P1.1-2 (1-1 in text): Conductivity of a dilute gas

Section 1.1.2 provides an approximation for the thermal conductivity of a monatomic gas at ideal gas conditions. Test the validity of this approximation by comparing the conductivity estimated using Eq. (1-18) to the value of thermal conductivity for a monotonic ideal gas (e.g., low pressure argon) provided by the internal function in EES. Note that the molecular radius, σ, is provided in EES by the Lennard-Jones potential using the function sigma_LJ.

a.) What is the value and units of the proportionality constant required to make Eq. (1-18) an equality?

Equation (1-18) is repeated below:

$$k \propto \frac{c_v}{\sigma^2}\sqrt{\frac{T}{MW}} \tag{1}$$

Equation (1) is written as an equality by including a constant of proportionality (C_k):

$$k = C_k \frac{c_v}{\sigma^2}\sqrt{\frac{T}{MW}} \tag{2}$$

Solving for C_k leads to:

$$C_k = \frac{k\,\sigma^2}{c_v}\sqrt{\frac{MW}{T}} \tag{3}$$

which indicates that C_k has units m-kg$^{1.5}$/s-kgmol^{05}-K$^{0.5}$.

The inputs are entered in EES for Argon at relatively low pressure (0.1 MPa) and 300 K.

```
"Problem 1.1-2"
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

T=300 [K]                                   "temperature"
F$='Argon'                                  "fluid"
P_MPa=0.1 [MPa]                             "pressure, in MPa"
P=P_MPa*convert(MPa, Pa)                    "pressure"
```

The conductivity, specific heat capacity, Lennard-Jones potential, and molecular weight of Argon (k, c_v, σ, and MW) are evaluated using EES' built-in funcions. Equation (3) is used to evaluate the proportionality constant.

```
k=conductivity(F$,T=T,P=P)              "conductivity"
cv=cv(F$,T=T,P=P)                       "specific heat capacity at constant volume"
MW=molarMass(F$)                        "molecular weight"
sigma=sigma_LJ(F$)                      "Lennard-Jones potential"
C_k=k*sigma^2*sqrt(MW/T)/cv             "constant of proportionality"
```

which leads to C_k = 2.619x10^{-24} m-kg$^{1.5}$/s-kgmol$^{0.5}$-K$^{0.5}$.

b.) Plot the value of the proportionality constant for 300 K argon at pressures between 0.01 and 100 MPa on a semi-log plot with pressure on the log scale. At what pressure does the approximation given in Eq. (1-18) begin to fail at 300 K for argon?

Figure 1 illustrates the constant of proportionality as a function of pressure for argon at 300 K. The approximation provided by Eq. (1-18) breaks down at approximately 1 MPa.

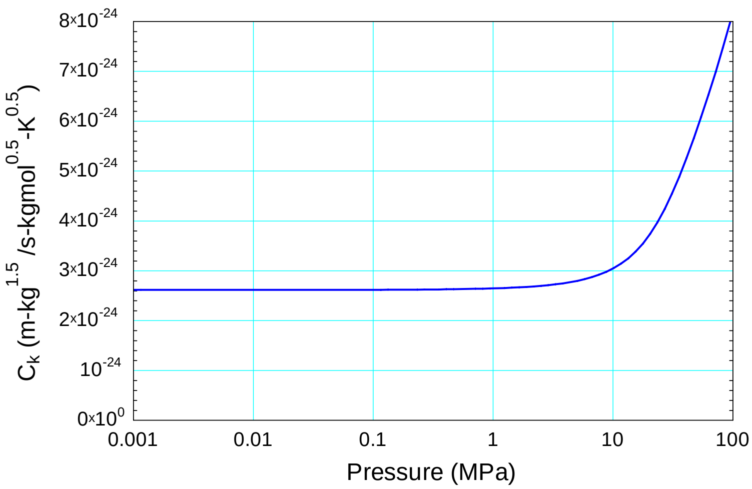

Figure 1: Constant of proportionality in Eq. (3) as a function of pressure for argon at 300 K.