Problem 1.2-16

The temperature distribution across a L = 0.3 m thick wall at a certain instant of time is given by $T = 900 - 900\,x - 500\,x^2$ where T is the temperature in the wall in °C at position x in m. The density, specific heat capacity and thermal conductivity of the wall are ρ = 2050 kg/m3, c = 0.96 kJ/kg-K and k = 1.13 W/m-K, respectively.

a) Calculate the rate of change of the average wall temperature.

The inputs are entered in EES:

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"P1.2-16"
$UnitSystem SI MASS  RAD PA K J
$Tabstops 0.2 0.4 0.6 0.8 3.5

"known"
k=1.13 [W/m-K]                                          "conductivity"
rho=2050 [kg/m^3]                                       "density"
c=0.96 [kJ/kg-K]*convert(kJ/kg-K,J/kg-K)                "specific heat capacity"
L=0.3 [m]                                               "thickness of the wall"
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A control volume that includes the wall is shown in Figure 1.

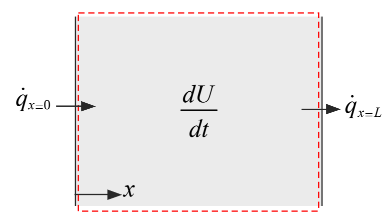

Figure 1: Energy balance on the wall.

The energy balance on the control volume is:

$$\dot{q}_{x=0} = \frac{dU}{dt} + \dot{q}_{x=L} \tag{1}$$

Fourier's law is used to compute the conduction heat transfer at each edge:

$$\dot{q}_{x=0} = -k\,A \left.\frac{dT}{dx}\right|_{x=0} \tag{2}$$

$$\dot{q}_{x=L} = -k\,A \left.\frac{dT}{dx}\right|_{x=L} \tag{3}$$

The energy storage is given by:

$$\frac{dU}{dt} = A\,L\,\rho\,c\,\frac{d\overline{T}}{dt} \tag{4}$$

where $\overline{T}$ is the average temperature in the wall. Substituting Eqs. (2), (3), and (4) into Eq. (1) leads to:

$$-k\,A\left.\frac{dT}{dx}\right|_{x=0} = A\,L\,\rho\,c\,\frac{d\overline{T}}{dt} - k\,A\left.\frac{dT}{dx}\right|_{x=L} \tag{5}$$

Solving for the rate of change of the average temperature leads to:

$$\frac{d\overline{T}}{dt} = \frac{k}{L\,\rho\,c}\left(\left.\frac{dT}{dx}\right|_{x=L} - \left.\frac{dT}{dx}\right|_{x=0}\right) \tag{6}$$

where the temperature gradients can be obtained by taking the derivative of the temperature distribution given in the problem statement:

```
dTdx0=-900 [C/m]                              "temperature gradient at x=0"
dTdxL=-900 [C/m] - 2*500 [C/m^2]*L            "temperature gradient at x=L"
dTbardt=k*(dTdxL-dTdx0)/(L*rho*c)             "rate of change of the average temperature"
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which leads to $\frac{d\overline{T}}{dt}$ = -0.000574 ºC/s.

b) The left surface of the wall (at x = 0) is exposed to air at T_∞ = 1000°C. Determine the average convection coefficient between the air and the wall surface at x = 0 m.

Newton's law of cooling defines the heat transfer coefficient according to:

$$\overline{h}\,A\left(T_\infty - T_{x=0}\right) = -k\,A\left.\frac{\partial T}{\partial x}\right|_{x=0} \tag{7}$$

solving for the heat transfer coefficient:

$$\overline{h} = \frac{-k\left.\frac{\partial T}{\partial x}\right|_{x=0}}{\left(T_\infty - T_{x=0}\right)} \tag{8}$$

```
T_infinity=1000 [C]                           "ambient temperature at x=0"
T0=900[C]                                     "temperature at x=0"
h_bar=-k*dTdx0/(T_infinity-T0)                "heat transfer coefficient"
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