Problem 1.7-2: Rotating Ring

Figure P1.7-2 illustrates a metal ring of radius R = 5.0 cm that is rotating with an angular velocity ω = 0.1 rad/s. During each rotation, the ring material passes from compartment #1 containing hot fluid at $T_{f,1}$ = 200ºC to compartment #2 containing fluid at $T_{f,2}$ = 20ºC. The heat transfer coefficient between the ring surface and the fluid in compartments #1 and #2 are $\overline{h}_1$ = 10 W/m^2-K and $\overline{h}_2$ = 20 W/m^2-K, respectively. The ring has a circular cross-section with diameter d = 1.0 mm. For this problem you can assume that d/R<<1. The metal has conductivity k = 100 W/m-K, density ρ = 2700 kg/m^3, and specific heat capacity c = 900 J/kg-K.

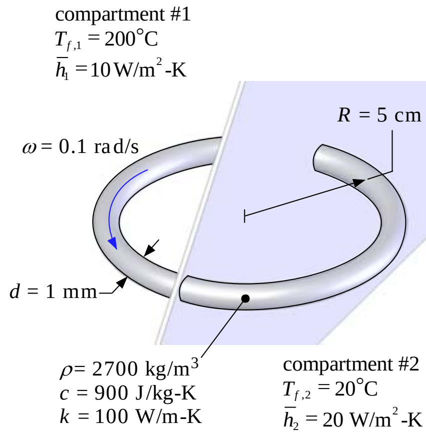


Figure P1.7-2: Metal ring rotates between two compartments.

a.) Is it appropriate to treat the ring as an extended surface?

The input parameters are entered into EES.

```
$UnitSystem SI MASS RAD PA K J
$Tabstops 0.2 0.4 0.6 3.5 in

"Inputs"
T_f_1=converttemp(C,K,200 [C])       "temperature of fluid in compartment #1"
T_f_2=converttemp(C,K,20 [C])        "temperature of fluid in compartment #2"
h_1=10 [W/m^2-K]                     "heat transfer coefficient in compartment #1"
h_2=20 [W/m^2-K]                     "heat transfer coefficient in compartment #2"
R=5.0 [cm]*convert(cm,m)             "radius of ring"
d=1.0 [mm]*convert(mm,m)             "diameter of ring"
omega=0.1 [rad/s]                    "rate of ring rotation"
rho=2700 [kg/m^3]                    "density"
c=900 [J/kg-K]                       "specific heat capacity"
k=100 [W/m-K]                        "thermal conductivity"
```

The Biot number associated with the ring in compartment #2 will be the largest (the heat transfer coefficient is larger in compartment #2) and therefore limit the applicability of the extended surface model. An appropriate Biot number is:

$$Bi = \frac{h_2\, d}{2\, k}$$

which is calculated in EES:

```
"Biot number calculation"
Bi=h_2*d/(2*k)                              "Biot number in compartment #2"
```

and found to be 0.0001; clearly there will be almost no temperature gradient within the ring between the center and the surface. The temperature of the ring can be considered to be a function only of the linear coordinate *s* that follows the circumference of the ring (see Figure 1).

b.) Develop an analytical model that can predict the temperature distribution in the ring.

A differential control volume for the ring is considered in Figure 2.

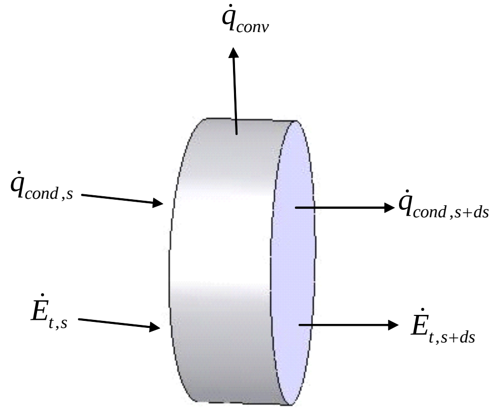


Figure 2: Differential control volume for ring material.

The energy balance for the differential control volume is:

$$\dot{q}_{cond,s} + \dot{E}_{t,s} = \dot{q}_{conv} + \dot{q}_{cond,s} + \frac{d\dot{q}_{cond}}{ds}\, ds + \dot{E}_{t,s} + \frac{d\dot{E}_t}{ds}\, ds$$

where $\dot{q}_{cond}$, $\dot{q}_{conv}$, and $\dot{E}_t$ are the energy transfers due to conduction, convection, and the motion of the material, respectively; the rate equations for these terms in compartment #1 are:

$$\dot{q}_{cond} = -k\, A_c \frac{dT_1}{ds}$$

$$\dot{q}_{conv} = p\, h_1\, ds\left(T_1 - T_{f,1}\right)$$

$$\dot{E}_t = \omega\, R\, A_c\, \rho\, c\, T_1$$

where A_c and p are the cross-sectional area and perimeter of the ring:

$$A_c = \frac{\pi\, d^2}{4}$$

$$p = \pi\, d$$

Combining these equations leads to:

$$0 = p\, h_1\, ds\left(T_1 - T_{f,1}\right) + \frac{d}{ds}\left[-k\, A_c\, \frac{dT_1}{ds}\right] ds + \frac{d}{ds}\left[\omega\, R\, A_c\, \rho\, c\, T_1\right] ds$$

which can be simplified:

$$k\, A_c\, \frac{d^2 T_1}{ds^2} - \omega\, R\, A_c\, \rho\, c\, \frac{dT_1}{ds} - p\, h_1\left(T_1 - T_{f,1}\right) = 0$$

or

$$\frac{d^2\theta_1}{ds^2} - \frac{\omega\, R}{\alpha}\frac{d\theta_1}{ds} - m_1^2\, \theta_1 = 0 \qquad (1)$$

where α is the thermal diffusivity of the ring material and m_1 is the fin parameter for compartment #1:

$$m_1 = \sqrt{\frac{p\, h_1}{k\, A_c}}$$

and θ_1 is the ring-to-fluid temperature difference in compartment #1:

$$\theta_1 = T_1 - T_{f,1}$$

Equation (1) is the governing differential equation for the ring material in compartment #1 and therefore applies from $s = \pi R$ to $s = 2\pi R$. A similar set of steps leads to a governing differential equation for the temperature of the ring material in compartment #2, T_2, that is valid from $s = 0$ to $s = \pi R$:

$$\frac{d^2\theta_2}{ds^2} - \frac{\omega\, R}{\alpha}\frac{d\theta_2}{ds} - m_2^2\, \theta_2 = 0 \qquad (2)$$

where m_2 is the fin parameter for compartment #2:

$$m_2 = \sqrt{\frac{p\,h_2}{k\,A_c}}$$

and θ_2 is the ring-to-fluid temperature difference in compartment #2:

$$\theta_2 = T_2 - T_{f,2}$$

The general solutions to the governing differential equations can be obtained using Maple or simply copied from Section 1.7.3. In compartment #1 the solution is:

$$T_1 = T_{f,1} + C_1 \exp\left(\lambda_1\, s\right) + C_2 \exp\left(\lambda_2\, s\right) \tag{3}$$

where

$$\lambda_1 = \frac{\omega\, R}{2\,\alpha} + \sqrt{\frac{1}{4}\left(\frac{\omega\, R}{\alpha}\right)^2 + m_1^2}$$

$$\lambda_2 = \frac{\omega\, R}{2\,\alpha} - \sqrt{\frac{1}{4}\left(\frac{\omega\, R}{\alpha}\right)^2 + m_1^2}$$

and in compartment #2 the solution is:

$$T_2 = T_{f,2} + C_3 \exp\left(\beta_1\, s\right) + C_4 \exp\left(\beta_2\, s\right) \tag{4}$$

where

$$\beta_1 = \frac{\omega\, R}{2\,\alpha} + \sqrt{\frac{1}{4}\left(\frac{\omega\, R}{\alpha}\right)^2 + m_2^2}$$

$$\beta_2 = \frac{\omega\, R}{2\,\alpha} - \sqrt{\frac{1}{4}\left(\frac{\omega\, R}{\alpha}\right)^2 + m_2^2}$$

These equations are entered in EES but they cannot be solved until the 4 undetermined constants, C_1 through C_4, are determined using the boundary conditions:

```
"Solution Parameters"
p=pi*d                                                  "Perimeter of ring"
A_c=pi*d^2/4                                            "Cross-sectional area of ring"
m_1=sqrt(p*h_1/(k*A_c))          "Fin constant in compartment #1"
m_2=sqrt(p*h_2/(k*A_c))          "Fin constant in compartment #2"
alpha=k/(rho*c)                              "Thermal diffusivity of ring material"
lambda_1=omega*R/(2*alpha)+sqrt((omega*R/alpha)^2/4+m_1^2)     "solution constants"
```

```
lambda_2=omega*R/(2*alpha)-sqrt((omega*R/alpha)^2/4+m_1^2)
beta_1=omega*R/(2*alpha)+sqrt((omega*R/alpha)^2/4+m_2^2)
beta_2=omega*R/(2*alpha)-sqrt((omega*R/alpha)^2/4+m_2^2)

"Solution"
theta_1=C_1*exp(lambda_1*s)+C_2*exp(lambda_2*s)     "ring-to-fluid temp. dif. in comp. #1"
T_1=T_f_1+theta_1                                   "ring temperature in comp. #1"
theta_2=C_3*exp(beta_1*s)+C_4*exp(beta_2*s)         "ring-to-fluid temp. dif. in comp. #2"
T_2=T_f_2+theta_2                                   "ring temperature in comp. #2"
```

There are 4 constants (2 for each 2nd order differential equation) and therefore 4 boundary conditions are required to provide a solution. There can be no discontinuity in the temperature at the interfaces of the compartments; therefore, the temperature of the material leaving compartment #1 is the same as the temperature of the material entering compartment #2. The same requirement applies for the material leaving compartment #2 and entering compartment #1.

$$T_1\big|_{s=\pi R} = T_2\big|_{s=\pi R} \tag{5}$$

$$T_1\big|_{s=2\pi R} = T_2\big|_{s=0} \tag{6}$$

which leads to:

$$T_{f,1} + C_1 \exp\left(\lambda_1 \pi R\right) + C_2 \exp\left(\lambda_2 \pi R\right) = T_{f,2} + C_3 \exp\left(\beta_1 \pi R\right) + C_4 \exp\left(\beta_2 \pi R\right)$$

$$T_{f,1} + C_1 \exp\left(\lambda_1 2\pi R\right) + C_2 \exp\left(\lambda_2 2\pi R\right) = T_{f,2} + C_3 + C_4$$

These boundary conditions are entered in EES:

```
"Boundary Conditions"
"Temperature continuity"
T_f_1+C_1*exp(lambda_1*pi*R)+C_2*exp(lambda_2*pi*R)=T_f_2+C_3*exp(beta_1*pi*R)+C_4*exp(beta
_2*pi*R)
T_f_1+C_1*exp(lambda_1*2*pi*R)+C_2*exp(lambda_2*2*pi*R)=T_f_2+C_3+C_4
```

The remaining boundary conditions are determined by an energy balance at the interfaces between the compartments. The rate of energy leaving compartment #2 at $s = \pi R$ must be equal to the rate of energy entering compartment #1 at $s = \pi R$ and also at $s = 0$ in compartment #2 and $s = 2\pi R$ in compartment #1. Energy is transferred through the ring due to motion of ring material as well as conduction; therefore, these energy balances become:

$$\omega R A_c \rho c T_1\big|_{s=\pi R} - k A \frac{dT_1}{ds}\bigg|_{s=\pi R} = \omega R A \rho c T_2\big|_{s=\pi R} - k A \frac{dT_2}{ds}\bigg|_{s=\pi R}$$

Equation (5) results in the 1st term on either side canceling. Since the temperature at the interface is continuous, the rate of energy transfer by ring motion must be the same at the interface regardless if it is determined from compartment #1 or compartment #2. Therefore, the temperature gradient at the interface must be continuous:

$$\left.\frac{dT_1}{ds}\right|_{s=\pi R} = \left.\frac{dT_2}{ds}\right|_{s=\pi R} \tag{7}$$

which leads to:

$$C_1\,\lambda_1 \exp\left(\lambda_1\,\pi\,R\right) + C_2\,\lambda_2 \exp\left(\lambda_2\,\pi\,R\right) = C_3\,\beta_1 \exp\left(\beta_1\,\pi R\right) + C_4\,\beta_2 \exp\left(\beta_2\,\pi R\right)$$

The temperature gradient at the other interface must also be continuous:

$$\left.\frac{dT_1}{ds}\right|_{s=2\pi R} = \left.\frac{dT_2}{ds}\right|_{s=0} \tag{8}$$

which leads to:

$$C_1\,\lambda_1 \exp\left(\lambda_1\,2\,\pi\,R\right) + C_2\,\lambda_2 \exp\left(\lambda_2\,2\,\pi\,R\right) = C_3\,\beta_1 + C_4\,\beta_2$$

These boundary conditions are entered in EES:

```
"Temperature gradient continuity"
C_1*lambda_1*exp(lambda_1*pi*R)+C_2*lambda_2*exp(lambda_2*pi*R)=C_3*beta_1*exp(beta_1*pi*R)
+C_4*beta_2*exp(beta_2*pi*R)
C_1*lambda_1*exp(lambda_1*2*pi*R)+C_2*lambda_2*exp(lambda_2*2*pi*R)=C_3*beta_1+C_4*beta_2
```

If the parameter s is set to some value (e.g., $s = 0$) then there are an equal number of equations and unknowns and it should be possible obtain the unknown constants; however, it is likely that solving the problem (Calculate/Solve) will lead to the Error dialog shown in Figure 3.

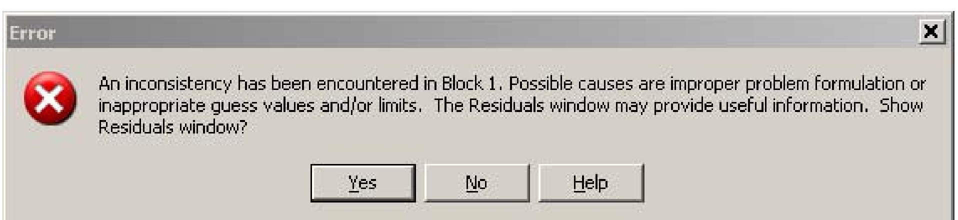


Figure 3: Error dialog.

The problem is that the initial guess values for the constants are sufficiently far from the actual values that the process of simultaneously solving the 4 non-linear coupled equations associated with the boundary conditions does not converge. Select Yes in the Error dialog to view the residuals, Figure 4. The Residuals window provides the most useful information available in the event that a calculation is not successful. The first column in the Residuals window indicates the Block number; a Block is a set of equations that must be solved together because they are dependent. Equations that are designated to be in block 0 are equations that contain a single unknown variable. The variable that is being solved for is shown in bold font. The equations in block 0 are solved in the order that they appear in the Residuals window, regardless of the order that they were entered in the Equations window. Also shown in the Residuals window are the relative and absolute values of the residuals for each equation. The absolute residual is the difference between the left and right side of the equations. The relative residual is the ratio of the absolute residual to the value terms on the left side of the equations (assuming that they are not identically zero). If the equation is properly solved, the residual should be zero, but it may be some small value depending on the stop criteria tolerances that have been set.

There are a total of 24 equations in 2 blocks in the Main program.

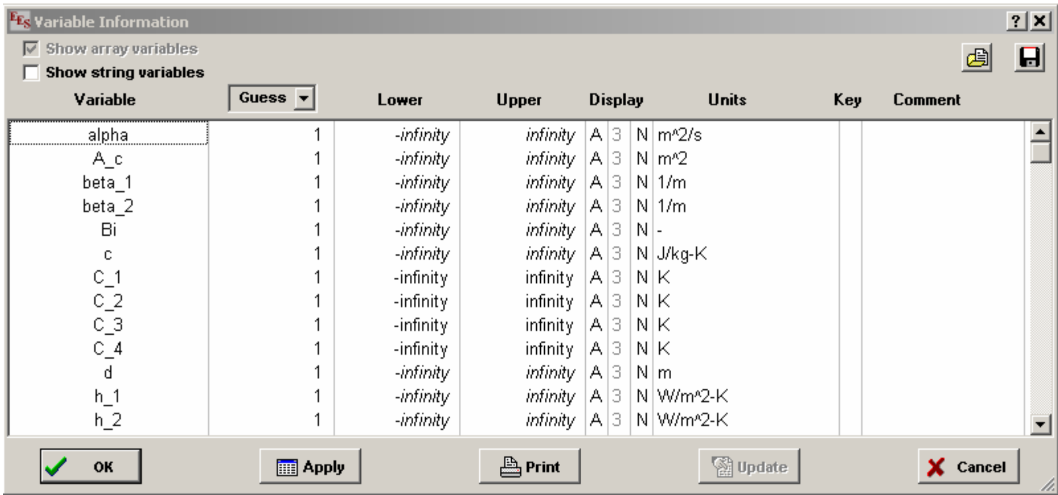
Block	Rel. Res.	Abs. Res.	Units	Calls	Equations
0	0.000E+00	0.000E+00	OK	1	**h_1**=10[0]
0	0.000E+00	0.000E+00	OK	1	**h_2**=20[0]
0	0.000E+00	0.000E+00	OK	1	**omega**=0.1[0]
0	0.000E+00	0.000E+00	OK	1	**rho**=2700[0]
0	0.000E+00	0.000E+00	OK	1	**c**=900[0]
0	0.000E+00	0.000E+00	OK	1	**k**=100[0]
0	0.000E+00	0.000E+00	OK	4	**T_f_1**=converttemp(C,K,200[900])
0	0.000E+00	0.000E+00	OK	4	**T_f_2**=converttemp(C,K,20[900])
0	0.000E+00	0.000E+00	OK	4	**R**=5.0[0]*convert(cm,m)
0	0.000E+00	0.000E+00	OK	4	**d**=1.0[0]*convert(mm,m)
0	0.000E+00	0.000E+00	OK	4	**Bi**=h_2*d/(2*k)
0	0.000E+00	0.000E+00	OK	4	**p**=pi*d
0	0.000E+00	0.000E+00	OK	4	**A_c**=pi*d^2/4
0	0.000E+00	0.000E+00	OK	4	**m_1**=sqrt(p*h_1/(k*A_c))
0	0.000E+00	0.000E+00	OK	4	**m_2**=sqrt(p*h_2/(k*A_c))
0	0.000E+00	0.000E+00	OK	4	**alpha**=k/(rho*c)
0	0.000E+00	0.000E+00	OK	4	**lambda_1**=omega*R/(2*alpha)+sqrt((omega*R/alpha)^2+m_1^2)
0	0.000E+00	0.000E+00	OK	4	**lambda_2**=omega*R/(2*alpha)-sqrt((omega*R/alpha)^2+m_1^2)
0	0.000E+00	0.000E+00	OK	4	**beta_1**=omega*R/(2*alpha)+sqrt((omega*R/alpha)^2+m_2^2)
0	0.000E+00	0.000E+00	OK	4	**beta_2**=omega*R/(2*alpha)-sqrt((omega*R/alpha)^2+m_2^2)
1	2.184E-02	3.528E+10	OK	203	T_f_1+ **C_1***exp(lambda_1*pi*R)+ **C_2***exp(lambda_2*pi*R)=T_f_2+
1	1.930E+22	5.782E+24	OK	276	T_f_1+ **C_1***exp(lambda_1*2*pi*R)+ **C_2***exp(lambda_2*2*pi*R)=T_
1	1.295E-02	3.881E+12	OK	204	**C_1***lambda_1*exp(lambda_1*pi*R)+ **C_2***lambda_2*exp(lambda_
1	3.322E+24	1.063E+27	OK	276	**C_1***lambda_1*exp(lambda_1*2*pi*R)+ **C_2***lambda_2*exp(lambd

A bold block number indicates that the equation was not solved to tolerance.

Variables shown in bold font are determined by the equation(s) in each block.

Figure 4: Residuals window.

Scanning down the Residuals window in Figure 4, it can be seen that the equations in block 1 were not solved correctly. Note that there are four equations in this block, representing a system of four equations and four unknowns, C_1 through C_4. (Note that part of the equations in the figure shown below have been cut off.) These four equations must be solved simultaneously and that is why EES has placed them in the same block. Armed with the knowledge of where the calculation is failing, it is often possible to provide better guess values and/or limits in order to make the calculation succeed. The Variable information window (Options/Variable Info) provides information about the guess value and limits for each variable (Figure 5); the default guess value for every variable is 1 and the limits are from –infinity to infinity.



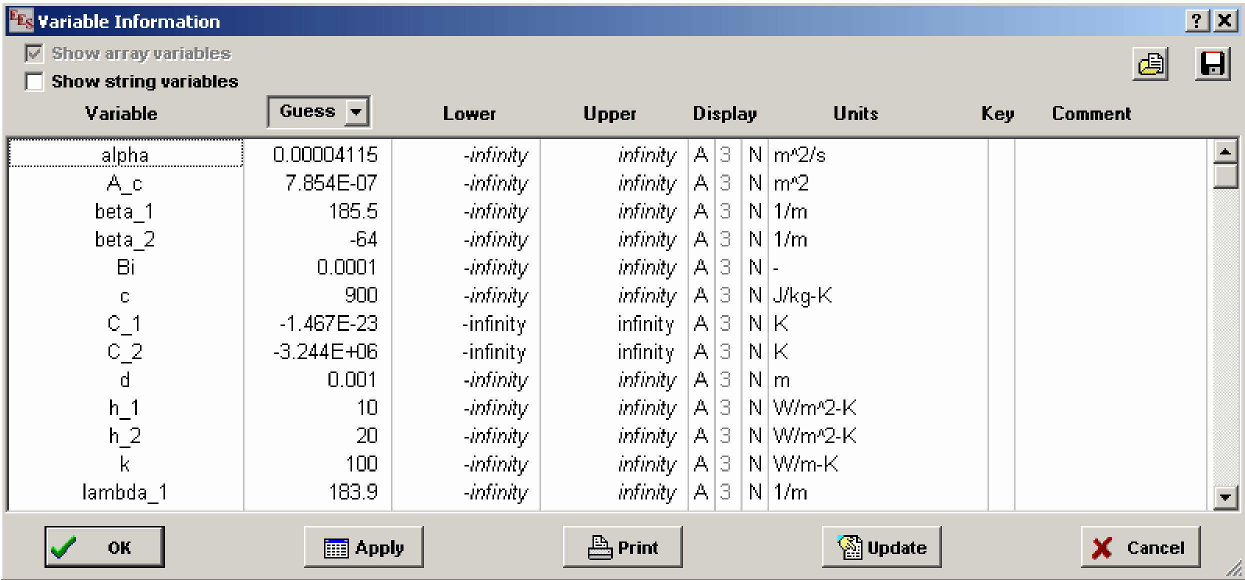
Variable	Guess	Lower	Upper	Display			Units	Key	Comment
alpha	1	-infinity	infinity	A	3	N	m^2/s		
A_c	1	-infinity	infinity	A	3	N	m^2		
beta_1	1	-infinity	infinity	A	3	N	1/m		
beta_2	1	-infinity	infinity	A	3	N	1/m		
Bi	1	-infinity	infinity	A	3	N	-		
c	1	-infinity	infinity	A	3	N	J/kg-K		
C_1	1	-infinity	infinity	A	3	N	K		
C_2	1	-infinity	infinity	A	3	N	K		
C_3	1	-infinity	infinity	A	3	N	K		
C_4	1	-infinity	infinity	A	3	N	K		
d	1	-infinity	infinity	A	3	N	m		
h_1	1	-infinity	infinity	A	3	N	W/m^2-K		
h_2	1	-infinity	infinity	A	3	N	W/m^2-K		

Figure 5: Variable Information window

Approximate values of the constants and their limits are not obvious for this problem, which makes it difficult to set more appropriate values. One technique for proceeding is simply to set the constants to a much larger or smaller value and see if that helps the problem to converge. A better technique might be to divide the problem into several smaller ones. For example, it is possible to consider only compartment #1 and set the constants so that the temperatures at the boundaries are equal to the fluid temperature in compartment #2. This is easily done by commenting out the 2nd set of boundary conditions and the right hand side of the 1st set (do not delete these boundary conditions as you will need them later.):

```
"Boundary Conditions"
"Temperature continuity"
T_f_1+C_1*exp(lambda_1*pi*R)+C_2*exp(lambda_2*pi*R)=T_f_2{+C_3*exp(beta_1*pi*R)+C_4*exp(beta_2*pi*R)}
T_f_1+C_1*exp(lambda_1*2*pi*R)+C_2*exp(lambda_2*2*pi*R)=T_f_2{+C_3+C_4}
"Temperature gradient continuity"
{C_1*lambda_1*exp(lambda_1*pi*R)+C_2*lambda_2*exp(lambda_2*pi*R)=C_3*beta_1*exp(beta_1*pi*R)+C_4*beta_2*exp(beta_2*pi*R)
C_1*lambda_1*exp(lambda_1*2*pi*R)+C_2*lambda_2*exp(lambda_2*2*pi*R)=C_3*beta_1+C_4*beta_2}
```

The new problem will solve, providing reasonable values for C_1 and C_2; now update the guess values for all of the variables (Calculate/Update Guess Values) and the variable information window will be updated to reflect the most recent values of each variable (Figure 6). Notice that, as we suspected, the values for C_1 and C_2 are very large and very small, respectively. Therefore a guess value of 1 was not appropriate for either variable.



Variable Information

Show array variables

Show string variables

Variable	Guess	Lower	Upper	Display			Units	Key	Comment
alpha	0.00004115	-infinity	infinity	A	3	N	m^2/s		
A_c	7.854E-07	-infinity	infinity	A	3	N	m^2		
beta_1	185.5	-infinity	infinity	A	3	N	1/m		
beta_2	-64	-infinity	infinity	A	3	N	1/m		
Bi	0.0001	-infinity	infinity	A	3	N	-		
c	900	-infinity	infinity	A	3	N	J/kg-K		
C_1	-1.467E-23	-infinity	infinity	A	3	N	K		
C_2	-3.244E+06	-infinity	infinity	A	3	N	K		
d	0.001	-infinity	infinity	A	3	N	m		
h_1	10	-infinity	infinity	A	3	N	W/m^2-K		
h_2	20	-infinity	infinity	A	3	N	W/m^2-K		
k	100	-infinity	infinity	A	3	N	W/m-K		
lambda_1	183.9	-infinity	infinity	A	3	N	1/m		

OK Apply Print Update Cancel

Figure 6: Variable Information window after updating guess values.

Uncomment the boundary conditions and solve the problem again; because the guess values for C_1 and C_2 are more appropriate, the equations will now solve with no problem. The solution can be plotted by making two parametric tables, the first with the variables s_bar, s and T_1_C (the solution in compartment #1 converted from K to °C for presentation) where s_bar is runs from 0 to 1 and the second with s_bar, s and T_2_C where s_bar runs from 1 to 2; the variable s_bar is defined as a dimensionless position.

```
s_bar=0                              "dimensionless position along ring"
s=s_bar*pi*R                         "position along ring"
T_1_C=converttemp(K,C,T_1)
T_2_C=converttemp(K,C,T_2)
```

c.) Plot the temperature distribution in the ring as a function of position along the circumference of the ring for various values of the ring rotation rate.

The temperature distribution through the ring is shown in Figure 7. Also shown in Figure 7 is the temperature distribution for several values of ring rotation rate. Notice that the solution for very low rotation limits to a pair of interacting, adiabatic tip fins and the symmetric nature of the resulting temperature distribution reflects this behavior. This solution represents a balance between conduction into the ring at the interface and convection to the surrounding fluids. As the rotation rate is increased the lines become progressively flatter. The limit of very high rotation rate would be a flat line; the energy transferred by ring rotation is sufficiently large that no temperature gradient is required. The ring would reach a uniform temperature where convection into the ring in the hot compartment is balanced by convection out in the cold compartment.

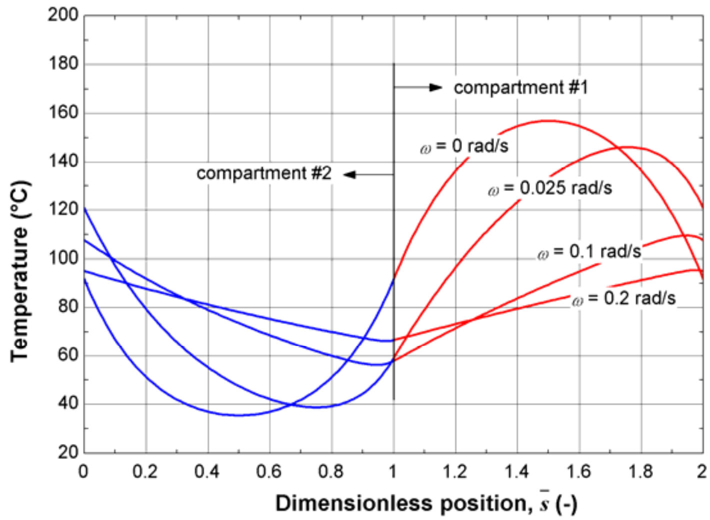


Figure 7: Temperature distribution as a function of dimensionless position for ω = 0.1 rad/s and ω = 0 rad/s.

d.) Compute the total rate of energy transferred between the two compartments.

The total rate of energy transferred between the compartments ($\dot{E}$, the energy transferred from compartment #2 to compartment #1) is related to conduction and energy carried by ring material motion at the two interfaces.

$$\dot{E} = \omega\, R\, A_c\, \rho\, c \left(T_2\big|_{s=\pi R} - T_2\big|_{s=0} \right) - k\, A_c \left[\left.\frac{dT_2}{dx}\right|_{s=\pi R} - \left.\frac{dT_2}{dx}\right|_{s=0} \right]$$

or

$$\begin{aligned} \dot{E} &= \omega\, R\, A_c\, \rho\, c \left\{ C_3 \left[\exp\left(\beta_1\, \pi\, R\right) - 1 \right] + C_4 \left[\exp\left(\beta_2\, \pi\, R\right) - 1 \right] \right\} \\ &- k\, A_c \left\{ C_3\, \beta_1 \left[\exp\left(\beta_1\, \pi\, R\right) - 1 \right] + C_4\, \beta_2 \left[\exp\left(\beta_2\, \pi\, R\right) - 1 \right] \right\} \end{aligned}$$

Figure 8 illustrates the rate of energy carried from compartment #2 to compartment #1 (the energy transferred is negative as the fluid in compartment #2 is colder). Notice that the two regimes of behavior previously discussed in the context of Figure 7 are evident; at low rates of rotation the energy is carried between compartments by conduction whereas at high rates of rotation energy is carried by ring motion.

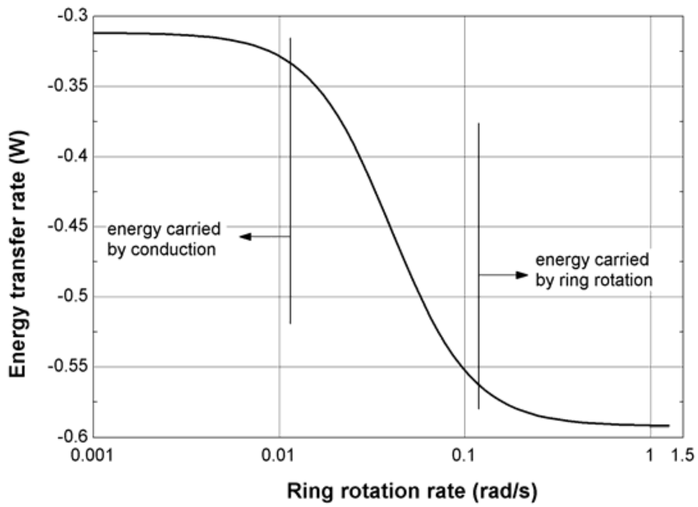


Figure 8: Energy transferred from compartment #2 to compartment #1 as a function of the rate of ring rotation.

e.) Determine the dimensionless parameter that characterizes the relative importance of energy transfer due to ring rotation and energy transfer by conduction. Plot the heat transfer rate between the two compartments as a function of this parameter (varied by changing the ring rotation rate).

The energy transferred between the compartments is ultimately transferred from/to the fluid by convection. However, the method of transfer between the compartments is both conduction and energy carried by ring rotation; one of these processes will typically be dominant. The ratio of the energy transfer caused by rotation to conduction can be used to form a governing dimensionless parameter that helps understand the problem and guide the solution. The use of dimensionless parameters based on the underlying physics is an important tool that allows many problems to be simplified and provides substantial insight into the characteristics of a solution. For this problem, the dimensionless parameter is:

$$\frac{\text{rotation}}{\text{conduction}} = \frac{\omega\, R\, A\, \rho\, c\left(T_{f,1} - T_{f,2}\right)}{1} \frac{R}{k\, A\left(T_{f,1} - T_{f,2}\right)} = \frac{\omega\, R^2}{\alpha}$$

It would have been best to calculate the value of the dimensionless parameter BEFORE we solved the problem. If $\omega R^2/\alpha$ is much less than unity (even if ω itself is not zero) then ring rotation can be neglected as it is dominated by conduction. On the other hand, if $\omega R^2/\alpha$ is much larger than unity (even if k is not zero) then the conduction terms in the governing equations could have been neglected and the problem would be simplified at least to solving 1st order rather than 2nd order equations. Figure 9 shows the energy transfer rate as a function of $\omega R^2/\alpha$ and demonstrates that the delineation between the regimes of behavior is unity for the appropriately defined dimensionless number.

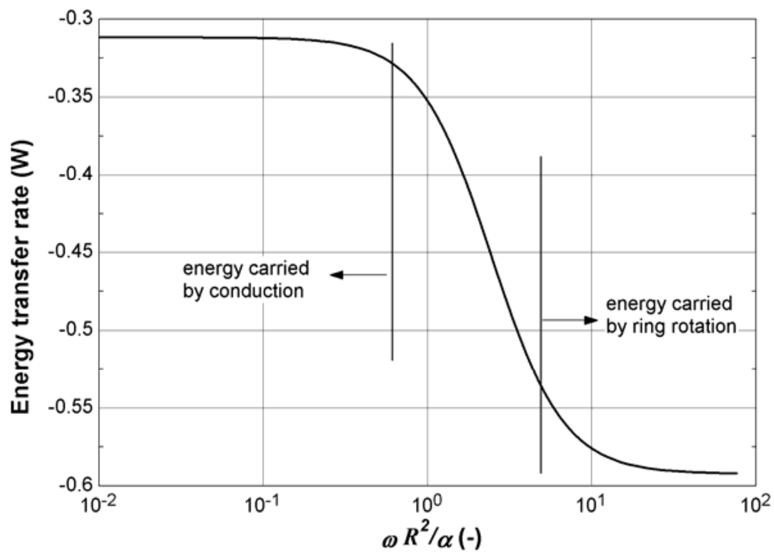


Figure 9: Energy transferred from compartment #2 to compartment #1 as a function of the dimensionless number $\omega R^2/\alpha$ which characterizes the rate of energy carried by ring motion to conduction.