Problem 1.2-8 (1-3 in text): Frozen Gutters

You have a problem with your house. Every spring at some point the snow immediately adjacent to your roof melts and runs along the roof line until it reaches the gutter. The water in the gutter is exposed to air at temperature less than 0°C and therefore freezes, blocking the gutter and causing water to run into your attic. The situation is shown in Figure P1.2-8.

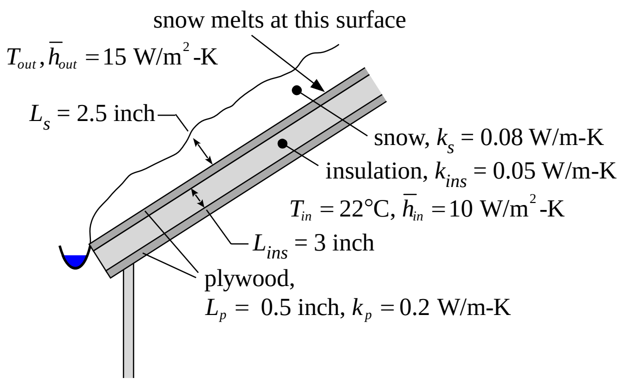

Figure P1.2-8: Roof of your house.

The air in the attic is at T_{in} = 22°C and the heat transfer coefficient between the inside air and the inner surface of the roof is $\overline{h}_{in}$ = 10 W/m^2-K. The roof is composed of a L_{ins} = 3.0 inch thick piece of insulation with conductivity k_{ins} = 0.05 W/m-K that is sandwiched between two L_p = 0.5 inch thick pieces of plywood with conductivity k_p = 0.2 W/m-K. There is an L_s = 2.5 inch thick layer of snow on the roof with conductivity k_s = 0.08 W/m-K. The heat transfer coefficient between the outside air at temperature T_{out} and the surface of the snow is $\overline{h}_{out}$ = 15 W/m^2-K. Neglect radiation and contact resistances for part (a) of this problem.

a.) What is the range of outdoor air temperatures where you should be concerned that your gutters will become blocked by ice?

The input parameters are entered in EES and converted to base SI units (N, m, J, K) in order to eliminate any unit conversion errors; note that units should still be checked as you work the problem but that this is actually a check on the unit consistency of the equations.

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"P1.2-8: Frozen Gutters"
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

T_in=converttemp(C,K,22)                    "temperature in your attic"
L_ins=3 [inch]*convert(inch,m)              "insulation thickness"
L_p=0.5 [inch]*convert(inch,m)              "plywood thickness"
k_ins=0.05 [W/m-K]                          "insulation conductivity"
k_p=0.2 [W/m-K]                             "plywood conductivity"
k_s=0.08 [W/m-K]                            "snow conductivity"
L_s=2.5 [inch]*convert(inch,m)              "snow thickness"
h_in=10 [W/m^2-K]
    "heat transfer coefficient between attic air and inner surface of roof"
h_out=15 [W/m^2-K]
    "heat transfer coefficient between outside air and snow"
A=1 [m^2]                                   "per unit area"
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The problem may be represented by the resistance network shown in Figure 2.

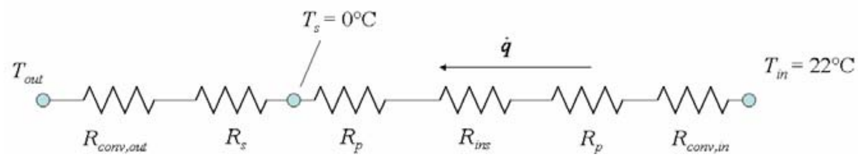

Figure 2: Resistance network representing the roof of your house.

The network includes resistances that correspond to convection with the inside and outside air:

$$R_{conv,out} = \frac{1}{h_{out}\ A} \tag{1}$$

$$R_{conv,in} = \frac{1}{h_{in}\ A} \tag{2}$$

where A is 1 m^2 in order to accomplish the problem on a per unit area basis. There are also conduction resistances associated with the insulation, plywood and snow:

$$R_{ins} = \frac{L_{ins}}{k_{ins}\ A} \tag{3}$$

$$R_p = \frac{L_p}{k_p\ A} \tag{4}$$

$$R_s = \frac{L_s}{k_s\ A} \tag{5}$$

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R_conv_out=1/(h_out*A)                    "outer convection resistance"
R_s=L_s/(k_s*A)                           "snow resistance"
R_p=L_p/(k_p*A)                           "plywood resistance"
R_ins=L_ins/(k_ins*A)                     "insulation resistance"
R_conv_in=1/(h_in*A)                      "inner convection resistance"
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Which leads to $R_{conv,out}$ = 0.07 K/W, R_s = 0.79 K/W, R_p = 0.06 K/W, R_{ins} = 1.52 K/W and $R_{conv,in}$ = 0.10 K/W. Therefore, the dominant effects for this problem are conduction through the insulation and the snow; the other effects (convection and the plywood conduction) are not terribly important since the largest resistances will dominate in a series network.

If the snow at the surface of the room is melting then the temperature at the connection between R_s and R_p must be T_s = 0°C (see Figure 2). Therefore, the heat transferred through the roof ($\dot{q}$ in Figure 2) must be:

$$\dot{q} = \frac{(T_{in} - T_s)}{R_{conv,in} + 2R_p + R_{ins}} \tag{6}$$

The temperature of the outside air must therefore be:

$$T_{out} = T_s - \dot{q}\left(R_s + R_{conv,out}\right) \tag{7}$$

```
T_s=converttemp(C,K,0)
    "roof-to-snow interface temperature must be melting point of water"
q_dot=(T_in-T_s)/(R_conv_in+2*R_p+R_ins)
    "heat transfer from the attic to the snow when melting point is reached"
T_out=T_s-q_dot*(R_s+R_conv_out)
    "outside temperature required to reach melting point at roof surface"
T_out_C=converttemp(K,C,T_out)                              "outside temperature in C"
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which leads to T_{out} = -10.8°C. If the temperature is below this then the roof temperature will be below freezing and the snow will not melt. If the temperature is above 0°C then the water will not refreeze upon hitting the gutter. Therefore, the range of temperatures of concern are -10.8°C < T_{out} < 0°C.

b.) Would your answer change much if you considered radiation from the outside surface of the snow to surroundings at T_{out}? Assume that the emissivity of snow is ε_s = 0.82.

The modified resistance network that includes radiation is shown in Figure 3.

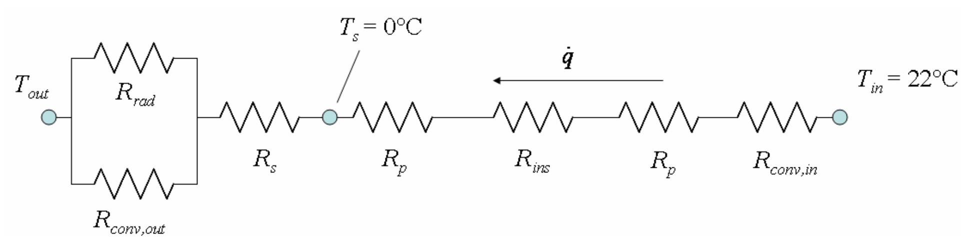

Figure 3: Resistance network representing the roof of your house and including radiation.

The additional resistance for radiation is in parallel with convection from the surface of the snow as heat is transferred from the surface by both mechanisms. The radiation resistance can be calculated approximately according to:

$$R_{rad} = \frac{1}{4\bar{T}^3 \varepsilon_s \sigma A} \tag{8}$$

where $\bar{T}$ is the average temperature of the surroundings and the snow surface. In order to get a quick idea of the magnitude of this resistance we can approximate $\bar{T}$ with its largest possible value (which will result in the largest possible amount of radiation); the maximum temperature of the snow is 0°C:

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e_s=0.82 [-]                                                "emissivity of snow"
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R_rad=1/(4*T_s^3*e_s*sigma#*A)                    "radiation resistance"
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which leads to R_{rad} = 0.26 K/W. Notice that R_{rad} is much larger than $R_{conv,out}$; the smallest resistance in a parallel combination dominates and therefore the impact of radiation will be minimal. Furthermore, $R_{conv,out}$ is not even a very important resistance in the original series circuit shown in Figure 2.