Problem 1.3-10

A current of 100 amps passes through a bare stainless-steel wire of D = 1.0 mm diameter. The thermal conductivity and electrical resistance per unit length of the wire are k = 15 W/m-K and R'_e = 0.14 ohm/m, respectively. The wire is submerged in an oil that is maintained at T_∞ = 30°C. The steady-state temperature at the center of the wire is measured to be 180°C, independent of axial position within the oil bath.

a) What is the temperature at the outer surface of the wire?

The inputs are entered in EES:

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$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

k=15 [W/m-K]                                  "conductivity of wire"
R\L=0.14 [ohm/m]                              "resistance per unit length of wire"
i=100 [amp]                                   "current"
D=1[mm]/2*convert(mm,m)                       "diameter"
T_c=converttemp(C,K,180 [C])                  "temperature at center of wire"
T_oil=converttemp(C,K,30 [C])                 "temperature of oil"
L=1 [m]                                       "per unit length basis"
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The rate of generation per unit volume is:

$$\dot{g}''' = \frac{I^2 R'_e L}{L \pi r_{out}^2} \tag{1}$$

where r_{out} is the outer radius of the wire:

$$r_{out} = \frac{D}{2} \tag{2}$$

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r_out=D/2                                     "outer radius"
g_dot```*(pi*r_out^2*L)=i^2*R\L*L             "rate of volumetric generation"
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The general solution for radial conduction with uniform volumetric generation is provided in Table 1-3:

$$T = -\frac{\dot{g}''' r^2}{4k} + C_1 \ln(r) + C_2 \tag{3}$$

$$\frac{dT}{dr} = -\frac{\dot{g}''' r}{2k} + \frac{C_1}{r} \tag{4}$$

In order for the temperature to remain bounded at r = 0, it is necessary that C_1 be zero. The temperature at the center is therefore:

$$T_c = C_2 \tag{5}$$

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T_c=C_2                                                   "solve for C_2"
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The temperature at the surface is given by:

$$T_s = -\frac{\dot{g}''' r_{out}^2}{4k} + C_1 \ln\left(r_{out}\right) + C_2 \tag{6}$$

```
T_s=-g_dot```*r_out^2/(4*k)+C_2                "surface temperature"
T_s_C=converttemp(K,C,T_s)                     "in C"
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which leads to T_s = 172.6ºC.

b) Estimate the convection coefficient between the submerged wire and the oil.

An interface energy balance at $r = r_{out}$ leads to:

$$\bar{h}\left(T_{oil} - T_s\right) = k \left.\frac{dT}{dr}\right|_{r=r_{out}} \tag{7}$$

where the temperature gradient is evaluated using Eq. (4):

$$\left.\frac{dT}{dr}\right|_{r=r_{out}} = -\frac{\dot{g}''' r_{out}}{2k} \tag{8}$$

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dT\dr_s=-g_dot```*r_out/(2*k)                  "surface temperature gradient"
h_bar*(T_oil-T_s)=k*dT\dr_s                    "interface energy balance"
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which leads to $\bar{h}$ = 6250 W/m^2-K.

c) A plastic material (k_p = 0.05 W/m-K) can be applied to the outer surface of the wire. Can the insulation result in a reduction of the center temperature? If so, what insulation thickness should be applied?

The resistance to conduction through the insulation is:

$$R_{ins} = \frac{\ln\left(\frac{r_{out} + th_{ins}}{r_{out}}\right)}{2\,\pi\,k_p\,L} \tag{9}$$

and the resistance to convection from the surface of the insulation is:

$$R_{conv} = \frac{1}{2\,\pi\, r_{out}\, L\, \overline{h}} \tag{10}$$

The total resistance between the surface of the wire and the oil is:

$$R_{total} = R_{conv} + R_{ins} \tag{11}$$

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k_p=0.05 [W/m-K]                        "conductivity of plastic"
R_ins=ln((r_out+th_ins)/r_out)/(2*pi*k_p*L)   "resistance to conduction through plastic"
R_conv=1/(h_bar*2*pi*L*(r_out+th_ins))  "resistance to convection from external surface"
R_total=R_ins+R_conv                    "total resistance from surface of wire"
```

Figure 1 illustrates R_{ins}, R_{conv}, and R_{total} as a function of the insulation thickness. Notice that increasing the insulation thickness reduces R_{conv} because there is more surface area for convection but increases R_{cond} because the length for conduction is longer. In some situations, the reduction in R_{conv} dominates the problem and therefore the total resistance may be reduced by adding insulation. However, in this case, there is no such region.

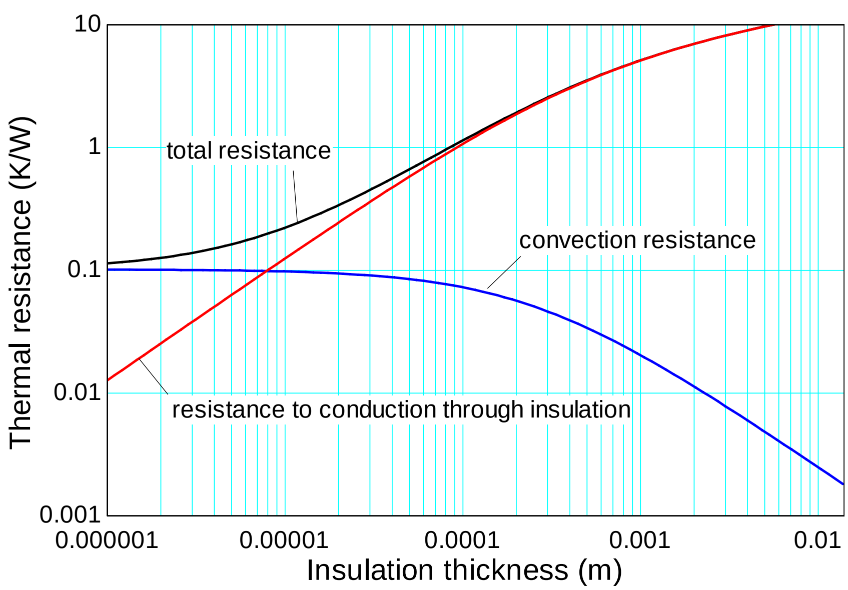

Figure 5: Resistance to convection, resistance to conduction through insulation, and total resistance from wire surface as a function of the insulation thickness.