

# INSTRUCTOR'S SOLUTIONS MANUAL

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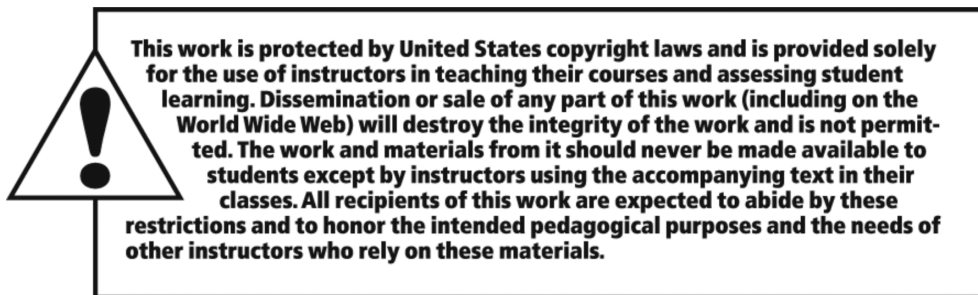
## GEOMETRY

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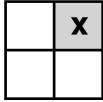
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# Chapter 1

## Section 1.2 Practice

1. Each figure is a square divided into four smaller squares, with one of the smaller squares shaded with an X in it. So, predict that the next figure is a square divided into four smaller squares, with the upper-right square shaded and marked, as sketched below.



2. a. The second number is the first plus 2, the third is the second minus 3, the fourth is the third plus 4, and the fifth is the fourth minus 5. So, predict that the sixth is the fifth plus 6, or 3.
- b. Each number is  $\frac{1}{3}$  of the previous number, so predict that the next number is  $\frac{1}{81}$ .
3. No. None of the angles are marked, so we may not conclude that they have the same measure.

## Vocabulary & Readiness Check 1.2

- defined term
- undefined term
- A statement that we prove is called a theorem.
- A statement that we accept as true but do not prove is called an axiom or a postulate.

## Exercise Set 1.2

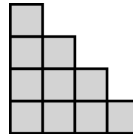
2. Each figure is a light brown rectangle divided into 1 more equal-sized section than the previous figure. A sketch of the next figure is provided.



4. Each figure is a square divided into 4 different colored squares. Each figure is the previous image rotated 90° clockwise. A sketch of the next figure is provided. G stands for green, R stands for red, B stands for blue, and Y stands for yellow.



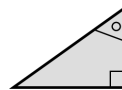
6. Each subsequent figure has 1 square added on top of the existing columns, and 1 square added to the right. The next figure should have 4 squares in the first column, 3 in the second column, 2 in the third column, and 1 in the fourth column. A sketch of the next figure is provided.



8. The figures are rectangles that alternate between tan and green. There is an X in a bottom corner of each tan rectangle, and in a top corner of each green rectangle. The X switches from left to right in figures with the same color. The next figure will be green with an X in the top right corner. A sketch of the next figure is provided.



10. The figures are green right triangles. The second figure is the first figure reflected vertically. The third figure is the second figure reflected horizontally. The next figure will be the third figure reflected vertically. A sketch of the next figure is provided.



12. Each number is 2 more than the previous number, so predict that the next two numbers are 2 and 4.

14. Each number has a denominator that is one larger than the denominator of the previous number when written as a fraction, so predict that the next two numbers are  $\frac{1}{5}$  and  $\frac{1}{6}$ .
16. Each number is 3 times the previous number, so predict that the next two numbers are 162 and 486.
18. Each number is half of the previous number, so predict that the next two numbers are  $\frac{1}{40}$  and  $\frac{1}{80}$ .
20. Add 1 to the first number to get the second number, and add 1 to the second number to get the third number. Subtract 1 from the third number to get the fourth number, and subtract 1 from the fourth number to get the fifth number. So, a pattern could indicate to add 1 to the fifth number to get the sixth number, and add 1 to the sixth number to get the second number. So, predict that the next two numbers are 0 and 1.
22. Yes. The figure has 3 sides that are connected, which is the requirement for being called a triangle.
24. No. The figure is a triangle because it has 3 connected sides, but no indication is provided that one of the angles is a right angle.
26. Yes. Both angles measure  $37^\circ$ , so they have the same measure.
28. No. No indication is provided that the segments are the same length.
30. Yes. Both angles have the middle ray as a side.
32. No. The opposite sides of the figure are not marked parallel, so it cannot be concluded that the figure is a parallelogram.
34. The student's number is not correct. Since the number that follows  $\frac{1}{4}$  is  $\frac{1}{6}$ , the denominator is not doubled to get the next denominator. Add 2 to the denominator to get

the next denominator. Therefore, the next number in the list is  $\frac{1}{8}$ .

36. The student's figure is not correct. In this pattern, the positions of the dot and the x are reversed to generate the next figure. Then the image is flipped vertically to generate the next figure. The correct figure should have the positions of the dot and the x reversed, as shown.

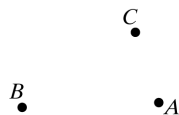


38. Neither axioms nor postulates are proven. They are accepted as true.

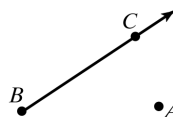
### Section 1.3 Practice

1. a. Two other ways to name  $\overline{EH}$  are  $\overline{HE}$  and line  $n$ .  
 b. There are many correct answers. Two other ways to name plane  $M$  are plane  $GED$  and plane  $GFD$ .  
 c. Points  $D$ ,  $E$ , and  $F$  lie on the same line, so they are collinear.  
 d. Points  $D$ ,  $E$ ,  $F$ , and  $G$  lie on the same plane, so they are coplanar.  
 e. There are many correct answers. For instance, points  $H$ ,  $E$ , and  $F$  do not lie on the same line.
2. a. The three segments are  $\overline{GJ}$  or  $\overline{JG}$ ,  $\overline{JQ}$  or  $\overline{QJ}$ , and  $\overline{GQ}$  or  $\overline{QG}$ .  
 b. The four rays are  $\overline{GJ}$  or  $\overline{GQ}$ ,  $\overline{JG}$ ,  $\overline{JQ}$ , and  $\overline{QG}$  or  $\overline{QJ}$ .  
 c. The opposite rays are  $\overline{JG}$  and  $\overline{JQ}$ .

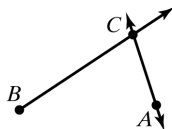
3. a.



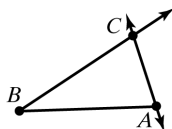
- b.



c.



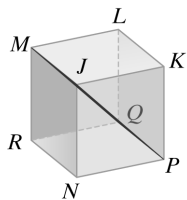
d.



4. Points  $W$ ,  $Z$ , and  $A$  are collinear and  $A$  is between  $W$  and  $Z$ . So,  $\overline{AW}$  and  $\overline{AZ}$  are opposite rays.  
Points  $Y$ ,  $X$ , and  $A$  are collinear and  $A$  is between  $Y$  and  $X$ . So,  $\overline{AY}$  and  $\overline{AX}$  are opposite rays.

5. The plane intersects in  $\overline{EH}$ .

6.



### Vocabulary & Readiness Check 1.3

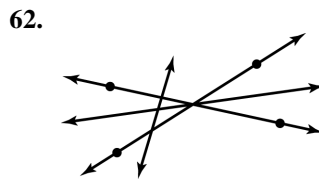
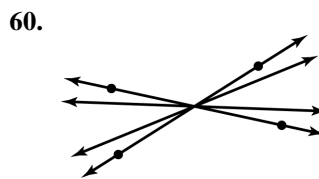
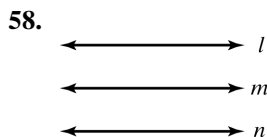
1. C. The symbol  $\overline{RS}$  means a segment  $RS$ .
2. A. The symbol  $\overline{RS}$  means a line  $RS$ .
3. D. The symbol  $\overline{RS}$  means a ray  $RS$ .
4. B. The symbol  $\overline{SR}$  means a ray  $SR$ .
5. A line extends in opposite directions without end.
6. A plane extends in two dimensions without end.
7. A ray consists of an endpoint and all points of a line on one side of the endpoint.
8. A segment consists of two endpoints and all the points between them.
9. Coplanar points lie on the same plane.

10. Collinear points lie on the same line.
11. A geometric figure is any nonempty subset of space.
12. Opposite rays are two rays that share an endpoint and form a line.
13. A point has a location but no dimension.
14. Space is the set of all points.

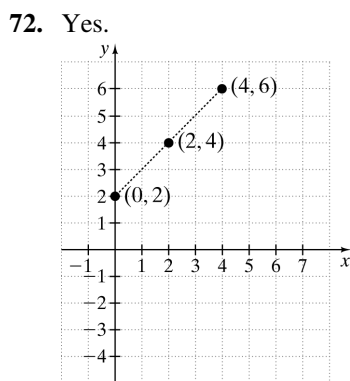
### Exercise Set 1.3

2.  $\overline{NT}$ ,  $\overline{TN}$ ,  $\overline{TR}$ ,  $\overline{RN}$ ,  $\overline{NR}$ , and line  $l$  are all possible answers.
4. There are many correct answers. Sample answers: plane  $TNJ$  and plane  $TRJ$
6. Points  $R$ ,  $N$ , and  $T$  lie on the same line, so they are collinear.
8. Points  $R$ ,  $N$ ,  $T$ , and  $J$  lie on the same plane, so they are coplanar.
10. The segments are  $\overline{AZ}$  or  $\overline{ZA}$ ,  $\overline{AX}$  or  $\overline{XA}$ ,  $\overline{XY}$  or  $\overline{YX}$ ,  $\overline{AY}$  or  $\overline{YA}$ ,  $\overline{XZ}$  or  $\overline{ZX}$  and  $\overline{YZ}$  or  $\overline{ZY}$ .
12. The rays are  $\overline{AZ}$  or  $\overline{AX}$ ,  $\overline{AY}$ ,  $\overline{XZ}$  or  $\overline{XY}$ ,  $\overline{YZ}$ ,  $\overline{ZA}$  or  $\overline{ZX}$  or  $\overline{ZY}$ ,  $\overline{YA}$  or  $\overline{YX}$ , and  $\overline{XA}$ .
14. The opposite rays with endpoint  $X$  are  $\overline{XA}$  and  $\overline{XZ}$  or  $\overline{XY}$ .
16. The only line is  $\overline{BC}$
18. The rays are  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CB}$ .
20. False.  $\overline{XP}$  and  $\overline{XN}$  are not opposite rays because  $P$ ,  $X$ , and  $N$  are not collinear.
22. True.  $\overline{XL}$  and  $\overline{XN}$  are opposite rays because  $\overline{XL}$  and  $\overline{XN}$  are two rays with the same endpoints going in opposite directions and all three points are collinear.

- 24. There are many correct answers. Sample Answer: Points  $A$ ,  $E$ , and  $B$  are not collinear.
- 26. There are many correct answers. Sample Answer: Points  $W$ ,  $A$ , and  $X$  are not collinear.
- 28. False. Point  $D$  lies on line  $l$ , not line  $m$ .
- 30. True. Point  $C$  does lie on line  $m$ .
- 32. True. Points  $D$ ,  $E$ , and  $B$  are collinear because they all lie on the same line, line  $l$ .
- 34. False. The fact that the points are collinear means that many planes pass through these three points, not just one.
- 36. The plane intersects in  $\overline{VW}$ .
- 38. The plane intersects in  $\overline{TX}$ .
- 40. Sample Answer: planes  $TXW$  and  $TSR$
- 42. Sample Answer: planes  $UXV$  and  $VWS$
- 44. Point  $G$
- 46. Point  $H$
- 48. Point  $E$
- 50. Point  $H$
- 52. Coplanar. All four points are on the same plane.
- 54. Coplanar. All four points are on the same plane.
- 56. Noncoplanar. All four points are not on the same plane.

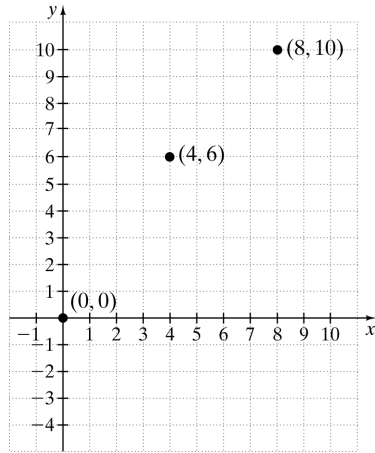


- 64. Sometimes. This is true if  $\overline{JL}$  is a ray where point  $K$  is on the ray  $\overline{JL}$ . This is false if  $\overline{KL}$  is a line where  $J$  is on that line between  $K$  and  $L$ . Then the two rays would have the same endpoints and be going in opposite directions.
- 66. Sometimes. This could be true like Exercise 52 where there are four points that all are on the same plane. This could be false like Exercise 56 where there are four points and one of them is on a different plane from the other three.
- 68. Never. Postulate 1.3-2 states that two distinct lines intersect at exactly one point.
- 70. The following times can also be represented as opposite rays on a clock: 1:35, 2:40, 3:45, 4:50, 5:55, 7:05, 8:10, 9:15, 10:20, 11:25, and 12:30.

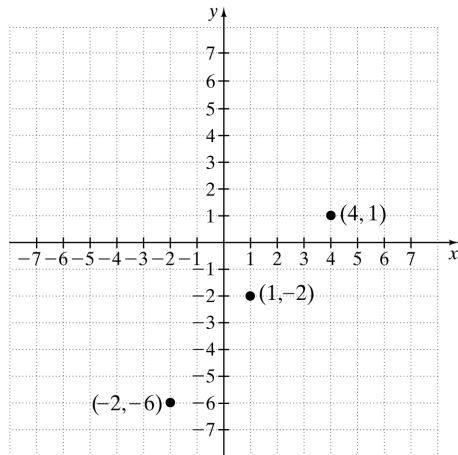




74. No.



76. No.



78. One line can have many planes pass through it. A single line can be the intersection of multiple planes, but there is no distinct number.

80. If two lines intersect then only one plane can contain both of those lines. Postulate 1.3-3 states that if two distinct planes intersect, then they intersect in exactly one line. This means that no two planes can contain two distinct lines. Using the diagram, the plane can be labeled  $ABC$ , which refers to a single plane.

## Section 1.4 Practice

1.  $NQ$  is the distance between points  $N$  and  $Q$ . Use the Ruler Postulate to find  $NQ$ . The coordinate of  $N$  is  $-2$ , and the coordinate of  $Q$  is  $13$ .

$$NQ = |13 - (-2)|$$

$$NQ = |16|$$

$$NQ = 16$$

2. a.  $\overline{AC}$  and  $\overline{DF}$  are marked the same, so  $\overline{AC} \cong \overline{DF}$  and  $AC = DF$ .  $\overline{AB}$  and  $\overline{DE}$  are marked the same, so  $\overline{AB} \cong \overline{DE}$  and  $AB = DE$ .  $\overline{BC}$  and  $\overline{EF}$  are marked the same, so  $\overline{BC} \cong \overline{EF}$  and  $BC = EF$ .

b.  $AC = DF$ , so if  $AC = 10$  cm,  
 $DF = 10$  cm.

c.  $CB = FE$ , so if  $CB = 25$  cm,  
 $FE = 25$  cm.

3. If  $BC = GD$ , then  $\overline{BC} \cong \overline{GD}$ .

$$BC = |(-1) - (-6)| = |5| = 5$$

$$GD = |8 - 3| = |5| = 5$$

Yes,  $BC = GD$ , so  $\overline{BC} \cong \overline{GD}$ .

4. Use the Segment Addition Postulate and algebra to find  $x$ .

$$QR + RS = QS$$

$$(12x - 3) + (8x + 2) = 79$$

$$20x - 1 = 79$$

$$20x = 80$$

$$\frac{20x}{20} = \frac{80}{20}$$

$$x = 4$$

Use the value of  $x$  to find  $QR$  and  $RS$ .

$$QR = 12x - 3 \quad RS = 8x + 2$$

$$= 12(4) - 3 \quad = 8(4) + 2$$

$$= 48 - 3 \quad = 34$$

$$= 45$$

5. Use the meaning of the midpoint to find the value of  $x$ .

$$QP = PR$$

$$11x - 8 = 10x - 1$$

$$x - 8 = -1$$

$$x = 7$$

Now, use the value of  $x$  to find  $\overline{QP}$  and  $\overline{PR}$ , which should be the same.

$$\begin{aligned} \overline{QP} &= 11x - 8 & \overline{PR} &= 10x - 1 \\ &= 11(7) - 8 & &= 10(7) - 1 \\ &= 77 - 8 & &= 70 - 1 \\ &= 69 & &= 69 \end{aligned}$$

To find  $\overline{QR}$ , add.

$$\begin{aligned} \overline{QR} &= \overline{QP} + \overline{PR} \\ &= 69 + 69 \\ &= 138 \end{aligned}$$

### Vocabulary & Readiness Check 1.4

- Two segments that have the same length are called congruent segments.
- The real number that corresponds to a point is called the coordinate of the point.
- Given three distinct points on a line, one of the points will always be between the other two.
- A point that divides a segment into two congruent segments is called a midpoint.
- If two segments are congruent, then their lengths are equal.
- The distance between two points is the absolute value of the difference of their coordinates.
- A line, ray, segment, or plane that intersects a segment at its midpoint is called a segment bisector.
- In geometry, to bisect a segment means to divide it into two equal parts.

### Exercise Set 1.4

- Yes;  $\overline{Q}$  is between  $\overline{R}$  and  $\overline{T}$  because  $\overline{Q}$  appears on  $\overline{RT}$ .
- $\overline{AB}$  is the distance between points  $A$  and  $B$ . Use the ruler postulate to find  $\overline{AB}$ .  

$$\overline{AB} = |-6 - (-8)|$$

$$\overline{AB} = |2|$$

$$\overline{AB} = 2$$
- $\overline{AD}$  is the distance between points  $A$  and  $D$ . Use the ruler postulate to find  $\overline{AD}$ .  

$$\overline{AD} = |3 - (-8)|$$

$$\overline{AD} = |11|$$

$$\overline{AD} = 11$$
- $\overline{VX}$  and  $\overline{UW}$  are marked the same, so  $\overline{VX} \cong \overline{UW}$  and  $VX = UW$ , which means since  $VX = 105$  cm, then  $UW = 105$  cm.
- $\overline{DE}$  and  $\overline{FG}$  are marked the same, so  $\overline{DE} \cong \overline{FG}$  and  $DE = FG$ , which means since  $DE = 7$  in., then  $FG = 7$  in.
- $\overline{QS}$  and  $\overline{RT}$  are marked the same, so  $\overline{QS} \cong \overline{RT}$  and  $QS = RT$ , which means since  $QS = 33$  ft, then  $RT = 33$  ft.
- If  $\overline{MP} = \overline{NQ}$ , then  $\overline{MP} \cong \overline{NQ}$ .  

$$\overline{MP} = |6 - (-3)| = |9| = 9$$

$$\overline{NQ} = |12 - 3| = |9| = 9$$
 Yes,  $\overline{MP} = \overline{NQ}$ , so  $\overline{MP} \cong \overline{NQ}$ .
- If  $\overline{LP} = \overline{MQ}$ , then  $\overline{LP} \cong \overline{MQ}$ .  

$$\overline{LP} = |6 - (-8)| = |14| = 14$$

$$\overline{MQ} = |12 - (-3)| = |15| = 15$$
 No,  $\overline{LP} \neq \overline{MQ}$ , so  $\overline{LP}$  is not congruent to  $\overline{MQ}$ .
- If  $\overline{ST} = 15$  and  $\overline{RT} = 40$ , then  $\overline{RS} = 40 - 15 = 25$ .

28. a. Use the Segment Addition Postulate and algebra to find  $x$ .

$$\begin{aligned}RS + ST &= RT \\(14x - 8) + (9x + 10) &= 232 \\23x + 2 &= 232 \\23x &= 230 \\\frac{23x}{23} &= \frac{230}{23} \\x &= 10\end{aligned}$$

- b. Use the value of  $x$  to find  $RS$  and  $ST$ .

$$\begin{aligned}RS &= 14x - 8 & ST &= 9x + 10 \\&= 14(10) - 8 & &= 9(10) + 10 \\&= 140 - 8 & &= 90 + 10 \\&= 132 & &= 100\end{aligned}$$

30. a. Use the meaning of the midpoint to find the value of  $y$ .

$$\begin{aligned}9y - 25 &= 4y \\5y - 25 &= 0 \\5y &= 25 \\\frac{5y}{5} &= \frac{25}{5} \\y &= 5\end{aligned}$$

- b. Use the value of  $y$  to find  $QC$  and  $CR$ , which should be the same.

$$\begin{aligned}QC &= 9y - 25 & CR &= 4y \\&= 9(5) - 25 & &= 4(5) \\&= 45 - 25 & &= 20 \\&= 20\end{aligned}$$

To find  $QR$ , add.

$$\begin{aligned}QR &= QC + CR \\&= 20 + 20 \\&= 40\end{aligned}$$

32.  $\overline{PT}$  and  $\overline{TQ}$  are marked the same, so  $\overline{PT} \cong \overline{TQ}$  and  $PT = TQ$ . Use this to solve for  $x$ .

$$\begin{aligned}PT &= TQ \\6x - 35 &= 2x - 3 \\4x &= 32 \\\frac{4x}{4} &= \frac{32}{4} \\x &= 8\end{aligned}$$

Use  $x$  to find  $PT$ .

$$\begin{aligned}PT &= 6x - 35 \\&= 6(8) - 35 \\&= 48 - 35 \\&= 13\end{aligned}$$

34.  $\overline{PT}$  and  $\overline{TQ}$  are marked the same, so  $\overline{PT} \cong \overline{TQ}$  and  $PT = TQ$ . Use this to solve for  $x$ .

$$\begin{aligned}PT &= TQ \\7x - 24 &= 6x - 2 \\x &= 22\end{aligned}$$

Use  $x$  to find  $PT$ .

$$\begin{aligned}PT &= 7x - 24 \\&= 7(22) - 24 \\&= 154 - 24 \\&= 130\end{aligned}$$

36. Find  $ZX$  and  $WY$ .

$$\begin{aligned}ZX &= |-7 - 1| & WY &= |5 - (-3)| \\&= |-8| & &= |8| \\&= 8 & &= 8\end{aligned}$$

Because  $ZX = WY$ ,  $\overline{ZX} \cong \overline{WY}$ .

38. You are given that  $AT = 7$ . Because  $\angle DEC$ , the coordinate of  $T$  must be either 7 or  $-7$ . The midpoint is halfway between these coordinates. Take half of the distance

from zero.  $\frac{7-0}{2} = 3\frac{1}{2}$ , so the coordinate of

the midpoint is either  $3\frac{1}{2}$  or  $-3\frac{1}{2}$ .

40. The coordinate of  $G$  must either be 6 units greater than or 6 units less than the coordinate of  $P$ .

$$\begin{aligned}10 + 6 &= 16 \\10 - 6 &= 4\end{aligned}$$

The possible coordinates of  $G$  are 4 and 16.

42.  $\overline{ED}$  and  $\overline{DB}$  are marked the same, so  $\overline{ED} \cong \overline{DB}$  and  $ED = DB$ . Use this to solve for  $x$ .

$$\begin{aligned}ED &= DB \\x + 4 &= 3x - 8 \\-2x + 4 &= -8 \\-2x &= -12 \\\frac{-2x}{-2} &= \frac{-12}{-2} \\x &= 6\end{aligned}$$

Use  $x$  to find  $ED$  and  $DB$ . They should be the same.

$$\begin{array}{rcl} ED & = & x + 4 \\ & = & 6 + 4 \\ & = & 10 \end{array} \qquad \begin{array}{rcl} DB & = & 3x - 8 \\ & = & 3(6) - 8 \\ & = & 18 - 8 \\ & = & 10 \end{array}$$

Add  $ED$  and  $DB$  to find  $EB$ .

$$\begin{array}{rcl} EB & = & ED + DB \\ & = & 10 + 10 \\ & = & 20 \end{array}$$

44. Mile markers for Macclenny and Tallahassee are shown, so take the absolute value of the difference to find the distance.

$$|199 - 335| = |-136| = 136$$

Divide the distance by the number of miles per hour to find the time.

$$\frac{136}{58} = 2\frac{20}{58} \text{ hours, or 2 hours and}$$

$$20\frac{20}{29} \text{ minutes.}$$

46. The highway sign gives the number of miles away each town is from the sign. If your friend is reading the sign, then your friend is the same distance away from the town as the sign. Because 80 is shown on the sign for Watertown, Watertown is 80 miles away, so your friend is incorrect.

48. a. Add  $GJ$  and  $HK$ , but because they overlap, you will need to subtract the distance that they share, which is  $JH$ .

$$\begin{array}{l} GK = GH - HJ + HK \\ GK = (2x + 3) - x + (4x - 3) \\ GK = 5x \end{array}$$

- b. Solve for  $x$  using  $GK = 30$ .

$$\begin{array}{l} GK = 5x \\ 30 = 5x \\ \frac{30}{5} = \frac{5x}{5} \\ 6 = x \end{array}$$

Subtract  $HJ$  from  $GJ$  to find  $GH$ .

$$\begin{array}{l} GH = GJ - HJ \\ = (2x + 3) - x \\ = x + 3 \\ = 6 + 3 \\ = 9 \end{array}$$

Subtract  $HJ$  from  $HK$  to find  $JK$ .

$$\begin{array}{l} JK = HK - HJ \\ = (4x - 3) - x \\ = 3x - 3 \\ = 3(6) - 3 \\ = 18 - 3 \\ = 15 \end{array}$$

50. The mile markers are like numbers on a number line. The number at each marker is the distance from the start of the road. This means that you can subtract the number on a mile marker from the number on another mile marker and take the absolute value to get the distance between the markers, just like finding the distance between two coordinates on a number line.

### Section 1.5 Practice

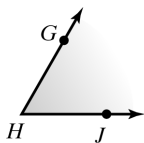
- $\angle NPQ, \angle QPN$
  - $\angle QPM$
- $m\angle TQN = |180 - 165|^\circ = 15^\circ$ , acute angle  
 $m\angle TQM = |180 - 45|^\circ = 135^\circ$ , obtuse angle  
 $m\angle TQR = |180 - 0|^\circ = 180^\circ$ , straight angle
- $\angle A \cong \angle D$ , so  $m\angle A = m\angle D$ .  $\angle C \cong \angle F$ , so  $m\angle C = m\angle F$ .
  - $m\angle A = 25^\circ$ , so  $m\angle D = 25^\circ$ .
  - $m\angle F = 120^\circ$ , so  $m\angle C = 120^\circ$ .
- $x = \frac{360^\circ}{6} = 60^\circ$
- Use the Angle Addition Postulate and solve for  $x$ .
 
$$\begin{array}{l} m\angle PQR + m\angle RQS = m\angle PQS \\ (4x - 10) + (11x + 10) = 180 \\ 15x = 180 \\ x = 12 \end{array}$$
 Substitute the value of  $x$  to find  $m\angle PQR$ .
 
$$\begin{array}{l} m\angle PQR = (4x - 10)^\circ \\ m\angle PQR = (4 \cdot 12 - 10)^\circ \\ m\angle PQR = 38^\circ \end{array}$$
 Substitute the value of  $x$  to find  $m\angle RQS$ .
 
$$\begin{array}{l} m\angle RQS = (11x + 10)^\circ \\ m\angle RQS = (11 \cdot 12 + 10)^\circ \\ m\angle RQS = 142^\circ \end{array}$$

## Vocabulary &amp; Readiness Check 1.5

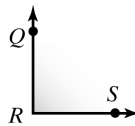
1. An instrument used to measure angles in degrees is called a protractor.
2. Two angles that have the same measure are called congruent angles.
3. An angle consists of two different rays with a common endpoint.
4. The rays of an angle are also called the sides of the angle.
5. The common endpoint of the rays of an angle is called the vertex of the angle.
6. If two angles are congruent, then their measures are equal.
7. An obtuse angle has a degree measure between  $90^\circ$  and  $180^\circ$ .
8. An acute angle has a degree measure between  $0^\circ$  and  $90^\circ$ .
9. A straight angle has a degree measure equal to  $180^\circ$ .
10. A right angle has a degree measure of  $90^\circ$ .

## Exercise Set 1.5

2.  $\angle ABC$ ,  $\angle 1$ ,  $\angle CBA$ ,  $\angle B$
4.  $\angle MKL$ ,  $\angle 3$ ,  $\angle LKM$ ,  $\angle K$
6.  $\angle DAF = |90 - 0|^\circ = 90^\circ$ ; right angle
8.  $\angle BAC = |25 - 0|^\circ = 25^\circ$ ; acute angle
10.  $\angle DAE = |90 - 70|^\circ = 20^\circ$ , acute angle
12. Sample of an acute angle,  $\angle GHJ$ .



14. Sample of a right angle,  $\angle QRS$ .



16.  $\angle FJH \cong \angle BJA$
18. If  $m\angle GHF = 130^\circ$ , then  $m\angle JBC = 130^\circ$ .
20. A clock face is a circle which is divided into 12 equal slices. Each slice represents an angle measuring  $\frac{360^\circ}{12} = 30^\circ$ . The angle measure of the clock hands at 5:00 is equal to  $5 \cdot 30^\circ = 150^\circ$ .
22. A clock face is a circle which is divided into 12 equal slices. Each slice represents an angle measuring  $\frac{360^\circ}{12} = 30^\circ$ . The angle measure of the clock hands at 11:00 is equal to  $1 \cdot 30^\circ = 30^\circ$ .
24.  $y = \frac{360^\circ}{5} = 72^\circ$
26.  $x = \frac{360^\circ}{12} = 30^\circ$
28. Use the Angle Addition Postulate and solve for  $x$ .  
 $m\angle YNZ + m\angle ZNM = m\angle YNM = 45^\circ$   
 $(6x - 2) + (2x - 1) = 45$   
 $8x - 3 = 45$   
 $8x = 48$   
 $x = 6$   
 Substitute the value of  $x$  to solve for  $m\angle YNZ$ .  
 $m\angle YNZ = (6x - 2)^\circ$   
 $m\angle YNZ = (6 \cdot 6 - 2)^\circ$   
 $m\angle YNZ = 34^\circ$   
 Substitute the value of  $x$  to solve for  $m\angle ZNM$ .  
 $m\angle ZNM = (2x - 1)^\circ$   
 $m\angle ZNM = (2 \cdot 6 - 1)^\circ$   
 $m\angle ZNM = 11^\circ$

30. Use the Angle Addition Postulate and solve for  $x$ .

$$m\angle ABD + m\angle DBC = m\angle ABC = 180^\circ$$

$$(5x - 3) + (3x + 23) = 180$$

$$8x + 20 = 180$$

$$8x = 160$$

$$x = 20$$

Substitute the value of  $x$  to solve for  $m\angle ABD$ .

$$m\angle ABD = (5x - 3)^\circ$$

$$m\angle ABD = (5 \cdot 20 - 3)^\circ$$

$$m\angle ABD = 97^\circ$$

Substitute the value of  $x$  to solve for  $m\angle DBC$ .

$$m\angle DBC = (3x + 23)^\circ$$

$$m\angle DBC = (3 \cdot 20 + 23)^\circ$$

$$m\angle DBC = 83^\circ$$

32. Use the Angle Addition Postulate and solve for  $x$ .

$$m\angle MRJ + m\angle JRK = m\angle MRK = 90^\circ$$

$$(8x + 1) + (x + 8) = 90$$

$$9x + 9 = 90$$

$$9x = 81$$

$$x = 9$$

Substitute the value of  $x$  to solve for  $m\angle MRJ$ .

$$m\angle MRJ = (8x + 1)^\circ$$

$$m\angle MRJ = (8 \cdot 9 + 1)^\circ$$

$$m\angle MRJ = 73^\circ$$

Substitute the value of  $x$  to solve for  $m\angle JRK$ .

$$m\angle JRK = (x + 8)^\circ$$

$$m\angle JRK = (9 + 8)^\circ$$

$$m\angle JRK = 17^\circ$$

34. The measure of the angle is  $90^\circ$  and it is a right angle.
36. The measure of the angle is  $88^\circ$  and it is an acute angle.

38.  $m\angle AOB = m\angle COD = 28^\circ$ ,  
 $m\angle BOC = (3x - 2)^\circ$ , and  $m\angle AOD = 6x^\circ$ .

Use the Angle Addition Postulate and solve for  $x$ .

$$m\angle AOB + m\angle BOC + m\angle COD = m\angle AOD$$

$$28 + (3x - 2) + 28 = 6x$$

$$3x = 54$$

$$x = 18$$

Substitute the value of  $x$  to solve for  $m\angle BOC$ .

$$m\angle BOC = (3x - 2)^\circ = 52^\circ$$

Substitute the value of  $x$  to solve for  $m\angle AOD$ .

$$m\angle AOD = 6x^\circ = 108^\circ$$

Check your work by adding the interior angles to be sure that their sum equals  $m\angle AOD$ .

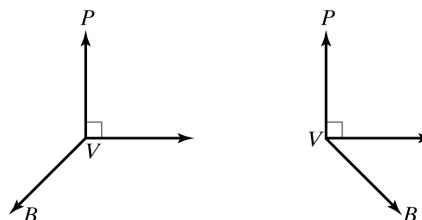
$$m\angle AOB + m\angle BOC + m\angle COD = m\angle AOD$$

$$28^\circ + 52^\circ + 28^\circ = 108^\circ = m\angle AOD$$

40. The clock face is a circle with 12 equal intervals. Each interval represents  $\frac{360^\circ}{12} = 30^\circ$ . The minute hand moves  $30^\circ$  every 5 minutes and the hour hand moves  $30^\circ$  every hour. At 8:40, the minute hand is on the 8 and the hour hand has moved  $\frac{2}{3}$  of the way from 8 to 9, so the clock hands are  $\frac{2}{3}$  of an interval apart. Since each interval represents  $30^\circ$ , at 8:40 the angle formed by the clock hands is  $\frac{2}{3} \cdot 30^\circ = 20^\circ$ .

42. The clock face is a circle with 12 equal intervals. Each interval represents  $\frac{360^\circ}{12} = 30^\circ$ . The minute hand moves  $30^\circ$  every 5 minutes and the hour hand moves  $30^\circ$  every hour. At 4:30, the minute hand is on the 6 and the hour hand has moved  $\frac{1}{2}$  of the way from 4 to 5, so the hands are 1.5 intervals apart. Since each interval represents  $30^\circ$ , at 4:30 the angle formed by the clock hands is  $\frac{1}{2} \cdot 30^\circ + 30^\circ = 45^\circ$ .

44. There are two ways to sketch  $\angle PVB = 135^\circ$ .



## Section 1.6 Practice

1.
  - a. True.  $\angle 8$  and  $\angle 5$  are two adjacent angles, and their noncommon sides form opposite rays.
  - b. True.  $\angle 8$  and  $\angle 6$  have sides that form opposite rays.
  - c. False.  $\angle 7$  and  $\angle 5$  are not adjacent angles.
  - d. False.  $\angle 7$  and  $\angle 6$  do not have sides that form opposite rays.
  
2. Find any angle pairs with measures that have a sum of  $90^\circ$ .
  - $\angle 8$  and  $\angle 7$  are complementary.
  - $\angle 7$  and  $\angle 6$  are complementary.
  - $\angle 6$  and  $\angle 5$  are complementary.
  - $\angle 8$  and  $\angle 5$  are complementary.
  
3. Find any angle pairs with measures that have a sum of  $180^\circ$ .
  - $\angle 5$  and  $\angle 6$  are supplementary.
  - $\angle 6$  and  $\angle 7$  are supplementary.
  - $\angle 7$  and  $\angle 8$  are supplementary.
  - $\angle 8$  and  $\angle 5$  are supplementary.
  
4.
  - a. Since  $\angle M$  and  $\angle C$  are supplementary, the sum of their measures is  $180^\circ$ .
 
$$m\angle M + m\angle C = 180^\circ$$

$$m\angle M + 16^\circ = 180^\circ$$

$$m\angle M = 164^\circ$$
  - b. Since  $\angle N$  and  $\angle C$  are complementary, the sum of their measures is  $90^\circ$ .
 
$$m\angle N + m\angle C = 90^\circ$$

$$m\angle N + 16^\circ = 90^\circ$$

$$m\angle N = 74^\circ$$
  
5.
  - a. Since  $m\angle LKM = m\angle MKN$ ,  
 $m\angle LKN = m\angle LKM + m\angle MKN$ .  
 $m\angle LKN = m\angle LKM + m\angle LKM$   
 $72^\circ = 2(m\angle LKM)$   
 $36^\circ = m\angle LKM$
  - b. Since  $m\angle LKM = m\angle MKN$ ,  
 $m\angle MKN = 36^\circ$ .
  
6. Since  $\overline{HL}$  bisects  $\angle KHJ$ ,  
 $m\angle KHL = m\angle LHJ$ .  
 Solve for  $x$ .  
 $m\angle KHL = m\angle LHJ$ .  

$$3x^\circ = (5x - 46)^\circ$$

$$-2x = -46$$

$$x = 23$$
 Use substitution to find the measure of each angle.  
 $m\angle KHL = 3x^\circ$   

$$= 3(23)^\circ$$

$$= 69^\circ$$
 $m\angle LHJ = (5x - 46)^\circ$   

$$= (5 \cdot 23 - 46)^\circ$$

$$= (115 - 46)^\circ$$

$$= 69^\circ$$
  
7.  $\angle DEA$  and  $\angle BEA$  form a linear pair, so the sum of their measures is  $180^\circ$ .  
 $m\angle DEA + m\angle BEA = 180^\circ$   
 $(5x - 6)^\circ + (2x + 46)^\circ = 180^\circ$   

$$7x + 40 = 180$$

$$7x = 140$$

$$x = 20$$
 $\angle DEC$  and  $\angle BEC$  form a linear pair.  
 $m\angle DEC + m\angle BEC = 180^\circ$   
 $(2y + 56)^\circ + (6y + 4)^\circ = 180^\circ$   

$$8y + 60 = 180$$

$$8y = 120$$

$$y = 15$$
 Use substitution to find the measure of each angle.  
 $m\angle DEA = (5x - 6)^\circ$   

$$= (5 \cdot 20 - 6)^\circ$$

$$= (100 - 6)^\circ$$

$$= 94^\circ$$
 $m\angle BEA = (2x + 46)^\circ$   

$$= (2 \cdot 20 + 46)^\circ$$

$$= (40 + 46)^\circ$$

$$= 86^\circ$$

$$\begin{aligned}
 m\angle DEC &= (2y + 56)^\circ \\
 &= (2 \cdot 15 + 56)^\circ \\
 &= (30 + 56)^\circ \\
 &= 86^\circ \\
 m\angle BEC &= (6y + 4)^\circ \\
 &= (6 \cdot 15 + 4)^\circ \\
 &= (90 + 4)^\circ \\
 &= 94^\circ
 \end{aligned}$$

## Vocabulary &amp; Readiness Check 1.6

- Two angles are complementary angles if their measures have a sum of  $90^\circ$ .
- Two angles are supplementary angles if their measures have a sum of  $180^\circ$ .
- Two angles are vertical angles if their sides form opposite rays.
- Two angles are adjacent angles if they share a common side and a common vertex, but have no interior points in common.
- Two adjacent angles form a linear pair if their noncommon sides are opposite rays.
- A ray that divides an angle into two adjacent congruent angles is called an angle bisector.

## Exercise Set 1.6

- $\angle 3$  and  $\angle 1$  are vertical angles.
- True.  $\angle 2$  and  $\angle 1$  are adjacent angles, and their noncommon sides form opposite rays.
- False.  $\angle 4$  and  $\angle 2$  are not adjacent angles.
- False.  $\angle 3$  and  $\angle 5$  do not have sides that form opposite rays.
- False.  $\angle 1$  and  $\angle 2$  do not have measures that add up to  $180^\circ$ .
- $\angle COE$  shares side  $\overline{OE}$  and is also a right angle.
- $\angle COD$  and  $\angle EOD$  form right angle  $\angle COE$ .
- $\angle AOD$  is vertical to  $\angle BOC$ .

- False.  $m\angle 1 + m\angle 4 \neq 90^\circ$
- True.  $m\angle 1 + m\angle 3 = 90^\circ$
- True.  $m\angle 3 + m\angle 4 = 90^\circ$
- False.  $m\angle 3 + m\angle 4 \neq 180^\circ$
- True.  $m\angle 4 + m\angle 1 = 180^\circ$
- The conclusion cannot be made from the given information. There is no indication that  $\overline{AC}$  is an angle bisector.
- The conclusion can be made from the given information, assuming that  $\overline{JD}$  is a line segment. Then  $\angle JCA$  and  $\angle ACD$  form a linear pair, which means the angles are supplementary, and the sum of their measures is  $180^\circ$ .
- The conclusion cannot be made from the given information. There is no indication that  $\overline{JC} \cong \overline{DC}$ .
- The conclusion can be made from the given information, assuming  $\overline{DE}$  and  $\overline{JF}$  are straight rays. Then,  $\angle EAF$  and  $\angle JAD$  have sides that form opposite rays.
- $\angle ACD$  and  $\angle ACF$  are a linear pair, so they are supplementary, and it is given that the sum of their measures is  $180^\circ$ . So,  $m\angle ACD = 180^\circ \div 2 = 90^\circ$ .  $m\angle ACD - m\angle ACB = m\angle BCD$ . Therefore,  $90^\circ - 65^\circ = m\angle BCD = 25^\circ$
- From Exercise 36,  $m\angle BCD = 25^\circ$ , and it is given that  $m\angle BCD = m\angle ECF$ .  $m\angle ECF + m\angle ACF = m\angle ACE$ , so  $25^\circ + 90^\circ = m\angle ACE = 115^\circ$ .
- Supplement:  $180^\circ - 70^\circ = 110^\circ$   
Complement:  $90^\circ - 70^\circ = 20^\circ$
- Supplement:  $180^\circ - 1^\circ = 179^\circ$   
Complement:  $90^\circ - 1^\circ = 89^\circ$
- Supplement:  $180^\circ - 175^\circ = 5^\circ$   
Complement: There is no complement because  $175^\circ > 90^\circ$ .



$$46. \quad m\angle GJH + m\angle DJG = m\angle DJH$$

$$29^\circ + 29^\circ = 58^\circ$$

$$m\angle DJH = 58^\circ$$

$$48. \quad m\angle SVT = m\angle RVS$$

$$m\angle SVT = 36^\circ$$

$$50. \quad \text{a.} \quad m\angle AQB = m\angle GQB$$

$$(8x - 5)^\circ = (9x - 13)^\circ$$

$$-5 = x - 13$$

$$8 = x$$

$$\text{b.} \quad m\angle AQB = (8x - 5)^\circ$$

$$(8 \cdot 8 - 5)^\circ = 59^\circ$$

$$\text{c.} \quad m\angle BQG = (9x - 13)^\circ$$

$$(9 \cdot 8 - 13)^\circ = 59^\circ$$

$$\text{d.} \quad m\angle AQG = m\angle AQB + m\angle BQG$$

$$= 59^\circ + 59^\circ$$

$$= 118^\circ$$

$$52. \quad \text{Solve for } x.$$

$$m\angle ABC = m\angle ABD + m\angle CBD$$

$$(4x - 12)^\circ = 24^\circ + 24^\circ$$

$$4x - 12 = 48$$

$$4x = 60$$

$$x = 15$$

Solve for  $m\angle ABC$  using the value of  $x$ .

$$m\angle ABC = (4x - 12)^\circ$$

$$= (4 \cdot 15 - 12)^\circ$$

$$= (60 - 12)^\circ$$

$$= 48^\circ$$

$$54. \quad \text{Solve for } x.$$

$$m\angle ABD = m\angle CBD$$

$$(3x + 20)^\circ = (6x - 16)^\circ$$

$$20 = 3x - 16$$

$$36 = 3x$$

$$12 = x$$

Solve for  $m\angle ABC$  using the value of  $x$ .

$$m\angle ABC = m\angle ABD + m\angle CBD$$

$$= (3x + 20)^\circ + (6x - 16)^\circ$$

$$= (9x + 4)^\circ$$

$$= (9 \cdot 12 + 4)^\circ$$

$$= (108 + 4)^\circ$$

$$= 112^\circ$$

$$56. \quad \text{a.} \quad \text{Since } \angle EFG \text{ and } \angle GFH \text{ form a linear pair, they are supplementary, and the sum of their measures is } 180^\circ.$$

$$m\angle EFG + m\angle GFH = 180^\circ$$

$$(2n + 21)^\circ + (4n + 15)^\circ = 180^\circ$$

$$6n + 36 = 180$$

$$6n = 144$$

$$n = 24$$

$$\text{b.} \quad m\angle EFG = (2n + 21)^\circ$$

$$= (2 \cdot 24 + 21)^\circ$$

$$= (48 + 21)^\circ$$

$$= 69^\circ$$

$$m\angle GFH = (4n + 15)^\circ$$

$$= (4 \cdot 24 + 15)^\circ$$

$$= (96 + 15)^\circ$$

$$= 111^\circ$$

$$\text{c.} \quad m\angle EFG + m\angle GFH = 180^\circ$$

$$69^\circ + 111^\circ = 180^\circ$$

$$58. \quad \text{Find } x.$$

$$m\angle FGJ + m\angle HGJ = 180^\circ$$

$$(2x + 4)^\circ + (7x - 4)^\circ = 180^\circ$$

$$9x^\circ = 180^\circ$$

$$x = 20$$

Substitute the value of  $x$  to find  $m\angle FGJ$  and  $m\angle HGJ$ .

$$m\angle FGJ = (2x + 4)^\circ$$

$$= (2 \cdot 20 + 4)^\circ$$

$$= (40 + 4)^\circ$$

$$= 44^\circ$$

$$m\angle HGJ = (7x - 4)^\circ$$

$$= (7 \cdot 20 - 4)^\circ$$

$$= (140 - 4)^\circ$$

$$= 136^\circ$$

Find  $y$ .

$$m\angle FGK + m\angle HGK = 180^\circ$$

$$(3y + 46)^\circ + (2y - 16)^\circ = 180^\circ$$

$$5y + 30 = 180$$

$$5y = 150$$

$$y = 30$$

Substitute the value of  $y$  to find  $m\angle FGK$  and  $m\angle HGK$ .

$$\begin{aligned} m\angle FGK &= (3y + 46)^\circ \\ &= (3 \cdot 30 + 46)^\circ \\ &= (90 + 46)^\circ \\ &= 136^\circ \\ m\angle HGK &= (2y - 16)^\circ \\ &= (2 \cdot 30 - 16)^\circ \\ &= (60 - 16)^\circ \\ &= 44^\circ \end{aligned}$$

60. Find  $x$ .

$$\begin{aligned} m\angle LWT + m\angle TWZ &= 180^\circ \\ (3x + 2)^\circ + (x - 26)^\circ &= 180^\circ \\ 4x - 24 &= 180 \\ 4x &= 204 \\ x &= 51 \end{aligned}$$

Substitute the value of  $x$  to find  $m\angle LWT$  and  $m\angle TWZ$ .

$$\begin{aligned} m\angle LWT &= (3x + 2)^\circ \\ &= (3 \cdot 51 + 2)^\circ \\ &= (153 + 2)^\circ \\ &= 155^\circ \\ m\angle TWZ &= (x - 26)^\circ \\ &= (51 - 26)^\circ \\ &= 25^\circ \end{aligned}$$

Find  $y$ .

$$\begin{aligned} m\angle LWV + m\angle VWZ &= 180^\circ \\ (2y + 1)^\circ + (12y + 11)^\circ &= 180^\circ \\ 14y + 12 &= 180 \\ 14y &= 168 \\ y &= 12 \end{aligned}$$

Substitute the value of  $y$  to find  $m\angle LWV$  and  $m\angle VWZ$ .

$$\begin{aligned} m\angle LWV &= (2y + 1)^\circ \\ &= (2 \cdot 12 + 1)^\circ \\ &= (24 + 1)^\circ \\ &= 25^\circ \\ m\angle VWZ &= (12y + 11)^\circ \\ &= (12 \cdot 12 + 11)^\circ \\ &= (144 + 11)^\circ \\ &= 155^\circ \end{aligned}$$

62. Let the angles be  $\angle A$  and  $\angle B$ , where  $m\angle B = (m\angle A - 20)^\circ$ . Since  $\angle A$  and  $\angle B$  are complementary, the sum of their measures is  $90^\circ$ .

Find  $m\angle A$ .

$$\begin{aligned} m\angle A + m\angle B &= 90^\circ \\ m\angle A + (m\angle A - 20) &= 90^\circ \\ 2(m\angle A) - 20 &= 90^\circ \\ 2(m\angle A) &= 110^\circ \\ m\angle A &= 55^\circ \end{aligned}$$

Find  $m\angle B$ .

$$\begin{aligned} m\angle A + m\angle B &= 90^\circ \\ 55^\circ + m\angle B &= 90^\circ \\ m\angle B &= 35^\circ \end{aligned}$$

64. The first student is correct. The correct equation is  $m\angle ABX = \frac{1}{2}m\angle ABC$ . The angle bisector splits  $\angle ABC$  in half to make  $\angle ABX$  and  $\angle CBX$ . The second student's equation states that one of the halves is twice as big as  $\angle ABC$ .

66. For this to occur,  $m\angle A = m\angle B$  and

$$\begin{aligned} m\angle A + m\angle B &= 180^\circ \\ m\angle A &= m\angle B \\ m\angle A + m\angle B &= 180^\circ \\ m\angle A + m\angle A &= 180^\circ \\ 2(m\angle A) &= 180^\circ \end{aligned}$$

$$m\angle A = 90^\circ, m\angle B = 90^\circ$$

The only instance where vertical angles are supplementary occurs when the angles are right angles.

68. Two angles are supplementary if the sum of their measures is  $180^\circ$ . All of the acute angles in the figure are supplementary to  $\angle JQM$ .

70. Two adjacent angles form a linear pair if their noncommon sides are opposite rays.  $\angle LMN$ ,  $\angle QMP$ , and  $\angle NMP$  form linear pairs with  $\angle LMQ$ .

## Section 1.7 Practice

1. Use the Midpoint Formula  $M = \frac{a+b}{2}$ ,

where  $a = -9$  and  $b = 4$ .

$$\begin{aligned} M &= \frac{a+b}{2} \\ &= \frac{-9+4}{2} \\ &= \frac{-5}{2} \\ &= -2.5 \end{aligned}$$

2. Let  $(x_1, y_1) = P(5, -2)$ , and  $(x_2, y_2) = Q(8, -6)$ .

$$\begin{aligned} \text{x-coordinate of } M: M &= \frac{a+b}{2} \\ &= \frac{5+8}{2} \\ &= \frac{13}{2} \\ &= 6.5 \end{aligned}$$

$$\begin{aligned} \text{y-coordinate of } M: M &= \frac{a+b}{2} \\ &= \frac{-2+(-6)}{2} \\ &= \frac{-8}{2} \\ &= -4 \end{aligned}$$

The midpoint of segment  $\overline{PQ}$  is  $M(6.5, -4)$ .

3. Let the coordinates of  $B$  be  $(x_2, y_2)$ .

$$(4, -9) = \left( \frac{-3+x_2}{2}, \frac{-5+y_2}{2} \right)$$

$$x_2: 4 = \frac{-3+x_2}{2}, \text{ and } y_2: -9 = \frac{-5+y_2}{2}$$

$$8 = -3+x_2 \qquad -18 = -5+y_2$$

$$11 = x_2 \qquad -13 = y_2$$

The coordinates of  $B$  are  $(11, -13)$ .

4. Let the coordinates of  $P$  be  $(x_1, y_1)$  and the coordinates of  $Q$  be  $(x_2, y_2)$ . Use the Distance Formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-2 - (-3)]^2 + (3 - 7)^2} \\ &= \sqrt{(1)^2 + (-4)^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17} \approx 4.1 \end{aligned}$$

## Vocabulary &amp; Readiness Check 1.7

- The midpoint of a line segment is a point exactly halfway between the two endpoints of the line segment.
- The Distance Formula is  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .
- The Midpoint Formula for a segment on the coordinate plane is  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ .
- The Midpoint Formula for a segment on a number line is  $M = \frac{a+b}{2}$ .

## Exercise Set 1.7

$$2. M = \frac{a+b}{2} = \frac{-9+6}{2} = \frac{-3}{2} = -1.5$$

$$4. M = \frac{a+b}{2} = \frac{-8+(-12)}{2} = \frac{-20}{2} = -10$$

$$\begin{aligned} 6. M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{3+7}{2}, \frac{9+11}{2} \right) \\ &= \left( \frac{10}{2}, \frac{20}{2} \right) \\ &= (5, 10) \end{aligned}$$

$$\begin{aligned}
 8. \quad M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left( \frac{-3 + 6}{2}, \frac{-4 + (-8)}{2} \right) \\
 &= \left( \frac{3}{2}, \frac{-12}{2} \right) \\
 &= (1.5, -6)
 \end{aligned}$$

$$\begin{aligned}
 10. \quad M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left( \frac{-2 + 2}{2}, \frac{5 + 15}{2} \right) \\
 &= \left( \frac{0}{2}, \frac{20}{2} \right) \\
 &= (0, 10)
 \end{aligned}$$

$$\begin{aligned}
 12. \quad M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 M &= \left( \frac{-\frac{2}{5} + \left(-\frac{2}{5}\right)}{2}, \frac{\frac{7}{15} + \left(-\frac{4}{15}\right)}{2} \right) \\
 &= \left( \frac{\left(-\frac{4}{5}\right)}{2}, \frac{\left(\frac{3}{15}\right)}{2} \right) \\
 &= \left( -\frac{4}{10}, \frac{3}{30} \right) \\
 &= \left( -\frac{2}{5}, \frac{1}{10} \right)
 \end{aligned}$$

14. Let the coordinates of  $S$  be  $(x_2, y_2)$ .

$$(5, -8) = \left( \frac{5 + x_2}{2}, \frac{-15 + y_2}{2} \right)$$

$$x_2: 5 = \frac{5 + x_2}{2}, \text{ and } y_2: -8 = \frac{-15 + y_2}{2}$$

$$10 = 5 + x_2 \qquad -16 = -15 + y_2$$

$$5 = x_2 \qquad -1 = y_2$$

The coordinates of  $S$  are  $(5, -1)$ .

16. Let the coordinates of  $S$  be  $(x_2, y_2)$ .

$$(5, -8) = \left( \frac{-2 + x_2}{2}, \frac{8 + y_2}{2} \right)$$

$$x_2: 5 = \frac{-2 + x_2}{2}, \text{ and } y_2: -8 = \frac{8 + y_2}{2}$$

$$10 = -2 + x_2 \qquad -16 = 8 + y_2$$

$$12 = x_2 \qquad -24 = y_2$$

The coordinates of  $S$  are  $(12, -24)$ .

$$\begin{aligned}
 18. \quad d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-8 - 10)^2 + (14 - 14)^2} \\
 &= \sqrt{(-18)^2 + (0)^2} \\
 &= \sqrt{324 + 0} \\
 &= \sqrt{324} \\
 &= 18
 \end{aligned}$$

$$\begin{aligned}
 20. \quad d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(0 - 0)^2 + (12 - 3)^2} \\
 &= \sqrt{(0)^2 + (9)^2} \\
 &= \sqrt{0 + 81} \\
 &= \sqrt{81} \\
 &= 9
 \end{aligned}$$

$$\begin{aligned}
 22. \quad d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(5 - 12)^2 + [12 - (-12)]^2} \\
 &= \sqrt{(-7)^2 + (24)^2} \\
 &= \sqrt{49 + 576} \\
 &= \sqrt{625} \\
 &= 25
 \end{aligned}$$

$$\begin{aligned}
 24. \quad d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-8 - 12)^2 + (18 - 6)^2} \\
 &= \sqrt{(-20)^2 + (12)^2} \\
 &= \sqrt{400 + 144} \\
 &= \sqrt{544} \\
 &= 4\sqrt{34} \approx 23.3
 \end{aligned}$$

26. Brookline has the coordinates (8, 2), and Charleston has the coordinates (4, 5).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 8)^2 + (5 - 2)^2} \\ &= \sqrt{(-4)^2 + (3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

28. Everett has the coordinates (-3, 5), and Fairfield has the coordinates (-8, -3).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-8 - (-3)]^2 + (-3 - 5)^2} \\ &= \sqrt{(-5)^2 + (-8)^2} \\ &= \sqrt{25 + 64} \\ &= \sqrt{89} \approx 9.3 \end{aligned}$$

30. Platform D has coordinates (20, 20), and Platform E has coordinates (30, -15).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(30 - 20)^2 + (-15 - 20)^2} \\ &= \sqrt{(10)^2 + (-35)^2} \\ &= \sqrt{100 + 1225} \\ &= \sqrt{1325} \approx 36.4 \text{ m} \end{aligned}$$

32. a.  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\begin{aligned} &= \sqrt{[5 - (-3)]^2 + [-7 - (-1)]^2} \\ &= \sqrt{(8)^2 + (-6)^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

b.  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$$\begin{aligned} &= \left( \frac{-3 + 5}{2}, \frac{-1 + (-7)}{2} \right) \\ &= \left( \frac{2}{2}, \frac{-8}{2} \right) \\ &= (1, -4) \end{aligned}$$

34. a.  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\begin{aligned} &= \sqrt{[1 - (-4)]^2 + [3 - (-2)]^2} \\ &= \sqrt{(5)^2 + (5)^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \approx 7.1 \end{aligned}$$

b.  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$$\begin{aligned} &= \left( \frac{-4 + 1}{2}, \frac{-2 + 3}{2} \right) \\ &= \left( \frac{-3}{2}, \frac{1}{2} \right) \\ &= (-1.5, 0.5) \end{aligned}$$

36. a.  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\begin{aligned} &= \sqrt{[-3 - (-5)]^2 + [-5 - (-3)]^2} \\ &= \sqrt{(2)^2 + (-2)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \approx 2.8 \end{aligned}$$

b.  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$$\begin{aligned} &= \left( \frac{-5 + (-3)}{2}, \frac{-3 + (-5)}{2} \right) \\ &= \left( \frac{-8}{2}, \frac{-8}{2} \right) \\ &= (-4, -4) \end{aligned}$$

38. Point A has coordinates  $(0,3)$ , and Point B has coordinates  $(-2,-2)$ .

$$\begin{aligned} \text{a. } d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 0)^2 + (-2 - 3)^2} \\ &= \sqrt{(-2)^2 + (-5)^2} \\ &= \sqrt{4 + 25} \\ &= \sqrt{29} \approx 5.4 \end{aligned}$$

$$\begin{aligned} \text{b. } M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{0 + (-2)}{2}, \frac{3 + (-2)}{2} \right) \\ &= \left( \frac{-2}{2}, \frac{1}{2} \right) \\ &= (-1, 0.5) \end{aligned}$$

40. Central Station has coordinates  $(0,3)$ , and South Station has coordinates  $(0,-4)$ .

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - 0)^2 + (-4 - 3)^2} \\ &= \sqrt{(0)^2 + (-7)^2} \\ &= \sqrt{0 + 49} \\ &= \sqrt{49} \\ &= 7 \text{ miles} \end{aligned}$$

42. Cedar Station has coordinates  $(-3,1)$ , and City Plaza Station has coordinates  $(0,0)$ .

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[0 - (-3)]^2 + (0 - 1)^2} \\ &= \sqrt{(3)^2 + (-1)^2} \\ &= \sqrt{9 + 1} \\ &= \sqrt{10} \approx 3.2 \text{ miles} \end{aligned}$$

44. Sycamore Station has coordinates  $(4,-6)$ , and Cedar Station has coordinates  $(-3,1)$ .

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3 - 4)^2 + [1 - (-6)]^2} \\ &= \sqrt{(-7)^2 + (7)^2} \\ &= \sqrt{49 + 49} \\ &= \sqrt{98} \approx 9.9 \text{ miles} \end{aligned}$$

46. a. If Point  $P$  is the midpoint, let the coordinates of  $R$  be  $(x_2, y_2)$ .

$$(-4, 6) = \left( \frac{2 + x_2}{2}, \frac{4 + y_2}{2} \right)$$

$$\begin{aligned} x_2: \quad -4 &= \frac{2 + x_2}{2}, \text{ and } y_2: \quad 6 = \frac{4 + y_2}{2} \\ -8 &= 2 + x_2 & 12 &= 4 + y_2 \\ -10 &= x_2 & 8 &= y_2 \end{aligned}$$

If  $P$  is the midpoint, the coordinates of  $R$  are  $(-10, 8)$ .

If Point  $Q$  is the midpoint, let the coordinates of  $R$  be  $(x_2, y_2)$ .

$$(2, 4) = \left( \frac{-4 + x_2}{2}, \frac{6 + y_2}{2} \right)$$

$$\begin{aligned} x_2: \quad 2 &= \frac{-4 + x_2}{2}, \text{ and } y_2: \quad 4 = \frac{6 + y_2}{2} \\ 4 &= -4 + x_2 & 8 &= 6 + y_2 \\ 8 &= x_2 & 2 &= y_2 \end{aligned}$$

If  $Q$  is the midpoint, the coordinates of  $R$  are  $(8, 2)$ .

If Point  $R$  is the midpoint, find the coordinates of  $R$ .

$$\begin{aligned} M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{-4 + 2}{2}, \frac{6 + 4}{2} \right) \\ &= \left( \frac{-2}{2}, \frac{10}{2} \right) \\ &= (-1, 5) \end{aligned}$$

If  $R$  is the midpoint, its coordinates are  $(-1, 5)$ .

- b. Find the possible lengths of  $\overline{RQ}$ .

If  $P$  is the midpoint:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[2 - (-10)]^2 + (4 - 8)^2} \\ &= \sqrt{(12)^2 + (-4)^2} \\ &= \sqrt{144 + 16} \\ &= \sqrt{160} \end{aligned}$$

If  $Q$  is the midpoint:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - 8)^2 + (4 - 2)^2} \\ &= \sqrt{(-6)^2 + (2)^2} \\ &= \sqrt{36 + 4} \\ &= \sqrt{40} \end{aligned}$$

If  $R$  is the midpoint:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[2 - (-1)]^2 + (4 - 5)^2} \\ &= \sqrt{(3)^2 + (-1)^2} \\ &= \sqrt{9 + 1} \\ &= \sqrt{10} \end{aligned}$$

In order for  $\overline{RQ}$  to have a length of  $\sqrt{160}$ ,  $P$  must be the midpoint. So,  $R$  must have the coordinates  $(-10, 8)$ .

48. The terms inside the distance formula

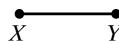
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  are squared, which always results in a positive answer. Then they are added, and the sum of two positive numbers is always positive. The square root of a positive number is always positive.

50. Your friend substituted the values in the Distance Formula incorrectly. Each subtraction is written with the wrong partner. It should say the following.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 1)^2 + (8 - 5)^2} \end{aligned}$$

### Section 1.8 Practice

1.  $\overline{RS}$  must be twice as long as  $\overline{XY}$  below.



Step 1:

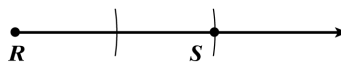
Draw a ray with endpoint  $R$ .

Step 2:

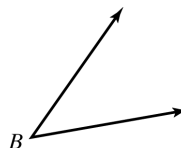
Open the compass to the length of  $\overline{XY}$ . With the same compass setting, put the compass point on  $R$  and draw an arc that intersects the ray.

Step 3:

Without changing the compass setting, put the compass point on the intersection of the ray and the arc and draw an arc that intersects the ray on the opposite side of the intersection as  $R$ . Label the point of intersection  $S$ .



2.  $\angle Q$  must be congruent to  $\angle B$  below.



Step 1:

Draw a ray with endpoint  $Q$ .

Step 2:

With the compass point on vertex  $B$ , draw an arc that intersects the sides of  $\angle B$ .

Step 3:

With the same compass setting, put the compass point on point  $Q$ . Draw an arc that intersects the ray.

Step 4:

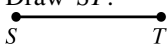
Open the compass to the distance between the points of intersection between the sides of  $\angle B$  and the arc from Step 2. Keeping the same compass setting, put the compass point on the intersection from Step 3. Draw an arc that intersects the arc.

Step 5:

Draw a ray from point  $Q$  through the intersection of the arcs.



3. Draw  $\overline{ST}$ .



Step 1:

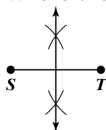
Put the compass point on point  $S$  and draw arcs above and below the segment. Be sure that the opening is greater than  $\frac{1}{2}ST$ .

Step 2:

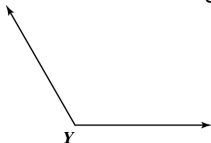
With the same compass setting, put the compass point on point  $T$  and draw arcs above and below the segment that intersect the arcs from Step 1.

Step 3:

Draw a line that passes through the points where the arcs intersect.



4. Draw an obtuse angle,  $\angle Y$ .



Step 1:

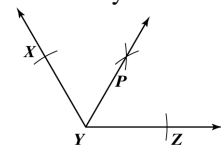
Put the compass point on vertex  $Y$ . Draw arcs that intersect the sides of the angle. Label the intersections  $X$  and  $Z$ .

Step 2:

Put the compass point on point  $X$  and draw an arc inside the angle. Then put the compass point on point  $Z$  without changing the compass setting, and draw an arc that intersects the last arc at a point  $P$ .

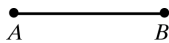
Step 3:

Draw a ray from vertex  $Y$  through point  $P$ .



Exercise Set 1.8

2.  $\overline{VW}$  must be twice as long as  $\overline{AB}$  below.



Step 1:

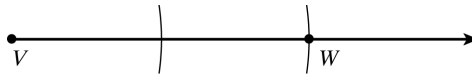
Draw a ray with endpoint  $V$ .

Step 2:

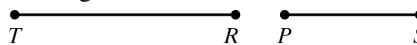
Open the compass to the length of  $\overline{AB}$ . With the same compass setting, put the compass point on  $V$  and draw an arc that intersects the ray.

Step 3:

Without changing the compass setting, put the compass point on the intersection of the ray and the arc and draw an arc that intersects the ray on the opposite side of the intersection as  $V$ . Label the point of intersection  $W$ .



4. The length of  $\overline{QJ}$  must be the difference of the lengths of  $\overline{TR}$  and  $\overline{PS}$  below.



Step 1:

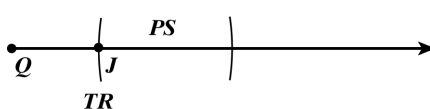
Draw a ray with endpoint  $Q$ .

Step 2:

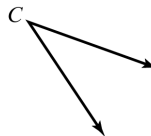
Open the compass to the length of  $\overline{TR}$ . Put the compass point on  $Q$  and draw an arc that intersects the ray.

Step 3:

Open the compass to the length of  $\overline{PS}$ . Put the compass point on the intersection of the ray and the arc and draw an arc that intersects the ray between the intersection and point  $Q$ . Label this point  $J$ .



6.  $m\angle F$  must be twice the measure of  $\angle C$ .



Step 1:

Draw a ray with endpoint  $F$ .

Step 2:

With the compass point on vertex  $C$ , draw an arc that intersects the sides of  $\angle C$ .

Step 3:

With the same compass setting, put the compass point on point  $F$ . Draw a large arc that intersects the ray.

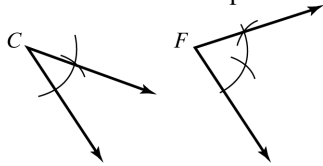
Step 4:

Open the compass to the distance between the points of intersection between the sides of  $\angle C$  and the arc from Step 2. Keeping the same compass setting, put the compass point on the intersection from Step 3. Draw an arc that intersects the arc above the ray.

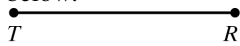


Step 5:  
 Keeping the same compass setting, put the compass point on the intersection from Step 4. Draw an arc that intersects the large arc above the ray.

Step 6:  
 Draw a ray from point  $F$  through the intersection from Step 5.



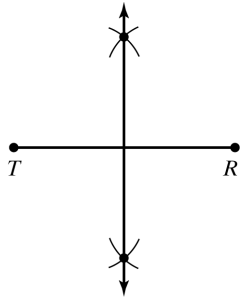
8. Construct the perpendicular bisector of  $\overline{TR}$  below.



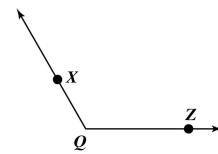
Step 1:  
 Put the compass point on point  $T$  and draw arcs above and below the segment. Be sure that the opening is greater than  $\frac{1}{2}TR$ .

Step 2:  
 With the same compass setting, put the compass point on point  $R$  and draw arcs above and below the segment intersecting the arcs from Step 1.

Step 3:  
 Draw a line that passes through the points where the arcs intersect.



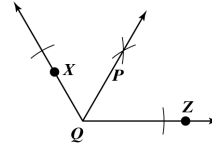
10. Draw an obtuse angle,  $\angle XQZ$ .



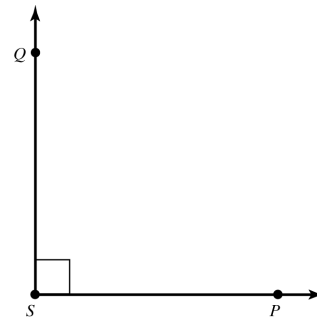
Step 1:  
 Put the compass point on vertex  $Q$ . Draw arcs that intersect the sides of the angle.

Step 2:  
 Put the compass point on one of the intersections and draw an arc inside the angle. Then put the compass point on the other intersection without changing the compass setting, and draw an arc that intersects the last arc.

Step 3:  
 Draw a ray from vertex  $Y$  to the point where the two arcs you drew in Step 2 intersect.



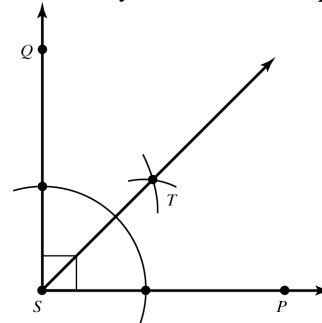
12. Draw a right angle,  $\angle PSQ$ .



Step 1:  
 Put the compass point on vertex  $S$ . Draw arcs that intersect the sides of the angle.

Step 2:  
 Put the compass point on the intersection of an arc and the side of the angle, and draw an arc inside the angle. Without changing the compass setting, put the compass point on the other intersection to draw an arc that intersects the last arc. Label this point  $T$ .

Step 3:  
 Draw a ray from vertex  $S$  to point  $T$ .



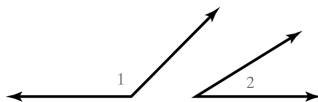
14. Construct the angle bisector of  $\angle A$ . You now have two angles with measures  $\frac{1}{2}m\angle A$ .

Pick one of the smaller angles and do the construction of the angle bisector for that angle. The measures of the two angles are half of the original angle, or  $\frac{1}{4}m\angle A$ .

16. There is only one point at which a segment can be cut in half, so a segment can only have one bisector.

18. In space, there is a third axis so there are many angles at which a perpendicular bisector can bisect the segment, so there are an infinite number of perpendicular bisectors for this segment in space.

20. Draw angles 1 and 2. Construct an angle congruent to  $\angle 2$  that shares a side with  $\angle 1$  so that the other side of the angle is in the interior of  $\angle 1$ .



Step 1:

Put the compass point on the vertex of  $\angle 2$ . Draw arcs that intersect each side of the angle. Label the points  $A$  and  $B$ .

Step 2:

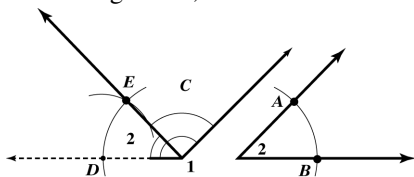
With the same compass setting, put the compass point on the vertex of  $\angle 1$  and draw an arc that intersects the side of angle 1. Label this point  $D$ .

Step 3:

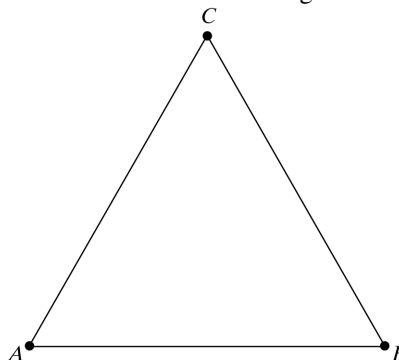
Set the compass to be the length between point  $A$  and point  $B$ . Put the compass on point  $D$ , and draw an arc that intersects the arc from the previous step. Label the point of intersection  $E$ .

Step 4:

Draw a ray from the vertex of  $\angle 1$  through point  $E$ . Label the angle enclosed by this ray and the leg of  $\angle 1$ ,  $\angle C$ .



22. Draw  $\triangle ABC$  with acute angles.



Step 1:

Put the compass point on vertex  $A$ , and draw an arc that intersects both sides of the triangle that meet at  $A$ . Label the points  $D$  and  $E$ .

Step 2:

Open the compass a little more than half the distance between  $D$  and  $E$ . Put the compass point on point  $D$  and draw an arc. With the same compass setting, draw an arc using the point  $E$  that intersects the other arc.

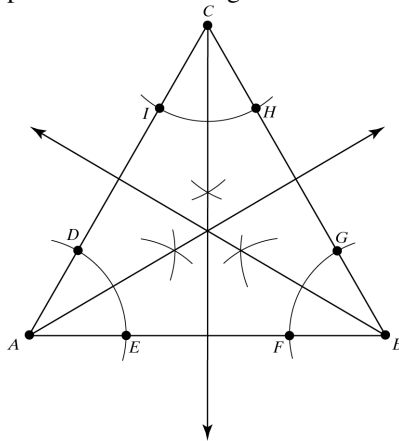
Step 3:

Draw the ray with endpoint  $A$  through the intersection of the two arcs.

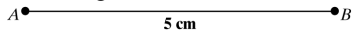
Step 4:

Repeat Steps 1–3 with the other vertices of the triangle to construct angle bisectors for  $\angle B$  and  $\angle C$ .

The angle bisectors of a triangle intersect at a point inside the triangle.



24. Draw segment  $\overline{AB}$  that is 5 cm long.



Step 1:

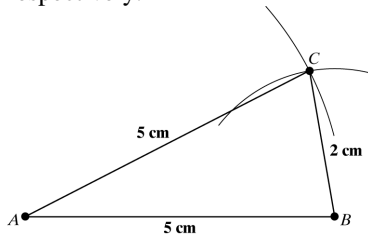
Set the compass length to the segment that is 5 cm long. Put the compass point on point  $A$  and draw an arc above  $\overline{AB}$ .

Step 2:

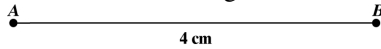
Set the compass length to the segment that is 2 cm long. Put the compass point on point  $B$  and draw an arc that intersects the previous arc. Label the intersection  $C$ .

Step 3:

Use a straightedge to draw  $\triangle ABC$ . The lengths of  $\overline{AC}$  and  $\overline{BC}$  are 5 cm and 2 cm respectively.



26. Use the segments below and draw segment  $\overline{AB}$  that is 4 cm long.

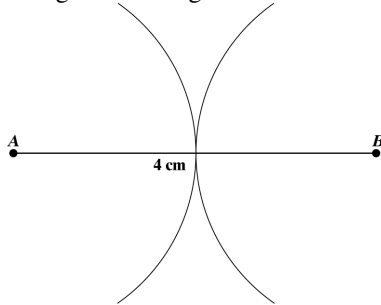


Step 1:

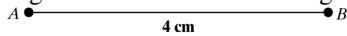
Open the compass to the length of the segment that is 2 cm long. Put the compass point at point  $A$  and draw a circle.

Step 2:

With the same compass setting, put the compass point on point  $B$  and draw a circle. The only place where the circles intersect is on  $\overline{AB}$ . So, it is not possible to construct a triangle with the given side measures.



28. Use the segments shown below and draw segment  $\overline{AB}$  that is 4 cm long.



Step 1:

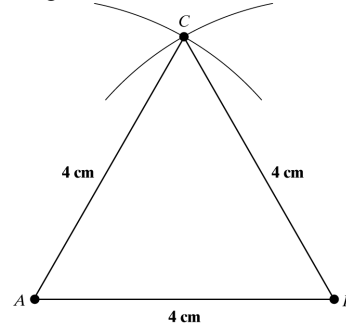
Open the compass to the length of the segment that is 4 cm long. Put the compass point at point  $A$  and draw an arc above  $\overline{AB}$ .

Step 2:

With the same compass setting, put the compass point on point  $B$  and draw an arc that intersects the previous arc. Label the intersection  $C$ .

Step 3:

Use a straightedge to draw  $\triangle ABC$ . The lengths of  $\overline{AC}$  and  $\overline{BC}$  are both 4 cm.



30. The answer is A. In order for the half circles to all line up correctly, the line at the bottom must be drawn first. Count the number of half circles. There are five. So to construct this, you would use a straight edge to draw the segment and then a compass to draw five half circles.

Chapter 1 Review

1. Each new image has a square added inside the smaller square, rotated by  $45^\circ$ .



2. Each shape inside the circle has one more side added to it from the previous one.



3. Each fraction is being multiplied by  $\frac{1}{2}$ .

$$\frac{1}{48}, \frac{1}{96}$$

4. Each number is decreasing by 5.  
20, 15

5. Answers may vary. Sample Answer:  $l$  and  $m$

6.  $\overline{QR}$

7. Answers may vary. Sample Answer:  $A, B, C$
8.  $T$
9. True; Postulate 1.3-1 states, "Through any two points, there is exactly one line." This means that two points are always collinear.
10. False;  $\overline{LM}$  has endpoint  $L$  and  $\overline{ML}$  has endpoint  $M$ . Thus, since the rays have two different endpoints they cannot be the same ray.
11.  $-2 - 5 = -7$ ,  $-2 + 5 = 3$ , so two possible coordinates are  $-7$  and  $3$ .
12. Use the Midpoint Formula.  

$$\frac{-2+3}{2} = \frac{1}{2}$$
13.  $AB = BC$   
 $3m + 5 = 4m - 10$   
 $m = 15$
14. Use the Segment Addition Postulate to find the value of  $a$ .  
 $a + a + 8 = 50$   
 $a = 21$   
 Substitute the value  $a$  to find  $XY$  and  $YZ$ .  
 $XY = a$  ;  $YZ = a + 8$   
 $= 21$        $= 29$
15. This is an acute angle.
16. This is a right angle.
17. Use the Angle Addition Postulate to find  $m\angle PQR$ .  
 $m\angle PQR + m\angle MQP = m\angle MQR$   
 $m\angle PQR = m\angle MQR - m\angle MQP$   
 $= 61^\circ - 25^\circ$   
 $= 36^\circ$
18.  $m\angle NQM = m\angle PQR$   
 $2x + 8 = x + 22$   
 $x = 14$
19.  $\angle ADB$  (or  $\angle BDA$ ) and  $\angle BDC$  (or  $\angle CDB$ ).
20. Answers may vary. Sample Answer:  $\angle ADC$  and  $\angle CDF$
21. Answers may vary. Sample Answer:  $\angle ADC$  and  $\angle EDF$
22. Answers may vary. Sample Answer:  $\angle ADB$  and  $\angle BDF$
23. Since the angles form a linear pair, the sum of their measures is  $180^\circ$ .  
 $(3x + 31) + (2x - 6) = 180$   
 $5x + 25 = 180$   
 $5x = 155$   
 $x = 31$
24. Since the angles are complementary, the sum of their measures is  $90^\circ$ .  
 $3x + (4x - 15) = 90$   
 $7x - 15 = 90$   
 $7x = 105$   
 $x = 15$
25. Use the Distance Formula.  
 $\sqrt{(0 - (-1))^2 + (4 - 5)^2} \approx 1.4$
26. Use the Distance Formula.  
 $\sqrt{(6 - (-1))^2 + (2 - (-1))^2} \approx 7.6$
27. Use the Midpoint Formula to find the  $x$ -coordinate.  
 $\frac{-3+3}{2} = \frac{0}{2} = 0$   
 Use the Midpoint Formula to find the  $y$ -coordinate.  
 $\frac{2+(-2)}{2} = \frac{0}{2} = 0$   
 So the coordinates of the midpoint are  $(0, 0)$ .
28. Use the Distance Formula.  
 $\sqrt{(-3 - 3)^2 + (2 - (-2))^2} \approx 7.2$

29. Solve the Midpoint Formula for  $x_2$ .

$$\frac{x_1 + x_2}{2} = m_x$$

$$x_1 + x_2 = 2m_x$$

$$x_2 = 2m_x - x_1$$

Now use the  $x$  values you have been given to solve for  $x_2$ .

$$x_2 = 2m_x - x_1$$

$$= 2(-1) - (-8)$$

$$= 6$$

Solve the Midpoint Formula for  $y_2$ .

$$\frac{y_1 + y_2}{2} = m_y$$

$$y_1 + y_2 = 2m_y$$

$$y_2 = 2m_y - y_1$$

Now use the  $y$  values you have been given to solve for  $y_2$ .

$$y_2 = 2m_y - y_1$$

$$= 2(1) - 4$$

$$= -2$$

So the coordinates of the endpoint  $K$  are  $(6, -2)$ .

30. Solve the Midpoint Formula for  $x_2$ .

$$\frac{x_1 + x_2}{2} = m_x$$

$$x_1 + x_2 = 2m_x$$

$$x_2 = 2m_x - x_1$$

Now use the  $x$  values you have been given to solve for  $x_2$ .

$$x_2 = 2m_x - x_1$$

$$= 2(5) - 9$$

$$= 1$$

Solve the Midpoint Formula for  $y_2$ .

$$\frac{y_1 + y_2}{2} = m_y$$

$$y_1 + y_2 = 2m_y$$

$$y_2 = 2m_y - y_1$$

Now use the  $y$  values you have been given to solve for  $y_2$ .

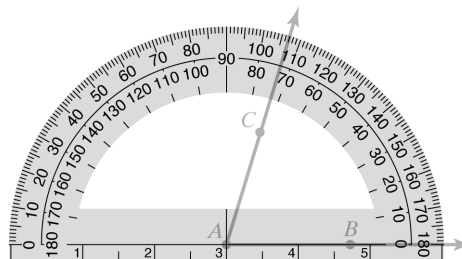
$$y_2 = 2m_y - y_1$$

$$= 2(-2) - (-5)$$

$$= 1$$

So the coordinate of the endpoint  $K$  is  $(1, 1)$ .

31. Holding your protractor on the paper, draw a straight line along the bottom of the protractor, shown below as  $\overline{AB}$ . Then mark a point  $C$  at  $73^\circ$  and draw a straight line from  $A$  to that point  $C$ .



Step 1:

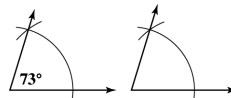
Use a straight edge to draw a ray. Put the compass point on the vertex of the angle and draw an arc that intersects both sides of the angle. Then put the compass point on the endpoint of the drawn ray and draw an arc using the same compass setting.

Step 2:

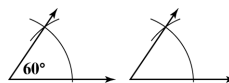
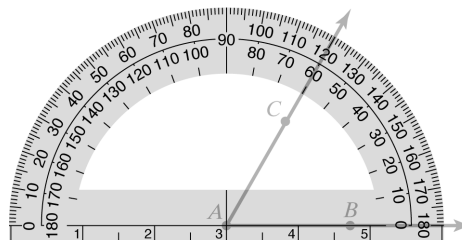
Set the compass to the distance between the two intersection points on the angle. Then put the compass point on the intersection of the ray and the arc from Step 1, and draw an arc using the same setting that intersects the arc from Step 1.

Step 3:

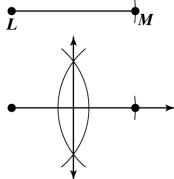
Use a straight edge to draw a ray from the endpoint of the ray from Step 1 through the intersection from Step 2.



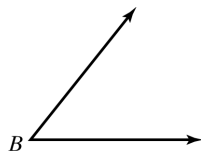
32. Holding your protractor on the paper, draw a straight line along the bottom of the protractor, shown below as  $\overline{AB}$ . Then mark a point  $C$  at  $60^\circ$  and draw a straight line from  $A$  to that point  $C$ .



33. Step 1:  
Put the compass point on point  $L$  and draw a long arc. Be sure the opening is greater than  $\frac{1}{2}LM$ . With the same compass setting, put the compass point on point  $M$  and draw another long arc.  
Step 2:  
Draw a straight line between the two arc intersections. Now this new line is the perpendicular bisector.

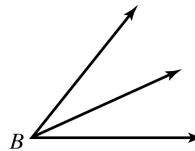


34. a. Sketch the angle.  
Step 1:  
Use a straight edge to draw a ray. Put the compass point on the vertex of the angle and draw an arc that intersects both sides of the angle. Then put the compass point on the endpoint of the drawn ray and draw an arc using the same compass setting.  
Step 2:  
Set the compass to the distance between the two intersection points on the angle. Then put the compass point on the intersection of the ray and the arc from Step 1, and draw an arc using the same setting that intersects the arc from Step 1.  
Step 3:  
Use a straight edge to draw a ray from the endpoint of the ray from Step 1 through the intersection from Step 2.



- b. Step 1:  
Start with the angle constructed in part a. Put the compass point on an intersection of the drawn arc and a side. Draw an arc inside the angle. Then move the compass point to the intersection of the drawn arc and the other side and draw another arc inside the angle.

- Step 2:  
Draw a ray from vertex  $B$  through the intersection of the arcs from Step 1.



35. Yes; All four points are located on plane  $AEF$ .
36. Yes; All four points are located on plane  $DCE$ .
37. No; Sample Answer: Points  $H$ ,  $G$ , and  $F$  are in plane  $HGF$  and point  $B$  is not in that plane.
38. No; Sample Answer: Points  $A$ ,  $E$ , and  $B$  are in plane  $AEB$  and point  $C$  is not in that plane.
39. a.  $\overline{BF}$   
b.  $\overline{EH}$
40. Answers may vary. Sample Answer:  $\overline{PR}$ ,  $\overline{RQ}$
41.  $\overline{PQ}$ ,  $\overline{PB}$
42. Answers may vary. Sample Answer:  $\angle CPQ$ ,  $\angle CPB$
43. Answers may vary. Sample Answer:  $D$ ,  $P$ ,  $C$
44. Answers may vary. Sample Answer:  $\overline{PA}$  and  $\overline{PR}$
45. Answers may vary. Sample Answer:  $\overline{AP}$ ,  $\overline{PR}$ ,  $\overline{RQ}$
46. Answers may vary. Sample Answer:  $\angle APC$  and  $\angle CPR$
47. Answers may vary. Sample Answer:  $\angle APD$  and  $\angle CPR$
48. a. Since the angles form a linear pair, the sum of their measures is  $180^\circ$ .  
 $m\angle ABC + m\angle CBD = 180$   
 $2x + 3x = 180$   
 $5x = 180$   
 $x = 36$

b.  $m\angle ABC = 2x$   
 $= 2(36)$   
 $= 72$   
 So,  $\angle ABC$  is acute.  
 $m\angle CBD = 3x$   
 $= 3(36)$   
 $= 108$   
 So,  $\angle CBD$  is obtuse.

49.  $|-1 - (-4)| = 3$

50.  $|3 - 0| = 3$

51.  $|6 - 3| = 3$

52.  $|6 - (-1)| = 7$

53. Use the fact that  $\overline{AC}$  is congruent to  $\overline{CD}$  to solve for  $x$ .  
 $4x + 5 = 3x + 8$   
 $x = 3$

Solve for  $\overline{AC}$ .

$\overline{AC} = 4(3) + 5 = 17$

Solve for  $\overline{CD}$ .

$\overline{CD} = 3(3) + 8 = 17$

Use the Segment Addition Postulate to find  $\overline{AD}$ .

$AC + CD = AD$

$17 + 17 = 34$

54. Use the Angle Addition Postulate to find  $m\angle FCB$ .

$m\angle FCB + m\angle BCD = m\angle FCD$

$m\angle FCB = m\angle FCD - m\angle BCD$

$= 130^\circ - 95^\circ$

$= 35^\circ$

55. Use the Angle Addition Postulate.

$m\angle FCA + m\angle FCE + m\angle ECD = 180$

$180 - m\angle FCA - m\angle ECD = m\angle FCE$

Substitute  $m\angle FCA$  for  $m\angle ECD$ .

$180 - m\angle FCA - m\angle FCA = m\angle FCE$

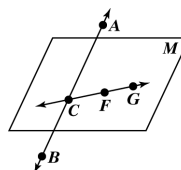
Solve for  $m\angle FCE$ .

$m\angle FCE = 180^\circ - 50^\circ - 50^\circ$

$= 80^\circ$

56. No;  $Q$  is not always the midpoint. It depends on the situation. If all three points are distinct and collinear then the answer is yes,  $Q$  is always the midpoint. If the three points are not distinct or they are not collinear then it does not mean that  $Q$  is the midpoint.

57. No;  $C$ ,  $F$ , and  $G$  are collinear, so  $\overline{AB}$  must intersect  $\overline{GF}$  at  $C$  in plane  $M$ .



## Chapter 1 Test

- Each new number has 6 being added to it. 31, 37
- Each new image has each section inside being cut in half. The number of sections doubles each time.



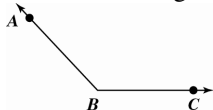
- $B$
- $C$
- $C$
- $C$
- $A$ ; Since  $\overline{MN}$  bisects  $\angle EMG$ , the measure of  $\angle EMG$  is twice the measure of  $\angle EMN$ .
- $\angle EFD$  and  $\angle AFC$  or  $\angle 2$  are vertical angles.
- $\angle EFD$  and  $\angle 3$  or  $\angle 1$  form a linear pair.
- The complementary angles are  $\angle DEF$  and  $\angle FEA$ .
- True
- False
- True
- True

- 15. False
- 16. right
- 17. obtuse
- 18. acute
- 19. straight
- 20.  $|3 - (-7)| = 10$

- 21. Not congruent.  
 $AC = |-1 - (-7)| = 6$   
 $BD = |3 - (-4)| = 7$

- 22. a.  $XM = MY$   
 $5x + 4 = 3x + 20$   
 $x = 8$
- b. Substitute the value of  $x$  to find the segment lengths.  
 $MY = 3(8) + 20$   
 $= 44$   
 $XM = MY = 44$   
 Use the Segment Addition Postulate to find  $XY$ .  
 $XY = XM + MY$   
 $= 44 + 44$   
 $= 88$

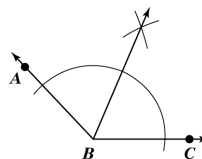
- 23. Draw obtuse angle  $\angle ABC$  shown below.



Step 1:  
Put the compass point on the vertex of the angle and draw an arc that intersects both sides.

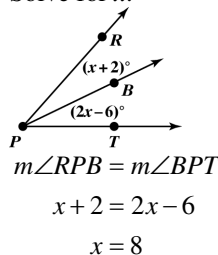
Step 2:  
Put the compass point on an intersection of the drawn arc and a side. Draw an arc inside the angle. Then move the compass point to the intersection of the drawn arc and the other side and draw another arc inside the angle.

Step 3:  
Draw a ray from the vertex through the intersection of the arcs from Step 2.



- 24. A, B, C
- 25. point B
- 26. a. 1 plane  
b. 1 plane
- 27. Answers may vary. Sample Answer: A, B, C, E
- 28. Use the Segment Addition Postulate and then solve for  $x$ .  
 $JH + HK = JK$   
 $(4x - 15) + (2x + 3) = 48$   
 $6x = 60$   
 $x = 10$
- 29.  $\overline{VW}$  is the perpendicular bisector of  $\overline{AY}$ .
- 30. If  $EW = 3.5$  then  $AY = \underline{7}$ .
- 31.  $\underline{E}$  is the midpoint of  $\overline{AY}$ .
- 32. Use the Angle Addition Postulate and the fact that the angles are complementary to find the value of  $y$ .  
 $m\angle BDJ + m\angle JDR = 90$   
 $(7y + 2) + (2y + 7) = 90$   
 $9y = 81$   
 $y = 9$

- 33. Solve for  $x$ .





Find  $m\angle RPB$  and  $m\angle BPT$ .

$$\begin{aligned} m\angle BPT &= (x + 2)^\circ \\ &= 10^\circ \end{aligned}$$

$$m\angle RPB = m\angle BPT = 10^\circ$$

Use the Angle Addition Postulate to find  $m\angle RPT$ .

$$\begin{aligned} m\angle RPT &= m\angle RPB + m\angle BPT \\ &= 10^\circ + 10^\circ \\ &= 20^\circ \end{aligned}$$

- 34.** Use the Midpoint formula to find the  $x$ -coordinate.

$$\frac{2 + (-1)}{2} = \frac{1}{2}$$

Use the Midpoint formula to find the  $y$ -coordinate.

$$\frac{4 + 7}{2} = \frac{11}{2}$$

So the coordinates of the midpoint are

$$\left(\frac{1}{2}, \frac{11}{2}\right).$$

- 35.** Solve the Midpoint Formula for  $x_2$ .

$$\frac{x_1 + x_2}{2} = m_x$$

$$x_1 + x_2 = 2m_x$$

$$x_2 = 2m_x - x_1$$

Now use the  $x$  values you have been given to solve for  $x_2$ .

$$x_2 = 2m_x - x_1$$

$$= 2(5) - 3$$

$$= 7$$

Solve the Midpoint Formula for  $y_2$ .

$$\frac{y_1 + y_2}{2} = m_y$$

$$y_1 + y_2 = 2m_y$$

$$y_2 = 2m_y - y_1$$

Now use the  $y$  values you have been given to solve for  $y_2$ .

$$y_2 = 2m_y - y_1$$

$$= 2(-2) - 4$$

$$= -8$$

So the coordinates of the endpoint  $C$  are

$$(7, -8).$$

- 36.** Use the Distance Formula.

$$\sqrt{(-9 - (-6))^2 + (8 - 0)^2} \approx 8.5$$