
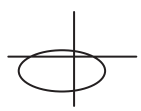
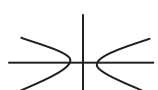
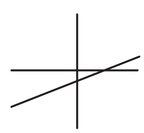
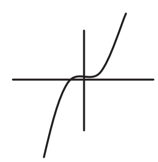


d) {(1,4),(1,6)} e) {(-2,-6),(0,-6)} f) {(-6,-7), (-2,-5), $\left(\frac{1}{2},\frac{2}{3}\right)$,(0.5, 3)}

2. Determine whether each relation is also a function.

a) $y = x + 3$ b) $y - x = 5$ c) $x = 3y^2$

3. Use the vertical line test to determine whether each graph is the graph of a function.

a)  b)  c)  d)  e) 

4. For each function, find the indicated values.

a) $f(x) = x - 2$; find $f(3), f(-1)$ b) $g(x) = 3x^2 - 4x + 1$; find $g(0), g(-2)$

5. Graph each linear function.

a) $f(x) = x$ b) $f(x) = -2x + 1$ c) $f(x) = 2x - 3$

Teaching Notes:

- For domain and range, students find it helpful to think of *x* values as inputs, and *y* values as outputs.
- Point out to students that equivalent domain or range elements that occur more than once only need to be listed once.
- Some students are very confused by function notation.
- Refer to the end of section exercises for application problems.
- Refer students to the ***Vertical Line Test*** chart in the text.

Answers: (graph answers at end of mini-lectures) 1a) domain {A,B,C}, range {1,2,3], function, b) domain {A,B,C}, range {1,3}, function, c) domain {A,B,C}, range {1,2,3}, not a function, d) domain {1}, range {4,6}, not a function, e) domain {-2,0}, range {-6}, function, f) domain {-6,-2,0.5 }, range {-7,-5, $\frac{2}{3}$,3},not a function; 2a) function, b) function, c) not a function; 3a) function, b) not a function, c) not a function, d) function, e) function; 4a) 1,-3, b) 1,21

Mini-Lecture 3.7

Finding Domains and Ranges from Graphs and Graphing Piecewise-Defined Functions

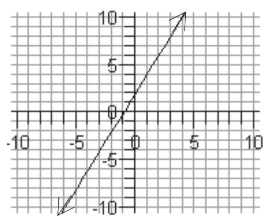
Learning Objectives:

1. Find the domain and range from a graph.
2. Graph piecewise-defined functions.

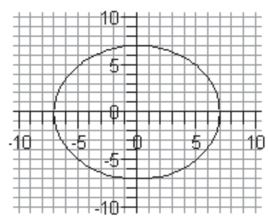
Examples:

1. Find the domain and range of each relation.

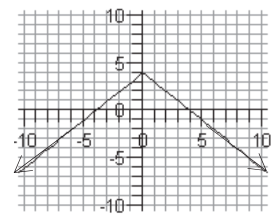
a)



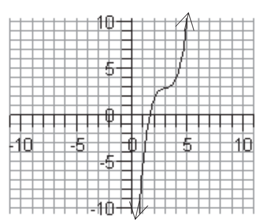
b)



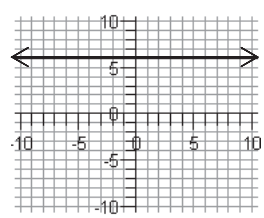
c)



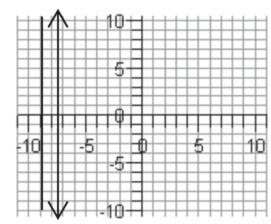
d)



e)



f)



2. Graph each piecewise-defined function.

a) $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x+2 & \text{if } x > 0 \end{cases}$ b) $g(x) = \begin{cases} 4x+3 & \text{if } x \le 1 \\ \frac{1}{3}x-2 & \text{if } x > 1 \end{cases}$

3. Graph each piecewise-defined function. Use the graph to determine the domain and range.

a) $g(x) = \begin{cases} x+2 & \text{if } x < 0 \\ -x+2 & \text{if } x \ge 0 \end{cases}$ b) $h(x) = \begin{cases} -2 & \text{if } x \le 0 \\ 2 & \text{if } x \ge 1 \end{cases}$

Teaching Notes:

- Refresh students' memories of domain and range by asking for volunteer to draw the graph of a relation and a function on the board, and then discuss the domain and range of each graph.
- Some students have trouble separating what is happening along the x-axis versus what is happening along the y-axis to arrive at domain and range values.

Answers: (graph answers at end of mini-lectures) 1a) domain $(-\infty,\infty)$, *range* $(-\infty,\infty)$, *b) domain (-7,7), range (-7,7), c) domain* $(-\infty,\infty)$, *range* $(-\infty,4)$, *d) domain* $(-\infty,\infty)$, *range* $(-\infty,\infty)$, *e) domain* $(-\infty,\infty)$, *range {6}, f) domain {-7}, range* $(-\infty,\infty)$; *3a) domain* $(-\infty,\infty)$, *range* $(-\infty,2]$, *b) domain* $(-\infty,0]\cup[1,\infty)$, *range {-2,2}*

Mini-Lecture 3.8

Shifting and Reflecting Graphs of Functions

Learning Objectives:

1. Graph common equations.
2. Vertical and horizontal shifts.
3. Reflect graphs.
4. Key vocabulary: *parabola, reflection*.

Examples:

1. Sketch the graph of each common function.

   a) $f(x) = x$ b) $f(x) = |x|$ c) $f(x) = x^2$ d) $f(x) = \sqrt{x}$

2. Sketch each pair of functions on one axes.

   a) $f(x) = x$, $g(x) = x + 2$ b) $f(x) = |x|$, $g(x) = |x| - 2$ c) $f(x) = |x|$, $g(x) = |x - 2|$

   d) $f(x) = |x|$, $g(x) = |x + 2|$ e) $f(x) = x^2$, $g(x) = (x-2)^2 + 1$ f) $f(x) = \sqrt{x}$, $g(x) = \sqrt{x+1} - 2$

3. Sketch each pair of functions on one axes.

   a) $f(x) = x$, $g(x) = -x$ b) $f(x) = |x|$, $g(x) = -|x|$

   c) $f(x) = \sqrt{x}$, $g(x) = -\sqrt{x-2}$ d) $f(x) = x^2$, $g(x) = -(x+2)^2 - 1$

Teaching Notes:

- Some students are confused at first about how to graph non-linear functions and need to be given the x-values to use to calculate ordered pairs.
- Most students find vertical shifts easy to understand.
- Some students are confused by the directions of a horizontal shift.
- Objectives 2 and 3 can be covered in a more timely manner if students are broken into groups and each group is given one type of common graph to focus on. Then the class can discuss the results and generalize to arrive at the shifting and reflecting properties.
- Refer students to the ***Common Graphs, Vertical Shifts, Horizontal Shifts,*** and ***Reflections About the x-axis*** charts in the text.

Answers: (graph answers at end of mini-lectures)

Mini-Lecture 4.1

Solving Systems of Linear Equations in Two Variables

Learning Objectives:

1. Determine whether an ordered pair is a solution of a system of two linear equations.
2. Solve a system of two equations by graphing.
3. Solve a system using substitution.
4. Solve a system using elimination.
5. Key vocabulary: *solution of a system, consistent, inconsistent, dependent*

Examples:

1. Determine whether the given ordered pair is a solution of the system.

   a) $\begin{array}{l} x+y=4 \\ x-y=2 \end{array}$; (3,1) b) $\begin{array}{l} y=4 \\ x=-3y \end{array}$; (-6,4) c) $\begin{array}{l} 2x+y=4 \\ -3x \quad =2y+8 \end{array}$; $\left(\frac{1}{2},3\right)$

2. Solve each system by graphing.

   a) $\begin{array}{l} x+y=4 \\ x-y=2 \end{array}$ b) $\begin{array}{l} 2x+4y=10 \\ 4x+3y=10 \end{array}$ c) $\begin{array}{l} y=-x+3 \\ 2x+2y=-1 \end{array}$

3. Use the substitution method to solve each system of equations.

   a) $\begin{array}{l} x+y=4 \\ x-y=2 \end{array}$ b) $\begin{array}{l} \frac{1}{4}x+\frac{1}{4}y=2 \\ x-y=2 \end{array}$ c) $\begin{array}{l} y=-3x+8 \\ 12x+4y=32 \end{array}$

4. Use the elimination method to solve each system of equations.

   a) $\begin{array}{l} x+y=4 \\ x-y=2 \end{array}$ b) $\begin{array}{l} x-6y=-9 \\ 8x-6y=-30 \end{array}$ c) $\begin{array}{l} x-4y=-8 \\ -6x-3y=-6 \end{array}$

   d) $\begin{array}{l} 3x+6y=3 \\ 2x+9y=-8 \end{array}$ e) $\begin{array}{l} 6x-8y=8 \\ 12x=16y+24 \end{array}$ f) $\begin{array}{l} -6x-4y=-2 \\ -12y=-6+18x \end{array}$

Teaching Notes:

- Help students visualize a system by graphing examples of the three possible results: one solution, no solution, ∞ solutions.
- Some students have trouble with the substitution method when fractions are involved.
- Most students prefer the addition method.
- Encourage students to check final answers.
- Many students have trouble drawing the conclusion of "no solution" or "infinite solutions" from the non-graphing methods.
- Refer students to the ***Possible Solutions to Systems of Two linear Equations,*** and ***Solving a System of Two Equations using the Substitution/Elimination Method*** charts in the text.

Answers: 1a) yes, b) no, c) no; 2a) (3,1), b) (1,2), c) ∅ ; 3a) (3,1), b) (5,3), c) {(x,y)|y=-3x+8}; 4a) (3,1), b) (-3,1), c) (0,2), d) (5,-2), e) ∅ , f) {(x,y)|-6x-4y=-2}

Mini-Lecture 4.2

Solving Systems of Linear Equations in Three Variables

Learning Objectives:

1. Solve a system of three equations in three variables.
2. Key vocabulary: *ordered triple.*

Examples:

1. Determine whether the given ordered pair is a solution of the system.

a) $\begin{aligned} x+y+z&=3\\ x-y+2z&=-1\\ 4x+y+z&=15 \end{aligned}$; (4,1,-2)

b) $\begin{aligned} x-y+z&=-5\\ x+y+z&=3\\ x+y-z&=7 \end{aligned}$; (1,-2,4)

2. Solve each system.

a) $\begin{aligned} x+y+z&=3\\ x-y+2z&=-1\\ 4x+y+z&=15 \end{aligned}$

b) $\begin{aligned} 5x+3y+z&=25\\ 3x-3y-z&=7\\ 4x+y+4z&=14 \end{aligned}$

c) $\begin{aligned} x+4y+2z&=-7\\ 5y+4z&=-15\\ z&=-5 \end{aligned}$

d) $\begin{aligned} x-y+4z&=3\\ 5x+z&=0\\ x+3y+z&=-9 \end{aligned}$

e) $\begin{aligned} \frac{2}{3}x-\frac{1}{2}y+2z&=-18\\ x-\frac{2}{3}y-\frac{1}{2}z&=-12\\ x-\frac{1}{2}y-z&=-8 \end{aligned}$

3. Try to solve each system. Explain your results.

a) $\begin{aligned} x-y+5z&=5\\ 5x+z&=0\\ -x+y-5z&=2 \end{aligned}$

b) $\begin{aligned} x-9y-z&=-3\\ 4x-36y-4z&=-12\\ 3x-27y-3z&=-9 \end{aligned}$

Teaching Notes:

- Students need to be extremely neat and organized to succeed with these.
- Most students prefer to use the elimination method repeatedly.
- Some students prefer to use the substitution method to eliminate the first variable whenever it is possible to do so without generating fractions.
- Most students have trouble visualizing these systems. Refer them to the figures of intersecting planes in the text.
- Refer students to the ***Solving a System of Three Linear Equations by the Elimination Method*** chart in the text.

Answers: 1a) yes, b) no; 2a) (4,1,-2), b) (4,2,-1), c) (-1,1,-5), d) (0,-3,0), e) (-6,12,-4); 3a) $\varnothing$, inconsistent system, b) $\{(x,y,z,)\,|\,x-9y-z=-3\}$, dependent system

Mini-Lecture 4.3

Systems of Linear Equations and Problem Solving

Learning Objectives:

1. Solve problems that can be modeled by a system of two linear equations.
2. Solve problems with cost and revenue functions.
3. Solve problems that can be modeled by a system of three linear equations.
4. Key vocabulary: *break-even point.*

Examples:

1. Use a system of two linear equations to solve each problem.

   a) One number is 5 less than a second number. Twice the second number is 2 less than 5 times the first. Find the two numbers.

   b) Two trucks leave a city and head in the same direction. After 7 hours the faster truck is 56 miles ahead of the slower truck. The slower truck has traveled 301 miles. Find the speed of the two trucks.

2. Given the cost function $C(x)$ and the revenue function $R(x)$, find the number of units x that must be sold to break even.

   a) $C(x) = 95x + 1100$, $R(x) = 105x$ b) $C(x) = 0.3x + 1400$, $R(x) = 1.3x$

   c) A gift box manufacturing company recently purchased \$1000 worth of new equipment to make gift boxes to sell. The cost of producing a package of gift boxes is \$0.50 and it is sold for \$3.00. Find the number of packages that must be sold for the company to break even.

3. Use a system of three linear equations to solve the problem.

   c) Find the values of a, b, and c such that the equation $y = ax^2 + bx + c$ has ordered pair solutions (-3, -37), (-2,-22), and (1,-1).

   d) A store sells tents, sleeping bags, and camp stools. A customer buys a tent, 2 sleeping bags, and 5 camp stools for \$138. The price of a tent is 9 times the price of a camp stool. The cost of a sleeping bag is \$13 more than the cost of a camp stool. Find the cost of each item.

Teaching Notes:

- Many students find these problems difficult.
- Some students find it very helpful to see a graph of the cost versus revenue problem.
- Encourage students to draw and label a diagram whenever possible.
- Remind students to check if their answers seem reasonable.

Answers: 1a) 4 and 9, b) 43 mph and 51 mph; 2a) 110 units, b) 1400 units, c) 400 packages; 3a) a=-2, b=5, c=-4, b) tent: \$63, sleeping bag: \$20, camp stool: \$7

Mini-Lecture 4.4
Solving Systems of Equations Using Matrices

Learning Objectives:

1. Use matrices to solve a system of two equations.
2. Use matrices to solve a system of three equations.
3. Key vocabulary: *matrix, element.*

Examples:

1. Create a matrix for each system. Do not solve the system.

a) $\begin{aligned} 2x + 3y &= 4 \\ 5x - 7y &= -1 \end{aligned}$ b) $\begin{aligned} x &= 5 \\ -2x + y &= 3 \end{aligned}$ c) $\begin{aligned} 4x - y &= 2 \\ 3y + 5 &= -1 \\ 6x + y - 3z &= 0 \end{aligned}$

2. Use matrices to solve each system of two linear equations.

a) $\begin{aligned} 4x + 5y &= -5 \\ 2x + 8y &= 14 \end{aligned}$ b) $\begin{aligned} 6x + y &= 15 \\ 6x + y &= 21 \end{aligned}$ c) $\begin{aligned} 5x + y &= 7 \\ 6x + 2y &= 6 \end{aligned}$

3. Use matrices to solve each system of three linear equations.

a) $\begin{aligned} 4x - y - 4z &= -7 \\ -8x + 5z &= -13 \\ 3y + z &= 16 \end{aligned}$ b) $\begin{aligned} 2x - y + 6z &= 12 \\ 6x + 7y + 5z &= 43 \\ 2x - 5y + z &= -1 \end{aligned}$ c) $\begin{aligned} 6x - 4y + 5z &= -20 \\ -18x + 12y - 15z &= 60 \\ 18x - 12y + 15z &= -60 \end{aligned}$

Teaching Notes:

- Point out the similarities between solving systems by matrices and solving by the elimination method.
- Remind students that the equations must be written in standard form before writing the corresponding matrix.
- Most students appreciate seeing how matrices can be solved using a calculator, and are amazed at how easy it is to solve a system of three equations with a calculator.
- Refer students to the ***Elementary Row Operations*** chart in the text.

Answers: 1a) $\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 5 & -7 & -1 \end{array}\right]$, *b)* $\left[\begin{array}{cc|c} 1 & 0 & 5 \\ -2 & 1 & 3 \end{array}\right]$, *c)* $\left[\begin{array}{ccc|c} 4 & -1 & 0 & 2 \\ 0 & 3 & 0 & -6 \\ 6 & 1 & -3 & 0 \end{array}\right]$; *2a) (-5,3), b)* $\varnothing$*, c) (2,-3); 3a) (6,3,7), b) (4,2,1), c) {(x,y,z)|6x-4y+5z=-20}*

Mini-Lecture 4.5

Systems of Linear Inequalities

Learning Objectives:

1. Graph a system of linear inequalities.

Examples:

1. Graph the solutions of each system of two linear inequalities.

a) $\begin{array}{l} y \geq 2x - 4 \\ y \leq -x + 1 \end{array}$ b) $\begin{array}{l} y \leq 2x - 1 \\ x + y > -4 \end{array}$

c) $\begin{array}{l} y \leq 2x + 1 \\ y < -3x \end{array}$ d) $\begin{array}{l} x + 3y > -6 \\ y < -2 \end{array}$

e) $\begin{array}{l} x \geq -2 \\ y \geq 6 \end{array}$

2. Graph the solutions of each system of three linear inequalities.

a) $\begin{array}{r} x + y \geq 1 \\ x - y \geq 1 \\ x \leq 4 \end{array}$ b) $\begin{array}{r} 2x + 3y \geq 6 \\ x - y \leq 3 \\ y \leq 2 \end{array}$ c) $\begin{array}{r} 2x + 3y \leq 6 \\ x - y \geq 3 \\ x \geq 1 \end{array}$

3. The equation that represents the traffic control and emergency vehicle response in a small city is $2P + 3F \leq 23$, where P is the number of police cars on active duty and F is the number of fire trucks that are out responding to a call. In order to comply with staffing limitations, the inequality $4P + 2F \leq 34$ is necessary. Because P and F cannot be negative, $P \geq 0$ and $F \geq 0$. Using the horizontal axis for P and the vertical axis for F, graph the system of four linear inequalities. If 2 police cars are on active duty and 7 fire trucks are out on calls, are all of the requirements satisfied?

Teaching Notes:

- Remind students to use a dashed line for < or > and a solid line for $\leq$ or $\geq$.
- Encourage students to use different colors for each line.
- Encourage students to check their graphs using a test point from the solution region.
- Refer students to the ***Graphing the Solution of a System of Linear Inequalities*** chart in the text.

Answers: (graph answers at end of mini-lectures) 3) no

Mini-Lecture 5.1

Polynomial Functions and Adding and Subtracting Polynomials

Learning Objectives:

1. Define vocabulary of polynomials.
2. Combine like terms.
3. Add polynomials.
4. Subtract polynomials.
5. Evaluate polynomial functions.
6. Key vocabulary: *term, constant, polynomial, monomial, binomial, trinomial, degree.*

Examples:

1. Find the degree of each polynomial, state how many terms it has, and indicate whether it's a monomial, binomial, or trinomial.

   a) $3x$ b) $9x^2$ c) $-2x^3+5$ d) x^2y^2-4x+3

2. Simplify each polynomial by combining like terms.

   a) $2x+3x$ b) $10y-8y$ c) $xy+3x-2xy$

   d) $-x+2x-6x^2-3x^2$ e) $-9y+8y+2y^5$ f) $-2xy^2+3x-x+8xy^2-\frac{3}{5}$

3. Add or subtract.

   a) $(-3y^2-2y+5)+(2y+7)$ b) $\left(2x^2-3x\right)+\left(-6x^2-7x\right)$

   c) $\begin{array}{r} 5x^2+3x-2 \\ \underline{+\left(7x^2-5x-3\right)} \end{array}$ d) $\left(2x-2\right)-\left(-x-2\right)$ e) $\begin{array}{r} -6x^2-3x+9 \\ \underline{-\qquad\left(7x+10\right)} \end{array}$

   f) $\left(3x^2+x+2\right)-\left(4x^2+x-5\right)$ g) $\left(7.2y^2-y+11.32\right)+\left(2.3y^2+4.8y-1.2\right)$

   h) $\left(\frac{2}{5}x^2-\frac{1}{3}x+\frac{1}{5}\right)-\left(\frac{3}{5}x^2+\frac{2}{3}x-\frac{4}{5}\right)$

4. If $P(x)=2x^2-3x+1$, find each function value,

   a) $P(1)$ b) $P(0)$ c) $P(-2)$ d) $P(-10)$

Teaching Notes:

- Most students find these polynomial operations easy.
- Tell students that identifying the degree of a polynomial is important for later work with factoring and solving equations.
- Remind students that this section is a review of distributing and collecting like terms.
- Some students forget to distribute the minus sign when lining up vertically.

Answers: 1a) 1,1 monomial, b) 2,1,monomial, c) 3,2,binomial, d) 4,3,trinomial; 2a) 5x, b) 2y, c) –xy+3x, d) x-9x^2*, e) –y+2y*^{5}*,* $6xy^2+2x-\frac{3}{5}$*; 3a) -3y*^{2}*+12, b) -4x*^{2}*-10x, c) 12x*^{2}*-2x-5, d) 3x, e) -6x*^{2}*-10x-1, f) –x*^{2}*+7, g) 9.5y*^{2}*+3.8y+10.12, h)* $-\frac{1}{5}x^2-x+1$*; 4a) 0, b) 1, c) 15, d) 231*

Mini-Lecture 5.2
Multiplying Polynomials

Learning Objectives:

1. Multiply any two polynomials.
2. Multiply binomials.
3. Square binomials.
4. Multiply the sum and difference of two terms.
5. Multiply three or more polynomials.
6. Evaluate polynomial functions.

Examples:

1. Multiply.

   a) $(2x)(4x)$ b) $(-6a^2)(5a^3)$ c) $(4.1\,x\,y^2z^{10})(6\,xy^5z)$ d) $2x(3x-4)$

   e) $-3y(5xy+2x)$ f) $-2b^2z(2z^2a+baz-b)$ g) $(x+3)(3x-4)$

   h) $\begin{array}{r} 4y-3 \\ \times\ 2y-2 \\ \hline \end{array}$ i) $\left(3y-\frac{1}{4}\right)\left(4y-\frac{1}{6}\right)$ j) $(3x^2-4y^2)(x^2-6y^2)$

   k) $(x+3)(2x^2-x+5)$ l) $(x+3)(x-2)(2x-1)$ m) $(y-2)^4$

2. Use special products to multiply.

   a) $(x+2)^2$ b) $(x-4)^2$ c) $(x+3)(x-3)$

   d) $(2xy-3b)(2xy+3b)$ e) $\left(5x-\frac{1}{2}\right)\left(5x+\frac{1}{2}\right)$ f) $\left[6-(2b-2)\right]^2$

3. If $f(x)=x^2-2x$, find the following.

   a) $f(a)$ b) $f(c)$ c) $f(a+b)$ d) $f(a-2)$

Teaching Notes:

- Encourage students to multiply binomials with FOIL mentally whenever possible. This will make factoring easier for them in future sections.
- In problem 2b), some students do not realize that $nm = mn$ and that they are therefore like terms.
- Many students distribute the exponent when squaring a binomial, even after repeated reminders to multiply the binomial by itself.
- Refer students to the ***Square of a Binomial*** and ***Product of the Sum and Difference of Two Terms*** charts in the text.

Answers: 1a) $8x^2$, b) $-30a^5$, c) $24.6x^2y^7z^{11}$, d) $6x^2-8x$, e) $-15xy^2-6xy$, f) $-4ab^2z^3-2ab^3z^2+2b^3z$, g) $3x^2+5x-12$, h) $8y^2-14y+6$, i) $12y^2-\frac{3}{2}y+\frac{1}{24}$, j) $3x^4-22x^2y^2+24y^4$, k) $2x^3+5x^2+2x+15$, l) $2x^3+x^2-13x+6$, m) $y^4-8y^3+24y^2-32y+16$; 2a) x^2+4x+4, b) $x^2-8x+16$, c) x^2-9, d) $4x^2y^2-9b^2$, e) $25x^2-\frac{1}{4}$, f) $64-32b+4b^2$; 3a) a^2-2a, b) c^2-2c, c) $a^2+2ab+b^2-2a-2b$, d) a^2-6a+8

Mini-Lecture 5.3

Dividing Polynomials and Synthetic Division

Learning Objectives:

1. Divide a polynomial by a monomial.
2. Divide by a polynomial.
3. Use synthetic division.
4. Use the remainder theorem to evaluate polynomials.

Examples:

1. Divide.

   a) $8x^4 - 4x^3$ by $4x^2$ b) $\dfrac{3x^3y + 9x^2y^2 - 3xy^3}{3xy}$ c) $\dfrac{8x^5y + 32x^4y - 16x^3y^2}{-4x^4y}$

2. Divide.

   a) $(x^2 + 12x + 35) \div (x + 5)$ b) $(5x^2 - 17x + 14) \div (x - 2)$
   c) $(-4x^3 - 8x^2 + 7x - 1) \div (2x - 1)$ d) $(20x + 12x^2 + 3) \div (-6x - 1)$
   e) $(x^2 + 16x + 60) \div (x + 7)$ f) $(8x^4 + 12x^3 - 2x) \div (2x^2 + x)$ g) $(x^4 + 16) \div (x - 2)$

3. Use synthetic division to divide.

   a) $\dfrac{x^2 - 4x - 45}{x + 5}$ b) $\dfrac{2x^2 - 9x - 35}{x - 7}$
   c) $\dfrac{-2x^3 - 6x^2 + 14x + 24}{x + 4}$ d) $\dfrac{x^4 + 16}{x - 2}$

4. Use the remainder theorem and synthetic division to find $P(2)$ if
   $P(x) = 8x^5 - 4x^4 + 15x^3 + 5x^2 - 7x$.

Teaching Notes:

- Remind students to check their answers by multiplying.
- Encourage students to write the intermediate step $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$ when dividing by a monomial.
- Most students understand dividing by a binomial better if a numerical long division problem is shown in parallel.
- Most students understand the synthetic division process after a couple of examples.
- Most students prefer synthetic division over long division.

Answers: 1a) $2x^2-x$, b) $x^2+3xy-y^2$, c) $-2x-8+\dfrac{4y}{x}$; 2a) $x+7$, b) $5x-7$, c) $-2x^2-5x+1$, d) $-2x-3$, e) $x+9-\dfrac{3}{x+7}$, f) $4x^2+4x-2$, g) $x^3+2x^2+4x+8+\dfrac{32}{x-2}$; 3a) $x-9$, b) $2x+5$, c) $-2x^2+2x+6$, d) $x^3+2x^2+4x+8+\dfrac{32}{x-2}$; 4) 318

Mini-Lecture 5.4

The Greatest Common Factor and Factoring by Grouping

Learning Objectives:

1. Identify the greatest common factor (GCF).
2. Factor out the greatest common factor (GCF) of a polynomial's terms.
3. Factor a polynomial by grouping.
4. Key vocabulary: *greatest common factor (GCF).*

Examples:

1. Find the greatest common factor of each list of monomials.

   a) $4, 24$ b) $15x, 20$ c) $15x^2, 20x$ d) $9x^2y, 27xy^2$

2. Factor out the greatest common factor.

   a) $16x-12$ b) $28x+28$ c) $5z-25xz^4$

   d) $18x+9x^2-6x^3$ e) $18a^3b-12ab+9ab^2-12a^2b$ f) $3x(y-5)+(y-5)$

3. Factor each polynomial by grouping.

   a) $xy+y+5x+5$ b) $2y-12-xy+6x$

   c) $xy+9x-7y-63$ d) $5xy-10x+7y-14$

4. Factor each polynomial.

   a) $16x^3-12x$ b) $-27xy^3+18x^4y$ c) $8a^2b^2c-12ab^2c-8ac+6a$

   d) $9y(z+2)-4(z+2)$ e) $4xy-8x+7y-14$ f) x^3+5x^2+x+5

Teaching Notes:

- Remind students to check their factoring answers by multiplication.
- Some students need to rewrite the coefficients in problem 2 in factored form in order to see the greatest common factor.
- Some students omit the 1 in the answer to Problem 2b).
- Many students are confused at first by factor by grouping problems where a negative sign must be factored out of the second grouping, as in problem 3b).

Answers: 1a) 4, b) 5, c) 5x, d) 9xy; 2a) 4(4x-3), b) 28(x+1), c) 5z(1-5xz^3), d) 3x(6+3x-2x^2), e) 3ab(6a^2-4+3b-4a), f) (y-5)(3x+1); 3a) (x+1)(y+5), b) (y-6)(2-x), c) (y+9)(x-7), d) (y-2)(5x+7); 4a) 4x(4x^2-3), b) 9xy(-3y^2+2x^3), c) 2a(4ab^{2}c-6b^{2}c-4c+3), d) (9y-4)(z+2), e) (y-2)(4y+7), f) (x+5)(x^2+1)

Mini-Lecture 5.5

Factoring Trinomials

Learning Objectives:

1. Factor trinomials of the form $x^2 + bx + c$.
2. Factor trinomials of the form $ax^2 + bx + c$ by the trial and check method.
3. Factor trinomials of the form $ax^2 + bx + c$ by grouping.
4. Factor by substitution.
5. Key vocabulary: *prime, perfect square trinomial.*

Examples:

1. Factor each trinomial.

   a) $x^2 + 3x + 2$ b) $x^2 + 6x + 8$ c) $x^2 - 6x + 8$ d) $x^2 + x - 2$

   e) $x^2 - x - 2$ f) $x^2 - 3x - 10$ g) $2x^2 + 4x - 48$ h) $3x^2 - 3x - 18$

   i) $x^2 + 15x + 16$ j) $x^2y^2 - 6xy^2 + 8y^2$ k) $x^5 + 4x^4 - 5x^3$

2. Factor each trinomial by the trial and check method.

   a) $4y^2 + 12y + 9$ b) $8x^2 - 18x + 9$ c) $6x^2 + 5x - 6$

   d) $7x^2 - 31x - 20$ e) $6x^2 + 27x - 15$ f) $6x^2y^2 - 7xy^2 - 20y^2$

3. Factor the trinomials in 2a), b), c), and d) using the grouping method.

4. Use substitution to factor each trinomial completely.

   a) $x^4 - 5x^2 - 6$ b) $9x^6 + 6x^3 - 8$ c) $(a+4)^2 + 7(a+4) + 12$

Teaching Notes:

- Some students can factor trinomials very quickly using the trial and check method.
- Some students become very frustrated with the trial and check method and appreciate seeing the grouping method because it provides a recipe that works for any non-prime polynomial.
- Remind students to always try to factor a GCF first.
- Refer to the end of section exercises for mixed practice.
- Refer students to the ***Factoring a Trinomial of the Form*** $ax^2 + bx + c$ and ***Factoring a Trinomial of the Form*** $ax^2 + bx + c$ ***by Grouping*** charts in the text.

Answers: 1a) (x+2)(x+1), b) (x+4)(x+2), c) (x-4)(x-2), d) (x+2)(x-1), e) (x-2)(x+1), f) (x-5)(x+2), g) 2(x+6)(x-4), h) 3(x-3)(x+2), i) prime, j) $y^2(x-4)(x-2)$*, k)* $x^3(x-4)(x-1)$*; 2a) (2y+3)(2y+3), b) (4x-3)(2x-3), c) (3x-2)(2x+3), d) (7x+4)(x-5), e) 3(2x-1)(x+5), f)* $y^2(2x-5)(3x+4)$*; 3a-d) same as 2a-d); 4a)* $(x^2-6)(x^2+1)$*, b)* $(3x^3+4)(3x^3-2)$*, c) (a+8)(a+7)*

Mini-Lecture 5.6

Factoring by Special Products

Learning Objectives:

1. Factor a perfect square trinomial.
2. Factor the difference of two squares.
3. Factor the sum or difference of two cubes.

Examples:

1. Factor completely or state the polynomial is prime.

   a) x^2+4x+4 b) $x^2-12x+36$ c) $9x^2+6x+1$

   d) $3x^2-12x+12$ e) $25x^2y^3-10xy^3-y^3$ f) $x^2+39xy+40y^2$

2. Factor completely.

   a) x^2-49 b) y^2-81 c) $\frac{1}{16}-25z^2$

   d) $(x+3)^2-64$ e) $3x^2-75$ f) $x^2+10x+25-x^4$

3. Factor completely.

   a) x^3+8 b) x^3+1 c) y^3-27 d) $64-x^3$

   e) p^3+8q^3 f) $x^3y^2+125y^2$ g) $a^3b^2-27b^2$ h) $54y^3+250$

4. Mixed practice.

   a) $64-x^2$ b) x^3+16x^2+64x c) $1000y^3-1$

   d) $x^2-6xy+9y^2$ e) $18x^2-98$ f) $(2x+3)^2-64$

Teaching Notes:

- Encourage students to always check if the first and last terms of a trinomial are perfect squares. If they are, then perfect square trinomial factoring may apply.
- Some students understand the difference of a square formula better if 2a) and 2b) are also done using trinomial factoring with a $0x$ middle term.
- Some students find the sum and difference of cubes formulas confusing at first and need to see many examples.
- Remind students to factor out a GCF whenever possible.

Answers: 1a) $(x+2)^2$, b) $(x-6)^2$, c) $(3x+1)^2$, d) $3(x-2)^2$, e) $y^3(5x-1)^2$, f) prime; 2a) $(x+7)(x-7)$, b) $(y+9)(y-9)$, c) $\left(\frac{1}{4}+5z\right)\left(\frac{1}{4}-5z\right)$, d) $(x+11)(x-5)$, e) $3(x+5)(x-5)$, f) $(x+5+x^2)(x+5-x^2)$; 3a) $(x+2)(x^2-2x+4)$, b) $(x+1)(x^2-x+1)$, c) $(y-3)(y^2+3y+9)$, d) $(4-x)(16+4x+x^2)$, e) $(p+2q)(p^2-2pq+4q^2)$, f) $y^2(x+5)(x^2-5x+25)$, g) $b^2(a-3)(a^2+3a+9)$, h) $2(3y+5)(9y^2-15y+25)$; 4a) $(8+x)(8-x)$, b) $x(x+8)^2$, c) $(10y-1)(100y^2+10y+1)$, d) $(x-3y)^2$, e) $2(3x+7)(3x-7)$, f) $(2x+11)(2x-5)$

Mini-Lecture 5.7

Solving Equations by Factoring and Solving Problems

Learning Objectives:

1. Solve polynomial equations by factoring.
2. Solve problems that can be modeled by polynomial equations.
3. Find the x-intercepts of a polynomial function.
4. Key vocabulary: *polynomial equation, quadratic equation, standard form, zero-factor property.*

Examples:

1. Solve each equation.

   a) $4 \bullet x = 0$ b) $3(x-5)=0$ c) $x(x+7)=0$ d) $(2x-3)(5x+4)=0$

   e) $x^2-11x+30=0$ f) $6x^2+13x+6=0$ g) $x^2+3x=70$

   h) $x\left(3x+4\right)=4$ i) $x(x-8)=x^2+5x$ j) $\dfrac{x^2}{56}+\dfrac{1}{8}=\dfrac{x}{7}$

   k) $(3x+2)(x-9)(5x-1)=0$ l) $x^3=25x$ m) $x^3+7x^2=18x$

   n) $x^5=64x^3$ o) $x^3-x=-3x^2+3$

2. Solve.

   a) One number exceeds another number by 6 and the product of the two numbers is 72. Find the numbers.
   b) A certain rectangle's length is 3 feet longer than its width. If the area of the rectangle is 70 square feet, find its dimensions.
   c) One leg of a right triangle is 14 inches longer than the smaller leg, and the hypotenuse is 16 inches longer than the smaller leg. Find the lengths of the sides of the triangle.

3. Find the x-intercepts of the function $f(x)=(x-7)(x+3)$.

Teaching Notes:

- Remind students to always put the equation in standard form before factoring.
- Some students try to use the zero-factor property before the equation is in standard form. For example in 1g) : $x^2+3x=70 \rightarrow x(x+3)=70 \rightarrow x=70,\ x+3=70$ … etc.
- Many students find the applied problems difficult and need to see more examples.
- Remind students to check whether their answers are reasonable for applied problems.
- Refer students to the ***Solving a Polynomial Equation by Factoring*** chart in the text.

Answers: 1a) {0}, b) {5}, c) {0,-7}, d) $\left\{\frac{3}{2},-\frac{4}{5}\right\}$*, e) {6,5}, f)* $\left\{-\frac{2}{3},-\frac{3}{2}\right\}$*, g) {-10,7}, h)* $\left\{-2,\frac{2}{3}\right\}$*, i) {0}, j) {1,7}; k)* $\left\{-\frac{2}{3},\frac{1}{5},9\right\}$*, l) {-5,0,5}, m) {-9,0,2}, n) {-8,0,8}, o) {-3,-1,1}; 2a) 6 and 12, or, -12 and -6, b) 10 ft by 7 ft, c) 10 in, 24 in, 26 in; 3) (7, 0) and (-3, 0)*

Mini-Lecture 6.1

Rational Functions and Multiplying and Dividing Rational Expressions

Learning Objectives:

1. Find the domain of a rational function.
2. Simplify rational expressions.
3. Multiply rational expressions.
4. Divide rational expressions.
5. Key vocabulary: *undefined, domain, simplify, reciprocal.*

Examples:

1. Find the domain of each rational expression.

   a) $f(x)=\dfrac{2+3x}{4}$ b) $g(x)=-\dfrac{6x+x^2}{5x}$ c) $h(x)=\dfrac{2-5x}{-7+x}$

   d) $p(x)=\dfrac{-9}{7x+5}$ e) $f(x)=\dfrac{x}{2x^2+x-3}$

2. Simplify each rational expression.

   a) $\dfrac{2x^2-6x}{2x}$ b) $\dfrac{x^2-81}{9+x}$ c) $\dfrac{x^2-10x+25}{x-5}$

   d) $\dfrac{x-8}{8-x}$ e) $\dfrac{y^2+5y+6}{y^2+10y+21}$ f) $\dfrac{y^3-64}{4y-16}$

3. Multiply and simplify.

   a) $\dfrac{3x-3}{x}\bullet\dfrac{8x^2}{5x-5}$ b) $\dfrac{24xy^2}{x^2-49}\bullet\dfrac{3x-21}{8x^2y^2}$ c) $\dfrac{x^2+7x+10}{x^2+8x+15}\bullet\dfrac{x^2+3x}{x^2-7x-18}$

4. Divide and simplify.

   a) $\dfrac{4x^2}{5}\div\dfrac{x^3}{40}$ b) $\dfrac{x^2+5x-6}{x^2+9x+18}\div\dfrac{x^2-1}{x^2+7x+12}$ c) $\dfrac{x^2-4x}{x^3-64}\div\dfrac{2x}{2x^2+8x+32}$

Teaching Notes:

- Many students need a review of simplifying, multiplying and dividing numerical fractions before attempting algebraic ones.
- Many students have trouble with problems where the factors in the numerator and denominator have opposite signs.
- Refer to the end-of-section exercises for applied problems.
- Refer students to the ***Simplifying/Multiplying/Dividing Rational Expressions*** charts in the text.

Answers: 1a) {x|x is a real number}, b) {x|x is a real number and x≠0}, c) {x|x is a real number and x≠7}, d) {x|x is a real number and $x\neq-\frac{5}{7}$*}, e) {x|x is a real number and* $x\neq-\frac{3}{2}$*, x≠1}; 2a) x-3, b) x-9, c) x-5, d) -1, e)* $\frac{y+2}{y+7}$*, f)* $\frac{y^2+4y+16}{4}$*; 3a)* $\frac{24x}{5}$*, b)* $\frac{9}{x(x+7)}$*, c)* $\frac{x}{x-9}$*; 4a)* $\frac{32}{5x}$*, b)* $\frac{x+4}{x+1}$*, c) 1*

Mini-Lecture 6.2

Adding and Subtracting Rational Expressions

Learning Objectives:

1. Add or subtract rational expressions with the same denominator.
2. Find the Least Common Denominator (LCD) of two or more rational expressions.
3. Add and subtract rational expressions with different denominators.
4. Key vocabulary: *least common denominator.*

Examples:

1. Add or subtract as indicated.

   a) $\frac{3}{x}+\frac{8}{x}$ b) $\frac{x^2}{x+3}-\frac{9}{x+3}$ c) $\frac{8x-5}{x^2+6x+8}+\frac{7-7x}{x^2+6x+8}$

2. Find the LCD of the rational expressions in each list.

   a) $\frac{3}{11},\frac{2}{7x}$ b) $\frac{6}{7y},\frac{3}{y^2}$

   c) $\frac{4}{x-3},\frac{9}{x+3}$ d) $\frac{6}{x^2-y^2},\frac{5}{x^2+2xy+y^2},\frac{1}{8}$

3. Add or subtract as indicated. If possible, simplify your answer.

   a) $\frac{5}{6y}-\frac{9}{5y}$ b) $\frac{7}{x^2}+\frac{3}{x}$ c) $\frac{6}{r}+\frac{8}{r-2}$

   d) $\frac{1}{x-4}-\frac{1}{4-x}$ e) $\frac{x+3}{x^2+4x-12}+\frac{3x+2}{x^2+14x+48}$

   f) $\frac{7x}{x+1}+\frac{8}{x-1}-\frac{14}{x^2-1}$ g) $\frac{10}{x^2+5x}+\frac{6}{x}-\frac{2}{x+5}$

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Teaching Notes:

- Most students need a review of adding, subtracting, and finding LCDs of numerical fractions before attempting algebraic ones.
- Many students find this section difficult.
- Some students need to see more examples for objective 2. Extra time spent here is well worth it and pays off with greater success in objective 3.
- Refer students to the ***Adding or Subtracting Rational Expressions with Common/Different Denominators*** and ***Finding the Least Common Denominator*** charts in the text.

Answers: 1a) $\frac{11}{x}$*, b) x-3, c)* $\frac{1}{x+4}$*; 2a) 77x, b)* $7y^2$*, c) (x-3)(x+3), d)* $8(x+y)^2(x-y)$*; 3a)* $\frac{-29}{30y}$*, b)* $\frac{7+3x}{x^2}$*, c)* $\frac{14r-12}{r(r-2)}$*,*
d) $\frac{2}{x-4}$*, e)* $\frac{4x^2+7x+20}{(x-2)(x+6)(x+8)}$*, f)* $\frac{7x-6}{x-1}$*, g)* $\frac{4x+40}{x(x+5)}$

Mini-Lecture 6.3
Simplifying Complex Fractions

Learning Objectives:

1. Simplify complex fractions by simplifying the numerator and denominator and then dividing.
2. Simplify complex fractions by multiplying by the Least Common Denominator (LCD).
3. Simplify expressions with negative exponents.
4. Key vocabulary: *complex rational expression.*

Examples:

1. Simplify each complex fraction by simplifying the numerator and denominator and then dividing.

a) $\dfrac{3+\frac{1}{8}}{4-\frac{5}{8}}$ b) $\dfrac{\frac{x}{x+4}}{\frac{4}{x+4}}$ c) $\dfrac{\frac{9}{a}+9}{\frac{9}{a}-9}$

d) $\dfrac{\frac{16x^2-25y^2}{xy}}{\frac{4}{y}-\frac{5}{x}}$ e) $\dfrac{\frac{2}{x}+\frac{9}{x^2}}{\frac{4}{x^2}-\frac{81}{x}}$ f) $\dfrac{\frac{4}{5-x}+\frac{5}{x-5}}{\frac{2}{x}+\frac{3}{x-5}}$

g) $\dfrac{\frac{4}{x+5}}{\frac{1}{x-5}-\frac{2}{x^2-25}}$ h) $\dfrac{\frac{3}{x+5}+\frac{9}{x+7}}{\frac{2x+11}{x^2+12x+35}}$

2. Simplify selected problems from 1a) through 1h) by multiplying by the least common denominator.

3. Simplify.

a) $\dfrac{x^{-2}+y^{-1}}{x^{-3}}$ b) $\dfrac{2x^{-1}+5y^{-1}}{-7x^{-2}-3y^{-2}}$ c) $\dfrac{-6x^{-1}+(6y)^{-1}}{x^{-2}}$

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Teaching Notes:

- Stronger students tend to prefer using the multiply by LCD method.
- Many students need to be reminded of how to deal with negative exponents before attempting objective 3.
- Refer to students to the ***Simplifying a Complex Fraction: Method 1/ Method 2*** charts in the text.

Answers: *1a)* $\frac{25}{27}$, *b)* $\frac{x}{4}$, *c)* $\frac{1+a}{1-a}$, *d) 4x+5y, e)* $\frac{2x+9}{4-81x}$, *f)* $\frac{x}{5x-10}$, *g)* $\frac{4x-20}{x+3}$, *h) 6; 2a-h) same as 1a-h);*
3a) $\frac{xy+x^3}{y}$, *b)* $\frac{2xy^2+5x^2y}{-7y^2-3x^2}$, *c)* $\frac{-36xy+x^2}{6y}$

Mini-Lecture 6.4
Solving Equations Containing Rational Expressions

Learning Objectives:

1. Solve equations containing rational expressions.
2. Key vocabulary: *equation versus expression, extraneous solutions.*

Examples:

1. Solve each equation and check the solution.

   a) $\frac{2}{5}y - \frac{1}{3}y = 5$ b) $\frac{3y+6}{5} = 1 + \frac{3}{4}y$

2. Solve each equation and check the solution.

   a) $\frac{14}{x} = 5 - \frac{1}{x}$ b) $\frac{x-5}{x+2} = \frac{12}{x+2}$

   c) $1 + \frac{1}{x} = \frac{20}{x^2}$ d) $\frac{4x+1}{2x-5} = \frac{6x-1}{3x-6}$

   e) $\frac{4}{3x} - \frac{1}{x+1} = \frac{1}{2x^2+2x}$ f) $\frac{x+6}{x^2+5x+4} - \frac{6}{x^2+2x+1} = \frac{x-6}{x^2+5x+4}$

3. Solve and check. If there is no solution, so indicate.

   a) $\frac{x}{x-5} - 2 = \frac{5}{x-5}$ b) $\frac{1}{x+5} + \frac{2}{x+3} = \frac{-2}{x^2+8x+15}$

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Teaching Notes:

- Remind students to always determine the unallowed values for x before solving a rational expression.
- Many students are confused by the concept of an extraneous solution. Show them a simple example such as :
  $x = 3 \rightarrow x \bullet x = 3 \bullet x \rightarrow x^2 = 3x \rightarrow x^2 - 3x = 0 \rightarrow x = 0, 3; x = 0$ is extraneous.
- Some students prefer to make equivalent fractions with a common denominator, and then set the numerators equal to each other.
- Refer students to the ***To Solve an Equation Containing Rational Expressions*** chart in the text.

Answers: 1a) {75}, b) $\left\{\frac{4}{3}\right\}$*; 2a) {3}, b) {17}, c) {-5,4}, d) {1}, e)* $\left\{-\frac{5}{2}\right\}$*, f) {2}; 3a)* $\varnothing$*, b)* $\varnothing$

Mini-Lecture 6.5

Rational Equations and Problem Solving

Learning Objectives:

1. Solve an equation containing rational expressions for a specified variable.
2. Solve number problems by writing equations containing rational expressions.
3. Solve problems modeled by proportions.
4. Solve problems about work.
5. Solve problems about distance, rate, and time.
6. Key vocabulary: *ratio, rate, proportion.*

Examples:

1. Solve each equation for the specific variable.

   a) $\frac{PV}{T} = \frac{pv}{t}$ for V b) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$ for c c) $P = \frac{A}{1+rt}$ for r

   d) $A = \frac{1}{2}h\,(B+b)$ for B e) $F = \frac{-GMm}{r^2}$ for M f) $S = \frac{a_1 - a_n r}{1-r}$ for a_1

2. Solve.

   a) ***Number*** Two times the reciprocal of a number equals 4 times the reciprocal of 5. Find the number.

   b) ***Proportion*** The ratio of the weight of an object on Earth to an object on Planet X is 4 to 9. If a person weighs 230 pounds on Earth, find his weight on planet X. Round to the nearest whole number.

   c) ***Work*** One pump can drain a pool in 9 minutes. When a second pump is also used, the pool only takes 6 minutes to drain. How long would it take the second pump to drain the pool if it were the only pump in use?

   d) ***Rate*** Alex can run 5 miles per hour on level ground on a still day. One windy day he runs 11 miles with the wind, and in the same amount of time runs 4 miles against the wind. What is the rate of the wind?

Teaching Notes:

- Many students find this section difficult.
- Most students need to set up a chart to solve work and rate problems. Refer them to the textbook examples for samples.
- Encourage students to check whether their solutions seem reasonable.
- Refer to students to the ***Solving an Equation for a Specified Variable*** chart in the text.

<u>Answers</u>: 1a) $V = \frac{pvT}{tP}$*, b)* $c = \frac{ab}{b+a}$*, c)* $r = \frac{A-P}{Pt}$*, d)* $B = \frac{2A-bh}{h}$*, e)* $M = -\frac{Fr^2}{Gm}$*, f)* $a_1 = S(1-r) + a_n r$ *; 2a)* $\frac{5}{2}$*, b) 518 pounds, c) 18 minutes, d)* $2\frac{1}{3}$ *mph*

Mini-Lecture 6.6

Variation and Problem Solving

Learning Objectives:

1. Solve problems involving direct variation.
2. Solve problems involving inverse variation.
3. Solve problems involving joint variation.
4. Solve problems involving combined variation.
5. Key vocabulary: *constant of variation or constant of proportionality.*

Examples:

1. Identify the following variation equations as direct, inverse, joint, or combined.

   a) $y = kx$ b) $y = \frac{k}{x}$ c) $y = kxz$ d) $y = \frac{km^2 p}{n^3}$

2. Find the constant of variation and the ***direct*** variation equation for each situation. Then solve as indicated.
   a) $y = 4$ when $x = 3$. Find y when $x = 9$.
   b) The amount of gas that a helicopter uses is directly proportional to the number of hours spent flying. The helicopter flies for 3 hours and uses 18 gallons of fuel. Find the number of gallons of fuel that the helicopter uses to fly for 5 hours.

3. Find the constant of variation and the ***inverse*** variation equation for each situation. Then solve as indicated.
   a) $y = 4$ when $x = 3$. Find y when $x = 6$.
   b) The amount of time it takes a swimmer to swim a race is inversely proportional to the swimmer's speed. A swimmer finishes a race in 50 seconds with a speed of 3 feet per second. Find the speed if it takes 25 seconds to finish the race.

4. Find the constant of variation and the ***joint*** or the ***combined*** variation equation for each situation. Then solve as indicated.
   a) r varies jointly as the square of s and the square of t. $r = 12$ when $s = 1$ and $t = 2$.
   b) x is directly proportional to y and inversely proportional to the cube of z. $x = 3$ when $y = 3$ and $z = 2$.
   c) The volume V of a given mass of gas varies directly as the temperature T and inversely as the pressure P. A measuring device is calibrated to give $V = 300\ in^3$ when $T = 250^\circ$ and $P = 10\ lb/in^2$. What is the volume on this device when the temperature is 370° and the pressure is $20\ lb/in^2$?

Teaching Notes:

- Most students will understand the concepts of direct and inverse variation better if real-life examples are discussed in a qualitative way for problem 1.
- Some students are confused by solving for the constant of variation and then using that constant in the original equation and solving for a different variable.

Answers: 1a) direct, b) inverse, c) joint, d) combined; 2a) $k = \frac{4}{3}, y = \frac{4}{3}x, y = 12$*, b) k=6, g=6h, 30 gallons of fuel; 3a)* $k = 12, y = \frac{12}{x}, y = 2$*, b) k=150,* $t = \frac{150}{s}$*, 6 feet per second; 4a) k=3,* $r = 3s^2t^2$*, b) k=8,* $x = \frac{8y}{z^3}$*, c) k=12,* $V = \frac{12T}{P}$*, 222* in^3

Mini-Lecture 7.1

Radical Expressions and Radical Functions

Learning Objectives:

1. Find square roots.
2. Approximate roots using a calculator.
3. Find cube roots.
4. Find n th roots.
5. Find $\sqrt[n]{a^n}$ when a is any real number.
6. Find function values and graph square and cube root functions.
7. Key vocabulary: *principal square root, negative square root, index, n th root.*

Examples:

1. Find each square root. Assume that all variables represent non-negative real numbers.

   a) $\sqrt{25}$ b) $\sqrt{\frac{1}{9}}$ c) $\sqrt{0.04}$ d) $-\sqrt{49}$

   e) $\sqrt{x^2}$ f) $\sqrt{4x^4}$ g) $\sqrt{16x^{10}}$ h) $-\sqrt{100x^{36}}$

2. Use a calculator to approximate each square root to three decimal places.

   a) $\sqrt{11}$ b) $\sqrt{37}$ c) $\sqrt{113}$ d) $\sqrt{205}$

3. Find each n th root. Assume that all variables represent non-negative real numbers.

   a) $\sqrt[3]{8}$ b) $\sqrt[3]{\frac{1}{64}}$ c) $\sqrt[3]{-27}$ d) $\sqrt[3]{x^6}$

   e) $\sqrt[3]{-64x^9y^{12}}$ f) $\sqrt[4]{16}$ g) $-\sqrt[4]{81}$ h) $\sqrt[4]{-81}$

   i) $\sqrt[4]{x^{16}}$ j) $\sqrt[5]{32}$ k) $\sqrt[5]{-32x^{20}}$ l) $\sqrt[4]{256x^{12}y^8}$

4. Simplify. Assume that the variables represent any real number.

   a) $\sqrt{(-6)^2}$ b) $\sqrt[3]{(-27)^3}$ c) $\sqrt{16x^2}$ d) $\sqrt[4]{(x-1)^4}$

5. If $f(x)=\sqrt[3]{x}+2$, solve as indicated

   a) Find $f(0)$ b) Find $f(-8)$ c) Find domain d) Graph $f(x)$

Teaching Notes:

- Some students think $\sqrt{4} = +2 \text{ or } -2$. Be sure to define *principal square root* early.
- Some students find higher-order radicals confusing at first.
- Many students are unsure when the absolute value symbol is needed in objective 4.
- Refer students to the ***Finding*** $\sqrt[n]{a^n}$ chart in the text.

Answers: (graph answers at end of mini-lectures) 1a) 5, b) $\frac{1}{3}$, c) 0.2, d) -7, e) x, f) $2x^2$, g) $4x^5$, h) $-10x^{18}$; 2a) 3.317, b) 6.083, c) 10.630, d) 14.318; 3a) 2, b) $\frac{1}{4}$, c) -3, d) x^2, e) $-4x^3y^4$, f) 2, g) -3, h) not a real number, i) x^4, j) 2, k) $-2x^4$, l) $4x^3y^2$; 4a) 6, b) -27, c) 4|x|, d) |x-1|; 5a) 2, b) 0, c) all real numbers

Mini-Lecture 7.2

Rational Exponents

Learning Objectives:

1. Understand the meaning of $a^{\frac{1}{n}}$, $a^{\frac{m}{n}}$, and $a^{-\frac{m}{n}}$.
2. Use rules for exponents to simplify expressions that contain rational exponents.
3. Use rational exponents to simplify radical expressions.

Examples:

1. Use radical notation to rewrite each expression. Simplify if possible.

   a) $25^{\frac{1}{2}}$ b) $8^{\frac{1}{3}}$ c) $\left(\frac{1}{49}\right)^{\frac{1}{2}}$ d) $(-8)^{\frac{1}{3}}$ e) $(16x^6)^{\frac{1}{2}}$

   Simplify if possible. Write final answers with positive exponents.

   f) $81^{\frac{3}{4}}$ g) $(-8)^{\frac{2}{3}}$ h) $8^{-\frac{2}{3}}$ i) $(-64)^{-\frac{4}{3}}$ j) $\dfrac{1}{x^{-\frac{2}{3}}}$ k) $\dfrac{3}{4x^{-\frac{5}{9}}}$

2. Use the properties of exponents to simplify each expression. Write with positive exponents.

   a) $x^{\frac{4}{3}}\,x^{\frac{5}{3}}$ b) $y^{\frac{5}{3}}\,y^{-\frac{1}{3}}$ c) $\dfrac{x^{\frac{3}{5}}}{x^{\frac{1}{10}}}$ d) $\left(81^{\frac{1}{4}}\,x^{\frac{2}{3}}\right)^3$

   e) $\dfrac{a^{\frac{3}{4}}\,a^{-\frac{1}{2}}}{a^{\frac{4}{3}}}$ f) $\dfrac{x^{\frac{10}{3}}}{\left(x^4\right)^{\frac{1}{3}}}$ g) $\dfrac{\left(3x^{\frac{1}{5}}\right)^4}{x^{\frac{3}{10}}}$ h) $\dfrac{\left(a^{-3}b^2\right)^{\frac{1}{8}}}{\left(a^{-2}b\right)^{-\frac{1}{4}}}$

3. Use rational exponents to simplify each radical. Assume that all variables represent positive real numbers.

   a) $\sqrt[12]{a^4}$ b) $\sqrt[4]{25}$ c) $\sqrt[4]{64x^2}$ d) $\sqrt[12]{a^6b^6}$

   Use rational exponents to write as a single radical expression.

   e) $\sqrt[3]{x}\bullet\sqrt{x}$ f) $\dfrac{\sqrt[8]{y}}{\sqrt[9]{y}}$ g) $\sqrt[12]{x}\bullet\sqrt[3]{x^2}$ h) $\sqrt[5]{2x}\bullet\sqrt[3]{y}$

Teaching Notes:

- Most students think rational exponents are easy once they see that the denominator is the root and the numerator is the power.
- Refer students to the ***Definition of*** $a^{\frac{1}{n}}$ / $a^{\frac{m}{n}}$ / $a^{-\frac{m}{n}}$ and ***Summary of Exponent Rules*** charts in text.

Answers: 1a) $\sqrt{25}=5$*, b)* $\sqrt[3]{8}=2$*, c)* $\sqrt{\frac{1}{49}}=\frac{1}{7}$*, d)* $\sqrt[3]{-8}=-2$*, e)* $\sqrt{16x^6}=4x^3$*, f) 27, g) 4, h)* $\frac{1}{4}$*, i)* $\frac{1}{256}$*, j)* $x^{\frac{2}{3}}$*, k)* $\frac{3x^{\frac{5}{9}}}{4}$*; 2a)* x^3*, b)* $y^{\frac{4}{3}}$*, c)* $x^{\frac{1}{2}}$*, d)* $27x^2$*, e)* $\frac{1}{a^{\frac{13}{12}}}$*, f)* x^2*, g)* $81x^{\frac{1}{2}}$*, h)* $\frac{b^{\frac{1}{2}}}{a^{\frac{7}{8}}}$*; 3a)* $\sqrt[3]{a}$*, b)* $\sqrt{5}$*, c)* $2\sqrt{2x}$*, d)* $\sqrt{ab}$*, e)* $\sqrt[6]{x^5}$*, f)* $\sqrt[72]{y}$*, g)* $\sqrt[4]{x^3}$*, h)* $\sqrt[15]{8x^3y^5}$

Mini-Lecture 7.3

Simplifying Radical Expressions

<u>Learning Objectives</u>:

1. Use the product rule for radicals.
2. Use the quotient rule for radicals.
3. Simplify radicals.
4. Use the distance and midpoint formulas.

<u>Examples</u>:

1. Use the product rule to multiply. Assume that all variables represent positive real numbers.
   a) $\sqrt{5} \bullet \sqrt{2}$ b) $\sqrt[3]{7} \bullet \sqrt[3]{9}$ c) $\sqrt{5x} \bullet \sqrt{3y}$ d) $\sqrt[4]{5x^3} \bullet \sqrt[4]{4}$

2. Use the quotient rule to simplify. Assume that all variables represent positive real numbers.
   a) $\sqrt{\dfrac{9}{64}}$ b) $\sqrt[4]{\dfrac{x}{16y^4}}$ c) $\sqrt[3]{\dfrac{2}{8x^9}}$ d) $\sqrt{\dfrac{x^{12}}{25y^8}}$ e) $-\sqrt[3]{\dfrac{125x}{y^9}}$

3. Simplify. Assume that all variables represent positive real numbers.
   a) $\sqrt{20}$ b) $\sqrt{48}$ c) $\sqrt{16x^2}$ d) $\sqrt{16x^3}$
   e) $\sqrt{90x^7y^8}$ f) $\sqrt[3]{54}$ g) $\sqrt[3]{x^4}$ h) $\sqrt[3]{-24x^8y^{10}}$
   i) $\sqrt[5]{-32x^4y^{10}}$ j) $\dfrac{\sqrt{80}}{\sqrt{4}}$ k) $\dfrac{\sqrt[3]{81}}{\sqrt[3]{3}}$ l) $\dfrac{\sqrt{x^7y^3}}{\sqrt{xy}}$
   m) $\dfrac{\sqrt[3]{40x^5y^9}}{\sqrt[3]{5x^2}}$ n) $\dfrac{\sqrt{50x^2}}{-5\sqrt{25x^{-2}}}$ o) $\dfrac{\sqrt[5]{729x^9y^3}}{\sqrt[5]{3x^2y^{-7}}}$

4. Find the distance between each pair of points.
   a) $(2,3)\ ;\ (-2,6)$ b) $(5,-7)\ ;\ (2,-1)$ c) $\left(3\sqrt{5},2\right)\ ;\ \left(7\sqrt{5},3\right)$

   Find the midpoint of each line segment whose endpoints are given.
   d) $(2,4)\ ;\ (4,3)$ e) $\left(-\dfrac{3}{4},-1\right);\left(-\dfrac{3}{2},-1\right)$ f) $\left(2\sqrt{5},-5\sqrt{5}\right)\ ;\ \left(5\sqrt{5},-2\sqrt{5}\right)$

<u>Teaching Notes</u>:

- Some students have trouble simplifying roots with non-perfect squares inside. Encourage them to write numbers as the product of the highest possible perfect square with another number.
- Some students need a lot of practice simplifying radicals with no variables before attempting those with variables.
- Remind students that the root divides the exponent for variables within radicals.
- Refer students to the ***Product / Quotient Rules, Distance Formula,*** and ***Midpoint Formula*** charts.

<u>Answers</u>: *1a)* $\sqrt{10}$ *, b)* $\sqrt[3]{63}$ *, c)* $\sqrt{15xy}$ *, d)* $\sqrt[4]{20x^3}$ *; 2a)* $\frac{3}{8}$*, b)* $\frac{\sqrt[4]{x}}{2y}$*, c)* $\frac{\sqrt[3]{2}}{2x^3}$*, d)* $\frac{x^6}{5y^4}$*, e)* $-\frac{5\sqrt[3]{x}}{y^3}$*; 3a)* $2\sqrt{5}$ *, b)* $4\sqrt{3}$ *, c) 4x, d)* $4x\sqrt{x}$ *, e)* $3x^3y^4\sqrt{10x}$ *, f)* $3\sqrt[3]{2}$ *, g)* $x\sqrt[3]{x}$ *, h)* $-2x^2y^3\sqrt[3]{3x^2y}$ *, i)* $-2y^2\sqrt[5]{x^4}$ *, j)* $2\sqrt{5}$ *, k) 3, l)* x^3y*, m)* $2xy^3$*, n)* $\frac{x^2\sqrt{2}}{-5}$*, o)* $3xy^2\sqrt[5]{x^2}$ *; 4a) 5 units, b)* $\sqrt{45} \approx 6.708$ *units, c) 9 units, d)* $(3,\frac{7}{2})$*, e)* $(-\frac{9}{8},-1)$*, f)* $\left(\frac{7\sqrt{5}}{2},-\frac{7\sqrt{5}}{2}\right)$

Mini-Lecture 7.4

Adding, Subtracting, and Multiplying Radical Expressions

Learning Objectives:

1. Add or subtract radical expressions.
2. Multiply radical expressions.
3. Key vocabulary: *like radicals.*

Examples:

1. Add or subtract as indicated. Assume that all variables represent positive real numbers.

a) $\sqrt{63}-\sqrt{7}$ b) $-3\sqrt{200}-5\sqrt{8}+9\sqrt{98}$ c) $\sqrt{300x^3}-x\sqrt{12x}$

d) $\sqrt[3]{8x}-\sqrt[3]{27x}$ e) $7\sqrt[3]{x^3y^{13}}+5xy\sqrt[3]{8y^{10}}$ f) $\dfrac{2\sqrt{2}}{3}+\dfrac{3\sqrt{2}}{5}$

g) $\dfrac{2x\sqrt{11}}{5}+\sqrt{\dfrac{11x^2}{100}}$ h) $10\sqrt[4]{x^7}-2x\sqrt[4]{x^3}$ i) $\sqrt{\dfrac{20}{x^2}}+\sqrt{\dfrac{5}{4x^2}}$

2. Multiply. Then simplify if possible. Assume that all variables represent positive real numbers.

a) $\sqrt{6}\left(\sqrt{5}+\sqrt{7}\right)$ b) $\sqrt{7}\left(\sqrt{11}+\sqrt{7}\right)$ c) $\left(\sqrt{7}-\sqrt{2}\right)^2$

d) $\sqrt{2x}\left(\sqrt{2}-\sqrt{x}\right)$ e) $\left(6\sqrt{y}+z\right)\left(3\sqrt{y}-1\right)$ f) $\left(\sqrt[3]{x}+5\right)\left(\sqrt[3]{x}+2\right)$

g) $\left(5\sqrt{3}+9\right)\left(6\sqrt{3}-4\right)$ h) $\left(\sqrt{x-4}+3\right)^2$ i) $\left(\sqrt[3]{x}+7\right)\left(\sqrt[3]{x}-7\sqrt{x}+2\right)$

Teaching Notes:

- Most students find objective 1 easy once they realize that adding / subtracting like radicals is analogous to adding / subtracting like terms.
- Some students are not sure how to handle a coefficient in front of a radical once the radical is simplified.
- Many students distribute the exponent in examples 2c) and 2h).

<u>*Answers*</u>*: 1a)* $2\sqrt{7}$ *, b)* $23\sqrt{2}$ *, c)* $8x\sqrt{3x}$ *, d)* $-\sqrt[3]{x}$ *, e)* $17xy^4\sqrt[3]{y}$ *, f)* $\dfrac{19\sqrt{2}}{15}$ *, g)* $\dfrac{x\sqrt{11}}{2}$ *, h)* $8x\sqrt[4]{x^3}$ *, i)* $\dfrac{5\sqrt{5}}{2x}$ *;*
2a) $\sqrt{30}+\sqrt{42}$ *, b)* $\sqrt{77}+7$ *, c)* $9-2\sqrt{14}$ *, d)* $2\sqrt{x}-x\sqrt{2}$ *, e)* $18y+(3z-6)\sqrt{y}-z$ *, f)* $\sqrt[3]{x^2}+7\sqrt[3]{x}+10$ *, g)* $54+34\sqrt{3}$ *,*
h) $x+5+6\sqrt{x-4}$ *,*   *i)* $\sqrt[3]{x^2}-7\sqrt[6]{x^5}+9\sqrt[3]{x}-49\sqrt{x}+14$

Mini-Lecture 7.5

Rationalizing Numerators and Denominators of Radical Expressions

Learning Objectives:

1. Rationalize denominators.
2. Rationalize denominators having two terms.
3. Rationalize numerators.
4. Key vocabulary: *rationalize, conjugate.*

Examples:

1. Rationalize each denominator. Assume that all variables represent positive real numbers.

a) $\dfrac{3}{\sqrt{5}}$ b) $\sqrt{\dfrac{1}{7}}$ c) $\dfrac{6}{\sqrt[3]{4}}$ d) $\dfrac{5}{\sqrt{18x}}$

e) $-\dfrac{7\sqrt{3}}{\sqrt{11}}$ f) $\sqrt{\dfrac{23a}{2b}}$ g) $\dfrac{\sqrt[3]{10x}}{\sqrt[3]{5y^4}}$ h) $\sqrt[4]{\dfrac{81}{49x^{19}}}$

2. Rationalize each denominator. Assume that all variables represent positive real numbers.

a) $\dfrac{2}{\sqrt{5}-4}$ b) $\dfrac{-6}{\sqrt{y}+3}$ c) $\dfrac{\sqrt{2}+\sqrt{4}}{\sqrt{3}+\sqrt{2}}$ d) $\dfrac{3\sqrt{x}-2}{3\sqrt{x}-\sqrt{y}}$

3. Rationalize each numerator. Assume that all variables represent positive real numbers.

a) $\sqrt{\dfrac{5}{2}}$ b) $\dfrac{\sqrt{2x^7}}{8}$ c) $\dfrac{\sqrt[3]{6x^2}}{\sqrt[3]{5y}}$ d) $\sqrt{\dfrac{16x^5y}{4z}}$

e) $\dfrac{\sqrt{13}+1}{2}$ f) $\dfrac{3-\sqrt{11}}{-4}$ g) $\dfrac{\sqrt{x}-1}{\sqrt{x}+1}$ h) $\dfrac{\sqrt{x}+2\sqrt{y}}{3\sqrt{x}}$

Teaching Notes:

- Some students need to see a few examples of why $\sqrt{a}\bullet\sqrt{a}=a$ before applying it to rationalizing a denominator.
- Most students can rationalize denominators easily for square roots.
- Some students have trouble figuring out what to multiply by when rationalizing higher roots and need a step-by-step procedure.

Answers: 1a) $\dfrac{3\sqrt{5}}{5}$, *b)* $\dfrac{\sqrt{7}}{7}$, *c)* $3\sqrt[3]{2}$, *d)* $\dfrac{5\sqrt{2x}}{6x}$, *e)* $-\dfrac{7\sqrt{33}}{11}$, *f)* $\dfrac{\sqrt{46ab}}{2b}$, *g)* $\dfrac{\sqrt[3]{2xy^2}}{y^2}$, *h)* $\dfrac{3\sqrt[4]{49x}}{7x^5}$; *2a)* $-\dfrac{2(\sqrt{5}+4)}{11}$, *b)* $\dfrac{-6\sqrt{y}+18}{y-9}$, *c)* $\sqrt{6}-2+2\sqrt{3}-2\sqrt{2}$, *d)* $\dfrac{9x+3\sqrt{xy}-6\sqrt{x}-2\sqrt{y}}{9x-y}$; *3a)* $\dfrac{5}{\sqrt{10}}$, *b)* $\dfrac{x^4}{4\sqrt{2x}}$, *c)* $\dfrac{6x}{\sqrt[3]{180xy}}$, *d)* $\dfrac{2x^3y}{\sqrt{xyz}}$, *e)* $\dfrac{6}{\sqrt{13}-1}$, *f)* $\dfrac{1}{2(3+\sqrt{11})}$, *g)* $\dfrac{x-1}{x+2\sqrt{x}+1}$, *h)* $\dfrac{x-4y}{3x-6\sqrt{xy}}$

Mini-Lecture 7.6

Radical Equations and Problem Solving

Learning Objectives:

1. Solve equations that contain radical expressions.
2. Use the Pythagorean Theorem to model problems.
3. Key vocabulary: *extraneous solution.*

Examples:

1. Solve. Check your solutions.

 a) $\sqrt{4x}=2$ b) $\sqrt{x+1}=7$ c) $\sqrt{3x}=-6$

 d) $\sqrt{5x+6}\ +2=8$ e) $\sqrt[3]{6x}=-4$ f) $\sqrt[3]{3x+4}-4=0$

 g) $x-\sqrt{16x-16}\ =-3$

2. Solve. Check your solutions.

 a) $\sqrt{4x+1}=3+\sqrt{x-2}$ b) $\sqrt{x+20}-\sqrt{x-4}=4$ c) $\sqrt{x}\ +3=\sqrt{x+21}$

 d) $\sqrt{4x-3}=\sqrt{x+6}$ e) $\sqrt{x+1}-\sqrt{x-1}=2$ f) $\sqrt[3]{7x-2}=\ \sqrt[3]{x+8}$

3. Solve.

 a) ***Triangle*** A triangle has sides of length 12m and 16m. Find the length of the hypotenuse.
 b) ***Triangle*** A triangle has a hypotenuse of length 25cm and one leg of length 15cm. Find the length of the other leg.
 c) ***Kite*** A kite is secured to a rope that is tied to the ground. A breeze blows the kite so that the rope is taught while the kite is directly above a flagpole that is 30ft from where the rope is staked down. Find the altitude of the kite if the rope is 110ft long.
 d) ***Voltage*** The maximum number of volts, E, that can be placed across a resistor is given by $E=\sqrt{PR}$, where P is the power in watts and R is resistance in ohms. If a 2 watt resistor can have at most 40 volts of electricity across it, find the number of ohms of resistance of this resistor.

Teaching Notes:

- Show students a simple example of an extraneous solution, such as:
 $x=3\rightarrow x^2=9\rightarrow x=\pm 3\rightarrow x=-3$ is extraneous.
- Encourage students to draw a diagram whenever possible for applied problems.
- Refer students to the ***Power Rule, Solving a Radical Equation,*** and ***Pythagorean Theorem*** charts in the text.

<u>Answers</u>: 1a) {1}, b) {48}, c) $\varnothing$, d) {6}, e) $\left\{-\frac{32}{3}\right\}$, f) {20}, g) {5}; 2a) {6,2}, b) {5}, c) {4}, d) {3}, e) $\varnothing$, f) $\left\{\frac{5}{3}\right\}$;
3a) 20 m, b) 20 cm, c) 105.83 ft, d) 800 ohms of resistance

Mini-Lecture 7.7

Complex Numbers

Learning Objectives:

1. Write square roots of negative numbers in the form *bi.*
2. Add or subtract complex numbers.
3. Multiply complex numbers.
4. Divide complex numbers.
5. Raise *i* to powers.
6. Key vocabulary: *imaginary number, complex number, complex conjugate.*

Examples:

1. Write using *i* notation.

   a) $\sqrt{-9}$ b) $\sqrt{-18}$ c) $-\sqrt{4}$ d) $5\sqrt{-20}$

   Write using *i* notation. Then multiply or divide as indicated.

   e) $\sqrt{-3} \bullet \sqrt{-7}$ f) $\sqrt{25} \bullet \sqrt{-1}$ g) $\sqrt{4} \bullet \sqrt{-64}$ h) $\dfrac{\sqrt{81}}{\sqrt{-6}}$

2. Add or subtract as indicated. Write your answers in $a + bi$ form.

   a) $(3-5i)+(2+4i)$ b) $(8-i)-(2-3i)$ c) $7-(9+3i)$

3. Multiply. Write your answers in $a + bi$ form.

   a) $6i \bullet 8i$ b) $-3i \bullet 5i$ c) $2i\ (4-9i)$

   d) $(2+i)(1+4i)$ e) $(\sqrt{2}-2i)(\sqrt{2}+2i)$ f) $(3-2i)^2$

4. Divide. Write your answers in $a + bi$ form.

   a) $\dfrac{2}{i}$ b) $\dfrac{3}{7i}$ c) $\dfrac{6}{2+3i}$ d) $\dfrac{3+2i}{4-3i}$

5. Find each power of *i.*

   a) i^3 b) i^4 c) i^5 d) i^6 e) i^{27} f) $(-2i)^5$

Teaching Notes:

- Most students find objectives 1 and 2 fairly straightforward.
- Encourage students to keep their work neat and organized to avoid errors with objectives 3 and 4.
- Some students have more success with problems 4c) and 4d) if they multiply the complex conjugates off to the side and then put the final result within the problem as they solve it.
- Refer students to the ***Sum or Difference of Complex Numbers,*** and ***Complex Conjugates*** charts.

Answers: 1a) 3i, b) $3i\sqrt{2}$ *, c) -2, d)* $10i\sqrt{5}$ *, e)* $-\sqrt{21}$ *, f) 5i, g) 16i, h)* $-\frac{3}{2}i\sqrt{6}$ *; 2a) 5-i, b) 6+2i, c) -2-3i; 3a) -48, b) 15, c) 18+8i, d) -2+9i, e) 6, f) 5-12i; 4a) -2i, b)* $-\frac{3}{7}i$ *, c)* $\frac{12}{13}-\frac{18}{13}i$ *, d)* $\frac{6}{25}+\frac{17}{25}i$ *; 5a) –i, b) 1, c) i, d) -1, e) –i, f) -32i*

Mini-Lecture 8.1

Solving Quadratic Equations by Completing the Square

Learning Objectives:

1. Use the square root property to solve quadratic equations.
2. Write perfect square trinomials.
3. Solve quadratic equations by completing the square.
4. Use quadratic equations to solve problems.
5. Key vocabulary: *quadratic equation, perfect square trinomial.*

Examples:

1. Use the square root property to solve each equation.

   a) $x^2 = 9$ b) $x^2 = 20$ c) $2x^2 + 72 = 0$ d) $4x^2 = 16$

   e) $(x-5)^2 = 25$ f) $(x+3)^2 = 11$ g) $(4x+1)^2 = 36$ h) $(5x-3)^2 = 48$

2. Add the proper constant to each binomial so that the resulting trinomial is a prefect square trinomial. Then factor the trinomial.

   a) $x^2 + 8x$ b) $y^2 - 12y$ c) $m^2 + 7m$

3. Solve each equation by completing the square.

   a) $x^2 + 4x = -3$ b) $x^2 - 2x = 35$ c) $x^2 + 20x + 30 = 0$

   d) $2x^2 - 5x = 3$ e) $2x^2 + 11x = -12$ f) $2x^2 + 5x - 3 = 0$

   g) $6x^2 + 10x + 2 = 0$ h) $4x^2 - 16x + 80 = 0$ i) $x^2 + x = -1$

4. The distance, $s(t)$, in feet traveled by a freely falling object is given by the function $s(t) = 16t^2$, where *t* is time in seconds. How long would it take for an object to fall to the ground from 576 feet high?

Teaching Notes:

- Many students forget the +/- when using the square root property.
- Most students are confused by completing the square at first and need to see many examples.
- Refer students to the ***Solving a Quadratic Equation in x by Completing the Square*** chart in text.

Answers: 1a) {3,-3}, b) $\{2\sqrt{5}, -2\sqrt{5}\}$*, c){6i,-6i}, d) {2,-2}, e) {10,0}, f)* $\{-3+\sqrt{11}, -3-\sqrt{11}\}$*, g)* $\left\{\frac{5}{4}, -\frac{7}{4}\right\}$*,*
h) $\left\{\frac{3+4\sqrt{3}}{5}, \frac{3-4\sqrt{3}}{5}\right\}$*; 2a) +16,* $(x+4)^2$*, b) +36,* $(y-6)^2$*, c)* $+\frac{49}{4}$*,* $\left(m+\frac{7}{2}\right)^2$*; 3a) {-3,-1}, b) {7,-5},*
c) $\{-10+\sqrt{70}, -10-\sqrt{70}\}$*, d)* $\left\{3, -\frac{1}{2}\right\}$*, e)* $\left\{-\frac{3}{2}, -4\right\}$*, f)* $\left\{\frac{1}{2}, -3\right\}$*, g)* $\left\{\frac{-5+\sqrt{13}}{6}, \frac{-5-\sqrt{13}}{6}\right\}$*, h) {2+4i,2-4i},*
i) $\left\{-\frac{1}{2}+i\frac{\sqrt{3}}{2}, -\frac{1}{2}-i\frac{\sqrt{3}}{2}\right\}$*; 4) t=6 seconds*

Mini-Lecture 8.2

Solving Quadratic Equations by Using the Quadratic Formula

Learning Objectives:

1. Solve quadratic equations by using the quadratic formula.
2. Determine the number and type of solutions of a quadratic equation by using the discriminant.
3. Solve problems modeled by quadratic equations.
4. Key vocabulary: *discriminant.*

Examples:

1. Use the quadratic formula to solve each equation.

   a) $x^2 + 5x + 6 = 0$ b) $x^2 + 4x - 7 = 0$ c) $3x^2 - 9x = -2$

   d) $5x^2 = -10x - 3$ e) $5x^2 = -8$ f) $9 + 3x(x-2) = 8$

   g) $\frac{x^2}{18} + x + \frac{35}{9} = 0$ h) $(x+8)(2x-9) = 2(x-1) - 72$

2. Use the discriminant to determine the number and types of solutions of each equation.

   a) $4x^2 - 8x + 4 = 0$ b) $6x^2 = 2x - 5$

   c) $x^2 + 8x + 7 = 0$ d) $10 - 5x^2 = 6x + 5$

3. Solve each equation by completing the square.

   c) ***Geometry*** A rectangular sign has an area of 21 square yards. Its length is 6 yards more than its width. Find the dimensions of the sign.
   d) ***Geometry*** The hypotenuse of a right triangle is 7 feet long. One leg of the triangle is 5 feet longer than the other leg. Find the perimeter of the triangle.
   e) ***Revenue*** The revenue for a small company is given by the quadratic function $r(t) = 14t^2 + 16t + 860$, where t is the number of years since 1998 and $r(t)$ is in thousands of dollars. Find the year in which the company's revenue will be \$1290 thousand. Round to the nearest whole year.

Teaching Notes:

- Encourage students to memorize the quadratic formula.
- Many students reduce final answers incorrectly. For example: $\frac{4 \pm \sqrt{5}}{8} \rightarrow \frac{1 \pm \sqrt{5}}{2}$.
- Some students prefer to always use the quadratic formula because it has no restrictions on when it can be used. Encourage them to master the other methods, which are often quicker and easier to apply.
- Refer students to the ***Discriminant*** chart in the text.

Answers: 1a) {-3,-2}, b) $\{-2+\sqrt{11}, -2-\sqrt{11}\}$*, c)* $\left\{\frac{9+\sqrt{57}}{6}, \frac{9-\sqrt{57}}{6}\right\}$*, d)* $\left\{\frac{-5+\sqrt{10}}{5}, \frac{-5-\sqrt{10}}{5}\right\}$*, e)* $\left\{\frac{2i\sqrt{10}}{5}, \frac{-2i\sqrt{10}}{5}\right\}$*, f)* $\left\{\frac{3+\sqrt{6}}{3}, \frac{3-\sqrt{6}}{3}\right\}$*, g)* $\{-9+\sqrt{11}, -9-\sqrt{11}\}$*, h)* $\left\{-\frac{1}{2}, -2\right\}$*; 2a) one real solution, b) two complex but not real solutions, c) two real solutions, d) two real solutions; 3a)* $-3+\sqrt{30}$ *yds by* $3+\sqrt{30}$ *yds, b)* $7+\sqrt{73}$ *ft, c) 2003*