Problem 1.3-7: Nuclear Fuel Element

Figure P1.3-7 illustrates a spherical, nuclear fuel element which consists of a sphere of fissionable material (fuel) with radius r_{fuel} = 5 cm and k_{fuel} = 1 W/m-K that is surrounded by a spherical shell of metal cladding with outer radius r_{clad} = 7 cm and k_{clad} = 300 W/m-K. The outer surface of the cladding is exposed to helium gas that is being heated by the reactor. The convection coefficient between the gas and the cladding surface is $\overline{h}_{gas}$ = 100 W/m^2-K and the temperature of the gas is T_{gas} = 500°C. Neglect radiation heat transfer from the surface.

Inside the fuel element, fission fragments are produced which have high velocities. The products collide with the atoms of the material and provide the thermal energy for the reactor. This process can be modeled as a volumetric source of heat generation in the material that is not uniform throughout the fuel. The volumetric generation ($\dot{g}'''$) can be approximated by the function:

$$\dot{g}''' = \dot{g}_e'''\left(\frac{r}{r_{fuel}}\right)^b$$

where $\dot{g}_e'''$ = 5x10^5 W/m^3 is the volumetric rate of heat generation at the edge of the sphere and b = 1.0; note that the parameter b is a dimensionless positive constant that characterizes how quickly the generation rate increases in the radial direction.

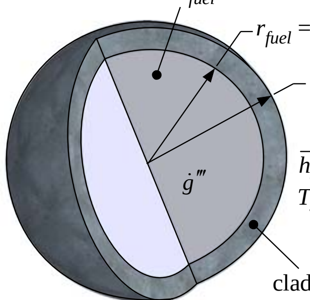

Figure P1.3-7: Spherical fuel element surrounded by cladding

a.) Enter the problem inputs into EES; be sure to set the units appropriately.

The inputs are entered according to:

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
r_fuel=5[cm]*convert(cm,m)                    "radius of fuel"
k_fuel=1 [W/m-K]                              "conductivity of fuel"
r_clad=7[cm]*convert(cm,m)                    "cladding radius"
k_clad=300 [W/m-K]                            "cladding conductivity"
h_gas=100 [W/m^2-K]                           "convection coefficient"
T_gas=converttemp(C,K,500)                    "gas temperature"
```

```
gve=5e5 [W/m^3]                                    "generation at the center"
b=1 [-]                                            "decay constant"
```

b.) Determine the governing differential equation that applies within the sphere (i.e., your differential equation should be valid for $0 < r < r_{fuel}$). The differential equation should include only those symbols given in the problem statement. Clearly show your steps.

A differential control volume is shown in Figure 2 and includes conduction at r and $r+dr$ at the inner and outer surfaces of the spherical shell as well as generation within the enclosed volume.

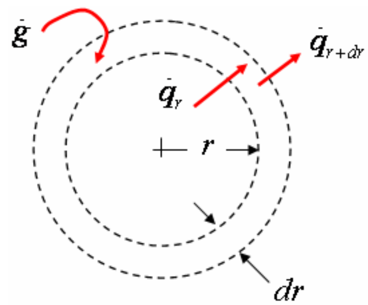

Figure 2: Differential control volume

The energy balance suggested by Figure 2 is:

$$\dot{q}_r + \dot{g} = \dot{q}_{r+dr} \tag{1}$$

The term at $r + dr$ can be expanded:

$$\dot{q}_{r+dr} = \dot{q}_r + \frac{d\dot{q}_r}{dr} dr \tag{2}$$

and substituted into Eq. (1):

$$\dot{q}_r + \dot{g} = \dot{q}_r + \frac{d\dot{q}_r}{dr} dr \tag{3}$$

and simplified:

$$\dot{g} = \frac{d\dot{q}_r}{dr} dr \tag{4}$$

The conduction is governed by Fourier's Law:

$$\dot{q}_r = -k_{fuel}\, 4\pi r^2 \frac{dT}{dr} \tag{5}$$

and the generation is the product of the volume and the local generation rate:

$$\dot{g} = 4\pi r^2\, dr\, \dot{g}_e'''\left(\frac{r}{r_{fuel}}\right)^b \tag{6}$$

The rate equations are substituted into Eq. (4):

$$4\pi r^2\, dr\, \dot{g}_e'''\left(\frac{r}{r_{fuel}}\right)^b = \frac{d}{dr}\left[-k_{fuel}\, 4\pi r^2 \frac{dT}{dr}\right] dr \tag{7}$$

which can be simplified:

$$\boxed{\frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] + r^2 \frac{\dot{g}_e'''}{k_{fuel}}\left(\frac{r}{r_{fuel}}\right)^b = 0} \tag{8}$$

Notice that it is not possible to cancel the r^2 term from each side of Eq. (7) because it appears within the differential on the right side.

c.) Enter the governing differential equation into Maple and use Maple to obtain a solution that includes two constants of integration.

The generation function and the governing differential equation are entered according to:

```
> gen:=gve*(r/r_fuel)^b;
```

$$gen := gve\left(\frac{r}{r\_fuel}\right)^b$$

```
> GDE:=diff(r^2*diff(T(r),r),r)+r^2*gen/k_fuel=0;
```

$$GDE := 2\, r\left(\frac{d}{dr}\mathrm{T}(r)\right) + r^2\left(\frac{d^2}{dr^2}\mathrm{T}(r)\right) + \frac{r^2\, gve\left(\frac{r}{r\_fuel}\right)^b}{k\_fuel} = 0$$

and solved using the dsolve command:

```
> Tr:=rhs(dsolve(GDE));
```

$$Tr := -\frac{\_C1}{r} - \frac{r^2\, gve\left(\frac{r}{r\_fuel}\right)^b}{k\_fuel\,(b^2 + 6 + 5\, b)} + \_C2$$

Notice the two constants of integration that must be determined using the boundary conditions.

d.) The boundary condition at the center of the sphere is that the temperature must remain finite; this should eliminate one of the constants of integration in your Maple solution. Which constant must be zero?

In order for the temperature to remain finite as r approaches 0, the constant C_1 must be zero.

e.) Determine a symbolic equation for the remaining boundary condition (the one at $r = r_{fuel}$) in terms of the temperature and temperature gradient evaluated at $r = r_{fuel}$.

An interface energy balance at $r = r_{fuel}$ includes conduction from the fuel and conduction into the cladding, as shown in Figure 3.

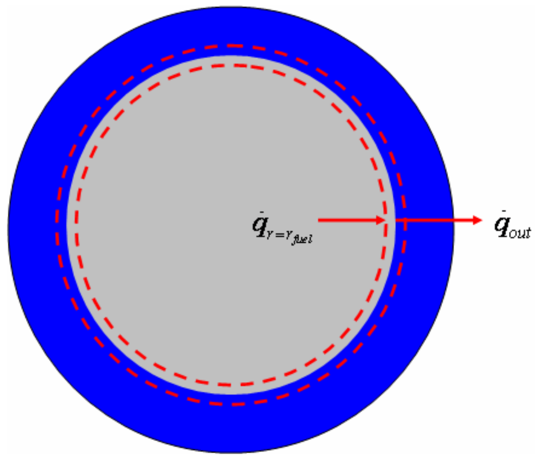

Figure 3: Interface balance at $r = r_{fuel}$

The energy balance suggested by Figure 3 is:

$$\dot{q}_{r=r_{ruel}} = \dot{q}_{out} \tag{9}$$

The conduction term on the left side of Eq. (9) is evaluated using Fourier's law:

$$\dot{q}_{r=r_{ruel}} = -k_{fuel}\, 4\,\pi\, r_{fuel}^2 \left.\frac{dT}{dr}\right|_{r=r_{fuel}} \tag{10}$$

while the heat transfer out of the cladding is driven by the difference between the temperature at interface between the fuel and the cladding and the temperature of the surrounding gas. The heat transfer is resisted by the sum of the conduction resistance of the cladding (R_{clad}):

$$R_{clad} = \frac{1}{4\,\pi\, k_{clad}}\left[\frac{1}{r_{fuel}} - \frac{1}{r_{clad}}\right] \tag{11}$$

and the convection resistance (R_{conv}):

$$R_{conv} = \frac{1}{4\,\pi\, r_{clad}^2\, h_{gas}} \tag{12}$$

so that:

$$\dot{q}_{out} = \frac{T_{r_{fuel}} - T_{gas}}{R_{clad} + R_{conv}} \tag{13}$$

Subsituting Eqs. (13) and (10) into Eq. (9) leads to:

$$\boxed{-k_{fuel}\, 4\,\pi\, r_{fuel}^{2} \left.\frac{dT}{dr}\right|_{r=r_{fuel}} = \frac{T_{r_{fuel}} - T_{gas}}{R_{clad} + R_{conv}}} \tag{14}$$

Equation (14) provides a single equation for the unknown constant of integration, C_2.

f.) Use the expressions from Maple to determine the required constant of integration within EES. Copy the solution for the temperature in the cladding from Maple and paste it into EES; modify the expression as necessary for compatibility (remember to eliminate the C1 term) and use it to generate plot of temperature vs radius within the cladding.

The solution in Maple is manipulated using the diff and eval commands:

```
> dTdr_rfuel:=eval(diff(Tr,r),r=r_fuel);
```

$$dTdr\_rfuel := \frac{\_C1}{r\_fuel^2} - \frac{2\; r\_fuel\; gve}{k\_fuel\;(b^2+6+5\,b)} - \frac{r\_fuel\; gve\; b}{k\_fuel\;(b^2+6+5\,b)}$$

```
> T_rfuel=eval(Tr,r=r_fuel);
```

$$T\_rfuel = -\frac{\_C1}{r\_fuel} - \frac{r\_fuel^2\; gve}{k\_fuel\;(b^2+6+5\,b)} + \_C2$$

The expressions are copied and pasted into EES; the constant C_1 is eliminated and the expressions are modified to be compatible with EES (the := is replaced with = and the _C2 is replaced with C2). Also, the _C1 portion of the expression for T_rfuel is deleted.

```
"Boundary condition expressions"
dTdr_rfuel =-2*r_fuel*gve/k_fuel/(b^2+6+5*b)-r_fuel*gve/k_fuel/(b^2+6+5*b)*b     "from Maple"
T_rfuel =-r_fuel^2*gve/k_fuel/(b^2+6+5*b)+C2                                     "from Maple"
```

The two resistance values must be calculated:

```
Rst_clad=(1/r_fuel-1/r_clad)/(4*pi*k_clad)      "conduction resistance of cladding"
Rst_conv=1/(4*pi*r_clad^2*h_gas)                "convection resistance"
```

Note that the use of R_clad to represent the cladding resistance would have resulted in problems because of the existence of the variable r_clad; EES is not case-sensitive. Finally, the boundary condition, Eq. (14), is programmed:

```
-4*pi*r_fuel^2*k_fuel*dTdr_rfuel=(T_rfuel-T_gas)/(Rst_clad+Rst_conv)     "boundary condition"
```

Solving the problem should provide a solution for C2 = -1916 K; note that the units should also be set and checked for all of your variables.

The Maple solution is cut and pasted into EES:

```
"Solution"
T=-r^2*gve/k_fuel/(b^2+6+5*b)*(r/r_fuel)^b+C2 "solution from Maple"
```

To facilitate plotting, the solution is converted from K to °C and the radial location is defined in terms of a dimensionless radial position (r_bar) that goes from 0 to 1 (therefore, if the radius of the fuel sphere changes in parametric studies it is not necessary to reset the parametric table).

```
r_bar=r/r_fuel
T_C=converttemp(K,C,T)
```

Figure 4 illustrates the temperature as a function of radius

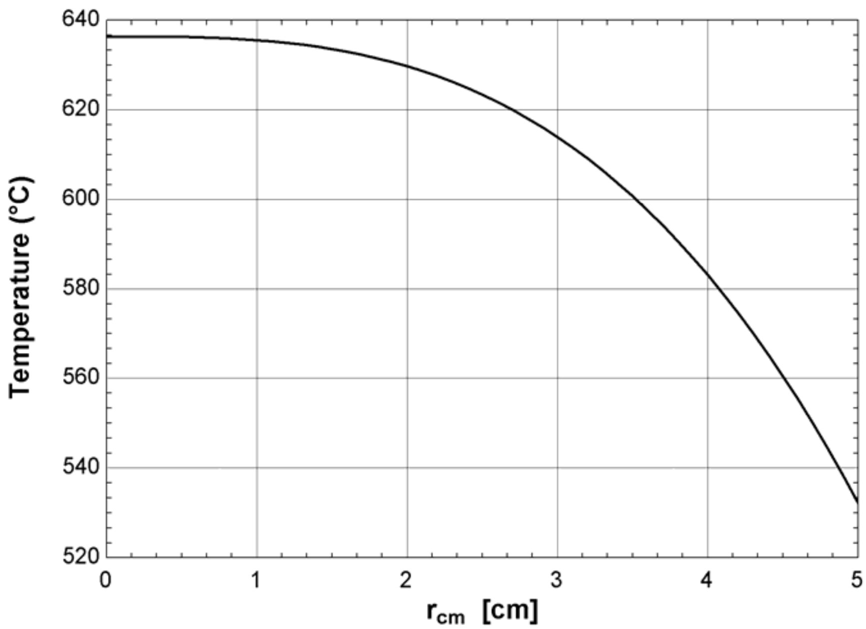

Figure 4: Temperature distribution in the fuel