**PART ONE—Introduction and General Principles**

**CHAPTER 1: Basic Ideas and Examples**

**and CHAPTER 2: How to Think about Strategic Games**

# Teaching Suggestions

The first two chapters spark the students’ interest and pre­pare the groundwork for the analysis to come. Chapter 1 uses several stories or examples to illustrate kinds of strategic games, and Chapter 2 begins to connect them to a framework of concepts and terminology. Some teachers may find it better to mingle the two—pausing after each story to define the concepts and terms relevant to it and then doing the same with the next story. Others may prefer to present the stories from Chapter 1 as a way of stimulating student interest and capturing student attention, following the first chapter’s presentation with a summary of the terminology and the classifications found in Chapter 2.

Much of Chapter 2 may be useful to students as a reference source—a reminder of the difference between decisions and games, the game-theoretic meaning of rationality, or what distinguishes cooperative games from noncooperative games. You will certainly want to define those terms and concepts that you will focus on during the semester, and em­phasize the connections between the stories and the technical jargon used in analyzing them. It is probably not necessary at this stage to go into great detail about each level of classi­fication, each term, and each assumption. A brief summary or discussion in the context of various stories will allow you to move forward; you can always return to specific concepts when they become crucial to your analysis later in the term.

# Stories to Motivate the Subject and the Main Ideas

Some will find that it is not a good idea to retell our stories from Chapter 1. Students can easily read them in advance, and depending on the culture of the institution at which you teach, many may do so. Others will find that you *can* retell our stories on the first day of class; while a few students may have started to read the book, the majority may be waiting for a syllabus (or even to buy the book). In either case, you can use our stories as the basis for variation or discussion. You can provide your own variations on the stories or ask students to come up with stories of their own.

Whether you tell our stories or your own, or have students think of their own, you can start the discussion of a story by posing questions such as: Was this a game with strategic interaction or only a decision problem? If a game, who were the strategically active players? What were the strategies available to the players—not merely what they did but what else they could have done? In light of their strategies, can we make sense of why they did what they did? And so on. This process can quickly build a framework for understanding strategies as complete plans of action, rollback, each player’s simultaneously thinking of what everyone else is doing, or even equilibrium. It is also a good way to make the transition from the stories to the concepts of Chapter 2.

Most students in an elementary course will not have an extensive background in economics, politics, or business studies. Therefore, motivating them by using examples from these disciplines to introduce the ideas of strategies and games may not work. We have chosen the stories in Chapter 1 to relate to the lives of the students—to their relations with parents, siblings, and friends or with sports, for example. If your class has some specific background, you should draw on it for sources of stories or cases. Thus, economics or business teachers may be able to use the OPEC cartel to motivate the prisoners’ dilemma, repeated play, and different strategic situations of large and small players, or a very simple version of a Keynesian low-level equilibrium trap (no firm invests and creates jobs, because none thinks that it can sell the output profitably, because incomes are low, because firms aren’t investing) to motivate the idea of lock-in equilibria in games with positive feedbacks. In courses more specifically targeted to political science students, teachers may be able to use campaign advertising or special-interest lobbying stories to introduce the prisoners’ dilemma.

Later chapters will develop examples from economics, politics, and other subject areas, in each case explaining the discipline-specific contexts to the extent necessary.

# Game Playing in Class

Playing a few well-designed games in class, and watching others play them, brings to life the concepts of strategy, backward induction, and Nash equilibrium far better than any amount of formal statement or problem-set drill. There are several games that are appropriate for use on the first or second day of class. These games are simple but can be used to convey important points about basic tools and concepts, including rollback analysis, multiple equilibria, and focal points., It may be appropriate to start the course with two classroom games before teaching or even mentioning any of these concepts. Games 1 and 2 go well together as do 5 and 6; other pairings can be devised to fit your own course. We generally choose one sequential-move game and one simultaneous-move game from the list below. The concepts relevant to each emerge naturally during the discussion of each game.

The list provided here is sufficiently long that you will not be able to use all the games on the first day of class. You will probably find that some of the games we discuss here are better suited for use later in the semester when you are covering the material relevant to the game. We have noted those games that are repeated later in this manual and the chapter in which they reappear.

Here is some general advice about playing games in class.

DO:

1. Use real money or food for prizes when you can. Even small rewards raise student interest and attention. You can also make students’ scores in the game count for a small fraction of the course credit or simply give credit for active participation.

2. Bring enough rolls of coins to class when playing games that involve using change; do not expect your students to have the right change. Similarly, if you are using food items like M&Ms or jelly beans as an alternative to actual currency, make sure that you have enough to accommodate all students; be aware of allergies.

3. Use games with stories, not just abstract trees or matrices.

4. Follow the game immediately with a discussion of the general concepts or methods of analysis that the game was supposed to illustrate. If there isn’t enough time for a good discussion, circulate an explanation or post it on the course Web site.

DON’T:

1. Make the game so complex that the main conceptual point is smothered.

2. Choose games where the outcome depends significantly on uncertainty rather than skill if the games count toward the course credit.,

Game 1—21 Flags

This is a simple Nim-like sequential-move game that we have adopted from Episode 6 of the *Survivor Thailand* television series that aired in fall 2002 (*Survivor* Season 5). The game is played between two players (or teams) who alternate in taking turns to remove some number of flags from a field of 21 available for play. Each time it is a player’s turn, he chooses to remove one, two, or three flags; the player to remove the last flag is the winner. For easier in-class play, you can use coins instead of flags. Lay them out on the glass of the overhead projector so the whole class can easily see what is going on.

In the *Survivor* show, the game was played as an “immunity challenge” between two “tribes.” The losing tribe had to vote out one of its members, weakening it for future competitions. In the specific context, this loss had a crucial effect on the eventual outcome of the game. Thus, a million dollars hinged on the ability to do the simple calculation. The players got almost all their moves wrong, so if you have access to a video of the show, you should consider showing it to your students. Seeing the video and then playing a similar game will be a good way for your students to learn the concepts.

The correct solution is simple. If you leave the other player (or team) with 4 flags, he must remove 1, 2, or 3, and then you can take the rest and win. To make sure of leaving the other player with four flags, at the turn before, you have to leave him facing 8 flags. Carrying the logic further, that means leaving 12, 16, and 20 flags on subsequent turns. Therefore, starting with 21 flags, the first player should remove 1, and then proceed to take 4 minus whatever the other takes at his immediately preceding turn.

Have several pairs of students play this game in front of the class. We have found that the first pair makes choices almost at random, but the second pair does better, figuring out one or perhaps two of the final rounds correctly. By the third or at most the fourth time, the players will have figured out the full backward induction.

You can then hold a brief discussion. You should nudge or guide it a little toward three conclusions. First, bring out the idea of backward induction, or the importance of solving sequential-move games backward from the final moves. Second, identify the idea of correct strategies that constitute a solution of the game; explain that you will soon give this solution a formal name, the rollback equilibrium. Finally, get to the idea that one can learn correct strategies by actually playing a game. With this conclusion comes the idea that if a game is played by experienced players, we might expect to observe correct strategies and equilibrium outcomes. This will give students some confidence in the concepts of backward induction and rollback equilibrium.

The last remark motivates a brief digression. Over the last decade, behavioral game theorists have made a valuable contribution to the stock of interesting games that can be played in classrooms. However, many of them come to the subject with a negative agenda, namely, to argue that everything in conventional game theory is wrong. Our experience suggests otherwise. To be sure, it takes time and experience to understand the rules of any game, and a lot of practice and experimentation to play it well. But students learn quite fast, often faster than their teachers. Some basic difficulties with the foundations of the subject undeniably remain and are interesting at a research level. But it is counterproductive to tell beginners that what they are about to learn is all wrong; it destroys their whole motivation to learn. We find it better to convey a sense of guarded optimism about the standard Nash theory, without pretending that it closes the subject, although we believe this to be the truth of the matter.

Game 2—Guessing Half of the Average

This is the well-known simultaneous-move game of the “generalized beauty contest.” Choose 10 students, and give them blank cards. Each student is to write his or her name on the card, and a number between 0 and 100. The cards will be collected and the numbers on them averaged. The student whose choice is closest to half of the average is the winner. These rules are explained in advance and in public.

The Nash equilibrium of this game is 0; it results from an iterated dominance argument. Since the average can never exceed 100, half of the average can never exceed 50. Therefore, any choice above 50 is dominated by 50. Then the average can never exceed 50, and so forth after each iteration. The first time the game is played, the winner is usually close to 25. This outcome fits Nagel’s observation (1995) that the outcome is as if the students expect the others to choose at random, averaging 50, and then choose half of that. Next, choose a different set of 10 students from the class (who have watched the outcome of the first group’s game). This second group chooses much smaller numbers, and the winner is close to 10 (as if one more round of the dominance calculation were performed) or even 5 or 6 (as if two more rounds were performed). The third group of 10 chooses much smaller numbers, including several zeros, and the winner’s choice is as low as 3 or 4. Incidentally, we have found that learning proceeds somewhat faster by watching others play than when the same group of 10 plays successively. Perhaps the brain does a better job of observation and interpretation if the ego is not engaged in playing the game.

Again, hold a brief discussion. The points to bring out are:

1. The logical concept of dominance, iterated elimination of dominated strategies, and the culmination in a Nash equilibrium.

2. Getting close to the Nash equilibrium by the experience of playing the game. Whether it is a crucial flaw of the theory that zero is rarely exactly attained or true that the theory gives a good approximation can be a point to be debated depending on the time available.

3. The idea that if you have good reason to believe that others will not be playing their Nash equilibrium strategies, then your optimal choice will also differ from your own Nash equilibrium strategy.

The discussion can also touch on the question of what would happen if the object of the game were to come closest to the average, not half of the average. That game is, of course, Keynes’s famous metaphor for the stock market, where everyone is trying to guess what everyone else is trying to guess. The game has multiple Nash equilibria, each sustained by its own bootstraps. Details of this are best postponed to a later point in the course where you cover multiple equilibria more systematically, but a quick mention in the first class provides students with an interesting economic application at the outset. You can also stress the importance of this game in their own lives. Part or even all their social security is likely to be in individual accounts. When they decide how to invest this sum, they will have to think about whether the historical pattern of returns and volatility of various assets persist when everyone makes the same decisions that they are now contemplating. This interaction between individual choice (strategy) and aggregate outcomes (equilibrium) comes naturally to someone who is trained to think game-theoretically, but others are often liable to forget the effect of everyone’s simultaneous choices. In the context of saving for retirement this can be very costly.

Game 3—Claim a Pile of Dimes (The Centipede Game)

This simple two-player game is similar to Game 1 (21 Flags) in terms of the concepts and issues that it highlights. In this game, two players, A and B, are chosen. The instructor places a dime on the table. Player A can say “Stop” or “Pass.” If Stop, then A gets the dime and the game is over. If Pass, then a second dime is added and it is B’s turn to say Stop or Pass. This goes on to the maximum of a dollar (five turns each). The players are told these rules in advance. As with the flag game, you can play this game five times in succession with different pairs of players for each game. Keep a record of where the game stops for each pair.

This game is discussed in the text (Chapter 3, Figure 3.10), but most students will not have read that far ahead at this stage. Our experience is that the simple, theoretical subgame-perfect equilibrium of immediate pickup is never observed. Most games go to 60 or 70 cents, but you do see the students thinking further ahead. Later pairs learn from observing the outcomes of earlier pairs, but the direction of this learning is not always the same. Sometimes they collude better; sometimes they get closer to the subgame-perfect outcome.

After the five pairs have played, hold a brief discussion. Ask students why they made the choices that they did. Develop the idea of rollback (or backward induction). Investigate why they did not achieve the rollback equilibrium. Did the players fail to figure it out, or did they understand it instinctively but have dif­ferent objective functions?

This game could also be played to motivate the ideas of rollback right before they are covered with the material in Chapter 3. If you prefer to cover simultaneous-move games first, then you might want to save this game until after you have completed that material. However, if you are following the order of the material in the book, rollback is likely to be the subject of your lectures within the first two weeks; you could use this game to motivate the following week’s lectures.

Game 4—The Tire Story (The Pure Coordination Game)

Another game that we have successfully played in the first lecture is based on the “We can’t take the exam; we had a flat tire” story from Section 2 of Chapter 1. Even if the students have read ahead, the discussion in the text makes it clear that there is no obvious focal answer to the question, “Which tire?”

Bring along a stack of index cards and, when you are ready to play this game, hand one card to each student. After relating the story, ask each student to pretend that she is one of those taking the exam and must answer the tire question on the card. Collect the cards and tabulate the answers on the board. Start a discussion about why different students chose different tires; focus on the difficulties of obtaining a focal equilibrium when players have different backgrounds or concerns. You can also relate the discussion back to the ma­terial in the text regarding the necessity of being prepared to face a strategically savvy opponent at any time.

Game 5—Single-Offer Bargaining (The Ultimatum Game)

Two players, A and B, are chosen. Player A offers a split of a dollar (whole dimes only). If B agrees, both get paid the agreed coins and the game is over. If B refuses, it is B’s turn, but now the sum is only 80 cents. If A accepts B’s offer, the two get paid the agreed coins. If A refuses, the game is over and neither gets anything.

Do this five times in succession with different pairs and the second-round totals falling successively to 70, 60, 50, and 40 cents. Keep a record of the successive outcomes.

Again, hold a brief discussion. The aim is to get students to start thinking about rollback and subgame perfectness and, if students understand these strategies but still don’t play them, why they don’t. Also, consider how the discrepancy changes with the second-round fraction.

Game 6—Divide a Dollar

This game asks pairs of students (or each student individ­ually, if you use a handout and match students with a random opponent after the fact) to divide a dollar between them. Each writes the amount of the dollar he wants—1 cent or $1 or some value in between. If the amounts requested for a given pair of students (or for two randomly selected student responses) add up to $1 exactly, then each student gets the amount requested. If the two amounts add up to anything other than $1, each player gets nothing. You can play this with real money if you can afford it; we have managed to play this game without real cash with perfectly acceptable results.

In actual play this is a game with discrete strategies, 100 for each player, or fewer if the choices are restricted to be multiples of nickels or dimes or even quarters. Thus, it is also relevant to (and discussed again in) Chapter 4. But you may prefer to conduct an analysis of the game, treating the choices as continuous variables, in which case the game could be placed in Chapter 5.

Discussion of the game will bring out the idea that a game can have multiple equilibria (any two values summing to $1 can make up an equilibrium) but that sometimes one of those multiple equilibria is focal. This game is in direct contrast, then, with the tire game described in Game 4 in which there are also multiple equilibria but none are as obvi­ously focal as 50 cents each is here. You can consider the various types of games in which multiple equilibria arise, including an assurance-type game, Battle of the Two Sexes, and Chicken (Chapter 4). In some cases, there are focal outcomes; in others, players may prefer to alternate among the different equilibria. You can lead from here into the idea of mixed strategies without much difficulty.

Game 7—Auctioning a Penny Jar (Winner’s Curse)

Show a jar of pennies; pass it around so each student can have a closer look and form an estimate of the contents. Show students a stack of 100 pennies to give them a better idea of what the jar might contain. While the jar is going around, explain the rules. Everyone submits a sealed bid; hand out blank cards and ask students to write their names and bids on the cards and return them. (This is also a good way for you to get to remember their names during the first meeting of the class or the section.) The winner will pay his bid and get money (paper and silver, not pennies) equal to that in the jar. Ties for a positive top bid split both prize and payment equally. When you explain the rules, emphasize that the winner must pay his bid on the spot in cash.

After you have collected and sorted the cards, write the whole distribution of bids on the board. Our experience is that if the jar contains approximately $5.00, the bids average to $3.50 (including a few zeros). Thus, the estimates are on the average conservative. But the winner usually bids about $6.00. Hold a brief discussion with the goal of getting across the idea of the winner’s curse.

The emphasis of this game is a concept relating to auc­tions that are not covered in the text until Chapter 15. It is a simple enough game to play early in the semester if you want to increase interest in the topics or hook additional students, but this game could be saved until it’s time to cover auctions.

Game 8—All-Pay Auction of $10

Everyone plays. Show the students a $10 bill, and announce that it is the prize; the known value of the prize guarantees that there is no winner’s curse. Hand out cards. Ask each student to write their name and a bid (in whole quarters). Col­lect the cards. The highest positive bid wins $10; if two or more tie with the highest positive bids, they share the $10 equally. *All* players pay the instructor what they bid, win or lose.

Be sure to emphasize before bids are submitted that you are using real money; students must pay their bids in cash on the spot. They can ensure they don’t lose money by writing $0.00. But if almost everyone does that, then someone can win with 25 cents and walk away with a tidy profit of $9.75.

Once you have collected the cards, write the distribution of bids on the board. Hold a brief discussion about the distribution and the value of the optimal bid. This game usually leads to gross overbidding; a profit of $50 in a class or section of 20 students is not uncommon. If that happens, you will have to find ways of returning the profit to the class; we have done this by having a party if the sum is large enough or by bringing cookies to the next meeting if the sum is small. Of course, do not announce this plan in advance.

This game is also covered in Chapter 15. If you play the game on the first day, you can lead up to at least some of the points made there, even though the analysis at this early stage cannot go anywhere close to that level. If you prefer to follow this game with a more in-depth discussion and the derivation of the formula for the optimal bid, then you will want to wait and play it when you get to Chapter 15.

# Movie Excerpts

Many movies contain scenes that illustrate some aspect of strategic interaction. You can screen these scenes in class as an introduction to that topic and let a discussion lead to a theoretical analysis of it. Most movies or excerpts from films are best shown later in the course, in conjunction with the particular theoretical ideas being developed. But one short scene worth showing at the outset, because many of your students will have seen the movie, is the poison scene from *The Princess Bride*.(The “poison death scene” is available in various versions on YouTube.) In it the hero (Westley) challenges one of the villains (Vizzini) to a duel of wits. Westley will poison one of two wine cups without Vizzini observing his action and set one in front of each of them. Vizzini will choose a cup for himself; Westley then must drink from the other cup. Vizzini goes through a whole sequence of arguments as to why Westley would or would not choose one cup or the other. Finally, he believes he knows which cup is safe and drinks from it. Westley drinks from the other. Just as Vizzini is laughing and advising Westley to “never go against a Sicilian when death is on the line,” Vizzini drops dead.

You will want to pause the video at this point to have a brief discussion. The students will quickly see that each of Vizzini’s arguments is inherently self-contradictory. If Westley thinks through to the same point that leads Vizzini to believe that a particular cup will contain the poison, then he should instead put the poison in the other cup. Any systematic action can be thought through and defeated by the other player. Therefore, the only correct strategy is to be unsystematic or random. This is a good way to motivate the idea of mixed strategies.

But this is not the main point of the story. Resume the video. The princess is surprised to find that Westley had put the poison in the cup he placed closer to himself. “They were both poisoned,” he replies, “I have been building up immunity to Iocaine for years.” Thus, the game being played was really one of asymmetric information; Vizzini did not know Westley’s payoffs and did not think the strategy of poisoning both cups was open to him. This introduces the idea at the outset of the course. Tell the students that it will be discussed in greater depth in Chapter 9.

Here is some general advice for screening movie excerpts.

DO:

1. Take suggestions from students about other movies or games you can use; their ideas may be more appealing to other students than yours. Always ask and discuss in advance just what strategic issue the excerpt illustrates.

2. Come prepared with the correct URL and notes indicating the exact time at which your excerpt begins in the relevant video clip. The effectiveness of any clip can be lost if students have to wait for you to find it.

DON’T:

1. Assume that students know the general plots of the movies from which you show excerpts. Prepare a brief explanation of the situation and the film’s characters as it pertains to your excerpt before you start the clip.

2. Show a whole movie or a long clip when only a small point pertains to strategy. The students will get distracted by the other incidental aspects.

# News and Current Affairs

You will find that there is no shortage of motivation and illustration for game theory. Just keep an eye on recent news events. These will give you several events of topical interest to tie in with your course. More than likely, you will find examples of various concepts throughout the semester that help you make even clearer your points about multiperson prisoners’ dilemmas (countries’ willingness to take actions to combat climate change), credibility of strategic moves (Trump’s threats), the importance of patience in bargaining (the Afghan Taliban saying, “The Americans have all the watches; we have all the time”), and so on. Negotiations between the government of Israel and the Palestinian authority and between the North Korean regime and its neighboring countries and the United States provide continuing opportunities for discussion. At a more trivial but perhaps more engaging level, so do incidents from the CBS reality show *Survivor*.

One way to increase your students’ willingness or desire to apply what they are learning to actual events is to provide a semester-long assignment requiring each of them to add to a class collection of real-world events amenable to analysis using the theory of games. It may take them a few weeks before they are able to do the appropriate analysis on their own, but you can add weekly event analyses to the collection until such time as they are able to take over that role. You do not have to restrict their examples to newsworthy events; many interesting and relevant examples can be culled from recent films or novels. The film *Waking Ned Devine*, for instance, has a wonderful example of a collective-action game and shows how individual incentives can be different from those of the group as a whole.

# Books and Web Sites

There are plenty of other sources for examples. The fol­lowing books have several cases and stories, varying greatly in their context and relevance, sometimes with an explicitly game-theoretic analysis and sometimes without:

Robert Axelrod, *The Evolution of Cooperation* (New York: Basic Books, 1984)

Steven J. Brams, *The Presidential Election Game* (New Haven, Conn.: Yale University Press, 1978).

Steven J. Brams, *Biblical Games* (Cambridge, Mass.: MIT Press, 1980).

Steven J. Brams, *Superpower Games* (New Haven, Conn.: Yale University Press, 1985).

Steven J. Brams and Alan D. Taylor, *Fair Division* (Cambridge: Cambridge University Press, 1996).

Adam Brandenburger and Barry Nalebuff, *Co-opetition* (New York: Doubleday, 1996).

Michael Chwe, *Jane Austen: Game Theorist* (Princeton, N.J.: Princeton University Press, 2013).

Avinash Dixit and Barry Nalebuff, *Thinking Strategically* (New York: W. W. Norton, 1991).

Avinash Dixit and Barry Nalebuff, *The Art of Strategy* (New York: W. W. Norton, 2008).

Paul Fisher, *Rock, Paper, Scissors: Game Theory in Everyday Life* (New York: Basic Books,   
 2008).

Erving Goffman, *The Presentation of Self in Everyday Life* (New York: Doubleday, 1959).

John Kay, *Why Firms Succeed* (New York: Oxford University Press, 1995).

Paul Kennedy, ed., *Grand Strategies in War and Peace* (New Haven, Conn.: Yale University Press, 1991).

David McAdams, *Game-Changer: Game Theory and the Art of Transforming Strategic Situations* (New York: W. W. Norton, 2014)

John McMillan, *Games, Strategies and Managers* (New York: Oxford University Press, 1996).

Howard Raiffa, *The Art and Science of Negotiation* (Cambridge, Mass.: Harvard University Press, 1982).

Matt Ridley, *The Origins of Virtue: Human Instincts and the Evolution of Cooperation* 1st Edition(New York: Penguin Books, 1998)

Thomas Schelling, *The Strategy of Conflict* (Cambridge, Mass.: Harvard University. Press, 1960)

Richard Thaler, *The Winner’s Curse: Paradoxes and Anomalies of Economic Life* (Princeton, N.J.: Princeton University Press, 1992).

In addition, there are several Web sites dealing with game theory from the perspectives of research, teaching, and history of thought. Here are a few URLs:

Marko Grobelnik, Robert A. Miller, and Vesna Prasnikar have some downloadable software (for matrix games and extensive-form games) for running classroom experiments at www.comlabgames.com/strategicplay.

For teaching and research resources and many additional links, see

* David Levine, UCLA, for a focus on economic theory: levine.sscnet.ucla.edu
* Alvin Roth, Stanford University, for a focus on laboratory experiments: web.stanford.edu/~alroth/alroth.html
* Mike Shor, University of Connecticut, for a wide variety of teaching-related resources, including many examples of game theory in movies, music, fiction, and more: www.gametheory.net

For ideas on in-class experiments, some of which are appropriate for use with topics covered in this text, see:

* ideas.repec.org/a/taf/jeduce/v31y2000i4p406-406.html
* classex.de
* gametheory.tau.ac.il
* veconlab.econ.virginia.edu/admin.htm

The first link takes you to a survey of non-computerized classroom games. The latter three sites allow your students to play games online and collect data from their responses for use in class discussion. Instructor registration is required.