PROBLEM 1.3-6: Heating a Pipe

You need to transport water through a pipe from one building to another in an arctic environment, as shown in Figure P1.3-6. The water leaves the building very close to freezing, at T_w = 5°C, and is exposed to a high velocity, very cold wind. The temperature of the surrounding air is T_a = -35°C and the heat transfer coefficient between the outer surface of the pipe and the air is $\overline{h}_a$ = 50 W/m^2-K. The pipe has an inner radius of $r_{h,in}$ = 2 inch and an outer radius of $r_{h,out}$ = 4 inch and is made of a material with a conductivity k_h = 5 W/m-K. The heat transfer coefficient between the water and the inside surface of the pipe is very large and therefore the inside surface of the pipe can be assumed to be at the water temperature. Neglect radiation from the external surface of the pipe.

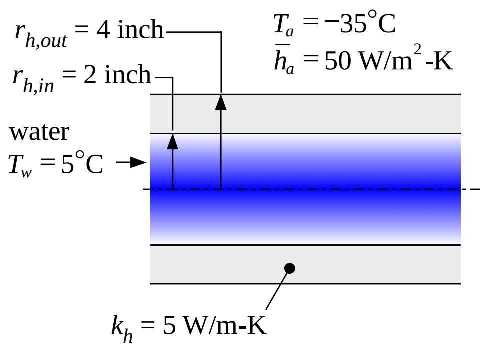


Figure P1.3-6: Heated pipe transporting near freezing water in an arctic environment.

a.) Determine the rate of heat lost from the water to the air for a unit length, L=1 m, of pipe.

The known information is converted to base SI units and entered in EES. The units for each variable are entered in the Variable Information window.

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
r_h_in=2 [inch]*convert(inch,m)                    "inner radius of plastic pipe"
r_h_out=4.0 [inch]*convert(inch,m)                 "outer radius of heater"
T_a=converttemp(C,K,-35)                           "air temperature"
T_w=converttemp(C,K,5)                             "water temperature"
h_a=50 [W/m^2-K]                                   "air to heater heat transfer coefficient"
k_h=5 [W/m-K]                                      "heater conductivity"
L=1 [m]                                            "unit length of pipe"
```

The resistance network that represents the situation includes a conduction resistance through the pipe (R_{cond}) and a convection resistance from the outer surface of the pipe (R_{conv}).

$$R_{cond} = \frac{\ln\left(\frac{r_{h,out}}{r_{h,in}}\right)}{2\,\pi\,k_h\,L} \tag{1}$$

and

$$R_{conv} = \frac{1}{h_a \, 2 \, \pi \, r_{h,out} \, L} \tag{2}$$

and the total heat loss is:

$$\dot{q} = \frac{\left(T_w - T_a\right)}{R_{cond} + R_{conv}} \tag{3}$$

These equations are entered in EES:

```
"Part a"
R_cond=ln(r_h_out/r_h_in)/(2*pi*k_h*L)          "conduction through the pipe"
R_conv=1/(h_a*2*pi*r_h_out*L)                   "convection from outer surface"
q_dot=(T_w-T_a)/(R_cond+R_conv)                 "heat loss"
```

The heat loss is 750 W per m of pipe.

b.) Plot the heat lost from the water as a function of the pipe outer radius from 0.06 m to 0.3 m (keep the same inner radius for this study). Explain the shape of your plot.

The value of r_h_out in the input section is commented (highlight the variable and select comment). A parametric table is created that includes the variables r_h_out and q_dot; the value of r_h_out in the table is varied from 0.01 m to 0.3 m (right click on the r_h_out column header and select Alter Values). The results in the parametric table are plotted in Figure 2.

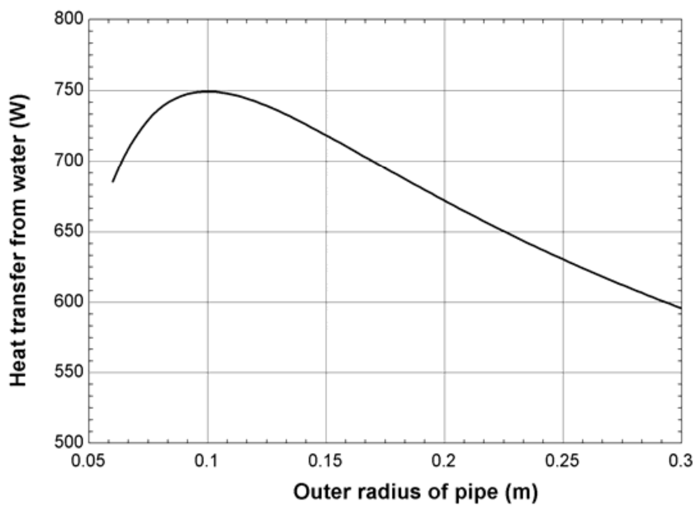


Figure 2: Heat transfer from the water as a function of the outer radius of the pipe.

The result can be understood by plotting the resistances R_conv, R_cond, and R_conv+R_cond as a function of the outer radius of the pipe (as shown in Figure 3). Not that you'll need to define a new variable, R_total:

```
R_total=R_cond+R_conv            "total resistance"
```

in order to make the plot.

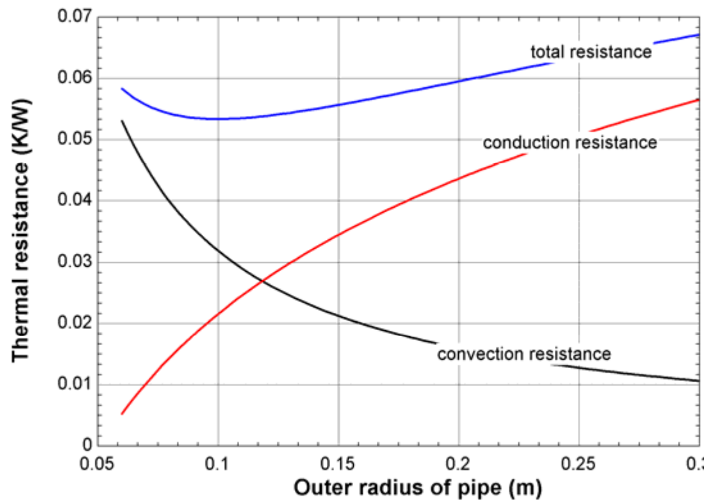


Figure 3: Thermal resistance as a function of the pipe outer radius.

Initially, the convection resistance drops more rapidly than the conduction resistance increases as the radius increases; therefore, initially the total resistance is reduced with radius and the heat loss increases. Eventually, the conduction resistance dominates the convection resistance and therefore the total resistance rises with outer radius and therefore the heat loss is reduced.

In order to reduce the heat loss from the water and therefore prevent freezing, you run current through the pipe material so that it generates thermal energy at with a uniform volumetric rate, $\dot{g}''' = 5\text{x}10^5\ \text{W/m}^3$.

c.) Develop an analytical model capable of predicting the temperature distribution within the pipe. Implement your model in EES.

The general solution for the temperature distribution in a cylindrical system with a constant rate of generation is:

$$T = -\frac{\dot{g}''' r^2}{4k} + C_1 \ln(r) + C_2 \tag{4}$$

where C_1 and C_2 are constants of integration that must come from the boundary conditions. The boundary conditions are a set temperature at $r_{h,in}$:

$$T_w = -\frac{\dot{g}''' r_{h,in}^2}{4k_h} + C_1 \ln(r_{h,in}) + C_2 \tag{5}$$

and an interface energy balance at $r_{h,out}$:

$$-k_h \left. \frac{dT}{dr} \right|_{r=r_{h,out}} = h_a \left(T_{r=r_{h,out}} - T_a \right) \tag{6}$$

The derivative of temperature with respect to radius can be found by manipulating Eq. (4) or in Table 3-1:

$$\frac{dT}{dr} = -\frac{\dot{g}''' \, r}{2\,k} + \frac{C_1}{r} \tag{7}$$

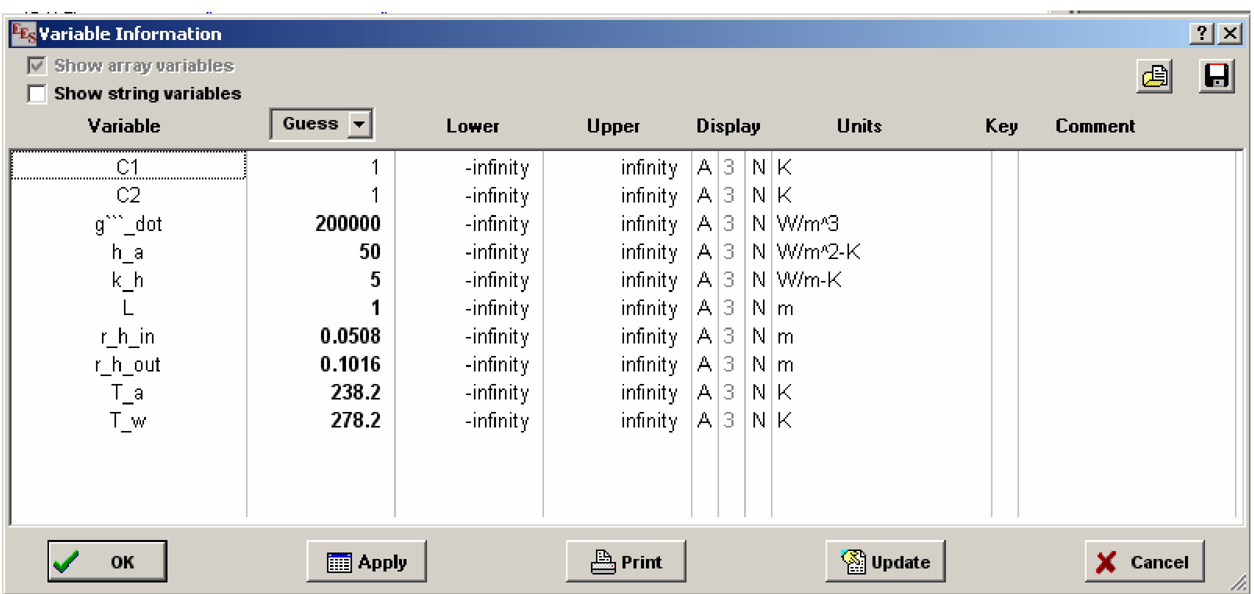
Substituting Eq. (7) and Eq. (4) into Eq. (6) leads to:

$$-k_h \left[-\frac{\dot{g}''' \, r_{h,out}}{2\,k_h} + \frac{C_1}{r_{h,out}} \right] = h_a \left(-\frac{\dot{g}''' \, r_{h,out}^2}{4\,k_h} + C_1 \ln\left(r_{h,out}\right) + C_2 - T_a \right) \tag{8}$$

Equations (5) and (8) are programmed in EES to obtain the constants C_1 and C_2.

```
g```_dot=5e5 [W/m^3]                                          "volumetric generation rate"
"boundary conditions"
T_w=-g```_dot*r_h_in^2/(4*k_h)+C1*ln(r_h_in)+C2
-k_h*(-g```_dot*r_h_out/(2*k_h)+C1/r_h_out)=h_a*(-g```_dot*r_h_out^2/(4*k_h)+C1*ln(r_h_out)+C2-T_a)
```

Note that the units of the constant should be set to K (Figure 4):



Variable	Guess	Lower	Upper	Display			Units	Key	Comment
C1	1	-infinity	infinity	A	3	N	K		
C2	1	-infinity	infinity	A	3	N	K		
g```_dot	200000	-infinity	infinity	A	3	N	W/m^3		
h_a	50	-infinity	infinity	A	3	N	W/m^2-K		
k_h	5	-infinity	infinity	A	3	N	W/m-K		
L	1	-infinity	infinity	A	3	N	m		
r_h_in	0.0508	-infinity	infinity	A	3	N	m		
r_h_out	0.1016	-infinity	infinity	A	3	N	m		
T_a	238.2	-infinity	infinity	A	3	N	K		
T_w	278.2	-infinity	infinity	A	3	N	K		

Figure 4: Variable Information window.

However, if you check units now you will obtain a unit error (Figure 5).

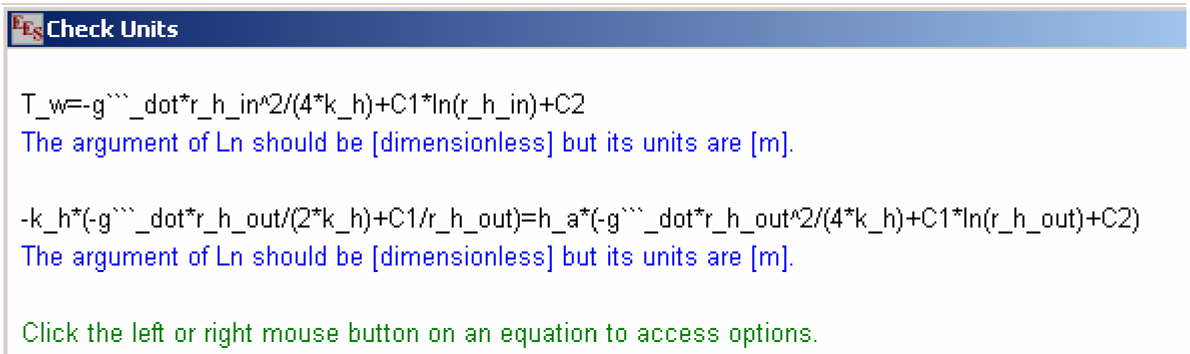


Figure 5: Unit error.

because the argument of the natural logarithm has units m. This cannot be helped; if the algebra associated with explicitly solving for each constant was followed through and these constants were substituted back into Eq. (4) then you would find that the arguments of the natural logarithm can be expressed as the ratio of two radii.

The solution, Eq. (4), is programmed in EES:

```
T=-g```_dot*r^2/(4*k_h)+C1*ln(r)+C2      "solution"
```

d.) Prepare a plot showing the temperature as a function of position within the pipe.

The radius is expressed in terms of a non-dimensional radius (r_bar) to facilitate making the parametric table (it is easier to vary r_bar from 0 to 1 than it is to vary r from r_h_in to r_h_out, particularly if you plan on varying r_h_in or r_h_out).

```
r_bar=(r-r_h_in)/(r_h_out-r_h_in)          "dimensionless radius used to make plots"
{r_bar=0}
T_C=converttemp(K,C,T)
```

The temperature distribution is shown in Figure 6.

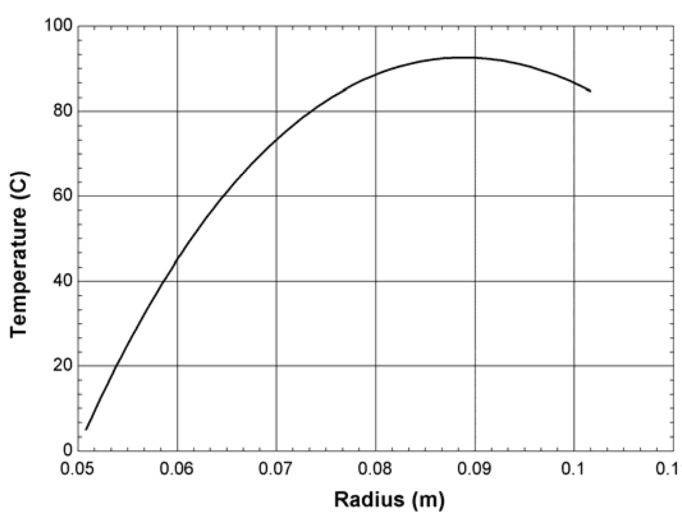


Figure 6: Temperature in pipe as a function of radius.

e.) Calculate the heat transfer from the water when you are heating the pipe.

The heat transfer from the water is obtained by applying Fourier's law at $r = r_{h,in}$:

$$\dot{q}_w = -k_h \, 2\,\pi \, r_{h,in} \, L \left.\frac{dT}{dr}\right|_{r=r_{h,in}} \tag{9}$$

Substituting Eq. (7) into Eq. (9):

$$\dot{q}_w = -k_h \, 2\,\pi \, r_{h,in} \, L \left[-\frac{\dot{g}''' \, r_{h,in}}{2\,k} + \frac{C_1}{r_{h,in}} \right] \tag{10}$$

```
q_dot_w=-k_h*(-g```_dot*r_h_in/(2*k_h)+C1/r_h_in)*2*pi*r_h_in*L          "heat transfer from the water"
```

The heat transfer rate from the water is -8336 W per m of pipe; that is, heat is transferred to the water, which is evident from the temperature distribution. While you don't want the water to freeze, you probably also don't want to heat it and therefore $\dot{g}'''$ =5e5 W/m^3 is probably too high.

f.) Determine the volumetric generation rate that is required so that there is no heat transferred from the water.

EES can provide this solution. Simply comment out the generation rate that you set and then specify that the heat transfer rate from the water is 0.

```
{g```_dot=5e5 [W/m^3]}                                    "volumetric generation rate"
"boundary conditions"
T_w=-g```_dot*r_h_in^2/(4*k_h)+C1*ln(r_h_in)+C2
-k_h*(-g```_dot*r_h_out/(2*k_h)+C1/r_h_out)=h_a*(-g```_dot*r_h_out^2/(4*k_h)+C1*ln(r_h_out)+C2)

T=-g```_dot*r^2/(4*k_h)+C1*ln(r)+C2                       "solution"
r_bar=(r-r_h_in)/(r_h_out-r_h_in)                         "dimensionless radius used to make plots"
r_bar=0
T_C=converttemp(K,C,T)

q_dot_w=-k_h*(-g```_dot*r_h_in/(2*k_h)+C1/r_h_in)*2*pi*r_h_in*L "heat transfer from the water"
q_dot_w=0
```

The volumetric generation rate that results in no heat transfer to the water is 4.12x10^4 W/m^3.