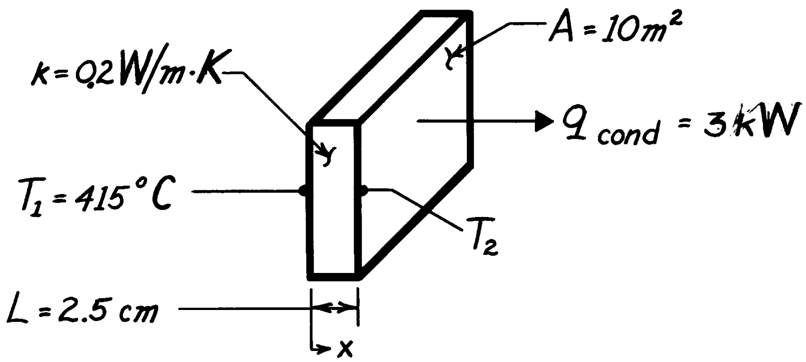
PROBLEM 1.1

KNOWN: Heat rate, q, through one-dimensional wall of area A, thickness L, thermal conductivity k and inner temperature, T_1.

FIND: The outer temperature of the wall, T_2.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The rate equation for conduction through the wall is given by Fourier's law,

$$q_{cond} = q_x = q_x'' \cdot A = -k\frac{dT}{dx} \cdot A = kA\frac{T_1 - T_2}{L}.$$

Solving for T_2 gives

$$T_2 = T_1 - \frac{q_{cond}L}{kA}.$$

Substituting numerical values, find

$$T_2 = 415^\circ C - \frac{3000W \times 0.025m}{0.2W/m \cdot K \times 10m^2}$$

$$T_2 = 415^\circ C - 37.5^\circ C$$

$$T_2 = 378^\circ C.$$ <

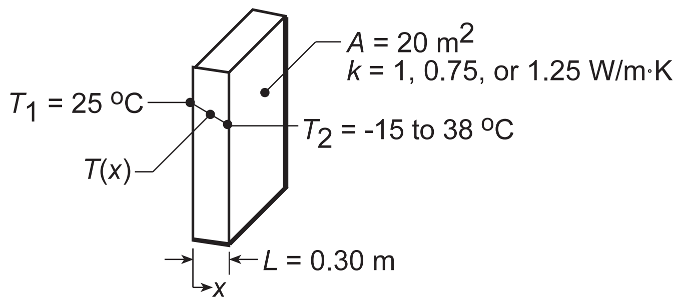
COMMENTS: Note direction of heat flow and fact that T_2 must be less than T_1.

PROBLEM 1.2

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of ambient air temperatures ranging from -15 to 38°C.

SCHEMATIC:



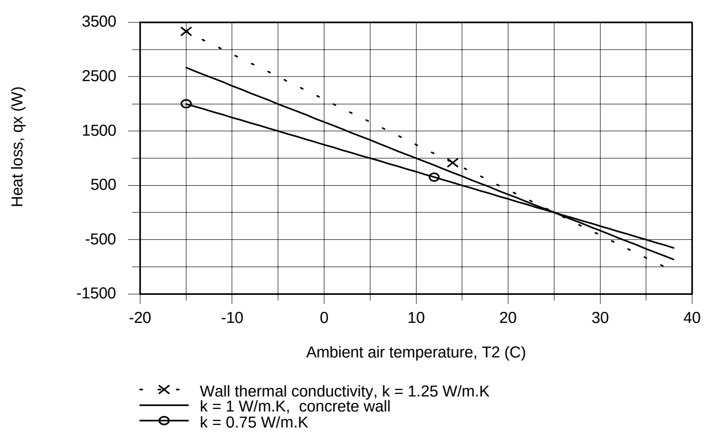
ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties, (4) Outside wall temperature is that of the ambient air.

ANALYSIS: From Fourier's law, it is evident that the gradient, $dT/dx = -q''_x/k$, is a constant, and hence the temperature distribution is linear, if q''_x and k are each constant. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is T_2 = -15°C are

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 1\,\text{W/m}\cdot\text{K}\,\frac{25^\circ\text{C} - \left(-15^\circ\text{C}\right)}{0.30\,\text{m}} = 133.3\,\text{W/m}^2 . \qquad (1)$$

$$q_x = q''_x \times A = 133.3\,\text{W/m}^2 \times 20\,\text{m}^2 = 2667\,\text{W} . \qquad (2) <$$

Combining Eqs. (1) and (2), the heat rate q_x can be determined for the range of ambient temperature, $-15 \le T_2 \le 38$°C, with different wall thermal conductivities, k.



For the concrete wall, k = 1 W/m·K, the heat loss varies linearily from +2667 W to -867 W and is zero when the inside and ambient temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

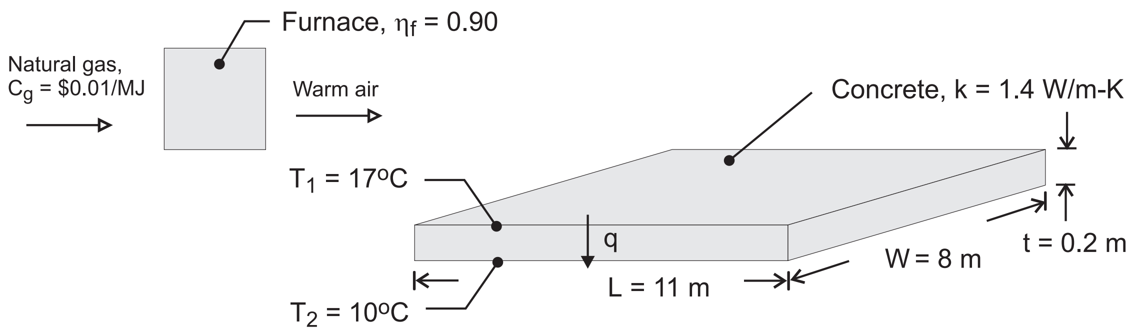
COMMENTS: Without steady-state conditions and constant k, the temperature distribution in a plane wall would not be linear.

PROBLEM 1.3

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,W/m\cdot K\,(11\,m \times 8\,m)\frac{7°C}{0.20\,m} = 4312\ W \qquad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,W \times \$0.01/MJ}{0.9 \times 10^6\,J/MJ}(24\,h/d \times 3600\,s/h) = \$4.14/d \qquad <$$

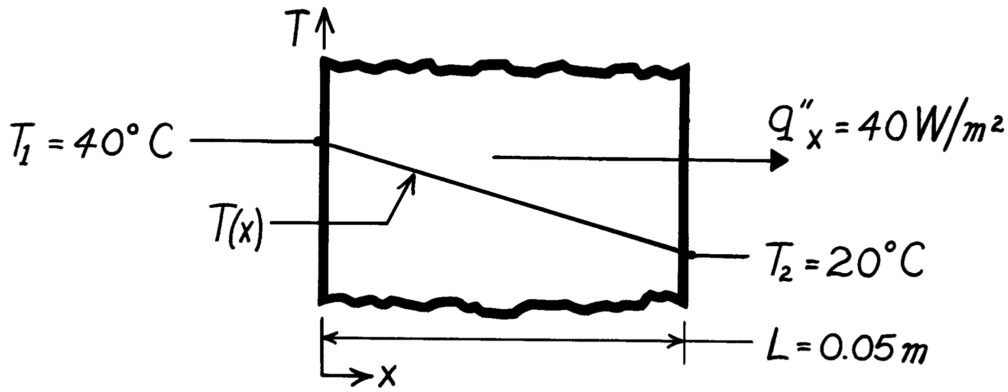
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.4

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05 m}{(40\text{-}20)^\circ C}$$

$$k = 0.10 \ W/m \cdot K. \qquad <$$

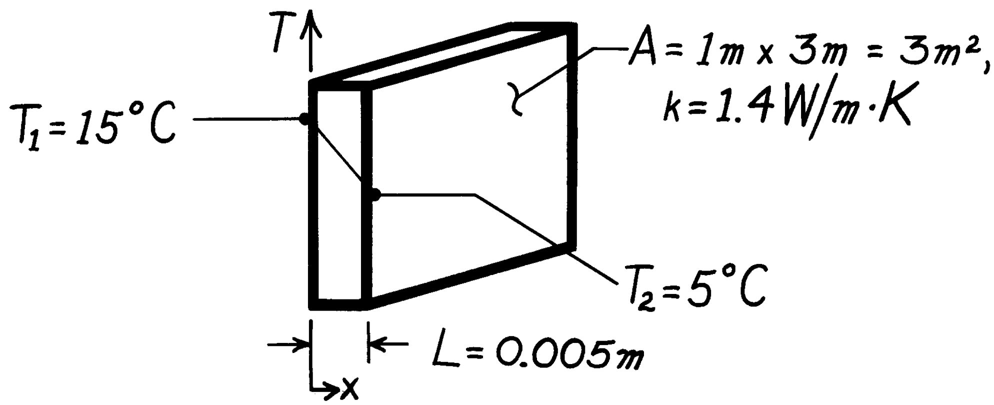
COMMENTS: Note that the $^\circ$C or K temperature units may be used interchangeably when evaluating a temperature difference.

PROBLEM 1.5

KNOWN: Inner and outer surface temperatures of a glass window of prescribed dimensions.

FIND: Heat loss through window.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing conditions the heat flux may be computed from Fourier's law, Eq. 1.2.

$$q''_x = k \frac{T_1 - T_2}{L}$$

$$q''_x = 1.4 \frac{W}{m \cdot K} \frac{(15\text{-}5)^\circ C}{0.005m}$$

$$q''_x = 2800 \ W/m^2.$$

Since the heat flux is uniform over the surface, the heat loss (rate) is

$$q = q''_x \times A$$

$$q = 2800 \ W/m^2 \times 3m^2$$

$$q = 8400 \ W.$$ <

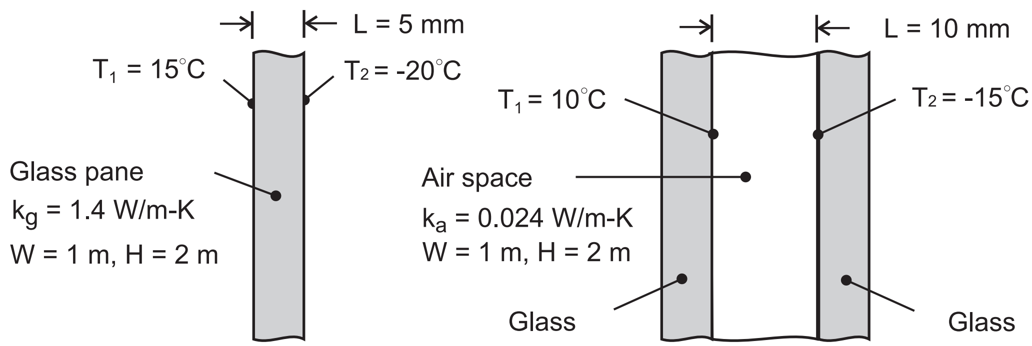
COMMENTS: A linear temperature distribution exists in the glass for the prescribed conditions.

PROBLEM 1.6

KNOWN: Width, height, thickness and thermal conductivity of a single pane window and the air space of a double pane window. Representative winter surface temperatures of single pane and air space.

FIND: Heat loss through single and double pane windows.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through glass or air, (2) Steady-state conditions, (3) Enclosed air of double pane window is stagnant (negligible buoyancy induced motion).

ANALYSIS: From Fourier's law, the heat losses are

Single Pane: $$q_g = k_g A \frac{T_1 - T_2}{L} = 1.4\ \text{W/m}\cdot\text{K}\left(2\text{m}^2\right)\frac{35\ ^\circ\text{C}}{0.005\text{m}} = 19,600\ \text{W}$$

Double Pane: $$q_a = k_a A \frac{T_1 - T_2}{L} = 0.024\left(2\text{m}^2\right)\frac{25\ ^\circ\text{C}}{0.010\ \text{m}} = 120\ \text{W}$$

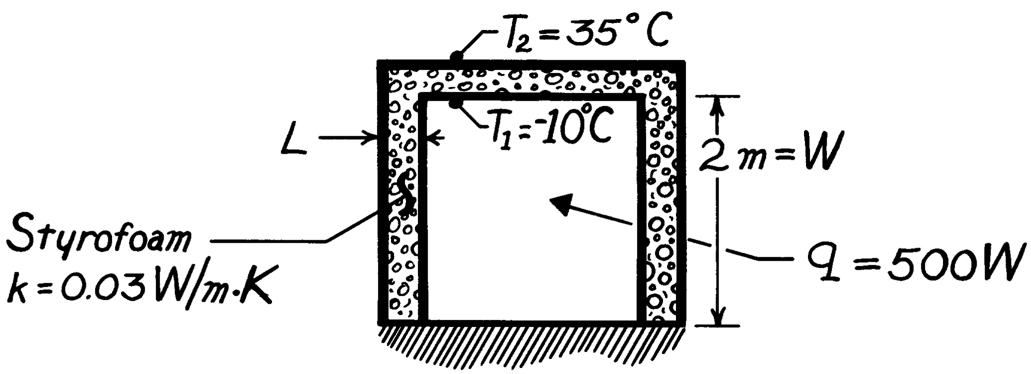
COMMENTS: Losses associated with a single pane are unacceptable and would remain excessive, even if the thickness of the glass were doubled to match that of the air space. The principal advantage of the double pane construction resides with the low thermal conductivity of air (~ 60 times smaller than that of glass). For a fixed ambient outside air temperature, use of the double pane construction would also increase the surface temperature of the glass exposed to the room (inside) air.

PROBLEM 1.7

KNOWN: Dimensions of freezer compartment. Inner and outer surface temperatures.

FIND: Thickness of styrofoam insulation needed to maintain heat load below prescribed value.

SCHEMATIC:



ASSUMPTIONS: (1) Perfectly insulated bottom, (2) One-dimensional conduction through 5 walls of area $A = 4m^2$, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: Using Fourier's law, Eq. 1.2, the heat rate is

$$q = q'' \cdot A = k \frac{\Delta T}{L} A_{total}$$

Solving for L and recognizing that $A_{total} = 5 \times W^2$, find

$$L = \frac{5\, k\, \Delta T\, W^2}{q}$$

$$L = \frac{5 \times 0.03\ \text{W/m} \cdot \text{K} \left[35 - (-10)\right]^\circ \text{C} \left(4\text{m}^2\right)}{500\ \text{W}}$$

$$L = 0.054\text{m} = 54\text{mm}.$$ <

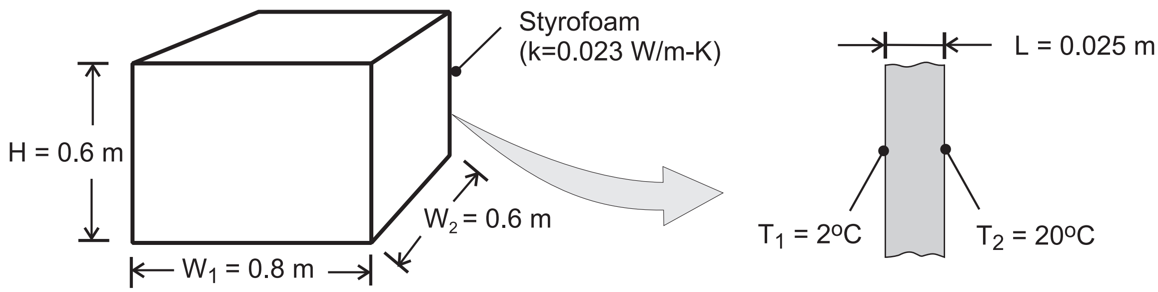
COMMENTS: The corners will cause local departures from one-dimensional conduction and a slightly larger heat loss.

PROBLEM 1.8

KNOWN: Dimensions and thermal conductivity of food/beverage container. Inner and outer surface temperatures.

FIND: Heat flux through container wall and total heat load.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through bottom wall, (3) Uniform surface temperatures and one-dimensional conduction through remaining walls.

ANALYSIS: From Fourier's law, Eq. 1.2, the heat flux is

$$q'' = k\frac{T_2 - T_1}{L} = \frac{0.023\ \text{W/m}\cdot\text{K}\ (20-2)^\circ\text{C}}{0.025\ \text{m}} = 16.6\ \text{W/m}^2 \qquad <$$

Since the flux is uniform over each of the five walls through which heat is transferred, the heat load is

$$q = q'' \times A_{total} = q''\left[H\left(2W_1 + 2W_2\right) + W_1 \times W_2\right]$$

$$q = 16.6\ \text{W/m}^2\left[0.6\text{m}\left(1.6\text{m} + 1.2\text{m}\right) + \left(0.8\text{m} \times 0.6\text{m}\right)\right] = 35.9\ \text{W} \qquad <$$

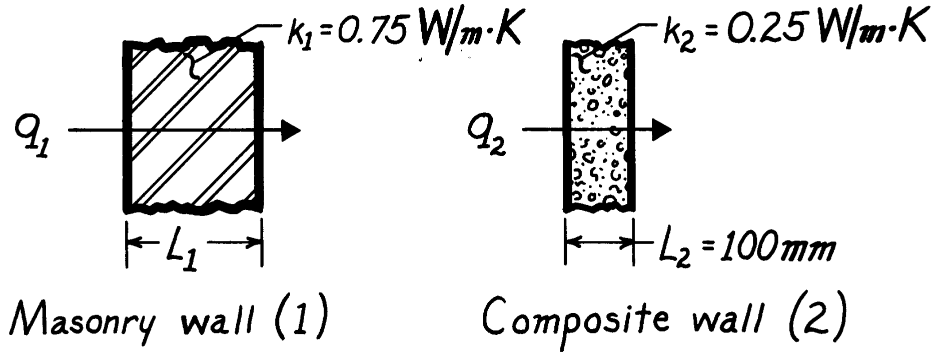
COMMENTS: The corners and edges of the container create local departures from one-dimensional conduction, which increase the heat load. However, for H, W_1, W_2 >> L, the effect is negligible.

PROBLEM 1.9

KNOWN: Masonry wall of known thermal conductivity has a heat rate which is 80% of that through a composite wall of prescribed thermal conductivity and thickness.

FIND: Thickness of masonry wall.

SCHEMATIC:



ASSUMPTIONS: (1) Both walls subjected to same surface temperatures, (2) One-dimensional conduction, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: For steady-state conditions, the conduction heat flux through a one-dimensional wall follows from Fourier's law, Eq. 1.2,

$$q'' = k \frac{\Delta T}{L}$$

where ΔT represents the difference in surface temperatures. Since ΔT is the same for both walls, it follows that

$$L_1 = L_2 \frac{k_1}{k_2} \cdot \frac{q_2''}{q_1''}.$$

With the heat fluxes related as

$$q_1'' = 0.8\, q_2''$$

$$L_1 = 100\text{mm}\ \frac{0.75\ \text{W/m}\cdot\text{K}}{0.25\ \text{W/m}\cdot\text{K}} \times \frac{1}{0.8} = 375\text{mm}. \qquad <$$

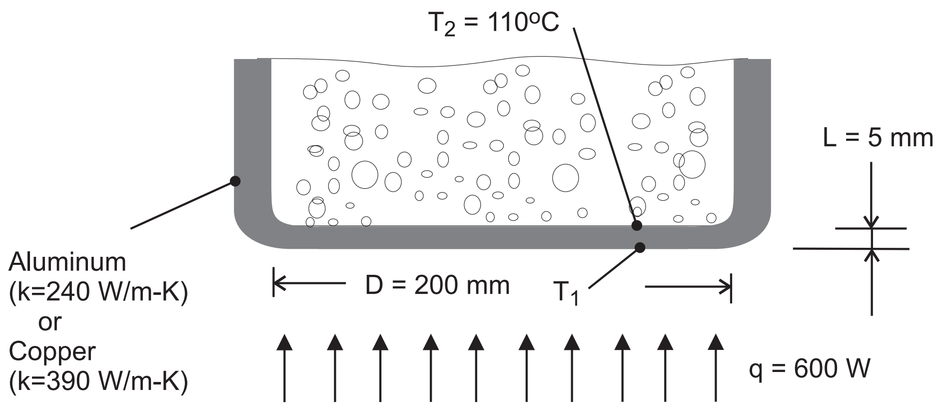
COMMENTS: Not knowing the temperature difference across the walls, we cannot find the value of the heat rate.

PROBLEM 1.10

KNOWN: Thickness, diameter and inner surface temperature of bottom of pan used to boil water. Rate of heat transfer to the pan.

FIND: Outer surface temperature of pan for an aluminum and a copper bottom.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through bottom of pan.

ANALYSIS: From Fourier's law, the rate of heat transfer by conduction through the bottom of the pan is

$$q = kA\frac{T_1 - T_2}{L}$$

Hence,

$$T_1 = T_2 + \frac{qL}{kA}$$

where $A = \pi D^2/4 = \pi(0.2\text{m})^2/4 = 0.0314\ \text{m}^2$.

Aluminum: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{240\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.40\ ^\circ\text{C}$$

Copper: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{390\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.25\ ^\circ\text{C}$$

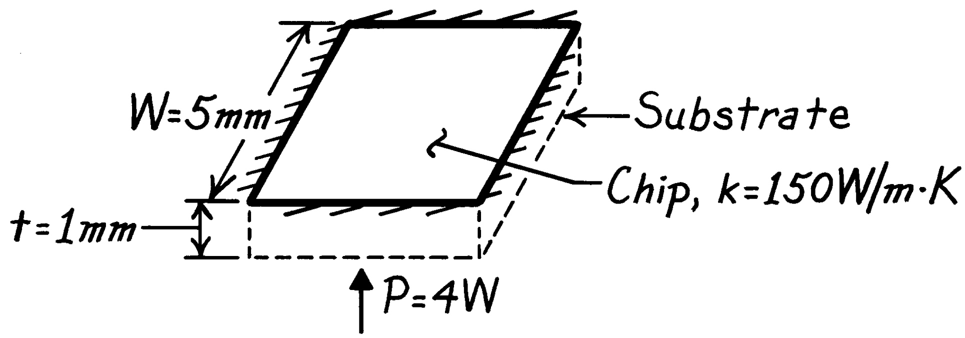
COMMENTS: Although the temperature drop across the bottom is slightly larger for aluminum (due to its smaller thermal conductivity), it is sufficiently small to be negligible for both materials. To a good approximation, the bottom may be considered *isothermal* at T ≈ 110 °C, which is a desirable feature of pots and pans.

PROBLEM 1.11

KNOWN: Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

FIND: Temperature drop across the chip.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform heat dissipation, (4) Negligible heat loss from back and sides, (5) One-dimensional conduction in chip.

ANALYSIS: All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, from Fourier's law,

$$P = q = kA\,\frac{\Delta T}{t}$$

or

$$\Delta T = \frac{t \cdot P}{kW^2} = \frac{0.001\text{ m} \times 4\text{ W}}{150\text{ W/m}\cdot\text{K}\left(0.005\text{ m}\right)^2}$$

$$\Delta T = 1.1^\circ\text{ C.}$$ <

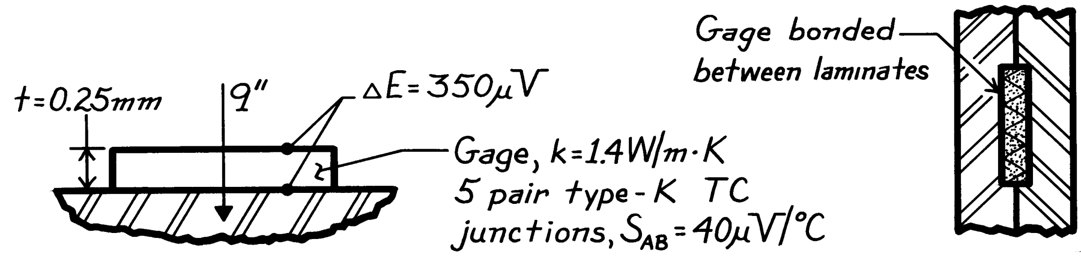
COMMENTS: For fixed P, the temperature drop across the chip decreases with increasing k and W, as well as with decreasing t.

PROBLEM 1.12

KNOWN: Heat flux gage with thin-film thermocouples on upper and lower surfaces; output voltage, calibration constant, thickness and thermal conductivity of gage.

FIND: (a) Heat flux, (b) Precaution when sandwiching gage between two materials.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat conduction in gage, (3) Constant properties.

ANALYSIS: (a) Fourier's law applied to the gage can be written as

$$q'' = k\frac{\Delta T}{\Delta x}$$

and the gradient can be expressed as

$$\frac{\Delta T}{\Delta x} = \frac{\Delta E / N}{S_{AB} t}$$

where N is the number of differentially connected thermocouple junctions, S_{AB} is the Seebeck coefficient for type K thermocouples (A-chromel and B-alumel), and $\Delta x = t$ is the gage thickness. Hence,

$$q'' = \frac{k\Delta E}{N S_{AB} t}$$

$$q'' = \frac{1.4\ \mathrm{W/m\cdot K} \times 350\times10^{-6}\ \mathrm{V}}{5\times 40\times10^{-6}\ \mathrm{V/^\circ C}\times 0.25\times10^{-3}\ \mathrm{m}} = 9800\ \mathrm{W/m^2}. \qquad <$$

(b) The major precaution to be taken with this type of gage is to match its thermal conductivity with that of the material on which it is installed. If the gage is bonded between laminates (see sketch above) and its thermal conductivity is significantly different from that of the laminates, one dimensional heat flow will be disturbed and the gage will read incorrectly.

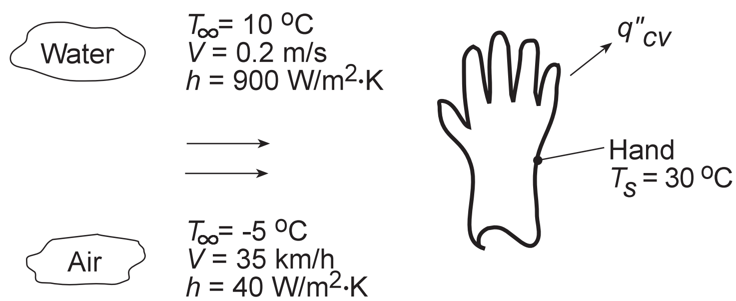
COMMENTS: If the thermal conductivity of the gage is lower than that of the laminates, will it indicate heat fluxes that are systematically high or low?

PROBLEM 1.13

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m^2 under normal room conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{air} = 40\,W/m^2\cdot K\left[30-(-5)\right]K = 1{,}400\,W/m^2 \qquad <$$

For the water stream:

$$q''_{water} = 900\,W/m^2\cdot K\left(30-10\right)K = 18{,}000\,W/m^2 \qquad <$$

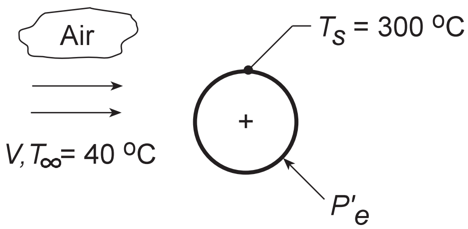
COMMENTS: The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m^2 which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

PROBLEM 1.14

KNOWN: Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

FIND: (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as $h = CV^n$, determine the parameters C and n.

SCHEMATIC:



V(m/s)	1	2	4	8	12
P'_e (W/m)	450	658	983	1507	1963
h ($W/m^2 \cdot K$)	22.0	32.2	48.1	73.8	96.1

ASSUMPTIONS: (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

ANALYSIS: (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newtons law of cooling on a per unit length basis,

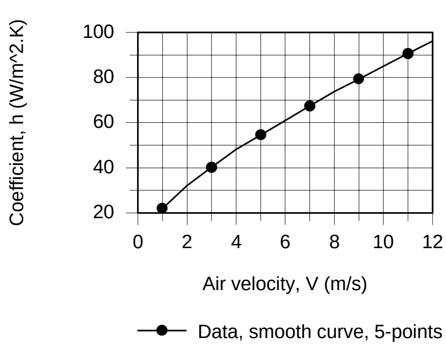
$$P'_e = h(\pi D)(T_s - T_\infty)$$

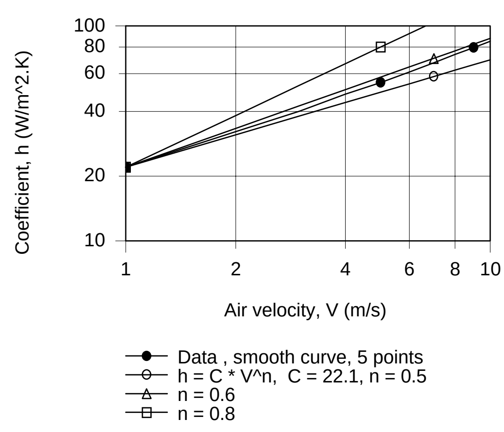
where P'_e is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table above, find

$$h = 450\,\mathrm{W/m} \Big/ \pi \times 0.025\,\mathrm{m}\,(300-40)^\circ\mathrm{C} = 22.0\,\mathrm{W/m^2{\cdot}K} \qquad <$$

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing C = 22.12 $W/m^2 \cdot K(s/m)^n$, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice. Hence, C = 22.12 and n = 0.6. <



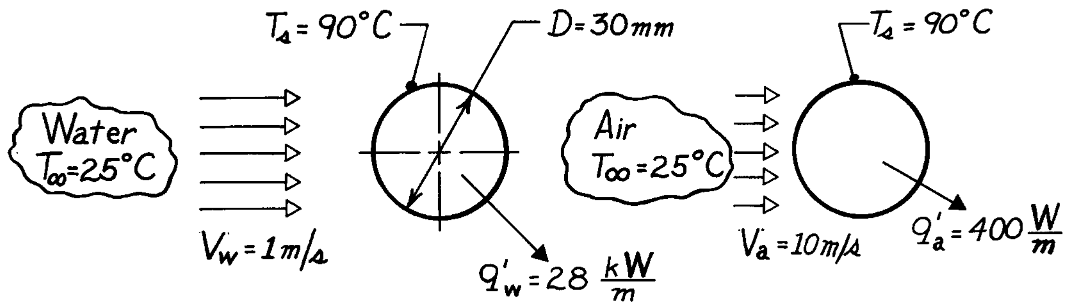


PROBLEM 1.15

KNOWN: Long, 30mm-diameter cylinder with embedded electrical heater; power required to maintain a specified surface temperature for water and air flows.

FIND: Convection coefficients for the water and air flow convection processes, h_w and h_a, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Flow is cross-wise over cylinder which is very long in the direction normal to flow.

ANALYSIS: The convection heat rate from the cylinder per unit length of the cylinder has the form

$$q' = h(\pi D)\ (T_s - T_\infty)$$

and solving for the heat transfer convection coefficient, find

$$h = \frac{q'}{\pi D\ (T_s - T_\infty)}.$$

Substituting numerical values for the water and air situations:

Water $$h_w = \frac{28 \times 10^3\ \text{W/m}}{\pi \times 0.030\text{m}\ (90\text{-}25)^\circ\ \text{C}} = 4{,}570\ \text{W/m}^2 \cdot \text{K}$$ <

Air $$h_a = \frac{400\ \text{W/m}}{\pi \times 0.030\text{m}\ (90\text{-}25)^\circ\ \text{C}} = 65\ \text{W/m}^2 \cdot \text{K}.$$ <

COMMENTS: Note that the air velocity is 10 times that of the water flow, yet

$$h_w \approx 70 \times h_a.$$

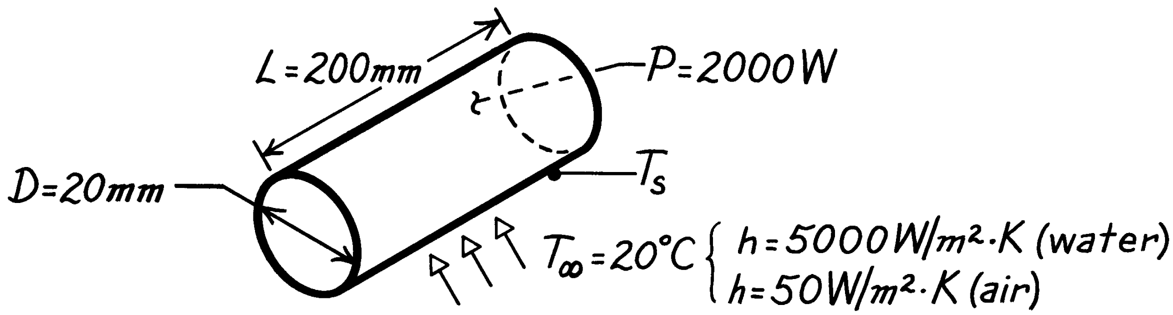
These values for the convection coefficient are typical for forced convection heat transfer with liquids and gases. See Table 1.1.

PROBLEM 1.16

KNOWN: Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

FIND: Heater surface temperatures in water and air.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

ANALYSIS: With P = q_{conv}, Newton's law of cooling yields

$$P = hA\left(T_S - T_\infty\right) = h\pi DL\left(T_S - T_\infty\right)$$

$$T_S = T_\infty + \frac{P}{h\pi DL}.$$

In water,

$$T_s = 20^\circ C + \frac{2000\ W}{5000\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 31.8^\circ C = 51.8^\circ C. \qquad <$$

In air,

$$T_s = 20^\circ C + \frac{2000\ W}{50\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 3183^\circ C = 3203^\circ C. \qquad <$$

COMMENTS: (1) Air is much less effective than water as a heat transfer fluid. Hence, the cartridge temperature is much higher in air, so high, in fact, that the cartridge would melt.

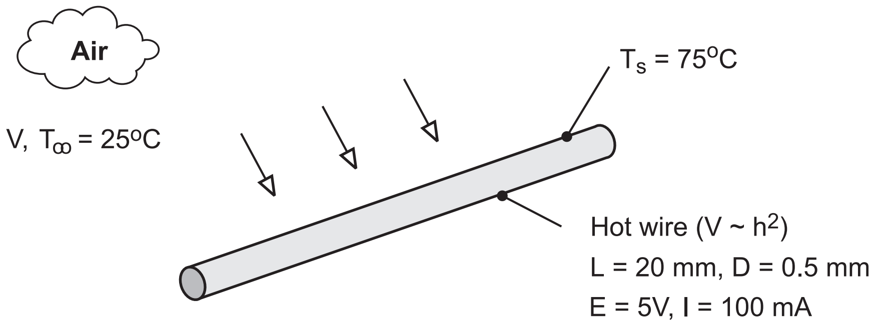
(2) In air, the high cartridge temperature would render radiation significant.

PROBLEM 1.17

KNOWN: Length, diameter and calibration of a hot wire anemometer. Temperature of air stream. Current, voltage drop and surface temperature of wire for a particular application.

FIND: Air velocity

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from the wire by natural convection or radiation.

ANALYSIS: If all of the electric energy is transferred by convection to the air, the following equality must be satisfied

$$P_{elec} = EI = hA\left(T_s - T_\infty\right)$$

where $A = \pi DL = \pi\left(0.0005\text{m} \times 0.02\text{m}\right) = 3.14 \times 10^{-5}\text{m}^2.$

Hence,

$$h = \frac{EI}{A\left(T_s - T_\infty\right)} = \frac{5\text{V} \times 0.1\text{A}}{3.14 \times 10^{-5}\text{m}^2\left(50\ ^\circ\text{C}\right)} = 318\ \text{W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5} h^2 = 6.25 \times 10^{-5}\left(318\ \text{W/m}^2 \cdot \text{K}\right)^2 = 6.3\ \text{m/s} \qquad <$$

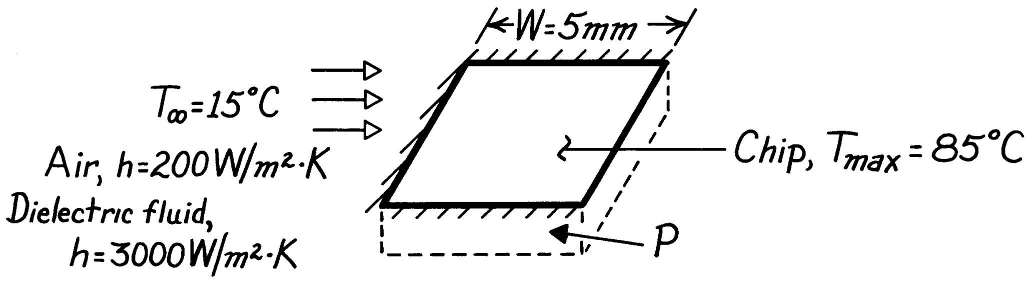
COMMENTS: The convection coefficient is sufficiently large to render buoyancy (natural convection) and radiation effects negligible.

PROBLEM 1.18

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

In *air*,

$$P_{max} = 200\ W/m^2\cdot K(0.005\ m)^2(85 - 15)\ °C = 0.35\ W. \qquad <$$

In the *dielectric liquid*

$$P_{max} = 3000\ W/m^2\cdot K(0.005\ m)^2(85\text{-}15)\ °C = 5.25\ W. \qquad <$$

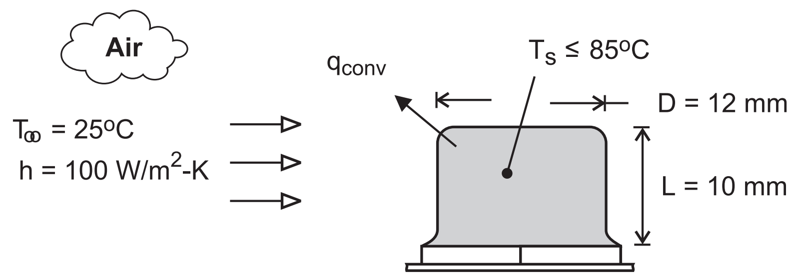
COMMENTS: Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

PROBLEM 1.19

KNOWN: Length, diameter and maximum allowable surface temperature of a power transistor. Temperature and convection coefficient for air cooling.

FIND: Maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through base of transistor, (3) Negligible heat transfer by radiation from surface of transistor.

ANALYSIS: Subject to the foregoing assumptions, the power dissipated by the transistor is equivalent to the rate at which heat is transferred by convection to the air. Hence,

$$P_{elec} = q_{conv} = hA\left(T_s - T_\infty\right)$$

where $A = \pi\left(DL + D^2/4\right) = \pi\left[0.012\text{m} \times 0.01\text{m} + \left(0.012\text{m}\right)^2/4\right] = 4.90 \times 10^{-4}\text{m}^2.$

For a maximum allowable surface temperature of 85°C, the power is

$$P_{elec} = 100\ \text{W/m}^2 \cdot \text{K}\left(4.90 \times 10^{-4}\text{m}^2\right)\ \left(85 - 25\right)^\circ \text{C} = 2.94\ \text{W} \qquad <$$

COMMENTS: (1) For the prescribed surface temperature and convection coefficient, radiation will be negligible relative to convection. However, conduction through the base could be significant, thereby permitting operation at a larger power.

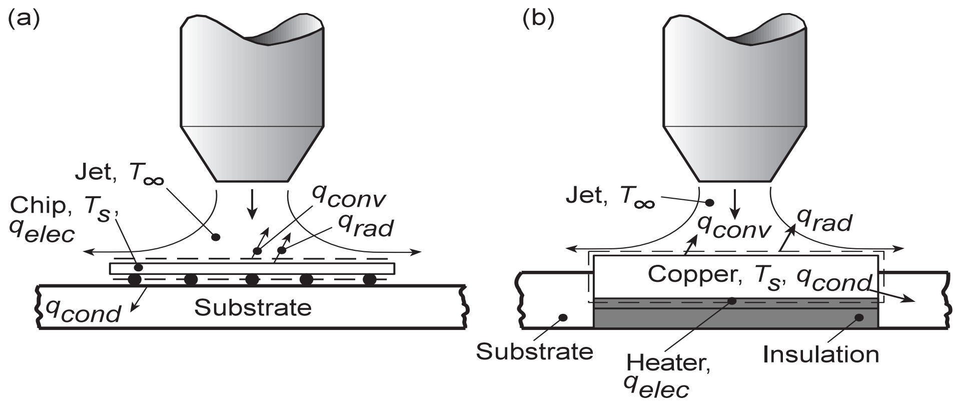
(2) The *local* convection coefficient varies over the surface, and *hot spots* could exist if there are locations at which the local value of *h* is substantially smaller than the prescribed average value.

PROBLEM 1.20

KNOWN: Air jet impingement is an effective means of cooling logic chips.

FIND: Procedure for measuring convection coefficients associated with a 10 mm × 10 mm chip.

SCHEMATIC:



ASSUMPTIONS: Steady-state conditions.

ANALYSIS: One approach would be to use the actual chip-substrate system, Case (a), to perform the measurements. In this case, the electric power dissipated in the chip would be transferred from the chip by radiation and conduction (to the substrate), as well as by convection to the jet. An energy balance for the chip yields $q_{elec} = q_{conv} + q_{cond} + q_{rad}$. Hence, with $q_{conv} = hA\left(T_s - T_\infty\right)$, where A = 100 mm^2 is the surface area of the chip,

$$h = \frac{q_{elec} - q_{cond} - q_{rad}}{A\left(T_s - T_\infty\right)} \tag{1}$$

While the electric power (q_{elec}) and the jet (T_∞) and surface (T_s) temperatures may be measured, losses from the chip by conduction and radiation would have to be estimated. Unless the losses are negligible (an unlikely condition), the accuracy of the procedure could be compromised by uncertainties associated with determining the conduction and radiation losses.

A second approach, Case (b), could involve fabrication of a heater assembly for which the conduction and radiation losses are controlled and minimized. A 10 mm × 10 mm copper block (k ~ 400 W/m·K) could be inserted in a poorly conducting substrate (k < 0.1 W/m·K) and a patch heater could be applied to the back of the block and insulated from below. If conduction to both the substrate and insulation could thereby be rendered negligible, heat would be transferred almost exclusively through the block. If radiation were rendered negligible by applying a low emissivity coating ($\varepsilon < 0.1$) to the surface of the copper block, virtually all of the heat would be transferred by convection to the jet. Hence, q_{cond} and q_{rad} may be neglected in equation (1), and the expression may be used to accurately determine h from the known (A) and measured (q_{elec}, T_s, T_∞) quantities.

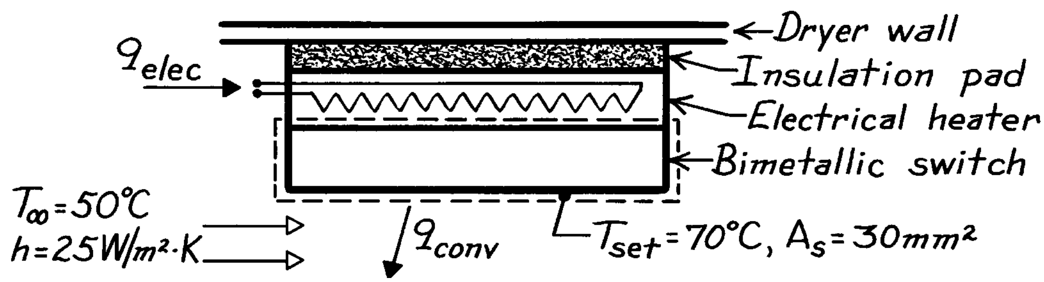
COMMENTS: Since convection coefficients associated with gas flows are generally small, concurrent heat transfer by radiation and/or conduction must often be considered. However, jet impingement is one of the more effective means of transferring heat by convection and convection coefficients well in excess of 100 W/m^2·K may be achieved.

PROBLEM 1.21

KNOWN: Upper temperature set point, T_{set}, of a bimetallic switch and convection heat transfer coefficient between clothes dryer air and exposed surface of switch.

FIND: Electrical power for heater to maintain T_{set} when air temperature is $T_\infty = 50°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Electrical heater is perfectly insulated from dryer wall, (3) Heater and switch are isothermal at T_{set}, (4) Negligible heat transfer from sides of heater or switch, (5) Switch surface, A_s, loses heat only by convection.

ANALYSIS: Define a control volume around the bimetallic switch which experiences heat input from the heater and convection heat transfer to the dryer air. That is,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$
$$q_{elec} - hA_s\left(T_{set} - T_\infty\right) = 0.$$

The electrical power required is,

$$q_{elec} = hA_s\left(T_{set} - T_\infty\right)$$

$$q_{elec} = 25\ W/m^2 \cdot K \times 30 \times 10^{-6}\ m^2\left(70 - 50\right)K = 15\ mW. \qquad <$$

COMMENTS: (1) This type of controller can achieve variable operating air temperatures with a single set-point, inexpensive, bimetallic-thermostatic switch by adjusting power levels to the heater.

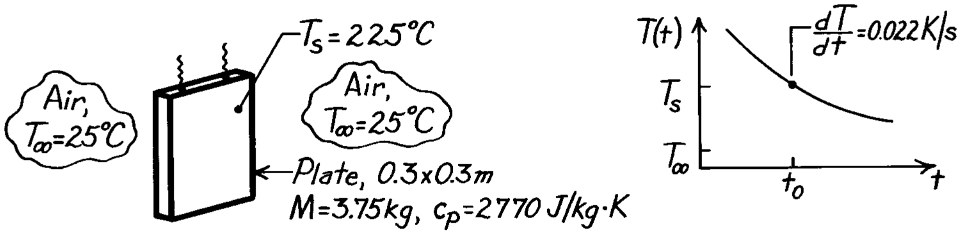
(2) Will the heater power requirement increase or decrease if the insulation pad is other than perfect?

PROBLEM 1.22

KNOWN: Hot vertical plate suspended in cool, still air. Change in plate temperature with time at the instant when the plate temperature is 225°C.

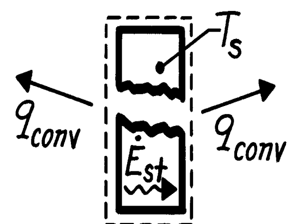
FIND: Convection heat transfer coefficient for this condition.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is isothermal and of uniform temperature, (2) Negligible radiation exchange with surroundings, (3) Negligible heat lost through suspension wires.

ANALYSIS: As shown in the cooling curve above, the plate temperature decreases with time. The condition of interest is for time t_o. For a control surface about the plate, the conservation of energy requirement is



$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-2hA_s\left(T_s - T_\infty\right) = Mc_p \frac{dT}{dt}$$

where A_s is the surface area of one side of the plate. Solving for h, find

$$h = \frac{Mc_p}{2A_s\left(T_s - T_\infty\right)} \frac{dT}{dt}$$

$$h = \frac{3.75\ \text{kg} \times 2770\ \text{J/kg}\cdot\text{K}}{2\times\left(0.3\times0.3\right)\text{m}^2\left(225-25\right)\text{K}} \times 0.022\ \text{K/s} = 6.4\ \text{W/m}^2\cdot\text{K}$$ <

COMMENTS: (1) Assuming the plate is very highly polished with emissivity of 0.08, determine whether radiation exchange with the surroundings at 25°C is negligible compared to convection.

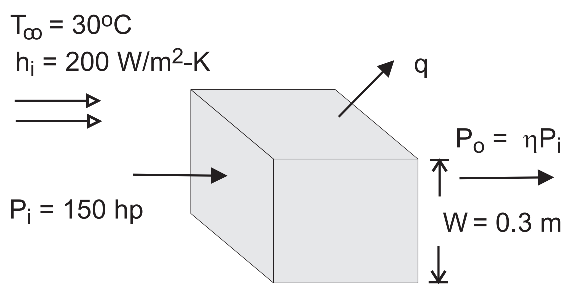
(2) We will later consider the criterion for determining whether the isothermal plate assumption is reasonable. If the thermal conductivity of the present plate were high (such as aluminum or copper), the criterion would be satisfied.

PROBLEM 1.23

KNOWN: Width, input power and efficiency of a transmission. Temperature and convection coefficient associated with air flow over the casing.

FIND: Surface temperature of casing.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Negligible radiation.

ANALYSIS: From Newton's law of cooling,

$$q = hA_s\left(T_s - T_\infty\right) = 6\,hW^2\left(T_s - T_\infty\right)$$

where the output power is $\eta\,P_i$ and the heat rate is

$$q = P_i - P_o = P_i\left(1-\eta\right) = 150\,\text{hp}\times 746\,\text{W/hp}\times 0.07 = 7833\,\text{W}$$

Hence,

$$T_s = T_\infty + \frac{q}{6\,hW^2} = 30^\circ\text{C} + \frac{7833\text{ W}}{6\times 200\text{ W/m}^2\cdot\text{K}\times\left(0.3\,\text{m}\right)^2} = 102.5^\circ\text{C}$$ <

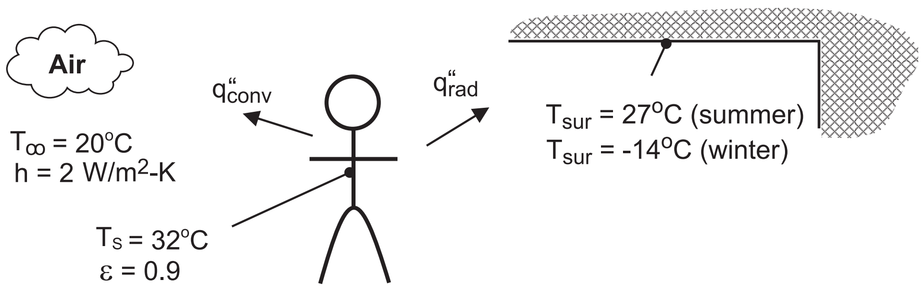
COMMENTS: There will, in fact, be considerable variability of the local convection coefficient over the transmission case and the prescribed value represents an average over the surface.

PROBLEM 1.24

KNOWN: Air and wall temperatures of a room. Surface temperature, convection coefficient and emissivity of a person in the room.

FIND: Basis for difference in comfort level between summer and winter.

SCHEMATIC:



ASSUMPTIONS: (1) Person may be approximated as a small object in a large enclosure.

ANALYSIS: Thermal comfort is linked to heat loss from the human body, and a *chilled* feeling is associated with excessive heat loss. Because the temperature of the room air is fixed, the different summer and winter comfort levels can not be attributed to convection heat transfer from the body. In both cases, the heat flux is

Summer and Winter: $q''_{conv} = h(T_s - T_\infty) = 2\ W/m^2 \cdot K \times 12\ °C = 24\ W/m^2$

However, the heat flux due to radiation will differ, with values of

Summer: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left(305^4 - 300^4\right) K^4 = 28.3\ W/m^2$

Winter: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 \left(305^4 - 287^4\right) K^4 = 95.4\ W/m^2$

There is a significant difference between winter and summer radiation fluxes, and the chilled condition is attributable to the effect of the colder walls on radiation.

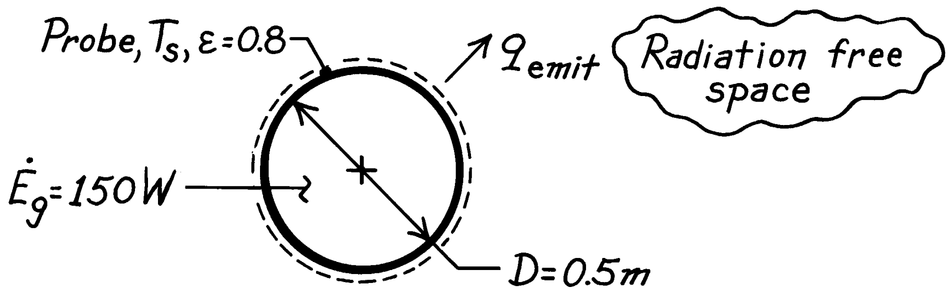
COMMENTS: For a representative surface area of A = 1.5 m^2, the heat losses are q_{conv} = 36 W, $q_{rad(summer)}$ = 42.5 W and $q_{rad(winter)}$ = 143.1 W. The winter time radiation loss is significant and if maintained over a 24 h period would amount to 2,950 kcal.

PROBLEM 1.25

KNOWN: Diameter and emissivity of spherical interplanetary probe. Power dissipation within probe.

FIND: Probe surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation incident on the probe.

ANALYSIS: Conservation of energy dictates a balance between energy generation within the probe and radiation emission from the probe surface. Hence, at any instant

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$\varepsilon A_s \sigma T_s^4 = \dot{E}_g$$

$$T_s = \left(\frac{\dot{E}_g}{\varepsilon \pi D^2 \sigma} \right)^{1/4}$$

$$T_s = \left(\frac{150\text{W}}{0.8\pi (0.5\text{ m})^2\, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4} \right)^{1/4}$$

$$T_s = 254.7\text{ K.}$$ <

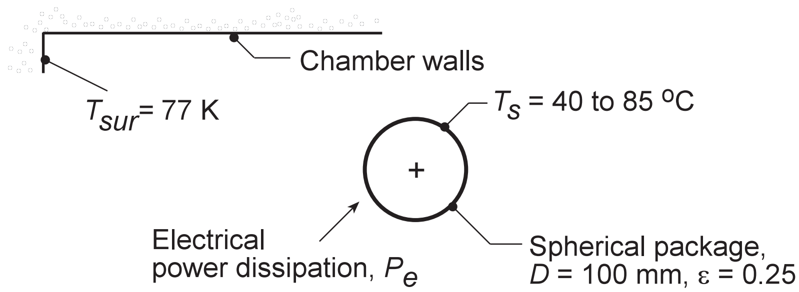
COMMENTS: Incident radiation, as, for example, from the sun, would increase the surface temperature.

PROBLEM 1.26

KNOWN: Spherical shaped instrumentation package with prescribed surface emissivity within a large space-simulation chamber having walls at 77 K.

FIND: Acceptable power dissipation for operating the package surface temperature in the range T_s = 40 to 85°C. Show graphically the effect of emissivity variations for 0.2 and 0.3.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform surface temperature, (2) Chamber walls are large compared to the spherical package, and (3) Steady-state conditions.

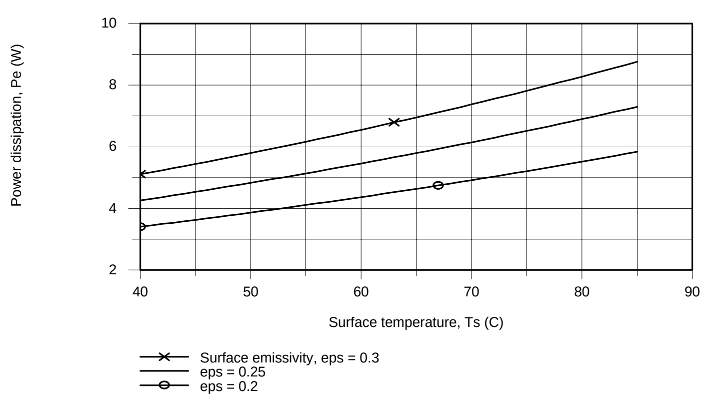
ANALYSIS: From an overall energy balance on the package, the internal power dissipation P_e will be transferred by radiation exchange between the package and the chamber walls. From Eq. 1.7,

$$q_{rad} = P_e = \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4 \right)$$

For the condition when T_s = 40°C, with $A_s = \pi D^2$ the power dissipation will be

$$P_e = 0.25 (\pi \times 0.10\,\text{m}) \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \times \left[(40 + 273)^4 - 77^4 \right] \text{K}^4 = 4.3\,\text{W} \qquad <$$

Repeating this calculation for the range 40 ≤ T_s ≤ 85°C, we can obtain the power dissipation as a function of surface temperature for the ε = 0.25 condition. Similarly, with 0.2 or 0.3, the family of curves shown below has been obtained.



COMMENTS: (1) As expected, the internal power dissipation increases with increasing emissivity and surface temperature. Because the radiation rate equation is non-linear with respect to temperature, the power dissipation will likewise not be linear with surface temperature.

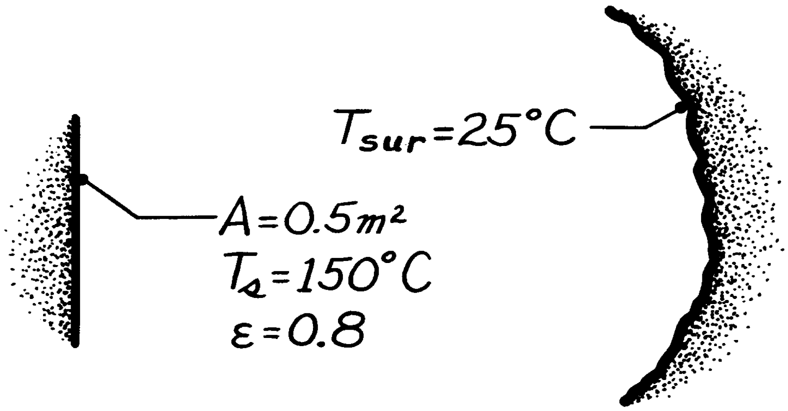
(2) What is the maximum power dissipation that is possible if the surface temperature is not to exceed 85°C? What kind of a coating should be applied to the instrument package in order to approach this limiting condition?

PROBLEM 1.27

KNOWN: Area, emissivity and temperature of a surface placed in a large, evacuated chamber of prescribed temperature.

FIND: (a) Rate of surface radiation emission, (b) Net rate of radiation exchange between surface and chamber walls.

SCHEMATIC:



ASSUMPTIONS: (1) Area of the enclosed surface is much less than that of chamber walls.

ANALYSIS: (a) From Eq. 1.5, the rate at which radiation is emitted by the surface is

$$q_{emit} = E \cdot A = \varepsilon A \sigma T_s^4$$

$$q_{emit} = 0.8\left(0.5\ m^2\right)\ 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ \left[(150 + 273)K\right]^4$$

$$q_{emit} = 726\ W. \quad <$$

(b) From Eq. 1.7, the *net* rate at which radiation is transferred *from* the surface to the chamber walls is

$$q = \varepsilon\ A\ \sigma\ \left(T_s^4 - T_{sur}^4\right)$$

$$q = 0.8\left(0.5\ m^2\right)\ 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ \left[(423K)^4\ -\ (298K)^4\right]$$

$$q = 547\ W. \quad <$$

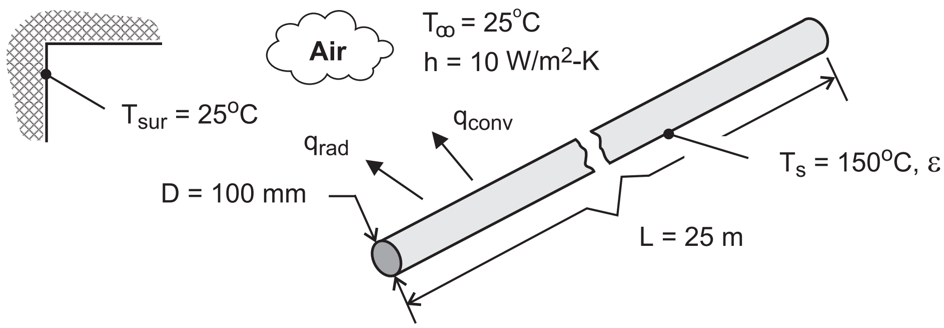
COMMENTS: The foregoing result gives the net heat loss from the surface which occurs at the instant the surface is placed in the chamber. The surface would, of course, cool due to this heat loss and its temperature, as well as the heat loss, would decrease with increasing time. Steady-state conditions would eventually be achieved when the temperature of the surface reached that of the surroundings.

PROBLEM 1.28

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1\text{m} \times 25\text{m}\right) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2\cdot\text{K}\left(150-25\right)\text{K} + 0.8\times5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1,250+1,095\right)\text{w/m}^2 = \left(9813+8592\right)\text{W} = 18,405\ \text{W} \qquad <$$

(b) The annual energy loss is

$$E = qt = 18,405\ \text{W}\times3600\ \text{s/h}\times24\text{h/d}\times365\ \text{d/y} = 5.80\times10^{11}\ \text{J}$$

With a furnace energy consumption of $E_f = E/\eta_f = 6.45\times10^{11}$ J, the annual cost of the loss is

$$C = C_g E_f = 0.01\ \$/\text{MJ}\times6.45\times10^5\text{MJ} = \$6450 \qquad <$$

COMMENTS: The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.

PROBLEM 1.29

KNOWN: Exact and approximate expressions for the linearized radiation coefficient, h_r and h_{ra}, respectively.

FIND: (a) Comparison of the coefficients with ε = 0.05 and 0.9 and surface temperatures which may exceed that of the surroundings (T_{sur} = 25°C) by 10 to 100°C; also comparison with a free convection coefficient correlation, (b) Plot of the relative error (h_r - r_{ra})/h_r as a function of the furnace temperature associated with a workpiece at T_s = 25°C having ε = 0.05, 0.2 or 0.9.

ASSUMPTIONS: (1) Furnace walls are large compared to the workpiece and (2) Steady-state conditions.

ANALYSIS: (a) The linearized radiation coefficient, Eq. 1.9, follows from the radiation exchange rate equation,

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$

If $T_s \approx T_{sur}$, the coefficient may be approximated by the simpler expression

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3 \qquad \bar{T} = \left(T_s + T_{sur}\right)/2$$

For the condition of ε = 0.05, T_s = T_{sur} + 10 = 35°C = 308 K and T_{sur} = 25°C = 298 K, find that

$$h_r = 0.05\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(308+298)\left(308^2+298^2\right)\mathrm{K}^3 = 0.32\,\mathrm{W/m^2\cdot K} \quad <$$

$$h_{r,a} = 4\times 0.05\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\left((308+298)/2\right)^3\mathrm{K}^3 = 0.32\,\mathrm{W/m^2\cdot K} \quad <$$

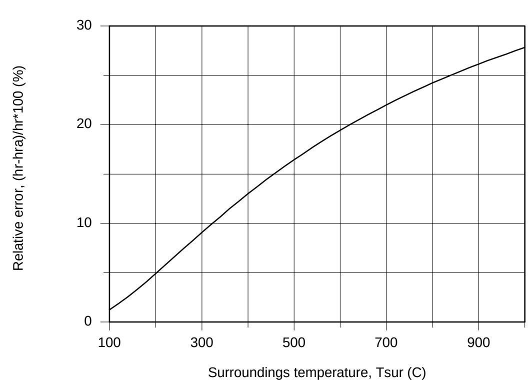
The free convection coefficient with T_s = 35°C and T_∞ = T_{sur} = 25°C, find that

$$h = 0.98\Delta T^{1/3} = 0.98\left(T_s - T_\infty\right)^{1/3} = 0.98(308-298)^{1/3} = 2.1\,\mathrm{W/m^2\cdot K} \quad <$$

For the range T_s - T_{sur} = 10 to 100°C with ε = 0.05 and 0.9, the results for the coefficients are tabulated below. For this range of surface and surroundings temperatures, the radiation and free convection coefficients are of comparable magnitude for moderate values of the emissivity, say ε > 0.2. The approximate expression for the linearized radiation coefficient is valid within 2% for these conditions.

(b) The above expressions for the radiation coefficients, h_r and $h_{r,a}$, are used for the workpiece at T_s = 25°C placed inside a furnace with walls which may vary from 100 to 1000°C. The relative error, (h_r - h_{ra})/h_r, will be independent of the surface emissivity and is plotted as a function of T_{sur}. For T_{sur} > 150°C, the approximate expression provides estimates which are in error more than 5%. The approximate expression should be used with caution, and only for surface and surrounding temperature differences of 50 to 100°C.

		Coefficients (W/m²·K)		
T_s (°C)	ε	h_r	$h_{r,a}$	h
35	0.05	0.32	0.32	2.1
	0.9	5.7	5.7	
135	0.05	0.51	0.50	4.7
	0.9	9.2	9.0	

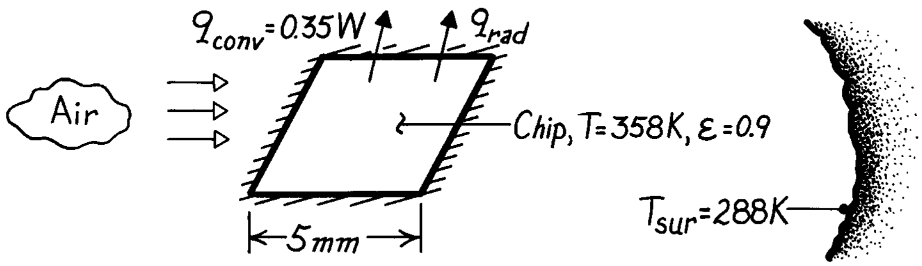


PROBLEM 1.30

KNOWN: Chip width, temperature, and heat loss by convection in air. Chip emissivity and temperature of large surroundings.

FIND: Increase in chip power due to radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between small surface and large enclosure.

ANALYSIS: Heat transfer from the chip due to net radiation exchange with the surroundings is

$$q_{rad} = \varepsilon W^2 \sigma \left(T^4 - T_{sur}^4 \right)$$

$$q_{rad} = 0.9(0.005\ \text{m})^2\ 5.67\times10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(358^4 - 288^4\right)\text{K}^4$$

$$q_{rad} = 0.0122\ \text{W}.$$

The percent increase in chip power is therefore

$$\frac{\Delta P}{P}\times100 = \frac{q_{rad}}{q_{conv}}\times100 = \frac{0.0122\ \text{W}}{0.350\ \text{W}}\times100 = 3.5\%.$$ <

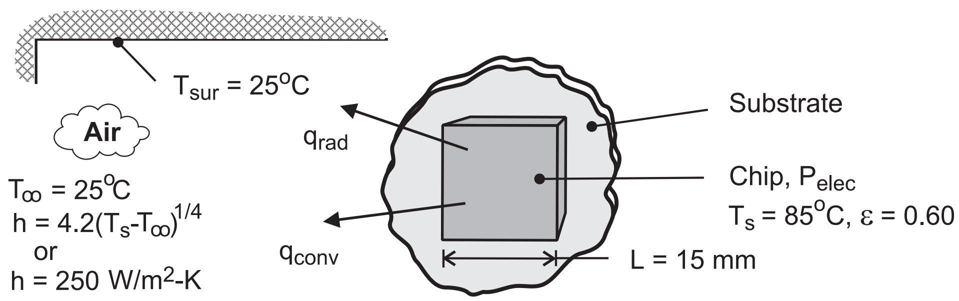
COMMENTS: For the prescribed conditions, radiation effects are small. Relative to convection, the effect of radiation would increase with increasing chip temperature and decreasing convection coefficient.

PROBLEM 1.31

KNOWN: Width, surface emissivity and maximum allowable temperature of an electronic chip. Temperature of air and surroundings. Convection coefficient.

FIND: (a) Maximum power dissipation for free convection with $h(W/m^2\cdot K) = 4.2(T - T_\infty)^{1/4}$, (b) Maximum power dissipation for forced convection with $h = 250\ W/m^2\cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between a small surface and a large enclosure, (3) Negligible heat transfer from sides of chip or from back of chip by conduction through the substrate.

ANALYSIS: Subject to the foregoing assumptions, electric power dissipation by the chip must be balanced by convection and radiation heat transfer from the chip. Hence, from Eq. (1.10),

$$P_{elec} = q_{conv} + q_{rad} = hA\left(T_s - T_\infty\right) + \varepsilon A\sigma\left(T_s^4 - T_{sur}^4\right)$$

where $A = L^2 = (0.015\text{m})^2 = 2.25\times10^{-4}\text{m}^2$.

(a) If heat transfer is by natural convection,

$$q_{conv} = C\ A\left(T_s - T_\infty\right)^{5/4} = 4.2\ W/m^2\cdot K^{5/4}\left(2.25\times10^{-4}m^2\right)(60K)^{5/4} = 0.158\ W$$

$$q_{rad} = 0.60\left(2.25\times10^{-4}m^2\right)5.67\times10^{-8}\ W/m^2\cdot K^4\left(358^4 - 298^4\right)K^4 = 0.065\ W$$

$$P_{elec} = 0.158\ W + 0.065\ W = 0.223\ W$$ <

(b) If heat transfer is by forced convection,

$$q_{conv} = hA\left(T_s - T_\infty\right) = 250\ W/m^2\cdot K\left(2.25\times10^{-4}m^2\right)(60K) = 3.375\ W$$

$$P_{elec} = 3.375\ W + 0.065\ W = 3.44\ W$$ <

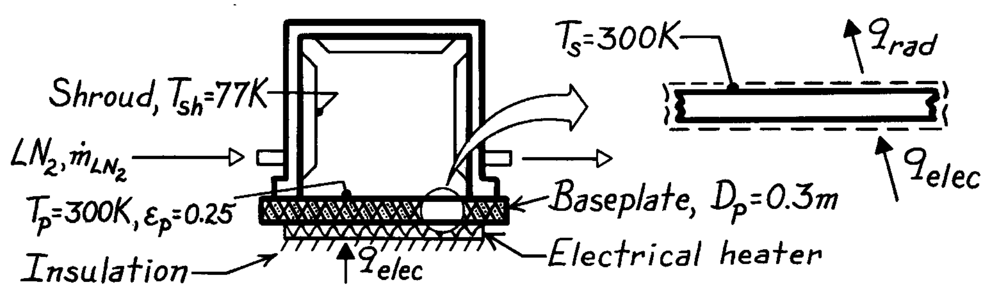
COMMENTS: Clearly, radiation and natural convection are inefficient mechanisms for transferring heat from the chip. For $T_s = 85°C$ and $T_\infty = 25°C$, the natural convection coefficient is $11.7\ W/m^2\cdot K$. Even for forced convection with $h = 250\ W/m^2\cdot K$, the power dissipation is well below that associated with many of today's processors. To provide acceptable cooling, it is often necessary to attach the chip to a highly conducting substrate and to thereby provide an additional heat transfer mechanism due to conduction from the back surface.

PROBLEM 1.32

KNOWN: Vacuum enclosure maintained at 77 K by liquid nitrogen shroud while baseplate is maintained at 300 K by an electrical heater.

FIND: (a) Electrical power required to maintain baseplate, (b) Liquid nitrogen consumption rate, (c) Effect on consumption rate if aluminum foil ($\varepsilon_p = 0.09$) is bonded to baseplate surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses from backside of heater or sides of plate, (3) Vacuum enclosure large compared to baseplate, (4) Enclosure is evacuated with negligible convection, (5) Liquid nitrogen (LN2) is heated only by heat transfer to the shroud, and (6) Foil is intimately bonded to baseplate.

PROPERTIES: Heat of vaporization of liquid nitrogen (given): 125 kJ/kg.

ANALYSIS: (a) From an energy balance on the baseplate,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{elec} - q_{rad} = 0$$

and using Eq. 1.7 for radiative exchange between the baseplate and shroud,

$$q_{elec} = \varepsilon_p A_p \sigma \left(T_p^4 - T_{sh}^4\right).$$

Substituting numerical values, with $A_p = \left(\pi D_p^2 / 4\right)$, find

$$q_{elec} = 0.25\left(\pi (0.3\text{ m})^2 / 4\right) 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(300^4 - 77^4\right)\text{K}^4 = 8.1\text{ W}. \quad <$$

(b) From an energy balance on the enclosure, radiative transfer heats the liquid nitrogen stream causing evaporation,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{rad} - \dot{m}_{LN2} h_{fg} = 0$$

where $\dot{m}_{LN2}$ is the liquid nitrogen consumption rate. Hence,

$$\dot{m}_{LN2} = q_{rad} / h_{fg} = 8.1\text{ W} / 125\text{ kJ/kg} = 6.48\times10^{-5}\text{ kg/s} = 0.23\text{ kg/h}. \quad <$$

(c) If aluminum foil ($\varepsilon_p = 0.09$) were bonded to the upper surface of the baseplate,

$$q_{rad,foil} = q_{rad}\left(\varepsilon_f / \varepsilon_p\right) = 8.1\text{ W}\left(0.09/0.25\right) = 2.9\text{ W}$$

and the liquid nitrogen consumption rate would be reduced by

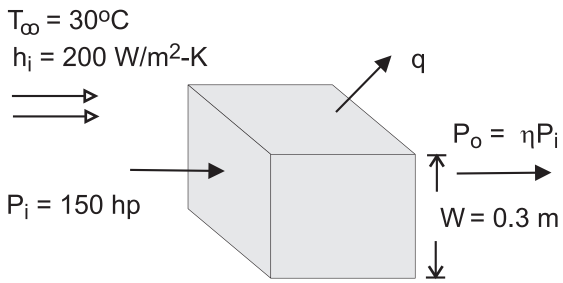
$$(0.25 - 0.09)/0.25 = 64\% \text{ to } 0.083\text{ kg/h}. \quad <$$

PROBLEM 1.33

KNOWN: Width, input power and efficiency of a transmission. Temperature and convection coefficient for air flow over the casing. Emissivity of casing and temperature of surroundings.

FIND: Surface temperature of casing.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Radiation exchange with large surroundings.

ANALYSIS: Heat transfer from the case must balance heat dissipation in the transmission, which may be expressed as $q = P_i - P_o = P_i (1 - \eta) = 150\text{ hp} \times 746\text{ W/hp} \times 0.07 = 7833\text{ W}$. Heat transfer from the case is by convection and radiation, in which case

$$q = A_s\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A_s = 6\ W^2$. Hence,

$$7833\,\text{W} = 6\left(0.30\,\text{m}\right)^2\left[200\,\text{W}/\text{m}^2\cdot\text{K}\left(T_s - 303\text{K}\right) + 0.8\times5.67\times10^{-8}\,\text{W}/\text{m}^2\cdot\text{K}^4\left(T_s^4 - 303^4\right)\text{K}^4\right]$$

A trial-and-error solution yields

$$T_s \approx 373\,\text{K} = 100°\text{C}$$ <

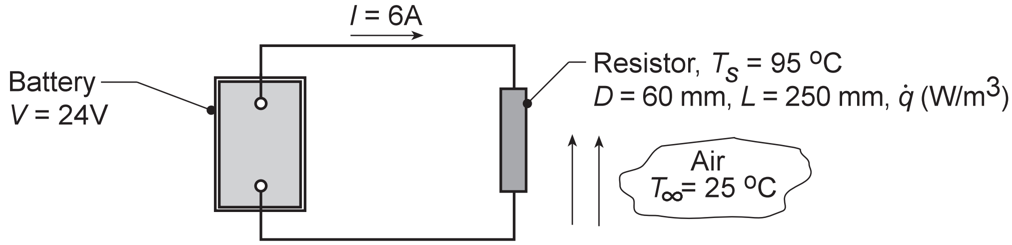
COMMENTS: (1) For $T_s \approx 373$ K, $q_{conv} \approx 7{,}560$ W and $q_{rad} \approx 270$ W, in which case heat transfer is dominated by convection, (2) If radiation is neglected, the corresponding surface temperature is T_s = 102.5°C.

PROBLEM 1.34

KNOWN: Resistor connected to a battery operating at a prescribed temperature in air.

FIND: (a) Considering the resistor as the system, determine corresponding values for $\dot{E}_{in}(W)$, $\dot{E}_g(W)$, $\dot{E}_{out}(W)$ and $\dot{E}_{st}(W)$. If a control surface is placed about the entire system, determine the values for $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$. (b) Determine the volumetric heat generation rate within the resistor, $\dot{q}$ (W/m^3), (c) Neglecting radiation from the resistor, determine the convection coefficient.

SCHEMATIC:



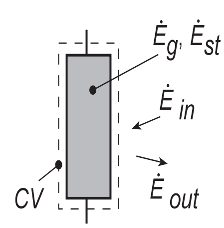
ASSUMPTIONS: (1) Electrical power is dissipated uniformly within the resistor, (2) Temperature of the resistor is uniform, (3) Negligible electrical power dissipated in the lead wires, (4) Negligible radiation exchange between the resistor and the surroundings, (5) No heat transfer occurs from the battery, (5) Steady-state conditions.

ANALYSIS: (a) Referring to Section 1.3.1, the conservation of energy requirement for a control volume at an instant of time, Eq 1.11a, is

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

where $\dot{E}_{in}$, $\dot{E}_{out}$ correspond to *surface* inflow and outflow processes, respectively. The energy generation term $\dot{E}_g$ is associated with conversion of some other energy form (chemical, electrical, electromagnetic or nuclear) to thermal energy. The energy storage term $\dot{E}_{st}$ is associated with changes in the internal, kinetic and/or potential energies of the matter in the control volume. $\dot{E}_g$, $\dot{E}_{st}$ are *volumetric* phenomena. The electrical power delivered by the battery is P = VI = 24V×6A = 144 W.

Control volume: Resistor.



$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,W$$

<

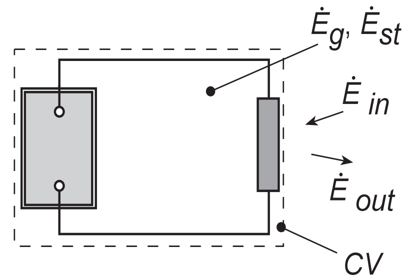
$$\dot{E}_g = 144\,W \qquad \dot{E}_{st} = 0$$

The $\dot{E}_g$ term is due to conversion of electrical energy to thermal energy. The term $\dot{E}_{out}$ is due to convection from the resistor surface to the air.

Continued...

PROBLEM 1.34 (Cont.)

Control volume: Battery-Resistor System.



$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,\mathrm{W} \qquad <$$
$$\dot{E}_g = 0 \qquad \dot{E}_{st} = -144\,\mathrm{W}$$

The $\dot{E}_{st}$ term represents the decrease in the chemical energy within the battery. The conversion of chemical energy to electrical energy and its subsequent conversion to thermal energy are processes internal to the system which are not associated with $\dot{E}_{st}$ or $\dot{E}_g$. The $\dot{E}_{out}$ term is due to convection from the resistor surface to the air.

(b) From the energy balance on the resistor with volume, $\forall = (\pi D^2/4)L$,

$$\dot{E}_g = \dot{q}\forall \qquad 144\,\mathrm{W} = \dot{q}\left(\pi\,(0.06\,\mathrm{m})^2/4\right)\times 0.25\,\mathrm{m} \qquad \dot{q} = 2.04\times 10^5\,\mathrm{W/m^3} \qquad <$$

(c) From the energy balance on the resistor and Newton's law of cooling with $A_s = \pi DL + 2(\pi D^2/4)$,

$$\dot{E}_{out} = q_{cv} = hA_s\left(T_s - T_\infty\right)$$

$$144\,\mathrm{W} = h\left[\pi\times 0.06\,\mathrm{m}\times 0.25\,\mathrm{m} + 2\left(\pi\times 0.06^2\,\mathrm{m}^2/4\right)\right](95-25)^\circ\,\mathrm{C}$$

$$144\,\mathrm{W} = h\,[0.0471 + 0.0057]\,\mathrm{m}^2\,(95-25)^\circ\,\mathrm{C}$$

$$h = 39.0\,\mathrm{W/m^2{\cdot}K} \qquad <$$

COMMENTS: (1) In using the conservation of energy requirement, Eq. 1.11a, it is important to recognize that $\dot{E}_{in}$ and $\dot{E}_{out}$ will always represent *surface* processes and $\dot{E}_g$ and $\dot{E}_{st}$, *volumetric* processes. The generation term $\dot{E}_g$ is associated with a *conversion* process from some form of energy to *thermal energy*. The storage term $\dot{E}_{st}$ represents the rate of change of *internal energy*.

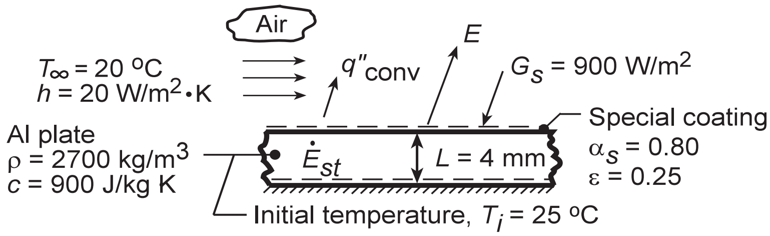
(2) From Table 1.1 and the magnitude of the convection coefficient determined from part (c), we conclude that the resistor is experiencing forced, rather than free, convection.

PROBLEM 1.35

KNOWN: Thickness and initial temperature of an aluminum plate whose thermal environment is changed.

FIND: (a) Initial rate of temperature change, (b) Steady-state temperature of plate, (c) Effect of emissivity and absorptivity on steady-state temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible end effects, (2) Uniform plate temperature at any instant, (3) Constant properties, (4) Adiabatic bottom surface, (5) Negligible radiation from surroundings, (6) No internal heat generation.

ANALYSIS: (a) Applying an energy balance, Eq. 1.11a, at an instant of time to a control volume about the plate, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$, it follows for a unit surface area.

$$\alpha_S G_S\left(1\,m^2\right) - E\left(1\,m^2\right) - q''_{conv}\left(1\,m^2\right) = (d/dt)(McT) = \rho\left(1\,m^2 \times L\right)c\,(dT/dt).$$

Rearranging and substituting from Eqs. 1.3 and 1.5, we obtain

$$dT/dt = (1/\rho Lc)\left[\alpha_S G_S - \varepsilon\sigma T_i^4 - h\left(T_i - T_\infty\right)\right].$$

$$dT/dt = \left(2700\,kg/m^3 \times 0.004\,m \times 900\,J/kg\cdot K\right)^{-1} \times$$

$$\left[0.8\times 900\,W/m^2 - 0.25\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(298\,K\right)^4 - 20\,W/m^2\cdot K\left(25-20\right)^\circ C\right]$$

$$dT/dt = 0.052^\circ\,C/s\,.$$ <

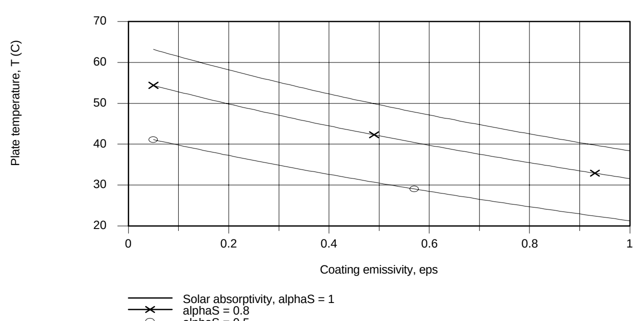
(b) Under steady-state conditions, $\dot{E}_{st}$ = 0, and the energy balance reduces to

$$\alpha_S G_S = \varepsilon\sigma T^4 + h\left(T - T_\infty\right) \tag{2}$$

$$0.8\times 900\,W/m^2 = 0.25\times 5.67\times 10^{-8}\,W/m^2\cdot K^4 \times T^4 + 20\,W/m^2\cdot K\left(T - 293\,K\right)$$

The solution yields T = 321.4 K = 48.4°C. <

(c) Using the IHT *First Law Model* for an *Isothermal Plane Wall*, parametric calculations yield the following results.



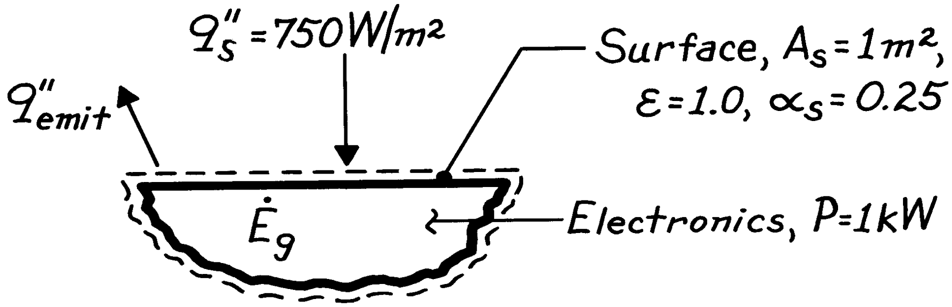
COMMENTS: The surface radiative properties have a significant effect on the plate temperature, which decreases with increasing ε and decreasing α_S. If a low temperature is desired, the plate coating should be characterized by a large value of ε/α_S. The temperature also decreases with increasing h.

PROBLEM 1.36

KNOWN: Surface area of electronic package and power dissipation by the electronics. Surface emissivity and absorptivity to solar radiation. Solar flux.

FIND: Surface temperature without and with incident solar radiation.

SCHEMATIC:



ASSUMPTIONS: Steady-state conditions.

ANALYSIS: Applying conservation of energy to a control surface about the compartment, at any instant

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0.$$

It follows that, with the solar input,

$$\alpha_S A_S q''_S - A_S E + P = 0$$
$$\alpha_S A_S q''_S - A_S \varepsilon \sigma T_S^4 + P = 0$$
$$T_S = \left(\frac{\alpha_S A_S q''_S + P}{A_S \varepsilon \sigma} \right)^{1/4}.$$

In the shade $(q''_S = 0)$,

$$T_S = \left(\frac{1000 \text{ W}}{1 \text{ m}^2 \times 1 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \right)^{1/4} = 364 \text{ K}. \quad <$$

In the sun,

$$T_S = \left(\frac{0.25 \times 1 \text{ m}^2 \times 750 \text{ W/m}^2 + 1000 \text{ W}}{1 \text{ m}^2 \times 1 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \right)^{1/4} = 380 \text{ K}. \quad <$$

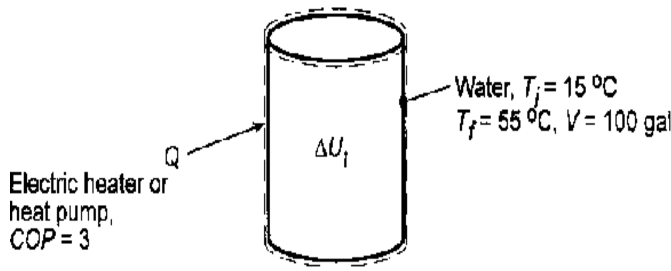
COMMENTS: In orbit, the space station would be continuously cycling between shade and sunshine, and a steady-state condition would not exist.

PROBLEM 1.37

KNOWN: Daily hot water consumption for a family of four and temperatures associated with ground water and water storage tank. Unit cost of electric power. Heat pump COP.

FIND: Annual heating requirement and costs associated with using electric resistance heating or a heat pump.

SCHEMATIC:



ASSUMPTIONS: (1) Process may be modelled as one involving heat addition in a closed system, (2) Properties of water are constant.

PROPERTIES: Table A-6, Water (T_{ave} = 308 K): $\rho = v_f^{-1}$ = 993 kg/m^3, $c_{p,f}$ = 4.178 kJ/kg·K.

ANALYSIS: From Eq. 1.11c, the daily heating requirement is $Q_{daily} = \Delta U_t = Mc\Delta T$ $= \rho Vc\left(T_f - T_i\right)$. With V = 100 gal/264.17 gal/m^3 = 0.379 m^3,

$$Q_{daily} = 993\text{kg}/\text{m}^3\left(0.379\,\text{m}^3\right)4.178\text{kJ/kg}\cdot\text{K}\left(40^\circ\text{C}\right) = 62,900\,\text{kJ}$$

The annual heating requirement is then, $Q_{annual} = 365\,\text{days}\left(62,900\,\text{kJ/day}\right) = 2.30\times10^7\text{ kJ}$, or, with 1 kWh = 1 kJ/s (3600 s) = 3600 kJ,

$$Q_{annual} = 6380\,\text{kWh} \qquad <$$

With electric resistance heating, $Q_{annual} = Q_{elec}$ and the associated cost, C, is

$$C = 6380\,\text{kWh}\left(\$0.08/\text{kWh}\right) = \$510 \qquad <$$

If a heat pump is used, $Q_{annual} = COP\left(W_{elec}\right)$. Hence,

$$W_{elec} = Q_{annual}/\left(COP\right) = 6380\text{kWh}/\left(3\right) = 2130\,\text{kWh}$$

The corresponding cost is

$$C = 2130\,\text{kWh}\left(\$0.08/\text{kWh}\right) = \$170 \qquad <$$

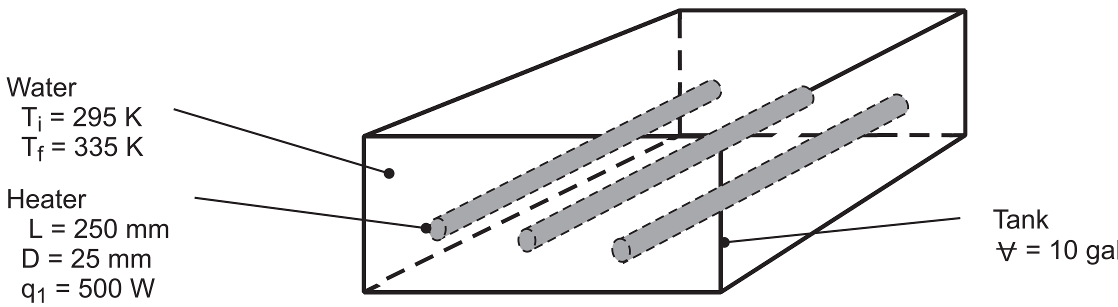
COMMENTS: Although annual operating costs are significantly lower for a heat pump, corresponding capital costs are much higher. The feasibility of this approach depends on other factors such as geography and seasonal variations in COP, as well as the time value of money.

PROBLEM 1.38

KNOWN: Initial temperature of water and tank volume. Power dissipation, emissivity, length and diameter of submerged heaters. Expressions for convection coefficient associated with natural convection in water and air.

FIND: (a) Time to raise temperature of water to prescribed value, (b) Heater temperature shortly after activation and at conclusion of process, (c) Heater temperature if activated in air.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss from tank to surroundings, (2) Water is *well-mixed* (at a uniform, but time varying temperature) during heating, (3) Negligible changes in thermal energy storage for heaters, (4) Constant properties, (5) Surroundings afforded by tank wall are large relative to heaters.

ANALYSIS: (a) Application of conservation of energy to a closed system (the water) at an instant, Eq. (1.11d), yields

$$\frac{dU}{dt} = Mc\frac{dT}{dt} = \rho \forall c\frac{dT}{dt} = q = 3q_1$$

Hence,
$$\int_0^t dt = \left(\rho \forall c/3q_1\right)\int_{T_i}^{T_f} dT$$

$$t = \frac{990\ \text{kg/m}^3 \times 10\text{gal}\left(3.79\times 10^{-3}\text{m}^3/\text{gal}\right)4180\text{J/kg}\cdot\text{K}}{3\times 500\ \text{W}}\left(335-295\right)\text{K} = 4180\ \text{s} \qquad <$$

(b) From Eq. (1.3a), the heat rate by convection from each heater is

$$q_1 = Aq_1'' = Ah\left(T_s - T\right) = \left(\pi DL\right)370\left(T_s - T\right)^{4/3}$$

Hence,

$$T_s = T + \left(\frac{q_1}{370\pi DL}\right)^{3/4} = T + \left(\frac{500\ \text{W}}{370\ \text{W/m}^2\cdot\text{K}^{4/3}\times\pi\times 0.025\ \text{m}\times 0.250\ \text{m}}\right)^{3/4} = \left(T+24\right)\text{K}$$

With water temperatures of $T_i \approx 295$ K and $T_f = 335$ K shortly after the start of heating and at the end of heating, respectively,

$T_{s,i} = 319$ K $\qquad$ $T_{s,f} = 359$ K $\qquad <$

Continued …..

PROBLEM 1.38 (Continued)

(c) From Eq. (1.10), the heat rate in air is

$$q_1 = \pi DL\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

Substituting the prescribed values of q_1, D, L, $T_\infty = T_{sur}$ and ε, an iterative solution yields

$$T_s = 830\ \mathrm{K}$$ <

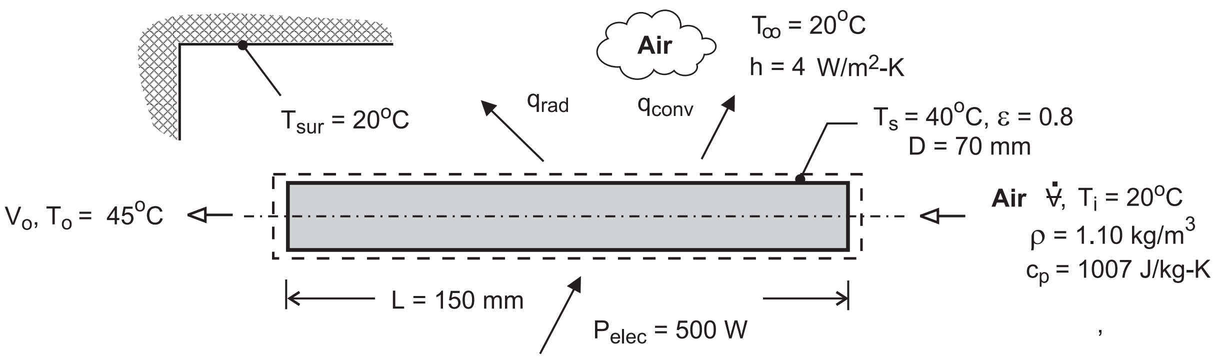
COMMENTS: In part (c) it is presumed that the heater can be operated at $T_s = 830$ K without experiencing burnout. The much larger value of T_s for air is due to the smaller convection coefficient. However, with q_{conv} and q_{rad} equal to 59 W and 441 W, respectively, a significant portion of the heat dissipation is effected by radiation.

PROBLEM 1.39

KNOWN: Power consumption, diameter, and inlet and discharge temperatures of a hair dryer.

FIND: (a) Volumetric flow rate and discharge velocity of heated air, (b) Heat loss from case.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible work done by fan, (5) Negligible heat transfer from casing of dryer to ambient air (Part (a)), (6) Radiation exchange between a small surface and a large enclosure (Part (b)).

ANALYSIS: (a) For a control surface about the air flow passage through the dryer, conservation of energy for an open system reduces to

$$\dot{m}(u+pv)_i - \dot{m}(u+pv)_o + q = 0$$

where u + pv = i and q = P_{elec}. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = P_{elec}$$

$$\dot{m} = \frac{P_{elec}}{c_p(T_o - T_i)} = \frac{500\text{ W}}{1007\text{ J/kg}\cdot\text{K}\left(25^\circ\text{C}\right)} = 0.0199\text{ kg/s}$$

$$\dot{\forall} = \frac{\dot{m}}{\rho} = \frac{0.0199\text{ kg/s}}{1.10\text{ kg/m}^3} = 0.0181\text{ m}^3/\text{s} \quad <$$

$$V_o = \frac{\dot{\forall}}{A_c} = \frac{4\dot{\forall}}{\pi D^2} = \frac{4\times 0.0181\text{ m}^3/\text{s}}{\pi(0.07\text{ m})^2} = 4.7\text{ m/s} \quad <$$

(b) Heat transfer from the casing is by convection and radiation, and from Eq. (1.10)

$$q = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right)$$

Continued …..

where $A_s = \pi DL = \pi(0.07 \text{ m} \times 0.15 \text{ m}) = 0.033 \text{ m}^2$. Hence,

$$q = 4\text{W/m}^2 \cdot \text{K}\left(0.033 \text{ m}^2\right)\left(20^\circ\text{C}\right) + 0.8 \times 0.033 \text{ m}^2 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4\left(313^4 - 293^4\right)\text{K}^4$$

$$q = 2.64 \text{ W} + 3.33 \text{ W} = 5.97 \text{ W}$$ <

The heat loss is much less than the electrical power, and the assumption of negligible heat loss is justified.

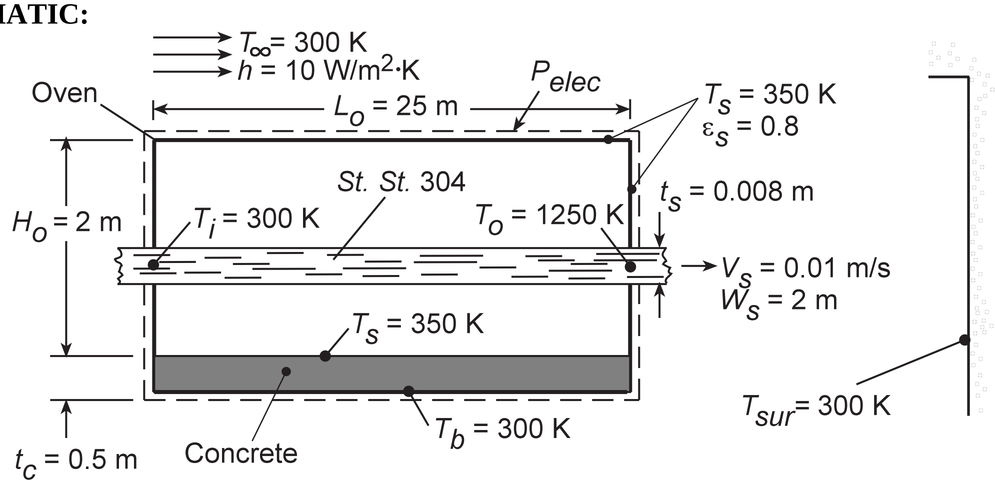
COMMENTS: Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the dryer, causing a reduction in the density. However, for the prescribed temperature rise, the change in ρ, and hence the effect on $\dot{\forall}$, is small.

PROBLEM 1.40

KNOWN: Speed, width, thickness and initial and final temperatures of 304 stainless steel in an annealing process. Dimensions of annealing oven and temperature, emissivity and convection coefficient of surfaces exposed to ambient air and large surroundings of equivalent temperatures. Thickness of pad on which oven rests and pad surface temperatures.

FIND: Oven operating power.

SCHEMATIC:



ASSUMPTIONS: (1) steady-state, (2) Constant properties, (3) Negligible changes in kinetic and potential energy.

PROPERTIES: Table A.1, St.St.304 $\left(\overline{T} = (T_i + T_o)/2 = 775\,K\right)$: ρ = 7900 kg/m³, c_p = 578 J/kg·K; Table A.3, Concrete, T = 300 K: k_c = 1.4 W/m·K.

ANALYSIS: The rate of energy addition to the oven must balance the rate of energy transfer to the steel sheet and the rate of heat loss from the oven. With $\dot{E}_{in} - \dot{E}_{out} -$ = 0, it follows that

$$P_{elec} + \dot{m}(u_i - u_o) - q = 0$$

where heat is transferred from the oven. With $\dot{m} = \rho V_s (W_s t_s)$, $(u_i - u_o) = c_p (T_i - T_o)$, and

$$q = (2H_oL_o + 2H_oW_o + W_oL_o) \times \left[h(T_s - T_\infty) + \varepsilon_s \sigma \left(T_s^4 - T_{sur}^4\right)\right] + k_c (W_oL_o)(T_s - T_b)/t_c,$$

it follows that

$$P_{elec} = \rho V_s (W_s t_s) c_p (T_o - T_i) + (2H_oL_o + 2H_oW_o + W_oL_o) \times$$
$$\left[h(T_s - T_o) + \varepsilon_s \sigma \left(T_s^4 - T_{sur}^4\right)\right] + k_c (W_oL_o)(T_s - T_b)/t_c$$

$$P_{elec} = 7900\,\text{kg/m}^3 \times 0.01\,\text{m/s}(2\,\text{m} \times 0.008\,\text{m})578\,\text{J/kg}\cdot\text{K}(1250 - 300)\text{K}$$
$$+(2\times 2\text{m}\times 25\text{m} + 2\times 2\text{m}\times 2.4\text{m} + 2.4\text{m}\times 25\text{m})[10\,\text{W/m}^2\cdot\text{K}(350-300)\text{K}$$
$$+0.8\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(350^4 - 300^4\right)\text{K}^4] + 1.4\,\text{W/m}\cdot\text{K}(2.4\text{m}\times 25\text{m})(350-300)\text{K/0.5m}$$

Continued.….

$$P_{elec} = 694,000\,\mathrm{W} + 169.6\,\mathrm{m}^2\,(500 + 313)\,\mathrm{W/m}^2 + 8400\,\mathrm{W}$$

$$= (694,000 + 84,800 + 53,100 + 8400)\,\mathrm{W} = 840\,\mathrm{kW}$$ <

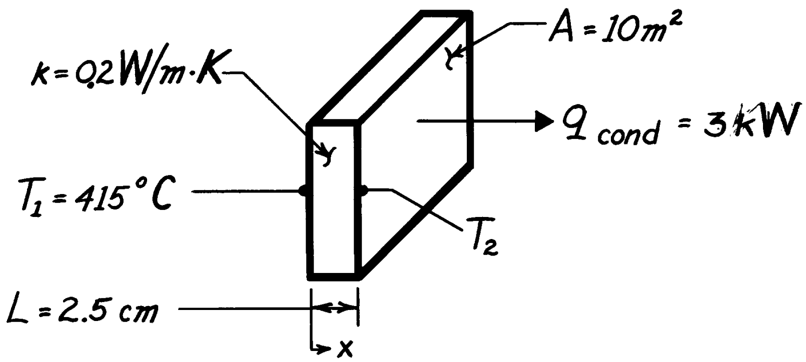
COMMENTS: Of the total energy input, 83% is transferred to the steel while approximately 10%, 6% and 1% are lost by convection, radiation and conduction from the oven. The convection and radiation losses can both be reduced by adding insulation to the side and top surfaces, which would reduce the corresponding value of T_s.

PROBLEM 1.1

KNOWN: Heat rate, q, through one-dimensional wall of area A, thickness L, thermal conductivity k and inner temperature, T_1.

FIND: The outer temperature of the wall, T_2.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The rate equation for conduction through the wall is given by Fourier's law,

$$q_{cond} = q_x = q_x'' \cdot A = -k\frac{dT}{dx} \cdot A = kA\frac{T_1 - T_2}{L}.$$

Solving for T_2 gives

$$T_2 = T_1 - \frac{q_{cond} L}{kA}.$$

Substituting numerical values, find

$$T_2 = 415^\circ C - \frac{3000W \times 0.025m}{0.2W/m \cdot K \times 10m^2}$$

$$T_2 = 415^\circ C - 37.5^\circ C$$

$$T_2 = 378^\circ C.$$ <

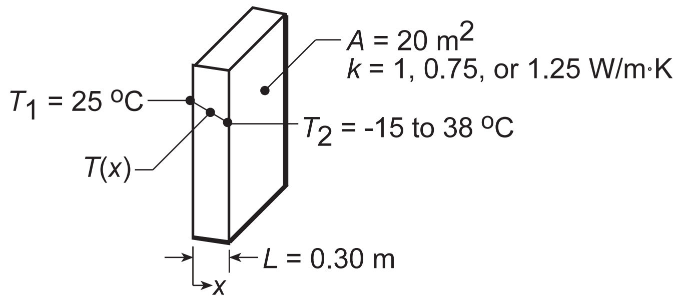
COMMENTS: Note direction of heat flow and fact that T_2 must be less than T_1.

PROBLEM 1.2

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of ambient air temperatures ranging from -15 to 38°C.

SCHEMATIC:



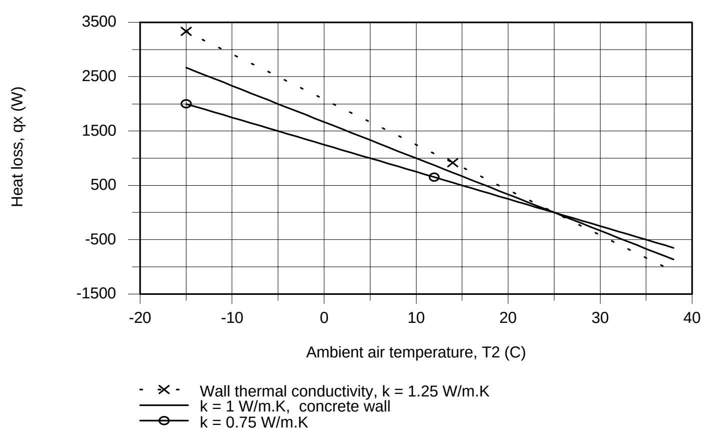
ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties, (4) Outside wall temperature is that of the ambient air.

ANALYSIS: From Fourier's law, it is evident that the gradient, $dT/dx = -q''_x/k$, is a constant, and hence the temperature distribution is linear, if q''_x and k are each constant. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is $T_2 = -15°C$ are

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 1\,W/m\cdot K\,\frac{25^\circ C - \left(-15^\circ C\right)}{0.30\,m} = 133.3\,W/m^2 . \qquad (1)$$

$$q_x = q''_x \times A = 133.3\,W/m^2 \times 20\,m^2 = 2667\,W . \qquad (2) <$$

Combining Eqs. (1) and (2), the heat rate q_x can be determined for the range of ambient temperature, $-15 \le T_2 \le 38°C$, with different wall thermal conductivities, k.



For the concrete wall, k = 1 W/m·K, the heat loss varies linearily from +2667 W to -867 W and is zero when the inside and ambient temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

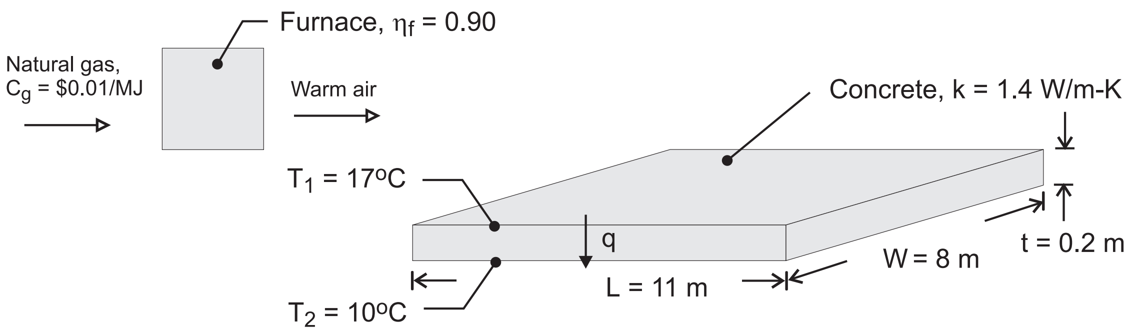
COMMENTS: Without steady-state conditions and constant k, the temperature distribution in a plane wall would not be linear.

PROBLEM 1.3

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,\text{W/m}\cdot\text{K}\left(11\,\text{m}\times 8\,\text{m}\right)\frac{7^\circ\text{C}}{0.20\,\text{m}} = 4312\ \text{W} \quad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,\text{W}\times \$0.01/\text{MJ}}{0.9\times 10^6\,\text{J/MJ}}\left(24\,\text{h/d}\times 3600\,\text{s/h}\right) = \$4.14/\text{d} \quad <$$

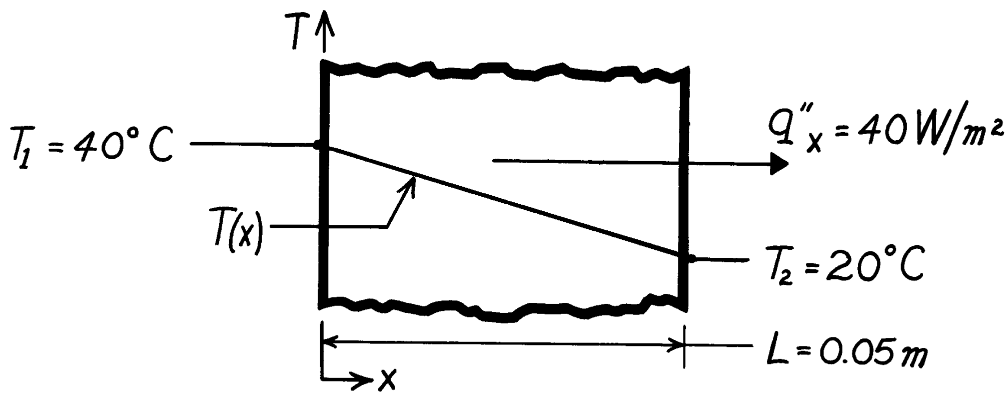
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.4

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^\circ C}$$

$$k = 0.10\ W/m\cdot K.$$ <

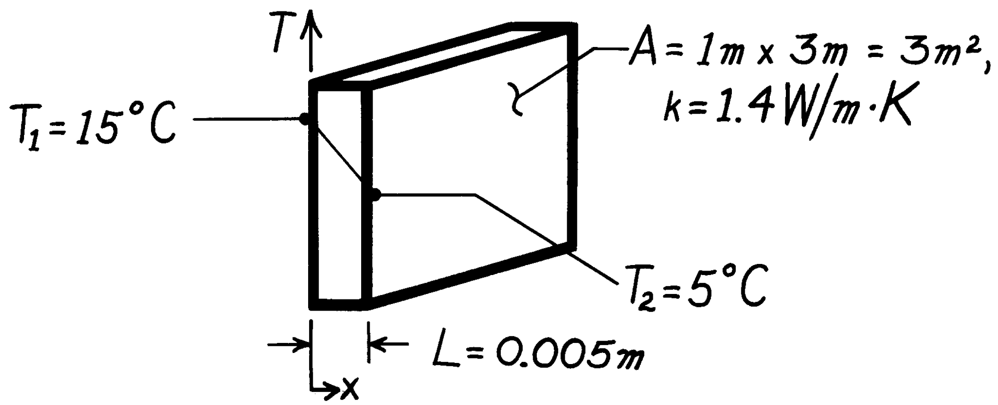
COMMENTS: Note that the °C or K temperature units may be used interchangeably when evaluating a temperature difference.

PROBLEM 1.5

KNOWN: Inner and outer surface temperatures of a glass window of prescribed dimensions.

FIND: Heat loss through window.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing conditions the heat flux may be computed from Fourier's law, Eq. 1.2.

$$q''_x = k \frac{T_1 - T_2}{L}$$

$$q''_x = 1.4 \frac{W}{m \cdot K} \frac{(15\text{-}5)^\circ C}{0.005m}$$

$$q''_x = 2800 \text{ W/m}^2.$$

Since the heat flux is uniform over the surface, the heat loss (rate) is

$$q = q''_x \times A$$

$$q = 2800 \text{ W/m}^2 \times 3m^2$$

$$q = 8400 \text{ W}.$$ <

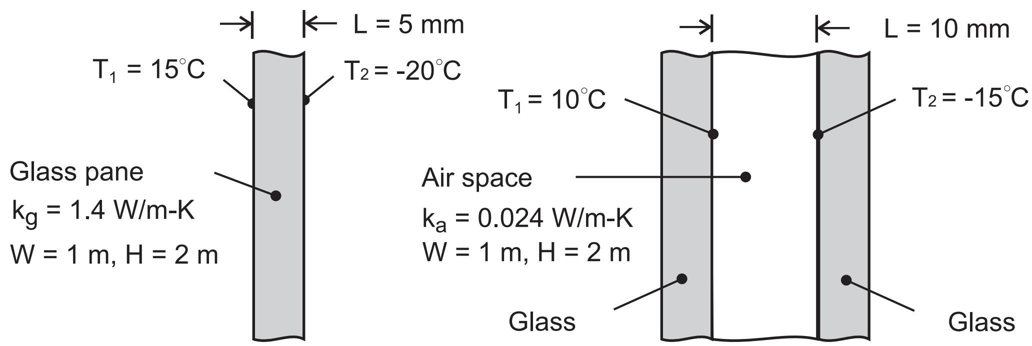
COMMENTS: A linear temperature distribution exists in the glass for the prescribed conditions.

PROBLEM 1.6

KNOWN: Width, height, thickness and thermal conductivity of a single pane window and the air space of a double pane window. Representative winter surface temperatures of single pane and air space.

FIND: Heat loss through single and double pane windows.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through glass or air, (2) Steady-state conditions, (3) Enclosed air of double pane window is stagnant (negligible buoyancy induced motion).

ANALYSIS: From Fourier's law, the heat losses are

Single Pane: $$q_g = k_g A \frac{T_1 - T_2}{L} = 1.4 \text{ W/m}\cdot\text{K}\left(2\text{m}^2\right)\frac{35\ ^\circ\text{C}}{0.005\text{m}} = 19,600 \text{ W}$$

Double Pane: $$q_a = k_a A \frac{T_1 - T_2}{L} = 0.024\left(2\text{m}^2\right)\frac{25\ ^\circ\text{C}}{0.010 \text{ m}} = 120 \text{ W}$$

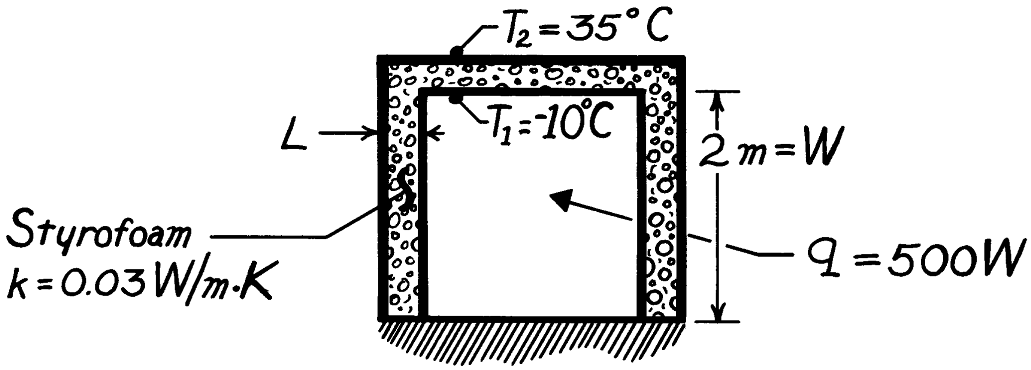
COMMENTS: Losses associated with a single pane are unacceptable and would remain excessive, even if the thickness of the glass were doubled to match that of the air space. The principal advantage of the double pane construction resides with the low thermal conductivity of air (~ 60 times smaller than that of glass). For a fixed ambient outside air temperature, use of the double pane construction would also increase the surface temperature of the glass exposed to the room (inside) air.

PROBLEM 1.7

KNOWN: Dimensions of freezer compartment. Inner and outer surface temperatures.

FIND: Thickness of styrofoam insulation needed to maintain heat load below prescribed value.

SCHEMATIC:



ASSUMPTIONS: (1) Perfectly insulated bottom, (2) One-dimensional conduction through 5 walls of area $A = 4m^2$, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: Using Fourier's law, Eq. 1.2, the heat rate is

$$q = q'' \cdot A = k \frac{\Delta T}{L} A_{total}$$

Solving for L and recognizing that $A_{total} = 5 \times W^2$, find

$$L = \frac{5 k \Delta T W^2}{q}$$

$$L = \frac{5 \times 0.03 \text{ W/m} \cdot \text{K} \left[35 - (-10)\right]^{\circ} \text{C} \left(4\text{m}^2\right)}{500 \text{ W}}$$

$$L = 0.054\text{m} = 54\text{mm}.$$ <

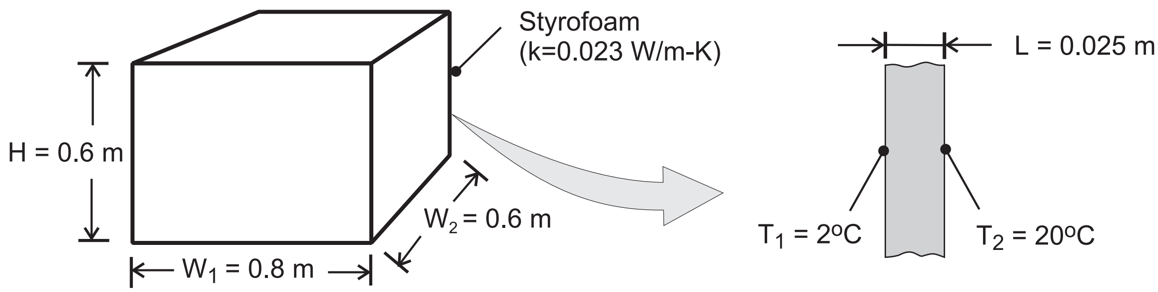
COMMENTS: The corners will cause local departures from one-dimensional conduction and a slightly larger heat loss.

PROBLEM 1.8

KNOWN: Dimensions and thermal conductivity of food/beverage container. Inner and outer surface temperatures.

FIND: Heat flux through container wall and total heat load.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through bottom wall, (3) Uniform surface temperatures and one-dimensional conduction through remaining walls.

ANALYSIS: From Fourier's law, Eq. 1.2, the heat flux is

$$q'' = k\frac{T_2 - T_1}{L} = \frac{0.023\ \text{W/m}\cdot\text{K}\ (20-2)^\circ\ \text{C}}{0.025\ \text{m}} = 16.6\ \text{W/m}^2 \qquad <$$

Since the flux is uniform over each of the five walls through which heat is transferred, the heat load is

$$q = q'' \times A_{total} = q''\left[H(2W_1 + 2W_2) + W_1 \times W_2\right]$$

$$q = 16.6\ \text{W/m}^2\left[0.6\text{m}(1.6\text{m} + 1.2\text{m}) + (0.8\text{m} \times 0.6\text{m})\right] = 35.9\ \text{W} \qquad <$$

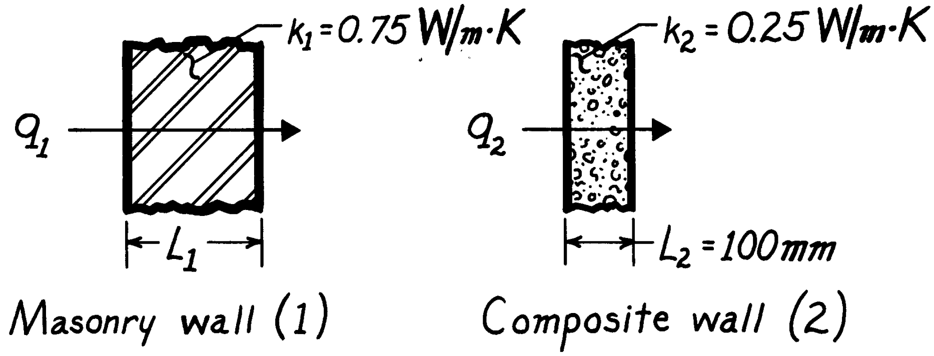
COMMENTS: The corners and edges of the container create local departures from one-dimensional conduction, which increase the heat load. However, for H, W_1, W_2 >> L, the effect is negligible.

PROBLEM 1.9

KNOWN: Masonry wall of known thermal conductivity has a heat rate which is 80% of that through a composite wall of prescribed thermal conductivity and thickness.

FIND: Thickness of masonry wall.

SCHEMATIC:



ASSUMPTIONS: (1) Both walls subjected to same surface temperatures, (2) One-dimensional conduction, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: For steady-state conditions, the conduction heat flux through a one-dimensional wall follows from Fourier's law, Eq. 1.2,

$$q'' = k \frac{\Delta T}{L}$$

where ΔT represents the difference in surface temperatures. Since ΔT is the same for both walls, it follows that

$$L_1 = L_2 \frac{k_1}{k_2} \cdot \frac{q_2''}{q_1''}.$$

With the heat fluxes related as

$$q_1'' = 0.8\, q_2''$$

$$L_1 = 100\text{mm} \frac{0.75\ \text{W/m}\cdot\text{K}}{0.25\ \text{W/m}\cdot\text{K}} \times \frac{1}{0.8} = 375\text{mm}. \qquad <$$

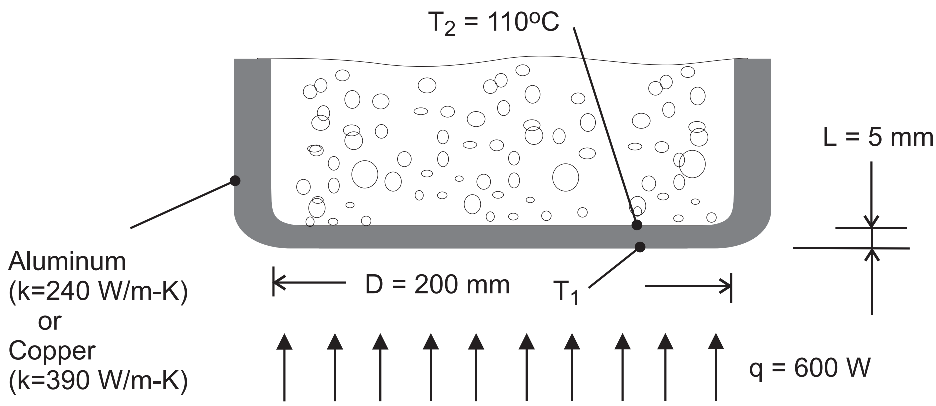
COMMENTS: Not knowing the temperature difference across the walls, we cannot find the value of the heat rate.

PROBLEM 1.10

KNOWN: Thickness, diameter and inner surface temperature of bottom of pan used to boil water. Rate of heat transfer to the pan.

FIND: Outer surface temperature of pan for an aluminum and a copper bottom.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through bottom of pan.

ANALYSIS: From Fourier's law, the rate of heat transfer by conduction through the bottom of the pan is

$$q = kA\frac{T_1 - T_2}{L}$$

Hence,

$$T_1 = T_2 + \frac{qL}{kA}$$

where $A = \pi D^2/4 = \pi(0.2\text{m})^2/4 = 0.0314\ \text{m}^2$.

Aluminum: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{240\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.40\ ^\circ\text{C}$$

Copper: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{390\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.25\ ^\circ\text{C}$$

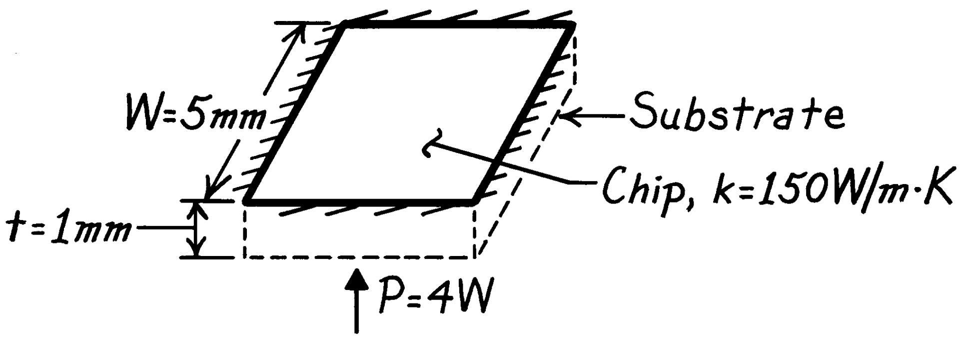
COMMENTS: Although the temperature drop across the bottom is slightly larger for aluminum (due to its smaller thermal conductivity), it is sufficiently small to be negligible for both materials. To a good approximation, the bottom may be considered *isothermal* at T ≈ 110 °C, which is a desirable feature of pots and pans.

PROBLEM 1.11

KNOWN: Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

FIND: Temperature drop across the chip.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Uniform heat dissipation, (4) Negligible heat loss from back and sides, (5) One-dimensional conduction in chip.

ANALYSIS: All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, from Fourier's law,

$$P = q = kA\,\frac{\Delta T}{t}$$

or

$$\Delta T = \frac{t \cdot P}{kW^2} = \frac{0.001\ \text{m} \times 4\ \text{W}}{150\ \text{W/m} \cdot \text{K}\left(0.005\ \text{m}\right)^2}$$

$$\Delta T = 1.1^\circ\ \text{C}.$$ <

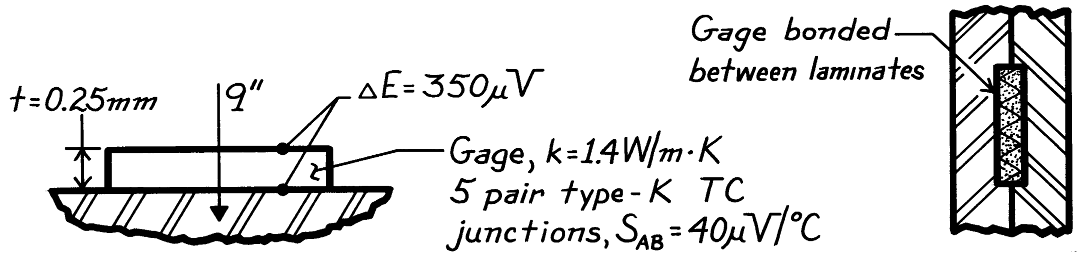
COMMENTS: For fixed P, the temperature drop across the chip decreases with increasing k and W, as well as with decreasing t.

PROBLEM 1.12

KNOWN: Heat flux gage with thin-film thermocouples on upper and lower surfaces; output voltage, calibration constant, thickness and thermal conductivity of gage.

FIND: (a) Heat flux, (b) Precaution when sandwiching gage between two materials.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat conduction in gage, (3) Constant properties.

ANALYSIS: (a) Fourier's law applied to the gage can be written as

$$q'' = k\frac{\Delta T}{\Delta x}$$

and the gradient can be expressed as

$$\frac{\Delta T}{\Delta x} = \frac{\Delta E / N}{S_{AB} t}$$

where N is the number of differentially connected thermocouple junctions, S_{AB} is the Seebeck coefficient for type K thermocouples (A-chromel and B-alumel), and $\Delta x = t$ is the gage thickness. Hence,

$$q'' = \frac{k\Delta E}{N S_{AB} t}$$

$$q'' = \frac{1.4\ \mathrm{W/m \cdot K} \times 350 \times 10^{-6}\ \mathrm{V}}{5 \times 40 \times 10^{-6}\ \mathrm{V/^\circ C} \times 0.25 \times 10^{-3}\ \mathrm{m}} = 9800\ \mathrm{W/m^2}. \qquad <$$

(b) The major precaution to be taken with this type of gage is to match its thermal conductivity with that of the material on which it is installed. If the gage is bonded between laminates (see sketch above) and its thermal conductivity is significantly different from that of the laminates, one dimensional heat flow will be disturbed and the gage will read incorrectly.

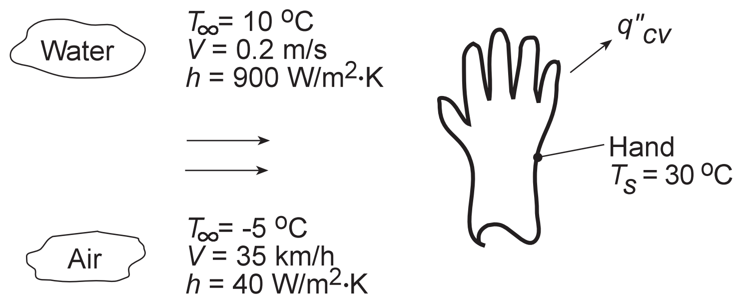
COMMENTS: If the thermal conductivity of the gage is lower than that of the laminates, will it indicate heat fluxes that are systematically high or low?

PROBLEM 1.13

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m^2 under normal room conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{air} = 40\,W/m^2\cdot K\left[30-(-5)\right]K = 1,400\,W/m^2 \qquad <$$

For the water stream:

$$q''_{water} = 900\,W/m^2\cdot K\left(30-10\right)K = 18,000\,W/m^2 \qquad <$$

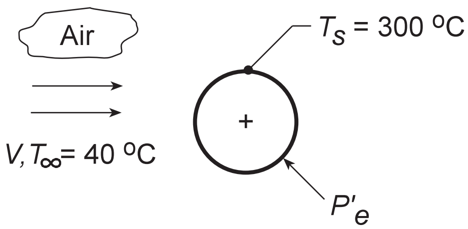
COMMENTS: The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m^2 which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

PROBLEM 1.14

KNOWN: Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

FIND: (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as $h = CV^n$, determine the parameters C and n.

SCHEMATIC:



V(m/s)	1	2	4	8	12
P'_e (W/m)	450	658	983	1507	1963
h (W/m^2·K)	22.0	32.2	48.1	73.8	96.1

ASSUMPTIONS: (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

ANALYSIS: (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newtons law of cooling on a per unit length basis,

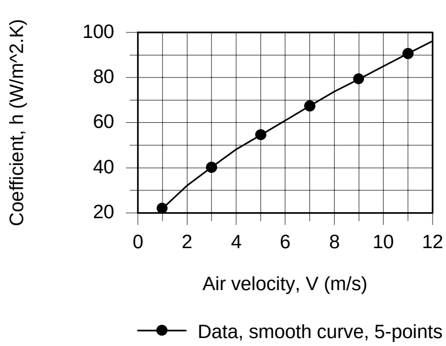
$$P'_e = h(\pi D)(T_s - T_\infty)$$

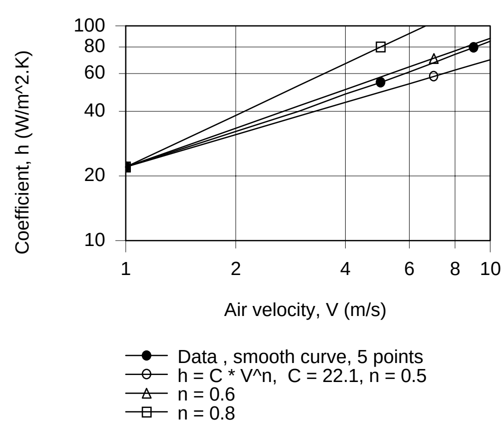
where P'_e is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table above, find

$$h = 450\,\text{W/m}\Big/\pi \times 0.025\,\text{m}(300-40)^\circ\text{C} = 22.0\,\text{W}\Big/\text{m}^2\cdot\text{K}$$ <

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing C = 22.12 W/m^2·K(s/m)^n, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice. Hence, C = 22.12 and n = 0.6. <



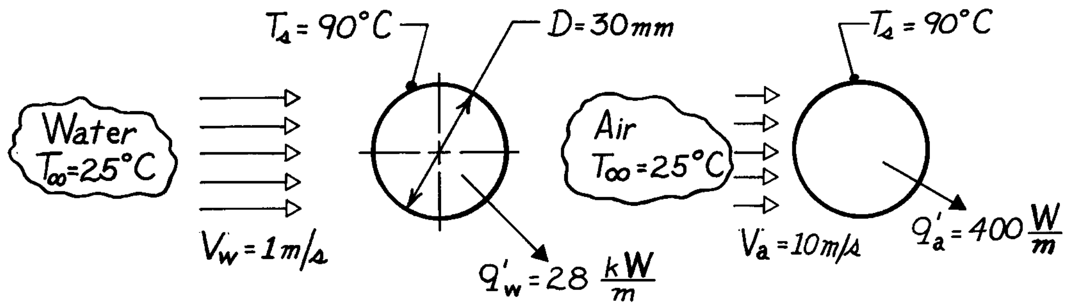


PROBLEM 1.15

KNOWN: Long, 30mm-diameter cylinder with embedded electrical heater; power required to maintain a specified surface temperature for water and air flows.

FIND: Convection coefficients for the water and air flow convection processes, h_w and h_a, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Flow is cross-wise over cylinder which is very long in the direction normal to flow.

ANALYSIS: The convection heat rate from the cylinder per unit length of the cylinder has the form

$$q' = h(\pi D)\,(T_s - T_\infty)$$

and solving for the heat transfer convection coefficient, find

$$h = \frac{q'}{\pi D\,(T_s - T_\infty)}.$$

Substituting numerical values for the water and air situations:

Water $$h_w = \frac{28 \times 10^3 \text{ W/m}}{\pi \times 0.030\text{m }(90\text{-}25)^\circ \text{ C}} = 4{,}570 \text{ W/m}^2\cdot\text{K}$$ <

Air $$h_a = \frac{400 \text{ W/m}}{\pi \times 0.030\text{m }(90\text{-}25)^\circ \text{ C}} = 65 \text{ W/m}^2\cdot\text{K}.$$ <

COMMENTS: Note that the air velocity is 10 times that of the water flow, yet

$$h_w \approx 70 \times h_a.$$

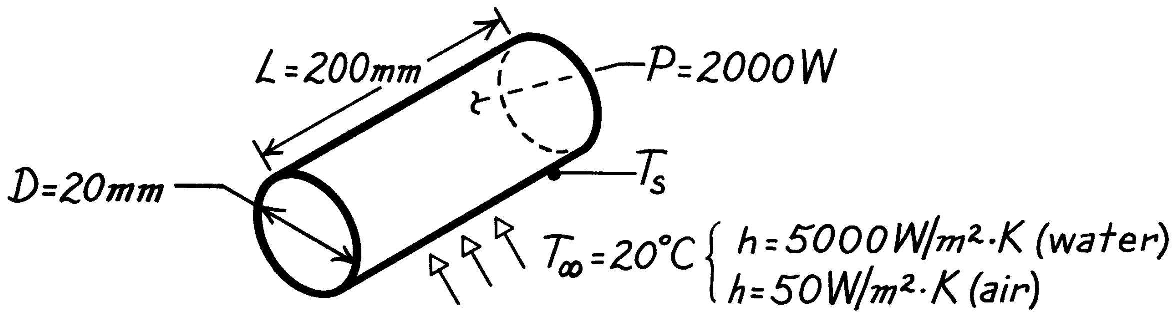
These values for the convection coefficient are typical for forced convection heat transfer with liquids and gases. See Table 1.1.

PROBLEM 1.16

KNOWN: Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

FIND: Heater surface temperatures in water and air.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

ANALYSIS: With P = q_{conv}, Newton's law of cooling yields

$$P = hA\left(T_S - T_\infty\right) = h\pi DL\left(T_S - T_\infty\right)$$

$$T_S = T_\infty + \frac{P}{h\pi DL}.$$

In water,

$$T_s = 20^\circ C + \frac{2000\ W}{5000\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 31.8^\circ C = 51.8^\circ C. \qquad <$$

In air,

$$T_s = 20^\circ C + \frac{2000\ W}{50\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 3183^\circ C = 3203^\circ C. \qquad <$$

COMMENTS: (1) Air is much less effective than water as a heat transfer fluid. Hence, the cartridge temperature is much higher in air, so high, in fact, that the cartridge would melt.

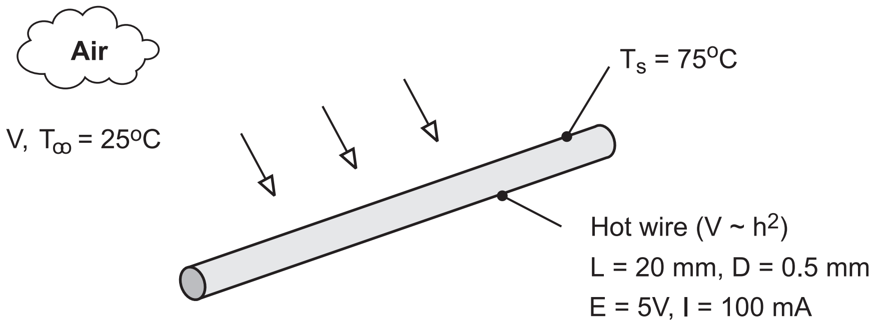
(2) In air, the high cartridge temperature would render radiation significant.

PROBLEM 1.17

KNOWN: Length, diameter and calibration of a hot wire anemometer. Temperature of air stream. Current, voltage drop and surface temperature of wire for a particular application.

FIND: Air velocity

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from the wire by natural convection or radiation.

ANALYSIS: If all of the electric energy is transferred by convection to the air, the following equality must be satisfied

$$P_{elec} = EI = hA\left(T_s - T_\infty\right)$$

where $A = \pi DL = \pi\left(0.0005\text{m} \times 0.02\text{m}\right) = 3.14 \times 10^{-5}\text{m}^2$.

Hence,

$$h = \frac{EI}{A\left(T_s - T_\infty\right)} = \frac{5\text{V} \times 0.1\text{A}}{3.14 \times 10^{-5}\text{m}^2\left(50\ ^\circ\text{C}\right)} = 318\ \text{W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5}h^2 = 6.25 \times 10^{-5}\left(318\ \text{W/m}^2 \cdot \text{K}\right)^2 = 6.3\ \text{m/s}$$ <

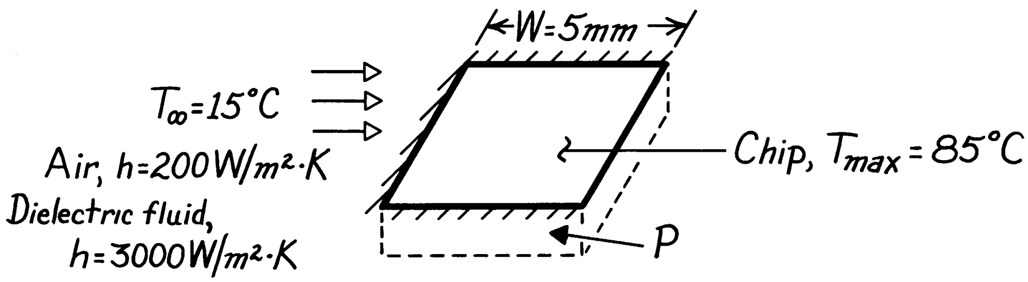
COMMENTS: The convection coefficient is sufficiently large to render buoyancy (natural convection) and radiation effects negligible.

PROBLEM 1.18

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

In *air*,

$$P_{max} = 200\ W/m^2 \cdot K(0.005\ m)^2(85 - 15)\ °C = 0.35\ W. \qquad <$$

In the *dielectric liquid*

$$P_{max} = 3000\ W/m^2 \cdot K(0.005\ m)^2(85\text{-}15)\ °C = 5.25\ W. \qquad <$$

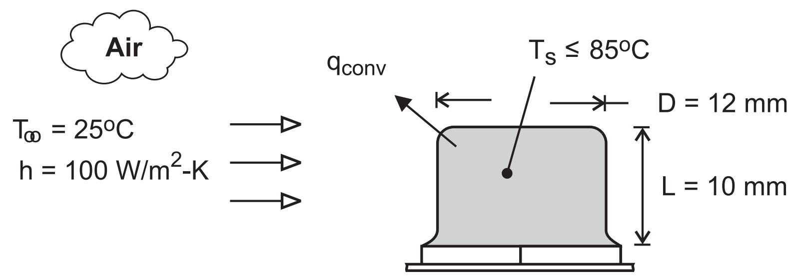
COMMENTS: Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

PROBLEM 1.19

KNOWN: Length, diameter and maximum allowable surface temperature of a power transistor. Temperature and convection coefficient for air cooling.

FIND: Maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer through base of transistor, (3) Negligible heat transfer by radiation from surface of transistor.

ANALYSIS: Subject to the foregoing assumptions, the power dissipated by the transistor is equivalent to the rate at which heat is transferred by convection to the air. Hence,

$$P_{elec} = q_{conv} = hA\left(T_s - T_\infty\right)$$

where $A = \pi\left(DL + D^2/4\right) = \pi\left[0.012\text{m} \times 0.01\text{m} + \left(0.012\text{m}\right)^2/4\right] = 4.90 \times 10^{-4}\,\text{m}^2.$

For a maximum allowable surface temperature of 85°C, the power is

$$P_{elec} = 100\ \text{W/m}^2 \cdot \text{K}\left(4.90 \times 10^{-4}\text{m}^2\right)\ \left(85 - 25\right)^\circ \text{C} = 2.94\ \text{W} \qquad <$$

COMMENTS: (1) For the prescribed surface temperature and convection coefficient, radiation will be negligible relative to convection. However, conduction through the base could be significant, thereby permitting operation at a larger power.

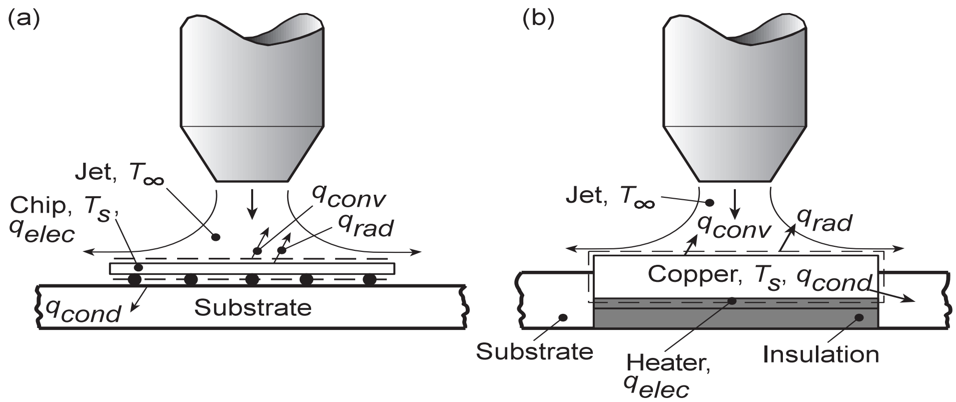
(2) The *local* convection coefficient varies over the surface, and *hot spots* could exist if there are locations at which the local value of *h* is substantially smaller than the prescribed average value.

PROBLEM 1.20

KNOWN: Air jet impingement is an effective means of cooling logic chips.

FIND: Procedure for measuring convection coefficients associated with a 10 mm × 10 mm chip.

SCHEMATIC:



ASSUMPTIONS: Steady-state conditions.

ANALYSIS: One approach would be to use the actual chip-substrate system, Case (a), to perform the measurements. In this case, the electric power dissipated in the chip would be transferred from the chip by radiation and conduction (to the substrate), as well as by convection to the jet. An energy balance for the chip yields $q_{elec} = q_{conv} + q_{cond} + q_{rad}$. Hence, with $q_{conv} = hA(T_s - T_\infty)$, where A = 100 mm^2 is the surface area of the chip,

$$h = \frac{q_{elec} - q_{cond} - q_{rad}}{A(T_s - T_\infty)} \qquad (1)$$

While the electric power (q_{elec}) and the jet (T_∞) and surface (T_s) temperatures may be measured, losses from the chip by conduction and radiation would have to be estimated. Unless the losses are negligible (an unlikely condition), the accuracy of the procedure could be compromised by uncertainties associated with determining the conduction and radiation losses.

A second approach, Case (b), could involve fabrication of a heater assembly for which the conduction and radiation losses are controlled and minimized. A 10 mm × 10 mm copper block (k ~ 400 W/m·K) could be inserted in a poorly conducting substrate (k < 0.1 W/m·K) and a patch heater could be applied to the back of the block and insulated from below. If conduction to both the substrate and insulation could thereby be rendered negligible, heat would be transferred almost exclusively through the block. If radiation were rendered negligible by applying a low emissivity coating (ε < 0.1) to the surface of the copper block, virtually all of the heat would be transferred by convection to the jet. Hence, q_{cond} and q_{rad} may be neglected in equation (1), and the expression may be used to accurately determine h from the known (A) and measured (q_{elec}, T_s, T_∞) quantities.

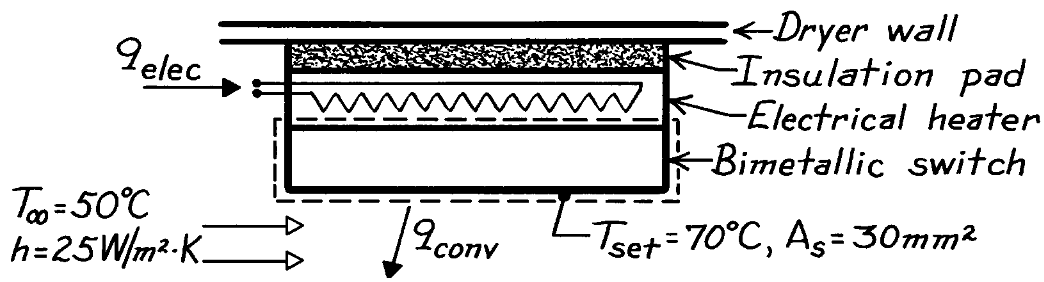
COMMENTS: Since convection coefficients associated with gas flows are generally small, concurrent heat transfer by radiation and/or conduction must often be considered. However, jet impingement is one of the more effective means of transferring heat by convection and convection coefficients well in excess of 100 W/m^2·K may be achieved.

PROBLEM 1.21

KNOWN: Upper temperature set point, T_{set}, of a bimetallic switch and convection heat transfer coefficient between clothes dryer air and exposed surface of switch.

FIND: Electrical power for heater to maintain T_{set} when air temperature is $T_\infty = 50°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Electrical heater is perfectly insulated from dryer wall, (3) Heater and switch are isothermal at T_{set}, (4) Negligible heat transfer from sides of heater or switch, (5) Switch surface, A_s, loses heat only by convection.

ANALYSIS: Define a control volume around the bimetallic switch which experiences heat input from the heater and convection heat transfer to the dryer air. That is,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$
$$q_{elec} - hA_s\left(T_{set} - T_\infty\right) = 0.$$

The electrical power required is,

$$q_{elec} = hA_s\left(T_{set} - T_\infty\right)$$

$$q_{elec} = 25 \text{ W/m}^2 \cdot \text{K} \times 30 \times 10^{-6} \text{ m}^2 \left(70 - 50\right)\text{K} = 15 \text{ mW}. \qquad <$$

COMMENTS: (1) This type of controller can achieve variable operating air temperatures with a single set-point, inexpensive, bimetallic-thermostatic switch by adjusting power levels to the heater.

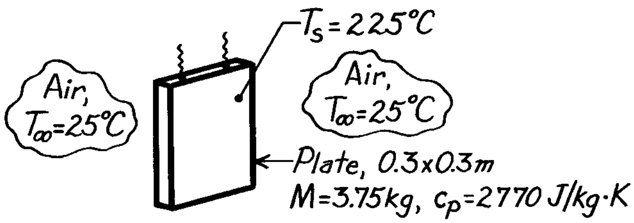
(2) Will the heater power requirement increase or decrease if the insulation pad is other than perfect?

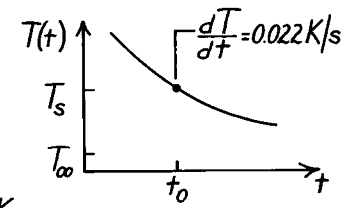
PROBLEM 1.22

KNOWN: Hot vertical plate suspended in cool, still air. Change in plate temperature with time at the instant when the plate temperature is 225°C.

FIND: Convection heat transfer coefficient for this condition.

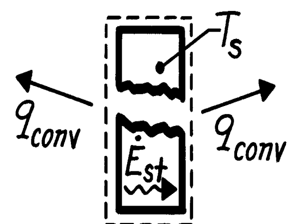
SCHEMATIC:





ASSUMPTIONS: (1) Plate is isothermal and of uniform temperature, (2) Negligible radiation exchange with surroundings, (3) Negligible heat lost through suspension wires.

ANALYSIS: As shown in the cooling curve above, the plate temperature decreases with time. The condition of interest is for time t_o. For a control surface about the plate, the conservation of energy requirement is



$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-2hA_s\left(T_s - T_\infty\right) = Mc_p\frac{dT}{dt}$$

where A_s is the surface area of one side of the plate. Solving for h, find

$$h=\frac{Mc_p}{2A_s\left(T_s - T_\infty\right)}\frac{dT}{dt}$$

$$h=\frac{3.75\ \text{kg}\times 2770\ \text{J/kg}\cdot\text{K}}{2\times(0.3\times 0.3)\text{m}^2\,(225-25)\text{K}}\times 0.022\ \text{K/s}=6.4\ \text{W/m}^2\cdot\text{K}$$ <

COMMENTS: (1) Assuming the plate is very highly polished with emissivity of 0.08, determine whether radiation exchange with the surroundings at 25°C is negligible compared to convection.

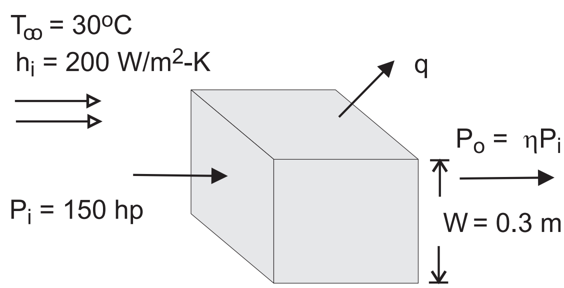
(2) We will later consider the criterion for determining whether the isothermal plate assumption is reasonable. If the thermal conductivity of the present plate were high (such as aluminum or copper), the criterion would be satisfied.

PROBLEM 1.23

KNOWN: Width, input power and efficiency of a transmission. Temperature and convection coefficient associated with air flow over the casing.

FIND: Surface temperature of casing.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Negligible radiation.

ANALYSIS: From Newton's law of cooling,

$$q = hA_s\left(T_s - T_\infty\right) = 6\,hW^2\left(T_s - T_\infty\right)$$

where the output power is $\eta\ P_i$ and the heat rate is

$$q = P_i - P_o = P_i\left(1-\eta\right) = 150\,\text{hp}\times 746\,\text{W/hp}\times 0.07 = 7833\,\text{W}$$

Hence,

$$T_s = T_\infty + \frac{q}{6\ hW^2} = 30°C + \frac{7833\ W}{6\times 200\ W/m^2\cdot K\times\left(0.3\,m\right)^2} = 102.5°C$$ <

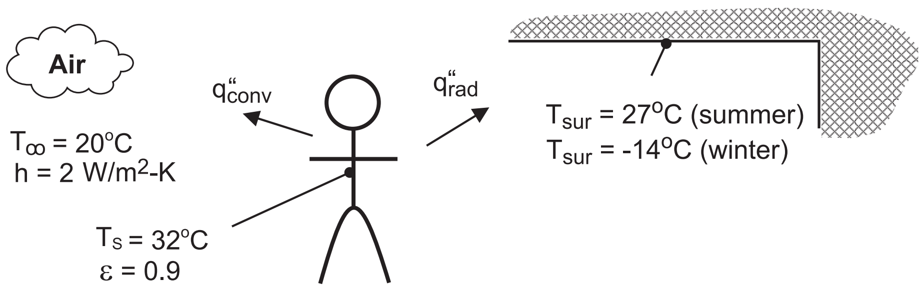
COMMENTS: There will, in fact, be considerable variability of the local convection coefficient over the transmission case and the prescribed value represents an average over the surface.

PROBLEM 1.24

KNOWN: Air and wall temperatures of a room. Surface temperature, convection coefficient and emissivity of a person in the room.

FIND: Basis for difference in comfort level between summer and winter.

SCHEMATIC:



ASSUMPTIONS: (1) Person may be approximated as a small object in a large enclosure.

ANALYSIS: Thermal comfort is linked to heat loss from the human body, and a *chilled* feeling is associated with excessive heat loss. Because the temperature of the room air is fixed, the different summer and winter comfort levels can not be attributed to convection heat transfer from the body. In both cases, the heat flux is

Summer and Winter: $q''_{conv} = h\left(T_s - T_\infty\right) = 2\ \mathrm{W/m^2 \cdot K} \times 12\ ^\circ\mathrm{C} = 24\ \mathrm{W/m^2}$

However, the heat flux due to radiation will differ, with values of

Summer: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\left(305^4 - 300^4\right)\mathrm{K}^4 = 28.3\ \mathrm{W/m^2}$

Winter: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\left(305^4 - 287^4\right)\mathrm{K}^4 = 95.4\ \mathrm{W/m^2}$

There is a significant difference between winter and summer radiation fluxes, and the chilled condition is attributable to the effect of the colder walls on radiation.

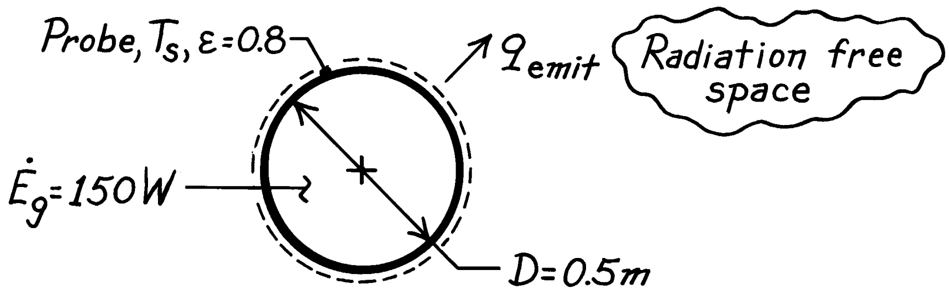
COMMENTS: For a representative surface area of A = 1.5 m^2, the heat losses are q_{conv} = 36 W, $q_{rad(summer)}$ = 42.5 W and $q_{rad(winter)}$ = 143.1 W. The winter time radiation loss is significant and if maintained over a 24 h period would amount to 2,950 kcal.

PROBLEM 1.25

KNOWN: Diameter and emissivity of spherical interplanetary probe. Power dissipation within probe.

FIND: Probe surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation incident on the probe.

ANALYSIS: Conservation of energy dictates a balance between energy generation within the probe and radiation emission from the probe surface. Hence, at any instant

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$\varepsilon A_s \sigma T_s^4 = \dot{E}_g$$

$$T_s = \left(\frac{\dot{E}_g}{\varepsilon \pi D^2 \sigma}\right)^{1/4}$$

$$T_s = \left(\frac{150\text{W}}{0.8\pi (0.5\text{ m})^2\, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\right)^{1/4}$$

$$T_s = 254.7\text{ K.}$$ <

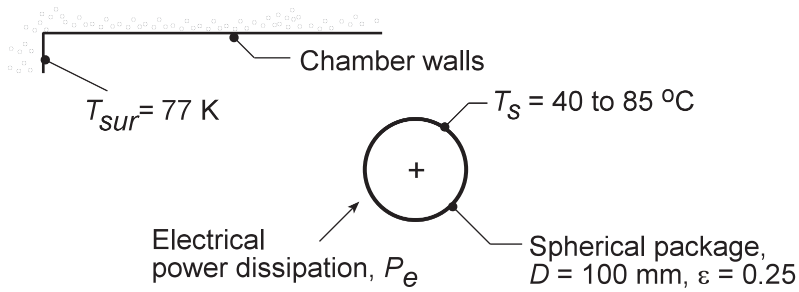
COMMENTS: Incident radiation, as, for example, from the sun, would increase the surface temperature.

PROBLEM 1.26

KNOWN: Spherical shaped instrumentation package with prescribed surface emissivity within a large space-simulation chamber having walls at 77 K.

FIND: Acceptable power dissipation for operating the package surface temperature in the range T_s = 40 to 85°C. Show graphically the effect of emissivity variations for 0.2 and 0.3.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform surface temperature, (2) Chamber walls are large compared to the spherical package, and (3) Steady-state conditions.

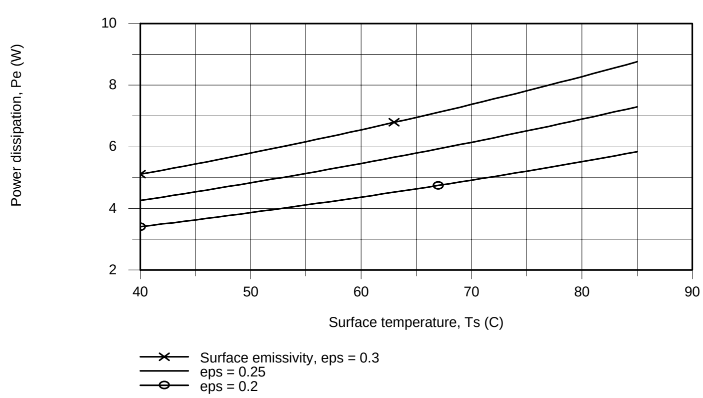
ANALYSIS: From an overall energy balance on the package, the internal power dissipation P_e will be transferred by radiation exchange between the package and the chamber walls. From Eq. 1.7,

$$q_{rad} = P_e = \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right)$$

For the condition when T_s = 40°C, with $A_s = \pi D^2$ the power dissipation will be

$$P_e = 0.25(\pi \times 0.10\,\text{m}) \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \times \left[(40+273)^4 - 77^4\right]\text{K}^4 = 4.3\,\text{W} \qquad <$$

Repeating this calculation for the range $40 \le T_s \le 85$°C, we can obtain the power dissipation as a function of surface temperature for the ε = 0.25 condition. Similarly, with 0.2 or 0.3, the family of curves shown below has been obtained.



COMMENTS: (1) As expected, the internal power dissipation increases with increasing emissivity and surface temperature. Because the radiation rate equation is non-linear with respect to temperature, the power dissipation will likewise not be linear with surface temperature.

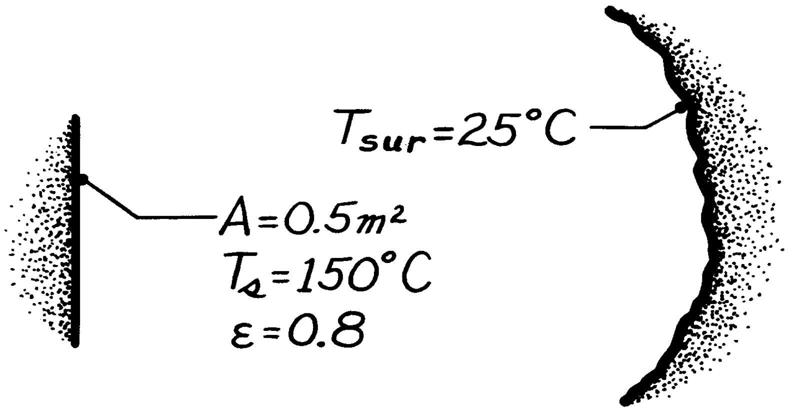
(2) What is the maximum power dissipation that is possible if the surface temperature is not to exceed 85°C? What kind of a coating should be applied to the instrument package in order to approach this limiting condition?

PROBLEM 1.27

KNOWN: Area, emissivity and temperature of a surface placed in a large, evacuated chamber of prescribed temperature.

FIND: (a) Rate of surface radiation emission, (b) Net rate of radiation exchange between surface and chamber walls.

SCHEMATIC:



ASSUMPTIONS: (1) Area of the enclosed surface is much less than that of chamber walls.

ANALYSIS: (a) From Eq. 1.5, the rate at which radiation is emitted by the surface is

$$q_{emit} = E \cdot A = \varepsilon A \sigma T_s^4$$

$$q_{emit} = 0.8\left(0.5\ m^2\right)5.67\times10^{-8}\ W/m^2\cdot K^4\left[(150+273)K\right]^4$$

$$q_{emit} = 726\ W. \qquad <$$

(b) From Eq. 1.7, the *net* rate at which radiation is transferred *from* the surface to the chamber walls is

$$q = \varepsilon A \sigma\left(T_s^4 - T_{sur}^4\right)$$

$$q = 0.8\left(0.5\ m^2\right)5.67\times10^{-8}\ W/m^2\cdot K^4\left[(423K)^4 - (298K)^4\right]$$

$$q = 547\ W. \qquad <$$

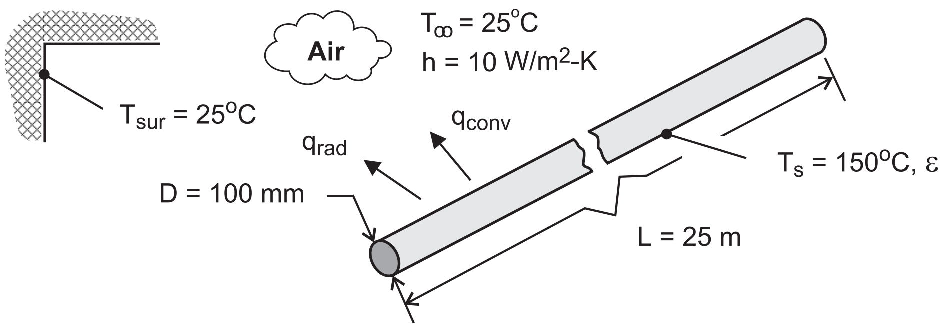
COMMENTS: The foregoing result gives the net heat loss from the surface which occurs at the instant the surface is placed in the chamber. The surface would, of course, cool due to this heat loss and its temperature, as well as the heat loss, would decrease with increasing time. Steady-state conditions would eventually be achieved when the temperature of the surface reached that of the surroundings.

PROBLEM 1.28

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1\text{m} \times 25\text{m}\right) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2 \cdot \text{K}\left(150 - 25\right)\text{K} + 0.8 \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1,250 + 1,095\right)\text{w/m}^2 = \left(9813 + 8592\right)\text{W} = 18,405\ \text{W}$$ <

(b) The annual energy loss is

$$E = qt = 18,405\ \text{W} \times 3600\ \text{s/h} \times 24\text{h/d} \times 365\ \text{d/y} = 5.80 \times 10^{11}\ \text{J}$$

With a furnace energy consumption of $E_f = E/\eta_f = 6.45 \times 10^{11}\ \text{J}$, the annual cost of the loss is

$$C = C_g E_f = 0.01\ \$/\text{MJ} \times 6.45 \times 10^5 \text{MJ} = \$6450$$ <

COMMENTS: The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.

PROBLEM 1.29

KNOWN: Exact and approximate expressions for the linearized radiation coefficient, h_r and h_{ra}, respectively.

FIND: (a) Comparison of the coefficients with $\varepsilon = 0.05$ and 0.9 and surface temperatures which may exceed that of the surroundings ($T_{sur} = 25°C$) by 10 to 100°C; also comparison with a free convection coefficient correlation, (b) Plot of the relative error $(h_r - r_{ra})/h_r$ as a function of the furnace temperature associated with a workpiece at $T_s = 25°C$ having $\varepsilon = 0.05$, 0.2 or 0.9.

ASSUMPTIONS: (1) Furnace walls are large compared to the workpiece and (2) Steady-state conditions.

ANALYSIS: (a) The linearized radiation coefficient, Eq. 1.9, follows from the radiation exchange rate equation,

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$

If $T_s \approx T_{sur}$, the coefficient may be approximated by the simpler expression

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3 \qquad \bar{T} = \left(T_s + T_{sur}\right)/2$$

For the condition of $\varepsilon = 0.05$, $T_s = T_{sur} + 10 = 35°C = 308$ K and $T_{sur} = 25°C = 298$ K, find that

$$h_r = 0.05\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(308+298)\left(308^2+298^2\right)K^3 = 0.32\,W/m^2\cdot K \qquad <$$

$$h_{r,a} = 4\times0.05\times5.67\times10^{-8}\,W/m^2\cdot K^4\left((308+298)/2\right)^3 K^3 = 0.32\,W/m^2\cdot K \qquad <$$

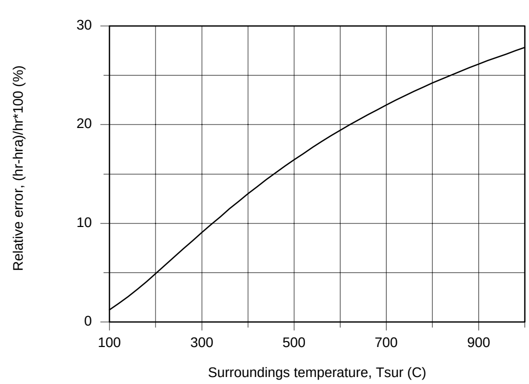
The free convection coefficient with $T_s = 35°C$ and $T_\infty = T_{sur} = 25°C$, find that

$$h = 0.98\Delta T^{1/3} = 0.98\left(T_s - T_\infty\right)^{1/3} = 0.98\left(308-298\right)^{1/3} = 2.1\,W/m^2\cdot K \qquad <$$

For the range $T_s - T_{sur} = 10$ to 100°C with $\varepsilon = 0.05$ and 0.9, the results for the coefficients are tabulated below. For this range of surface and surroundings temperatures, the radiation and free convection coefficients are of comparable magnitude for moderate values of the emissivity, say $\varepsilon > 0.2$. The approximate expression for the linearized radiation coefficient is valid within 2% for these conditions.

(b) The above expressions for the radiation coefficients, h_r and $h_{r,a}$, are used for the workpiece at $T_s = 25°C$ placed inside a furnace with walls which may vary from 100 to 1000°C. The relative error, $(h_r - h_{ra})/h_r$, will be independent of the surface emissivity and is plotted as a function of T_{sur}. For $T_{sur} > 150°C$, the approximate expression provides estimates which are in error more than 5%. The approximate expression should be used with caution, and only for surface and surrounding temperature differences of 50 to 100°C.

		Coefficients ($W/m^2\cdot K$)		
T_s (°C)	ε	h_r	$h_{r,a}$	h
35	0.05	0.32	0.32	2.1
	0.9	5.7	5.7	
135	0.05	0.51	0.50	4.7
	0.9	9.2	9.0	

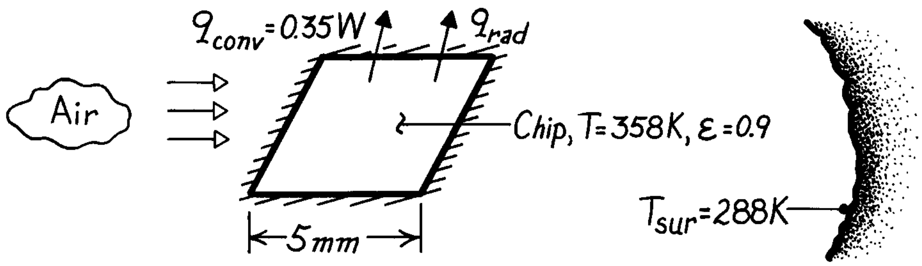


PROBLEM 1.30

KNOWN: Chip width, temperature, and heat loss by convection in air. Chip emissivity and temperature of large surroundings.

FIND: Increase in chip power due to radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between small surface and large enclosure.

ANALYSIS: Heat transfer from the chip due to net radiation exchange with the surroundings is

$$q_{rad} = \varepsilon W^2 \sigma \left(T^4 - T_{sur}^4\right)$$

$$q_{rad} = 0.9(0.005\text{ m})^2\, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4 \left(358^4 - 288^4\right)\text{K}^4$$

$$q_{rad} = 0.0122\text{ W}.$$

The percent increase in chip power is therefore

$$\frac{\Delta P}{P}\times100 = \frac{q_{rad}}{q_{conv}}\times100 = \frac{0.0122\text{ W}}{0.350\text{ W}}\times100 = 3.5\%.$$ <

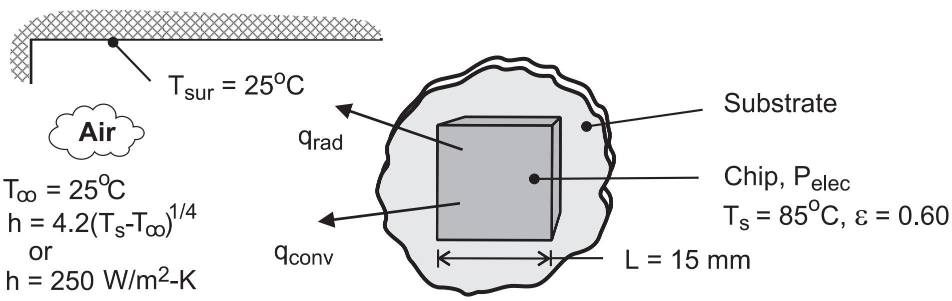
COMMENTS: For the prescribed conditions, radiation effects are small. Relative to convection, the effect of radiation would increase with increasing chip temperature and decreasing convection coefficient.

PROBLEM 1.31

KNOWN: Width, surface emissivity and maximum allowable temperature of an electronic chip. Temperature of air and surroundings. Convection coefficient.

FIND: (a) Maximum power dissipation for free convection with $h(W/m^2 \cdot K) = 4.2(T - T_\infty)^{1/4}$, (b) Maximum power dissipation for forced convection with $h = 250\ W/m^2 \cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between a small surface and a large enclosure, (3) Negligible heat transfer from sides of chip or from back of chip by conduction through the substrate.

ANALYSIS: Subject to the foregoing assumptions, electric power dissipation by the chip must be balanced by convection and radiation heat transfer from the chip. Hence, from Eq. (1.10),

$$P_{elec} = q_{conv} + q_{rad} = hA(T_s - T_\infty) + \varepsilon A \sigma \left(T_s^4 - T_{sur}^4\right)$$

where $A = L^2 = (0.015m)^2 = 2.25 \times 10^{-4} m^2$.

(a) If heat transfer is by natural convection,

$$q_{conv} = C\,A(T_s - T_\infty)^{5/4} = 4.2\ W/m^2 \cdot K^{5/4}\left(2.25\times10^{-4} m^2\right)(60K)^{5/4} = 0.158\ W$$

$$q_{rad} = 0.60\left(2.25\times10^{-4} m^2\right)5.67\times10^{-8}\ W/m^2 \cdot K^4\left(358^4 - 298^4\right)K^4 = 0.065\ W$$

$$P_{elec} = 0.158\ W + 0.065\ W = 0.223\ W$$ <

(b) If heat transfer is by forced convection,

$$q_{conv} = hA(T_s - T_\infty) = 250\ W/m^2 \cdot K\left(2.25\times10^{-4} m^2\right)(60K) = 3.375\ W$$

$$P_{elec} = 3.375\ W + 0.065\ W = 3.44\ W$$ <

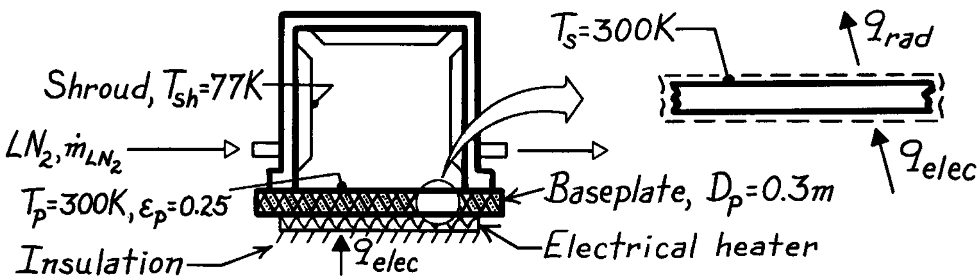
COMMENTS: Clearly, radiation and natural convection are inefficient mechanisms for transferring heat from the chip. For $T_s = 85°C$ and $T_\infty = 25°C$, the natural convection coefficient is $11.7\ W/m^2 \cdot K$. Even for forced convection with $h = 250\ W/m^2 \cdot K$, the power dissipation is well below that associated with many of today's processors. To provide acceptable cooling, it is often necessary to attach the chip to a highly conducting substrate and to thereby provide an additional heat transfer mechanism due to conduction from the back surface.

PROBLEM 1.32

KNOWN: Vacuum enclosure maintained at 77 K by liquid nitrogen shroud while baseplate is maintained at 300 K by an electrical heater.

FIND: (a) Electrical power required to maintain baseplate, (b) Liquid nitrogen consumption rate, (c) Effect on consumption rate if aluminum foil ($\varepsilon_p = 0.09$) is bonded to baseplate surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses from backside of heater or sides of plate, (3) Vacuum enclosure large compared to baseplate, (4) Enclosure is evacuated with negligible convection, (5) Liquid nitrogen (LN2) is heated only by heat transfer to the shroud, and (6) Foil is intimately bonded to baseplate.

PROPERTIES: Heat of vaporization of liquid nitrogen (given): 125 kJ/kg.

ANALYSIS: (a) From an energy balance on the baseplate,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{elec} - q_{rad} = 0$$

and using Eq. 1.7 for radiative exchange between the baseplate and shroud,

$$q_{elec} = \varepsilon_p A_p \sigma \left(T_p^4 - T_{sh}^4\right).$$

Substituting numerical values, with $A_p = \left(\pi D_p^2/4\right)$, find

$$q_{elec} = 0.25\left(\pi(0.3\text{ m})^2/4\right)5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(300^4 - 77^4\right)\text{K}^4 = 8.1\text{ W}. \qquad <$$

(b) From an energy balance on the enclosure, radiative transfer heats the liquid nitrogen stream causing evaporation,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{rad} - \dot{m}_{LN2} h_{fg} = 0$$

where $\dot{m}_{LN2}$ is the liquid nitrogen consumption rate. Hence,

$$\dot{m}_{LN2} = q_{rad}/h_{fg} = 8.1\text{ W}/125\text{ kJ/kg} = 6.48\times10^{-5}\text{ kg/s} = 0.23\text{ kg/h}. \qquad <$$

(c) If aluminum foil ($\varepsilon_p = 0.09$) were bonded to the upper surface of the baseplate,

$$q_{rad,foil} = q_{rad}\left(\varepsilon_f/\varepsilon_p\right) = 8.1\text{ W}\left(0.09/0.25\right) = 2.9\text{ W}$$

and the liquid nitrogen consumption rate would be reduced by

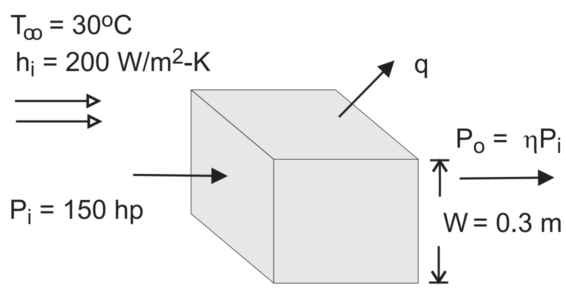
(0.25 - 0.09)/0.25 = 64% to 0.083 kg/h. <

PROBLEM 1.33

KNOWN: Width, input power and efficiency of a transmission. Temperature and convection coefficient for air flow over the casing. Emissivity of casing and temperature of surroundings.

FIND: Surface temperature of casing.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Radiation exchange with large surroundings.

ANALYSIS: Heat transfer from the case must balance heat dissipation in the transmission, which may be expressed as $q = P_i - P_o = P_i (1 - \eta) = 150 \text{ hp} \times 746 \text{ W/hp} \times 0.07 = 7833$ W. Heat transfer from the case is by convection and radiation, in which case

$$q = A_s \left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A_s = 6\ W^2$. Hence,

$$7833\,\text{W} = 6(0.30\,\text{m})^2\left[200\,\text{W/m}^2\cdot\text{K}\left(T_s - 303\text{K}\right) + 0.8\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_s^4 - 303^4\right)\text{K}^4\right]$$

A trial-and-error solution yields

$$T_s \approx 373\,\text{K} = 100°\text{C}$$ $<$

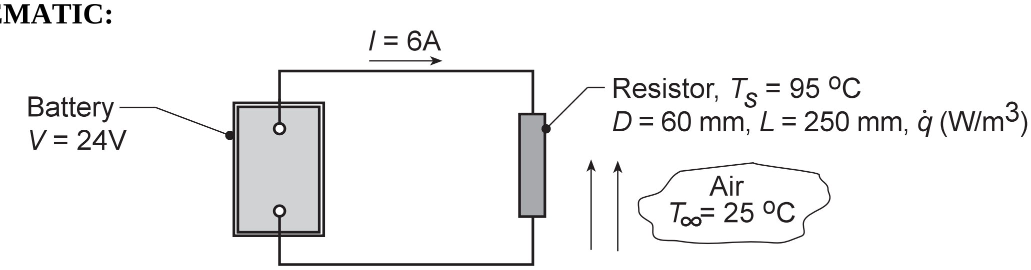
COMMENTS: (1) For $T_s \approx 373$ K, $q_{conv} \approx 7{,}560$ W and $q_{rad} \approx 270$ W, in which case heat transfer is dominated by convection, (2) If radiation is neglected, the corresponding surface temperature is T_s = 102.5°C.

PROBLEM 1.34

KNOWN: Resistor connected to a battery operating at a prescribed temperature in air.

FIND: (a) Considering the resistor as the system, determine corresponding values for $\dot{E}_{in}\,(W)$, $\dot{E}_g\,(W)$, $\dot{E}_{out}\,(W)$ and $\dot{E}_{st}\,(W)$. If a control surface is placed about the entire system, determine the values for $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$. (b) Determine the volumetric heat generation rate within the resistor, $\dot{q}$ (W/m^3), (c) Neglecting radiation from the resistor, determine the convection coefficient.

SCHEMATIC:



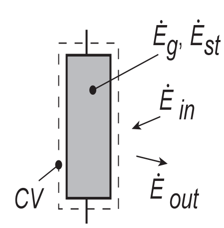
ASSUMPTIONS: (1) Electrical power is dissipated uniformly within the resistor, (2) Temperature of the resistor is uniform, (3) Negligible electrical power dissipated in the lead wires, (4) Negligible radiation exchange between the resistor and the surroundings, (5) No heat transfer occurs from the battery, (5) Steady-state conditions.

ANALYSIS: (a) Referring to Section 1.3.1, the conservation of energy requirement for a control volume at an instant of time, Eq 1.11a, is

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

where $\dot{E}_{in}$, $\dot{E}_{out}$ correspond to *surface* inflow and outflow processes, respectively. The energy generation term $\dot{E}_g$ is associated with conversion of some other energy form (chemical, electrical, electromagnetic or nuclear) to thermal energy. The energy storage term $\dot{E}_{st}$ is associated with changes in the internal, kinetic and/or potential energies of the matter in the control volume. $\dot{E}_g$, $\dot{E}_{st}$ are *volumetric* phenomena. The electrical power delivered by the battery is P = VI = 24V×6A = 144 W.

Control volume: Resistor.



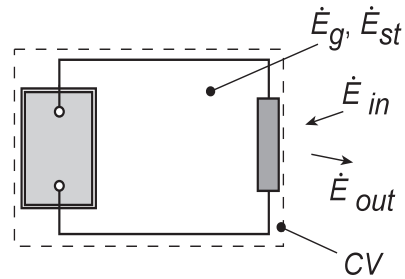
$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,\mathrm{W}$$

<

$$\dot{E}_g = 144\,\mathrm{W} \qquad \dot{E}_{st} = 0$$

The $\dot{E}_g$ term is due to conversion of electrical energy to thermal energy. The term $\dot{E}_{out}$ is due to convection from the resistor surface to the air.

Continued...



Control volume: Battery-Resistor System.

$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,W \qquad <$$
$$\dot{E}_g = 0 \qquad \dot{E}_{st} = -144\,W$$

The $\dot{E}_{st}$ term represents the decrease in the chemical energy within the battery. The conversion of chemical energy to electrical energy and its subsequent conversion to thermal energy are processes internal to the system which are not associated with $\dot{E}_{st}$ or $\dot{E}_g$. The $\dot{E}_{out}$ term is due to convection from the resistor surface to the air.

(b) From the energy balance on the resistor with volume, $\forall = (\pi D^2/4)L$,

$$\dot{E}_g = \dot{q}\forall \qquad 144\,W = \dot{q}\left(\pi\,(0.06\,m)^2/4\right)\times 0.25\,m \qquad \dot{q} = 2.04\times10^5\,W/m^3 \qquad <$$

(c) From the energy balance on the resistor and Newton's law of cooling with $A_s = \pi DL + 2(\pi D^2/4)$,

$$\dot{E}_{out} = q_{cv} = hA_s\left(T_s - T_\infty\right)$$

$$144\,W = h\left[\pi\times0.06\,m\times0.25\,m + 2\left(\pi\times0.06^2\,m^2/4\right)\right](95-25)^\circ\,C$$

$$144\,W = h\left[0.0471+0.0057\right]m^2\,(95-25)^\circ\,C$$

$$h = 39.0\,W/m^2{\cdot}K \qquad <$$

COMMENTS: (1) In using the conservation of energy requirement, Eq. 1.11a, it is important to recognize that $\dot{E}_{in}$ and $\dot{E}_{out}$ will always represent *surface* processes and $\dot{E}_g$ and $\dot{E}_{st}$, *volumetric* processes. The generation term $\dot{E}_g$ is associated with a *conversion* process from some form of energy to *thermal energy*. The storage term $\dot{E}_{st}$ represents the rate of change of *internal energy*.

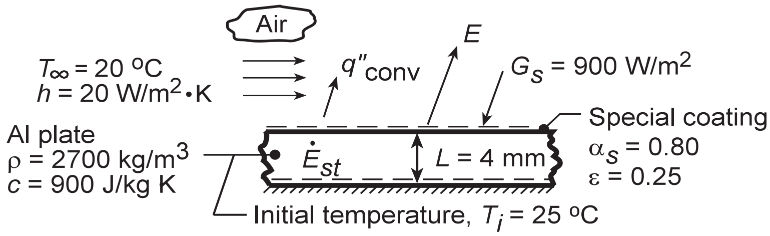
(2) From Table 1.1 and the magnitude of the convection coefficient determined from part (c), we conclude that the resistor is experiencing forced, rather than free, convection.

PROBLEM 1.35

KNOWN: Thickness and initial temperature of an aluminum plate whose thermal environment is changed.

FIND: (a) Initial rate of temperature change, (b) Steady-state temperature of plate, (c) Effect of emissivity and absorptivity on steady-state temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible end effects, (2) Uniform plate temperature at any instant, (3) Constant properties, (4) Adiabatic bottom surface, (5) Negligible radiation from surroundings, (6) No internal heat generation.

ANALYSIS: (a) Applying an energy balance, Eq. 1.11a, at an instant of time to a control volume about the plate, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$, it follows for a unit surface area.

$$\alpha_S G_S \left(1\,m^2\right) - E\left(1\,m^2\right) - q''_{conv}\left(1\,m^2\right) = (d/dt)(McT) = \rho\left(1\,m^2 \times L\right)c\,(dT/dt).$$

Rearranging and substituting from Eqs. 1.3 and 1.5, we obtain

$$dT/dt = (1/\rho Lc)\left[\alpha_S G_S - \varepsilon\sigma T_i^4 - h\left(T_i - T_\infty\right)\right].$$

$$dT/dt = \left(2700\,kg/m^3 \times 0.004\,m \times 900\,J/kg\cdot K\right)^{-1} \times$$

$$\left[0.8 \times 900\,W/m^2 - 0.25 \times 5.67 \times 10^{-8}\,W/m^2\cdot K^4\,(298\,K)^4 - 20\,W/m^2\cdot K\,(25-20)^\circ C\right]$$

$$dT/dt = 0.052^\circ\,C/s\,.$$ <

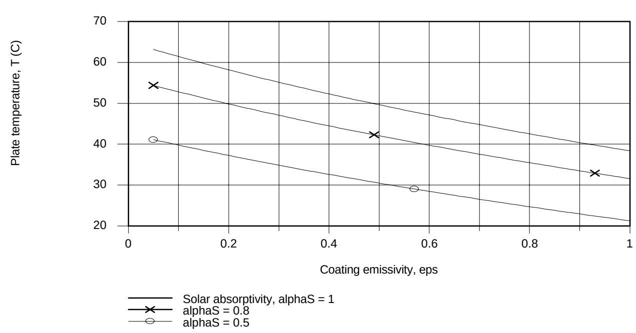
(b) Under steady-state conditions, $\dot{E}_{st}$ = 0, and the energy balance reduces to

$$\alpha_S G_S = \varepsilon\sigma T^4 + h\left(T - T_\infty\right) \qquad (2)$$

$$0.8 \times 900\,W/m^2 = 0.25 \times 5.67 \times 10^{-8}\,W/m^2\cdot K^4 \times T^4 + 20\,W/m^2\cdot K\,(T - 293\,K)$$

The solution yields T = 321.4 K = 48.4°C. <

(c) Using the IHT *First Law Model* for an *Isothermal Plane Wall*, parametric calculations yield the following results.



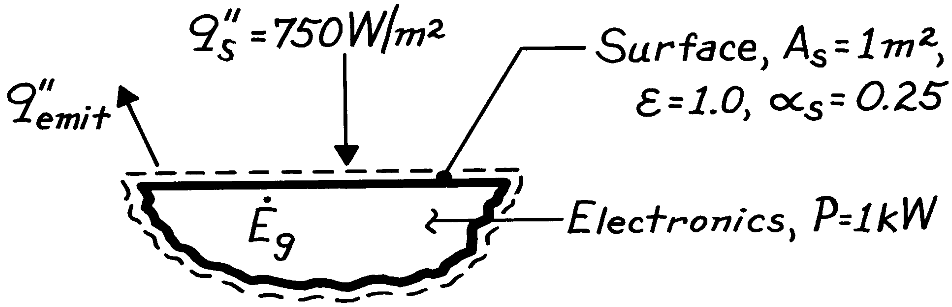
COMMENTS: The surface radiative properties have a significant effect on the plate temperature, which decreases with increasing ε and decreasing α_S. If a low temperature is desired, the plate coating should be characterized by a large value of ε/α_S. The temperature also decreases with increasing h.

PROBLEM 1.36

KNOWN: Surface area of electronic package and power dissipation by the electronics. Surface emissivity and absorptivity to solar radiation. Solar flux.

FIND: Surface temperature without and with incident solar radiation.

SCHEMATIC:



ASSUMPTIONS: Steady-state conditions.

ANALYSIS: Applying conservation of energy to a control surface about the compartment, at any instant

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0.$$

It follows that, with the solar input,

$$\alpha_S A_S q_S'' - A_S E + P = 0$$
$$\alpha_S A_S q_S'' - A_S \varepsilon\sigma T_S^4 + P = 0$$
$$T_S = \left(\frac{\alpha_S A_S q_S'' + P}{A_S \varepsilon\sigma}\right)^{1/4}.$$

In the shade $(q_S'' = 0)$,

$$T_S = \left(\frac{1000\ \text{W}}{1\ \text{m}^2 \times 1 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 364\ \text{K}.$$ <

In the sun,

$$T_S = \left(\frac{0.25 \times 1\ \text{m}^2 \times 750\ \text{W/m}^2 + 1000\ \text{W}}{1\ \text{m}^2 \times 1 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 380\ \text{K}.$$ <

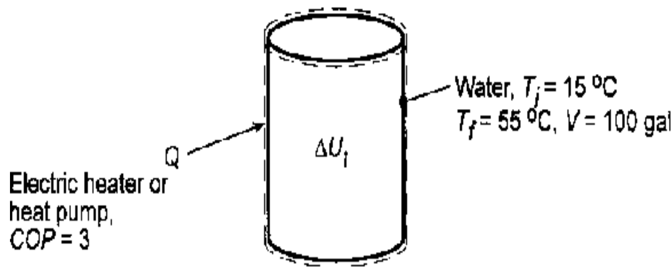
COMMENTS: In orbit, the space station would be continuously cycling between shade and sunshine, and a steady-state condition would not exist.

PROBLEM 1.37

KNOWN: Daily hot water consumption for a family of four and temperatures associated with ground water and water storage tank. Unit cost of electric power. Heat pump COP.

FIND: Annual heating requirement and costs associated with using electric resistance heating or a heat pump.

SCHEMATIC:



ASSUMPTIONS: (1) Process may be modelled as one involving heat addition in a closed system, (2) Properties of water are constant.

PROPERTIES: Table A-6, Water (T_{ave} = 308 K): $\rho = v_f^{-1}$ = 993 kg/m^3, $c_{p,f}$ = 4.178 kJ/kg·K.

ANALYSIS: From Eq. 1.11c, the daily heating requirement is $Q_{daily} = \Delta U_t = Mc\Delta T$ $= \rho Vc(T_f - T_i)$. With V = 100 gal/264.17 gal/m^3 = 0.379 m^3,

$$Q_{daily} = 993\,\text{kg/m}^3\left(0.379\,\text{m}^3\right)4.178\,\text{kJ/kg}\cdot\text{K}\left(40^\circ\text{C}\right) = 62{,}900\,\text{kJ}$$

The annual heating requirement is then, $Q_{annual} = 365\,\text{days}(62{,}900\,\text{kJ/day}) = 2.30\times10^7$ kJ , or, with 1 kWh = 1 kJ/s (3600 s) = 3600 kJ,

$$Q_{annual} = 6380\,\text{kWh}$$ <

With electric resistance heating, $Q_{annual} = Q_{elec}$ and the associated cost, C, is

$$C = 6380\,\text{kWh}(\$0.08/\text{kWh}) = \$510$$ <

If a heat pump is used, $Q_{annual} = COP(W_{elec})$. Hence,

$$W_{elec} = Q_{annual}/(COP) = 6380\,\text{kWh}/(3) = 2130\,\text{kWh}$$

The corresponding cost is

$$C = 2130\,\text{kWh}(\$0.08/\text{kWh}) = \$170$$ <

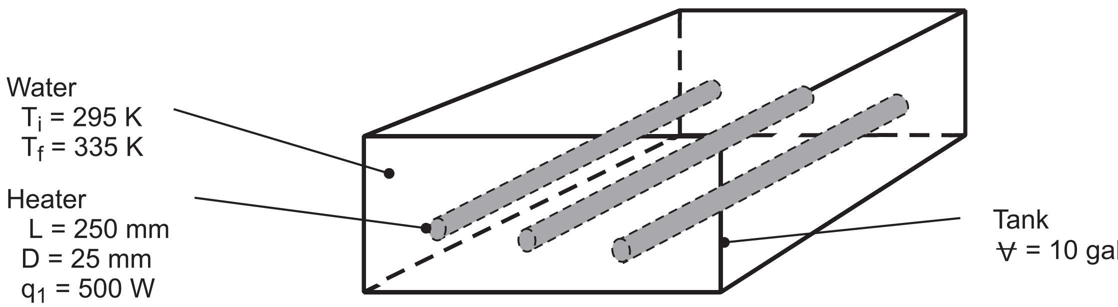
COMMENTS: Although annual operating costs are significantly lower for a heat pump, corresponding capital costs are much higher. The feasibility of this approach depends on other factors such as geography and seasonal variations in COP, as well as the time value of money.

PROBLEM 1.38

KNOWN: Initial temperature of water and tank volume. Power dissipation, emissivity, length and diameter of submerged heaters. Expressions for convection coefficient associated with natural convection in water and air.

FIND: (a) Time to raise temperature of water to prescribed value, (b) Heater temperature shortly after activation and at conclusion of process, (c) Heater temperature if activated in air.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss from tank to surroundings, (2) Water is *well-mixed* (at a uniform, but time varying temperature) during heating, (3) Negligible changes in thermal energy storage for heaters, (4) Constant properties, (5) Surroundings afforded by tank wall are large relative to heaters.

ANALYSIS: (a) Application of conservation of energy to a closed system (the water) at an instant, Eq. (1.11d), yields

$$\frac{dU}{dt} = Mc\frac{dT}{dt} = \rho\forall c\frac{dT}{dt} = q = 3q_1$$

Hence,
$$\int_0^t dt = \left(\rho\forall c/3q_1\right)\int_{T_i}^{T_f} dT$$

$$t = \frac{990\ \text{kg/m}^3 \times 10\text{gal}\left(3.79\times10^{-3}\text{m}^3/\text{gal}\right)4180\text{J/kg}\cdot\text{K}}{3\times500\ \text{W}}\left(335-295\right)\text{K} = 4180\ \text{s} \qquad <$$

(b) From Eq. (1.3a), the heat rate by convection from each heater is

$$q_1 = Aq_1'' = Ah\left(T_s - T\right) = \left(\pi DL\right)370\left(T_s - T\right)^{4/3}$$

Hence,

$$T_s = T + \left(\frac{q_1}{370\pi DL}\right)^{3/4} = T + \left(\frac{500\ \text{W}}{370\ \text{W/m}^2\cdot\text{K}^{4/3}\times\pi\times0.025\ \text{m}\times0.250\ \text{m}}\right)^{3/4} = \left(T+24\right)\text{K}$$

With water temperatures of $T_i \approx 295$ K and $T_f = 335$ K shortly after the start of heating and at the end of heating, respectively,

$T_{s,i} = 319$ K $\qquad$ $T_{s,f} = 359$ K $\qquad <$

Continued …..

PROBLEM 1.38 (Continued)

(c) From Eq. (1.10), the heat rate in air is

$$q_1 = \pi DL\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

Substituting the prescribed values of q_1, D, L, $T_\infty = T_{sur}$ and ε, an iterative solution yields

$T_s = 830$ K <

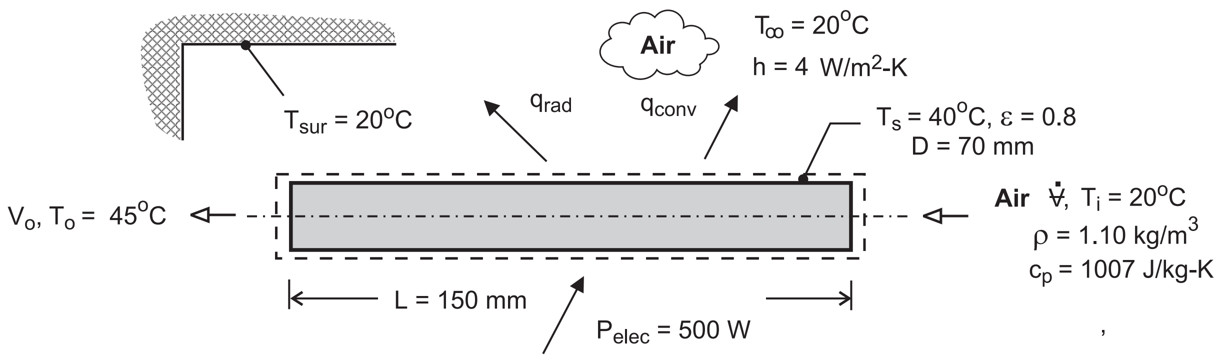
COMMENTS: In part (c) it is presumed that the heater can be operated at $T_s = 830$ K without experiencing burnout. The much larger value of T_s for air is due to the smaller convection coefficient. However, with q_{conv} and q_{rad} equal to 59 W and 441 W, respectively, a significant portion of the heat dissipation is effected by radiation.

PROBLEM 1.39

KNOWN: Power consumption, diameter, and inlet and discharge temperatures of a hair dryer.

FIND: (a) Volumetric flow rate and discharge velocity of heated air, (b) Heat loss from case.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible work done by fan, (5) Negligible heat transfer from casing of dryer to ambient air (Part (a)), (6) Radiation exchange between a small surface and a large enclosure (Part (b)).

ANALYSIS: (a) For a control surface about the air flow passage through the dryer, conservation of energy for an open system reduces to

$$\dot{m}(u + pv)_i - \dot{m}(u + pv)_o + q = 0$$

where u + pv = i and q = P_{elec}. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = P_{elec}$$

$$\dot{m} = \frac{P_{elec}}{c_p(T_o - T_i)} = \frac{500\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\left(25^\circ\text{C}\right)} = 0.0199\ \text{kg/s}$$

$$\dot{\forall} = \frac{\dot{m}}{\rho} = \frac{0.0199\ \text{kg/s}}{1.10\ \text{kg/m}^3} = 0.0181\ \text{m}^3/\text{s} \qquad <$$

$$V_o = \frac{\dot{\forall}}{A_c} = \frac{4\dot{\forall}}{\pi D^2} = \frac{4\times 0.0181\ \text{m}^3/\text{s}}{\pi(0.07\ \text{m})^2} = 4.7\ \text{m/s} \qquad <$$

(b) Heat transfer from the casing is by convection and radiation, and from Eq. (1.10)

$$q = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right)$$

Continued …..

where $A_s = \pi DL = \pi(0.07\text{ m} \times 0.15\text{ m}) = 0.033\text{ m}^2$. Hence,

$$q = 4\text{W/m}^2 \cdot \text{K}\left(0.033\text{ m}^2\right)\left(20^\circ\text{C}\right) + 0.8 \times 0.033\text{ m}^2 \times 5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4\left(313^4 - 293^4\right)\text{K}^4$$

$$q = 2.64\text{ W} + 3.33\text{ W} = 5.97\text{ W}$$ <

The heat loss is much less than the electrical power, and the assumption of negligible heat loss is justified.

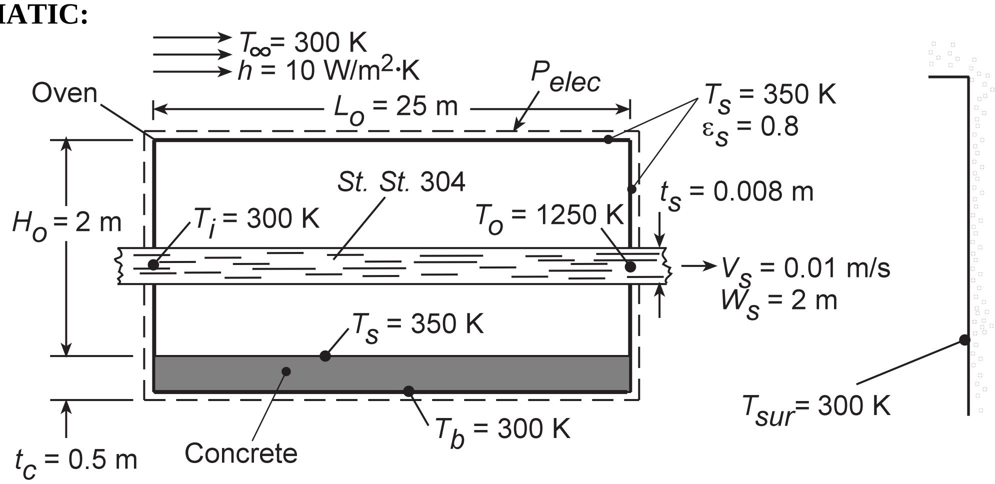
COMMENTS: Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the dryer, causing a reduction in the density. However, for the prescribed temperature rise, the change in ρ, and hence the effect on $\dot{\forall}$, is small.

PROBLEM 1.40

KNOWN: Speed, width, thickness and initial and final temperatures of 304 stainless steel in an annealing process. Dimensions of annealing oven and temperature, emissivity and convection coefficient of surfaces exposed to ambient air and large surroundings of equivalent temperatures. Thickness of pad on which oven rests and pad surface temperatures.

FIND: Oven operating power.

SCHEMATIC:



ASSUMPTIONS: (1) steady-state, (2) Constant properties, (3) Negligible changes in kinetic and potential energy.

PROPERTIES: Table A.1, St.St.304 $\left(\overline{T} = \left(T_i + T_o\right)/2 = 775\,K\right)$: ρ = 7900 kg/m^3, c_p = 578 J/kg·K; Table A.3, Concrete, T = 300 K: k_c = 1.4 W/m·K.

ANALYSIS: The rate of energy addition to the oven must balance the rate of energy transfer to the steel sheet and the rate of heat loss from the oven. With $\dot{E}_{in} - \dot{E}_{out} -$ = 0, it follows that

$$P_{elec} + \dot{m}\left(u_i - u_o\right) - q = 0$$

where heat is transferred from the oven. With $\dot{m} = \rho V_s \left(W_s t_s\right)$, $\left(u_i - u_o\right) = c_p\left(T_i - T_o\right)$, and

$$q = \left(2H_oL_o + 2H_oW_o + W_oL_o\right)\times\left[h\left(T_s - T_\infty\right) + \varepsilon_s\sigma\left(T_s^4 - T_{sur}^4\right)\right] + k_c\left(W_oL_o\right)\left(T_s - T_b\right)/t_c,$$

it follows that

$$P_{elec} = \rho V_s\left(W_s t_s\right)c_p\left(T_o - T_i\right) + \left(2H_oL_o + 2H_oW_o + W_oL_o\right)\times$$
$$\left[h\left(T_s - T_o\right) + \varepsilon_s\sigma\left(T_s^4 - T_{sur}^4\right)\right] + k_c\left(W_oL_o\right)\left(T_s - T_b\right)/t_c$$

$$P_{elec} = 7900\,kg/m^3 \times 0.01\,m/s\left(2\,m \times 0.008\,m\right)578 J/kg\cdot K\left(1250 - 300\right)K$$
$$+\left(2\times 2m\times 25m + 2\times 2m\times 2.4m + 2.4m\times 25m\right)[10 W/m^2\cdot K\left(350 - 300\right)K$$
$$+0.8\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(350^4 - 300^4\right)K^4] + 1.4 W/m\cdot K\left(2.4m\times 25m\right)\left(350 - 300\right)K/0.5m$$

Continued.….

PROBLEM 1.40 (Cont.)

$$P_{elec} = 694,000\text{W} + 169.6\text{m}^2 (500 + 313)\text{W/m}^2 + 8400\text{W}$$

$$= (694,000 + 84,800 + 53,100 + 8400)\text{W} = 840\text{kW}$$ <

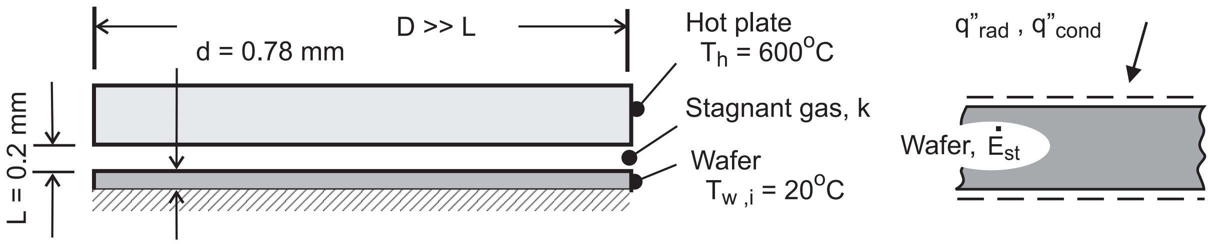
COMMENTS: Of the total energy input, 83% is transferred to the steel while approximately 10%, 6% and 1% are lost by convection, radiation and conduction from the oven. The convection and radiation losses can both be reduced by adding insulation to the side and top surfaces, which would reduce the corresponding value of T_s.

PROBLEM 1.41

KNOWN: Hot plate-type wafer thermal processing tool based upon heat transfer modes by conduction through gas within the gap and by radiation exchange across gap.

FIND: (a) Radiative and conduction heat fluxes across gap for specified hot plate and wafer temperatures and gap separation; initial time rate of change in wafer temperature for each mode, and (b) heat fluxes and initial temperature-time change for gap separations of 0.2, 0.5 and 1.0 mm for hot plate temperatures $300 < T_h < 1300°C$. Comment on the relative importance of the modes and the influence of the gap distance. Under what conditions could a wafer be heated to 900°C in less than 10 seconds?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions for flux calculations, (2) Diameter of hot plate and wafer much larger than gap spacing, approximating plane, infinite planes, (3) One-dimensional conduction through gas, (4) Hot plate and wafer are blackbodies, (5) Negligible heat losses from wafer backside, and (6) Wafer temperature is uniform at the onset of heating.

PROPERTIES: Wafer: $\rho = 2700$ kg/m^3, c = 875 J/kg·K; Gas in gap: k = 0.0436 W/m·K.

ANALYSIS: (a) The radiative heat flux between the hot plate and wafer for $T_h = 600°C$ and $T_w = 20° C$ follows from the rate equation,

$$q''_{rad} = \sigma\left(T_h^4 - T_w^4\right) = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left((600+273)^4 - (20+273)^4\right)\mathrm{K^4} = 32.5\,\mathrm{kW/m^2} \quad <$$

The conduction heat flux through the gas in the gap with L = 0.2 mm follows from Fourier's law,

$$q''_{cond} = k\frac{T_h - T_w}{L} = 0.0436\,\mathrm{W/m\cdot K}\,\frac{(600-20)\,\mathrm{K}}{0.0002\ \mathrm{m}} = 126\ \mathrm{kW/m^2} \quad <$$

The initial time rate of change of the wafer can be determined from an energy balance on the wafer at the instant of time the heating process begins,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad\qquad \dot{E}''_{st} = \rho c\, d\left(\frac{dT_w}{dt}\right)_i$$

where $\dot{E}''_{out} = 0$ and $\dot{E}''_{in} = q''_{rad}$ or q''_{cond}. Substituting numerical values, find

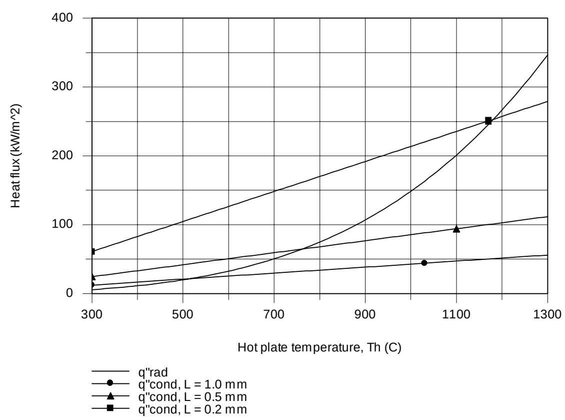
$$\left.\frac{dT_w}{dt}\right)_{i,rad} = \frac{q''_{rad}}{\rho c d} = \frac{32.5\times10^3\ \mathrm{W/m^2}}{2700\,\mathrm{kg/m^3}\times875\ \mathrm{J/kg\cdot K}\times0.00078\ \mathrm{m}} = 17.6\ \mathrm{K/s} \quad <$$

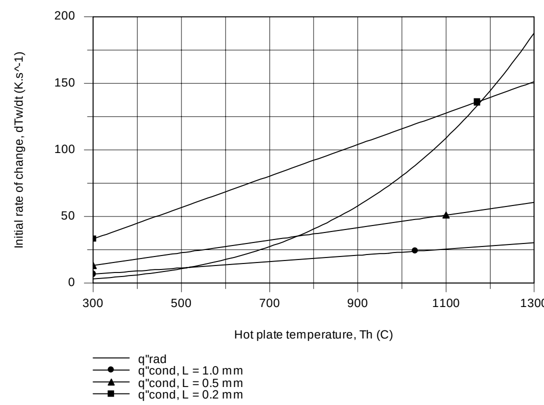
$$\left.\frac{dT_w}{dt}\right)_{i,cond} = \frac{q''_{cond}}{\rho c d} = 68.4\ \mathrm{K/s} \quad <$$

Continued …..

PROBLEM 1.41 (Cont.)

(b) Using the foregoing equations, the heat fluxes and initial rate of temperature change for each mode can be calculated for selected gap separations L and range of hot plate temperatures T_h with T_w = 20°C.





In the left-hand graph, the conduction heat flux increases linearly with T_h and inversely with L as expected. The radiative heat flux is independent of L and highly non-linear with T_h, but does not approach that for the highest conduction heat rate until T_h approaches 1200°C.

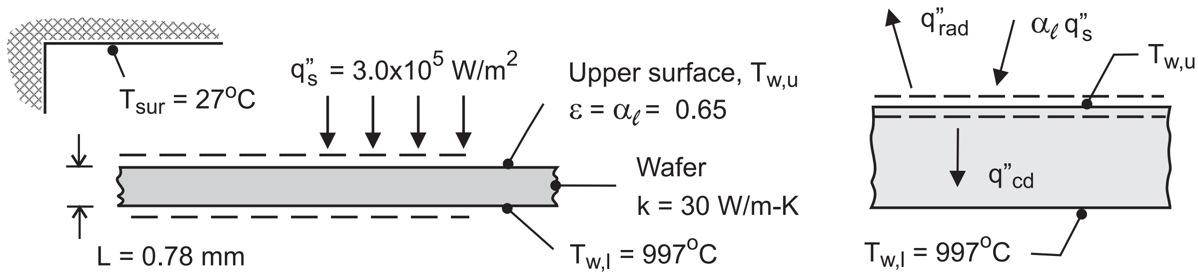
The general trends for the initial temperature-time change, $(dT_w/dt)_i$, follow those for the heat fluxes. To reach 900°C in 10 s requires an average temperature-time change rate of 90 K/s. Recognizing that (dT_w/dt) will decrease with increasing T_w, this rate could be met only with a very high T_h and the smallest L.

PROBLEM 1.42

KNOWN: Silicon wafer, radiantly heated by lamps, experiencing an annealing process with known backside temperature.

FIND: Whether temperature difference across the wafer thickness is less than 2°C in order to avoid damaging the wafer.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wafer, (3) Radiation exchange between upper surface of wafer and surroundings is between a small object and a large enclosure, and (4) Vacuum condition in chamber, no convection.

PROPERTIES: Wafer: k = 30 W/m·K, $\varepsilon = \alpha_\ell = 0.65$.

ANALYSIS: Perform a surface energy balance on the upper surface of the wafer to determine $T_{w,u}$. The processes include the absorbed radiant flux from the lamps, radiation exchange with the chamber walls, and conduction through the wafer.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha_\ell q''_s - q''_{rad} - q''_{cd} = 0$$

$$\alpha_\ell q''_s - \varepsilon\sigma\left(T^4_{w,u} - T^4_{sur}\right) - k\frac{T_{w,u} - T_{w,\ell}}{L} = 0$$

$$0.65\times 3.0\times 10^5\,\mathrm{W/m^2} - 0.65\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(T^4_{w,u} - (27+273)^4\right)\mathrm{K}^4$$

$$-30\,\mathrm{W/m\cdot K}\left[T_{w,u} - (997+273)\right]\mathrm{K}/0.00078\ \mathrm{m} = 0$$

$$T_{w,u} = 1273\,\mathrm{K} = 1000°\mathrm{C}$$ <

COMMENTS: (1) The temperature difference for this steady-state operating condition, $T_{w,u} - T_{w,l}$, is larger than 2°C. Warping of the wafer and inducing slip planes in the crystal structure could occur.

(2) The radiation exchange rate equation requires that temperature must be expressed in kelvin units. Why is it permissible to use kelvin or Celsius temperature units in the conduction rate equation?

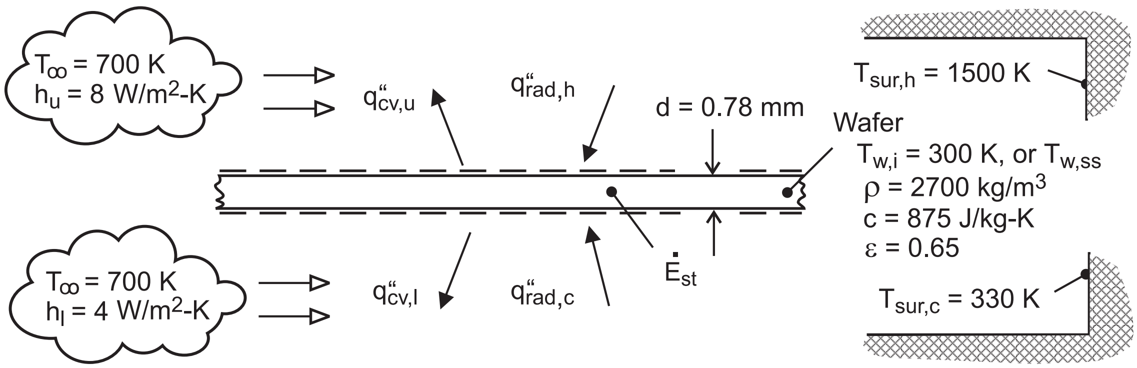
(3) Note how the surface energy balance, Eq. 1.12, is represented schematically. It is essential to show the control surfaces, and then identify the rate processes associated with the surfaces. Make sure the directions (in or out) of the process are consistent with the energy balance equation.

PROBLEM 1.43

KNOWN: Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of the wafer temperature corresponding to the wafer temperature $T_{w,i} = 300\,\text{K}$, and (b) Steady-state temperature reached if the wafer remains in this position. How significant is convection for this situation? Sketch how you'd expect the wafer temperature to vary as a function of vertical distance.

SCHEMATIC:



ASSUMPTIONS: (1) Wafer temperature is uniform, (2) Transient conditions when wafer is initially positioned, (3) Hot and cool zones have uniform temperatures, (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone), and (4) Negligible heat losses from wafer to mounting pin holder.

ANALYSIS: The energy balance on the wafer illustrated in the schematic above includes convection from the upper (u) and lower (l) surfaces with the ambient gas, radiation exchange with the hot- and cool-zone (chamber) surroundings, and the rate of energy storage term for the transient condition.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$q''_{rad,h} + q''_{rad,c} - q''_{cv,u} - q''_{cv,l} = \rho c d \frac{dT_w}{dt}$$

$$\varepsilon\sigma\left(T_{sur,h}^4 - T_w^4\right) + \varepsilon\sigma\left(T_{sur,c}^4 - T_w^4\right) - h_u\left(T_w - T_\infty\right) - h_l\left(T_w - T_\infty\right) = \rho c d \frac{dT_w}{dt}$$

(a) For the initial condition, the time rate of temperature change of the wafer is determined using the energy balance above with $T_w = T_{w,i} = 300\,\text{K}$,

$$0.65 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(1500^4 - 300^4\right)\text{K}^4 + 0.65 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(330^4 - 300^4\right)\text{K}^4$$

$$-8\,\text{W/m}^2 \cdot \text{K}\left(300 - 700\right)\text{K} - 4\,\text{W/m}^2 \cdot \text{K}\left(300 - 700\right)\text{K} =$$

$$2700\,\text{kg/m}^3 \times 875\,\text{J/kg} \cdot \text{K} \times 0.00078\,\text{m}\left(dT_w/dt\right)_i$$

$$\left(dT_w/dt\right)_i = 104\,\text{K/s}$$ <

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, $T_w = T_{w,ss}$.

Continued …..

PROBLEM 1.43 (Cont.)

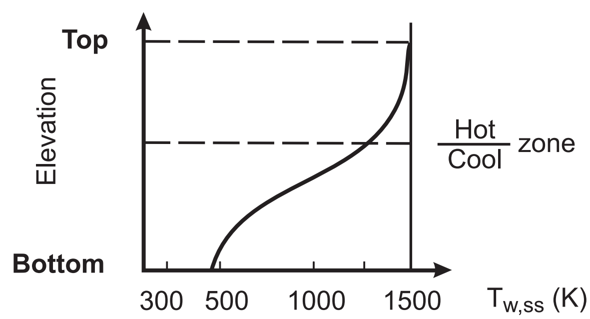
$$0.65\sigma\left(1500^4 - T_{w,ss}^4\right)K^4 + 0.65\sigma\left(330^4 - T_{w,ss}^4\right)K^4$$

$$-8\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K - 4\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K = 0$$

$$T_{w,ss} = 1251\ K$$ <

To determine the relative importance of the convection processes, re-solve the energy balance above ignoring those processes to find $\left(dT_w/dt\right)_i = 101\ K/s$ and $T_{w,ss} = 1262\ K$. We conclude that the radiation exchange processes control the initial time rate of temperature change and the steady-state temperature.

If the wafer were elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone chamber. Conversely, by lowering the wafer, the upper surface would experience less radiant exchange with the hot zone chamber, and its temperature would decrease. The temperature-distance trend might appear as shown in the sketch.

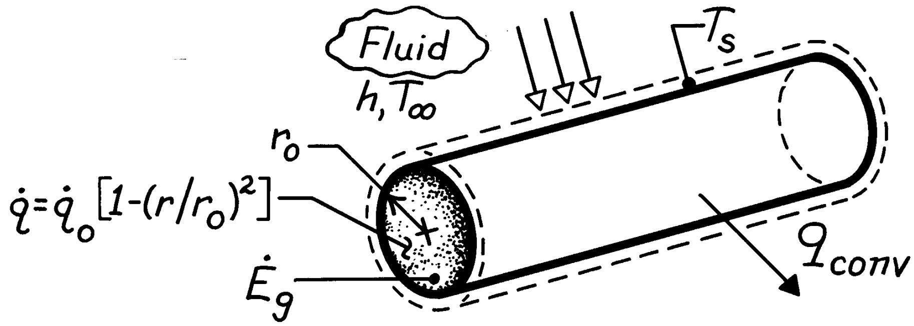


PROBLEM 1.44

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Total energy generation rate and surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature drop across thin container wall.

ANALYSIS: The rate of energy generation is

$$\dot{E}_g = \int \dot{q} dV = \dot{q}_o \int_0^{r_o} \left[1-(r/r_o)^2\right] 2\pi r L dr$$

$$\dot{E}_g = 2\pi L \dot{q}_o \left(r_o^2/2 - r_o^2/4\right)$$

or per unit length,

$$\dot{E}'_g = \frac{\pi \dot{q}_o r_o^2}{2}.$$ <

Performing an energy balance for a control surface about the container yields, at an instant,

$$\dot{E}'_g - \dot{E}'_{out} = 0$$

and substituting for the convection heat rate per unit length,

$$\frac{\pi \dot{q}_o r_o^2}{2} = h(2\pi r_o)(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}.$$ <

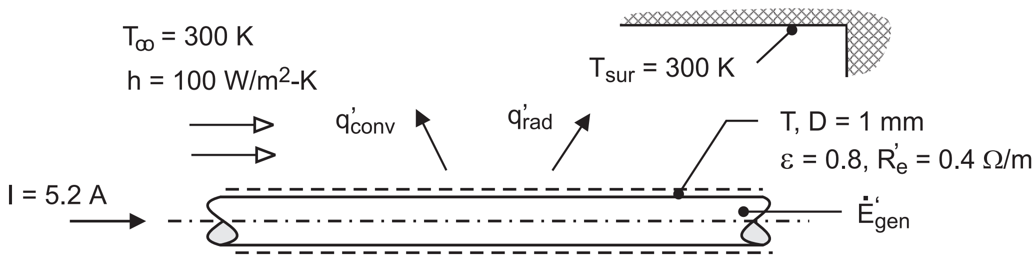
COMMENTS: The temperature within the radioactive wastes increases with decreasing r from T_s at r_o to a maximum value at the centerline.

PROBLEM 1.45

KNOWN: Rod of prescribed diameter experiencing electrical dissipation from passage of electrical current and convection under different air velocity conditions. See Example 1.3.

FIND: Rod temperature as a function of the electrical current for $0 \le I \le 10$ A with convection coefficients of 50, 100 and 250 $W/m^2 \cdot K$. Will variations in the surface emissivity have a significant effect on the rod temperature?

SCHEMATIC:



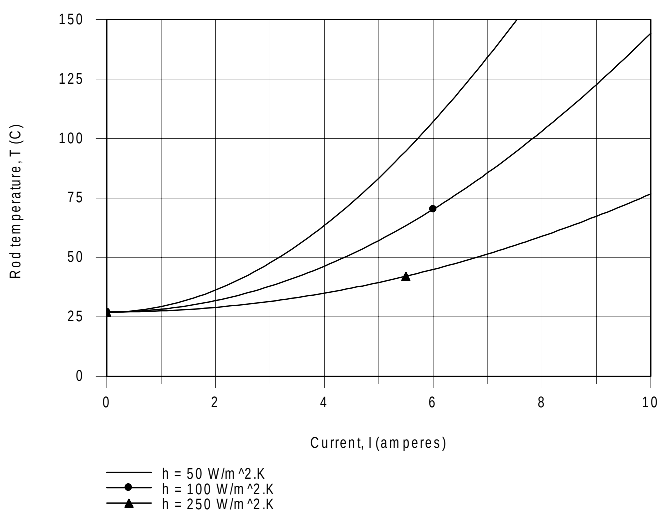
ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform rod temperature, (3) Radiation exchange between the outer surface of the rod and the surroundings is between a small surface and large enclosure.

ANALYSIS: The energy balance on the rod for steady-state conditions has the form,

$$q'_{conv} + q'_{rad} = \dot{E}'_{gen}$$

$$\pi Dh\left(T - T_\infty\right) + \pi D\varepsilon\sigma\left(T^4 - T_{sur}^4\right) = I^2 R'_e$$

Using this equation in the Workspace of IHT, the rod temperature is calculated and plotted as a function of current for selected convection coefficients.



COMMENTS: (1) For forced convection over the cylinder, the convection heat transfer coefficient is dependent upon air velocity approximately as $h \sim V^{0.6}$. Hence, to achieve a 5-fold change in the convection coefficient (from 50 to 250 $W/m^2 \cdot K$), the air velocity must be changed by a factor of nearly 15.

Continued …..

PROBLEM 1.45 (Cont.)

(2) For the condition of I = 4 A with h = 50 W/m^2·K with T = 63.5°C, the convection and radiation exchange rates per unit length are, respectively, $q'_{cv} = 5.7\ W/m$ and $q'_{rad} = 0.67\ W/m$. We conclude that convection is the dominate heat transfer mode and that changes in surface emissivity could have only a minor effect. Will this also be the case if h = 100 or 250 W/m^2·K?

(3) What would happen to the rod temperature if there was a "loss of coolant" condition where the air flow would cease?

(4) The Workspace for the IHT program to calculate the heat losses and perform the parametric analysis to generate the graph is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results. It is also good practice to show plots in *customary* units, that is, the units used to prescribe the problem. As such the graph of the rod temperature is shown above with Celsius units, even though the calculations require temperatures in kelvins.

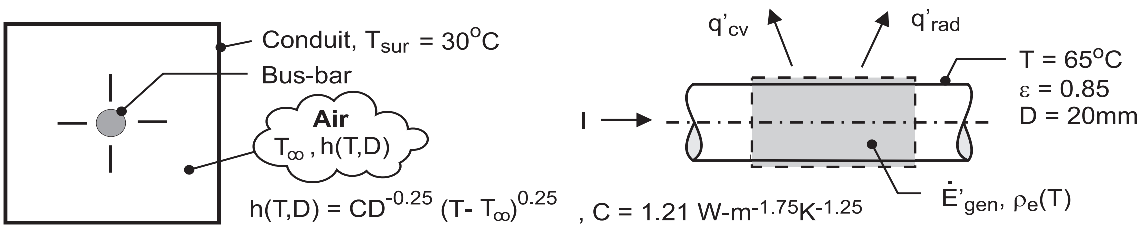
```
// Energy balance; from Ex. 1.3, Comment 1
-q'cv - q'rad + Edot'g = 0
q'cv = pi*D*h*(T - Tinf)
q'rad = pi*D*eps*sigma*(T^4 - Tsur^4)
sigma = 5.67e-8

// The generation term has the form
Edot'g = I^2*R'e
qdot = I^2*R'e / (pi*D^2/4)

// Input parameters
D = 0.001
Tsur = 300
T_C = T – 273          // Representing temperature in Celsius units using _C subscript
eps = 0.8
Tinf = 300
h = 100
//h = 50               // Values of coefficient for parameter study
//h = 250
I = 5.2                // For graph, sweep over range from 0 to 10 A
//I = 4                // For evaluation of heat rates with h = 50 W/m^2.K
R'e = 0.4

/*  Base case results: I = 5.2 A with h = 100 W/m^2.K, find T = 60 C (Comment 2 case).
Edot'g      T       T_C     q'cv    q'rad   qdot            D       I       R'e
            Tinf    Tsur    eps     h       sigma
10.82       332.6   59.55   10.23   0.5886  1.377E7         0.001   5.2     0.4
            300     300     0.8     100     5.67E-8   */

/* Results: I = 4 A with h = 50 W/m^2.K, find q'cv =  5.7 W/m and q'rad = 0.67 W/m
Edot'g      T       T_C     q'cv    q'rad   qdot    D       I       R'e
      Tinf  Tsur    eps     h       sigma
6.4         336.5   63.47   5.728   0.6721  8.149E6 0.001   4       0.4
      300   300     0.8     50      5.67E-8     */
```

PROBLEM 1.46

KNOWN: Long bus bar of prescribed diameter and ambient air and surroundings temperatures. Relations for the electrical resistivity and free convection coefficient as a function of temperature.

FIND: (a) Current carrying capacity of the bus bar if its surface temperature is not to exceed 65°C; compare relative importance of convection and radiation exchange heat rates, and (b) Show graphically the operating temperature of the bus bar as a function of current for the range $100 \le I \le 5000$ A for bus-bar diameters of 10, 20 and 40 mm. Plot the ratio of the heat transfer by convection to the total heat transfer for these conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Bus bar and conduit are very long in direction normal to page, (3) Uniform bus-bar temperature, (4) Radiation exchange between the outer surface of the bus bar and the conduit is between a small surface and a large enclosure.

PROPERTIES: Bus-bar material, $\rho_e = \rho_{e,o}\left[1+\alpha\left(T-T_o\right)\right]$, $\rho_{e,o} = 0.0171\mu\Omega\cdot m$, $T_o = 25°C$, $\alpha = 0.00396\,K^{-1}$.

ANALYSIS: An energy balance on the bus-bar for a unit length as shown in the schematic above has the form

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q'_{rad} - q'_{conv} + I^2R'_e = 0$$

$$-\varepsilon\pi D\sigma\left(T^4 - T_{sur}^4\right) - h\pi D\left(T - T_\infty\right) + I^2\rho_e / A_c = 0$$

where $R'_e = \rho_e / A_c$ and $A_c = \pi D^2/4$. Using the relations for $\rho_e(T)$ and $h(T,D)$, and substituting numerical values with T = 65°C, find

$$q'_{rad} = 0.85\pi\left(0.020m\right)\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(\left[65+273\right]^4 - \left[30+273\right]^4\right)K^4 = 223\,W/m \quad <$$

$$q'_{conv} = 7.83\,W/m^2\cdot K\ \pi\left(0.020m\right)\left(65-30\right)K = 17.2\,W/m \quad <$$

where $h = 1.21\,W\cdot m^{-1.75}\cdot K^{-1.25}\left(0.020m\right)^{-0.25}\left(65-30\right)^{0.25} = 7.83\,W/m^2\cdot K$

$$I^2R'_e = I^2\left(198.2\times 10^{-6}\Omega\cdot m\right)/\pi\left(0.020\right)^2 m^2/4 = 6.31\times 10^{-5} I^2\ W/m$$

where $\rho_e = 0.0171\times 10^{-6}\Omega\cdot m\left[1 + 0.00396\,K^{-1}\left(65-25\right)K\right] = 198.2\,\mu\Omega\cdot m$

The maximum allowable current capacity and the ratio of the convection to total heat transfer rate are

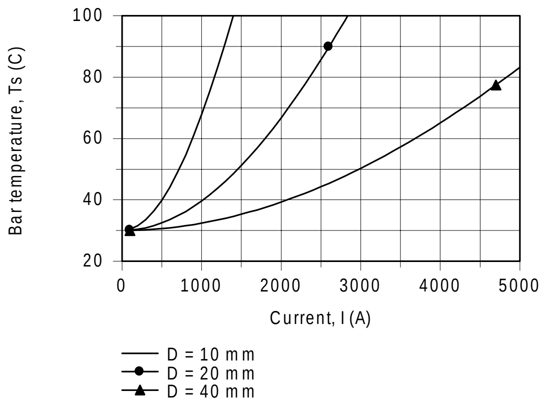
$$I = 1950\,A \qquad q'_{cv}/\left(q'_{cv}+q'_{rad}\right) = q'_{cv}/q'_{tot} = 0.072 \quad <$$

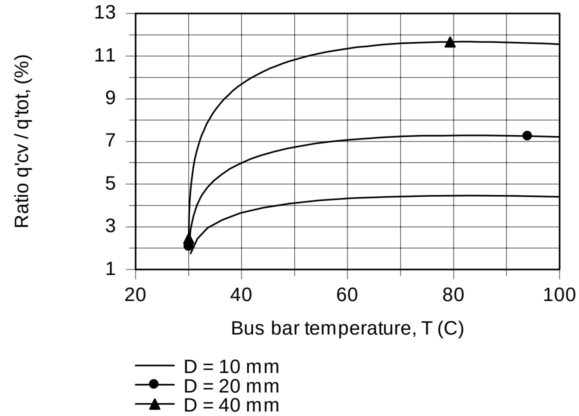
For this operating condition, convection heat transfer is only 7.2% of the total heat transfer.

(b) Using these equations in the Workspace of IHT, the bus-bar operating temperature is calculated and plotted as a function of the current for the range $100 \le I \le 5000$ A for diameters of 10, 20 and 40 mm. Also shown below is the corresponding graph of the ratio (expressed in percentage units) of the heat transfer by convection to the total heat transfer, q'_{cv}/q'_{tot}.

Continued …..

PROBLEM 1.46 (Cont.)





COMMENTS: (1) The trade-off between current-carrying capacity, operating temperature and bar diameter is shown in the first graph. If the surface temperature is not to exceed 65°C, the maximum current capacities for the 10, 20 and 40-mm diameter bus bars are 960, 1950, and 4000 A, respectively.

(2) From the second graph with q'_{cv} / q'_{tot} vs. T, note that the convection heat transfer rate is always a small fraction of the total heat transfer. That is, radiation is the dominant mode of heat transfer. Note also that the convection contribution increases with increasing diameter.

(3) The Workspace for the IHT program to perform the parametric analysis and generate the graphs is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

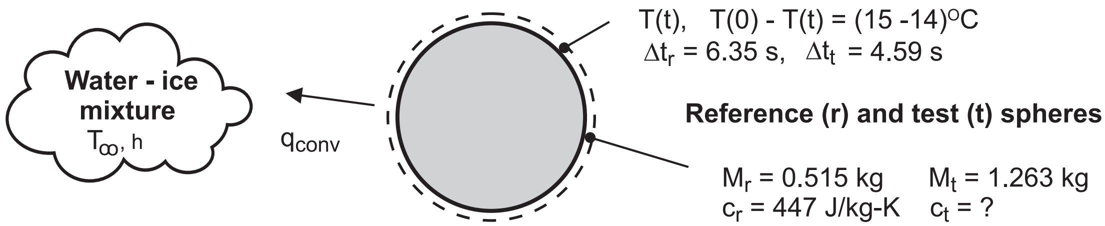
```
/* Results: base-case conditions, Part (a)
I         R'e       cvovertot  hbar     q'cv     q'rad    rhoe       D      Tinf_C   Ts_C
          Tsur_C    eps
1950      6.309E-5  7.171      7.826    17.21    222.8    1.982E-8   0.02   30       65
          30        0.85 */

// Energy balance, on a per unit length basis; steady-state conditions
// Edot'in - Edot'out + Edot'gen = 0
-q'cv - q'rad + Edot'gen = 0
q'cv = hbar * P * (Ts - Tinf)
P = pi * D
q'rad = eps * sigma * (Ts^4 - Tsur^4)
sigma = 5.67e-8
Edot'gen = I^2 * R'e
R'e = rhoe / Ac
rhoe = rhoeo * (1 + alpha * (Ts - To) )
To = 25 + 273
Ac = pi * D^2 / 4

// Convection coefficient
hbar = 1.21 * (D^-0.25) * (Ts - Tinf)^0.25          // Compact convection coeff. correlation
// Convection vs. total heat rates
cvovertot = q'cv / (q'cv + q'rad) * 100

// Input parameters
D = 0.020
// D = 0.010                  // Values of diameter for parameter study
// D = 0.040
// I = 1950                   // Base case condition unknown
rhoeo = 0.01711e-6
alpha = 0.00396
Tinf_C  = 30
Tinf = Tinf_C + 273
Ts_C = 65                     // Base case condition to determine current
Ts = Ts_C + 273
Tsur_C = 30
Tsur = Tsur_C + 273
eps = 0.85
```

PROBLEM 1.47

KNOWN: Elapsed times corresponding to a temperature change from 15 to 14°C for a reference sphere and test sphere of unknown composition suddenly immersed in a stirred water-ice mixture. Mass and specific heat of reference sphere.

FIND: Specific heat of the test sphere of known mass.

SCHEMATIC:



ASSUMPTIONS: (1) Spheres are of equal diameter, (2) Spheres experience temperature change from 15 to 14°C, (3) Spheres experience same convection heat transfer rate when the time rates of surface temperature are observed, (4) At any time, the temperatures of the spheres are uniform, (5) Negligible heat loss through the thermocouple wires.

PROPERTIES: Reference-grade sphere material: c_r = 447 J/kg K.

ANALYSIS: Apply the conservation of energy requirement at an instant of time, Eq. 1.11a, after a sphere has been immersed in the ice-water mixture at T_∞.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-q_{conv} = Mc\frac{dT}{dt}$$

where $q_{conv} = h\,A_s\left(T - T_\infty\right)$. Since the temperatures of the spheres are uniform, the change in energy storage term can be represented with the time rate of temperature change, dT/dt. The convection heat rates are equal at this instant of time, and hence the change in energy storage terms for the reference (r) and test (t) spheres must be equal.

$$M_r\ c_r\ \left.\frac{dT}{dt}\right)_r = M_t\ c_t\ \left.\frac{dT}{dt}\right)_t$$

Approximating the instantaneous differential change, dT/dt, by the difference change over a short period of time, ΔT/Δt, the specific heat of the test sphere can be calculated.

$$0.515\ \text{kg}\times 447\,\text{J/kg}\cdot\text{K}\ \frac{(15-14)\text{K}}{6.35\,\text{s}} = 1.263\,\text{kg}\times c_t\times\frac{(15-14)\text{K}}{4.59\,\text{s}}$$

$$c_t = 132\ \text{J/kg}\cdot\text{K}$$ <

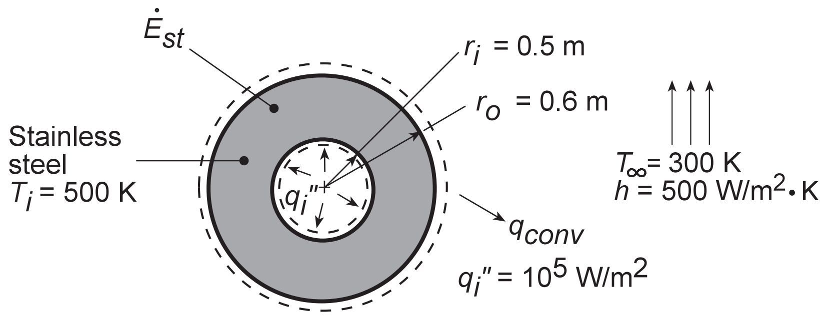
COMMENTS: Why was it important to perform the experiments with the reference and test spheres over the same temperature range (from 15 to 14°C)? Why does the analysis require that the spheres have uniform temperatures at all times?

PROBLEM 1.48

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: *Table A.1*, Stainless Steel, AISI 302: ρ = 8055 kg/m^3, c_p = 510 J/kg·K.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$. Identifying relevant processes and solving for dT/dt,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho\frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p\frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q_i'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right].$$

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right)_i = \frac{3\left[10^5\frac{W}{m^2}(0.5m)^2 - 500\frac{W}{m^2\cdot K}(0.6m)^2(500-300)K\right]}{8055\frac{kg}{m^3}510\frac{J}{kg\cdot K}\left[(0.6)^3 - (0.5)^3\right]m^3}$$

$$\left.\frac{dT}{dt}\right)_i = -0.089\,K/s.$$ <

(b) Under steady-state conditions with $\dot{E}_{st}$ = 0, it follows that

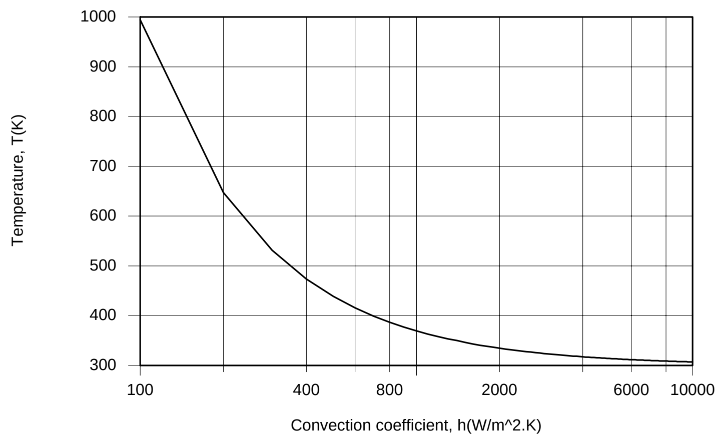
$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

Continued …..

PROBLEM 1.48 (Cont.)

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300\text{K} + \frac{10^5\,\text{W/m}^2}{500\,\text{W/m}^2\cdot\text{K}}\left(\frac{0.5\text{m}}{0.6\text{m}}\right)^2 = 439\text{K} \qquad <$$

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Hollow Sphere*. As shown below, there is a sharp increase in temperature with decreasing values of h < 1000 $\text{W/m}^2\cdot\text{K}$. For T > 380 K, boiling will occur at the canister surface, and for T > 410 K a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in h and increase in T.



Although the canister remains well below the melting point of stainless steel for h = 100 $\text{W/m}^2\cdot\text{K}$, boiling should be avoided, in which case the convection coefficient should be maintained at h > 1000 $\text{W/m}^2\cdot\text{K}$.

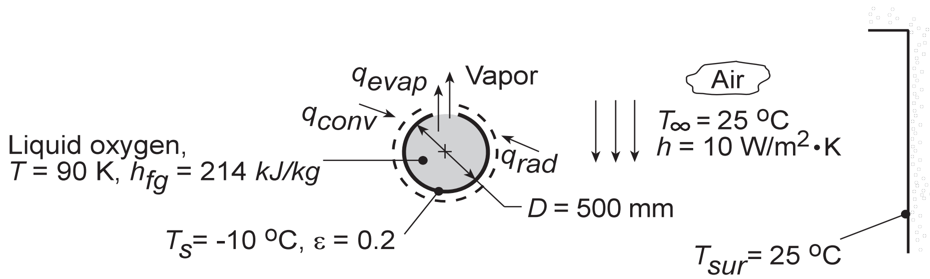
COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is $\theta = (S/R)\left(1 - e^{-Rt}\right) + \theta_i e^{-Rt}$, where $\theta \equiv T - T_\infty$, $S \equiv 3q_i'' r_i^2 / \rho c_p \left(r_o^3 - r_i^3\right)$, $R = 3hr_o^2/\rho c_p \left(r_o^3 - r_i^3\right)$. Note results for $t \to \infty$ and for S = 0.

PROBLEM 1.49

KNOWN: Boiling point and latent heat of liquid oxygen. Diameter and emissivity of container. Free convection coefficient and temperature of surrounding air and walls.

FIND: Mass evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of container outer surface equals boiling point of oxygen.

ANALYSIS: (a) Applying an energy balance to a control surface about the container, it follows that, at any instant,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \text{or} \qquad q_{conv} + q_{rad} - q_{evap} = 0 .$$

The evaporative heat loss is equal to the product of the mass rate of vapor production and the heat of vaporization. Hence,

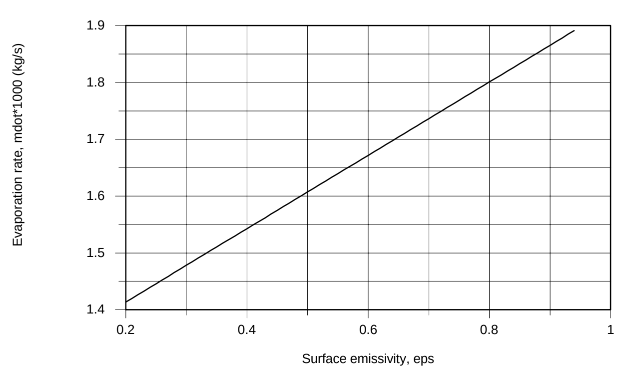
$$\left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]A_s - \dot{m}_{evap}h_{fg} = 0 \qquad (1)$$

$$\dot{m}_{evap} = \frac{\left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]\pi D^2}{h_{fg}}$$

$$\dot{m}_{evap} = \frac{\left[10\,\mathrm{W/m^2\cdot K}\left(298-263\right)\mathrm{K} + 0.2\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(298^4 - 263^4\right)\mathrm{K}^4\right]\pi\left(0.5\,\mathrm{m}\right)^2}{214\,\mathrm{kJ/kg}}$$

$$\dot{m}_{evap} = \frac{\left(350+35.2\right)\mathrm{W/m^2}\left(0.785\ \mathrm{m^2}\right)}{214\,\mathrm{kJ/kg}} = 1.41\times 10^{-3}\ \mathrm{kg/s} . \qquad <$$

(b) Using the energy balance, Eq. (1), the mass rate of vapor production can be determined for the range of emissivity 0.2 to 0.94. The effect of increasing emissivity is to increase the heat rate into the container and, hence, increase the vapor production rate.



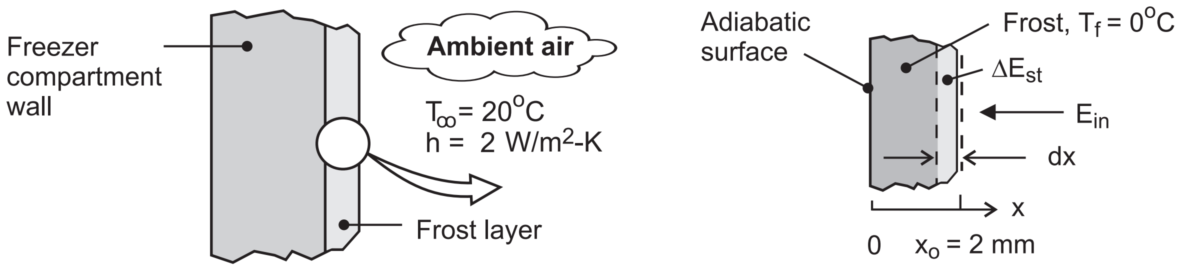
COMMENTS: To reduce the loss of oxygen due to vapor production, insulation should be applied to the outer surface of the container, in order to reduce q_{conv} and q_{rad}. Note from the calculations in part (a), that heat transfer by convection is greater than by radiation exchange.

PROBLEM 1.50

KNOWN: Frost formation of 2-mm thickness on a freezer compartment. Surface exposed to convection process with ambient air.

FIND: Time required for the frost to melt, t_m.

SCHEMATIC:



ASSUMPTIONS: (1) Frost is isothermal at the fusion temperature, T_f, (2) The water melt falls away from the exposed surface, (3) Negligible radiation exchange at the exposed surface, and (4) Backside surface of frost formation is adiabatic.

PROPERTIES: Frost, $\rho_f = 770\ \text{kg/m}^3$, $h_{sf} = 334\ \text{kJ/kg}$.

ANALYSIS: The time t_m required to melt a 2-mm thick frost layer may be determined by applying an energy balance, Eq 1.11b, over the differential time interval dt and to a differential control volume extending inward from the surface of the layer dx. From the schematic above, the energy *in* is the convection heat flux over the time period dt and the change in energy storage is the latent energy change within the control volume, $A_s \cdot dx$.

$$E_{in} - E_{out} = E_{st}$$

$$q''_{conv} A_s dt = dU_{\ell at}$$

$$h\,A_s\left(T_\infty - T_f\right)dt = -\rho_f\,A_s\,h_{sf}\,dx$$

Integrating both sides of the equation and defining appropriate limits, find

$$h\left(T_\infty - T_f\right)\int_0^{t_m} dt = -\rho_f\,h_{sf}\int_{x_o}^{0} dx$$

$$t_m = \frac{\rho_f\,h_{sf}\,x_o}{h\left(T_\infty - T_f\right)}$$

$$t_m = \frac{700\ \text{kg/m}^3 \times 334\times10^3\ \text{J/kg} \times 0.002\text{m}}{2\ \text{W/m}^2\cdot\text{K}\left(20-0\right)\text{K}} = 11{,}690\ \text{s} = 3.2\ \text{hour} \qquad <$$

COMMENTS: (1) The energy balance could be formulated intuitively by recognizing that the total heat *in* by convection during the time interval $t_m\left(q''_{cv}\cdot t_m\right)$ must be equal to the total latent energy for melting the frost layer $\left(\rho\,x_o h_{sf}\right)$. This equality is directly comparable to the derived expression above for t_m.

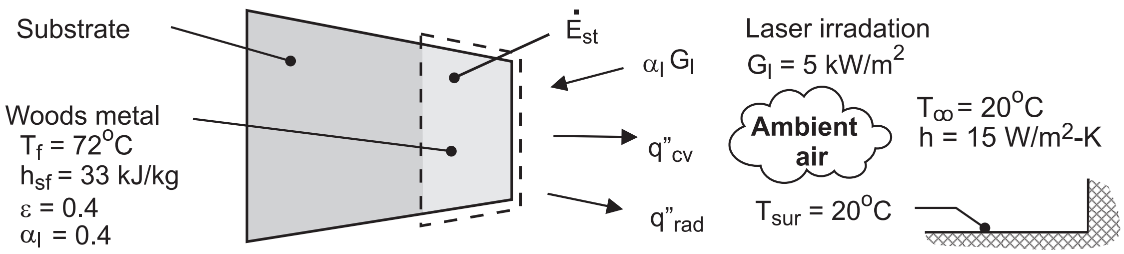
(2) Explain why the energy storage term in the analysis has a negative sign, and the limits of integration are as shown. *Hint*: Recall from the formulation of Eq. 1.11b, that the storage term represents the change between the final and initial states.

PROBLEM 1.51

KNOWN: Vertical slab of Woods metal initially at its fusion temperature, T_f, joined to a substrate. Exposed surface is irradiated with laser source, $G_\ell \left(W/m^2\right)$.

FIND: Instantaneous rate of melting per unit area, $\dot{m}''_m$ (kg/s·m^2), and the material removed in a period of 2 s, (a) Neglecting heat transfer from the irradiated surface by convection and radiation exchange, and (b) Allowing for convection and radiation exchange.

SCHEMATIC:



ASSUMPTIONS: (1) Woods metal slab is isothermal at the fusion temperature, T_f, and (2) The melt runs off the irradiated surface.

ANALYSIS: (a) The instantaneous rate of melting per unit area may be determined by applying an energy balance, Eq 1.11a, on the metal slab at an instant of time neglecting convection and radiation exchange from the irradiated surface.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad\qquad \alpha_\ell G_\ell = \frac{d}{dt}\left(-M'' h_{sf}\right) = -h_{sf}\frac{dM''}{dt}$$

where $dM''/dt = \dot{m}''_m$ is the time rate of change of mass in the control volume. Substituting values,

$$0.4\times 5000\,W/m^2 = -33,000\,J/kg\times\dot{m}''_m \qquad \dot{m}''_m = -60.6\times10^{-3}\,kg/s\cdot m^2 \qquad <$$

The material removed in a 2s period per unit area is

$$M''_{2s} = \dot{m}''_m\cdot\Delta t = 121\ g/m^2 \qquad <$$

(b) The energy balance considering convection and radiation exchange with the surroundings yields

$$\alpha_\ell G_\ell - q''_{cv} - q''_{rad} = -h_{sf}\,\dot{m}''_m$$

$$q''_{cv} = h\left(T_f - T_\infty\right) = 15\,W/m^2\cdot K\left(72-20\right)K = 780\,W/m^2$$

$$q''_{rad} = \varepsilon\sigma\left(T_f^4 - T_\infty^4\right) = 0.4\times5.67\times10^{-8}\,W/m^2\cdot K\left(\left[72+273\right]^4 - \left[20+273\right]^4\right)K^4 = 154\,W/m^2$$

$$\dot{m}''_m = -32.3\times10^{-3}\,kg/s\cdot m^2 \qquad M_{2s} = 64\,g/m^2 \qquad <$$

COMMENTS: (1) The effects of heat transfer by convection and radiation reduce the estimate for the material removal rate by a factor of two. The heat transfer by convection is nearly 5 times larger than by radiation exchange.

(2) Suppose the work piece were horizontal, rather than vertical, and the melt puddled on the surface rather than ran off. How would this affect the analysis?

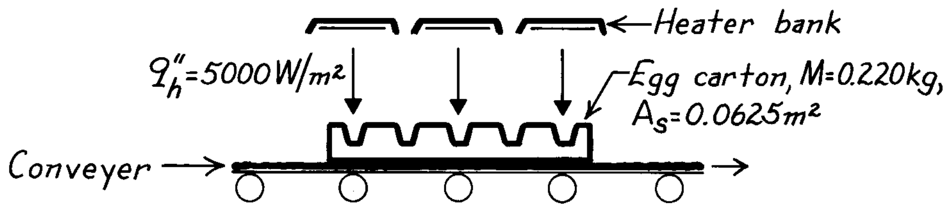
(3) Lasers are common heating sources for metals processing, including the present application of melting (heat transfer with phase change), as well as for heating work pieces during milling and turning (laser-assisted machining).

PROBLEM 1.52

KNOWN: Hot formed paper egg carton of prescribed mass, surface area and water content exposed to infrared heater providing known radiant flux.

FIND: Whether water content can be reduced from 75% to 65% by weight during the 18s period carton is on conveyor.

SCHEMATIC:

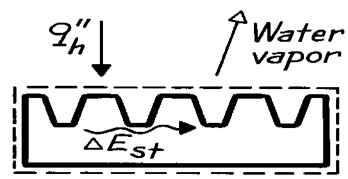


ASSUMPTIONS: (1) All the radiant flux from the heater bank is absorbed by the carton, (2) Negligible heat loss from carton by convection and radiation, (3) Negligible mass loss occurs from bottom side.

PROPERTIES: Water (given): h_{fg} = 2400 kJ/kg.

ANALYSIS: Define a control surface about the carton, and write the conservation of energy requirement for an interval of time, Δt,

$$E_{in} - E_{out} = \Delta E_{st} = 0$$



where E_{in} is due to the absorbed radiant flux, q_h'', from the heater and E_{out} is the energy leaving due to evaporation of water from the carton. Hence.

$$q_h'' \cdot A_s \cdot \Delta t = \Delta M \cdot h_{fg}.$$

For the prescribed radiant flux q_h'',

$$\Delta M = \frac{q_h'' A_s \Delta t}{h_{fg}} = \frac{5000\ W/m^2 \times 0.0625\ m^2 \times 18s}{2400\ kJ/kg} = 0.00234\ kg.$$

The chief engineer's requirement was to remove 10% of the water content, or

$$\Delta M_{req} = M \times 0.10 = 0.220\ kg \times 0.10 = 0.022\ kg$$

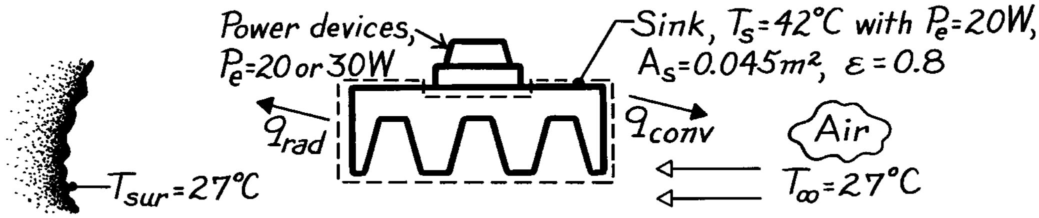
which is nearly an order of magnitude larger than the evaporative loss. Considering heat losses by convection and radiation, the actual water removal from the carton will be less than ΔM. Hence, the purchase should not be recommended, since the desired water removal cannot be achieved. <

PROBLEM 1.53

KNOWN: Average heat sink temperature when total dissipation is 20 W with prescribed air and surroundings temperature, sink surface area and emissivity.

FIND: Sink temperature when dissipation is 30 W.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) All dissipated power in devices is transferred to the sink, (3) Sink is isothermal, (4) Surroundings and air temperature remain the same for both power levels, (5) Convection coefficient is the same for both power levels, (6) Heat sink is a small surface within a large enclosure, the surroundings.

ANALYSIS: Define a control volume around the heat sink. Power dissipated within the devices is transferred into the sink, while the sink loses heat to the ambient air and surroundings by convection and radiation exchange, respectively.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_e - hA_s\left(T_s - T_\infty\right) - A_s\varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0. \tag{1}$$

Consider the situation when $P_e = 20$ W for which $T_s = 42°C$; find the value of h.

$$h = \left[P_e / A_s - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right] / \left(T_s - T_\infty\right)$$

$$h = \left[20\text{ W}/0.045\text{ m}^2 - 0.8\times5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(315^4 - 300^4\right)\text{K}^4\right] / \left(315 - 300\right)\text{K}$$

$$h = 24.4\text{ W/m}^2\cdot\text{K}.$$

For the situation when $P_e = 30$ W, using this value for h with Eq. (1), obtain

$$30\text{ W - }24.4\text{ W/m}^2\cdot\text{K}\times0.045\text{ m}^2\left(T_s - 300\right)\text{K}$$
$$-0.045\text{ m}^2\times0.8\times5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(T_s^4 - 300^4\right)\text{K}^4 = 0$$

$$30 = 1.098\left(T_s - 300\right) + 2.041\times10^{-9}\left(T_s^4 - 300^4\right).$$

By trial-and-error, find

$$T_s \approx 322\text{ K} = 49^\circ\text{C}.$$ <

COMMENTS: (1) It is good practice to express all temperatures in kelvin units when using energy balances involving radiation exchange.

(2) Note that we have assumed A_s is the same for the convection and radiation processes. Since not all portions of the fins are completely exposed to the surroundings, $A_{s,rad}$ is less than $A_{s,conv} = A_s$.

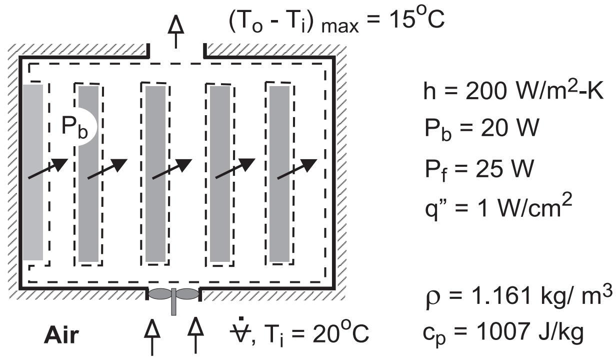
(3) Is the assumption that the heat sink is isothermal reasonable?

PROBLEM 1.54

KNOWN: Number and power dissipation of PCBs in a computer console. Convection coefficient associated with heat transfer from individual components in a board. Inlet temperature of cooling air and fan power requirement. Maximum allowable temperature rise of air. Heat flux from component most susceptible to thermal failure.

FIND: (a) Minimum allowable volumetric flow rate of air, (b) Preferred location and corresponding surface temperature of most thermally sensitive component.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible heat transfer from console to ambient air, (5) Uniform convection coefficient for all components.

ANALYSIS: (a) For a control surface about the air space in the console, conservation of energy for an open system, Eq. (1.11e), reduces to

$$\dot{m}(u+pv)_i - \dot{m}(u+pv)_o + q - \dot{W} = 0$$

where $u + pv = i$, $q = 5P_b$, and $\dot{W} = -P_f$. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = 5\ P_b + P_f$$

For a maximum allowable temperature rise of 15°C, the required mass flow rate is

$$\dot{m} = \frac{5\ P_b + P_f}{c_p(T_o - T_i)} = \frac{5\times 20\ W + 25\ W}{1007\ J/kg\cdot K\left(15\ ^\circ C\right)} = 8.28\times 10^{-3}\,kg/s$$

The corresponding volumetric flow rate is

$$\forall = \frac{\dot{m}}{\rho} = \frac{8.28\times 10^{-3}\,kg/s}{1.161\ kg/m^3} = 7.13\times 10^{-3}\,m^3/s \qquad <$$

(b) The component which is most susceptible to thermal failure should be mounted at the bottom of one of the PCBs, where the air is coolest. From the corresponding form of Newton's law of cooling, $q'' = h(T_s - T_i)$, the surface temperature is

$$T_s = T_i + \frac{q''}{h} = 20^\circ C + \frac{1\times 10^4\ W/m^2}{200\ W/m^2\cdot K} = 70^\circ C \qquad <$$

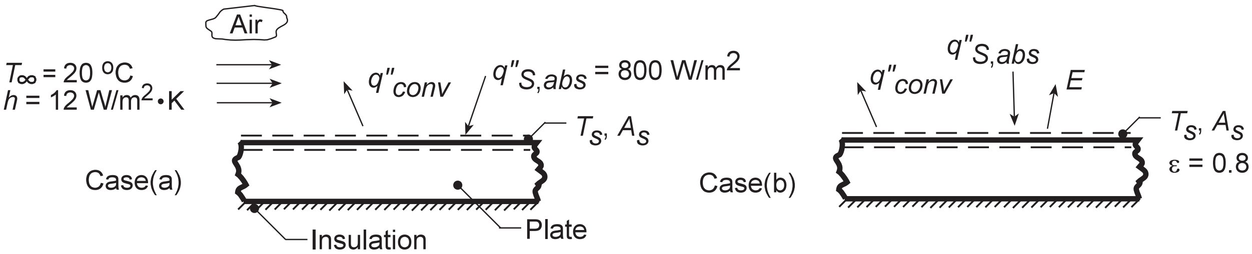
COMMENTS: (1) Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the console, causing a reduction in the density. However, for the prescribed temperature rise, the change in ρ, and hence the effect on $\dot{\forall}$, is small. (2) If the thermally sensitive component were located at the top of a PCB, it would be exposed to warmer air (T_o = 35°C) and the surface temperature would be T_s = 85°C.

PROBLEM 1.55

KNOWN: Top surface of car roof absorbs solar flux, $q''_{S,abs}$, and experiences for case (a): convection with air at T_∞ and for case (b): the same convection process and radiation emission from the roof.

FIND: Temperature of the plate, T_s, for the two cases. Effect of airflow on roof temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer to auto interior, (3) Negligible radiation from atmosphere.

ANALYSIS: (a) Apply an energy balance to the control surfaces shown on the schematic. For an instant of time, $\dot{E}_{in} - \dot{E}_{out}$ = 0. Neglecting radiation emission, the relevant processes are convection between the plate and the air, q''_{conv}, and the absorbed solar flux, $q''_{S,abs}$. Considering the roof to have an area A_S,

$$q''_{S,abs} \cdot A_S - hA_S\left(T_S - T_\infty\right) = 0$$

$$T_S = T_\infty + q''_{S,abs}/h$$

$$T_S = 20^\circ C + \frac{800 W/m^2}{12 W/m^2 \cdot K} = 20^\circ C + 66.7^\circ C = 86.7^\circ C$$ <

(b) With radiation emission from the surface, the energy balance has the form

$$q''_{S,abs} \cdot A_S - q_{conv} - E \cdot A_S = 0$$

$$q''_{S,abs} A_S - hA_S\left(T_S - T_\infty\right) - \varepsilon A_S \sigma T_S^4 = 0.$$

Substituting numerical values, with temperature in absolute units (K),

$$800\frac{W}{m^2} - 12\frac{W}{m^2 \cdot K}\left(T_S - 293K\right) - 0.8 \times 5.67 \times 10^{-8}\frac{W}{m^2 \cdot K^4} T_S^4 = 0$$

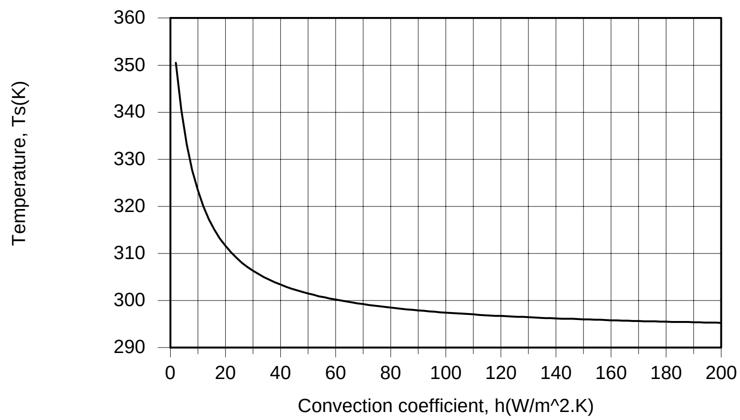
$$12 T_S + 4.536 \times 10^{-8} T_S^4 = 4316$$

It follows that T_s = 320 K = 47°C. <

Continued.….

PROBLEM 1.55 (Cont.)

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Plane Wall*. As shown below, the roof temperature depends strongly on the velocity of the auto relative to the ambient air. For a convection coefficient of h = 40 $W/m^2 \cdot K$, which would be typical for a velocity of 55 mph, the roof temperature would exceed the ambient temperature by less than 10°C.



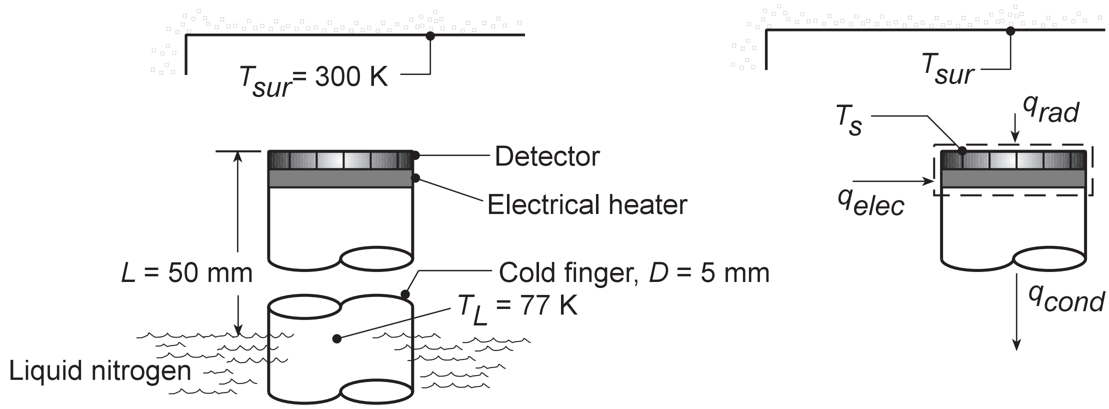
COMMENTS: By considering radiation emission, T_s decreases, as expected. Note the manner in which q''_{conv} is formulated using Newton's law of cooling; since q''_{conv} is shown leaving the control surface, the rate equation must be $h(T_S - T_\infty)$ and not $h(T_\infty - T_S)$.

PROBLEM 1.56

KNOWN: Detector and heater attached to cold finger immersed in liquid nitrogen. Detector surface of $\varepsilon = 0.9$ is exposed to large vacuum enclosure maintained at 300 K.

FIND: (a) Temperature of detector when no power is supplied to heater, (b) Heater power (W) required to maintain detector at 195 K, (c) Effect of finger thermal conductivity on heater power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through cold finger, (3) Detector and heater are very thin and isothermal at T_s, (4) Detector surface is small compared to enclosure surface.

PROPERTIES: Cold finger (given): $k = 10$ W/m·K.

ANALYSIS: Define a control volume about detector and heater and apply conservation of energy requirement on a rate basis, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \tag{1}$$

where

$$\dot{E}_{in} = q_{rad} + q_{elec}; \qquad \dot{E}_{out} = q_{cond} \tag{2,3}$$

Combining Eqs. (2,3) with (1), and using the appropriate rate equations,

$$\varepsilon A_s \sigma \left(T_{sur}^4 - T_s^4\right) + q_{elec} = k A_s \left(T_s - T_L\right)/L. \tag{4}$$

(a) Where $q_{elec} = 0$, substituting numerical values

$$0.9 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4} \left(300^4 - T_s^4\right)\mathrm{K}^4 = 10\,\mathrm{W/m \cdot K}\left(T_s - 77\right)\mathrm{K}/0.050\,\mathrm{m}$$

$$5.103 \times 10^{-8}\left(300^4 - T_s^4\right) = 200\left(T_s - 77\right)$$

$$T_s = 79.1\,\mathrm{K}$$ <

Continued.….

PROBLEM 1.56 (Cont.)

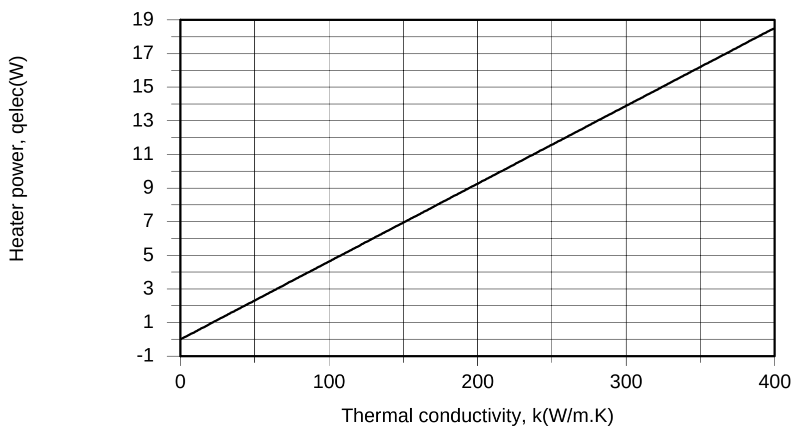
(b) When T_s = 195 K, Eq. (4) yields

$$0.9\times[\pi(0.005\,\text{m})^2/4]\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(300^4-195^4\right)\text{K}^4+q_{elec}$$

$$=10\,\text{W/m}\cdot\text{K}\times[\pi(0.005\,\text{m})^2/4]\times(195-77)\,\text{K}/0.050\,\text{m}$$

$$q_{elec}=0.457\,\text{W}=457\,\text{mW}$$ <

(c) Calculations were performed using the *First Law Model* for a *Nonisothermal Plane Wall*. With net radiative transfer to the detector fixed by the prescribed values of T_s and T_{sur}, Eq. (4) indicates that q_{elec} increases linearly with increasing k.



Heat transfer by conduction through the finger material increases with its thermal conductivity. Note that, for k = 0.1 W/m·K, q_{elec} = -2 mW, where the minus sign implies the need for a heat *sink*, rather than a heat source, to maintain the detector at 195 K. In this case q_{rad} exceeds q_{cond}, and a heat sink would be needed to dispose of the difference. A conductivity of k = 0.114 W/m·K yields a precise balance between q_{rad} and q_{cond}. Hence to circumvent heaving to use a heat sink, while minimizing the heater power requirement, k should exceed, but remain as close as possible to the value of 0.114 W/m·K. Using a graphite fiber composite, with the fibers oriented normal to the direction of conduction, Table A.2 indicates a value of k ≈ 0.54 W/m·K at an average finger temperature of $\bar{T}$ = 136 K. For this value, q_{elec} = 18 mW

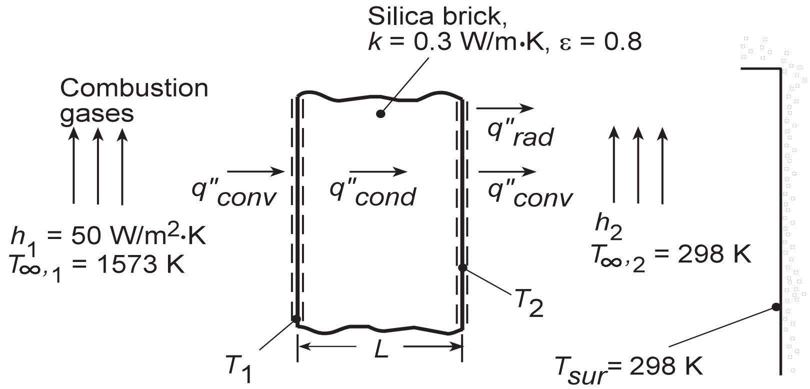
COMMENTS: The heater power requirement could be further reduced by decreasing ε.

PROBLEM 1.57

KNOWN: Conditions at opposite sides of a furnace wall of prescribed thickness, thermal conductivity and surface emissivity.

FIND: Effect of wall thickness and outer convection coefficient on surface temperatures. Recommended values of L and h_2.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation exchange at surface 1, (4) Surface 2 is exposed to large surroundings.

ANALYSIS: The unknown temperatures may be obtained by simultaneously solving energy balance equations for the two surface. At surface 1,

$$q''_{conv,1} = q''_{cond}$$

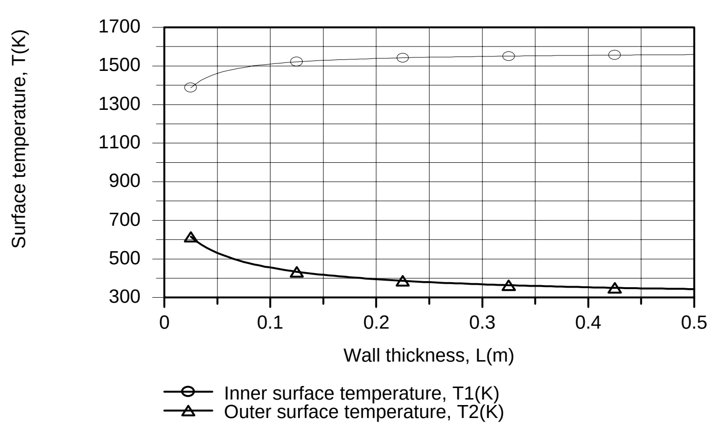
$$h_1\left(T_{\infty,1} - T_1\right) = k\left(T_1 - T_2\right)/L \tag{1}$$

At surface 2,

$$q''_{cond} = q''_{conv} + q''_{rad}$$

$$k\left(T_1 - T_2\right)/L = h_2\left(T_2 - T_{\infty,2}\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right) \tag{2}$$

Using the IHT *First Law Model* for a *Nonisothermal Plane Wall*, we obtain

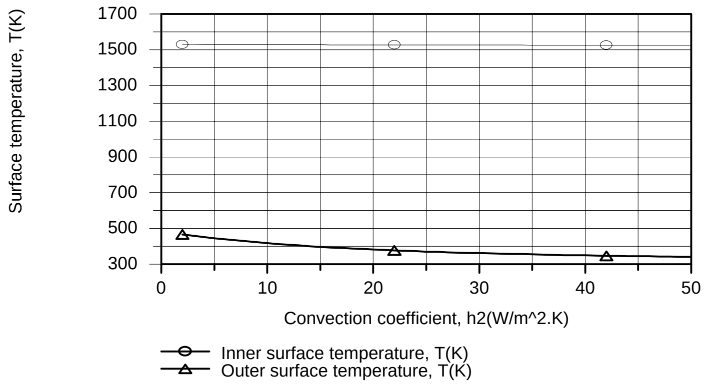


Continued …..

PROBLEM 1.57 (Cont.)

Both q''_{cond} and T_2 decrease with increasing wall thickness, and for the prescribed value of h_2 = 10 W/m^2·K, a value of L ≥ 0.275 m is needed to maintain $T_2 \leq 373$ K = 100 °C. Note that inner surface temperature T_1, and hence the temperature difference, $T_1 - T_2$, increases with increasing L.

Performing the calculations for the prescribed range of h_2, we obtain



For the prescribed value of L = 0.15 m, a value of $h_2 \geq 24$ W/m^2·K is needed to maintain $T_2 \leq 373$ K. The variation has a negligible effect on T_1, causing it to decrease slightly with increasing h_2, but does have a strong influence on T_2.

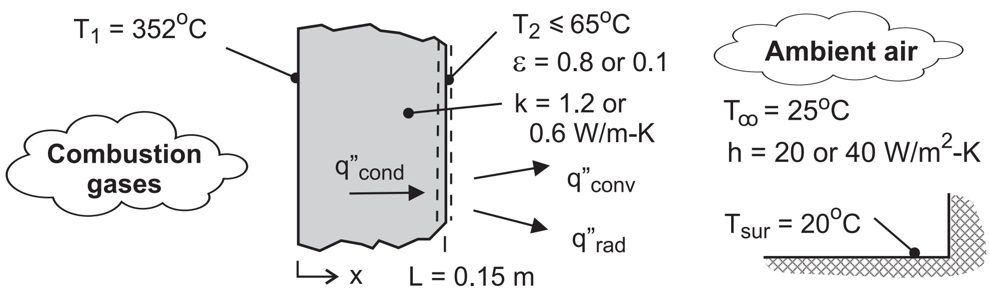
COMMENTS: If one wishes to avoid use of active (forced convection) cooling on side 2, reliance will have to be placed on free convection, for which $h_2 \approx 5$ W/m^2·K. The minimum wall thickness would then be L = 0.40 m.

PROBLEM 1.58

KNOWN: Furnace wall with inner surface temperature T_1 = 352°C and prescribed thermal conductivity experiencing convection and radiation exchange on outer surface. See Example 1.5.

FIND: (a) Outer surface temperature T_2 resulting from decreasing the wall thermal conductivity k or increasing the convection coefficient h by a factor of two; benefit of applying a low emissivity coating ($\varepsilon < 0.8$); comment on the effectiveness of these strategies to reduce risk of burn injury when $T_2 \le 65$°C; and (b) Calculate and plot T_2 as a function of h for the range $20 \le h \le 100$ W/m^2·K for three materials with k = 0.3, 0.6, and 1.2 W/m·K; what conditions will provide for safe outer surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Radiation exchange is between small surface and large enclosure, (4) Inner surface temperature remains constant for all conditions.

ANALYSIS: (a) The surface (x = L) energy balance is

$$k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right)$$

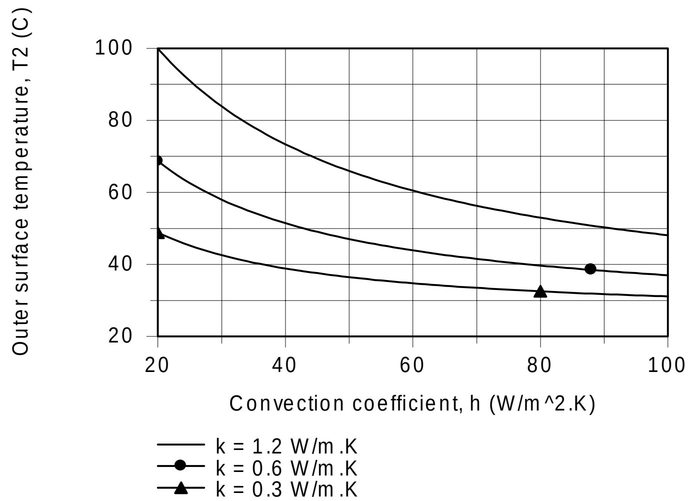
With T_1 = 352°C, the effects of parameters h, k and ε on the outer surface temperature are calculated and tabulated below.

Conditions	$k\left(W/m\cdot K\right)$	$h\left(W/m^2\cdot K\right)$	ε	$T_2\left(°C\right)$
Example 1.5	1.2	20	0.8	100
Decrease k by ½	0.6	20	0.8	69
Increase h by 2	1.2	40	0.8	73
Change k and h	0.6	40	0.8	51
Decrease ε	1.2	20	0.1	115

(b) Using the energy balance relation in the Workspace of IHT, the outer surface temperature can be calculated and plotted as a function of the convection coefficient for selected values of the wall thermal conductivity.

Continued …..

PROBLEM 1.58 (Cont.)



COMMENTS: (1) From the parameter study of part (a), note that decreasing the thermal conductivity is more effective in reducing T_2 than is increasing the convection coefficient. Only if both changes are made will T_2 be in the safe range.

(2) From part (a), note that applying a low emissivity coating is not beneficial. Did you suspect that before you did the analysis? Give a physical explanation for this result.

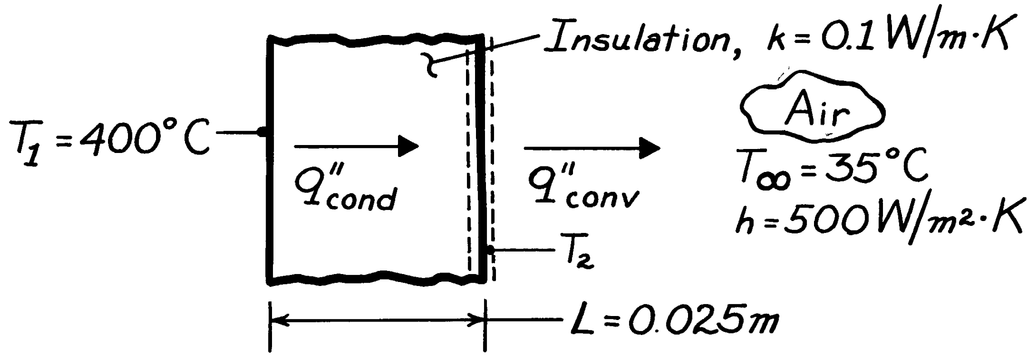
(3) From the parameter study graph we conclude that safe wall conditions ($T_2 \le 65°C$) can be maintained for these conditions: with k = 1.2 W/m·K when h > 55 $W/m^2·K$; with k = 0.6 W/m·K when h > 25 $W/m^2·K$; and with k = 0.3 W/m·K when h > 20 W/m·K.

PROBLEM 1.59

KNOWN: Inner surface temperature, thickness and thermal conductivity of insulation exposed at its outer surface to air of prescribed temperature and convection coefficient.

FIND: Outer surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the insulation, (3) Negligible radiation exchange between outer surface and surroundings.

ANALYSIS: From an energy balance at the outer surface at an instant of time,

$$q''_{cond} = q''_{conv}.$$

Using the appropriate rate equations,

$$k\frac{(T_1 - T_2)}{L} = h(T_2 - T_\infty).$$

Solving for T_2, find

$$T_2 = \frac{\frac{k}{L}T_1 + h\,T_\infty}{h + \frac{k}{L}} = \frac{\frac{0.1\ \text{W/m}\cdot\text{K}}{0.025\text{m}}\left(400^\circ\text{C}\right) + 500\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(35^\circ\text{C}\right)}{500\frac{\text{W}}{\text{m}^2\cdot\text{K}} + \frac{0.1\ \text{W/m}\cdot\text{K}}{0.025\text{m}}}$$

$$T_2 = 37.9^\circ\text{C}.$$ <

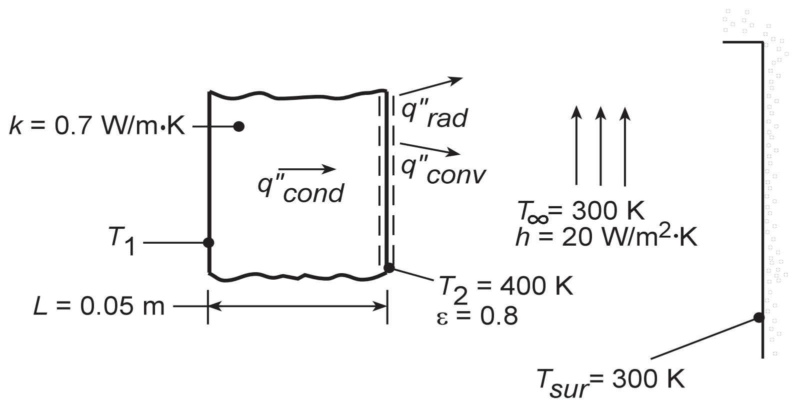
COMMENTS: If the temperature of the surroundings is approximately that of the air, radiation exchange between the outer surface and the surroundings will be negligible, since T_2 is small. In this case convection makes the dominant contribution to heat transfer from the outer surface, and assumption (3) is excellent.

PROBLEM 1.60

KNOWN: Thickness and thermal conductivity, k, of an oven wall. Temperature and emissivity, ε, of front surface. Temperature and convection coefficient, h, of air. Temperature of large surroundings.

FIND: (a) Temperature of back surface, (b) Effect of variations in k, h and ε.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Radiation exchange with large surroundings.

ANALYSIS: (a) Applying an energy balance, Eq. 1.13, at an instant of time to the front surface and substituting the appropriate rate equations, Eqs. 1.2, 1.3a and 1.7, find

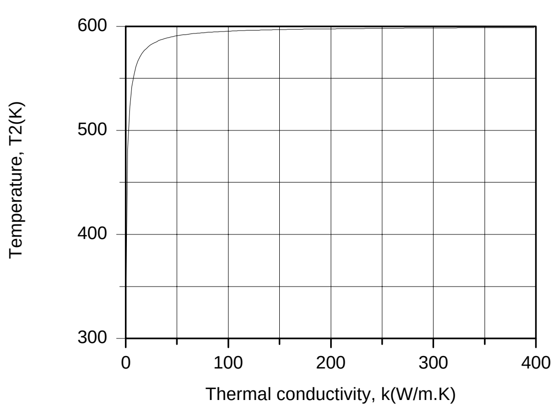
$$k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right).$$

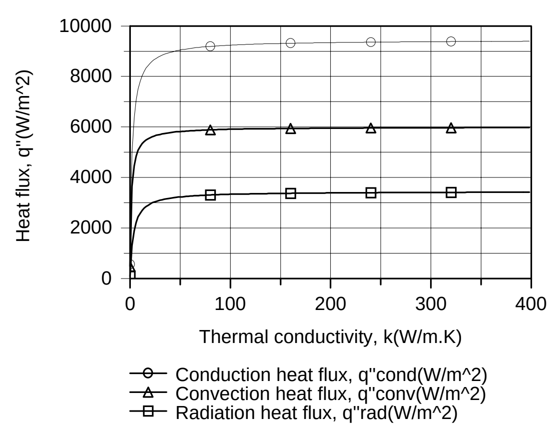
Substituting numerical values, find

$$T_1 - T_2 = \frac{0.05\,m}{0.7\,W/m\cdot K}\left[20\frac{W}{m^2\cdot K}100\,K + 0.8\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\left[\left(400\,K\right)^4 - \left(300\,K\right)^4\right]\right] = 200\,K\,.$$

Since T_2 = 400 K, it follows that T_1 = 600 K. <

(b) Parametric effects may be evaluated by using the IHT *First Law* Model for a *Nonisothermal Plane* Wall. Changes in k strongly influence conditions for k < 20 W/m·K, but have a negligible effect for larger values, as T_2 approaches T_1 and the heat fluxes approach the corresponding limiting values

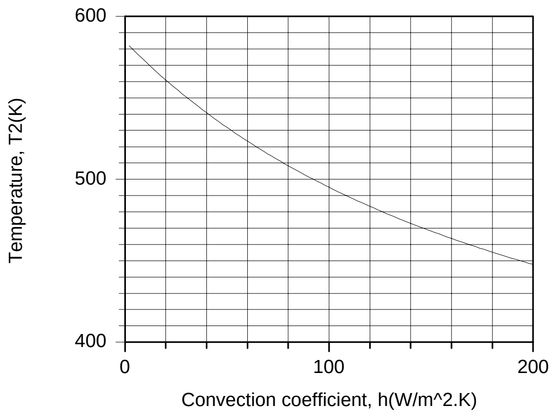


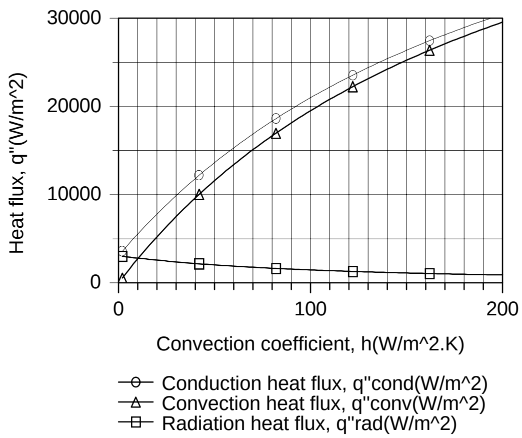


PROBLEM 1.60 (Cont.)

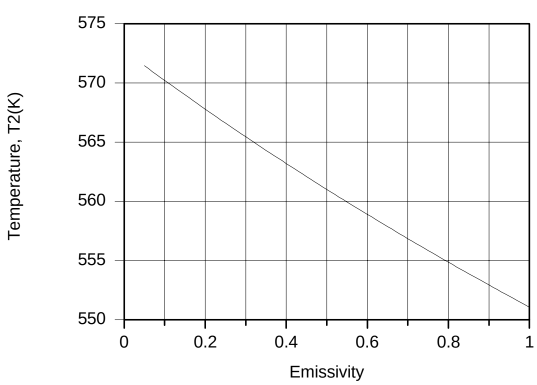
The implication is that, for k > 20 W/m·K, heat transfer by conduction in the wall is extremely efficient relative to heat transfer by convection and radiation, which become the *limiting* heat transfer processes. Larger fluxes could be obtained by increasing ε and h and/or by decreasing T_∞ and T_{sur}.

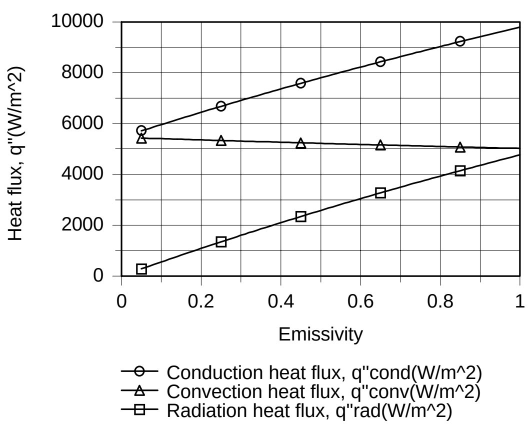
With increasing h, the front surface is cooled more effectively (T_2 decreases), and although q''_{rad} decreases, the reduction is exceeded by the increase in q''_{conv}. With a reduction in T_2 and fixed values of k and L, q''_{cond} must also increase.





The surface temperature also decreases with increasing ε, and the increase in q''_{rad} exceeds the reduction in q''_{conv}, allowing q''_{cond} to increase with ε.





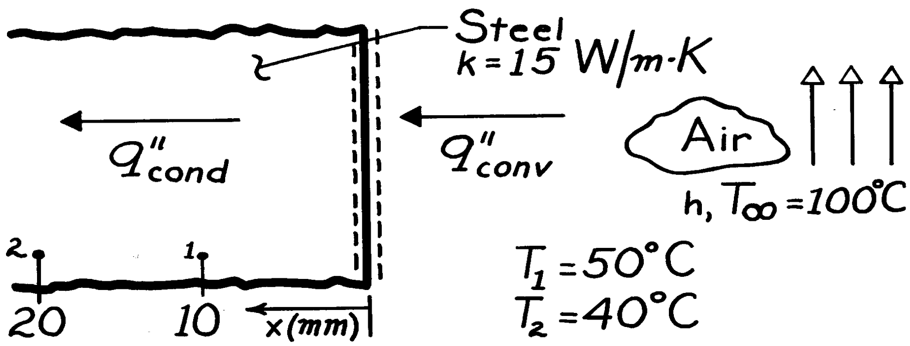
COMMENTS: Conservation of energy, of course, dictates that, irrespective of the prescribed conditions, $q''_{cond} = q''_{conv} + q''_{rad}$.

PROBLEM 1.61

KNOWN: Temperatures at 10 mm and 20 mm from the surface and in the adjoining airflow for a thick steel casting.

FIND: Surface convection coefficient, h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in the x-direction, (3) Constant properties, (4) Negligible generation.

ANALYSIS: From a surface energy balance, it follows that

$$q''_{cond} = q''_{conv}$$

where the convection rate equation has the form

$$q''_{conv} = h\left(T_\infty - T_0\right),$$

and q''_{cond} can be evaluated from the temperatures prescribed at surfaces 1 and 2. That is, from Fourier's law,

$$q''_{cond} = k\frac{T_1 - T_2}{x_2 - x_1}$$

$$q''_{cond} = 15\frac{W}{m\cdot K}\frac{(50-40)^\circ C}{(20\text{-}10)\times 10^{-3} m} = 15{,}000\ W/m^2.$$

Since the temperature gradient in the solid must be linear for the prescribed conditions, it follows that

$T_0 = 60°C$.

Hence, the convection coefficient is

$$h = \frac{q''_{cond}}{T_\infty - T_0}$$

$$h = \frac{15{,}000\ W/m^2}{40^\circ C} = 375\ W/m^2\cdot K. \qquad <$$

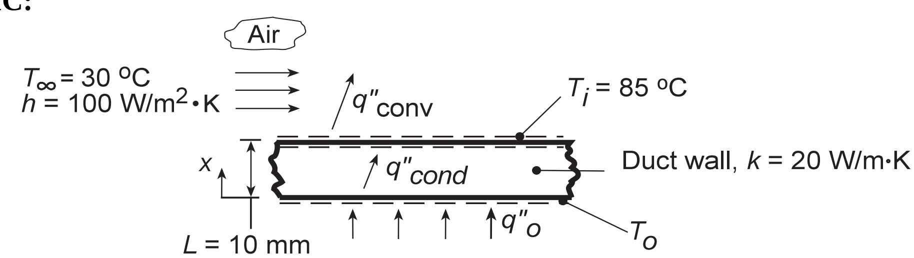
COMMENTS: The accuracy of this procedure for measuring h depends strongly on the validity of the assumed conditions.

PROBLEM 1.62

KNOWN: Duct wall of prescribed thickness and thermal conductivity experiences prescribed heat flux q''_o at outer surface and convection at inner surface with known heat transfer coefficient.

FIND: (a) Heat flux at outer surface required to maintain inner surface of duct at T_i = 85°C, (b) Temperature of outer surface, T_o, (c) Effect of h on T_o and q''_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Constant properties, (4) Backside of heater perfectly insulated, (5) Negligible radiation.

ANALYSIS: (a) By performing an energy balance on the wall, recognize that $q''_o = q''_{cond}$. From an energy balance on the top surface, it follows that $q''_{cond} = q''_{conv} = q''_o$. Hence, using the convection rate equation,

$$q''_o = q''_{conv} = h\left(T_i - T_\infty\right) = 100\ \mathrm{W/m^2 \cdot K}\left(85-30\right)^\circ \mathrm{C} = 5500\,\mathrm{W/m^2}. \qquad <$$

(b) Considering the duct wall and applying Fourier's Law,

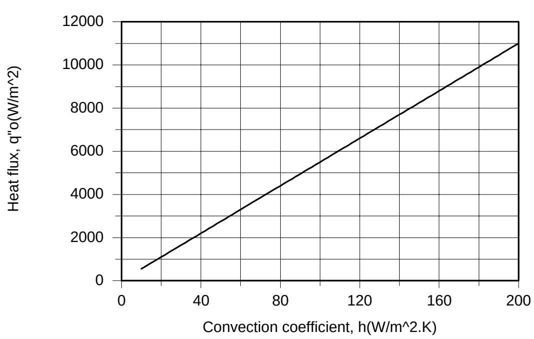
$$q''_o = k\frac{\Delta T}{\Delta X} = k\frac{T_o - T_i}{L}$$

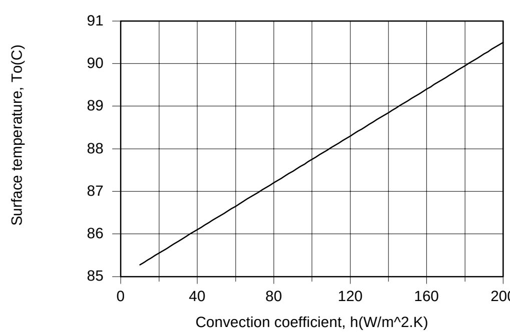
$$T_o = T_i + \frac{q''_o L}{k} = 85^\circ\mathrm{C} + \frac{5500\,\mathrm{W/m^2} \times 0.010\,\mathrm{m}}{20\,\mathrm{W/m \cdot K}} = \left(85+2.8\right)^\circ \mathrm{C} = 87.8^\circ \mathrm{C}. \qquad <$$

(c) For T_i = 85°C, the desired results may be obtained by simultaneously solving the energy balance equations

$$q''_o = k\frac{T_o - T_i}{L} \qquad \text{and} \qquad k\frac{T_o - T_i}{L} = h\left(T_i - T_\infty\right)$$

Using the IHT *First Law* Model for a *Nonisothermal Plane Wall*, the following results are obtained.





Since q''_{conv} increases linearly with increasing h, the applied heat flux q''_o must be balanced by an increase in q''_{cond}, which, with fixed k, T_i and L, necessitates an increase in T_o.

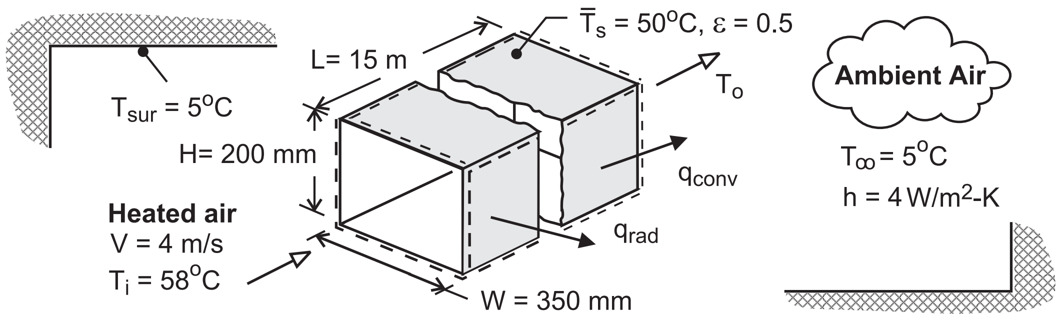
COMMENTS: The temperature difference across the wall is small, amounting to a maximum value of $\left(T_o - T_i\right)$ = 5.5°C for h = 200 W/m^2·K. If the wall were thinner (L < 10 mm) or made from a material with higher conductivity (k > 20 W/m·K), this difference would be reduced.

PROBLEM 1.63

KNOWN: Dimensions, average surface temperature and emissivity of heating duct. Duct air inlet temperature and velocity. Temperature of ambient air and surroundings. Convection coefficient.

FIND: (a) Heat loss from duct, (b) Air outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Radiation exchange between a small surface and a large enclosure.

ANALYSIS: (a) Heat transfer from the surface of the duct to the ambient air and the surroundings is given by Eq. (1.10)

$$q = hA_s\left(T_s - T_\infty\right) + \varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right)$$

where A_s = L (2W + 2H) = 15 m (0.7 m + 0.5 m) = 16.5 m^2. Hence,

$$q = 4\ \text{W/m}^2\cdot\text{K}\times 16.5\ \text{m}^2\left(45^\circ\text{C}\right) + 0.5\times 16.5\ \text{m}^2\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(323^4 - 278^4\right)\text{K}^4$$

$$q = q_{conv} + q_{rad} = 2970\ \text{W} + 2298\ \text{W} = 5268\ \text{W}$$ <

(b) With i = u + pv, $\dot{W} = 0$ and the third assumption, Eq. (1.11e) yields,

$$\dot{m}\left(i_i - i_o\right) = \dot{m}c_p\left(T_i - T_o\right) = q$$

where the sign on q has been reversed to reflect the fact that heat transfer is *from* the system. With $\dot{m} = \rho V A_c = 1.10\ \text{kg/m}^3 \times 4\ \text{m/s}\left(0.35\text{m}\times 0.20\text{m}\right) = 0.308\ \text{kg/s}$, the outlet temperature is

$$T_o = T_i - \frac{q}{\dot{m}c_p} = 58^\circ\text{C} - \frac{5268\ \text{W}}{0.308\ \text{kg/s}\times 1008\ \text{J/kg}\cdot\text{K}} = 41^\circ\text{C}$$ <

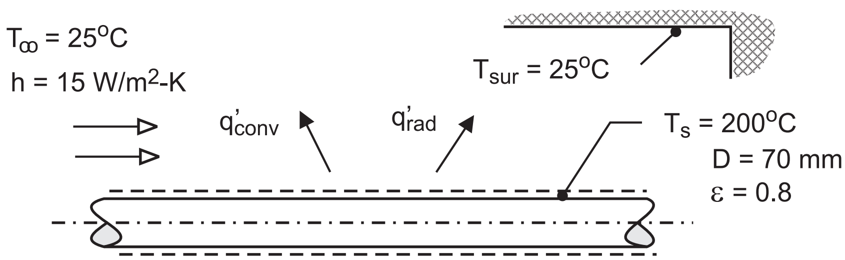
COMMENTS: The temperature drop of the air is large and unacceptable, unless the intent is to use the duct to heat the basement. If not, the duct should be insulated to insure maximum delivery of thermal energy to the intended space(s).

PROBLEM 1.64

KNOWN: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures. See Example 1.2.

FIND: (a) Which option to reduce heat loss to the room is more effective: reduce by a factor of two the convection coefficient (from 15 to 7.5 W/m$^2\cdot$K) or the emissivity (from 0.8 to 0.4) and (b) Show graphically the heat loss as a function of the convection coefficient for the range $5 \le h \le 20$ W/m$^2\cdot$K for emissivities of 0.2, 0.4 and 0.8. Comment on the relative efficacy of reducing heat losses associated with the convection and radiation processes.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between pipe and the room is between a small surface in a much larger enclosure, (3) The surface emissivity and absorptivity are equal, and (4) Restriction of the air flow does not alter the radiation exchange process between the pipe and the room.

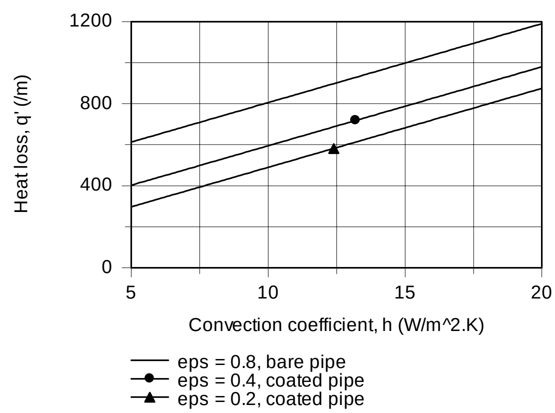
ANALYSIS: (a) The heat rate from the pipe to the room per unit length is

$$q' = q'/L = q'_{conv} + q'_{rad} = h(\pi D)(T_s - T_\infty) + \varepsilon(\pi D)\sigma\left(T_s^4 - T_{sur}^4\right)$$

Substituting numerical values for the two options, the resulting heat rates are calculated and compared with those for the conditions of Example 1.2. We conclude that both options are comparably effective.

Conditions	$h\left(W/m^2\cdot K\right)$	ε	$q'(W/m)$
Base case, Example 1.2	15	0.8	998
Reducing h by factor of 2	7.5	0.8	788
Reducing ε by factor of 2	15	0.4	709

(b) Using IHT, the heat loss can be calculated as a function of the convection coefficient for selected values of the surface emissivity.



Continued …..

PROBLEM 1.64 (Cont.)

COMMENTS: (1) In Example 1.2, Comment 3, we read that the heat rates by convection and radiation exchange were comparable for the base case conditions (577 vs. 421 W/m). It follows that reducing the key transport parameter (h or ε) by a factor of two yields comparable reductions in the heat loss. Coating the pipe to reduce the emissivity might to be the more practical option as it may be difficult to control air movement.

(2) For this pipe size and thermal conditions (T_s and T_∞), the minimum possible convection coefficient is approximately 7.5 $W/m^2 \cdot K$, corresponding to free convection heat transfer to quiescent ambient air. Larger values of h are a consequence of forced air flow conditions.

(3) The Workspace for the IHT program to calculate the heat loss and generate the graph for the heat loss as a function of the convection coefficient for selected emissivities is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

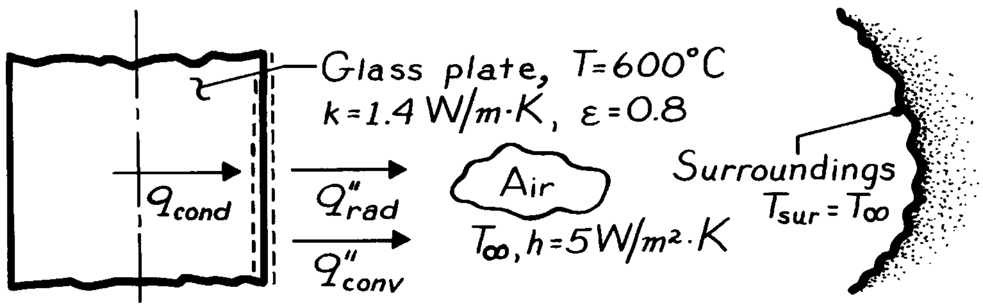
```
// Heat loss per unit pipe length; rate equation from Ex. 1.2
q' = q'cv + q'rad
q'cv = pi*D*h*(Ts - Tinf)
q'rad = pi*D*eps*sigma*(Ts^4 - Tsur^4)
sigma = 5.67e-8

// Input parameters
D = 0.07
Ts_C = 200        // Representing temperatures in Celsius units using _C subscripting
Ts = Ts_C +273
Tinf_C = 25
Tinf = Tinf_C + 273
h = 15            // For graph, sweep over range from 5 to 20
Tsur_C = 25
Tsur = Tsur_C + 273
eps = 0.8
//eps = 0.4       // Values of emissivity for parameter study
//eps = 0.2

/* Base case results
Tinf   Ts     Tsur   q'       q'cv    q'rad   D      Tinf_C  Ts_C   Tsur_C
       eps    h      sigma
298    473    298    997.9    577.3   420.6   0.07   25      200    25
       0.8    15     5.67E-8    */
```

PROBLEM 1.65

KNOWN: Conditions associated with surface cooling of plate glass which is initially at 600°C. Maximum allowable temperature gradient in the glass.

FIND: Lowest allowable air temperature, T_∞

SCHEMATIC:



ASSUMPTIONS: (1) Surface of glass exchanges radiation with large surroundings at $T_{sur} = T_\infty$, (2) One-dimensional conduction in the x-direction.

ANALYSIS: The maximum temperature gradient will exist at the surface of the glass and at the instant that cooling is initiated. From the surface energy balance, Eq. 1.12, and the rate equations, Eqs. 1.1, 1.3a and 1.7, it follows that

$$-k\frac{dT}{dx} - h\left(T_s - T_\infty\right) - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0$$

or, with $(dT/dx)_{max} = -15°C/mm = -15{,}000°C/m$ and $T_{sur} = T_\infty$,

$$-1.4\frac{W}{m\cdot K}\left[-15{,}000\frac{\circ C}{m}\right] = 5\frac{W}{m^2\cdot K}\left(873 - T_\infty\right)K$$

$$+0.8\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\left[873^4 - T_\infty^4\right]K^4.$$

T_∞ may be obtained from a trial-and-error solution, from which it follows that, for $T_\infty = 618K$,

$$21{,}000\frac{W}{m^2} \approx 1275\frac{W}{m^2} + 19{,}730\frac{W}{m^2}.$$

Hence the lowest allowable air temperature is

$$T_\infty \approx 618K = 345^\circ C.$$ <

COMMENTS: (1) Initially, cooling is determined primarily by radiation effects.

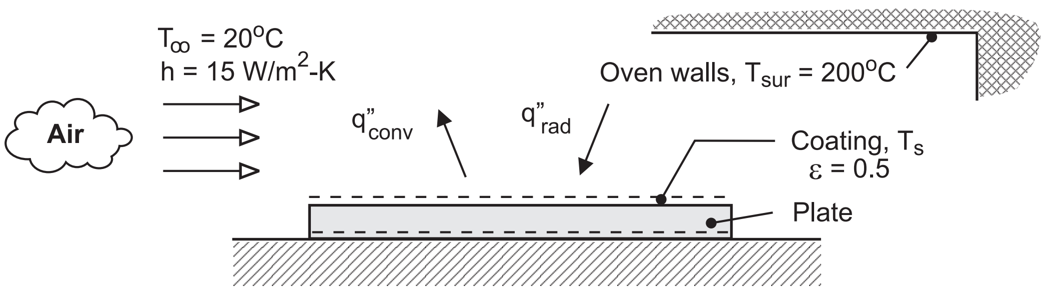
(2) For fixed T_∞, the surface *temperature gradient* would *decrease* with *increasing* time into the cooling process. Accordingly, T_∞ could be decreasing with increasing time and still keep within the maximum allowable temperature gradient.

PROBLEM 1.66

KNOWN: Hot-wall oven, in lieu of infrared lamps, with temperature T_{sur} = 200°C for heating a coated plate to the cure temperature. See Example 1.6.

FIND: (a) The plate temperature T_s for prescribed convection conditions and coating emissivity, and (b) Calculate and plot T_s as a function of T_{sur} for the range $150 \le T_{sur} \le 250$°C for ambient air temperatures of 20, 40 and 60°C; identify conditions for which acceptable curing temperatures between 100 and 110°C may be maintained.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from back surface of plate, (3) Plate is small object in large isothermal surroundings (hot oven walls).

ANALYSIS: (a) The temperature of the plate can be determined from an energy balance on the plate, considering radiation exchange with the hot oven walls and convection with the ambient air.

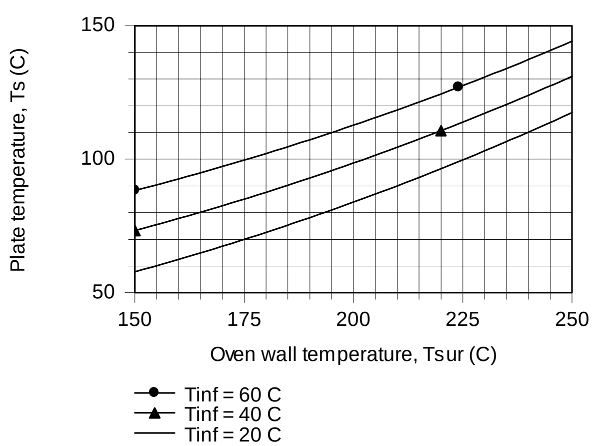
$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \qquad \text{or} \qquad q''_{rad} - q''_{conv} = 0$$

$$\varepsilon\sigma\left(T_{sur}^4 - T_s^4\right) - h\left(T_s - T_\infty\right) = 0$$

$$0.5 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\left([200+273]^4 - T_s^4\right)\mathrm{K}^4 - 15\ \mathrm{W/m^2 \cdot K}\left(T_s - [20+273]\right)\mathrm{K} = 0$$

$$T_s = 357\ \mathrm{K} = 84°\mathrm{C} \qquad <$$

(b) Using the energy balance relation in the Workspace of IHT, the plate temperature can be calculated and plotted as a function of oven wall temperature for selected ambient air temperatures.



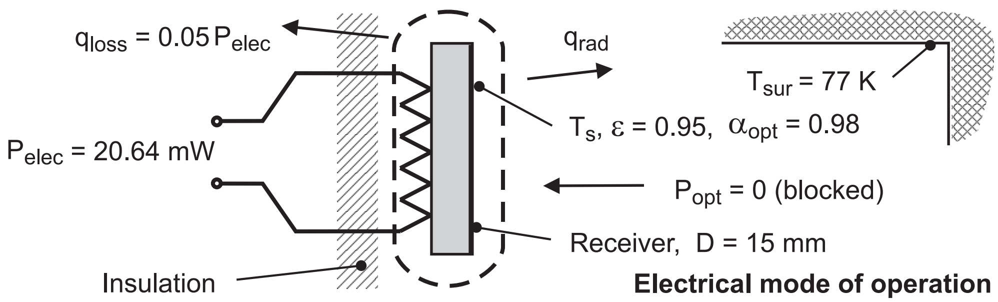
COMMENTS: From the graph, acceptable cure temperatures between 100 and 110°C can be maintained for these conditions: with T_∞ = 20°C when $225 \le T_{sur} \le 240$°C; with T_∞ = 40°C when $205 \le T_{sur} \le 220$°C; and with T_∞ = 60°C when $175 \le T_{sur} \le 195$°C.

PROBLEM 1.67

KNOWN: Operating conditions for an electrical-substitution radiometer having the same receiver temperature, T_s, in electrical and optical modes.

FIND: Optical power of a laser beam and corresponding receiver temperature when the indicated electrical power is 20.64 mW.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Conduction losses from backside of receiver negligible in optical mode, (3) Chamber walls form large isothermal surroundings; negligible effects due to aperture, (4) Radiation exchange between the receiver surface and the chamber walls is between small surface and large enclosure, (5) Negligible convection effects.

PROPERTIES: Receiver surface: ε = 0.95, α_{opt} = 0.98.

ANALYSIS: The schematic represents the operating conditions for the *electrical mode* with the optical beam blocked. The temperature of the receiver surface can be found from an energy balance on the receiver, considering the electrical power input, conduction loss from the backside of the receiver, and the radiation exchange between the receiver and the chamber.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_{elec} - q_{loss} - q_{rad} = 0$$

$$P_{elec} - 0.05\,P_{elec} - \varepsilon\,A_s\sigma\left(T_s^4 - T_{sur}^4\right) = 0$$

$$20.64\times10^{-3}\,\text{W}\,(1-0.05) - 0.95\left(\pi 0.015^2/4\right)\text{m}^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_s^4 - 77^4\right)\text{K}^4 = 0$$

$$T_s = 213.9\ \text{K} \qquad <$$

For the *optical mode* of operation, the optical beam is incident on the receiver surface, there is no electrical power input, and the receiver temperature is the same as for the electrical mode. The optical power of the beam can be found from an energy balance on the receiver considering the absorbed beam power and radiation exchange between the receiver and the chamber.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_{opt}\,P_{opt} - q_{rad} = 0.98\,P_{opt} - 19.60\,\text{mW} = 0$$

$$P_{opt} = 19.99\,\text{mW} \qquad <$$

where q_{rad} follows from the previous energy balance using T_s = 213.9K.

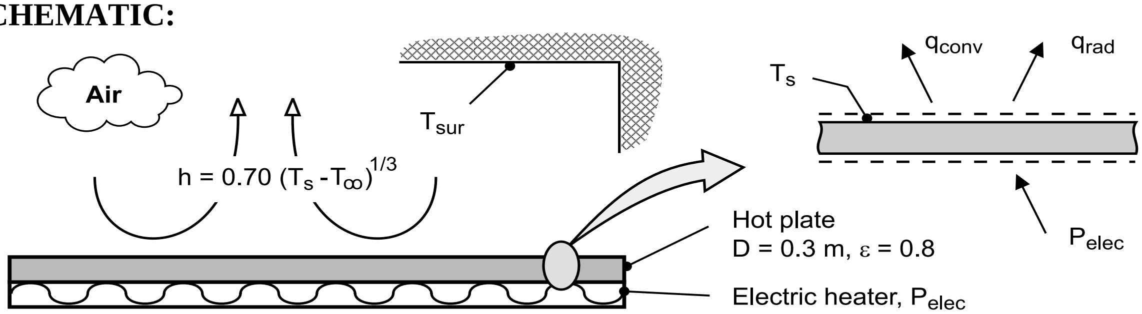
COMMENTS: Recognizing that the receiver temperature, and hence the radiation exchange, is the same for both modes, an energy balance could be directly written in terms of the absorbed optical power and equivalent electrical power, $\alpha_{opt}\,P_{opt} = P_{elec} - q_{loss}$.

PROBLEM 1.68

KNOWN: Surface temperature, diameter and emissivity of a hot plate. Temperature of surroundings and ambient air. Expression for convection coefficient.

FIND: (a) Operating power for prescribed surface temperature, (b) Effect of surface temperature on power requirement and on the relative contributions of radiation and convection to heat transfer from the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is of uniform surface temperature, (2) Walls of room are large relative to plate, (3) Negligible heat loss from bottom or sides of plate.

ANALYSIS: (a) From an energy balance on the hot plate, $P_{elec} = q_{conv} + q_{rad} = A_p\,(q''_{conv} + q''_{rad})$.

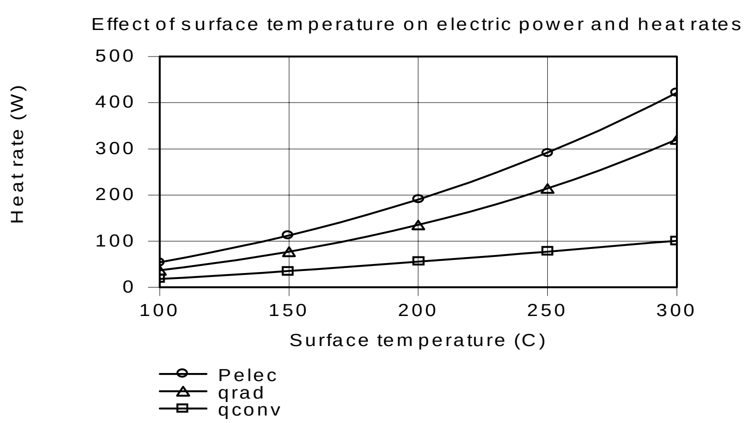
Substituting for the area of the plate and from Eqs. (1.3a) and (1.7), with $h = 0.70\,(T_s - T_\infty)^{1/3}$, it follows that

$$P_{elec} = \left(\pi D^2/4\right)\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

$$P_{elec} = \pi\left(0.3\text{m}\right)^2/4\left[0.70\left(175\right)^{4/3} + 0.8\times 5.67\times 10^{-8}\left(473^4 - 298^4\right)\right]\ \text{W/m}^2$$

$$P_{elec} = 0.0707\ \text{m}^2\left[685\ \text{W/m}^2 + 1913\ \text{W/m}^2\right] = 48.4\ \text{W} + 135.2\ \text{W} = 190.6\ \text{W} \qquad <$$

(b) As shown graphically, both the radiation and convection heat rates, and hence the requisite electric power, increase with increasing surface temperature.



However, because of its dependence on the fourth power of the surface temperature, the increase in radiation is more pronounced. The significant relative effect of radiation is due to the small convection coefficients characteristic of natural convection, with $3.37 \le h \le 5.2\ \text{W/m}^2\cdot\text{K}$ for $100 \le T_s < 300°\text{C}$.

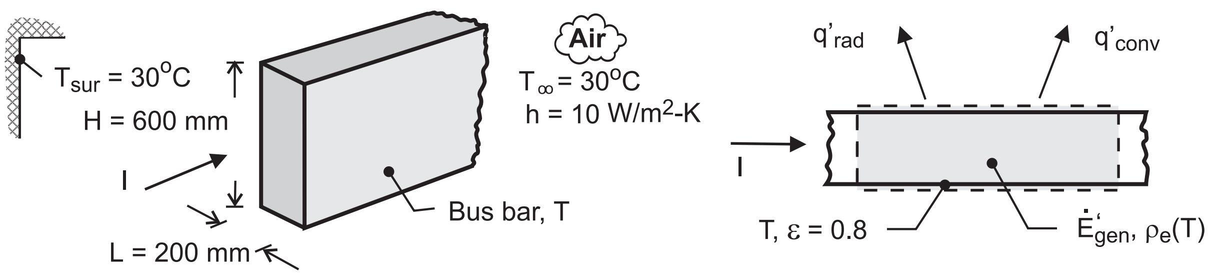
COMMENTS: Radiation losses could be reduced by applying a low emissivity coating to the surface, which would have to maintain its integrity over the range of operating temperatures.

PROBLEM 1.69

KNOWN: Long bus bar of rectangular cross-section and ambient air and surroundings temperatures. Relation for the electrical resistivity as a function of temperature.

FIND: (a) Temperature of the bar with a current of 60,000 A, and (b) Compute and plot the operating temperature of the bus bar as a function of the convection coefficient for the range $10 \le h \le 100$ $W/m^2 \cdot K$. Minimum convection coefficient required to maintain a safe-operating temperature below 120°C. Will increasing the emissivity significantly affect this result?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Bus bar is long, (3) Uniform bus-bar temperature, (3) Radiation exchange between the outer surface of the bus bar and its surroundings is between a small surface and a large enclosure.

PROPERTIES: Bus-bar material, $\rho_e = \rho_{e,o}\left[1+\alpha\left(T-T_o\right)\right]$, $\rho_{e,o} = 0.0828\,\mu\Omega\cdot m$, $T_o = 25°C$, $\alpha = 0.0040\,K^{-1}$.

ANALYSIS: (a) An energy balance on the bus-bar for a unit length as shown in the schematic above has the form

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0 \qquad -q'_{rad} - q'_{conv} + I^2 R'_e = 0$$

$$-\varepsilon P \sigma\left(T^4 - T_{sur}^4\right) - hP\left(T - T_\infty\right) + I^2 \rho_e / A_c = 0$$

where $P = 2(H+W)$, $R'_e = \rho_e / A_c$ and $A_c = H \times W$. Substituting numerical values,

$$-0.8 \times 2(0.600+0.200)\text{m} \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T^4 - [30+273]^4\right)\text{K}^4$$

$$-10\,\text{W/m}^2\cdot\text{K} \times 2(0.600+0.200)\text{m}\left(T - [30+273]\right)\text{K}$$

$$+(60{,}000\,\text{A})^2\left\{0.0828\times10^{-6}\,\Omega\cdot\text{m}\left[1 + 0.0040\,\text{K}^{-1}\left(T - [25+273]\right)\text{K}\right]\right\}/(0.600\times0.200)\,\text{m}^2 = 0$$

Solving for the bus-bar temperature, find $\quad T = 426\ K = 153°C.$ <

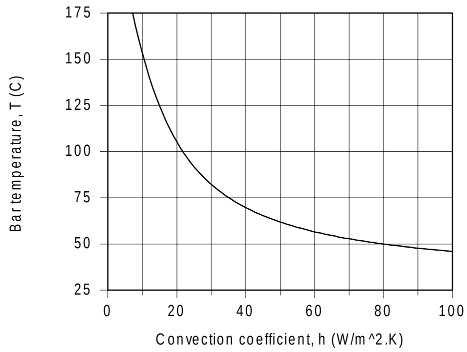
(b) Using the energy balance relation in the Workspace of IHT, the bus-bar operating temperature is calculated as a function of the convection coefficient for the range $10 \le h \le 100$ $W/m^2 \cdot K$. From this graph we can determine that to maintain a safe operating temperature below 120°C, the minimum convection coefficient required is

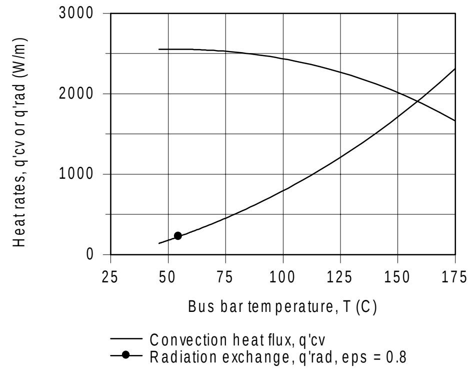
$$h_{min} = 16\ \text{W/m}^2\cdot\text{K}.$$ <

Continued …..

PROBLEM 1.69 (Cont.)

Using the same equations, we can calculate and plot the heat transfer rates by convection and radiation as a function of the bus-bar temperature.





Note that convection is the dominant mode for low bus-bar temperatures; that is, for low current flow. As the bus-bar temperature increases toward the safe-operating limit (120°C), convection and radiation exchange heat transfer rates become comparable. Notice that the relative importance of the radiation exchange rate increases with increasing bus-bar temperature.

COMMENTS: (1) It follows from the second graph that increasing the surface emissivity will be only significant at higher temperatures, especially beyond the safe-operating limit.

(2) The Workspace for the IHT program to perform the parametric analysis and generate the graphs is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

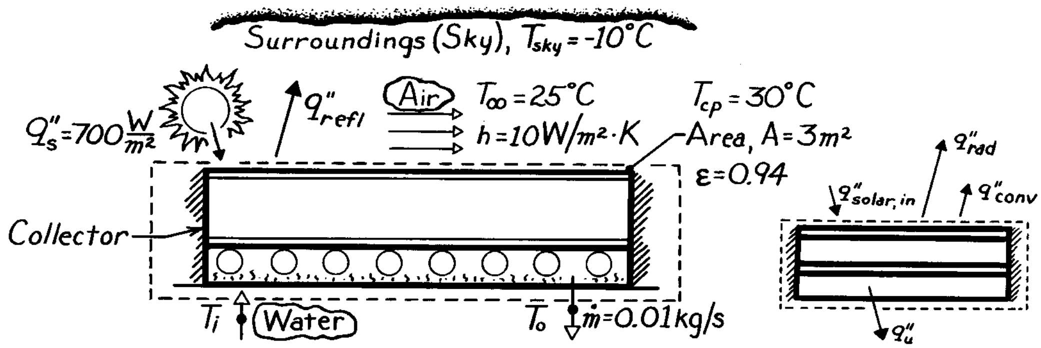
```
/* Results for base case conditions:
Ts_C   q'cv    q'rad    rhoe       H      I      Tinf_C   Tsur_C   W      alpha
       eps     h
153.3  1973    1786     1.253E-7   0.6    6E4    30       30       0.2    0.004
       0.8     10   */

// Surface energy balance on a per unit length basis
-q'cv - q'rad + Edot'gen = 0
q'cv = h * P * (Ts - Tinf)
P = 2 * (W + H)            // perimeter of the bar experiencing surface heat transfer
q'rad = eps * sigma * (Ts^4 - Tsur^4) * P
sigma = 5.67e-8
Edot'gen = I^2 * Re'
Re' = rhoe / Ac
rhoe = rhoeo * ( 1 + alpha * (Ts - Teo))
Ac = W * H

// Input parameters
I = 60000
alpha = 0.0040             // temperature coefficient, K^-1; typical value for cast aluminum
rhoeo = 0.0828e-6          // electrical resistivity at the reference temperature, Teo; microohm-m
Teo = 25 + 273             // reference temperature, K
W = 0.200
H = 0.600
Tinf_C = 30
Tinf = Tinf_C + 273
h = 10
eps = 0.8
Tsur_C = 30
Tsur = Tsur_C + 273
Ts_C = Ts - 273
```

PROBLEM 1.70

KNOWN: Solar collector designed to heat water operating under prescribed solar irradiation and loss conditions.

FIND: (a) Useful heat collected per unit area of the collector, q''_u, (b) Temperature rise of the water flow, $T_o - T_i$, and (c) Collector efficiency.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses out sides or back of collector, (3) Collector area is small compared to sky surroundings.

PROPERTIES: *Table A.6,* Water (300K): c_p = 4179 J/kg·K.

ANALYSIS: (a) Defining the collector as the control volume and writing the conservation of energy requirement on a per unit area basis, find that

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}.$$

Identifying processes as per above right sketch,

$$q''_{solar} - q''_{rad} - q''_{conv} - q''_u = 0$$

where $q''_{solar} = 0.9\ q''_S$; that is, 90% of the solar flux is absorbed in the collector (Eq. 1.6). Using the appropriate rate equations, the useful heat rate per unit area is

$$q''_u = 0.9\ q''_S - \varepsilon\sigma\left(T^4_{cp} - T^4_{sky}\right) - h\left(T_s - T_\infty\right)$$

$$q''_u = 0.9 \times 700 \frac{W}{m^2} - 0.94 \times 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}\left(303^4 - 263^4\right)K^4 - 10 \frac{W}{m^2 \cdot K}(30 - 25)^\circ C$$

$$q''_u = 630\ W/m^2 - 194\ W/m^2 - 50\ W/m^2 = 386\ W/m^2. \quad <$$

(b) The total useful heat collected is $q''_u \cdot A$. Defining a control volume about the water tubing, the useful heat causes an enthalpy change of the flowing water. That is,

$$q''_u \cdot A = \dot{m} c_p \left(T_i - T_o\right) \qquad \text{or}$$

$$\left(T_i - T_o\right) = 386\ W/m^2 \times 3 m^2 / 0.01 kg/s \times 4179 J/kg \cdot K = 27.7^\circ C. \quad <$$

(c) The efficiency is $\eta = q''_u / q''_S = \left(386\ W/m^2\right)/\left(700\ W/m^2\right) = 0.55$ or 55%. $<$

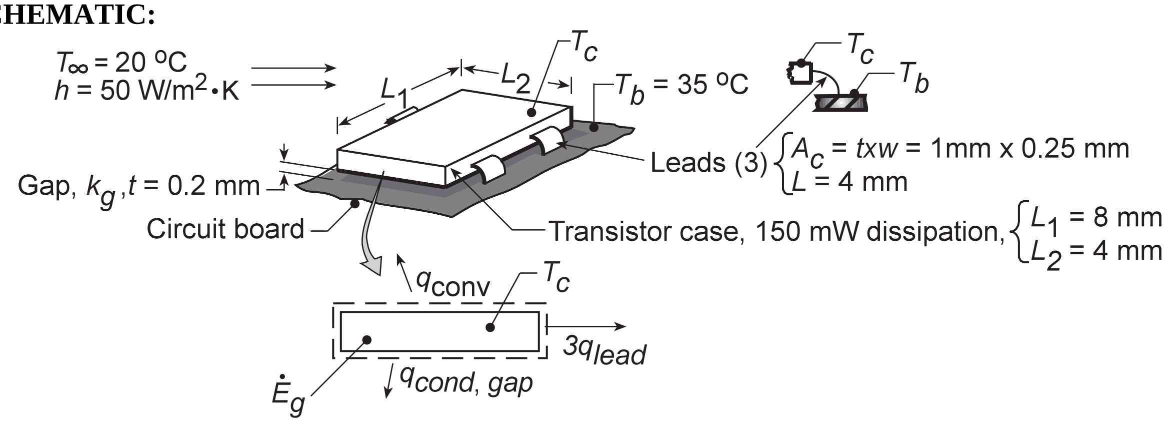
COMMENTS: Note how the sky has been treated as large surroundings at a uniform temperature T_{sky}.

PROBLEM 1.71

KNOWN: Surface-mount transistor with prescribed dissipation and convection cooling conditions.

FIND: (a) Case temperature for mounting arrangement with air-gap and conductive paste between case and circuit board, (b) Consider options for increasing $\dot{E}_g$, subject to the constraint that T_c = 40°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Transistor case is isothermal, (3) Upper surface experiences convection; negligible losses from edges, (4) Leads provide conduction path between case and board, (5) Negligible radiation, (6) Negligible energy generation in leads due to current flow, (7) Negligible convection from surface of leads.

PROPERTIES: (Given): Air, $k_{g,a}$ = 0.0263 W/m·K; Paste, $k_{g,p}$ = 0.12 W/m·K; Metal leads, k_ℓ = 25 W/m·K.

ANALYSIS: (a) Define the transistor as the system and identify modes of heat transfer.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \Delta\dot{E}_{st} = 0$$

$$-q_{conv} - q_{cond,gap} - 3q_{lead} + \dot{E}_g = 0$$

$$-hA_s\left(T_c - T_\infty\right) - k_g A_s \frac{T_c - T_b}{t} - 3k_\ell A_c \frac{T_c - T_b}{L} + \dot{E}_g = 0$$

where $A_s = L_1 \times L_2$ = 4 × 8 mm² = 32 × 10⁻⁶ m² and A_c = t × w = 0.25 × 1 mm² = 25 × 10⁻⁸ m².

Rearranging and solving for T_c,

$$T_c = \left\{hA_sT_\infty + \left[k_gA_s/t + 3\left(k_\ell A_c/L\right)\right]T_b + \dot{E}_g\right\} / \left[hA_s + k_gA_s/t + 3\left(k_\ell A_c/L\right)\right]$$

Substituting numerical values, with the *air-gap condition* ($k_{g,a}$ = 0.0263 W/m·K)

$$T_c = \left\{50\,\mathrm{W/m^2 \cdot K} \times 32\times10^{-6}\,\mathrm{m^2} \times 20^\circ\mathrm{C} + \left[\left(0.0263\,\mathrm{W/m\cdot K} \times 32\times10^{-6}\,\mathrm{m^2}/0.2\times10^{-3}\,\mathrm{m}\right)\right.\right.$$

$$\left.\left. +3\left(25\,\mathrm{W/m\cdot K}\times25\times10^{-8}\,\mathrm{m^2}/4\times10^{-3}\,\mathrm{m}\right)\right]35^\circ\mathrm{C}\right\} / \left[1.600\times10^{-3} + 4.208\times10^{-3} + 4.688\times10^{-3}\right]\mathrm{W/K}$$

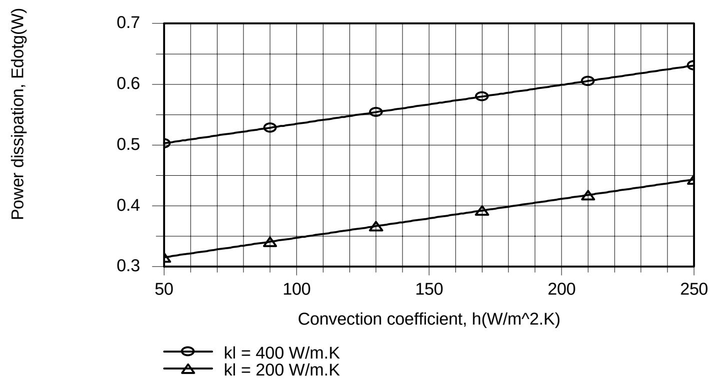
$$T_c = 47.0^\circ\mathrm{C}.$$ <

Continued.….

PROBLEM 1.71 (Cont.)

With the *paste condition* ($k_{g,p}$ = 0.12 W/m·K), T_c = 39.9°C. As expected, the effect of the conductive paste is to improve the coupling between the circuit board and the case. Hence, T_c decreases.

(b) Using the keyboard to enter model equations into the workspace, IHT has been used to perform the desired calculations. For values of k_ℓ = 200 and 400 W/m·K and convection coefficients in the range from 50 to 250 W/m^2·K, the energy balance equation may be used to compute the power dissipation for a maximum allowable case temperature of 40°C.



As indicated by the energy balance, the power dissipation increases linearly with increasing h, as well as with increasing k_ℓ. For h = 250 W/m^2·K (enhanced air cooling) and k_ℓ = 400 W/m·K (copper leads), the transistor may dissipate up to 0.63 W.

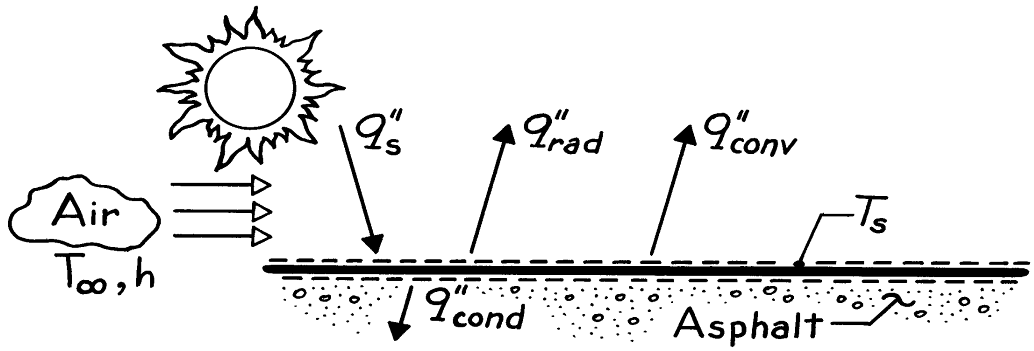
COMMENTS: Additional benefits may be derived by increasing heat transfer across the gap separating the case from the board, perhaps by inserting a highly conductive material in the gap.

PROBLEM 1.72(a)

KNOWN: Solar radiation is incident on an asphalt paving.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes shown on the schematic include:

q''_S Incident solar radiation, a large portion of which $q''_{S,abs}$, is absorbed by the asphalt surface,

q''_{rad} Radiation emitted by the surface to the air,

q''_{conv} Convection heat transfer from the surface to the air, and

q''_{cond} Conduction heat transfer from the surface into the asphalt.

Applying the surface energy balance, Eq. 1.12,

$$q''_{S,abs} - q''_{rad} - q''_{conv} = q''_{cond}.$$

COMMENTS: (1) q''_{cond} and q''_{conv} could be evaluated from Eqs. 1.1 and 1.3, respectively.

(2) It has been assumed that the pavement surface temperature is higher than that of the underlying pavement and the air, in which case heat transfer by conduction and convection are from the surface.

(3) For simplicity, radiation incident on the pavement due to atmospheric emission has been ignored (see Section 12.8 for a discussion). Eq. 1.6 may then be used for the absorbed solar irradiation and Eq. 1.5 may be used to obtain the emitted radiation q''_{rad}.

(4) With the rate equations, the energy balance becomes

$$q''_{S,abs} - \varepsilon\, \sigma\, T_s^4 - h\left(T_s - T_\infty\right) = -k \left.\frac{dT}{dx}\right]_s.$$

PROBLEM 1.72(b)

KNOWN: Physical mechanism for microwave heating.

FIND: Comparison of (a) cooking in a microwave oven with a conventional radiant or convection oven and (b) a microwave clothes dryer with a conventional dryer.

(a) Microwave cooking occurs as a result of volumetric thermal energy generation *throughout* the food, without heating of the food container or the oven wall. Conventional cooking relies on radiant heat transfer from the oven walls and/or convection heat transfer from the air space to the surface of the food and subsequent heat transfer by conduction to the core of the food. Microwave cooking is more efficient and is achieved in less time.

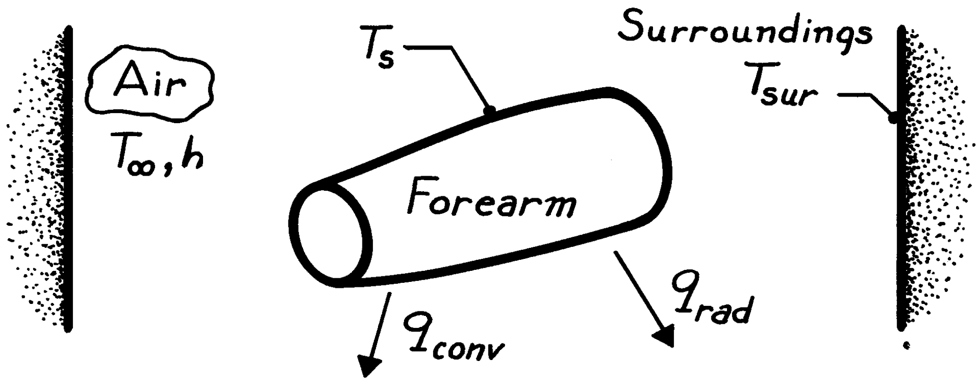
(b) In a microwave dryer, the microwave radiation would heat the water, but not the fabric, directly (the fabric would be heated indirectly by energy transfer from the water). By heating the water, energy would go directly into evaporation, unlike a conventional dryer where the walls and air are first heated electrically or by a gas heater, and thermal energy is subsequently transferred to the wet clothes. The microwave dryer would still require a rotating drum and air flow to remove the water vapor, but is able to operate more efficiently and at lower temperatures. For a more detailed description of microwave drying, see *Mechanical Engineering*, March 1993, page 120.

PROBLEM 1.72(c)

KNOWN: Surface temperature of exposed arm exceeds that of the room air and walls.

FIND: Relevant heat transfer processes.

SCHEMATIC:



Neglecting evaporation from the surface of the skin, the only relevant heat transfer processes are:

q_{conv}	Convection heat transfer from the skin to the room air, and
q_{rad}	Net radiation exchange between the surface of the skin and the surroundings (walls of the room).

You are not imagining things. Even though the room air is maintained at a fixed temperature ($T_\infty = 15°C$), the inner surface temperature of the outside walls, T_{sur}, will decrease with decreasing outside air temperature. Upon exposure to these walls, body heat loss will be larger due to increased q_{rad}.

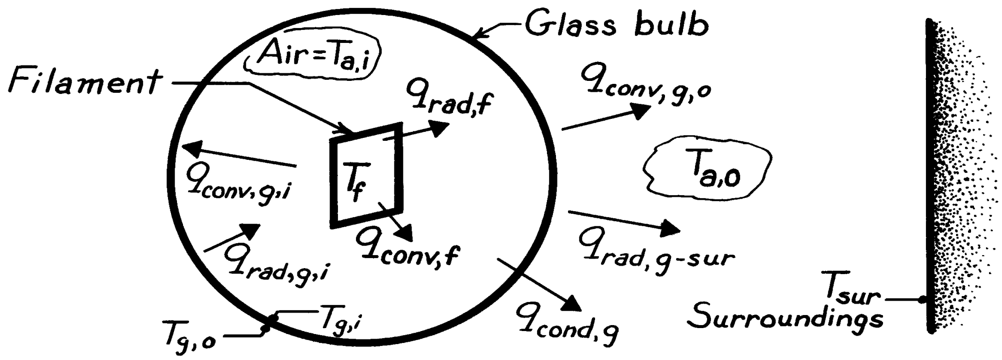
COMMENTS: The foregoing reasoning assumes that the thermostat measures the true room air temperature and is shielded from radiation exchange with the outside walls.

PROBLEM 1.72(d)

KNOWN: Tungsten filament is heated to 2900 K in an air-filled glass bulb.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes associated with the filament and bulb include:

$q_{rad,f}$	Radiation emitted by the tungsten filament, a portion of which is transmitted through the glass,
$q_{conv,f}$	Free convection from filament to air of temperature $T_{a,i} < T_f$,
$q_{rad,g,i}$	Radiation emitted by inner surface of glass, a small portion of which is intercepted by the filament,
$q_{conv,g,i}$	Free convection from air to inner glass surface of temperature $T_{g,i} < T_{a,i}$,
$q_{cond,g}$	Conduction through glass wall,
$q_{conv,g,o}$	Free convection from outer glass surface to room air of temperature $T_{a,o} < T_{g,o}$, and
$q_{rad,g\text{-}sur}$	Net radiation heat transfer between outer glass surface and surroundings, such as the walls of a room, of temperature $T_{sur} < T_{g,o}$.

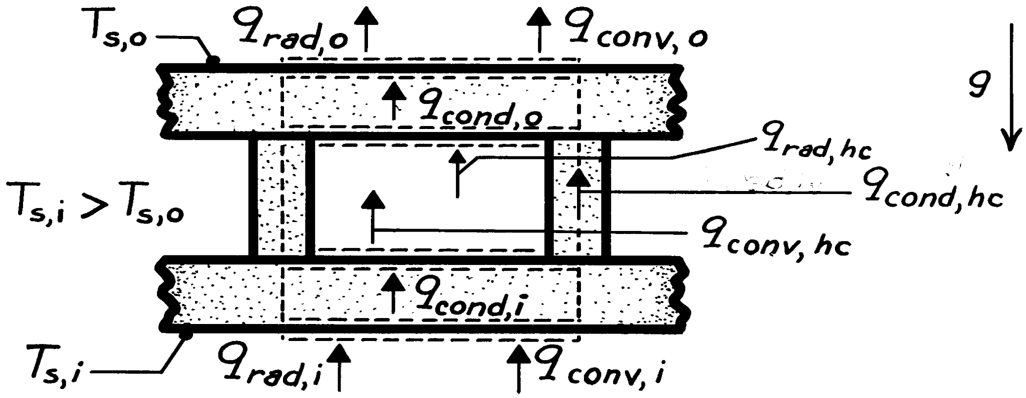
COMMENTS: If the glass bulb is evacuated, no convection is present within the bulb; that is, $q_{conv,f} = q_{conv,g,i} = 0$.

PROBLEM 1.72(e)

KNOWN: Geometry of a composite insulation consisting of a honeycomb core.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The above schematic represents the cross section of a single honeycomb cell and surface slabs. Assumed direction of gravity field is downward. Assuming that the bottom (inner) surface temperature exceeds the top (outer) surface temperature $\left(T_{s,i} > T_{s,o}\right)$, heat transfer is in the direction shown.

Heat may be transferred to the inner surface by convection and radiation, whereupon it is transferred through the composite by

$q_{cond,i}$	Conduction through the inner solid slab,
$q_{conv,hc}$	Free convection through the cellular airspace,
$q_{cond,hc}$	Conduction through the honeycomb wall,
$q_{rad,hc}$	Radiation between the honeycomb surfaces, and
$q_{cond,o}$	Conduction through the outer solid slab.

Heat may then be transferred from the outer surface by convection and radiation. Note that for a single cell under steady state conditions,

$$q_{rad,i} + q_{conv,i} = q_{cond,i} = q_{conv,hc} + q_{cond,hc}$$

$$+ q_{rad,hc} = q_{cond,o} = q_{rad,o} + q_{conv,o}.$$

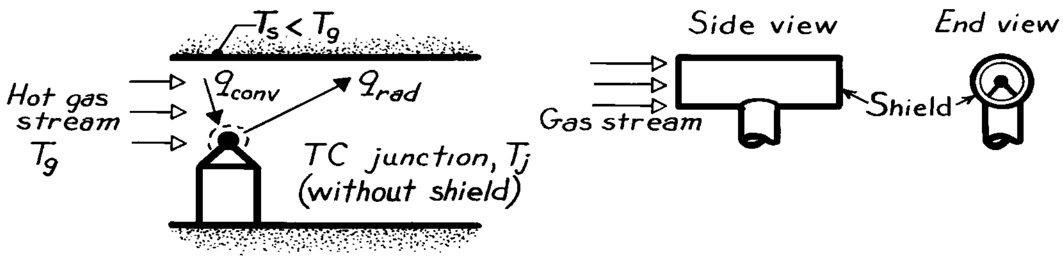
COMMENTS: Performance would be enhanced by using materials of low thermal conductivity, k, and emissivity, ε. Evacuating the airspace would enhance performance by eliminating heat transfer due to free convection.

PROBLEM 1.72(f)

KNOWN: A thermocouple junction is used, with or without a radiation shield, to measure the temperature of a gas flowing through a channel. The wall of the channel is at a temperature much less than that of the gas.

FIND: (a) Relevant heat transfer processes, (b) Temperature of junction relative to that of gas, (c) Effect of radiation shield.

SCHEMATIC:



ASSUMPTIONS: (1) Junction is small relative to channel walls, (2) Steady-state conditions, (3) Negligible heat transfer by conduction through the thermocouple leads.

ANALYSIS: (a) The relevant heat transfer processes are:

q_{rad} Net radiation transfer from the junction to the walls, and

q_{conv} Convection transfer from the gas to the junction.

(b) From a surface energy balance on the junction,

$$q_{conv} = q_{rad}$$

or from Eqs. 1.3a and 1.7,

$$h\,A\left(T_j - T_g\right) = \varepsilon\,A\,\sigma\left(T_j^4 - T_s^4\right).$$

To satisfy this equality, it follows that

$$T_s < T_j < T_g.$$

That is, the junction assumes a temperature between that of the channel wall and the gas, thereby sensing a temperature which is less than that of the gas.

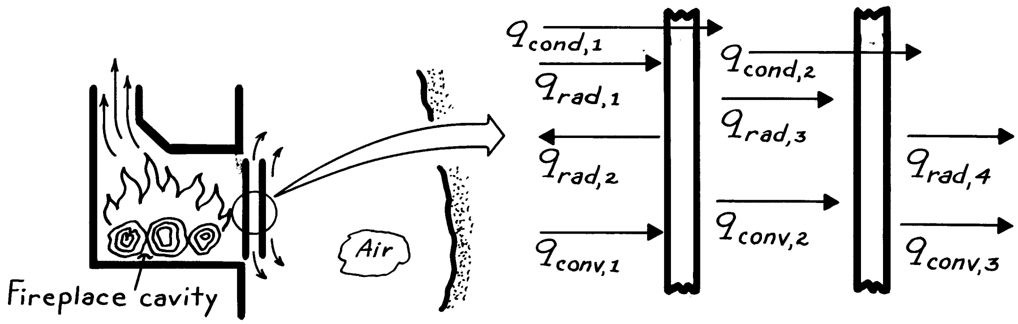
(c) The measurement error $\left(T_g - T_j\right)$ is reduced by using a radiation shield as shown in the schematic. The junction now exchanges radiation with the shield, whose temperature must exceed that of the channel wall. The radiation loss from the junction is therefore reduced, and its temperature more closely approaches that of the gas.

PROBLEM 1.72(g)

KNOWN: Fireplace cavity is separated from room air by two glass plates, open at both ends.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant heat transfer processes associated with the double-glazed, glass fire screen are:

$q_{rad,1}$	Radiation from flames and cavity wall, portions of which are absorbed and transmitted by the two panes,
$q_{rad,2}$	Emission from inner surface of inner pane to cavity,
$q_{rad,3}$	Net radiation exchange between outer surface of inner pane and inner surface of outer pane,
$q_{rad,4}$	Net radiation exchange between outer surface of outer pane and walls of room,
$q_{conv,1}$	Convection between cavity gases and inner pane,
q_{conv2}	Convection across air space between panes,
$q_{conv,3}$	Convection from outer surface to room air,
$q_{cond,1}$	Conduction across inner pane, and
$q_{cond,2}$	Conduction across outer pane.

COMMENTS: (1) Much of the luminous portion of the flame radiation is transmitted to the room interior.

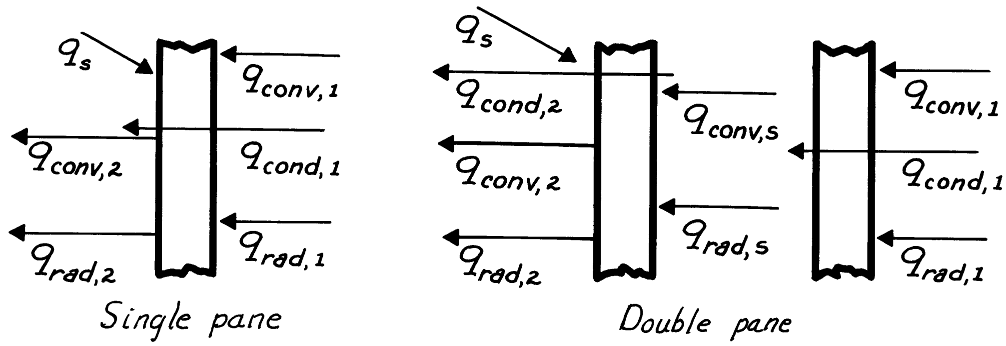
(2) All convection processes are buoyancy driven (free convection).

PROBLEM 1.73(a)

KNOWN: Room air is separated from ambient air by one or two glass panes.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes associated with single (above left schematic) and double (above right schematic) glass panes include.

$q_{conv,1}$	Convection from room air to inner surface of first pane,
$q_{rad,1}$	Net radiation exchange between room walls and inner surface of first pane,
$q_{cond,1}$	Conduction through first pane,
$q_{conv,s}$	Convection across airspace between panes,
$q_{rad,s}$	Net radiation exchange between outer surface of first pane and inner surface of second pane (across airspace),
$q_{cond,2}$	Conduction through a second pane,
$q_{conv,2}$	Convection from outer surface of single (or second) pane to ambient air,
$q_{rad,2}$	Net radiation exchange between outer surface of single (or second) pane and surroundings such as the ground, and
q_S	Incident solar radiation during day; fraction transmitted to room is smaller for double pane.

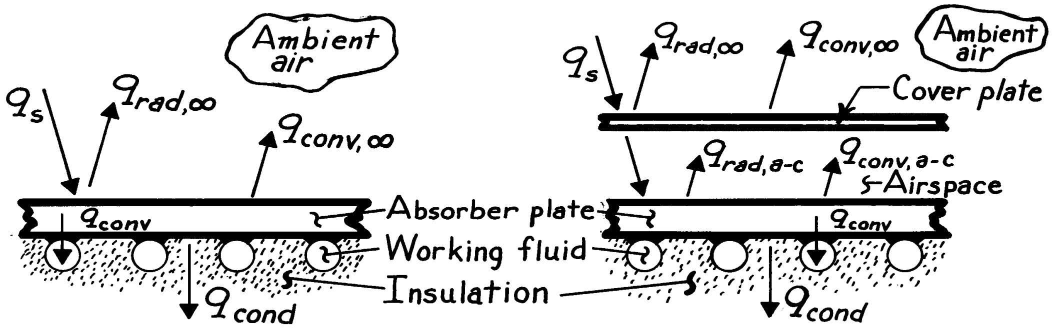
COMMENTS: Heat loss from the room is significantly reduced by the double pane construction.

PROBLEM 1.73(b)

KNOWN: Configuration of a flat plate solar collector.

FIND: Relevant heat transfer processes with and without a cover plate.

SCHEMATIC:



The relevant processes without (above left schematic) and with (above right schematic) include:

q_S	Incident solar radiation, a large portion of which is absorbed by the absorber plate. Reduced with use of cover plate (primarily due to reflection off cover plate).
$q_{rad,\infty}$	Net radiation exchange between absorber plate or cover plate and surroundings,
$q_{conv,\infty}$	Convection from absorber plate or cover plate to ambient air,
$q_{rad,a-c}$	Net radiation exchange between absorber and cover plates,
$q_{conv,a-c}$	Convection heat transfer across airspace between absorber and cover plates,
q_{cond}	Conduction through insulation, and
q_{conv}	Convection to working fluid.

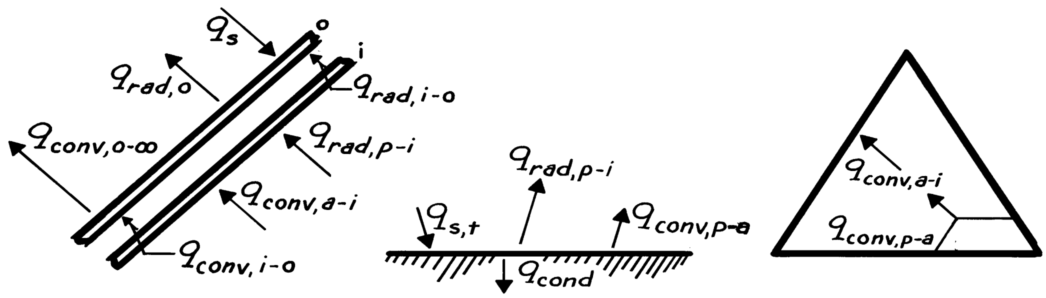
COMMENTS: The cover plate acts to significantly reduce heat losses by convection and radiation from the absorber plate to the surroundings.

PROBLEM 1.73(c)

KNOWN: Configuration of a solar collector used to heat air for agricultural applications.

FIND: Relevant heat transfer processes.

SCHEMATIC:



Assume the temperature of the absorber plates exceeds the ambient air temperature. At the *cover plates*, the relevant processes are:

$q_{conv,a-i}$	Convection from inside air to inner surface,
$q_{rad,p-i}$	Net radiation transfer from absorber plates to inner surface,
$q_{conv,i-o}$	Convection across airspace between covers,
$q_{rad,i-o}$	Net radiation transfer from inner to outer cover,
$q_{conv,o-\infty}$	Convection from outer cover to ambient air,
$q_{rad,o}$	Net radiation transfer from outer cover to surroundings, and
q_S	Incident solar radiation.

Additional processes relevant to the *absorber plates* and *airspace* are:

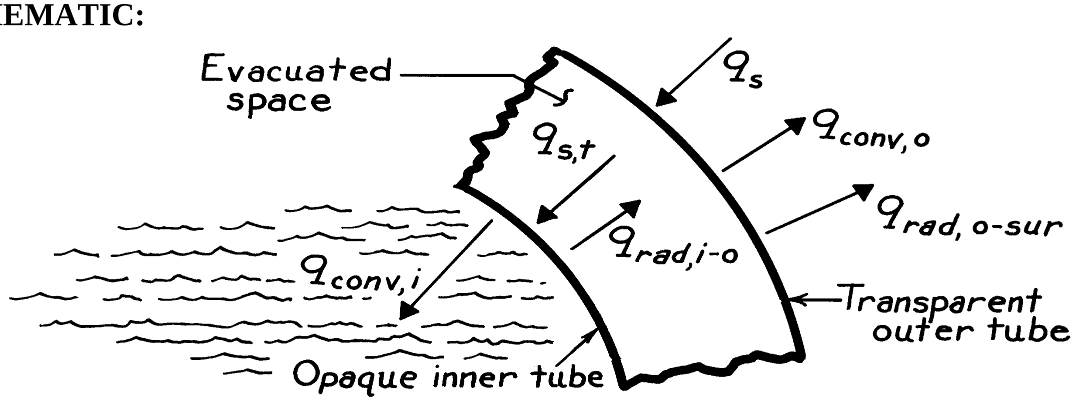
$q_{S,t}$	Solar radiation transmitted by cover plates,
$q_{conv,p-a}$	Convection from absorber plates to inside air, and
q_{cond}	Conduction through insulation.

PROBLEM 1.73(d)

KNOWN: Features of an evacuated tube solar collector.

FIND: Relevant heat transfer processes for one of the tubes.

SCHEMATIC:



The relevant heat transfer processes for one of the evacuated tube solar collectors includes:

q_S	Incident solar radiation including contribution due to reflection off panel (most is transmitted),
$q_{conv,o}$	Convection heat transfer from outer surface to ambient air,
$q_{rad,o-sur}$	Net rate of radiation heat exchange between outer surface of outer tube and the surroundings, including the panel,
$q_{S,t}$	Solar radiation transmitted through outer tube and incident on inner tube (most is absorbed),
$q_{rad,i-o}$	Net rate of radiation heat exchange between outer surface of inner tube and inner surface of outer tube, and
$q_{conv,i}$	Convection heat transfer to working fluid.

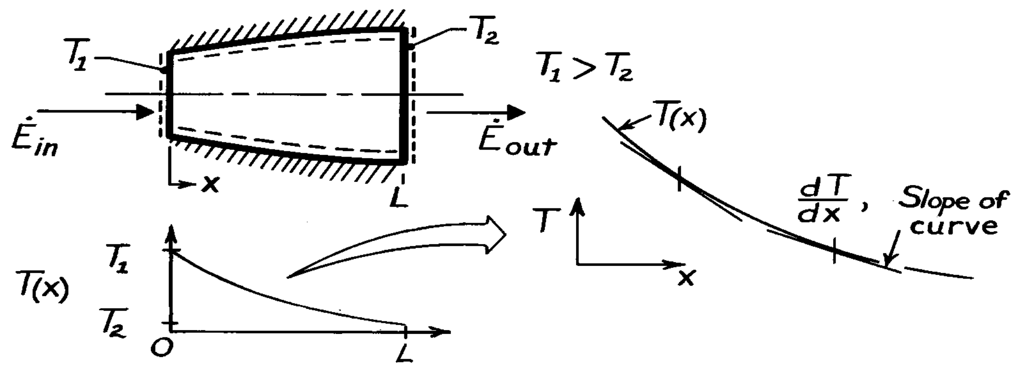
There is also conduction heat transfer through the inner and outer tube walls. If the walls are thin, the temperature drop across the walls will be small.

PROBLEM 2.1

KNOWN: Steady-state, one-dimensional heat conduction through an axisymmetric shape.

FIND: Sketch temperature distribution and explain shape of curve.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: Performing an energy balance on the object according to Eq. 1.11a, $\dot{E}_{in} - \dot{E}_{out} = 0,$ it follows that

$$\dot{E}_{in} - \dot{E}_{out} = q_x$$

and that $q_x \neq q_x(x)$. That is, the heat rate within the object is everywhere constant. From Fourier's law,

$$q_x = -kA_x \frac{dT}{dx},$$

and since q_x and k are both constants, it follows that

$$A_x \frac{dT}{dx} = \text{Constant.}$$

That is, the product of the cross-sectional area normal to the heat rate and temperature gradient remains a constant and independent of distance x. It follows that since A_x increases with x, then dT/dx must decrease with increasing x. Hence, the temperature distribution appears as shown above.

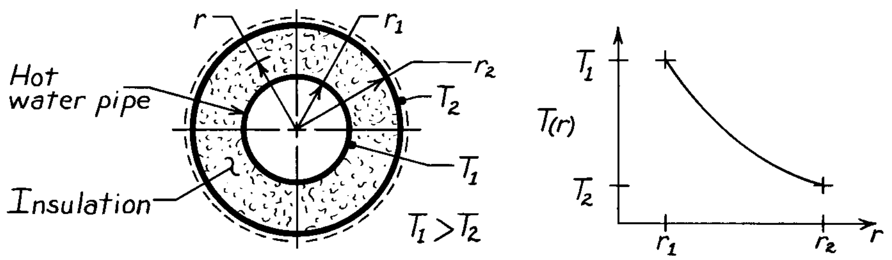
COMMENTS: (1) Be sure to recognize that dT/dx is the slope of the temperature distribution. (2) What would the distribution be when $T_2 > T_1$? (3) How does the heat flux, q_x'', vary with distance?

PROBLEM 2.2

KNOWN: Hot water pipe covered with thick layer of insulation.

FIND: Sketch temperature distribution and give brief explanation to justify shape.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (radial) conduction, (3) No internal heat generation, (4) Insulation has uniform properties independent of temperature and position.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional (cylindrical) radial system has the form

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi r\ell)\frac{dT}{dr}$$

where $A_r = 2\pi r\ell$ and ℓ is the axial length of the pipe-insulation system. Recognize that for steady-state conditions with no internal heat generation, an energy balance on the system requires $\dot{E}_{in} = \dot{E}_{out}$ since $\dot{E}_g = \dot{E}_{st} = 0.$ Hence

$$q_r = \text{Constant.}$$

That is, q_r is independent of radius (r). Since the thermal conductivity is also constant, it follows that

$$r\left[\frac{dT}{dr}\right] = \text{Constant.}$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius, r, remains constant throughout the insulation. For our situation, the temperature distribution must appear as shown in the sketch.

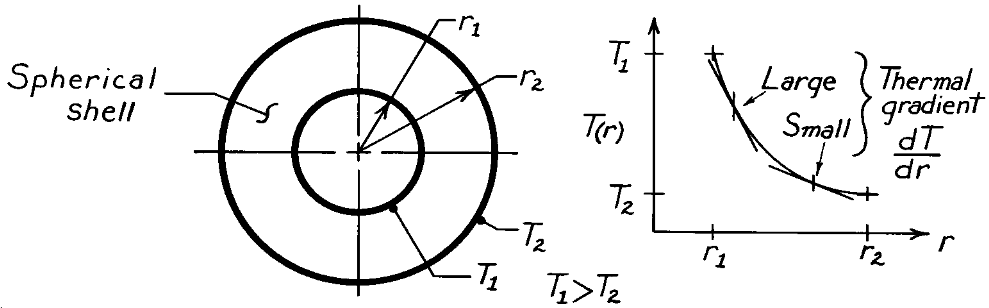
COMMENTS: (1) Note that, while q_r is a constant and independent of r, q''_r is not a constant. How does $q''_r(r)$ vary with r? (2) Recognize that the radial temperature gradient, dT/dr, decreases with increasing radius.

PROBLEM 2.3

KNOWN: A spherical shell with prescribed geometry and surface temperatures.

FIND: Sketch temperature distribution and explain shape of the curve.

SCHEMATIC:



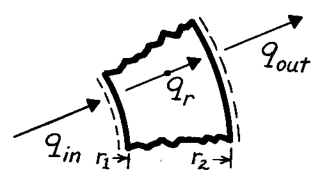
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial (spherical coordinates) direction, (3) No internal generation, (4) Constant properties.

ANALYSIS: Fourier's law, Eq. 2.1, for this one-dimensional, radial (spherical coordinate) system has the form

$$q_r = -k\,A_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$

where A_r is the surface area of a sphere. For steady-state conditions, an energy balance on the system yields $\dot{E}_{in} = \dot{E}_{out}$, since $\dot{E}_g = \dot{E}_{st} = 0$. Hence,

$$q_{in} = q_{out} = q_r \neq q_r(r).$$



That is, q_r is a constant, independent of the radial coordinate. Since the thermal conductivity is constant, it follows that

$$r^2\left[\frac{dT}{dr}\right] = \text{Constant}.$$

This relation requires that the product of the radial temperature gradient, dT/dr, and the radius squared, r^2, remains constant throughout the shell. Hence, the temperature distribution appears as shown in the sketch.

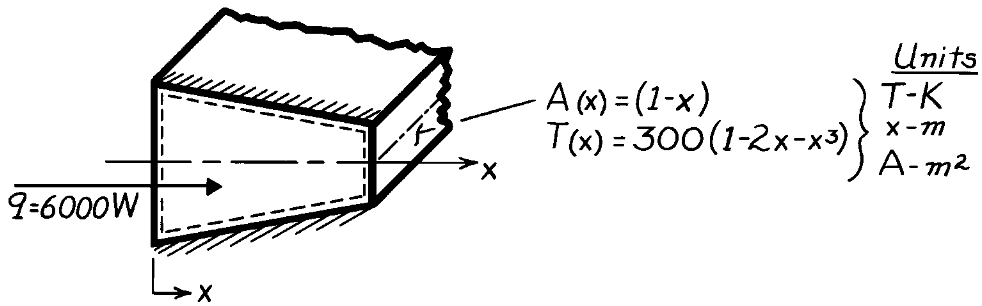
COMMENTS: Note that, for the above conditions, $q_r \neq q_r(r)$; that is, q_r is everywhere constant. How does q_r'' vary as a function of radius?

PROBLEM 2.4

KNOWN: Symmetric shape with prescribed variation in cross-sectional area, temperature distribution and heat rate.

FIND: Expression for the thermal conductivity, k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation.

ANALYSIS: Applying the energy balance, Eq. 1.11a, to the system, it follows that, since $\dot{E}_{in} = \dot{E}_{out}$,

$$q_x = \text{Constant} \neq f(x).$$

Using Fourier's law, Eq. 2.1, with appropriate expressions for A_x and T, yields

$$q_x = -k\, A_x \frac{dT}{dx}$$
$$6000\text{W} = -k \cdot (1\text{-x})\text{m}^2 \cdot \frac{d}{dx}\left[300\left(1-2x\text{-}x^3\right)\right]\frac{\text{K}}{\text{m}}.$$

Solving for k and recognizing its units are W/m·K,

$$k = \frac{-6000}{(1\text{-x})\left[300\left(-2-3x^2\right)\right]} = \frac{20}{(1-x)\left(2+3x^2\right)}. \qquad <$$

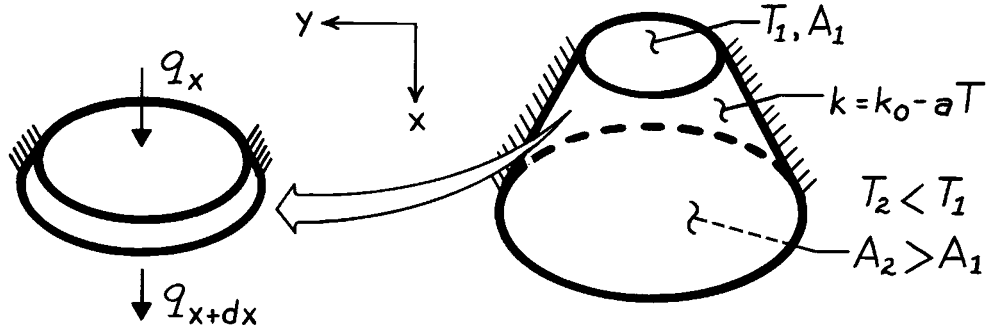
COMMENTS: (1) At x = 0, k = 10W/m·K and k → ∞ as x → 1. (2) Recognize that the 1-D assumption is an approximation which becomes more inappropriate as the area change with x, and hence two-dimensional effects, become more pronounced.

PROBLEM 2.5

KNOWN: End-face temperatures and temperature dependence of k for a truncated cone.

FIND: Variation with axial distance along the cone of q_x, q_x'', k, and dT/dx.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x (negligible temperature gradients along y), (2) Steady-state conditions, (3) Adiabatic sides, (4) No internal heat generation.

ANALYSIS: For the prescribed conditions, it follows from conservation of energy, Eq. 1.11a, that for a differential control volume, $\dot{E}_{in} = \dot{E}_{out}$ or $q_x = q_{x+dx}$. Hence

q_x is independent of x.

Since A(x) *increases* with *increasing* x, it follows that $q_x'' = q_x / A(x)$ *decreases* with *increasing* x. Since T *decreases* with *increasing* x, k *increases* with *increasing* x. Hence, from Fourier's law, Eq. 2.2,

$$q_x'' = -k \frac{dT}{dx},$$

it follows that | dT/dx | *decreases* with increasing x.

PROBLEM 2.6

KNOWN: Temperature dependence of the thermal conductivity, k(T), for heat transfer through a plane wall.

FIND: Effect of k(T) on temperature distribution, T(x).

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: From Fourier's law and the form of k(T),

$$q''_x = -k\,\frac{dT}{dx} = -\left(k_o + aT\right)\frac{dT}{dx}. \qquad (1)$$

The shape of the temperature distribution may be inferred from knowledge of $d^2T/dx^2 = d(dT/dx)/dx$. Since q''_x is independent of x for the prescribed conditions,

$$\frac{dq''_x}{dx} = -\frac{d}{dx}\left[\left(k_o + aT\right)\frac{dT}{dx}\right] = 0$$

$$-\left(k_o + aT\right)\frac{d^2T}{dx^2} - a\left[\frac{dT}{dx}\right]^2 = 0.$$

Hence,

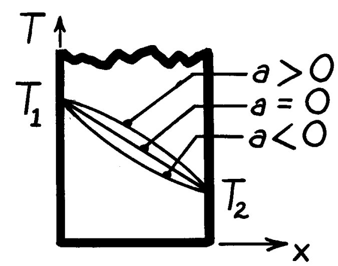
$$\frac{d^2T}{dx^2} = \frac{-a}{k_o + aT}\left[\frac{dT}{dx}\right]^2 \qquad \text{where} \begin{cases} k_o + aT = k > 0 \\ \left[\dfrac{dT}{dx}\right]^2 > 0 \end{cases}$$

from which it follows that for

$$a > 0: \ d^2T/dx^2 < 0$$

$$a = 0: \ d^2T/dx^2 = 0$$

$$a < 0: \ d^2T/dx^2 > 0.$$



COMMENTS: The shape of the distribution could also be inferred from Eq. (1). Since T decreases with increasing x,

a > 0: k decreases with increasing x = > | dT/dx | increases with increasing x

a = 0: $k = k_o$ = > dT/dx is constant

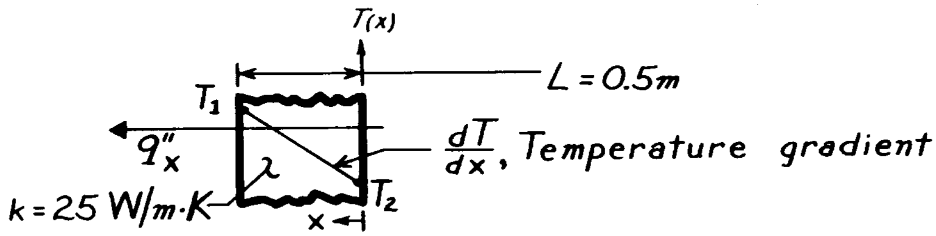
a < 0: k increases with increasing x = > | dT/dx | decreases with increasing x.

PROBLEM 2.7

KNOWN: Thermal conductivity and thickness of a one-dimensional system with no internal heat generation and steady-state conditions.

FIND: Unknown surface temperatures, temperature gradient or heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat flow, (2) No internal heat generation, (3) Steady-state conditions, (4) Constant properties.

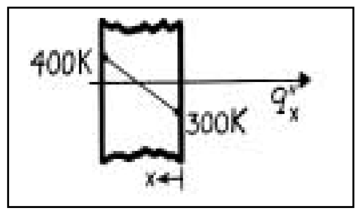
ANALYSIS: The rate equation and temperature gradient for this system are

$$q_x'' = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_1 - T_2}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities can be determined.

(a) $$\frac{dT}{dx} = \frac{(400-300)\text{K}}{0.5\text{m}} = 200\ \text{K/m}$$

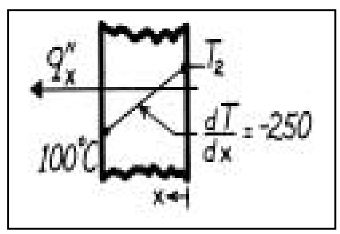
$$q_x'' = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times 200\frac{\text{K}}{\text{m}} = -5000\ \text{W/m}^2.$$ <



(b) $$q_x'' = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-250\frac{\text{K}}{\text{m}}\right] = 6250\ \text{W/m}^2$$

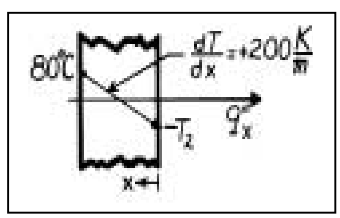
$$T_2 = T_1 - L\left[\frac{dT}{dx}\right] = 1000^\circ\text{C-}0.5\text{m}\left[\text{-}250\frac{\text{K}}{\text{m}}\right]$$

$$T_2 = 225^\circ\text{C}.$$ <



(c) $$q_x'' = -25\frac{\text{W}}{\text{m}\cdot\text{K}} \times 200\frac{\text{K}}{\text{m}} = -5000\ \text{W/m}^2$$

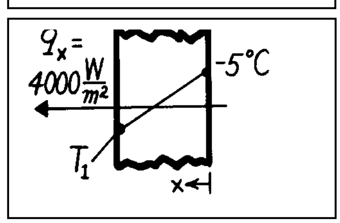
$$T_2 = 80^\circ\text{C-}0.5\text{m}\left[200\frac{\text{K}}{\text{m}}\right] = -20^\circ\text{C}.$$ <



(d) $$\frac{dT}{dx} = -\frac{q_x''}{k} = -\frac{4000\ \text{W/m}^2}{25\ \text{W/m}\cdot\text{K}} = -160\frac{\text{K}}{\text{m}}$$

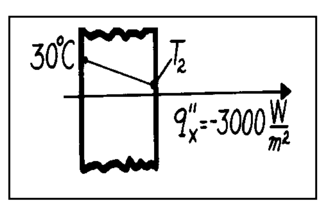
$$T_1 = L\left[\frac{dT}{dx}\right] + T_2 = 0.5\text{m}\left[\text{-}160\frac{\text{K}}{\text{m}}\right] + \left(-5^\circ\text{C}\right)\sqrt{a^2+b^2}$$

$$T_1 = -85^\circ\text{C}.$$ <



(e) $$\frac{dT}{dx} = -\frac{q_x''}{k} = -\frac{\left(-3000\ \text{W/m}^2\right)}{25\ \text{W/m}\cdot\text{K}} = 120\frac{\text{K}}{\text{m}}$$

$$T_2 = 30^\circ\text{C-}0.5\text{m}\left[120\frac{\text{K}}{\text{m}}\right] = -30^\circ\text{C}.$$

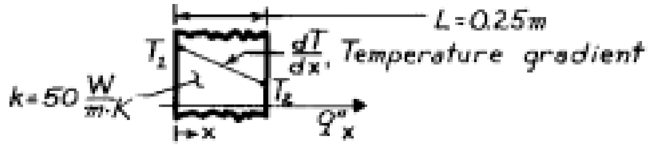


PROBLEM 2.8

KNOWN: One-dimensional system with prescribed thermal conductivity and thickness.

FIND: Unknowns for various temperature conditions and sketch distribution.

SCHEMATIC:



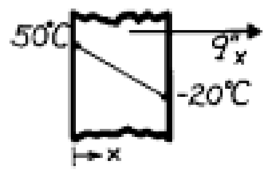
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) Constant properties.

ANALYSIS: The rate equation and temperature gradient for this system are

$$q_x'' = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_2 - T_1}{L}. \tag{1,2}$$

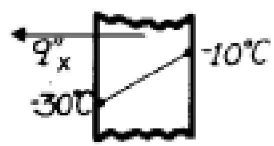
Using Eqs. (1) and (2), the unknown quantities for each case can be determined.

(a)
$$\frac{dT}{dx} = \frac{(-20-50)\text{K}}{0.25\text{m}} = -280 \text{ K/m}$$
$$q_x'' = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-280\frac{\text{K}}{\text{m}}\right] = 14.0 \text{ kW/m}^2.$$



$<$

(b)
$$\frac{dT}{dx} = \frac{(-10-(-30))\text{K}}{0.25\text{m}} = 80 \text{ K/m}$$
$$q_x'' = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[80\frac{\text{K}}{\text{m}}\right] = -4.0 \text{ kW/m}^2.$$



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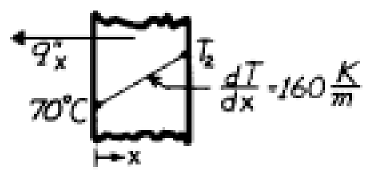
(c)
$$q_x'' = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[160\frac{\text{K}}{\text{m}}\right] = -8.0 \text{ kW/m}^2$$
$$T_2 = L\cdot\frac{dT}{dx} + T_1 = 0.25\text{m} \times \left[160\frac{\text{K}}{\text{m}}\right] + 70^\circ\text{C}.$$
$$T_2 = 110^\circ\text{C}.$$

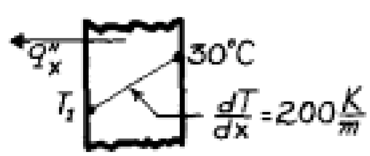


$<$

(d)
$$q_x'' = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[-80\frac{\text{K}}{\text{m}}\right] = 4.0 \text{ kW/m}^2$$
$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 40^\circ\text{C} - 0.25\text{m}\left[-80\frac{\text{K}}{\text{m}}\right].\in$$
$$T_1 = 60^\circ\text{C}.$$

$<$

(e)
$$q_x'' = -50\frac{\text{W}}{\text{m}\cdot\text{K}} \times \left[200\frac{\text{K}}{\text{m}}\right] = -10.0 \text{ kW/m}^2$$
$$T_1 = T_2 - L\cdot\frac{dT}{dx} = 30^\circ\text{C} - 0.25\text{m}\left[200\frac{\text{K}}{\text{m}}\right] = -20^\circ\text{C}.$$



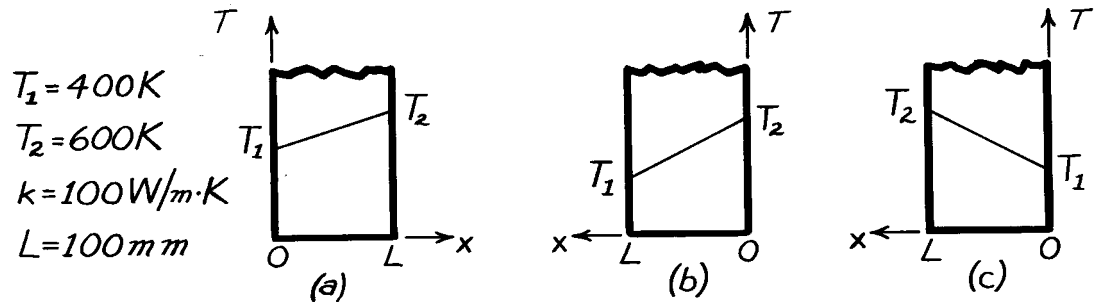
$<$

PROBLEM 2.9

KNOWN: Plane wall with prescribed thermal conductivity, thickness, and surface temperatures.

FIND: Heat flux, q''_x, and temperature gradient, dT/dx, for the three different coordinate systems shown.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat flow, (2) Steady-state conditions, (3) No internal generation, (4) Constant properties.

ANALYSIS: The rate equation for conduction heat transfer is

$$q''_x = -k\frac{dT}{dx}, \tag{1}$$

where the temperature gradient is constant throughout the wall and of the form

$$\frac{dT}{dx} = \frac{T(L) - T(0)}{L}. \tag{2}$$

Substituting numerical values, find the temperature gradients,

(a) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(600 - 400)K}{0.100m} = 2000 \text{ K/m}$$ <

(b) $$\frac{dT}{dx} = \frac{T_1 - T_2}{L} = \frac{(400 - 600)K}{0.100m} = -2000 \text{ K/m}$$ <

(c) $$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(600 - 400)K}{0.100m} = 2000 \text{ K/m.}$$ <

The heat rates, using Eq. (1) with k = 100 W/m·K, are

(a) $$q''_x = -100\frac{W}{m \cdot K} \times 2000 \text{ K/m} = \text{-200 kW/m}^2$$ <

(b) $$q''_x = -100\frac{W}{m \cdot K}(-2000 \text{ K/m}) = +200 \text{ kW/m}^2$$ <

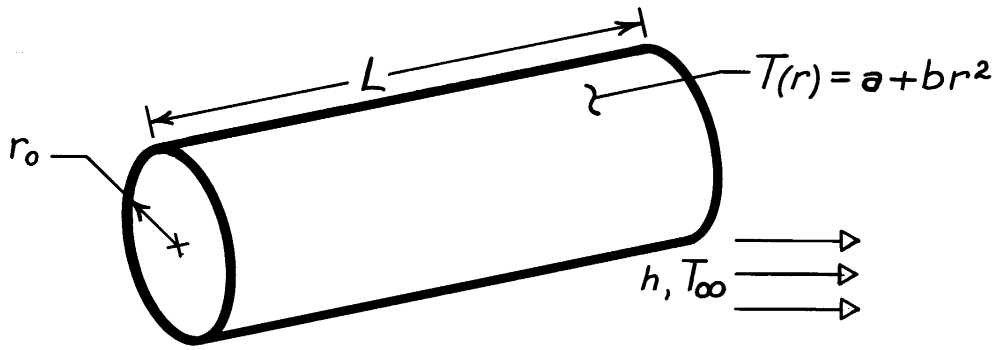
(c) $$q''_x = -100\frac{W}{m \cdot K} \times 2000 \text{ K/m} = \text{-200 kW/m}^2$$ <

PROBLEM 2.10

KNOWN: Temperature distribution in solid cylinder and convection coefficient at cylinder surface.

FIND: Expressions for heat rate at cylinder surface and fluid temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The heat rate from Fourier's law for the radial (cylindrical) system has the form

$$q_r = -kA_r \frac{dT}{dr}.$$

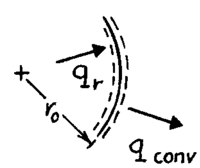
Substituting for the temperature distribution, $T(r) = a + br^2$,

$$q_r = -k(2\pi rL)\,2br \;=\; -4\pi kbLr^2.$$

At the outer surface ($r = r_o$), the conduction heat rate is

$$q_{r=r_o} = -4\pi kbLr_o^2. \qquad <$$

From a surface energy balance at $r = r_o$,



$$q_{r=r_o} = q_{conv} = h(2\pi r_o L)\left[T(r_o) - T_\infty\right],$$

Substituting for $q_{r=r_o}$ and solving for T_∞,

$$T_\infty \;=\; T(r_o) + \frac{2kbr_o}{h}$$

$$T_\infty \;=\; a + br_o^2 + \frac{2kbr_o}{h}$$

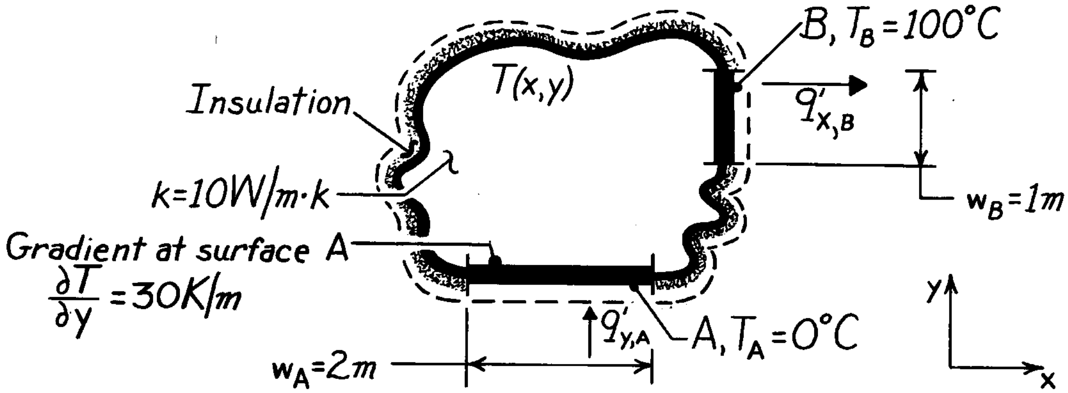
$$T_\infty \;=\; a + br_o\left[r_o + \frac{2k}{h}\right]. \qquad <$$

PROBLEM 2.11

KNOWN: Two-dimensional body with specified thermal conductivity and two isothermal surfaces of prescribed temperatures; one surface, A, has a prescribed temperature gradient.

FIND: Temperature gradients, $\partial T/\partial x$ and $\partial T/\partial y$, at the surface B.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) No heat generation, (4) Constant properties.

ANALYSIS: At the surface A, the temperature gradient in the x-direction must be zero. That is, $(\partial T/\partial x)_A = 0$. This follows from the requirement that the heat flux vector must be normal to an isothermal surface. The heat rate at the surface A is given by Fourier's law written as

$$q'_{y,A} = -k \cdot w_A \left.\frac{\partial T}{\partial y}\right]_A = -10\frac{W}{m \cdot K} \times 2m \times 30\frac{K}{m} = -600 W/m.$$

On the surface B, it follows that

$$(\partial T/\partial y)_B = 0$$ <

in order to satisfy the requirement that the heat flux vector be normal to the isothermal surface B. Using the conservation of energy requirement, Eq. 1.11a, on the body, find

$$q'_{y,A} - q'_{x,B} = 0 \qquad \text{or} \qquad q'_{x,B} = q'_{y,A}.$$

Note that,

$$q'_{x,B} = -k \cdot w_B \left.\frac{\partial T}{\partial x}\right]_B$$

and hence

$$(\partial T/\partial x)_B = \frac{-q'_{y,A}}{k \cdot w_B} = \frac{-(-600\ W/m)}{10\ W/m \cdot K \times 1m} = 60\ K/m.$$ <

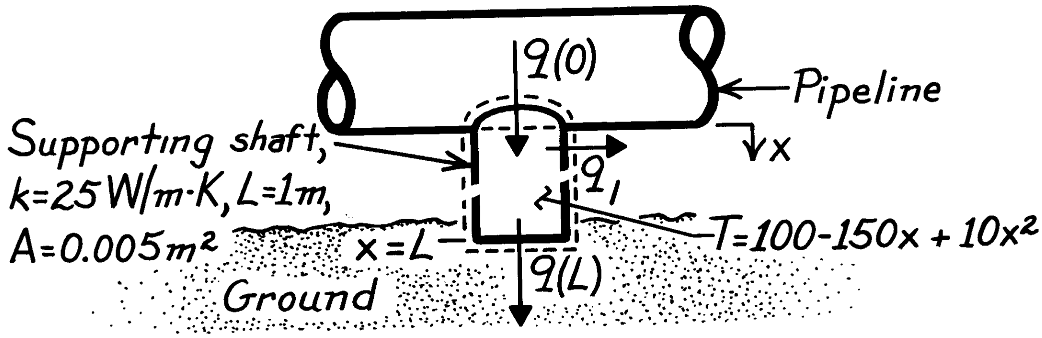
COMMENTS: Note that, in using the conservation requirement, $q'_{in} = +q'_{y,A}$ and $q'_{out} = +q'_{x,B}$.

PROBLEM 2.12

KNOWN: Length and thermal conductivity of a shaft. Temperature distribution along shaft.

FIND: Temperature and heat rates at ends of shaft.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

ANALYSIS: Temperatures at the top and bottom of the shaft are, respectively,

$$T(0) = 100°C \qquad T(L) = -40°C. \qquad <$$

Applying Fourier's law, Eq. 2.1,

$$q_x = -kA\frac{dT}{dx} = -25\ \text{W/m}\cdot\text{K}\left(0.005\ \text{m}^2\right)\left(-150+20x\right)^\circ\text{C/m}$$
$$q_x = 0.125\left(150\ -\ 20x\right)\text{W}.$$

Hence,

$$q_x(0) = 18.75\ \text{W} \qquad q_x(L) = 16.25\ \text{W}. \qquad <$$

The difference in heat rates, $q_x(0) > q_x(L)$, is due to heat losses q_ℓ from the side of the shaft.

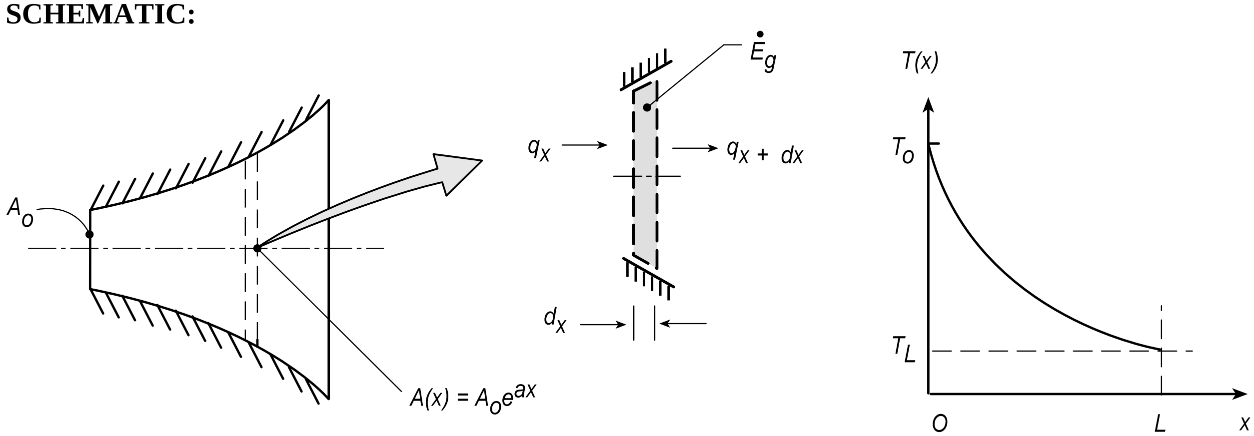
COMMENTS: Heat loss from the side requires the existence of temperature gradients over the shaft cross-section. Hence, specification of T as a function of only x is an approximation.

PROBLEM 2.13

KNOWN: A rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$ where A_o and a are constants.

FIND: (a) Expression for the conduction heat rate, $q_x(x)$; use this expression to determine the temperature distribution, T(x); and sketch of the temperature distribution, (b) Considering the presence of volumetric heat generation rate, $\dot{q} = \dot{q}_o \exp(-ax)$, obtain an expression for $q_x(x)$ when the left face, x = 0, is well insulated.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the rod, (2) Constant properties, (3) Steady-state conditions.

ANALYSIS: Perform an energy balance on the control volume, A(x)·dx,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$q_x - q_{x+dx} + \dot{q} \cdot A(x) \cdot dx = 0$$

The conduction heat rate terms can be expressed as a Taylor series and substituting expressions for $\dot{q}$ and A(x),

$$-\frac{d}{dx}(q_x) + \dot{q}_o \exp(-ax) \cdot A_o \exp(ax) = 0 \qquad (1)$$

$$q_x = -k \cdot A(x) \frac{dT}{dx} \qquad (2)$$

(a) With no internal generation, $\dot{q}_o = 0$, and from Eq. (1) find

$$-\frac{d}{dx}(q_x) = 0 \qquad <$$

indicating that the heat rate is constant with x. By combining Eqs. (1) and (2)

$$-\frac{d}{dx}\left(-k \cdot A(x) \frac{dT}{dx}\right) = 0 \qquad \text{or} \qquad A(x) \cdot \frac{dT}{dx} = C_1 \qquad (3) <$$

Continued...

PROBLEM 2.13 (Cont.)

That is, the product of the cross-sectional area and the temperature gradient is a constant, independent of x. Hence, with $T(0) > T(L)$, the temperature distribution is exponential, and as shown in the sketch above. Separating variables and integrating Eq. (3), the general form for the temperature distribution can be determined,

$$A_o \exp(ax) \cdot \frac{dT}{dx} = C_1$$

$$dT = C_1 A_o^{-1} \exp(-ax)\, dx$$

$$T(x) = -C_1 A_o a \exp(-ax) + C_2 \qquad <$$

We could use the two temperature boundary conditions, $T_o = T(0)$ and $T_L = T(L)$, to evaluate C_1 and C_2 and, hence, obtain the temperature distribution in terms of T_o and T_L.

(b) With the internal generation, from Eq. (1),

$$-\frac{d}{dx}(q_x) + \dot{q}_o A_o = 0 \qquad \text{or} \qquad q_x = \dot{q}_o A_o x \qquad <$$

That is, the heat rate increases linearly with x.

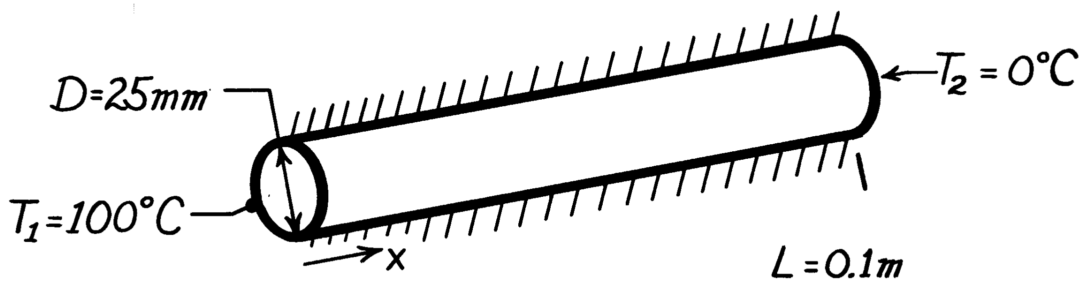
COMMENTS: In part (b), you could determine the temperature distribution using Fourier's law and knowledge of the heat rate dependence upon the x-coordinate. Give it a try!

PROBLEM 2.14

KNOWN: Dimensions and end temperatures of a cylindrical rod which is insulated on its side.

FIND: Rate of heat transfer associated with different rod materials.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction along cylinder axis, (2) Steady-state conditions, (3) Constant properties.

PROPERTIES: The properties may be evaluated from *Tables A-1* to *A-3* at a mean temperature of 50°C = 323K and are summarized below.

ANALYSIS: The heat transfer rate may be obtained from Fourier's law. Since the axial temperature gradient is linear, this expression reduces to

$$q = kA\frac{T_1 - T_2}{L} = k\frac{\pi(0.025\text{m})^2}{4}\frac{(100-0)°\text{C}}{0.1\text{m}} = 0.491(\text{m}\cdot°\text{C})\cdot k$$

	Cu (pure)	Al (2024)	St.St. (302)	SiN	Oak	Magnesia (85%)	Pyrex	
k(W/m·K)	401	177	16.3	14.9	0.19	0.052	1.4	
q(W)	197	87	8.0	7.3	0.093	0.026	0.69	<

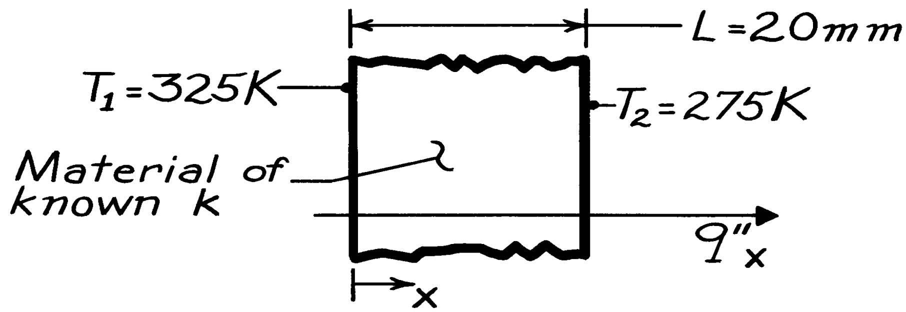
COMMENTS: The k values of Cu and Al were obtained by linear interpolation; the k value of St.St. was obtained by linear extrapolation, as was the value for SiN; the value for magnesia was obtained by linear interpolation; and the values for oak and pyrex are for 300 K.

PROBLEM 2.15

KNOWN: One-dimensional system with prescribed surface temperatures and thickness.

FIND: Heat flux through system constructed of these materials: (a) pure aluminum, (b) plain carbon steel, (c) AISI 316, stainless steel, (d) pyroceram, (e) teflon and (f) concrete.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No heat generation, (4) Constant thermal properties.

PROPERTIES: The thermal conductivity is evaluated at the average temperature of the system, T = $(T_1+T_2)/2$ = (325+275)K/2 = 300K. Property values and table identification are shown below.

ANALYSIS: For this system, Fourier's law can be written as

$$q''_x = -k\frac{dT}{dx} = -k\frac{T_2 - T_1}{L}.$$

Substituting numerical values, the heat flux is

$$q''_x = -k\frac{(275-325)K}{20\times10^{-3}m} = +2500\frac{K}{m}\cdot k$$

where q''_x will have units W/m^2 if k has units W/m·K. The heat fluxes for each system follow.

Material	Table	Thermal conductivity k(W/m·K)	Heat flux q''_x (kW/m^2)	
(a) Pure Aluminum	A-1	237	593	<
(b) Plain carbon steel	A-1	60.5	151	
(c) AISI 316, S.S.	A-1	13.4	33.5	
(d) Pyroceram	A-2	3.98	9.95	
(e) Teflon	A-3	0.35	0.88	
(f) Concrete	A-3	1.4	3.5	

COMMENTS: Recognize that the thermal conductivity of these solid materials varies by more than two orders of magnitude.

PROBLEM 2.16

KNOWN: Different thicknesses of three materials: rock, 18 ft; wood, 15 in; and fiberglass insulation, 6 in.

FIND: The insulating quality of the materials as measured by the R-value.

PROPERTIES: *Table A-3* (300K):

Material	Thermal conductivity, W/m·K
Limestone	2.15
Softwood	0.12
Blanket (glass, fiber 10 kg/m^3)	0.048

ANALYSIS: The R-value, a quantity commonly used in the construction industry and building technology, is defined as

$$R \equiv \frac{L(\text{in})}{k\left(\text{Btu}\cdot\text{in}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)}.$$

The R-value can be interpreted as the thermal resistance of a 1 ft^2 cross section of the material. Using the conversion factor for thermal conductivity between the SI and English systems, the R-values are:

Rock, Limestone, 18 ft:

$$R=\frac{18\text{ ft}\times 12\dfrac{\text{in}}{\text{ft}}}{2.15\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W/m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=14.5\ \left(\text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

Wood, Softwood, 15 in:

$$R=\frac{15\text{ in}}{0.12\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu}/\text{h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W}/\text{m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=18\ \left(\text{Btu}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

Insulation, Blanket, 6 in:

$$R=\frac{6\text{ in}}{0.048\dfrac{\text{W}}{\text{m}\cdot\text{K}}\times 0.5778\dfrac{\text{Btu}/\text{h}\cdot\text{ft}\cdot{}^\circ\text{F}}{\text{W}/\text{m}\cdot\text{K}}\times 12\dfrac{\text{in}}{\text{ft}}}=18\ \left(\text{Btu}/\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}\right)^{-1}$$

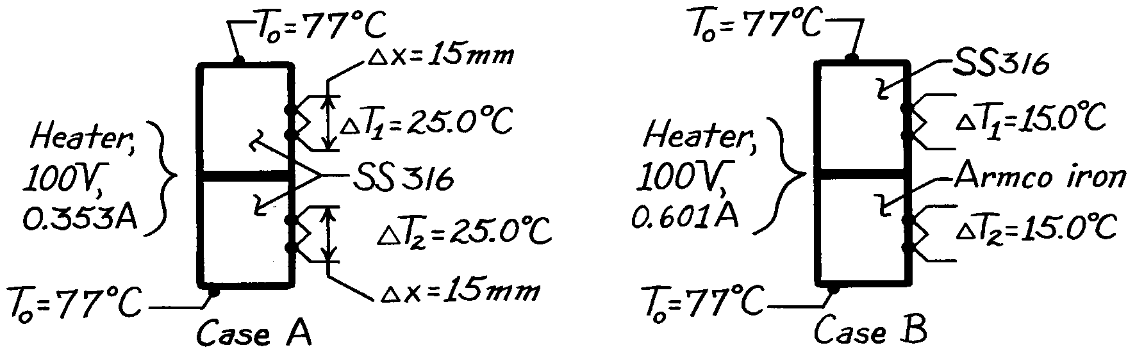
COMMENTS: The R-value of 19 given in the advertisement is reasonable.

PROBLEM 2.17

KNOWN: Electrical heater sandwiched between two identical cylindrical (30 mm dia. × 60 mm length) samples whose opposite ends contact plates maintained at T_o.

FIND: (a) Thermal conductivity of SS316 samples for the prescribed conditions (A) and their average temperature, (b) Thermal conductivity of Armco iron sample for the prescribed conditions (B), (c) Comment on advantages of experimental arrangement, lateral heat losses, and conditions for which $\Delta T_1 \neq \Delta T_2$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer in samples, (2) Steady-state conditions, (3) Negligible contact resistance between materials.

PROPERTIES: *Table A.2*, Stainless steel 316 $(\bar{T} = 400\ \text{K})$: $k_{ss} = 15.2\ \text{W/m}\cdot\text{K}$; Armco iron $(\bar{T} = 380\ \text{K})$: $k_{iron} = 71.6\ \text{W/m}\cdot\text{K}$.

ANALYSIS: (a) For Case A recognize that half the heater power will pass through each of the samples which are presumed identical. Apply Fourier's law to a sample

$$q = kA_c\frac{\Delta T}{\Delta x}$$

$$k = \frac{q\Delta x}{A_c\Delta T} = \frac{0.5(100\text{V}\times 0.353\text{A})\times 0.015\ \text{m}}{\pi(0.030\ \text{m})^2/4\times 25.0^\circ\text{C}} = 15.0\ \text{W/m}\cdot\text{K}. \qquad <$$

The total temperature drop across the length of the sample is $\Delta T_1(L/\Delta x)$ = 25°C (60 mm/15 mm) = 100°C. Hence, the heater temperature is T_h = 177°C. Thus the average temperature of the sample is

$$\bar{T} = (T_o + T_h)/2 = 127^\circ\text{C} = 400\ \text{K} \qquad <$$

.

We compare the calculated value of k with the tabulated value (see above) at 400 K and note the good agreement.

(b) For Case B, we assume that the thermal conductivity of the SS316 sample is the same as that found in Part (a). The heat rate through the Armco iron sample is

Continued …..

PROBLEM 2.17 (CONT.)

$$q_{iron} = q_{heater} - q_{ss} = 100\text{V} \times 0.601\text{A} - 15.0\ \text{W/m}\cdot\text{K} \times \frac{\pi(0.030\ \text{m})^2}{4} \times \frac{15.0^\circ\text{C}}{0.015\ \text{m}}$$

$$q_{iron} = (60.1 - 10.6)\text{W} = 49.5\ \text{W}$$

where

$$q_{ss} = k_{ss} A_c \Delta T_2 / \Delta x_2.$$

Applying Fourier's law to the iron sample,

$$k_{iron} = \frac{q_{iron}\Delta x_2}{A_c \Delta T_2} = \frac{49.5\ \text{W} \times 0.015\ \text{m}}{\pi(0.030\ \text{m})^2 / 4 \times 15.0^\circ\text{C}} = 70.0\ \text{W/m}\cdot\text{K}. \qquad <$$

The total drop across the iron sample is 15°C(60/15) = 60°C; the heater temperature is (77 + 60)°C = 137°C. Hence the average temperature of the iron sample is

$$\overline{T} = (137\ +\ 77)^\circ\text{C}/2 = 107^\circ\text{C} = 380\ \text{K}. \qquad <$$

We compare the computed value of k with the tabulated value (see above) at 380 K and note the good agreement.

(c) The principal advantage of having two identical samples is the assurance that all the electrical power dissipated in the heater will appear as equivalent heat flows through the samples. With only one sample, heat can flow from the backside of the heater even though insulated.

Heat leakage out the lateral surfaces of the cylindrically shaped samples will become significant when the sample thermal conductivity is comparable to that of the insulating material. Hence, the method is suitable for metallics, but must be used with caution on nonmetallic materials.

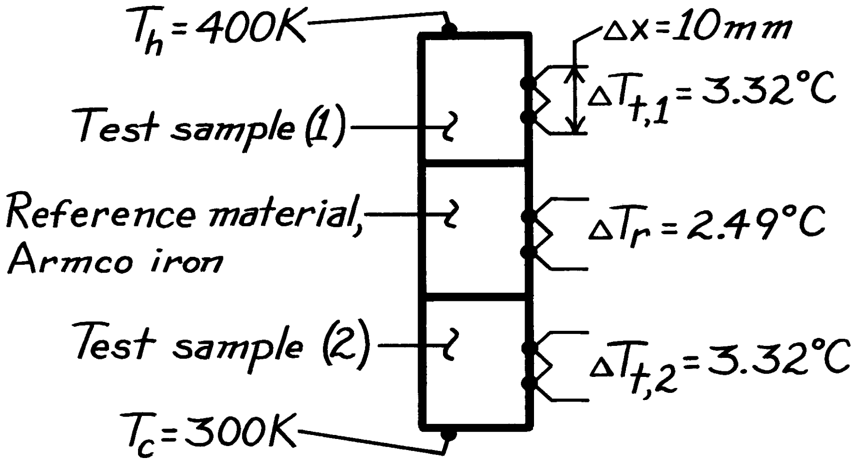
For any combination of materials in the upper and lower position, we expect $\Delta T_1 = \Delta T_2$. However, if the insulation were improperly applied along the lateral surfaces, it is possible that heat leakage will occur, causing $\Delta T_1 \neq \Delta T_2$.

PROBLEM 2.18

KNOWN: Comparative method for measuring thermal conductivity involving two identical samples stacked with a reference material.

FIND: (a) Thermal conductivity of test material and associated temperature, (b) Conditions for which $\Delta T_{t,1} \neq \Delta T_{t,2}$

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer through samples and reference material, (3) Negligible thermal contact resistance between materials.

PROPERTIES: *Table A.2*, Armco iron $(\overline{T} = 350\text{ K})$: $k_r = 69.2\text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) Recognizing that the heat rate through the samples and reference material, all of the same diameter, is the same, it follows from Fourier's law that

$$k_t \frac{\Delta T_{t,1}}{\Delta x} = k_r \frac{\Delta T_r}{\Delta x} = k_t \frac{\Delta T_{t,2}}{\Delta x}$$

$$k_t = k_r \frac{\Delta T_r}{\Delta T_t} = 69.2\text{ W/m}\cdot\text{K} \frac{2.49°\text{C}}{3.32°\text{C}} = 51.9\text{ W/m}\cdot\text{K}. \quad <$$

We should assign this value a temperature of 350 K. <

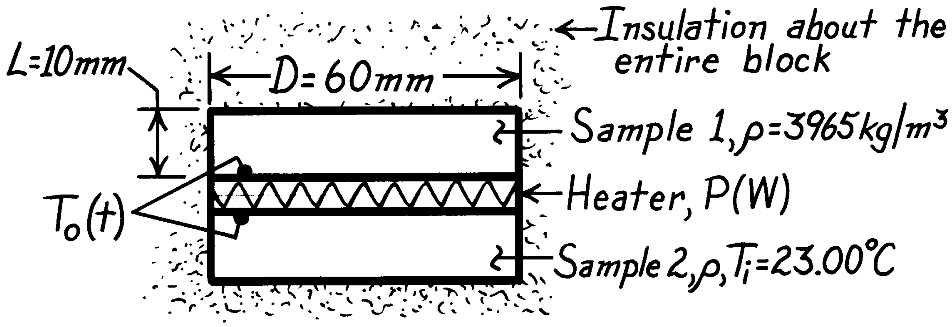
(b) If the test samples are identical in every respect, $\Delta T_{t,1} \neq \Delta T_{t,2}$ if the thermal conductivity is highly dependent upon temperature. Also, if there is heat leakage out the lateral surface, we can expect $\Delta T_{t,2} < \Delta T_{t,1}$. Leakage could be influential, if the thermal conductivity of the test material were less than an order of magnitude larger than that of the insulating material.

PROBLEM 2.19

KNOWN: Identical samples of prescribed diameter, length and density initially at a uniform temperature T_i, sandwich an electric heater which provides a uniform heat flux q_o'' for a period of time Δt_o. Conditions shortly after energizing and a long time after de-energizing heater are prescribed.

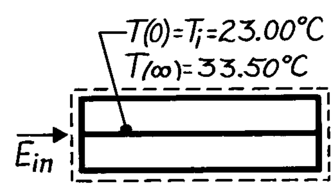
FIND: Specific heat and thermal conductivity of the test sample material. From these properties, identify type of material using Table A.1 or A.2.

SCHEMATIC:



ASSUMPTIONS: (1) One dimensional heat transfer in samples, (2) Constant properties, (3) Negligible heat loss through insulation, (4) Negligible heater mass.

ANALYSIS: Consider a control volume about the samples and heater, and apply conservation of energy over the time interval from t = 0 to ∞



$$E_{in} - E_{out} = \Delta E = E_f - E_i$$

$$P\Delta t_o - 0 = Mc_p\left[T(\infty) - T_i\right]$$

where energy inflow is prescribed by the Case A power condition and the final temperature T_f by Case B. Solving for c_p,

$$c_p = \frac{P\Delta t_o}{M\left[T(\infty) - T_i\right]} = \frac{15\ \text{W} \times 120\ \text{s}}{2 \times 3965\ \text{kg/m}^3\left(\pi \times 0.060^2/4\right)\text{m}^2 \times 0.010\ \text{m}\left[33.50 - 23.00\right]^\circ\text{C}}$$

$$c_p = 765\ \text{J/kg}\cdot\text{K}$$ <

where $M = \rho V = 2\rho(\pi D^2/4)L$ is the mass of both samples. For Case A, the transient thermal response of the heater is given by

Continued …..

$$T_o(t) - T_i = 2q_o''\left[\frac{t}{\pi\rho c_p k}\right]^{1/2}$$

$$k = \frac{t}{\pi\rho c_p}\left[\frac{2q_o''}{T_o(t) - T_i}\right]^2$$

$$k = \frac{30\ \text{s}}{\pi \times 3965\ \text{kg/m}^3 \times 765\ \text{J/kg}\cdot\text{K}}\left[\frac{2 \times 2653\ \text{W/m}^2}{(24.57\ -\ 23.00)^\circ\text{C}}\right]^2 = 36.0\ \text{W/m}\cdot\text{K} \qquad <$$

where

$$q_o'' = \frac{P}{2A_s} = \frac{P}{2\left(\pi D^2/4\right)} = \frac{15\ \text{W}}{2\left(\pi \times 0.060^2/4\right)\text{m}^2} = 2653\ \text{W/m}^2.$$

With the following properties now known,

$\rho = 3965\ \text{kg/m}^3$ $\qquad c_p = 765\ \text{J/kg}\cdot\text{K}$ $\qquad k = 36\ \text{W/m}\cdot\text{K}$

entries in Table A.1 are scanned to determine whether these values are typical of a metallic material. Consider the following,

- metallics with low ρ generally have higher thermal conductivities,
- specific heats of both types of materials are of similar magnitude,
- the low k value of the sample is typical of poor metallic conductors which generally have much higher specific heats,
- more than likely, the material is nonmetallic.

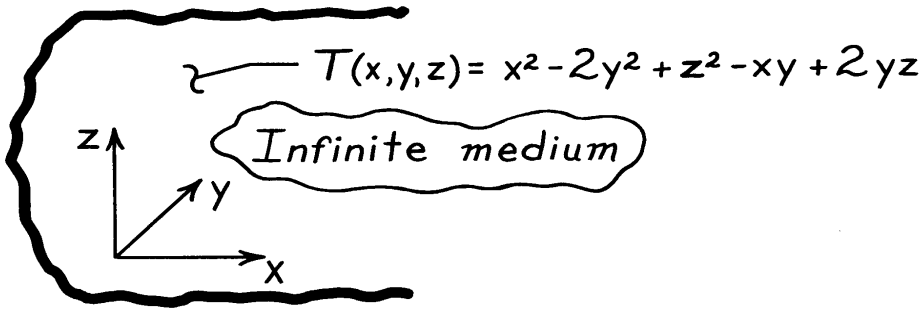
From Table A.2, the second entry, polycrystalline aluminum oxide, has properties at 300 K corresponding to those found for the samples. $<$

PROBLEM 2.20

KNOWN: Temperature distribution, T(x,y,z), within an infinite, homogeneous body at a given instant of time.

FIND: Regions where the temperature changes with time.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties of infinite medium and (2) No internal heat generation.

ANALYSIS: The temperature distribution throughout the medium, at any instant of time, must satisfy the heat equation. For the three-dimensional cartesian coordinate system, with constant properties and no internal heat generation, the heat equation, Eq. 2.15, has the form

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}. \quad (1)$$

If T(x,y,z) satisfies this relation, conservation of energy is satisfied at every point in the medium. Substituting T(x,y,z) into the Eq. (1), first find the gradients, $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial z$.

$$\frac{\partial}{\partial x}(2x - y)+\frac{\partial}{\partial y}(-4y - x + 2z)+\frac{\partial}{\partial z}(2z + 2y)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Performing the differentiations,

$$2-4+2=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Hence,

$$\frac{\partial T}{\partial t}=0$$

which implies that, at the prescribed instant, the temperature is everywhere independent of time.

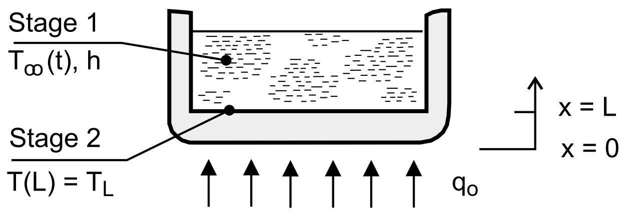
COMMENTS: Since we do not know the initial and boundary conditions, we cannot determine the temperature distribution, T(x,y,z), at any future time. We can only determine that, for this special instant of time, the temperature will not change.

PROBLEM 2.21

KNOWN: Diameter D, thickness L and initial temperature T_i of pan. Heat rate from stove to bottom of pan. Convection coefficient h and variation of water temperature $T_\infty(t)$ during Stage 1. Temperature T_L of pan surface in contact with water during Stage 2.

FIND: Form of heat equation and boundary conditions associated with the two stages.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in pan bottom, (2) Heat transfer from stove is uniformly distributed over surface of pan in contact with the stove, (3) Constant properties.

ANALYSIS:

Stage 1

Heat Equation: $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions: $$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_o'' = \frac{q_o}{\left(\pi D^2/4\right)}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty(t)\right]$$

Initial Condition: $$T(x,0) = T_i$$

Stage 2

Heat Equation: $$\frac{d^2 T}{dx^2} = 0$$

Boundary Conditions: $$-k\left.\frac{dT}{dx}\right|_{x=0} = q_o''$$

$$T(L) = T_L$$

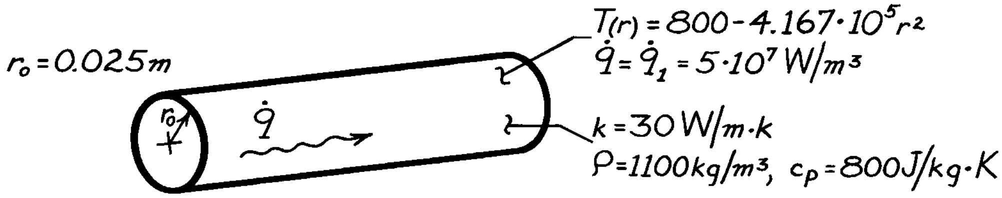
COMMENTS: Stage 1 is a transient process for which $T_\infty(t)$ must be determined separately. As a first approximation, it could be estimated by neglecting changes in thermal energy storage by the pan bottom and assuming that all of the heat transferred from the stove acted to increase thermal energy storage within the water. Hence, with $q \approx Mc_p\, dT_\infty/dt$, where M and c_p are the mass and specific heat of the water in the pan, $T_\infty(t) \approx (q/Mc_p)\, t$.

PROBLEM 2.22

KNOWN: Steady-state temperature distribution in a cylindrical rod having uniform heat generation of $\dot{q}_1 = 5\times10^7\ \text{W/m}^3$.

FIND: (a) Steady-state centerline and surface heat transfer rates per unit length, q_r'. (b) Initial time rate of change of the centerline and surface temperatures in response to a change in the generation rate from $\dot{q}_1$ to $\dot{q}_2 = 10^8\ \text{W/m}^3$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the r direction, (2) Uniform generation, and (3) Steady-state for $\dot{q}_1 = 5\times10^7\ \text{W/m}^3$.

ANALYSIS: (a) From the rate equations for cylindrical coordinates,

$$q_r'' = -k\frac{\partial T}{\partial r} \qquad q = -kA_r\frac{\partial T}{\partial r}.$$

Hence,

$$q_r = -k(2\pi rL)\frac{\partial T}{\partial r}$$

or

$$q_r' = -2\pi kr\frac{\partial T}{\partial r}$$

where ∂T/∂r may be evaluated from the prescribed temperature distribution, T(r).

At r = 0, the gradient is (∂T/∂r) = 0. Hence, from Eq. (1) the heat rate is

$$q_r'(0) = 0. \qquad <$$

At r = r_o, the temperature gradient is

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -2\left[4.167\times10^5\ \frac{K}{m^2}\right](r_o) = -2\left(4.167\times10^5\right)(0.025m)$$

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = -0.208\times10^5\ \text{K/m}.$$

Continued …..

Hence, the heat rate at the outer surface ($r = r_o$) per unit length is

$$q'_r(r_o) = -2\pi[30\ \text{W/m}\cdot\text{K}](0.025\text{m})\left[-0.208\times10^5\ \text{K/m}\right]$$

$$q'_r(r_o) = 0.980\times10^5\ \text{W/m}. \qquad <$$

(b) Transient (time-dependent) conditions will exist when the generation is changed, and for the prescribed assumptions, the temperature is determined by the following form of the heat equation, Eq. 2.20

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right]+\dot{q}_2 = \rho c_p\frac{\partial T}{\partial t}$$

Hence

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right]+\dot{q}_2\right].$$

However, initially (at t = 0), the temperature distribution is given by the prescribed form, $T(r) = 800 - 4.167\times10^5 r^2$, and

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] = \frac{k}{r}\frac{\partial}{\partial r}\left[r\left(-8.334\times10^5\cdot r\right)\right]$$

$$= \frac{k}{r}\left(-16.668\times10^5\cdot r\right)$$

$$= 30\ \text{W/m}\cdot\text{K}\left[-16.668\times10^5\ \text{K/m}^2\right]$$

$$= -5\times10^7\ \text{W/m}^3\ (\text{the original } \dot{q} = \dot{q}_1).$$

Hence, everywhere in the wall,

$$\frac{\partial T}{\partial t} = \frac{1}{1100\ \text{kg/m}^3\times800\ \text{J/kg}\cdot\text{K}}\left[-5\times10^7+10^8\right]\ \text{W/m}^3$$

or

$$\frac{\partial T}{\partial t} = 56.82\ \text{K/s}. \qquad <$$

COMMENTS: (1) The value of $(\partial T/\partial t)$ will decrease with increasing time, until a new steady-state condition is reached and once again $(\partial T/\partial t) = 0$.

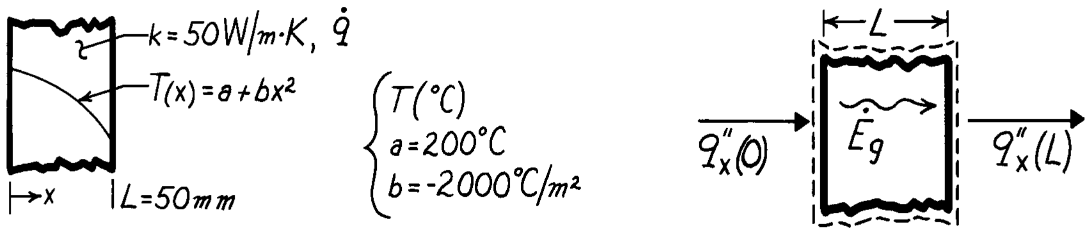
(2) By applying the energy conservation requirement, Eq. 1.11a, to a unit length of the rod for the steady-state condition, $\dot{E}'_{in} - E'_{out} + \dot{E}'_{gen} = 0$. Hence $q'_r(0) - q'_r(r_o) = -\dot{q}_1\left(\pi r_o^2\right)$.

PROBLEM 2.23

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.15 re-written as

$$\dot{q} = -k \frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q} = -k \frac{d}{dx}\left[\frac{d}{dx}\left(a + bx^2\right)\right] = -k\frac{d}{dx}\left[2bx\right] = -2bk$$

$$\dot{q} = -2\left(-2000^\circ C/m^2\right) \times 50\ W/m\cdot K = 2.0\times 10^5\ W/m^3. \qquad <$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x) = -k\left.\frac{dT}{dx}\right]_x.$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x) = -k\frac{d}{dx}\left[a + bx^2\right] = -2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0) = 0 \qquad <$$

$$q''_x(L) = -2kbL = -2\times 50 W/m\cdot K\left(-2000^\circ C/m^2\right)\times 0.050m$$

$$q''_x(L) = 10{,}000\ W/m^2. \qquad <$$

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(0) - q''_x(L) + \dot{q}L = 0$$

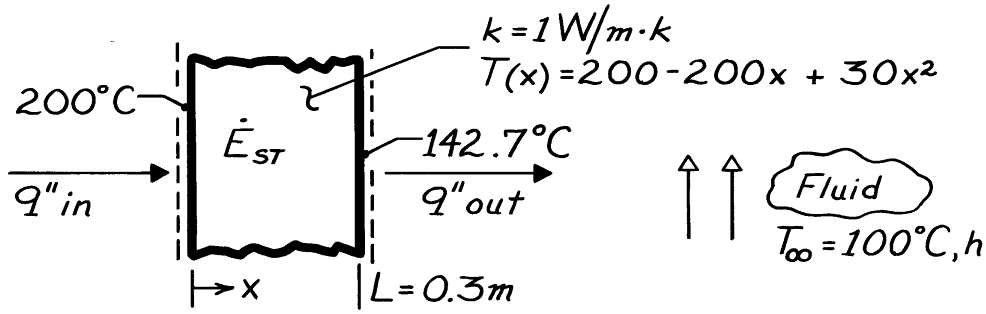
$$\dot{q} = \frac{q''_x(L) - q''_x(0)}{L} = \frac{10{,}000\ W/m^2 - 0}{0.050m} = 2.0\times 10^5 W/m^3.$$

PROBLEM 2.24

KNOWN: Wall thickness, thermal conductivity, temperature distribution, and fluid temperature.

FIND: (a) Surface heat rates and rate of change of wall energy storage per unit area, and (b) Convection coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant k.

ANALYSIS: (a) From Fourier's law,

$$q''_x = -k\frac{\partial T}{\partial x} = (200 - 60x)\cdot k$$

$$q''_{in} = q''_{x=0} = 200\frac{°C}{m}\times 1\frac{W}{m\cdot K} = 200\ W/m^2$$ <

$$q''_{out} = q''_{x=L} = (200 - 60\times 0.3)°C/m\ \times\ 1\ W/m\cdot K = 182\ W/m^2.$$ <

Applying an energy balance to a control volume about the wall, Eq. 1.11a,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$\dot{E}''_{st} = q''_{in} - q''_{out} = 18\ W/m^2.$$ <

(b) Applying a surface energy balance at x = L,

$$q''_{out} = h\left[T(L) - T_\infty\right]$$

$$h = \frac{q''_{out}}{T(L) - T_\infty} = \frac{182\ W/m^2}{(142.7 - 100)°C}$$

$$h = 4.3\ W/m^2\cdot K.$$ <

COMMENTS: (1) From the heat equation,

$$(\partial T/\partial t) = (k/\rho c_p)\ \partial^2 T/\partial x^2 = 60(k/\rho c_p),$$

it follows that the temperature is increasing with time at every point in the wall.

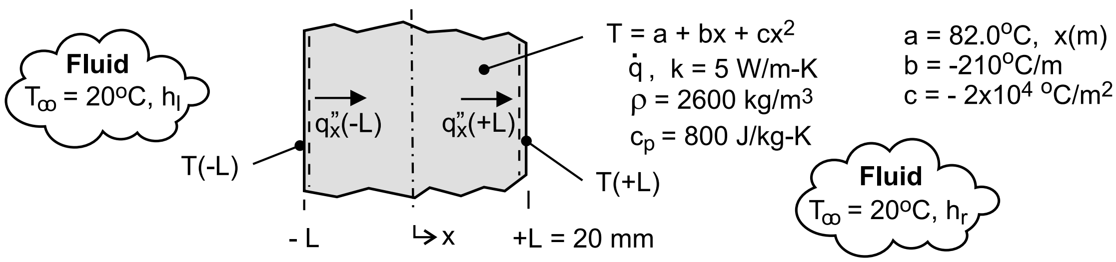
(2) The value of h is small and is typical of free convection in a gas.

PROBLEM 2.25

KNOWN: Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation $\dot{q}$ while convection occurs at both of its surfaces.

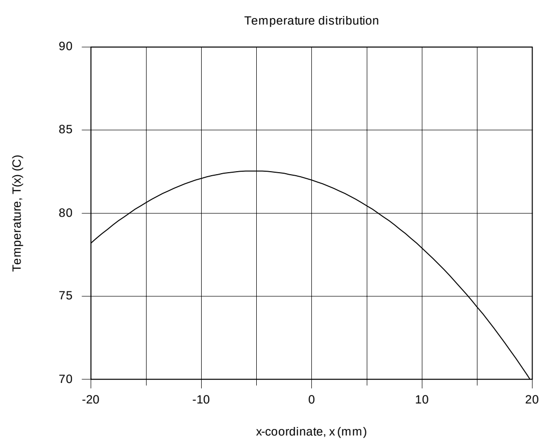
FIND: (a) Sketch the temperature distribution, T(x), and identify significant physical features, (b) Determine $\dot{q}$, (c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces x = L and x = +L, (e) Obtain an expression for the heat flux distribution, $q''_x(x)$; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ($\dot{q}$ = 0), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with $\dot{q} = 0$; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

ANALYSIS: (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane, T(-5.25 mm) = 83.3°C, (3) the gradient at the x = +L surface is greater than at x = -L. Find also that T(-L) = 78.2°C and T(+L) = 69.8°C for use in part (d).



(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, Eq. 2.15, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x)=a+bx+cx^2$$

$$\frac{d}{dx}(0+b+2cx)+\frac{\dot{q}}{k}=(0+2c)+\frac{\dot{q}}{k}=0$$

Continued …..

$$\dot{q} = -2ck = -2\left(-2\times10^{4}{}^\circ C/m^2\right)5\,W/m\cdot K = 2\times10^5\ W/m^3 \qquad <$$

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q''_x(x) = -k\frac{dT}{dx} \qquad \text{where} \qquad T(x) = a + bx + cx^2$$

$$q''_x(-L) = -k\left[0 + b + 2cx\right]_{x=-L} = -\left[b - 2cL\right]k$$

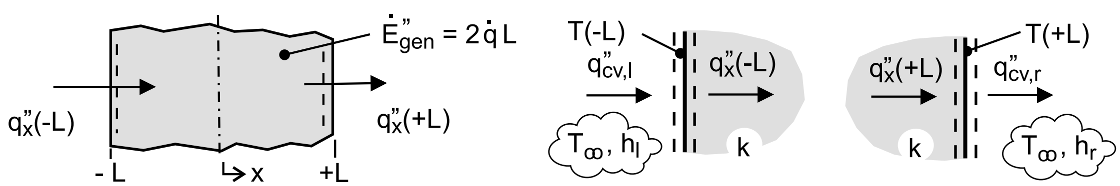
$$q''_x(-L) = -\left[-210^\circ C/m - 2\left(-2\times10^{4}{}^\circ C/m^2\right)0.020m\right]\times 5\,W/m\cdot K = -2950\,W/m^2 \qquad <$$

$$q''_x(+L) = -(b + 2cL)k = +5050\,W/m^2 \qquad <$$

From an overall energy balance on the wall as shown in the sketch below, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$,

$$+q''_x(-L) - q''_x(+L) + 2\dot{q}L \overset{?}{=} 0 \quad \text{or} \quad -2950\,W/m^2 - 5050\,W/m^2 + 8000\,W/m^2 = 0$$

where $2\dot{q}L = 2\times2\times10^5\ W/m^3\times0.020\ m = 8000\,W/m^2$, so the equality is satisfied



Part (c) Overall energy balance　　　Part (d) Surface energy balances

(d) The convection coefficients, h_l and h_r, for the left- and right-hand boundaries (x = -L and x= +L, respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for T(-L) and T(+L).

$$q''_{cv,\ell} = q''_x(-L)$$

$$h_l\left[T_\infty - T(-L)\right] = h_l\left[20 - 78.2\right]K = -2950\,W/m^2 \qquad h_l = 51\,W/m^2\cdot K \qquad <$$

$$q''_{cv,r} = q''_x(+L)$$

$$h_r\left[T(+L) - T_\infty\right] = h_r\left[69.8 - 20\right]K = +5050\,W/m^2 \qquad h_r = 101\,W/m^2\cdot K \qquad <$$

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q''_x(x) = -k\frac{dT}{dx} = -k\left[0 + b + 2cx\right]$$

$$q''_x(x) = -5\,W/m\cdot K\left[-210^\circ C/m + 2\left(-2\times10^{4}{}^\circ C/m^2\right)\right]x = 1050 + 2\times10^5 x \qquad <$$

Continued …..

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location where $q''_x(x_{max}) = 0$,

$$x_{max} = -\frac{1050\,W/m^2}{2\times10^5\ W/m^3} = -5.25\times10^{-3} m = -5.25\,mm \qquad <$$

(f) If the source of the heat generation is suddenly deactivated so that $\dot{q} = 0$, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \rho\, c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still $T(x) = a + bx + cx^2$. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}''_{st} = k\frac{\partial}{\partial x}[0 + b + 2cx] = k[0 + 2c] = 5\,W/m\cdot K\times2\left(-2\times10^4 °C/m^2\right) = -2\times10^5\ W/m^3 \quad <$$

(g) With no heat generation, the wall will eventually ($t \to \infty$) come to equilibrium with the fluid, $T(x,\infty) = T_\infty = 20°C$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.11b. The "initial" state is that corresponding to the steady-state temperature distribution, T_i, and the "final" state has $T_f = 20°C$. We've used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \quad E''_{in} = 0.$$

$$-E''_{out} = \rho\, c_p 2L\left(T_f - T_\infty\right) - \rho\, c_p \int_{-L}^{+L}\left(T_i - T_\infty\right)dx$$

$$E''_{out} = \rho\, c_p \int_{-L}^{+L}\left[a + bx + cx^2 - T_\infty\right]dx = \rho\, c_p \left[ax + bx^2/2 + cx^3/3 - T_\infty x\right]_{-L}^{+L}$$

$$E''_{out} = \rho\, c_p\left[2aL + 0 + 2cx^3/3 - 2T_\infty L\right]$$

$$E''_{out} = 2600\,kg/m^3\times800\,J/kg\cdot K\left[2\times82°C\times0.020m + 2\left(-2\times10^4 °C/m^2\right)\right.$$
$$\left.(0.020m)^3/3 - 2(20°C)0.020m\right]$$

$$E''_{out} = 4.94\times10^6\ J/m^2 \qquad <$$

COMMENTS: (1) In part (a), note that the temperature gradient is larger at x = + L than at x = - L. This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

Continued …..

(2) In evaluating the conduction heat fluxes, $q''_x(x)$, it is important to recognize that this flux is in the positive x-direction. See how this convention is used in formulating the energy balance in part (c).

(3) It is good practice to represent energy balances with a schematic, clearly defining the system or surface, showing the CV or CS with dashed lines, and labeling the processes. Review again the features in the schematics for the energy balances of parts (c & d).

(4) Re-writing the heat diffusion equation introduced in part (b) as

$$-\frac{d}{dx}\left(-k\frac{dT}{dx}\right)+\dot{q}=0$$

recognize that the term in parenthesis is the heat flux. From the differential equation, note that if the differential of this term is a constant $(\dot{q}/k)$, then the term must be a linear function of the x-coordinate. This agrees with the analysis of part (e).

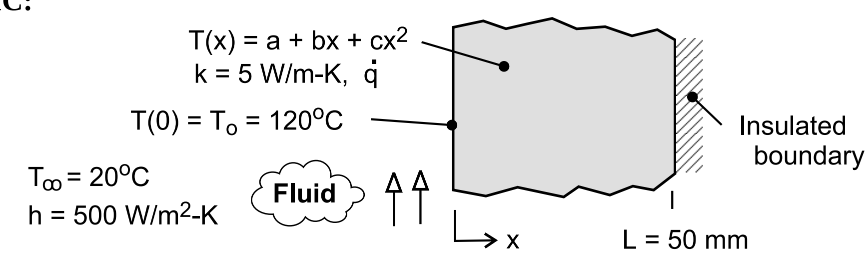
(5) In part (f), we evaluated $\dot{E}_{st}$, the rate of energy change stored in the wall at the instant the volumetric heat generation was deactivated. Did you notice that $\dot{E}_{st}=-2\times10^5\ W/m^3$ is the same value of the deactivated $\dot{q}$? How do you explain this?

PROBLEM 2.26

KNOWN: Steady-state conduction with uniform internal energy generation in a plane wall; temperature distribution has quadratic form. Surface at x=0 is prescribed and boundary at x = L is insulated.

FIND: (a) Calculate the internal energy generation rate, $\dot{q}$, by applying an overall energy balance to the wall, (b) Determine the coefficients a, b, and c, by applying the boundary conditions to the prescribed form of the temperature distribution; plot the temperature distribution and label as Case 1, (c) Determine new values for a, b, and c for conditions when the convection coefficient is halved, and the generation rate remains unchanged; plot the temperature distribution and label as Case 2; (d) Determine new values for a, b, and c for conditions when the generation rate is doubled, and the convection coefficient remains unchanged (h = 500 W/m^2·K); plot the temperature distribution and label as Case 3.

SCHEMATIC:



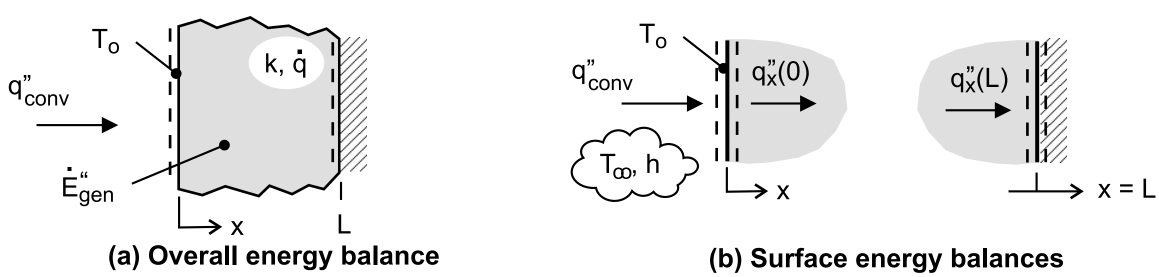
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties and uniform internal generation, and (3) Boundary at x = L is adiabatic.

ANALYSIS: (a) The internal energy generation rate can be calculated from an overall energy balance on the wall as shown in the schematic below.

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_{gen} = 0 \qquad \text{where} \qquad \dot{E}''_{in} = q''_{conv}$$

$$h\left(T_\infty - T_o\right) + \dot{q}\,L = 0 \tag{1}$$

$$\dot{q} = -h\left(T_\infty - T_o\right)/L = -500\,\mathrm{W/m^2 \cdot K}\left(20-120\right)^\circ\mathrm{C}/0.050\ \mathrm{m} = 1.0\times10^6\ \mathrm{W/m^3} \quad <$$



(a) Overall energy balance **(b) Surface energy balances**

(b) The coefficients of the temperature distribution, T(x) = a + bx + cx^2, can be evaluated by applying the boundary conditions at x = 0 and x = L. See Table 2.1 for representation of the boundary conditions, and the schematic above for the relevant surface energy balances.

Boundary condition at x = 0, convection surface condition

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conv} - q''_x(0) = 0 \qquad \text{where} \qquad q''_x(0) = -k\left.\frac{dT}{dx}\right)_{x=0}$$

$$h\left(T_\infty - T_o\right) - \left[-k\left(0 + b + 2cx\right)_{x=0}\right] = 0$$

Continued …..

$$b = -h\left(T_\infty - T_o\right)/k = -500\,W/m^2\cdot K\left(20-120\right)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \quad <$$

Boundary condition at x = L, adiabatic or insulated surface

$$\dot{E}_{in} - \dot{E}_{out} = -q''_x\left(L\right) = 0 \qquad \text{where} \qquad q''_x\left(L\right) = -k\left.\frac{dT}{dx}\right)_{x=L}$$

$$k\left[0+b+2cx\right]_{x=L} = 0 \tag{3}$$

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.050m\right) = -1.0\times10^5\,K/m^2 \quad <$$

Since the surface temperature at x = 0 is known, T(0) = T_o = 120°C, find

$$T\left(0\right) = 120°C = a + b\cdot0 + c\cdot0 \qquad \text{or} \qquad a = 120°C \tag{4} \quad <$$

Using the foregoing coefficients with the expression for T(x) in the Workspace of IHT, the temperature distribution can be determined and is plotted as Case 1 in the graph below.

(c) Consider Case 2 when the convection coefficient is halved, h_2 = h/2 = 250 W/m^2·K, $\dot{q} = 1\times10^6$ W/m^3 and other parameters remain unchanged except that $T_o \neq 120°C$. We can determine a, b, and c for the temperature distribution expression by repeating the analyses of parts (a) and (b).

Overall energy balance on the wall, see Eqs. (1,4)

$$a = T_o = \dot{q}L/h + T_\infty = 1\times10^6\,W/m^3\times0.050m/250\,W/m^2\cdot K + 20°C = 220°C \quad <$$

Surface energy balance at x = 0, see Eq. (2)

$$b = -h\left(T_\infty - T_o\right)/k = -250\,W/m^2\cdot K\left(20-220\right)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \quad <$$

Surface energy balance at x = L, see Eq. (3)

$$c = -b/2L = -1.0\times10^4\,K/m/\left(2\times0.050m\right) = -1.0\times10^5\,K/m^2 \quad <$$

The new temperature distribution, T_2 (x), is plotted as Case 2 below.

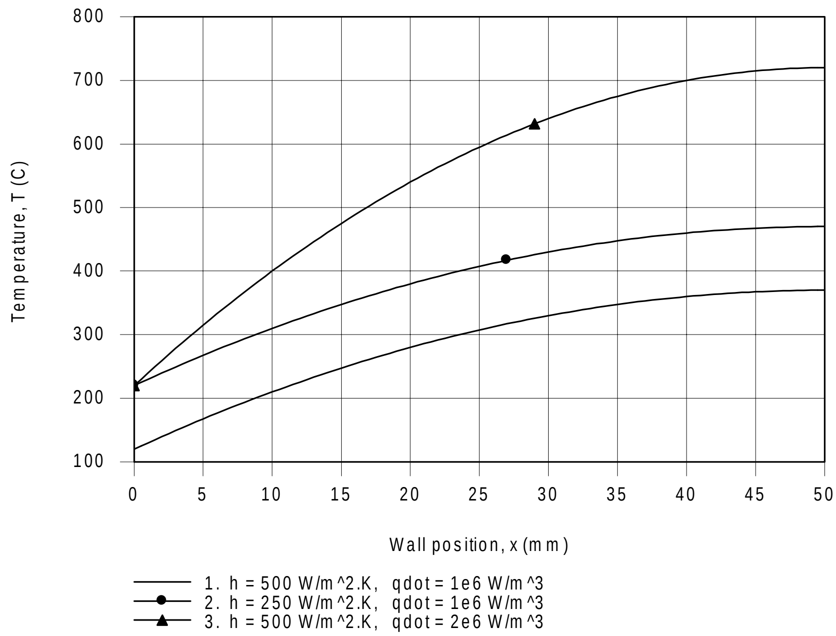
(d) Consider Case 3 when the internal energy volumetric generation rate is doubled, $\dot{q}_3 = 2\dot{q} = 2\times10^6\,W/m^3$, h = 500 W/m^2·K, and other parameters remain unchanged except that $T_o \neq 120°C$. Following the same analysis as part (c), the coefficients for the new temperature distribution, T (x), are

$$a = 220°C \qquad b = 2\times10^4\,K/m \qquad c = -2\times10^5\,K/m^2 \quad <$$

and the distribution is plotted as Case 3 below.

Continued …..

PROBLEM 2.26 (Cont.)



COMMENTS: Note the following features in the family of temperature distributions plotted above. The temperature gradients at x = L are zero since the boundary is insulated (adiabatic) for all cases. The shapes of the distributions are all quadratic, with the maximum temperatures at the insulated boundary.

By halving the convection coefficient for Case 2, we expect the surface temperature T_o to increase relative to the Case 1 value, since the same heat flux is removed from the wall $(\dot{q}L)$ but the convection resistance has increased.

By doubling the generation rate for Case 3, we expect the surface temperature T_o to increase relative to the Case 1 value, since double the amount of heat flux is removed from the wall $(2\dot{q}L)$.

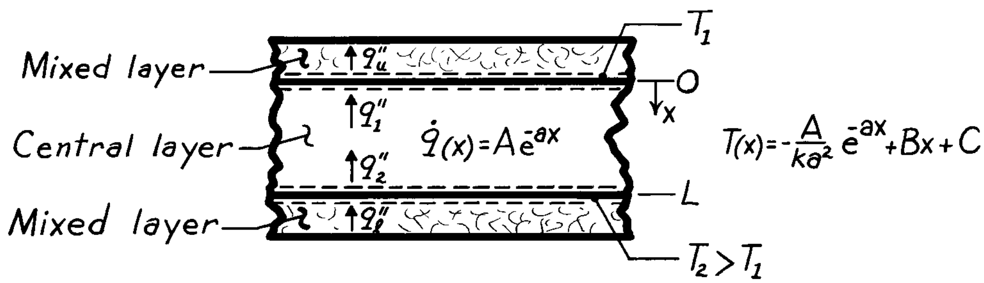
Can you explain why T_o is the same for Cases 2 and 3, yet the insulated boundary temperatures are quite different? Can you explain the relative magnitudes of T(L) for the three cases?

PROBLEM 2.27

KNOWN: Temperature distribution and distribution of heat generation in central layer of a solar pond.

FIND: (a) Heat fluxes at lower and upper surfaces of the central layer, (b) Whether conditions are steady or transient, (c) Rate of thermal energy generation for the entire central layer.

SCHEMATIC:



ASSUMPTIONS: (1) Central layer is stagnant, (2) One-dimensional conduction, (3) Constant properties

ANALYSIS: (a) The desired fluxes correspond to conduction fluxes in the central layer at the lower and upper surfaces. A general form for the conduction flux is

$$q''_{cond} = -k\frac{\partial T}{\partial x} = -k\left[\frac{A}{ka}e^{-ax} + B\right].$$

Hence,

$$q''_l = q''_{cond(x=L)} = -k\left[\frac{A}{ka}e^{-aL} + B\right] \quad q''_u = q''_{cond(x=0)} = -k\left[\frac{A}{ka} + B\right].$$ <

(b) Conditions are steady if $\partial T/\partial t = 0$. Applying the heat equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad -\frac{A}{k}e^{-ax} + \frac{A}{k}e^{-ax} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Hence conditions are *steady* since

$$\partial T/\partial t = 0 \qquad (\text{for all } 0 \le \times \le L).$$ <

(c) For the central layer, the energy generation is

$$\dot{E}''_g = \int_0^L \dot{q}\,dx = A\int_0^L e^{-ax}\,dx$$

$$\dot{E}_g = -\frac{A}{a}e^{-ax}\Big|_0^L = -\frac{A}{a}\left(e^{-aL} - 1\right) = \frac{A}{a}\left(1 - e^{-aL}\right).$$ <

Alternatively, from an overall energy balance,

$$q''_2 - q''_1 + \dot{E}''_g = 0 \qquad \dot{E}''_g = q''_1 - q''_2 = \left(-q''_{cond(x=0)}\right) - \left(-q''_{cond(x=L)}\right)$$

$$\dot{E}_g = k\left[\frac{A}{ka} + B\right] - k\left[\frac{A}{ka}e^{-aL} + B\right] = \frac{A}{a}\left(1 - e^{-aL}\right).$$

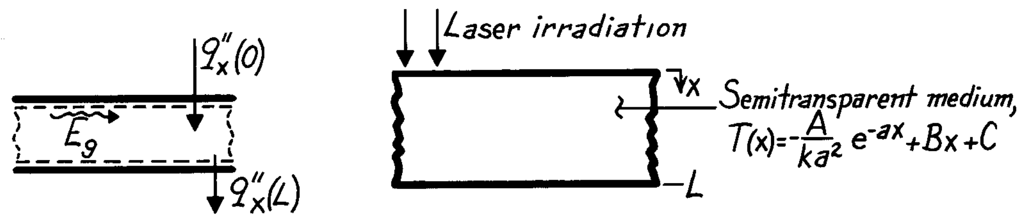
COMMENTS: Conduction is in the negative x-direction, necessitating use of minus signs in the above energy balance.

PROBLEM 2.28

KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) Heat generation rate $\dot{q}(x)$, (c) Expression for absorbed radiation per unit surface area in terms of A, a, B, C, L, and k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $\dot{q}(x)$.

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q''_x = -k\left[\frac{dT}{dx}\right] = -k\left[-\frac{A}{ka^2}(-a)e^{-ax} + B\right]$$

Front Surface, x=0: $$q''_x(0) = -k\left[+\frac{A}{ka}\cdot 1 + B\right] = -\left[\frac{A}{a} + kB\right]$$ <

Rear Surface, x=L: $$q''_x(L) = -k\left[+\frac{A}{ka}e^{-aL} + B\right] = -\left[\frac{A}{a}e^{-aL} + kB\right].$$ <

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d}{dx}\left(\frac{dT}{dx}\right)$$

$$\dot{q}(x) = -k\frac{d}{dx}\left[+\frac{A}{ka}e^{-ax} + B\right] = Ae^{-ax}.$$ <

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

recognize that $\dot{E}_g$ represents the absorbed irradiation. On a unit area basis

$$\dot{E}''_g = -\dot{E}''_{in} + \dot{E}''_{out} = -q''_x(0) + q''_x(L) = +\frac{A}{a}\left(1 - e^{-aL}\right).$$ <

Alternatively, evaluate $\dot{E}''_g$ by integration over the volume of the medium,

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L Ae^{-ax}dx = -\frac{A}{a}\left[e^{-ax}\right]_0^L = \frac{A}{a}\left(1 - e^{-aL}\right).$$

PROBLEM 2.29

KNOWN: Steady-state temperature distribution in a one-dimensional wall of thermal conductivity, $T(x) = Ax^3 + Bx^2 + Cx + D$.

FIND: Expressions for the heat generation rate in the wall and the heat fluxes at the two wall faces (x = 0,L).

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Homogeneous medium.

ANALYSIS: The appropriate form of the heat diffusion equation for these conditions is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d^2T}{dx^2}.$$

Hence, the generation rate is

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right] = -k\frac{d}{dx}\left[3Ax^2 + 2Bx + C + 0\right]$$

$$\dot{q} = -k\left[6Ax + 2B\right] \qquad <$$

which is linear with the coordinate x. The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x = -k\frac{dT}{dx} = -k\left[3Ax^2 + 2Bx + C\right]$$

using the expression for the temperature gradient derived above. Hence, the heat fluxes are:

Surface x=0:

$$q''_x(0) = -kC \qquad <$$

Surface x=L:

$$q''_x(L) = -k\left[3AL^2 + 2BL + C\right]. \qquad <$$

COMMENTS: (1) From an overall energy balance on the wall, find

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_g = 0$$

$$q''_x(0) - q''_x(L) + \dot{E}''_g = (-kC) - (-k)\left[3AL^2 + 2BL + C\right] + \dot{E}''_g = 0$$

$$\dot{E}''_g = -3AkL^2 - 2BkL.$$

From integration of the volumetric heat rate, we can also find $\dot{E}''_g$ as

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L -k\left[6Ax + 2B\right]dx = -k\left[3Ax^2 + 2Bx\right]_0^L$$

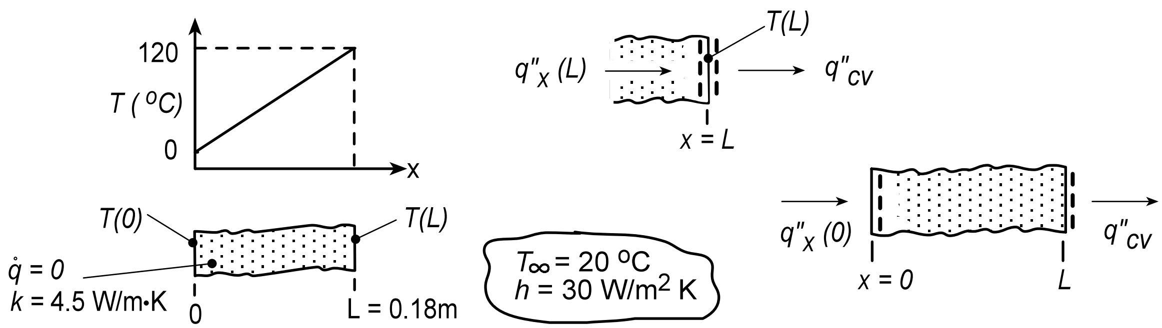
$$\dot{E}''_g = -3AkL^2 - 2BkL.$$

PROBLEM 2.30

KNOWN: Plane wall with no internal energy generation.

FIND: Determine whether the prescribed temperature distribution is possible; explain your reasoning. With the temperatures T(0) = 0°C and T_∞ = 20°C fixed, compute and plot the temperature T(L) as a function of the convection coefficient for the range $10 \le h \le 100$ W/m^2·K.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal energy generation, (3) Constant properties, (4) No radiation exchange at the surface x = L, and (5) Steady-state conditions.

ANALYSIS: (a) Is the prescribed temperature distribution possible? If so, the energy balance at the surface x = L as shown above in the Schematic, must be satisfied.

$$\dot{E}_{in} - \dot{E}_{out} \;?=?\; 0 \qquad\qquad q''_x(L) - q''_{cv} \;?=?\; 0 \qquad (1,2)$$

where the conduction and convection heat fluxes are, respectively,

$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = -k\frac{T(L)-T(0)}{L} = -4.5\,\mathrm{W/m\cdot K}\times(120-0)^\circ\mathrm{C}/0.18\,\mathrm{m} = -3000\,\mathrm{W/m^2}$$

$$q''_{cv} = h\left[T(L) - T_\infty\right] = 30\,\mathrm{W/m^2\cdot K}\times(120-20)^\circ\mathrm{C} = 3000\,\mathrm{W/m^2}$$

Substituting the heat flux values into Eq. (2), find (-3000) - (3000) $\neq$ 0 and therefore, the temperature distribution is not possible.

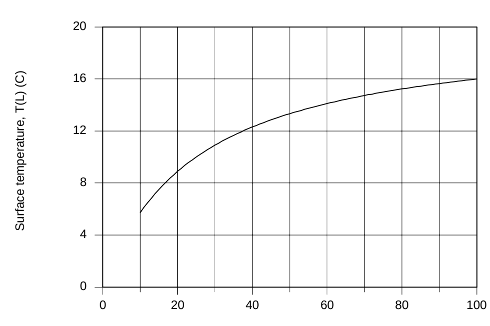
(b) With T(0) = 0°C and T_∞ = 20°C, the temperature at the surface x = L, T(L), can be determined from an overall energy balance on the wall as shown above in the Schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q''_x(0) - q''_{cv} = 0 \qquad -k\frac{T(L)-T(0)}{L} - h\left[T(L) - T_\infty\right] = 0$$

$$-4.5\,\mathrm{W/m\cdot K}\left[T(L) - 0^\circ\mathrm{C}\right]/0.18\,\mathrm{m} - 30\,\mathrm{W/m^2\cdot K}\left[T(L) - 20^\circ\mathrm{C}\right] = 0$$

T(L) = 10.9°C <

Using this same analysis, T(L) as a function of the convection coefficient can be determined and plotted. We don't expect T(L) to be linearly dependent upon h. Note that as h increases to larger values, T(L) approaches T_∞. To what value will T(L) approach as h decreases?

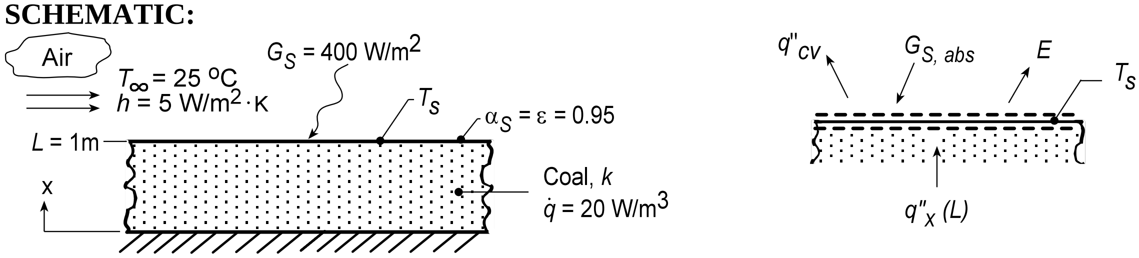


PROBLEM 2.31

KNOWN: Coal pile of prescribed depth experiencing uniform volumetric generation with convection, absorbed irradiation and emission on its upper surface.

FIND: (a) The appropriate form of the heat diffusion equation (HDE) and whether the prescribed temperature distribution satisfies this HDE; conditions at the bottom of the pile, x = 0; sketch of the temperature distribution with labeling of key features; (b) Expression for the conduction heat rate at the location x = L; expression for the surface temperature T_s based upon a surface energy balance at x = L; evaluate T_S and T(0) for the prescribed conditions; (c) Based upon typical daily averages for G_S and h, compute and plot T_S and T(0) for (1) h = 5 W/m^2·K with $50 \le G_S \le 500$ W/m^2, (2) G_S = 400 W/m^2 with $5 \le h \le 50$ W/m^2·K.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Uniform volumetric heat generation, (3) Constant properties, (4) Negligible irradiation from the surroundings, and (5) Steady-state conditions.

PROPERTIES: *Table A.3*, Coal (300K): k = 0.26 W/m.K

ANALYSIS: (a) For one-dimensional, steady-state conduction with uniform volumetric heat generation and constant properties the heat diffusion equation (HDE) follows from Eq. 2.16,

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad (1) \quad <$$

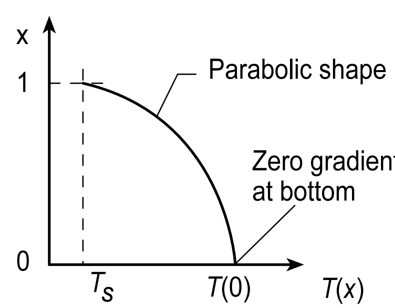
Substituting the temperature distribution into the HDE, Eq. (1),

$$T(x)=T_S+\frac{\dot{q}L^2}{2k}\left(1-\frac{x^2}{L^2}\right) \qquad \frac{d}{dx}\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]+\frac{\dot{q}}{k}\overset{?}{=}0 \qquad (2,3)$$

we find that it does indeed satisfy the HDE for all values of x. <

From Eq. (2), note that the temperature distribution must be quadratic, with maximum value at x = 0. At x = 0, the heat flux is

$$q''_x(0)=-k\left.\frac{dT}{dx}\right)_{x=0}=-k\left[0+\frac{\dot{q}L^2}{2k}\left(0-\frac{2x}{L^2}\right)\right]_{x=0}=0$$



so that the gradient at x = 0 is zero. Hence, the bottom is insulated.

(b) From an overall energy balance on the pile, the conduction heat flux at the surface must be

$$q''_x(L)=\dot{E}''_g=\dot{q}L \quad <$$

Continued...

PROBLEM 2.31 (Cont.)

From a surface energy balance per unit area shown in the Schematic above,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad\qquad q''_x(L) - q''_{cv} + G_{S,abs} - E = 0$$

$$\dot{q}L - h\left(T_S - T_\infty\right) + 0.95G_S - \varepsilon\sigma T_S^4 = 0 \tag{4}$$

$$20\,\mathrm{W/m^3} \times 1\,\mathrm{m} - 5\,\mathrm{W/m^2{\cdot}K}\left(T_s - 298\,\mathrm{K}\right) + 0.95 \times 400\,\mathrm{W/m^2} - 0.95 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2{\cdot}K^4}\,T_s^4 = 0$$

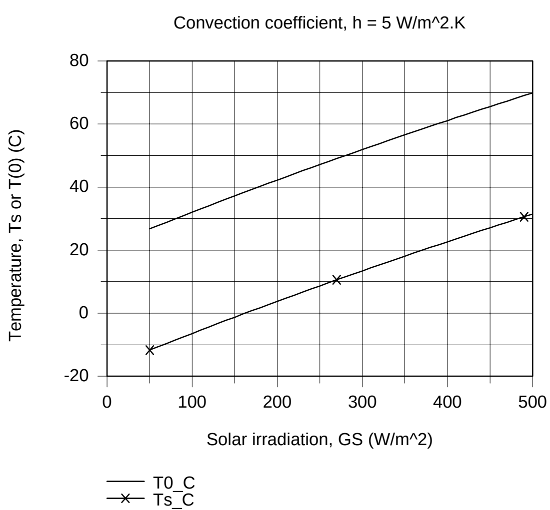
$$T_S = 295.7\ \mathrm{K} = 22.7^\circ\mathrm{C}$$ <

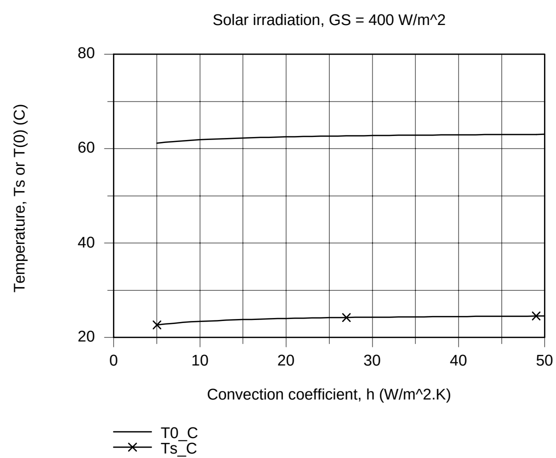
From Eq. (2) with x = 0, find

$$T(0) = T_S + \frac{\dot{q}L^2}{2k} = 22.7^\circ\mathrm{C} + \frac{30\,\mathrm{W/m^2} \times (1\,\mathrm{m})^2}{2 \times 0.26\,\mathrm{W/m \cdot K}} = 61.1^\circ\mathrm{C} \tag{5}$$ <

where the thermal conductivity for coal was obtained from Table A.3.

(c) Two plots are generated using Eq. (4) and (5) for T_s and T(0), respectively; (1) with h = 5 $W/m^2{\cdot}K$ for $50 \le G_S \le 500$ W/m^2 and (2) with G_S = 400 W/m^2 for $5 \le h \le 50$ $W/m^2{\cdot}K$.





From the T vs. h plot with G_S = 400 W/m^2, note that the convection coefficient does not have a major influence on the surface or bottom coal pile temperatures. From the T vs. G_S plot with h = 5 $W/m^2{\cdot}K$, note that the solar irradiation has a very significant effect on the temperatures. The fact that T_s is less than the ambient air temperature, T_∞, and, in the case of very low values of G_S, below freezing, is a consequence of the large magnitude of the emissive power E.

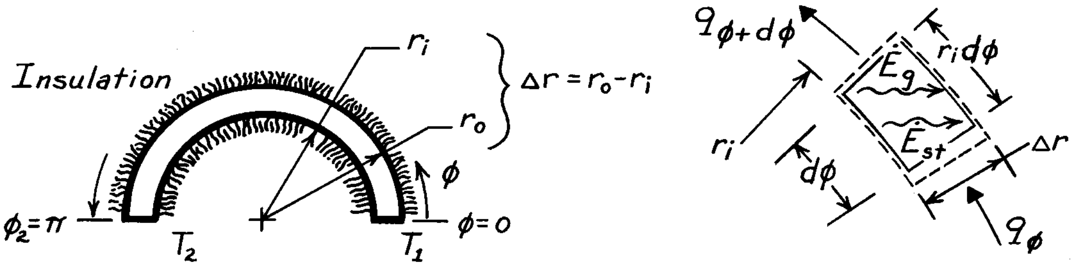
COMMENTS: In our analysis we ignored irradiation from the sky, an environmental radiation effect you'll consider in Chapter 12. Treated as large isothermal surroundings, $G_{sky} = \sigma T_{sky}^4$ where T_{sky} = -30°C for very clear conditions and nearly air temperature for cloudy conditions. For low G_S conditions we should consider G_{sky}, the effect of which will be to predict higher values for T_s and T(0).

PROBLEM 2.32

KNOWN: Cylindrical system with negligible temperature variation in the r,z directions.

FIND: (a) Heat equation beginning with a properly defined control volume, (b) Temperature distribution $T(\phi)$ for steady-state conditions with no internal heat generation and constant properties, (c) Heat rate for Part (b) conditions.

SCHEMATIC:



ASSUMPTIONS: (1) T is independent of r,z, (2) $\Delta r = (r_o - r_i) << r_i$.

ANALYSIS: (a) Define the control volume as $V = r_i d\phi \cdot \Delta r \cdot L$ where L is length normal to page. Apply the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} \qquad q_\phi - q_{\phi+d\phi} + \dot{q}V = \rho V c \frac{\partial T}{\partial t} \tag{1,2}$$

where

$$q_\phi = -k(\Delta r \cdot L)\frac{\partial T}{r_i \partial \phi} \qquad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi. \tag{3,4}$$

Eqs. (3) and (4) follow from Fourier's law, Eq. 2.1, and from Eq. 2.7, respectively. Combining Eqs. (3) and (4) with Eq. (2) and canceling like terms, find

$$\frac{1}{r_i^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \dot{q} = \rho c \frac{\partial T}{\partial t}. \tag{5} <$$

Since temperature is independent of r and z, this form agrees with Eq. 2.20.

(b) For steady-state conditions with $\dot{q} = 0$, the heat equation, (5), becomes

$$\frac{d}{d\phi}\left[k\frac{dT}{d\phi}\right] = 0. \tag{6}$$

With constant properties, it follows that $dT/d\phi$ is constant which implies $T(\phi)$ is linear in ϕ. That is,

$$\frac{dT}{d\phi} = \frac{T_2 - T_1}{\phi_2 - \phi_1} = +\frac{1}{\pi}(T_2 - T_1) \quad \text{or} \quad T(\phi) = T_1 + \frac{1}{\pi}(T_2 - T_1)\phi. \tag{7,8} <$$

(c) The heat rate for the conditions of Part (b) follows from Fourier's law, Eq. (3), using the temperature gradient of Eq. (7). That is,

$$q_\phi = -k(\Delta r \cdot L)\frac{1}{r_i}\left[+\frac{1}{\pi}(T_2 - T_1)\right] = -k\left[\frac{r_o - r_i}{\pi r_i}\right]L(T_2 - T_1). \tag{9} <$$

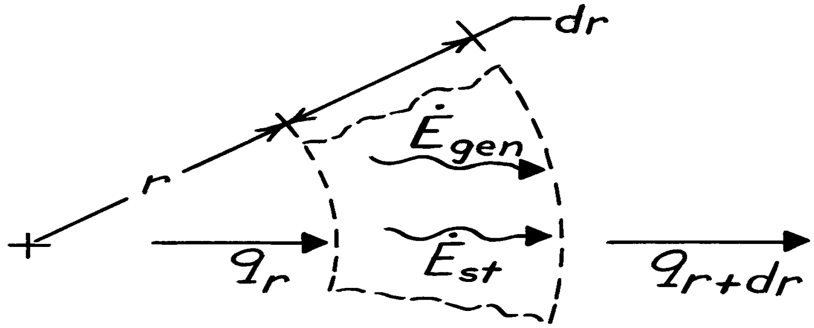
COMMENTS: Note the expression for the temperature gradient in Fourier's law, Eq. (3), is $\partial T/r_i\partial\phi$ not $\partial T/\partial\phi$. For the conditions of Parts (b) and (c), note that q_ϕ is independent of ϕ; this is first indicated by Eq. (6) and confirmed by Eq. (9).

PROBLEM 2.33

KNOWN: Heat diffusion with internal heat generation for one-dimensional cylindrical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has volume, $V = A_r \cdot dr = 2\pi r \cdot dr \cdot 1$, with unit thickness normal to page. Using the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this one-dimensional coordinate system is

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \times 2\pi r \cdot 1 \times \frac{\partial T}{\partial r}.$$

At the outer surface, r+dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 2\pi r \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k2\pi r \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 2\pi r dr = \rho \cdot 2\pi r dr \cdot c_p \frac{\partial T}{\partial t}$$

Dividing by the factor $2\pi r\ dr$, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$ <

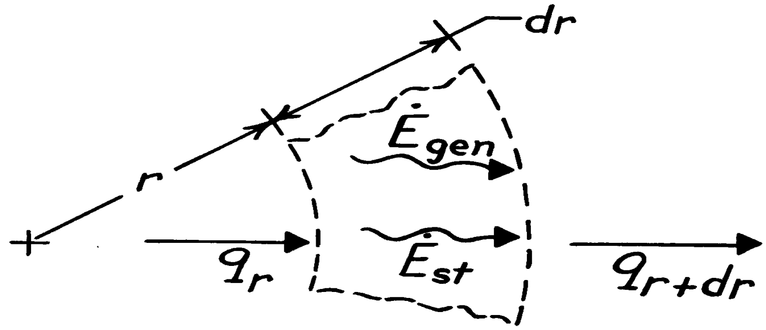
COMMENTS: (1) Note how the result compares with Eq. 2.20 when the terms for the ϕ,z coordinates are eliminated. (2) Recognize that we did not require $\dot{q}$ and k to be independent of r.

PROBLEM 2.34

KNOWN: Heat diffusion with internal heat generation for one-dimensional spherical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has the volume, $V = A_r \cdot dr = 4\pi r^2 dr$. Using the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this coordinate system has the form

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}.$$

At the outer surface, r+dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k \cdot 4\pi r^2 \cdot \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 4\pi r^2 dr = \rho \cdot 4\pi r^2 dr \cdot c_p \frac{\partial T}{\partial t}.$$

Dividing by the factor $4\pi r^2 dr$, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2 \frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$ <

COMMENTS: (1) Note how the result compares with Eq. 2.23 when the terms for the θ,ϕ directions are eliminated.

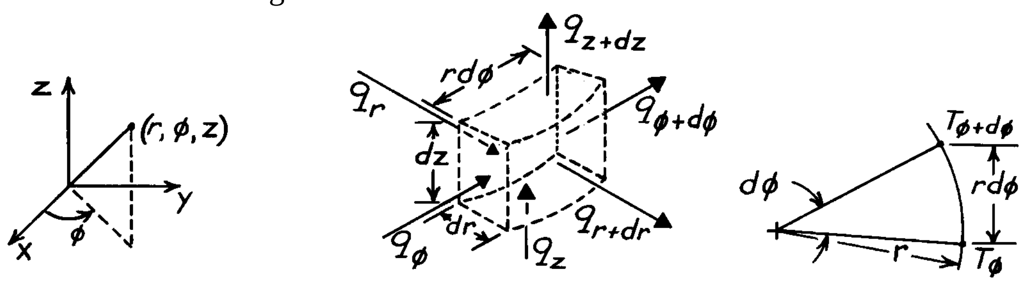
(2) Recognize that we did not require $\dot{q}$ and k to be independent of the coordinate r.

PROBLEM 2.35

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,z) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See also Fig. 2.9.



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Consider the differential control volume identified above having a volume given as V = dr·rdϕ·dz. From the conservation of energy requirement,

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_z - q_{z+dz} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}(dr \cdot rd\phi \cdot dz) \qquad \dot{E}_g = \rho Vc\partial T/\partial t = \rho(dr \cdot rd\phi \cdot dz)c\, \partial T/\partial t. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial \phi}(q_\phi)d\phi, \quad q_{z+dz} = q_z + \frac{\partial}{\partial z}(q_z)dz. \qquad (4,5,6)$$

Using Fourier's law, the expressions for the conduction heat rates are

$$q_r = -kA_r\partial T/\partial r = -k(rd\phi \cdot dz)\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\partial\phi = -k(dr \cdot dz)\partial T/r\partial\phi \qquad (8)$$

$$q_z = -kA_z\partial T/\partial z = -k(dr \cdot rd\phi)\partial T/\partial z. \qquad (9)$$

Note from the above, right schematic that the gradient in the ϕ-direction is ∂T/r∂ϕ and not ∂T/∂ϕ. Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1),

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial\phi}(q_\phi)d\phi - \frac{\partial}{\partial z}(q_z)dz + \dot{q}\; dr \cdot rd\phi \cdot dz = \rho(dr \cdot rd\phi \cdot dz)c\frac{\partial T}{\partial t}. \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial r}\left[-k(rd\phi \cdot dz)\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial \phi}\left[-k(drdz)\frac{\partial T}{r\partial\phi}\right]d\phi - \frac{\partial}{\partial z}\left[-k(dr \cdot rd\phi)\frac{\partial T}{\partial z}\right]dz$$

$$+\dot{q}\; dr \cdot rd\phi \cdot dz = \rho(dr \cdot rd\phi \cdot dz)c\frac{\partial T}{\partial t}. \qquad (11)$$

Dividing Eq. (11) by the volume of the CV, Eq. 2.20 is obtained.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}$$

<

PROBLEM 2.36

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,θ) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See Figure 2.10.

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: The differential control volume is V = dr·rsinθdϕ·rdθ, and the conduction terms are identified in Figure 2.10. Conservation of energy requires

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_\theta - q_{\theta+d\theta} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] \qquad \dot{E}_{st} = \rho Vc\frac{\partial T}{\partial t} = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t}. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial\phi}(q_\phi)d\phi, \quad q_{\theta+d\theta} = q_\theta + \frac{\partial}{\partial\theta}(q_\theta)d\theta. \qquad (4,5,6)$$

From Fourier's law, the conduction heat rates have the following forms.

$$q_r = -kA_r\partial T/\partial r = -k[r\sin\theta d\phi \cdot rd\theta]\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\sin\theta\partial\phi = -k[dr \cdot rd\theta]\partial T/r\sin\theta\partial\phi \qquad (8)$$

$$q_\theta = -kA_\theta\partial T/r\partial\theta = -k[dr \cdot r\sin\theta d\phi]\partial T/r\partial\theta. \qquad (9)$$

Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1), the energy balance becomes

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial_\phi}(q_\phi)d\phi - \frac{\partial}{\partial\theta}(q_\theta)d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial\theta}\left[-k[r\sin\theta d\phi \cdot rd\theta]\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k[dr \cdot rd\theta]\frac{\partial T}{r\sin\theta\partial\phi}\right]d\phi$$

$$-\frac{\partial}{\partial\theta}\left[-k[dr \cdot r\sin\theta d\phi]\frac{\partial T}{r\partial\theta}\right]d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \qquad (11)$$

Dividing Eq. (11) by the volume of the control volume, V, Eq. 2.23 is obtained.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[k\sin\theta\frac{\partial T}{\partial\theta}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}. \qquad <$$

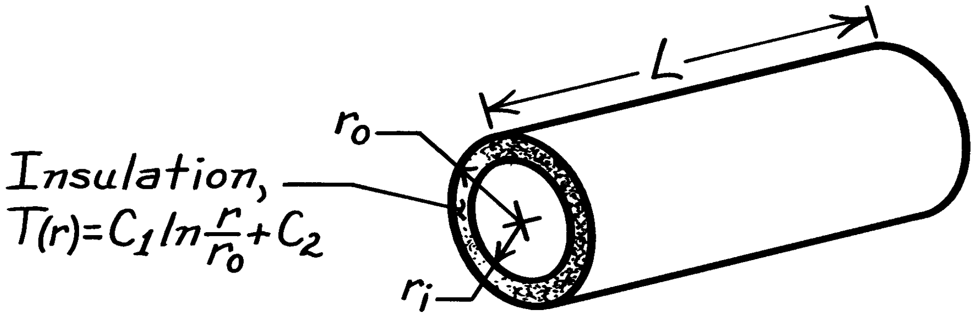
COMMENTS: Note how the temperature gradients in Eqs. (7) - (9) are formulated. The numerator is always ∂T while the denominator is the dimension of the control volume in the specified coordinate direction.

PROBLEM 2.37

KNOWN: Temperature distribution in steam pipe insulation.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.20, the heat equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{C_1}{r}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.19, the radial component of the heat flux is

$$q''_r=-k\frac{\partial T}{\partial r}=-k\frac{C_1}{r}.$$

Hence, q''_r decreases with increasing $r\left(q''_r \alpha 1/r\right)$. <

At any radial location, the heat rate is

$$q_r=2\pi rLq''_r=-2\pi kC_1L$$

Hence, q_r is independent of r. <

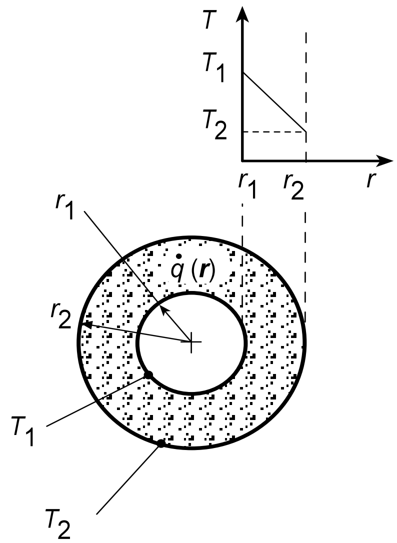
COMMENTS: The requirement that q_r is invariant with r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q''_r varies inversely with r.

PROBLEM 2.38

KNOWN: Inner and outer radii and surface temperatures of a long circular tube with internal energy generation.

FIND: Conditions for which a linear radial temperature distribution may be maintained.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties.

ANALYSIS: For the assumed conditions, Eq. 2.20 reduces to

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\dot{q}=0$$

If $\dot{q}$ = 0 or $\dot{q}$ = constant, it is clearly impossible to have a linear radial temperature distribution. However, we may use the heat equation to infer a special form of $\dot{q}$ (r) for which dT/dr is a constant (call it C_1). It follows that

$$\frac{k}{r}\frac{d}{dr}(r C_1)+\dot{q}=0$$

$$\dot{q}=-\frac{C_1 k}{r}$$ <

where $C_1 = (T_2 - T_1)/(r_2 - r_1)$. Hence, if the generation rate varies inversely with radial location, the radial temperature distribution is linear.

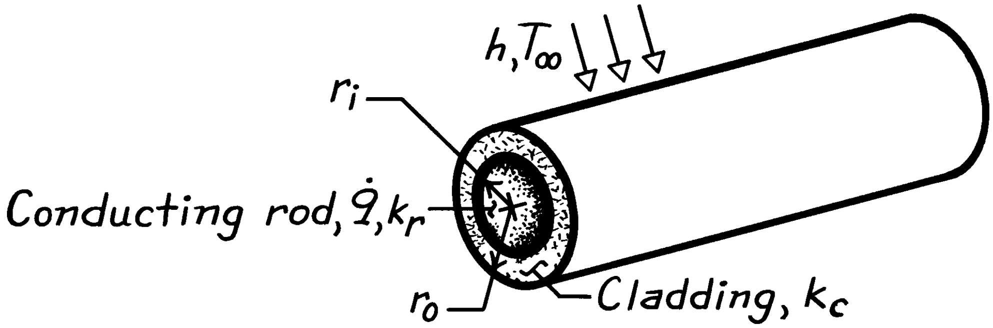
COMMENTS: Conditions for which $\dot{q} \propto (1/r)$ would be unusual.

PROBLEM 2.39

KNOWN: Radii and thermal conductivity of conducting rod and cladding material. Volumetric rate of thermal energy generation in the rod. Convection conditions at outer surface.

FIND: Heat equations and boundary conditions for rod and cladding.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: From Equation 2.20, the appropriate forms of the heat equation are

Conducting Rod:

$$\frac{k_r}{r}\frac{d}{dr}\left(r\frac{dT_r}{dt}\right)+\dot{q}=0$$ <

Cladding:

$$\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0.$$ <

Appropriate boundary conditions are:

(a) $dT_r / dr|_{r=0} = 0$ <

(b) $T_r(r_i) = T_c(r_i)$ <

(c) $k_r \dfrac{dT_r}{dr}|_{r_i} = k_c \dfrac{dT_c}{dr}|_{r_i}$ <

(d) $k_c \dfrac{dT_c}{dr}|_{r_o} = h\left[T_c(r_o) - T_\infty\right]$ <

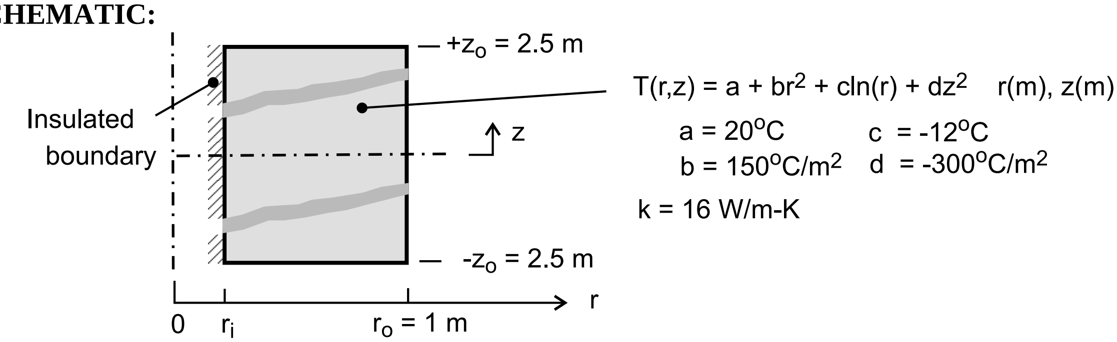
COMMENTS: Condition (a) corresponds to symmetry at the centerline, while the interface conditions at $r = r_i$ (b,c) correspond to requirements of thermal equilibrium and conservation of energy. Condition (d) results from conservation of energy at the outer surface.

PROBLEM 2.40

KNOWN: Steady-state temperature distribution for hollow cylindrical solid with volumetric heat generation.

FIND: (a) Determine the inner radius of the cylinder, r_i, (b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}$, (c) Determine the axial distribution of the heat flux at the outer surface, $q''_r(r_o, z)$, and the heat rate at this outer surface; is the heat rate *in* or *out* of the cylinder; (d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q''_z(r, +z_o)$ and $q''_z(r, -z_o)$, and the corresponding heat rates; are the heat rates *in* or *out* of the cylinder; (e) Determine the relationship of the surface heat rates to the heat generation rate; is an overall energy balance satisfied?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction with constant properties and volumetric heat generation.

ANALYSIS: (a) Since the inner boundary, r = r_i, is adiabatic, then $q''_r(r_i, z) = 0$. Hence the temperature gradient in the r-direction must be zero.

$$\left.\frac{\partial T}{\partial r}\right)_{r_i} = 0 + 2br_i + c/r_i + 0 = 0$$

$$r_i = +\left(-\frac{c}{2b}\right)^{1/2} = \left(-\frac{-12°C}{2\times 150°C/m^2}\right)^{1/2} = 0.2 \text{ m}$$ <

(b) To determine $\dot{q}$, substitute the temperature distribution into the heat diffusion equation, Eq. 2.20, for two-dimensional (r,z), steady-state conduction

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[0 + 2br + c/r + 0\right]\right) + \frac{\partial}{\partial z}\left(0 + 0 + 0 + 2dz\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\left[4br + 0\right] + 2d + \frac{\dot{q}}{k} = 0$$

$$\dot{q} = -k\left[4b - 2d\right] = -16\,W/m\cdot K\left[4\times 150°C/m^2 - 2\left(-300°C/m^2\right)\right]$$

$$\dot{q} = 0\,W/m^3$$ <

(c) The heat flux and the heat rate at the outer surface, r = r_o, may be calculated using Fourier's law. Note that the sign of the heat flux in the positive r-direction is negative, and hence the heat flow is *into* the cylinder.

$$q''_r(r_{o}, z) = -k\left.\frac{\partial T}{\partial r}\right)_{r_o} = -k\left[0 + 2br_o + c/r_o + 0\right]$$

Continued …..

$$q_r''(r_{o,}z) = -16\,\text{W/m}\cdot\text{K}\left[2\times150°\text{C/m}^2\times1\text{ m} - 12°\text{C}/1\text{ m}\right] = -4608\,\text{W/m}^2 \qquad <$$

$$q_r(r_o) = A_r\,q_r''(r_{o,}z) \qquad \text{where} \qquad A_r = 2\pi r_o(2z_o)$$

$$q_r(r_o) = -4\pi\times1\text{ m}\times2.5\text{ m}\times4608\,\text{W/m}^2 = -144,765\,\text{W} \qquad <$$

(d) The heat fluxes and the heat rates at end faces, z = + z_o and – z_o, may be calculated using Fourier's law. The direction of the heat rate *in* or *out* of the end face is determined by the sign of the heat flux in the positive z-direction.

At the upper end face, z = + z_o: heat rate is *out* of the cylinder $<$

$$q_z''(r,+z_o) = -k\left.\frac{\partial T}{\partial z}\right)_{z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q_z''(r,+z_o) = -16\,\text{W/m}\cdot\text{K}\times2\left(-300°\text{C/m}^2\right)2.5\text{ m} = +24,000\,\text{W/m}^2 \qquad <$$

$$q_z(+z_o) = A_z\,q_z''(r,+z_o) \qquad \text{where} \qquad A_z = \pi\left(r_o^2 - r_i^2\right)$$

$$q_z(+z_o) = \pi\left(1^2 - 0.2^2\right)\text{m}^2\times24,000\,\text{W/m}^2 = +72,382\,\text{W} \qquad <$$

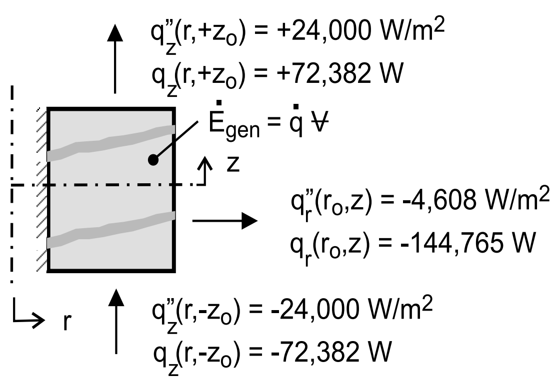
At the lower end face, z = - z_o: heat rate is *out* of the cylinder $<$

$$q_z''(r,-z_o) = -k\left.\frac{\partial T}{\partial z}\right)_{-z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q_z''(r,-z_o) = -16\,\text{W/m}^2\cdot\text{K}\times2(-300°\text{C/m})(-2.5\text{ m}) = -24,000\,\text{W/m}^2 \qquad <$$

$$q_z(-z_o) = -72,382\,\text{W} \qquad <$$

(e) The heat rates from the surfaces and the volumetric heat generation can be related through an overall energy balance on the cylinder as shown in the sketch.



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad \text{where} \qquad \dot{E}_{gen} = \dot{q}\forall = 0$$

$$\dot{E}_{in} = -q_r(r_o) = -(-144,765\,\text{W}) = +144,765\,\text{W} \qquad <$$

$$\dot{E}_{out} = +q_z(z_o) - q_z(-z_o) = \left[72,382 - (-72,382)\right]\text{W} = +144,764\,\text{W} \qquad <$$

The overall energy balance is satisfied.

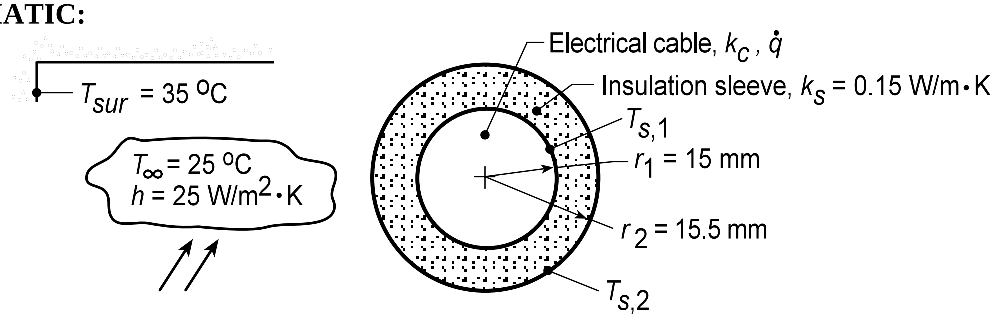
COMMENTS: When using Fourier's law, the heat flux q_z'' denotes the heat flux in the positive z-direction. At a boundary, the sign of the numerical value will determine whether heat is flowing into or out of the boundary.

PROBLEM 2.41

KNOWN: An electric cable with an insulating sleeve experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that prescribed temperature distributions for the cable and insulating sleeve satisfy their appropriate heat diffusion equations; sketch temperature distributions labeling key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the sleeve, q_r', in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the cable to obtain an alternative expression for q_r' in terms of $\dot{q}$ and r_1; (c) Apply surface energy balance around the outer surface of the sleeve to obtain an expression for which $T_{s,2}$ can be evaluated; (d) Determine $T_{s,1}$, $T_{s,2}$, and T_o for the specified geometry and operating conditions; and (e) Plot $T_{s,1}$, $T_{s,2}$, and T_o as a function of the outer radius for the range $15.5 \le r_2 \le 20$ mm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Uniform volumetric heat generation in cable, (3) Negligible thermal contact resistance between the cable and sleeve, (4) Constant properties in cable and sleeve, (5) Surroundings large compared to the sleeve, and (6) Steady-state conditions.

ANALYSIS: (a) The appropriate forms of the heat diffusion equation (HDE) for the insulation and cable are identified. The temperature distributions are valid if they satisfy the relevant HDE.

Insulation: The temperature distribution is given as

$$T(r) = T_{s,2} + \left(T_{s,1} - T_{s,2}\right)\frac{\ln(r/r_2)}{\ln(r_1/r_2)} \tag{1}$$

and the appropriate HDE (radial coordinates, SS, $\dot{q}$ = 0), Eq. 2.20,

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r\left[0 + \left(T_{s,1} - T_{s,2}\right)\frac{1/r}{\ln(r_1/r_2)}\right]\right) = \frac{d}{dr}\left(\frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)}\right) ?=?0$$

Hence, the temperature distribution satisfies the HDE. <

Cable: The temperature distribution is given as

$$T(r) = T_{s,1} + \frac{\dot{q}r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right) \tag{2}$$

and the appropriate HDE (radial coordinates, SS, $\dot{q}$ uniform), Eq. 2.20,

Continued...

PROBLEM 2.41 (Cont.)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k_c}=0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\left[0+\frac{\dot{q}r_1^2}{4k_c}\left(0-\frac{2r}{r_1^2}\right)\right]\right)+\frac{\dot{q}}{k_c}?=?0$$

$$\frac{1}{r}\frac{d}{dr}\left(-\frac{\dot{q}r_1^2}{4k_c}\frac{2r^2}{r_1^2}\right)+\frac{\dot{q}}{k_c}?=?0$$

$$\frac{1}{r}\left(-\frac{\dot{q}r_1^2}{4k_c}\frac{4r}{r_1^2}\right)+\frac{\dot{q}}{k_c}?=?0$$

Hence the temperature distribution satisfies the HDE. <

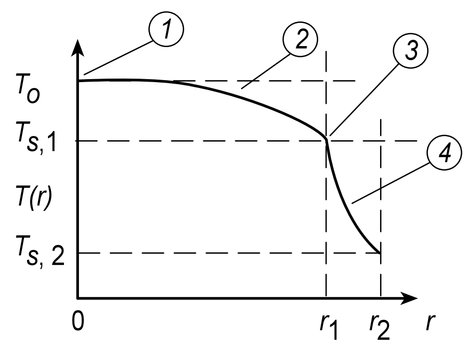
The temperature distributions in the cable, $0 \le r \le r_1$, and sleeve, $r_1 \le r \le r_2$, and their key features are as follows:

(1) Zero gradient, symmetry condition,

(2) Increasing gradient with increasing radius, r, because of $\dot{q}$,

(3) Discontinuous T(r) across cable-sleeve interface because of different thermal conductivities,

(4) Decreasing gradient with increasing radius, r, since heat rate is constant.



(b) Using Fourier's law for the radial-cylindrical coordinate, the heat rate through the *insulation* (sleeve) per unit length is

$$q_r' = -kA_r'\frac{dT}{dr} = -k2\pi r\frac{dT}{dr}$$ <

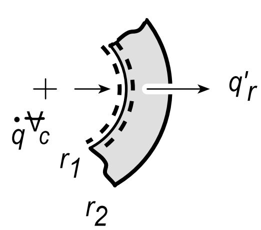
and substituting for the temperature distribution, Eq. (1),

$$q_r' = -k_s 2\pi r\left[0+\left(T_{s,1}-T_{s,2}\right)\frac{1/r}{\ln\left(r_1/r_2\right)}\right] = 2\pi k_s\frac{\left(T_{s,1}-T_{s,2}\right)}{\ln\left(r_2/r_1\right)} \quad (3)$$ <

Applying an energy balance to a control surface placed around the cable,

$$\dot{E}_{in}-\dot{E}_{out}=0$$

$$\dot{q}\forall_c'-q_r'=0$$



where $\dot{q}\forall_c$ represents the dissipated electrical power in the cable

Continued...

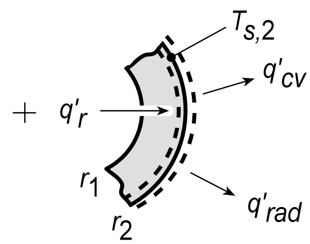
PROBLEM 2.41 (Cont.)

$$\dot{q}\left(\pi r_1^2\right) - q_r' = 0 \qquad \text{or} \qquad q_r' = \pi \dot{q} r_1^2 \qquad (4) <$$

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_r' - q_{cv}' - q_{rad}' = 0$$



$$\pi \dot{q} r_1^2 - h\left(2\pi r_2\right)\left(T_{s,2} - T_\infty\right) - \varepsilon\left(2\pi r_2\right)\sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5) <$$

This relation can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider a cable-sleeve system with the following prescribed conditions:

r_1 = 15 mm	k_c = 200 W/m·K	h = 25 W/m^2·K	ε = 0.9
r_2 = 15.5 mm	k_s = 0.15 W/m·K	T_∞ = 25°C	T_{sur} = 35°C

For 250 A with R_e' = 0.005 Ω/m, the volumetric heat generation rate is

$$\dot{q} = I^2 R_e' / \forall_c' = I^2 R_e' \Big/ \left(\pi r_1^2\right)$$

$$\dot{q} = \left(250\,\text{A}\right)^2 \times 0.005\,\Omega/\text{m} \Big/ \left(\pi \times 0.015^2\,\text{m}^2\right) = 4.42 \times 10^5\ \text{W/m}^3$$

Substituting numerical values in appropriate equations, we can evaluate $T_{s,1}$, $T_{s,2}$ and T_o.

Sleeve outer surface temperature, $T_{s,2}$: Using Eq. (5),

$$\pi \times 4.42 \times 10^5\ \text{W/m}^3 \times (0.015\text{m})^2 - 25\ \text{W/m}^2\cdot\text{K} \times \left(2\pi \times 0.0155\text{m}\right)\left(T_{s,2} - 298\text{K}\right)$$
$$-0.9 \times \left(2\pi \times 0.0155\text{m}\right) \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(T_{s,2}^4 - 308^4\right)\text{K}^4 = 0$$

$$T_{s,2} = 395\,\text{K} = 122^\circ\text{C} \qquad <$$

Sleeve-cable interface temperature, $T_{s,1}$: Using Eqs. (3) and (4), with $T_{s,2}$ = 395 K,

$$\pi \dot{q} r_1^2 = 2\pi k_s \frac{\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$\pi \times 4.42 \times 10^5\ \text{W/m}^3 \times \left(0.015\,\text{m}\right)^2 = 2\pi \times 0.15\ \text{W/m}\cdot\text{K}\,\frac{\left(T_{s,1} - 395\,\text{K}\right)}{\ln\left(15.5/15.0\right)}$$

$$T_{s,1} = 406\,\text{K} = 133^\circ\text{C} \qquad <$$

Continued...

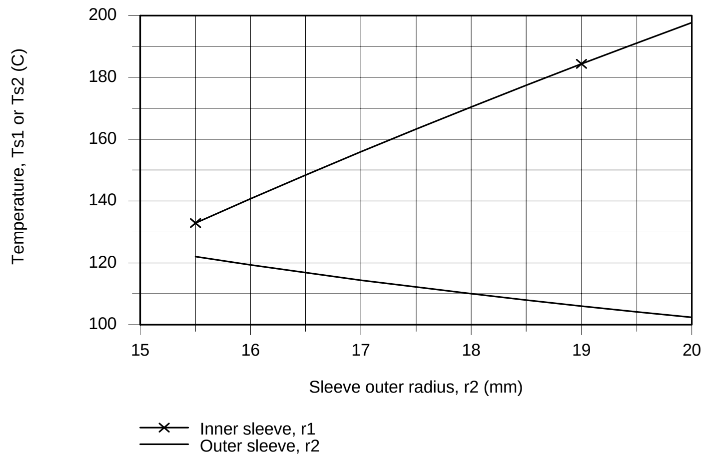
PROBLEM 2.41 (Cont.)

Cable centerline temperature, T_o: Using Eq. (2) with $T_{s,1}$ = 133°C,

$$T_o = T(0) = T_{s,1} + \frac{\dot{q} r_1^2}{4k_c}$$

$$T_o = 133^\circ\text{C} + 4.42\times10^5\ \text{W/m}^3 \times (0.015\,\text{m})^2 \Big/ (4\times 200\ \text{W/m}\cdot\text{K}) = 133.1^\circ\text{C}$$ <

(e) With all other conditions remaining the same, the relations of part (d) can be used to calculate T_o, $T_{s,1}$ and $T_{s,2}$ as a function of the sleeve outer radius r_2 for the range $15.5 \le r_2 \le 20$ mm.



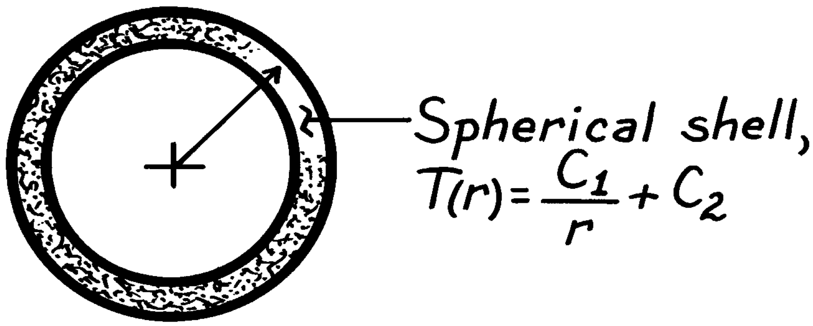
On the plot above T_o would show the same behavior as $T_{s,1}$ since the temperature rise between cable center and its surface is 0.12°C. With increasing r_2, we expect $T_{s,2}$ to decrease since the heat flux decreases with increasing r_2. We expect $T_{s,1}$ to increase with increasing r_2 since the thermal resistance of the sleeve increases.

PROBLEM 2.42

KNOWN: Temperature distribution in a spherical shell.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.23, the heat equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{C_1}{r^2}\right) = 0.$$

Hence, steady-state conditions exist. <

From Equation 2.22, the radial component of the heat flux is

$$q_r'' = -k\frac{\partial T}{\partial r} = -k\frac{C_1}{r^2}.$$

Hence, q_r'' decreases with increasing $r^2 \left(q_r'' \alpha 1/r^2\right)$. <

At any radial location, the heat rate is

$$q_r = 4\pi r^2 q_r'' = 4\pi k C_1.$$

Hence, q_r is independent of r. <

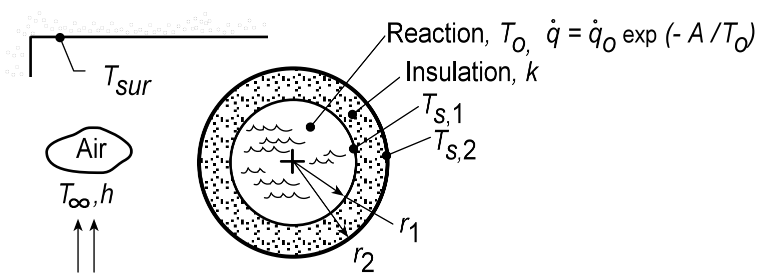
COMMENTS: The fact that q_r is independent of r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r^2.

PROBLEM 2.43

KNOWN: Spherical container with an exothermic reaction enclosed by an insulating material whose outer surface experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that the prescribed temperature distribution for the insulation satisfies the appropriate form of the heat diffusion equation; sketch the temperature distribution and label key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the insulation layer, q_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the container and obtain an alternative expression for q_r in terms of $\dot{q}$ and r_1; (c) Apply a surface energy balance around the outer surface of the insulation to obtain an expression to evaluate $T_{s,2}$; (d) Determine $T_{s,2}$ for the specified geometry and operating conditions; (e) Compute and plot the variation of $T_{s,2}$ as a function of the outer radius for the range $201 \le r_2 \le 210$ mm; explore approaches for reducing $T_{s,2} \le 45°C$ to eliminate potential risk for burn injuries to personnel.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial spherical conduction, (2) Isothermal reaction in container so that $T_o = T_{s,1}$, (2) Negligible thermal contact resistance between the container and insulation, (3) Constant properties in the insulation, (4) Surroundings large compared to the insulated vessel, and (5) Steady-state conditions.

ANALYSIS: The appropriate form of the heat diffusion equation (HDE) for the insulation follows from Eq. 2.23,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad (1) <$$

The temperature distribution is given as

$$T(r) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right] \qquad (2)$$

Substitute T(r) into the HDE to see if it is satisfied:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-(r_1/r_2)}\right]\right) ?=?\, 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(+\left(T_{s,1}-T_{s,2}\right)\frac{r_1}{1-(r_1/r_2)}\right) ?=?\, 0 \qquad <$$

and since the expression in parenthesis is independent of r, T(r) does indeed satisfy the HDE. The temperature distribution in the insulation and its key features are as follows:

Continued...