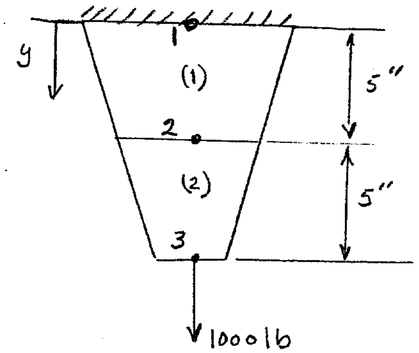
$\boxed{1.1}$

(a) Using two elements



$A(y) = 0.25 - 0.0125y$

$A_1 = 0.25 \text{ in}^2 \qquad A_2 = 0.1875 \text{ in}^2$

$A_3 = 0.125 \text{ in}^2$

$$K_{eq} = \frac{(A_{i+1} + A_i)E}{2\ell}$$

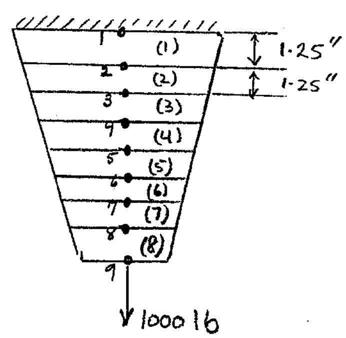
$$K_1 = \frac{(0.25 + 0.1875)(10.4 \times 10^6)}{2(5)} = 455{,}000 \ \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{(0.1875 + 0.125)(10.4 \times 10^6)}{2(5)} = 325{,}000 \ \frac{\text{lb}}{\text{in}}$$

$$\begin{array}{cc} & \begin{array}{ccc} 1 & \quad 0 & \quad \end{array} \\ 10^3 & \begin{bmatrix} \cancel{455} & \cancel{-455} & \\ \underset{0}{\cancel{-455}} & 455+325 & -325 \\ & -325 & 325 \end{bmatrix} \end{array} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$u_1 = 0,\quad \underline{\underline{u_2 = 0.002197 \text{ in}}} \qquad \underline{\underline{u_3 = 0.005274 \text{ in}}}$ ⇦

(b)



$A_1 = 0.25 \text{ in}^2$

$A_2 = 0.234375 \text{ in}^2$

$A_3 = 0.21875 \text{ in}^2$

$A_4 = 0.203125 \text{ in}^2$

$A_5 = 0.1875 \text{ in}^2$

$A_6 = 0.171875 \text{ in}^2$

$A_7 = 0.15625 \text{ in}^2$

$A_8 = 0.140625 \text{ in}^2$

$A_9 = 0.125 \text{ in}^2$

$\boxed{\begin{array}{l}1.1\\ \text{Cont.}\end{array}}$

$K_1 = 2015000 \ \frac{lb}{in}$ $\quad K_2 = 1885000 \ \frac{lb}{in}$ $\quad K_3 = 1755000 \ \frac{lb}{in}$

$K_4 = 1625000 \ \frac{lb}{in}$ $\quad K_5 = 1495000 \ \frac{lb}{in}$ $\quad K_6 = 1365000 \ \frac{lb}{in}$

$K_7 = 1235000 \ \frac{lb}{in}$ $\quad K_8 = 1105000 \ \frac{lb}{in}$

$$\begin{array}{ccccccccc} 1 & 0 & & & & & & & \end{array}$$

$$\begin{bmatrix} \cancel{2015000} & \cancel{-2015000} & & & & & & & \\ \cancel{-2015000}\ 0 & 2015000+1885000 & -1885000 & & & & & & \\ & -1885000 & 1885000+1755000 & -1755000 & & & & & \\ & & -1755000 & 1755000+1625000 & -1625000 & & & & \\ & & & -1625000 & 1625000+1495000 & -1495000 & & & \\ & & & & -1495000 & 1495000+1365000 & -1365000 & & \\ & & & & & -1365000 & 1365000+1235000 & -1235000 & \\ & & & & & & -1235000 & 1235000+1105000 & -1105000 \\ & & & & & & & -1105000 & 1105000 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \end{Bmatrix}$$

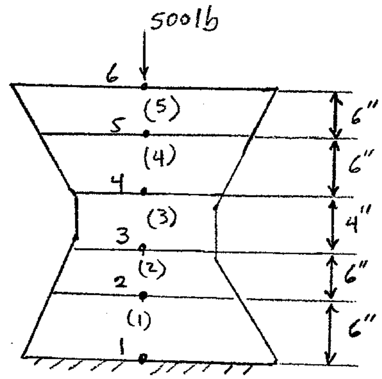
$u_1 = 0,\quad u_2 = 0.00049628,\quad u_3 = 0.0010268,\quad u_4 = 0.0015965$

$u_5 = 0.0022119,\quad u_6 = 0.0028808,\quad u_7 = 0.0036134,\quad u_8 = 0.0044232$

$u_9 = 0.0053281$ in.

y (in)	exact defl.	Two-element	eight element
0	0	0	0
1.25	0.00049645		0.00049628
2.5	0.0010272		0.0010268
3.75	0.0015972		0.0015965
5.0	0.0022129	0.002197	0.0022119
6.25	0.0028822		0.0028808
7.5	0.0036154		0.0036134
8.75	0.0044259		0.0044232
10.0	0.0053319	0.005274	0.0053281

$\boxed{1.2}$



$$K = \frac{A_{avg.}E}{l}$$

$$K_1 = K_5 = \frac{\left(\frac{10.5+12}{2}\right)(3)(3.27\times10^{6})}{6}$$

$$K_1 = K_5 = 18{,}393{,}750 \ \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_4 = \frac{\left(\frac{10.5+9}{2}\right)(3)(3.27\times10^{6})}{6} = 15{,}941{,}250 \ \frac{\text{lb}}{\text{in}}$$

$$K_3 = \frac{(9)(3)(3.27\times10^{6})}{4} = 22{,}072{,}500 \ \frac{\text{lb}}{\text{in}}$$

$$\begin{array}{c}
\begin{array}{cccccc} 1 & 0 & & & & \end{array} \\
\left[\begin{array}{cccccc}
\cancel{18393750} & \cancel{-18393750} & & & & \\
\cancel{-18393750}\ 0 & 18393750 + 15941250 & -15941250 & & & \\
 & -15941250 & 15941250 + 22072500 & -22072500 & & \\
 & & -22072500 & 22072500 + 15941250 & -15941250 & \\
 & & & -15941250 & 15941250 + 18393750 & -18393750 \\
 & & & & -18393750 & 18393750
\end{array}\right]
\end{array}
\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -500 \end{Bmatrix}$$

$$\{u\} = \begin{Bmatrix} 0 \\ -2.71831\times10^{-5} \\ -5.85483\times10^{-5} \\ -8.12009\times10^{-5} \\ -1.12566\times10^{-4} \\ -1.39749\times10^{-4} \end{Bmatrix} \text{in} \Leftarrow$$

$$\sigma = \left(\frac{u_{i+1}-u_i}{l}\right)E$$

$$\sigma^{(1)} = \frac{(-2.71831\times10^{-5}-0)(3.27\times10^{6})}{6} = \underline{\underline{14.8 \text{ psi C}}}$$

$$\sigma^{(5)} = \frac{(-1.39749+1.12566)\times10^{-4}\times(3.27\times10^{6})}{6} = \underline{\underline{14.8 \text{ psi C}}}$$

as a check:

$$\sigma^{(1)} = \sigma^{(5)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5+12}{2}\right)(3)} = 14.8 \text{ psi C}$$

$$\sigma^{(2)} = \frac{(-5.85483+2.71831)\times10^{-5}\times(3.27\times10^{6})}{6} = \underline{\underline{17.1 \text{ psi C}}}$$

$$\sigma^{(4)} = \frac{(-1.12566\times10^{-4}+8.12009\times10^{-5})(3.27\times10^{6})}{6} = \underline{\underline{17.1 \text{ psi C}}}$$

as a check:

$$\sigma^{(2)} = \sigma^{(4)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5+9}{2}\right)(3)} = 17.1 \text{ psi}$$

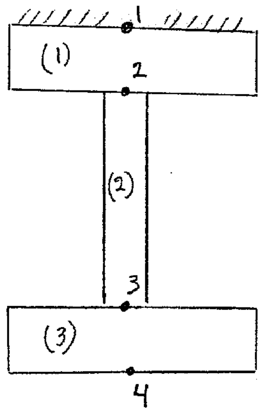
$$\sigma^{(3)} = \frac{(-8.12009\times10^{-5}+5.85483)\times10^{-5}\times(3.27\times10^{6})}{4}$$

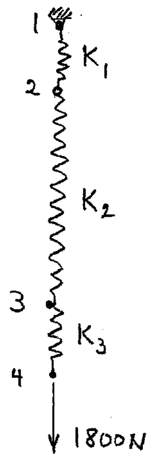
$$\sigma^{(3)} = \underline{\underline{18.5 \text{ psi C}}}$$

as a check:

$$\sigma^{(3)} = \frac{F}{A} = \frac{500}{(9)(3)} = 18.5 \text{ psi}$$

$\boxed{1.3}$





$$K = \frac{AE}{\ell}$$

$$K_1 = K_3 = \frac{(0.08)(0.006)(68.9 \times 10^9)}{0.025}$$

$$K_1 = K_3 = 1.32288 \times 10^9 \ \frac{N}{m}$$

$$K_2 = \frac{(0.02)(0.006)(68.9 \times 10^9)}{0.1}$$

$$K_2 = 0.08268 \times 10^9 \ \frac{N}{m}$$

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 0 & & \end{array} \\ 10^9 & \begin{bmatrix} \cancel{1.32288} & \cancel{-1.32288} & & \\ \cancel{-1.32288} & 1.32288+0.08268 & -0.08268 & \\ & -0.08268 & 0.08268+1.32288 & -1.32288 \\ & & -1.32288 & 1.32288 \end{bmatrix} \end{array} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1800 \end{Bmatrix}$$

$$\begin{bmatrix} 1.40556 & -0.08268 & 0 \\ -0.08268 & 1.40556 & -1.32288 \\ 0 & -1.32288 & 1.32288 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1800 \times 10^{-9} \end{Bmatrix}$$

$\underline{\underline{u_2 = 1.360668 \times 10^{-6} \ m}}$ $\quad$ $\underline{\underline{u_3 = 2.313135 \times 10^{-5} m}}$ $\quad$ $\underline{\underline{u_4 = 2.449202 \times 10^{-5} \ m}}$

$$\sigma^{(1)} = E \ \frac{u_2 - u_1}{\ell} = (68.9 \times 10^9)\left(\frac{1.360668 \times 10^{-6} - 0}{0.025}\right) = 3750000 = \underline{\underline{3.75 \ MPa}}$$

as a check: $\sigma^{(1)} = \frac{F}{A} = \frac{1800 \ N}{(0.08)(0.006)} = \underline{\underline{3.75 \ MPa}}$

$$\sigma^{(2)} = E \ \frac{u_3 - u_2}{\ell} = (68.9 \times 10^9)\left(\frac{2.313135 \times 10^{-5} - 1.360668 \times 10^{-6}}{0.1}\right) = \underline{\underline{15 \ MPa}}$$

as a check $\sigma^{(2)} = \frac{1800 \ N}{(0.02)(0.006)} = \underline{\underline{15 \ MPa}}$

$$\sigma^{(3)} = E \ \frac{u_4 - u_3}{\ell} = (68.9 \times 10^9) \ \frac{(2.449202 - 2.313135) \times 10^{-5}}{0.025} = \underline{\underline{3.75 \ MPa}}$$

$\boxed{1.4}$

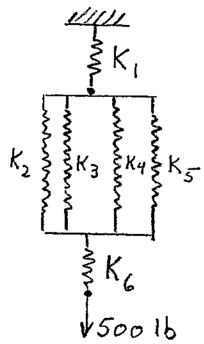
$$K_1 = K_6 = \frac{(4)(0.125)(28\times10^6)}{2}$$

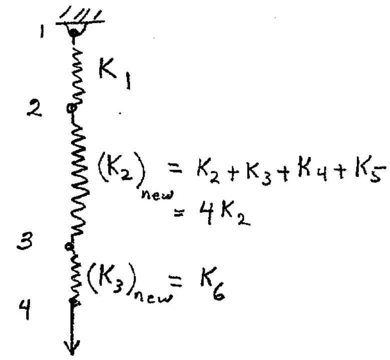
$$K_1 = K_6 = 7\times10^6 \ \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_3 = K_4 = K_5 = \frac{(0.625)(0.125)(28\times10^6)}{8}$$

$$K_2 = 273438 \ \frac{\text{lb}}{\text{in}}$$

$$(K_2)_{new} = (4)(273438) = 1093750 \ \frac{\text{lb}}{\text{in}}$$





$$\begin{bmatrix} \cancel{7\times10^6}\ 1 & 0 \quad \cancel{-7\times10^6}\ 0 & & \\ \cancel{-7\times10^6}\ 0 & 7\times10^6+1093750 & -1093750 & \\ & -1093750 & 1093750+7\times10^6 & -7\times10^6 \\ & & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$\begin{bmatrix} 8093750 & -1093750 & 0 \\ -1093750 & 8093750 & -7\times10^6 \\ 0 & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500 \end{Bmatrix}$$

$u_2 = \underline{\underline{7.142857\times10^{-5} \text{ in}}}$ $\quad u_3 = \underline{\underline{5.285714\times10^{-4} \text{ in}}}$ $\quad u_4 = \underline{\underline{6\times10^{-4} \text{ in}}}$

$$\sigma^{(1)} = E\left(\frac{u_2-u_1}{l}\right) = 28\times10^6\left(\frac{7.142857\times10^{-5} \text{ in}}{2}\right) = \underline{\underline{1000 \ \frac{\text{lb}}{\text{in}^2}}}$$

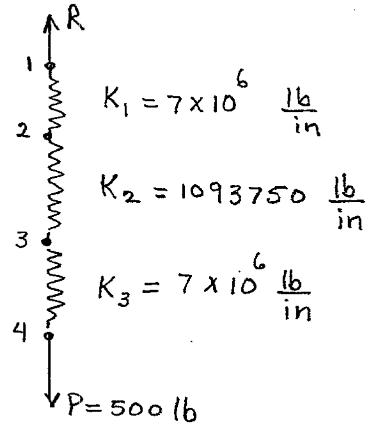
as a check: $\sigma^{(1)} = \frac{F}{A} = \frac{500}{(4)(0.125)} = 1000 \ \frac{\text{lb}}{\text{in}^2}$

$$\sigma^{(2)} = E\left(\frac{u_3-u_2}{l}\right) = 28\times10^6\left(\frac{5.285714\times10^{-4}-7.142857\times10^{-5}}{8}\right) = \underline{\underline{1600 \ \frac{\text{lb}}{\text{in}^2}}}$$

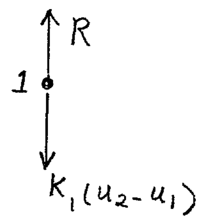
as a check: $\sigma^{(2)} = \frac{500}{(2.5)(0.125)} = 1600 \ \frac{\text{lb}}{\text{in}^2}$

$$\sigma^{(3)} = E\left(\frac{u_4-u_3}{l}\right) = 28\times10^6\left(\frac{6\times10^{-4}-5.285714\times10^{-4}}{2}\right) = \underline{\underline{1000 \ \frac{\text{lb}}{\text{in}^2}}}$$

$\boxed{1.5}$

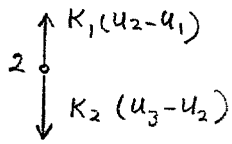


node 1:



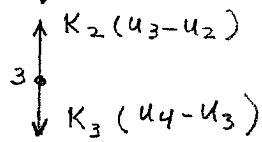
$$R - K_1(u_2 - u_1) = 0$$

node 2:



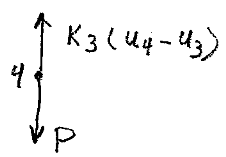
$$K_1(u_2 - u_1) - K_2(u_3 - u_2) = 0$$

node 3:



$$K_2(u_3 - u_2) - K_3(u_4 - u_3) = 0$$

node 4:

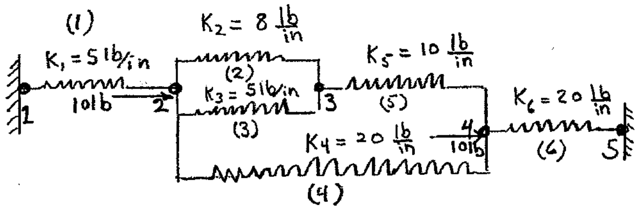


$$K_3(u_4 - u_3) - P = 0$$

In matrix form:

$$\begin{bmatrix} K_1 & -K_1 & & \\ -K_1 & K_1 + K_2 & -K_2 & \\ & -K_2 & K_2 + K_3 & -K_3 \\ & & K_3 & K_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -R \\ 0 \\ 0 \\ P \end{Bmatrix}$$

1.6



Size of the global matrix: 5x5

$$[K]^{(1)} = \begin{array}{cc} ① & ② \end{array} \quad \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \begin{array}{c} ① \\ ② \end{array} \qquad [K]^{(2)} = \begin{array}{cc} ② & ③ \end{array} \quad \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{array}{c} ② \\ ③ \end{array}$$

$$[K]^{(3)} = \begin{array}{cc} ② & ③ \end{array} \quad \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \begin{array}{c} ② \\ ③ \end{array} \qquad [K]^{(4)} = \begin{array}{cc} ② & ④ \end{array} \quad \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{array}{c} ② \\ ④ \end{array}$$

$$[K]^{(5)} = \begin{array}{cc} ③ & ④ \end{array} \quad \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{array}{c} ③ \\ ④ \end{array} \qquad [K]^{(6)} = \begin{array}{cc} ④ & ⑤ \end{array} \quad \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{array}{c} ④ \\ ⑤ \end{array}$$

$$[K]^{(G)} = \begin{bmatrix} 5 & -5 & & & \\ -5 & 5+8+5+20 & -8-5 & -20 & \\ & -8-5 & 8+5+10 & -10 & \\ & -20 & -10 & 20+10+20 & -20 \\ & & & -20 & 20 \end{bmatrix}$$

applying B.Cs and loads; $u_1 = u_5 = 0$ $\quad F_2 = 10\,lb, \quad F_4 = 10\ lb$

$$\begin{bmatrix} 38 & -13 & -20 \\ -13 & 23 & -10 \\ -20 & -10 & 50 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 10 \end{Bmatrix}$$

$\underline{\underline{u_2 = 0.962\ in}} \qquad u_3 = \underline{\underline{0.874\ in}} \qquad u_4 = \underline{\underline{0.760\,in}}$

node 1:

$R_1 \longleftarrow \bullet_1 \longrightarrow K_1(u_2 - u_1) \qquad R_1 = 5\,(0.962 - 0) = \underline{\underline{4.8\ lb}}$

node 5

$K(u_4 - u_5) \longrightarrow \bullet_5 \longleftarrow R_5 \qquad R_5 = 20\,(0.760 - 0) = \underline{\underline{15.2\ lb}}$

note also as a check, $\quad R_1 + R_5 = \sum F_{external} \qquad 4.8 + 15.2 = 10 + 10$

$\boxed{1.7}$

$$
150\begin{bmatrix}
\overset{\frac{1}{150}}{\cancel{5.88}} & \overset{0}{\cancel{5.88}} & & & & & & \\
-5.88 & 5.88+2.27 & -2.27 & & & & & \\
 & -2.27 & 2.27+10 & -10 & & & & \\
 & & -10 & 10+0.581 & -0.581 & & & \\
 & & & -0.581 & 0.581+0.781 & -0.781 & & \\
 & & & & -0.781 & 0.781+2.22 & -2.22 & \\
 & & & & & -2.22 & 2.22+1.47 & -1.47 \\
 & & & & & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{150}}{\cancel{1.47}}
\end{bmatrix}
\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{Bmatrix}
=
\begin{Bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}
$$

$$
\{T\} = \begin{Bmatrix} 10 \\ 12.04 \\ 17.31 \\ 18.51 \\ 39.12 \\ 54.46 \\ 59.85 \\ 68 \end{Bmatrix} {}^\circ F \quad \Leftarrow
$$

$$q = UA\,\Delta T$$

$$q = (1.47)(150)(68-59.85) = \underline{\underline{1800\ \frac{Btu}{hr}}} \quad \Leftarrow$$

or as another example:

$$q = (0.781)(150)(54.46-39.12) = 1800\ \frac{Btu}{hr}$$

or

$$q = \frac{1}{\sum R_{esistance}} A\left(T_{in}-T_{out}\right) = \frac{(150)(68-10)}{0.17+0.44+0.1+1.72+1.28+0.45+0.68} = 1800\ \frac{Btu}{hr}$$

$\boxed{1.8}$

$$150\begin{bmatrix} \overset{\frac{1}{150}}{\cancel{5.88}} & \overset{0}{\cancel{-5.88}} & & & & & \\ -5.88 & 5.88+1.23 & -1.23 & & & & \\ & -1.23 & 1.23+0.76 & -0.76 & & & \\ & & -0.76 & 0.76+0.053 & -0.053 & & \\ & & & -0.053 & 0.053+2.22 & -2.22 & \\ & & & & -2.22 & 2.22+1.47 & -1.47 \\ & & & & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{150}}{\cancel{1.47}} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 20.37 \\ 22.12 \\ 24.95 \\ 65.57 \\ 66.54 \\ 68 \end{Bmatrix} {}^\circ F \Longleftarrow$$

$$q = UA\,\Delta T$$

as an example:

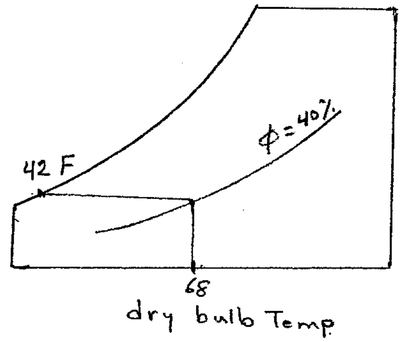
$$q = (1.47)(150)(68-66.54) = \underline{\underline{320\ \frac{Btu}{hr}}}$$

or

$$q = \frac{1}{\sum R_{esistance}} A\,(T_{in} - T_{out}) = \frac{(150)(68-20)}{0.17+0.81+1.32+19+0.45+0.68} = 320\ \frac{Btu}{hr}$$

$\boxed{1.9}$

With the help of a psychometric chart, using a dry bulb Temp. of $68^\circ F$ and $\phi = 40\%$, we identify condensation temperature to be $42^\circ F$. Thus condensation will occur between surfaces 4 and 5.



$\boxed{1.10}$

$$1000\begin{bmatrix} \overset{1/1000}{\cancel{1.47}} & \overset{0}{\cancel{-1.47}} & & & \\ -1.47 & 1.47+0.053 & -0.053 & & \\ & -0.053 & 0.053+2.22 & -2.22 & \\ & & -2.22 & 2.22+1.47 & -1.47 \\ & & & \underset{0}{\cancel{-1.47}} & \underset{1/1000}{\cancel{1.47}} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 15 \\ 16.81 \\ 66.99 \\ 68.19 \\ 70 \end{Bmatrix} {}^\circ F \Leftarrow$$

$$q = UA\,\Delta T$$

as an example:

$$q = (1.47)(1000)(70-68.19) = \underline{\underline{2660 \ \frac{Btu}{hr}}} \Leftarrow$$

or

$$q = (1.47)(1000)(16.81-15) = 2660 \ \frac{Btu}{hr}$$

$\boxed{1.11}$

$$22.5\begin{bmatrix} \overset{\frac{1}{22.5}}{\cancel{5.88}} & \overset{0}{\cancel{-5.88}} & & \\ -5.88 & 5.88+2.56 & -2.56 & \\ & -2.56 & 2.56+1.47 & -1.47 \\ & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{22.5}}{\cancel{1.47}} \end{bmatrix}\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 26.85 \\ 42.59 \\ 70 \end{Bmatrix} \ ^\circ F$$

$$q = U_1 A \Delta T = (5.88)(22.5)(26.85-20) = 910 \ \frac{Btu}{hr}$$

or as another example:

$$q = U_2 A \Delta T = (2.56)(22.5)(42.59-26.85) = 910 \ \frac{Btu}{hr}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A \ (T_{in} - T_{out}) = \frac{1}{(0.17+0.39+0.68)}(22.5)(70-20)$$

$$q = 910 \ \frac{Btu}{hr}$$

$\boxed{1.12}$

$$(A_1)_c = (A_5)_c = \left(\frac{12+10.5}{2}\right)(3) - (3)\frac{\pi}{4}(0.5)^2 = 33.161 \text{ in}^2$$

$$(A_2)_c = (A_4)_c = \frac{(10.5+9)}{2}(3) - 3\,\frac{\pi}{4}(0.5)^2 = 28.661 \text{ in}^2$$

$$(A_3)_c = (9)(3) - 3\,\frac{\pi}{4}(0.5)^2 = 26.411 \text{ in}^2$$

$$(K_1)_c = (K_5)_c = \frac{(33.161)(3.27\times10^6)}{6} = 18{,}072{,}745 \ \frac{\text{lb}}{\text{in}}$$

$$(K_2)_c = (K_4)_c = \frac{(28.661)(3.27\times10^6)}{6} = 15{,}620{,}245 \ \frac{\text{lb}}{\text{in}}$$

$$(K_3)_c = \frac{(26.411)(3.27\times10^6)}{4} = 21{,}590{,}992 \ \frac{\text{lb}}{\text{in}}$$

$$(K_1)_s = (K_2)_s = (K_4)_s = (K_5)_s = \frac{(3)(\frac{\pi}{4})(0.5)^2(29\times10^6)}{6} = 2{,}847{,}068 \ \frac{\text{lb}}{\text{in}}$$

$$(K_3)_s = \frac{(3)(\frac{\pi}{4})(0.5)^2(29\times10^6)}{4} = 4{,}270{,}602 \ \frac{\text{lb}}{\text{in}}$$

The Combined stiffnesses:

$$K_1 = K_5 = 18{,}072{,}745 + 2{,}847{,}068 = 20{,}919{,}813$$

$$K_2 = K_4 = 15{,}620{,}245 + 2{,}847{,}068 = 18{,}467{,}313$$

$$K_3 = 21{,}590{,}992 + 4{,}270{,}602 = 25{,}861{,}594$$

$$\begin{array}{cccccc} 1 & 0 & & & & \\ \end{array}$$

$$\begin{bmatrix} \cancel{20{,}919{,}813} & \cancel{-20{,}919{,}813} & & & & \\ \cancel{-20{,}919{,}813}\ 0 & 20{,}919{,}813 + 18{,}467{,}313 & -18{,}467{,}313 & & & \\ & -18{,}467{,}313 & 18{,}467{,}313 + 25{,}861{,}594 & -25{,}861{,}594 & & \\ & & -25{,}861{,}594 & 25{,}861{,}594 + 18{,}467{,}313 & -18{,}467{,}313 & \\ & & & -18{,}467{,}313 & 18{,}467{,}313 + 20{,}919{,}813 & -20{,}919{,}813 \\ & & & & -20{,}919{,}813 & 20{,}919{,}813 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1000 \end{Bmatrix}$$

$u_1 = 0$ $\quad u_2 = -4.78016\times10^{-5}$ in $\quad u_3 = -1.01951\times10^{-4}$ in

$\underline{u_4 = -1.40618\times10^{-4}}$ $\quad \underline{u_5 = -1.94768\times10^{-4} \text{ in}}$ $\quad \underline{u_6 = -2.42570\times10^{-4} \text{ in}}$

1.12 Cont.

$$\sigma^{(1)}_{Concrete} = E_c \left(\frac{u_2 - u_1}{\ell}\right) = \frac{(3.27\times 10^{6})(-4.78016\times 10^{-5})}{6} = \underline{\underline{26 \frac{lb}{in^2} \quad C}}$$

$$\sigma^{(1)}_{Steel} = E_s \left(\frac{u_2 - u_1}{\ell}\right) = \underline{\underline{231 \frac{lb}{in^2}}} \quad C$$

as a check:

$$\sigma^{(1)}_{concrete}(A_1)_c + \sigma^{(1)}_s (A_1)_s = (26)(33.161) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = \underline{\underline{1000 \text{ lb}}}$$

$$\sigma^{(2)}_c = E_c\left(\frac{u_3 - u_2}{\ell}\right) = \frac{(3.27\times 10^{6})(-1.01951\times 10^{-4} + 4.78016\times 10^{-5})}{6} = \underline{\underline{30 \frac{lb}{in^2} \quad C}}$$

$$\sigma^{(2)}_s = 231 \frac{lb}{in^2} \quad C$$

check: $\sigma^{(2)}_c (A_2)_c + \sigma^{(2)}_s (A_2)_s = (30)(28.661) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000 \text{ lb}$

$$\sigma^{(3)}_c = E_c\left(\frac{u_4 - u_3}{\ell}\right) = \frac{(3.27\times 10^{6})(-1.40618\times 10^{-4} + 1.01951\times 10^{-4})}{4} = \underline{\underline{32 \frac{lb}{in^2} \quad C}}$$

$$\sigma^{(3)}_s = E_s\left(\frac{u_4 - u_3}{\ell}\right) = \frac{(29\times 10^{6})(-1.40618\times 10^{-4} + 1.01951\times 10^{-4})}{4} = \underline{\underline{280 \frac{lb}{in^2} \quad C}}$$

check: $\sigma^{(3)}_c (A_3)_c + \sigma^{(3)}_s (A_3)_s = (32)(26.411) + (280)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000 \text{ lb}$

$$\sigma^{(4)}_c = E_c\left(\frac{u_5 - u_4}{\ell}\right) = \frac{(3.27\times 10^{6})(-1.94768\times 10^{-4} + 1.40618\times 10^{-4})}{6} = \underline{\underline{30 \frac{lb}{in^2} \quad C}}$$

$$\sigma^{(4)}_s = 231 \frac{lb}{in^2} \quad C$$

$$\sigma^{(5)}_c = E_c\left(\frac{u_6 - u_5}{\ell}\right) = \frac{(3.27\times 10^{6})(-2.42570 + 1.94768)\times 10^{-4}}{6} = \underline{\underline{26 \frac{lb}{in^2} \quad C}}$$

$$\sigma^{(5)}_s = 231 \frac{lb}{in^2} \quad C$$

note $\sigma^{(1)}_c = \sigma^{(5)}_c$ and $\sigma^{(2)}_c = \sigma^{(4)}_c$ as expected.

$\boxed{1.13}$

$$\Lambda^{(e)} = \frac{A_{avg}\,E}{2\,l}\,(u_{i+1} - u_i)^2$$

$$\Lambda_{total} = \Lambda^{(1)} + \Lambda^{(2)} + \Lambda^{(3)} + \Lambda^{(4)} + \Lambda^{(5)}$$

$$\Lambda^{(1)} = \Lambda^{(5)} = \frac{(A_1)_c\,E_c}{2l}(u_{i+1} - u_i)^2 + \frac{(A_1)_s\,E_s}{2\,l}(u_{i+1} - u_i)^2$$

$$= \frac{(A_1)_c\,E_c + (A_1)_s\,E_s}{2\,l}(u_2 - u_1)^2$$

$$\Lambda^{(1)} = \left[\frac{(33.161)(3.27\times10^6) + 3(\frac{\pi}{4})(0.5)^2(29\times10^6)}{(2)(6)}\right](-4.78016\times10^{-5})^2 = 0.024 \text{ lb.in}$$

or $\Lambda^{(5)} = \Big[$ // $\Big]\left[(-2.4257 + 1.94768)\times10^{-4}\right]^2 = 0.024$ ✓

$$\Lambda^{(2)} = \frac{(A_2)_c\,E_c + (A_2)_s\,E_s}{2\,l}(u_3 - u_2)$$

$$= \left[\frac{(28.661)(3.27\times10^6) + 3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(6)}\right]\left[(-1.01951\times10^{-4} + 4.78016\times10^{-5})\right]^2 = 0.027$$

or $\Lambda^{(4)} = \Big[$ // $\Big]\left[(-1.94768 + 1.40618)\times10^{-4}\right]^2 = 0.027$ ✓

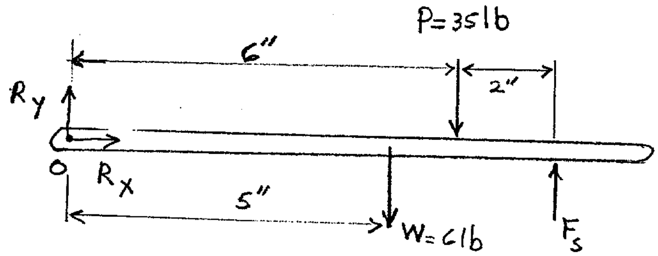
$$\Lambda^{(3)} = \frac{(A_3)_c\,E_c + (A_3)_s\,E_s}{2\,l}(u_4 - u_3)$$

$$\Lambda^{(3)} = \left[\frac{(26.411)(3.27\times10^6) + 3\,\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(4)}\right]\left[(-1.40618 + 1.01951)\times10^{-4}\right]^2$$

$$= 0.019 \text{ lb.in}$$

$$\Lambda_{total} = (2)(0.024) + 0.019 + 2(0.027) = \underline{\underline{0.121 \text{ lb.in}}} \Leftarrow$$

1.14



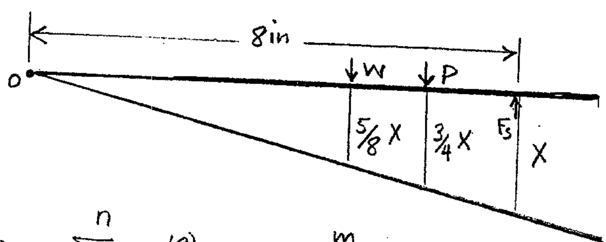
(a) $\circlearrowleft + \; \Sigma M_o = 0$

$$(-6\ \text{lb})(5\ \text{in}) + (35\ \text{lb})(6\ \text{in}) + F_s(8\ \text{in}) = 0$$

$$F_s = 30\ \text{lb}$$

$F_s = KX$ $\qquad$ $30\ \text{lb} = 60\ X$ $\qquad$ $\underline{\underline{X = 0.5\ \text{in}}}$

(b)



$$\frac{\partial \pi}{\partial u_i} = \frac{\partial}{\partial u_i}\sum_{i=1}^{n} \Lambda^{(e)} - \frac{\partial}{\partial u_i}\sum_{i=1}^{m} F_i u_i = 0$$

$$\frac{\partial}{\partial X}\left(\frac{1}{2}KX^2\right) - \frac{\partial}{\partial X}\left(35\left(\frac{3}{4}X\right)\right) - (6)\left(\frac{5}{8}X\right) = 0$$

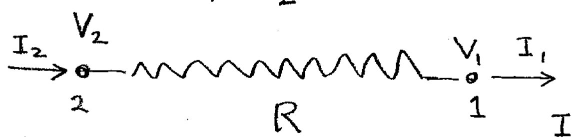
$$KX - (35)\left(\frac{3}{4}\right) - (6)\left(\frac{5}{8}\right) = 0$$

$$X = \frac{(35)\left(\frac{3}{4}\right) + (6)\left(\frac{5}{8}\right)}{60}$$

$$\underline{\underline{X = 0.5\ \text{in}}}$$

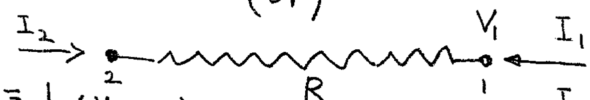
1.15

$I_1 = I_2$



$I_2 = \frac{1}{R}(V_2 - V_1)$ $\qquad$ $I_1 = \frac{1}{R}(V_2 - V_1)$

(or)



$I_2 = \frac{1}{R}(V_2 - V_1)$ $\qquad$ $I_1 = \frac{1}{R}(V_1 - V_2)$

Because of the fact that charge is conserved in a circuit (Kirchhoff's current law), at any time, the algebraic sum of the currents entering any node must be zero. Thus we can write

$$I_1 = \frac{1}{R}(V_1 - V_2)$$

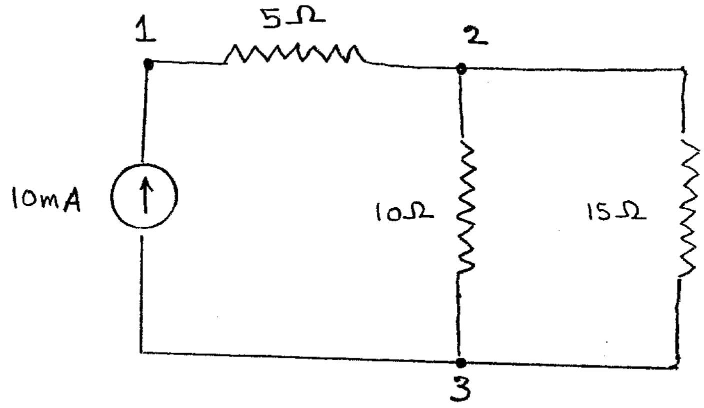
$$I_2 = \frac{1}{R}(V_2 - V_1)$$

Note $\quad I_1 + I_2 = 0$

In matrix form:

$$\frac{1}{R}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$$

1.16



$$[K]^{(1)} = \frac{1}{5}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(2)} = \frac{1}{10}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(3)} = \frac{1}{15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$; Apply $V_3 = 0$ as a boundary condition

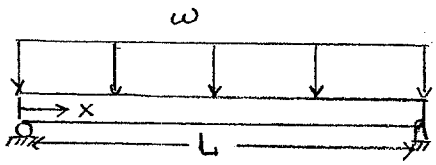
$$\begin{bmatrix} \frac{1}{5} & -\frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{10}+\frac{1}{15} & -\frac{1}{10}-\frac{1}{15} \\ 0 & \cancel{-\frac{1}{10}-\frac{1}{15}}\ 0 & \cancel{\frac{1}{10}+\frac{1}{15}}\ 1 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0.01 \\ 0 \\ 0 \end{Bmatrix}$$

$V_1 = 0.11$ $\qquad$ $V_2 = 0.06$

$\underline{\underline{V_1 - V_2 = 0.05 \text{ Volts}}}$ ⇐

$\underline{\underline{V_2 - V_3 = 0.06 \text{ Volts}}}$ ⇐

1-17



$$\frac{d^2Y}{dx^2} = \frac{M(x)}{EI} = \frac{\omega x(L-x)}{2EI}$$

$$\frac{dY}{dx} = \frac{1}{2EI}\left(\frac{\omega x^2 L}{2} - \frac{\omega x^3}{3}\right) + C_1$$

$$Y = \frac{1}{2EI}\left(\frac{\omega x^3 L}{6} - \frac{\omega x^4}{12}\right) + C_1 x + C_2$$

Applying Boundary Conditions:

$Y = 0$ @ $x = 0$ $\qquad C_2 = 0$

$Y = 0$ @ $x = L$ $\qquad C_1 = -\dfrac{\omega L^3}{24EI}$

$$Y_{exact} = -\frac{\omega x}{24EI}\left(x^3 - 2Lx^2 + L^3\right)$$

(a) Note that the assumed Solution satisfies the boundary Conditions. $Y = C_1\left[\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)\right]$

$$\frac{dY}{dx} = C_1\left[\frac{2x}{L^2} - \frac{1}{L}\right]$$

$$\frac{d^2Y}{dx^2} = C_1\left(\frac{2}{L^2}\right)$$

$$\frac{2C_1}{L^2} - \frac{\omega x(L-x)}{2EI} = R$$

We may force the error function to equal zero at $x = L/2$

$$\frac{2C_1}{L^2} - \frac{\omega \frac{L}{2}(L - \frac{L}{2})}{2EI} = 0 \rightarrow C_1 = \frac{\omega L^4}{16EI}$$

$$Y = \frac{\omega L^4}{16EI}\left[\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)\right]$$

1.17 Cont.

(b) $$\int_0^L R\,dx = 0$$

$$\int_0^L \left[\frac{2C_1}{L^2} - \frac{\omega x(L-x)}{2EI}\right] dx = 0$$

$$\frac{2C_1}{L^2}L - \frac{\omega}{2EI}\left(\frac{L^3}{2} - \frac{L^3}{3}\right) = 0 \longrightarrow C_1 = \frac{\omega L^4}{24EI}$$

$$Y = \frac{\omega L^4}{24EI}\left(\left(\frac{X}{L}\right)^2 - \left(\frac{X}{L}\right)\right)$$

For W24x104 Beam $I = 3100\ \text{in}^4$; $E = 29\times10^6\ \frac{\text{lb}}{\text{in}^2}$

Exact

$$Y_{max} = \frac{-5\omega L^4}{384EI} = \frac{-5\left(5000\frac{\text{lb}}{\text{ft}}\times\frac{1\text{ft}}{12\text{in}}\right)\left(20\text{ft}\times\frac{12\text{in}}{\text{ft}}\right)^4}{(384)\left(29\times10^6\frac{\text{lb}}{\text{in}^2}\right)(3100\ \text{in}^4)} = -0.20\ \text{in}$$

Collocation

$$Y_{max} = -\frac{\omega L^4}{64EI} = \frac{-\left(5000\frac{\text{lb}}{\text{ft}}\times\frac{1\text{ft}}{12\text{in}}\right)\left(20\text{ft}\times\frac{12\text{in}}{\text{ft}}\right)^4}{(64)\left(29\times10^6\frac{\text{lb}}{\text{in}^2}\right)(3100\text{in}^4)} = -0.24\ \text{in}$$

Subdomain

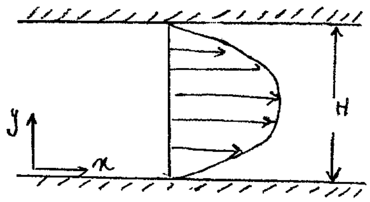
$$Y_{max} = \frac{-\omega L^4}{96EI} = \frac{-\left(5000\frac{\text{lb}}{\text{ft}}\times\frac{1\text{ft}}{12\text{in}}\right)\left(20\text{ft}\times\frac{12\text{in}}{\text{ft}}\right)^4}{(96)\left(29\times10^6\frac{\text{lb}}{\text{in}^2}\right)(3100\ \text{in}^4)} = -0.16\ \text{in}$$

1.18

See Section 1.7

1.19

$$\mu \frac{d^2u}{dy^2} = \frac{dP}{dx}$$



$$\frac{du}{dy} = \frac{1}{\mu}\frac{dP}{dx}\,y + c_1$$

$$u(y) = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + c_1 y + C_2$$

applying B.Cs $u(0)=0$ and $u(H)=0$

$$0 = 0 + 0 + C_2 \qquad \text{and} \qquad 0 = \frac{1}{\mu}\frac{dP}{dx}\frac{H^2}{2} + c_1 H$$

$$C_2 = 0 \qquad \text{and} \qquad c_1 = -\frac{H}{2\mu}\frac{dP}{dx}$$

$$\underline{\underline{u(y) = -\frac{1}{2\mu}\frac{dP}{dx}(Hy - y^2)}}$$

note because Pressure drops in the direction of flow, $\frac{dP}{dx} < 0$ in velocity equation.

(a) $$\mu \frac{d^2u}{dy^2} = \frac{dP}{dx} \qquad u_{assumed} = c_1 \sin\left(\frac{\pi y}{H}\right)$$

note the assumed solution Satisfies the boundary Conditions.

$$\frac{du}{dy} = \frac{d}{dy}\left(c_1 \sin\left(\frac{\pi y}{H}\right)\right) = c_1 \frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)$$

$$\frac{d^2u}{dy^2} = \frac{d}{dy}\left(c_1 \frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)\right) = -c_1\left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi y}{H}\right)$$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi y}{H}\right)\right] - \frac{dP}{dx} = \mathcal{R}$$

We may force the error function to equal zero at $y = \frac{H}{2}$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi}{H}\frac{H}{2}\right)\right] - \frac{dP}{dx} = 0$$

$$c_1 = -\frac{H^2}{\mu\pi^2}\frac{dP}{dx} \qquad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{\mu\pi^2}\frac{dP}{dx}\left(\sin\left(\frac{\pi y}{H}\right)\right)}}$$

1.19 Cont.

(b) $$\int_0^H R\,dy = 0$$

$$\int_0^H \left[\mu\left[-C_1\left(\frac{\pi}{H}\right)^2 \sin\frac{\pi y}{H}\right] - \frac{dp}{dx}\right] dy = 0$$

$$\mu C_1\left(\frac{\pi}{H}\right)^2 \left[\frac{1}{\frac{\pi}{H}} \cos\frac{\pi y}{H}\right]_0^H - \frac{dp}{dx} H = 0$$

$$\mu C_1 \frac{\pi}{H}(-2) = H\frac{dp}{dx}$$

$$C_1 = -\frac{H^2}{2\mu\pi}\frac{dp}{dx} \qquad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{2\mu\pi}\frac{dp}{dx}\left[\sin\left(\frac{\pi y}{H}\right)\right]}}$$

For Comparison, let's use:

$$\mu = 0.02\ \frac{N\cdot s}{m^2}\ ; \quad H = 0.01\ mm = 1\times10^{-5}\ m\ ; \quad \frac{dp}{dx} = -1\times10^{8}\ \frac{Pa}{m}$$

evaluating max vel. @ $y = \frac{H}{2}$

Exact

$$u_{max} = -\frac{1}{2\mu}\frac{dp}{dx}\left(H\left(\frac{H}{2}\right) - \left(\frac{H}{2}\right)^2\right) = -\frac{1}{2\mu}\frac{dp}{dx}\frac{H^2}{4} = -\frac{(-1\times10^{8})(1\times10^{-5})^2}{(8)(0.02)} = \underline{\underline{0.06\ m/s}}$$

Collocation method

$$u_{max} = -\frac{H^2}{\mu\pi^2}\frac{dp}{dx}\left(\sin\left(\frac{\pi\, H/2}{H}\right)\right) = -\frac{H^2}{\mu\pi^2}\frac{dp}{dx} = -\frac{(1\times10^{-5})^2(-1\times10^{8})}{(0.02)\,\pi^2} = \underline{\underline{0.05\ m/s}}$$

Subdomain method

$$u_{max} = -\frac{H^2}{2\mu\pi}\frac{dp}{dx}\left(\sin\left(\frac{\pi\, H/2}{H}\right)\right) = -\frac{H^2}{2\mu\pi}\frac{dp}{dx} = -\frac{(1\times10^{-5})^2(-1\times10^{8})}{(2)(0.02)\,\pi} = \underline{\underline{0.08\ m/s}}$$

1.20

(a) Galerkin method

$$\int_0^H \sin\left(\frac{\pi y}{H}\right)\left[\mu\left(-c_1\left(\frac{\pi}{H}\right)^2 \sin\frac{\pi y}{H}\right) - \frac{dp}{dx}\right] dy = 0$$

$$-\mu c_1 \left(\frac{\pi}{H}\right)^2 \int_0^H \sin^2\left(\frac{\pi y}{H}\right) dy = \frac{dp}{dx}\int_0^H \sin\left(\frac{\pi y}{H}\right) dy$$

$$-\mu c_1\left(\frac{\pi}{H}\right)^2\left[\frac{1}{2}y - \frac{1}{4\left(\frac{\pi}{H}\right)}\sin\left(\frac{2\pi}{H}y\right)\right]_0^H = \frac{dp}{dx}\left[-\frac{1}{\frac{\pi}{H}}\cos\left(\frac{\pi}{H}y\right)\right]_0^H$$

$$-\mu c_1\left(\frac{\pi}{H}\right)^2\left(\frac{1}{2}H\right) = -\frac{H}{\pi}\frac{dp}{dx}(-2)$$

$$c_1 = -\frac{4H^2}{\mu\pi^3}\frac{dp}{dx} \qquad \text{then,} \qquad \underline{\underline{u(y) = -\frac{4H^2}{\mu\pi^3}\frac{dp}{dx}\sin\left(\frac{\pi y}{H}\right)}}$$

(b) least-squares method

$$\int_0^H R\frac{\partial R}{\partial c_1}dy = \int_0^H \overbrace{\sin\left(\frac{\pi y}{H}\right)\left[\mu\left(-c_1\left(\frac{\pi}{H}\right)^2\sin\frac{\pi y}{H} - \frac{dp}{dx}\right]\right.}^{R}\overbrace{\left[-\mu\left(\frac{\pi}{H}\right)^2\sin\frac{\pi y}{H}\right]}^{\frac{\partial R}{\partial c_1}}dy = 0$$

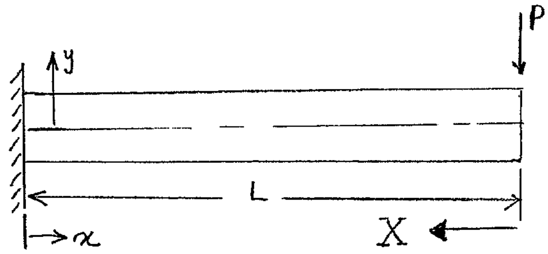
results in:

$$c_1 = -\frac{4H^2}{\mu\pi^3}\frac{dp}{dx} \qquad \text{and} \qquad \underline{\underline{u(y) = -\frac{4H^2}{\mu\pi^3}\frac{dp}{dx}\sin\left(\frac{\pi y}{H}\right)}}$$

using values from Problem 1.19 for Comparison

$$u_{max} = -\frac{4H^2}{\mu\pi^3}\frac{dp}{dx} = \frac{-4(1\times10^{-5})^2(-1\times10^{8})}{(0.02)\pi^3} = \underline{\underline{0.06\ m/s}}$$

1.21



$$\frac{d^2y}{dX^2} = -\frac{PX}{EI}$$

$$\frac{dy}{dX} = -\frac{P}{EI}\frac{X^2}{2} + c_1$$

$$y = -\frac{P}{EI}\frac{X^3}{6} + c_1 X + c_2$$

B.Cs ① $y(L) = 0$, ② $\left.\frac{dy}{dX}\right|_{X=L} = 0$

② $0 = -\frac{P}{EI}\frac{L^2}{2} + c_1$ $\qquad c_1 = \frac{P}{EI}\frac{L^2}{2}$

① $0 = -P\frac{L^3}{6} + \frac{P}{EI}\frac{L^2}{2}L + c_2$ $\qquad c_2 = \frac{P}{EI}\left(-\frac{1}{3}L^3\right)$

$$y_{exact} = \frac{P}{6EI}\left(-X^3 + 3L^2X - 2L^3\right)$$

In terms of x: Substitute for $X = L - x$ and simplify

$$y_{exact} = \frac{P}{6EI}\left(-(L-x)^3 + 3L^2(L-x) - 2L^3\right)$$

$$y_{exact} = \frac{P}{6EI}\left(x^3 - 3L^2x\right)$$

$$EI\frac{d^2y}{dx^2} + P(L-x) = 0$$

B.Cs $y(0) = 0$ and $\left.\frac{dy}{dx}\right|_{x=0} = 0$

Let us assume: $y_{assumed} = c_1x^2 + c_2x^3$

note, the assumed solution satisfies the boundary conditions.

1.21 Cont

$$\frac{dy}{dx} = 2C_1 x + 3C_2 x^2$$

$$\frac{d^2y}{dx^2} = 2C_1 + 6C_2 x$$

then Residual R becomes:

$$R = EI\,(2C_1 + 6C_2 x) + P(L - x)$$

Subdomain method:

$$\int_0^{L/2} R\,dx = 0 \qquad \int_0^{L/2} \left[EI\,(2C_1 + 6C_2 x) + P(L-x)\right] dx = 0 \qquad (1)$$

$$\int_{L/2}^{L} R\,dx = 0 \qquad \int_{L/2}^{L} \left[EI\,(2C_1 + 6C_2 x) + P(L-x)\right] dx = 0 \qquad (2)$$

Integrating (1):

$$EI\left(2C_1 (L/2) + 6C_2 \frac{(L/2)^2}{2}\right) + P\left(L\left(\frac{L}{2}\right) - \frac{(L/2)^2}{2}\right) = 0$$

Simplifying:

$$2C_1 + \tfrac{3}{2} C_2 L = -\frac{3PL}{4EI} \qquad (3)$$

Integrating (2) and simplifying:

$$C_1 + \frac{9}{4} C_2 L = -\frac{PL}{8EI} \qquad (4)$$

Solving (3) and (4) Simultaneously:

$$\underline{\underline{C_1 = -\frac{PL}{2EI}}} \quad \text{and} \quad \underline{\underline{C_2 = \frac{P}{6EI}}}$$

$$y = \frac{P}{6EI}\left(x^3 - 3Lx^2\right)$$

note, because the assumed solution has the Same functional form as the exact solution, the Coefficients are exact.

$\boxed{\begin{array}{c}1.21 \\ \text{Cont.}\end{array}}$

Galerkin method:

$$\int_0^L x^2 R\, dx = 0 \qquad \int_0^L x^2\left[EI(2C_1 + 6C_2) + P(L-x)\right]dx = 0 \quad (1)$$

$$\int_0^L x^3 R\, dx = 0 \qquad \int_0^L x^3\left[EI(2C_1 + 6C_2) + P(L-x)\right]dx = 0 \quad (2)$$

Integrating and simplifying (1) and (2), we get:

$$\begin{cases} \frac{2}{3} C_1 + \frac{3}{2} C_2 L = -\frac{PL}{12EI} & (3) \\ \frac{1}{2} C_1 + \frac{6}{5} C_2 L = -\frac{PL}{20EI} & (4) \end{cases}$$

Solving (3) and (4) simultaneously

$$C_1 = -\frac{PL}{2EI} \quad \text{and} \quad C_2 = \frac{P}{6EI}$$

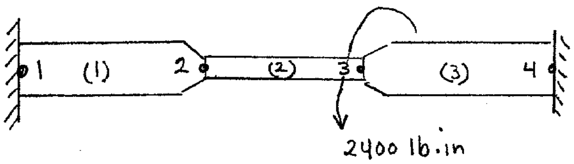
$$y = \frac{P}{6EI}(x^3 - 3Lx^2)$$

$\boxed{1.22}$

$J_1 = J_3 = \frac{1}{2}\pi r^4$

$J_1 = J_3 = \frac{1}{2}\pi\left(\frac{1.5}{2}\right)^4 = 0.497\ in^4$

$J_2 = \frac{1}{2}\pi\left(\frac{1}{2}\right)^4 = 0.0982\ in^4$



$$[K]^{(1)} = [K]^{(3)} = \frac{JG}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{(0.497)(9.8\times10^6)}{(2)(12)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}202942 & -202942\\ -202942 & 202942\end{bmatrix}$$

$$[K]^{(2)} = \frac{(0.0982)(11.2\times10^6)}{(1.5)(12)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}61102 & -61102\\ -61102 & 61102\end{bmatrix}$$

$$\begin{array}{c}\begin{array}{cccc}1 & 0 & & \end{array}\\ \begin{bmatrix}\cancel{202942} & \cancel{-202942} & & \\ \cancel{-202942}\ 0 & 202942+61102 & -61102 & \\ & -61102 & 61102+202942 & \cancel{-202942}\ 0\\ & & \cancel{-202942}\ 0 & \cancel{202942}\ 1\end{bmatrix}\end{array}\begin{Bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{Bmatrix} = \begin{Bmatrix}0\\ 0\\ 2400\\ 0\end{Bmatrix}$$

$\theta_1 = 0$ $\quad \underline{\underline{\theta_2 = 0.002222\ rad}}$ $\quad \underline{\underline{\theta_3 = 0.009604\ rad}}$ $\quad \theta_4 = 0$

$R_4 = K_3(\theta_3 - 0) = 202942 \times 0.009604 = \underline{\underline{1949\ lb.in}}$

$R_1 = K_1(\theta_2 - 0) = 202942 \times 0.002222 = \underline{\underline{451\ lb.in}}$

note they add up to 2400 lb.in

$\boxed{1.23}$

$$\begin{bmatrix}264044 & -61102\\ -61102 & 264044\end{bmatrix}\begin{Bmatrix}\theta_2\\ \theta_3\end{Bmatrix} = \begin{Bmatrix}1200\\ 1200\end{Bmatrix}$$

$\underline{\underline{\theta_2 = \theta_3 = 0.005913\ rad}}$ $\Leftarrow$

$\underline{\underline{R_1 = R_4}} = K_1(\theta_2 - 0) = K_3(\theta_3 - 0) = 202942 \times 0.005913 = \underline{\underline{1200\ lb.in}}$

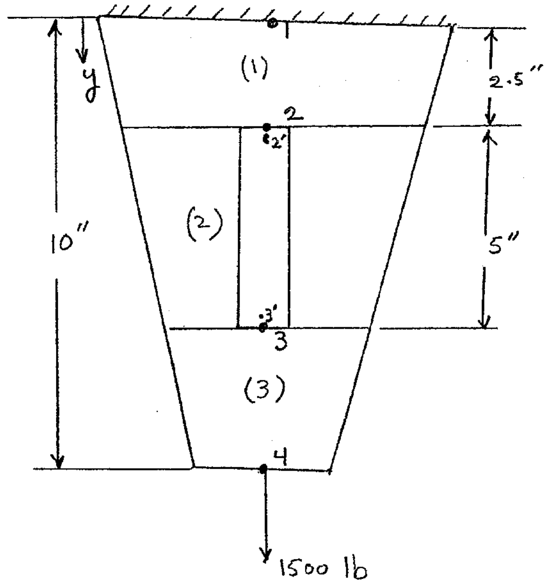
$\boxed{1.24}$

$A_1 = 0.5\ in^2$

$A_2 = 0.4375\ in^2 \qquad A_{2'} = 0.4375 - (0.5)(0.125) = 0.375\ in^2$

$A_3 = 0.3125\ in^2 \qquad A_{3'} = 0.3125 - (0.5)(0.125) = 0.25\ in^2$

$A_4 = 0.25\ in^2$



$$K = \frac{A_{avg} E}{l}$$

$$K_1 = \frac{\left(\frac{0.5+0.4375}{2}\right)(10.6\times10^6)}{2.5} = 1,987,500\ \frac{lb}{in}$$

$$K_2 = \frac{\left(\frac{0.375+0.25}{2}\right)(10.6\times10^6)}{5} = 662,500\ \frac{lb}{in}$$

$$K_3 = \frac{\left(\frac{0.3125+0.25}{2}\right)(10.6\times10^6)}{2.5} = 1,192,500\ \frac{lb}{in}$$

without the hole:

$A(y) = (4 - 0.2y)(0.125)$
$= 0.5 - 0.025y$

$$\begin{bmatrix} \overset{1}{\cancel{1987500}} & \overset{0}{\cancel{-1,987,500}} & & \\ \underset{0}{\cancel{-1,987,500}} & 1,987,500 + 662500 & -662500 & \\ & -662500 & 662500 + 1192500 & -1192500 \\ & & -1192500 & 1192500 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1500 \end{Bmatrix}$$

$u_1 = 0 \qquad \underline{\underline{u_2 = 7.5472\times10^{-4}\ in}} \qquad \underline{\underline{u_3 = 0.003020\ in}} \qquad \underline{\underline{u_4 = 0.004277\ in}}$

$$\sigma^{(1)} = (10.6\times10^6)\left(\frac{7.5472\times10^{-4}}{2.5}\right) = 3200\ \frac{lb}{in^2}$$

as a check: $\sigma^{(1)} = \dfrac{1500}{\frac{0.5+0.4375}{2}} = 3200\ \dfrac{lb}{in^2}$

$$\sigma^{(2)} = (10.6\times10^6)\left(\frac{0.00302 - 7.5472\times10^{-4}}{5}\right) = 4800\ \frac{lb}{in^2}$$

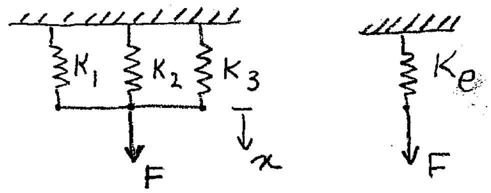
check: $\sigma^{(2)} = \dfrac{1500}{\left(\frac{0.375+0.25}{2}\right)} = 4800\ \dfrac{lb}{in^2}$

$$\sigma^{(3)} = (10.6\times10^6)\left(\frac{0.004277 - 0.00302}{2.5}\right) = 5300\ \frac{lb}{in^2}$$

check: $\sigma^{(3)} = \dfrac{1500}{\left(\frac{0.3125+0.25}{2}\right)} = 5300$ psi

$\boxed{1.25}$

Springs in Parallel

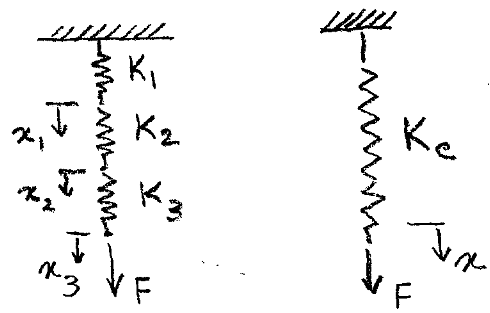


$$F = F_1 + F_2 + F_3$$

$$K_e x = K_1 x + K_2 x + K_3 x$$

$$\underline{\underline{K_e = K_1 + K_2 + K_3}}$$

Springs in Series



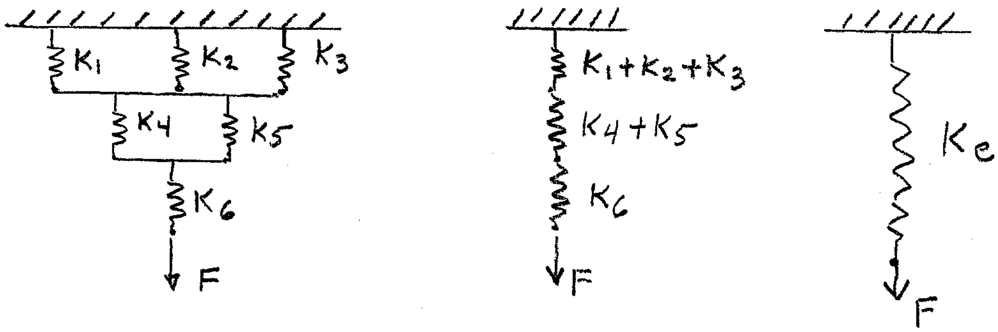
$$x = x_1 + x_2 + x_3$$

$$\frac{F}{K_e} = \frac{F}{K_1} + \frac{F}{K_2} + \frac{F}{K_3}$$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

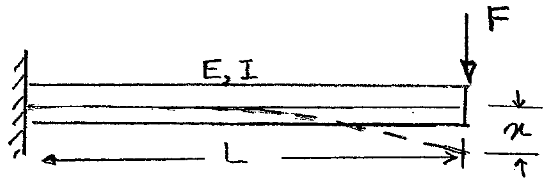
$$\underline{\underline{K_e = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}}}$$

1.26



$$K_e = \frac{1}{\frac{1}{K_1+K_2+K_3} + \frac{1}{K_4+K_5} + \frac{1}{K_6}}$$

1.27



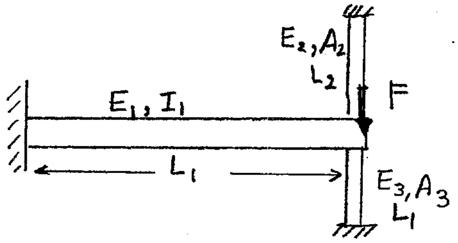
From Table 4.1

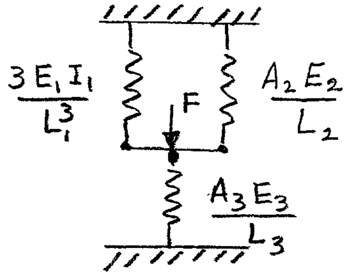
$$X = \frac{FL^3}{3EI}$$

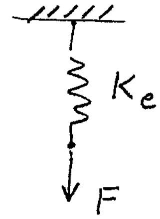
$$F = \frac{3EI}{L^3}x$$

$$K_e = \frac{3EI}{L^3}$$

1.28



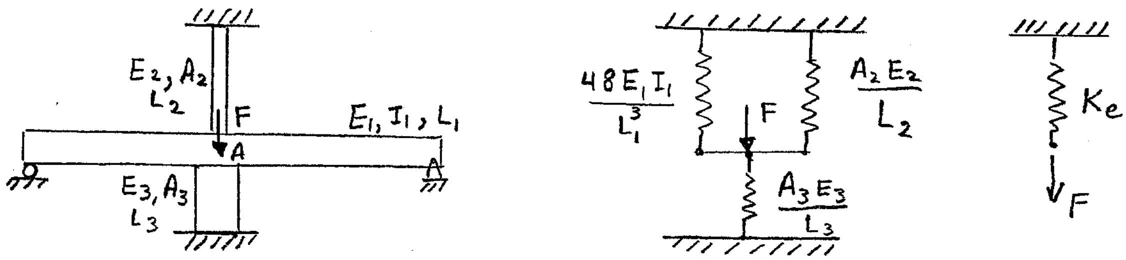




$$K_e = \frac{3 E_1 I_1}{L_1^3} + \frac{A_2 E_2}{L_2} + \frac{A_3 E_3}{L_3}$$

Note, deflection of each spring is the same, therefore, springs are in parallel.

1.29



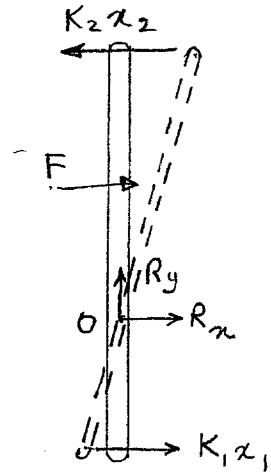
From Table 4.1, the K_e for the beam is:

$$(K_e)_{beam} = \frac{48E_1I_1}{L_1^3}$$

$$K_e = \frac{48E_1I_1}{L_1^3} + \frac{A_2E_2}{L_2} + \frac{A_3E_3}{L_3}$$

Note, deflection of each spring is the same, therefore, springs are in parallel.

$\boxed{1.30}$



$\circlearrowleft + \; \Sigma M_o = 0$

$$-FL + (K_2 x_2)(2L) + (K_1 x_1)(L) = 0$$

Note $X_2 = 2X_1$ and simplify

$$(2K_2 X_1)(2L) + K_1 x_1 L = FL$$

$$\underline{\underline{x_1 = \frac{F}{4K_2 + K_1}}} \Leftarrow$$

$$\Pi = \sum_{e=1}^{n} \Lambda^{(e)} - \Sigma F_i u_i$$

$$\Pi = \frac{1}{2} K_2 x_2^2 + \frac{1}{2} K_1 x_1^2 - F x_1 = \frac{1}{2} K_2 (2x_1)^2 + \frac{1}{2} K_1 x_1^2 - F x_1$$

$$\Pi = 2K_2 x_1^2 + \frac{1}{2} K_1 x_1^2 - F x_1$$

$$\frac{\partial \Pi}{\partial x_1} = 4K_2 x_1 + K_1 x_1 - F = 0$$

$$\underline{\underline{x_1 = \frac{F}{4K_2 + K_1}}} \Leftarrow$$