PROBLEM 1.2-1: Composite Wall

A plane wall is a composite of a low conductivity material (with thickness L_1 and conductivity k_1) and a high conductivity material (with thickness $L_2 = L_1$ and conductivity k_2). The edge of the wall at $x = 0$ is at temperature T_1 and the edge at $x = L_1 + L_2$ has temperature T_2, as shown in Figure P1.2-1(a). T_1 is greater than T_2. The wall is at steady-state and the temperature distribution in the wall is one-dimensional in x.

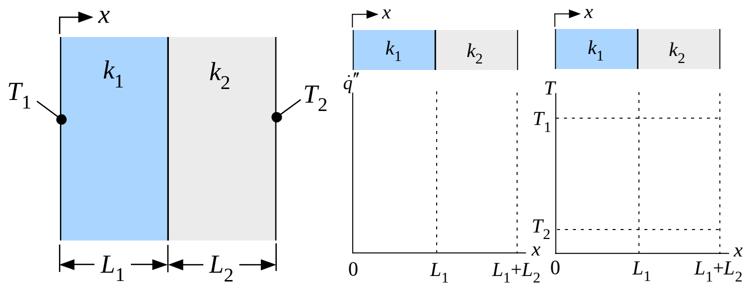

Figure P1.2-1: (a) Composite wall with $k_1 < k_2$, and (b) sketch of heat flux and temperature.

a.) Sketch the heat flux ($\dot{q}''$) and temperature (T) as a function of position within the wall on the axes in Fig. 1.2-1(b). Make sure that your sketch reflects the fact that (1) the wall is at steady state, and (2) $k_1 < k_2$.

If the process is at steady state, then I can draw a control volume that extends from one surface to any location x in the material, as shown in Figure 2.

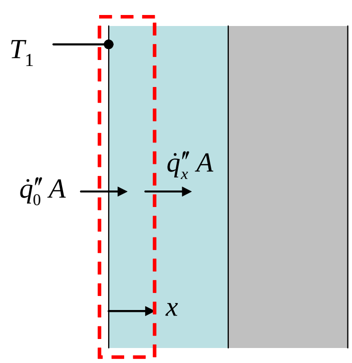

Figure 2: Control volume for solution

An energy balance on the control volume leads to:

$$\dot{q}_0'' A = \dot{q}_x'' A \tag{1}$$

Equation (1) shows that the heat fux at any location x must be constant. The heat flux associated with conduction is governed by Fourier's law:

$$\dot{q}_x'' = -k \frac{dT}{dx} \tag{2}$$

Solving Eq. (2) for the temperature gradient leads to:

$$\frac{dT}{dx} = -\frac{\dot{q}''_x}{k} \tag{3}$$

The numerator of Eq. (3), the heat flux, is constant while the denominator changes depending on whether you are in material 1 or material 2. In the low conductivity material 1, the temperature gradient will be higher than in the high conductivity material 2. Within each material, the temperature gradient must be constant (i.e., the temperature must be linear with *x*). The solution is shown in Figure 3.

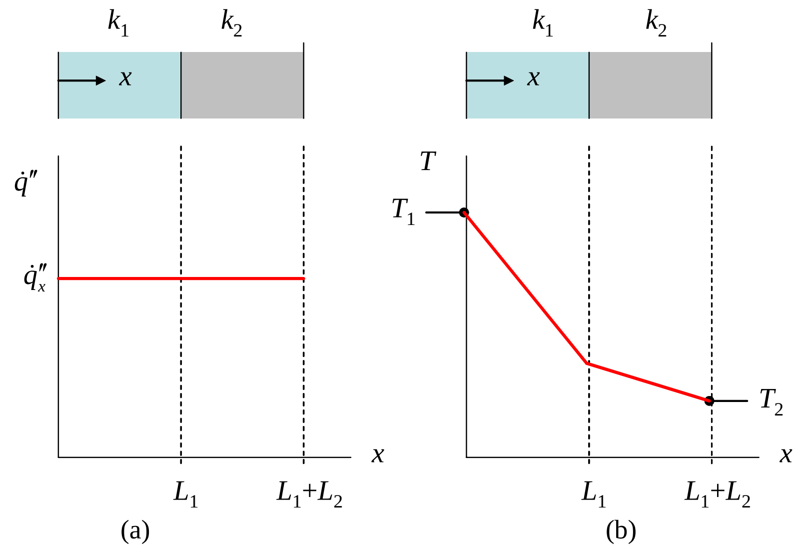

Figure 3: (a) Heat transfer rate and **(b)** temperature as a function of position within wall.