Problem 1.4-4: Storing Hay in a Barn

If you bale hay without allowing it to dry sufficiently then the hay bales will contain a lot of water. Besides making the bales heavy and therefore difficult to put in the barn, the water in the hay bails causes an exothermic chemical reaction to occur within the bale (i.e., the hay is rotting). The chemical reaction proceeds at a rate that is related to temperature and the bales may be thermally isolated (they are placed in a barn and surrounded by other hay bales); as a result, the hay can become very hot and even start a barn fire. Figure P1.4-4 illustrates a cross-section of a barn wall with hay stacked against it.

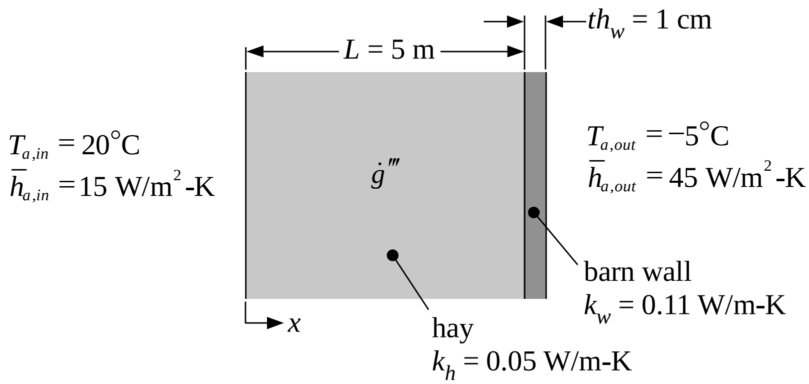

Figure P1.4-4: Barn wall with hay.

The air within the barn is maintained at $T_{a,in}$ = 20°C and the heat transfer coefficient between the air and the inner surface of the hay is $\overline{h}_{a,in}$ = 15 W/m^2-K. The outside air is at $T_{a,out}$ = -5°C with $\overline{h}_{a,out}$ = 45 W/m^2-K. Neglect radiation from the surfaces in this problem. The barn wall is composed of wood (k_w = 0.11 W/m-K) and is th_w = 1 cm thick. The hay has been stacked L = 5 m thick against the wall. Hay is a composite structure composed of plant fiber and air. However, hay can be modeled as a single material with an effective conductivity k_h = 0.05 W/m-K. The volumetric generation of the hay due to the chemical reaction is given by:

$$\dot{g}''' = 1.5\left[\frac{\text{W}}{\text{m}^3}\right]\left[\exp\left(\frac{T}{320[K]}\right)\right]^{0.5}$$

where T is temperature in K.

a.) Develop a numerical model that can predict the temperature distribution within the hay.

The input information is entered in EES and a function is used to define the volumetric generation:

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

function gen(T)
    "volumetric heat generation in wall"
    "Input - T, temperature [K]"
    "Output - gen, volumetric rate of heat generation [W/m^3]"
```

```
     gen=1.5 [W/m^3]*sqrt(exp(T/320 [K]))
end

"Inputs"
T_a_in=converttemp(C,K,20)                    "temperature of air within barn"
h_a_in=15 [W/m^2-K]                           "internal heat transfer coefficient"
T_a_out=converttemp(C,K,-5)                   "temperature of air outside barn"
h_a_out=45 [W/m^2-K]                          "external heat transfer coefficient"
k_w=0.11 [W/m-K]                              "conductivity of barn wall"
th_w=1.0 [cm]*convert(cm,m)                   "barn wall thickness"
L=5.0 [m]                                     "thickness of hay"
k_h=0.05 [W/m-K]                              "conductivity of hay"
A = 1 [m^2]                                   "per unit area of wall"
```

Nodes are distributed uniformly throughout the computational domain (which consists only of the hay, not the barn wall), the location of each node (x_i) is:

$$x_i = \frac{(i-1)}{(N-1)} L \quad i = 1..N \tag{1}$$

where N is the number of nodes used for the simulation. The distance between adjacent nodes (Δx) is:

$$\Delta x = \frac{L}{N-1} \tag{2}$$

```
"Setup grid"
N=10 [-]                                      "number of nodes"
duplicate i=1,N
     x[i]=(i-1)*L/(N-1)                       "position of each node"
end
Deltax=L/(N-1)                                "distance between adjacent nodes"
```

A control volume is defined around each node and an energy balance is written for each control volume. The control volume for an arbitrary, internal node (i.e., a node that is not placed on the edge of the hay) experiences conduction heat transfer passing through the internal surface ($\dot{q}_{LHS}$), conduction heat transfer passing through the external surface ($\dot{q}_{RHS}$), and heat generation within the control volume ($\dot{g}$). A steady-state energy balance for the control volume is shown in Fig. 2 and leads to:

$$\dot{q}_{LHS} + \dot{q}_{RHS} + \dot{g} = 0 \tag{3}$$

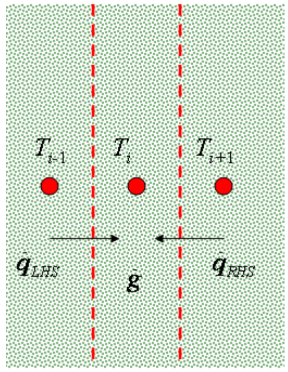

Figure 2: An internal control volume

Each of the terms in the energy balance in Eq. (3) must be modeled using a rate equation. Conduction through the inner surface is driven by the temperature difference between nodes *i*-1 and *i* through the material that lies between these nodes.

$$\dot{q}_{LHS} = \frac{k_h\, A\left(T_{i-1} - T_i\right)}{\Delta x} \tag{4}$$

where *A* is the area of the wall (assumed to be 1 m^2, corresponding to doing the problem on a per unit area of wall basis). The conduction into the outer surface is:

$$\dot{q}_{RHS} = \frac{k_h\, A\left(T_{i+1} - T_i\right)}{\Delta x} \tag{5}$$

The generation is the product of the volume of the control volume and the volumetric generation rate, which is approximately:

$$\dot{g} = \dot{g}'''_{T_i}\, A\, \Delta x \tag{6}$$

where $\dot{g}'''$ must be evaluated at the nodal temperature T_i. Substituting Eqs. (4) through (6) into Eq. (3) leads to:

$$\frac{k_h\, A\left(T_{i-1} - T_i\right)}{\Delta x} + \frac{k_h\, A\left(T_{i+1} - T_i\right)}{\Delta x} + \dot{g}'''_{T_i} A\, \Delta x = 0 \quad \text{for } i = 2...(N-1) \tag{7}$$

Figure 3 illustrates the control volume associated with the node that is placed on the outer surface of the hay (i.e., node *N*).

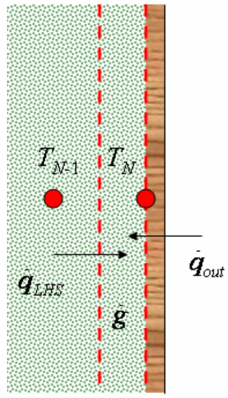

Figure 3: Control volume for node *N* located on hay outer surface

The energy balance for the control volume associated with node *N* is:

$$\dot{q}_{LHS} + \dot{g} + \dot{q}_{out} = 0 \tag{8}$$

where the conduction term is:

$$\dot{q}_{LHS} = \frac{k_h \, A\left(T_{N-1} - T_N\right)}{\Delta x}, \tag{9}$$

the generation term is:

$$\dot{g} = \dot{g}'''_{T_N} \, A\frac{\Delta x}{2}, \tag{10}$$

(note the factor of 2 corresponding to half the volume), and the heat transfer to the external air is:

$$\dot{q}_{conv} = \frac{\left(T_{a,out} - T_N\right)}{R_w + R_{conv,out}} \tag{11}$$

where

$$R_w = \frac{th_w}{k_w \, A} \tag{12}$$

and

$$R_{conv,out} = \frac{1}{h_{a,out} \, A} \tag{13}$$

Substituting Eqs. (9) through (11) into Eq. (8) leads to:

$$k_h \frac{A\left(T_{N-1}-T_N\right)}{\Delta x}+\dot{g}_{T_N}''' A \frac{\Delta x}{2}+\frac{\left(T_{a,out}-T_N\right)}{R_w+R_{conv,out}}=0 \tag{14}$$

A similar procedure applied to the control volume associated with node 1 leads to:

$$k_h \frac{A\left(T_2-T_1\right)}{\Delta x}+\dot{g}_{T_1}''' A \frac{\Delta x}{2}+h_{a,in} A\left(T_{a,in}-T_1\right)=0 \tag{15}$$

Equations (7), (14), and (15) represent N equations in an equal number of unknowns; the solution of these equations provides the numerical solution.

```
"Internal control volumes"
duplicate i=2,(N-1)
     k_h*A*(T[i-1]-T[i])/Deltax+k_h*A*(T[i+1]-T[i])/Deltax+gen(T[i])*A*Deltax=0
end
R_w=th_w/(k_w*A)                                  "conduction resistance of barn wall"
R_conv_out=1/(h_a_out*A)                          "convection resistance to external air"
k_h*A*(T[N-1]-T[N])/Deltax+gen(T[N])*A*Deltax/2+(T_a_out-T[N])/(R_w+R_conv_out)=0
                                                  "Node N"
k_h*A*(T[2]-T[1])/Deltax+gen(T[1])*A*Deltax/2+h_a_in*A*(T_a_in-T[1])=0
                                                 "Node 1"
```

If the EES program is solved then the temperature distribution will be placed in the Arrays window.

b.) Prepare a plot that shows the temperature distribution as a function of position in the hay.

The information in the Arrays table is used to prepare the plot shown in Figure 4.

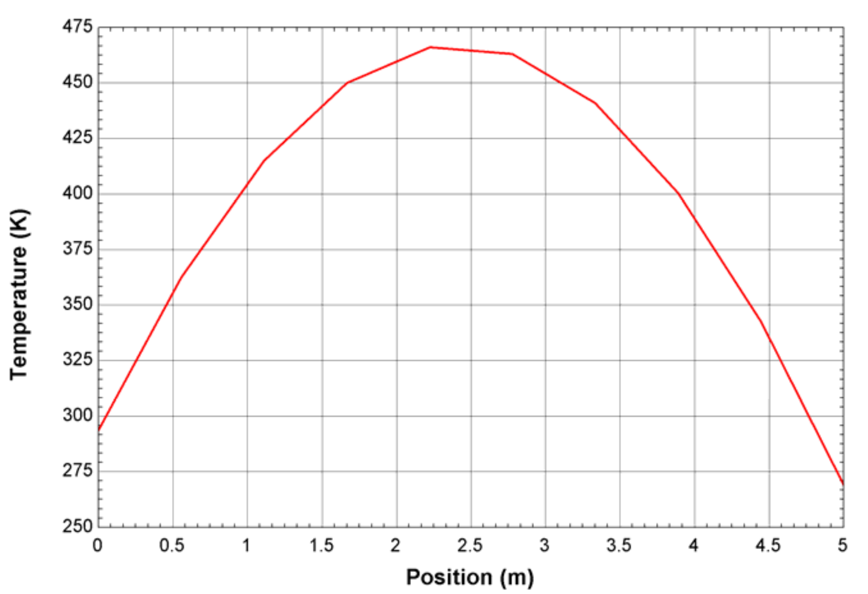

Figure 4: Temperature as a function of position in the wall.

c.) Prepare a plot that shows that you are using a sufficient number of nodes in your numerical solution.

The most relevant result of the calculation is the maximum temperature within the wall.

```
T_max=MAX(T[1..N])     "Maximum temperature in the wall"
```

Comment out the number of nodes assignment:

```
{N=10 [-]}
```

and prepare a parametric table that contains `N` and `T_max` (Figure 5).

Table 1

1..8	N [-]	T_{max}
Run 1	2	293.5
Run 2	5	469.3
Run 3	10	466.2
Run 4	20	466.8
Run 5	50	466.7
Run 6	100	466.8
Run 7	200	466.8
Run 8	500	466.8

Figure 5: Parametric table

The information in the parametric table is used to create Figure 6 which shows the maximum temperature as a function of the number of nodes; Figure 6 suggests that 50 nodes should be used to obtain a numerically convergent solution.

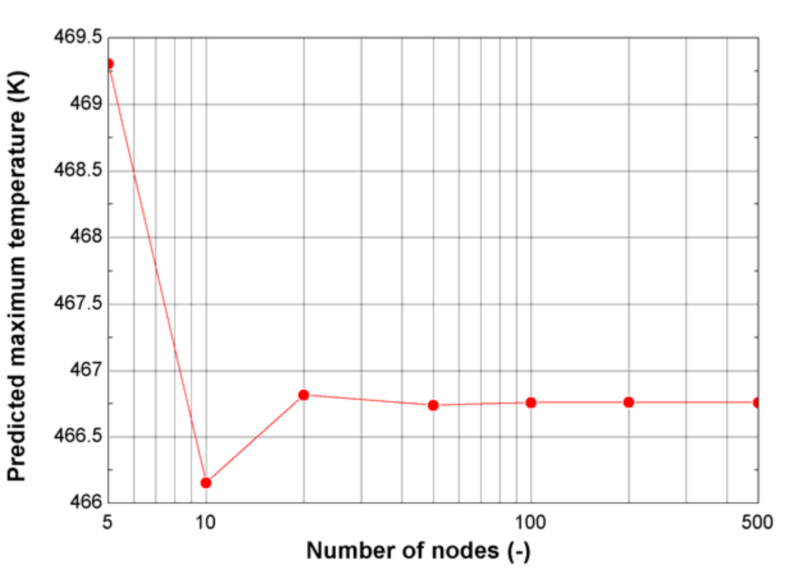

Figure 6: Predicted maximum temperature as a function of the number of nodes.

d.) Verify that your solution is correct by comparing it with an analytical solution in an appropriate limit. Prepare a plot that overlays your numerical solution and the analytical solution in this limit.

There are a few limits; the easiest one would be to turn the generation off (i.e., set it to zero). Alternatively, set the generation rate to a constant value (e.g., 1 W/m^3) and obtain the analytical solution. Modify the function:

```
function gen(T)
	"volumetric heat generation in wall"
	"Input - T, temperature [K]"
	"Output - gen, volumetric rate of heat generation [W/m^3]"

	gen=1.0 [W/m^3] {1.5 [W/m^3]*sqrt(exp(T/320 [K]))}

end
```

The general solution for a plane wall subjected to a constant generation rate was provided in Table 3-1:

$$T = -\frac{\dot{g}'''}{2\,k_h}x^2 + C_1\,x + C_2 \tag{16}$$

The boundary condition at $x = L$ is:

$$-k_h\,A\left.\frac{dT}{dx}\right|_{x=L} = \frac{\left(T_{x=L} - T_{a,out}\right)}{R_w + R_{conv,out}} \tag{17}$$

where the temperature gradient can also be obtained from Table 3-1:

$$\left.\frac{dT}{dx}\right|_{x=L} = \left[-\frac{\dot{g}'''}{k_h}x + C_1\right]_{x=L} = -\frac{\dot{g}'''}{k_h}L + C_1 \tag{18}$$

and

$$T_{x=L} = \left[-\frac{\dot{g}'''}{2\,k_h}x^2 + C_1\,x + C_2\right]_{x=L} = -\frac{\dot{g}'''}{2\,k_h}L^2 + C_1\,L + C_2 \tag{19}$$

```
"Analytical solution for constant generation"
g```_dot=gen(300 [K])                                  "obtain the rate of generation"
-k_h*A*dTdx_L=(T_L-T_a_out)/(R_w+R_conv_out)           "boundary condition at x=L"
dTdx_L=-g```_dot*L/k_h+C_1                             "temperature gradient at x=L"
T_L=-g```_dot*L^2/(2*k_h)+C_1*L+C_2                    "temperature at x=L"
```

The boundary condition at $x = 0$ is:

$$h_{a,in}\,A\left(T_{a,in}-T_{x=0}\right)=-k_h\,A\left.\frac{dT}{dx}\right|_{x=0} \tag{20}$$

where the temperature gradient can also be obtained from Table 3-1:

$$\left.\frac{dT}{dx}\right|_{x=0}=\left[-\frac{\dot{g}'''}{k_h}x+C_1\right]_{x=0}=C_1 \tag{21}$$

and

$$T_{x=0}=\left[-\frac{\dot{g}'''}{2\,k_h}x^2+C_1\,x+C_2\right]_{x=0}=C_2 \tag{22}$$

```
h_a_in*A*(T_a_in-T_0)=-k_h*A*dTdx_0          "boundary condition at x=0"
dTdx_0=C_1                                    "temperature gradient at x=0"
T_0=C_2                                       "temperature at x=0"
```

Solving the problem shows that C_1 = 45.0 K/m and C_2 = 293.3 K. The solution at each node is obtained:

```
duplicate i=1,N
    T_an[i]=-g```_dot*x[i]^2/(2*k_h)+C_1*x[i]+C_2
end
```

Figure 7 illustrates the analytical and numerical solutions and shows that they agree.

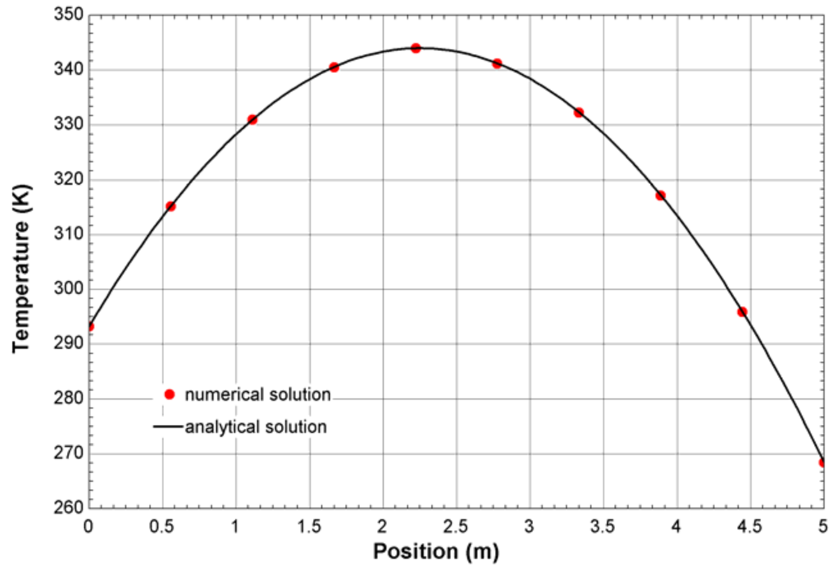

Figure 7: Numerical and analytical solutions in the limit that $\dot{g}'''$ is constant.

Temperatures above T_{fire}= 200°F are considered to be a fire hazard and temperatures above T_d = 140°F will result in a degradation of the hay to the point where it is not usable.

e.) What is the maximum allowable thickness of hay (L_{max}) based on keeping the maximum temperature below T_{fire}?

You can either manually adjust L until the variable T_{max} is equal to T_{fire} or simply set T_{max} and comment out the assignment of the variable L and let EES automatically determine the correct value (note that you need to return the generation function to its original state).

```
{L=5.0 [m]}                         "thickness of hay"
T_fire=converttemp(F,K,200)                               "combustion temperature"
T_max=T_fire
```

Which leads to L_{max} = 3.615 m.

f.) If $L = L_{max}$ from (e) then how much of the hay will remain usable (what percent of the hay is lost to heat degradation)?

Figure 8 illustrates the temperature distribution for L = 3.615 m and shows the extent of the region of the heat damaged hay.

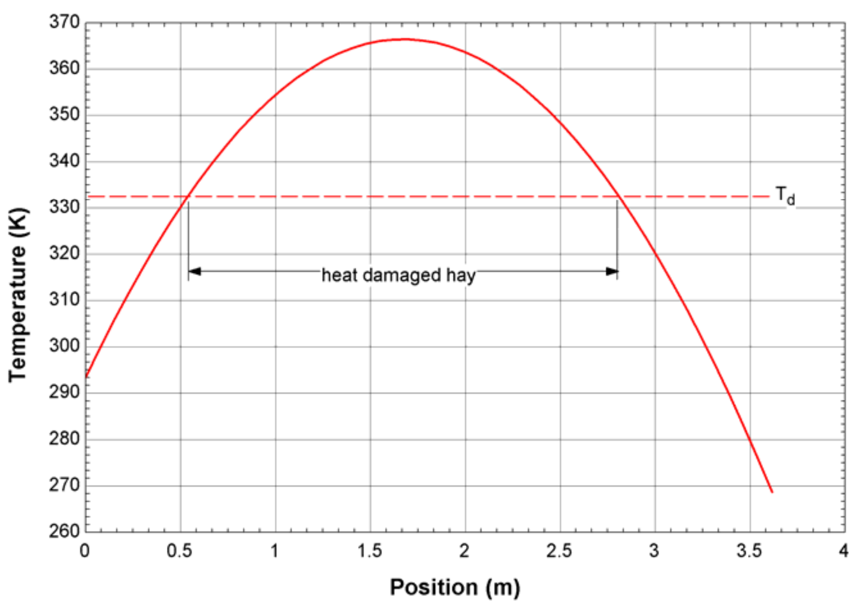

Figure 8: Temperature distribution for L = 3.615 m.

The region of usable hay extends from 0 to 0.63 m and from 2.73 m to 3.62 m. Therefore, only 43% of the hay will be useable when it is removed from the bar. Note that simple calculations like this can be done easily using the Calculator function in EES (select Calculator from the Windows menu). The calculator environment includes all of the variables from the last run of EES. Therefore, typing ?L returns 3.615 (Figure 9).

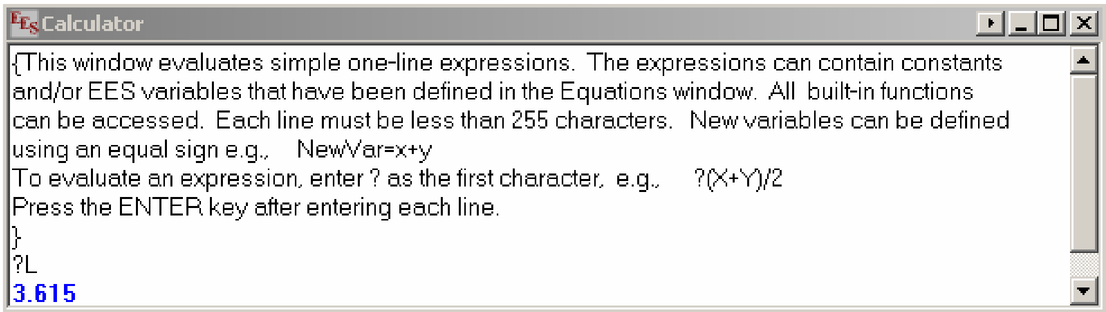

Figure 9: Calculator window.

To calculate the efficiency of the storage process, use the Calculator window as shown in Figure 10.

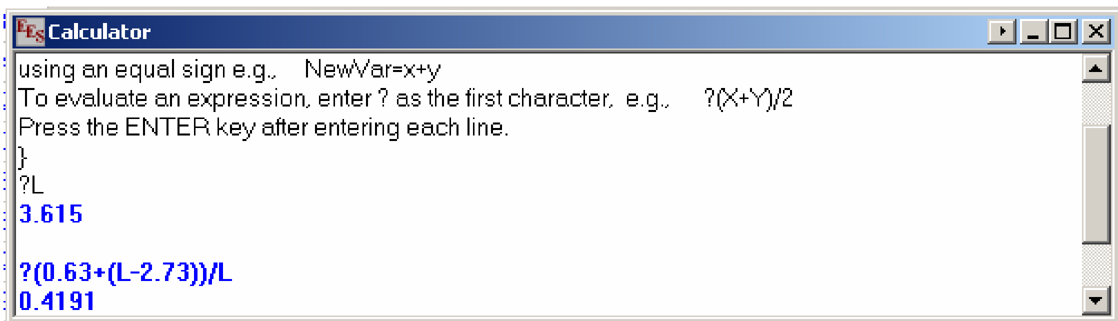

Figure 10: Calculator window.