Problem 1.3-2: Composite Wall with Generation

A plane wall is composed of two materials, A and B, with the same conductivity k and thickness L, as shown in Figure P1.3-2. The left side of material A is adiabatic (i.e., well insulated) and the right side of material B is held at a temperature T_L. There is no volumetric generation in material A but material B experiences a uniform rate of volumetric generation of thermal energy, $\dot{g}'''$.

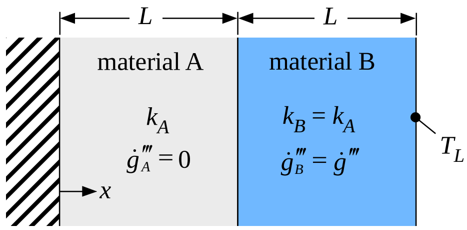

Figure P1.3-2: Plane wall composed of materials A and B.

a.) Sketch the rate of heat transfer ($\dot{q}$) as a function of position within the wall. Note that the sketch should be qualitatively correct but cannot be quantitative as you have not been given any numbers for the problem.

The control volume shown in Figure 2 can be used to evaluate the heat transfer from $0 < x < L$.

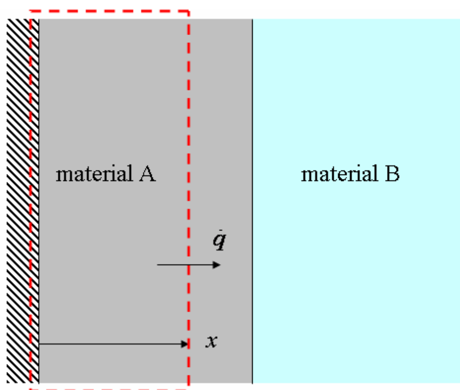

Figure 2: Control volume.

The heat transfer rate must be zero until $x = L$:

$$\dot{q} = 0 \quad \text{for } 0 < x < L \tag{1}$$

The control volume shown in Figure 3 can be used to evaluate the heat transfer from $L < x < 2\,L$.

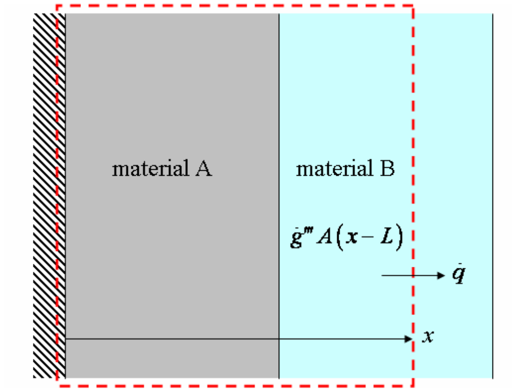

Figure 3: Control volume.

The heat transfer rate must increase linearly from $x = L$ to $x = 2\,L$:

$$\dot{q} = \dot{g}''' A(x - L) \quad \text{for } L < x < 2L \tag{2}$$

The heat transfer rate is sketched in Figure 4.

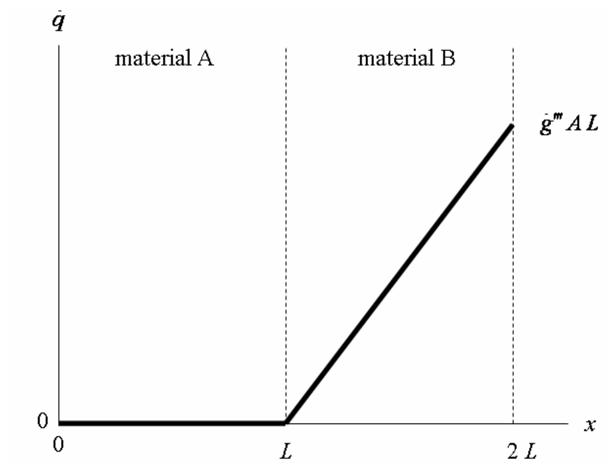

Figure 4: Rate of heat transfer as a function of position in the wall

b.) Sketch the temperature as a function of position within the wall. Again, be sure that your sketch has the correct qualitative features.

The temperature gradient is given by Fourier's law:

$$\frac{dT}{dx} = -\frac{\dot{q}}{k} \tag{3}$$

Therefore, the temperature gradient will be zero within material A and then become increasingly negative as you move towards positive x within material B. Figure 5 shows a sketch of the temperature distribution.

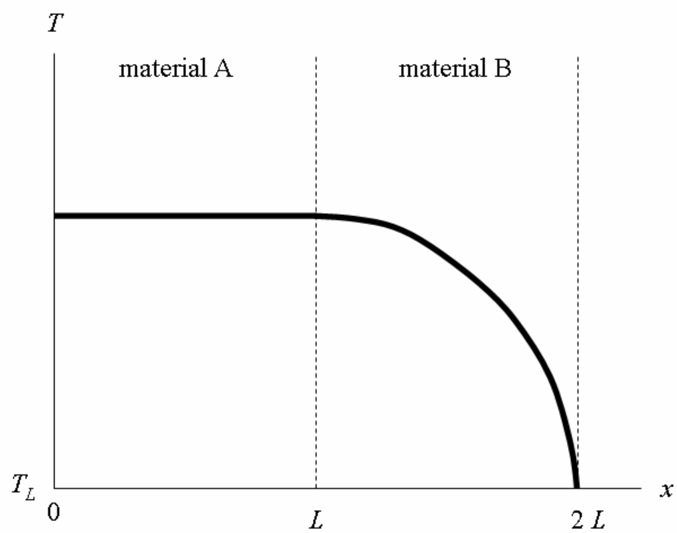

Figure 5: Temperature as a function of position in the wall