Problem 1.3-11: Nuclear Fuel Element

Figure P1.3-11 illustrates a spherical, nuclear fuel element which consists of a sphere of fissionable material (fuel) with radius r_{fuel} = 5 cm and k_{fuel} = 2 W/m-K that is surrounded by a spherical shell of metal cladding with outer radius r_{clad} = 7 cm and k_{clad} = 0.25 W/m-K. The outer surface of the cladding is exposed to fluid that is being heated by the reactor. The convection coefficient between the fluid and the cladding surface is $\overline{h}$ = 50 W/m^2-K and the temperature of the fluid is T_∞ = 500°C. Neglect radiation heat transfer from the surface.

Inside the fuel element, thermal energy is being generated for the reactor. This process can be modeled as a volumetric source of heat generation in the material that is not uniform throughout the fuel. The volumetric generation ($\dot{g}'''$) can be approximated by the function:

$$\dot{g}''' = \frac{\beta}{r}$$

where β = 5x10^3 W/m^2.

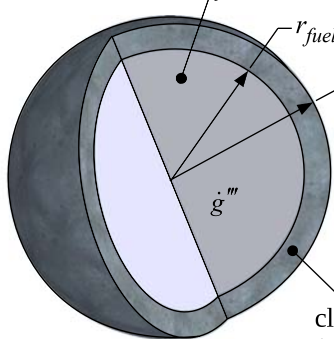

Figure P1.3-11: Spherical fuel element surrounded by cladding

a.) Determine an analytical solution for the temperature distribution within the fuel element. Implement your solution in EES and plot the temperature as a function of radius for 0 < r < r_{fuel}.

The inputs are entered according to:

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$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"
r_fuel=5 [cm]*convert(cm,m)                    "radius of fuel element"
k_fuel=2 [W/m-K]                               "conductivity of fuel element"
r_clad=7 [cm]*convert(cm,m)                    "radius of cladding"
k_clad=0.25 [W/m-K]                            "conductivity of cladding"
h_bar=50 [W/m^2-K]                             "heat transfer coefficient"
T_infinity=converttemp(C,K,500[C])             "temperature of fluid"
beta=5e3 [W/m^2]                               "constant for volumetric generation"
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A differential control volume is shown in Figure 2 and includes conduction at *r* and *r*+*dr* at the inner and outer surfaces of the spherical shell as well as generation within the enclosed volume.

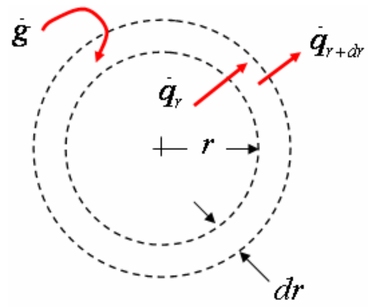

Figure 2: Differential control volume

The energy balance suggested by Figure 2 is:

$$\dot{q}_r + \dot{g} = \dot{q}_{r+dr} \tag{1}$$

The term at $r + dr$ can be expanded:

$$\dot{q}_{r+dr} = \dot{q}_r + \frac{d\dot{q}_r}{dr} dr \tag{2}$$

and substituted into Eq. (1):

$$\dot{q}_r + \dot{g} = \dot{q}_r + \frac{d\dot{q}_r}{dr} dr \tag{3}$$

and simplified:

$$\dot{g} = \frac{d\dot{q}_r}{dr} dr \tag{4}$$

The conduction is governed by Fourier's Law:

$$\dot{q}_r = -k_{fuel}\, 4\pi r^2 \frac{dT}{dr} \tag{5}$$

and the generation is the product of the volume and the local generation rate:

$$\dot{g} = 4\pi r^2\, dr\, \dot{g}''' = 4\pi r^2\, dr \frac{\beta}{r} \tag{6}$$

The rate equations, Eqs. (5) and (6), are substituted into Eq. (4):

$$4\pi\, r\, dr\, \beta = \frac{d}{dr}\left[-k_{fuel}\, 4\pi\, r^2 \frac{dT}{dr} \right] dr \tag{7}$$

which can be simplified:

$$\frac{d}{dr}\left[r^2 \frac{dT}{dr} \right] = -\frac{r\,\beta}{k_{fuel}} \tag{8}$$

Equation (8) can be separated and integrated:

$$\int d\left[r^2 \frac{dT}{dr} \right] = \int -\frac{r\,\beta}{k_{fuel}}\, dr \tag{9}$$

which leads to:

$$r^2 \frac{dT}{dr} = -\frac{r^2\,\beta}{2\,k_{fuel}} + C_1 \tag{10}$$

where C_1 is a constant of integration. Equation (10) can be separated and integrated:

$$\int dT = \int \left(-\frac{\beta_o}{2\,k_{fuel}} + \frac{C_1}{r^2} \right) dr \tag{11}$$

which leads to:

$$T = -\frac{\beta}{2\,k_{fuel}}\, r - \frac{C_1}{r} + C_2 \tag{12}$$

where C_2 is the second constant of integration. The boundary condition at $r = 0$ requires that the temperature remain finite and therefore $C_1 = 0$.

$$T = -\frac{\beta_o}{2\,k_{fuel}}\, r + C_2 \tag{13}$$

The boundary condition at $r = r_{fuel}$ is obtained using an interface balance, as show in Figure 3. The interface energy balance includes conduction from the fuel and heat transfer into the cladding.

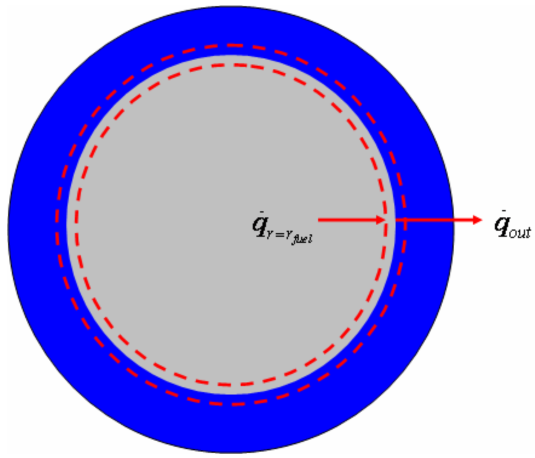

Figure 3: Interface balance at *r* = *r*

The energy balance suggested by Figure 3 is:

$$\dot{q}_{r=r_{ruel}} = \dot{q}_{out} \tag{14}$$

The conduction term on the left side of Eq. (14) is evaluated using Fourier's law:

$$\dot{q}_{r=r_{ruel}} = -k_{fuel}\, 4\,\pi\, r_{fuel}^2 \left.\frac{dT}{dr}\right|_{r=r_{fuel}} \tag{15}$$

while the heat transfer out of the cladding is driven by the difference between the temperature at interface between the fuel and the cladding and the temperature of the surrounding gas. The heat transfer is resisted by the sum of the conduction resistance of the cladding ($R_{cond,clad}$):

$$R_{cond,clad} = \frac{1}{4\,\pi\, k_{clad}}\left[\frac{1}{r_{fuel}} - \frac{1}{r_{clad}}\right] \tag{16}$$

and the convection resistance (R_{conv}):

$$R_{conv} = \frac{1}{4\,\pi\, r_{clad}^2\, \overline{h}} \tag{17}$$

so that:

$$\dot{q}_{out} = \frac{T_{r=r_{fuel}} - T_\infty}{R_{cond,clad} + R_{conv}} \tag{18}$$

Subsituting Eqs. (18) and (15) into Eq. (14) leads to:

$$-k_{fuel}\,4\,\pi\,r_{fuel}^2\,\frac{dT}{dr}\bigg|_{r=r_{fuel}} = \frac{\left(T_{r=r_{fuel}} - T_{\infty}\right)}{R_{cond,clad} + R_{conv}} \tag{19}$$

Equation (19) provides a single equation for the unknown constant of integration, C_2. Substituting Eq. (13) into Eq. (19) leads to:

$$-k_{fuel}\,4\,\pi\,r_{fuel}^2\left(-\frac{\beta}{2\,k_{fuel}}\right) = \frac{\left(-\frac{\beta}{2\,k_{fuel}}\,r_{fuel} + C_2 - T_{\infty}\right)}{R_{cond,clad} + R_{conv}} \tag{20}$$

The resistances are computed according to Eqs. (16) and (17) and the constant C_2 is computed according to Eq. (20).

```
R_cond_clad=(1/r_fuel-1/r_clad)/(4*pi*k_clad)        "resistance to conduction through cladding"
R_conv=1/(4*pi*r_clad^2*h_bar)                       "resistance to convection from cladding"
k_fuel*4*pi*r_fuel^2*(beta/(2*k_fuel))=(-beta*r_fuel/(2*k_fuel)+C_2-T_infinity)/(R_cond_clad+R_conv)
                                                     "boundary condition"
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The temperature distribution is obtained using Eq. (13).

```
r=0 [m]                                              "radius"
T=-beta*r/(2*k_fuel)+C_2                             "temperature distribution"
T_C=converttemp(K,C,T)                               "in C"
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Figure 4 illustrates the temperature in the sphere as a function of position.

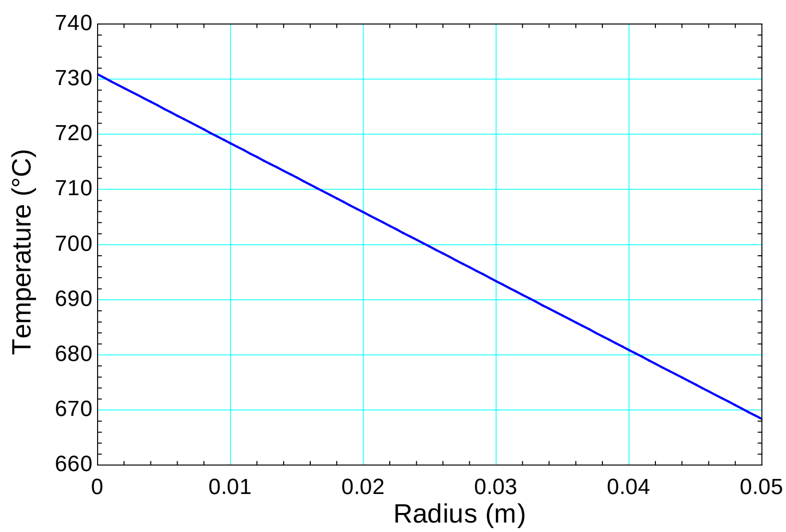

Figure 4: Temperature as a function of radius.

b.) The maximum allowable temperature in the fuel element is T_{max} = 1100ºC. What is the maximum value of β that can be used? What is the associated total rate that heat is transferred to the gas?

The maximum temperature occurs at r = 0. According to Eq. (13), the temperature at r = 0 is:

$$T_{r=0} = C_2 \tag{21}$$

The guess values are updated and the specified value of β is commented out. The temperature at the center is specified to be T_{max}:

```
{beta=5e3 [W/m^2]}                          "constant for volumetric generation"
T_max_s=converttemp(C,K,1100 [C])  "maximum allowable temperature"
T_max=C_2                                       "maximum temperature in fuel"
T_max_s=T_max                                 "adjust beta so that center temperature is equal to T_max_s"
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which leads to β = 1.3x10^4 W/m^2. The rate of heat transfer is given by Eq. (18):

$$\dot{q}_{out} = \frac{T_{r=r_{fuel}} - T_\infty}{R_{cond,clad} + R_{conv}} = \frac{-\dfrac{\beta_o}{2\,k_{fuel}} r_{fuel} + C_2 - T_\infty}{R_{cond,clad} + R_{conv}} \tag{22}$$

```
q_dot=(-beta*r_fuel/(2*k_fuel)+C_2-T_infinity)/(R_cond_clad+R_conv) "heat transfer from fuel"
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which leads to $\dot{q}_{out}$ = 204.1 W.

c.) You are designing the fuel elements. You can vary r_{fuel} and β. The cladding must always be 2 cm thick (that is r_{clad} = r_{fuel} + 2 cm). The constraint is that the fuel temperature cannot exceed T_{max} = 1100ºC and the design target (the figure of merit to be maximized) is the rate of heat transfer per unit volume of material (fuel and cladding). What values of r_{fuel} and β are optimal?

The volume of the fuel and the cladding is:

$$V = \frac{4}{3} \pi\, r_{clad}^3 \tag{23}$$

and therefore the heat transfer per unit volume can be determined.

```
V=4*pi*r_clad^3/3                                   "volume of fuel"
q_dot\V=q_dot/V                                     "heat transfer per volume"
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The cladding radius is specified based on the fuel radius:

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r_clad=r_fuel+2 [cm]*convert(cm,m)            "radius of cladding"
{r_clad=7 [cm]*convert(cm,m)                   "radius of cladding"}
```

The fuel radius is varied in a parametric table and the heat transfer per unit volume as a function of fuel radius is shown in Figure 5.

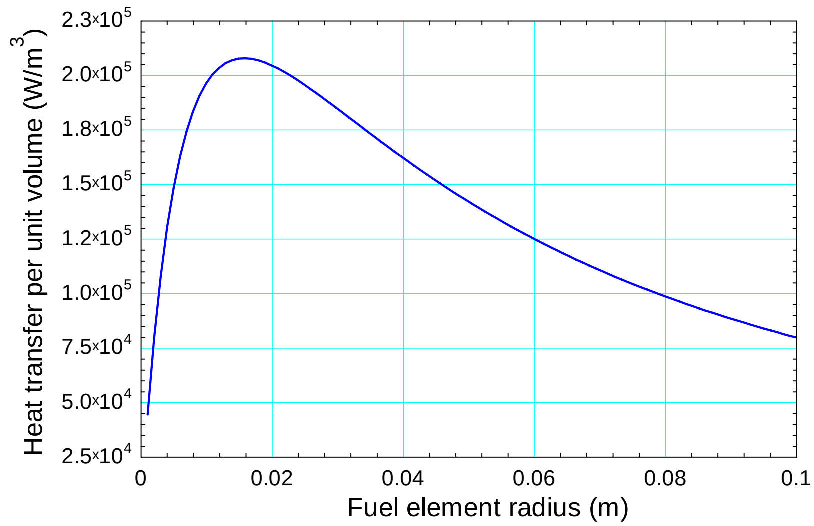

Figure 5: Heat transfer per unit volume as a function of fuel element radius.

Figure 5 shows that the optimal value of r_{fuel} is approximately 1.6 cm. A more exact value can be obtained using the Min/Max feature from the Calculate menu. The optimal design is r_{fuel} = 1.57 cm with $\beta = 2.6\text{x}10^4$ W/m^2.