Problem 1.4-6

A current lead must be designed to carry current to a cryogenic superconducting magnet, as shown in Figure P1.4-6.

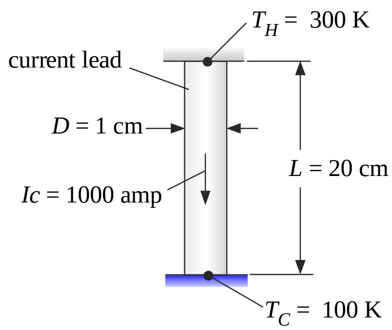

Figure P1.4-6: Current lead.

The current lead carries Ic = 1000 amp and therefore experiences substantial generation of thermal energy due to ohmic dissipation. The electrical resistivity of the lead material depends on temperature according to:

$$\rho_e = 17\text{x}10^{-9}\ [\text{ohm-m}] + \left(T - 300\ [\text{K}]\right) 5\text{x}10^{-11} \left[\frac{\text{ohm-m}}{\text{K}}\right] \tag{1}$$

The length of the current lead is L = 20 cm and the diameter is D = 1 cm. The hot end of the lead (at x = 0) is maintained at $T_{x=0} = T_H$ = 300 K and the cold end (at $x = L$) is maintained at $T_{x=L} = T_C$ = 100 K. The conductivity of the lead material is k = 400 W/m-K. The lead is installed in a vacuum chamber and therefore you may assume that the external surfaces of the lead (the outer surface of the cylinder) are adiabatic.

a.) Develop a numerical model in EES that can predict the temperature distribution within the current lead. Plot the temperature as a function of position.

The input information is entered in EES and a function is used to define the electrical resistivity according to Eq. (1):

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

function rho_e(T)
"Input:
T - temperature (K)

Output:
rho_e - electrical resistivity (ohm-m)"
    rho_e=17e-9 [ohm-m]+(T-300 [K])*5e-11 [ohm-m/K]
end

"Inputs"
L=20 [cm]*convert(cm,m)                          "length of current lead"
T_H=300 [K]                                      "hot end temperature"
T_C=100 [K]                                      "cold end temperature"
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```
Ic=1000 [amp]                          "current"
k=400 [W/m-K]                          "conductivity of lead"
D_cm=1 [cm]                            "diameter of lead, in cm"
D=D_cm*convert(cm,m)                   "diameter of lead"
```

Nodes are distributed uniformly throughout the computational, the location of each node (x_i) is:

$$x_i = \frac{(i-1)}{(N-1)} L \quad i = 1..N \tag{2}$$

where N is the number of nodes used for the simulation. The distance between adjacent nodes (Δx) is:

$$\Delta x = \frac{L}{N-1} \tag{3}$$

```
N=21 [-]                    "number of nodes"
duplicate i=1,N
    x[i]=L*(i-1)/(N-1)      "axial position"
end
Dx=L/(N-1)                  "distance between adjacent nodes"
```

A control volume is defined around each node and an energy balance is written for each control volume. The control volume for an arbitrary, internal node (i.e., a node that is not placed on the edge of the hay) experiences conduction heat transfer passing through the top surface ($\dot{q}_{top}$), conduction heat transfer passing through the bottom surface ($\dot{q}_{bottom}$), and heat generation within the control volume ($\dot{g}$). A steady-state energy balance for an internal control volume:

$$\dot{q}_{LHS} + \dot{q}_{RHS} + \dot{g} = 0 \tag{4}$$

Each of the terms in the energy balance in Eq. (4) must be modeled using a rate equation. Conduction through the inner surface is driven by the temperature difference between nodes i-1 and i through the material that lies between these nodes.

$$\dot{q}_{top} = \frac{k\, A_c \left(T_{i-1} - T_i\right)}{\Delta x} \tag{5}$$

where A_c is the cross-sectional area of the lead:

$$A_c = \frac{\pi\, D^2}{4} \tag{6}$$

The conduction through the bottom surface is:

$$\dot{q}_{bottom} = \frac{k\,A_c\left(T_{i+1}-T_i\right)}{\Delta x} \tag{7}$$

The generation is the product of the electrial resistance of the material in the control volume and the current squared:

$$\dot{g} = \rho_{e,T=T_i}\frac{\Delta x}{A_c}Ic^2 \tag{8}$$

Substituting Eqs. (5), (7), and (8) into Eq. (4) leads to:

$$\frac{k\,A_c\left(T_{i-1}-T_i\right)}{\Delta x}+\frac{k\,A_c\left(T_{i+1}-T_i\right)}{\Delta x}+\rho_{e,T=T_i}\frac{\Delta x}{A_c}Ic^2 = 0 \quad \text{for } i = 2...(N-1) \tag{9}$$

```
A_c=pi*D^2/4                                              "cross-sectional area of lead"
duplicate i=2,(N-1)
     k*A_c*(T[i-1]-T[i])/Dx+k*A_c*(T[i+1]-T[i])/Dx+rho_e(T[i])*Dx*Ic^2/A_c=0
          "energy balance on internal nodes"
end
```

The temperatures of nodes 1 and N are specified:

$$T_1 = T_H \tag{10}$$

$$T_N = T_H \tag{11}$$

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T[1]=T_H                         "hot end temperature"
T[N]=T_C                         "cold end temperature"
```

Figure 2 illustrates the temperature as a function of position.

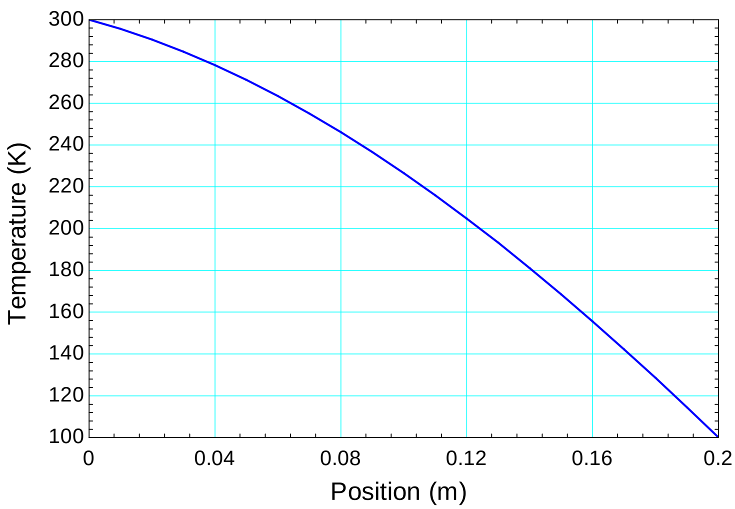

Figure 2: Temperature as a function of position.

b.) Determine the rate of energy transfer into the superconducting magnet at the cold end of the current lead. This parasitic must be removed in order to keep the magnet cold and therefore must be minimized in the design of the current lead.

An energy balance on node N leads to:

$$\dot{q}_C = \frac{k\,A_c\left(T_{N-1} - T_N\right)}{\Delta x} + \rho_{e,T=T_N}\frac{\Delta x}{2\,A_c}Ic^2 \tag{12}$$

```
q_dot=k*A_c*(T[N-1]-T[N])/Dx+rho_e(T[N])*Dx*Ic^2/(2*A_c)        "parasitic to cold end"
```

which leads to $\dot{q}_C$ = 45.7 W.

c.) Prepare a plot showing the rate of energy transfer into the magnet as a function of the number of nodes used in your model.

Figure 3 illustrates $\dot{q}_C$ as a function of N.

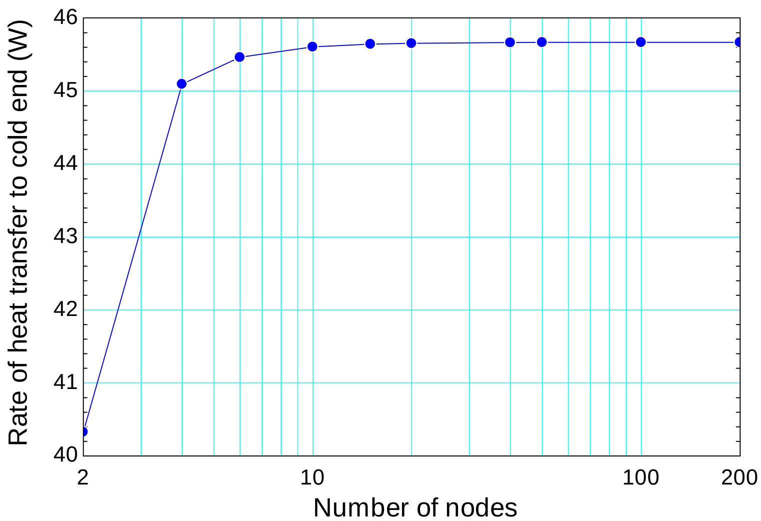

Figure 3: Rate of heat transfer to the cold end as a function of the number of nodes.

d.) Plot the rate of heat transfer to the cold end as a function of the diameter of the lead. You should see a minimum value and therefore an optimal diameter - explain why this occurs.

Figure 4 illustrates the rate of heat transfer to the cold end as a function of the diameter of the lead. At very low diameters the ohmic dissipation is large because the electrical resistance is high and therefore the parasitic is large. At very large diameters, the thermal resistance of the lead is large therefore the parasitic is large. The optimal diameter is around 0.9 cm and balances these effects.

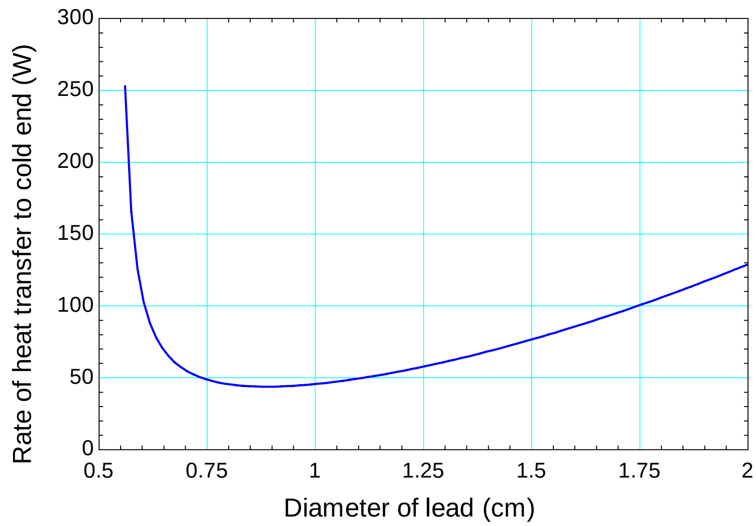

Figure 4: Rate of heat transfer to cold end as a function of the diameter of the lead.

e.) Prepare a plot showing the optimal diameter and minimized rate of heat transfer to the cold end as a function of the current that must be carried by the lead. You may want to use the Min/Max Table selection from the Calculate menu to accomplish this.

Figure 5 illustrates the minimized rate of heat transfer to the cold end and the optimal diameter as function of the level of current.

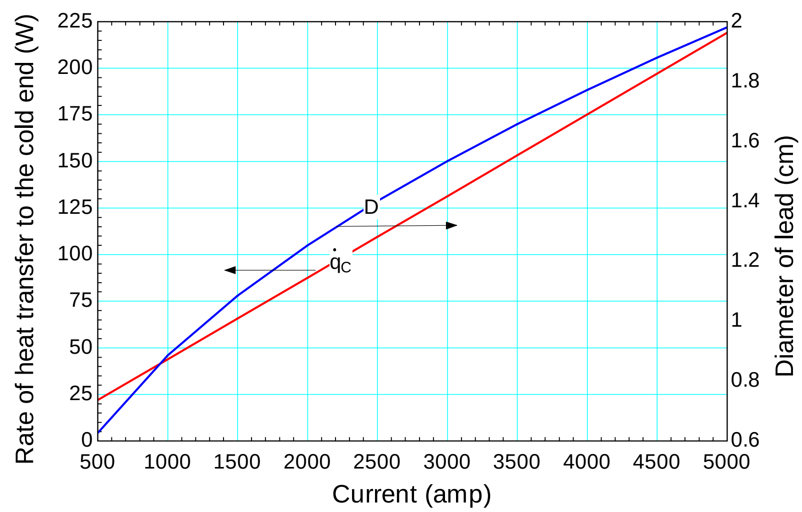

Figure 5: Minimized rate of heat transfer and optimal lead diameter as a function of current.