Problem 1.8-3: Cryogenic Thermal Standoff

It is often necessary to provide a fluid outlet port that will allow very cold gas to escape from a cryogenic facility; for example, the boil-off of liquid nitrogen or liquid helium from within a vacuum vessel must be allowed to vent to the atmosphere. The seal between the base of the flange that contains the fluid passage and the surrounding vessel is usually made with an o-ring; these seals are convenient in that they are hermetic and easily demountable. However, most convenient o-ring materials do not retain their ductility at temperatures much below 0°C and therefore it is important that the o-ring be kept at or above this temperature so that it continues to provide a good seal; if the o-ring "freezes" then the cryogenic facility will lose its vacuum. The o-ring temperature is maintained at an appropriate level using a thermal standoff, as shown in Figure P1.8-3

The fluid passage is attached to the flange via a separate, slightly larger tube made of a stainless steel with thermal conductivity k = 15 W/m-K. This thermal standoff has an outer radius, r_{ts} = 1 cm, thickness t_{ts} = 1 mm, and length L_{ts} = 5 cm. The cryogenic gas is at 77 K and you may assume that the point x = 0 in Figure P1.8-3 is at T_{cold} = 77 K. The inside of the tube is exposed to a vacuum and you may assume that it experiences negligible radiation heat transfer and no convective heat transfer. The outside of the tube is exposed to air at T_{air} = 20°C with heat transfer coefficient, $\overline{h}$ = 7 W/m^2-K. The bottom of the tube (x = L_{ts}) is welded to the flange. The flange has an outer radius r_{fl} = 8 cm and a thickness t_{fl} = 1.5 mm. The inside of the flange is exposed to vacuum and therefore, for the purposes of this problem, adiabatic. The outside of the flange is exposed to the same 20°C air with the same 7 W/m^2-K heat transfer coefficient.

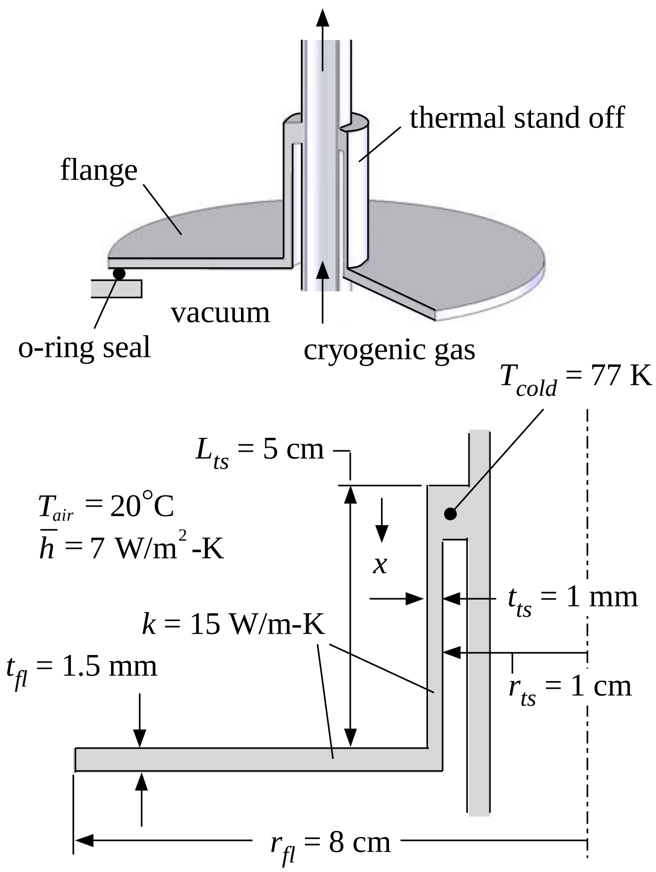

Figure P1.8-3: Cryogenic thermal standoff.

a.) Is it appropriate to treat the thermal standoff and the flange as extended surfaces? That is, can the temperature within the stand-off be treated as being only a function of x and the temperature in the flange only a function of r?

The input parameters are entered in EES:

```
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"
k=15 [W/m-K]                        "thermal conductivity"
r_ts=1.0 [cm]*convert(cm,m)         "outer radius of thermal standoff"
t_ts=1.0 [mm]*convert(mm,m)         "thickness of thermal standoff"
L_ts=5.0 [cm]*convert(cm,m)         "length of thermal standoff"
T_cold=77 [K]                       "cryogenic gas temperature"
T_air=converttemp(C,K,20 [C])       "air temperature"
h=7 [W/m^2-K]                       "heat transfer coefficient"
r_fl=8.0 [cm]*convert(cm,m)         "radius of flange"
t_fl=1.5 [mm]*convert(mm,m)         "thickness of flange"
```

The Biot numbers that must be calculated for the standoff and the flange (Bi_{ts} and Bi_{fl}, respectively) are:

$$Bi_{ts} = \frac{h\,t_{ts}}{k}$$

and

$$Bi_{fl} = \frac{h\,t_{fl}}{k}$$

These are computed in EES:

```
Bi_ts=t_ts*h/k       "Biot number for thermal standoff"
Bi_fl=t_fl*h/k       "Biot number for flange"
```

and found to be very small.

b.) Develop an analytical model of the problem that can predict the temperature distribution as a function of *x* in the thermal stand-off and as a function of *r* in the flange.

An energy balance on the differential control volume within the thermal standoff leads to:

$$\dot{q}_x = \dot{q}_{x+dx} + \dot{q}_{conv}$$

or, after making the usual simplifications:

$$0 = -\frac{d}{dx}\left[2\,\pi\,r_{ts}\,t_{ts}\,k\,\frac{dT_{ts}}{dx}\right]dx + 2\,\pi\,r_{ts}\,dx\,h\left(T_{ts} - T_{air}\right)$$

which becomes the governing differential equation for a constant cross-sectional area fin that was solved in Section 1.6:

$$\frac{d^2\theta_{ts}}{dx^2} - m_{ts}^2\,\theta_{ts} = 0 \qquad (1)$$

where

$$m_{ts} = \sqrt{\frac{h}{t_{ts}\,k}}$$

and

$$\theta_{ts} = T_{ts} - T_{air}$$

Equation (1) is solved by exponentials (or equivalently by sinh and cosh):

$$\theta_{ts} = C_1 \exp\left(m_{ts}\, x\right) + C_2 \exp\left(-m_{ts}\, x\right) \tag{2}$$

where C_1 and C_2 are determined by the boundary coefficients.

An energy balance on the differential control volume within the flange (see Figure 1) is:

$$\dot{q}_r = \dot{q}_{r+dr} + \dot{q}_{conv}$$

or, after making the usual simplifications:

$$0 = -\frac{d}{dr}\left[2\,\pi\, r\, t_{fl}\, k\, \frac{dT_{fl}}{dr}\right] dr + 2\,\pi\, r\, dr\left(T_{fl} - T_{air}\right)$$

which becomes Bessel's equation:

$$\frac{d}{dr}\left[r\,\frac{d\theta_{fl}}{dr}\right] - m_{fl}^2\, r\, \theta_{fl} = 0 \tag{3}$$

where

$$m_{fl} = \sqrt{\frac{h}{t_{fl}\, k}}$$

and

$$\theta_{fl} = T_{fl} - T_{air}$$

Equation (3) is solved by 0^{th} order modified Bessel functions:

$$\theta_{fl} = C_3\, \mathrm{BesselI}\left(0, m_{fl}\, r\right) + C_4\, \mathrm{BesselK}\left(0, m_{fl}\, r\right) \tag{4}$$

where C_3 and C_4 are determined by the boundary coefficients.

There must be four boundary conditions; two for each of the 2^{nd} order differential equations. The temperature at the top of the thermal stand-off is specified:

$$T_{ts,x=0} = T_{cold}$$

or, substituting into Eq. (2):

$$C_1 + C_2 = T_{cold} - T_{air}$$

The temperature at the bottom of the thermal standoff must be equal to the temperature at the inner edge of the flange:

$$T_{ts,x=L_{ts}} = T_{fl,r=r_{ts}}$$

or

$$C_1 \exp\left(m_{ts}\, L_{ts}\right) + C_2 \exp\left(-m_{ts}\, L_{ts}\right) = C_3 \,\mathrm{BesselI}\left(0, m_{fl}\, r_{ts}\right) + C_4 \,\mathrm{BesselK}\left(0, m_{fl}\, r_{ts}\right)$$

The rate of heat transfer out of the bottom of the thermal standoff must be equal to the rate that heat that is transferred into the inner edge of the flange:

$$-k\, 2\,\pi\, r_{ts}\, t_{ts} \left.\frac{dT_{ts}}{dx}\right|_{L_{ts}} = -k\, 2\,\pi\, r_{ts}\, t_{fl} \left.\frac{dT_{fl}}{dr}\right|_{r_{ts}}$$

or

$$t_{ts} \left.\frac{d\theta_{ts}}{dx}\right|_{L_{ts}} = t_{fl} \left.\frac{d\theta_{fl}}{dr}\right|_{r_{ts}}$$

whic leads to:

$$t_{ts}\left[C_1\, m_{ts} \exp\left(m_{ts}\, L_{ts}\right) - C_2\, m_{ts} \exp\left(-m_{ts}\, L_{ts}\right)\right] = t_{fl}\left[C_3\, m_{fl}\,\mathrm{BesselI}\left(1, m_{fl}\, r_{ts}\right) - C_4\, m_{fl}\,\mathrm{BesselK}\left(1, m_{fl}\, r_{ts}\right)\right]$$

Finally, the outer edge of the flange is assumed to be adiabatic (the small amount of convection from the edge can be neglected with little error):

$$-k\, 2\,\pi\, r_{fl}\, t_{fl} \left.\frac{dT_{fl}}{dr}\right|_{r_{fl}} = 0$$

or

$$\left.\frac{d\theta_{fl}}{dr}\right|_{r_{fl}} = 0$$

which leads to:

$$C_3\, m_{fl}\,\mathrm{BesselI}\left(1, m_{fl}\, r_{fl}\right) - C_4\, m_{fl}\,\mathrm{BesselK}\left(1, m_{fl}\, r_{fl}\right) = 0$$

The boundary condition equations are entered into EES:

```
"Solution parameters"
m_ts=sqrt(h/(t_ts*k))
m_fl=sqrt(h/(t_fl*k))

"Boundary conditions"
C_1+C_2=T_cold-T_air
C_1*exp(m_ts*L_ts)+C_2*exp(-m_ts*L_ts)=C_3*BesselI(0,m_fl*r_ts)+C_4*BesselK(0,m_fl*r_ts)
t_ts*(C_1*m_ts*exp(m_ts*L_ts)-C_2*m_ts*exp(-m_ts*L_ts))=t_fl*(C_3*m_fl*BesselI(1,m_fl*r_ts)-
C_4*m_fl*BesselK(1,m_fl*r_ts))
C_3*m_fl*BesselI(1,m_fl*r_fl)-C_4*m_fl*BesselK(1,m_fl*r_fl)=0
```

The solution is provided in EES using two parametric tables for the thermal standoff and flange temperature distributions. The first table is titled 'Thermal Stand off' and includes values of dimensionless position, $\bar{x}$, ranging from 0 to 1; where:

$$\bar{x} = \frac{x}{L_{ts}}$$

The coordinate, s, is also computed; s goes from 0 to L_{ts} + (r_{fl} – r_{ts}) as you move from the top of the thermal stand off to the edge of the flange. The use of s provides a convenient method for looking at the entire temperature distribution in the thermal standoff and flange.

$$s = x$$

The 'Thermal Stand off' table includes columns that contain the values of θ_{ts} and T_{ts}. The solution for the temperature within the thermal stand off is only calculated if you are running the parametric table entitled 'Thermal Stand off'. The selection of equations that are to be solved with each parametric table is facilitated by using the $IF PARAMETRICTABLE directive to check which table is being calculated. The equations located between the $IF and $ENDIF in the EES code below are executed only if the 'Thermal Stand off' parametric table is being calculated.

```
$IF PARAMETRICTABLE='Thermal Stand off'
        x_bar=x/L_ts
        s=x
        theta_ts=C_1*exp(m_ts*x)+C_2*exp(-m_ts*x)
        Temp_ts=theta_ts+T_air
        Temp_ts_C=converttemp(K,C,Temp_ts)
$ENDIF
```

A second parametric table entitled 'Flange' is includes values of dimensionless position in the flange, $\bar{r}$:

$$r = \frac{\left(r - r_{ts}\right)}{\left(r_{fl} - r_{ts}\right)}$$

and the coordinate, s, which is computed in the flange as:

$$s = r - r_{ts} + L$$

The table 'Flange' also includes the values θ_{fl} and T_{fl}. The required commands are included in a separate $IF statement:

```
$IF PARAMETRICTABLE='Flange'
	r_bar=(r-r_ts)/(r_fl-r_ts)
	s=L_ts+r-r_ts
	theta_fl=C_3*BesselI(0,m_fl*r)+C_4*BesselK(0,m_fl*r)
	Temp_fl=theta_fl+T_air
	Temp_fl_C=converttemp(K,C,Temp_fl)
$ENDIF
```

By sequentially running parametric table 'Thermal Stand off' and 'Flange' it is possible to determine the entire temperature distribution; the result is shown in Figure 2.

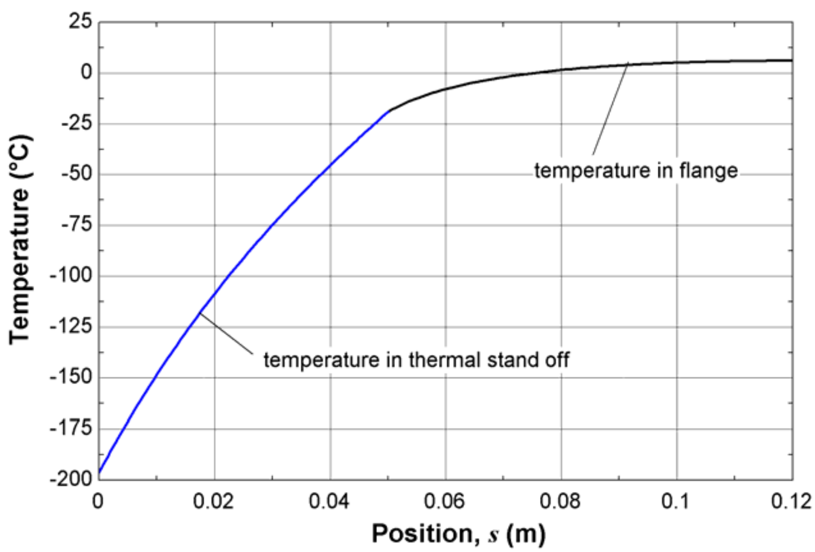

Figure 2: Temperature as a function of the coordinate *s*.

Notice that the solution satisfies each of the boundary conditions. The temperature at the top of the stand off is equal to 77 K and the temperature gradient at the edge of the flange is zero. The temperature at the intersection of the flange and the thermal stand off is continuous but there is a discontinuity in the temperature gradient related to the fact that the flange is slightly thicker (and therefore has a lower temperature gradient for the same conduction heat transfer) than the thermal stand off.

The value of the thermal stand off is clear. If the o-ring seal is placed towards the outer radius of the flange then Figure 2 shows that the temperature will remain above freezing and therefore the o-ring will continue to function. Using the EES model it is possible to evaluate alternative, more effective designs (i.e., thermal stand off geometries that keep the temperature at the outer edge of the flange higher). Figure 3 illustrates the temperature at the edge of the flange (i.e., $T_{fl,r=r_{fl}}$) as a function of the thermal stand-off thickness for various values of its length.

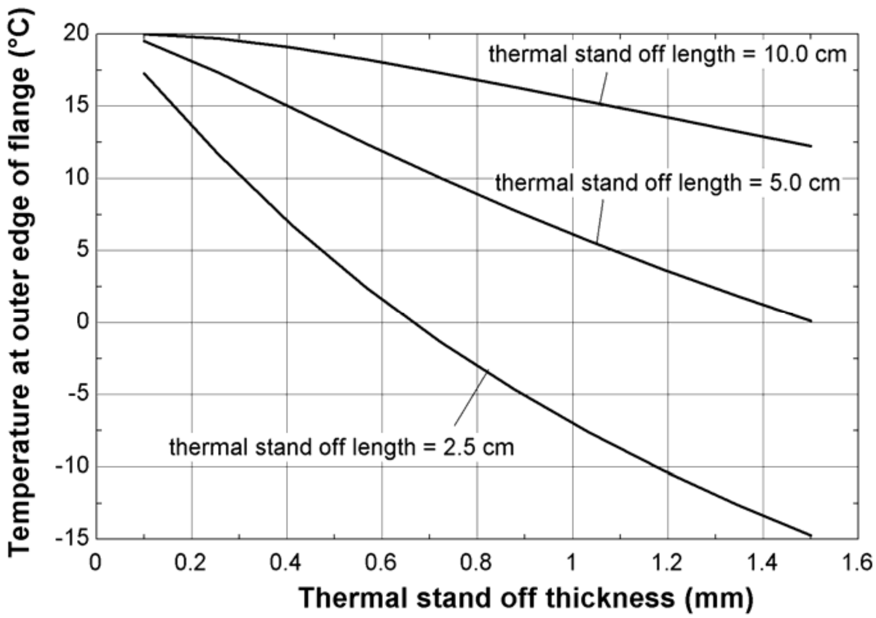

Figure 3: Temperature at the edge of the flange as a function of the thermal stand off thickness for various values of the thermal stand off length.

Note that either increasing the thermal stand off length or decreasing its thickness will tend to make it a less efficient fin and therefore increase the temperature gradient due to conduction. It turns out that a good thermal stand off is a bad fin, isolating the tip of the fin (i.e., the flange) from the base of the fin (i.e., the cryogenic temperature).