Problem 1.2-5

Figure P1.2-5 illustrates a wafer that is being developed in an optical lithography process.

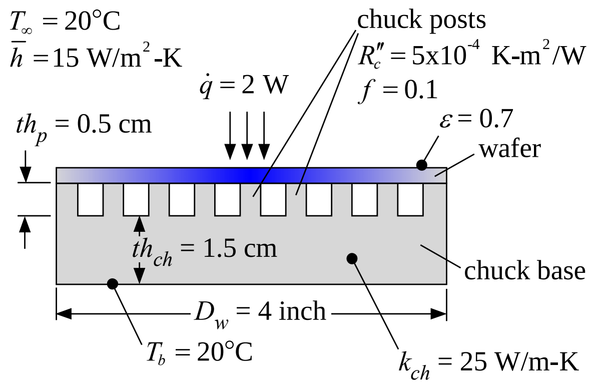


Figure P1.2-5: Wafer being developed in an optical lithography process.

The energy required to develop the resist is deposited at a rate of $\dot{q}$ = 2 W near the center of the upper side of the wafer. The wafer has diameter D_w = 4 inch and is made of a conductive material; therefore, you may assume that the wafer is isothermal. The wafer is cooled by convection and radiation to the surroundings at T_∞ as well as conduction to the chuck. The surrounding air is at T_∞ = 20ºC and the heat transfer coefficient is $\bar{h}$ = 15 W/m^2-K. The emissivity of the wafer surface is ε = 0.7. The chuck is made out of a single piece of material with conductivity k_{ch} = 25 W/m-K and consists of a base that is th_{ch} = 1.5 cm thick and an array of posts that are th_p = 0.5 cm tall. The area of the base of the chuck is the same as the area of the wafer. The posts occupy f = 10% of the chuck area and the wafer rests on the top of the posts. There is an area specific contact resistance of R''_c = 5x10^{-4} K-m^2/W between the bottom of the wafer and the top of the posts. The bottom surface of the chuck base is maintained at T_b = 20 ºC.

a.) What is the temperature of the wafer at steady-state?

The inputs are entered in EES:

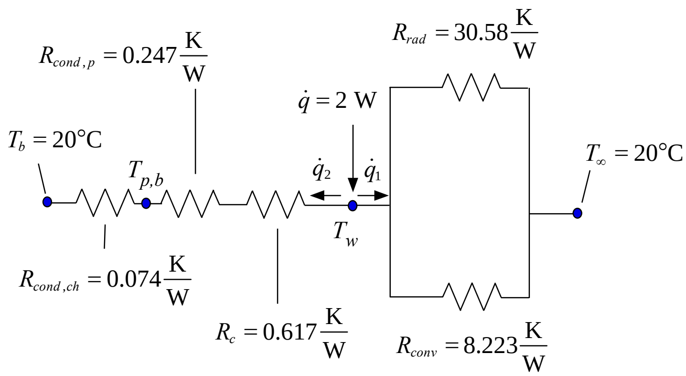
```
"Problem 1.2-5"
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
D_w=4.0 [inch]*convert(inch,m)                    "diameter of wafer"
e=0.7 [-]                                         "emissivity of wafer"
h_bar=15 [W/m^2-K]                                "heat transfer coefficient"
q_dot=2 [W]                                       "power"
th_ch=1.5 [cm]*convert(cm,m)                      "chuck base thickness"
k_ch=25 [W/m-K]                                   "chuck conductivity"
R``_c=5e-4 [K-m^2/W]                              "contact resistance"
th_p=0.5 [cm]*convert(cm,m)                       "post height"
f = 0.1 [-]                                       "fraction of post coverage"
T_infinity_C=20[C]                                "ambient temperature in C"
T_infinity=converttemp(C,K,T_infinity_C)          "ambient temperature"
T_b_C=20 [C]                                      "chuck base temperature in C"
```

```
T_b=converttemp(C,K,T_b_C)                                    "chuck base temperature"
```

Note that the inputs are converted to base SI units and the units for each variable are set in the Variables Information window.

The resistance network used to represent this problem is shown in Figure P1.2-5-2:



The resistances include:
$R_{cond,ch}$ = conduction through chuck base
$R_{cond,p}$ = conduction through posts
R_c = contact resistance
R_{rad} = radiation resistance
R_{conv} = convection resistance

Figure P1.2-5-2: Resistance network.

In order to compute the resistance to radiation, it is necessary to guess a value of the wafer temperature (T_w) and subsequently comment out this guess in order to close up the solution. A reasonable value is chosen:

```
T_w=300 [K]          "guess for wafer temperature - will be commented out"
```

The cross-sectional area of the wafer is:

$$A_w = \frac{\pi\, D_w^2}{4} \tag{1}$$

The resistance to convection from the top surface of the wafer is:

$$R_{conv} = \frac{1}{A_w\, \bar{h}} \tag{2}$$

```
A_w=pi*D_w^2/4                                                "wafer area"
R_conv=1/(A_w*h_bar)                                          "convection resistance"
```

The equations should be solved and the units set as you move through the problem (rather than at the end); this prevents the accumulation of small errors that are difficult to debug. The resistance to radiation is:

$$R_{rad} = \frac{1}{A_w\,\varepsilon\left(T_w^2 + T_\infty^2\right)\left(T_w + T_\infty\right)} \tag{3}$$

```
R_rad=1/(A_w*sigma#*e*(T_w^2+T_infinity^2)*(T_w+T_infinity))  "radiation resistance"
```

The contact resistance is:

$$R_c = \frac{R''_c}{A_w\,f} \tag{4}$$

Notice that the factor f in the denominator accounts for the contact area between the posts and the wafer.

```
R_c=R``_c/(A_w*f)        "contact resistance"
```

The resistance to conduction through the posts is:

$$R_{cond,p} = \frac{th_p}{k_{ch}\,A_w\,f} \tag{5}$$

and the resistance to conduction through the base is:

$$R_{cond,ch} = \frac{th_{ch}}{k_{ch}\,A_w} \tag{6}$$

```
R_cond_p=th_p/(k_ch*A_w*f)              "resistance to conduction through posts"
R_cond_ch=th_ch/(k_ch*A_w)              "resistance to conduction through chuck"
```

The rate of heat transfer by radiation and convection ($\dot{q}_1$) and through the chuck ($\dot{q}_2$) are computed:

$$\dot{q}_1 = \frac{\left(T_w - T_\infty\right)}{\left(\frac{1}{R_{conv}} + \frac{1}{R_{rad}}\right)^{-1}} \tag{7}$$

$$\dot{q}_2 = \frac{\left(T_w - T_b\right)}{R_c + R_{cond,p} + R_{cond,ch}} \tag{8}$$

```
q_dot_1=(T_w-T_infinity)/(1/R_conv+1/R_rad)^(-1)    "rate of heat transfer by convection and radiation"
q_dot_2=(T_w-T_b)/(R_c+R_cond_p+R_cond_ch)          "rate of heat transfer to chuck"
```

Because we guessed a value for T_w, it is not likely that $\dot{q}_1$ and $\dot{q}_2$ sum to the applied power to the wafer, as required by an energy balance:

$$\dot{q} = \dot{q}_1 + \dot{q}_2 \tag{9}$$

In order to finish the solution it is necessary to vary T_w until an energy balance is satisfied. EES automates this process; however, it will work best if it starts from a good set of guess values. Therefore, select Update Guesses from the Calculate menu. Then comment out the assumed value of T_w:

```
{T_w=300 [K]}          "guess for wafer temperature - will be commented out"
```

and enter the energy balance:

```
q_dot=q_dot_1+q_dot_2                    "energy balance"
T_w_C=converttemp(K,C,T_w)               "wafer temperature in C"
```

which leads to T_w = 294.8 K (21.64°C).

b.) Prepare a plot showing the wafer temperature as a function of the applied power, $\dot{q}$.

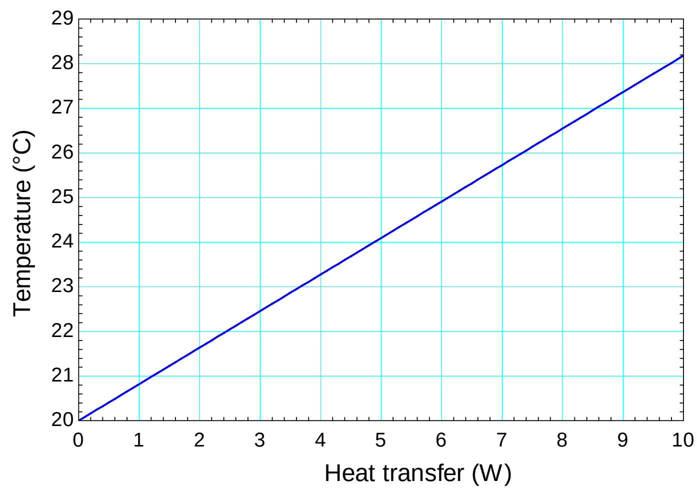


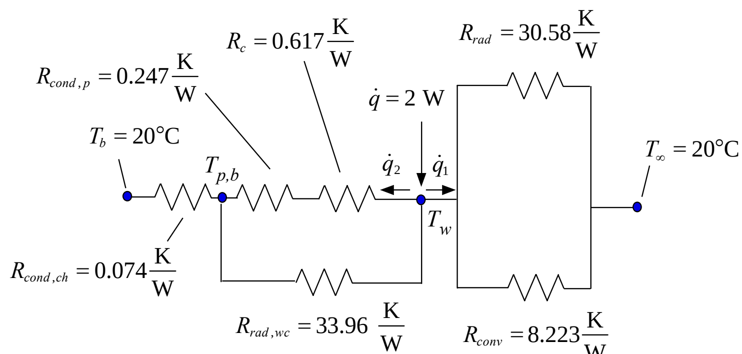
Figure P1.2-5-3: Wafer temperature as a function of applied power.

c.) What are the dominant heat transfer mechanisms for this problem? What aspects of the problem are least important?

The values of the resistances at the nominal conditions given in the problem statement are shown in Figure P1.2-5-2. The value of the radiation and convection resistances are both large relative to the sum of resistances between T_w and T_b and therefore these mechanisms are not likely to play an important role in the problem. The resistance to conduction through the base of the chuck is small relative to the resistance to conduction through the posts and the contact resistance; therefore, conduction through the chuck base is not very important. The dominant resistance in the problem is the contact resistance and the resistance to conduction through the posts is also important.

d.) Radiation between the underside of the wafer and the top of the chuck base was ignored in the analysis; is this an important mechanism for heat transfer? Assume that the chuck surface is black and justify your answer.

The resistance network, modified to include the resistance to radiation from the bottom of the wafer to the top of the chuck, is shown in Figure P1.2-5-4.



The resistances include:
$R_{cond,ch}$ = conduction through chuck base
$R_{cond,p}$ = conduction through posts
R_c = contact resistance
R_{rad} = radiation resistance
R_{conv} = convection resistance
$R_{rad,wc}$ = radiation resistance from top of chuck to bottom of wafer

Figure P1.2-5-4: Resistance network, including radiation from the wafer bottom.

The temperature of the top of the chuck is estimated using our previous solution:

$$T_{p,b} = T_w - \dot{q}_1\left(R_c + R_{cond,p}\right) \tag{10}$$

and used to estimate the resistance to radiation from the top of the chuck to the bottom of the wafer:

$$R_{rad,wc} = \frac{1}{(1-f)\,A_w\,\varepsilon\left(T_w^2 + T_{p,b}^2\right)\left(T_w + T_{p,b}\right)} \tag{11}$$

```
T_p_b=T_w-q_dot_2*(R_c+R_cond_p)   "temperature of the top surface of chuck"
R_rad_wc=1/(A_w*(1-f)*sigma#*e*(T_w^2+T_p_b^2)*(T_w+T_p_b))
                                   "radiation resistance between bottom of wafer and top of chuck"
```

which leads to $R_{rad,wc}$ = 33.96 K/W. Because $R_{rad,wc}$ is in series with R_c and $R_{cond,p}$ and much larger than the sum of these resistances it is not very important to the problem.

e.) In an effort to maintain the wafer temperature at T_w= 20ºC, you decide to try to reduce and control the chuck base temperature, T_b. What temperature do you need to reduce T_b to in order that T_w= 20ºC? If you can only control T_b to within ±0.5 K then how well can you control T_w?

The specified chuck temperature is commented out and instead the wafer temperature is specified:

```
{T_b_C=20 [C]}                         "chuck base temperature in C"
T_w_C=20 [C]                           "specified wafer temperature"
```

which leads to T_b = 291.3 K (18.13ºC). In order to evaluate the impact of a ±0.5 K fluctuation of T_b on T_w, the required value of T_b is specified and the value of T_w is again commented out:

```
T_b_C=18.13 [C]        "chuck base temperature in C"
{T_w_C=20 [C]          "specified wafer temperature"}
```

which leads to T_w = 293.2 K (20ºC), as expected. Now the value of T_b is elevated by 0.5 K in order to determine the impact on T_w:

```
T_b_C=18.13 [C] + 0.5 [K]        "chuck base temperature in C"
```

which leads to T_w = 293.6 K (20.44ºC). Therefore, the ±0.5 K uncertainty in T_b leads to a ±0.44 K uncertainty in T_w.

f.) Perform the same analysis you carried out in (e), but this time evaluate the merit of controlling the surrounding temperature, T_∞, rather than the chuck temperature. What are the advantages and disadvantages associated with controlling T_∞?

The chuck temperature is returned to 20ºC:

```
T_b_C=20 [C]           "chuck base temperature in C"
```

The specified surrounding temperature is commented out and instead the wafer temperature is specified:

```
{T_infinity_C=20[C]}                   "ambient temperature in C"
T_w_C=20 [C]                           "specified wafer temperature"
```

which leads to T_∞ = 280.0 K (6.835ºC); clearly the ambient temperature would need to be reduced by much more than the chuck temperature due to the weaker interaction between the wafer and the surroundings. This is a disadvantage of using the ambient temperature to control the wafer temperature.

In order to evaluate the impact of a ±0.5 K fluctuation of T_∞ on T_w, the required value of T_∞ is specified and the value of T_w is again commented out:

```
T_infinity_C=6.835 [C]   "ambient temperature in C"
```

```
{T_w_C=20 [C]                "specified wafer temperature"}
```

which leads to T_w = 293.2 K (20ºC), as expected. Now the value of T_∞ is elevated by 0.5 K in order to determine the impact on T_w:

```
T_infinity_C=6.835 [C]+0.5 [K]    "ambient temperature in C"
```

which leads to T_w = 293.2 K (20.06ºC). Therefore, the ±0.5 K uncertainty in T_∞ leads to a ±0.06 K uncertainty in T_w. This is an advantage of using T_∞ to control the wafer temperature and is also related to the relatively weak thermal interaction between T_∞ and T_w.