Problem 1.9-7: Flat Plate Solar Collector

Figure P1.9-7 illustrates a flat plate solar collector.

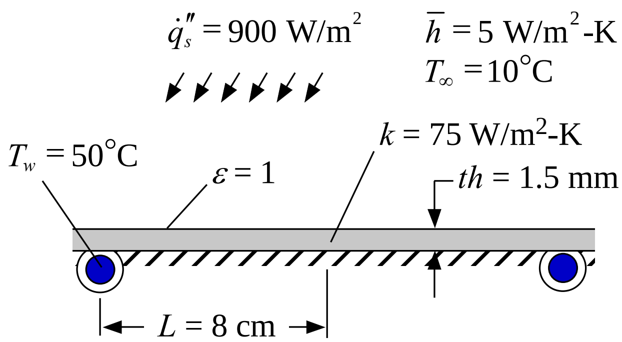

Figure 1.9-7: Flat plate solar collector.

The collector consists of a flat plate that is th = 1.5 mm thick with conductivity k = 75 W/m-K. The plate is insulated on its back side and experiences a solar flux of $\dot{q}''_s$ = 900 W/m^2 which is all absorbed. The surface is exposed to convection and radiation to the surroundings. The emissivity of the surface is ε = 1. The heat transfer coefficient is $\bar{h}$ = 5 W/m^2-K and the surrounding temperature is T_∞ = 10ºC. The temperature of the water is T_w = 50ºC. The center-to-center distance between adjacent tubes is 2 L where L = 8 cm.

a.) Develop a numerical model that can predict the rate of energy transfer to the water per unit length of collector.

The inputs are entered in EES:

```
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"
th=1.5 [mm]*convert(mm,m)                    "thickness"
k=75 [W/m-K]                                 "conductivity of collector plate"
e=1 [-]                                      "emissivity of collector plate"
q``_s=900 [W/m^2]                            "solar flux"
T_infinity=converttemp(C,K,10[C])            "ambient temperature"
h_bar=5 [W/m^2-K]                            "heat transfer coefficient"
L=8 [cm]*convert(cm,m)                       "half-width between tubes"
T_w=converttemp(C,K,50 [C])                  "water temperature"
W=1 [m]                                      "per unit length of collector"
```

Nodes are placed along the length of the collector. Only the region from 0 < x < L is considered due to the symmetry of the system. Therefore, the position of each node is:

$$x_i = \frac{L\left(i-1\right)}{\left(N-1\right)} \quad \text{for } i = 1..N \tag{1}$$

The distance between adjacent nodes is:

$$\Delta x = \frac{L}{(N-1)} \tag{2}$$

```
N=21 [-]                                                    "number of nodes"
Dx=L/(N-1)
duplicate i=1,N
   x[i]=L*(i-1)/(N-1)                                       "location of each node"
end
```

Energy balances on each of the internal nodes leads to:

$$\dot{q}_{LHS,i} + \dot{q}_{RHS,i} + \dot{q}_{s,i} + \dot{q}_{conv,i} + \dot{q}_{rad,i} = 0 \text{ for } i = 2..(N-1) \tag{3}$$

where

$$\dot{q}_{LHS,i} = \frac{k\,W\,th}{\Delta x}\left(T_{i-1} - T_i\right) \tag{4}$$

$$\dot{q}_{RHS,i} = \frac{k\,W\,th}{\Delta x}\left(T_{i+1} - T_i\right) \tag{5}$$

$$\dot{q}_{s,i} = W\,\Delta x\,\dot{q}_s'' \tag{6}$$

$$\dot{q}_{s,i} = W\,\Delta x\,\overline{h}\left(T_\infty - T_i\right) \tag{7}$$

$$\dot{q}_{rad,i} = W\,\Delta x\,\varepsilon\,\sigma\left(T_\infty^4 - T_i^4\right) \tag{8}$$

```
"internal nodes"
duplicate i=2,(N-1)
   q_dot_LHS[i]=k*W*th*(T[i-1]-T[i])/Dx              "conduction from left hand side node"
   q_dot_RHS[i]=k*W*th*(T[i+1]-T[i])/Dx              "conduction from right hand side node"
   q_dot_s[i]=W*Dx*q``_s                             "absorbed solar radiation"
   q_dot_conv[i]=W*Dx*h_bar*(T_infinity-T[i])        "convection"
   q_dot_rad[i]=W*Dx*e*sigma#*(T_infinity^4-T[i]^4)  "radiation"
   q_dot_LHS[i]+q_dot_RHS[i]+q_dot_s[i]+q_dot_conv[i]+q_dot_rad[i]=0 "energy balance"
end
```

The temperature of node 1 is assumed to be equal to the water temperature (neglecting any resistance to convection on the water-side):

$$T_1 = T_w \tag{9}$$

An energy balance on node N leads to:

$$\dot{q}_{LHS,N} + \dot{q}_{s,N} + \dot{q}_{conv,N} + \dot{q}_{rad,N} = 0 \tag{10}$$

where

$$\dot{q}_{LHS,N} = \frac{k\,W\,th}{\Delta x}\left(T_{N-1} - T_N\right) \tag{11}$$

$$\dot{q}_{s,N} = \frac{W\,\Delta x}{2}\dot{q}''_s \tag{12}$$

$$\dot{q}_{s,N} = \frac{W\,\Delta x}{2}\bar{h}\left(T_\infty - T_N\right) \tag{13}$$

$$\dot{q}_{rad,N} = \frac{W\,\Delta x}{2}\varepsilon\,\sigma\left(T_\infty^4 - T_N^4\right) \tag{14}$$

```
"node 1"
T[1]=T_w

"node N"
q_dot_LHS[N]=k*W*th*(T[N-1]-T[N])/Dx                    "conduction from left hand side node"
q_dot_s[N]=W*Dx*q``_s/2                                 "absorbed solar radiation"
q_dot_conv[N]=W*Dx*h_bar*(T_infinity-T[N])/2            "convection"
q_dot_rad[N]=W*Dx*e*sigma#*(T_infinity^4-T[N]^4)/2      "radiation"
q_dot_LHS[N]+q_dot_s[N]+q_dot_conv[N]+q_dot_rad[N]=0   "energy balance"
```

The temperature distribution within the collector is shown in Figure 2:

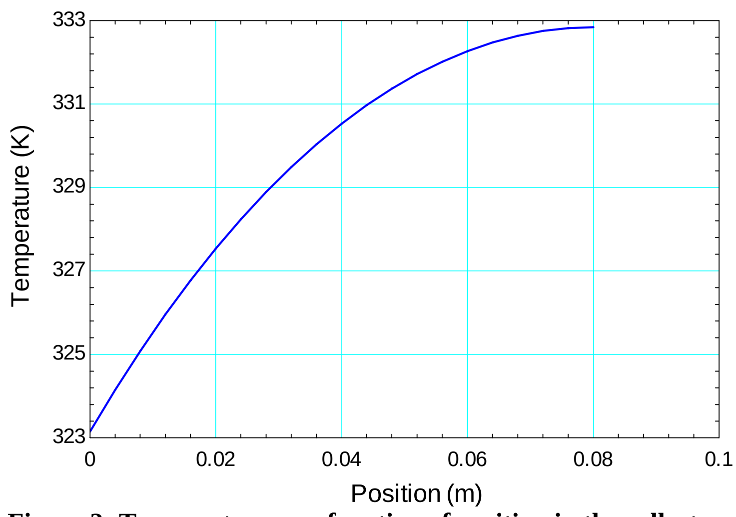

Figure 2: Temperature as a function of position in the collector.

An energy balance on node 1 provides the rate of energy transfer to the water:

$$\dot{q}_{water} = \dot{q}_{RHS,1} + \dot{q}_{s,1} + \dot{q}_{conv,1} + \dot{q}_{rad,1} \tag{15}$$

where

$$\dot{q}_{RHS,1} = \frac{k\,W\,th}{\Delta x}\left(T_2 - T_1\right) \tag{16}$$

$$\dot{q}_{s,1} = \frac{W\,\Delta x}{2}\dot{q}_s'' \tag{17}$$

$$\dot{q}_{s,1} = \frac{W\,\Delta x}{2}\bar{h}\left(T_\infty - T_1\right) \tag{18}$$

$$\dot{q}_{rad,1} = \frac{W\,\Delta x}{2}\varepsilon\,\sigma\left(T_\infty^4 - T_1^4\right) \tag{19}$$

```
"node 1"
q_dot_RHS[1]=k*W*th*(T[2]-T[1])/Dx                  "conduction from right hand side node"
q_dot_s[1]=W*Dx*q``_s/2                             "absorbed solar radiation"
q_dot_conv[1]=W*Dx*h_bar*(T_infinity-T[1])/2        "convection"
q_dot_rad[1]=W*Dx*e*sigma#*(T_infinity^4-T[1]^4)/2  "radiation"
q_dot_water=q_dot_RHS[1]+q_dot_s[1]+q_dot_conv[1]+q_dot_rad[1] "energy balance"
```

which leads to $\dot{q}_{water}$ = 28.9 W.

b.) Determine the efficiency of the collector; efficiency is defined as the ratio of the energy delivered to the water to the solar energy incident on the collector.

The efficiency is calculated according to:

$$\eta = \frac{\dot{q}_{water}}{L\,W} \tag{20}$$

```
eta=q_dot_water/(L*W*q``_s)                "efficiency"
```

which leads to η = 0.402.

c.) Plot the efficiency as a function of the number of nodes used in the solution.

Figure 3 illustrates the efficiency as a function of the number of nodes and shows that at least 20-30 nodes are required for numerical convergence.

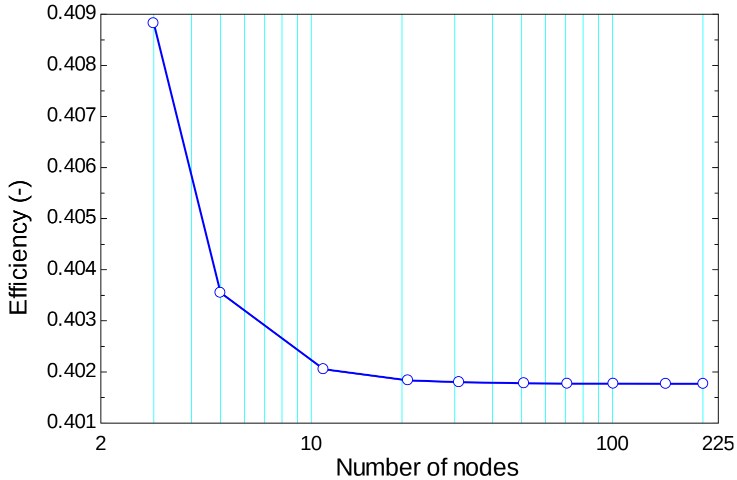

Figure 3: Efficiency as a function of the number of nodes.

d.) Plot the efficiency as a function of T_w - T_∞. Explain your plot.

Figure 4 illustrates the efficiency as a function of the water-to-ambient temperature difference. When the water temperature is low then the losses are low (but not zero, because the temperature of the copper plate is elevated by conduction). As the water temperature increases, the temperature of the plate increases and therefore the losses increase and efficiency drops. The drop in efficiency is dramatic for this type of unglazed collector and therefore the collector may be suitable for providing water heating for swimming pools (at low water temperature) but probably is not suitable for providing domestic hot water (at high water temperature).

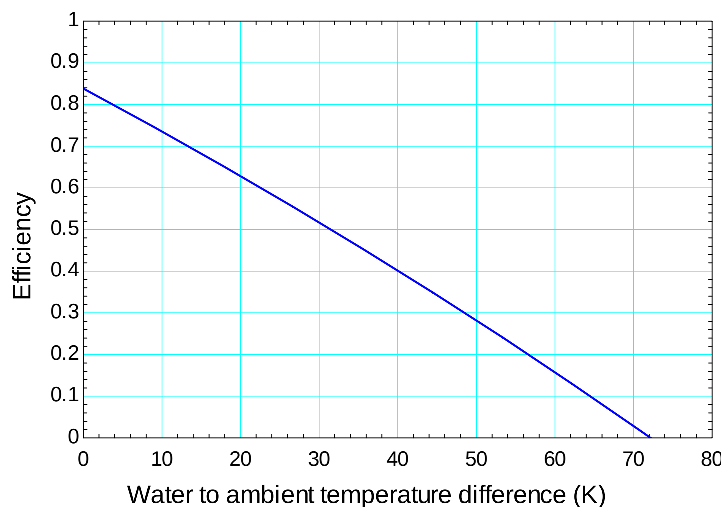

Figure 4: Solar collector efficiency as a function of the water-to-ambient temperature difference.