Problem P1.2-11 (1-4 in text)

Figure P1.2-11(a) illustrates a composite wall. The wall is composed of two materials (A with k_A = 1 W/m-K and B with k_B = 5 W/m-K), each has thickness L = 1.0 cm. The surface of the wall at x = 0 is perfectly insulated. A very thin heater is placed between the insulation and material A; the heating element provides $\dot{q}'' = 5000\ \text{W/m}^2$ of heat. The surface of the wall at x = 2L is exposed to fluid at $T_{f,in}$ = 300 K with heat transfer coefficient $\overline{h}_{in}$ = 100 W/m^2-K.

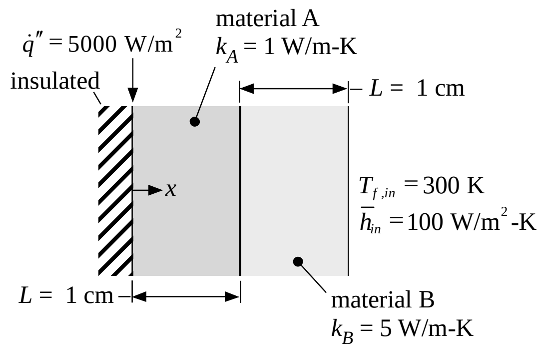


Figure P1.2-11(a): Composite wall with a heater.

You may neglect radiation and contact resistance for parts (a) through (c) of this problem.

a.) Draw a resistance network to represent this problem; clearly indicate what each resistance represents and calculate the value of each resistance.

The input parameters are entered in EES:

```
“P1.2-11: Heater"
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
q_flux=100 [W/m^2]                    "heat flux provided by the heater"
L = 1.0 [cm]*convert(cm,m)            "thickness of each layer"
k_A=1.0 [W/m-K]                       "conductivity of material A"
k_B=5.0 [W/m-K]                       "conductivity of material B"
T_f_in=300 [K]                        "fluid temperature at inside surface"
h_in=100 [W/m^2-K]                    "heat transfer on inside surface"
A=1 [m^2]                             "per unit area"
```

The resistance network that represents the problem shown in Figure 2 is:

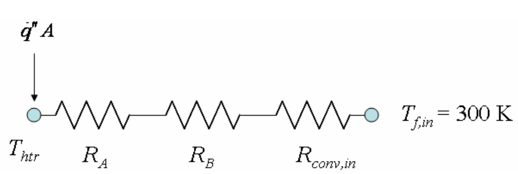


Figure 2: Resistance network.

The resistances due to conduction through materials A and B are:

$$R_A = \frac{L}{k_A\,A} \tag{1}$$

$$R_B = \frac{L}{k_B\,A} \tag{2}$$

where A is the area of the wall, taken to be 1 m^2 in order to carry out the analysis on a per unit area basis. The resistance due to convection is:

$$R_{conv,in} = \frac{1}{h_{in}\,A} \tag{3}$$

```
"part (a)"
R_A=L/(k_A*A)                                    "resistance to conduction through A"
R_B=L/(k_B*A)                                    "resistance to conduction through B"
R_conv_in=1/(h_in*A)
  "resistance to convection on inner surface"
```

which leads to R_A = 0.01 K/W, R_B = 0.002 K/W, and $R_{conv,in}$ = 0.01 K/W.

b.) Use your resistance network from (a) to determine the temperature of the heating element.

The resistance network for this problem is simple; the temperature drop across each resistor is equal to the product of the heat transferred through the resistor and its resistance. In this simple case, all of the heat provided by the heater must pass through materials A, B, and into the fluid by convection so these resistances are in series. The heater temperature (T_{htr}) is therefore:

$$T_{htr} = T_{f,in} + \left(R_A + R_B + R_{conv,in}\right)\dot{q}''\,A \tag{4}$$

```
T_htr=T_f_in+(R_A+R_B+R_conv_in)*q_flux*A   "heater temperature"
```

which leads to T_{htr} = 410 K.

c.) Sketch the temperature distribution on the axes provided below. Make sure that the sketch is consistent with your solution from (b).

The temperatures at $x = L$ and $x = 2L$ can be computed according to:

$$T_{x=L} = T_{f,in} + \left(R_B + R_{conv,in}\right)\dot{q}''\,A \tag{5}$$

$$T_{x=2L} = T_{f,in} + R_{conv,in}\,\dot{q}''\,A \tag{6}$$

```
T_L=T_f_in+(R_B+R_conv_in)*q_flux*A              "temperature at x=L"
T_2L=T_f_in+R_conv_in*q_flux*A                   "temperature at x=2L"
```

which leads to $T_{x=L}$ = 360 K and $T_{x=2L}$ = 350 K. The temperature distribution is sketched on the axes in Figure 3.

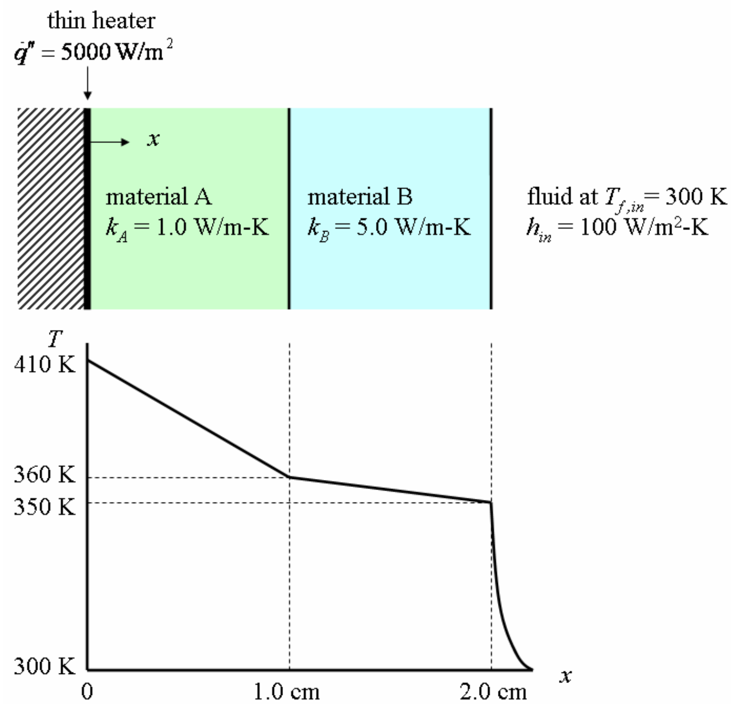


Figure 3: Sketch of temperature distribution.

Notice that the temperature drop through the two larger resistances (R_A and R_B) are much larger than the temperature drop across the small resistance, R_B.

Figure P1.2-11(b) illustrates the same composite wall shown in Figure P1.2-11(a), but there is an additional layer added to the wall, material C with k_C = 2.0 W/m-K and L = 1.0 cm.

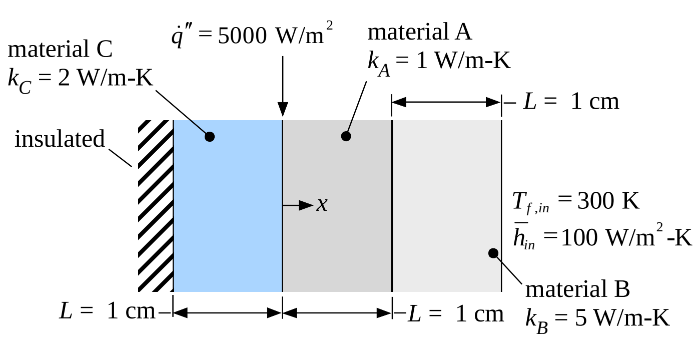
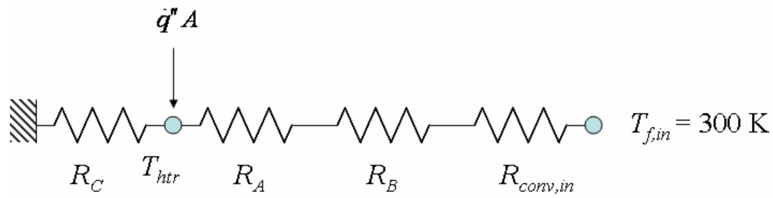


Figure P1.2-11(b): Composite wall with Material C.

Neglect radiation and contact resistance for parts (d) through (f) of this problem.

d.) Draw a resistance network to represent the problem shown in Figure P1.2-11(b); clearly indicate what each resistance represents and calculate the value of each resistance.

There is an additional resistor corresponding to conduction through material C, R_C, as shown below:



Notice that the boundary condition at the end of R_C corresponds to the insulated wall; that is, no heat can be transferred through this resistance. The resistance to conduction through material C is:

$$R_C = \frac{L}{k_C\, A} \tag{7}$$

```
"part (b)"
k_C=2.0 [W/m-K]                     "conductivity of material C"
R_C=L/(k_C*A)                       "resistance to conduction through C"
```

which leads to R_C = 0.005 K/W.

e.) Use your resistance network from (d) to determine the temperature of the heating element.

Because there is no heat transferred through R_C, all of the heat must still go through materials A and B and be convected from the inner surface of the wall. Therefore, the answer is not changed from part (b), T_{htr} = 410 K.

f.) Sketch the temperature distribution on the axes provided below. Make sure that the sketch is consistent with your solution from (e).

The answer is unchanged from part (c) except that there is material to the left of the heater. However, no heat is transferred through material C and therefore there is no temperature gradient in the material.

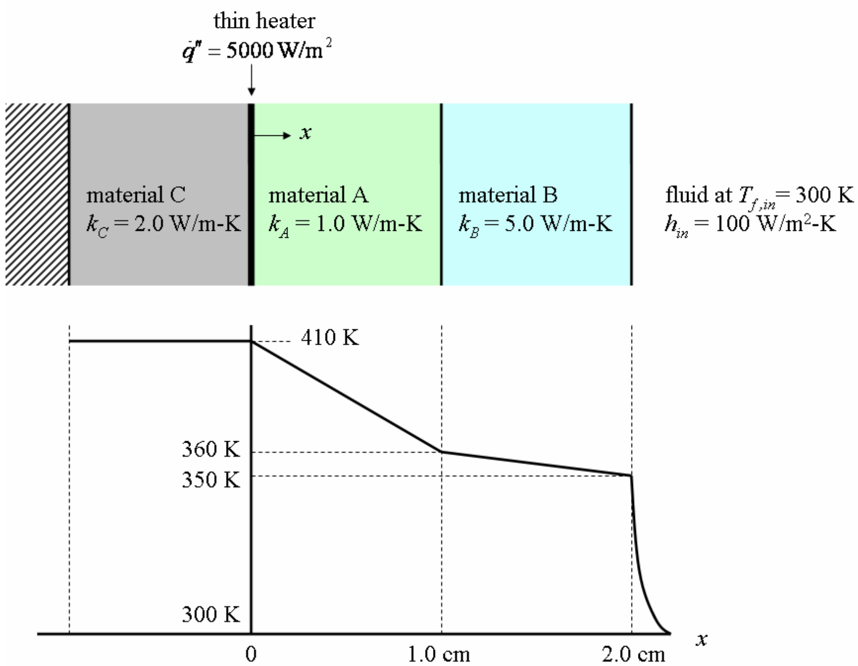


Figure P1.2-11(c) illustrates the same composite wall shown in Figure P1.2-11(b), but there is a contact resistance between materials A and B, $R''_c = 0.01\text{ K-m}^2/\text{W}$, and the surface of the wall at $x = -L$ is exposed to fluid at $T_{f,out}$ = 400 K with a heat transfer coefficient $\bar{h}_{out}$ = 10 W/m^2-K.

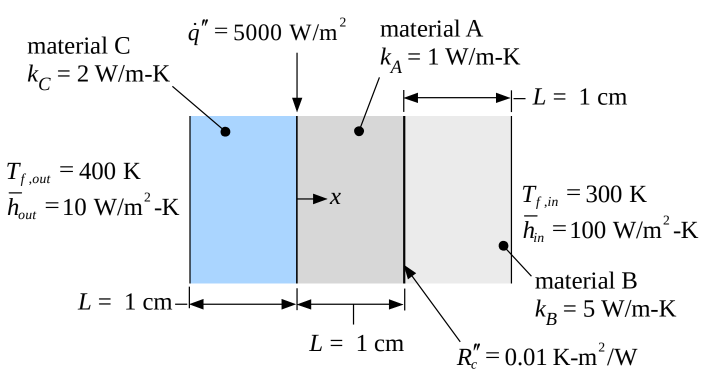
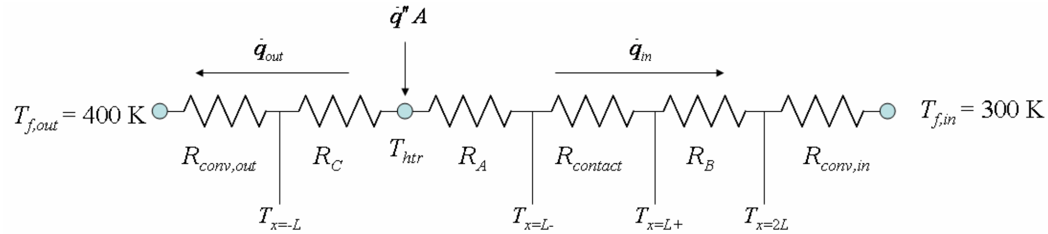


Figure P1.2-11(c): Composite wall with convection at the outer surface and contact resistance.

Neglect radiation for parts (g) through (i) of this problem.

g.) Draw a resistance network to represent the problem shown in Figure P1.2-11(c); clearly indicate what each resistance represents and calculate the value of each resistance.

The additional resistances associated with contact resistance and convection to the fluid at the outer surface are indicated. Notice that the boundary condition has changed; heat provided by the heater has two paths ($\dot{q}_{out}$ and $\dot{q}_{in}$) and so the problem is not as easy to solve.



The additional resistances are computed according to:

$$R_{conv,out} = \frac{1}{h_{out}\, A} \tag{8}$$

$$R_{contact} = \frac{R''_c}{A} \tag{9}$$

```
"part (c)"
R``_c=0.01 [K-m^2/W]                "area specific contact resistance"
h_out=10 [W/m^2-K]                  "heat transfer coefficient"
T_f_out=400 [K]                     "fluid temperature on outside surface"
R_contact=R``_c/A                   "contact resistance"
R_conv_out=1/(h_out*A)
    "convection resistance on outer surface"
```

which leads to $R_{contact}$ = 0.01 K/W and $R_{conv,out}$ = 0.1 K/W.

h.) Use your resistance network from (j) to determine the temperature of the heating element.

It is necessary to carry out an energy balance on the heater:

$$\dot{q}''\, A = \dot{q}_{in} + \dot{q}_{out} \tag{10}$$

The heat transfer rates can be related to T_{htr} according to:

$$\dot{q}_{in} = \frac{\left(T_{htr} - T_{f,in}\right)}{R_A + R_{contact} + R_B + R_{conv,in}} \tag{11}$$

$$\dot{q}_{out} = \frac{\left(T_{htr} - T_{f,out}\right)}{R_C + R_{conv,out}} \tag{12}$$

These are 3 equations in 3 unknowns, T_{htr}, $\dot{q}_{out}$ and $\dot{q}_{in}$, and therefore can be solved simultaneously in EES (note that the previous temperature calculations from part (b) must be commented out):

```
{T_htr=T_f_in+(R_A+R_B+R_conv_in)*q_flux*A          "heater temperature"
T_L=T_f_in+(R_B+R_conv_in)*q_flux*A                 "temperature at x=L"
```

```
T_2L=T_f_in+R_conv_in*q_flux*A                              "temperature at x=2L"}
q_flux*A=q_dot_in+q_dot_out                                 "energy balance on the heater"
q_dot_in=(T_htr-T_f_in)/(R_A+R_contact+R_B+R_conv_in)       "heat flow to inner fluid"
q_dot_out=(T_htr-T_f_out)/(R_C+R_conv_out)                  "heat flow to outer fluid"
```

which leads to T_{htr} = 446 K. The other intermediate temperatures shown on the resistance diagram can be computed:

$$T_{x=L-} = T_{htr} - R_A\,\dot{q}_{in} \tag{13}$$

$$T_{x=L+} = T_{htr} - \left(R_A + R_{contact}\right)\dot{q}_{in} \tag{14}$$

$$T_{x=2L} = T_{htr} - \left(R_A + R_{contact} + R_B\right)\dot{q}_{in} \tag{15}$$

$$T_{x=-L} = T_{htr} - R_C\,\dot{q}_{out} \tag{16}$$

```
"intermediate temperatures"
T_Lm=T_htr-R_A*q_dot_in
T_Lp=T_htr-(R_A+R_contact)*q_dot_in
T_2L=T_htr-(R_A+R_contact+R_B)*q_dot_in
T_mL=T_htr-R_C*q_dot_out
```

which leads to $T_{x=L-}$ = 400.4 K, $T_{x=L+}$ = 354.7 K, $T_{x=2L}$ = 345.6 K, and $T_{x=-L}$ = 443.8 K.

i.) Sketch the temperature distribution on the axes provided below.