Problem 1.3-4: Cylinder Boundary Conditions

A cylinder with conductivity k experiences a uniform rate of volumetric generation $\dot{g}'''$, as shown in Figure P1.3-4. The cylinder experiences 1-D, steady state conduction heat transfer in the radial direction and therefore the general solution to the ordinary differential equation for temperature (T) is:

$$T = -\frac{\dot{g}''' r^2}{4k} + C_1 \ln\left(r\right) + C_2 \tag{1}$$

where r is the radial location and C_1 and C_2 are undetermined constants. At the inner radius of the cylinder ($r = r_{in}$), a heater applies a uniform rate of heat transfer, $\dot{q}_{in}$. At the outer radius of the cylinder ($r = r_{out}$), the temperature is fixed at T_{out}. The length of the cylinder is L. Write the two algebraic equations that can be solved in order to obtain the constants C_1 and C_2. Your equations must contain only the following symbols in the problem statement: $\dot{q}_{in}$, T_{out}, k, r_{in}, r_{out}, L, $\dot{g}'''$, C_1, and C_2. Do not solve these equations.

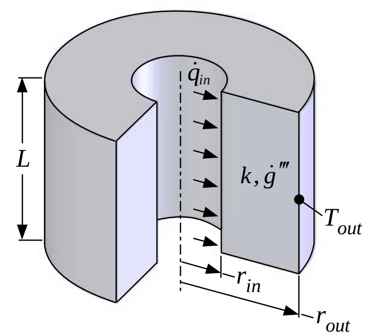

Figure P1.3-4: Cylinder with uniform volumetric generation.

At the outer surface, the temperature is specified and therefore the boundary condition is:

$$T_{out} = -\frac{\dot{g}''' r_{out}^2}{4k} + C_1 \ln\left(r_{out}\right) + C_2 \tag{2}$$

At the inner surface, the temperature is not specified and therefore it is necessary to do an energy balance on this interface, as shown in Figure 2.

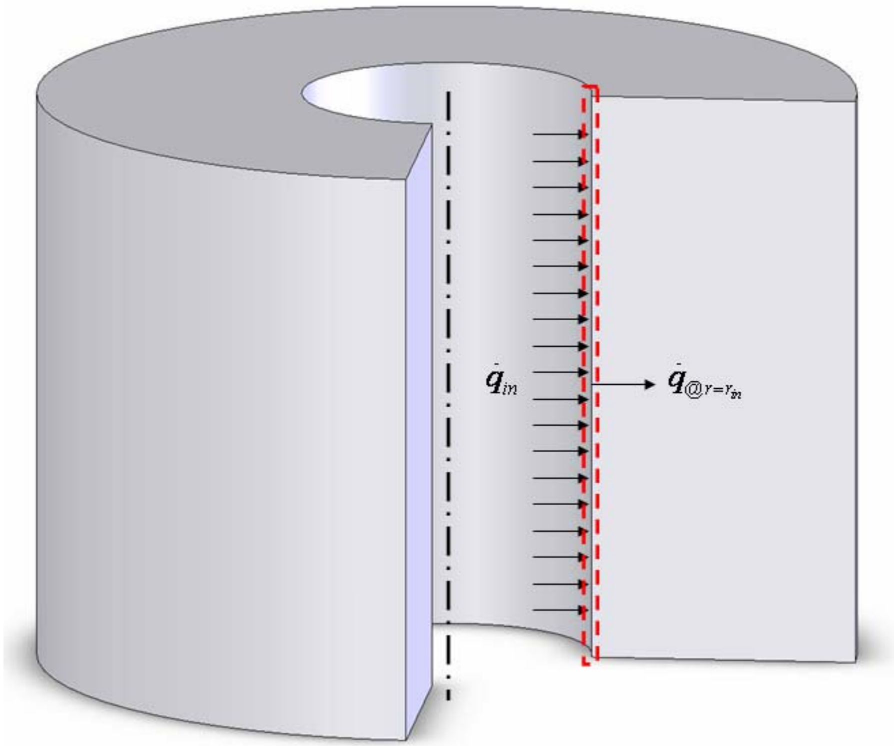

Figure 2: Interface balance at $r = r_{in}$.

The interface energy balance is:

$$\dot{q}_{in} = \dot{q}_{@r=r_{in}} \tag{3}$$

Substituting Fourier's law for $\dot{q}_{@r=r_{in}}$ leads to:

$$\dot{q}_{in} = -k\,2\,\pi\,r_{in}\,L\left.\frac{dT}{dr}\right|_{r=r_{in}} \tag{4}$$

Substituting the general solution, Eq. (1), into Eq. (4) leads to:

$$\dot{q}_{in} = -k\,2\,\pi\,r_{in}\,L\left[-\frac{\dot{g}'''\,r_{in}}{2\,k}+\frac{C_1}{r_{in}}\right] \tag{5}$$