Problem 1.2-20
Figure P1.2-20 illustrates a temperature sensor that is mounted in a pipe and used to measure the temperature of a flow of air.

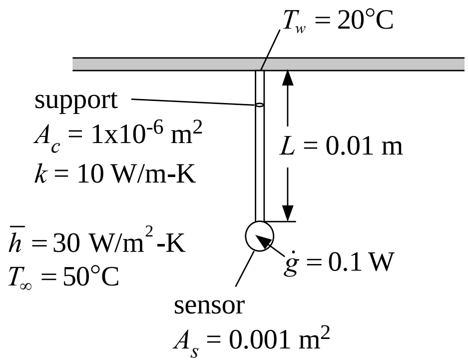

Figure P1.2-20: Temperature sensor.

The operation of the sensor leads to the dissipation of $\dot{g}$ = 0.1 W of electrical power. This power is either convected to the air at T_∞ = 50°C or conducted along the support to the wall at T_w = 20°C. Treat the support as conduction through a plane wall (i.e., neglect convection from the edges of the support). The heat transfer coefficient between the air and the sensor is $\bar{h}$ = 30 W/m^2-K. The surface area of the sensor is A_s = 0.001 m^2. The support has cross-sectional area A_c = 1x10^{-6} m^2, length L = 0.01 m, and conductivity k = 10 W/m-K.

a.) What is the temperature of the temperature sensor?

A resistance network representation of this problem is shown in Figure 2.

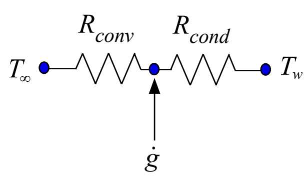

Figure 2: Resistance network.

The resistance to conduction through the support is:

$$R_{cond} = \frac{L}{k\,A_c} = \frac{0.01\text{ m}}{}\left|\frac{\text{K-m}}{10\text{ W}}\right|\frac{}{1\text{x}10^{-6}\text{ m}^2} = 1000\,\frac{\text{K}}{\text{W}} \tag{1}$$

The resistance to convection from the sensor surface is:

$$R_{conv} = \frac{1}{\bar{h}\,A_s} = \frac{\text{m}^2\text{-K}}{30\text{ W}}\left|\frac{}{0.001\,\text{m}^2}\right. = 33.3\,\frac{\text{K}}{\text{W}} \tag{2}$$

An energy balance on the sensor leads to:

$$\dot{g} = \frac{\left(T_s - T_\infty\right)}{R_{conv}} + \frac{\left(T_s - T_w\right)}{R_{cond}} \tag{3}$$

Solving for T_s leads to:

$$T_s = \frac{\dot{g} + \dfrac{T_\infty}{R_{conv}} + \dfrac{T_w}{R_{cond}}}{\dfrac{1}{R_{conv}} + \dfrac{1}{R_{cond}}} = \frac{0.1\,\text{W} + \dfrac{323.2\text{ K}}{33.3\,\text{K/W}} + \dfrac{293.2\text{ K}}{1000\,\text{K/W}}}{\dfrac{1}{33.3\text{ K/W}} + \dfrac{1}{1000\,\text{K/W}}} = 325.4\,\text{K} \tag{4}$$

b.) What is the error associated with the sensor measurement (i.e., what is the difference between the sensor and the air temperature)? Is the error primarily due to self-heating of the sensor associated with $\dot{g}$ or due to the thermal communication between the sensor and with the wall? Justify your answer.

The error is T_s - T_∞ = 2.26 K. The error is dominated by self-heating rather than mounting error. The mounting error would cause the sensor temperature to be less than the fluid temperature whereas the self-heating causes the temperature of the sensor to be elevated relative to the fluid temperature.

c.) Radiation has been neglected for this problem. If the emissivity of the sensor surface is ε = 0.02, then assess whether radiation is truly negligible.

The radiation resistance is:

$$\begin{aligned} R_{rad} &= \frac{1}{A_s\,\varepsilon\,\sigma\left(T_s^2 + T_w^2\right)\left(T_s + T_w\right)} \\ &= \frac{}{0.001\,\text{m}^2}\left|\frac{}{0.02}\right|\frac{\text{m}^2\text{ K}^4}{5.67\text{x}10^{-8}\text{ W}}\left|\frac{}{\left(325.4^2 + 293.2^2\right)\left(325.4 + 293.2\right)\text{K}^3}\right. \\ &= 7432\,\frac{\text{K}}{\text{W}} \end{aligned} \tag{5}$$

The radiation occurs in parallel with convection and conduction and is large relative to either of these resistances, therefore it is probably negligible.