**Problem 1.6-5**

A cylindrical bracket that is $L$ = 4 cm long with diameter $D$ = 5 mm extends between a wall at $T_H$ = 100°C (at $x$= 0) and a wall at $T_C$ = 20°C (at $x$= $L$). The conductivity of the bracket is $k$ = 25 W/m$^2$-K. The cylinder is surrounded by gas at $T_\infty$ = 200°C and the heat transfer coefficient is $\overline{h}$ = 250 W/m$^2$-K.

a.) Is an extended surface approximation appropriate for this problem? Justify your answer.

The inputs are entered in EES:

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
D=5 [mm]*convert(mm,m)                          "diameter of strut"
L=4 [cm]*convert(cm,m)                          "length of strut"
k=25 [W/m-K]                                    "conductivity of strut"
h_bar=250 [W/m^2-K]                             "heat transfer coefficient"
T_H=converttemp(C,K,100 [C])                    "hot end temperature"
T_C=converttemp(C,K,20 [C])                     "cold end temperature"
T_infinity=converttemp(C,K,200 [C])             "ambient temperature"
```

The Biot number is:

$$Bi = \frac{\overline{h}\, D}{k} \tag{1}$$

```
Bi=h_bar*D/k                                    "Biot number"
```

which leads to $Bi$ = 0.05, justifying the extended surface approximation.

b.) Assume that your answer to (a) was yes. Develop an analytical model in EES. Plot the temperature as a function of position within the bracket.

The development of the governing differential equation and the derivation of the general solution proceeds as discussed in Section 1.6.2 and leads to:

$$T = C_1 \exp\left(m\, x\right) + C_2 \exp\left(-m\, x\right) + T_\infty \tag{2}$$

where $C_1$ and $C_2$ are undetermined constants and $m$ is the fin constant:

$$m = \sqrt{\frac{per\, \overline{h}}{k\, A_c}} \tag{3}$$

where $per$ is the perimeter of the bracket and $A_c$ is the cross-sectional area of the bracket.

$$per = \pi\, D \tag{4}$$

$$A_c = \pi \frac{D^2}{4} \tag{5}$$

```
per=pi*D                                  "perimeter"
A_c=pi*D^2/4                              "cross-sectional area"
m=sqrt(per*h_bar/(k*A_c))                 "fin constant"
```

The boundary conditions at $x = 0$ and $x = L$ lead to:

$$T_H = C_1 + C_2 + T_\infty \tag{6}$$

$$T_C = C_1 \exp(m\,L) + C_2 \exp(-m\,L) + T_\infty \tag{7}$$

Equations (6) and (7) are entered in EES in order to determine $C_1$ and $C_2$:

```
T_H=C_1+C_2+T_infinity                    "boundary condition at x=0"
T_C=C_1*exp(m*L)+C_2*exp(-m*L)+T_infinity "boundary condition at x=L"
```

and the solution is entered in EES:

```
x=0 [m]                                   "axial position"
T=C_1*exp(m*x)+C_2*exp(-m*x)+T_infinity   "solution"
T_degC=converttemp(K,C,T)                 "in C"
```

Figure 1 illustrates the temperature as a function of position in the bracket.

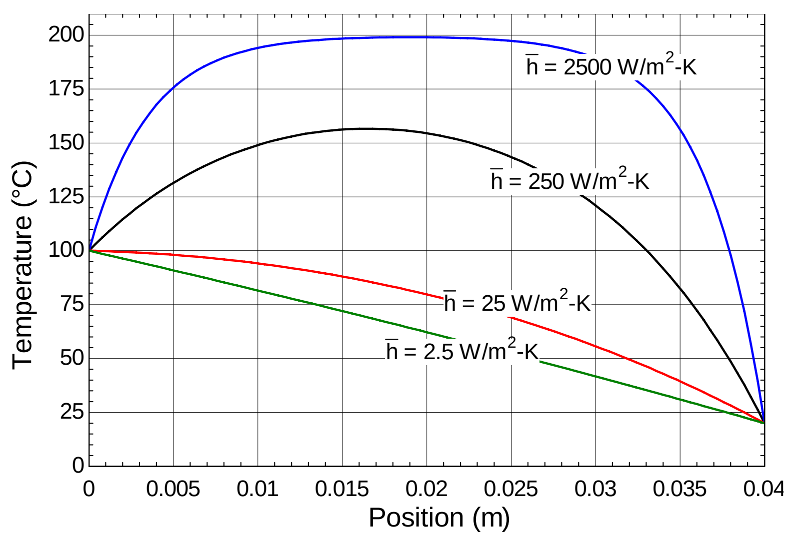


**Figure 1: Temperature as a function of position for various values of the heat transfer coefficient.**

c.) Overlay on your plot from (b) the temperature as a function of position with $\bar{h}$ = 2.5, 25 and 2500 W/m$^2$-K. Explain the shape of your plots.

The requested plots are shown in Figure 1. As the heat transfer coefficient increases, the resistance between the bracket and the surrounding gas:

$$R_{conv} = \frac{1}{\overline{h}\ per\ L} \tag{8}$$

diminishes while the resistance to conduction along the bracket:

$$R_{cond} = \frac{L}{k\ A_c} \tag{9}$$

does not change.

```
R_cond=L/(k*A_c)                     "conduction resistance from T_H to T_C"
R_conv=1/(h_bar*per*L)               "convection resistance from surface to ambient"
R_cond\R_conv=R_cond/R_conv          "ratio of conduction to convection resistances"
```

At $\overline{h}$ = 2500 W/m$^2$-K, $R_{cond}/R_{conv}$ = 128 and therefore the bracket material very quickly equilibrates with the gas (see Figure 1). At $\overline{h}$ = 2.5 W/m$^2$-K, $R_{cond}/R_{conv}$ = 0.128 and therefore convection is not very important and the bracket material temperature distribution is nearly linear (i.e., the situation is close to conduction through a plane wall).

d.) Plot the heat transfer from the wall at $T_H$ into the bracket (i.e., the heat transfer into the bracket at $x$ = 0) as a function of $\overline{h}$. Explain the shape of your plot.

The heat transfer into the bracket at $x$ = 0 is:

$$\dot{q}_H = -k\ A_c \left( \frac{dT}{dx} \right)_{x=0} \tag{10}$$

Substituting Eq. (2) into Eq. (10) leads to:

$$\dot{q}_H = -k\ A_c \left( m\,C_1 - m\,C_2 \right) \tag{11}$$

```
q_dot_H=-k*(m*C_1-m*C_2)*A_c          "heat transfer rate at x=0"
```

Figure 2 illustrates the rate of heat transfer as a function of $\overline{h}$ and shows that as $\overline{h}$ approaches zero the rate of heat transfer approaches a constant value, consistent with conduction through a plane wall:

$$\dot{q}_{H,\overline{h}\to 0} = \frac{k\ A_c}{L}\left( T_H - T_C \right) \tag{12}$$

As $\bar{h}$ becomes large, the heat transfer is reduced and eventually changes sign as heat is transferred into the wall from the warmer gas.

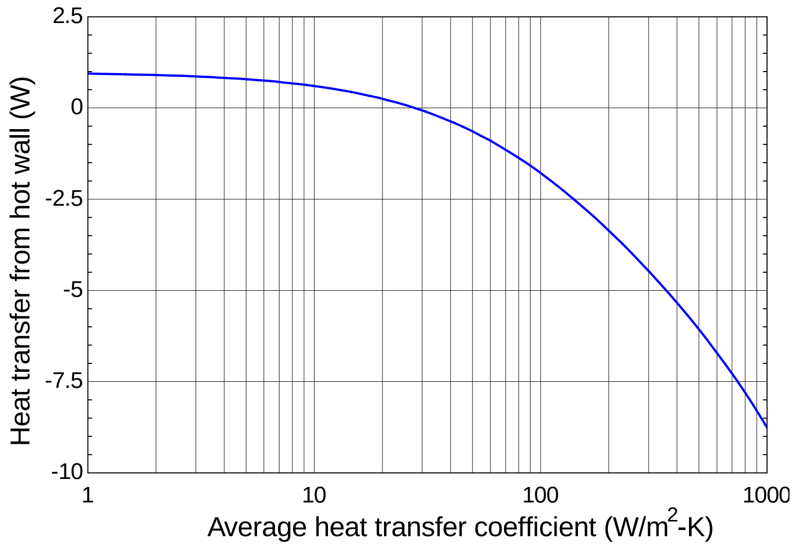


**Figure 2: Heat transfer from hot wall as a function of the heat transfer coefficient.**