Problem 1.9-5
An A triangular fin is shown in Figure P1.9-5.

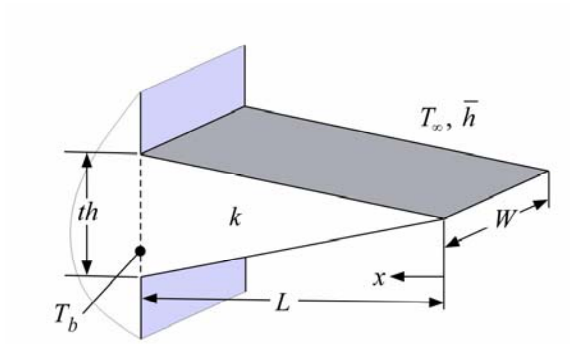

Figure P1.9-5: Wedge fin

The fin infinitely long (in the *z*-direction) and can be treated as an extended surface. The thickness of the fin base is th = 1 cm and the length is L = 10 cm. The conductivity of the material is k = 24 W/m-K. The base temperature is T_b = 140°C and the ambient temperature is T_∞ = 25°C. The heat transfer coefficient is $\bar{h}$ = 15 W/m^2-K. The width of the fin, W, is much larger than its length.
a.) Develop a numerical model of the fin.

The inputs are entered in EES:

```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
L=10 [cm]*convert(cm,m)                    "length of fin"
th=1 [cm]*convert(cm,m)                    "base thickness"
k=24 [W/m-K]                               "conductivity"
T_infinity=converttemp(C,K,25 [C])         "ambient temperature"
T_b=converttemp(C,K,140 [C])               "base temperature"
h_bar=15 [W/m^2-K]                         "heat transfer coefficient"
W=1 [m]                                    "per unit width of fin"
```

The nodes are positioned along the fin according to:

$$x_i = \frac{(i-1)L}{(N-1)} \quad \text{for } i = 1..N \tag{1}$$

where N is the number of nodes. The distance between adjacent nodes is:

$$\Delta x = \frac{L}{(N-1)} \tag{2}$$

The cross-sectional area for conduction at each node is:

$$A_{c,i} = \frac{th\,W\,x_i}{L} \quad \text{for } i = 1..N \tag{3}$$

```
"Let x=0 at the tip"
N=11 [-]                                        "number of nodes"
duplicate i=1,N
     x[i]=L*(i-1)/(N-1)                         "position"
     Ac[i]=x[i]*th*W/L                          "area"
end
Dx=L/(N-1)                                      "distance between nodes"
```

The total surface area available for convection is:

$$A_s = 2\,W\sqrt{L^2 + \left(\frac{th}{2}\right)^2} \tag{4}$$

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A_s=2*sqrt(L^2+(th/2)^2)*W                      "surface area"
```

The temperature at the base is fixed:

$$T_N = T_b \tag{5}$$

Energy balances on the internal nodes are:

$$A_s\,\frac{\Delta x}{L}\,\bar{h}\left(T_\infty - T_i\right) + k\,\frac{\left(A_{c,i} + A_{c,i+1}\right)}{2\,\Delta x}\left(T_{i+1} - T_i\right) + k\,\frac{\left(A_{c,i} + A_{c,i-1}\right)}{2\,\Delta x}\left(T_{i-1} - T_i\right) = 0 \quad \text{for } i = 2..\left(N-1\right) \tag{6}$$

An energy balance on the node at the tip is:

$$A_s\,\frac{\Delta x}{2\,L}\,\bar{h}\left(T_\infty - T_i\right) + k\,\frac{\left(A_{c,1} + A_{c,2}\right)}{2\,\Delta x}\left(T_2 - T_1\right) = 0 \tag{7}$$

```
T[N]=T_b                                        "base temperature"
"internal node energy balances"
duplicate i=2,(N-1)
     A_s*Dx/L*h_bar*(T_infinity-T[i])+k*(Ac[i]+Ac[i+1])*(T[i+1]-T[i])/(2*Dx)+k*(Ac[i]+Ac[i-1])*(T[i-1]-
T[i])/(2*Dx)=0
end
A_s*Dx/L*h_bar*(T_infinity-T[1])/2+k*(Ac[1]+Ac[2])*(T[2]-T[1])/(2*Dx)=0
```

The solution is converted to Celsius.

```
duplicate i=1,N
     T_C[i]=converttemp(K,C,T[i])
end
```

b.) Plot the temperature distribution within the fin.

Figure 2 illustrates the temperature as a function of position (recall that x is measured from the tip of the fin).

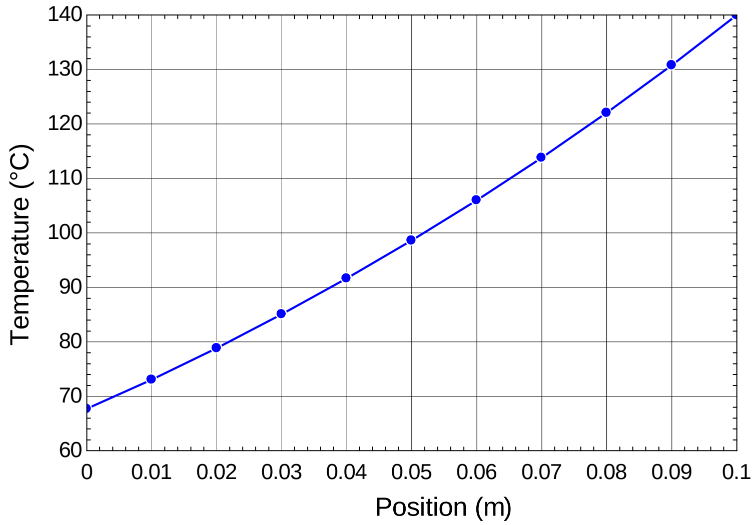

Figure 2: Temperature distribution in the fin.

c.) Determine the fin efficiency. Compare your answer with the fin efficiency obtained from the EES function eta_fin_straight_triangular.

The rate of heat transfer to the fin base is obtained by carrying out an energy balance on node N.

$$\dot{q} = A_s \frac{\Delta x}{2\,L} \bar{h} \left(T_N - T_\infty\right) + k \frac{\left(A_{c,N} + A_{c,N-1}\right)}{2\,\Delta x} \left(T_N - T_{N-1}\right) \tag{8}$$

The maximum possible heat transfer is:

$$\dot{q}_{max} = A_s \, \bar{h} \left(T_b - T_\infty\right) \tag{9}$$

The fin efficiency is:

$$\eta = \frac{\dot{q}}{\dot{q}_{max}} \tag{10}$$

```
q_dot=A_s*Dx*h_bar*(T[N]-T_infinity)/(2*L)+k*(Ac[N]+Ac[N-1])*(T[N]-T[N-1])/(2*Dx)
      "actual heat transfer rate"
q_dot_max=A_s*h_bar*(T_b-T_infinity)                    "maximum possible heat transfer rate"
eta=q_dot/q_dot_max                                     "fin efficiency"
```

which leads to η = 0.6561. The EES function eta_fin_straight_triangular is also determined:

```
eta_EES=eta_fin_straight_triangular(th,L,h_bar,k)       "fin efficiency from EES"
```

which leads to $\eta_{EES} = 0.6556$.

d.) Plot the fin efficiency as a function of the number of nodes used in the solution.

Figure 3 illustrates the predicted efficiency as a function of the number of nodes and suggests that you must use at least 10 nodes.

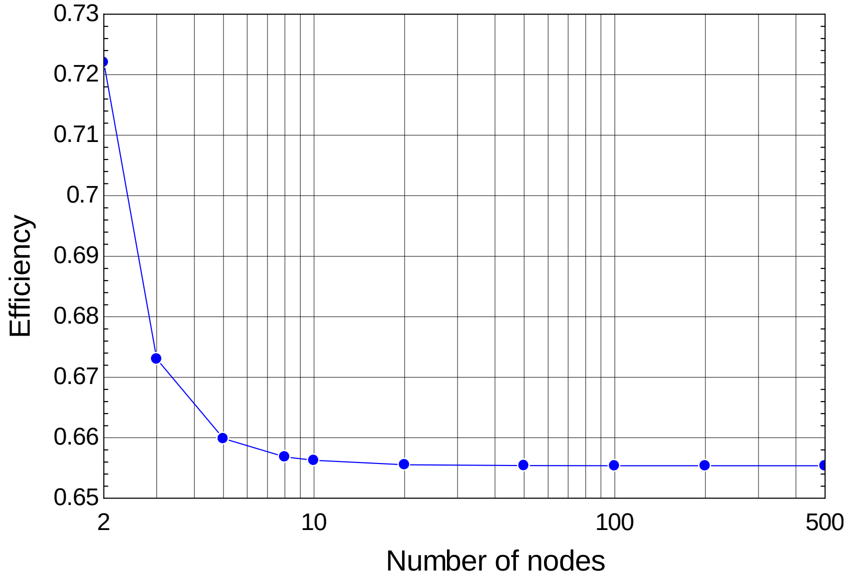

Figure 3: Predicted efficiency as a function of number of nodes.