SOLUTIONS MANUAL FOR

Heat Exchangers: Selection, Rating, and Thermal Design

Third Edition

by

Sadik Kakaç
Anchasa Pramuanjaroenkij
Hongtan Liu

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Problem 2.1

Starting from Eq. (2.22), show that for a parallelflow heat exchanger, Eq. (2.26a) becomes

$$\frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \exp\left[-\left(\frac{1}{C_h} + \frac{1}{C_c}\right)UA\right]$$

SOLUTION:

The heat transferred across the area dA is:

$$\delta Q = U\left(T_h - T_c\right)dA \qquad (1)$$

The heat transfer rate can also be written as the change in enthalpy of each fluid (with the correct sign) between the area A and A+dA:

* for the hot fluid ($dT_h<0$)

$$\delta Q = -\dot{m}_h c_{p,h} dT_h \qquad (2)$$

* for the cold fluid ($dT_c>0$)

$$\delta Q = \dot{m}_c c_{p,c} dT_c \qquad (3)$$

The notion of heat capacity can be introduced as:

$$C = \dot{m} c_p \qquad (4)$$

This parameter represents the rate of heat transferred by a fluid when its temperature varies with one degree.

The equation (2) and (3) give:

$$\delta Q = -C_h dT_h = C_c dT_c \qquad (5)$$

Equations (1) and (5) give:

$$\frac{dT_h}{T_h - T_c} = -\frac{U}{C_h} dA \qquad (6)$$

$$\frac{dT_c}{T_h - T_c} = -\frac{U}{C_c} dA \qquad (7)$$

Subtracting equation (7) from (6):

$$\frac{d(T_h - T_c)}{T_h - T_c} = \left(\frac{1}{C_c} - \frac{1}{C_h}\right) UdA \qquad (8)$$

Considering the overall heat transfer coefficient U=constant, equation (8) can be integrated:

$$\ln\left(T_h - T_c\right) = \left(\frac{1}{C_c} - \frac{1}{C_h}\right) UA + \ln B \qquad (9)$$

$$T_h - T_c = B\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right) UA\right] \qquad (10)$$

The constant of integration, K is obtained from the boundary condition at the inlet:

at A=0, $T_h - T_c = T_{h1} - T_{c2}$ (11)

$$K = T_{h1} - T_{c2} \tag{12}$$

Introducing equation (12) in (10) we have:

$$\frac{T_h - T_c}{T_{h1} - T_{c2}} = \exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] \tag{13}$$

At the outlet the heat transfer area is A_t=A and T_h-T_c=T_{h2}-T_{c2} and:

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = e^{-\left(\frac{1}{C_h} + \frac{1}{C_c}\right)UA} \tag{14}$$

Problem 2.2

Show that for a parallel flow heat exchanger the variation of the hot fluid temperature along the heat exchanger is given by

$$\frac{T_h - T_{h1}}{T_{h1} - T_{c1}} = \frac{-C_c}{C_h + C_c}\left\{1 - e^{-\left(\frac{1}{C_h}+\frac{1}{C_c}\right)UA}\right\}$$

Obtain a similar expression for the variation of the cold fluid temperature along the heat exchanger. Also show that for $A \to \infty$, the temperature will be equal to mixing-cup temperature of the fluids which is given by

$$T_\infty = \frac{C_h T_{h1} + C_c T_{c1}}{C_h + C_c}$$

SOLUTION:

The heat transferred across the area dA is:

$$\delta Q = U(T_h - T_c)dA \qquad (1)$$

The heat transfer rate can also be written as the change in enthalpy of each fluid (with the correct sign) between the area A and A+dA:

* for the hot fluid ($dT_h<0$)

$$\delta Q = -\dot{m}_h c_{p,h} dT_h \qquad (2)$$

* for the cold fluid ($dT_c>0$)

$$\delta Q = \dot{m}_c c_{p,c} dT_c \qquad (3)$$

The notion of heat capacity can be introduced as:

$$C = \dot{m} c_p \qquad (4)$$

Equation (2) and (3) give:

$$\delta Q = -C_h dT_h = C_c dT_c \qquad (5)$$

Equations (1) and (5) give:

$$\frac{dT_h}{T_h - T_c} = -\frac{U}{C_h}dA \qquad (6)$$

$$\frac{dT_c}{T_h - T_c} = -\frac{U}{C_c}dA \qquad (7)$$

Subtracting equation (7) from (6):

$$\frac{d(T_h - T_c)}{T_h - T_c} = \left(\frac{1}{C_c} - \frac{1}{C_h}\right)UdA \qquad (8)$$

Considering the overall heat transfer coefficient U=constant, equation (8) can be integrated:

$$\ln\left(T_h - T_c\right) = \left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA + \ln B \qquad (9)$$

$$T_h - T_c = B\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] \qquad (10)$$

The constant of integration, K is obtained from the boundary condition at the inlet:

at A=0, $T_h - T_c = T_{h1} - T_{c2}$ (11)

$$K = T_{h1} - T_{c2} \qquad (12)$$

Introducing equation (12) in (10) we have:

$$\frac{T_h - T_c}{T_{h1} - T_{c2}} = \exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] \qquad (13)$$

From equation (10) it can be observed that the temperature difference T_h-T_c is an exponential function of surface area A, and T_h-$T_c \rightarrow 0$ when $A \rightarrow 0$. The variation of the hot fluid temperature and that of the cold fluid temperature can be obtained separately. By multiplying equations (6) and (13):

$$\frac{dT_h}{T_{h1} - T_{c2}} = -\frac{U}{C_h}\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right]dA \qquad (14)$$

Integrating:

$$\frac{T_h}{T_{h1} - T_{c1}} = -\frac{U}{C_h}\frac{\exp\left[-\left(\frac{1}{C_h} + \frac{1}{C_c}\right)UA\right]}{-\frac{C_h + C_c}{C_h C_c}U} + B \qquad (15)$$

$$\frac{T_h}{T_{h1} - T_{c2}} = \frac{C_c}{C_c - C_h}\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] + B \qquad (16)$$

The constant of integration, B is obtained from the boundary condition:

at A=0, T_h=T_{h1}, and

$$B = \frac{T_{h1}}{T_{h1} - T_{c2}} - \frac{C_c}{C_c - C_h} \qquad (17)$$

From (16) and (17) we have:

$$\frac{T_h - T_{h1}}{T_{h1} - T_{c1}} = \frac{-C_c}{C_h + C_c}\left\{1 - \exp\left[-\left(\frac{1}{C_h} + \frac{1}{C_c}\right)UA\right]\right\} \qquad (18)$$

From equations (7) and (13) following the same procedure we obtain:

$$\frac{T_c - T_{c1}}{T_{h1} - T_{c1}} = \frac{C_h}{C_h + C_c}\left\{1 - e^{-\left(\frac{1}{C_h} + \frac{1}{C_c}\right)UA}\right\} \qquad (19)$$

Equation (10) shows that for $A \rightarrow \infty$, $T_h = T_c = T_\infty$.

The value of T_∞ can be calculated, for example, from equation (19):

$$T_{\infty} = T_{c1} + \frac{C_c}{C_h + C_c}\left(T_{h1} - T_{c1}\right) \tag{20}$$

$$T_{\infty} = \frac{C_h T_{h1} + C_c T_{c1}}{C_h + C_c} \tag{21}$$

Problem 2.3

Show that the variation of the hot and cold fluid temperature along a counterflow heat exchanger is given by

$$\frac{T_h - T_{h1}}{T_{h1} - T_{c2}} = \frac{C_c}{C_c - C_h}\left\{\exp\left[\left(\frac{1}{Cc} - \frac{1}{C_h}\right)UA\right] - 1\right\}$$

and

$$\frac{T_c - T_{c2}}{T_{h1} - T_{c2}} = \frac{C_h}{C_c - C_h}\left\{\exp\left[\left(\frac{1}{Cc} - \frac{1}{C_h}\right)UA\right] - 1\right\}$$

SOLUTION:

$$\frac{dT_h}{T_h - T_c} = -\frac{U}{C_h}dA \qquad (1)$$

$$\frac{dT_c}{T_h - T_c} = -\frac{U}{C_c}dA \qquad (2)$$

Subtracting equation (2) from (1):

$$\frac{d(T_h - T_c)}{T_h - T_c} = \left(\frac{1}{C_c} - \frac{1}{C_h}\right)UdA \qquad (3)$$

Integrating for constant values of U, C_c and C_h we have

$$\ln(T_h - T_c) = \left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA + \ln B$$

$$T_h - T_c = B\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] \qquad (4)$$

where B the constant of integration results from the boundary condition:

at A=0, $T_h - T_c = T_{h1} - T_{c2}$

$$B = T_{h1} - T_{c2} \qquad (5)$$

Introducing equation (5) in (4):

$$\frac{T_h - T_c}{T_{h1} - T_{c2}} = \exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] \qquad (6)$$

Examining the evolution of T_h and T_c separately by multiplying equations (1) and (6), (2) and (6) respectively, we have:

$$\frac{dT_h}{T_{h1}-T_{c2}} = -\frac{U}{C_h}\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]dA \tag{7.1}$$

$$\frac{dT_c}{T_{h1}-T_{c2}} = -\frac{U}{C_c}\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]dA \tag{7.2}$$

Integrating:

$$\frac{T_h}{T_{h1}-T_{c2}} = -\frac{U}{C_h}\frac{\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]}{\left(\frac{1}{C_c}-\frac{1}{C_h}\right)U}+B$$

$$\frac{T_h}{T_{h1}-T_{c2}} = \frac{C_c}{C_c-C_h}\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]+B \tag{8.1}$$

$$\frac{T_c}{T_{h1}-T_{c2}} = -\frac{U}{C_c}\frac{\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]}{\left(\frac{1}{C_c}-\frac{1}{C_h}\right)U}+B'$$

$$\frac{T_c}{T_{h1}-T_{c2}} = \frac{C_h}{C_c-C_h}\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]+B' \tag{8.2}$$

For A=0, $T_h=T_{h1}$, $T_c = T_{c2}$ and:

$$\frac{T_{h1}}{T_{h1}-T_{c2}} = \frac{C_c}{C_c-C_h}+B$$

$$B = \frac{T_{h1}}{T_{h1}-T_{c2}}-\frac{C_c}{C_c-C_h} \tag{9.1}$$

$$\frac{T_{c2}}{T_{h1}-T_{c2}} = \frac{C_h}{C_c-C_h}+B'$$

$$B = \frac{T_{c2}}{T_{h1}-T_{c2}}-\frac{C_h}{C_c-C_h} \tag{9.2}$$

Substituting (9.1) in (8.1), (9.2) in (8.2), respectively:

$$\frac{T_h-T_{h1}}{T_{h1}-T_{c2}} = \frac{C_c}{C_c-C_h}\left\{\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]-1\right\} \tag{10.1}$$

$$\frac{T_c-T_{c2}}{T_{h1}-T_{c2}} = \frac{C_h}{C_c-C_h}\left\{\exp\left[\left(\frac{1}{C_c}-\frac{1}{C_h}\right)UA\right]-1\right\} \tag{10.2}$$

Problem 2.4

From problem 2.3, show that for the case $C_h<C_c$, $\frac{d^2T_h}{dA^2} > 0$ and $\frac{d^2T_c}{dA^2} > 0$, and therefore temperature curves are convex and for the case $C_h>C_c$, $\frac{d^2T_h}{dA^2} < 0$, and $\frac{d^2T_c}{dA^2} < 0$, therefore, the temperature curves are concave (see Figure 2.6).

<u>SOLUTION:</u>

The hot fluid has a smaller heat capacity than the cold fluid, that is why it is the one who "commands the transfer"

Differentiating equation (10.1) in problem 2.3:

$$dT_h = d(T_h - T_c)$$

$$\frac{dT_h}{dA} = (T_{h1} - T_{c2})\left(-\frac{1}{C_h}\right)U\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right]$$

$$\frac{d^2T_h}{dA^2} = (T_{h1} - T_{c2})\left(-\frac{1}{C_h}\right)\left(\frac{1}{C_c} - \frac{1}{C_h}\right)U^2\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right]$$

$$\frac{d^2T_h}{dA^2} = \frac{(T_{h1} - T_{c2})(C_c - C_h)}{C_c C_h^2}U^2\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] > 0 \qquad (1)$$

Similarly, from equation (10.2):

$$\frac{dT_c}{dA} = (T_{h1} - T_{c2})\left(-\frac{1}{C_c}\right)U\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right]$$

$$\frac{d^2T_c}{dA^2} = \frac{(T_{h1} - T_{c2})(C_c - C_h)}{C_c^2 C_h}U^2\exp\left[\left(\frac{1}{C_c} - \frac{1}{C_h}\right)UA\right] > 0 \qquad (2)$$

Since, the second derivatives with respect to area of both T_h and T_c are positive as seen in equations (1) and (2), both the temperature curves are convex.

Problem 2.5

Show that when the heat capacities of hot and cold fluids are equal (C_c=C_h=C), the variation of the hot and cold fluid temperature along a counter flow heat exchanger are linear with the surface area as:

$$\frac{T_c - T_{c_2}}{T_{h_1} - T_{c_2}} = \frac{T_h - T_{h_1}}{T_{h_1} - T_{c_2}} = -\frac{UA}{C}$$

SOLUTION:

When the two fluids have the same heat capacity, from equation (6) in problem 2.3:

$$\delta Q = U(T_h - T_c)dA = -C_h dT_h \qquad (1)$$

In equation (10.2) in problem 2.3 when $C_c \rightarrow C_h$ we have:

$$\frac{T_c - T_{c2}}{T_{h1} - T_{c2}} = \lim \frac{C_h}{C_c - C_h}\left(e^{\frac{C_h - C_c}{C_c C_h}UA} - 1 \right) = \lim\left(-C_h \frac{UA}{C_c C_h} \right) = -\frac{UA}{C_c} \qquad (2)$$

Similarly, from equation (10.1) in problem 2.3:

$$\frac{d(T_h - T_c)}{T_h - T_c} = -\frac{U}{C_c}dA\,, \qquad \text{When } C_c \rightarrow C_h \qquad (3)$$

But C_c=C_h=C and from (2) and (3):

$$\ln(T_h - T_c) = -\frac{UA}{C_h} + \ln D \qquad (4)$$

Problem 2.6

Assume that in a condenser, there will be no-subcooling and condensate leaves the condenser at saturation temperature, T_h. Show that variation of the coolant temperature along the condenser is given by

$$\frac{T_c - T_{c1}}{T_h - T_{c1}} = 1 - \exp\left[-\frac{UA}{C_c}\right]$$

SOLUTION:

The heat transferred along a surface element dA is:

$$\delta Q = U(T_h - T_c)dA = -C_h dT_h \qquad (1)$$

Because T_h = constant in a condenser, we can write:

$$dT_h = d(T_h - T_c) \qquad (2)$$

Using equations (1) and (2):

$$\frac{d(T_h - T_c)}{T_h - T_c} = -\frac{U}{C_c}dA\ , \qquad (3)$$

Integrating:

$$\ln(T_h - T_c) = -\frac{UA}{C_h} + \ln D$$

$$T_h - T_c = B\exp\left(-\frac{U}{C_c}A\right) \qquad (4)$$

The constant of integration, B can be calculated with the boundary condition:

$T_c = T_{c1}$, for A=0.

$T_h - T_{c1} = B \qquad (5)$

The temperature distribution for the cold fluid can be obtained by introducing (5) in (4) as:

$$T_h - T_c = (T_h - T_{c1})\exp\left[-\frac{UA}{C_c}\right]$$

$$\frac{T_c - T_{c1}}{T_h - T_{c1}} = 1 - \exp\left(-\frac{UA}{C_c}\right)$$

Problem 2.7

In a boiler (evaporator), the temperature of hot gases decreases from T_{h1} to T_{h2}, while boiling occurs at a constant temperature T_c. Obtain an expression, as in Problem 2.6, for the variation of hot fluid temperature with the surface area.

SOLUTION:

The rate of heat transfer δQ across the heat transfer area dA can be expressed as:

$$\delta Q = U(T_h - T_c)dA = -C_h dT_h \quad (1)$$

In an evaporator T_c = constant and

$$dT_h = d(T_h - T_c) \quad (2)$$

From equations (1) and (2):

$$\frac{d(T_h - T_c)}{T_h - T_c} = -\frac{U}{C_h}dA \quad (3)$$

$$\ln(T_h - T_c) = -\frac{UA}{C_h} + \ln D$$

$$T_h - T_c = D\exp\left(-\frac{UA}{C_h}\right) \quad (4)$$

The boundary condition at A=0 gives the value of the constant D:

at A=0 $\quad T_h = T_{h1}$

$$T_{h1} - T_c = D \quad (5)$$

Introducing (5) in (4):

$$T_h - T_c = (T_{h1} - T_c)\exp\left(-\frac{U}{C_h}A\right) \quad (6)$$

Rearranging:

$$1 - \frac{T_h - T_c}{T_{h1} - T_c} = 1 - \exp\left(-\frac{U}{C_h}A\right)$$

$$\frac{T_h - T_{h1}}{T_{h1} - T_c} = -\left[1 - \exp\left(-\frac{U}{C_h}A\right)\right] \quad (7)$$

Problem 2.8

Show that Eq. (2.46) is also applicable for $C_h>C_c$, that is $C^*=C_c/C_h$.

SOLUTION:

From Eq. (2.26b)

$$T_{h2}-T_{c1}=(T_{h1}-T_{c2})\exp\left[UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right)\right] \qquad (1)$$

For the case $C_h>C_c$, $C_c=C_{\min}$, $C_h=C_{\max}$,

$$T_{h2}-T_{c1}=(T_{h1}-T_{c2})\exp\left[\frac{UA}{C_{\min}}\left(1-\frac{C_c}{C_h}\right)\right] \qquad (2)$$

$$=(T_{h1}-T_{c2})\exp[NTU(1-C^*)]$$

From heat balance equation

$$C_c(T_{c2}-T_{c1})=C_h(T_{h1}-T_{h2}) \qquad (3)$$

or

$$C^*(T_{c2}-T_{c1})=(T_{h1}-T_{h2}) \qquad (4)$$

The heat exchanger efficiency

$$\varepsilon=\frac{Q}{Q_{\max}}=\frac{C_{\min}(T_{c2}-T_{c1})}{C_{\min}(T_{h1}-T_{c1})}=\frac{T_{c2}-T_{c1}}{T_{h1}-T_{c1}}$$

$$=\frac{(T_{c2}-T_{c1})(1-C^*)}{(T_{h1}-T_{c1})(1-C^*)}$$

$$=\frac{T_{c2}-T_{c1}-C^*(T_{c2}-T_{c1})}{T_{h1}-T_{c1}-C^*(T_{h1}-T_{c1})} \qquad (5)$$

$$=\frac{T_{c2}-T_{c1}-T_{h1}+T_{h2}}{T_{h2}-T_{c1}-C^*(T_{h1}-T_{c2})-T_{h2}+T_{h1}-C^*T_{c2}+C^*T_{c1}}$$

$$=\frac{T_{h2}-T_{c1}-(T_{h1}-T_{c2})}{T_{h2}-T_{c1}-C^*(T_{h1}-T_{c2})}$$

or

$$\varepsilon = \frac{1 - \dfrac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}}{1 - C^* \dfrac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}} \tag{6}$$

$$= \frac{1 - \exp[-NTU(1 - C^*)]}{1 - C^* \exp[-NTU(1 - C^*)]}$$

This proves that for $C_h > C_c$, Eq. (2.46) can also be derived from Eq. (2.16b).

Problem 2.9

Obtain the expression for exchanger heat transfer effectiveness, ε, for parallel flow given by Eq. (2.47).

SOLUTION:

From Eq. (2.26c)

$$T_{h2} - T_{c2} = (T_{h1} - T_{c1}) \exp\left[-UA\left(\frac{1}{C_c} + \frac{1}{C_h} \right) \right] \qquad (1)$$

Assume $C_h > C_c$, $C_c = C_{\min}$, $C_h = C_{\max}$,

$$T_{h2} - T_{c2} = (T_{h1} - T_{c1}) \exp\left[-\frac{UA}{C_{\min}}\left(1 + \frac{C_c}{C_h} \right) \right] \qquad (2)$$

$$= (T_{h1} - T_{c1}) \exp[-NTU(1 + C^*)]$$

From heat balance equation

$$C_c(T_{c2} - T_{c1}) = C_h(T_{h1} - T_{h2}) \qquad (3)$$

or

$$C^*(T_{c2} - T_{c1}) = (T_{h1} - T_{h2}) \qquad (4)$$

The heat exchanger efficiency

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_{\min}(T_{c2} - T_{c1})}{C_{\min}(T_{h1} - T_{c1})} = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}}$$

$$= \frac{(T_{c2} - T_{c1})(1 + C^*)}{(T_{h1} - T_{c1})(1 + C^*)}$$

$$= \frac{(T_{c2} - T_{c1})\left(1 + \dfrac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}} \right)}{(T_{h1} - T_{c1})(1 + C^*)} \qquad (5)$$

$$= \frac{T_{c2} - T_{c1} + T_{h1} - T_{h2}}{(T_{h1} - T_{c1})(1 + C^*)}$$

$$= \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{(T_{h1} - T_{c1})(1 + C^*)}$$

$$= \frac{1 - \exp[-NTU(1 + C^*)]}{1 + C^*}$$

This proves that for $C_h > C_c$, Eq. (2.47) can be derived from Eq. (2.16c). For case $C_h < C_c$, similar result can also be obtained.

Problem 2.10

5,000 kg/hr of water will be heated from 20°C to 35°C by hot water at 140°C. A 15°C hot water temperature drop is allowed. A number of double-pipe heat exchangers with annuli and pipes each connected in series will be used. Hot water flows through the inner tube. The thermal conductivity of the material is 50 W/m.K.

Fouling factors: R_{fi} 0.000176 m^2.K/W

R_{fo} = 0.000352 m^2.K/W.

Inner tube diameters: ID = 0.0525m, OD = 0.0603m

Annulus diameters: ID = 0.0779m, OD = 0.0889m.

The heat transfer coefficients in the inner tube and in the annulus are 4620 W / $m^2 \cdot$ K and 1600 W / $m^2 \cdot$ K, respectively. Calculate the overall heat transfer coefficient and the surface area of the heat exchanger for both parallel and counter flow arrangements.

GIVEN:

-A double pipe heat exchanger, with hot water flows through the inner tube.

-Cold water inlet temperature (T_{c1}) = 20°C

-Cold water outlet temperature (T_{c2}) = 35°C

-Cold water mass flow rate ($\dot{m}_h$) = 5000 kg/hr = 1.3889 kg/s

-Hot Water inlet temperature (T_{h1}) = 140°C

-Hot water temperature drop (ΔT_h) = 15°C

-Thermal conductivity of tube material (k_w) = 50 W/m.K

-Heat transfer coefficient in the inner tube (h_i) = 4620 W/m^2.K

-Heat transfer coefficient in the annulus (h_o) = 1600 W/m^2.K

-Fouling factors: (R_{fi}) = 0.000176 m2.K/W

(R_{fo}) = 0.000352 m2.K/W

-Inner tube diameters : (ID) = 0.0525 m, (OD) = 0.0603 m

-Annulus diameters: (ID) = 0.0779 m, (OD) = 0.0889 m

FIND:

a. Overall heat transfer coefficient (U_o)

b. Surface area (A).

SOLUTION:

a.

The total thermal resistance R_t can be expressed as: [eq. (2.11)]

$$R_t = \frac{1}{UA} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o}$$

$$= \frac{1}{h_iA_i} + \frac{\ln\left(r_o/r_i\right)}{2\pi kL} + \frac{R_{fi}}{A_i} + \frac{R_{fo}}{A_o} + \frac{1}{A_oh_o}$$

$$\frac{1}{U_o} = \frac{A_o}{h_iA_i} + \frac{A_o \cdot \ln\left(r_o/r_i\right)}{2\pi kL} + \frac{A_oR_{fi}}{A_i} + R_{fo} + \frac{1}{h_o}$$

$$= \frac{d_o}{h_id_i} + \frac{d_o \cdot \ln\left(r_o/r_i\right)}{2k} + \frac{d_oR_{fi}}{d_i} + R_{fo} + \frac{1}{h_o}$$

$$= \frac{0.0603}{4620 \times 0.0525} + \frac{0.0603 \times \ln\left(\frac{0.0603}{0.0525}\right)}{2 \times 50} + \frac{0.000176 \times 0.0603}{0.0525} + 0.000352 + \frac{1}{1600}$$

$$= \; 0.001511 \quad W/m^2.K$$

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right) = 17.222 \times 4.179 \times (50 - 20) = 2159.12 \quad kW$$

U_o is the overall heat transfer coefficient based on outer surface area, i.e.

$$T_{h2} = T_{h1} - \frac{\dot{m}_c c_{p,c}}{\dot{m}_h c_{p,h}}(T_{c2} - T_{c1})$$

$$= 100 - \frac{62000 \times 4.179}{80000 \times 4.22} \times 30 = 76.98 \quad ^oC$$

b.

Heat balance equation:

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right)$$

$c_{p,c}$ = 4.179 kJ/kg.K (from table B.2 in appendix B)

$$\therefore \; Q = 1.3889 \times 4179 \times (35 - 20) = 87063.2 \;\; W$$

for parallel flow:

$$\Delta T_m = \frac{120 - 90}{\ln\frac{120}{90}} = 104.28 \quad ^oC$$

for counter flow:

$$\Delta T_m = \Delta T_1 = \Delta T_2 = 105 \quad {}^{o}C$$

So,

$$A_o = \frac{Q}{U_o \Delta T_m} = \frac{87063.2}{661.7 \times 104.28} = 1.262 \ \ m^2 \qquad \text{for parallel flow.}$$

$$A_o = \frac{Q}{U_o \Delta T_m} = \frac{87063.2}{661.7 \times 105} = 1.253 \ \ m^2 \qquad \text{for counter flow.}$$

Problem 2.11

Water at a rate of 45,500 kg/hr is heated from 80°C to 150°C in a shell-and-tube heat-exchanger having two shell passes and eight tube passes with a total surface area of 925m². Hot exhaust gases having approximately the same thermal physical properties as air enter at 350°C and exit at 175°C. Determine the overall heat transfer coefficient based on the outside surface area.

GIVEN:

-A shell-and-tube heat exchanger having two shell passes and eight tube passes.

-Cold water inlet temperature (T_{c1}) = 80°C

-Cold water outlet temperature (T_{c2}) = 150°C

-Cold water mass flow rate ($\dot{m}_h$) = 45,500 kg/hr = 12.6389 kg/s

-Hot gas inlet temperature (T_{h1}) = 350°C

-Hot gas outlet temperature (T_{h2}) = 175°C

-Total surface area (A) = 925 m²

FIND:

Overall heat transfer coefficient (U)

SOLUTION:

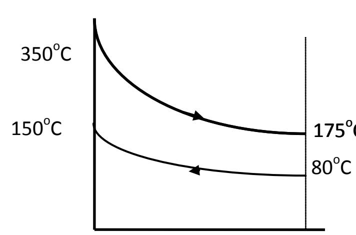
The heat balance equation:

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right) = \dot{m}_h c_{p,h}\left(T_{h2} - T_{h1}\right)$$

$c_{p,c}$ = 4.227 kJ/(kg.K) (at average temperature of $\frac{80+150}{2} = 115^{o}\,C$)

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right) = 17.222 \times 4.179 \times (50 - 20) = 2159.12 \quad kW$$

$$\Delta T_{lm,cf} = \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}} = \frac{200 - 95}{\ln\frac{200}{95}} = 141.05 \quad {}^{o}C$$



$$P = \frac{\Delta T_c}{T_{h1} - T_{c1}} = \frac{70}{350 - 80} = 0.26$$

$$R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}} = \frac{175}{70} = 2.5$$

From Figure 2.8, F=0.96

$$Q = UAF\Delta T_{lm,cf}$$

$$\therefore \ \mathrm{U} = \frac{\mathrm{Q}}{\mathrm{AF} \cdot \Delta \mathrm{T_m}} = \frac{3739.5 \times 10^3}{925 \times 0.96 \times 141.05} = 29.86 \ \ \mathrm{W/(m^2 \cdot K)}$$

Problem 2.12

A shell-and-tube heat exchanger given in Problem 2.11 is used to heat 62,000 kg / hr of water from 20°C to about 50°C. Hot water at 100°C is available. Determine how the heat transfer rate and the water outlet temperature vary with the hot water mass flow rate. Calculate the heat transfer rates and the outlet temperatures for hot water flow rates:

a. **80,000 kg / hr**

b. **40,000 kg / hr**

GIVEN:

-A shell-and-tube heat exchanger having two shell passes and eight tube passes.

-Cold water inlet temperature (T_{c1}) = 20°C

-Cold water outlet temperature (T_{c2}) = 50°C

-Cold water mass flow rate ($\dot{m}_h$) = 62,000 kg/hr = 17.222 kg/s

-Hot Water inlet temperature (T_{h1}) = 100°C

-Total surface area (A) = 925 m^2

FIND:

Heat transfer rates (Q) and outlet temperature of hot water (T_{h2}) for mass flow rate of

a. 80,000 kg/hr

b. 40,000 kg/hr

SOLUTION:

The heat balance equation:

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right) = \dot{m}_h c_{p,h}\left(T_{h2} - T_{h1}\right)$$

$$c_{p,c} = 4.179 \text{ kJ/(kg.K)} \quad (T = 35^{o}C)$$
$$c_{p,h} = 4.22 \text{ kJ/(kg.K)} \quad (T = 100^{o}C)$$

a. $\dot{m}_h = 80{,}000 \quad \text{kg / hr}$

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right) = 17.222 \times 4.179 \times (50 - 20) = 2159.12 \quad kW$$

$$T_{h2} = T_{h1} - \frac{\dot{m}_c c_{p,c}}{\dot{m}_h c_{p,h}}(T_{c2} - T_{c1})$$

$$= 100 - \frac{62000 \times 4.179}{80000 \times 4.22} \times 30 = 76.98 \quad {}^oC$$

b. $\dot{m}_h = 40{,}000 \quad kg / hr$

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right) = 17.222 \times 4.179 \times (50 - 20) = 2159.12 \quad kW$$

$$T_{h2} = T_{h1} - \frac{\dot{m}_c c_{p,c}}{\dot{m}_h c_{p,h}}(T_{c2} - T_{c1})$$

$$= 100 - \frac{62000 \times 4.179}{40000 \times 4.22} \times 30 = 53.95 \quad {}^oC$$

Problem 2.13

Water at a flow rate of 5,000 kg/hr (c_p=4182 J/kg.K) is heated from 10°C to 35°C in an oil cooler by engine oil having an inlet temperature of 65°C (c_p= 2072 J/kg.K) with a flow rate of 6,000 kg/hr. Take the overall heat transfer coefficient to be 3,500 W/m^2.K. What are the areas required for:

a. Parallel flow

b. Counterflow

GIVEN:

-Cold water inlet temperature (T_{c1}) = 10°C

-Cold water outlet temperature (T_{c2}) = 35°C

-Cold water mass flow rate ($\dot{m}_h$) = 5,000 kg/hr = 1.389 kg/s

-Hot Water inlet temperature (T_{h1}) = 65°C

-Hot water mass flow rate ($\dot{m}_h$) = 6,000 kg/hr = 1.667 kg/s

-Overall heat coefficient (U) = 3,500 W/m^2.K

-Specific heat of cold water ($c_{p,c}$) = 4182 J/kg.K

-Specific heat of hot water ($c_{p,h}$) = 2072 J/kg.K

FIND:

Heat transfer area (A) required for:

a. parallel flow;

b. counter flow.

SOLUTION:

The heat balance equation:

$$Q = \dot{m}_c c_{p,c}\left(T_{c2} - T_{c1}\right) = \dot{m}_h c_{p,h}\left(T_{h2} - T_{h1}\right)$$

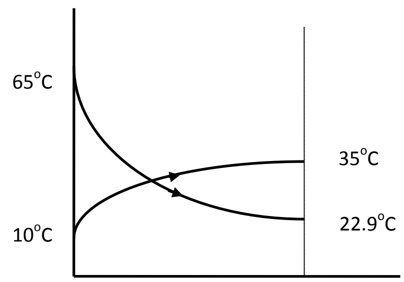
$$Q = \dot{m}_h c_{p,h}\left(T_{h2} - T_{h1}\right) = \frac{9.4}{3600} \times 1060 \times (616 - 232) = 1063 \quad W$$

$$T_{c2} = T_{c1} + \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} (T_{h2} - T_{h1})$$

$$= 16 + \frac{1063}{\left(\frac{0.3 \times 10^{-3} \times 999}{60}\right) \times 4180} = 67 \quad ^oC$$

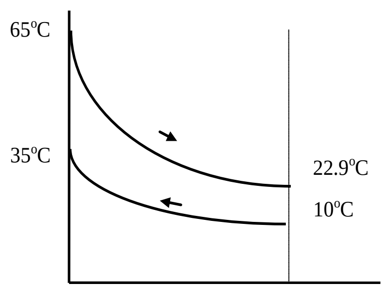
a. Parallel flow:

The parallel flow is not an acceptable solution, because of the temperatures cross.



Impossible case

b. counter flow:



$$\Delta T_{lm,cf} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{(65 - 35) - (22.9 - 10)}{\ln \frac{65 - 35}{22.9 - 10}} = 20.3 \quad ^oC$$

$$A = \frac{Q}{U \cdot \Delta T_{lm,cf}} = \frac{145.208 \times 10^3}{3500 \times 20.3} = 2.04 \ m^2$$

Problem 2.14

In order to cool a mass flow rate of 9.4 kg / h of air from 616°C to 232°C, it is passed through the inner tube of double-pipe heat exchanger with counterflow, which is 1.5 m long with an outer diameter of the inner tube of 2 cm.

a. Calculate the heat transfer rate. For air, $c_{p,h}$ = 1060 J/kg.K

b. The cooling water enters the annular side at 16°C with a mass flow rate of 0.3 L/min.

Calculate the exit temperature of the water. For water, $c_{p,c}$ = 4180 J/kg.K

c. Determine the effectiveness of this heat exchanger, NTU. The overall heat transfer coefficient based on the outside heat transfer surface area is 38.5 W/m^2.K. Calculate the surface area of the heat exchanger and number of double-pipe heat exchangers.

GIVEN:

-Hot air inlet temperature (T_{h1}) = 616°C

-Hot air outlet temperature (T_{h2}) = 232°C

-Hot air mass flow rate ($\dot{m}_h$) = 9.4 kg/hr = 0.002611 kg/s

-Specific heat of hot air ($c_{p,h}$) = 1060 J/kg.K

-Cooling water inlet temperature (T_{c1}) = 16°C

-Specific heat of cooling water ($c_{p,c}$) = 4180 J/kg.K

-Hot water mass flow rate ($\dot{m}_h$) = 0.3 l/min

-The overall heat transfer coefficient of the hot fluid is (U) = 38.5 W/m^2.K

FIND:

a. Heat transfer rate Q

b. Exit temperature of the water T

c. Effectiveness of the heat exchanger ε, NTU, and heat transfer surface area A