

Problem 1.9-2: Bracket (revisited)

Reconsider the disk-shaped bracket that was discussed in Problem P1.8-7. You have decided to generate a numerical model of the bracket that has three nodes, positioned as shown in Figure P1.9-2.

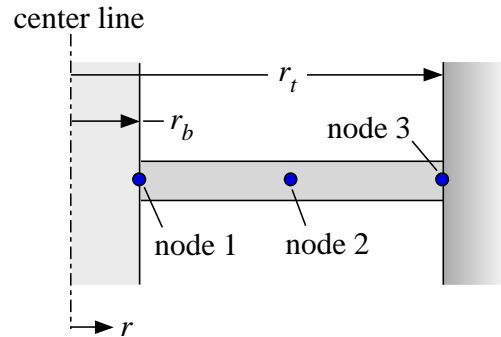


Figure P1.9-2: A 3-node numerical model of the disk-shaped bracket.

- a.) Derive a system of algebraic equations that can be solved in order to predict the temperatures at each of the three nodes in Figure P1.9-2 (T_1 , T_2 , T_3). Your equations should include only those symbols defined in the problem statement as well as the radial locations of the three nodes (r_1 , r_2 , and r_3). Do not solve these equations.

The temperature at node 3 is specified:

$$\boxed{T_3 = T_t} \quad (1)$$

An energy balance on node 2 is shown in Figure P1.9-2(b).

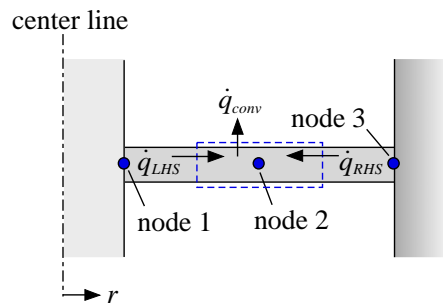


Figure P1.9-2(b): Energy balance on node 2.

and leads to:

$$\dot{q}_{LHS} + \dot{q}_{RHS} = \dot{q}_{conv} \quad (2)$$

or

$$2\pi k th \frac{(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)} + 2\pi k th \frac{(T_3 - T_2)}{\ln\left(\frac{r_3}{r_2}\right)} = 2\pi \bar{h} \left[\left(\frac{r_2 + r_3}{2} \right)^2 - \left(\frac{r_2 + r_1}{2} \right)^2 \right] (T_2 - T_\infty) \quad (3)$$

An energy balance on node 1 is shown in Figure P1.9-2(c).

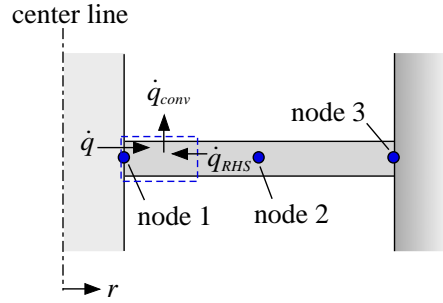


Figure P1.9-2(c): Energy balance on node 1.

and leads to:

$$\dot{q} + \dot{q}_{RHS} = \dot{q}_{conv} \quad (4)$$

or

$$\dot{q} + 2\pi k th \frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} = 2\pi \bar{h} \left[\left(\frac{r_1 + r_2}{2} \right)^2 - r_1^2 \right] (T_1 - T_\infty) \quad (5)$$