Problem 1.3-1: Composite Wall with Generation

A plane wall is composed of two materials, A and B, with the same thickness, L, as shown in Figure P1.3-1. The same, spatially uniform, volumetric rate of generation is present in both materials ($\dot{g}''' = \dot{g}'''_A = \dot{g}'''_B$) and the wall is at steady state. The conductivity of material B is twice that of material A ($k_B = 2\,k_A$). The left side of the wall is adiabatic and the right side is maintained at a temperature $T_{x=2L} = T_o$.

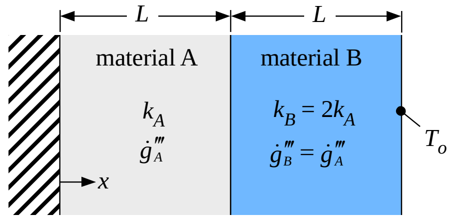

Figure P1.3-1: Plane wall composed of materials A and B.

a.) Sketch the rate of heat transfer as a function of position within the wall ranging from $x = 0$ (the left face of material A) to $x = 2L$ (the right face of material B). Note that the sketch should be qualitatively correct, but cannot be quantitative as you have not been given any numbers for the problem.

The heat transfer rate must increase linearly from zero at x=0; to see this, consider the energy balance on the control volume shown in Figure 2.

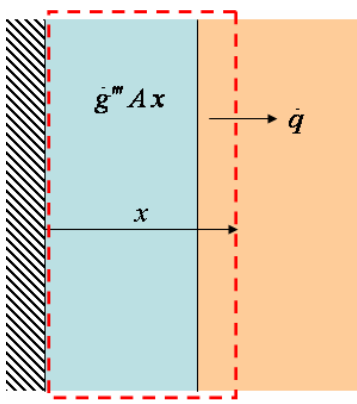

Figure 2: Control volume.

The heat transfer rate at any position x must be:

$$\dot{q} = \dot{g}''' A\,x \tag{1}$$

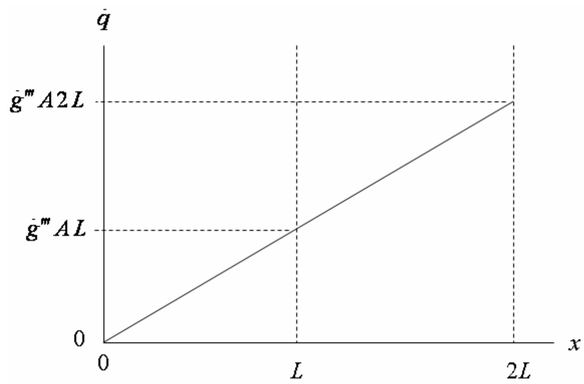

Figure 3: Rate of heat transfer as a function of position in the wall

b.) Sketch the temperature as a function of position within the wall. Again, be sure that your sketch has the correct qualitative features.

The temperature gradient is, according to Fourier's law:

$$\frac{dT}{dx} = -\frac{\dot{q}}{k} \tag{2}$$

Therefore, the temperature gradient will become increasingly negative as you move towards positive x; however, there will be a step change in the temperature gradient at the interface between materials A and B (i.e., at x=L). The conductivity of material B is twice that of A and so the temperature gradient at x=L^+ will be half that of x=L^-.

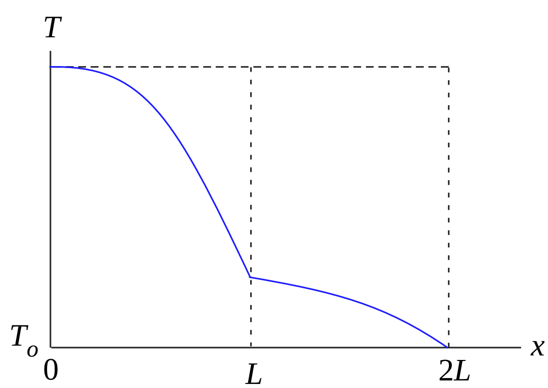

Figure 4: Temperature as a function of position in the wall