PROBLEM 1.3-8 (1-8 in text): Hay Temperature

Freshly cut hay is not really dead; chemical reactions continue in the plant cells and therefore a small amount of heat is released within the hay bale. This is an example of the conversion of chemical to thermal energy and can be thought of as thermal energy generation. The amount of thermal energy generation within a hay bale depends on the moisture content of the hay when it is baled. Baled hay can become a fire hazard if the rate of volumetric generation is sufficiently high and the hay bale sufficiently large so that the interior temperature of the bale reaches 170°F, the temperature at which self-ignition can occur. Here, we will model a round hay bale that is wrapped in plastic to protect it from the rain. You may assume that the bale is at steady state and is sufficiently long that it can be treated as a one-dimensional, radial conduction problem. The radius of the hay bale is R_{bale} = 5 ft and the bale is wrapped in plastic that is t_p = 0.045 inch thick with conductivity k_p = 0.15 W/m-K. The bale is surrounded by air at T_∞ = 20°C with $\overline{h}$ = 10 W/m^2-K. You may neglect radiation. The conductivity of the hay is k = 0.04 W/m-K.

a.) If the volumetric rate of thermal energy generation is constant and equal to $\dot{g}'''$ = 2 W/m^3 then determine the maximum temperature in the hay bale.

The inputs are entered in EES:

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

R_bale=5 [ft]*convert(ft,m)                  "hay bale radius"
t_p=0.045 [inch]*convert(inch,m)             "plastic thickness"
k_p= 0.15 [W/m-K]                            "plastic conductivity"
h=10 [W/m^2-K]                               "heat transfer coefficient"
T_infinity=converttemp(C,K,20 [C])           "ambient temperature"
L=1 [m]                                      "per unit length of bale"
k = 0.04 [W/m-K]                             "conductivity of hay"
g_dot_v=2 [W/m^3]                            "volumetric heat generation"
```

This is an example of a one-dimensional steady conduction problem with constant volumetric generation and therefore the formulae provided in Table 1-3 can used directly. The general solution is:

$$T = -\frac{\dot{g}''' r^2}{4\,k} + C_1 \ln\left(r\right) + C_2 \tag{1}$$

where C_1 and C_2 are constants selected to enforce the boundary conditions. The boundary condition at the center of the bale is either that the temperature remain bounded or that the temperature gradient be zero; either will lead to C_1 = 0. An energy balance at the outer edge of the hay bale leads to:

$$-k\,2\,\pi\,R_{bale}\,L \left.\frac{dT}{dr}\right|_{r=R_{bale}} = \frac{T_{r=R_{bale}} - T_\infty}{R_p + R_{conv}} \tag{2}$$

where R_p and R_{conv} are the thermal resistances associated with conduction through the plastic and convection from the outer surface of the bale, respectively:

$$R_p = \frac{t_p}{k_p\, 2\,\pi\, R_{bale}\, L} \tag{3}$$

$$R_{conv} = \frac{1}{h\, 2\,\pi\, R_{bale}\, L} \tag{4}$$

where L = 1 m for a problem that is done on a unit length basis. The temperature gradient and temperature at the outer radius of the bale are obtained using Eq. (1) with C_1 = 0:

$$\left.\frac{dT}{dr}\right|_{r=R_{bale}} = -\frac{\dot{g}'''\, R_{bale}}{2\,k} \tag{5}$$

$$T_{r=R_{bale}} = -\frac{\dot{g}'''\, R_{bale}^2}{4\,k} + C_2 \tag{6}$$

Placing equations (2) through (6) into EES allows the constants of integration to be determined:

```
R_p=t_p/(k_p*2*pi*R_bale*L)
    "thermal resistance associated with conduction through plastic"
R_conv=1/(2*pi*R_bale*L*h)
    "thermal resistance associated with convection"
dTdr_Rbale=-g_dot_v*R_bale/(2*k)                        "temperature gradient at outer edge"
T_Rbale=-g_dot_v*R_bale^2/(4*k)+C_2                     "temperature at outer edge"
-k*2*pi*R_bale*L*dTdr_Rbale=(T_Rbale-T_infinity)/(R_p+R_conv) "interface energy balance"
```

The maximum temperature in the bale occurs at the center; according to Eq. (1) with C_1 = 0, this temperature is given by:

$$T = -\frac{\dot{g}'''\, r^2}{4\,k} + C_1 \ln\left(r\right) + C_2 \tag{7}$$

```
T_max=C_2                                               "maximum bale temperature"
T_max_F=converttemp(K,F,T_max)                          "maximum bale temperature in F"
```

The maximum temperature in the hay bale is 322.3 K or 120.6°F.

b.) Prepare a plot showing the maximum temperature in the hay bale as a function of the hay bale radius. How large can the hay bale be before there is a problem with self-ignition?

A parametric table is generated that contains the variables T_max_F and R_bale and used to generate Figure 1.

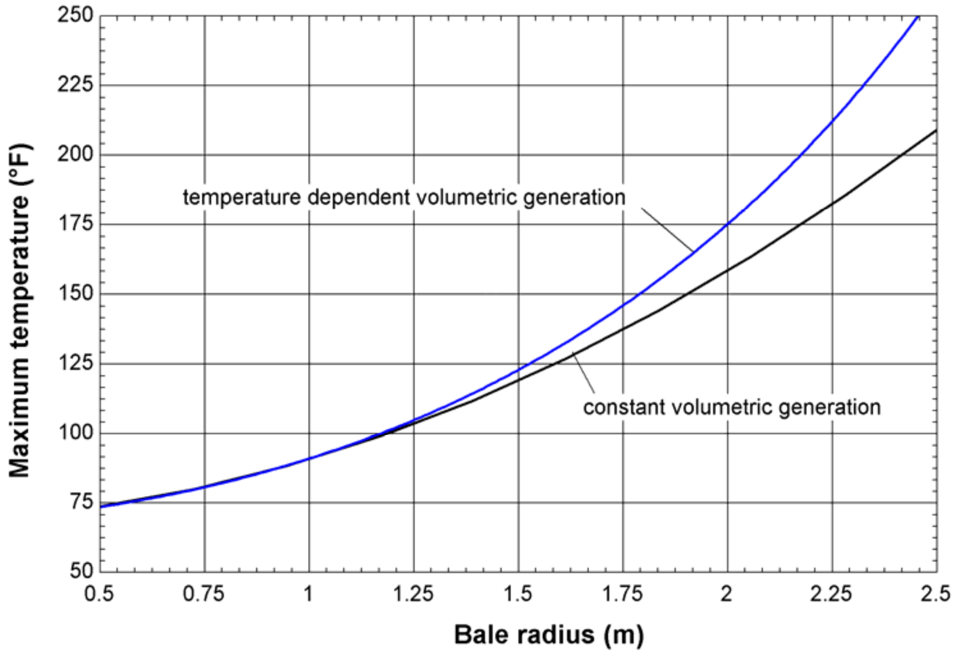

Figure 1: Maximum temperature as a function of the bale radius.

Note that a hay bale larger than approximately 2.1 m will result in a hay fire.

Prepare a model that can consider temperature-dependent volumetric generation. Increasing temperature tends to increase the rate of chemical reaction and therefore increases the rate of generation of thermal energy according to: $\dot{g}''' = a + b\,T$ where a = -1 W/m^3 and b = 0.01 W/m^3-K and T is in K.

c.) Enter the governing equation into Maple and obtain the general solution (i.e., a solution that includes two constants).

The governing differential equation is obtained as discussed in Section 1.3:

$$\dot{g}'''\,r = \frac{d}{dr}\left(-k\,r\,\frac{dT}{dr}\right) \tag{8}$$

This ordinary differential equation is entered in Maple:

```
> restart;
> ODE:=(a+b*T(r))*r=diff(-k*r*diff(T(r),r),r);
```

$$ODE := (a + b\,\mathrm{T}(r))\,r = -k\left(\frac{d}{dr}\,\mathrm{T}(r)\right) - k\,r\left(\frac{d^2}{dr^2}\,\mathrm{T}(r)\right)$$

and solved:

```
> Ts:=dsolve(ODE);
```

$$Ts := \mathrm{T}(r) = \mathrm{BesselJ}\left(0, \sqrt{\frac{b}{k}}\, r\right) \_C2 + \mathrm{BesselY}\left(0, \sqrt{\frac{b}{k}}\, r\right) \_C1 - \frac{a}{b}$$

Note that the solution is given in the form of Bessel functions;

$$T = -C_2\,\mathrm{BesselJ}\left(0, \sqrt{\frac{b}{k}} r\right) + C_1\,\mathrm{BesselY}\left(0, \sqrt{\frac{b}{k}} r\right) - \frac{a}{b} \tag{9}$$

Even though we have not yet learned about Bessel functions, we can manipulate this solution within Maple.

d.) Use the boundary conditions to obtain values for the two constants in your general solution (hint: one of the two constants must be zero in order to keep the temperature at the center of the hay bale finite). You should obtain a symbolic expression for the boundary condition in Maple that can be evaluated in EES.

In part (a) we could not take the natural logarithm of 0 in Eq. (7) and therefore C_1 was zero. A similar thing happens with the Bessel functions. We can evaluate the limits of the two Bessel functions as $r \rightarrow 0$:

```
> limit(BesselJ(0,r),r=0);
```

$$1$$

```
> limit(BesselY(0,r),r=0);
```

$$-\infty$$

The BesselY function becomes infinite and therefore C_1 in Eq. (9) must be 0.

```
> Ts:=subs(_C1=0,Ts);
```

$$Ts := \mathrm{T}(r) = \mathrm{BesselJ}\left(0, \sqrt{\frac{b}{k}}\, r\right) \_C2 - \frac{a}{b}$$

The boundary condition at the outer surface of the hay does not change; the temperature and temperature gradient at R_{bale} can be evaluated symbolically using Maple:

```
> dTdr_Rbale:=eval(diff(rhs(Ts),r),r=R_bale);
```

$$dTdr\_Rbale := -\mathrm{BesselJ}\left(1, \sqrt{\frac{b}{k}}\, R\_bale\right) \sqrt{\frac{b}{k}}\, \_C2$$

```
> T_Rbale:=eval(rhs(Ts),r=R_bale);
```

$$T\_Rbale := \mathrm{BesselJ}\left(0, \sqrt{\frac{b}{k}}\, R\_bale\right) \_C2 - \frac{a}{b}$$

These symbolic expressions are cut and paste into EES and used to replace Eqs. (5) and (6) and provide a new constant C_2:

```
{g_dot_v=2 [W/m^3]                                    "volumetric heat generation"}
a=-1 [W/m^3]
    "coefficients for volumetric generation function"
b=0.01 [W/m^3-K]

R_p=t_p/(k_p*2*pi*R_bale*L)
    "thermal resistance associated with conduction through plastic"
R_conv=1/(2*pi*R_bale*L*h)
    "thermal resistance associated with convection"
{dTdr_Rbale=-g_dot_v*R_bale/(2*k)                     "temperature gradient at outer edge"
T_Rbale=-g_dot_v*R_bale^2/(4*k)+C_2                   "temperature at outer edge"}
dTdr_Rbale = -BesselJ(1,(b/k)^(1/2)*R_bale)*(b/k)^(1/2)*C_2
    "symbolic expressions cut and paste from Maple"
T_Rbale = BesselJ(0,(b/k)^(1/2)*R_bale)*C_2-1/b*a
-k*2*pi*R_bale*L*dTdr_Rbale=(T_Rbale-T_infinity)/(R_p+R_conv)  "interface energy balance"
```

The maximum temperature is the temperature at the center of the bale; this is evaluated using Maple:

```
> T_max=eval(rhs(Ts),r=0);
```

$$T\_max = \_C2 - \frac{a}{b}$$

and copied and pasted into EES:

```
{T_max=C_2                                            "maximum bale temperature"}
T_max = C_2-1/b*a
    "symbolic expression cut and paste from Maple"
T_max_F=converttemp(K,F,T_max)                         "maximum bale temperature in F"
```

e.) Overlay on your plot from part (b) a plot of the maximum temperature in the hay bale as a function of bale radius when the volumetric generation is a function of temperature.

The result is shown in Figure 1.