Solution Manual for

Fundamentals of Electromagnetics for Electrical and Computer Engineering

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CHAPTER 1

1.1. (a) Total distance = $1 + \frac{1}{2} + \frac{1}{4} + \cdots = \frac{1}{1-\frac{1}{2}} = 2$ m

(b) Distance north = $1 - \frac{1}{4} + \frac{1}{16} - \cdots = \frac{1}{1+\frac{1}{4}} = 0.8$ m

Distance east = $\frac{1}{2} - \frac{1}{8} + \frac{1}{32} - \cdots = \frac{1}{2}\left(1 - \frac{1}{4} + \frac{1}{16} - \cdots\right) = 0.4$ m

$\therefore$ Final position is (0.8, 0.4)

(c) Straight line distance = $\sqrt{(0.8)^2 + (0.4)^2} = 0.8944$ m

1.2. $\mathbf{A} + \mathbf{B} + \mathbf{C} = 2\mathbf{a}_1 + 3\mathbf{a}_2 + 2\mathbf{a}_3$ — (1)

$2\mathbf{A} + \mathbf{B} - \mathbf{C} = \mathbf{a}_1 + 3\mathbf{a}_2$ — (2)

$\mathbf{A} - 2\mathbf{B} + 3\mathbf{C} = 4\mathbf{a}_1 + 5\mathbf{a}_2 + \mathbf{a}_3$ — (3)

(1) + (2) $\rightarrow 3\mathbf{A} + 2\mathbf{B} = 3\mathbf{a}_1 + 16\mathbf{a}_2 + 2\mathbf{a}_3$ — (4)

(2) $\times$ 3 + (3) $\rightarrow 7\mathbf{A} + \mathbf{B} = 7\mathbf{a}_1 + 14\mathbf{a}_2 + \mathbf{a}_3$ — (5)

[(5) $\times$ 2 – (4)] $\div$ 11 $\rightarrow \mathbf{A} = \mathbf{a}_1 + 2\mathbf{a}_2$ — (6)

(5) – (6) $\times$ 7 $\rightarrow$ $\mathbf{B} = \mathbf{a}_3$ — (7)

(1) – (6) – (7) $\rightarrow$ $\mathbf{C} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$ — (8)

1.3. $(\mathbf{A} + \mathbf{B}) \bullet (\mathbf{A} - \mathbf{B}) = \mathbf{A} \bullet \mathbf{A} - \mathbf{A} \bullet \mathbf{B} + \mathbf{B} \bullet \mathbf{A} - \mathbf{B} \bullet \mathbf{B} = A^2 - B^2$

$(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} - \mathbf{B}) = \mathbf{A} \times \mathbf{A} - \mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} - \mathbf{B} \times \mathbf{B} = 2\mathbf{B} \times \mathbf{A}$

For $\mathbf{A} = 3\mathbf{a}_1 - 5\mathbf{a}_2 + 4\mathbf{a}_3$ and $\mathbf{B} = \mathbf{a}_1 + \mathbf{a}_2 - 2\mathbf{a}_3$,

$\mathbf{A} + \mathbf{B} = 4\mathbf{a}_1 - 4\mathbf{a}_2 + 2\mathbf{a}_3$, $\mathbf{A} - \mathbf{B} = 2\mathbf{a}_1 - 6\mathbf{a}_2 + 6\mathbf{a}_3$,

$A^2 = 9 + 25 + 16 = 50$, and $B^2 = 1 + 1 + 4 = 6$

$(\mathbf{A} + \mathbf{B}) \bullet (\mathbf{A} - \mathbf{B}) = 8 + 24 + 12 = 44 = A^2 - B^2$

$$(\mathbf{A}+\mathbf{B})\times(\mathbf{A}-\mathbf{B}) = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 4 & -4 & 2 \\ 2 & -6 & 6 \end{vmatrix} = -12\mathbf{a}_x - 20\mathbf{a}_y - 16\mathbf{a}_z$$

$$= 2\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 1 & 1 & -2 \\ 3 & -5 & 4 \end{vmatrix} = 2\mathbf{B}\times\mathbf{A}$$

1.4. $\mathbf{B}\times\mathbf{C} = -4\mathbf{a}_x + 2\mathbf{a}_y + 8\mathbf{a}_z$, $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = 8\mathbf{a}_x + 16\mathbf{a}_y$

$\mathbf{C}\times\mathbf{A} = -\mathbf{a}_x - 2\mathbf{a}_y + 7\mathbf{a}_z$, $\mathbf{B}\times(\mathbf{C}\times\mathbf{A}) = -12\mathbf{a}_x - 8\mathbf{a}_y - 4\mathbf{a}_z$

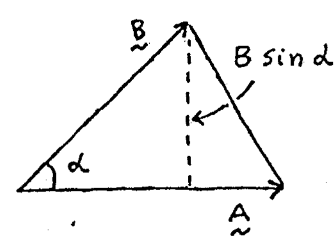
$\mathbf{A}\times\mathbf{B} = \mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$, $\mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 4\mathbf{a}_x - 8\mathbf{a}_y + 4\mathbf{a}_z$

$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = 0$

In fact, this quantity is zero for any **A**, **B**, and **C**.

1.5. Area $= \frac{1}{2}AB\sin\alpha = \frac{1}{2}|\mathbf{A}\times\mathbf{B}|$

For the points (1, 2, 1), (–3, –4, 5), and (2, –1, –3),



$\mathbf{A} = 4\mathbf{a}_x + 6\mathbf{a}_y - 4\mathbf{a}_z$

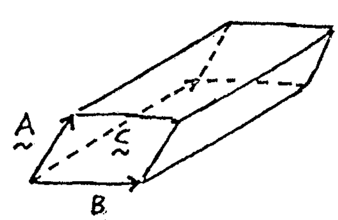
$\mathbf{B} = 5\mathbf{a}_x + 3\mathbf{a}_y - 8\mathbf{a}_z$

$\mathbf{A}\times\mathbf{B} = -36\mathbf{a}_x + 12\mathbf{a}_y - 18\mathbf{a}_z$

$\therefore$ Area $= \frac{1}{2}\sqrt{(-36)^2 + (12)^2 + (-18)^2} = 21$ units.

1.6. Area of the base = $|\mathbf{B}\times\mathbf{C}|$

Height of parallelepiped = Projection of **A** onto the normal to the base



$= \mathbf{A}\cdot\frac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}$

$\therefore$ Volume of parallelepiped = Area of base × height = $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C}$

For $\mathbf{A} = 4\mathbf{a}_x$, $\mathbf{B} = 2\mathbf{a}_x + \mathbf{a}_y + 3\mathbf{a}_z$, and $\mathbf{C} = 2\mathbf{a}_y + 6\mathbf{a}_z$, $\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = 0$.

Hence, volume of the parallelepiped is zero. The three vectors lie in a plane.

1.7. The vector **A** must be perpendicular to both $(-\mathbf{a}_y + 2\mathbf{a}_z)$ and $(\mathbf{a}_x - 2\mathbf{a}_z)$.

Hence $\mathbf{A} = C(-\mathbf{a}_y + 2\mathbf{a}_z) \times (\mathbf{a}_x - 2\mathbf{a}_z) = C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$ where C is a constant. To find C, we note that $\mathbf{a}_x \times \mathbf{A} = \mathbf{a}_x \times C(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = 2\mathbf{a}_z - \mathbf{a}_y$

$\therefore C = 1$ and $\mathbf{A} = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$.

Verification: $\mathbf{a}_y \times \mathbf{A} = \mathbf{a}_y \times (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = \mathbf{a}_x - 2\mathbf{a}_z$.

1.8. Vector from $A(5, 0, 3)$ to $B(3, 3, 2) = -2\mathbf{a}_x + 3\mathbf{a}_y - \mathbf{a}_z$

Vector from $C(6, 2, 4)$ to $D(3, 3, 6) = -3\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$

Component of **AB** along **CD** $= \mathbf{AB} \bullet \dfrac{\mathbf{CD}}{CD} = \dfrac{6+3-2}{\sqrt{9+1+4}} = 1.8708$

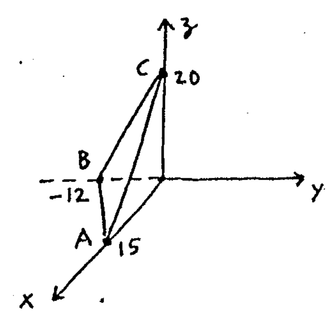
1.9. Writing the equation for the plane as $\dfrac{x}{15} - \dfrac{y}{12} + \dfrac{z}{20} = 1$, we find the intercepts on the x, y, and z-axes to be at 15, –12, and 20, respectively. Thus

$\mathbf{R}_{AB} = -15\mathbf{a}_x - 12\mathbf{a}_y$

$\mathbf{R}_{AC} = -15\mathbf{a}_x + 20\mathbf{a}_z$

$\mathbf{R}_{AC} \times \mathbf{R}_{AB} = 240\mathbf{a}_x - 300\mathbf{a}_y + 180\mathbf{a}_z$

$$\mathbf{a}_n = \frac{\mathbf{R}_{AC} \times \mathbf{R}_{AB}}{|\mathbf{R}_{AC} \times \mathbf{R}_{AB}|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y + 3\mathbf{a}_z}{5\sqrt{2}}$$



Distance from origin to the plane $= 15\mathbf{a}_x \bullet \mathbf{a}_n = 6\sqrt{2}$.

1.10. For $y = 2x$, $z = 4y$, we have $dy = 2\,dx$, $dz = 4\,dy = 8\,dx$.

$\therefore d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = dx\,\mathbf{a}_x + 2\,dx\,\mathbf{a}_y + 8\,dx\,\mathbf{a}_z$

$= (\mathbf{a}_x + 2\mathbf{a}_y + 8\mathbf{a}_z)\,dx$, independent of the point.

1.11. For $x = y = z^2$, we have $dx = dy = 2z\,dz$.

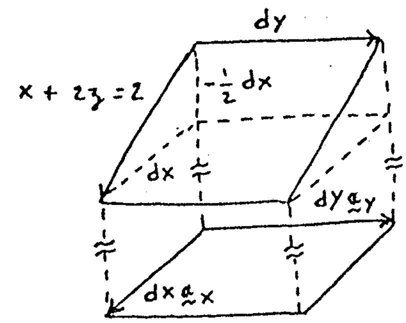
At the point (4, 4, 2), $dx = dy = 4\,dz$

$\therefore d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z = 4\,dz\,\mathbf{a}_x + 4\,dz\,\mathbf{a}_y + dz\,\mathbf{a}_z$

$= (4\mathbf{a}_x + 4\mathbf{a}_y + \mathbf{a}_z)\,dz$

1.12. Differential length vector having projection $dy\,\mathbf{a}_y = dy\,\mathbf{a}_y$

Differential length vector having projection $dx\,\mathbf{a}_x$ is



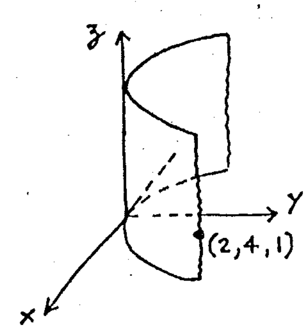
$$dx\,\mathbf{a}_x + dz\,\mathbf{a}_z = dx\,\mathbf{a}_x - \frac{1}{2}\,dx\,\mathbf{a}_z$$

$$= \left(\mathbf{a}_x - \frac{1}{2}\mathbf{a}_z\right) dx,$$

since for $x + 2z = 2$, $dz = -\frac{1}{2}\,dx$, independent of the point.

$$\therefore\ d\mathbf{S} = \left(\mathbf{a}_x - \frac{1}{2}\mathbf{a}_z\right) dx \times dy\,\mathbf{a}_y = \left(\frac{1}{2}\mathbf{a}_x + \mathbf{a}_z\right) dx\,dy\,.$$

1.13. One vector tangential to the surface is $dz\,\mathbf{a}_z$. Another tangential vector is given by



$$d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y$$
$$= dx\,\mathbf{a}_x + 2x\,dx\,\mathbf{a}_y$$
$$= (\mathbf{a}_x + 4\mathbf{a}_y)\,dx$$

$\therefore$ Vector normal to the plane $= (\mathbf{a}_x + 4\mathbf{a}_y)\,dx \times dz\,\mathbf{a}_z$

$$= (4\mathbf{a}_x - \mathbf{a}_y)\,dx\,dz$$

Unit vector normal to the plane $= \dfrac{4\mathbf{a}_x - \mathbf{a}_y}{\sqrt{17}}$.

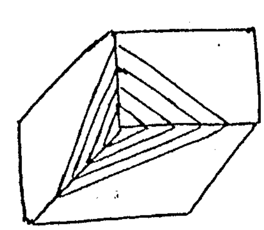
1.14. Denoting $h(x, y)$ to be the height field, we have

$x^2 + y^2 + h^2 = 4,\ x^2 + y^2 \le 4$

or, $h = \sqrt{4 - x^2 - y^2}\ ,\ x^2 + y^2 \le 4$.

1.15. The number field is $x + y + z$.

$\therefore$ Constant magnitude surfaces are the planes $x + y + z =$ constant.

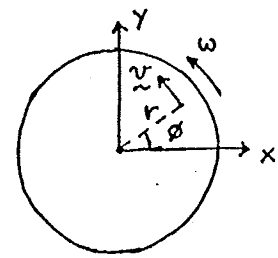


1.16. $\mathbf{d}(x, y, z) = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

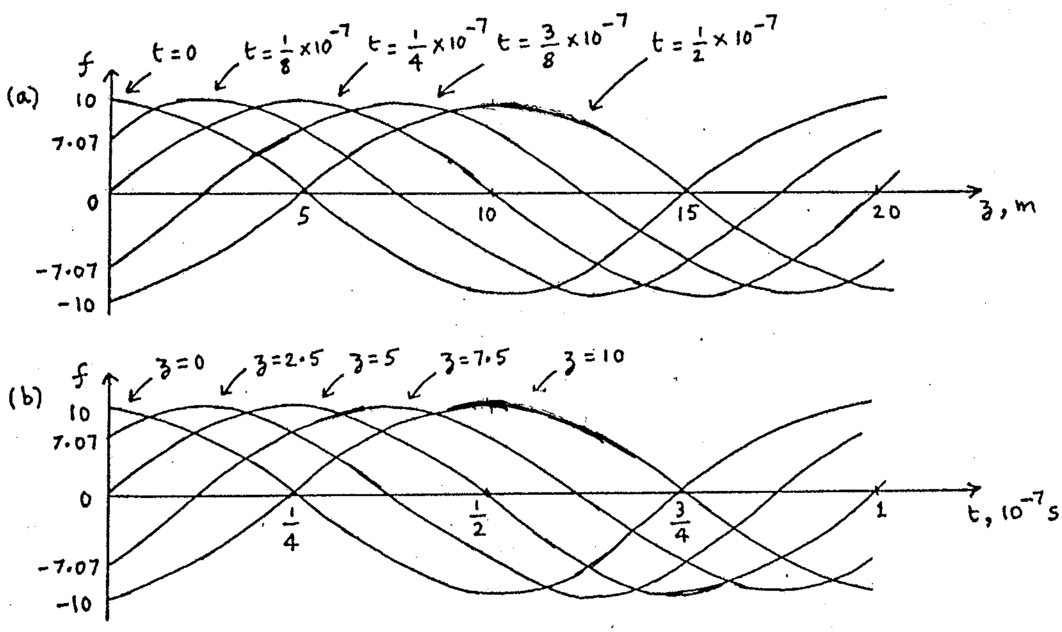
Constant magnitude surfaces are $x^2 + y^2 + z^2$ = constant, and hence are spherical surfaces centered at the corner. Direction lines are radial lines emanating from the corner.

1.17. $\mathbf{v} = -r\omega \sin \phi\, \mathbf{a}_x + r\omega \cos \phi\, \mathbf{a}_y$

$= \omega(-y\mathbf{a}_x + x\mathbf{a}_y)$

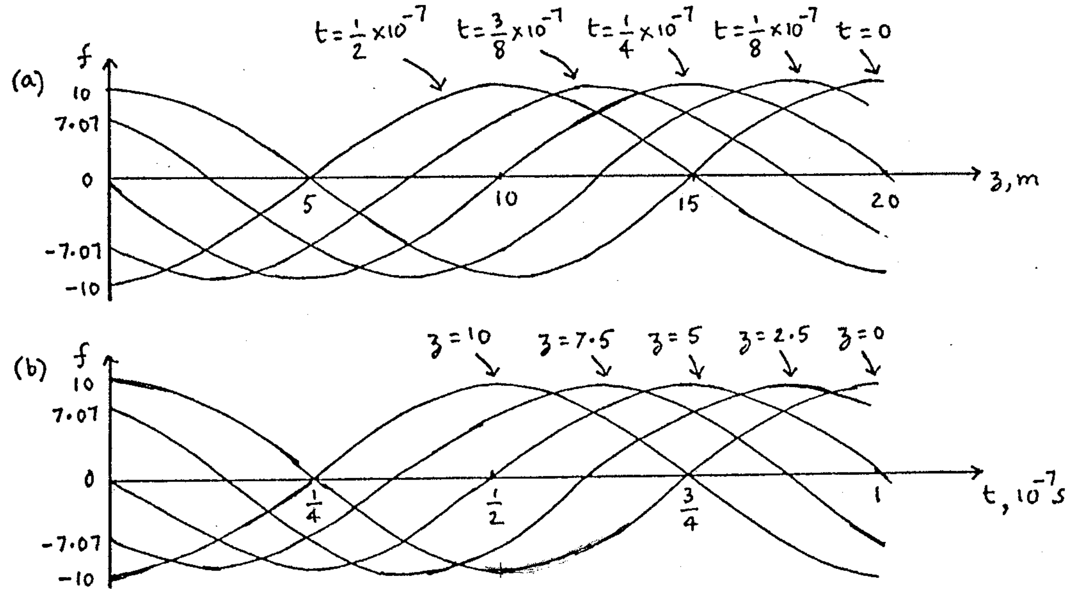


1.18. $f(z, t) = 10 \cos (2\pi \times 10^7 t - 0.1\, \pi z)$



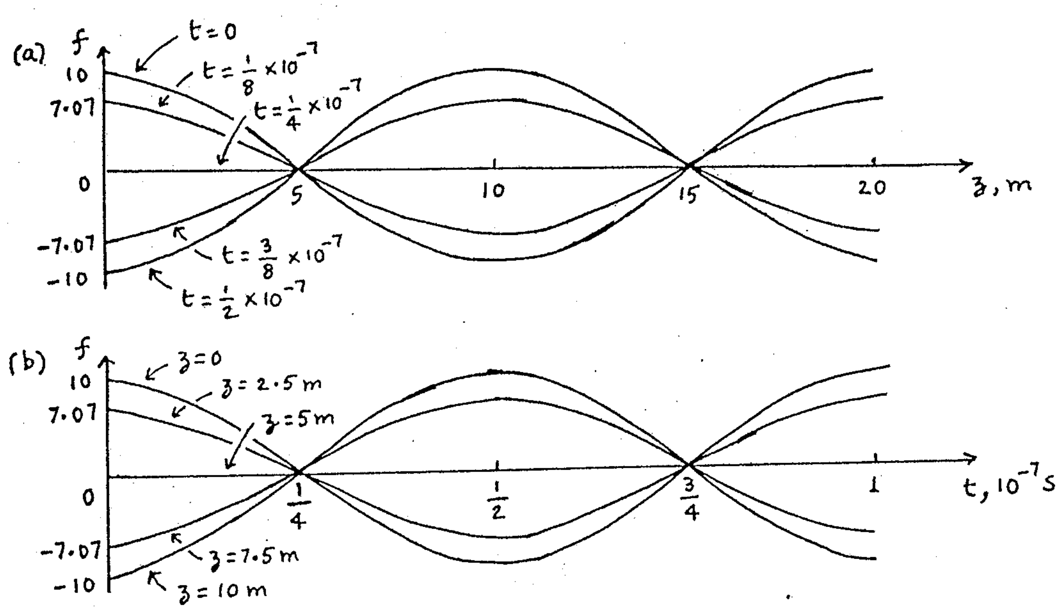
$f(z, t)$ represents a traveling wave progressing with time in the positive z-direction.

1.19. $f(z, t) = 10\cos(2\pi \times 10^7 t + 0.1\pi z)$



$f(z, t)$ represents a traveling wave progressing with time in the negative z-direction.

1.20. $f(z, t) = 10\cos 2\pi \times 10^7 t \cos 0.1\pi z$



$f(z, t)$ represents a standing wave.

1.21. (a) The two components are in phase; hence, linear polarization.

(b) The two components are perpendicular in direction, differ in phase by 90° and equal in amplitude; hence, circular polarization.

(c) The two components are perpendicular in direction, differ in phase by 90° but unequal in amplitude; hence elliptical polarization.

1.22. $\mathbf{F}_1$ and $\mathbf{F}_2$ differ in phase by 90°.

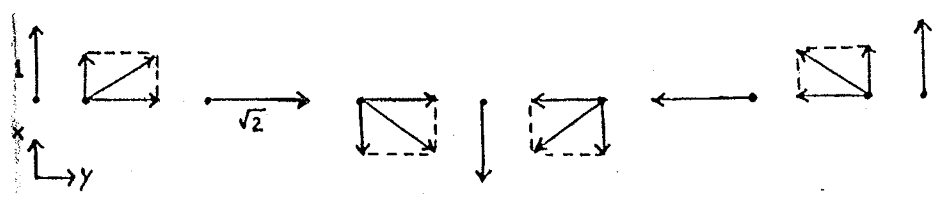
$$|\mathbf{F}_1| = \sqrt{3+1}\,\cos\omega t = 2\cos\omega t;\ |\mathbf{F}_2| = \sqrt{\frac{1}{4}+\frac{3}{4}+3}\,\sin\omega t = 2\sin\omega t\,.$$

$\therefore$ $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in amplitude.

$$\mathbf{F}_1\cdot\mathbf{F}_2 = -\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=0.\ \ \therefore\ \mathbf{F}_1 \text{ is perpendicular to } \mathbf{F}_2.$$

Thus $\mathbf{F}_1 + \mathbf{F}_2$ is circularly polarized.

1.23.

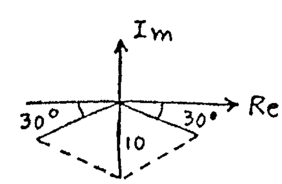


The polarization is elliptical with major axis in the y-direction, minor axis in the x-direction, and eccentricity equal to $\sqrt{2}$.

1.24. $10\cos(\omega t - 30°) + 10\cos(\omega t + 210°)$

$$\begin{array}{ccccc} \downarrow & & \downarrow & & \\ 10\,e^{-j30°} & + & 10\,e^{j210°} & = & 10\,e^{-j90°} \end{array}$$

$\therefore$ The sum is $10\cos(\omega t - 90°) = 10\sin\omega t$.

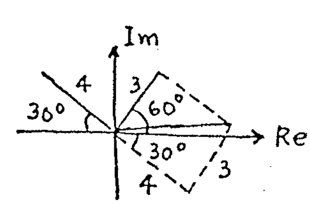


1.25. $3\cos(\omega t + 60°) - 4\cos(\omega t + 150°)$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 3\,e^{j60°} & - & 4\,e^{j150°} \end{array}$$

$= 5\,e^{j(60°-53.13°)} = 5e^{j6.87°}$

$\rightarrow 5\cos(\omega t + 6.87°)$.



1.26. Replacing $\frac{di}{dt}$ by $j10^6\bar{I}$, i by $\bar{I}$, and 13 cos 10^6t by $13e^{j0°}$, we have

$$5\times10^{-6}\times j10^6\bar{I}+12\bar{I}=13e^{j0°}$$

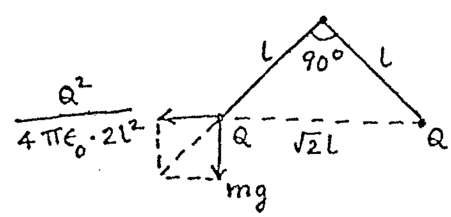
$$\text{or, } (12+j5)\bar{I}=13, \bar{I}=\frac{13}{12+j5}=\frac{13}{13e^{j22.62°}}=1e^{-j22.62°}$$

Thus $i = 1\cos(10^6t - 22.62°) = 1\cos(10^6t - 0.126\pi)$

1.27. From the construction shown,

$$\frac{Q^2}{4\pi\varepsilon_0\cdot 2l^2}\Big/mg=\tan 45°=1$$

$$\text{or, } Q=\sqrt{8\pi\varepsilon_0 l^2 mg}$$



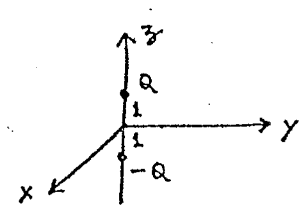
1.28. (a) At the point (0, 0, 100),

$$\mathbf{E}=\frac{Q}{4\pi\varepsilon_0(99)^2}\mathbf{a}_z+\frac{-Q}{4\pi\varepsilon_0(101)^2}\mathbf{a}_z$$

$$=\frac{Q}{4\pi\varepsilon_0}\frac{101^2-99^2}{99^2\times101^2}\mathbf{a}_z$$

$$=\frac{Q}{4\pi\varepsilon_0}\frac{(100+1)^2-(100-1)^2}{(100-1)^2\times(100+1)^2}\mathbf{a}_z=\frac{Q}{4\pi\varepsilon_0}\frac{400}{(100^2-1)^2}\mathbf{a}_z$$

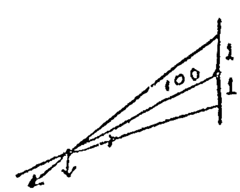
$$\approx\frac{Q}{4\pi\varepsilon_0}\frac{400}{100^4}\mathbf{a}_z=\frac{Q}{100^3\pi\varepsilon_0}\mathbf{a}_z$$



(b) At the point (100, 0, 0)

$$\mathbf{E}=-\frac{2Q}{4\pi\varepsilon_0(100^2+1^2)^{3/2}}\mathbf{a}_z$$

$$\approx-\frac{Q}{2\pi\varepsilon_0(100^3)}\mathbf{a}_z$$



1.29. $$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{9^{3/2}} + \frac{2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z}{9^{3/2}} + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{9^{3/2}}\right.$$

$$+ \frac{\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z}{6^{3/2}} + \frac{2\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{6^{3/2}} + \frac{\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{6^{3/2}}$$

$$\left. + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{12^{3/2}} + \frac{\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{3^{3/2}}\right]$$

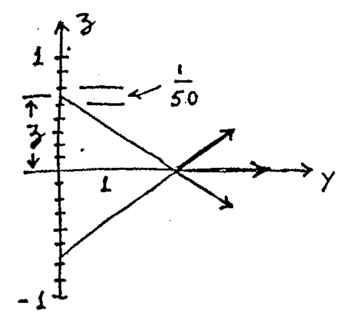
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{5}{(3)^3} + \frac{4}{(\sqrt{6})^3} + \frac{2}{(\sqrt{12})^3} + \frac{1}{(\sqrt{3})^3}\right](\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$$

$$= \frac{Q}{4\pi\varepsilon_0}(0.18519 + 0.27217 + 0.04811 + 0.19245)(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$$

$$= \frac{0.0555\,Q}{\varepsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \text{ N/C.}$$

1.30. For the *i*th segment,

$$z = \frac{2i-1}{100} \text{ and charge} = \frac{10^{-3}}{50} \text{ C.}$$



$$\therefore\ \mathbf{E} = \frac{2}{4\pi\varepsilon_0}\sum_{i=1}^{50}\frac{10^{-3}}{50}\frac{1}{\left(z^2+1\right)^{3/2}}\mathbf{a}_y$$

$$= \frac{10^{-5}}{\pi\varepsilon_0}\sum_{i=1}^{50}\left[10^{-4}(2i-1)^2+1\right]^{-3/2}\mathbf{a}_y$$

1.31. For the *i*th segment, $z = \frac{2i-1}{100}$, charge density $= 10^{-3}\,\frac{2i-1}{100}$ C/m,

and charge $= 10^{-3}\,\frac{2i-1}{100}\cdot\frac{1}{50} = \frac{10^{-6}}{5}(2i-1)$ C.

$$\therefore\ \mathbf{E} = \frac{2}{4\pi\varepsilon_0}\sum_{i=1}^{50}\frac{10^{-6}(2i-1)}{5}\frac{1}{\left(z^2+1\right)^{3/2}}\mathbf{a}_y$$

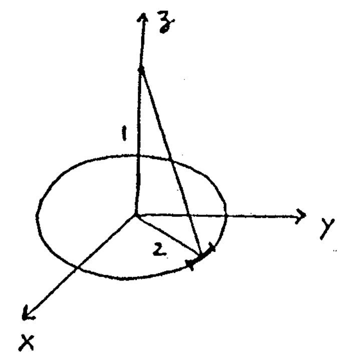
$$= \frac{10^{-7}}{\pi\varepsilon_0}\sum_{i=1}^{50}(2i-1)\left[10^{-4}(2i-1)^2+1\right]^{-3/2}\mathbf{a}_y$$

1.32. Dividing the circular ring into n segments and using the symmetry of the field about the z-axis, we obtain

$$\mathbf{E} = \sum_{n} \frac{2\pi(2)\times 10^{-3}}{n\cdot 4\pi\varepsilon_0}\cdot\frac{1}{\left(2^2+1^2\right)^{3/2}}\mathbf{a}_z$$

$$= \frac{4\pi\times 10^{-3}}{4\pi\varepsilon_0\cdot 5^{3/2}}\mathbf{a}_z = \frac{0.08944\times 10^{-3}}{\varepsilon_0}\mathbf{a}_z$$

$$= 1.012\times 10^{7}\mathbf{a}_z \text{ N/C.}$$



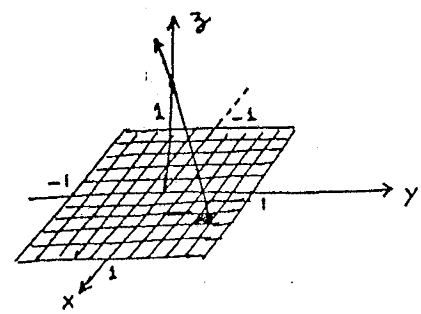
1.33. For the (ij)th area,

$$x = \frac{2i-1}{100},\ y = \frac{2j-1}{100},\text{ and}$$

$$\text{charge} = \frac{4}{10000}\times 10^{-3} = 4\times 10^{-7}\,\text{C}$$

$$\therefore\ \mathbf{E} = \frac{4}{4\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\frac{4\times 10^{-7}}{(x^2+y^2+1)^{3/2}}\mathbf{a}_z$$

$$= \frac{4\times 10^{-7}}{\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\left[10^{-4}(2i-1)^2+10^{-4}(2j-1)^2+1\right]^{-3/2}\mathbf{a}_z$$



1.34. For the (ij)th area, $x = \dfrac{2i-1}{100},\ y = \dfrac{2j-1}{100}$,

$$\text{charge density} = 10^{-3}\left(\frac{2i-1}{100}\right)\left(\frac{2j-1}{100}\right)^2 = 10^{-9}(2i-1)(2j-1)^2\ \text{C/m}^2$$

$$\text{charge} = \frac{4}{10000}\times 10^{-9}(2i-1)(2j-1)^2 = 4\times 10^{-13}(2i-1)(2j-1)^2\ \text{C}$$

$$\therefore\ \mathbf{E} = \frac{4}{4\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\frac{4\times 10^{-13}(2i-1)(2j-1)^2}{(x^2+y^2+1)^{3/2}}\mathbf{a}_z$$

$$= \frac{4\times 10^{-13}}{\pi\varepsilon_0}\sum_{i=1}^{50}\sum_{j=1}^{50}\frac{(2i-1)(2j-1)^2}{\left[10^{-4}(2i-1)^2+10^{-4}(2j-1)^2+1\right]^{3/2}}\mathbf{a}_z$$

1.35. (a) $\mathbf{J} = Ne\mathbf{v} = \dfrac{Ne^2}{m\omega}\, E_0 \sin \omega t\, \mathbf{a}_x$

$$= \frac{10^{12} \times 1.6021^2 \times 10^{-38}}{9.1083 \times 10^{-31} \times 2\pi \times 10^7} \times 10^{-3} \sin 2\pi \times 10^7 t\, \mathbf{a}_x$$

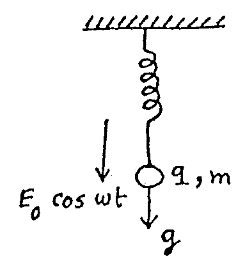
$$= 0.4485 \times 10^{-6} \sin 2\pi \times 10^7 t\, \mathbf{a}_x \ \ \mathrm{A/m^2}$$

(b) $\Delta I = \mathbf{J} \bullet \Delta\mathbf{S} = 0.4485 \times 10^{-6} \sin 2\pi \times 10^7 t\, \mathbf{a}_x \bullet 0.01\,(\mathbf{a}_x + \mathbf{a}_y)$

$$= 0.4485 \times 10^{-8} \sin 2\pi \times 10^7 t \ \ \mathrm{A}$$

1.36. Denoting x to be the displacement,

we write the equation of motion to be



$$m\frac{d^2x}{dt^2} = mg - kx + qE_0 \cos \omega t$$

$$\text{or, } m\frac{d^2x}{dt^2} + kx = mg + qE_0 \cos \omega t$$

The steady state solution consists of two parts.

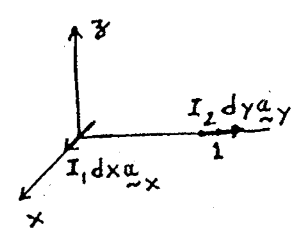
One is $x_1 = \dfrac{mg}{k}$ due to mg. To find the second part x_2, we write

$$(j\omega)^2 m\bar{x}_2 + k\bar{x}_2 = qE_0 e^{j0}, \text{ or, } (k - \omega^2 m)\bar{x}_2 = qE_0$$

$$\bar{x}_2 = \frac{qE_0}{k - \omega^2 m}. \text{ Thus } x = x_1 + x_2 = \frac{mg}{k} + \frac{qE_0}{k - \omega^2 m} \cos \omega t$$

$$\therefore \text{ Velocity} = \frac{dx}{dt} = -\frac{qE_0\omega}{k - \omega^2 m} \sin \omega t$$

1.37. $d\mathbf{F}_1 = I_1\, dx\, \mathbf{a}_x \times \left[\dfrac{\mu_0 I_2\, dy\, \mathbf{a}_y \times (-\mathbf{a}_y)}{4\pi(1)^2}\right]$



$$= 0$$

$$d\mathbf{F}_2 = I_2\, dy\, \mathbf{a}_y \times \left[\frac{\mu_0 I_1\, dx\, \mathbf{a}_x \times \mathbf{a}_y}{4\pi(1)^2}\right]$$

$$= I_2\, dy\, \mathbf{a}_y \times \frac{\mu_0}{4\pi} I_1\, dx\, \mathbf{a}_z = \frac{\mu_0}{4\pi} I_1 I_2\, dx\, dy\, \mathbf{a}_x$$

1.38. (a) For (0, 1, 1), $\mathbf{a}_R = \dfrac{-\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{\sqrt{3}}$, $R = \sqrt{3}$, and

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\,dx\,(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{3} \times \frac{(-\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)}{\sqrt{3}}$$

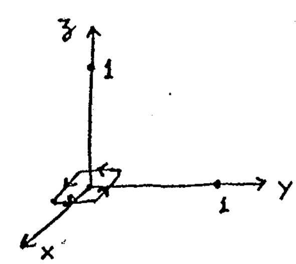
$$= \frac{\mu_0 I\,dx}{4\sqrt{3}\pi}(-\mathbf{a}_y + \mathbf{a}_z)$$

(b) For (2, 2, 2), $\mathbf{a}_R = \dfrac{\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{3}}$, $R = 3$, and

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\,dx\,(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{9} \times \frac{(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{3}$$

$$= 0$$

1.39. (a) At (0, 0, 1), the components of **B** perpendicular to the z-axis cancel, whereas the z components add. Thus



$$\mathbf{B} = 4\left[\frac{\mu_0}{4\pi} \frac{0.01\mathbf{a}_y \times (-0.005\mathbf{a}_x + \mathbf{a}_z)}{(1 + 0.005^2)^{3/2}} \bullet \mathbf{a}_z\right]\mathbf{a}_z$$

$$\approx \left[\frac{\mu_0}{\pi}(0.00005\mathbf{a}_z + 0.01\mathbf{a}_x) \bullet \mathbf{a}_z\right]\mathbf{a}_z = \frac{5 \times 10^{-5}\mu_0}{\pi}\mathbf{a}_z$$

(b) At the point (0, 1, 0),

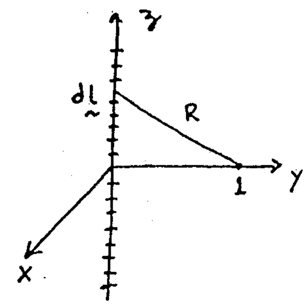
$$\mathbf{B} = \frac{\mu_0}{4\pi}\left[-\frac{0.01}{(1-0.005)^2}\mathbf{a}_z + \frac{0.01}{(1+0.005)^2}\mathbf{a}_z\right.$$

$$\left. + \frac{0.01\mathbf{a}_y \times (-0.005\mathbf{a}_x + \mathbf{a}_y)}{(1+0.005^2)^{3/2}} - \frac{0.01\mathbf{a}_y \times (0.005\mathbf{a}_x + \mathbf{a}_y)}{(1+0.005^2)^{3/2}}\right]$$

$$= \frac{\mu_0}{4\pi}\left[-\frac{0.01 \times 4 \times 1 \times 0.005}{(1-0.005^2)^2}\mathbf{a}_z + \frac{2 \times 0.01 \times 0.005}{(1+0.005^2)^{3/2}}\mathbf{a}_z\right]$$

$$\approx -\frac{10^{-4}\mu_0}{4\pi}\mathbf{a}_z$$

1.40. For the ith element, $d\mathbf{l} = \frac{1}{50}\mathbf{a}_z$,



$$R = \sqrt{1 + \left(\frac{2i-1}{100}\right)^2},$$

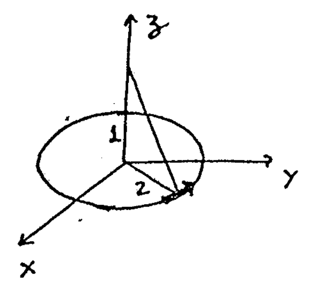
$$\mathbf{a}_R = \frac{1}{R}\left[\mathbf{a}_y - \left(\frac{2i-1}{100}\right)\mathbf{a}_z\right], \text{ and}$$

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{1}{50}\mathbf{a}_z \times \left[\mathbf{a}_y - \left(\frac{2i-1}{100}\right)\mathbf{a}_z\right]\frac{1}{R^3}$$

$$= -\frac{\mu_0 I}{200\pi R^3}\mathbf{a}_x$$

$$\mathbf{B} = 2\sum_{i=1}^{50} d\mathbf{B} = -\frac{\mu_0 I}{100\pi}\sum_{i=1}^{50}\left[1 + 10^{-4}(2i-1)^2\right]^{-3/2}\mathbf{a}_x$$

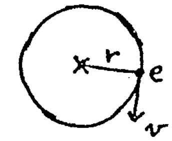
1.41. Dividing the loop into n segments and using the symmetry of the field about the z-axis, we obtain



$$\mathbf{B} = \sum_n \frac{\mu_0 I}{4\pi}\frac{2\pi(2)}{n}\frac{2}{\left(2^2 + 1^2\right)^{3/2}}\mathbf{a}_z$$

$$= \frac{8\pi\mu_0 I}{4\pi \times 5^{3/2}}\mathbf{a}_z = 0.179\mu_0 I\,\mathbf{a}_z$$

1.42. Equating the magnetic force to the centripetal force, we have



$$evB_0 = \frac{mv^2}{r}, \text{ or, } r = \frac{mv}{eB_0}$$

$$\omega = \frac{v}{r} = \frac{eB_0}{m}$$

$$\text{Orbital frequency} = \frac{\omega}{2\pi} = \frac{eB_0}{2\pi m}$$

For $B_0 = 5 \times 10^{-5}$,

orbital frequency = $\frac{1.7578\times10^{11}}{2\pi}\times5\times10^{-5}=1.3988\times10^{6}$ Hz

= 1.3988 MHz.

1.43. $\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})=0$

$\therefore\ \mathbf{E}=-\mathbf{v}\times\mathbf{B}$

$=-v_0(3\mathbf{a}_x-\mathbf{a}_y+2\mathbf{a}_z)\times B_0(\mathbf{a}_x+2\mathbf{a}_y-4\mathbf{a}_z)$

$=-v_0B_0(14\mathbf{a}_y+7\mathbf{a}_z)$

1.44. (a) $q\mathbf{E}+qv_0\mathbf{a}_x\times\mathbf{B}=0$ — (1)

$q\mathbf{E}+qv_0\mathbf{a}_y\times\mathbf{B}=0$ — (2)

$q\mathbf{E}+qv_0(\mathbf{a}_x+\mathbf{a}_y)\times\mathbf{B}=-qE_0\mathbf{a}_z$ — (3)

(1) + (2) – (3) → $q\mathbf{E}=qE_0\mathbf{a}_z$

$\therefore\ \mathbf{E}=E_0\mathbf{a}_z$

(1) – (2) → $(\mathbf{a}_x-\mathbf{a}_y)\times\mathbf{B}=0$

$\therefore\ \mathbf{B}=C(\mathbf{a}_x-\mathbf{a}_y)$ where C is a constant

To find C, we use (1). Thus,

$qE_0\mathbf{a}_z+qv_0\mathbf{a}_z\times C(\mathbf{a}_x-\mathbf{a}_y)=0$

or, $qE_0\mathbf{a}_z-qv_0C\mathbf{a}_z=0$

$$C=\frac{E_0}{v_0}$$

Thus, $\mathbf{E}=E_0\mathbf{a}_z$ and $\mathbf{B}=\frac{E_0}{v_0}(\mathbf{a}_x-\mathbf{a}_y)$

(b) For $\mathbf{v}=v_0(\mathbf{a}_x-\mathbf{a}_y)$,

$$\mathbf{F}=qE_0\mathbf{a}_z+qv_0(\mathbf{a}_x-\mathbf{a}_y)\times\frac{E_0}{v_0}(\mathbf{a}_x-\mathbf{a}_y)$$

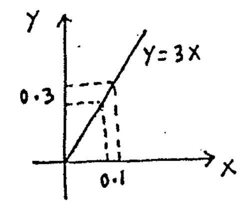
$=qE_0\mathbf{a}_z$

CHAPTER 2

2.1. $d\mathbf{l} = 0.1\mathbf{a}_x + 0.3\mathbf{a}_y$, $\mathbf{F} = [(i-1)0.1]^2\mathbf{a}_y$, $\mathbf{F} \bullet d\mathbf{l} = 0.003(i-1)^2$

$$\int_{(0,0,0)}^{(1,3,0)} \mathbf{F} \bullet d\mathbf{l} \approx \sum_{i=1}^{10} 0.003(i-1)^2$$

$$= 0.003(0 + 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81) = 0.855$$



2.2. $d\mathbf{l} = \frac{1}{n}\mathbf{a}_x + \frac{3}{n}\mathbf{a}_y, \mathbf{F} = \left[(i-1)\frac{1}{n}\right]^2 \mathbf{a}_y, \mathbf{F} \bullet d\mathbf{l} = \frac{3}{n^3}(i-1)^2$

$$\int_{(0,0,0)}^{(1,3,0)} \mathbf{F} \bullet d\mathbf{l} \approx \sum_{i=1}^{n} \frac{3}{n^3}(i-1)^2 = \frac{3}{n^3} \cdot \frac{2n^3 - 3n^2 + n}{6} = 1 - \frac{3}{2n} + \frac{1}{2n^2}$$

n	5	10	100	∞
$1 - \frac{3}{2n} + \frac{1}{2n^2}$	0.82	0.855	0.98505	1

2.3. For the straight line path $y = 3x$, $z = 0$, $dy = 3\,dx$, and $dz = 0$.

$d\mathbf{l} = dx\,\mathbf{a}_x + 3\,dx\,\mathbf{a}_y$, $\mathbf{F} \bullet d\mathbf{l} = x^2\mathbf{a}_y \bullet (dx\,\mathbf{a}_x + 3\,dx\,\mathbf{a}_y) = 3x^2dx$

$$\int_{(0,0,0)}^{(1,3,0)} \mathbf{F} \bullet d\mathbf{l} \approx \int_0^1 3x^2 dx = \left[x^3\right]_0^1 = 1$$

2.4. (a) For $y = x$, $z = 0$, $dy = dx$, $dz = 0$, $d\mathbf{l} = dx\,\mathbf{a}_x + dx\,\mathbf{a}_y$,

$\mathbf{E} = x\mathbf{a}_x + x\mathbf{a}_y$, $\mathbf{E} \bullet d\mathbf{l} = x\,dx + x\,dx = 2\,x\,dx$

$$\int_{(0,0,0)}^{(1,1,0)} \mathbf{E} \bullet d\mathbf{l} \approx \int_0^1 2x\,dx = \left[x^2\right]_0^1 = 1$$

(b) From (0, 0, 0) to (1, 0, 0), $y = 0$, $z = 0$, $dy = dz = 0$, $d\mathbf{l} = dx\,\mathbf{a}_x$,

$\mathbf{E} = x\mathbf{a}_y$, $\mathbf{E} \bullet d\mathbf{l} = 0$, $\int_{(0,0,0)}^{(1,0,0)} \mathbf{E} \bullet d\mathbf{l} = 0$

From (1, 0, 0) to (1, 1, 0), $x = 1$, $z = 0$, $dx = dz = 0$, $d\mathbf{l} = dy\,\mathbf{a}_y$,

$\mathbf{E} = y\mathbf{a}_x + \mathbf{a}_y$, $\mathbf{E} \bullet d\mathbf{l} = dy$, $\int_{(1,0,0)}^{(1,1,0)} \mathbf{E} \bullet d\mathbf{l} = \int_0^1 dy = 1$

$$\therefore \int_{(0,0,0)}^{(1,1,0)} \mathbf{E} \bullet d\mathbf{l} = \int_{(0,0,0)}^{(1,0,0)} \mathbf{E} \bullet d\mathbf{l} + \int_{(1,0,0)}^{(1,1,0)} \mathbf{E} \bullet d\mathbf{l} = 0 + 1 = 1$$

(c) $\mathbf{E} \bullet d\mathbf{l} = (y\mathbf{a}_x + x\mathbf{a}_y) \bullet (dx\, \mathbf{a}_x + dy\, \mathbf{a}_y + dz\, \mathbf{a}_z) = y\, dx + x\, dy = d(xy)$

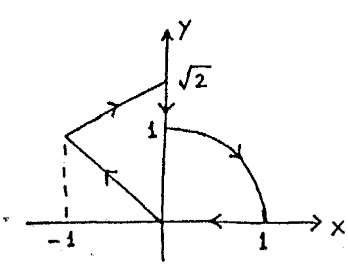
$\int_{(0,0,0)}^{(1,1,0)} \mathbf{E} \bullet d\mathbf{l} = \int_{(0,0,0)}^{(1,1,0)} d(xy) = [xy]_{(0,0,0)}^{(1,1,0)} = 1 - 0 = 1$, independent of the path followed from (0, 0,) to (1, 1, 0).

2.5. $$\oint_C d\mathbf{l} = \oint_C (dx\, \mathbf{a}_x + dy\, \mathbf{a}_y + dz\, \mathbf{a}_z)$$

$$= \left[\oint_C dx\right]\mathbf{a}_x + \left[\oint_C dy\right]\mathbf{a}_y + \left[\oint_C dz\right]\mathbf{a}_z = 0$$

$$\oint_C \mathbf{F} \bullet d\mathbf{l} = \mathbf{F} \bullet \oint_C d\mathbf{l} = 0$$

2.6. From (0, 0, 0) to (–1, 1, 0):



$y = -x$, $dy = -dx$, $d\mathbf{l} = dx\, \mathbf{a}_x - dx\, \mathbf{a}_y$

$\mathbf{F} = -x\mathbf{a}_x - x\mathbf{a}_y$, $\mathbf{F} \bullet d\mathbf{l} = 0$

$$\int_{(0,0,0)}^{(-1,1,0)} \mathbf{F} \bullet d\mathbf{l} = 0$$

From (–1, 1 0) to (0, $\sqrt{2}$, 0):

$y = (\sqrt{2} - 1)x + \sqrt{2}$, $dy = (\sqrt{2} - 1)\, dx$, $d\mathbf{l} = dx\, \mathbf{a}_x + (\sqrt{2} - 1)\, dx\, \mathbf{a}_y$,

$\mathbf{F} = [(\sqrt{2} - 1)x + \sqrt{2}\,]\mathbf{a}_x - x\mathbf{a}_y$, $\mathbf{F} \bullet d\mathbf{l} = \sqrt{2}\, dx$

$$\int_{(-1,1,0)}^{(0,\sqrt{2},0)} \mathbf{F} \bullet d\mathbf{l} = \int_{-1}^{0} \sqrt{2}\, dx = \sqrt{2}\,.$$

From (0, $\sqrt{2}$, 0) to (0, 1, 0): $x = 0$, $dx = 0$, $d\mathbf{l} = dy\, \mathbf{a}_y$,

$\mathbf{F} = y\mathbf{a}_x$, $\mathbf{F} \bullet d\mathbf{l} = 0$, $\int_{(0,\sqrt{2},0)}^{(0,1,0)} \mathbf{F} \bullet d\mathbf{l} = 0\,.$

From (0, 1, 0) to (1, 0, 0): $x^2 + y^2 = 1$, $dy = -\frac{x}{y}\ dx$,

$d\mathbf{l} = dx\ \mathbf{a}_x - \frac{x}{\sqrt{1-x^2}}\ dx\ \mathbf{a}_y$, $\mathbf{F} = \sqrt{1-x^2}\ \mathbf{a}_x - x\mathbf{a}_y$,

$\mathbf{F} \bullet d\mathbf{l} = \frac{1}{\sqrt{1-x^2}}\ dx$, $\int_{(0,1,0)}^{(1,0,0)} \mathbf{F} \bullet d\mathbf{l} = \int_0^1 \frac{1}{\sqrt{1-x^2}}\ dx = \frac{\pi}{2}$

From (1, 0, 0) to (0, 0, 0): $y = 0$, $dy = 0$, $dl = dx\ \mathbf{a}_x$, $\mathbf{F} = -x\mathbf{a}_y$,

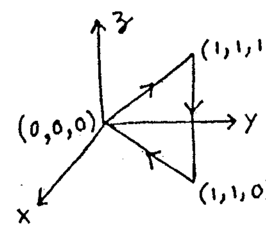
$\mathbf{F} \bullet d\mathbf{l} = 0$, $\int_{(1,0,0)}^{(0,0,0)} \mathbf{F} \bullet d\mathbf{l} = 0$

Thus $\oint \mathbf{F} \bullet d\mathbf{l} = 0 + \sqrt{2} + 0 + \frac{\pi}{2} + 0 = \sqrt{2} + \frac{\pi}{2} = 2.985$.

2.7. From (0, 0, 0) to (1, 1, 1): $x = y = z$, $dx = dy = dz$

$d\mathbf{l} = dx\ \mathbf{a}_x + dx\ \mathbf{a}_y + dx\ \mathbf{a}_z$,

$\mathbf{F} = x^2\mathbf{a}_x + x^2\mathbf{a}_y + x^2\mathbf{a}_z$, $\mathbf{F} \bullet d\mathbf{l} = 3x^2 dx$



$\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \bullet d\mathbf{l} = \int_0^1 3x^2 dx = \left[x^3\right]_0^1 = 1$

From (1, 1, 1) to (1, 1, 0): $y = x = 1$, $dy = dx = 0$,

$d\mathbf{l} = dz\ \mathbf{a}_z$, $\mathbf{F} = \mathbf{a}_x + z\mathbf{a}_y + z\mathbf{a}_z$, $\mathbf{F} \bullet d\mathbf{l} = z\ dz$

$\int_{(1,1,1)}^{(1,1,0)} \mathbf{F} \bullet d\mathbf{l} = \int_1^0 z\ dz = -\frac{1}{2}$.

From (1, 1, 0) to (0, 0, 0): $z = 0$, $y = x$, $dz = 0$, $dy = dx$,

$d\mathbf{l} = dx\ \mathbf{a}_x + dx\ \mathbf{a}_y$, $\mathbf{F} = x^2\mathbf{a}_x$, $\mathbf{F} \bullet d\mathbf{l} = x^2 dx$.

$\int_{(1,1,0)}^{(0,0,0)} \mathbf{F} \bullet d\mathbf{l} = \int_1^0 x^2 dx = -\frac{1}{3}$.

Thus $\oint_C \mathbf{F} \bullet d\mathbf{l} = 1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

2.8. For the ijth square, $x = (2i - 1)\ 0.05$ and $y = (2j - 1)\ 0.05$;

$\mathbf{B} = \frac{(2i-1)^2}{400} e^{-0.05(2j-1)}\mathbf{a}_z$, $d\mathbf{S} = \frac{1}{100}\mathbf{a}_z$

$$\mathbf{B} \bullet d\mathbf{S} = \frac{10^{-4}}{4}(2i-1)^2 e^{-0.05(2j-1)}$$

$$\psi = \sum_{i=1}^{10}\sum_{j=1}^{10}\frac{10^{-4}}{4}(2i-1)^2 e^{-0.05(2j-1)}$$

$$= \frac{10^{-4}}{4}e^{0.05}\sum_{i=1}^{10}(2i-1)^2\sum_{j=1}^{10}e^{-0.1j}$$

$$= \frac{10^{-4}}{4}e^{0.05}(1330)\frac{e^{-0.1}-e^{-1.1}}{1-e^{-0.1}} = 0.21009$$

2.9. For the ijth square, $x = \dfrac{2i-1}{2n}$, $y = \dfrac{2j-1}{2n}$, $d\mathbf{S} = \dfrac{1}{n^2}\mathbf{a}_z$,

$$\mathbf{B} = \left(\frac{2i-1}{2n}\right)^2 e^{-\frac{2j-1}{2n}}\mathbf{a}_z, \mathbf{B} \bullet d\mathbf{S} = \frac{1}{4n^4}(2i-1)^2 e^{-\frac{2j-1}{2n}}$$

$$\psi = \frac{1}{4n^4}\sum_{i=1}^{n}(2i-1)^2\sum_{j=1}^{n}e^{-\frac{2j-1}{2n}}$$

$$= \frac{1}{4n^4}\sum_{i=1}^{n}(4i^2-4i+1)\sum_{j=1}^{n}e^{-(j-1)\frac{1}{n}}\, e^{-\frac{1}{2n}}$$

$$= \frac{1}{4n^4}\left[\frac{4(2n^3+3n^2+n)}{6} - \frac{4(n^2+n)}{2} + n\right]\left(\frac{1-e^{-1}}{1-e^{-1/n}}\right)e^{-\frac{1}{2n}}$$

$$= \left(\frac{4n^2-1}{12n^3}\right)\left(\frac{1-e^{-1}}{1-e^{-1/n}}\right)e^{-\frac{1}{2n}}$$

n	5	10	100	∞
ψ	0.20825	0.21009	0.21070	0.21071

2.10. $\mathbf{B} \bullet d\mathbf{S} = x^2e^{-y}\mathbf{a}_z \bullet dx\, dy\, \mathbf{a}_z = x^2e^{-y}\, dx\, dy$

$$\psi = \int_{y=0}^{1}\int_{x=0}^{1} x^2e^{-y}dx\, dy = \frac{1}{3}(1-e^{-1}) = 0.21071$$

2.11. We note that $\mathbf{A} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$ is directed radially away from the origin and hence is normal to the hemispherical surface everywhere on the surface. Also $|\mathbf{A}| = \sqrt{x^2 + y^2 + z^2}$ is constant on the hemispherical surface and is equal to 2. Thus

$$\int \mathbf{A} \bullet d\mathbf{S} = |\mathbf{A}| \times \int dS = 2 \text{ (area of the hemispherical surface)}$$

$$= 2 \times 2\pi(2)^2 = 16\pi$$

2.12. Consider the volume bounded by the surface S to be comprised of a number of infinitesimal rectangular boxes. Then the total vector area of the surface is equal to the sum of the vector areas of the surfaces of the individual boxes since the contributions to the sum from the interior surfaces cancel. Now, since the total vector area of each infinitesimal box is zero, it follows that the vector area of the arbitrary closed surface S is zero, that is, $\oint_S d\mathbf{S} = 0$ for any S. Then $\oint_S \mathbf{A} \bullet d\mathbf{S} = \mathbf{A} \bullet \oint_S d\mathbf{S} = 0$.

2.13. For the surface $x = 0$, $\mathbf{J} = (y - 3)\mathbf{a}_y + (2 + z)\mathbf{a}_z$, $d\mathbf{S} = -dy\, dz\, \mathbf{a}_x$,

$\mathbf{J} \bullet d\mathbf{S} = 0$, $\int \mathbf{J} \bullet d\mathbf{S} = 0$.

For the surface $x = 1$, $\mathbf{J} = 3\mathbf{a}_x + (y - 3)\mathbf{a}_y + (2 + z)\mathbf{a}_z$, $d\mathbf{S} = dy\, dz\, \mathbf{a}_x$,

$\mathbf{J} \bullet d\mathbf{S} = 3\, dy\, dz$, $\int \mathbf{J} \bullet d\mathbf{S} = \int_{z=0}^{3} \int_{y=0}^{2} 3\, dy\, dz = 18$.

For the surface $y = 0$, $\mathbf{J} = 3x\, \mathbf{a}_x - 3\mathbf{a}_y + (2 + z)\mathbf{a}_z$, $d\mathbf{S} = -dz\, dx\, \mathbf{a}_y$,

$\mathbf{J} \bullet d\mathbf{S} = 3\, dz\, dx$, $\int \mathbf{J} \bullet d\mathbf{S} = \int_{x=0}^{1} \int_{z=0}^{3} 3\, dz\, dx = 9$.

For the surface $y = 2$, $\mathbf{J} = 3x\, \mathbf{a}_x - \mathbf{a}_y + (2 + z)\mathbf{a}_z$, $d\mathbf{S} = dz\, dx\, \mathbf{a}_y$,

$\mathbf{J} \bullet d\mathbf{S} = -dz\, dx$, $\int \mathbf{J} \bullet d\mathbf{S} = \int_{x=0}^{1} \int_{z=0}^{3} -dz\, dx = -3$.

For the surface $z = 0$, $\mathbf{J} = 3x\, \mathbf{a}_x + (y - 3)\mathbf{a}_y + 2\mathbf{a}_z$, $d\mathbf{S} = -dx\, dy\, \mathbf{a}_z$,

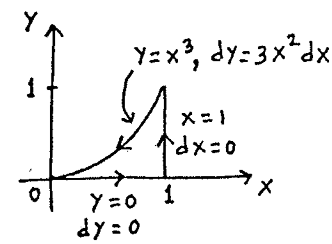
$\mathbf{J} \bullet d\mathbf{S} = -2\, dx\, dy$, $\int \mathbf{J} \bullet d\mathbf{S} = \int_{y=0}^{2} \int_{x=0}^{1} -2\, dx\, dy = -4$.

For the surface $z = 3$, $\mathbf{J} = 3x\mathbf{a}_x + (y-3)\mathbf{a}_y + 5\mathbf{a}_z$, $d\mathbf{S} = dx\,dy\,\mathbf{a}_z$,

$$\mathbf{J} \bullet d\mathbf{S} = 5\,dx\,dy, \quad \int \mathbf{J} \bullet d\mathbf{S} = \int_{y=0}^{2} \int_{x=0}^{1} 5\,dx\,dy = 10\,.$$

$$\therefore \oint \mathbf{J} \bullet d\mathbf{S} = 0 + 18 + 9 - 3 - 4 + 10 = 30\text{ A}\,.$$

2.14. From $\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \bullet d\mathbf{S}$



and noting that the normal to the surface is directed toward the positive-z side in accordance with the right-hand screw rule, we have time rate of decrease of magnetic flux

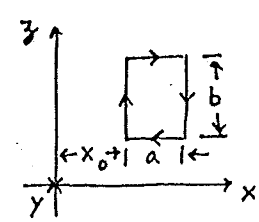
$$= -\frac{d}{dt} \int \mathbf{B} \bullet d\mathbf{S} = \oint_C \mathbf{E} \bullet d\mathbf{l}$$

$$= \int_{x=0}^{1} -x\mathbf{a}_y \cos\omega t \bullet dx\,\mathbf{a}_x + \int_{y=0}^{1} (y\mathbf{a}_x - \mathbf{a}_y)\cos\omega t \bullet dy\,\mathbf{a}_y$$

$$+ \int_{x=1}^{0} (x^3\mathbf{a}_x - x\mathbf{a}_y)\cos\omega t \bullet (dx\,\mathbf{a}_x + 3x^2 dx\,\mathbf{a}_y)$$

$$= 0 + \int_{y=0}^{1} -\cos\omega t\,dy + \int_{x=1}^{0} -2x^3 \cos\omega t\,dx = -\frac{1}{2}\cos\omega t\,.$$

2.15. $\oint_S \mathbf{B} \bullet d\mathbf{S} = \int \frac{B_0}{x}\mathbf{a}_y \bullet dz\,dx\,\mathbf{a}_y$



$$= \int_{x=x_0}^{x_0+a} \int_{z=z_0}^{z_0+b} \frac{B_0}{x}\,dz\,dx$$

$$= B_0 b\,[(\ln(x_0 + a) - \ln x_0]$$

$$\therefore \oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \bullet d\mathbf{S} = -\frac{d}{dt}\{B_0 b[\ln(x_0 + a) - \ln x_0]\}$$

$$= -B_0 b\left(\frac{1}{x_0 + a} - \frac{1}{x_0}\right)\frac{dx_0}{dt} = -B_0 b v_0\left(\frac{1}{x_0 + a} - \frac{1}{x_0}\right).$$

From the motional *emf* concept, the induced *emf* is

$\left(v_0 \frac{B_0}{x_0} b - v_0 \frac{B_0}{x_0 + a} b \right)$, which agrees with the above result.

2.16. $\int_S \mathbf{B} \bullet d\mathbf{S} = \int_{x=x_0}^{x_0+a} \int_{z=z_0}^{z_0+b} \frac{B_0}{x} \cos \omega t \; dz \, dx$

$$= B_0 b \cos \omega t \cdot [\ln (x_0 + a) - \ln x_0]$$

$$\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \bullet d\mathbf{S} = B_0 b \omega \sin \omega t \cdot \ln \frac{x_0 + a}{x_0}$$

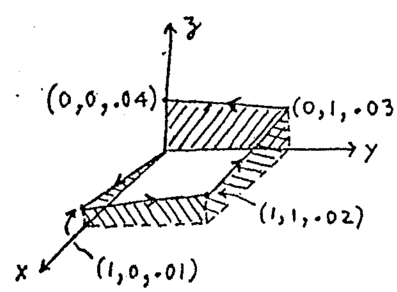
2.17. $\int_S \mathbf{B} \bullet d\mathbf{S} = B_0 b \cos \omega t \cdot [\ln (x_0 + a) - \ln x_0]$

$$\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \bullet d\mathbf{S}$$

$$= B_0 b \omega \sin \omega t \cdot \ln \frac{x_0 + a}{x_0} - B_0 b \cos \omega t \cdot \left(\frac{1}{x_0 + a} - \frac{1}{x_0} \right) \frac{dx_0}{dt}$$

$$= B_0 b \omega \ln \frac{x_0 + a}{x_0} \sin \omega t - B_0 b \, v_0 \left(\frac{1}{x_0 + a} - \frac{1}{x_0} \right) \cos \omega t.$$

2.18. Considering the surface bounded by the closed path to be comprised of the four crosshatched surfaces plus the surface in the *xy*-plane, we obtain

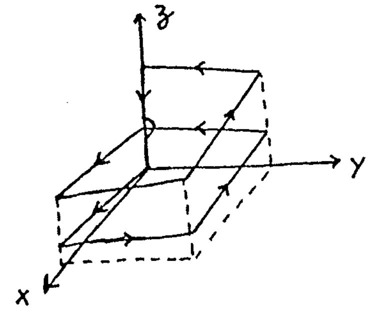


$$\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \bullet d\mathbf{S}$$

$$= -\frac{d}{dt} \int_{x=0}^{1} \int_{y=0}^{1} B_0 \cos \omega t \, \mathbf{a}_z \bullet dx \, dy \, \mathbf{a}_z = B_0 \omega \sin \omega t$$

since the four crosshatched surfaces do not contribute to the flux enclosed.

2.19. From considerations similar to those employed in Prob. 2.18, we obtain

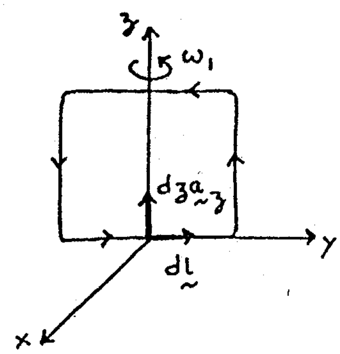


$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 2B_0\omega \sin \omega t .$$

2.20. $d\mathbf{S} = d\mathbf{l} \times dz\, \mathbf{a}_z$

$= dl\, (\cos\, \omega_1 t\, \mathbf{a}_y - \sin\, \omega_1 t\, \mathbf{a}_x) \times dz\, \mathbf{a}_z$

$= dl\, dz\, (\cos\, \omega_1 t\, \mathbf{a}_x + \sin\, \omega_1 t\, \mathbf{a}_y)$



$\mathbf{B} \cdot d\mathbf{S} = B_0\, dl\, dz \cos\, \omega_1 t \cos\, \omega_2 t$

$$\int \mathbf{B} \cdot d\mathbf{S} = B_0 A \cos\, \omega_1 t \cos\, \omega_2 t$$

$$= \frac{B_0 A}{2}[\cos\,(\omega_1 + \omega_2)t + \cos\,(\omega_1 - \omega_2)t]$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{S}$$

$$= \frac{1}{2} B_0 A(\omega_1 + \omega_2) \sin\,(\omega_1 + \omega_2)t + \frac{1}{2} B_0 A(\omega_1 - \omega_2) \sin\,(\omega_1 - \omega_2)t .$$

2.21. Using $d\mathbf{S}$ from Prob. 2.20, we obtain

$\mathbf{B} \cdot d\mathbf{S} = B_0\, dl\, dz\, (\cos^2\, \omega_1 t + \sin^2\, \omega_1 t) = B_0\, dl\, dz$

$$\int \mathbf{B} \cdot d\mathbf{S} = B_0 A$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot dS = -\frac{d}{dt}(B_0 A) = 0 .$$

Note that the *emf* is zero because **B** is circularly polarized with the rotation in the same sense as and with the same angular velocity as that of the loop so that the flux enclosed is a constant.

2.22. Using $d\mathbf{S}$ from Prob. 2.20, we obtain

$\mathbf{B} \cdot d\mathbf{S} = B_0\, dl\, dz\, (\cos^2\, \omega_1 t - \sin^2\, \omega_1 t)$

$= B_0\, dl\, dz \cos\, 2\omega_1 t$

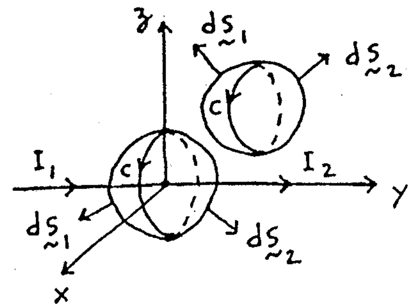
$$\int \mathbf{B} \cdot d\mathbf{S} = B_0 A \cos 2\omega_1 t$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{S} = 2B_0 A\, \omega_1 \sin 2\omega_1 t\,.$$

2.23. (a) $\oint_C \mathbf{H} \cdot d\mathbf{l} = 0 + \frac{d}{dt} \int_{S_2} \mathbf{D} \cdot d\mathbf{S}_2$

$$= 0 - \frac{d}{dt} \int_{S_1} \mathbf{D} \cdot d\mathbf{S}_1$$

$$\frac{d}{dt} \oint_{S_1+S_2} \mathbf{D} \cdot d\mathbf{S} = 0$$

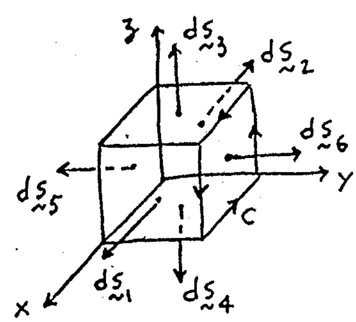


(b) $\oint_C \mathbf{H} \cdot d\mathbf{l} = I_2 + \frac{d}{dt} \int_{S_2} \mathbf{D} \cdot d\mathbf{S}_2$

$$= I_1 - \frac{d}{dt} \int_{S_1} \mathbf{D} \cdot d\mathbf{S}_1$$

$$\frac{d}{dt} \oint_{S_1+S_2} \mathbf{D} \cdot d\mathbf{S} = I_1 - I_2$$

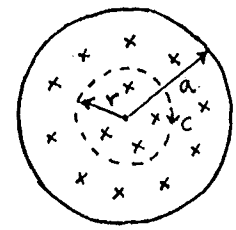
2.24. Considering the closed path C shown in the figure and noting that $\int \mathbf{J} \cdot d\mathbf{S}$ is zero for the surfaces S_1, S_2, S_3, S_4, and S_5 but equal to $\int_{x=0}^{1} \int_{z=0}^{1} \cos \omega t\, dx\, dz$ or, $\cos \omega t$ for the surface S_6, we have



$$\oint_C \mathbf{H} \cdot d\mathbf{l} = 0 - \frac{d}{dt}\left[\int_{S_1} \mathbf{D} \cdot d\mathbf{S}_1 + \int_{S_2} \mathbf{D} \cdot d\mathbf{S}_2 + \int_{S_3} \mathbf{D} \cdot d\mathbf{S}_3 + \int_{S_4} \mathbf{D} \cdot d\mathbf{S}_4 + \int_{S_5} \mathbf{D} \cdot d\mathbf{S}_5 \right] = \cos \omega t + \frac{d}{dt} \int_{S_6} \mathbf{D} \cdot d\mathbf{S}_6$$

Thus $\frac{d}{dt} \oint_S \mathbf{D} \cdot d\mathbf{S} = -\cos \omega t\,.$

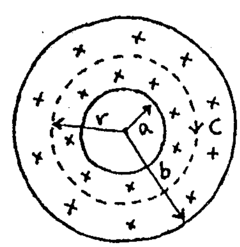
2.25. We first note from symmetry considerations that **H** is directed circular to the axis of the wire and is dependent only on the radial distance r. Applying Ampere's circuital law without the displacement current term to a circular path of radius r, we then obtain



$$2\pi rH = \begin{cases} J_0 \pi r^2 & \text{for } r < a \\ J_0 \pi a^2 & \text{for } r > a \end{cases}$$

$$H = \begin{cases} J_0 r/2 & \text{for } r < a \\ J_0 a^2/2r & \text{for } r > a \end{cases}$$

2.26. From symmetry considerations, we first note that **H** is directed circular to the axis of the wire and is dependent on the radial distance r. Applying Ampere's circuital law without the displacement current term to a circular path of radius r, we then obtain

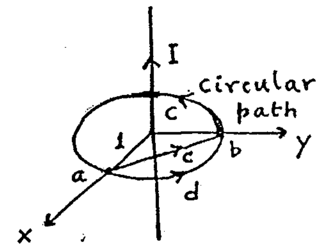


$$2\pi rH = \begin{cases} 0 & \text{for } r < a \\ J_0(\pi r^2 - \pi a^2) & \text{for } a < r < b \\ J_0(\pi b^2 - \pi a^2) & \text{for } r > b \end{cases}$$

$$H = \begin{cases} 0 & \text{for } r < a \\ \dfrac{J_0}{2r}(r^2 - a^2) & \text{for } a < r < b \\ \dfrac{J_0}{2r}(b^2 - a^2) & \text{for } r > b \end{cases}$$

2.27. (a) From Ampere's circuital law,

$$\oint_C \mathbf{H} \bullet d\mathbf{l} = I$$



Then from symmetry considerations,

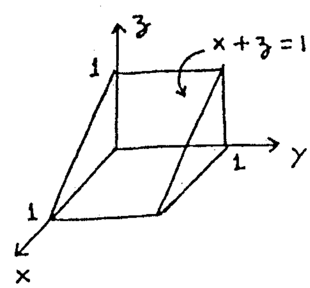
$$\int_{adb} \mathbf{H} \bullet d\mathbf{l} = \frac{I}{4}.$$

(b) From Ampere's circuital law, $\oint_{adbca} \mathbf{H} \bullet d\mathbf{l} = 0$, or,

$$\int_{adb} \mathbf{H} \bullet d\mathbf{l} + \int_b^a \mathbf{H} \bullet d\mathbf{l} = 0, \text{ or, } \int_{adb} \mathbf{H} \bullet d\mathbf{l} - \int_a^b \mathbf{H} \bullet d\mathbf{l} = 0$$

$$\therefore \int_a^b \mathbf{H} \bullet d\mathbf{l} = \int_{adb} \mathbf{H} \bullet d\mathbf{l} = \frac{I}{4}$$

2.28. Charge enclosed = $\oint_S \mathbf{D} \bullet d\mathbf{S}$, where



S is the surface of the box. The only contribution to the surface integral comes from the surface $y = 1$, since $\mathbf{D} = y\mathbf{a}_y$ is zero on the surface $y = 0$ and is parallel to the remaining three surfaces. Thus

$$Q = \int_{x=0}^{1} \int_{z=0}^{1-x} 1\mathbf{a}_y \bullet dx\, dz\, \mathbf{a}_y = \int_{x=0}^{1} \int_{z=0}^{1-x} dx\, dz$$

$$= \int_0^1 (1-x)\, dx = \frac{1}{2}\text{ C}.$$

2.29. Displacement flux = $\oint_S \mathbf{D} \bullet d\mathbf{S} = \int_V \rho\, dv$

$$= \int_{x=0}^{1} \int_{y=0}^{1} \int_{z=0}^{1} x\, e^{-x^2}\, dx\, dy\, dz = \frac{1}{2}(1-e^{-1}) = 0.31606\text{ C}.$$