Problem 1.2-18

Urea formaldehyde foam with conductivity k_{foam} = 0.020 Btu/hr-ft-F is commonly used as an insulation material in building walls. The major advantage of foam insulation is that it can be installed in existing walls by injection through a small hole. In a particular case, foam is to be installed in a wall consisting of a $th_{plaster}$ = 5/8 inch thick sheet of plaster board with conductivity $k_{plaster}$ = 0.028 Btu/hr-ft-F, a th_{as} = 3.5 inch air space and a th_{brick} = 2.5 inch layer of brick with conductivity k_{brick} = 0.038 Btu/hr-ft-F. Before the air gap is filled with foam, there is natural convection associated with buoyancy induced air motion. The equivalent thermal resistance of the air gap on a per unit area basis due to the natural convection is R''_{ag} = 0.95 hr-ft^2-F/Btu. The convection coefficients for the inner and outer surface of the wall are $\bar{h}_i$ = 1.5 Btu/hr-ft^2-F and $\bar{h}_o$ = 3.5 Btu/hr-ft^2-F, respectively.

a) The R-value of a wall is the resistance of the wall on a per unit area basis, expressed in units ft^2-F-hr/Btu. Calculate the R-value of the wall before the foam insulation is applied.

The inputs are entered in EES:

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$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"known"
k_brick=0.038 [Btu/hr-ft-F]*convert(Btu/hr-ft-F,W/m-K)            "brick conductivity"
k_plaster=0.028 [Btu/hr-ft-F]*convert(Btu/hr-ft-F,W/m-K)          "plaster conductivity"
k_foam=0.020 [Btu/hr-ft-F]*convert(Btu/hr-ft-F,W/m-K)             "formaldehyde foam conductivity"
th_plaster=(5/8) [in]*convert(in,m)                               "thickness of plaster"
th_as=3.5 [in]*convert(in,m)                                      "thickness of air space"
th_brick=2.5 [in]*convert(in,m)                                   "thickness of brick"
R``_ag=0.95 [hr-ft^2-F/Btu]*convert(hr-ft^2-F/Btu,K-m^2/W)
     "resistance of air gap without foam per unit area"
h_bar_i=1.5 [Btu/hr-ft^2-F]*convert(Btu/hr-ft^2-F,W/m^2-K)        "inside heat transfer coefficient"
h_bar_o=3.5 [Btu/hr-ft^2-F]*convert(Btu/hr-ft^2-F,W/m^2-K)        "outside heat transfer coefficient"
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The resistance per unit area without foam is:

$$R''_{total,nofoam} = \frac{1}{\bar{h}_i} + \frac{th_{plaster}}{k_{plaster}} + R''_{ag} + \frac{th_{brick}}{k_{brick}} + \frac{1}{\bar{h}_o} \tag{1}$$

```
"No foam"
R``_conv_i=1/h_bar_i                  "resistance to convection on inner surface per unit area"
R``_plaster=th_plaster/k_plaster      "resistance to conduction through plaster per unit area"
R``_brick=th_brick/k_brick            "resistance to conduction through brick per unit area"
R``_conv_o=1/h_bar_o                  "resistance to convection on outer surface per unit area"
R``_total_nofoam=R``_conv_i+R``_plaster+R``_ag+R``_brick+R``_conv_o "total resistance per unit area"
R_value_nofoam=R``_total_nofoam*convert(K-m^2/W,ft^2-F-hr/Btu)"R-value of wall without foam"
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which leads to an R-value of 9.245 ft^2-F-hr/Btu.

b) Calculate the R-value of the wall after insulation is applied. Assume that the insulating foam completely fills the air gap.

The resistance per unit area with foam is:

$$R''_{total,\,foam} = \frac{1}{\overline{h}_i} + \frac{th_{plaster}}{k_{plaster}} + \frac{th_{as}}{k_{foam}} + \frac{th_{brick}}{k_{brick}} + \frac{1}{\overline{h}_o} \tag{2}$$

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"Foam, no shrinkage"
R``_foam=th_as/k_foam                          "resistance to conduction through foam per unit area"
R``_total_foam=R``_conv_i+R``_plaster+R``_foam+R``_brick+R``_conv_o "total resistance per unit area"
R_value_foam=R``_total_foam*convert(K-m^2/W,ft^2-F-hr/Btu)   "R-value of wall with foam"
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which leads to an R-value of 22.88 ft^2-F-hr/Btu.

c) Foam insulation ordinarily shrinks after it is installed by an amount dependent upon conditions such as the outdoor temperature. Calculate the R-value of the wall assuming that the foam shrinks by 3%. Assume that the air in the gap between the foam and the plaster and the foam and the brick is stagnant.

The resistance per unit area with the foam considering shrinkage is:

$$R''_{total,\,foam} = \frac{1}{\overline{h}_i} + \frac{th_{plaster}}{k_{plaster}} + \left(1 - shrinkage\right)\frac{th_{as}}{k_{foam}} + shrinkage\,\frac{th_{as}}{k_a} + \frac{th_{brick}}{k_{brick}} + \frac{1}{\overline{h}_o} \tag{3}$$

where k_a is the thermal conductivity of air (evaluated at 20°C):

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"Foam, shrinkage"
Shrinkage=0.03 [-]                                "amount of shrinkage"
R``_foam_s=(1-Shrinkage)*th_as/k_foam             "resistance to conduction through foam per unit area"
k_a=conductivity(Air,T=converttemp(C,K,20 [C]))   "conductivity of air"
R``_air_s=Shrinkage*th_as/k_a                     "resistance to conduction through air per unit area"
R``_total_foam_s=R``_conv_i+R``_plaster+R``_foam_s+R``_air_s+R``_brick+R``_conv_o
    "total resistance per unit area"
R_value_foam_s=R``_total_foam_s*convert(K-m^2/W,ft^2-F-hr/Btu) "R-value of wall with foam"
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which leads to an R-value of 23.04 ft^2-F-hr/Btu. Note that the R-value actually improved with shrinkage because the stagnant air is less conductive than the foam. However, if the shrinkage is more extreme then natural convection will cause the resistance of the air to drop and the R-value to be reduced.